[Practical Assessment, Research, and Evaluation](https://scholarworks.umass.edu/pare)

Volume 20 *Volume 20, 2015* Article 2

2015

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Practical Assessment, Research & Evaluation

A peer-reviewed electronic journal.



Volume 20, Number 2, January 2015 ISSN 1531-7714

# What is Rotating in Exploratory Factor Analysis?

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Exploratory factor analysis (EFA) is one of the most commonly-reported quantitative methodology in the social sciences, yet much of the detail regarding what happens during an EFA remains unclear. The goal of this brief technical note is to explore what "rotation" is, what exactly is rotating, and why we use rotation when performing EFAs. Some commentary about the relative utility and desirability of different rotation methods concludes the narrative.

Those of us who regularly use exploratory factor analysis (EFA), one of the most commonly-used statistical techniques reported in the social sciences literature (e.g., Fabrigar, Wegener, MacCallum, & Strahan, 1999; Osborne, Costello, & Kellow, 2008), know that rotation happens, that there are different types of rotation, and hopefully that the goal of all rotation methods is to clarify results. But what exactly is rotating during an EFA? The goal of this article is to answer that simple question.

Let us look "under the hood" at the mysterious inner workings of EFA.[1]

## Basics of exploratory factor analysis

Exploratory factor analysis is a statistical tool used for many purposes. It was originally developed in the early 1900s to attempt to establish intelligence as a unitary or multidimensional construct (Spearman, 1904), and is a general-purpose dimension reduction tool with many applications. In the modern social sciences, it is perhaps most frequently used to explore the psychometric properties of an instrument or scale. Exploratory factor analysis examines all the pairwise relationships between individual variables (e.g., items on a scale) and seeks to extract latent factors from the measured variables. During the 110 years since Spearman's seminal work in this area, few statistical techniques have been so widely used and, often, subject to misperceptions (see, for example, Costello & Osborne, 2005; Osborne, Costello, & Kellow, 2008).

**Principal components analysis (PCA)** is a computationally simplified version of the general class of dimension reduction analyses. PCA computes the analysis without regard to the underlying latent structure of the variables, using all the variance in the manifest variables. PCA was developed decades ago when analyses were mostly computed by hand and thus shortcuts that did not substantially diminish the outcome were valuable. Now with popular statistical software packages and the readily available processing power in even the cheapest laptop computers, PCA is probably not necessary. It is also not considered a true method of factor analysis and there is disagreement among statisticians about when it should be used, if at all. There are some situations where PCA might be an appropriate option, but more often than not, researchers use PCA when EFA would be appropriate and preferable (for example, see Ford, MacCallum, & Tait, 1986; Gorusch, 1983; Widaman, 1993). It is still the default dimension reduction procedure in many statistical analysis software packages despite it usually not being the desirable choice. We will not discuss PCA further, as the goal of the paper is to focus on EFA and rotation.

[1] In this article the discussion is limited to exploratory factor analysis as there is no rotation analogue in confirmatory factor analysis.

**Exploratory factor analysis.** The second method of dimension reduction is common factor analysis or exploratory factor analysis. Common factor analysis recognizes that model variance contains both shared and unique variance across variables. EFA examines only the shared variance from the model each time a factor is created, while allowing the unique variance and error variance to remain in the model.

The general process for conducting exploratory factor analysis is briefly outlined in Table 1. Many of these steps are discussed in depth elsewhere (Osborne et al., 2008; Osborne & Fitzpatrick, 2012; Thompson, 2004). Because this is intended to be a brief technical note, let us focus on step #4, rotation.

Table 1. Steps To Follow When Conducting EFA

1. Data cleaning
2. Deciding on extraction method to use
3. Deciding how many factors to retain
4. Deciding on a method of rotation (if desired)
5. Interpretation of results (return to #3 if solution is not ideal)
6. Replication

## A pedagogical example

To illustrate the points in this paper, I will use data from a study on engineering majors conducted several years ago. For this example, we will examine two scales that should be minimally correlated: Engineering problem solving and interest in engineering. The items from the relevant subscales are listed below:

*Engineering problem solving items*:[2]

How well did you feel prepared for:

1. Defining what the problem really is
2. Searching for and collecting information needed to solve the problem
3. Thinking up potential solutions to the problem
4. Detailing how to implement the solution to the problem
5. Assessing and passing judgment on a possible or planned solution to the problem
6. Comparing and contrasting two solutions to the problem on a particular dimension such as cost
7. Selecting one idea or solution to the problem from among those considered
8. Communicating elements of the solution in sketches, diagrams, lists, and written or oral reports

*Interest in engineering*:[3]

1. I find many topics in engineering to be interesting
2. Solving engineering problems is interesting to me
3. Engineering fascinates me
4. I am interested in solving engineering problems
5. Learning new topics in engineering is interesting to me
6. I find engineering intellectually stimulating

In this example I will use principal axis factoring extraction, specifying two factors to be extracted, and will request a factor loading plot.[4] In this example, the analysis produces two strong factors (extracted eigenvalues of 7.42 and 3.28, which accounts for 76.42% of the variance). The extracted (un-rotated) factor matrix is presented in Table 2, below. I also present the scree plot from this analysis showing that the two-factor solution is strongly supported.

As you can see in Table 2, although we expect two very clear factors, the factor loadings are not immediately identifiable as two separate factors prior to rotation. To be sure, looking only at Factor 1 loadings, all fourteen items seem to be similar. It is only in combination with the loadings on Factor 2 where the two factors separate. If one plots each item in two-dimensional space (Factor 1 on the X axis, and Factor 2 on the Y axis), we see clearly the separation, as presented in Figure 1, below:

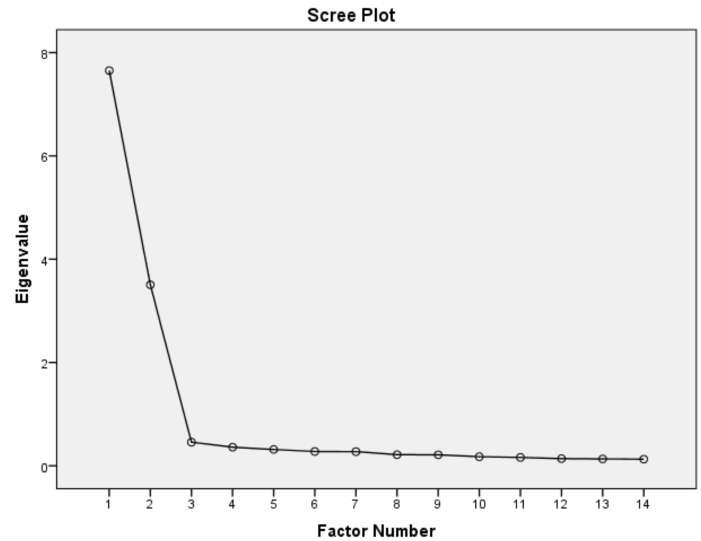
[2] Assessed on a seven point Likert type scale anchored by "did not prepare me at all" to "prepared me a lot"

[3] Assessed on a seven point Likert type scale anchored by "strongly disagree" and "strongly agree"

[4] In many analyses of these data, no other eigenvalue rose above 0.50 in value and thus for the purposes of this increasingly inaccurately labeled *brief* technical note is discounted as a legitimate possibility.

Table 2. Unrotated Factor Matrix and scree plot

| | Factor | |
|---|---|---|
| | 1 | 2 |
| EngProbSolv1 | .759 | -.389 |
| EngProbSolv2 | .703 | -.418 |
| EngProbSolv3 | .784 | -.392 |
| EngProbSolv4 | .798 | -.416 |
| EngProbSolv5 | .811 | -.375 |
| EngProbSolv6 | .795 | -.369 |
| EngProbSolv7 | .804 | -.360 |
| EngProbSolv8 | .763 | -.299 |
| INTERESTeng1 | .630 | .521 |
| INTERESTeng2 | .660 | .630 |
| INTERESTeng3 | .669 | .627 |
| INTERESTeng4 | .668 | .609 |
| INTERESTeng5 | .657 | .607 |
| INTERESTeng6 | .647 | .578 |




Extraction Method: Principal Axis Factoring.

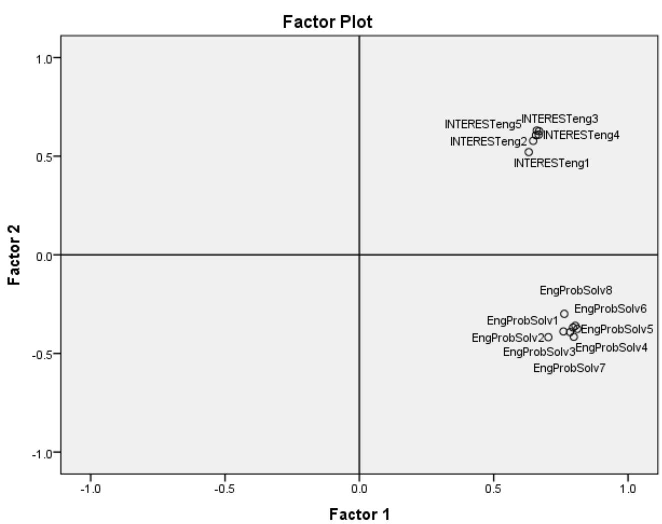



Figure 1: Unrotated solution from initial extraction

## What exactly is rotation, and what is being rotated?

Unrotated results from a factor analysis – as presented above- is not easy to interpret, although the plot helps. Simply put, rotation was developed not long after factor analysis to help researchers clarify and simplify the results of a factor analysis. Indeed, early methods were subjective and graphical in nature (Thurstone, 1938) because the calculations were labor intensive. Later scholars attempted to make rotation less subjective or exploratory (e.g., Horst, 1941), leading to initial algorithms such as Quartimax (Carroll, 1953) and Varimax (Kaiser, 1958).[5]

Quite simply, we use the term "rotation" because, historically and conceptually, the axes are being rotated so that the clusters of items fall as closely as possible to them.[6] Looking at Figure 1, for example, if you imagine rotating the axes so that they intersect the centroid of each cluster variables, you get the essence of rotation.

As you can see in Figure 2 (and Table 3), following rotation of the axes (or items), the items now fall closely about each axis line. This has the effect of making the factor loading pattern much clearer as one of the two pairs of coordinates for each item tends to be close to 0.00, as you can see in Table 3. In this example analysis, the factors were correlated $r = 0.37$.

[5] Note that rotation *does not* alter the basic aspects of the analysis, such as the amount of variance extracted from the items. Indeed, although eigenvalues might change as factor loadings are adjusted by rotation, the overall percentage of variance accounted for will remain constant.

[6] Alternatively, you could imagine rotating each cluster of items toward the axis. It really works out to be functionally the same.

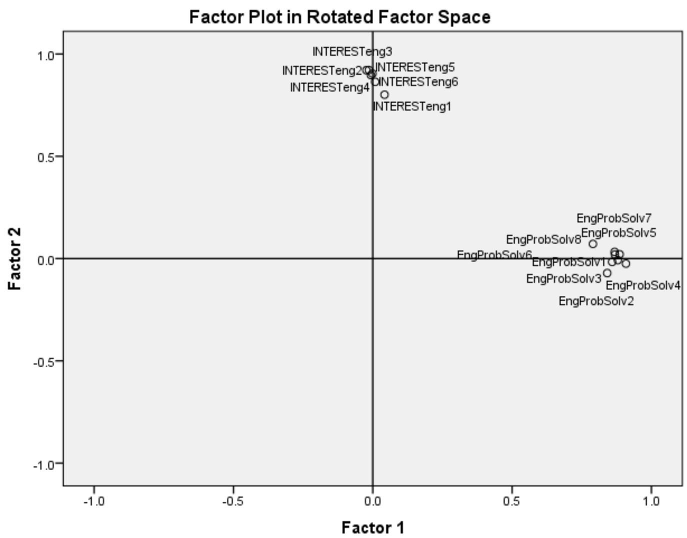



Figure 2: Rotated factor solution following Principal Axis Factoring extraction and oblique (oblimin) rotation

Table 3. Pattern Matrix[7]

| | Factor | |
|---|---|---|
| | 1 | 2 |
| EngProbSolv1 | .859 | -.016 |
| EngProbSolv2 | .841 | -.071 |
| EngProbSolv3 | .879 | -.008 |
| EngProbSolv4 | .909 | -.025 |
| EngProbSolv5 | .886 | .021 |
| EngProbSolv6 | .869 | .020 |
| EngProbSolv7 | .868 | .033 |
| EngProbSolv8 | .790 | .072 |
| INTERESTeng1 | .042 | .801 |
| INTERESTeng2 | -.023 | .921 |
| INTERESTeng3 | -.014 | .922 |
| INTERESTeng4 | -.001 | .904 |
| INTERESTeng5 | -.007 | .897 |
| INTERESTeng6 | .009 | .864 |

Extraction Method: Principal Axis Factoring.
Rotation Method: Oblimin with Kaiser Normalization.

Most statistical packages will allow small loadings to be suppressed following rotation, so that the results become even more obvious and immediately apparent.

[7] When an oblique rotation is performed, where factors are allowed to correlate, the factor loadings are contained in a table called the "pattern matrix." More on this below.

In Table 4, below, loadings less than 0.10 are suppressed, allowing easy visual confirmation that the EFA with oblique rotation produced our expected result.

Table 4. Pattern Matrix with loadings < 0.10 suppressed

| | Factor | |
|---|---|---|
| | 1 | 2 |
| EngProbSolv1 | .859 | |
| EngProbSolv2 | .841 | |
| EngProbSolv3 | .879 | |
| EngProbSolv4 | .909 | |
| EngProbSolv5 | .886 | |
| EngProbSolv6 | .869 | |
| EngProbSolv7 | .868 | |
| EngProbSolv8 | .790 | |
| INTERESTeng1 | | .801 |
| INTERESTeng2 | | .921 |
| INTERESTeng3 | | .922 |
| INTERESTeng4 | | .904 |
| INTERESTeng5 | | .897 |
| INTERESTeng6 | | .864 |

Extraction Method: Principal Axis Factoring.
Rotation Method: Oblimin with Kaiser Normalization.

## Different types of rotations

There are many choices of rotation method, depending on what software you are using. Each uses slightly different algorithms or methods to achieve the same broad goal- simplification of the factor structure. Rotation methods fall into two broad categories: orthogonal and oblique (referring to the angle maintained between the X and Y axes). Orthogonal rotations produce factors that are uncorrelated (i.e., maintain a 90° angle between axes); oblique methods allow the factors to correlate (i.e., allow the X and Y axes to assume a different angle than 90°). Traditionally, researchers have been guided to orthogonal rotation because (the argument went) uncorrelated factors are more easily interpretable. There is also an argument in favor of orthogonal rotation as the mathematics are simpler, and that made a significant difference during much of the 20th century when EFA was performed by hand calculations or much more limited computing power. Orthogonal rotations are generally the default setting in most statistical computing packages.

There does not seem to be a compelling reason for modern researchers to default to orthogonal rotations. In the social sciences (and many other sciences, such as biomedical sciences) we generally expect some correlation among factors, since behavior is rarely partitioned into neatly packaged units that function independently of one another. Therefore using orthogonal rotation potentially results in a less useful solution where factors are correlated. Remembering that EFA is an exploratory technique (not a confirmatory technique), we should be looking for the clearest solution possible. Further, there does not appear to be a drawback to using oblique rotation even if the factors are truly uncorrelated. Oblique rotations do not force factors to be correlated, and so in that instance, the factors would be allowed to assume a correlation of zero, and the solution would be the same as that of an orthogonal rotation.

Oblique rotation output is only slightly more complex than orthogonal rotation output, but should yield either identical or superior results to that of orthogonal rotations. In SPSS output the *rotated factor matrix* is interpreted after orthogonal rotation; the rotated factor matrix represents both the loadings and the correlations between the variables and factors. In contrast, when using oblique rotation the *pattern matrix* is examined for factor/item loadings and the *factor correlation matrix* reveals any correlation between the factors. The pattern matrix holds the loadings (which are of most interest), and each row of the pattern matrix can be thought of as a regression equation where the standardized observed variable is expressed as a function of the factors, with loadings as the regression coefficients. The *structure matrix* holds the correlations between the variables and the factors, which are generally of less interest in exploratory applications (e.g., Gorusch, 1983).

There are a variety of choices in each category. Varimax rotation is by far the most orthogonal rotation, likely because it is the default in many software packages, but also because it was developed as an incremental improvement upon prior algorithms quartimax, and equamax. There is no widely preferred method of oblique rotation; all tend to produce similar results (Fabrigar et al., 1999), and it seems generally fine to use the default settings in software packages. Common oblique rotations you will see include: direct oblimin, quartimin, and promax.

The mathematical algorithms for each rotation are different, and beyond the scope of this brief technical note. Note that for all rotations, the goal is the same: simplicity and clarity of factor loadings. For details on how they achieve these goals, you should refer to the manual for your statistical software (e.g., IBM SPSS base statistics manual p. 97,[8] or Gorusch, 1983; for a good overview of the technical details of different versions of varimax rotation, see Forina, Armanino, Lanteri, & Leardi, 1989).

## Do orthogonal and oblique rotations produce noticeable differences?

Orthogonal and oblique rotations will produce virtually identical solutions in the unlikely case where factors are perfectly uncorrelated. As the correlation between latent variables diverges from r = 0.00, then the oblique solution will produce increasingly clearer results. Looking at the same data after orthogonal (Varimax) rotation (Figure 3 and Table 5, below), one

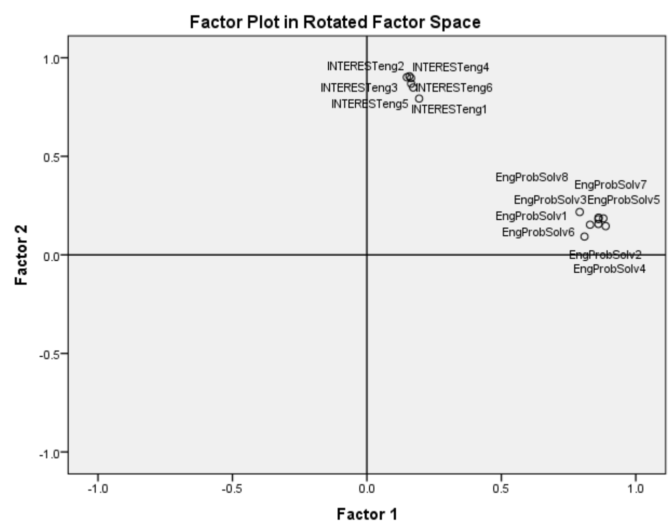



Figure 3: Same data using orthogonal rotation (Varimax)

can see that this outcome still provides a similar conclusion, but with a slightly less clear solution. This is because these factors are modestly correlated, but the mandate to maintain a $90^{o}$ angle between axes means

[8] Retrieved from ftp://public.dhe.ibm.com/software/analytics/spss/documentation/statistics/22.0/en/client/Manuals/IBM_SPSS_Statistics_Base.pdf

that the centroids of the clusters cannot move closer to the axis lines. In this case, the difference is not great, but noticeable in Table 5, where again loadings less than 0.10 were suppressed. You can see that only one loading in Table 5 is less than the suppression cutoff of 0.10, leaving a less clear result than the oblique rotation from Table 4. This is a small but clear example of the higher efficacy of oblique rotations to create clear patterns of results in EFA where the factors are indeed correlated.

Table 5. Rotated Factor Matrix with loadings < 0.10 suppressed

| | Factor | |
|---|---|---|
| | 1 | 2 |
| EngProbSolv1 | .830 | .153 |
| EngProbSolv2 | .809 | |
| EngProbSolv3 | .861 | .157 |
| EngProbSolv4 | .888 | .146 |
| EngProbSolv5 | .879 | .185 |
| EngProbSolv6 | .861 | .181 |
| EngProbSolv7 | .862 | .189 |
| EngProbSolv8 | .791 | .218 |
| INTERESTeng1 | .194 | .793 |
| INTERESTeng2 | .149 | .901 |
| INTERESTeng3 | .158 | .906 |
| INTERESTeng4 | .164 | .897 |
| INTERESTeng5 | .165 | .868 |
| INTERESTeng6 | .172 | .849 |

Extraction Method: Maximum Likelihood.
Rotation Method: Varimax with Kaiser Normalization.

## Summary

The enigmatic and widely-used technique of exploratory factor analysis is a complex class of procedures with many options. Many of us, even those with decades of experience using EFA, remain unclear on some of the nuances and details of what exactly is happening "under the hood" when we perform this analysis. Rotation is literally a rotation of X- and Y-axes in order to align clusters of variables plotted in two-dimensional space with the axis lines, which has the effect of clarifying the loading patterns in tables (making larger numbers larger, and smaller numbers closer to zero).9 Recall that the original goal of EFA was to reduce groups of variables to conceptually important latent variables. Given this, the overriding goal of EFA must be to make sense of data. A clear, easily- interpreted structure to the EFA is the goal, therefore.

Many authors have written on guidelines for extraction and rotation of factors, focusing on eigenvalues, scree plots, parallel analysis, replication, and so on. It is my belief that the over-arching value has to be theoretical framework and an easily-interpretable factor structure. Absent this, which we use to make sense of data, none of the technical details seem important.

In this brief technical note I used two-dimensional plots as an example. If an instrument is uni-dimensional no rotation is possible. If an instrument is three- (or more) dimensional, then items are plotted in multidimensional space, and three (or more) axes are rotated within this multidimensional space with the same goals.

I also suggest that in the modern era of high-power computing, orthogonal rotations are probably not a best practice, as oblique rotations can accurately model uncorrelated and correlated factors, whereas orthogonal rotations cannot handle correlated factors as effectively. Thus, there is little cost to using oblique rotations regardless of the underlying relatedness of the factors.

[9] Or, if you prefer, rotation of the clusters of items to more closely align with the axis lines.

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## Acknowledgements and Author Notes

An early version of this paper was presented as a talk at the 2013 American Statistical Association Conference on Statistical Practice in New Orleans, LA. Many thanks to Elizabeth Hoag-Carhart (Old Dominion University) who helped with that presentation.

Readers interested in receiving a free PDF of the author's new book (Best Practices in Exploratory Factor Analysis) can email the author with PAREonline in the subject to request a copy.

## Citation:

Osborne, Jason W. (2015). What is Rotating in Exploratory Factor Analysis? *Practical Assessment, Research & Evaluation*, 20(2). Available online: http://pareonline.net/getvn.asp?v=20&n=2

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