2008

Semileptonic decays of the light J(P)=1/2(+) ground state baryon octet

A Faessler

T Gutsche

BR Holstein
holstein@physics.umass.edu

MA Ivanov

JG Korner

See next page for additional authors

Authors
A Faessler, T Gutsche, BR Holstein, MA Ivanov, JG Korner, and VE Lyubovitskij

Semileptonic decays of the light $J^P = 1/2^+$ ground state baryon octet

Amand Faessler[1], Thomas Gutsche[1], Barry R. Holstein[2],
Mikhail A. Ivanov[3], Jürgen G. Körner[4], Valery E. Lyubovitskij[1]*

[1] *Institut für Theoretische Physik, Universität Tübingen, Auf der Morgenstelle 14, D–72076 Tübingen, Germany*

[2] *Department of Physics–LGRT, University of Massachusetts, Amherst, MA 01003 USA*

[3] *Bogoliubov Laboratory of Theoretical Physics, Joint Institute for Nuclear Research, 141980 Dubna, Russia*

[4] *Institut für Physik, Johannes Gutenberg-Universität, D–55099 Mainz, Germany*

(Dated: November 6, 2008)

We calculate the semileptonic baryon octet–octet transition form factors using a manifestly Lorentz covariant quark model approach based on the factorization of the contribution of valence quarks and chiral effects. We perform a detailed analysis of SU(3) breaking corrections to the hyperon semileptonic decay form factors. We present complete results on decay rates and asymmetry parameters including lepton mass effects for the rates.

PACS numbers: 12.39.Fe, 12.39.Ki, 13.30.Ce, 14.20.Dh, 14.20.Jn
Keywords: chiral symmetry, effective Lagrangian, relativistic quark model, nucleon and hyperon vector and axial form factors

* On leave of absence from Department of Physics, Tomsk State University, 634050 Tomsk, Russia

I. INTRODUCTION

The analysis of the semileptonic decays of the baryon octet $B_i \to B_j e\bar{\nu}_e$ presents an opportunity to shed light on the Cabibbo–Kobayashi–Maskawa (CKM) matrix element V_{us}. At zero momentum transfer, the weak baryon matrix elements for the $B_i \to B_j e\bar{\nu}_e$ transitions are determined by just two constants — the vector coupling $F_1^{B_iB_j}$ and its axial counterpart $G_1^{B_iB_j}$. In the limit of exact SU(3) symmetry $F_1^{B_iB_j}$ and $G_1^{B_iB_j}$ are expressed in terms of basic parameters — the vector couplings are given in terms of well–known Clebsch–Gordan coefficients which are fixed due to the conservation of the vector current (CVC), while the axial couplings are given in terms of the familiar SU(3) octet axial–vector couplings F and D. The Ademollo–Gatto theorem (AGT) [1] protects the vector form factors from leading SU(3)–breaking corrections generated by the mass difference of strange and nonstrange quarks—the first nonvanishing breaking effects begin at second order in symmetry–breaking. As emphasized in Ref. [2], the vanishing of the first–order correction to the vector hyperon form factors $F_1^{B_iB_j}$ presents an opportunity to determine V_{us} from the direct measurement of $V_{us}F_1^{B_iB_j}$. The axial form factor, on the other hand, contains symmetry–breaking corrections already at first order. We note that the experimental data on baryon semileptonic decays [3] are well described by the Cabibbo theory [4], which assumes SU(3) invariance of the strong interactions. However, for a precise determination of V_{us} one needs to include the leading and very likely also the subleading SU(3) breaking corrections.

The theoretical analysis of SU(3) breaking corrections to hyperon semileptonic decay form factors has been performed in various approaches, including quark and soliton models, the $1/N_c$ expansion of QCD, chiral perturbation theory (ChPT), lattice QCD, *etc.* (for an overview and references see [5]). In Ref. [5] we have suggested the use of a quark-based approach, which offers the possibility to consistently include chiral corrections (both SU(3)–symmetric and SU(3)–breaking) to the baryon semileptonic form factors. By matching the baryon matrix elements to the corresponding quantities derived in baryon ChPT we reproduced the chiral expansion of physical quantities (e.g. mass, magnetic moments, slopes and the axial charge of the nucleon) at the order of accuracy at which we worked. In the valence quark calculation of the baryon matrix elements we employed a simple generic ansatz for the spatial form of the quark wave function [6, 7].

In the present paper we evaluate the baryon matrix elements within a Lorentz and gauge invariant constituent quark model [8, 9]. Note that in Refs. [10, 11] we have studied the electromagnetic properties of the baryon octet and the $\Delta(1230)$–resonance in an analogous approach. In particular, we developed an approach based on a nonlinear chirally symmetric Lagrangian which involves constituent quarks *and* chiral fields. In a first step, this Lagrangian was used to dress the constituent quarks with a cloud of light pseudoscalar mesons and other (virtual) heavy states using the calculational technique of infrared dimensional regularization (IDR) [12]. Then, within a formal chiral expansion, we evaluated the dressed transition operators relevant for the interaction of quarks with external fields in the presence of a virtual meson cloud. In a next step, these dressed operators were used to calculate baryon matrix elements. (A simpler and more phenomenological quark model based on similar ideas regarding the dressing of constituent quarks by the meson cloud has been developed in Refs. [7].) In the present paper we improve the quantitative determination of valence quark effects by resorting to a specific relativistic quark model [8, 11] describing the internal quark dynamics. This procedure will allow us to generate predictions for all six form factors showing up in the matrix elements of the semileptonic decays of the baryon octet. With the explicit form factors together with radiative corrections, we present predictions for the corresponding decay widths and asymmetries.

The paper is structured as follows. First, in Section II, we discuss the basic notions of our approach which is directly connected to our previous work in Refs. [5, 10, 11]. That is, we derive a chiral Lagrangian motivated by baryon ChPT [12, 13], and write it in terms of quark and mesonic degrees of freedom. Using constituent quarks dressed with a cloud of light pseudoscalar mesons and other mesons heavier than the pseudoscalar mesons, we derive dressed transition operators within the chiral expansion, which are in turn used in a Lorentz and gauge invariant quark model [8] explicitly including internal quark dynamics to calculate baryon matrix elements. In Section III we derive specific expressions for the vector and axial baryon semileptonic decay constants, while in Section IV we present the numerical analysis of the axial nucleon charge and the vector and axial vector hyperon semileptonic form factors. Finally, in Section V we summarize our results.

II. APPROACH

A. Matrix elements of semileptonic decays of the baryon octet

In Refs. [5, 10, 11] we have developed a Lorentz covariant quark approach which allowed us to study light baryon properties based on the inclusion of chiral effects in a consistent fashion by matching the quark model approach to the predictions of ChPT. In particular, our results for various baryon properties (static properties and form factors in the Euclidean region) derived in [5, 10, 11] using this approach satisfy the low–energy theorems and identities dictated by the infrared singularities of QCD (see, *e.g.*, the detailed discussion in Refs. [5, 10] and a brief overview in Section II C).

The main idea is to include chiral effects in the transition quark operators, which are then sandwiched between the respective baryon states. We have developed a technique which allows us to explicitly generate chiral corrections associated with the small scale $\lambda \sim m_q$, where m_q is the constituent quark mass, together with effects of the internal dynamics of the valence quarks. In particular, as a first step, we dress the bare valence quark operators by a cloud of pseudoscalar mesons and states heavier than the pseudoscalar mesons in a straightforward manner by the use of an effective chirally–invariant Lagrangian (see the explicit forms in Refs. [5, 10, 11] and the relevant expressions for the calculation of semileptonic form factors below). In particular, the Lagrangian which dynamically generates the dressing of the constituent quarks by the mesonic degrees of freedom, consists of two basic pieces $\mathcal{L}_q$ and $\mathcal{L}_U$:

$$\mathcal{L}_{qU} \,=\, \mathcal{L}_q + \mathcal{L}_U\,, \qquad \mathcal{L}_q \,=\, \mathcal{L}_q^{(1)} + \mathcal{L}_q^{(2)} + \mathcal{L}_q^{(3)} + \mathcal{L}_q^{(4)} + \cdots\,, \qquad \mathcal{L}_U \,=\, \mathcal{L}_U^{(2)} + \cdots\,. \tag{1}$$

The superscript (i) attached to $\mathcal{L}_U^{(i)}$ and $\mathcal{L}_q^{(i)}$ denotes the low energy dimension of the Lagrangian:

$$\mathcal{L}_U^{(2)} \;=\; \frac{F^2}{4}\langle u_\mu u^\mu + \chi_+\rangle\,, \qquad \mathcal{L}_q^{(1)} \;=\; \bar q \left[i\not{D} - m + \frac{1}{2}\, g \not{u}\,\gamma^5 \right] q\,, \tag{2a}$$

$$\mathcal{L}_q^{(2)} \;=\; \frac{C_3^q}{2}\,\langle u_\mu u^\mu\rangle\, \bar q\, q \,+\, \frac{C_4^q}{4}\, \bar q\, i\,\sigma^{\mu\nu}\,[u_\mu, u_\nu]\, q + \frac{C_6^q}{8m}\, \bar q\,\sigma^{\mu\nu} F_{\mu\nu}^+\, q \,+\, \cdots\,, \tag{2b}$$

$$\mathcal{L}_q^{(3)} \;=\; \frac{D_{16}^q}{2}\, \bar q \not{u}\,\gamma^5\, q\,\langle\,\chi_+\rangle + \frac{D_{17}^q}{8}\, \bar q\,\{\not{u}\,\gamma^5\,, \hat\chi_+\}\, q + \frac{iD_{18}^q}{2}\, \bar q \gamma^\mu\gamma^5\,[D_\mu, \chi_-]\, q + \frac{D_{22}^q}{2}\, \bar q \gamma^\mu\gamma^5\,[D^\nu, F_{\mu\nu}^-]\, q \,+\, \cdots\,, \tag{2c}$$

$$\mathcal{L}_q^{(4)} \;=\; \frac{E_6^q}{2}\langle\chi_+\rangle\, \bar q\, \sigma^{\mu\nu} F_{\mu\nu}^+\, q + \frac{E_7^q}{4} \bar q\, \sigma^{\mu\nu}\{F_{\mu\nu}^+ \hat\chi_+\}\, q + \frac{E_8^q}{2} \bar q\, \sigma^{\mu\nu}\langle F_{\mu\nu}^+ \hat\chi_+\rangle\, q + \cdots\,, \tag{2d}$$

where the symbols $\langle\;\rangle$, $[\;]$ and $\{\;\}$ occurring in Eq. (2) denote the trace over flavor matrices, commutator, and anticommutator, respectively. In Eq. (2) we display only the terms involved in the calculation of semileptonic vector and axial vector quark coupling constants.

We use the following notation. q, $U = u^2 = \exp(i\phi/F)$ are the quark and chiral fields, respectively, where ϕ is the octet of pseudoscalar fields and F is the octet decay constant, $\sigma_{\mu\nu} = i/2[\gamma_\mu, \gamma_\nu]$, $u_\mu = i\{u^\dagger, \nabla_\mu u\}$. D_μ and ∇_μ are the covariant derivatives acting on the quark and chiral fields, respectively, including external vector (v_μ) and axial (a_μ) fields, $F_{\mu\nu}^\pm = u^\dagger F_{\mu\nu}^R u \pm u F_{\mu\nu}^L u^\dagger$ is the stress tensor involving v_μ and a_μ, $\chi_\pm = u^\dagger \chi u^\dagger \pm u \chi^\dagger u$ and $\hat\chi_+ = \chi_+ - \langle\chi_+\rangle/3$ with $\chi = 2B\mathcal{M} + \ldots$, where B is the quark vacuum condensate parameter and $\mathcal{M} = \mathrm{diag}\{\hat m, \hat m, \hat m_s\}$ is the mass matrix of current quarks (We work in the isospin symmetry limit with $\hat m_u = \hat m_d = \hat m = 7$ MeV. The mass of the strange quark $\hat m_s$ is related to the nonstrange one via $\hat m_s \simeq 25\,\hat m$).

The parameters $m = 420$ MeV and $g = 0.9$ denote the constituent quark mass and axial charge in the chiral limit (*i.e.*, they are counted as quantities of order $\mathcal{O}(1)$ in the chiral expansion). C_i^q, D_i^q and E_i^q are the SU(3) quark second–, third– and fourth–order low–energy constants (LEC's). We denote the SU(3) quark LEC's by capital letters in order to distinguish them from the SU(2) LEC's c_i^q, d_i^q and e_i^q. Also, for the quark LEC's we use the additional superscript "q" to differentiate them from the analogous ChPT LEC's: C_i, D_i, E_i in SU(3) and c_i, d_i, e_i in SU(2). For the numerical analysis we will use: $M_\pi = 139.57$ MeV, $M_K = 493.677$ MeV (the charged pion and kaon masses), $M_\eta = 547.51$ MeV and $F = (F_\pi + F_K)/2$ in SU(3) with $F_\pi = 92.4$ MeV and $F_K/F_\pi = 1.22$. Using the Lagrangian (2) we can calculate the semileptonic vector and axial vector quark couplings including chiral corrections following the procedure discussed in detail in Refs. [5, 10, 11]. In Appendix A we list the results for the semileptonic quark couplings $f_{1,2,3}^{du}$, $f_{1,2,3}^{su}$, $g_{1,2,3}^{du}$ and $g_{1,2,3}^{su}$ up to order $\mathcal{O}(p^4)$ in the three–flavor picture.

In Refs. [10, 11] we illustrated the dressing technique in the case of the electromagnetic quark operator. We performed a detailed analysis of the electromagnetic properties of the baryon octet and of the $\Delta \to N\gamma$ transition. In Ref. [5] we extended this technique to the case of vector and axial vector quark operators, deriving master formulae for the calculation of the semileptonic form factors of baryons including the effects of valence quarks together with chiral corrections. Below we briefly review the derivation of these master formulae, which will be the starting point for the present paper.

First, we define the bare vector and axial vector quark transition operators constructed from quark fields of flavor i and j as:

$$J_{\mu,\,V}(q) = \int d^4x\, e^{-iqx}\, j_{\mu,\,V}(x)\,, \qquad j_{\mu,V}(x) = \bar q_j(x)\,\gamma_\mu\, q_i(x)\,, \tag{3a}$$

$$J_{\mu,\,A}(q) = \int d^4x\, e^{-iqx}\, j_{\mu,\,A}(x)\,, \qquad j_{\mu,A}(x) = \bar q_j(x)\,\gamma_\mu\,\gamma_5\, q_i(x)\,. \tag{3b}$$

Next, using the chiral Lagrangian derived in Ref. [5], we construct the vector/axial vector currents with quantum numbers of the bare quark currents which include mesonic degrees of freedom. These currents are then projected on the corresponding (initial and final) quark states in order to evaluate dressed vector $f_k^{ij}(q^2)$ and axial vector $g_k^{ij}(q^2)$ $(k = 1, 2, 3)$ quark form factors which encode the chiral corrections. Finally, using the dressed quark form factors in momentum space we can determine their Fourier–transform in coordinate space.

In the one-body approximation the dressed quark operators $j^{\rm dress}_{\mu,\,V(A)}(x)$ and their Fourier transforms $J^{\rm dress}_{\mu,\,V(A)}(q)$ have the forms (for an extension which also includes the two–body quark–quark interactions see Ref. [5])

$$\begin{aligned} j^{\rm dress}_{\mu,\,V}(x) &= f_1^{ij}(-\partial^2)\,[\bar q_j(x)\gamma_\mu q_i(x)] \,+\, \frac{f_2^{ij}(-\partial^2)}{m_i}\,\partial^\nu\,[\bar q_j(x)\sigma_{\mu\nu}q_i(x)] \,-\, \frac{f_3^{ij}(-\partial^2)}{m_i}\,i\,\partial_\mu\,[\bar q_j(x)q_i(x)]\,, && \text{(4a)}\\ J^{\rm dress}_{\mu,\,V}(q) &= \int d^4x\,e^{-iqx}\,\bar q_j(x)\left[\gamma_\mu\,f_1^{ij}(q^2) \,+\, \frac{i\sigma_{\mu\nu}\,q^\nu}{m_i}\,f_2^{ij}(q^2) \,+\, \frac{q_\mu}{m_i}\,f_3^{ij}(q^2)\right]q_i(x)\,, && \text{(4b)}\end{aligned}$$

and

$$\begin{aligned} j^{\rm dress}_{\mu,\,A}(x) &= g_1^{ij}(-\partial^2)\,[\bar q_i(x)\gamma_\mu\gamma_5 q_j(x)] \,+\, \frac{g_2^{ij}(-\partial^2)}{m_i}\,\partial^\nu\,[\bar q_j(x)\sigma_{\mu\nu}\gamma_5 q_i(x)] \,-\, \frac{g_3^{ij}(-\partial^2)}{m_i}\,i\,\partial_\mu\,[\bar q_j(x)\gamma_5 q_i(x)]\,, && \text{(5a)}\\ J^{\rm dress}_{\mu,\,A}(q) &= \int d^4x\,e^{-iqx}\,\bar q_j(x)\left[\gamma_\mu\,\gamma_5\,g_1^{ij}(q^2) \,+\, \frac{i\sigma_{\mu\nu}q^\nu}{m_i}\,\gamma_5\,g_2^{ij}(q^2) \,+\, \frac{q_\mu}{m_i}\,\gamma_5\,g_3^{ij}(q^2)\right]q_i(x)\,, && \text{(5b)}\end{aligned}$$

where $m_{i(j)}$ denotes the dressed constituent quark mass of the $i(j)$–th flavor generated by the corresponding chiral Lagrangian (for details see Ref. [10]); $f^{ij}_{1,2,3}(q^2)$ and $g^{ij}_{1,2,3}(q^2)$ denote the quark-level vector and axial vector $i \to j$ flavor changing form factors. Up to and including the third order in the chiral expansion, the tree and loop diagrams which contribute to the dressed vector $J^{\rm dress}_{\mu,\,V}(q)$ and axial vector $J^{\rm dress}_{\mu,\,A}(q)$ operators, respectively, are displayed in Figs.1 and 2 of Ref. [5]. In Appendix A we present our results for the semileptonic vector $f_k^{ij} = f_k^{ij}(0)$ and axial $g_k^{ij} = g_k^{ij}(0)$ couplings at the order of accuracy at which we work – up to order $\mathcal{O}(p^4)$ in the three–flavor picture including chiral corrections (both SU(3)–symmetric and SU(3)–breaking). For simplicity we restrict our approach to the isospin symmetry limit in our consideration.

In order to calculate the vector and axial vector current transitions between baryons we sandwich the dressed quark operators between the relevant baryon states. The master formulae are:

$$\langle B_j(p')|\,J^{\rm dress}_{\mu,\,V(A)}(q)\,|B_i(p)\rangle = (2\pi)^4\,\delta^4(p'-p-q)\,M^{B_iB_j}_{\mu,\,V(A)}(p,p')\,, \tag{6}$$

$$\begin{aligned} M^{B_iB_j}_{\mu,\,V}(p,p') &= \sum_{k=1}^3 f_k^{ij}(q^2)\,\langle B_j(p')|\,V^{ij}_{\mu,k}(0)\,|B_i(p)\rangle \\ &= \bar u_{B_j}(p')\Big\{\gamma_\mu\,F_1^{B_iB_j}(q^2) \,+\, \frac{i\sigma_{\mu\nu}q^\nu}{m_{B_i}}\,F_2^{B_iB_j}(q^2) + \frac{q_\mu}{m_{B_i}}\,F_3^{B_iB_j}(q^2)\Big\}u_{B_i}(p)\;, && \text{(7a)}\\ M^{B_iB_j}_{\mu,\,A}(p,p') &= \sum_{k=1}^3 g_k^{ij}(q^2)\,\langle B_j(p')|\,A^{ij}_{\mu,k}(0)\,|B_i(p)\rangle \\ &= \bar u_{B_j}(p')\Big\{\gamma_\mu\,\gamma_5\,G_1^{B_iB_j}(q^2) \,+\, \frac{i\,\sigma_{\mu\nu}q^\nu}{m_{B_i}}\,\gamma_5\,G_2^{B_iB_j}(q^2) + \frac{q_\mu}{m_{B_i}}\,\gamma_5\,G_3^{B_iB_j}(q^2)\Big\}u_{B_i}(p)\,, && \text{(7b)}\end{aligned}$$

where $B_i(p)$ denotes the baryon state and $u_{B_i}(p)$ is the baryon spinor normalized according to

$$\langle B_i(p')|B_i(p)\rangle = 2E_{B_i}\,(2\pi)^3\,\delta^3(\vec p-\vec p\,')\,, \qquad \bar u_{B_i}(p)u_{B_i}(p) = 2m_{B_i}\,. \tag{8}$$

The baryon energy and its mass are denoted by $E_{B_i} = \sqrt{m^2_{B_i}+\vec p^{\,2}}$ and m_{B_i}. The index $i(j)$ attached to the baryon state indicates the flavor of the quark involved in the semileptonic transition, and $F_k^{B_iB_j}(q^2)$ and $G_k^{B_iB_j}(q^2)$ with $k = 1, 2, 3$ are the vector and axial vector semileptonic form factors of the baryons.

The main idea of the above relations is to express the matrix elements of the dressed quark operators in terms of the matrix elements of the bare vector and axial vector quark operators $V^{ij}_{\mu,k}(0)$ and $A^{ij}_{\mu,k}(0)$, respectively, where

$$V^{ij}_{\mu,k}(0) = \bar q_j(0)\Gamma^V_{\mu,k}q_i(0)\,, \qquad\qquad A^{ij}_{\mu,k}(0) = \bar q_j(0)\Gamma^A_{\mu,k}q_i(0)\,, \tag{9}$$

with

$$\Gamma^V_{\mu,1} = \gamma_\mu\,, \qquad \Gamma^V_{\mu,2} = \frac{i\sigma_{\mu\nu}q^\nu}{m_i}\,, \qquad \Gamma^V_{\mu,3} = \frac{q_\mu}{m_i}\,,$$
$$\Gamma^A_{\mu,1} = \gamma_\mu\gamma_5\,, \quad \Gamma^A_{\mu,2} = \frac{i\sigma_{\mu\nu}q^\nu}{m_i}\gamma_5\,, \quad \Gamma^A_{\mu,3} = \frac{q_\mu}{m_i}\gamma_5\,. \tag{10}$$

Next we specify the expansion of the bare matrix elements $\langle B_j(p')|\, V^{ij}_{\mu,k}(0)\,|B_i(p)\rangle$ and $\langle B_j(p')|\, A^{ij}_{\mu,k}(0)\,|B_i(p)\rangle$ in terms of the form factors $V^{B_iB_j}_{lk}(q^2)$ and $A^{B_iB_j}_{lk}(q^2)$ with $(l = 1, 2, 3)$ encoding the effects of the internal dynamics of valence quarks:

$$\langle B_j(p')|\, V^{ij}_{\mu,k}(0)\,|B_i(p)\rangle \;=\; \bar u_{B_j}(p')\left(\gamma_\mu\, V^{B_iB_j}_{1k}(q^2) \,+\, \frac{i\sigma_{\mu\nu}q^\nu}{m_{B_i}}\, V^{B_iB_j}_{2k}(q^2) + \frac{q_\mu}{m_{B_i}}\, V^{B_iB_j}_{3k}(q^2)\right) u_{B_i}(p)\,, \tag{11a}$$
$$\langle B_j(p')|\, A^{ij}_{\mu,k}(0)\,|B_i(p)\rangle \;=\; \bar u_{B_j}(p)\left(\gamma_\mu\gamma_5\, A^{B_iB_j}_{1k}(q^2) \,+\, \frac{i\sigma_{\mu\nu}q^\nu}{m_{B_i}}\gamma_5\, A^{B_iB_j}_{2k}(q^2) + \frac{q_\mu}{m_{B_i}}\gamma_5\, A^{B_iB_j}_{3k}(q^2)\right) u_{B_i}(p)\,. \tag{11b}$$

Combining chiral effects (encoded in the chiral form factors $f^{ij}_k(q^2)$ and $g^{ij}_k(q^2)$) and valence quarks effects (encoded in the form factors $V^{B_iB_j}_{lk}(q^2)$ and $A^{B_iB_j}_{lk}(q^2)$) the expressions for the vector and axial vector form factors $F^{B_iB_j}_k$ and $G^{B_iB_j}_1$, which govern the semileptonic transitions between octet baryons, are defined as:

$$F^{B_iB_j}_1(q^2) = \sum_{k=1}^{2} f^{ij}_k(q^2)\, V^{B_iB_j}_{1k}(q^2)\,, \qquad G^{B_iB_j}_1(q^2) = \sum_{k=1}^{2} g^{ij}_k(q^2)\, A^{B_iB_j}_{1k}(q^2)\,, \tag{12a}$$
$$F^{B_iB_j}_2(q^2) = \sum_{k=1}^{2} f^{ij}_k(q^2)\, V^{B_iB_j}_{2k}(q^2)\,, \qquad G^{B_iB_j}_2(q^2) = \sum_{k=1}^{2} g^{ij}_k(q^2)\, A^{B_iB_j}_{2k}(q^2)\,, \tag{12b}$$
$$F^{B_iB_j}_3(q^2) = \sum_{k=1}^{3} f^{ij}_k(q^2)\, V^{B_iB_j}_{3k}(q^2)\,, \qquad G^{B_iB_j}_3(q^2) = \sum_{k=1}^{3} g^{ij}_k(q^2)\, A^{B_iB_j}_{3k}(q^2)\,, \tag{12c}$$

Note that the operators $V(A)^{ij}_{\mu,3}(0)$ are proportional to q_μ, and therefore do not generate contributions to the baryon form factors $F^{B_iB_j}_{1,2}(q^2)$ and $G^{B_iB_j}_{1,2}(q^2)$. Further simplifications occur when we consider the semileptonic coupling constants of baryons at maximal recoil $q^2 = 0$. For the couplings encoding valence quark effects we get the following constraints due to Lorentz covariance and gauge invariance:

$$V^{B_iB_j}_{12}(0) = A^{B_iB_j}_{12}(0) = 0\,, \quad V^{B_iB_j}_{31}(0) = \mathcal{O}(m_{B_i} - m_{B_j})\,, \quad V^{B_iB_j}_{32}(0) = \mathcal{O}(m_{B_i} - m_{B_j})\,. \tag{13}$$

It is seen that the $V^{B_iB_j}_{31}(0)$ and $V^{B_iB_j}_{32}(0)$ couplings start at the first order in SU(3) breaking. In the case of the couplings $f^{ij}_k = f^{ij}_k(0)$ and $g^{ij}_k = g^{ij}_k(0)$ encoding the chiral effects we have the following results (see details in Appendix A):

1) The vector coupling f^{du}_1 governing the $d \to u$ transition is trivial and equal to unity — $f^{du}_1 = 1$, because we work in the isospin symmetry limit. In the case of the $s \to u$ transition, the corresponding vector coupling f^{su}_1 contains symmetry breaking corrections of second order in SU(3) — $\mathcal{O}((M_K^2 - M_\pi^2)^2)$ and $\mathcal{O}((M_K^2 - M_\eta^2)^2)$. Note that this is nothing but the statement of the Ademollo–Gatto theorem (AGT) which asserts that the coupling f^{su}_1 is protected from *first–order* symmetry breaking corrections.

2) The coupling f^{du}_3 vanishes due to isospin invariance, while the coupling f^{su}_3 starts at first order in SU(3) breaking — $f^{su}_3 = \mathcal{O}(M_K^2 - M_\pi^2)$.

3) The axial vector couplings g^{ij}_2 are either equal to zero (e.g. the coupling g^{du}_2 governing the $d \to u$ transition) or vanish at the order of accuracy that we are working at (e.g. the coupling g^{su}_2 governing the $s \to u$ transition).

The set of Eqs. (6)–(12) contains our main result: we separate the effects of the internal dynamics of the valence quarks contained in the matrix elements of the bare quark operators $V(A)^{ij}_{\mu,k}(0)$ and the effects dictated by chiral dynamics which are encoded in the relativistic form factors $f^{ij}_k(q^2)$ and $g^{ij}_k(q^2)$. Due to the factorization of chiral effects and the effects of the internal dynamics of the valence quarks the calculation of the form factors $f(g)^{ij}_k(q^2)$ which encode the chiral dynamics, on one side, and the matrix elements of $V(A)^{ij}_{\mu,k}(0)$ which encodes the effects of the valence quarks, on the other side, can be done independently. The evaluation of the matrix elements $V(A)^{ij}_{\mu,k}(0)$ is not restricted to small momenta squared and, therefore, can shed light on baryon form factors at higher (Euclidean) momentum squared in comparison with ChPT. In particular, as a first step, we employ a formalism motivated by the ChPT Lagrangian for the calculation of $f(g)_k^{ij}(q^2)$ which is formulated in terms of constituent quark degrees of freedom. The evaluation of the matrix elements of the bare quark operators can then be relegated to quark models based on specific assumptions on the internal quark dynamics, hadronization, and confinement. Note that Eqs. (6)–(12) are valid for the calculations of dressed vector and axial vector quark operators of *any* flavor content. In Ref. [5] we calculated the vector and axial vector coupling constants $F_1^{B_iB_j}(0)$ and $G_1^{B_iB_j}(0)$. Here we extend our analysis to all six coupling constants $F_i^{B_iB_j}(0)$ and $G_i^{B_iB_j}(0)$ $(i = 1, 2, 3)$.

B. Evaluation of the matrix elements of the valence quark operators

In this section we discuss the calculation of the baryonic matrix elements

$$\langle B(p')|\, V^{ij}_{\mu,k}(0)\, |B(p)\rangle \qquad \text{and} \qquad \langle B(p')|\, A^{ij}_{\mu,k}(0)\, |B(p)\rangle \tag{14}$$

induced by the bare quark operators (9). We will consistently employ the relativistic three-quark model (RQM) [8, 9] to compute the matrix elements (14). The RQM was previously successfully applied to the study of the properties of baryons containing light and heavy quarks [8, 9]. The main advantages of this approach are: Lorentz and gauge invariance, a small number of parameters, and the modelling of effects of strong interactions at large ($\sim$ 1 fm) distances. Various properties of light and heavy baryons in electromagnetic, strong and weak decays have been successfully analyzed within this RQM [8, 9] where the effects of valence quarks have been consistently taken into account. Here we extend this approach to evaluate the effects of valence quarks in the semileptonic decays of the baryon octet.

Let us begin by briefly reviewing the basic notions of the RQM approach [8, 9]. The RQM is based on an interaction Lagrangian describing the coupling between baryons and their constituent quarks. The coupling of a baryon $B(q_1q_2q_3)$ to its constituent quarks q_1, q_2 and q_3 is described by the Lagrangian

$$\mathcal{L}^{\rm str}_{\rm int}(x) = g_B \bar{B}(x) \int\!\! dx_1\!\! \int\!\! dx_2\!\! \int\!\! dx_3\, F(x,x_1,x_2,x_3)\, J_B(x_1,x_2,x_3) \,+\, \text{h.c.} \tag{15}$$

where $J_B(x_1,x_2,x_3)$ is a three-quark current with the quantum numbers of the relevant baryon B [14, 15]. One has

$$J_B(x_1,x_2,x_3) \;=\; \epsilon^{a_1a_2a_3}\,\Gamma_1\, q_1^{a_1}(x_1)\, q_2^{a_2}(x_2) C\,\Gamma_2\, q_3^{a_3}(x_3)\,, \tag{16}$$

where $\Gamma_{1,2}$ are Dirac structures, $C = \gamma^0\gamma^2$ is the charge conjugation matrix and $a_i (i = 1, 2, 3)$ are color indices. In Appendix B we list the relevant three-quark currents for the baryon octet. The choice of light baryon three-quark currents has been discussed in detail in Refs. [14, 15].

The function F is related to the scalar part of the Bethe-Salpeter amplitude and characterizes the finite size of the baryon. In the following we use a specific form for the vertex function [8, 9]

$$F(x,x_1,x_2,x_3) \;=\; N\; \delta^4(x - \sum_{i=1}^{3} w_i x_i)\; \Phi\biggl(\sum_{i<j}(x_i-x_j)^2\biggr) \tag{17}$$

where Φ is the correlation function of the three constituent quarks with masses m_1, m_2, m_3 and $N = 9$ is a normalization factor. With the variable w_i defined by $w_i = m_i/(m_1 + m_2 + m_3)$ the function Φ depends only on the relative Jacobi coordinates (ξ_1, ξ_2) via $\Phi(\xi_1^2 + \xi_2^2)$, where

$$\begin{aligned} x_1 &= x - \frac{\xi_1}{\sqrt{2}}\,(w_2+w_3) \,+\, \frac{\xi_2}{\sqrt{6}}\,(w_2-w_3)\,, \\ x_2 &= x + \frac{\xi_1}{\sqrt{2}}\,w_1 \,-\, \frac{\xi_2}{\sqrt{6}}\,(w_1+2w_3)\,, \\ x_3 &= x + \frac{\xi_1}{\sqrt{2}}\,w_1 \,+\, \frac{\xi_2}{\sqrt{6}}\,(w_1+2w_2)\,, \end{aligned} \tag{18}$$

and $x = \sum\limits_{i=1}^{3} w_i x_i$ is the center of mass (CM) coordinate. Expressed in terms of the relative Jacobi coordinates and the center of mass coordinate, the Fourier transform of the vertex function reads [8, 9]:

$$\Phi(\xi_1^2+\xi_2^2) = \int \frac{d^4p_1}{(2\pi)^4} \int \frac{d^4p_2}{(2\pi)^4} e^{-ip_1\xi_1 - ip_2\xi_2}\, \widetilde{\Phi}(-p_1^2-p_2^2)\,. \tag{19}$$

The baryon-quark coupling constants g_B are determined by the compositeness condition [8, 9] (see also [16, 17]), which implies that the renormalization constant of the hadron wave function is set equal to zero:

$$Z_B = 1 - \Sigma'_B(m_B) = 0 \tag{20}$$

where $\Sigma'_B(m_B) = g_B^2 \Pi'_B(m_B)$ is the first derivative of the baryon mass operator described by the diagram in Fig.1, and m_B is the baryon mass. To clarify the physical meaning of Eq.(20) we first want to remind the reader that the renormalization constant $Z_B^{1/2}$ can also be interpreted as the matrix element between the physical and the corresponding bare state. For $Z_B = 0$ it then follows that the physical state does not contain the bare one and is described as a bound state. The interaction Lagrangian Eq. (15) and the corresponding free components describe both the constituents (quarks) and the physical particles (hadrons), which are taken to be the bound states of the constituents. As a result of the interaction, the physical particle is dressed, *i.e.* its mass and its wave function have to be renormalized. The condition $Z_B = 0$ also effectively excludes the constituent degrees of freedom from the physical space and thereby guarantees that there is no double counting for the physical observable under consideration. In this picture the constituent quarks exist in virtual states only. One of the corollaries of the compositeness condition is the absence of a direct interaction of the dressed charged particle with the electromagnetic and the weak gauge boson field. Taking into account both the tree-level diagram and the diagrams with the self-energy and counter-term insertions into the external legs (that is the tree-level diagram times $(Z_B - 1)$) one obtains a common factor Z_B which is equal to zero [17].

The quantities of interest—the matrix elements (14)—are described by the triangle diagram in Fig.2(a). In case of the matrix elements $\langle B(p')|\, V^{ij}_{\mu,1}(0)\, |B(p)\rangle$ and $\langle B(p')|\, A^{ij}_{\mu,1}(0)\, |B(p)\rangle$ we need to include two additional so-called "bubble" diagrams in Figs.2(b) and 2(c) which guarantee gauge invariance of the matrix elements (see details in Refs. [8, 9] and [18, 19]). In particular, the "bubble" diagrams are generated by the non-local coupling of the baryon to the constituent quarks and the external gauge field which arises after gauging of the non-local strong interaction Lagrangian (15) containing the vertex function (17). In Appendix C we present more details of how to restore gauge invariance in the non-local strong interaction Lagrangian (15) through the "bubble" diagrams in Figs.2(b) and 2(c). Note that the contributions of the bubble diagrams Figs.2(b) and 2(c) to the matrix elements $\langle B(p')|\, V^{ij}_{\mu,1}(0)\, |B(p)\rangle$ and $\langle B(p')|\, A^{ij}_{\mu,1}(0)\, |B(p)\rangle$ are suppressed. In the present application the bubble diagrams contribute less than 5 % in magnitude compared to the contribution of the triangle diagram in Fig.2(a).

In the evaluation of the quark-loop diagrams we use the free fermion propagator for the constituent quark [8, 9]:

$$i\, S_q(x-y) = \langle 0|T\, q(x)\, \bar q(y)|0\rangle \;=\; \int \frac{d^4k}{(2\pi)^4 i}\, e^{-ik(x-y)}\; \tilde S_q(k) \tag{21}$$

where $\tilde S_q(k) = (m_q - \not k - i\epsilon)^{-1}$ is the usual free fermion propagator in momentum space. The appearance of unphysical imaginary parts in Feynman diagrams can be avoided by postulating the condition that the baryon mass must be less than the sum of the constituent quark masses $M_B < \sum_i m_{q_i}$.

In the next step we have to specify the vertex function $\tilde\Phi$, which characterizes the finite size of the baryons and the internal quark dynamics. In principle, its functional form can be calculated from the solutions of the Bethe-Salpeter equation for baryon bound states [20]. In Refs. [21] it was found that, using various forms for the vertex function, the basic hadron observables are relatively insensitive to the specific details of the functional form of the hadron-quark vertex form factor. Using this observation as a guiding principle, we select a simple Gaussian form for the vertex function $\tilde\Phi$ (any choice for $\tilde\Phi$ is appropriate as long as it falls off sufficiently fast in the ultraviolet region of Euclidean space to render the Feynman diagrams ultraviolet finite). We shall employ the Gaussian form

$$\tilde\Phi(k_{1E}^2, k_{2E}^2) \doteq \exp(-18\; [k_{1E}^2 + k_{2E}^2]/\Lambda_B^2)\,, \tag{22}$$

where k_{1E} and k_{2E} are Euclidean momenta and Λ_B is a size parameter which parametrizes the distribution of quarks inside a given baryon. In previous papers [8, 9] we have determined a set of parameters for the light baryons

$$m_u = m_d = 420 \;\; \text{MeV}\,, \quad m_s = 570 \;\; \text{MeV}\,, \quad \Lambda_B = 0.75 - 1.25 \;\; \text{GeV} \tag{23}$$

which gives very satisfactory agreement with a wide class of experimental data. Note that most of the results are not sensitive to the actual values of Λ_B in the above range. We present some sample results of this approach in Table 1. These are the magnetic moments of the baryon octet and the nucleon electromagnetic radii generated with $m_u = m_d = 420$ MeV, $m_s = 570$ MeV and $\Lambda_B = 1.25$ GeV. We show the contributions both of the valence quarks (3q) and of the meson cloud. In the present paper we present a corresponding analysis for the semileptonic coupling constants of the baryon octet using this same set of model parameters.

C. Connection with chiral perturbation theory

As stressed earlier, results for the baryon properties obtained using this approach [5, 10] satisfy the low-energy theorems and identities dictated by the infrared singularities of QCD [12],[13],[22]-[25]. As a result we can relate the parameters of our approach to those of ChPT. In particular, we have analyzed the chiral expansion of the following properties of the nucleon: mass, magnetic moments, charge radii, the πN σ–term, axial charge and πNN coupling constant in SU(2). We have also extended our results to SU(3) including kaon and η-meson degrees of freedom.

The results are:

1. Nucleon mass and πN σ–term.

$$m_N = \overset{\circ}{m}_N - 4c_1 M^2 - \frac{3\,\overset{\circ}{g}{}_A^2\, M^3}{32\pi F^2} + k_1 M^4 \ln\frac{M}{\overset{\circ}{m}_N} + k_2 M^4 + \mathcal{O}(M^5)\,, \tag{24a}$$

$$\sigma_{\pi N} = -4c_1 M^2 - \frac{9\,\overset{\circ}{g}{}_A^2\, M^3}{64\pi F^2} + \sigma_1 M^4 \ln\frac{M}{\overset{\circ}{m}_N} + \sigma_2 M^4 + \mathcal{O}(M^5)\,, \tag{24b}$$

where

$$\begin{aligned}
k_1 &= \frac{1}{2}\sigma_1 = -\frac{3}{32\pi^2 F^2\,\overset{\circ}{m}_N}\left(\overset{\circ}{g}{}_A^2 - 8c_1\,\overset{\circ}{m}_N + c_2\,\overset{\circ}{m}_N + 4c_3\,\overset{\circ}{m}_N\right),\\
k_2 &= \bar{e}_1 - \frac{3}{128\pi^2 F^2\,\overset{\circ}{m}_N}\left(2\,\overset{\circ}{g}{}_A^2 - c_2\,\overset{\circ}{m}_N\right),\\
\sigma_2 &= 2\bar{e}_1 - \frac{3}{64\pi^2 F^2\,\overset{\circ}{m}_N}\left(\overset{\circ}{g}{}_A^2 - 8c_1\,\overset{\circ}{m}_N + 4c_3\,\overset{\circ}{m}_N\right),\\
\bar{e}_1 &= e_1 - \frac{3\bar{\lambda}}{2F^2\,\overset{\circ}{m}_N}\left(\overset{\circ}{g}{}_A^2 - 8c_1\,\overset{\circ}{m}_N + c_2\,\overset{\circ}{m}_N + 4c_3\,\overset{\circ}{m}_N\right),
\end{aligned} \tag{25}$$

and

$$\lambda(\mu) = \frac{\mu^{d-4}}{(4\pi)^2}\left(\frac{1}{d-4} - \frac{1}{2}(\ln 4\pi + \Gamma'(1) + 1)\right), \qquad \bar{\lambda} = \lambda(\overset{\circ}{m}_N)\,. \tag{26}$$

2. Magnetic moments and charge radii.

$$\begin{aligned}
\mu_p &= -\frac{\overset{\circ}{g}{}_A^2}{8\pi}\,\frac{M}{F^2}\,\overset{\circ}{m}_N + \ldots\,,\\
\langle r^2\rangle_p^E &= -\frac{1 + 5\,\overset{\circ}{g}{}_A^2}{16\,\pi^2\,F^2}\ln\frac{M}{\overset{\circ}{m}_N} + \ldots\,,\\
\langle r^2\rangle_p^M &= \frac{\overset{\circ}{g}{}_A^2}{16\,\pi\,F^2\,\mu_p}\,\frac{\overset{\circ}{m}_N}{M} + \ldots\,,
\end{aligned} \tag{27}$$

3. Axial charge $g_A = G_1^{np}(0)$, πNN coupling constant and induced pseudoscalar form factor $g_P(q^2) = 2G_3^{np}(q^2)$.

$$g_A = \overset{\circ}{g}_A\left(1 + \frac{4\bar{d}_{16}M^2}{\overset{\circ}{g}_A} - \frac{\overset{\circ}{g}{}_A^2\, M^2}{16\pi^2 F^2} + \frac{M^3}{24\pi\,\overset{\circ}{m}_N F^2}\left(3 + 3\,\overset{\circ}{g}{}_A^2 - 4c_3\,\overset{\circ}{m}_N + 8c_4\,\overset{\circ}{m}_N\right) + \mathcal{O}(M^4)\right) \tag{28a}$$

$$\begin{aligned}
g_{\pi N} &= \frac{\overset{\circ}{g}_A \overset{\circ}{m}_N}{F}\left(1 - \frac{\bar{l}_4 M^2}{F^2} - \frac{4c_1 M^2}{\overset{\circ}{m}_N} + (4\bar{d}_{16} - 2d_{18})\frac{M^2}{\overset{\circ}{g}_A} - \frac{\overset{\circ}{g}{}_A^2\, M^2}{16\pi^2 F^2}\right.\\
&+ \left.\frac{M^3}{96\pi\,\overset{\circ}{m}_N\,F^2}(12 + 3\,\overset{\circ}{g}{}_A^2 - 16c_3\,\overset{\circ}{m}_N + 32c_4\,\overset{\circ}{m}_N) + \mathcal{O}(M^4)\right) = \frac{g_A m_N}{F_\pi}(1 + \Delta_{\rm GT})\,,
\end{aligned} \tag{28b}$$

$$g_P(q^2) = 4m_N F_\pi \frac{g_{\pi N}}{M_\pi^2 - q^2} - \frac{2}{3}m_N^2 g_A \langle r_A^2\rangle + \mathcal{O}(p^2) \tag{28c}$$

where $\langle r_A^2\rangle$ is the axial mean–square radius, $\Delta_{\rm GT} = -2d_{18}M^2/\,\overset{\circ}{g}_A + \mathcal{O}(M^4)$ is the correction [23] to the Goldberger-Treiman (GT) relation [26] which vanishes in the chiral limit (in full equivalence with the prediction of ChPT). Note that the correction $\Delta_{\rm GT}$ is related to the so-called Goldberger-Treiman discrepancy [27] $\Delta_{\rm D} = 1 - (m_N g_A / F_\pi g_{\pi N})$ via [25]: $\Delta_{\rm GT} = \Delta_{\rm D}/(1-\Delta_{\rm D})$. In Eqs. (24)-(28) we use the standard notation for the parameters of the ChPT Lagrangian: M represents the pion mass to leading–order in the chiral expansion, F_π is the leptonic decay constant (F is its value in the chiral limit), $\overset{\circ}{g}_A$ and $\overset{\circ}{m}_N$ are the axial charge and mass of the nucleon in the chiral limit; l_i, c_i, d_i and e_i are the low-energy constants (LEC's) with an overline indicating that the corresponding LEC's are renormalized.

In order to reproduce the above model–independent results we need to fulfill the following matching conditions between the parameters and LECs of the ChPT Lagrangian and our chiral quark–level Lagrangian (for the quark level LEC's we use the additional superscript "q" to differentiate them from the analogous ChPT LEC's) :

$$\frac{\overset{\circ}{m}_N}{m} = \left(\frac{\overset{\circ}{g}_A}{g}\right)^2 = R^2\,, \tag{29a}$$

$$-4c_1 M^2 = (\hat{m} - 4c_1^q M^2) R^2\,, \tag{29b}$$

$$8c_1 - c_2 - 4c_3 - \frac{\overset{\circ}{g}_A^{\,2}}{\overset{\circ}{m}_N} = \left(8c_1^q - c_2^q - 4c_3^q - \frac{\overset{\circ}{g}_A^{\,2}}{\overset{\circ}{m}_N}\right) R^2\,, \tag{29c}$$

$$\bar{e}_1 - \frac{3}{64\,\pi^2\,F^2}\left(\frac{2\,\overset{\circ}{g}_A^{\,2}}{\overset{\circ}{m}_N} - c_2\right) = \left(\bar{e}_1^q - \frac{3}{64\,\pi^2\,F^2}\left(\frac{2\,\overset{\circ}{g}_A^{\,2}}{\overset{\circ}{m}_N} - c_2^q\right)\right) R^2\,, \tag{29d}$$

$$c_3 - 2c_4 = c_3^q - 2c_4^q + \frac{3}{4\,\overset{\circ}{m}_N}(1-R^2)\,, \tag{29e}$$

$$\bar{d}_{16} - \frac{\overset{\circ}{g}_A^{\,3}}{64\pi^2 F^2} = \left(\bar{d}_{16}^q - \frac{g^3}{64\pi^2 F^2}\right) R\,, \tag{29f}$$

$$d_{18} = d_{18}^q\; R\,, \tag{29g}$$

$$d_{22} = d_{22}^q\; R + \;\overset{\circ}{g}_A\;\frac{Q}{R}\,, \tag{29h}$$

where $R = A_{11}^{np}(0)$ and $Q = (A_{11}^{np}(0))' = dA_{11}^{np}(q^2)/dq^2|_{q^2=0}$. In addition we deduce the following constraints on the form factors encoding valence quark effects: $A_{33}^{np}(0) = R^3$ and $A_{13}^{np}(0) = -2m_N^2 Q$.

III. RATES AND ASYMMETRY PARAMETERS IN SEMILEPTONIC DECAYS OF BARYONS

In this section we present detailed theoretical expressions [28]-[30] for the decay rates and asymmetry parameters in semileptonic baryon decays.

The decay width is given by the expression [28]

$$\Gamma(B_i \to B_j l \nu_l) = \frac{G_F^2}{384\pi^3 m_{B_i}^3}\;|V_{\rm CKM}|^2\,(1+\delta_{\rm rad})\int\limits_{m_l^2}^{\Delta^2} ds\;(1-m_l^2/s)^2\;\sqrt{(\Sigma^2 - s)(\Delta^2 - s)}\;N(s) \tag{30}$$

where

$$\begin{aligned} N(s) \;=\;& F_1^2(s)(\Delta^2(4s - m_l^2) + 2\Sigma^2\Delta^2(1 + 2m_l^2/s) - (\Sigma^2 + 2s)(2s + m_l^2)) \\ +\;& F_2^2(s)(\Delta^2 - s)(2\Sigma^2 + s)(2s + m_l^2)/m_{B_i}^2 + 3F_3^2(s)m_l^2(\Sigma^2 - s)s/m_{B_i}^2 \\ +\;& 6F_1(s)F_2(s)(\Delta^2 - s)(2s + m_l^2)\Sigma/m_{B_i} - 6F_1(s)F_3(s)m_l^2(\Sigma^2 - s)\Delta/m_{B_i} \\ +\;& G_1^2(s)(\Sigma^2(4s - m_l^2) + 2\Sigma^2\Delta^2(1 + 2m_l^2/s) - (\Delta^2 + 2s)(2s + m_l^2)) \\ +\;& G_2^2(s)(\Sigma^2 - s)(2\Delta^2 + s)(2s + m_l^2)/m_{B_i}^2 + 3G_3^2(s)m_l^2(\Delta^2 - s)s/m_{B_i}^2 \\ -\;& 6G_1(s)G_2(s)(\Sigma^2 - s)(2s + m_l^2)\Delta/m_{B_i} + 6G_1(s)G_3(s)m_l^2(\Delta^2 - s)\Sigma/m_{B_i}\,. \end{aligned} \tag{31}$$

We have introduced the notation: $s = q^2$, $\Sigma = m_{B_i} + m_{B_j}$, $\Delta = m_{B_i} - m_{B_j}$, $\beta = (m_{B_i} - m_{B_j})/m_{B_i}$. The factor $\delta_{\rm rad}$ represents the effect of radiative corrections [29] (see Table 2), $G_F = 1.16637 \times 10^{-5}$ GeV^{-2} is the Fermi coupling constant, and m_l is the leptonic (electron or muon) mass. For the corresponding CKM matrix elements $V_{\rm CKM} = V_{ud}$ or V_{us} we use the central values from [3]: $V_{ud} = 0.97377$ and $V_{us} = 0.225$. Also we assume that the form factors are real.

Next we simplify the master formula (30), integrating over s and including terms up to $\mathcal{O}(\beta^7)$ where $\beta = \Delta/m_{B_i}$ is the SU(3) breaking parameter. (In this case the term proportional to G_3^2 can be omitted because it already starts at order $\mathcal{O}(\beta^8)$.) Also, we include the momentum dependence of the leading form factors $F_1(s)$ and $G_1(s)$ and neglect the momentum dependence of the others. We expand the form factors $F_1(s), G_1(s)$ to first order in s:

$$F_1(s) = F_1(0)(1 + \frac{s}{6}\langle r^2_{F_1}\rangle + \mathcal{O}(s^2))\,, \qquad G_1(s) = G_1(0)(1 + \frac{s}{6}\langle r^2_{G_1}\rangle + \mathcal{O}(s^2))\,, \tag{32}$$

where $\langle r^2_{F_1}\rangle$ and $\langle r^2_{G_1}\rangle$ are the "charge" radii of the F_1 and G_1 form factors calculated within our approach (*cf.* the numerical results in Sec. IV). In addition we retain finite lepton masses. These approximations are sufficient for both the $n \to pe^-\bar\nu_e$ decay and for the muonic decay modes of hyperons. We also retain terms containing the form factors F_3 and G_3. Although their effects are proportional to m_l^2 they may give a measurable contribution for muonic modes (see also the discussion in Ref. [30, 31]).

At the order of accuracy to which we work the result for the decay width reads (exact formulas can be found in [29, 30]):

$$\begin{aligned}\Gamma(B_i \to B_j l\nu_l) \;&=\; \frac{G_F^2}{60\pi^3}\,|V_{\rm CKM}|^2\,\Delta^5\,(1+\delta_{\rm rad})\biggl\{(F_1^2 + 3G_1^2)(1-\frac{3}{2}\beta)\,R_0(x) + \beta^2\biggl(\frac{6}{7}F_1^2\,R_{F_1}(x) + \frac{12}{7}G_1^2\,R_{G_1}(x)\\ &+\; \frac{4}{7}F_2^2\,R_{F_2}(x) + \frac{12}{7}G_2^2\,R_{G_2}(x) + F_3^2\,R_{F_3}(x) + \frac{6}{7}F_1F_2\,R_{F_{12}}(x) + G_1G_3\,R_{G_{13}}(x)\biggr)\\ &-\; 4\beta(1-\frac{3}{2}\beta)(F_1F_3\,R_{F_{13}}(x) + G_1G_2\,R_{G_{12}}(x))\biggr\} + \mathcal{O}(\beta^8)\,,\end{aligned} \tag{33}$$

where $F_i = F_i(0)$, $G_i = G_i(0)$ and $x = m_l/\Delta$. Here the functions $R_i(x)$ take into account the charged lepton mass m_l (see their expressions in Appendix D). In the calculation of the asymmetry parameters we restrict ourselves to the electron modes. The expressions for the electron–neutrino $\alpha_{e\nu_e}$, electron α_e, neutrino α_{ν_e} and emitted baryon α_B asymmetries to the order of accuracy at which we are working are given in [29].

IV. NUMERICAL RESULTS

In this section we present our numerical results for the semileptonic decays of the baryon octet—coupling constants, decay widths and asymmetry parameters. First, we calculate the vector $V_{i1}^{B_iB_j}$ and axial vector $A_{i1}^{B_iB_j}$ couplings representing the contribution of the pure valence quarks to the semileptonic form factors of the baryons $F_i^{B_iB_j}$ and $G_i^{B_iB_j}$, *i.e.*, when $f_1^{ij} \equiv 1$, $g_1^{ij} \equiv 1$ and $f_{2,3}^{ij} = g_{2,3}^{ij} = 0$. This limiting case corresponds to the projection of the nonrenormalized weak quark current $j_{\mu,V-A} = \bar q_j\gamma_\mu(1-\gamma_5)q_i$ between the respective baryon states. Our results for $V_{i1}^{B_iB_j}$ and $A_{i1}^{B_iB_j}$ are displayed in Tables 3 and 4. In Table 3, for comparison, we also present the predictions of the naive SU(6) model for the couplings $V_{11}^{B_iB_j}$ and $A_{11}^{B_iB_j}$.

Combining the contributions of the valence quarks and chiral effects we then derive the full expressions for the semileptonic couplings constants $F_i^{B_iB_j}$ and $G_i^{B_iB_j}$. The resulting forms are listed in Tables 5, 6 and 7. For convenience, we present the results for the leading (Fermi) $F_1^{B_iB_j} = f_1^{ij}V_{11}^{B_iB_j}$ and (Gamow-Teller) $G_1^{B_iB_j} = g_1^{ij}A_{11}^{B_iB_j}$ couplings in the form of a product of their SU(3) symmetric value together with a multiplicative factor $1 + \delta_{V,A}^{B_iB_j}$ which includes the SU(3) breaking correction $\delta_{V,A}^{B_iB_j}$. (We remind the reader that the quark couplings $f_{2,3}^{ij}$ and $g_{2,3}^{ij}$ do not contribute to the leading baryon couplings $F_1^{B_iB_j}$ and $G_1^{B_iB_j}$.) Note that the axial vector couplings g_1^{du} and g_1^{su} defining the $d \to u$ and $s \to u$ flavor transitions, respectively, are expressed in terms of the unknown LEC's C_i^q and D_i^q. We fix the value of these couplings to be $g_1^{du} = 0.874$ and $g_1^{su} = 0.855$ in order to reproduce the experimental data on the semileptonic decay widths as well as the ratio $G_1/F_1 = 1.2695$ in $n \to p + e^- + \bar\nu_e$ decay.

The nucleon axial charge in the SU(3) limit (*cf.* Appendix A)—$g_A^{\rm SU_3}$—is given by

$$g_A^{\rm SU_3} = 1.258 \tag{34}$$

while the SU(3) breaking parameters, $\delta_V^{B_iB_j}$ and $\delta_A^{B_iB_j}$ are found to have the form:

$$\begin{aligned}
\delta_V^{\Lambda p} &= -0.069 \text{ (val)} \ + \ 0.070 \text{ (ch)} \ = 0.001\,, \\
\delta_V^{\Sigma n} &= -0.061 \text{ (val)} \ + \ 0.070 \text{ (ch)} \ = 0.009\,, \\
\delta_V^{\Xi\Lambda} &= -0.048 \text{ (val)} \ + \ 0.070 \text{ (ch)} \ = 0.022\,, \\
\delta_V^{\Xi\Sigma} &= -0.028 \text{ (val)} \ + \ 0.070 \text{ (ch)} \ = 0.042\,,
\end{aligned} \tag{35}$$

and

$$\begin{aligned}
\delta_A^{np} &= 0 \text{ (val)} \ + \ 0.009 \text{ (ch)} \ = 0.009\,, \\
\delta_A^{\Sigma\Lambda} &= 0.024 \text{ (val)} \ + \ 0.009 \text{ (ch)} \ = 0.033\,, \\
\delta_A^{\Lambda p} &= -0.030 \text{ (val)} \ - \ 0.013 \text{ (ch)} \ = -0.043\,, \\
\delta_A^{\Sigma n} &= 0.091 \text{ (val)} \ - \ 0.013 \text{ (ch)} \ = 0.078\,, \\
\delta_A^{\Xi\Lambda} &= 0.066 \text{ (val)} \ - \ 0.013 \text{ (ch)} \ = 0.053\,, \\
\delta_A^{\Xi\Sigma} &= 0.0085 \text{ (val)} \ - \ 0.013 \text{ (ch)} \ = -0.0045
\end{aligned} \tag{36}$$

where have denoted the contributions of valence quarks and chiral effects by the round brackets (val) and (ch), respectively.

Note that the SU(3) breaking corrections to the vector couplings $g_V^{B_iB_j}$ begin at second order, in accord with the Ademollo-Gatto theorem (AGT) [1] (see discussion in Appendix E), while corrections to the axial couplings $g_A^{B_iB_j}$ begin at first order. In this regard, if one works to first order in symmetry breaking, our results must be expressible in terms of a model-independent representation for the axial couplings derived in terms of the SU(3) symmetric couplings D and F plus four SU(3)-breaking parameters H_i [1, 32] (*cf.* the discussion in Ref. [5])—

$$\begin{aligned}
g_A^{np} &= D + F + \frac{2}{3}(H_2 - H_3)\,, \\
g_A^{\Lambda p} &= -\sqrt{\frac{3}{2}}\left(F + \frac{D}{3} + \frac{1}{9}(H_1 - 2H_2 - 3H_3 - 6H_4)\right), \\
g_A^{\Sigma^- n} &= D - F - \frac{1}{3}(H_1 + H_3)\,, \\
g_A^{\Sigma^- \Lambda} &= \sqrt{\frac{2}{3}}\left(D + \frac{1}{3}(H_1 + H_2 + 3H_4)\right), \\
g_A^{\Xi^- \Lambda} &= \sqrt{\frac{3}{2}}\left(F - \frac{D}{3} + \frac{1}{9}(2H_1 - H_2 - 3H_3 + 6H_4)\right), \\
g_A^{\Xi^- \Sigma^0} &= \sqrt{\frac{1}{2}}\left(D + F - \frac{1}{3}(H_2 - H_3)\right), \\
g_A^{\Xi^0 \Sigma^+} &= D + F - \frac{1}{3}(H_2 - H_3)\,.
\end{aligned} \tag{37}$$

Such a representation is indeed found to hold in our model with the values

$$D = 0.7505\,, \quad F = 0.5075 \tag{38}$$

for the SU(3) symmetric couplings, and

$$H_1 = -0.050\,, \quad H_2 = 0.011\,, \quad H_3 = -0.006\,, \quad H_4 = 0.037 \tag{39}$$

for the SU(3) breaking terms. The components of $\delta_A^{B_iB_j}$ which are *first* order in symmetry breaking—$\delta_A^{B_iB_j(1)}$—are proportional to the couplings H_i via:

$$\begin{aligned}
\delta_A^{np(1)} &= -2\delta_A^{\Xi\Sigma(1)} = \frac{2(H_2 - H_3)}{3(D+F)}\,, \\
\delta_A^{\Lambda p(1)} &= \frac{H_1 - 2H_2 - 3H_3 - 6H_4}{3(D+3F)}\,, \\
\delta_A^{\Sigma n(1)} &= -\frac{H_1 + H_3}{3(D-F)}\,, \\
\delta_A^{\Sigma\Lambda(1)} &= \frac{H_1 + H_2 + 3H_4}{3D}\,, \\
\delta_A^{\Xi\Lambda(1)} &= \frac{2H_1 - H_2 - 3H_3 + 6H_4}{3(3F-D)}\,.
\end{aligned} \tag{40}$$

From Eq. (40) one obtains a sum rule which relates the corrections $\delta_A^{np(1)} = -2\delta_A^{\Xi\Sigma(1)}$ to a linear combination of the four remaining $SU(3)$ breaking $\delta_A^{(1)}$–parameters together with the SU(3)-symmetric couplings F and D:

$$\delta_A^{np(1)} = -2\delta_A^{\Xi\Sigma(1)} = \frac{2}{3}\Big(\frac{D-3F}{D+F}\delta_A^{\Xi\Lambda(1)} + \frac{D+3F}{D+F}\delta_A^{\Lambda p(1)} + \frac{3(D-F)}{D+F}\delta_A^{\Sigma n(1)} + \frac{4D}{D+F}\delta_A^{\Sigma\Lambda(1)}\Big)\,. \tag{41}$$

The SU(3) LEC's from the chiral Lagrangian (2) can now be determined. Three of the four couplings C_3^q, C_4^q, $\bar{D}_{16}^q$ and $\bar{D}_{16}^q$ can be fixed by use of three constraints: the value of the nucleon axial charge in the SU(3) limit $g_A^{\rm SU_3} = D + F = 1.258$ together with the values of the axial quark couplings $g_1^{du} = 0.874$ and $g_1^{su} = 0.855$. Keeping, *e.g.*, $\bar{D}_{17}^q$ undetermined we can relate the remaining three LEC's via:

$$C_3^q = -0.319 \text{ GeV}^{-1}\ \bar{D}_{17}^q\,, \quad C_4^q = -0.451 \text{ GeV}^{-1}\ \bar{D}_{17}^q\,, \quad \bar{D}_{16}^q = 0.397\ \bar{D}_{17}^q\,. \tag{42}$$

In turn, the couplings $C_6^q = -1.476$, $\bar{E}_7^q = 0.086$ GeV^{-3}, $\bar{E}_8^q = 0.532$ GeV^{-3} are fixed from the description of magnetic moments of the baryon octet, while $\bar{E}_6^q = 1.868$ GeV^{-3} is found from the induced pseudoscalar form factor of the nucleon. The coupling $D_{22}^q = 0.006$ GeV^{-2} is determined by fitting the slope of the form factor G_1^{np}: $\langle r_{G_1}^2\rangle = 0.45$ fm^2. Finally, the coupling $D_{18}^q = -0.548$ GeV^{-2} is fixed by the fitting the central value of the induced pseudoscalar coupling of the nucleon $g_p = (M_\mu/m_N)G_1^{np}(q^2 = -0.88M_\mu^2) \simeq 8.25$ predicted by ChPT [24, 25] together with the value of the pion–nucleon coupling constant $g_{\pi N} = 13.10$. It should be noted that the LEC's C_6^q, $\bar{E}_7^q$, $\bar{E}_8^q$, D_{18}^q and D_{22}^q are unimportant for reproducing the semileptonic decay widths because they make no contribution to the leading baryon coupling constants $F_1^{B_iB_j}$ and $G_1^{B_iB_j}$.

Of particular interest is the decay $\Sigma^- \to ne^-\bar{\nu}_e$ for which we predict $G_1/F_1 = -0.260$ and $(G_1 - 0.237G_2)/F_1 = -0.278$ (see Table 6). The latter result underestimates the experimental value $-0.327 \pm 0.007 \pm 0.019$. However, this ratio was extracted by neglecting the q^2 dependence of the form factors F_1 and G_1 in the decay $\Sigma^- \to ne^-\bar{\nu}_e$ decay. We find (see the discussion below) that inclusion of the q^2 dependence brings about agreement with the data for both electron and muon decay widths of the decay $\Sigma^- \to n\,l^-\bar{\nu}_l$.

In Table 7 we present our results for the nonleading baryon semileptonic couplings $F_{2,3}$ and $G_{2,3}$. One can see that the pseudoscalar couplings $G_3^{B_iB_j}$ are dominated by the corresponding pion or kaon pole contribution. (Here the leading contribution of the pole term is shown in brackets.) We also display the induced pseudoscalar coupling constant of the nucleon g_p, which is fixed by the LEC D_{18}^q. In Table 8 we compare our results for the ratios $F_2^{B_iB_j}/F_1^{B_iB_j}$: i) with the predictions of the simple Cabibbo model in terms of the nucleon magnetic moments and baryon octet masses, ii) with the calculations performed in the $1/N_c$ expansion of QCD [33], and iii) with the results found in the SU(3) chiral quark-soliton model (χQSM) [34]. Because of SU(2) invariance, we exactly reproduce the result of the Cabibbo model for the ratio F_2^{np}/F_1^{np} in neutron β–decay, while for the other modes we find SU(3) breaking deviations. Our result for the ratio $F_2^{\Sigma n}/F_1^{\Sigma n} = -0.962$ compares well: i) with the experimental data (0.97 ± 0.14), ii) with the results of the $1/N_c$ expansion of QCD [33] (-1.02), iii) with the results found in the χQSM model [34] (-0.96), and iv) with calculations done in quenched lattice QCD [35] (-0.85 ± 0.45). Also, we have quite reasonable agreement for $F_2^{B_iB_j}/F_1^{B_iB_j}$ with the results of the $1/N_c$ expansion [33] and with those of the χQSM approach for the remaining semileptonic modes.

Finally, we would like to stress that our results for the various semileptonic couplings of the decay mode $\Sigma^- \to ne^-\bar{\nu}_e$ are in good agreement with the predictions of the lattice approach [35]. In Table 9 we give a detailed comparison with the results of Ref. [35] using our conventions for the semileptonic matrix elements.

It is useful to parametrize our predictions for the weak magnetic couplings F_2 in terms of SU(3) symmetric couplings together with first order SU(3) symmetry-breaking parameters. As stressed in Ref. [2] there is an ambiguity in expressing the SU(3) limit that clearly indicates the relevance of the first-order correction. It means that if in analogy to Eq. (37) we introduce a set of parameters $\{F^{F_2}, D^{F_2}, H_i^{F_2}\}$ [1] then we should apply it to $F_2^{B_iB_j}(0)$ or to $\frac{m_N}{m_{B_i}}F_2^{B_iB_j}(0)$. The second choice, $\frac{m_N}{m_{B_i}}F_2^{B_iB_j}(0)$, is traditionally preferred (See discussion in [2]. The difference is that we in addition multiply $F_2^{B_iB_j}(0)$ by the nucleon mass m_N to deal with dimensionless coupling). Otherwise the SU(3) breaking corrections will be overestimated. Within our model, we determine values for these parameters:

$$D^{F_2} = 1.237\,, \quad F^{F_2} = 0.563\,, \quad H_1^{F_2} = -0.246\,, \quad H_2^{F_2} = 0.096\,, \quad H_3^{F_2} = 0.021\,, \quad H_4^{F_2} = 0.030\,. \tag{43}$$

Also, we can check the consistency of our results with the model-independent predictions for the second-class coupling constants $\mathcal{F}^{B_iB_j} = \frac{m_N}{m_{B_i}}F_3^{B_iB_j}, \frac{m_N}{m_{B_i}}G_2^{B_iB_j}$ to first order in SU(3) breaking, which can be parametrized in terms of three SU(3) symmetry–breaking parameters $H_i^{\mathcal{F}}$ (see details in [1]):

$$\begin{aligned}
&\mathcal{F}^{np} = 0\,,\\
&\mathcal{F}^{\Lambda p} = \frac{1}{\sqrt{6}}\Big(- H_1^{\mathcal{F}} + 2H_2^{\mathcal{F}} + 2H_3^{\mathcal{F}}\Big)\,,\\
&\mathcal{F}^{\Sigma^- n} = -H_1^{\mathcal{F}}\,,\\
&\mathcal{F}^{\Sigma^- \Lambda} = -\sqrt{\frac{2}{3}}H_3^{\mathcal{F}}\,,\\
&\mathcal{F}^{\Xi^- \Lambda} = \frac{1}{\sqrt{6}}\Big(2H_1^{\mathcal{F}} - H_2^{\mathcal{F}} - 2H_3^{\mathcal{F}}\Big)\,,\\
&\mathcal{F}^{\Xi^- \Sigma^0} = -\sqrt{\frac{1}{2}}H_2^{\mathcal{F}}\,,\\
&\mathcal{F}^{\Xi^0 \Sigma^+} = -H_2^{\mathcal{F}}\,,
\end{aligned} \tag{44}$$

Using Eq. (44) one can derive the following sum rules for the amplitudes $\mathcal{F}^{B_iB_j}$:

$$\mathcal{F}^{\Lambda p} = \frac{1}{\sqrt{6}}(\mathcal{F}^{\Sigma^- n} - 2\mathcal{F}^{\Xi^0\Sigma^+}) - \mathcal{F}^{\Sigma^-\Lambda}\,, \tag{45a}$$

$$\mathcal{F}^{\Xi^-\Lambda} = -\frac{1}{\sqrt{6}}(2\mathcal{F}^{\Sigma^- n} - \mathcal{F}^{\Xi^0\Sigma^+}) + \mathcal{F}^{\Sigma^-\Lambda}\,, \tag{45b}$$

$$-\sqrt{6}(\mathcal{F}^{\Lambda p} + \mathcal{F}^{\Xi^-\Lambda}) = \mathcal{F}^{\Xi^0\Sigma^+} + \mathcal{F}^{\Sigma^- n}\,. \tag{45c}$$

(Note that the sum rule (45c) was originally derived in [1].) When we restrict our calculation to first-order SU(3) breaking terms, we indeed fulfill the sum rules (45) and for the SU(3)-breaking parameters we obtain $H_i^{\mathcal{F}}$:

$$H_1^{F_3} = 0.032\,, \quad H_2^{F_3} = -0.028\,, \quad H_3^{F_3} = -0.011\,, \quad H_1^{G_2} = 0.047\,, \quad H_2^{G_2} = -0.035\,, \quad H_3^{G_2} = -0.009\,. \tag{46}$$

Next we turn to the discussion of the semileptonic decay widths. We present our results in Table 10: i) total width Γ including all six couplings $F_{1,2,3}$ and $G_{1,2,3}$, leading q^2 dependence of F_1 and G_1 form factors and radiative corrections; ii) predictions $\Gamma(F_1, G_1)$ are the results *without* inclusion of the subleading semileptonic form factors $F_{2,3}$ and $G_{2,3}$; iii) predictions $\Gamma(F_1(0), G_1(0))$ are the total widths without inclusion of the subleading semileptonic form factors $F_{2,3}$ and $G_{2,3}$ *and* of the q^2 dependence in the form factors F_1 and G_1; iv) predictions Γ^0 are total results without radiative corrections. For comparison we present the results of a pure SU(3) fit where we include only the F_1 and G_1 coupling constants omitting the q^2 dependence of F_1 an G_1 form factors and subleading form factors $F_{2,3}$ and $G_{2,3}$. The values of F_1 and G_1 are given by the Cabibbo model [2] where G_1 is expressed in terms of the SU(3) couplings F and D. We fix F and D via $F = 0.470$ and $D = 0.800$. One can observe that the contribution of the subleading coupling constants $F_{2,3}$ and $G_{2,3}$ to the semileptonic decay width of the baryon octet is negligible. On the other hand, inclusion of q^2 dependence of the leading form factors F_1 and G_1 makes a significant difference for the $\Lambda \to p$, $\Sigma \to n$ and $\Xi \to \Lambda$ decay modes. As stressed above, this q^2 dependence inclusion substantially improves agreement with the data for both decays $\Sigma^- \to n\, l^- \bar\nu_l$ $(l = e, \mu)$. Specifically, the q^2 dependence yields a contribution of 0.78×10^6 s^{-1} (12%) to the decay width of $\Sigma^- \to n e^- \bar\nu_e$ transition and 0.61×10^6 s^{-1} (19%) to the decay width of $\Sigma^- \to n\mu^-\bar\nu_\mu$ transition.

Another interesting point of discussion – the rate ratio $R^0_{e\mu} = \Gamma(\Xi^0 \to \Sigma^+ e^- \bar\nu_e)/\Gamma(\Xi^0 \to \Sigma^+\mu^-\bar\nu_\mu)$ which has recently been measured by the KTeV Collaboration ($R^0_{e\mu} = 55.6^{+22.2}_{-16.7}$ [36]). Using a much larger data sample the NA48 Collaboration has published a preliminary value of ($R^0_{e\mu} = 114.1 \pm 19.4$ [37]). Our result $R^0_{e\mu} = 114.81$ nearly coincides with the central value of the NA48 Collaboration and is close to the theoretical prediction of Ref. [30]—$R^0_{e\mu} = 118.71$. Note, that for the corresponding ratio of the Ξ^- hyperon we find $R^-_{e\mu} = \Gamma(\Xi^- \to \Sigma^0 e^- \bar\nu_e)/\Gamma(\Xi^- \to \Sigma^0 \mu^- \bar\nu_\mu) = 77.61$.

For comparison, we present the χ^2/dof for our total results (the first column of Table 10) and the SU(3) fit: $\chi^2/8$ dof = 1.4 [this paper] and $\chi^2/8$ dof = 2.4 [SU(3) fit]. (We exclude from the χ^2 analysis the results for the neutron β decay and the poorly known data for the muonic modes of the cascade hyperons Ξ.)

As mentioned earlier, we include the momentum dependence of the $F_1(q^2)$ and $G_1(q^2)$ form factors up to first order in q^2. The slopes for F_1 and G_1 form factors calculated in our approach are found to be:

$$\langle r^2_{F_1}\rangle = \left\{ \begin{array}{l} 0.66 \text{ fm}^2\,, \;\; n \to p \\ 0.51 \text{ fm}^2\,, \;\; \Lambda \to p \\ 0.59 \text{ fm}^2\,, \;\; \Sigma \to n \\ 0.50 \text{ fm}^2\,, \;\; \Xi \to \Lambda \\ 0.43 \text{ fm}^2\,, \;\; \Xi \to \Sigma \end{array} \right. \qquad \text{and} \qquad \langle r^2_{G_1}\rangle = \left\{ \begin{array}{l} 0.45 \text{ fm}^2\,, \;\; n \to p \\ 0.32 \text{ fm}^2\,, \;\; \Lambda \to p \\ 0.40 \text{ fm}^2\,, \;\; \Sigma \to n \\ 0.41 \text{ fm}^2\,, \;\; \Sigma \to \Lambda \\ 0.30 \text{ fm}^2\,, \;\; \Xi \to \Lambda \\ 0.28 \text{ fm}^2\,, \;\; \Xi \to \Sigma \end{array} \right. . \tag{47}$$

These predictions for the radii of the F_1 and G_1 form factors are consistent both with data and with the results of alternative theoretical approaches. In particular, the electroproduction and the neutrino experiments which involve $d \to u$ transitions are well fitted using dipole formulas which give $\langle r^2_{F_1}\rangle = 0.66$ fm^2 and $\langle r^2_{G_1}\rangle = 0.40$ fm^2 for the slopes of the F_1 and G_1 form factors [38]. For the $s \to u$ modes one expects smaller radii $\langle r^2_{F_1}\rangle = 0.50$ fm^2 and $\langle r^2_{G_1}\rangle = 0.30$ fm^2, respectively (see discussion in [29, 38]). For example, the authors of [30] find slopes of $\langle r^2_{F_1}\rangle = 0.42$ fm^2 and $\langle r^2_{G_1}\rangle = 0.23$ fm^2 for the $\Xi \to \Sigma$ transition using a generalized vector dominance ansatz for the form factors. In Refs. [34, 39] the F_1 form factor radii have been calculated in the framework of ChPT and of the χQSM model. Our results are in qualitative agreement with the full covariant result of ChPT [39], while the χQSM approach [34] gives somewhat higher values for the corresponding slopes:

$$\langle r^2_{F_1}\rangle = \left\{ \begin{array}{l} 0.44 \pm 0.06 \text{ fm}^2 \text{ (ChPT)};\; 0.72 \text{ fm}^2 \;(\chi\text{QSM})\,, \;\; \Lambda \to p \\ 0.51 \pm 0.05 \text{ fm}^2 \text{ (ChPT)};\; 0.60 \text{ fm}^2 \;(\chi\text{QSM})\,, \;\; \Sigma \to n \\ 0.45 \pm 0.03 \text{ fm}^2 \text{ (ChPT)};\; 0.66 \text{ fm}^2 \;(\chi\text{QSM})\,, \;\; \Xi \to \Lambda \\ 0.46 \pm 0.07 \text{ fm}^2 \text{ (ChPT)};\; 0.80 \text{ fm}^2 \;(\chi\text{QSM})\,, \;\; \Xi \to \Sigma \end{array} \right. . \tag{48}$$

We do not include the q^2 dependence of the F_1 form factor in the $\Sigma \to \Lambda$ transition, since it vanishes on account of the assumed degeneracy of the u and d quark masses.

Our approach generates a very reasonable description of the baryon semileptonic data with only two parameters—the axial couplings g_1^{du} and g_1^{su} responsible for the $d \to u$ and $s \to u$ transitions, which are in turn expressed in terms of the parameters of the chiral Lagrangian (see Appendix A). We remind the reader that the parameters controlling the valence quark contributions to the semileptonic properties of baryons—the constituent quark masses $m_u = m_d = 420$ MeV, $m_s = 570$ MeV and the size parameter $\Lambda_B = 1.25$ GeV—have been previously fixed via the analysis of electromagnetic properties of the baryon octet [8, 11]. Also, the same set of parameters ($m_u = m_d, m_s, \Lambda_B$) has been successfully used in the analysis of strong, electromagnetic and weak decays of charm and bottom baryons with light baryons in the final state [8]. In Table 11 we present the decay rates of hyperons divided by the squared CKM matrix elements in order to remove the uncertainty related to the values of V_{ud} and V_{us}. Finally, in Table 12 we display the predictions for the asymmetry parameters in the electron modes.

V. SUMMARY

In this paper we have analyzed the semileptonic decay properties (coupling constants, decay widths and asymmetry parameters) of the baryon octet using a manifestly Lorentz covariant quark approach including chiral and SU(3) symmetry breaking effects.

Our main results are summarized as follows:

– We have derived results for the six couplings governing the semileptonic decays of the baryon octet, revealing both chiral and SU(3) symmetry–breaking corrections;

– We presented a numerical analysis of the decay rates and asymmetry parameters in the semileptonic decays of the baryon octet.

Our results provide a generally improved representation of hyperon semileptonic decay over the conventional SU(3)-symmetric (Cabibbo) analysis. We hope that the results of this paper can be used to reliably extract a value of the CKM matrix element V_{us} from semileptonic hyperon decay data along the lines of [2].

Acknowledgments

This work was supported by the DFG under Contract No. FA67/31-1, No. FA67/31-2, and No. GRK683. B.R.H. is supported by the US National Science Foundation under Grant No. PHY 05-53304. M.A.I. appreciates the partial support of the Heisenberg-Landau program and DFG grant KO 1069/12-1. This research is also part of the EU Integrated Infrastructure Initiative Hadronphysics project under contract number RII3-CT-2004-506078 and President grant of Russia "Scientific Schools" No. 871.2008.2.

APPENDIX A: CHIRAL EXPANSION OF THE VECTOR AND AXIAL VECTOR QUARK COUPLINGS

In this Appendix we list the results for the semileptonic vector and axial quark couplings including chiral corrections (both SU(3)–symmetric and SU(3)–breaking). The corresponding SU(3) chiral quark Lagrangian $\mathcal{L}_{qU}$ is specified in Sec.II. Below we list the results for the semileptonic quark couplings $f^{du}_{1,2,3}$, $f^{su}_{1,2,3}$, $g^{du}_{1,2,3}$ and $g^{su}_{1,2,3}$ up to order $\mathcal{O}(p^4)$ in the three–flavor picture.

1. Vector quark couplings.

a) Couplings f_1^{du} and f_1^{su}:

The vector coupling governing the $d \to u$ transition is trivial and equal to unity — $f_1^{du} = 1$, because we work in the isospin limit. In the case of the $s \to u$ transition, the corresponding vector coupling f_1^{su} contains symmetry breaking corrections of second order in SU(3) — $\mathcal{O}((M_K - M_\pi)^2)$ and $\mathcal{O}((M_K - M_\eta)^2)$. Note, that the Ademollo–Gatto theorem (AGT) protects the coupling f_1^{su} from *first–order* symmetry breaking corrections. The result for the f_1^{su} is

$$f_1^{su} = 1 - \frac{3}{16}\biggl((1+3g^2)(H_{\pi K} + H_{\eta K}) + 3g^2(G_{\pi K} + G_{\eta K})\biggr) = 1 + \delta f_1^{su} \,. \tag{A1}$$

Here $\delta f_1^{su} = 0.07$ is the $SU(3)$ breaking correction. The $\mathcal{O}(p^2)$ functions H_{ab} and G_{ab}, which show up in the context of ChPT [see, *e.g.*, Refs. [39, 40]], are defined as

$$H_{ab} \;=\; \frac{1}{(4\pi F)^2}\biggl(M_a^2 + M_b^2 - \frac{2M_a^2 M_b^2}{M_a^2 - M_b^2}\ln\frac{M_a^2}{M_b^2}\biggr) = \mathcal{O}((M_a^2 - M_b^2)^2)\,, \tag{A2a}$$

$$G_{ab} \;=\; -\frac{1}{(4\pi F)^2}\frac{2\pi}{3m}\frac{(M_a - M_b)^2}{M_a + M_b}(M_a^2 + 3M_a M_b + M_b^2) = \mathcal{O}((M_a^2 - M_b^2)^2)\,. \tag{A2b}$$

b) Couplings f_2^{du} and f_2^{su}:

The coupling f_2^{du} is expressed through the linear combination of diagonal couplings f_2^u and f_2^d relevant for $u \to u$ and $d \to d$ transitions:

$$f_2^{du} \;=\; \frac{1}{2}(f_2^u - f_2^d) = f_2^{\mathrm{SU}_3} + \delta f_2^{du}\,, \tag{A3a}$$

$$f_2^u \;=\; \frac{4}{3} f_2^{\mathrm{SU}_3} + \delta f_2^u\,, \tag{A3b}$$

$$f_3^d \;=\; -\frac{2}{3} f_2^{\mathrm{SU}_3} + \delta f_2^d\,, \tag{A3c}$$

where

$$f_2^{\mathrm{SU}_3} = C_6^q\biggl(\frac{1}{2} - \frac{3g^2\bar M^2}{32\pi^2 F^2}\biggr) + 12m\bar E_6^q \bar M^2 - \frac{3g^2\bar M m}{16\pi^2 F^2}\biggl(\pi + \frac{\bar M}{m}\biggr) + \mathcal{O}(\bar M^3) \tag{A4}$$

is the SU(3) symmetric term, and δf_2^{du}, δf_2^u and δf_2^d are the SU(3) breaking terms. The first–order terms read:

$$\delta f_2^u \;=\; h_2^u(M_K^2 - M_\pi^2) + \mathcal{O}((M_K^2 - M_\pi^2)^2)\,, \tag{A5a}$$

$$\delta f_2^d \;=\; -2\delta f_2^u - \frac{16}{3}m(\bar E_7^q - \bar E_8^q)(M_K^2 - M_\pi^2)\,, \tag{A5b}$$

$$\delta f_2^{du} \;=\; \frac{1}{2}(\delta f_2^u - \delta f_2^d)\,, \tag{A5c}$$

$$h_2^u \;=\; C_6^q \frac{g^2}{48\pi^2 F^2} - \frac{16}{9}m(2\bar E_7^q + 3\bar E_8^q) + \frac{g^2 m}{48\pi^2 F^2 \bar M}\biggl(\pi + \frac{2\bar M}{m}\biggr) + \mathcal{O}(\bar M)\,. \tag{A5d}$$

The coupling f_2^{su} is given by

$$f_2^{su} = f_2^{\rm SU_3} + \delta f_2^{su} \tag{A6}$$

where

$$\begin{aligned} \delta f_2^{su} &= (M_K^2 - M_\pi^2) h_2^{su} + \mathcal{O}((M_K^2 - M_\pi^2)^2)\,, & \text{(A7a)} \\ h_2^{su} &= -C_6^q \frac{g^2}{64\pi^2 F^2} + \frac{8}{3} m \bar{E}_7^q - \frac{g^2 m}{64\pi^2 F^2 \bar{M}} \biggl(\pi + \frac{2\bar{M}}{m}\biggr) + \mathcal{O}(\bar{M})\,. & \text{(A7b)} \end{aligned}$$

Here, for convenience, we define the so–called SU(3) symmetric octet mass $\bar{M}$ of pseudoscalar mesons as $\bar{M}^2 = 2\bar{m}B$ with $\bar{m} = (m_u + m_d + m_s)/3 = (2\hat{m} + m_s)/3$. Also c_i^q, d_i^q and $C_i^q, \bar{D}_i^q$ are the SU(2) and SU(3) quark low-energy constants (LEC's). The overline on top of the LEC's denotes renormalized quantities (see definitions in Ref. [5]).

c) Couplings f_3^{du} and f_3^{su}:

The coupling f_3^{du} vanishes due to isospin invariance, while the coupling f_3^{su} starts at the first order in SU(3) breaking:

$$f_3^{su} = \frac{g^2 m^2}{96\pi^2 F^2} \frac{M_K^2 - M_\pi^2}{\bar{M}^2} \biggl(1 - \frac{3\pi}{2}\frac{\bar{M}}{m} - 4\frac{\bar{M}^2}{m^2} + \mathcal{O}(\bar{M}^2)\biggr) + \mathcal{O}((M_K^2 - M_\pi^2)^2)\,. \tag{A8}$$

2. Axial vector quark couplings.

a) Couplings g_1^{du} and g_1^{su}:

The expressions for the axial vector couplings g_1^{du} and g_1^{su} responsible for the $d \to u$ and $s \to u$ transitions are as follows:

$$\begin{aligned} g_1^{du} &= g_1^{\rm SU_3} + \delta g_1^{du}\,, & \text{(A9a)} \\ g_1^{su} &= g_1^{\rm SU_3} + \delta g_1^{su}\,, & \text{(A9b)} \end{aligned}$$

where

$$g_1^{\rm SU_3} = g\biggl(1 - \frac{7g^2\bar{M}^2}{48\pi^2 F^2} + \frac{\bar{M}^3}{48\pi m F^2}\biggl(9 + \frac{23}{2}g^2 - 8C_3^q m + 24C_4^q m\biggr)\biggr) + 6\bar{M}^2\bar{D}_{16}^q + \mathcal{O}(\bar{M}^4) \tag{A10}$$

is the SU(3) symmetric term, δg_1^{du} and δg_1^{su} are the SU(3) breaking terms. Let us display the first–order terms:

$$\begin{aligned} \delta g_1^{du} &= h_1^{du}(M_K^2 - M_\pi^2) + \mathcal{O}((M_K^2 - M_\pi^2)^2)\,, & \text{(A11a)} \\ \delta g_1^{su} &= h_1^{su}(M_K^2 - M_\pi^2) + \mathcal{O}((M_K^2 - M_\pi^2)^2)\,, & \text{(A11b)} \\ h_1^{du} &= -2h_1^{su} + \frac{g}{48\pi^2 F^2}(9 + 23g^2) & \\ &= \frac{g}{96\pi^2 F^2}\biggl(9 + \frac{59}{3}g^2\biggr) - \frac{g\bar{M}}{96\pi m F^2}\biggl(9 + \frac{11}{2}g^2 - 16C_3^q m + 24C_4^q m\biggr) - \frac{2}{3}\bar{D}_{17}^q + \mathcal{O}(\bar{M}^2)\,. & \text{(A11c)} \end{aligned}$$

b) Couplings g_2^{du} and g_2^{su}:

The coupling g_2^{du} vanishes in the isospin limit, while the coupling g_2^{su} is zero at order of accuracy we are working at.

c) Couplings g_3^{du} and g_3^{su}:

The couplings g_3^{du} and g_3^{su} are related to the couplings g_1^{du} and g_1^{su} via:

$$\begin{aligned} g_3^{du} &= 2m^2\biggl(\frac{g_1^{du}}{M_\pi^2} - D_{22}^q - 2D_{18}^q\biggr)\,, & \text{(A12a)} \\ g_3^{su} &= 2m^2\biggl(\frac{g_1^{su}}{M_K^2} - D_{22}^q - 2D_{18}^q\biggr)\,. & \text{(A12b)} \end{aligned}$$

The SU(3) LEC's are fixed by: $C_6^q = -1.476$, $\bar{E}_7^q = 0.086$ GeV^{-3}, $\bar{E}_8^q = 0.532$ GeV^{-3} from the description of the baryon octet magnetic moments, $\bar{E}_6^q = 1.868$ from the description of the induced pseudoscalar form factor of the nucleon. The coupling $D_{22}^q = 0.006$ GeV^{-2} is fixed by fitting the slope of the form factor G_1^{np}: $\langle r_{G_1}^2 \rangle = 0.45$ fm^2. The coupling $D_{18}^q = -0.548$ GeV^{-2} is fixed by fitting the central value of the induced pseudoscalar coupling of the nucleon $g_p = (M_\mu/m_N)G_1^{np}(q^2 = -0.88M_\mu^2) \simeq 8.25$ predicted by ChPT [24, 25] and the value of the pion–nucleon coupling constant $g_{\pi N} = 13.10$.

APPENDIX B: THREE-QUARK BARYON CURRENTS AND FIERZ IDENTITIES

In this Appendix we specify the baryonic currents used in the main text following the approach of [14, 15]. The three-quark currents of the baryon octet are (we restrict ourselves to the so-called *vector* currents obtained in the SU(3) limit and without inclusion of terms with derivatives):

$$\begin{aligned}
J_p &= \varepsilon^{a_1a_2a_3}\gamma^\mu\gamma^5 d^{a_1}u^{a_2}C\gamma_\mu u^{a_3}\,, \\
J_n &= -\varepsilon^{a_1a_2a_3}\gamma^\mu\gamma^5 u^{a_1}d^{a_2}C\gamma_\mu d^{a_3}\,, \\
J_{\Sigma^+} &= \varepsilon^{a_1a_2a_3}\gamma^\mu\gamma^5 s^{a_1}u^{a_2}C\gamma_\mu u^{a_3}\,, \\
J_{\Sigma^0} &= \sqrt{2}\,\varepsilon^{a_1a_2a_3}\gamma^\mu\gamma^5 s^{a_1}u^{a_2}C\gamma_\mu d^{a_3}\,, \\
J_{\Sigma^-} &= \varepsilon^{a_1a_2a_3}\gamma^\mu\gamma^5 s^{a_1}d^{a_2}C\gamma_\mu d^{a_3}\,, \\
J_{\Xi^-} &= -\varepsilon^{a_1a_2a_3}\gamma^\mu\gamma^5 d^{a_1}s^{a_2}C\gamma_\mu s^{a_3}\,, \\
J_{\Xi^0} &= -\varepsilon^{a_1a_2a_3}\gamma^\mu\gamma^5 u^{a_1}s^{a_2}C\gamma_\mu s^{a_3}\,, \\
J_{\Lambda^0} &= \sqrt{\frac{2}{3}}\,\varepsilon^{a_1a_2a_3}\gamma^\mu\gamma^5(u^{a_1}d^{a_2}C\gamma_\mu s^{a_3} - d^{a_1}u^{a_2}C\gamma_\mu s^{a_3})\,.
\end{aligned} \tag{B1}$$

where $C = \gamma_0\gamma_2$ is the charge conjugation matrix.

When generating matrix elements it is convenient to use Fierz transformations and corresponding identities in order to interchange the quark fields. First we specify five possible spin structures $J^{\alpha\beta,\rho\sigma} = \Gamma_1^{\alpha\beta} \otimes (C\Gamma_2)^{\rho\sigma}$ defining the Fierz transformation of the baryon currents:

$$\begin{aligned}
P &= I \otimes C\gamma_5\,, \\
S &= \gamma_5 \otimes C\,, \\
A &= \gamma^\mu \otimes C\gamma_\mu\gamma_5\,, \\
V &= \gamma^\mu\gamma^5 \otimes C\gamma_\mu\,, \\
T &= \sigma^{\mu\nu}\gamma^5 \otimes C\sigma_{\mu\nu}\,.
\end{aligned} \tag{B2}$$

The Fierz transformation of the structures $J = \{P, S, A, V, T\}$ read

$$\begin{aligned}
P &= \frac{1}{4}\biggl(\tilde{P} + \tilde{S} - \tilde{A} + \tilde{V} + \frac{1}{2}\tilde{T}\biggr)\,, \\
S &= \frac{1}{4}\biggl(\tilde{P} + \tilde{S} + \tilde{A} - \tilde{V} + \frac{1}{2}\tilde{T}\biggr)\,, \\
A &= -\tilde{P} + \tilde{S} - \frac{1}{2}\biggl(\tilde{A} + \tilde{V}\biggr)\,, \\
V &= \tilde{P} - \tilde{S} - \frac{1}{2}\biggl(\tilde{A} + \tilde{V}\biggr)\,, \\
T &= 3(\tilde{P} + \tilde{S}) - \frac{1}{2}\tilde{T}\,.
\end{aligned} \tag{B3}$$

Viewing the Fierz transformation in terms of a Fierz matrix $\mathcal{F}$ one can check that $\mathcal{F}^2 = 1$. Using Eqs. (B3) one can derive useful identities

$$\begin{aligned}
2(P - S) - A + V &= 2(\tilde{P} - \tilde{S}) - \tilde{A} + \tilde{V}\,, \\
6(P + S) + T &= 6(\tilde{P} + \tilde{S}) + \tilde{T}\,, \\
V &= 2(P - S) - A - 2\tilde{V}\,, \\
T &= 6(P + S) - 2\tilde{T}\,.
\end{aligned} \tag{B4}$$

The symbol $\tilde{}$ is used to denote Fierz-transformed matrices according to $\tilde{J}^{\alpha\sigma,\rho\beta} = \Gamma_1^{\alpha\sigma} \otimes (C\Gamma_2)^{\rho\beta}$ where α, β, ρ and σ are Dirac indices.

APPENDIX C: GAUGING AND MATRIX ELEMENTS OF THE $n \to pW^-_{off-shell}$ TRANSITION

In this section we discuss the issue of gauge invariance in the context of the calculation of the baryonic matrix elements $\langle B(p')|\, V^{ij}_{\mu,1}(0)\, |B(p)\rangle$ and $\langle B(p')|\, A^{ij}_{\mu,1}(0)\, |B(p)\rangle$. The nonlocal structure of the strong interaction Lagrangian leads to the breaking of local symmetries, which can be restored using minimal substitution. In our approach we use an equivalent method suggested by Mandelstam [18] based on multiplying the quark fields with path-ordered exponentials—*gauge exponentials*. As a result of gauging the strong interaction Lagrangian (15) the conventional triangle diagram in Fig.2a has to be supplemented by the two additional diagrams in Figs.2b and 2c. In our previous papers we have concentrated on electromagnetic processes. For the present application we extend this procedure to the electroweak interactions. Following Terning [19] we can show that the Mandelstam method is equivalent to minimal substitution. Introducing the doublet of left fermions, L, (without specifying the number of generations), the free Lagrangian (kinetic term) for L is:

$$\mathcal{L}^L_0(x) \;=\; \bar L(x) i\not\partial_x L(x) \to \int dy \bar L(x)\delta^4(x-y) i\not\partial_y \Big[\mathcal{P}\exp\Big(\int\limits_x^y dz^\mu \Gamma^L_\mu(z)\Big) L(y)\Big]$$
$$= \int dy \bar L(x)\delta^4(x-y)\mathcal{P}\exp\Big(\int\limits_x^y dz^\mu \Gamma^L_\mu(z)\Big) i\not D^L_y L(y) = \bar L(x) i\not D^L_x L(x) \qquad \text{(C1)}$$

where $D^L_\mu = \partial_\mu + \Gamma^L_\mu\,,\quad \Gamma^L_\mu = -\frac{ig_W}{2}\vec W_\mu\,\vec\tau - \frac{ig'_W}{2} Y_L B_\mu$.

By analogy, the Mandelstam method works for the right singlet fields R

$$\mathcal{L}^R_0(x) \;=\; \bar R(x) i\not\partial_x R(x) \to \int dy \bar R(x)\delta^4(x-y) i\not\partial_y \Big[\mathcal{P}\exp\Big(\int\limits_x^y dz^\mu \Gamma^R_\mu(z)\Big) R(y)\Big]$$
$$= \int dy \bar R(x)\delta^4(x-y)\mathcal{P}\exp\Big(\int\limits_x^y dz^\mu \Gamma^R_\mu(z)\Big) i\not D^R_y R(y) = \bar R(x) i\not D^R_x R(x) \qquad \text{(C2)}$$

where $D^R_\mu = \partial_\mu + \Gamma^R_\mu\,,\quad \Gamma^R_\mu = -\frac{ig'_W}{2} Y_R B_\mu$. We employ the standard notation: W^i_μ (i=1,2,3) and B_μ are the gauge bosons, g_W and g'_W are the corresponding coupling constants (to distinguish them from the axial charge of the quark we attach the subscript W), Y_L and Y_R are the hypercharges of the left and right quarks, respectively. The set of the physical states of the gauge bosons ($W^\pm$, Z^0, A) is connected to the set (W^i, B) via

$$W^\pm_\mu = \frac{1}{\sqrt2}(W^1_\mu \mp iW^2_\mu)\,,\, Z^0_\mu = \cos\theta_W W^3_\mu - \sin\theta_W B_\mu\,,\quad A_\mu = \sin\theta_W W^3_\mu + \cos\theta_W B_\mu\,, \qquad \text{(C3)}$$

where θ_W is the Weinberg angle which relates the electromagnetic coupling constant e and the couplings g_W and g'_W via $e = g_W \sin\theta_W = g'_W \sin\theta_W$. The quantities Γ^L_μ and Γ^R_μ in terms of ($W^\pm$, Z^0, A) fields are given by

$$\Gamma^L_\mu \;=\; -\frac{ig_W}{\sqrt2}(W^+_\mu\tau^+ + W^-_\mu\tau^-) - ie\tan\theta_W Z^0_\mu\Big(\frac{\tau_3}{2\sin^2\theta_W} - Q\Big) - ieQA_\mu\,, \qquad \text{(C4a)}$$
$$\Gamma^R_\mu \;=\; \frac{ie}{2}\tan\theta_W Z^0_\mu - ieQA_\mu\,. \qquad \text{(C4b)}$$

In the case of the strong baryon–three–quark interaction Lagrangian it is not necessary to rewrite the Lagrangian in terms of left quark doublets and right singlets. Instead we merely substitute each quark field q by its left-handed $q_L = (1-\gamma_5)q/2$ and right–handed $q_R = (1+\gamma_5)q/2$ components. Then we proceed with the gauging of the theory. We only need to know the gauging for the quarks of specific flavor and handedness—*e.g.*, for the left–handed u_L, d_L

and s_L and the right-handed $q_R = u_R$, d_R and s_R quarks the gauging is

$$u_L(y) \to \mathcal{P}\exp\Big(\int_x^y dz^\mu \Gamma^L_\mu(z)\Big)_{11} u_L(y) + \mathcal{P}\exp\Big(\int_x^y dz^\mu \Gamma^L_\mu(z)\Big)_{12} d'_L(y)\,, \tag{C5a}$$

$$d_L(y) \to \mathcal{P}\exp\Big(\int_x^y dz^\mu \Gamma^L_\mu(z)\Big)_{21} u'_L(y) + \mathcal{P}\exp\Big(\int_x^y dz^\mu \Gamma^L_\mu(z)\Big)_{22} d'_L(y)\,, \tag{C5b}$$

$$s_L(y) \to \mathcal{P}\exp\Big(\int_x^y dz^\mu \Gamma^L_\mu(z)\Big)_{21} c'_L(y) + \mathcal{P}\exp\Big(\int_x^y dz^\mu \Gamma^L_\mu(z)\Big)_{22} s'_L(y)\,, \tag{C5c}$$

$$q_R(y) \to \mathcal{P}\exp\Big(\int_x^y dz^\mu \Gamma^R_\mu(z)\Big) q_R(y) \tag{C5d}$$

where (ij) are pairs of flavor indices. The mixed left-handed quark fields are defined as:

$$\begin{aligned} u'_L &= V^\dagger_{ud} u_L + V^\dagger_{cd} c_L + V^\dagger_{td} t_L\,, \\ d'_L &= V_{ud} d_L + V_{us} s_L + V_{ub} b_L\,, \\ c'_L &= V^\dagger_{us} u_L + V^\dagger_{cs} c_L + V^\dagger_{ts} t_L\,, \\ s'_L &= V_{cd} d_L + V_{cs} s_L + V_{cb} b_L\,. \end{aligned} \tag{C6}$$

In the derivation of Eqs. (C5b) and (C5c) we have used the unitarity condition $\sum\limits_k V_{ik} V^\dagger_{jk} = \delta_{ij}$ for the CKM matrix elements, which leads to the useful identities:

$$\begin{aligned} d_L &= d'_L V^\dagger_{ud} + s'_L V^\dagger_{cd} + b'_L V^\dagger_{td}\,, \\ s_L &= d'_L V^\dagger_{us} + s'_L V^\dagger_{cs} + b'_L V^\dagger_{ts}\,, \\ b_L &= d'_L V^\dagger_{ub} + s'_L V^\dagger_{cb} + b'_L V^\dagger_{tb}\,. \end{aligned} \tag{C7}$$

In the present manuscript we restrict our considerations to semileptonic processes (*i.e.*, processes with a single intermediate off–shell charged weak gauge boson $W^\pm$). Therefore, we expand the gauge exponentials and keep only the term linear in $W^\pm$ which gives a correction to the weak current (in addition to the standard term which comes from the gauging of the free quark Lagrangian). This is a rather important point. The use of nonlocal Lagrangians automatically requires an extension of the conventional currents dictated by the local symmetries. In addition we have an extra piece from "gauging" the strong Lagrangian which contains derivatives acting on quark fields.

For illustration we derive the weak current which governs the $n \to pW^-$ transition. The first contribution comes from "gauging" the free Lagrangian:

$$J^\mu_1(x) = \frac{g_W}{\sqrt{2}} V_{ud}\ \bar{u}_L(x)\ \gamma^\mu\ d_L(x) = \frac{g_W}{2\sqrt{2}} V_{ud}\ \bar{u}(x)\ O^\mu\ d(x) \tag{C8}$$

where $O^\mu = \gamma^\mu(1-\gamma^5)$.

To derive the contribution due to "gauging" the strong interaction Lagrangian we take the three–quark currents of the proton and neutron and proceed as follows:

- We express the quark fields in terms of left– and right–handed fields. One obtains:

$$\begin{aligned} J_p &= \varepsilon^{a_1a_2a_3}\,\gamma^\mu\gamma^5\,(d^{a_1}_L + d^{a_1}_R)\,(u^{a_2}_L C\gamma_\mu u^{a_3}_R \;+\; u^{a_2}_R C\gamma_\mu u^{a_3}_L)\,, \\ J_n &= -\varepsilon^{a_1a_2a_3}\,\gamma^\mu\gamma^5\,(u^{a_1}_L + u^{a_1}_R)\,(d^{a_2}_L C\gamma_\mu d^{a_3}_R \;+\; d^{a_2}_R C\gamma_\mu d^{a_3}_L)\,. \end{aligned}$$

- We perform the gauging using the master formulas (C5) and after some simple algebra we derive the "nonlocal" contributions to the weak current associated with the $d \to u$ flavor exchange:

$$J^\mu_2(x) = \int dy \frac{\delta \mathcal{L}^{weak}_{BB'}(y)}{\delta W^+_\mu(x)} \tag{C9}$$

where

$$\begin{aligned}\mathcal{L}_{np}^{weak}(x) &= \frac{g_W g_N}{\sqrt{2}} V_{ud}\,\bar{p}(x) \int dx_{123}\, F(x,x_1,x_2,x_3)\,\varepsilon^{a_1a_2a_3}\,\gamma^\mu\gamma^5\, d^{a_1}(x_1)d^{a_2}(x_2)C\gamma_\mu(1+\gamma_5)u^{a_3}(x_3)\, I(x_2,x,W^+)\\ &- \frac{g_W g_N}{\sqrt{2}} V_{ud}^\dagger\,\bar{n}(x) \int dx_{123}\, F(x,x_1,x_2,x_3)\,\varepsilon^{a_1a_2a_3}\,\gamma^\mu\gamma^5\, u^{a_1}(x_1)u^{a_2}(x_2)C\gamma_\mu(1+\gamma_5)d^{a_3}(x_3)\, I(x_2,x,W^-)\\ &+ \text{H.c.} \end{aligned} \quad \text{(C10)}$$

Using the Fierz transformation (see Appendix B) the Lagrangian $\mathcal{L}_{np}^{weak}$ can be written in a more convenient form

$$\begin{aligned}\mathcal{L}_{np}^{weak}(x) &= -\frac{g_W g_N}{2\sqrt{2}} V_{ud}\,\bar{p}(x) \int dx_{123}\, F(x,x_1,x_2,x_3)\,\varepsilon^{a_1a_2a_3}\,\gamma^\mu(1+\gamma^5)\, u^{a_1}(x_1)d^{a_2}(x_2)C\gamma_\mu d^{a_3}(x_3)\, I(x_2,x,W^+)\\ &+ \frac{g_W g_N}{2\sqrt{2}} V_{ud}^\dagger\,\bar{n}(x) \int dx_{123}\, F(x,x_1,x_2,x_3)\,\varepsilon^{a_1a_2a_3}\,\gamma^\mu(1+\gamma^5)\, d^{a_1}(x_1)u^{a_2}(x_2)C\gamma_\mu u^{a_3}(x_3)\, I(x_2,x,W^-)\\ &+ \text{H.c.} \end{aligned} \quad \text{(C11)}$$

where $\int dx_{123} = \int dx_1 \int dx_2 \int dx_3$ and $I(x_2,x,W^\pm) = \int\limits_x^{x_2} dz^\mu W_\mu^\pm(z)$.

- We remind the reader that the function $F(x,x_1,x_2,x_3)$ is related to the scalar part of the Bethe-Salpeter amplitude and characterizes the finite size of the baryon. We use a particular form for the vertex function defined in Eq. (17).
- The current $J_1^\mu(x)$ generates the triangle diagram (the left diagram in Fig.1) contributing to the $n \to pW^-$ transition, while the current $J_2^\mu(x)$ generates the bubble diagrams (the central and right diagram in Fig.1). By analogy one can derive the currents which govern the other six modes.
- A crucial check of our gauging procedure is to check the vector and axial-vector Ward-Takahashi identities (WTI) involving matrix elements of the $n \to pW^-$ transition. In general, for an off-shell neutron and proton with momentum p and p', respectively, and the momentum transfer $q = p' - p$, it is convenient to write down the corresponding weak matrix elements associated with the vector and axial vector current in the form (here and in the following we omit the weak coupling g and the CKM matrices in the matrix elements):

$$\Lambda_\mu^V(p,p') = \Lambda_\mu^{V;\perp}(p,p') + \frac{q_\mu}{q^2}\Big[\Sigma_N(p') - \Sigma_N(p)\Big] \quad \text{(C12)}$$

and

$$\Lambda_\mu^A(p,p') = \Lambda_\mu^{A;\perp}(p,p') - \frac{q_\mu}{q^2}\Big[\gamma^5\,\Sigma_N(p) + \Sigma_N(p')\,\gamma^5\Big] + \frac{q_\mu}{q^2}\Big[2m_q\,\Lambda_P(p,p')\Big]\,. \quad \text{(C13)}$$

Here, $\Lambda_\mu^{V;\perp}(p,p')$ and $\Lambda_\mu^{A;\perp}(p,p')$ are the contributions to the vector and axial vector matrix elements orthogonal to the W-boson (or leptonic pair) momenta; $\Sigma_N(p)$ is the nucleon mass operator and $\Lambda_P(p,p')$ is the pseudoscalar nucleon vertex function.

Then, the vector and axial vector WTI are satisfied according to

$$q^\mu\;\Lambda_\mu^V(p,p') = \Sigma_N(p') - \Sigma_N(p) \quad \text{(C14a)}$$

$$q^\mu\;\Lambda_\mu^A(p,p') = -\gamma^5\,\Sigma_N(p) - \Sigma_N(p')\,\gamma^5 + 2\,m_q\,\Lambda_P(p,p')\,. \quad \text{(C14b)}$$

In our derivation we have made use of the quark-level identities

$$\begin{aligned} S_{q_2}(k+p')\gamma_\mu S_{q_1}(k+p) &= S_{q_2}(k+p')\gamma_\mu^\perp S_{q_1}(k+p) + \frac{q_\mu}{q^2}[S_{q_2}(k+p') - S_{q_1}(k+p)]\\ &+ \frac{q_\mu}{q^2}(m_{q_2} - m_{q_1})S_{q_2}(k+p')S_{q_1}(k+p)\,, \end{aligned} \quad \text{(C15a)}$$

$$\begin{aligned} S_{q_2}(k+p')\gamma_\mu\gamma_5 S_{q_1}(k+p) &= S_{q_2}(k+p')(\gamma_\mu\gamma_5)^\perp S_{q_1}(k+p) - \frac{q_\mu}{q^2}[\gamma_5 S_{q_1}(k+p) + S_{q_2}(k+p')\gamma_5]\\ &+ \frac{q_\mu}{q^2}(m_{q_1} + m_{q_2})S_{q_2}(k+p')\gamma_5 S_{q_1}(k+p) \end{aligned} \quad \text{(C15b)}$$

which lead to the vector and axial vector WTI on the quark level:

$$q^\mu\; S_{q_2}(k+p')\gamma_\mu S_{q_1}(k+p) \;=\; S_{q_2}(k+p') - S_{q_1}(k+p) \;+\; (m_{q_2}-m_{q_1}) S_{q_2}(k+p') S_{q_1}(k+p)\,, \tag{C16a}$$

$$q^\mu\; S_{q_2}(k+p')\gamma_\mu\gamma_5 S_{q_1}(k+p) \;=\; -S_{q_2}(k+p')\gamma_5 - \gamma_5 S_{q_1}(k+p) \;+\; (m_{q_1}+m_{q_2}) S_{q_2}(k+p')\gamma_5 S_{q_1}(k+p)\,. \tag{C16b}$$

We have introduced the notation $\Gamma^\perp_\mu = \Gamma^\nu(g_{\mu\nu} - q_\mu q_\nu/q^2)$ for the so-called Dirac matrices orthogonal to the transverse momentum q. All three diagrams contribute to $\Lambda^{V;\perp}_\mu(p,p')$ and $\Lambda^{A;\perp}_\mu(p,p')$

$$\Lambda^{V-A;\perp}_\mu(p,p') \;=\; \Lambda^{V-A;\perp}_{\mu,\,\Delta}(p,p') \;+\; \Lambda^{V-A;\perp}_{\mu,\,\circ_L}(p,p') \;+\; \Lambda^{V-A;\perp}_{\mu,\,\circ_R}(p,p') \tag{C17}$$

where

$$\begin{aligned}
\Lambda^{V-A;\perp}_{\mu,\,\Delta}(p,p') \;&=\; -\alpha_N \int dk_{123} \tilde\Phi(z_0)\,\tilde\Phi[z_0+z_2(q)] \\
&\times\; \Gamma_{1f} S_q(k_1^+)\gamma^\beta\gamma^5\,\mathrm{tr}[\Gamma_{2f} S_q(k_2^+ + q) O^\perp_\mu S_q(k_2^+)\gamma_\beta S_q(-k_3^+)] && \text{(C18a)}\\
\Lambda^{V-A;\perp}_{\mu,\,\circ_L}(p,p') \;&=\; \alpha_N \int dk_{123}\;\; L^\perp_{2\mu}\,\tilde\Phi(z_0) \int\limits_0^1 dt\,\tilde\Phi'[z_0 + t z_2(-q)] \\
&\times\; \gamma^\alpha\gamma^5 S_q(k_1'^+)\gamma^\beta(1+\gamma^5)\,\mathrm{tr}[\gamma_\alpha S_q(k_2'^+)\gamma_\beta S_q(-k_3'^+)]\,, && \text{(C18b)}\\
\Lambda^{V-A;\perp}_{\mu,\,\circ_R}(p,p') \;&=\; \alpha_N \int dk_{123}\;\; L^\perp_{2\mu}\,\tilde\Phi(z_0) \int\limits_0^1 dt\,\tilde\Phi'[z_0 + t z_2(q)] \\
&\times\; \gamma^\alpha(1+\gamma^5) S_q(k_1^+)\gamma^\beta\gamma^5\,\mathrm{tr}[\gamma_\alpha S_q(k_2^+)\gamma_\beta S_q(-k_3^+)]\,. && \text{(C18c)}
\end{aligned}$$

Here $\Gamma_{1f}\otimes\Gamma_{2f} \;=\; \gamma^\alpha\gamma^5\otimes\gamma_\alpha - \gamma^\alpha\otimes\gamma_\alpha\gamma_5 + 2I\otimes\gamma_5 - 2\gamma_5\otimes I$.
The expressions for $\Sigma_N(p)$ and $\Lambda_P(p,p')$ are given by

$$\Sigma_N(p) = -\alpha_N \int dk_{123}\tilde\Phi^2(z_0)\gamma^\alpha\gamma^5 S_q(k_1^+)\gamma^\beta\gamma^5\,\mathrm{tr}[\gamma_\alpha S_q(k_2^+)\gamma_\beta S_q(-k_3^+)] \tag{C19}$$

and

$$\Lambda_P(p,p') = -\alpha_N \int dk_{123}\tilde\Phi(z_0)\,\tilde\Phi[z_0+z_2(q)]\Gamma_{1f} S_q(k_1^+)\gamma^\beta\gamma^5\,\mathrm{tr}[\Gamma_{2f} S_q(k_2^+ + q)\gamma_5 S_q(k_2^+)\gamma_\beta S_q(-k_3^+)]\,. \tag{C20}$$

We have used the notations from our paper on magnetic moments of heavy baryons [9]:

$$\begin{aligned}
\alpha_B \;&=\; 6\,g_B^2\,, \quad k_i^+ \;=\; k_i + p\omega_i\,, \quad k_i'^+ \;=\; k_i + p'\omega_i\,, \quad z_0 \;=\; -6(k_1^2+k_2^2+k_3^2)\\
dk_{123} \;&=\; \frac{d^4k_1 d^4k_2 d^4k_3}{(2\pi)^8 i^2}\,\delta^4(k_1+k_2+k_3)\,, \quad L_i \;=\; 12(k_i - \sum_{j=1}^3 k_j\omega_j)\,, && \text{(C21)}\\
z_1(q) \;&=\; -12q^2(\omega_2^2+\omega_2\omega_3+\omega_3^2) - L_1 q\,,\\
z_2(q) \;&=\; -12q^2(\omega_1^2+\omega_1\omega_3+\omega_3^2) - L_2 q\,,\\
z_3(q) \;&=\; -12q^2(\omega_1^2+\omega_1\omega_2+\omega_2^2) - L_3 q\,.
\end{aligned}$$

By analogy one can derive the matrix elements $\langle B(p')|\,V^{ij}_{\mu,1}(0)\,|B(p)\rangle$ and $\langle B(p')|\,A^{ij}_{\mu,1}(0)\,|B(p)\rangle$ for the other six modes.

APPENDIX D: FUNCTIONS $R_i(x)$

In this Appendix we write down the functions $R_i(x = m_l/\Delta)$:

$$
\begin{aligned}
R_0(x) &= \sqrt{1-x^2}\biggl(1 - \frac{9}{2}x^2 - 4x^4\biggr) + \frac{15}{4}x^4 \ln\frac{1+\sqrt{1-x^2}}{1-\sqrt{1-x^2}}\,, \\
R_{F_1}(x) &= R^0_{F_1}(x) + \frac{m^2_{B_i}}{9}\langle r^2_{F_1}\rangle R^{q^2}_{F_1}(x)\,, \\
R^0_{F_1}(x) &= \sqrt{1-x^2}\biggl(1 - \frac{45}{8}x^2 - \frac{37}{4}x^4 + \frac{3}{4}x^6\biggr) + \frac{105}{16}x^4 \ln\frac{1+\sqrt{1-x^2}}{1-\sqrt{1-x^2}}\,, \\
R^{q^2}_{F_1}(x) &= \sqrt{1-x^2}\biggl(1 + 4x^2 + \frac{271}{4}x^4 + 6x^6\biggr) - \frac{105}{4}x^4(1+\frac{x^2}{2}) \ln\frac{1+\sqrt{1-x^2}}{1-\sqrt{1-x^2}}\,, \\
R_{G_1}(x) &= R^0_{G_1}(x) + \frac{5m^2_{B_i}}{18}\langle r^2_{G_1}\rangle R^{q^2}_{G_1}(x)\,, \\
R^0_{G_1}(x) &= \sqrt{1-x^2}\biggl(1 - \frac{83}{16}x^2 - \frac{173}{24}x^4 + \frac{11}{24}x^6\biggr) + \frac{175}{32}x^4 \ln\frac{1+\sqrt{1-x^2}}{1-\sqrt{1-x^2}}\,, \\
R^{q^2}_{G_1}(x) &= \sqrt{1-x^2}\biggl(1 - \frac{8}{5}x^2 + \frac{319}{20}x^4 + \frac{2}{5}x^6\biggr) - \frac{21}{4}x^4(1+\frac{x^2}{2}) \ln\frac{1+\sqrt{1-x^2}}{1-\sqrt{1-x^2}}\,, \\
R_{F_2}(x) &= R_{F_{12}}(x^2) = \sqrt{1-x^2}\biggl(1 - \frac{19}{4}x^2 + \frac{87}{8}x^4 + 6x^6\biggr) - \frac{105}{16}x^6 \ln\frac{1+\sqrt{1-x^2}}{1-\sqrt{1-x^2}}\,, \\
R_{F_3}(x) &= x^2 R_0(x)\,, \qquad R_{G_2}(x) \;=\; (1-x^2)^{7/2}\,, \\
R_{F_{13}}(x) &= \frac{5}{4}x^2\sqrt{1-x^2}\biggl(1 + \frac{13}{2}x^2\biggr) - \frac{15}{4}x^4(1+\frac{x^2}{4}) \ln\frac{1+\sqrt{1-x^2}}{1-\sqrt{1-x^2}}\,, \\
R_{G_{12}}(x) &= \sqrt{1-x^2}\biggl(1 - \frac{13}{4}x^2 + \frac{33}{8}x^4\biggr) - \frac{15}{16}x^6 \ln\frac{1+\sqrt{1-x^2}}{1-\sqrt{1-x^2}}\,, \\
R_{G_{13}}(x) &= \frac{3}{2}x^2\sqrt{1-x^2}\biggl(1 + \frac{83}{6}x^2 + \frac{8}{3}x^4\biggr) - \frac{15}{2}x^4(1+\frac{3}{4}x^2) \ln\frac{1+\sqrt{1-x^2}}{1-\sqrt{1-x^2}}\,.
\end{aligned}
\tag{D1}
$$

APPENDIX E: CHECK OF THE ADEMOLLO–GATTO THEOREM (AGT)

As stressed above, the Ademollo–Gatto theorem (AGT) [1] protects the vector form factors from leading SU(3)–breaking corrections generated by the mass difference of strange and nonstrange quarks. The first nonvanishing breaking effects start at second order in symmetry–breaking. To demonstrate that this theorem is fulfilled in our approach we consider a strangeness-changing flavor transition $B_i \to B_j e \bar\nu_e$. The corresponding matrix element at $q = p' - p = 0$ is written as

$$M^{B_iB_j}_{\mu,V}(p,p) = \bar u_{B_j}(p)\gamma_\mu\, F^{B_iB_j}_1(0) u_{B_i}(p)\ , \tag{E1}$$

where the vector coupling constant $F^{B_iB_j}_1(0)$ is defined as

$$F^{B_iB_j}_1(0) = f^{su}_1\, V^{B_iB_j}_1\ . \tag{E2}$$

Note that we have already proved (see [5]) that the vector form factor f^{su}_1 obeys the AGT. Therefore, we merely need to demonstrate that the same is true for the form factor $V^{B_iB_j}_1$ encoding valence quark effects—the valence quark vector form factor. In other words, due to the factorization of chiral effects and the effects of valence quarks, *both* form factors – f^{su}_1 and $V^{B_iB_j}_1$ should obey the AGT. The quantity $V^{B_iB_j}_1$ is expressed in terms of the baryon-three-quark coupling constants $g_{B_i} = g_B(m_{B_i}, m_{1i})$ and $g_{B_j} = g_B(m_{B_j}, m_{1j})$, the Clebsch–Gordan coefficients $C_V^{B_iB_j}$ and the structure integral $I_{B_iB_j} = I(m_{B_i}, m_{B_j}, m_{1i}, m_{1j})$, according to the contributions from the diagrams in Fig.2:

$$V_1^{B_iB_j} = g_{B_i} g_{B_j} C_V^{B_iB_j} I_{B_iB_j} \tag{E3}$$

where $m_i = m_s$ and $m_j = m$ are the masses of strange and nonstrange quarks. In the above formulae we do not display the dependence on the spectator quark masses m_2 and m_3. Note that the coupling constant g_{B_i} is related to the structure integral $I_{B_iB_j}$ as $g_{B_i}^2 = 1/I_{B_iB_i}$.

Next, using the transformation of the matrix element $M_{\mu,V}^{B_iB_j}(p,p)$ under hermitian conjugation

$$\biggl(M_{\mu,V}^{B_iB_j}(p,p)\biggr)^\dagger = \bar u_{B_i}(p)\gamma_\mu F_1^{B_iB_j}(0) u_{B_j}(p) = M_{\mu,V}^{B_jB_i}(p,p) = \bar u_{B_j}(p)\gamma_\mu F_1^{B_jB_i}(0) u_{B_i}(p)\,, \tag{E4}$$

we deduce the condition $I_{B_iB_j} = I_{B_jB_i}$ which means that the structure integral $I(m_{B_i}, m_{B_j}, m_{1i}, m_{1j})$ is symmetric under the transformations $m_{B_i} \leftrightarrow m_{B_j}$, $m_{1i} \leftrightarrow m_{1j}$:

$$I(m_{B_i}, m_{B_j}, m_{1i}, m_{1j}) = I(m_{B_j}, m_{B_i}, m_{1j}, m_{1i})\,. \tag{E5}$$

Using the latter constraint, we express the structure integral $I_{B_iB_j}$ through the coupling constants g_{B_i} and g_{B_j}, i.e. one has

$$\begin{aligned} I_{B_iB_j} &= \frac{1}{2}\biggl(I_{B_iB_j} + I_{B_jB_i}\biggr) = \frac{1}{2}\biggl(I_{B_iB_i} + I_{B_jB_j} + \mathcal{O}(\delta^2_{B_iB_j}, \delta^2_{ij}, \delta_{B_iB_j}\delta_{ij})\biggr) \\ &= \frac{1}{2}\biggl(\frac{1}{g^2_{B_i}} + \frac{1}{g^2_{B_j}} + \mathcal{O}(\delta^2)\biggr) \end{aligned} \tag{E6}$$

where the parameters $\delta_{B_iB_j} = m_{B_i} - m_{B_j} = \mathcal{O}(\delta)$ and $\delta_{ij} = m_{1i} - m_{1j} = \mathcal{O}(\delta)$ are of first order in SU(3) breaking. Using the expansion (E6) we then obtain

$$V_1^{B_iB_j} = \frac{C_V^{B_iB_j}}{2}\biggl(\frac{g_{B_i}}{g_{B_j}} + \frac{g_{B_j}}{g_{B_i}} + \mathcal{O}(\delta^2)\biggr)\,. \tag{E7}$$

Finally, expanding $g_{B_i}/g_{B_j} + g_{B_j}/g_{B_i}$ in terms of the difference $g_{B_i} - g_{B_j} \sim O(\delta)$

$$\frac{g_{B_i}}{g_{B_j}} + \frac{g_{B_j}}{g_{B_i}} = 2 + \frac{(g_{B_i} - g_{B_j})^2}{g^2_{B_i}} + \mathcal{O}((g_{B_i} - g_{B_j})^3) = 2 + \mathcal{O}(\delta^2) \tag{E8}$$

we prove the Ademollo-Gatto theorem

$$V_1^{B_iB_j} = C_V^{B_iB_j}(1 + \mathcal{O}(\delta^2))\,. \tag{E9}$$

[1] M. Ademollo and R. Gatto, Phys. Rev. Lett. **13**, 264 (1964).

[2] N. Cabibbo, E. C. Swallow and R. Winston, Phys. Rev. Lett. **92**, 251803 (2004) [arXiv:hep-ph/0307214]; Ann. Rev. Nucl. Part. Sci. **53**, 39 (2003) [arXiv:hep-ph/0307298].

[3] W. M. Yao *et al.* [Particle Data Group], J. Phys. G **33**, 1 (2006).

[4] N. Cabibbo, Phys. Rev. Lett. **10**, 531 (1963).

[5] A. Faessler, T. Gutsche, B. R. Holstein and V. E. Lyubovitskij, Phys. Rev. D **77**, 114007 (2008) [arXiv:0712.3437 [hep-ph]].

[6] J. F. Donoghue, E. Golowich and B. R. Holstein, Camb. Monogr. Part. Phys. Nucl. Phys. Cosmol. **2**, 1 (1992).

[7] V. E. Lyubovitskij, T. Gutsche and A. Faessler, Phys. Rev. C **64**, 065203 (2001) [arXiv: hep-ph/0105043]; V. E. Lyubovitskij, T. Gutsche, A. Faessler and E. G. Drukarev, Phys. Rev. D **63**, 054026 (2001) [arXiv:hep-ph/0009341]; V. E. Lyubovitskij, T. Gutsche, A. Faessler and R. Vinh Mau, Phys. Lett. B **520**, 204 (2001) [arXiv:hep-ph/0108134]; V. E. Lyubovitskij, P. Wang, T. Gutsche and A. Faessler, Phys. Rev. C **66**, 055204 (2002) [arXiv:hep-ph/0207225]; T. Inoue, V. E. Lyubovitskij, T. Gutsche and A. Faessler, Phys. Rev. C **69**, 035207 (2004) [arXiv:hep-ph/0311275].

[8] M. A. Ivanov, M. P. Locher and V. E. Lyubovitskij, Few Body Syst. **21**, 131 (1996); M. A. Ivanov, V. E. Lyubovitskij, J. G. Körner and P. Kroll, Phys. Rev. D **56**, 348 (1997) [arXiv:hep-ph/9612463]; M. A. Ivanov, J. G. Körner, V. E. Lyubovitskij and A. G. Rusetsky, Phys. Rev. D **60**, 094002 (1999) [arXiv:hep-ph/9904421]; A. Faessler, T. Gutsche, M. A. Ivanov, J. G. Körner and V. E. Lyubovitskij, Phys. Lett. B **518**, 55 (2001) [arXiv:hep-ph/0107205].

[9] A. Faessler, T. Gutsche, M. A. Ivanov, J. G. Körner, V. E. Lyubovitskij, D. Nicmorus and K. Pumsa-ard, Phys. Rev. D **73**, 094013 (2006) [arXiv:hep-ph/0602193].
[10] A. Faessler, T. Gutsche, V. E. Lyubovitskij and K. Pumsa-ard, Phys. Rev. D **73**, 114021 (2006) [arXiv:hep-ph/0511319]; Prog. Part. Nucl. Phys. **55**, 12 (2005); AIP Conf. Proc. **884**, 43 (2007).
[11] A. Faessler, T. Gutsche, B. R. Holstein, V. E. Lyubovitskij, D. Nicmorus and K. Pumsa-ard, Phys. Rev. D **74**, 074010 (2006) [arXiv:hep-ph/0608015]; arXiv:hep-ph/0612246.
[12] T. Becher and H. Leutwyler, Eur. Phys. J. C **9**, 643 (1999) [arXiv:hep-ph/9901384].
[13] J. Gasser, M. E. Sainio and A. Švarc, Nucl. Phys. B **307**, 779 (1988); E. Jenkins and A. V. Manohar, Phys. Lett. B **255**, 558 (1991); V. Bernard, N. Kaiser, J. Kambor and U. G. Meissner, Nucl. Phys. B **388**, 315 (1992); P. J. Ellis and H. B. Tang, Phys. Rev. C **57**, 3356 (1998) [arXiv:hep-ph/9709354]; N. Fettes, U. G. Meissner and S. Steininger, Nucl. Phys. A **640**, 199 (1998) [arXiv:hep-ph/9803266]; B. Kubis and U. G. Meissner, Nucl. Phys. A **679**, 698 (2001) [arXiv:hep-ph/0007056]; M. R. Schindler, J. Gegelia and S. Scherer, Phys. Lett. B **586**, 258 (2004) [arXiv:hep-ph/0309005]; T. Fuchs, J. Gegelia and S. Scherer, Eur. Phys. J. A **19**, 35 (2004) [arXiv:hep-ph/0309234].
[14] B. L. Ioffe, Z. Phys. C **18**, 67 (1983); B. L. Ioffe and A. V. Smilga, Phys. Lett. B **133**, 436 (1983); Y. Chung, H. G. Dosch, M. Kremer and D. Schall, Phys. Lett. B **102**, 175 (1981); L. J. Reinders, H. Rubinstein and S. Yazaki, Phys. Rept. **127**, 1 (1985).
[15] G. V. Efimov, M. A. Ivanov and V. E. Lyubovitskij, Sov. J. Nucl. Phys. **48**, 126 (1988) [Yad. Fiz. **48**, 198 (1988)]; Few Body Syst. **6**, 17 (1989) [Acta Phys. Austriaca **6**, 17 (1989)].
[16] S. Weinberg, Phys. Rev. **130**, 776 (1963); A. Salam, Nuovo Cim. **25**, 224 (1962); K. Hayashi, M. Hirayama, T. Muta, N. Seto and T. Shirafuji, Fortsch. Phys. **15**, 625 (1967).
[17] G. V. Efimov and M. A. Ivanov, *The Quark Confinement Model of Hadrons*, (IOP Publishing, Bristol & Philadelphia, 1993).
[18] S. Mandelstam, Annals Phys. **19**, 1 (1962).
[19] J. Terning, Phys. Rev. D **44**, 887 (1991).
[20] M. A. Ivanov, J. G. Körner, V. E. Lyubovitskij and A. G. Rusetsky, Phys. Rev. D **59**, 074016 (1999) [arXiv:hep-ph/9809254]; R. Alkofer, A. Holl, M. Kloker, A. Krassnigg and C. D. Roberts, Few Body Syst. **37**, 1 (2005) [arXiv:nucl-th/0412046].
[21] I. V. Anikin, M. A. Ivanov, N. B. Kulimanova and V. E. Lyubovitskij, Z. Phys. C **65**, 681 (1995).
[22] J. Kambor and M. Mojzis, JHEP **9904**, 031 (1999) [arXiv:hep-ph/9901235]; J. Schweizer, Diploma thesis, University of Bern (2000).
[23] T. Becher and H. Leutwyler, JHEP **0106**, 017 (2001) [arXiv:hep-ph/0103263].
[24] V. Bernard, L. Elouadrhiri and U. G. Meissner, J. Phys. G **28**, R1 (2002) [arXiv:hep-ph/0107088].
[25] M. R. Schindler, T. Fuchs, J. Gegelia and S. Scherer, Phys. Rev. C **75**, 025202 (2007) [arXiv:nucl-th/0611083].
[26] M. L. Goldberger and S. B. Treiman, Phys. Rev. **110**, 1178 (1958); Y. Nambu, Phys. Rev. Lett. **4**, 380 (1960).
[27] H. Pagels, Phys. Rev. **179**, 1337 (1969).
[28] H. Pietschmann, Acta Phys. Austriaca Suppl. **12**, 1 (1974).
[29] A. Garcia, P. Kielanowski, Lect. Notes Phys. **222**, 1 (1985).
[30] A. Kadeer, J. G. Korner and U. Moosbrugger, arXiv:hep-ph/0511019.
[31] R. Flores-Mendieta, Phys. Rev. D **70**, 114036 (2004) [arXiv:hep-ph/0410171].
[32] A. Garcia, Phys. Rev. D **10**, 884 (1974); X. Song, P. K. Kabir and J. S. McCarthy, Phys. Rev. D **54**, 2108 (1996) [arXiv:hep-ph/9602422].
[33] R. Flores-Mendieta, E. E. Jenkins and A. V. Manohar, Phys. Rev. D **58**, 094028 (1998) [arXiv:hep-ph/9805416].
[34] T. Ledwig, A. Silva, H. C. Kim and K. Goeke, arXiv:0806.4072 [hep-ph].
[35] D. Guadagnoli, V. Lubicz, M. Papinutto and S. Simula, Nucl. Phys. B **761**, 63 (2007) [arXiv:hep-ph/0606181].
[36] E. Abouzaid *et al.* [KTeV Collaboration], Phys. Rev. Lett. **95**, 081801 (2005) [arXiv:hep-ex/0504055].
[37] C. Lazzeroni [NA48 Collaboration], PoS **HEP2005**, 096 (2006).
[38] M. Bourquin *et al.* [Bristol-Geneva-Heidelberg-Orsay-Rutherford-Strasbourg Collaboration], Z. Phys. C **12**, 307 (1982).
[39] A. Lacour, B. Kubis and U. G. Meissner, JHEP **0710**, 083 (2007) [arXiv:0708.3957 [hep-ph]].
[40] H. Leutwyler and M. Roos, Z. Phys. C **25**, 91 (1984); J. Gasser and H. Leutwyler, Nucl. Phys. B **250**, 517 (1985); A. Krause, Helv. Phys. Acta **63**, 3 (1990); J. Anderson and M. A. Luty, Phys. Rev. D **47**, 4975 (1993) [arXiv:hep-ph/9301219]; N. Kaiser, Phys. Rev. C **64**, 028201 (2001) [arXiv:nucl-th/0105043]; G. Villadoro, Phys. Rev. D **74**, 014018 (2006) [arXiv:hep-ph/0603226].

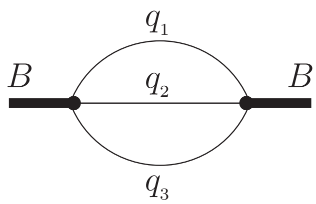

Fig. 1. *Baryon mass operator. Bold and thin lines refer to the baryons and quarks, respectively. Quarks are labeled by the indices* $k = 1, 2, 3$.

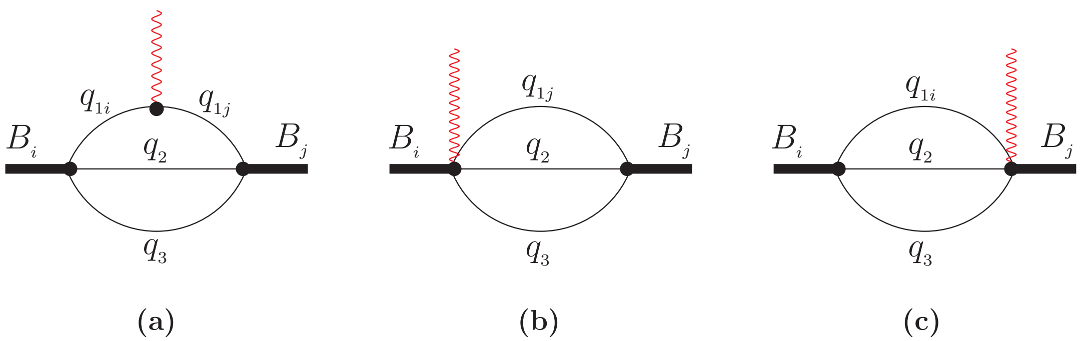

Fig. 2. *Diagrams contributing to the matrix elements of the bare quark operators* $V^{ij}_{\mu,k}(0)$ *and* $A^{ij}_{\mu,k}(0)$, $k = 1, 2, 3$: *triangle (a), bubble (b) and (c). Bold, thin and wiggly lines refer to the baryons, quarks and external weak field, respectively. Quarks participating in the quark flavor transition* $q_i \to q_j$ *are labeled by the indices* $1i$ *and* $1j$, *while the spectator quarks – by the indices* 2 *and* 3. *Initial and final baryons are labeled by the indices* i *and* j.

Table 1. Magnetic moments of the baryon octet (in units of the nuclear magneton μ_N) and nucleon electromagnetic radii (in units of fm^2).

Quantity	Our results [11]			Experiment [3]
	Valence quarks	Meson cloud	Total	
μ_p	2.530	0.263	2.793	2.793
μ_n	-1.530	-0.383	-1.913	-1.913
μ_Λ	-0.575	-0.038	-0.613	-0.613 ± 0.004
μ_{Σ^+}	2.336	0.196	2.532	2.458 ± 0.010
μ_{Σ^-}	-0.942	-0.327	-1.269	-1.160 ± 0.025
μ_{Ξ^0}	-1.240	-0.096	-1.336	-1.250 ± 0.014
μ_{Ξ^-}	-0.599	0.033	-0.566	-0.6507 ± 0.0025
$\lvert\mu_{\Sigma^0\Lambda}\rvert$	1.273	0.293	1.566	1.61 ± 0.08
$\langle r^2\rangle^p_E$	0.700	0.078	0.778	0.767 ± 0.012
$\langle r^2\rangle^n_E$	-0.0628	-0.0542	-0.117	-0.1161 ± 0.0022
$\langle r^2\rangle^p_M$	0.637	0.118	0.755	0.731 ± 0.060
$\langle r^2\rangle^n_M$	0.618	0.099	0.717	0.762 ± 0.019

Table 2. Numerical values for the radiative corrections in % (taken from Ref. [29]).

Decay mode	$\delta_{\rm rad}$	$\delta^{e\nu_e}_{\rm rad}$	$\delta^{e}_{\rm rad}$	$\delta^{\nu_e}_{\rm rad}$	$\delta^{B}_{\rm rad}$
$n \to p e^- \bar\nu_e$	6.96	1.98	1.98	2.10	2.10
$\Lambda \to p e^- \bar\nu_e$	4.17	1.99	1.99	2.10	2.10
$\Sigma^- \to n e^- \bar\nu_e$	1.85	1.98	1.98	2.10	2.10
$\Sigma^+ \to \Lambda e^+ \nu_e$	2.25	1.99	1.99	2.10	2.10
$\Sigma^- \to \Lambda e^- \bar\nu_e$	2.22	1.99	1.99	2.10	2.10
$\Xi^- \to \Lambda e^- \bar\nu_e$	1.95	1.98	1.98	2.10	2.10
$\Xi^- \to \Sigma^0 e^- \bar\nu_e$	2.10	1.99	1.99	2.10	2.10
$\Xi^0 \to \Sigma^+ e^- \bar\nu_e$	4.36	1.99	1.99	2.10	2.10
$\Lambda \to p \mu^- \bar\nu_\mu$	6.78				
$\Sigma^- \to n \mu^- \bar\nu_\mu$	1.88				
$\Xi^- \to \Sigma^0 \mu^- \bar\nu_\mu$	2.12				
$\Xi^0 \to \Sigma^+ \mu^- \bar\nu_\mu$	6.78				

Table 3. Couplings $V_{11}^{B_iB_j}$ and $A_{11}^{B_iB_j}$.

Mode	Our results		SU(6) quark model	
	$V_{11}^{B_iB_j}$	$A_{11}^{B_iB_j}$	$V_{11}^{B_iB_j}$	$A_{11}^{B_iB_j}$
$n \to p$	1	1.452	1	$\frac{5}{3}$
$\Lambda \to p$	−1.146	−1.039	$-\sqrt{\frac{3}{2}} = -1.225$	$-\sqrt{\frac{3}{2}} = -1.225$
$\Sigma^- \to n$	−0.943	0.307	−1	$\frac{1}{3} = 0.333$
$\Sigma^- \to \Lambda$	−0.002	0.724	0	$\sqrt{\frac{2}{3}} = 0.816$
$\Xi^- \to \Lambda$	1.170	0.388	$\sqrt{\frac{3}{2}} = 1.225$	$\frac{1}{\sqrt{6}} = 0.408$
$\Xi^- \to \Sigma^0$	0.689	1.035	$\frac{1}{\sqrt{2}} = 0.707$	$\frac{5}{3\sqrt{2}} = 1.179$
$\Xi^0 \to \Sigma^+$	0.975	1.464	1	$\frac{5}{3} = 1.667$

Table 4. Couplings $V_{21,31}^{B_iB_j}$ and $A_{21,31}^{B_iB_j}$.

Mode	$V_{21}^{B_iB_j}$	$V_{31}^{B_iB_j}$	$A_{21}^{B_iB_j}$	$A_{31}^{B_iB_j}$
$n \to p$	1.530	0	0	2.850
$\Lambda \to p$	−0.840	−0.093	−0.042	−1.431
$\Sigma^- \to n$	0.802	−0.288	−0.047	1.467
$\Sigma^- \to \Lambda$	1.180	−0.034	0.034	2.517
$\Xi^- \to \Lambda$	0.009	0.231	0.061	−0.048
$\Xi^- \to \Sigma^0$	1.235	0.014	0.006	2.374
$\Xi^0 \to \Sigma^+$	1.747	0.019	0.009	3.357

Table 5. Semileptonic decay constants of baryons $F_1^{B_iB_j}$ and $G_1^{B_iB_j}$.

Decay mode	$F_1^{B_iB_j}$	$G_1^{B_iB_j}$
$n \to p$	1	$g_A = 1.258\,(1+\delta_A^{np}) = 1.2695$
$\Lambda \to p$	$-\sqrt{\frac{3}{2}}(1+\delta_V^{\Lambda p}) = -1.226$	$-0.928\,(1+\delta_A^{\Lambda p}) = -0.888$
$\Sigma^- \to n$	$-(1+\delta_V^{\Sigma n}) = -1.009$	$0.243\,(1+\delta_A^{\Sigma n}) = 0.262$
$\Sigma^- \to \Lambda$	-0.002	$0.613\,(1+\delta_A^{\Sigma\Lambda}) = 0.633$
$\Xi^- \to \Lambda$	$\sqrt{\frac{3}{2}}(1+\delta_V^{\Xi\Lambda}) = 1.252$	$0.315\,(1+\delta_A^{\Xi\Lambda}) = 0.332$
$\Xi^- \to \Sigma^0$	$\frac{1}{\sqrt{2}}(1+\delta_V^{\Xi\Sigma}) = 0.737$	$0.890\,(1+\delta_A^{\Xi\Sigma}) = 0.885$
$\Xi^0 \to \Sigma^+$	$1+\delta_V^{\Xi\Sigma} = 1.042$	$1.258\,(1+\delta_A^{\Xi\Sigma}) = 1.252$

Table 6. Ratios $G_1^{B_iB_j}/F_1^{B_iB_j}$.

Decay mode	Our results	Data [3]
$n \to p$	1.2695	1.2695 ± 0.0029
$\Lambda \to p$	0.724	0.718 ± 0.015
$\Sigma^- \to n,\ \ G_1/F_1$	-0.260	-0.34 ± 0.017
$\Sigma^- \to n,\ \ (G_1 - 0.237G_2)/F_1$	-0.278	$-0.327 \pm 0.007 \pm 0.019$
$\Xi^- \to \Lambda$	0.265	0.25 ± 0.05
$\Xi^- \to \Sigma^0$	1.20	
$\Xi^0 \to \Sigma^+$	1.20	$1.20 \pm 0.04 \pm 0.03$

Table 7. Semileptonic decay constants of baryons $F_{2,3}^{B_iB_j}$ and $G_{2,3}^{B_iB_j}$.
Here $\mu_\pi = 0.13957$ and $\mu_K = 0.493677$ are the dimensionless masses of π and K mesons.

Decay mode	$F_2^{B_iB_j}$	$G_2^{B_iB_j}$	$F_3^{B_iB_j}$	$G_3^{B_iB_j}$
$n \to p$	1.853	0	0	$\frac{2.187}{\mu_\pi^2}$ $\left(\frac{2.271}{\mu_\pi^2}\right)$ $g_p = 8.25$
$\Lambda \to p$	-1.226	-0.072	-0.067	$-\frac{1.647}{\mu_K^2}$ $\left(-\frac{2.035}{\mu_K^2}\right)$
$\Sigma^- \to n$	0.971	-0.078	-0.055	$\frac{0.536}{\mu_K^2}$ $\left(\frac{0.663}{\mu_K^2}\right)$
$\Sigma^- \to \Lambda$	1.206	0.013	0.016	$\frac{1.645}{\mu_\pi^2}$ $\left(\frac{1.735}{\mu_\pi^2}\right)$
$\Xi^- \to \Lambda$	0.162	0.076	0.052	$\frac{1.002}{\mu_K^2}$ $\left(\frac{1.403}{\mu_K^2}\right)$
$\Xi^- \to \Sigma^0$	1.770	0.037	0.035	$\frac{2.783}{\mu_K^2}$ $\left(\frac{3.631}{\mu_K^2}\right)$
$\Xi^0 \to \Sigma^+$	2.503	0.052	0.050	$\frac{3.936}{\mu_K^2}$ $\left(\frac{5.137}{\mu_K^2}\right)$

Table 8. Ratios $F_2^{B_iB_j}/F_1^{B_iB_j}$.

Decay mode	Cabibbo model [2]	$1/N_c$ expansion [33]	χQSM [34]	Our results
$n \to p$	$\frac{1}{2}(\mu_p - \mu_n - 1) = 1.853$	1.85	1.57	1.853
$\Lambda \to p$	$\frac{m_\Lambda}{2m_N}(\mu_p - 1) = 1.066$	0.90	0.71	1
$\Sigma^- \to n$	$\frac{m_{\Sigma^-}}{m_N}(\mu_p + 2\mu_n - 1) = -1.297$	-1.02	-0.96	-0.962
$\Sigma^- \to \Lambda \quad (F_2)$	$-\frac{m_{\Sigma^-}}{2m_N}\sqrt{\frac{3}{2}}\,\mu_n = 1.490$	1.17	1.24	1.206
$\Xi^- \to \Lambda$	$-\frac{m_{\Xi^-}}{2m_N}(\mu_p + \mu_n - 1) = 0.085$	0.06	0.02	0.129
$\Xi^- \to \Sigma^0$	$\frac{m_{\Xi^-}}{2m_N}(\mu_p - \mu_n - 1) = 2.609$	1.85	2.02	2.402
$\Xi^0 \to \Sigma^+$	$\frac{m_{\Xi^0}}{2m_N}(\mu_p - \mu_n - 1) = 2.597$	1.85		2.402

Table 9. Results for the $\Sigma^- \to n e^- \bar{\nu}_e$ decay.

Quantity	Lattice approach [35]	Our results
F_1	$-0.988 \pm 0.029_{\rm lattice} \pm 0.040_{\rm HBChPT}$	-1.009
G_1/F_1	-0.287 ± 0.052	-0.260
$(G_1 - 0.237 G_2)/F_1$	-0.37 ± 0.08	-0.278
F_2/F_1	-0.85 ± 0.45	-0.962
F_3/F_1	0.24 ± 0.12	0.055
G_2/F_1	0.35 ± 0.15	0.077
G_3/F_1	-3.42 ± 1.85	-2.180

Table 10. Decay widths Γ (in units of 10^6 s^{-1}, for neutron decay in units of 10^{-3} s^{-1}).

Decay mode	Our results				SU(3) fit	Data [3]
	Γ	$\Gamma(F_1, G_1)$	$\Gamma(F_1(0), G_1(0))$	Γ^0		
$n \to p e^- \bar{\nu}_e$	1.12	1.12	1.12	1.05	1.12	1.129 ± 0.001
$\Lambda \to p e^- \bar{\nu}_e$	3.28	3.26	3.10	3.15	3.16	3.16±0.06
$\Lambda \to p \mu^- \bar{\nu}_\mu$	0.57	0.56	0.51	0.53	0.52	0.60±0.13
$\Sigma^- \to n e^- \bar{\nu}_e$	6.50	6.50	5.72	6.39	6.19	6.88±0.24
$\Sigma^- \to n \mu^- \bar{\nu}_\mu$	3.15	3.15	2.54	3.09	2.74	3.0±0.2
$\Sigma^+ \to \Lambda e^+ \nu_e$	0.26	0.26	0.26	0.25	0.27	0.25±0.06
$\Sigma^- \to \Lambda e^- \bar{\nu}_e$	0.43	0.43	0.43	0.42	0.45	0.39±0.02
$\Xi^- \to \Lambda e^- \bar{\nu}_e$	3.35	3.35	3.15	3.28	2.80	3.35±0.37
$\Xi^- \to \Lambda \mu^- \bar{\nu}_\mu$	0.96	0.96	0.85	0.94	0.76	$2.1^{+2.1}_{-1.3}$
$\Xi^- \to \Sigma^0 e^- \bar{\nu}_e$	0.52	0.51	0.50	0.51	0.51	0.53±0.10
$\Xi^- \to \Sigma^0 \mu^- \bar{\nu}_\mu$	0.0067	0.0067	0.0064	0.0065	0.0064	< 0.05
$\Xi^0 \to \Sigma^+ e^- \bar{\nu}_e$	0.93	0.93	0.91	0.89	0.91	0.93±0.14
$\Xi^0 \to \Sigma^+ \mu^- \bar{\nu}_\mu$	0.0081	0.0081	0.0078	0.0076	0.0078	0.02±0.01

Table 11. Predictions for $\Gamma/|V_{\rm CKM}|^2$ (in units of 10^7 s^{-1}).

Decay mode	$\Gamma/\lvert V_{\rm CKM}\rvert^2$	Decay mode	$\Gamma/\lvert V_{\rm CKM}\rvert^2$
$\Lambda \to p e^- \bar\nu_e$	6.48	$\Xi^- \to \Lambda e^- \bar\nu_e$	6.62
$\Lambda \to p \mu^- \bar\nu_\mu$	1.13	$\Xi^- \to \Lambda \mu^- \bar\nu_\mu$	1.90
$\Sigma^- \to n e^- \bar\nu_e$	12.84	$\Xi^- \to \Sigma^0 e^- \bar\nu_e$	1.03
$\Sigma^- \to n \mu^- \bar\nu_\mu$	6.22	$\Xi^- \to \Sigma^0 \mu^- \bar\nu_\mu$	0.013
$\Sigma^+ \to \Lambda e^+ \nu_e$	0.027	$\Xi^0 \to \Sigma^+ e^- \bar\nu_e$	1.84
$\Sigma^- \to \Lambda e^- \bar\nu_e$	0.045	$\Xi^0 \to \Sigma^+ \mu^- \bar\nu_\mu$	0.016

Table 12. Asymmetry parameters.

Decay mode	$\alpha_{e\nu_e}$	α_e	α_{ν_e}	α_B
$n \to p e^- \bar\nu_e$	−0.08	−0.10	0.99	−0.48
$\Lambda \to p e^- \bar\nu_e$	−0.01	0.02	0.92	−0.60
$\Sigma^- \to n e^- \bar\nu_e$	0.42	−0.50	−0.32	0.65
$\Sigma^+ \to \Lambda e^+ \nu_e$	−0.39	−0.68	0.63	0.06
$\Sigma^- \to \Lambda e^- \bar\nu_e$	−0.40	−0.69	0.63	0.07
$\Xi^- \to \Lambda e^- \bar\nu_e$	0.54	0.23	0.57	−0.54
$\Xi^- \to \Sigma^0 e^- \bar\nu_e$	−0.19	−0.18	0.96	−0.46
$\Xi^0 \to \Sigma^+ e^- \bar\nu_e$	−0.18	−0.17	0.92	−0.45