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Technical Change, Income Distribution, and Profitability in Multisector Linear Economies

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Abstract

This paper analyzes the effect of technical change on income distribution and profitability by comparing the long-run outcomes defined by a uniform profit rate in a multisector linear economy. We study three scenarios with (i) fixed real wage; (ii) fixed profit rate; or (iii) fixed wage-profit ratio, and show that any viable capital-using and labor-saving technical change *itself* (in the absence of power change) would bring about a fall in the rate of profit. Profit rate would not rise unless the technical change is so power-biased against the working-class that the wage-profit ratio can not be maintained. Our result conclusively supports the argument of the falling rate of profit due to a rising organic composition of capital as an underlying economic force.

Keywords: Technical Change; Falling Rate of Profit; Okishio Theorem;
JEL Codes: B51, D33, D57

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1 Introduction

The history of capitalism has seen the dramatic technological revolutions and the permanent improvement of the living standard, on the one hand, the constant conflicts over the distribution of income and the cyclical recessions and crisis on the other. To understand these dynamics, we should no doubt consider the institutional and political factors since technology evolves endogenously and distribution is profoundly shaped by policies (Acemoglu & Robinson, 2012; Bowles, 2012). At the same time, it is equally important to study the effects of technical change isolated from other factors as an underlying force behind the dynamics.

In particular, we focus on the effects of technological innovations on the functional income distribution and profitability, two important aspects of understanding the dynamics of capitalism. First, the change in the distribution of income among labor and capital has a direct hand in the shaping of inequality. It has a strong and positive link with personal income distribution, as shown by historical cross-country data (Bengtsson & Waldenström, 2018; Roine & Waldenström, 2015). Second, the rate of profit has significant impacts on investments and stability and thus relevant to the cyclical recessions and crisis (Basu & Das, 2017; Duménil & Lévy, 1993b, 2003; Glyn, 1997).

The model presented in this paper is flexible enough to explore the relationship among technical change, income distribution and profitability in different cases with (i) fixed real wage; (ii) fixed profit rate; or (iii) fixed wage-profit ratio. This uniform framework allows us to compare and incorporate the insights from different approaches following Okishio (1961), Sraffa (1960) and Marx (1993).

By exploring the effects of technical change in these three scenarios, we show that the capital-using and labor-saving (CU-LS) technical innovations adopted by capitalists do not increase the profit rate unless they are power-biased against the working-class such that the wage-profit ratio can not be maintained. In other words, the CU-LS technical change itself in the absence of power change would bring about a fall in the rate of profit.

We follow the convention of long-period analysis by considering multi-sector linear economies and focus on the long-run outcome defined by a uniform rate of profit. First, a multi-sector model is more appropriate in capturing the changes in relative prices than a one-good model, as argued by Piketty,

> The one-good, perfect competition model is not a very satisfactory model, to say the least. In practice, the right model to think about rising capital–income ratios and capital shares is a multisector model (with a large role played by capital-intensive sectors such as real estate and energy, and substantial movements in relative prices) with important variations in bargaining power over time. (Piketty, 2015, p. 81)

Second, the assumption on linear technology is consistent with the recent empirical evidence questioning the substitutability between capital and labor as well as the existence of a neoclassical production function (Basu, 2010; Campbell & Tavani, 2019; Foley, Michl, & Tavani, 2019; Zambelli, 2018). Finally, we focus on the long-run outcome since it can be embedded in the general equilibrium framework as a "stronger conception of equilibrium" to which the intertemporal equilibrium over infinite horizon converges (Duménil & Lévy, 1985, p. 328; Dana, Florenzano, Le Van, & Lévy, 1989a, 1989b).

Then we compare the long-run outcomes and its associated properties before and after the capital-using and labor-saving technical change that is cost-reducing under current prices. Two associated properties are considered — the sectoral wage-profit ratio and sectoral organic composition of capital (OCC) defined as the ratio of the cost of material inputs to the total cost in each sector.

In Okishio's scenario with fixed real wage, additional to the well-known result that profit rate rises, we show that the wage-profit ratio falls in the sector with technical change. This effect is also observed in the Sraffian case with fixed profit rate, even though real wages rise. These results are not trivial since the change in the wage-profit ratio is influenced by the movement of relative prices, which is quite complicated in the multi-sector model. In the Marxian scenario with fixed wage-profit ratio, we show that the rate of profit falls. Therefore, we can conclude that CU-LS technical innovation adopted by capitalists reduces the rate of profit if it is power-neutral in the sense that the distributional situation remains the same.

Moreover, we observe that the organic composition of capital rises in the sector with viable CU-LS technical change in all three scenarios, which helps us highlight the mechanism of falling profit rate due to rising OCC. The CU-LS technical change leads to a rising OCC, or equivalently, reduce total wage relative to the total costs. If the distributional situation remains the same, that is, the wage-profit ratio is constant, then profit is also reduced relative to the total cost, and thus profit rate falls by definition. Such an underlying force applies to all cases even when the technical change is power-biased, with the effect on profit rate being countered or amplified depending on how the power relation is changed.

1.1 Related Literature

This paper contributes to several different strands of literature.

First, it is directly connected to the long-standing controversy around Marx's "law of tendential fall in the rate of profit" (Marx, 1993). Marx argues that technical change in capitalism tends to bring about a falling profit rate due to the rising organic composition of capital. However, the Okishio (1961)'s Theorem and its following analytic results cast doubts on the internal consistency of Marx's argument under the condition of competitive capitalism (Roemer, 1977; van Parijs, 1980). The clear-cut result in this paper provides conclusive support for Marx's insight.

It is well-known that one could observe a falling profit rate by modifying some assumptions in Okishio's setting. For example, Shaikh (1980, 2016) develops a model of so-called real competition in which capitalist focus on profit margin instead of the profit rate, while Skott (1992) and Michl (1994) provide models with imperfect competition. Under competitive conditions, if real wage rises sufficiently, it is also possible to have a falling profit rate (Laibman, 1982; Okishio, 2001; Roemer, 1978). However, according to van Parijs (1980)'s obituary for Marx's argument, to relax the assumption of fixed real wage does not help in the construction of a theory of falling profit rate due to rising OCC — profit rate falls with rising real wage because of the growth in the working-class power as addressed by the profit-squeeze argument, not the rising OCC. Therefore, falling profit rate due to rising OCC is "not even a *possibility* under the condition of competitive capitalism" (van Parijs, 1980, p. 1).

Challenging to this view, the main result in this paper (Theorem 1) shows that to observe a falling profit rate wage need not rise too much to squeeze profit but just grow slightly to maintain the wage-profit ratio.

Roemer (1978) obtains similar result in a simple two-goods two-sector model with adjustment in real wage to keep a constant wage share, while Laibman (1982) shows that the change in profit rate is indeterminate in general in the same two-goods two-sector model but with a fixed rate of exploitation in reference to some consumption bundle. Franke (1999) presents a model with fixed capital stock and shows that wage share falls if the rate of profit remains fixed, which is different from our result in two aspects. First, Franke considers the aggregated wage share at the balanced growth path instead of the sectoral wage-profit ratio. Second, Franke's result holds only for the *ad hoc* technical change with increasing fixed capital stock and uniformly decreasing labor input in all sectors. Julius (2009) also observes a falling profit rate after technical change when the bargaining power is unchanged, but only the existence of such technical change is shown, which differs from our result with any CU-LS technical change.

Second, this paper also contributes to the large body of literature on the long-term trends of the real wage, income distribution, and profitability. Instead of taking Marx's general law as a failed empirical prediction like Acemoglu and Robinson (2015), our theoretical analysis identifies the falling profit rate due to rising OCC as a underlying economic force, which could be helpful in understanding the long-term trend of the profit rate to fall discovered in recent evidence (Basu & Manolakos, 2013; Basu & Vasudevan, 2013; Trofimov, 2018). Moreover, the three benchmarks we discuss can be used to characterize different regimes in the co-evolution of real wage, distribution, and profitability in the history of capitalism (Allen, 2008, 2019; Bengtsson & Waldenström, 2018; Cvrcek, 2013; Duménil & Lévy, 1993a, 2002, 2016, 2018; Karabarbounis & Neiman, 2014; Kotz & Basu, 2019; Roine & Waldenström, 2015; van Zanden, 1999; Wolff, 2001, 2003, 2010). To define the regime of profitability using the real wage rate is suggested in Foley (1986, pp. 136–139) and further elaborated by Basu (2018) who defines the Marx-Okishio threshold in a one-good model[1]. The three benchmarks and four regimes developed in this paper can be taken as a generalization of their one-good model result.

2 Model

Economic Environment. Consider the economy with linear technology (A, L) where a_{ij} represents the amount of good i needed in the production of one unit of good j, and L_j is the direct labor required in the production of one unit of good j. Suppose that (A, L) satisfies the following assumption[2].

Assumption 1. The nonnegative matrix A is productive, indecomposable, and labor is indispensable. That is, for the matrix $A \geq 0$,

(i) $\exists x \geq 0$, $x - Ax \geq 0$;

(ii) $\forall i, j, \exists t, (A)^t_{ij} > 0$; and

(iii) $L > 0$.

[1] I thank Deepankar Basu for the suggestion of the literature.

[2] For $a, b \in \mathbb{R}$, we use $\geq$ and $\leq$ to denote the weak inequality. For $a = (a_1, \dots, a_n), b = (b_1, \dots, b_n)$, $a \geqq b$ means $a_i \geq b_i$ for all i; $a \geq b$ means $a \geqq b$ and $a \neq b$; $a > b$ means $a_i > b_i$ for all i.

Let $p = (p_1, \ldots, p_n)$ be the price vector and w_i the wage rate in industry i. Assume that w_i is determined by class struggle in that industry so that the wage rates could be different among industries and adjust with technological progress, as the technology adopted in the industry would shape power structure within the process of production (e.g., Braverman, 1998; Guy & Skott, 2008, 2015). Let $W = \text{diag}\{w_1, \ldots, w_n\}$ be the diagonal matrix of wages, and denote the economy by $\mathcal{E}(A, L)$ with W. With this notation, the economy with constant real wage can be taken as a special case with $w_i = pb, \forall i$ where b is the subsistence wage, denoted by $\mathcal{E}(A, L, b)$.

Long-run Outcomes. We focus on the long-run outcome with a uniform rate of profit π such that

$$p = (1 + \pi)(pA + WL) \tag{1}$$

and define the wage-profit ratio γ by

$$\gamma_j = \frac{w_j L_j}{\pi(pA^j + w_j L_j)}, \forall j \tag{2}$$

Definition 1. Given the economy $\mathcal{E}(A, L)$ with W, the profile (p, π, γ) defined by (1) and (2) is called the *long-run outcome*.

The price at the long-run outcome is also called the price of production, and the uniform rate of profit is conventionally justified as the result of the movement of capital among industries by competition. We focus on the long-run outcome without assuming any characteristics of the consumption since the intertemporal equilibrium generally converges to the price of production (Dana et al., 1989b; Duménil & Lévy, 1985; Hahn, 1982).

Let $\Gamma = \text{diag}\{\gamma_1, \ldots, \gamma_n\}$, then (2) can be rewritten as $WL = \pi(pA + WL)\Gamma$. By (1) we have $pA + WL = p/(1 + \pi)$. Therefore,

$$WL = \frac{\pi}{1 + \pi} p\Gamma, \text{ that is, } w_i L_i = \frac{\pi}{1 + \pi} p_i \gamma_i, \forall i \tag{3}$$

Together with (1), we get

$$p = p[(1 + \pi)A + \pi\Gamma] \tag{4}$$

Technical Change. Now consider a technical change in sector i from (A^i, L_i) to (A^{*i}, L_i^*) and denote the new technology by (A^*, L^*). Specifically, we will focus on the viable capital-using and labor-saving technical change defined as follows.

Definition 2. Consider a technical change in sector i from (A^i, L_i) to (A^{*i}, L_i^*). It is *viable* if it is cost-reducing under current prices

$$pA^{*i} + w_i L_i^* < pA^i + w_i L_i.$$

It is *capital-using* and *labor-saving* (CU-LS) if

$$A^{i*} \geqq A^i, \quad L_i^* < L_i.$$

The concept of viable technical change is first introduced by Okishio (1961) and widely adopted as a criterion of technical choice in the related literature though not in all (e.g., Shaikh, 1978, 1980). And the definition of CU-LS technical change following Morishima (1973, p. 137) and Roemer (1977) is believed to capture the technological innovations that replace labor with machine and enhance the labor productivity, or in Marx's terminology, the technical change with a raising "technical composition of capital" (Marx, 1993, p. 244; Saad-Filho, 2019, p. 91). This type of technical change is relevant not just to the history of capitalism (Marquetti, 2003), but also to the worldwide trend of automation currently (e.g., Autor & Salomons, 2018; Dinlersoz & Wolf, 2018; Frey & Osborne, 2017).

Define the organic composition of capital (OCC) in sector i by

$$q_i = \frac{pA^i}{pA^i + w_i L_i}$$

which is the counterpart of $q = c/(c+v)$ following Sweezy (1942) where c and v are the constant capital and variable capital in terms of value respectively. Then the OCC after technical change is[3]

$$q_i^* = \frac{p^* A^{*i}}{p^* A^{*i} + w_i^* L_i^*}$$

Given the long-run outcome (p, π, γ), we have

$$\pi\gamma_i = \frac{w_i L_i}{pA^i + w_i L_i} < 1$$

and

$$q_i = 1 - \pi\gamma_i \tag{5}$$

3 The Effects of Technical Change on Income Distribution and Profitability

Next, we explore the consequences of technical change on income distribution and profitability, by comparing the long-run outcome after the technical change, (p^*, π^*, γ^*), with the initial one (p, π, γ). However, the long-run outcome after the technical change cannot be determined just by the technology (A^*, L^*) itself, and additional conditions should be introduced (Mandler, 1999; Yoshihara & Kwak, 2019).

We will discuss the following three cases as benchmarks named after Okishio, Sraffa and Marx respectively[4], by assuming

[3]Some authors argue that in a dynamic context it is better to define the organic composition of capital using the prices/values at the initial period (e.g., Saad-Filho, 1993), and our definition here should be termed as the value composition of capital. However, the different use of terminology will not change our analysis and the main results in this paper.

[4]We do not claim that our discussions below are authoritative to Okishio, Sraffa or Marx, given the controversial interpretations of their thoughts and the different viewpoints at different stages of their academic career (e.g., Garegnani, 2005; Heinrich, 2013; Okishio, 1961, 2001; Reuten, 2004; Reuten & Thomas, 2011; Sinha, 2014, 2016). What they really meant may be an interesting issue in the history of economic thought, which is not our concern here.

(i) Okishio's scenario with fixed real wage b;

(ii) Sraffian scenario with fixed profit rate π;

(iii) Marxian scenario with fixed wage-profit ratio γ.

The first one is adopted in Okishio's seminal paper and its following discussions (Okishio, 1961; Roemer, 1977, 1979). The second one is conventionally assumed in the literature on the choice of technique in Sraffian tradition (e.g., Bidard, 1990; Pasinetti, 1977; Sraffa, 1960). The last one assuming a fixed distributional situation is inspired by Marx's argument on falling profit-rate due to the rising OCC, "given that the rate of surplus-value, or the level of exploitation of labour by capital, remains the same" (Marx, 1993, p. 318). Instead of assuming a constant rate of exploitation and working with the value system like Laibman (1982) and Bidard (2004, pp. 73–76), we fix the wage-profit ratio or equivalently the wage share (Franke, 1999; Roemer, 1978).

Fixed Real Wage. Okishio (1961) shows that the profit rate would rise after any viable technical change if real wage is fixed. Moreover, the converse is also true: given fixed real wage, if the technical change leads to a rising profit rate, then it must be viable (Dietzenbacher, 1989).

Proposition 1. *Let (p, π, γ) be the long-run outcome of the economy $\mathcal{E}(A, L, b)$. Consider a technical change in sector i from (A, L) to (A^*, L^*), and let (p^*, π^*, γ^*) be the long-run outcome of $\mathcal{E}(A^*, L^*, b)$. Then $\pi^* > \pi$ if and only if the technical change is viable.*

Proof. For the 'if' part, see Roemer (1981, p. 98) or Bowles (1981) for a simple proof. For the 'only if' part, see Dietzenbacher (1989, Theorem 2, p. 41). □

Next, we look at the distributional consequence of technical change by examining the change in the wage-profit ratio γ_i. Since the ratio is measured in term of prices, we first establish the effect of technical change on relative prices.

Lemma 3.1. *Let (p, π, γ) be the long-run outcome of the economy $\mathcal{E}(A, L, b)$. Consider a viable technical change in sector i from (A, L) to (A^*, L^*), and let (p^*, π^*, γ^*) be the long-run outcome of $\mathcal{E}(A^*, L^*, b)$. Then the price in sector i decreases the most, that is,*

$$\frac{p_i^*}{p_i} < \frac{p_j^*}{p_j}, \forall j \neq i$$

Proof. Suppose, on the contrary, that there exists $i' \neq i$ such that

$$\frac{p_{i'}^*}{p_{i'}} \leqq \frac{p_j^*}{p_j}, \forall j,$$

Then if we choose good i' as the numeraire, the price increases after the technical change. Precisely, let $\bar{p} = p/p_{i'}$ and $\bar{p}^* = p^*/p_{i'}^*$, then we have $\bar{p}^* \geq \bar{p}$. However, we have

$$(1 + \pi^*)\bar{p}^*(A^{i'} + bL_{i'}) = \bar{p}_{i'}^* = 1 = \bar{p}_{i'} = (1 + \pi)\bar{p}(A^{i'} + bL_{i'}) \Rightarrow \pi^* < \pi$$

contradicted with Proposition 1. □

Lemma 3.1 shows that after the viable technical change in sector i, the price of good i decreases relative to that of any other goods[5], with which we could establish the following proposition showing the falling wage-profit ratio as a result of viable CU-LS technical change.

Proposition 2. *Let (p, π, γ) be a long-run outcome of the economy $\mathcal{E}(A, L, b)$. Consider a viable CU-LS technical change in sector i from (A, L) to (A^*, L^*). Then for the outcome (p^*, π^*, γ^*) of the economy $\mathcal{E}(A^*, L^*, b)$, we have falling wage-profit ratio in that sector, $\gamma_i^* < \gamma_i$. Moreover, the organic composition of capital rises, that is, $q_i^* > q_i$.*

Proof. It is sufficient to show that $\pi^*\gamma_i^* < \pi\gamma_i$ since $q_i = 1 - \pi\gamma_i$ and $\pi^* > \pi$. Since the wage-profit ratio is invariant with different normalization of the prices, we choose good i as numeraire and denote the prices by $\bar{p}$ and $\bar{p}^*$, then we have $\bar{p}^* \geq \bar{p}$ by Lemma 3.1. By (3), we have

$$\pi\gamma_i = (1+\pi)\bar{p}bL_i$$
$$\pi^*\gamma_i^* = (1+\pi^*)\bar{p}^*bL_i^*$$

By the price equation (1), we have

$$(1+\pi)\bar{p}(A^i + bL_i) = \bar{p}_i = 1 = \bar{p}_i^* = (1+\pi^*)\bar{p}^*(A^{*i} + bL_i)$$

and therefore

$$(1+\pi)\bar{p}bL_i - (1+\pi^*)\bar{p}^*bL_i^* = (1+\pi^*)\bar{p}^*A^{i^*} - (1+\pi)\bar{p}A^i > 0$$

since $\pi^* > \pi$ by Proposition 1, $\bar{p}^* \geq \bar{p}$ and $A^{*i} \geqq A^i$ be the definition of CU-LS technical change. Therefore, we establish $\pi^*\gamma_i^* < \pi\gamma_i$.

$\square$

Proposition 2 shows that workers are relatively worse off after the viable CU-LS technical change with constant real wage. This distributional consequence can be confirmed if we use labor value $v = vA + L$ and the rate of exploitation $e = (1 - vb)/(vb)$ to measure the distributional situation. Since a viable CU-LS technical change must be progressive in the sense that $v^* < v$ as shown in Roemer (1977), we have $e^* > e$. In other words, workers would receive less labor time in exchange for one unit of labor supply. This mechanism is well-established and recently incorporated into an intertemporal framework to generate persistent exploitation (Galanis, Veneziani, & Yoshihara, 2019), and Proposition 2 can be seen as the same mechanism after the transformation from values to prices. The same is true with the rising OCC. We show that OCC in terms of prices in sector i rises after any viable CU-LS technical change, which is also true with the OCC in terms of value as shown by Roemer (1979).

Fixed Profit Rate. Assume that wage rate is uniform among sectors, $W = wI$, and normalize the uniform wage to one, then the Sraffian system can be taken as the special

[5] Indeed, a stronger inequality $\frac{p_i^*}{p_i} \leq \frac{1+\pi}{1+\pi^*}\frac{p_j^*}{p_j}, \forall j$ can be established (Dietzenbacher, 1988, 1989). However, for our purpose here to show Proposition 2, Lemma 3.1 is sufficient.

case $\mathcal{E}(A, L)$ with I.[6] In such a system, it is well-known that real wage increases after viable technical change with fixed profit rate (e.g., Sraffa, 1960, Ch XII; Pasinetti, 1977, pp. 158–9).

Proposition 3. *Let (p, π, γ) be the long-run outcome of the economy $\mathcal{E}(A, L)$ with I. Consider a viable technical change from (A, L) to (A^*, L^*), and let (p^*, π, γ^*) be the long-run outcome of the economy $\mathcal{E}(A^*, L^*)$ with I, then $p^* < p$.*

Proof. See Fujimoto, Herrero, and Villar (1983, Proposition 2, p. 125) for a proof based on the so-called "Le Chatelier-Samuelson Principle" (Fujimoto, 1980), or Dietzenbacher (1988, Theorem 4.3, p. 388) for a direct proof with wages paid post factum. □

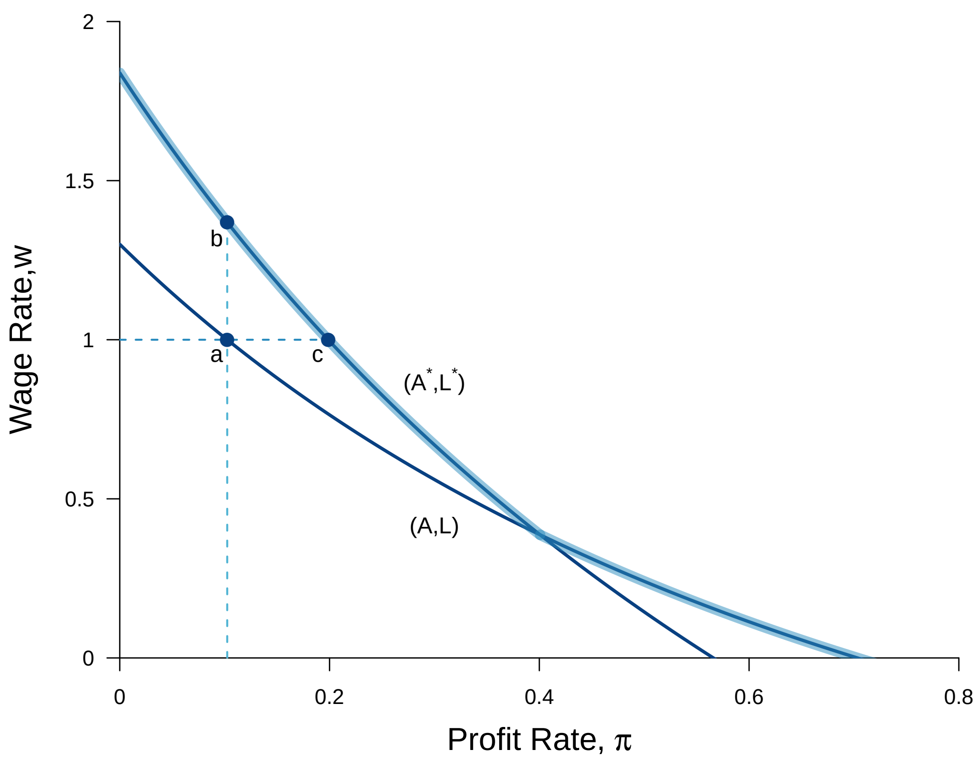

Figure 1: The $w - \pi$ Frontier with Techncial Change. Point **a** represents the initial outcome with (A, L), **b** the outcome after technical change to (A^*, L^*) with fixed profit rate and **c** the outcome after technical change with fixed real wage.

Proposition 3 shows that prices in terms of wage would fall after the viable technical change — all goods become cheaper when wage rate is normalized to be one. It implies

[6] Another conventional assumption in Sraffian tradition is that wage is paid *post factum*, while in Marxian tradition, wage is typically assumed to be advanced at the beginning. However, the choice between these two assumptions has no essential impact on the results because it is equivalent to pay w in advanced or $(1 + \pi)w$ post factum (see e.g., Bidard, 2004, p. 39; Abraham-Frois & Berrebi, 1997, p. 55).

that real wage rises no matter which bundle of goods is chosen as the standard. Or using the device of the so-called 'wage-profit' frontier, the viable technical change would push the frontier outward and thus wage rises if the the profit rate is fixed as shown in Figure 1. Point **a** represents the initial long-run outcome with (A, L), and point **b** the outcome after technical change with fixed profit rate[7].

However, the implication of the fixed profit rate and rising wage rate could be misleading with respect to the change in the distributional situation. It seems to indicate a relative improvement of the workers, which is not true in the sector with technical change, as shown in the following proposition.

Proposition 4. *Let (p, π, γ) be the long-run outcome of the economy $\mathcal{E}(A, L)$ with I. Consider a viable technical change from (A, L) to (A^*, L^*), and let (p^*, π, γ^*) be the long-run outcome of the economy $\mathcal{E}(A^*, L^*)$ with I, then $\gamma_j^* > \gamma_j$ for all $j \neq i$, but $\gamma_i^* < \gamma_i$ and thus $q_i^* > q_i$.*

Proof. By Proposition 3, we have $p^* < p$, and thus $p^*A^j < pA^j$. Then for any $j \neq i$, we have

$$\gamma_j = \frac{L_j}{\pi(pA^j + L_j)} < \frac{L_j}{\pi(p^*A^j + L_j)} = \gamma_j^*$$

For i, suppose on the contrary that $\gamma_i^* \geq \gamma_i$, then $\Gamma^* \geq \Gamma$, and thus

$$[(1+\pi)A^* + \pi\Gamma^*] \geq [(1+\pi)A + \pi\Gamma]$$

However, by (4) we have

$$p = p[(1+\pi)A + \pi\Gamma]$$
$$p^* = p^*[(1+\pi)A^* + \pi\Gamma^*]$$

which implies that both matrices $[(1+\pi)A + \pi\Gamma]$ and $[(1+\pi)A^* + \pi\Gamma^*]$ have the same Frobenius root, contradicted. Therefore, we have $\gamma_i^* < \gamma_i$, and then by the fixed profit rate, $q_i^* = 1 - \pi\gamma_i^* > 1 - \pi\gamma_i = q_i$. □

Proposition 4 shows that with the fixed profit rate, the wage-profit ratio in the sector with technical change would fall, while that in other sectors would rise. First, $\gamma_i^* < \gamma_i$ means that the workers in the sector with technical change is relatively worse off even though the real wage increases. Therefore, it could be misleading to analyze the distributional effect of technical change using the wage-profit frontier, or the so-called "technological frontier of the income distribution possibilities" (Pasinetti, 1977, p. 161)[8]. Indeed, as shown in Figure 1, from point **a** to **c** as shown in Okishio's scenario,

[7] The wage-profit frontier can be derived from (1) by choosing any bundle of goods δ as standard. From $p = (1+\pi)wL[I - (1+\pi)A]^{-1}$ and $p\delta = 1$, we have

$$w = \frac{1}{(1+\pi)L[I - (1+\pi)A]^{-1}\delta}$$

If $\delta = b$ in Okishio's theorem, then point **c** in Figure 1 represents the outcome after technical change with fixed real wage.

[8] The wage-profit frontier reflects the distributional conflict when the technique is fixed. This is because with fixed technique surplus is fixed, and thus the increase in real wage must associate with the decrease in profit. However, with technical progress, the surplus is changing so that the fixed profit rate and an increase in real wage are not sufficient to capture the change in distribution.

workers are relatively worse off, while from **c** to **b** workers are relatively improved, so the composite effect from **a** to **b** is not clear just from the figure.

Moreover, the wage-profit ratio rises in all sectors except the one with technical change, which implies that the aggregated wage-profit ratio in the economy as a whole, as the weighted average of the ratios in different sectors, could be either increasing or decreasing. For example, Franke (1999) considers the effect of the *ad hoc* technical changes with labor-saving in proportion in every sector, that is, $L^* = \alpha L$ where $\alpha < 1$, and shows that the aggregated wage share would fall along the balanced-growth path.

Finally, it should be noted that Proposition 4 holds for any viable technical change, including those that are not CU-LS. However, if we only focus on the viable CU-LS technical change, and assume that workers will choose the consumption bundle in a fixed proportion, that is, $b_i = \beta_i \bar{b}_i$ such that $w_i = pb_i$, then Proposition 3 implies the increase in real wage, $\beta_i^* > \beta_i$, and thus a falling rate of exploitation in each sector, $e_i^* < e_i, \forall i$, since the viable CU-LS technical change is progressive (Roemer, 1977).

Fixed Wage-Profit Ratio. If the technical change does not change the power relationship between capitalists and workers, then with the new technology the bargaining between the two classes should be able to find a new wage rate such that the wage-profit ratio remains the same. This existence of the new wage rate is shown in the following proposition.

Proposition 5. *Let (p, π, γ) be the long-run outcome of the economy $\mathcal{E}(A, L)$ with W. Consider a technical change from (A, L) to (A^*, L^*), then there exists a new diagonal matrix W^* such that (p^*, π^*, γ) is the long-run outcome of the economy $\mathcal{E}(A^*, L^*)$ with W^*.*

Proof. By (4) we have

$$p = p[(1+\pi)A + \pi\Gamma] \leq p[(1+\pi)A^* + \pi\Gamma] \tag{6}$$

then by Perron-Frobenius Theorem, there exists (p^*, π^*) such that

$$p^* = p^*[(1+\pi^*)A^* + \pi^*\Gamma]$$

and define W^* by

$$w_j^* = \frac{\gamma_j \pi^* p^* A^{*j}}{(1-\gamma_j \pi^*)L_j^*}, \forall j$$

Then (p^*, π^*, γ) is the long-run outcome of the economy $\mathcal{E}(A^*, L^*)$ with W^*. □

With the existence of the long-run outcome with constant wage-profit ratio after technical change established in Proposition 5, we could assume a fixed wage-profit ratio γ to explore the effect of technical change on profitability.

Theorem 1. *Let (p, π, γ) be the long-run outcome of the economy $\mathcal{E}(A, L)$ with W. Consider a viable CU-LS technical change from (A, L) to (A^*, L^*) and let (p^*, π^*, γ) be the new long-run outcome of the economy $\mathcal{E}(A^*, L^*)$ with W^*. Then $\pi^* < \pi$ and $q_i^* > q_i$.*

Proof. Let the matrix $M(\pi) = [(1+\pi)A^* + \pi\Gamma]$ and $\rho(\pi)$ be its Frobenius root. Then $\rho(\pi)$ are increasing since $M(\pi)$ is indecomposable and increasing in π. By (6), we have

$$p \leq p[(1+\pi)A^* + \pi\Gamma] \Rightarrow p \leq pM(\pi) \Rightarrow \rho(\pi) > 1$$

and

$$p^* = p^*[(1+\pi^*)A^* + \pi^*\Gamma] \Rightarrow p^* = p^*M(\pi^*) \Rightarrow \rho(\pi^*) = 1$$

Therefore we have $\pi^* < \pi$, and then $q_i^* = 1 - \pi^*\gamma_i > 1 - \pi\gamma_i = q_i$. □

Note that we obtain a falling profit-rate by allowing real wage to rise, which seems trivial since the Okishio's Theorem has already implied that "unless the rate of real wages rises sufficiently, the technological innovations adopted by capitalist do not reduce the general rate of profit" (Okishio, 1961, p. 95). Moreover, it has been argued that when constructing a theory of falling profit-rate due to rising OCC we can not relax the assumption of the fixed real wage; otherwise the falling profit rate is not a consequence of the rising OCC but the result of the increasing working-class power, which falls into the realm of the profit-squeeze approach (van Parijs, 1980, p. 4). However, Theorem 1 shows that in order to observe a falling profit-rate after the technical change, real wage need not rise too much to squeeze profit but just grow sufficiently to maintain the wage-profit ratio. Therefore, it is possible to have a consistent theory of falling profit-rate due to the rising OCC under the condition of competitive capitalism distinguished from the profit-squeeze argument.

Indeed, the intuition behind Theorem 1 is exactly Marx's insight on falling profit rate due to rising OCC, though he makes the argument in terms of value in an aggregated level (Marx, 1993, Ch 13). Following Sweezy (1942, p. 68), we can formulate Marx's argument as follows. If the rate of exploitation $e = s/v$ is fixed, and the organic composition of capital $q = c/(c+v)$ rises, then the general rate of profit r would fall since

$$r = \frac{s}{c+v} = \frac{s}{v}\frac{v}{c+v} = \frac{s}{v}(1 - \frac{c}{c+v}) = e(1-q)$$

Theorem 1 essentially translates this idea into the world with prices. The CU-LS technical change tends to reduce total wage relative to the total costs. Then by the constant wage-profit ratio, profit is also reduced relative to the total cost, and thus profit rate falls by definition. Therefore, Theorem 1 verifies Marx's insight on the tendency of the profit rate to fall.

4 Discussion

The main results on viable CU-LS technical change in the three benchmarks are summarized in Table 1. In the Okishio's scenario with fixed real wage, profit rate rises while the wage-profit ratio falls. If we fixed the profit rate, then the wage-profit ratio falls in the Sraffian case. In contrast, in the Marxian benchmark with fixed wage-profit ratio, profit rate falls. Moreover, as shown in the second column in Table 1, the OCC in sector i rises in all cases, which is not obvious since the change in q_i depends on the complicated behaviors of the relative prices (e.g., Bidard & Ehrbar, 2007; Dietzenbacher, 1988, 1989; Fujimoto et al., 1983, 1985).

By comparing the last two columns, we can see that in order to avoid a falling profit rate, the wage-profit ratio must fall either in the Sraffian case or the Okishio's scenario. Indeed, from the proof of Theorem 1 we have falling profit rate if the wage-profit ratio is not falling in any sector, that is, $\gamma^* \geqq \gamma$. Therefore, we can conclude that any capital-using and labor-saving technological innovations adopted by capitalists do not increase the rate of profit, unless they are power-biased against the working-class such that the wage-profit ratio can not be maintained. Technical change *itself* (that is, *in the absence* of power change) does bring about a falling rate of profit as the result of a rising OCC.[9]

Our result supports Marx's argument of the falling rate of profit due to a rising organic composition of capital in the sense that his argument is consistent and that there is a mechanism driving profit rate to fall in competitive capitalism. However, it does not mean that we must be at the Marxian scenario since technical change can be power-biased (Guy & Skott, 2008, 2015).

Figure 2: The three benchmarks, four regimes, and crisis tendencies. The axis represents the real wage after technical change. It is the same as before at Okishio's benchmark.

To make it clear, in Figure 2 we show along the axis of real wages (measured using some consumption bundle) the three benchmarks and different crisis tendencies[10]. The region on the left of the Sraffa's benchmark corresponds to the technological innovations that are so power-biased against the workers that wage-profit ratio falls and profit rate rises. Real wage grows slightly (regime II) or even falls (regime I) as observed in the early industrial revolution (e.g., Allen, 2019; Cvrcek, 2013; Humphries & Schneider, 2019). And thus these two regimes are associated with the crisis tendency called underconsumption or realization problem. For those technological innovations on the right of the Sraffian benchmark, the profit rate falls due to the rising OCC. Even though real wage is rising, the wage-profit ratio could fall (regime III). While in regime IV in

[9] In the line of Okishio's setting, Roemer correctly refutes the claim that "Technical change *itself* (that is, *in the absence* of real wage change) can bring about a falling rate of profit in consequence of a rising OCC" (Roemer, 1979, p. 385).

[10] On different crisis tendencies in Marxian economics, see Basu (2018) for a recent review.

Table 1: Effects of viable CU-LS technical change in sector i on income distribution and profitability.

	Composition of Capital (q_i)	Wage-Profit Ratio (γ_i)	Profit Rate (π)
Okishio's scenario	↑	↓	↑
Sraffian scenario	↑	↓	—
Marxian scenario	↑	—	↓

the right of the Marxian benchmark, technical innovation with the power change pro working-class leads to a rising wage-profit ratio and a falling profit rate by profit squeeze superposed on the effect of rising OCC.

Moreover, our results are just a theoretical analysis of the consequences of technical change isolated from other factors that would influence the income distribution and profitability such as demographic, institutional and political factors (Acemoglu & Robinson, 2012, 2015; Bowles, 2012). It remains an empirical question to study the actual regime of any particular period of capitalism (Basu & Manolakos, 2013; Basu & Vasudevan, 2013; Duménil & Lévy, 2016, 2018; Kotz & Basu, 2019).

Finally, note that the driving force to bring about a falling rate of profit due to the rising OCC exists not just in the Marxian scenario in which technical change is power-neutral but in all cases. The assumption of a fixed wage-profit ratio is just used to isolate the effect of rising OCC from that of the change in power associated with the technical change. Indeed, the effect of rising OCC is countered by power change in Okishio's scenario while strengthened in the profit-squeeze case.

5 Concluding Remarks

In this paper, we study the effects of viable capital-using and labor-saving technical change on income distribution and profitability, by considering three benchmarks with fixed real wage, constant profit rate, or unchanged distributional situation. We show any capital-using and labor-saving technological innovations adopted by capitalists do not increase the rate of profit unless they are power-biased against the working-class such that the wage-profit ratio can not be maintained. Technical change in the absence of power change would bring about a falling rate of profit due to rising OCC.

For further study, we could investigate the time series of the real wage, profit rate, and wage-profit ratio, to identify different regimes and the regime shifts to examine whether the actual dynamics of capitalism corresponds to the four regimes(**Hamilton2016**).

In addition to the empirical study on the identification of regimes and regime shifts, there are several possible extensions to our study. First, as in Okishio (1961)'s and Roemer (1978)'s setting, we could consider the economy with two departments — one for consumption goods, the other for capital goods, and assume that the consumption goods are not involved as inputs in the production of capital goods. Mathematically, it just means that we relax the assumption on indecomposability and therefore only weak inequality holds. For example, in the Marxian scenario with fixed wage-profit ratio, we have profit rate does not rise after CU-LS technical change, $\pi^* \leq \pi$. Specifically, if the technical change occurs in the department of capital goods, then the profit rate falls, while if it occurs in the department of consumption goods, then the profit rate remains the same.

Also, instead of focusing on the economy with single production and pure circulating capital, we could consider the economy with joint production. In general, it is also possible to have a falling profit rate with fixed real wage and joint production (Salvadori, 1981; Woods, 1984), while the Okishio's theorem can be extended to the case with pure fixed capital or joint production within the von Neumann's framework (Nakatani, 1980; Roemer, 1979; Woods, 1985). Indeed, the crucial assumption to generalize the Okishio's theorem is the existence of a positive standard commodity (Bidard, 1988). With this assumption, whether the argument on falling profit rate due to rising OCC could be generalized remains an open question[11].

Moreover, the theoretical analysis of distribution and profitability is incomplete in the sense that it leaves out the impacts of technical change on the labor market in the process of accumulation. In particular, further study could incorporate the effects of (i) the increase in the demand for labor by capital accumulation; (ii) the increase in the supply of labor, or the so-called "relative surplus population or industrial reserve army" (Marx, 1992, p. 781), generated by the capital-using and labor-saving technical change.

Finally, the model could be extended to capture the endogenous evolution of technology in two steps. First, technical innovation resulted from R&D could be modeled as a random process following Duménil and Lévy (1995, 2003) but augmented with the bargaining power, (A, L, α) where α is the bargaining power of capitalist in the Nash bargaining. After the innovations randomly emerged, the farsighted capitalist would adopt the new technique taking into account not just the cost but also the change in power. Furthermore, assume R&D investments endogenously respond to wage share (Zamparelli, 2015), then capital accumulation with tightening labor market would induce capital-using and labor-saving technological innovations.

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