Capital inflows, sustained investment surges, and the role of external economies of scale in a developing economy

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Abstract

Standard open economy macro models with unemployment predict a contractionary short-run effect of international capital inflows. Empirical evidence, on the other hand, often associates such inflows with short-term booms, and developing country policy makers frequently go out of their way to welcome foreign capital. Employing a portfolio balance framework, this paper distinguishes between international financial (i.e., bond) and “real” (i.e., equity) flows to explore the different consequences for capital accumulation that may follow over the medium run. The presence of external economies of scale generates multiple equilibria, and different kinds of capital flows may push investment in one direction or the other for sustained periods of time.

JEL classifications: F21, F32, F43, O11,

Key words: Capital flows; economies of scale; investment surges; real exchange rate.

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1 Introduction and Motivation

Are foreign capital inflows good or bad for developing economies? This question has received much attention over the last few decades as financial liberalization has led to increasing volumes of international debt and equity flows. This paper focuses on the effect of different kinds of capital flows on investment in the short run and over time. I find that, in the presence of external economies of scale, and given sufficient responsiveness of investment to profitability conditions, equity inflows are much more likely than bond inflows to result in sustained surges of investment that lead to a higher steady state capital stock. Moreover, potential balance of payments problems are less likely to arise from equity inflows. These results have implications for the theory and practice of international capital controls, and for the management of the “Dutch disease” problem.

What does the basic workhorse fixed (or sticky) price open economy Mundell-Fleming model have to say about the consequences of capital inflows? In a single country set-up, and with a flexible exchange rate and an exogenously determined supply of money, capital inflows lead to a real appreciation and reduced output as the tradable sector shrinks following declining competitiveness. Thus, capital inflows, at least in the short run, result in lower employment and income. Literature originating from the field of development economics points out the trade-offs between possible technological advancement on the one hand and Dutch disease-related concerns on the other.

Policy makers, by contrast, largely tend to be enthusiastic about capital inflows, and often go out of their way to attract them, both to relieve balance of payments-related pressures and for longer-run economic growth and efficiency. While memories of speculative attacks and currency crisis may lead to calls for caution, on the whole capital inflows are often seen as salutary and harbingers of better days. This enthusiasm is not unwarranted: Reinhart and Reinhart (2008) document capital inflow “bonanzas,” and find that, in developing countries these are associated with economic booms albeit ones accompanied by procyclical fiscal policies, and currency appreciation.

Crucial to these debates is the issue described by Corden (1994)(p. 8) as the “real appreciation problem,” that is the tendency of capital inflows to cause private sector booms and appreciate the real exchange rate, negatively impacting tradable sector output and generating current account deficits. While it is easy to understand why capital inflows lead to appreciation, there are other important channels through which inflows could affect real economy variables over the short-to-medium-run, and different kinds of capital flows could shape these channels differently.

In sum, the conflicting consequences of capital inflows for growth and investment have been much debated, leading Reinhart and Reinhart (1998) to describe these as a “mixed blessing.” This empirical and theoretical ambivalence about the effects of capital inflows has led some recent contributions to distinguish between different kinds of capital flows such as equity versus bond

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1See, for example, Prasad et al. (2007) and Reinhart and Reinhart (2008).
inflows, short-term versus long-term inflows, etc. For example, Blanchard and Chamon (2016) recently showed, in a short-run fix price framework that the effects of capital flows on output may depend on whether these are bond flows or equity flows. While both kinds of inflows result in real appreciation, the former lead to higher returns to equity while the latter have the opposite effect. Their econometric analysis, based on a sample of 181 countries, finds that while bond flows have a negative effect on annual GDP growth, non-bond flows have a positive effect. Empirical evidence on the determinants of sustained investment accelerations provided by Libman et al. (2019) suggests that while other kinds of financial inflows reduce the probability of a country experiencing an episode of high investment, FDI inflows may be an exception. The study also finds, based on nearly 190 episodes of sustained investment accelerations across the world,\footnote{That is investment surges that last at least 8 years.} that while the typical episode is initially accompanied by external account deficits, such deficits tend to vanish in their later stages. The present paper helps explain this empirical evidence.

I approach the issue by analyzing a model that incorporates interactions between financial markets and the real sector. The financial markets, modelled using a portfolio balance framework, trade domestic and foreign bonds, domestic equity, and money. The goods sector consists of two sub-sectors, one that produces a non-tradable good, while the other produces a tradable good, the price of which is determined in international markets. The economy, therefore has a dependent economy flavor with surplus labor in the Arthur Lewis sense.\footnote{See Lewis (1954).}

In line with Rodrik (2008) and Skott and Ros (1997), the tradable manufacturing sector is “special” in the sense it is the locus of capital accumulation and is characterized by external economies of scale (due to Arrow type “learning-by-doing” or other externalities). The presence of increasing returns to scale gives rise to multiple equilibria and different kinds of capital inflows may facilitate or hinder movement towards a high capital stock equilibrium. Most importantly, while both kinds of capital inflows generate Dutch disease-like currency appreciation, only equity inflows set in motion a compensating mechanism in the shape of lower cost of issuing equity that could – given adequate investment responsiveness – set off an episode of sustained accumulation culminating in a higher steady state capital stock.

In its combination of the portfolio balance model with the real side of the economy and the focus on distinguishing between equity and bond inflows the present paper is close to Blanchard and Chamon (2016). However, that paper focuses on the short-run in a one good economy. Flows at a given point in time naturally turn into stock changes over time, which has consequences beyond the short-run. I analyze these medium-run consequences. Moreover, unlike Blanchard and Chamon (2016), I focus on development issues by introducing surplus labor, distinguishing between the tradable/modern sector and the non-tradable one, incorporating capital accumulation and economies of scale in the former, and including balance of payment issues. The emphasis on increasing
returns to scale and sectoral differences characterizes much of traditional development theory as well as the writings of heterodox economists like Kaldor and the ‘structuralist’ school. More recently ‘new’ growth theory has rediscovered the potential significance of increasing returns.

In the structuralist/Post Keynesian theoretical tradition, some studies such as Burgstaller and Savendra-Rivano (1984), Burgstaller and Savendra-Rivano (1984), Dutt (1997) and Dutt (1998) have analyzed the effects of foreign direct investment flows on the real economy. These papers do not, however, incorporate a financial sector or the tradable/non-tradable good distinction. The present paper, on the other hand, can be seen as one where asset return/price movements provide the primum mobile that sets crucial mechanisms into motion.

The next section lays out the model, developing both the financial and good markets, and analyzing the short-run comparative static properties with the help of relevant thought experiments. Section 3 carries out the dynamic analysis, comparing the effects of equity and bond inflows on financial and real variables, and exploring the evolution of the capital stock and the balance of payments over time and in the presence of multiple equilibria. Section 4 concludes.

2 The Theoretical Framework

This section develops the basic framework and integrates the asset and goods markets to build the foundations for the later dynamic analysis.

2.1 The Financial Sector

Consider a financial sector with three domestic assets: (1) money ($M$), (2) domestic bonds ($B$), and (3) equity, i.e., claims on physical capital ($K$). One unit of equity is issued to purchase each machine. Firms issue claims on real capital, while owners of wealth employ their savings ($S$) to hold this equity. All commodity prices are fixed. Money pays no nominal return while the returns to domestic bonds and equity are denoted by $r$ and $r_K$, respectively. In addition, asset owners hold foreign bonds ($B^*$), the returns to which are determined in international markets and represented by $r^*$. All assets are imperfect substitutes for each other. Money is only held domestically while bonds and equity are held both domestically and abroad. Subscripts $d$ and $f$ denote domestic and foreign ownership, respectively.

I assume that there is no transactions demand for money, and the central bank can fix demand for money by setting the interest rate. Also, in order to keep things as simple as possible while deriving the comparative static relationships that typically emerge from the portfolio balance framework, I assume static expectations regarding both the exchange rate and the return to holding equity. Bonds are short term so that their capital value is essentially independent of the interest rate. Under these conditions, the asset demand functions for domestic asset holders can be written as:
\[ M = -\alpha r \quad (1) \]

\[ B_d = \left[ b_0 + \beta_1 (r - r_K) + \beta_2 \left( r - r^* - \frac{1}{e} + \sigma_B \right) \right] (W - M) \quad (2) \]

\[ K_d = \left[ k_0 + \beta_1 (r_K - r) + \beta_3 \left( r_K - r^* - \frac{1}{e} + \sigma_K \right) \right] (W - M) \quad (3) \]

\[ eB_d^* = \left[ f_0 + \beta_2 \left( r^* + \frac{1}{e} - r - \sigma_B \right) + \beta_3 \left( r^* + \frac{1}{e} - r_K - \sigma_K \right) \right] (W - M) \quad (4) \]

where \( b_0, k_0, f_0 \), and \( \beta_i \) (\( i = 1, 2, 3 \)) are parameters, \( e \) is the nominal and, with fixed prices normalized to unity, real exchange rate, \( \sigma_B \) and \( \sigma_K \) are demand shift parameters, and the subscript \( d \) indicates that the assets are domestically owned. The \( \beta \) parameters capture the relevant degree of asset substitutabilities. The real exchange rate is measured as the price of foreign goods relative to domestic ones. All asset stocks are expressed in real terms. I have ignored capital gains or losses on equity, although including these will generally not have any qualitative effect on the analysis.\(^4\) The return to holding foreign bonds is determined in the international market and domestic conditions play no role in determining equilibrium values in that market. Total domestic wealth is defined in the standard manner.

\[ W \equiv M + B_d + eB_d^* + K_d \quad (5) \]

Notice that the budget constraint requires that \( b_0 + k_0 + f_0 = 1 \). As in the standard portfolio specification, asset holdings are homogeneous in wealth. Apart from the introduction of (the more standard) non-linear demand functions, and the absence of the assumption that all assets are equally substitutable, the specifications are quite similar, up until this point, to Blanchard and Chamon (2016). One more departure, detailed next, is required to set the stage for the later dynamic analysis in Section 3.

Foreign holdings of the two non-monetary domestic assets, indicated by the subscript \( f \), are a constant proportion of the respective domestic holdings.

\[ B_f = \psi B_d \quad (6) \]

\[ K_f = \mu K_d \quad (7) \]

\(^4\)Suppose investors expect a steady stream of dividends, denoted by \( \pi^e \) over the lifetime of the equities. With static expectations and an infinite time horizon, the capital gains will be captured by the term \( \sigma eK_d \) instead of \( K_d \) on the left hand side of equation (3) and in the wealth term on the right hand side of eqs. (1)-(4). Incorporating this term does not qualitatively affect the comparative static results for equilibrium \( r_K \) or \( e \), with one exception. The sign of \( de/d\psi \) becomes ambiguous.
The underlying motivation here is to ensure the analytical tractability of the later dynamic analysis. With foreign asset holding tied to the domestic one, I only need to keep track of the total stock of individual assets, and not their distribution among domestic and foreign asset holders.

Equation (4) provides one market clearing condition. The conditions for the other two assets are as follows. Starting with the equity market,

\[ K = K_d + K_f \]

Or, substituting from eqs. (3), (7), and (5), and letting \( \Lambda \equiv k_0 + \beta_1 (r_K - r) + \beta_3 (r_K - r^* - \frac{1}{e} + \sigma_K) \), \( \Gamma \equiv b_0 + \beta_1 (r - r_K) + \beta_2 (r - r^* - \frac{1}{e} + \sigma_B) \), and \( \Theta \equiv f_0 + \beta_2 (r^* + \frac{1}{e} - r - \sigma_B) + \beta_3 (r^* + \frac{1}{e} - r_K - \sigma_K) \) denote the initial share of wealth dedicated to holding equity, domestic bonds, and foreign bonds,

\[
\left[ k_0 + \beta_1 (r_K - r) + \beta_3 \left( r_K - r^* - \frac{1}{e} + \sigma_K \right) \right] (W - M) - \frac{1}{1 + \mu} K = 0
\]

Moving next to the domestic bond market:

\[ B = B_d + B_f \]

Or, substituting from eqs. (2), (6), and (5),

\[
\left[ b_0 + \beta_1 (r - r_K) + \beta_2 \left( r - r^* - \frac{1}{e} + \sigma_B \right) \right] (W - M) - \frac{1}{1 + \psi} B = 0
\]

The system consists of three asset market clearing conditions. One of these is redundant by Walras’s law. Any 2 of these equations can, therefore, be used to solve for comparative static changes in \( r_K \) and \( e \) (recall that the central bank sets the interest rate). I use the domestic bond and equity market clearing conditions – eqs. (8) and (9) – to derive the comparative static solutions. Throughout, I’ll plausibly assume that: (1) domestic assets are better substitutes for each other than for foreign bonds, and that foreign bonds are closer substitutes for domestic bonds than for equity, i.e., \( \beta_1 > \beta_2 > \beta_3 \), and (2) that, as a result,

\[ \beta_1 \Theta, \beta_2 \Lambda > \beta_3 \Gamma \]  

(10)

This assumption will help resolve sign ambiguity for two partials below. Let’s next turn to relevant comparative static thought experiments.

**Increased availability of foreign bonds**

Increased volumes of foreign bonds available to domestic residents means an excess supply of such bonds, and as a result, greater
demand for domestic assets. The real exchange must appreciate, and given the satisfaction of condition (10), the returns to equity must fall, in order to divert demand away from domestic assets and towards foreign bonds. In formal terms,

$$\frac{dr_K}{dB^*_d} = -\frac{\beta_2 \Lambda - \beta_3 \Gamma}{\Delta_1} \frac{W - M}{e} < 0$$ \hspace{1cm} (11)$$

$$\frac{de}{dB^*_d} = -\frac{\beta_1 (1 - \Theta) + \beta_3 \Gamma}{\Delta_1} e(W - M) < 0$$ \hspace{1cm} (12)$$

where $\Delta_1 = \{(\beta_1 + \beta_3) \beta_2 + \beta_1 \beta_3 \frac{W - M}{e} + [(\beta_1 + \beta_3) \Gamma + \beta_1 \Lambda] B^*_d \} (W - M) > 0$.

The real exchange rate unambiguously appreciates. Intuitively, an increase in the supply of foreign bonds requires a decline in the relative returns to equity and a real appreciation to shift demand towards foreign bonds and remove their excess supply.

**Increased supply of equity**

If, by contrast, it is domestic equity that becomes available in greater volumes, then the effect on returns to equity is positive. A higher level of $r_K$ and a real depreciation are required to clear the equity market and remove the excess demand for other assets that increased wealth generates. In mathematical terms,

$$\frac{dr_K}{dK} = \frac{[\beta_2 (1 - \Lambda) + \beta_3 \Gamma] \frac{W - M}{e} + \Gamma B^*_d}{\Delta_1} \frac{1}{1 + \mu} > 0$$ \hspace{1cm} (13)$$

$$\frac{de}{dK} = \frac{\beta_1 \Theta - \beta_3 \Gamma (W - M)}{\Delta_1} \frac{1}{1 + \mu} > 0$$ \hspace{1cm} (14)$$

Signing the comparative static for $e$ requires satisfaction of condition (10).

**Shift in foreign preferences toward domestic equity**

Suppose foreign investors develop a stronger desire to hold domestic equity. This creates excess demand for equity, lowering returns. The lower $r_K$, along with real appreciation, helps clear the market by diverting demand to other assets.

$$\frac{dr_K}{d\mu} = -\frac{[\beta_2 (1 - \Lambda) + \beta_3 \Gamma] \frac{W - M}{e} + \Gamma B^*_d}{\Delta_1} \frac{K}{(1 + \mu)^2} < 0$$ \hspace{1cm} (15)$$

$$\frac{de}{d\mu} = -\frac{(\Theta \beta_1 - \beta_3 \Gamma) (W - M)}{\Delta_1} \frac{K}{(1 + \mu)^2} < 0$$ \hspace{1cm} (16)$$

**Shift in foreign preferences toward domestic bonds**

Greater foreign preference for domestic bonds has the opposite effect on $r_K$ to when the preference shift is toward equity. This is because the excess demand
for bonds now requires a rise in \( r_K \) to clear the market. The effect on the real exchange rate, i.e., a real appreciation, is qualitatively similar in the two cases.

\[
\frac{dr_K}{d\psi} = \frac{[(\Delta \beta_2 + \beta_3(1 - \Gamma))\frac{W-M}{e} + \Lambda B_d^*}{(1 + \psi)^2} B > 0 \tag{17}
\]

\[
\frac{de}{d\psi} = \frac{[(\beta_1 \Theta + (1 - \Gamma)\beta_3)(W - M)\frac{B}{(1 + \psi)^2}} < 0 \tag{18}
\]

Two key results from this section need to be highlighted here as these play an important role later. One is that, though both kinds of shifts in foreign demand towards domestic assets generate real appreciation – an outcome that has received considerable attention both from policy makers and researchers – the effect on the returns to equity, and hence on investment (as defined below) is quite different. Second, accumulation of foreign bonds and domestic equity have qualitatively contrasting consequences for returns to equity and the real exchange rate under our assumptions.

In order to make the analysis more transparent, subsequent analysis proceeds by representing the comparative static results derived from the asset markets, i.e., eqs. (11)-(18), in linear form. Thus,

\[
r_K = r_0 + r_1K - r_2B_d^* - r_3\mu + r_4\psi; \quad r_i > 0, i = 0, 1, 2, 3, 4 \tag{19}
\]

and,

\[
e = e_0 + e_1K - e_2B_d^* - e_3\mu - e_4\psi; \quad e_i > 0, i = 0, 1, 2, 3, 4 \tag{20}
\]

### 2.2 The Goods Market

I consider a small, open developing economy with two sectors: a modern industrial sector that produces a tradable good represented by \( T \), while the output of the traditional sector, represented by \( N \), is non-tradable. The price of the tradable good, \( P_T \), is internationally given and normalized to unity. Production in the tradable sector utilizes capital and labor, and is subject to external economies of scale. Individual firms are price takers in the labor market, and the real product wage \( w_T \) (\( \equiv W_T / eP_T \)) is fixed. Following a general specification common in the new growth theory, the production function in the traded goods sector is given by a standard Cobb-Douglas function that incorporates the existence of increasing returns to scale at the sectoral level. Thus, for the representative firm in the traded goods sector:

\[
Y_T = A_T \bar{K}^\gamma K^\alpha L_T^{1-\alpha} \tag{21}
\]

where \( Y_T \) is the output of the tradable good, \( K \) and \( L_T \) represent capital and labor in the tradable sector, and \( A_T \) is the exogenously given level of technology. The economy-wide average capital stock, \( \bar{K} \), equals the capital stock of the representative firm in equilibrium, and the parameter \( \gamma \) captures the presence of external, sector-level, economies of scale. For reasons that will become
clear in Section 3, the existence of a well-behaved dynamic system requires that \( \gamma < \alpha \), i.e., there is an upper bound on the degree of external economies. With firms acting as wage-takers, the level of employment is given by competitive conditions:

\[
L_T = \left[ \frac{(1 - \alpha)A_T}{w_T} \right]^{\frac{1}{\alpha}} K^{\frac{\gamma - \alpha}{\alpha}}
\]  
(22)

Plugging this level of employment back into equation (21) yields:

\[
Y_T = f K^{\frac{\gamma + \alpha}{\alpha}}
\]

where \( f = \left( \frac{1 - \alpha}{w_T} \right)^{\frac{1 - \alpha}{\alpha}} A_T^{\frac{1}{\alpha}} \) to avoid clutter. Total profits, \( R_T \), in terms of the non-tradable good, can be derived from eqs. (22) and (23) directly:

\[
R_T = eY_T - w_T L_T = e\alpha f K^{\frac{\gamma + \alpha}{\alpha}}
\]

(24)

The output of non-tradables, \( Y_N \), employs a fixed factor (land) and labor, and is subject to diminishing returns to labor.

\[
Y_N = A_N L_N^\delta; \; \delta < 1
\]

(25)

The functional distribution is determined by factor productivity at the margins. The real wage in terms of non-tradables, \( w_N \), and total profits, \( R_N \), can be derived as follows:

\[
w_N = \delta A_N L_N^{\delta - 1}
\]

(26)

\[
R_N = (1 - \delta) A_N L_N^\delta
\]

(27)

It may be useful to point out here that there is no assumption of wage equalization between the two sectors. Although a dramatic simplification, it can be justified by the short to medium-run nature of the analysis here and is further mitigated by the consideration that the two sectors may require different kinds of labor, especially in developing economies where the distinction between the modern/industrial/tradable sector – with wage bargaining and efficiency wages – and the traditional, largely non-tradable informal sector gives rise to dual labor markets. Forces for wage equalization may also be absent or weak in the short run due to legal barriers such as the Chinese hukou system. In any event, my focus here is limited to the interaction between different kinds of asset flows and the goods market over the short- to medium-run. Distributional issues, although interesting, are of tangential concern here.

In order to specify an equilibrium condition for the non-tradable sector, we need to define consumption behavior. Assuming a Cobb-Douglas utility
function, and denoting the consumption share of tradables with $\varepsilon$, and the respective consumption levels by $C_N$ and $C_T$,\(^5\)

$$\frac{C_N}{C_T} = \frac{1 - \varepsilon}{\varepsilon} e \quad (28)$$

We are now in a position to solve for the level of non-tradable output that clears that sector. The non-tradable output is used for consumption only so that the equilibrium condition is given by

$$Y_N = C_N$$

or, from equation (25) and the definition of sectoral factor incomes:

$$A_N L_N^\delta = (1 - \varepsilon)(1 - s)(w_N L_N + R_N + e w_T L_T + R_T) \quad (29)$$

where $s$ denotes the saving rate out of income, while the right hand side expresses the demand for the non-tradable good. As suggested by Metzler (1951), it makes sense in a portfolio set-up to assume that savers have a target level of wealth, and that the propensity to save out of current income will vary negatively with current wealth. Thus,

$$s = s(W); \ s' < 0 \quad (30)$$

Models in the Post Keynesian tradition often assume differences in saving behavior between functional income groups. I abstract away from those differences since again, these are of tangential interest here, although I return to this issue when relevant.

Substituting from eqs. (22), (24), (26), and (27) into equation (29), and normalizing all quantity variables by the level of the capital stock, $K$,

$$\frac{A_N L_N^\delta}{K} = \frac{(1 - \varepsilon)(1 - s)}{1 - (1 - \varepsilon)(1 - s)} e F K^\gamma \quad (31)$$

Non-tradable output is demand-led. It is decreasing in the saving rate and increasing in the real exchange rate (since an increase in $e$, i.e., a rise in the relative price of tradables leads to substitution toward non-tradables). Also, due to the demand generated by the tradable sector expansion, non-tradable output increases with an increase in the capital stock and a decline in the real product wage $w_T$. This latter feature will be mitigated, but not eliminated, if we introduce a higher saving rate for owners of capital relative to workers. Finally, notice that, in the absence of external economies of scale, i.e., if $\gamma = 0$, expansion of the capital stock will leave normalized tradable output unchanged (except for

\(^5\)That is, specifying the maximization problem as follows:

$$\max_{C_T, C_N} U(C_N, C_T) = C_T^\gamma C_N^{1-\varepsilon}$$

subject to

$$C_N + e C_T = Y$$

where $Y$ stands for real income in terms of non-tradables at any point in time.
through the wealth effect on savings). The capital stock term explicitly appears on the right hand side entirely due to the presence of increasing returns.

The equilibrium in the tradable sector is defined by the price of tradables being internationally given. The trade balance passively reflects the difference between output and expenditure in the tradable sector. I specify an independent investment function in line with the Keynesian-Kaleckian family of models. Investment, $I$, is affected positively by the profit rate, $r_T$, and negatively by the cost of issuing equity, which is also the real return to equity, $r_K$.\footnote{One could also add the interest rate on bonds as an argument. However, given our assumption that this interest rate is exogenously set by the central bank, it is not of much interest here and will not affect the analysis.}

\[
\frac{I}{K} = \dot{K} = \pi(r_T - r_K) = \pi \left( \frac{R_T}{cK} - r_K \right) \\
= \pi \left( \alpha f K \ddot{z} - r_K \right)
\]

where $\pi$ is a parameter capturing the speed of adjustment or the sensitivity of investment to the gap between the profit rate and the cost of issuing equity, hats or circumflexes over variables denote the growth rates of the associated variables, and the second line makes use of equation (24). The growth of the capital stock is declining in the real product wage and increasing, thanks to economies of scale, in the capital stock. Anything that affects the returns to equity (see section 2.1) also impacts investment. A relevant example in our case would be a shift in foreign asset preferences between equity and bonds that lead to capital inflows of one type or the other.

Based on equations (28) and (31), it is now straightforward to derive reduced form expressions for the consumption of tradables and the gap between this variable and output of tradables (both normalized by $K$).

\[
\frac{C_T}{K} = \frac{\varepsilon(1 - s)}{1 - (1 - \varepsilon)(1 - s)} f K \ddot{z} 
\]

\[
\frac{Y_T}{K} - \frac{C_T}{K} = \frac{s}{1 - (1 - \varepsilon)(1 - s)} f K \ddot{z}
\]

The output-consumption gap is increasing in the saving rate, the capital stock, and the level of technology, and declining in the real product wage. Again, the variable $K$ owes its explicit presence on the right hand side to external economies of scale.

Finally, the trade balance (as a proportion of the capital stock) is given by:

\[
\frac{TB}{K} = \frac{Y_T}{K} - \frac{C_T}{K} - \frac{I}{K}
\]

so that, incorporating investment income yields the current account ($CA$):
The current account, after setting the exogenous international interest rate to unity, and substitution from eqs. (6), (7), (32), and (34), can be expressed after some manipulation in the following form:

\[
\frac{CA}{K} = -\frac{FA}{K} = \frac{Y_T}{K} - \frac{C_T}{K} - \frac{I}{K} - r_fK_f - r_f\frac{B_f}{K} + \varepsilon\frac{B_d^*}{K} \quad (36)
\]

where the term in the square brackets on the right hand side is the gap between the investment and saving rates when the cost of issuing equity is zero. This gap is likely to be positive at that low cost of issuing equity.

A higher level of capital stock increases both saving, and via the profit rate, investment. The current account is a positive function of the saving rate and the real exchange rate, while the sign on the return to equity is ambiguous. A positive effect requires that investment be sufficiently sensitive to the cost of equity, i.e., \( r > \frac{\mu}{1+\mu} \). I assume investment to be sufficiently responsive here and throughout the reminder of the analysis. Intuitively, an increase in \( r \) lowers investment, thus working to create a current account surplus, but also raises equity remittances by foreign owners, which has the opposite effect on the current account. Assuming \( r > \frac{\mu}{1+\mu} \) means assuming that the investment effect dominates the profit repatriation effect.

One may note here that the level of the real exchange rate only affects the current account balance through inward remittances. This will change if I keep the real wage fixed in terms of a composite basket of (tradable and non-tradable) goods so that the exchange rate directly affects distribution, profitability, and investment. Such a change will, however, make the analysis more complicated while adding little of direct interest.

Rather than assuming balanced trade in the short run, I specify the more plausible assumption in the next section that the current account is balanced in the steady state.

2.3 More short-run effects of changes in asset preferences and supplies

This may be a good point to pause and take a broader look at the structure of the short-run set-up developed so far. Section 2.1 developed the asset market structure. The central bank sets the interest rate while the real exchange rate and returns to equity are determined by the relative supply of and demand for various assets. For example, section 2.1 tells us that a shift in investor preference towards claims on capital lowers returns to equity while a similar shift towards domestic bonds has the opposite effect. It also tells us that both shifts lead to real appreciation. Once these variables have been determined in the asset markets, the currency account and the level of investment are determined in the
goods market by these variables in conjunction with profitability, as specified in section 2.2. The level of profitability, in turn, is determined by the level of the capital stock, among other variables, thanks to economies of scale.

Let’s turn next to some more comparative statics to pull things together and lay the foundation for the dynamic analysis in the next section. For the purpose of this comparative static analysis section only, I will set the initial values of the asset stocks so that $K = B = B_d^*$.  

2.3.1 A shift in preferences towards domestic equity

To continue the exploration begun in section 2.1, consider the effect of a rise in $\mu$, but now on investment and the current account. We need to utilize eqs. (19), (20), (32) and (37).

$$\frac{d(I/K)}{d\mu} = \pi r_3 > 0$$

and,

$$\frac{d(CA)}{d\mu} = -\left(\frac{\pi - \mu}{1 + \mu}\right) r_3 + \frac{1}{(1 + \mu)^2} r_K - e_3 < 0$$

Intuitively, an increase in $\mu$ lowers the cost of issuing equity thus raising investment, increases profit remittances at a given level of $r_K$, and also leads to real appreciation. Starting with a balanced current account, all three effects work to create a current account deficit.

2.3.2 A shift in preferences towards domestic bonds

Now let's consider the effect of an increase in demand for domestic bonds.

$$\frac{d(I/K)}{d\psi} = -\pi r_4 < 0$$

and,

$$\frac{d(CA)}{d\psi} = \left(\frac{\pi - \mu}{1 + \mu}\right) r_4 - \frac{r}{(1 + \psi)^2} - e_4 \geq 0$$

The reduced investment resulting from higher cost of equity on the one hand, and real appreciation and increased investment income outflows on the other, have opposing effects on the current account.

2.3.3 Increased supply of domestic equity

Recall from Section 2.1 that, in this case, the return to equity must rise and the real exchange rate has to depreciate in order to clear the market for equities. What about investment and the current account?
\[
\frac{d(I/K)}{dK} = \pi \left[ \gamma F K \frac{\gamma}{\alpha} - r_1 \right] \geq 0
\]

and,
\[
\frac{d(CA)}{dK} = -\left\{ \frac{\gamma}{\alpha} \left[ \frac{s}{1 - (1 - s)(1 - \varepsilon)} \right] - \frac{\varepsilon s'}{1 - (1 - s)(1 - \varepsilon)^2} F K \tilde{z} + \left( \pi - \frac{\mu}{1 + \mu} \right) r_1 + e_1 \right\} \geq 0
\]

where \( s' < 0 \) is the Metzler wealth effect on savings. Investment may rise or fall. This is because both the profit rate and the cost of issuing equity increase with \( K \) in the goods sector. To give a preview of the next section, the presence of economies of scale means that the net effect depends on the level of the capital stock. At low levels of \( K \), the effect of external economies will dominate and investment rises.\(^7\) As the capital stock increases, the positive effect on investment weakens and the overall sign turns negative.

The sign of the effect on the current account could be negative at low levels of capital stock, owing to the increase in investment caused by the presence of economies of scale. This effect is increasingly offset as the level of the capital stock rises and the real exchange rate depreciates. At sufficiently higher levels of the capital stock, the sign is positive.

### 2.3.4 Increased supply of foreign bonds

Greater availability of foreign bonds requires a fall in return to equity and an appreciation to clear that market. The former effect ensures higher investment which, combined with the appreciation and decline in saving due to the wealth effect result, in turn, in a current account deficit.

\[
\frac{d(I/K)}{dB_d^2} = \pi r_2 > 0
\]

and,
\[
\frac{d(CA)}{dB_d^2} = -\left[ -\frac{\varepsilon s'}{1 - (1 - s)(1 - \varepsilon)^2} F K \tilde{z} + \left( \pi - \frac{\mu}{1 + \mu} \right) r_2 \right] - e_2 < 0
\]

The next section builds on these short-run comparative static outcomes to analyze the dynamics of the capital stock and the current account.

\(^{7}\) At \( K = 0 \), the positive profitability effect becomes infinite.
3 Accumulation and external account evolution over time

The exchange rate, returns to equity, non-tradable sector employment, consumption, savings, investment, and the current account are endogenous in the short-run while the asset stocks are pre-determined variables that evolve gradually over time. The analysis in the previous section helped lay the foundations for exploring the dynamics of capital stock and net international investment position evolution. To maintain analytical tractability, I assume that the stock of domestic bonds is given over the time period under consideration. This, along with the assumption of constant goods prices and tradable sector wages makes the analysis more plausibly “medium-run” rather than long-run in nature.

The equation for the evolution of the capital stock, i.e., equation (32) has already been discussed. Another equation of motion follows from the expression for the current account, but needs to be derived in a bit more detail since we did not consider the gradual evolution of asset stocks in Section 2.3. The balance of payments (BP) identity with a flexible exchange rate implies that any current account imbalance must be offset by a financial account imbalance in the opposite direction. Utilizing the current account expression from equation (37), this yields:

\[
\frac{s}{1 - (1 - \varepsilon)(1 - s)} F K \frac{dz}{dt} - \pi \left( \alpha F K \frac{dz}{dt} - r_K \right) - r_K \frac{\mu}{1 + \mu} K - \frac{r}{1 + \psi} B + eB_d^* \\
= \dot{B}_d - K_f - \dot{B}_f
\]

(38)

The left and right hand sides of the equation represent the current account and the (negative of the) financial account, respectively, and dots over variables represent time derivatives. Employing eqs. (6) and (7) leads, after some manipulation, to:

\[
\dot{B}_d = \left[ -\Omega F K \frac{dz}{dt} + \frac{\pi - \mu}{1 + \mu} r_K - r \frac{\psi}{1 + \psi} B + eB_d^* \right] K
\]

(39)

where \( \Omega = \left[ \frac{\pi \alpha}{1 + \mu} - \frac{s}{1 - (1 - \varepsilon)(1 - s)} \right] \). This equation describes the evolution of domestic holding of foreign bonds, which is increasing in the saving rate, the returns to equity, and the exchange rate. The discussion in section 2.3 discussed the underlying intuition.

Before I proceed, I will make one more simplification that helps avoid uninteresting detours. Notice that the term \( K \) makes an appearance in the denominator of the last two terms in the square brackets. Recall also, that \( e \) itself is a function of \( B_d^* \), so that the term \( eB_d^* \) adds another non-linearity. For the remainder of this analysis, I will ignore any pure magnitude effects that arise from these normalized stocks (e.g., the terms \( K^2 \) in the denominator whenever

Alternatively, one could postulate that the speed of issuance of domestic bonds is very slow relative to the evolution of other asset stocks.
I differentiate with respect to \( K \). These terms do not reflect any interesting economic mechanisms.

To summarize, the dynamic system consisting of the two state variables, \( K \) and \( B_d \), can be represented succinctly by the following equations:

\[
\dot{K} = f(K, B_d; \mu, \psi); \quad f_1, f_4 < 0, \quad f_2, f_3 > 0
\]

\[
\dot{B}_d^* = h(K, B_d^*; \mu, \psi); \quad h_2, h_3 < 0, \quad h_1 > 0; \quad h_4 \geq 0
\]

The signs of the partials assume that \( K > 1 \) (see the Appendix for detailed expressions).

The steady state is characterized by constant stocks of all financial assets. Equation (38) highlights the implication that the current account is balanced in the steady state. Moreover, along with the constant capital stock, it implies that the returns to equity, the level of tradable and non-tradable output, and the exchange rate too are constant in the steady state.

The slopes of the two isoclines in \( K - B_d^* \) can be derived from eqs. (32) and (39). Formally, employing the linear expressions (19) and (20),

\[
\frac{\partial B_d^*}{\partial K} \bigg|_{K=0} = \frac{r_1 - \gamma F K^{\frac{\gamma - \alpha}{\alpha}}}{r_2}
\]

\[
\frac{\partial B_d^*}{\partial \beta_1^*} \bigg|_{\beta_1^*=0} = \frac{(\pi - \mu) (2 \alpha \Omega + \Omega' / K) K^{\frac{\gamma - \alpha}{\alpha}}}{(1 + \mu) r_2 + \frac{B_d^*}{K} e_2 - \frac{B_d^*}{K} e_1 - \frac{B_d^*}{K} e_2}
\]

where \( \Omega' = \frac{\pi - \mu}{1 - (1 - \epsilon)(1 - \alpha)} < 0 \). Consider first the slope of the \( \dot{K} = 0 \) isocline. An increase in the level of foreign bond holdings causes \( r_K \) to decline, resulting in higher investment. The level of the capital stock must increase in order to restore investment to its original level through the effect on \( r_K \). However, notice that, there is an offsetting effect of the increase in capital from higher goods market profitability due to external economies. Given that \( \gamma < \alpha \), this effect is large at low levels of \( K \), but decreases rapidly thereafter. For example, when \( K = 1 \), the isocline is positively-sloped if \( r_1 - \gamma F > 0 \). The isocline, in the range where it is positively-sloped, is concave in \( K \).

Turning to the \( \dot{B}_d^* = 0 \) isocline, increased holdings of foreign bonds leads to a current account deficit through lower savings (the wealth effect), higher investment, and real appreciation. An increase in \( K \) is required to restore balance through higher returns to equity and real depreciation. Again, there is an offsetting effect due to the presence of economies: rising capital stock levels increase profitability and investment, and lower saving through the wealth effect. Again these effects are large at very low levels of \( K \) and, in the range where the isocline is positively-sloped, the isocline gets less steep as \( K \) increases.

As is typical for systems with external economies, several configurations are possible. As long as both isoclines are positively-sloped, however, it can be shown that the system has a saddle point solution when the \( B_d^* = 0 \) isocline is
steeper and a stable one when it is flatter than the other isocline. The Appendix shows the conditions under which the two isoclines are positively-sloped and the slope of the $B_d = 0$ isocline becomes flatter more rapidly than that of the $K = 0$ isocline, so that if the isoclines intersect twice, the $B_d = 0$ isocline is likely to be steeper when the capital stock is low and flatter when the capital stock is high. I treat this as our configuration of interest. Intuitively, in the case of both isoclines, an increase in the capital stock raises $r_K$ and, relevant only to the current account, $e$ which then requires an increase in the holdings of foreign bonds to restore zero change. However, in the case of the current account, there is the additional wealth effect on savings which, by reducing the rise in $B_d$ required for each increment of $K$ reduces the slope as the stocks of both assets increase (i.e., as we move rightward). This additional effect is what helps ensure that the $B_d = 0$ isocline flattens faster than the other one.

Figure 1 illustrates the system with the help of a phase diagram. The presence of economies of scale gives rise to two equilibria in our set-up, one with a low level of capital stock ($K_L$), and the other with a higher level ($K_H$). \[9\]

\[10\] Notice that the wealth effect appears both in the numerator and the denominator of equation (43).

\[11\] It is worth noting here that, in the absence of economies of scale, the two equations of motion will be linear in our set-up, and the two isoclines will, therefore be straight lines with (at most) one intersection.
3.1 Increased equity inflows

How does a switch in foreign investor sentiment towards domestic equity influence the evolution of the system? As discussed earlier in section 2.3.1, the effect of such a change is to boost investment and create a current account deficit. This means, in turn, that the level of the capital stock must be higher to restore investment to its original level and remove the current account deficit. Thus, in terms of Figure 2, the effect is to shift both isoclines rightward. Formally, the magnitudes of the curve shifts in the horizontal direction are:

\[
\frac{\partial K}{\partial \mu} \bigg|_{K=0} = \frac{r_3}{r_1 - \gamma FK^{\frac{\alpha}{\alpha-\gamma}}} > 0
\]

\[
\frac{\partial K}{\partial \mu} \bigg|_{\mu=0} = \frac{\pi \Omega + (1+\pi)r_K}{(1+\mu)^2} + \frac{\pi - \mu}{1+\mu} r_3 + \epsilon_3 B_d^r K > 0
\]

Let’s turn our focus next to the transitional dynamics. In intuitive terms, the initial shift in asset demand lowers the return to equity and the real exchange rate (i.e. appreciation). This means that investment picks up and the net foreign asset position declines as the economy experiences current account deficits. The subsequent transitional dynamics depend on the responsiveness of investment (i.e., the speed of adjustment, \(\pi\)). If we continue to assume sufficient sensitivity so that the capital stock adjusts faster than holdings of foreign bonds, the system follows the path labelled \(T_1\) in the figure. To understand why, it helps to focus on the “horse race” between the profit rate and the returns to equity. Since investment is responsive, so that \(K\) rises rapidly, economies of scale ensure that the profit rate rises and the exchange rate depreciates. At the same time, since \(B_d^r\) declines relatively slowly, the upward push on \(r_K\) is weak. The profit rate, therefore, rises relative to the cost of equity, and positive investment continues. The rise in \(r_K\) and real depreciation accompanying the rising capital stock then, over time, help stanch the decline in net investment position until the net financial outflows reverse. Beyond this point, the economy accumulates foreign bonds, and this, along with the accompanying real appreciation and rise in returns to equity guide the economy to the new high capital stock steady state. The economy has experienced a sustained spurt of investment and current account deficits followed by surpluses along the path. This is consistent with the empirical findings of Libman et al. (2019), who report that the trade balance initially declines and then recovers during episodes of sustained investment surges.

In the case where investment responds weakly to profitability conditions, the outcome is a trajectory such as \(T_2\). In this case, the initial decline in the returns to equity barely affects the capital stock so that economies of scale do not come into play in a significant manner. In response to initial investment, holdings of foreign bonds decline rapidly, and so \(r_K\) rises faster than \(r\), winning the horse race, and resulting in the capital stock declining after a while. As the capital stock declines, the real exchange rate appreciates, further magnifying the loss of
international bond holdings. Thus, in the case where the speed of adjustment of the capital stock is relatively low, a corner solution eventuates.

3.2 Increased bond inflows

What if the switch in preferences is toward domestic bonds rather than equity? In this case, contrary to that of equity inflows, the initial impact on investment is negative through the returns to equity channel.

In terms of curve shifts, the effect is to shift the $\dot{K} = 0$ leftward and up while the effect on the $\dot{B}_d = 0$ isocline is ambiguous. Formally, the magnitude of the curve shifts in the horizontal direction are given by:

$$\frac{\partial K}{\partial \psi} \bigg|_{\dot{K}=0} = -\frac{r_4}{r_1 - \gamma F K \frac{\mu}{\alpha}} < 0$$

$$\frac{\partial K}{\partial \psi} \bigg|_{\dot{B}_d=0} = -\frac{\left(\frac{\pi - \mu}{1+\mu}\right) r_4 - \frac{B_d^*}{K} e_4 - \frac{1}{(1+\psi)^2} \frac{r B}{K}}{r_1 + e_1 \frac{B_d^*}{K} - \left(\frac{2}{\alpha} \Omega + \Omega' K\right) F K \frac{\mu}{\alpha}} \geq 0$$

Looking at the numerator of the second expression, clearly there are two cases: (1) when the effect of bond inflows is sufficiently weaker on the returns to equity than on the real exchange rate, so that $\left(\frac{\pi - \mu}{1+\mu}\right) r_4 < e_4 \frac{B_d^*}{K} + \frac{1}{(1+\psi)^2} \frac{r B}{K}$, the $\dot{B}_d = 0$ isocline shifts down and to the right, while, (2) in the opposite case where $\left(\frac{\pi - \mu}{1+\mu}\right) r_4 > e_4 \frac{B_d^*}{K} + \frac{1}{(1+\psi)^2} \frac{r B}{K}$, the $\dot{B}_d = 0$ isocline shifts up and leftward.
Why? To understand the intuition, let’s consider the cases individually. Keep in mind that in both cases, the bond inflows initially cause a real appreciation and a rise in the return to equity. This means that, in both cases, investment initially declines.

**CASE 1:**

\[
\left( \frac{\pi - \mu}{1 + \rho} \right) r_4 < e_4 \frac{B^*_4}{K} + \frac{1}{(1 + \psi)^2} \frac{rB}{K}.
\]

This case, which is reminiscent of the “Dutch disease” phenomenon, is captured by Figure 3. Since in this case the impact of the bond inflows is greater on \( e \), the initial decline in the capital stock is accompanied by the loss of foreign bond holdings. As \( K \) declines, so does the profit rate (thanks to economies of scale), and the initial appreciation is magnified, leading to further loss of foreign bond holdings. A corner solution results.

**CASE 2:**

\[
\left( \frac{\pi - \mu}{1 + \rho} \right) r_4 > e_4 \frac{B^*_4}{K} + \frac{1}{(1 + \psi)^2} \frac{rB}{K}.
\]

This case, captured by Figure 4, is more complicated in that at least two kinds of trajectories are possible. Recall that now the initial impact of bond inflows is greater on \( r_K \), so that the resulting large initial decline in investment results in a current account surplus rather than a deficit, and therefore, contrary to case 1, a rise in foreign bond holdings. In the “virtuous” scenario, represented by the trajectory labelled \( T_2 \) in Figure 4, investment is not sensitive to the initial rise in \( r_K \). This lack of responsiveness means that the rise in foreign bond holdings and the resulting decline in \( r_K \) could reach a point beyond which investment recovers and the economy ends up at the higher capital stock equilibrium. The assumption that we have made up until now, that investment
is sensitive to profitability, however, precludes this scenario. Under this latter assumption, represented by the path $T_1$, investment continues to fall until the appreciation caused by the decline in the capital stock turns the initial current account surplus into a deficit. From thereon, both the stocks of physical capital and foreign bond holdings decline.

Thus, contrary to the case where foreign preferences shifted towards domestic equity, investment responsiveness precludes the possibility of attaining the higher capital stock equilibrium. Moreover, even in the virtuous case, the trajectory followed by the capital stock is that of decline followed by strong recovery rather than a consistent and sustained investment surge caused by capital inflows.

![Diagram](image)

**Figure 4:** The effect of bond inflows when the effect on the returns to equity is strong

## 4 Concluding Remarks

All capital inflows are not created equal. This paper has explored one reason why this may be the case by connecting the impact effects in the financial markets to mechanisms that spring into action over time in the goods sector. I analyzed the effects of two kinds of real appreciation-inducing capital flows, one that reduce the cost of issuing equity and the other that increase it. Given adequate investment responsiveness, and in the presence of learning externalities in a developing country context, different kinds of inflows can lead to very different outcomes in the real sector over time. Consistent with recent empirical evidence, one family of flows facilitates achievement of the higher capital stock steady state following sustained investment surges while the other is likely to
hinder such surges. Some inflows set into motion forces that offset Corden’s “real appreciation” problem while others magnify it.

The analysis has policy implications beyond narrow academic interest. Policy makers often resort to sterilization, exchange rate management, or capital controls to curb the real appreciation that follows foreign capital inflows. Given their different consequences for the real economy over time, sterilization is likely to be needed more for some kinds of inflows than for others. Furthermore, since as shown here different kinds of capital flows have different implications for the cost of issuing equity for investment, capital management techniques may work better if employed with these nuances in mind.

Developing country policy makers typically welcome capital inflows for short-run cyclical motives while worrying about the potential consequences over time in the form of Dutch disease-linked problems. This paper underlines the argument that the design of policy responses should take into account the implications for development and structural change independent of, and in addition to, those of exchange rate-related issues.

5 Appendix

The partial derivatives for eqs. (40) and (41)

Evaluated around the steady state,

\[ f_1 = \frac{\partial K}{\partial K} = \pi \gamma F K^{\frac{\gamma}{\alpha}} - r_1 \]

\[ f_2 = \frac{\partial K}{\partial B_d^2} = \pi r_2 \]

\[ f_3 = \frac{\partial K}{\partial B_d^3} = \pi r_3 \]

\[ f_4 = \frac{\partial K}{\partial B_d^4} = -\pi r_4 \]

\[ h_1 = \frac{\partial B_d^2}{\partial K} = \left( \frac{\pi - \mu}{1 + \mu} \right) r_1 + B_d^2 e_1 - \left( \frac{\gamma}{\alpha} \Omega + \Omega' K \right) F K^{\frac{\gamma}{\alpha}} \]

\[ h_2 = \frac{\partial B_d^3}{\partial K} = - \left( \frac{\pi - \mu}{1 + \mu} \right) r_2 - B_d^3 K e_2 - \Omega' F K^{\frac{\gamma}{\alpha}} \]

\[ h_3 = \frac{\partial B_d^4}{\partial K} = - \left( \frac{\pi + (1 + \pi) \gamma}{(1 + \mu)^2} \right) r_3 - B_d^4 K e_3 \]

\[ h_4 = \frac{\partial B_d^5}{\partial K} = \left( \frac{\pi - \mu}{1 + \mu} \right) r_4 - B_d^5 K e_4 - \frac{1}{(1 + \mu)^2} r_d^2 \]

Relative slope conditions

Let’s consider the conditions under which the configuration captured by Figure 1 represents the system. At \( K = 0 \), both isoclines have a slope of negative infinity (recall that \( \gamma < \alpha \)). At \( K = 1 \), that is, at low levels of \( K \), the conditions for the slope of two isoclines to be positive can be derived from eqs. (42) and (43), and expressed respectively as:

\[ r_1 > \gamma F \]  

(A.1)

\[ \left( \frac{\pi - \mu}{1 + \mu} \right) r_1 + e_1 B_d^* > \left( \frac{\gamma}{\alpha} \Omega + \Omega' \right) F \]  

(A.2)
Both conditions require that the returns to equity be relatively sensitive to the supply of this asset, i.e., $r_1$ be relatively large.

Next, given that these conditions are satisfied, the slope of the $\dot{K} = 0$ isocline is flatter than the other one at low levels of $K$ if,

$$\frac{r_1 - \gamma F K^\frac{\omega - \alpha}{\alpha}}{r_2} < \frac{-\left(\frac{\gamma}{\alpha} \Omega + \Omega' K\right) F K^\frac{\omega - \alpha}{\alpha} + \left(\frac{\pi - \mu}{1 + \mu}\right) r_1 + B_d^2 e_1}{\Omega' F K^\frac{\omega}{\alpha} + \left(\frac{\pi - \mu}{1 + \mu}\right) r_2 + B_d^2 e_2}$$

which simplifies at $K = 1$ to:

$$\left(\frac{\pi - \mu}{1 + \mu}\right) \gamma F + B_d^2 e_1 - \left(\frac{\gamma}{\alpha} \Omega + \Omega'\right) F > (\Omega' + B_d^2 e_2) \left(\frac{r_1 - \gamma F}{r_2}\right)$$  \hspace{1cm} (A.3)

The expression in the second set of parentheses on the right hand side is the slope of the $\dot{K} = 0$ isocline. That the left hand side expression be positive is the condition for the $B_d^2 = 0$ isocline to be positively sloped with the term $r_1$ replaced by $\gamma F$. Since a positive slope for the $\dot{K} = 0$ requires that $r_1 > \gamma F$, (A.1) and (A.3) are sufficient to ensure that, at low levels of $K$: (1) the two isoclines are positively-sloped, and (2) the the $\dot{K} = 0$ is flatter.

Finally, we can derive the slopes of both isoclines as the capital stock approaches infinity.

$$\lim_{K \to \infty} \frac{\partial B_d^2}{\partial K}\bigg|_{B_d^2 = 0} = \lim_{K \to \infty} \frac{r_1 - \gamma F K^\frac{\omega - \alpha}{\alpha}}{r_2} = \frac{r_1}{r_2} > 0$$  \hspace{1cm} (44)

$$\lim_{K \to \infty} \frac{\partial B_d^2}{\partial K}\bigg|_{B_d^2 = 0} = \lim_{K \to \infty} \frac{-\left(\frac{\gamma}{\alpha} \Omega + \Omega' K\right) F K^\frac{\omega - \alpha}{\alpha} + \left(\frac{\pi - \mu}{1 + \mu}\right) r_1 + e_1}{\Omega' F K^\frac{\omega}{\alpha} + \left(\frac{\pi - \mu}{1 + \mu}\right) r_2 + e_2} = -1$$  \hspace{1cm} (45)

Thus, the slope of the $\dot{K} = 0$ isocline is greater as the capital stock approaches infinity; indeed the slope of the other isoline turns negative. By implication, the slope of the $\dot{K} = 0$ is unambiguously greater than that of the other isocline at sufficiently high levels of $K$ where the $B_d^2 = 0$ isocline is still positively-sloped.

References


