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Hyun Woong Park
Denison University, Department of Economics

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A Marxian Model of Market for Money Capital: Profit Rate, Interest Rate, and Leverage Ratio

Hyun Woong Park*

Abstract

In this paper, I develop a Marxian model of market for money capital populated by capitalists equipped with equal money capital endowment but with heterogeneous linear production technology. Due to a maximization of return on equity, capitalists with relatively weak technology, yielding profit rate lower than interest rate, become a money capitalist (lender) and capitalists with relatively strong technology, yielding profit rate greater than interest rate, become an industrial capitalist (borrower). The equilibrium interest rate is derived by the associated demand and supply relation. In this context, Marx's notion of the role of credit system in an expanded reproduction of capital is understood in terms of an efficient reallocation of funds through credit market. From this setup of the model follow two essential relationships Marx establishes between the average profit rate and the interest rate: (i) that the profit (rate) sets a maximum limit of interest (rate), and (ii) that the two rates are correlated. Lastly, depending on the financial sector's leverage ratio, which is supported by its intermediation technology, the financial sector may be more or less profitable than the industrial sector. This result suggests that one aspect of the industrial-banking capitalists antagonism surrounding the division of profit into interest and profit of enterprise lies in the banking capitalists' strenuous efforts towards continuous innovations in financial intermediation technology.

Keywords: Money capital, interest-bearing capital, loanable capital, credit, interest rate

JEL Classification: B51

*Denison University, Economics Department: parkhw@denison.edu

1 Introduction

In Part V of *Capital* Volume 3, Marx suggests that loanable, interest-bearing capital—the culmination of capital—is paid out of surplus value (profits), produced by industrial capital, and that, however, the rate at which it is paid is determined purely by demand and supply relation without any reference to the law of value. Based on these contradictory aspects of interest, Marx analyzes that the interest rate is ultimately governed by the average profit rate. In this paper, I present a model of market for money capital to study within the Marxian framework the determination of interest rate and its relationship with capital profitability at individual, aggregate and sectoral levels.

The model describes an economy in one period without uncertainty, where only one good is produced and capitalists are equipped with equal endowment but with different linear technology. The technology heterogeneity provides a basis for explaining a divergence of capitalist class into industrial capitalists (borrowers) and money capitalists (lenders). More specifically, the capitalists equipped with less advanced technology, generating the profit rate smaller than a given interest rate, find it optimal to manage their endowment as loanable, interest-bearing capital, whereas the capitalists equipped with more advanced technology, generating the profit rate greater than the given interest rate, find it optimal to manage their endowment as industrial capital, financing it not only by own capital but also by borrowing. From this setup, the demand for and supply of money capital can be identified to determine the equilibrium interest rate.

From the fact that capitalists with less advanced technology lend their endowment to capitalists with more advanced technology, who therefore eventually manage the given total endowment as industrial capital, the first result of the model is obtained. That is, in the case without uncertainty, the average profit rate, or the profit rate of total social capital, in an economy with credit market is greater than that in an economy without credit market. This result demonstrates that Marx's notion of the role of credit system in an expanded reproduction of capital can be understood in terms of one of the standard results in economy theory about efficient reallocation of resources through credit market.

In Marx's discussion of interest-bearing capital and interest, two long-run relationships emerge between the average profit rate and the interest rate: (i) profit (rate) sets a maximum limit of interest (rate), and (ii) unless the share of interest payment out of gross profit is unreasonably volatile, interest rate moves in step with profit rate. My model formally replicates both of these relationships between the two rates. The first relationship holds since only capitalists equipped with more advanced technology, which generates the profit rate greater than the interest rate, operate industrial capital. As a way to verify the second relationship, it is shown that the two rates respond in the same direction to a shift in each of the model parameters.

In the benchmark model summarized thus far, capitalists decide to become a money

capitalist not because they have a skill in managing loanable, interest-bearing capital but because they are equipped with weak production technology. Consequently, the financial sector, lacking financial intermediation technology, cannot borrow; it is only industrial sector that uses leverage to amplify profitability. For this reason, the industrial sector is more profitable than the financial sector.

In an extension of the benchmark model, banking capitalists equipped with financial intermediation technology are introduced, who can raise funds at a cost lower than the interest rate they charge on lending. It is shown that the financial sector profitability is also amplified through borrowing and when the financial sector leverage ratio is sufficiently high, the financial sector may possibly be more profitable than the industrial sector. This result demonstrates that banking capitalists' strenuous effort towards an innovation in financial technology, which enables expanding the bank balance sheet by borrowing more at a lower cost, is one important aspect of what Marx describes as an antagonism between industrial capitalists and money-banking capitalists surrounding the division of profit into interest and profit of enterprise.

There are three important considerations in dealing with credit in a Marxian framework. First, Marx distinguishes between commercial credit and monetary credit. The former is used for transactions of consumption goods between capitalists and workers and for transactions of circulating capital goods between capitalists, whereas the latter is used for transactions of fixed capital goods between capitalists. The model presented in this paper considers only monetary credit, which includes direct lending and bank loan.

Second, Marx identifies two important sources of demand for credit. On one hand, there is a demand for credit to finance an expanded reproduction of capital, especially in a period when economic prospect warrants an expansion of production scale. On the other hand, capitalists also demand for credit to make debt payment, which usually occurs *en masse* during the downturn of business cycle. In the former, it is a demand for money as capital, i.e. money form of capital that could initiate the circuit of capital, whereas in the latter it is a demand for money as money, or money as means of payment. The model presented in this paper considers only the former type of credit demand, i.e. demand for money capital, not for means of payment. In the sense that credit analyzed in this paper is monetary credit and the credit demand arises as a demand for money capital, the model presented in this paper is a model of market for money capital, or loan capital, rather than a market for credit.

Third, Marx discusses two roles of credit in the capitalist system. On one hand, credit plays an essential role in financing an expanded reproduction of capital, which is why the credit system develops along with the development of capitalist production. On the other hand, an accumulation of interest-bearing capital tends to deviate from, and be independent of, an accumulation of industrial capital, in which case the credit system would make the system fragile and unstable. In particular, referring to the latter, Marx suggests that the credit system precipitating an outbreak of contradictions of capitalist production. The

benchmark model presented in this paper focuses on the first role of credit and leaves the second role for a future research.

This paper is in line with those, such as Harris (1976), Panico (1980), Lianos (1987), and Saros (2013), etc., that explore the general law of the determination of interest rate in Marxian theory against a view that there is no such law due to Marx's suggestion that the natural rate of interest does not exist. These papers commonly identify, as in this paper, two essential elements of Marxian theory of interest, i.e. (i) a (minimum and maximum) range of interest (rate) derived from the fact that interest is paid out of profit, and (ii) secular and cyclical correlations between the interest rate and the rate of profit. In particular, the last three papers present a formal model that determines the interest rate. Panico (1980) extends the standard Leontief input-output model to incorporate a banking sector and hence interest rates. Lianos (1987)'s model studies secular and cyclical relationships among the profit rate, the rate of profit of enterprise, and the interest rate. Saros (2013)'s model derives the general interest rate from the circuit of capital that includes bank capital.

These formal models compare to the model presented in this paper in a couple of ways. On one hand, Lianos (1987) and Saros (2013) have the profit rate taken as given in determining the interest rate. In contrast, in my paper, not only the interest rate but also the profit rate are endogenously determined within the model and their correlation is verified by examining the responses of each of the two rates to a permanent shift in the model parameters. Moreover, while all of these papers discussed thus far mention the demand and supply relation as the main mechanism of the determination of interest rate in Marxian theory, none of them explicitly considers it in their models. In contrast, my model explicitly identifies the demand for and supply of money capital and derives the equilibrium interest rate from the equality between demand and supply.

On the other hand, the central mechanism of determining the equilibrium interest rate in Panico (1980) and Saros (2013) is the equalization of profit rates across sectors, including both industrial and financial sectors. According Roemer (1981)'s Marxian general equilibrium model, in the case of imperfect entry or non-constant-return-to-scale heterogeneous production sets, profit maximization and the existence of credit market make the profit rates equalized across all capitalists. In comparison, in my one-good model, capitalists are equipped with equal endowment but with different linear technology, and maximize the return on equity; due to the linearity, each borrowing capitalist's optimal levels of debt and capital stock are unbounded and the profit rates are never equalized.¹

Hein (2006) attempts to combine Marxian theory of money and interest with a Kaleckian model of growth and distribution. Interpreting Marx's monetary theory as similar to the horizontalist version of endogenous money theory, Hein (2006) incorporates the interest rate as an exogenous variable into both saving function and investment function. Accordingly,

¹In order to limit debt and capital stock to some finite level, I take a simplified approach, i.e. taking the leverage ratio of each borrowing capitalist as exogenously given.

the standard solutions of the Kaleckian model are slightly modified to be dependent on the interest rate as well. By identifying Marx's monetary theory with the extremist version of endogenous money theory, Hein (2006) leaves no room for explaining the determination of the interest rate within the model.

Fine (1985) and Lapavitsas (1997) provide a methodological support for this paper. Fine (1985) characterizes commercial credit as financing simply income expenditure and hence transactions between a capitalist and a capitalist or other non-capitalist agent, whereas characterizing monetary credit as financing capitalist production and its expansion and hence transactions between capitalists. Lapavitsas (1997) highlights the generation of demand for and supply of money capital from the circuit of capital.

While my paper deals with the determination of interest rate and its relation to the profit only in a long-run, Cipolla (1997) closely examines Marxian theory of interest rate dynamics over the business cycle. Dymski (1990) provides a useful taxonomy of various economic theories of money and credit according to whether uncertainty is considered or not and whether production is self-financing or finance-constrained; in the case of finance-constrained, a further distinction is made according to whether credit expansion is dependent or independent of accumulation. According to Dymski (1990)'s taxonomy, my model can be categorized as no uncertainty/finance-constrained/accumulation-dependent.

The rest of the paper is organized as follows. In section 2, I present the basic setup of the benchmark model where all capitalists are equipped with production technology only and demonstrate a divergence of the capitalist class into industrial and money capitalists according to technology heterogeneity. In section 3, the equilibrium in the market for money capital is examined. In section 4, the average profit rates of capital at aggregate and sectoral levels and their relationship with the interest rate are discussed. In section 5, the benchmark model is extended to introduce banking capitalists equipped with financial intermediation technology.

2 A divergence of the capitalist class

The basic setup of the model is as follows. The model economy is in one period and only one good is produced. There is a continuum of capitalists who only care about the consumption at the end of the period. At the beginning of the period, all capitalists are endowed with the same amount of equity wealth W , which can be consumed or used as industrial capital or lent out to other capitalists.² In this sense that the endowment can be used for various purposes, I will conveniently consider it as money capital and call the market for lending

²Since endowment is equal across capitalists, it can be normalized to one. However, to trace the role of endowment in the model, let us proceed without normalization.

and borrowing it a market for money capital.³ The capitalists are differentiated from one another by technology. The technology heterogeneity can be considered as an outcome of various institutional settings, such as patent, that hinder technology spill-overs. Each of the capitalists has a fixed-coefficient technology characterized by capital-labor ratio, or capital intensity, and labor productivity, each denoted by τ and b , respectively. While these technology variables vary across capitalists, output-capital ratio, denoted by a , is assumed to be the same and the turnover time is also the same across all capitalists as one period.

For a capitalist operating capital stock by the amount K , the labor employment is, by definition, $L = \frac{K}{\tau}$. Also by definition, the following relation holds:

$$b = a\tau \tag{1}$$

Given output-capital ratio, a change in capital-labor ratio has a positive impact on labor productivity. This relation reflects stylized facts reported in the literature on historical trend of technical change as characterized mainly by increases in capital intensity and labor productivity; see, for instance, Foley et al. (2019). Note that according to Marxian theory, the essential character of technical change in a capitalist economy is described by a rise in organic composition of capital—which is reflected in τ in my model—driven by capital-using and labor-saving technical change, thereby raising labor productivity and lowering output-capital ratio. In this respect, the model will more closely match the so-called Marx-biased technical change if a is treated also as varying in a negative relation to τ .⁴ The only reason why a is taken as fixed is to make the aggregation of individual capitalists easier as will be discussed later.

The distribution of τ across capitalists is reflected in the probability density function $f(\tau)$ and the cumulative distribution function $F(\tau)$; the latter indicates the fraction of capitalists with technology characterized by capital-labor ratio less than τ . Since labor productivity is a monotonic function of capital-labor ratio, it holds $f(b) = f(\tau)$ and $F(b) = F(\tau)$. With the infinite number of capitalists normalized into one, $F(b)$ can be interpreted as the total number of capitalists with labor productivity below b and $1 - F(b)$ as the total number of capitalists with labor productivity above b .

Wage rate is given as w identically across capitalist firms.⁵ To make the model more realistic, let us suppose the uniform wage rate is a positive function of the average labor productivity. Formally,

$$w = w \left(\int_0^\infty b dF(b) \right), w' > 0 \tag{2}$$

³Although there is a theoretical issue of how to define money in a one-good, one-period model, it is not essential for the results of the model.

⁴In this case, equation (1) will be revised to $b = a(\tau)\tau$ where $a' < 0$.

⁵Instead of equal wage rate among the capitalists, an alternative approach is to assume an equal exploitation rate.

In this setup, for a capitalist equipped with technology of b and operating the capital stock of K at its full, or normal, rate of utilization, the profit rate, denoted by r , is computed as

$$r \equiv \frac{Y - wL}{K} = a \left(1 - \frac{w}{b} \right) \quad (3)$$

Since a and w are equal across capitalists, each capitalist's distinctive r is a function of b , i.e. $r = r(b; a, w)$. In this sense, each capitalist can be distinctively described as being equipped with b -technology or, equivalently, $r(b)$ -technology.

The profit rate defined in equation (3) measures the profitability of capital regardless of how the capital is financed. When capital structure—i.e. equity vs. debt—is taken into account, the return on equity (ROE) is a more relevant measure of profitability as will be discussed below. To make a clear distinction, let us call the one defined in equation (3), the profit rate of capital. In case the profit rate of capital is sufficiently high, the capitalist can consider expanding the scale of capital by borrowing. Otherwise, instead of using it as industrial capital, the capitalist may find it more profitable to manage her endowment as interest-bearing capital by lending it in the credit market. To examine this process more concretely, let us introduce a credit market.

Credit market is assumed to be organized as direct finance between borrowers and lenders of loanable capital without the help of financial intermediaries.⁶ The lending and borrowing activities can be considered as buying and issuing bonds. Suppose a capitalist, equipped with technology of labor productivity b that generates the profit rate r , can either borrows $B > 0$ or lend $B < 0$ at an on-going interest rate, denoted by i . The capitalist will make a portfolio decision—between productive investment (K) yielding r and financial investment ($B < 0$) yielding i —and a capital structure decision—regarding the amount of borrowing in financing the productive investment—in a way that maximizes profitability.

When capital is financed by a mix of own fund and debt, a relevant measure of profitability is return on equity (ROE)—or, following Marx's terminology, rate of profit of enterprise—denoted by r^e , which is computed by dividing net earnings, after interest payment, by equity.

$$r^e \equiv \frac{rK - iB}{W} = r + (r - i) \frac{B}{W} \quad (4)$$

where the second equality uses the budge constraint $K = W + B$.⁷

It can be easily seen from equation (4) that i is a threshold in making an decision whether technology yielding r is profitable enough to be activated or not. On one hand, for a capitalist equipped with technology generating the capital profit rate greater than the borrowing cost, i.e. $r > i$, it is optimal to be an industrial capitalist and use the entire endowment as industrial capital, additionally financed by borrowing, i.e. $B > 0$, as much

⁶In order to focus on the relationship between industrial capitalists as a borrower and money-capitalists as a lender, all the issues related to financial intermediation are assumed away.

⁷Equation (4) can be rewritten as $r^e = i + (r - i) \frac{K}{W}$.

as possible to expand the scale of capitalist production, $K > W$, to the largest possible extent. The industrial capitalist achieves the ROE greater than her capital profit rate, i.e. $r^e > r$. That is, when the technology is sufficiently profitable, the capital profit rate can be amplified through leverage. On the other hand, for a capitalist whose technology yields the capital profit rate less than the borrowing cost, i.e. $r < i$, it is optimal to be a money capitalist and lend all the endowment, i.e. $-B = W$ and hence $K = 0$, in which case the profit from capital is zero, $r = 0$, and the ROE is equal to the interest rate, $r^e = i$. Lastly, a capitalist whose technology yields the profit rate equal to the borrowing cost, i.e. $r = i$, will be indifferent between operating industrial capital or interest-bearing capital since in either case the ROE will be identical with her profit rate of capital, $r^e = r$.

This provides an explanation of a divergence of the capitalist class into industrial and money capitalists. Since interest earning is a return from lending, which is an investment option alternative to capitalist production, i reflects an opportunity cost of industrial capital. This is why i sets the minimum value of r required for operating industrial capital to be profitable. The capital profit rate below i is considered not profitable enough. In this sense, the interest rate sets a barrier for becoming an industrial capitalist. On the other hand, since the difference in capital profit rates lies in the difference in labor productivity, the divergence condition in terms of profit rate differential can be restated as a divergence condition in terms of labor productivity differential. Substituting the definition of r expressed in equation (3) into its critical value, i.e. $r = i$, the critical value of b , denoted by \hat{b} , is obtained as follows:

$$\hat{b} = \frac{aw}{a - i} \quad (5)$$

which is the minimum level of labor productivity required for managing a given equity as industrial capital to be profitable. Any $b < \hat{b}$ is considered too weak a technology to be activated profitably. In this sense, \hat{b} can be considered as technology barrier, corresponding to the interest rate barrier.

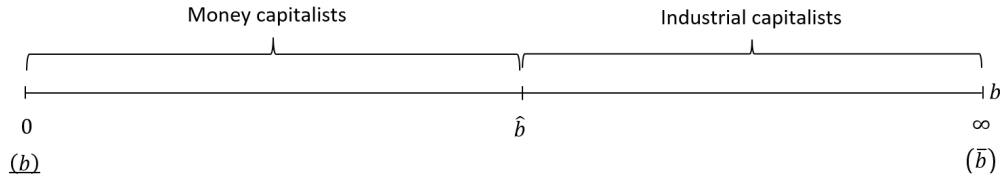
The divergence of the capitalist class can be expressed in terms of either the interest rate barrier or labor productivity barrier as summarized in the following lemma.

Lemma 1 *A capitalist, equipped with equity W and $r(b)$ -technology, facing wage rate w and interest rate i , makes the following optimal decision:*

- (i) *If $b > \hat{b}$ ($r > i$), then $B > 0$ and hence $K > W$, i.e. the capitalist becomes a industrial capitalist, and earns the ROE of $r^e > r > i$.*
- (ii) *If $b < \hat{b}$ ($r < i$), then $B < 0$, where $-B = W$, and hence $K = 0$, i.e. the capitalist lends the entire equity and becomes a money capitalist, earning the ROE of $r^e = i > r$;*

The result in lemma 1 is visualized in figure 1. Once the capitalist class diverges into industrial and money capitalists as in lemma 1, the former forms the industrial sector and the latter the financial sector.

Figure 1: The range of labor productivity, b , and the technology threshold, \hat{b} . The levels of b in parentheses are in case assumption 1 is adopted.



One of the important messages of lemma 1 is that in an economy with technology heterogeneity, the credit system makes a Pareto improvement within the capitalist class.⁸ In the absence of the credit system, all capitalists have no other option but to manage their money capital endowment as industrial capital, using the given technology, in which case each capitalist would achieve the ROE simply equal to her capital profit rate, i.e. $r^e = r$. However, when there is a credit market, all the capitalists, whether with strong or weak technology, can improve the profitability, achieving the ROE greater than the capital profit rate. It can be confirmed from lemma 1 that both industrial capitalists and money capitalists have $r^e > r$. Lemma 1 confirms a standard result in economic theory on the efficient allocation of resources through credit market in a one-good model with technology heterogeneity but without any kinds of financial frictions.

Understanding how \hat{b} behaves is important as it determines the fractions of industrial capitalists and money capitalists, which in turn determines the equilibrium interest rate through the demand and supply mechanism in the credit market as will be discussed shortly. Let us consider, for instance, an increase in i . According to the definition of \hat{b} in equation (5), the result is an increase in \hat{b} . Since interest earning from lending is an opportunity cost of capitalist production, the increase in i implies a technology requirement for operating industrial capitalist profitably is now stricter. Given a distribution of b , some of the industrial capitalists will now find it optimal to switch to a money capitalist. Eventually, the fraction of industrial capitalists decreases and the fraction of money capitalists increases. A change in \hat{b} in response to a change in w or a can be understood in a similar way.⁹

Assumptions implicit in the model are worth mentioning. First, according to the definition of \hat{b} , it may take a negative value when $a < i$. In order to rule out this degenerate case, it is assumed that $a \geq i$, implying that the general state of technology in the model economy is sufficiently advanced. Second, it is assumed that the market for money capital is the only credit market. Therefore, the financial sector does not borrow; it is only industrial

⁸Whether this result still holds for the overall economy, including both capitalist class and working class, is another crucial issue.

⁹Due to the identity, $b = a\tau$, discussed in equation (1), when examining how \hat{b} responds to a shift in a , an implicit assumption must be that τ also shifts in a way that leaves b unchanged; otherwise, a change in a brings about a proportional change in b , thereby changing any given distribution of b .

capitalists who rely on leverage and expand the balance sheet to amplify capital profitability. In an extension in section 5, an additional credit market is introduced where the financial sector can raise funds at a cost lower than i .

Third, since the model economy is a simple one-period world without uncertainty, there is no need for liquidity. As a consequence, the industrial capitalists use all their endowment for capitalist production and the money capitalists lend all their endowment. Neither keeps any of their endowment as cash. This assumption of certain world simplifies the analysis, especially when identifying the total demand for and total supply of credit to determine the equilibrium interest rate in the next section.

3 The equilibrium in the market for money capital

Since the production technology is linear and there is no uncertainty in the model, it would be optimal for both lenders and borrowers to let a borrower, i.e. a capitalist with $b > \hat{b}$, to borrow without limit since the borrower's productive activity will always yield $r > i$ for sure and will not fail to make debt payment; consequently, the lenders whose technology would yield $r < i$ will earn interests at the rate of i for sure. However, the endowment of money capitalists is the sole source of the supply of credit, which therefore is limited by the number of money capitalists.

Suppose that all industrial capitalists have the same level of leverage ratio, defined as asset over equity, denoted by λ^K . Since capitalists have the same endowment W , all industrial capitalists would have equal capital,

$$K = \lambda^K W \tag{6}$$

and equal borrowing,

$$B = (\lambda^K - 1)W \tag{7}$$

While it is commonly considered that leverage ratios are negatively affected by interest rates, the empirical studies on the relation between the two are mixed, the results depending on firms' capital structure policy, firm size, capital buffer, etc. In this context, it could be more reasonable to assume that firms are more responsive to expected profitability than to borrowing costs. As evidenced during the post-2008 Financial Crisis periods, even when interest rates are low, firms hesitate to borrow for capital formation if they expect a low profitability in the future.¹⁰

Once the borrowing by an industrial capitalist, reflected in λ^K , is given, the ROE can be rewritten as

$$r^e = \lambda^K r - (\lambda^K - 1)i \tag{8}$$

¹⁰During the quantitative easing period following the crisis, firms did borrow more but the proceeds were directed to asset market in search of capital gains. This type of borrowing is not considered here since there is no asset market in the model.

The total credit demand, on the one hand, is the sum of the demands for borrowing by all the industrial capitalists, i.e. capitalists with technology of $b > \hat{b}$. Then the total demand for credit, denoted by D , is

$$D = \int_{\hat{b}}^{\infty} B dF(b) \quad (9)$$

where \hat{b} and B are defined in equations (5) and (7), respectively. The negative relationship between the interest rate and the total demand for credit can be easily confirmed. A rise (fall) in i leads to an increase (decrease) in \hat{b} , thereby lowering (raising) the number of industrial capitalists who are willing to borrow to finance productive activities; this consequently reduces (raises) D ; hence, a downward-sloping demand curve.

Similarly, the total supply of credit is the sum of the supply of money capital by all money capitalists, i.e. the capitalists with technology of $b < \hat{b}$. Since money capitalists simply lend all of their initial endowment, the total credit supply, denoted by S , is

$$S = \int_0^{\hat{b}} W dF(b) \quad (10)$$

The positive relationship between the interest rate and the total credit supply can also be easily verified. An increase (a decrease) in i raises (lowers) \hat{b} , thereby increasing (reducing) the number of money-capitalists and eventually leading to an increase (a decrease) in S ; hence an upward-sloping supply curve.

Note that a change in i imparts an impact on the total demand for and total supply of credit through affecting the technology threshold, \hat{b} , thereby changing the (relative) numbers of industrial capitalists and money capitalists.

Finally, the interest rate that clears the money capital market is determined by the equilibrium condition, $D = S$. The model does not yield an analytical solution unless some assumption is made about the distribution of b . One possible approach is to adopt the following assumption.

Assumption 1 b is uniformly distributed over $[\underline{b}, \bar{b}]$, where $\underline{b} = \bar{b} - \theta$, $\theta \geq 0$, and $\underline{b} > w$.

Assumption 1 states that there is a continuum of technologies, the weakest technology being characterized by labor productivity, \underline{b} , while the strongest technology characterized by \bar{b} with the difference between the two being θ . $\underline{b} > w$ implies that even the weakest technology yields a positive profit rate of capital.¹¹ The simplest way to interpret the fact that b is uniformly distribution is to imagine that there is one capitalist for each technology. Note that $\theta = 0$ implies all capitalists have the same \bar{b} -technology. More precisely, it describes a

¹¹This assumption is not essential for the results of the model and hence can be dispensed with. Rather, it strengthens the results, especially lemma 1, as it allows a possibility of the case where some capitalists, even when their technology yields a positive profit rate, may find it more profitable to lend their money capital.

situation where all capitalists adopt the most advanced technology, which would be possible when there are no barriers that prevent technology spillover.

Under assumption 1, the equilibrium condition in the money capital market is expressed as

$$\int_{\hat{b}}^{\bar{b}} (\lambda^K - 1)W\theta^{-1}db = \int_{\underline{b}}^{\hat{b}} W\theta^{-1}db \quad (11)$$

Since \hat{b} is a function of i as expressed in equation (5), the market-clearing interest rate can be obtained as follows by solving equation (11):

$$i^* = a \left(1 - \frac{w}{\hat{b}^*} \right) > 0 \quad (12)$$

where

$$\hat{b}^* = \bar{b} - \frac{\theta}{\lambda^K} > 0 \quad (13)$$

is the equilibrium level of technology threshold.¹² It is an essential aspect of the model that \hat{b}^* guarantees the fractions of industrial capitalists and money capitalists required to establish an equilibrium in the money capital market. Since the amount of borrowing of each industrial capitalist is fixed (at $(\lambda^K - 1)W$) and the amount of lending by each money capitalist is also fixed (at W), the total demand for and total supply of credit will exactly match with each other only when the ratio between the industrial and money capitalists is at a certain level, which is reflected in \hat{b}^* . More importantly, \hat{b}^* depends nothing else but the distribution parameters of b and the industrial capitalists' leverage ratio, λ^K .

With \hat{b}^* , the equilibrium levels of the fractions of industrial capitalists and money capitalists, respectively, are defined and computed as follows.

$$\begin{aligned} 1 - F(\hat{b}^*) &\equiv \int_{\hat{b}^*}^{\bar{b}} \theta^{-1}db = \frac{1}{\lambda^K} \\ F(\hat{b}^*) &\equiv \int_{\underline{b}}^{\hat{b}^*} \theta^{-1}db = \frac{\lambda^K - 1}{\lambda^K} \end{aligned} \quad (14)$$

Note that the fractions of the industrial and money capitalists clearing the money capital market is a function of λ^K only. Since money capitalists are assumed to lend their entire endowment without any borrowing, the only way to have each industrial capitalist borrow more, for instance, and thus increase λ^K , is to have a decrease in the fraction of industrial capitalists and an increase in the fraction of money capitalist, as can be verified from equation (14). However, it does not necessarily mean a shrinkage of the size of industrial sector.

To see this, let us measure the size of each sector by its total asset. The industrial sector's total asset is the product between the fraction of industrial capitalists and each industrial capital's industrial capital, $K \equiv \lambda^K W$, whereas the financial sector's total asset is, similarly,

¹²The inequalities in equations (12) and (13) are due to $\lambda^K \geq 1$, by definition, and assumption 1.

Table 1: The impacts a shift in the industrial sector’s leverage ratio, λ^K , have on the fraction of capitalists, size (total asset), and capital concentration in each sector

	Fraction of capitalists	Size (total assets)	Capital concentration
Industrial sector	$\frac{d1/\lambda^K}{d\lambda^K} < 0$	$\frac{dW}{d\lambda^K} = 0$	$\frac{\partial \lambda^K W}{\partial \lambda^K} > 0$
Financial sector	$\frac{d(\lambda^K - 1)/\lambda^K}{d\lambda^K} > 0$	$\frac{\partial(\lambda^K - 1)/\lambda^K W}{\partial \lambda^K} > 0$	$\frac{dW}{d\lambda^K} = 0$

the product between the fraction of money capitalists and each money capitalist’s interest-bearing capital, W . Denoting the former by A^K and the latter by A^M , the total assets of industrial and financial sectors are, respectively, obtained as follows.

$$\begin{aligned}
 A^K &= \frac{1}{\lambda^K} \lambda^K W = W \\
 A^M &= \frac{\lambda^K - 1}{\lambda^K} W
 \end{aligned}
 \tag{15}$$

Since the total money capital initially endowed within the capitalist class as a whole is eventually reallocated to industrial capitalists through the money capital market, the total asset of the industrial sector is equal to the total money capital endowment and is independent of the fraction of industrial capitalists. Consequently, a decrease in the fraction of industrial capitalist implies an intensification of concentration of capital within the industrial sector. In this context, an increase in λ^K has no impact on the size of the industrial sector, but intensifies capital concentration and hence the scale of production within that sector.

In contrast, since money capitalists are assumed to have no debt, the balance sheet size of each of them remains equal to her initial endowment, W , and is independent of the industrial sector’s leverage ratio. Accordingly, the shift in λ^K has no impact on capital concentration within the financial sector. Interestingly, however, since a rise in λ^K , for example, requires an increase in the fraction of money capitalists, it necessarily expands the size of the financial sector in terms of total asset. Table 1 summarizes the impacts a shift in the industrial sector’s leverage ratio have on the fraction of capitalists, size, and capital concentration in each sector. It is worth mentioning that capital concentration of the financial sector does not have any meaningful consequences on the model results since all money capitalists are assumed to have the homogeneous financial technology. This is in contrast to the industrial sector characterized by heterogeneity in production technology, due to which a change in the capital concentration in that sector makes a difference.

It will be useful for later discussions to consider two extreme cases regarding the industrial sector’s borrowing behaviors. If industrial capitalists do not borrow, i.e. $\lambda^K = 1$, then as there is no need for money capitalists and, consequently, the fraction of industrial capitalists

would be one, whereas the fraction of money capitalists would be zero. That is, the barrier for becoming an industrial capitalist is pressed down to its lower bound. Formally, $\lambda^K = 1$ yields $\hat{b}^* = \underline{b}$ and $F(\underline{b}) = 0$. Consider the other extreme case where the industrial capitalists' leverage ratio approximates infinity, i.e. $\lambda^K \rightarrow \infty$. Since money capitalists lend all of their money capital, the only case this could be possible is when there is a negligibly small fraction of industrial capitalists, to whom the entire endowment of all money capitalists are lent. That is, the barrier for becoming an industrial capitalist is pushed up to its upper bound. At a limit, there would exist only one industrial capitalist, which is a negligible number relative to a continuum of capitalists, and, accordingly, her leverage ratio would be infinite. Formally, $\lambda^K \rightarrow \infty$ yields $\hat{b}^* = \bar{b}$ and $F(\bar{b}) = 1$. According to the discussion related to table 1, it corresponds to the case where the capital concentration and hence the scale of production in the industrial sector are infinite.

Of these two extreme cases of the industrial sector leverage ratio, $\lambda^K = 1$ is effectively equivalent to an absence of credit market, whereas $\lambda^K \rightarrow \infty$ is effectively equivalent to an absence of technology heterogeneity. As the latter case will be particularly important for discussions below, to understand its implication, let us consider the implication of $\theta = 0$, i.e., all capitalists being equipped with the same technology. The equilibrium in this case is described in the following corollary.

Corollary 1 *Consider the model economy under assumption 1. When there is no technology heterogeneity, i.e. $\theta = 0$, so that all capitalists are equipped with the same $r(\bar{b})$ -technology, in which case the technology barrier will obviously be set at $\hat{b}^* = \bar{b}$ (but the concept of technology barrier becomes meaningless when all capitalists have the same technology), the equilibrium is characterized by the following three, each of which implying one another:*

- (i) *The equilibrium interest rate will be established at $i^* = r(\bar{b})$.*
- (ii) *All capitalists achieve the same equilibrium ROE of $r^{e*} = r = i^*$.*
- (iii) *All capitalists are indifferent between managing their money capital endowment as industrial capital and interest-bearing capital, and therefore the fractions of industrial and financial capitalists are indeterminate.*

Proof. While corollary 1 immediately follows from equations (12) and (13) along with lemma 1, part (i) can be intuitively proved as follows. When $i > r$, it is optimal for all capitalists to manage their money capital as interest-bearing capital, in which case there will be a free fall in i . By the same logic, the interest rate will spike when $i < r$. From these, it follows that $i = r$ is the equilibrium when $\theta = 0$. ■

Note that in both cases of $\lambda^K \rightarrow \infty$ and $\theta = 0$, the equilibrium technology barrier is pushed up to its upper limit, $\hat{b}^* = \bar{b}$. However, an important difference is that in the case

of $\lambda^K \rightarrow \infty$, there is only one, or a negligibly small fraction of, industrial capitalist with \bar{b} -technology, operating the total capital, whereas in the case of $\theta = 0$, an indeterminate number of industrial capitalists equipped with the same \bar{b} -technology operate the total capital. Despite the difference in the number of industrial capitalists operating the given total capital, as long as technology is linear, as assumed in the model, the two cases yield the same profit rate of total capital. In this sense that $\lambda^K \rightarrow \infty$ is effectively equivalent to $\theta = 0$ in terms of the profit rate of total social capital, the situation where *either* $\theta = 0$ *or* $\lambda^K \rightarrow \infty$ holds will be described as technology heterogeneity being effectively absent; accordingly, the latter's the complementary set, i.e. the situation where *both* $\theta > 0$ *and* $\lambda^K < \infty$ hold, can be considered as the case with technology being effectively heterogeneous. The following lemma introduces a related, technical relation, that will be useful in proving analytical results below.

Lemma 2 *Consider the model economy under assumption 1. It holds that $\frac{\bar{b}}{\bar{b}^*} - 1 \geq \ln \frac{\bar{b}}{\bar{b}^*}$, where the equality holds when technology heterogeneity is effectively absent, i.e. when either $\theta = 0$ or $\lambda^K = \infty$ holds.*

Proof. It will be useful for the proof to note that for any $x \in \mathbb{R}$, $x - 1 \geq \ln x$ is true, with the equality holding when $x = 1$. Now, from equation (13), $\lambda^K \geq 1$ by definition, and assumption 1, it can be verified that $\frac{\bar{b}}{\bar{b}^*} \geq 1$ holds; hence $\frac{\bar{b}}{\bar{b}^*} \in \mathbb{R}$. As a consequence, $\frac{\bar{b}}{\bar{b}^*} - 1 \geq \ln \frac{\bar{b}}{\bar{b}^*}$ holds. In addition, according to equation (13), $\frac{\bar{b}}{\bar{b}^*} = 1$ is the case when either $\theta = 0$ or $\lambda^K = \infty$ holds. ■

There is no economic meaning of the (in)equality in lemma 2, which however will be essential in proving the analytical results presented below.

One of the uniqueness of the model is that since borrowing by each industrial capitalist and lending by each money capitalist are held constant—the former as $(\lambda^K - 1)W$ and the latter as W —an equilibration process takes place through adjusting the fraction of each of the two groups of capitalists, instead of adjusting the amounts of lending and borrowing by each of individual industrial and money capitalists. To verify this process more concretely, the following proposition examine how the equilibrium interest rate responds to a permanent shift in each of the key parameters, w , \bar{b} , and λ^K .

Proposition 1 *In the model economy under assumption 1,*

$$(i) \quad \frac{\partial i^*}{\partial w} = -\frac{a}{\bar{b}^*} < 0$$

$$(ii) \quad \frac{\partial i^*}{\partial \bar{b}} \gtrless 0 \iff \frac{\partial w / \partial \bar{b}}{w / \bar{b}} \leq \frac{\bar{b}}{\bar{b}^*}, \text{ where } \frac{\bar{b}}{\bar{b}^*} \geq 1, \text{ with the latter holding as equality when the technology heterogeneity is effectively absent.}$$

$$(iii) \quad \frac{\partial i^*}{\partial \lambda^K} = \frac{aw\theta}{\lambda^{K2}\bar{b}^{*2}} > 0$$

Part (i) of proposition 1 considers a shift in the uniform wage rate. First note that an increase in w , for instance, does not affect \hat{b}^* , and hence the equilibrium fractions of industrial and money capitalists remain unchanged as can be verified in equations (13) and (14). Out of equilibrium, however, the increase in w leads to a rise in \hat{b} as can be seen in equation (5), which will raise the fraction of money capitalists, $F(\hat{b})$, and reduce the fraction of industrial capitalists, $1 - F(\hat{b})$, consequently creating an excess supply of credit; the latter presses down i and hence \hat{b} . This process continues until \hat{b} is brought down back to \hat{b}^* , at which point i is settled at a new, lower equilibrium rate. Accordingly, there is a negative relation between the wage rate and the interest rate as stated in part (i).

Part (ii) of proposition 1 considers a technical change. In the model, an increase in \bar{b} , for instance, with n held constant represents an economy-wide technology innovation since it reflects a parallel shift of the entire range $[\underline{b}, \bar{b}]$, implying a labor productivity improvement of all capitalists by the same degree and hence an increase in the average labor productivity.¹³ In contrast to a change in w , the increase in \bar{b} raises the equilibrium level of technology barrier, \hat{b}^* , and hence i^* rises as well. Note, however, that the equilibrium fractions of industrial capitalists and money capitalists remain unchanged since they are a function of λ^K only. What is going on here is that following the shift of the entire range $[\underline{b}, \bar{b}]$, the equilibrium thresholds \hat{b}^* and i^* are also shifting in parallel in a way that the equilibrium fractions of industrial capitalists and money capitalists remain the same.

On the other hand, the economy-wide improvement in labor productivity generates another effect of raising the wage rate as assumed in equation (2) that the uniform wage rate is a positive function of the average labor productivity. This second effect lowers the interest rate as suggested by part (i) of proposition 1. The overall result of the improvement in the average labor productivity depends on the responsiveness of the wage rate. The result of part (ii) suggests that in case the technology heterogeneity is effectively absent, an overall consequence of a technological innovation is a decrease in the equilibrium interest rate if the wage rate increases elastically in response, i.e. the labor productivity elasticity of wage being greater than one. However, when technology is effectively heterogeneous, an overall consequence of the technological innovation is a decrease in the equilibrium interest rate only when the wage rate is *sufficiently* elastic in the sense that the threshold of the elasticity of wage is greater than one; otherwise, the economy-wide technology innovation could lead to an increase in the equilibrium interest rate even when the wage rate is elastic.

Note that the degree of elasticity of wage required to offset the interest rate-raising effect of a technology innovation is greater when technology is effectively heterogeneous. This will be explained below in section 4 in relation to the consequence of technology innovation on the profit rate of total capital. Here, let us consider its implication. Since the responsiveness of wage to a labor productivity improvement is affected by, among others, class struggle

¹³Since a (the capital productivity) is held constant, the increase in labor productivity is, according to equation (1), a result of an increase in τ (the capital-labor ratio).

and the strength of labor union, the result of part (ii) can be considered as suggesting that in the economy with a strong labor union, a period of technological innovations is likely to be accompanied by lower interest rates, whereas in the economy with a weak labor union, the period of technology innovations is likely to be accompanied by higher interest rates.

Part (iii) of proposition 1 suggests that the leverage ratio of industrial capitalists is positively correlated with the equilibrium interest rate. The result is quite intuitive, but the underlying mechanism within the model needs some explanation. Note that all money capitalists are assumed to lend their entire money capital endowment without any borrowing. Therefore, as discussed earlier, having each industrial capitalist borrow more, while maintaining the same amount of lending by each money capitalist, requires an increase in the barrier i^* , and hence \widehat{b}^* , as it brings about the necessary change in the fractions of each sector in equilibrium, i.e. a decrease in the fraction of industrial capitalists and an increase in the fraction of money capitalists; it should be additionally noted that in these changes, as discussed in table 1, the industrial sector's size remains unchanged while its degree of capital concentration further intensifies, and the financial sector's size expands while its degree of capital concentration remains the same.

4 The average rates of profit

Let us now examine capital profitability both at an aggregate level and sectoral level to study the profit rate of total, social capital and compare the profitability of industrial and financial sectors.

4.1 The profit rate of total capital

Since the profit rate of total social capital, or the average profit rate of capital, is defined as total profits divided by total capital, and since it is only capitalists with technology of $b > \widehat{b}$ who operate capital, the profit rate of total social capital, denoted by R , is obtained as

$$R \equiv \frac{\int_{\widehat{b}}^{\bar{b}} r(b)K\theta^{-1}db}{\int_{\widehat{b}}^{\bar{b}} K\theta^{-1}db} \quad (16)$$

Compared to the formal models that examine the relation between the average profit rate and the interest rate, such as Lianos (1987) and Saros (2013), where the average profit rate is taken as given, in my model both rates are determined endogenously. More importantly, the dynamics of the average profit rate is not entirely independent of the interest rate; note from equation (16) that a shift in i leads to a change in \widehat{b} , thereby altering the composition of capitalists class and hence the distribution of the given total social capital among capitalists with different technology. This result of the interest rate directly affecting the profit rate

of total capital applies only out of equilibrium since, as will be shown shortly, both rates in equilibrium are determined by the parameters of the model. However, it reveals an important aspect of the model that finance matters for an overall dynamics of capital profitability, which will also be more clarified in the discussions below.

Solving the integrals and using the definitions of r and K along with the equilibrium levels of i^* and \widehat{b}^* yields the equilibrium level of the average profit rate of capital as follows:

$$R^* = a \left(1 - \frac{w}{\frac{\bar{b}-\widehat{b}^*}{\ln \bar{b}-\ln \widehat{b}^*}} \right) > 0 \quad (17)$$

Two observations about R^* are noteworthy. First, $R^* > 0$ always holds due to $\lambda^K \geq 1$, by definition, and $\underline{b} > w$ in assumption 1. Considering the fact that it is the industrial capitalists with more advanced technology who operate capital, the average profit rate of capital being positive is not surprising. Second, from the definition $r \equiv a \left(1 - \frac{w}{b} \right)$ and the fact that $\frac{\partial r}{\partial b} > 0$, it follows that the industrial capitalists with technology of $b \in \left[\widehat{b}^*, \frac{\bar{b}-\widehat{b}^*}{\ln \bar{b}-\ln \widehat{b}^*} \right)$ yield the profit rate of capital less than the average rate, i.e. $r < R^*$, while the industrial capitalists with technology of $b \in \left(\frac{\bar{b}-\widehat{b}^*}{\ln \bar{b}-\ln \widehat{b}^*}, \bar{b} \right]$ yield the profit rate of capital greater than the average rate, i.e. $r > R^*$. A industrial capitalist with labor productivity equal to $\frac{\bar{b}-\widehat{b}^*}{\ln \bar{b}-\ln \widehat{b}^*}$ yields $r = R^*$.¹⁴

How does the equilibrium profit rate of total capital responds to a permanent shift in the model parameters? The following proposition summarizes the results, which can be compared to the comparative statistics of the equilibrium interest rate in proposition 1.

Proposition 2 *In the model economy under assumption 1,*

$$(i) \quad \frac{\partial R^*}{\partial w} = -\frac{a}{\frac{\bar{b}-\widehat{b}^*}{\ln \bar{b}-\ln \widehat{b}^*}} < 0$$

$$(ii) \quad \frac{\partial R^*}{\partial \bar{b}} \geq 0 \iff \frac{\partial w/\partial \bar{b}}{w/\bar{b}} \leq \frac{\frac{\bar{b}}{b^*}-1}{\ln \frac{\bar{b}}{b^*}}, \text{ where } \frac{\frac{\bar{b}}{b^*}-1}{\ln \frac{\bar{b}}{b^*}} \geq 1, \text{ with the latter holding as equality when the technology heterogeneity is effectively absent.}$$

$$(iii) \quad \frac{\partial R^*}{\partial \lambda^K} \geq 0, \text{ where the equality holds when the technology heterogeneity is effectively absent.}$$

Proof. Parts (ii) and (iii) are proved immediately by relying on lemma 2. ■

Part (i) of proposition 2 describes a negative correlation between the wage rate and the profit rate, and the unique mechanism of the model requires some explanation. First recall

¹⁴Note that $\widehat{b}^* \leq \frac{\bar{b}-\widehat{b}^*}{\ln \bar{b}-\ln \widehat{b}^*} \leq \bar{b}$ holds due to lemma 2.

from equation (13) that a change in w has no impact on \widehat{b}^* . Therefore, the same fraction of industrial capitalists with the same technology distribution operate the same amount of total capital before and after the change in w . Accordingly, in the case of an increase in w , for instance, each of those industrial capitalists will experience a reduction in r , and the average of these decreased r 's will definitely be less than the average before the increase in w .

Part (ii) examines a consequence of an economy-wide technological innovation. Recall that, in the context of assumption 1, a shift in \bar{b} with θ held constant represents a change in labor productivity of all capitalists by the same degree and hence a change in the average labor productivity. According to the result in part (ii), how R^* responds depends on the elasticity of wage rate, which is assumed to be a positive function of the average labor productivity; see equation (2).¹⁵ More specifically, a technology innovation leads to a decrease in R^* if w is elastic, i.e. the elasticity of w greater than one, only when technology is effectively heterogeneous—in the sense of both $\theta > 0$ and $\lambda^K < \infty$ hold. If the technology heterogeneity is effectively absent—in the sense of either $\theta = 0$ or $\lambda^K = \infty$ holds—the technological innovation leads to a decrease in R^* only when w is *sufficiently* elastic; otherwise, an elastic increase in w in response to the technology innovation may not be strong enough to offset the profit rate-enhancing effect of the technology innovation and hence R^* can eventually rise.

Note that the degree of elasticity of wage needed to offset the profit rate-enhancing effect of a technology innovation is stronger when technology is effectively heterogeneous than when the technology heterogeneity is effectively absent. To see why, compare, on the one hand, R^* in the former case where the given total capital is reallocated to, and operated by, industrial capitalists ranging over $[\widehat{b}^*, \bar{b}]$ and, on the other hand, R^* in the latter case where the same given total capital is reallocated to, and operated by, a single industrial capitalist with the most advanced \bar{b} -technology. It can be verified that the direct effect of an across-the-board-improvement in labor productivity on R^* is greater in the former case. This is because r is a concave function of b , i.e. the smaller the b , the greater the (marginal) effect an improvement in b has on r . This explains the intuition behind the two different cases in part (ii). Note that this result may change if the homogeneous technology in the case of technology heterogeneity being effectively absent is not the most advanced one but one below it, i.e. $b \in [\underline{b}, \bar{b}]$.

Part (iii) of proposition 2 states that the industrial sector's uniform leverage ratio has a positive impact on the average profit rate of capital. As discussed above, each of industrial capitalists borrowing more is possible only when the fraction of industrial capitalists falls and the fraction of money capitalists rises, which occurs when \widehat{b}^* increases. When the barrier gets raised, the industrial capitalists at the bottom tiers are no more profitable and

¹⁵If w is assumed to be constant, an increase in \bar{b} unambiguously raises R^* , which is the result of a one-sector version of the Okishio Theorem.

hence fall out, converting to a money capitalist. This implies a further capital reallocation from less efficient capitalists to more efficient capitalists. Since money capitalists lend all of their capital, which therefore is finally operated by industrial capitalists, the volume of total social capital remains unchanged however the capitalist class is divided into the two groups. Accordingly, capital concentration and thus the scale of production are intensified in the industrial sector. In all, an increase in λ^K implies that each industrial capitalist in a smaller, but more efficient group operates a greater volume of capital. The consequence of this can be nothing but an increase in the average profit rate of capital, R^* , as suggested by part (iii).

The main driver here is the efficient reallocation of resources through credit market from less efficient capitalists to more efficient capitalists. Accordingly, if there is no technology difference among the capitalists, such reallocation would not operate any longer and hence a change in λ^K will have no impact on R^* , which is what the second part of proposition 2, (iii) suggests.

One of the essential aspects in Marx's discussion of credit is that the credit system is necessary for an expanded reproduction of capital and that it develops alongside the development of capitalist production. The result in part (iii) of proposition 2 provides a useful insight in formalizing a contribution of the credit system to capital profitability, which is the source of accumulation and growth. Imagine a counter-factual economy without a credit system so that there is no money capitalists and all capitalists manage their money capital endowment as industrial capital. The average profit rate of capital in this economy, denoted by R^X , is defined and computed as follows.¹⁶

$$R^{X*} \equiv \frac{\int_{\underline{b}}^{\bar{b}} rW\theta^{-1}db}{\int_{\underline{b}}^{\bar{b}} W\theta^{-1}db} = a \left(1 - \frac{w}{\frac{\bar{b} - (\bar{b} - \theta)}{\ln \bar{b} - \ln (\bar{b} - \theta)}} \right) \quad (18)$$

By comparing R^{X*} with R^* , the contribution of the credit system can be verified as in the following theorem.

Theorem 1 *Consider the model economy under assumption 1 without a credit market.*

- (i) *If technology is heterogeneous, opening a credit market necessarily strengthens the profit rate of total capital unless the industrial sector's borrowing is zero. Formally, $R^* > R^{X*}$ if both $\theta > 0$ and $\lambda^K > 1$ hold.*
- (ii) *If technology is homogeneous, opening a credit market does not alter the average profit rate of capital. Formally, $R^* = R^{X*}$ if either $\theta = 0$ or $\lambda^K = 1$ holds.*

¹⁶Since an equilibrium in the model refers to a credit market equilibrium, but since the counter-factual economy is without credit market, there is no distinction between R^X and R^{X*} .

Proof. Part (i) can be proved using the following three facts. First, $\lambda^K \in [1, \infty]$ by definition. Second, due to equations (17) and (18), $R^* = R^{X^*}$ holds when $\lambda^K = 1$. Third, according to part (iii) of proposition 2, on the one hand, $\frac{\partial R^*}{\partial \lambda^K} \geq 0$ holds with inequality in case of both $\theta > 0$ and $\lambda^K < \infty$, whereas it holds with equality in case either $\theta = 0$ or $\lambda^K = \infty$. On the other hand, $\frac{\partial R^{X^*}}{\partial \lambda^K} = 0$ can be verified from equation (18).

In all, from these three, it necessarily follows that $R^* > R^{X^*}$ holds if both $\theta > 0$ and $\lambda^K > 1$ simultaneously, which proves part (i).

Together with the second fact above, the following proves part (ii): For any $x, y \in \mathbb{R}$, it holds that $\lim_{y \rightarrow x} \frac{x-y}{\ln x - \ln y} = x$. Using this, it can be verified that $\lim_{\theta \rightarrow 0} R^* = \lim_{\theta \rightarrow 0} R^{X^*}$. ■

While lemma 1 proves the contribution of credit market to capital profitability from the perspective of individual capitalists, theorem 1 proves it from the perspective of total social capital. The intuition is clear. The credit market enables capitals to be reallocated from capitalists with less advanced technology to capitalists with more advanced technology. The consequence of this cannot but be an increase in the average profit rate of capital. Here, technology heterogeneity is essential since when all capitalists have the same technology, the capital reallocation would not make any difference.

4.2 The relation between profit rate and interest rate

The basic principle of Marxian theory of credit and interest is that the source of interests paid for loanable, interest-bearing capital is surplus value produced by industrial capitals. From this, Marx establishes two fundamental relationships between the profit rate and the interest rate. First, the profit (rate) sets a maximum limit of the interest (rate). Second, insofar as the share of interests out of total profits is not drastically volatile, but is relatively stable over the long-run period, the interest rate will more or less move in step with the average rate of profit, although the former can deviate from the latter along the business cycle fluctuations. Let us verify each of these relations in the model developed thus far.

First note that in the literature, the first relation is taken as one of the distinctions of Marxian theory from the standard economic theory, which conceptualizes the profit rate and the interest rate as essentially the same. For instance, Fine (1985) explains the difference between the two rates from the existence of barriers between industrial capital and financial capital. From the perspective of circuit of capital framework, Lapavitsas (1997) suggests that the difference between the rates is due to the “systematically different location of industrial and interest-bearing capital relative to the circuit of the total social capital” (p.105). Lapavitsas (1997) further comments that “It is evident, however, that the complete demonstration of the tendency of the rate of interest normally to lie below the rate of profit requires a considerably more complex analysis than these general considerations” (p.98). The model developed thus far provides such complete demonstration as summarized in the following

theorem.

Theorem 2 *In the model economy under assumption 1, it holds that $R^* \geq i^*$, where the equality holds when technology heterogeneity is effectively absent i.e. if either $\theta = 0$ or $\lambda^K = \infty$.*

Proof. Using equations (12) and (17), $R^* \geq i^*$ can be rearranged into $\frac{\bar{b}}{\bar{b}^*} - 1 \geq \ln \frac{\bar{b}}{\bar{b}^*}$. The conditions for each of the equality and inequality of the latter, according to lemma 2, are exactly identical with those in theorem 2, which therefore completes the proof.

The case of $R^* = i^*$ can also be directly proved by verifying $\lim_{\theta \rightarrow 0} R^* = \lim_{\lambda^K \rightarrow \infty} R^* = a \left(1 - \frac{w}{b}\right)$ and $\lim_{\theta \rightarrow 0} i^* = \lim_{\lambda^K \rightarrow \infty} i^* = a \left(1 - \frac{w}{b}\right)$. ■

The reason why the profit rate of total capital is greater than the interest rate in the model is because it is only those capitalists with more advanced technology, satisfying $r > i$, who operate capital. Therefore, it is not surprising that the average of these r 's exceeds i in equilibrium, whereas it equals i in equilibrium, i.e. $R^* = i^*$, when the technology heterogeneity is effectively absent. The latter result is a direct consequence of corollary 1, according to which when all capitalists have the same technology and hence the same profit rate of capital, the equilibrium interest rate will be set equal to the capital profit rate, i.e. $i^* = r$.

Next, regarding the second relation about the co-movement between the two rates, formal models found in the literature usually take the profit rate as given and derive the interest rate as a positive function of the profit rate; e.g., see Lianos (1987) and Saros (2013). In comparison, in the model of this paper, not only the interest rate but also the profit rate—more precisely, the average profit rate—are determined endogenously within the model. In this setting, the co-movement between the two rates can be verified by confirming that the two rates respond to a permanent shift in the model parameters in the same direction. This can be done by comparing propositions 1 and 2. From this, the following theorem is derived.

Theorem 3 *In the model economy under assumption 1, a variation of R^* , in response to a permanent shift in the model parameters, i.e., w , \bar{b} , and λ^K , leads to a variation of i^* in the same direction.*

Marx also considers short-run business cycle behaviors of the interest rate where moving in the opposite direction from the profit rate is possible. For instance, when the economy is in a stage of expansion marked by a strong capital profitability, the interest rate tends to be very low; in contrast, the economy that is starting to fall or in recession with very weak capital profitability, is likely to experience a spike in the interest rate. However, since the source of interest payments is profit, the deviation cannot be sustained for a prolonged period, and the long-run development of the interest rate is ultimately guided by the profit

rate. As a way to highlight this, Marx suggests that there is a tendency for the rate of interest to fall.¹⁷

The relationships of between the profit rate and the interest rate Marx established and confirmed in theorems 2 and 3 have an empirical support. Using the U.S. data for 1896-2009, Valle Alejandro and Mendieta Muñoz (2012) find that both nominal and real interest rates—both short-term and long-term—consistently lie below the normal profit rate during the whole period with a few exceptional years, and that the long-term real interest rate and the normal profit rate moved along the similar trends with the correlation of 0.14.

4.3 The sectoral rates of profit

The average ROE of the industrial sector, denoted by R^K , is industrial capitalists' total net profit—after interest payments—divided by their total endowment:

$$R^K \equiv \frac{\int_{\hat{b}}^{\bar{b}} (r(b)K - iB)\theta^{-1}db}{\int_{\hat{b}}^{\bar{b}} W\theta^{-1}db} = \lambda^K R - (\lambda^K - 1)i \quad (19)$$

where the second equality is obtained by solving the integrals, using the definitions of \hat{b} , r , K , and B in equations (5), (3), (6), and (7). The average ROE of the financial sector, denoted by R^F , is total interests paid by industrial capitalists divided by total endowment of money capitalists:¹⁸

$$R^F \equiv \frac{\int_{\hat{b}}^{\bar{b}} iB\theta^{-1}db}{\int_{\hat{b}}^{\bar{b}} W\theta^{-1}db} \quad (20)$$

When the money capital market is in equilibrium, each of the sectoral average ROE becomes, respectively,

$$R^{K*} = \lambda^K R^* - (\lambda^K - 1)i^* \quad (21)$$

$$R^{F*} = i^* \quad (22)$$

¹⁷To quote the statements related to the discussions here: “Since we have seen that the level of the profit rate stands in inverse proportion to the development of capitalist production, it follows that the higher or lower rate of interest in a country stands in the same inverse proportion to the level of industrial development, particularly in so far as the variation in the rate of interest expresses an actual variation in the profit rate. We shall see later on that this need by no means always be the case. In this sense one can say that interest is governed by profit, and more precisely by the general rate of profit. And this kind of regulation applies even to its average. At all events, the average rate of profit should be considered as ultimately determining the maximum limit of the interest” (Marx, 1981, p. 481).

¹⁸In contrast to R^K in equation (19), solving equation (20) does not yield any meaning result, which therefore is omitted here. However, its equilibrium level has a meaningful expression as shown in equation (22).

which are derived by substituting \widehat{b}^* and i^* into equations (19) and (20). Using equations (21) and (22), a comparison can be made between the two sectors' profit rates in relation to the average profit rate of capital and the interest rate as in the following theorem.

Theorem 4 *In the model economy under assumption 1, it holds that $R^{K^*} \geq R^* \geq R^{F^*}$, where the equalities hold in case the technology heterogeneity is effectively absent.*

Note that theorem 4 is an aggregate version of lemma 1 and corollary 1. The financial sector's average ROE is exactly equal to the interest rate, which necessarily follows from $r^e = i$ for each of money capitalists. This is due to the basic setting of the model that assumes money capitalists lend all of their equity endowment and do not borrow. In contrast, the industrial sector's average ROE is not only greater than the interest rate; it is also greater than the average profit rate of capital. This result also necessarily follows from $r^e > r > i$ for each of industrial capitalists.

The industrial capitalists are able to use leverage to amplify the underlying capital profit rate, r , whereas this is ruled out for the money capitalists, since the latter are capitalists who have chosen to become money capitalists not because they have an expertise or technology specialized in managing interest-bearing capital, but simply because they have weak production technology. This setup of the model has placed the money capitalists in a position to passively accept the fallback option, i.e. lending. The financial sector in the real world economies, however, is characterized by specialized financial intermediation technology. As one possible way of extending the benchmark model presented thus far, in the next section, I introduce financial intermediation technology.

5 An extension: financial intermediation

Not only in Marxian theory in particular but also in economic theory in general, financial intermediation is essentially about borrowing at a low interest rate and lending at a high interest rate, reaping the difference between the interest receipt and the interest payment as bank profit. In this section, the benchmark model is extended to incorporate financial technology, which enables the financial sector to borrow, and possibly achieving profitability stronger than that of the industrial sector.

5.1 A divergence of the capitalist class

In addition to a continuum of capitalists equipped with homogeneous endowment and heterogeneous production technology, there are now what Marx called bankers, or banking capitalists, equipped with financial intermediation technology. More specifically, bankers have specialized skills and expertise in managing loanable, interest-bearing capital, such as monitoring and screening borrowers, collecting interest and principal, etc. In contrast to the

case of capitalists equipped with production technology, the financial technology distributed among bankers is homogeneous and bankers do not have initial endowment.¹⁹

The financial intermediation technology enables the banking capitalists to raise funds from those without the financial technology at an interest rate lower than the rate they charge on lending. Let us denote each of these interest rates, respectively, by i^D and i^L with $i^D < i^L$. Since the financial technology is homogeneous, these interest rates apply equally across bankers. To simplify the analysis, it is assumed that i^D is set by the bankers, whereas i^L is determined by the demand and supply relation in the bank loan market.

The capitalists who decide to operate part or all of their endowment as interest-bearing capital now use the intermediation service, instead of lending themselves directly. In addition to the money capitals obtained from money capitalists, suppose the banking capitalists can raise funds elsewhere. The sources of the additional funding could be savings of workers, government, or even foreign countries, etc. Note that an essential aspect of innovations in financial technology is raising funds without affecting the cost by creating various new types of debt securities. Accordingly, an increase in the additional funding, other than from money capitalists, can be considered as an outcome of financial innovations.

Let us suppose that the funds the bankers raised from money capitalists are in the form of equity, whereas the funds raised elsewhere is in the form of debt. Since money capitalists share the ownership in the financial intermediaries, money capitalists and banking capitalists can be grouped into financial sector. To simplify the analysis, let us assume that the shareholders of the banks are rewarded by the same rate as their creditors, which implies that the cost of debt and the cost of equity are equal to each other at i^D .

In this setup, when a capitalist with production technology borrows B , and invest the total funds, $W + B$, in a productive activity by K and in a bank by M , the return on equity is computed as

$$r^e \equiv \frac{rK + i^D M - i^L B}{W} = \frac{(r - i^D)K - i^L B}{W} + i^D \quad (23)$$

where the second equality uses the budget constraint $K + M = B + W$. The capitalist makes a portfolio decision and a capital structure decision in a way that maximizes the ROE. More specifically, for a capitalist with technology yielding $r < i^D$, it is optimal to invest the entire endowment at a bank, i.e. $K = 0$ and $M = W$, without any borrowing, $B = 0$; in this case, the capitalist's ROE is $r^e = i^D$. For a capitalist with technology yielding $r > i^D$, it is optimal to operate industrial capital of the largest possible scale without any financial investment, i.e. $K > 0$ and $M = 0$. However, since the scale of industrial capital beyond the endowment is financed by borrowing the optimal scale of industrial capital and hence the optimal borrowing depends on the marginal benefit and marginal cost of borrowing. That is, on the one hand, for a capitalist with $i^D < r < i^L$, it is optimal to operate industrial capital without any borrowing, i.e. $B = 0$ and $K = W$, achieving the ROE of $r^e = r$; on the other

¹⁹These assumptions are not essential to the main results of the model, but only simplify the analysis.

hand, for a capitalist with $r > i^L$, the optimal scales of industrial capital and borrowing are unbounded, and the corresponding ROE is $r^e = r + (r - i^L)\frac{B}{W}$, i.e. $r^e > r$ and r^e increasing as B rises.

In this optimal decision making process, there are two interest rate barriers instead of one as in the benchmark model; one is i^D , which is the threshold for deciding whether to make a productive or financial investment, and the other is i^L , which is the threshold for deciding whether to borrow or not once a decision is made to engage in productive activity. Accordingly, there are now two corresponding technology barriers, each of which is obtained as follows, respectively.

$$\hat{b}^D = \frac{aw}{a - i^D}, \quad \hat{b}^L = \frac{aw}{a - i^L} \quad (24)$$

Production technology with $b < \hat{b}^D$ is considered too weak to be activated profitably; production technology with $\hat{b}^D < b < \hat{b}^L$ is considered strong enough to be activated profitably but not as strong to warrant leveraged investment; labor productivity of $b > \hat{b}^L$ is considered as sufficiently strong to be activated profitably additionally financed by leverage.

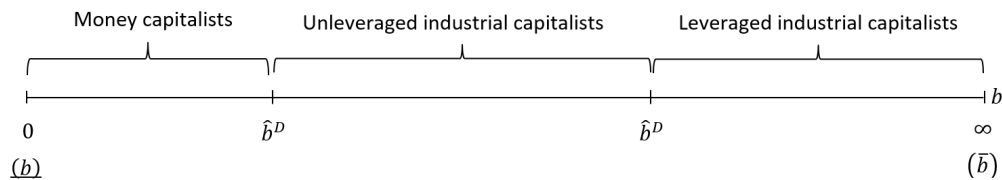
Lemma 3 summarizes the divergence of the capitalist class equipped with production technology in the extended model with financial intermediation.

Lemma 3 *A capitalist equipped with endowment W and $r(b)$ -technology makes the following optimal decisions regarding portfolio and capital structure:*

- (i) *If $b < \hat{b}^D$ ($r < i^D$), then $M = W$ and hence $K = 0$, i.e. the capitalist becomes a money capitalist, and earns the ROE of $r^e = i^D$.*
- (ii) *If $b > \hat{b}^D$ ($r > i^D$), then $M = 0$, i.e. the capitalist become an industrial capitalist, and*
 - a. *if $b < \hat{b}^L$ ($r < i^L$), then $B = 0$, eventually $r^e = r$,*
 - b. *if $b > \hat{b}^L$ ($r > i^L$), then $B > 0$, eventually $r^e = r + (r - i^L)\frac{B}{W}$.*

The result of lemma 3 is visualized in figure 2. It can be seen that in comparison to the benchmark model in figure 1, the capitalist class equipped with production technology now diverges into money capitalists, unleveraged industrial capitalists, and leveraged industrial capitalists. From this, another important difference follows. In the benchmark model, on the one hand, a change in the fraction of capitalists supplying money capital automatically leads to a change in the fraction of capitalists demanding money capital, and vice versa. In the extended model, on the other hand, this is not true since there are now two distinct thresholds. This result has an important consequence on the equilibrium analysis below.

Figure 2: The range of labor productivity, b , and the technology thresholds, \hat{b}^D and \hat{b}^L .



5.2 The equilibrium in the market for money capital

Suppose the leverage ratio of the banking sector—or, financial sector including money capitalists sharing the ownership of the banks—is λ^F . Since the total equity of the banking sector equals the sum of endowment of capitalists with technology of $b < \hat{b}^D$, the total supply of bank loan is

$$S = \lambda^F \int_0^{\hat{b}^D} W dF(b) \quad (25)$$

There are two ways through which the banking capitalists can increase the loan supply. First is to adjust the return paid on the funds, equity and debt, they raise. Consider, for instance, the banking sector increasing i^D . According to lemma 3, the industrial capitalists in the bottom tier will find it not profitable any longer to operate industrial capital and convert to a money capitalist, thereby increasing the fraction of money capitalists and hence equity of the banking sector. This implies, with the banking sector's leverage ratio held constant, an increase in the banking sector's debt as well. With the increase in both debt and equity, the balance sheet of the banking sector expands, implying an increase in the loan supply.

While the first way of expanding the banking sector balance sheet is costly as it requires increasing i^D , there is an alternative way which does not require raising the funding cost. That is, the banking capitalists can raise additional funds costlessly by creating various new types of debt securities, which could attract new debt-holders without increasing the interest rate on the debt; consequently, the banking sector's leverage ratio rises. An important aspect of financial innovations is to create new debt securities, which allow banks to raise leverage without affecting the borrowing cost. This second way of increasing the loan supply through innovations in financial technology in the model is reflected in an increase in λ^F .

While the banking capitalists' leverage ratio affects the bank loan supply, the industrial capitalists' leverage ratio affects the demand for bank loan. However, there is one important difference from the benchmark model related to the fact that now not all, but only some of the industrial capitalists borrow. As a result, the industrial sector's leverage ratio is not equal to that of an individual leveraged industrial capitalist. To make a distinction, let us denote the former by λ^K and the latter by λ^k . Accordingly, the total demand for bank loan, the sum of demands by capitalists equipped with production technology of $b > \hat{b}^L$, is

expressed as

$$D = \int_{\widehat{b}^L}^{\infty} (\lambda^k - 1)WdF(b) \quad (26)$$

The market-clearing loan interest rate is obtained as a solution to $S = D$. Similar to the benchmark model, for the purpose of obtaining an analytical solution let us adopt assumption 1, which describes a uniform distribution of b over $[\underline{b}, \bar{b}]$. Accordingly, the equilibrium of the market for bank loan is established as follows:

$$\lambda^F \int_{\underline{b}}^{\widehat{b}^D} W\theta^{-1}db = \int_{\widehat{b}^L}^{\bar{b}} (\lambda^k - 1)W\theta^{-1}db \quad (27)$$

Solving equation (27) for i^L yields the market-clearing lending interest rate as

$$i^{L*} = a \left(1 - \frac{w}{\widehat{b}^{L*}} \right) > 0 \quad (28)$$

where

$$\widehat{b}^{L*} = \bar{b} - \frac{\widehat{b}^D - \underline{b}}{\frac{\lambda^k - 1}{\lambda^F}} > 0 \quad (29)$$

While the demand and supply relations determine i^L and hence \widehat{b}^L as above, bankers choose i^D and by doing so, also choose \widehat{b}^D according to equation (24). In this setup, when adopting assumption 1, it needs to be ensured that both \widehat{b}^D and \widehat{b}^{L*} lie in a reasonable range. For this, two additional assumptions are adopted. The first is

Assumption 2 $i^D > r(\underline{b})$

which is equivalent to $\widehat{b}^D > \underline{b}$. Assumption 2 implies that banking capitalists set i^D above the capital profit rate of the least advanced technology since otherwise, no capitalist would invest endowment in banks. In addition, to ensure $i^D < i^{L*}$, the following assumption is also adopted.

Assumption 3 $\lambda^k > 1 + \lambda^F \left(\frac{\widehat{b}^D - \underline{b}}{\underline{b} - \widehat{b}^D} \right)$

which also ensures $b^D < b^{L*}$. Assumption 3 implies that in equilibrium each individual leveraged industrial capitalist must take out some positive level bank loan if $\widehat{b}^D > \underline{b}$, i.e. if banks raise funds to supply loans, as is the case due to assumption 2; otherwise, the bank loan market cannot clear. Assumptions 2 and 3 together ensure $\underline{b} < \widehat{b}^D < \widehat{b}^{L*} < \bar{b}$.²⁰

How would the market-clearing lending interest rate respond to a permanent shift in the model parameters? In conducting the comparative statistic analysis of i^{L*} , let us focus on the financial parameters, i^D , λ^F , and λ^k . Proposition 3 summarizes the results.

Proposition 3 (i) $\frac{\partial i^{L*}}{\partial i^D} < 0$, (ii) $\frac{\partial i^{L*}}{\partial \lambda^F} < 0$, (iii) $\frac{\partial i^{L*}}{\partial \lambda^k} > 0$

²⁰Recall that in the benchmark model, assumption 1 was enough to ensure $\underline{b} \leq \widehat{b}^* \leq \bar{b}$.

Parts (i) and (ii) examine a shift in the banking sector's funding cost and leverage ratio, respectively, which are the parameters bankers can control to adjust the loan supply. As discussed above, raising either of the two leads to an increase in the loan supply through increasing bank funding; raising i^D increases both equity and debt, whereas raising λ^F increases debt. With the leverage ratio of each leveraged industrial capital held constant, the increase in the supply of bank loan must be followed by an expansion of the number of leveraged industrial capitalists so that the bank loan market clears. This happens when there is a reduction in the barrier for becoming a leveraged industrial capitalist; hence a decrease in i^{L*} (and \widehat{b}^{L*}). This is what parts (i) and (ii) of proposition 3 suggest.

Part (iii) examines a shift in λ^k . Consider a case of, for instance, having each leveraged industrial capitalists borrow more. Since i^D (and hence \widehat{b}^D) and λ^F are held constant, implying that the supply of bank loan is fixed, the only way to clear the market is to reduce the number of leveraged industrial capitalists. This happens when the barrier for becoming a leveraged industrial capitalist is raised; hence an increase in i^{L*} (and \widehat{b}^{L*}).

As the loan interest rate is the source of bank profits, the results in proposition 3 have a direct implication on the banking sector's profitability. In particular, comparing parts (i) and (ii) shows that the two different ways of increasing the bank loan supply yield the same result of lowering i^{L*} , which is as expected. However, as will be shown below, they affect the banking sector's profitability differently.

Before proceeding, let us examine the industrial sector's average leverage ratio, which now is not equal to the leverage ratio of individual leveraged industrial capitalists but is endogenously determined. It is defined as the sum of industrial capitals operated by unleveraged industrial capitalists (with technology of $b \in [\widehat{b}^D, \widehat{b}^L]$) and by leveraged industrial capitalists (with technology of $b \in [\widehat{b}^L, \bar{b}]$), divided by their total endowment. In equilibrium, it is computed as

$$\lambda^{K*} \equiv \frac{\int_{\widehat{b}^D}^{\widehat{b}^{L*}} W\theta^{-1}db + \int_{\widehat{b}^{L*}}^{\bar{b}} \lambda^k W\theta^{-1}db}{\int_{\widehat{b}^D}^{\widehat{b}^{L*}} W\theta^{-1}db + \int_{\widehat{b}^{L*}}^{\bar{b}} W\theta^{-1}db} = \frac{\lambda^k(1 + \lambda^F) - 1}{\lambda^k + \lambda^F - 1} \quad (30)$$

It can be immediately verified that $\frac{\partial \lambda^{K*}}{\partial \lambda^k} > 0$ and $\frac{\partial \lambda^{K*}}{\partial \lambda^F} > 0$, which are as expected. That is, when individual leveraged capitalists borrow more or when the banking sector borrows more, the industrial sector's average leverage ratio will increase.

5.3 The average rates of profit

The profit rate of total social capital, or the average profit rate of capital, is defined as total profits produced by leveraged and unleveraged industrial capitalists divided by the sum of

industrial capitals operated by them.

$$R \equiv \frac{\int_{\hat{b}^D}^{\hat{b}^L} r(b)WdF(b) + \int_{\hat{b}^L}^{\infty} r(b)KdF(b)}{\int_{\hat{b}^D}^{\hat{b}^L} WdF(b) + \int_{\hat{b}^L}^{\infty} KdF(b)} \quad (31)$$

Under assumption 1, using the definitions of $r(b)$ and K , the equilibrium average profit rate of capital is computed as follows.

$$R^* = a \cdot \frac{(\hat{b}^{L*} - \hat{b}^D) - w(\ln \hat{b}^{L*} - \ln \hat{b}^D) + \lambda^k[(\bar{b} - \hat{b}^{L*}) - w(\ln \bar{b} - \ln \hat{b}^{L*})]}{(\hat{b}^{L*} - \hat{b}^D) + \lambda^k(\bar{b} - \hat{b}^{L*})} \quad (32)$$

Referencing equation (17), it can be verified that R^* in equation (32) is a weighted average of the average profit rate of capital of unleveraged industrial capitalists and the average profit rate of capital of leveraged industrial capitalists, where the weights are the share of each group's capital stock.

One of the important results of the benchmark model was that finance matters for the profit rate of total social capital. In part (iii) of proposition 2, it was shown that the leverage ratio of industrial capitalists affects the rate at which the total social capital yields profits. The underlying mechanism was that the credit relation affects the way in which a given total social capital is distributed among capitalists with different technology.

The similar result holds in this extended model with financial intermediation, which can be verified by comparative statistic analyses of R^* with respect to the three financial parameters of the model, i.e. i^D , λ^k , and λ^F . The results are much more complicated than the simple result of part (iii) of proposition 2 and therefore are not reported here due to space limit. However, the main idea of a change in any of the financial parameters affecting the profit rate of total social capital can be briefly summarized as follows. A shift in any of i^D , λ^k , and λ^F changes either of the two barriers, \hat{b}^D and \hat{b}^L and, consequently, there would be a change in the composition of the capitalist class, i.e. money capitalists versus industrial capitalists, and leveraged versus and unleveraged industrial capitalists as suggested in lemma 3 and visualized in figure 2. These changes lead to a change not only in the total social capital, by affecting the banking sector's equity and debt; they also reshape the distribution of it among capitalists equipped with different production technology, thereby eventually changing the profit rate of total social capital.

In what follows, let us focus on the sectoral profitability. Similar to equations (19) and (20), the industrial sector's average ROE and the financial sector's average ROE are, respectively, defined as follows.

$$R^K \equiv \frac{\int_{\hat{b}^D}^{\hat{b}^L} r(b)WdF(b) + \int_{\hat{b}^L}^{\infty} [r(b)K - i^L B]dF(b)}{\int_{\hat{b}^D}^{\hat{b}^L} WdF(b) + \int_{\hat{b}^L}^{\infty} WdF(b)} \quad (33)$$

$$R^F \equiv \frac{\int_{\widehat{b}^L}^{\infty} B i^L dF(b) - i^D (\lambda^F - 1) \int_0^{\widehat{b}^D} W dF(b)}{\int_0^{\widehat{b}^D} W dF(b)} \quad (34)$$

Under assumption 1, using the definitions of $r(b)$, B , and K , these sectoral profitability in equilibrium are rearranged into, respectively,

$$R^{K*} = R^* \left(\frac{(\widehat{b}^{L*} - \widehat{b}^D) + \lambda^k (\bar{b} - \widehat{b}^{L*})}{\bar{b} - \widehat{b}^D} \right) - i^{L*} (\lambda^k - 1) \left(\frac{\bar{b} - \widehat{b}^{L*}}{\bar{b} - \widehat{b}^D} \right) \quad (35)$$

$$R^{F*} = \lambda^F i^{L*} - (\lambda^F - 1) i^D \quad (36)$$

Two results are derived using the sectoral profitability. The first is related to proposition 3, which examines how the equilibrium loan interest rate responds to a permanent shift in the financial parameters. Since i^L is the source of bank profits, it remains to be seen how the financial sector profitability would be affected. The results are summarized in the following proposition.

Proposition 4 (i) $\frac{\partial R^{F*}}{\partial i^D} < 0$, (ii) $\frac{\partial R^{F*}}{\partial \lambda^F} > 0$, (iii) $\frac{\partial R^{F*}}{\partial \lambda^k} > 0$

First consider the two parameters, i^D and λ^F , the banking capitalists have under control to adjust either debt or equity in financing the supply of loan. As shown in proposition 3, an increase in each of the two parameters yields the same result, i.e. a reduction in i^{L*} since both of them raise the loan supply. However, according to proposition 4, they affect the financial sector profitability differently. In case the banking capitalists raise i^D to attract additional equity and debt, while maintaining the same leverage ratio, thereby expanding the balance sheet by supplying more loans, since i^{L*} falls, it is as expected that R^{F*} falls as well, as suggested in part (i), because the interest rate spread and hence net interest margin are squeezed.

In contrast, consider the case where the banking capitalists increase the loan supply by raising the leverage ratio through financial innovations, which allow them to borrow more without raising i^D . Although i^{L*} is reduced due to the increase in the loan supply, as long as it does not fall below i^D —which is ruled out by assumption 3—the banking capitalists can lend more by increasing the leverage and enjoy the profitability-amplifying effect of leverage. This is what part (ii) suggests.

According to part (iii), the industrial sector leverage has a positive impact on the financial sector's profitability. For instance, an increase in λ^k —which raises λ^{K*} —makes i^{L*} rise as shown in proposition 3, (ii), which means, with i^D held constant, a widening of the interest rate spread and hence net interest margin. Consequently, the average ROE of the financial sector rises.

One of the central messages of proposition 4 is that the banking capitalists can strengthen their profitability by relying on innovations in financial technology—reflected in an increase

in λ^F —which allow them to borrow more at zero additional cost and thus magnify the profitability. The bankers have another strategy to raise funds, i.e. raising i^D , which attracts additional debt and equity to flow into the banking sector, which in turn can be used in supplying more loans and reaping more interest earnings. However, this alternative strategy squeezes the net interest margin and hence has a negative impact on the financial sector’s average ROE. Accordingly, the financial sector strenuously pursues continuous financial innovations to maximize its profitability.

This is the central difference from the benchmark model, where the financial sector lacks financial technology and hence cannot use leverage to amplify its profit rate, due to which the financial sector’s average ROE was equal to the interest rate and was less than that of the industrial sector. In contrast, this result will not hold anymore once the financial sector is equipped with financial intermediation technology and rely on financial innovations to stretch the profitability as in this extended model. This can be verified by comparing R^{F*} with i^{L*} and R^{K*} . The result is summarized in the following theorem.

Theorem 5 (i) $R^{F*} > i^{L*}$, (ii) $R^{K*} \gtrless R^{F*} \iff \lambda^F \lesseqgtr \frac{R^{K*}-i^D}{i^{L*}-i^D}$

Part (i) suggests that the financial sector’s average ROE is greater than the loan interest rate, the source of bank earnings. This result compares with theorem 4 of the benchmark model, according to which the financial sector’s ROE was equal to the interest rate. The central factor that allows the financial sector to amplify the profitability, making a difference from the benchmark case, is its intermediation technology, which enables borrowing at a cost lower than the loan interest rate. In addition, the profitability amplification is intensified through innovations in financial technology, reflected in an increase in λ^K . In this relation, part (ii) suggests that when the financial sector’s leverage ratio is sufficiently large it may be more profitable than the industrial sector.²¹

The result in theorem 5 can be interpreted as suggesting that one important aspect of what Marx described as a conflict between functioning capitalists and bankers surrounding the division of gross profit into interest and profit of enterprise is the strenuous effort on the part of the bankers towards continuous financial innovations.

6 Conclusion

According to the Dymski (1990)’s taxonomy of economic theories of money and credit, briefly mentioned in section 1, the model presented in this paper can be categorized as having the following aspects. First, there is no uncertainty and capitalists—both industrial and financial—obtain non-stochastic returns. Second, the optimal levels of industrial capital and

²¹In interpreting the result of part (ii) it needs to keep in mind that both R^{L*} and i^{L*} are a function of λ^F and i^D .

interest-bearing capital are greater than the endowment of industrial capitalists and money capitalists, respectively; in this sense, both industrial capital and interest-bearing capital are finance-constrained and hence credit system is essential. Third, however, the credit is extended only for the purpose of financing capital accumulation; the possibility of credit expansion for speculations, independently of capital accumulation, which could end up with major disruption, is ruled out.

Bearing these in mind, the benchmark model presented in this paper can be extended in various ways, drawing upon related modeling strategies in the literature on banking and finance. A couple of ideas are briefly sketched here. First, the timing of the model can be altered from one-period to, at least, two-period, thereby distinguishing between short-term and long-term. In this setup, following Holmstrom and Tirole (1998), industrial capital's turnover cycle of production and realization can be supposed to take two periods, whereas industrial capitalists face expenditure needs in one period such as wage expenditure, operating costs, or repayment of short-term debt. Similarly, a maturity mismatch in banking capitalists' balance sheet can also be explicitly formalized as in Diamond and Dybvig (1983)'s classic model. Accordingly, there will emerge the demand for money, or liquidity, by both industrial and banking capitalists, which was absent in my model, and, consequently, it will be possible to explicitly formalize Marx's important distinction between money as money (means of payment) and money as money capital.

In addition, uncertainty can be introduced by making the return from industrial capital stochastic, which in turn will make the returns for banking capitalists uncertain as well. Or, in a three-period model, a liquidity shock arriving in an intermediate period can be formalized, following the tradition of Diamond and Dybvig (1983). In case the probability distribution of the shock is public information and capitalists are risk-neutral, a realization of downside risks will generate a liquidity crisis or the flight to quality, resembling a type of crisis Marx dramatically describes as a moment when the economy all together questions the moneyness of credit and flock to what is considered as the true money.

Incorporating these into the model of this paper will enable grasping broader dimensions of Marxian theory of credit in relation to accumulation and crisis. Park (2019) is part of this ongoing research.

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