Living on Empty

Roger Schwarzschild

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Section 1

Following the account in Barwise & Cooper (B&C), one arrives at the following sentence=truth-value pairs:

(1) Every negative number that is greater than 7 is prime. (TRUE)
(2) Some negative number that is greater than 7 is prime. (FALSE)
(3) The smallest negative number that is greater than 7 is prime. (UNDEFINED)
(4) Four negative numbers that are greater than 7 are prime. (FALSE)

In each of these examples the set A denoted by the common noun head of the subject NP (henceforth 'the base set'), is empty. (Note: I have used mathematical examples in order to avoid questions about whether A will always be empty through time.) The problem is that our intuitions are much more uncertain than the assignments in (1) and (2) would indicate, especially given that we are allowing quantifiers to be undefined as in (3). (I am assuming that if Q is undefined than X ϵ Q is also undefined and therefore sentence (3) is undefined. Perhaps B&C would disagree.) It is not that I would make the reverse assignments, its just that I don't find them intuitively obvious. I think things do not in general improve when one considers some of the other determiners such as "most", "few", and the natural numbers. In fact with some determiners the base set may be non-empty and still yield no clear intuitions, for example when A has two members and is used with "most":

(4) Most of the Senators from Utah voted against the bill.

At this point one might contend that I am merely pointing out well-known conditions for the use of these determiners which are knowingly suspended by B&C for the purpose of giving an account of the semantics. The problem is that these intuitions seem to play a central role in their analysis. If one has unclear intuitions about these sentences why is it the case that intuitions are so much clearer with respect to things that depend on them in B&C's account?

In the next section I will review two parts of the B&C analysis that demonstrate the importance of these intuitions. In the following section I will suggest a way to alter the B&C account slightly such that the troublesome examples are replaced by clearer ones as the starting point for their analysis.

Section 2
First example.

There-insertion

The terms "strong" and "weak" borrowed from Milsark (1977) are defined by B&C as follows:

> "A determiner is 'positive strong' (or 'negative strong' resp.) if for every model $M = \langle E, \| \; \| \rangle$ and every A, a subset of E, if the quantifier $\|D\|$ (A) is defined then A is a member of $\|D\|$ (A) (Or A is not a member of $\|D\|$ (A), resp.) . If D is not positive or negative strong then D is 'weak'. "

The problem with this definition is that in the cases in which A is non-empty, a sentence translated as "A is/is not a member of $\|D\|$(A), is strange when D is weak. Thus assuming there are plenty of ducks all of the following are true according to B&C:

(5) Some ducks are ducks.
(6) A duck is a duck.
(7) Three ducks are ducks.
(8) Many ducks are ducks.
(9) No ducks are not ducks.
(10) Few ducks are not ducks.

Furthermore B&C's definition of strong actually distinguishes determiners based on the most unintuitive cases, for example when A is null in "$\|$some$\|$(A)". This is in contrast to the clear judgments of the there-insertion sentences that are being explained.

Second example. The intersection condition.

> "DEFINITION. D satisfies the "intersection condition" if for all models $M = \langle E, \| \; \| \rangle$ and all A and X, subsets of E,
>
> $$X \in \|D\| \text{ (A)} \quad \text{iff} \quad X \in \|D\| \text{ (A} \cap \text{X)}$$
>
> PROPOSITION. Strong determiners do not satisfy the intersection condition. " B&C, page 190

First of all, note again that the relevant data is not extremely intuitive:

(12) Every man that loves Mary, loves Mary. (VALID)
(13) Most men that love Mary, love Mary. (VALID)
(14) Some men that love Mary, love Mary. (CONTING.)
(15) No man that loves Mary, loves Mary. (CONTING.)

According to B&C, (14) is true and (15) is false, just in case there is someone that loves Mary, though this does not matter for (12) and (13). The hardest one for me to accept is that (15) is not contradictory, given that (12) is valid. Again, its not that I would make different assignments it just that things are not as clear for me.

In the appendix, B&C show that a determiner satisfies the definition of intersection if and only if it is symmetric. A determiner D is symmetric if: $A \in \|D\| (B)$ iff $B \in \|D\| (A)$.

Now whether or not a determiner is symmetric can be tested without resorting to quantifiers living on empty:

(16) a. Every man is an astronaut.
b. Every astronaut is a man.
(17) a. Most men are astronauts.
b. Most astronauts are men.
(18) a. Some man is an astronaut.
b. Some astronaut is a man.
(19) a. No man is an astronaut.
b. No astronaut is a man.

The a. sentence clearly implies and is implied by the b. sentence in (18) and (19) but not in (16) and (17).

The way B&C have presented their analysis, the judgments for (16-19) follow in some sense from those for (12 -15), yet the former are unclear and the latter not.

I think it is particularly instructive to see how B&C deal with the definite determiners, **the** and **both** , in their discussion of intersection. What is interesting for us is that these determiners are undefined for certain sets even on B&C's account.

To begin with, they make the claim that **the** and **both** are non-intersective, since the following are valid:

(20) Both men that love Mary, love Mary. (= 32)
(21) The man that loves Mary, loves Mary. (= 33)
(22) The three men that love Mary, love Mary. (= 32)

But given their definitions of **the** and **both**, each of these sentences are undefined in some models hence not valid. One might suggest, that B&C intend for us to ignore cases in which the quantifier is undefined. But that will lead to other problems as follows. (20-22) show that **both** and **the** are non-intersective, given that we are ignoring sentences with undefined quantifiers. B&C show that a determiner satisfies the intersection condition iff it is symmetric. Now consider the following pair:

(23) Both astronauts are men.
(24) Both men are astronauts.

Assuming that both (23) and (24) are defined, they are both true. Likewise for the following pair:

(25) The boy is a pitcher.
(26) The pitcher is a boy.

If both (25) and (26) are defined, then both are true. The conclusion then, is that if we only consider sentences where the quantifier is defined we arrive at the conclusion that **the** and **both** are symmetric, which contradicts our conclusion that they are non-intersective.

Even with the uncertainty that one encounters in trying to categorize partially undefined determiners, B&C both explicitly and implicitly write as if their categorization was in accord with intuition. I think the fundamental intuition is that definites do not go in there-insertion sentences, this is testable. The other properties: **strong, non-intersective** and **non-symmetric** are all correlated with there-insertion facts in B&C's system, so their categorization of definite determiners is in effect correct.
Before leaving this section, I want to point out two more places in the body of the article where I believe B&C have made some implicit assumptions about how definites would behave if they were defined everywhere.
The first is in their assertion and discussion of the above proposition correlating 'strong' with 'non-intersective'. The proposition, as stated, applies to all strong determiners. In fact, in the appendix under **C6**, they only prove this for "proper strong determiners", where a "proper determiner" has the property of being defined for all base-sets. In fact, their proof relies on this fact. In the body of the article they seemed to have assumed that if the definite determiners were proper, they would continue to be strong.

This type of implicit assumption explains the way they chose to state the semantics of the definite determiners. Observe the following three definitions:

S5 b) $\|$**every**$\|$ is the function which assigns to each A, a subset of E, the family
$\|$**every**$\|$ (A) = {X subset of E | A subset of X }

d) For each natural number n, $\|$**n**$\|$, and $\|$**the n**$\|$ are functions on sets defined by:

$\|$**n**$\|$ (A) = { X subset of E | $|X \cap A| \geq n$ }

$\|$**the n**$\|$ (A) = $\|$every$\|$(A) if $|A| = n$ and is undefined otherwise.

"The n" is defined here in terms of "every". In fact, "every" could be replaced in this definition with "n" and the function for $\|$the n$\|$ would remain the same. They apparently chose to think of "the n" as a restriction on "every" because "every" is strong, as opposed to "n" which is weak. Again, the idea is that if "the n" was expanded to be defined everywhere it would look like "every" and not "n".

Section 3

The first step in the B&C analysis is to examine each determiner D, evaluating $\|$D$\|$(A) for all values of A. In doing this, they assume that $\|$D$\|$(A) is defined for all values of A, for

all determiners other than both, the n and neither. The remainder of their analysis is essentially based on the results of this examination.

I agree in some sense with the functions that B&C have assigned to the various determiners. But I do not believe that one can assign a total function to any determiner merely by checking intuitions for all arguments. In particular, I think as the base set A gets smaller it eventually hits a point at which our intuitions fail for sentences involving $\|D\|(A)$.

I have explored the possibility of somehow 'deriving' the values for quantifiers when the base is ϕ (the empty set) from one or more properties that we know the particular determiner has in cases where our intuitions are sound. At this point I'm a little skeptical about whether this can be done. In this section I will present my most recent attempt.

Recall the 'lives-on-A' property which holds for all natural language determiners: $X \in \|D\|(A)$ iff $X \cap A \in Q$. Given the 'lives-on-A' property:

$$Y \in \|D\| (\phi) \quad \text{iff} \quad Y \cap \phi \in \|D\| (\phi) \quad \text{iff} \quad \phi \in \|D\| (\phi)$$

This means that for any determiner D, $\|D\| (\phi)$ is either the empty set (and therefore does not contain the empty set) or $\|D\| (\phi)$ is Pow(E), the power set of the universe.

So now, the question becomes for a given determiner D, which of two values does it have for a null base set. Next we need the following definition from B&C:

DEFINITION. A determiner D is **symmetric** if for all A, B
$$B \in D(A) \quad \text{iff} \quad A \in D(B)$$

I like this property because it usually can be assigned with certainty in cases where the base is non-null.

Now if a determiner is symmetric then for all sets Y,

$$\phi \in D(Y) \quad \text{iff} \quad Y \in D(\phi)$$

which means that we can tell for a symmetric determiner what happens when the base is empty by checking whether the empty set is a member of any quantifier formed from that determiner. This gives good results for "some", "no", "three", "exactly three", and possibly "few".

1. a. Some cat is a dog. <---> Some dog is a cat. (TRUE)
   b. Some cat is a dinosaur. (FALSE)
   c. $\|some\|(\phi) = \phi$.

2. a. No cat is a mammal. <---> No mammal is a cat. (TRUE)
   b. No cat is a dinosaur. (TRUE)
   c. $\|No\|(\phi) = Pow(E)$

3\. a. (Exactly) three cats are dogs. <---> (Exactly) three dogs are cats. (TRUE)
b. (Exactly) three cats are dinosaurs. (FALSE)
c. $\|$(Exactly) three$\|(\phi) = \phi$.

4\. a. Few cats are dogs. <---> Few dogs are cats. (TRUE)
b. Few cats (if any) are dinosaurs. (TRUE)
c. $\|$Few$\|(\phi)$ = Pow(E).

This last one is admittedly suspect. First of all, it is easy to construct apparent counterexamples to the claim that "Few" is symmetric. Though, I think these counterexamples are weakened if you add phrases like "relative to ..." or "as compared with ...". The other thing is that saying that 4b is true seems to get us back into the valley of the unintuitive. "Many" is essentially like "few":

5\. a. Many cats are dogs. <---> Many dogs are cats. (TRUE)
b. Many cats are dinosaurs. (FALSE)
c. $\|$Many$\|(\phi) = \phi$.

Now we need to consider the determiners, "most" and "every". To begin with note that neither is symmetric:

Every prime number is odd --//--> Every odd number is prime.
Most Senators are men --//--> Most men are Senators.

This means that we cannot determine what happens to "most" and "every" at base ϕ based on symmetry. We could just say that "most" and "every" are undefined for ϕ. This would not affect their classification as 'strong' quantifiers since A ϵ $\|$D$\|$(A) would be true for all A where D is defined for both "most" and "every". This might also explain the observation made by B&C that "most" and "every" are hard to interpret when the base is null as opposed to the determiners considered so far. In the case of "every" there is however another way to go. We observe that "every" is anti-persistent: if A is a subset of B, then if <u>every B is an X</u> then <u>every A is an X</u>. If Y ϵ $\|$every$\|$ (X) for any Y and any non-empty X, then since ϕ is a subset of X, Y ϵ $\|$every$\|(\phi)$, hence $\|$every$\|(\phi)$ is non-empty. So the value of $\|$every$\|(\phi)$ is derivable from more intuitive sentences and its value conforms to B&C's value. I can't think of a way to derive $\|$most$\|(\phi)$ and so I will assume it is undefined.
Before turning to the definite determiners, let me say a word about how these 'derivations' fit into the B&C account. The simplest thing to say is that I have arrived at the B&C functions for the determiners by more intuitive means. The rest of their account stays the same. There may be another possibility. I have laid the property of symmetry as the basis for categorizing determiners with respect, for example, to the strong/weak distinction. Perhaps we can dispense with this distinction and give an account directly in

terms of symmetry. In fact, this is what Keenan and Stavi do in their account of there-insertion. It might be possible to rework other parts of the article using symmetry as the basis for all or most effects.

The definites are problematic for this way of doing things for the same reason that they were for B&C. If we consider only cases where the intuitions are sharp, the definite determiners will seem symmetric. For example:

1. The five cats have rabies.
2. The five animals with rabies are cats.

If the quantifiers in both 1 and 2 are defined then 1 is true if and only if 2 is true. I have a suggestion. We separate the use of **the** from the cardinality presupposition. This is possible in case where the head noun is plural and there is no number after **the**, for example:

1. The cats have rabies.
2. The animals with rabies are cats.

Both 2 and 1 could be defined but have different truth values. Also notice, that using examples such as these we can show that when defined, **the** is anti-persistent, for example, The animals have rabies implies that The cats have rabies. This situation is similar to that with the determiner **all**. It looks non-symmetric and anti-persistent unless you look at examples with numbers, such as All five cats have rabies.

This story will be of no use for **both** and **neither** unless we derive them somehow from **the**.

References

Barwise J. & Cooper, R. (1981) "Generalized Quantifiers and Natural Language", **Linguistics and Philosophy** 4, 159-219.

Keenan, E. & Stavi, J. (1986) "A Semantic Characterization of Natural Language Determiners", **Linguistics and Philosophy** 9, 253-326.