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Abstract

The Marxian labour theory of value considers labour as the only substance of value. The generalized commodity exploitation theorem (GCET) purports to demonstrate that many other commodities can be substances of value. This note argues that the GCET is based on two conceptual flaws: (a) failure to distinguish labour and labour-power; and (b) failure to distinguish labour-power and other commodities. Once these flaws are corrected, it is easy to show that commodities cannot function as the substances of value. Only labour can be the substance of value.
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1 Introduction

The Marxian labour theory of value rests on the argument that labour is the *only* substance of value (Marx, 1992, chapter 1). A strand of Analytical Marxist thinking has challenged this basic argument with what can be called the generalized commodity exploitation theorem (GCET). The crux of the GCET is the claim that there is nothing special in labour so far as

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it can be considered the substance of value. Other commodities can as well function as a substance of value. Moreover, claims the GCET, it can also be demonstrated that when some other commodity is chosen as the candidate substance of value, it is necessarily exploited in a capitalist economy. Hence, labour does not have a special function to play even in a theory of exploitation (Roemer, 1982, 1985).[1]

Clear expositions of the GCET can be found in Gintis and Bowles (1981) and Roemer (1982, appendix 6.1). In this paper, I will critically evaluate the GCET. The main finding of this critical scrutiny is that the GCET rests on two conceptual flaws. The first problem is that the GCET does not properly distinguish between labour and labour-power - a distinction that is key in Marxian political economy. The second conceptual flaw is that the GCET does not correctly distinguish between labour-power and other commodities - another key idea in Marxist economics. Both these conceptual flaws manifest themselves as a conflict between two reasonable intuitions about value: value as the sum of direct and indirect amounts of the substance of value, and value of output as the sum of values added or transferred by inputs. These flaws nullify the claim of the GCET that any commodity, and not just labour, can function as the substance of value. Hence, this critique, if it is valid, restores labour as the *only* substance of value.

This argument in this paper is related to, but also different from, earlier attempts to provide a critique of the GCET. Two contributions are worth mentioning in this regard. Fujimoto and Fujita (2008) provide a refutation of the GCET by arguing that the result is just a numerical representation of the productiveness of the 'complete matrix' captured by the Hawkins-Simon condition.[2] Such a technical condition does not have anything to do with exploitation, which is a social phenomenon, they rightly note. Yoshihara and Veneziani (2013) argue in a similar vein that exploitation should be understood in social and not in technological terms - which is what the GCET does. Drawing on their previous work, they offer a new definition of exploitation that undermines the implication of the GCET for exploitation

[1]For an excellent survey of the debate on exploitation theory since the seminal contribution of Nobuo Okishio in the 1960s, see Yoshihara (2017). I take the terminology of calling these results the generalized commodity exploitation theorem from Yoshihara (2017). On the other hand, Fujimoto and Fujita (2008) call it the commodity exploitation theorem.

[2]The so-called complete matrix is discussed below.

theory.[3]

Both of these, and similar, contributions are important in that they offer critiques of the GCET. But both papers accept the key claim that the notion of commodity value, e.g. steel value, is meaningful and use it in their own arguments. In this paper, I question this acceptance, and hence I offer a more foundational critique of the GCET. I argue that the notion of commodity value, as defined in Gintis and Bowles (1981) and Roemer (1982, appendix 6.1), is conceptually flawed. Hence, the notion of exploitation that is defined with that conceptually flawed notion is *ipso facto* flawed.

The implication of the argument in this paper goes beyond the theory of exploitation and helps formulate the question of how to properly define value. I show that the attempt to define the value of commodities using some commodity as a substance of value runs into conceptual conundrums. It gives rise to a conflict between two common intuitions about value. One option to avoid these conceptual conundrums is to retain the conceptual distinction between labour-power and labour, as the classical Marxist tradition does, and use labour as the substance of value. The rest of this paper is organized as follows: in the next section, I use a simple example of an economy with two goods to explain the argument; in the following section, I present a formal treatment in a multisectoral economy; I conclude the discussion in the last section.

2 An Example

Let me present the whole argument using a simple example discussed in Gintis and Bowles (1981, pp. 19). The essential points of the argument can be understood from the example, but for the sake of completeness, a formal treatment is presented in the following section.

2.1 The Set-up

The economy in this example has two goods, food (F) and jewelry (J), where F is a *basic* and J is not. What is a basic?

> We define an input into production to be a *basic* if it enters directly or indirectly (via its being an input into another produc-

[3]For references to this literature see Yoshihara (2017).

> tion input) into the production of all commodities. (Gintis and Bowles, 1981, pp. 18).

The technology for producing the two goods are specified as follows:

> Food is used to produce all goods, and is in the wage bundle, while jewelry is consumed only by nonworkers and is not used in production (i.e. it is not a basic). Specifically, suppose 1/2 bushel of F and 1/2 hour of labour are used to produce one bushel of F, and 1/2 bushel of food is in the wage bundle. Also suppose 1/4 bushel of food and one hour of labour is used to produce one unit of J. (Gintis and Bowles, 1981, pp. 19).

Let us represent the technology for production in a way that will allow us to compute values of the commodities: for production of F, we have

$$\frac{1}{2}\mathrm{F} + \frac{1}{2}\mathrm{L} \Rightarrow 1\mathrm{F}, \tag{1}$$

and for production of J, we have,

$$\frac{1}{4}\mathrm{F} + 1\mathrm{L} \Rightarrow 1\mathrm{J}. \tag{2}$$

Note, also, that the real wage bundle contains $(1/2)F$.

2.2 Labour Values: Definition and Two Intuitions

Let Λ_F, Λ_J and Λ_L represent the labour values of food, jewelry and labour-power. From the specification of the technology for producing F in (1) and J in (2), and the fact that the real wage bundle contains 1/2 units of food, we see that their labour values must be determined by the following equations,

$$\frac{\Lambda_F}{2} + \frac{1}{2} = \Lambda_F.$$

and

$$\frac{\Lambda_F}{4} + 1 = \Lambda_J,$$

and

$$\Lambda_L = \frac{\Lambda_F}{2}.$$

These equation captures two different intuitions about the concept of labour value. The first intuition is related to the definition of labour value: *the labour value of one unit of F, Λ_F, is the sum of the direct and indirect amounts of labour needed to produce each unit of F.* The amount of direct labour is 1/2 and the amount of indirect labour is $\Lambda_F/2$, the latter because each unit of F has Λ_F amount of labour value. The same reasoning gives us the value of jewelry, Λ_J, as the sum of $\Lambda_F/4$ (indirect labour) and 1 (direct labour). Finally, the real wage bundle has 1/2 units of F. So, the value of labour-power is $\Lambda_F/2 = 1/2$. In the production of labour-power, there is only an indirect input, food (F), because the labour that is used in the production of labour-power falls outside the domain of capitalist commodity production.

A different intuition can also be seen in the above equation, i.e. we can read it as an accounting relationship: *value of one unit of the output is equal to the sum of the values added and/or transferred by the inputs.* The value of one unit of F, the output, is Λ_F (the right hand side). What about the value on the input side of the equation? The non-labour input, 1/2 units of F, has $\Lambda_F/2$ units of value, and this is *transferred to* the output. The labour input, i.e. labour, does not have value. Labour-power, the commodity purchased by the capitalist, has value, and when it is used in production, we get labour, which adds value to the object of labour. So, in accounting terms, the left hand side is the sum of the value of 1/2 units of F, $\Lambda_F/2$, and the value *added by* 1/2 units of labour (which is by definition equal to 1/2). The same accounting reasoning applies to the equation determining the value of jewelry, or the equation determining the value of labour-power.

It is worth pausing and thinking about an important conceptual point. The distinction between value added and value transferred is crucial in the Marxist framework.[4] Commodities, when used in production, can transfer their value to the output. A commodity cannot add value. Labour, since it is not a commodity, can add value. Hence, the distinction between labour-power (the commodity purchased by the capitalist) and labour (the input in capitalist commodity production) has both qualitative and quantitative implications.

On solving the three equations, we get: $\Lambda_F = 1, \Lambda_J = 5/4, \Lambda_L = 1/2$. Two points about the these calculations are worth highlighting. First, the input represented by L in (1) and (2) is labour, and not labour-power. It is

[4]This distinction is also crucial in the construction of national income and product accounts.

important to keep the distinction in mind: labour-power is the ability to do useful work; labour is what we get when that ability is actually used. In a capitalist society, labour-power is the commodity that is sold on the labour market, not labour. In the production process, labour-power is used, which means labour is added to the objects of labour. And that is the source of value - the labour - in a labour value accounting system. This also means that the unit in which value is measured is hours of laobur time. Second, when we calculate the value of the commodity labour-power, we are using the standard interpretation definition. That is why we are calculating its value as the value of the real wage bundle.[5] Moreover, it is important to keep in mind that we are calculating the value of labour-power (the commodity purchased by the capitalist); we are not calculating the value of labour (the input into production). It is important to keep in mind that labour has no value.

2.3 Food Values: First Intuition

Since food is a basic, whereas jewelry is not, the former can be used as a substance of value, claims Gintis and Bowles (1981) and Roemer (1982, appendix 6.1). This means that we can meaningfully define the value of all commodities in terms of food. Let μ_F, μ_J, μ_L denote the 'food values' of food, jewelry and labour-power.

We would now like to calculate the food values of all commodities according to the first intuition discussed above: the food value of any commodity is the sum of the direct and indirect amounts of food used in producing 1 unit of the commodity. Using the technology for food production given in (1), we can implement this definition through the following value determination equation for food as,

$$\frac{1}{2} + \frac{\mu_L}{2} = \mu_F. \tag{3}$$

Here the direct input of food is $1/2$ and the indirect input of food is $\mu_L/2$ because the food value of labour-power is μ_L.[6] In a similar way, we can

[5] If we followed the New Interpretation, we would define the value of labour-power differently. But that difference in not relevant in this argument - which is about a strand of Marxist thinking within the standard interpretation.

[6] In the Analytical Marxist tradition, there is a common confusion between labour-power and labour *so far as value calculations are concerned*. Let us ignore this issue for the moment. We will come to the deeper problem related to this confusion below.

implement the definition of the food value of jewelry, using information about the technology of jewelry production given in (2), as,

$$\frac{1}{4} + \mu_L = \mu_J. \tag{4}$$

Since the wage bundle has 1/2 bushel of food, we finally get the value equation for labour-power as,

$$\mu_L = \frac{1}{2}. \tag{5}$$

Using (5) to substitute in (3) and solving for the food value of food gives us, $\mu_F = 3/4$. Using the same method in (4), we get $\mu_J = 3/4$. Hence, we have solved for the 'food value' of every commodity, and they are all positive numbers: $\mu_F = 3/4, \mu_J = 3/4, \mu_L = 1/2$. Two more points about this calculation of food values are worth highlighting.

First, while carrying out these calculations Gintis and Bowles (1981) seem to be using food (the basic) as the value *numéraire*. Setting up the "food theory of value", they note explicitly:

> We now *define the value of food as unity*, and the value of jewelry and labour as the amount of F directly and indirectly embodied in them. (Gintis and Bowles, 1981, pp. 19, emphasis added).

Here we can see the confusion between labour and labour-power come up again: even though labour does not have value, the authors want to compute its value in terms of F. Let us ignore this and look at the other issue, i.e. the notion of the *numéraire*. In classical theory of prices, choosing a *numéraire* means choosing to express all relative prices in terms of a given commodity. This is implemented by choosing the price of the *numéraire* as unity (Pasinetti, 1977, pp. 57–58). Thus, Gintis and Bowles (1981) are choosing food as the food value *numéraire* by defining "the value of food as unity". This would mean that $\mu_F = 1$, and the calculation of $\mu_F = 3/4$ would therefore be redundant. Gintis and Bowles (1981) do not seem to recognize this contradiction and do not comment on it.

Second, if we ignore the confusion relating to the use of the value *numéraire* and accept the calculation of food values, then we see that the food value of food is less than 1 (because $\mu_F = 3/4$). This is interpreted by Gintis and Bowles (1981) as demonstrating that food (the basic, in this case) is exploited. As I have pointed out above, this claim about exploitation of the commodity, food, can be criticized on other theoretical grounds (Fujimoto and Fujita, 2008; Yoshihara and Veneziani, 2013).

2.4 Food Values: Second Intuition

Let us now calculate the food values of the three commodities using the second, i.e. accounting, intuition: food value of any commodity must be equal to the food value added and/or transferred by the inputs. Starting with the production of food, we see that food value of the two inputs are $\mu_F/2$ (for food) and $\mu_L/2$ (for labour-power). Both commodities transfer their value to the output. Hence the value determination equation, using the accounting intuition, becomes,

$$\frac{\mu_F}{2} + \frac{\mu_L}{2} = \mu_F. \tag{6}$$

The same reasoning gives us the following as the value determining equation for jewelry,

$$\frac{\mu_F}{4} + \mu_L = \mu_J. \tag{7}$$

Finally, the value of labour-power is determined as follows:

$$\mu_L = \frac{\mu_F}{2}. \tag{8}$$

Using (8) to substitute in (6) and solving for the food value of food gives us, $\mu_F = 0$. Using (8), we get, $\mu_L = 0$. Using these magnitudes in (7), we get $\mu_J = 0$. Hence, we have solved for the 'food value' of every commodity, and they are all 0: $\mu_F = 0, \mu_J = 0, \mu_L = 0$.

2.5 Food Value: Conflict of Intuitions

The above discussion highlights an important point. When we try to compute the 'food value' of commodities, the two intuitions about value are in conflict. If we use the intuition that the food value of any commodity is the sum of direct and indirect amount of food required to produce one unit of any commodity, we see that every commodity has a positive magnitude of food value. On the other hand, if we use the accounting intuition of value, i.e. the value of output should be equal to the sum of value added and/or transferred by inputs, then we get a very different answer: every commodity has 0 food value.

The reason we did not face this conflict of intuitions in the case of labour value is that we distinguished between labour-power (the commodity purchased by the capitalist) and labour (the input in production) - and this

highlights the first conceptual flaw in the Analytical Marxist attempt to use commodities as substances of value. We did not use the value of labour-power in the determination of labour values of commodities. Instead, we used the value added by labour. In fact, it is the difference between the value of labour-power and the value added by labour that allowed Marx to identify the source of surplus value in the exploitation of the working class.

Can we appeal to the same logic to escape the conflict of intuitions in the case of food value (or any other commodity value)? To repeat one more time: labour-power is the capacity to do useful work; labour is the outcome when that capacity is used. As a qualitative matter, no such distinction can exist for any other commodity, including food. It is meaningless to try to define 'food-power' as distinct from food. This is because the capacity of food to be used is not distinct from the commodity food. Food, or any other commodity, does not have the will, consciousness or incentive to resist its use in whatsoever way its possessor wants. In the case of labour-power that is not the case. The possessor of labour-power, the worker, can and does, resist the way her employer, the capitalist, uses her labouring capacity. Hence, while it is meaningful to distinguish labour-power (the labouring capacity) from labour, it is meaningless to do so for any other commodity.

Gintis and Bowles (1981, pp. 7) would not agree with this argument because for them it is meaningful to define something called 'lathe-power' of the commodity lathe - and this highlights the second conceptual flaw in the Analytical Marxist attempt to use commodities as substances of value.

> But clearly every commodity has an abstract form as a commodity and a concrete form as a physical entity engaged in production. A lathe can be considered a union of lathe-power, its abstract potential to perform useful functions, and as lathing, the concrete activity of the lathe engaged in production. (Gintis and Bowles, 1981, pp. 7).

The difference only makes sense if by purchasing the former, i.e. lathe-power, the capitalist does *not* automatically get the latter, lathing. But that is clearly impossible unless we are ready to impute consciousness and will to the lathe.

From a quantitative angle, we had used the fact that labour, the input in production, has no value, but labour-power, the commodity purchased by the capitalist does have value, and that the value added by labour was greater than the value of labour-power. To repeat this logic in the case of food, we

would have to say that food, as an input in production, has no food value, but food, as the commodity purchased by the capitalist does have food value, and the magnitude of the latter is μ_F. Furthermore, we would have to say that each unit of food, used as an input in production, adds one unit of food value, which is different from the food value of each unit of food that was purchased by the capitalist - since the latter has a food value of μ_F. Clearly, these are absurd propositions - because we are talking about the same entity, food, in both cases.

3 Commodity Value: Formal Treatment

3.1 Basic Set-up

The above argument about the conceptual problems of using commodities as the substance of value can be demonstrated in a more formal setting. Consider a capitalist economy with n sectors, each producing a single commodity using labour and all commodities. There are no joint products and there is no fixed capital. The technical conditions of production in each sector is taken as given and is captured by the $n \times n$ matrix of input-output coefficients,

$$\boldsymbol{A} = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ & & \vdots & \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{bmatrix}$$

and the corresponding $1 \times n$ vector of direct labour inputs

$$\boldsymbol{L} = \begin{bmatrix} L_1 & L_2 & \cdots & L_n \end{bmatrix}$$

where a_{ij} is the physical magnitude of the i-th commodity used to produce 1 unit of the j-th commodity, and L_j is the quantity of direct labour used to produce 1 unit of commodity j. Let Let $\boldsymbol{b}$ denote the $n \times 1$ vector of the real wage bundle per hour of labour-power sold, and let the augmented input matrix be defined as,

$$\boldsymbol{M} = \boldsymbol{A} + \boldsymbol{b}\boldsymbol{L},$$

and assume that $\boldsymbol{M}$ is nonnegative and productive, as is standard in the literature (Roemer, 1981).[7]

[7]The non-negative matrix $\boldsymbol{M}$ is said to be productive if there exists a nonnegative vector, $\boldsymbol{x} \geq \boldsymbol{0}$, such that $\boldsymbol{x} > \boldsymbol{M}\boldsymbol{x}$. If $\boldsymbol{M}$ is productive, then $(\boldsymbol{I}_n - \boldsymbol{M})^{-1} > \boldsymbol{0}$ (Pasinetti,

I will follow Roemer (1982) and define values in terms of commodity 1 (the commodity is called 'steel'). Let the $1 \times (n+1)$ vector,

$$\boldsymbol{\mu} = \begin{bmatrix} \mu_1 & \mu_2 & \cdots & \mu_n & \mu_{n+1} \end{bmatrix},$$

denote the *steel values* of all commodities, and where, in particular, μ_{n+1} is the steel value of labour-power. Following Roemer (1982), we display the technology as follows,

$$\boldsymbol{\mathcal{A}} = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} & b_1 \\ a_{21} & a_{22} & \cdots & a_{2n} & b_2 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} & b_n \\ L_1 & L_2 & \cdots & L_n & 0 \end{bmatrix},$$

where the first row and first column refer to steel, the chosen substance of value.[8] In this representation of technology, Roemer (1982) notes that

> [the] j^{th} column of this array lists the inputs of all $(n+1)$ commodities used as inputs into production of the j^{th} good - and this statement is true also for $j = n+1$, namely, the commodity labour power. The i^{th} row lists the inputs of the i^{th} good into the various processes - and this statement is true also for $i = n+1$, namely the input of labour. (Roemer, 1982, pp. 187).

In the above array, the $(n+1) \times (n+1)$ entry is 0 because labour is not an input into the production of labour-power, i.e. labour-power is produced outside the framework of capitalist commodity production.

3.2 Steel Value: First Intuition

Let us calculate steel values according to the first intuition: value of one unit of any commodity is the sum of the direct and indirect amount of steel needed to (re)produce it. For the first n commodities, we have

$$\mu_j = a_{1j} + (\mu_2 a_{2j} + \cdots + \mu_n a_{nj} + \mu_{n+1} L_j), \quad j = 1, 2, \ldots, n, \tag{9}$$

1977, appendix).

[8] This matrix is called the 'complete matrix' in Bródy (1970), as noted by Fujimoto and Fujita (2008).

where a_{1j} is the direct amount of steel and $\mu_2 a_{2j} + \cdots + \mu_n a_{nj} + \mu_{n+1} L_j$ is the indirect amount of steel needed to produce 1 unit of the j-th commodity. For the $n+1$-st commodity, i.e. labour-power, we have,

$$\mu_{n+1} = b_1 + (\mu_2 b_2 + \cdots + \mu_n b_n)\,, \tag{10}$$

because the real wage bundle has $b_1, \ldots, b_n$ amounts of the first n commodities. Bringing the $n+1$ equations together and writing in matrix form, we get

$$\begin{bmatrix} \mu_1 & \mu_2 & \cdots & \mu_n & \mu_{n+1} \end{bmatrix} \begin{bmatrix} a_{11} & 0 & \cdots & 0 & 0 \\ -a_{21} & 1-a_{22} & \cdots & -a_{2n} & -b_2 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ -a_{n1} & -a_{n2} & \cdots & 1-a_{nn} & -b_n \\ -L_1 & -L_2 & \cdots & -L_n & -1 \end{bmatrix} = \begin{bmatrix} a_{11} \\ a_{12} \\ \vdots \\ a_{1n} \\ b_1 \end{bmatrix}$$

which can be used to solve for nontrivial solutions of $\boldsymbol{\mu}$ because, in general, the matrix multiplying $\boldsymbol{\mu}$ on the left hand side is invertible and the coefficient vector on the right hand side is not the zero vector. Moreover, one can show that $\mu_1 < 1$, which then leads Roemer (1982) to claim that steel is exploited.

It is worth noting the same confusion about the use of the value *numéraire* in the above argument that I had pointed out for the example in Gintis and Bowles (1981). Setting up the calculation of steel value, Roemer states: "We now compute values using commodity 1 as the value numeraire (say, steel)." Using the standard meaning of the term *numéraire*, this should mean that $\mu_1 = 1$. Hence, the calculation of the magnitude of μ_1 is redundant and the demonstration that $\mu_1 < 1$ is a clear contradiction.

3.3 Steel Value: Second Intuition

Let us now write the steel value determining equations using the second, i.e. accounting, intuition: value of one unit of the output is equal to the sum of the values added and/or transferred by the inputs. For $j = 1, 2, \ldots, n$, a_{1j} units of the first commodity, a_{2j} amount of the second commodity, ..., a_{nj} amount of the n-th commodity, and L_j units of labour are needed to produce 1 unit of the j-th commodity. Since each unit of commodity 1 has a steel value of μ_1, a_{1j} units of the first commodity transfer $\mu_1 a_{1j}$ units of steel value to the output; similarly, each unit of the second commodity has a steel value of μ_2, so that a_{2j} amount of the second commodity transfer $\mu_2 a_{2j}$ amount of

steel value to the output, and so on. Hence, for the first n commodities, we have

$$\mu_j = (\mu_1 a_{1j} + \mu_2 a_{2j} + \cdots + \mu_n a_{nj}) + \mu_{n+1} L_j, \quad j = 1, 2, \ldots, n. \tag{11}$$

Each unit of the $(n+1)$-st commodity, labour-power, is produced with $b_1, b_2, \ldots, b_n$ units of commodity 1 through n. Hence, the steel value of labour-power is given by

$$\mu_{n+1} = (\mu_1 b_1 + \mu_2 b_2 + \cdots + \mu_n b_n)\,. \tag{12}$$

Using (12) to replace μ_{n+1} in (11) and rearranging terms, we have,

$$\mu_j = \mu_1 (a_{1j} + b_1 L_j) + \cdots + \mu_n (a_{nj} + b_n L_j)\,, \quad j = 1, 2, \ldots, n.$$

In matrix form, the above equation system becomes,

$$\tilde{\boldsymbol{\mu}} = \tilde{\boldsymbol{\mu}} \boldsymbol{M}, \tag{13}$$

where $\boldsymbol{M} = \boldsymbol{A} + \boldsymbol{bL}$ is the augmented input matrix, and $\tilde{\boldsymbol{\mu}}$ is the $1 \times n$ vector formed from the first n elements of $\boldsymbol{\mu}$. For (13) to have a non-trivial solution, we need the determinant of $(\boldsymbol{I} - \boldsymbol{M})$ to be zero. But that is not possible because the matrix $\boldsymbol{M}$ is productive. Hence, there is no non-trivial solution vector of steel values of commodities, $\tilde{\boldsymbol{\mu}}$. The only possible solution, when $\boldsymbol{M}$ is productive, is $\tilde{\boldsymbol{\mu}} = \boldsymbol{0}$. Since the value of the $(n+1)$-st commodity is determined by the value of the first n, as can be seen in (12), this implies that $\boldsymbol{\mu} = \boldsymbol{0}$. Thus, the 'steel value' of all commodities, including labour-power, is 0.

If we had instead used the first intuition of value, i.e. as the sum of direct and indirect amount of steel, then we would have arrived at positive steel values of all commodities, $\boldsymbol{\mu} > \boldsymbol{0}$, as demonstrated above. The argument here is not that one or the other intuition is the correct one. The argument is that the two intuitions are in conflict when we attempt to define steel value or any other commodity value. To overcome the conflict among the two reasonable intuitions, we must give up the attempt to use commodities as substances of value.

4 Conclusion

The Marxian labour theory of value is built on the premise that labour is the *only* substance of value. This understanding comes from a deeper

understanding of the role of labour in the history of humankind, and of the role of the process of exchange in enforcing a social division of labour in a system of independent, private producers. After developing it, the labour theory of value is used by Marx to demonstrate the exploitative nature of capitalism.

A strand of Marxist thinking, which I will call Analytical Marxist for lack of a better term, developed a whole set of arguments in the 1970s and 1980s that was interpreted as raising serious doubts on the labour theory of value. One argument in this broader class of Analytical Marxist thinking is the generalized commodity exploitation theorem, which claims that, first, there can be multiple substances of value, and second, that any commodity which can function as a substance of value can also provide a basis for a value theory, i.e. profits will be positive if and only if that commodity (which is the substance of value) is exploited (Gintis and Bowles, 1981; Roemer, 1982).

In this paper, I have argued that the GCET is based on two conceptual flaws, and once those flaws are dealt with, the theorem collapses. The conceptual flaws are: (a) the failure to distinguish labour and labour-power so far as value determination is concerned; and (b) the inability to distinguish between the commodity labour-power and all other commodities. When we try to define any 'commodity value', these flaws manifest themselves in the conflict between two reasonable intuitions about value: (a) value of a commodity as the sum of direct and indirect amounts of the substance of value; (b) value of output as the sum of the values added and/or transferred by inputs. One consistent way to avoid this conflict between two reasonable intuitions is to use the Marxian labour theory of value - which rests on the premise that labour is the only substance of value.

The critique of the GCET that I have developed in this paper is related to, but also different from, earlier work that has been critical of the GCET. Fujimoto and Fujita (2008) argue that the GCET is just a restatement of the assumption that the matrix, $\boldsymbol{\mathcal{A}}$, satisfies the Hawkins-Simon condition. Note that if the matrix $\boldsymbol{\mathcal{A}}$, satisfies the Hawkins-Simon condition, then *any* nonnegative vector of surplus, or net output, of the $n+1$ items captured in the matrix $\boldsymbol{\mathcal{A}}$ can be produced, i.e. the production system represented by $\boldsymbol{\mathcal{A}}$ is productive and viable.[9] This is technically correct, but I would like to

[9]If *all* the principal minors of $\boldsymbol{I}-\boldsymbol{\mathcal{A}}$ are positive, then $\boldsymbol{\mathcal{A}}$ satisfies the Hawkins-Simon condition (Hawkins and Simon, 1949). It was later proved by Georgescu-Roegen (1966) that a necessary and sufficient condition for the Hawkins-Simon condition is that the *leading* principal minors be positive (Dasgupta, 1984, footnote 1.).

argue that there is a serious difficulty with regard to interpretation. Note, first, that there is a discrepancy between the last row and column of $\boldsymbol{\mathcal{A}}$: while the last row captures the magnitudes of labour input for producing commodities, the last column provides information about the commodities needed to reproduce each unit of labour-power. Since, labour-power and labour are different, we are no longer recording the use and production of the same entity in the $n+1$ row and $n+1$ column. This is unlike the case for the commodities captured by the first n rows and n columns - where the i-th row captures the use, and the i-th column captures the production of the same commodity. Second, since labour is not produced (but labour-power is), it is meaningless to talk about productiveness of the technology so far as the labour input is concerned. Hence, I would argue against interpreting the $\boldsymbol{\mathcal{A}}$ in any substantive way - and let it just function as a technical matrix.

Yoshihara and Veneziani (2013) extend the argument to an economy with a convex production set, of which the Leontief technology is a special case. In addition to arguing that the GCET is merely the numerical representation of the productiveness of the economy, they offer an interesting additional result: while labour exploitation is necessary and sufficient for positive profits, commodity exploitation is not necessary and sufficient for positive profits (Yoshihara and Veneziani, 2013, Theorem 1). This is an important result and undermines the theoretical bite of the GCET, assuming that the notion of commodity value is meaningful. The argument in this paper raises doubts about the validity of the latter.

While deriving this result, Yoshihara and Veneziani (2013) have to define 'commodity value' for any commodity, k. My argument against defining commodity values, e.g. food value or steel value, has been presented above for an economy with a Leontief technology. Since the Leontief specification of technology is a special case of their more general convex production sets, my argument, if valid, raises doubts about the conceptual validity - in the precise sense in which it has been discussed in this paper - of their result for general convex cone technologies. Of course, it is possible that the conceptual problem about the definition of commodity values discussed in this paper would not be valid for technologies that are represented by the *complement* of Leontief technology with respect to the general convex cone technology used in Yoshihara and Veneziani (2013). But investigation of this issue is beyond the scope of this paper.

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