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Supermultipliers, ‘endogenous autonomous demand’ and Functional Finance^a

by

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Abstract: Autonomous demand is seen as a stabilizing force and a driver of long-run growth in Sraffian supermultiplier models. Government consumption is the most plausible source of long-run autonomous demand. But an active fiscal policy guided by principles of functional finance can produce more powerful stabilization, avoid overheating and excessive utilization rates, and secure faster adjustments of the growth rate towards its target level. Recent attempts to endogenize autonomous demand effectively undermine the existence of a supermultiplier but show strong similarities with an earlier literature on feedback effects from the labor markets to aggregate demand.

^a We thank two anonymous referees and an associate editor for comments and suggestions on our first attempt. The revisions in this version have, we believe, improved the paper substantially. One of the referees expressed strong disagreement with our analysis; disagreements may remain, but the comments forced us to expand and clarify our argument.

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1. Introduction

Keynesian models of short-run equilibrium describe output as being determined by the product of autonomous demand and a demand multiplier. The components of autonomous demand typically include variables that are independent of current output but influenced by past levels of output; current output levels will affect the future levels of these variables. Other components of autonomous demand could, at least in principle, be completely exogenous, and movements in these variables could drive long-run growth. They could also, it has been argued, stabilize an economy in which investment is subject to Harrodian instability.

The literature on ‘Sraffian supermultipliers’ (SSM) has singled out several such potentially autonomous components, including capitalist consumption, residential investment, exports, and government consumption.¹ Descriptively it is questionable whether any of these components can be viewed as autonomous in the long run. They are also -- with the exception of government consumption -- extremely volatile. Even if they were autonomous, it is therefore hard to see how these components could stabilize an economy that is subject to Harrodian instability.²

Government consumption could in principle be autonomous, at least within a certain range: in the absence of binding supply-side constraints, policy makers could decide to raise government consumption at a fixed proportional rate every year. Mature economies may face labor constraints, but that is not the case for dual economies, and long-run capital constraints would be removed endogenously if the economy converges to a steady growth path with utilization at the normal (or desired) rate.³

¹ The Sraffian supermultiplier was introduced by Serrano (1995 a,b), Bortis (1997) and Dejuan (2005). Interest in the approach ballooned more recently following contributions by Freitas and Serrano (2015), Allain (2015) and Lavoie (2016). Skott (2017, 2019), Nikiforos (2018) and Oreiro, Silva and Santos (2020) are among the critics. Special issue of *Metroeconomica* (2019, vol 2) and *Review of Keynesian Economics* (2020, Vol.8, n.3) have been devoted to the Sraffian supermultiplier and the role of autonomous demand. As pointed out by Dutt (2019) and Palley (2019), the terminology is misleading; there is nothing particularly Sraffian about supermultipliers and autonomous demand.

² Volatility does not in itself exclude a component from being autonomous in the long run. Residential investment, for instance, is highly volatile but its trajectory could, in principle, be completely exogenous without any influence of interest rates and income.

³ An economy is mature if the long-run rate of growth is constrained by the labor supply in efficiency units. Maturity does not imply ‘full employment’; France, Japan or the US are mature in the sense that fast growth of aggregate demand at, say, 10 percent annually would lead to labor shortages within a few years. Dual economies -- including almost all developing economies -- have large amounts of hidden unemployment, and labor constraints do not prevent a prolonged period with Chinese-style growth rates.

There is a broad post-Keynesian consensus that aggregate demand policy can influence long-run growth and, second, that it may be needed to stabilize economic growth. The distinctiveness of the SSM analysis is the focus on a particular policy: the analysis suggests that maintaining a constant growth of government consumption will stabilize the economy at a growth rate that is equal to the growth in government consumption. This claim becomes interesting if policy makers do in fact maintain a constant growth rate of government consumption or if, alternatively, a case can be made that policy makers should follow such a policy. If neither of these conditions is met, the emphasis should be on how fiscal policy is or should be adjusted to fit economic circumstances, rather than on the implications of what would be an empirically irrelevant and undesirable set of policies.

The descriptive case for an exogenous trajectory of government consumption is weak, and to our knowledge no one has tried to make this case. For example, an increase in residential investment or in household consumption can be financed in the short and medium run by an increase in the level of household's indebtedness without an increase in the household's current income; but sustainability of these debts over time depends on a growing wage income which creates a link between residential investment and income in the long run.⁴ But even if the descriptive case is weak, perhaps a prescriptive case could be made. This article considers two well-known benchmark models of autonomous demand, Allain (2015) and Serrano et al. (2019). The detailed specifications of the models are quite different, but the dynamic properties are surprisingly similar:

The Harrodian forces, first, must be very weak in order for the SSM policy to stabilize the economy. Weak Harrodian forces, second, imply that accumulation rates adjust slowly to deviations of actual from desired utilization rates and that, consequently, the adjustment process towards the target rate of growth must be slow. The adjustment speed is needlessly retarded, however, by a policy that relies on the long-run effects of an increase in the growth rate of government consumption (the growth rate of autonomous consumption). The stimulus to accumulation only comes gradually as the utilization rate responds to the rise in the level of government consumption; an increase of two percentage points in the growth rate of government consumption is large from a long-run perspective, but the effects on the utilization rate are small in the short and medium run. For any significant increase in the targeted growth rate, third, the SSM policy generates a transition path with prolonged periods

⁴ The weakness of the descriptive case is discussed by Nikiforos (2018), Skott (2019b) and Oreiro, Silva and Santos (2020).

of very high utilization rates. Thus, the SSM policy may come up against binding capital constraints, and one would expect overheating and inflationary pressures (as well as balance of payments problems in open economies) long before the economy hits any such absolute capacity constraints. The SSM policy, finally, determines the long-run share of government consumption as a by-product of the growth process. It is not obvious why one would want to determine the share of resources going to health care, education, and other public services in this way. These results are illustrated numerically in section 4.

The weaknesses of an SSM policy regime would not be important if there were no alternative policy options. But keeping the growth rate of government consumption constant is just one policy option. Active Keynesian policy can adjust the fiscal and monetary instruments in light of current conditions and objectives. Thus, if the aim is to raise capital accumulation, policy can be adjusted to boost accumulation as quickly as possible, while taking into account capital constraints (constraints on the utilization rate) and the dangers of overheating. This active policy will not maintain a fixed growth rate of government consumption.

Lerner's (1943) principle of functional finance is usually applied to mature economies with a well-defined notion of full employment. In these economies, Lerner argued, fiscal and monetary policy should be adjusted to achieve full employment and a target level of investment. In a growth context, these short-run objectives translate into targets for the level and growth rate of output and for the capital intensity of production (Ryoo and Skott 2013, Skott 2016). In dual economies, the main supply side constraint comes from the capital stock rather than the supply of labor, and full employment (in the modern sector) is not a feasible short-run target. Policy makers have to define a growth target for the modern sector, weighing the benefits of fast accumulation against the cost of foregoing current consumption (Skott 2020). Once a target for the growth rate has been defined, aggregate demand policy is left to steer the economy to -- and then stabilize it at -- a growth path with accumulation at the target rate and utilization rate at the desired rate. In both mature and dual economies functional finance mandates the continuous adjustment of the policy instruments to achieve the chosen targets.

Our depiction of the autonomous-demand literature on economic growth may seem to ignore or misrepresent some recent developments. While most papers consider the effects of autonomous demand components that grow at an exogenously given rate, some

contributions have endogenized the growth rate of autonomous demand.⁵ This endogenization dilutes the distinctive contribution of the original SSM literature (which we view as a positive development). If ‘autonomous demand’ is endogenous in the long run, notions of supermultipliers and long-run autonomous demand as drivers of growth cease to be helpful. Instead, it becomes more fruitful to focus on the feedback effects – including via economic policy -- that can serve to stabilize the economy and align the natural and warranted growth rates in mature economies. Both Allain (2019, 2020) and Nomaler et al. (2020) introduce feedback effects of this kind, and in this respect their analysis has affinities with strands of (post-) Keynesian and (neo-) Marxian theory; examples include Skott (1989) and Flaschel and Chiarella (2000).⁶

Section 2 discusses some terminological issues relating to the meaning of the terms ‘autonomous demand’ and ‘supermultiplier’. This section also considers the relation between models that have endogenized autonomous demand and the literature on feedback effects from the labor market to aggregate demand. Section 3 outlines the two benchmark models of autonomous demand. Section 4 describes and simulates our two policy regimes: an ‘SSM regime’ with a constant growth rate of government consumption and a ‘functional-finance regime’ with a state-dependent fiscal policy. Section 5 offers a few concluding comments.

2. Autonomous demand

2.1. Short-run versus long-run autonomy

Short-run Keynesian models are centered around an equilibrium condition for the goods market

$$Y = F(Y, Z); \quad 1 > F_Y > 0, F(0, Z) > 0, F(Y^{max}, Z) < Y^{max}$$

where Z is a vector of variables that are independent of the current value of output. In a simple linear version and assuming a univariate Z , this equilibrium condition can be written

$$Y = aY + bZ$$

⁵ Dutt (2019), Palley (2019), Freitas and Christianes (2020), Nah and Lavoie (2020) and Hein and Woodgate (2020) are recent contributions with constant growth rates of autonomous demand; Brochier and Macedo Silva (2019), Allain (2019, 2020) and Nomaler et al. (2020) endogenize the autonomous component of aggregate demand.

⁶ Interesting contributions by Fazzari et al. (2013, 2020) also have affinities with neo-Marxian theory and functional finance. Both papers use the autonomous-demand terminology but emphasize the supply-side limits on economic growth and leave open the possibility that autonomous demand – even if it were to grow at a constant rate – may not stabilize the economy.

or

$$Y = \frac{b}{1-a}Z = mZ$$

Equilibrium output is given in this equation as the product of a multiplier $(b/(1-a))$ and ‘autonomous demand’ Z .

If the trajectory of Z is exogenously given and the multiplier is constant, the short-run results extends to the long run: the trajectory of Z determines the trajectory of Y , and autonomous demand ‘drives long-run growth’.

A modified version of this result applies to the case in which the Z vector contains some components that are influenced by past values of output. As a simple example, let⁷

$$Y = C + I$$

$$I = \gamma_1 Y + \gamma_2 K \quad (1)$$

$$C = c_0 e^{\alpha t} + c_1 Y + c_2 K \quad (2)$$

We now have

$$Y = m[c_0 e^{\alpha t} + (c_2 + \gamma_2)K] \quad (3)$$

$m = 1/(1 - c_1 - \gamma_1)$ is the short-run multiplier, and short-run Keynesian stability requires that $1 - c_1 - \gamma_1 > 0$. It is readily seen that if $\alpha - \gamma_2 > \gamma_1(c_2 + \gamma_2)/(1 - c_1 - \gamma_1)$, then this economy will converge to a steady growth path with⁸

$$Y = \frac{1}{1 - c_1 - \gamma_1 - \gamma_1(c_2 + \gamma_2)/(\alpha - \gamma_2 + \delta)} c_0 e^{\alpha t} = m^{sm} Z \quad (4)$$

$m^{sm} = \frac{1}{1 - c_1 - \gamma_1 - \gamma_1(c_2 + \gamma_2)/(\alpha - \gamma_2)}$ is the supermultiplier, and $Z = c_0 e^{\alpha t}$ is the component of demand that is autonomous in the long run and that drives economic growth.

⁷ The investment function describes the accumulation rate as a function of the output capital ratio. With a constant value of γ_2 it represents a benchmark Kaleckian specification. Freitas and Serrano (2015) also use the investment function (1) but with $\gamma_2 = 0$ and an added Harroddian dynamics for the investment propensity γ_1 . For simplicity, we take γ_1 and γ_2 to be constant in this example.

⁸ If capital depreciates at the rate δ , the growth rate of Z/K is given by

$$\left(\frac{\dot{Z}}{Z}\right) = \dot{Z} - \hat{K} = \alpha - \gamma_1 \frac{Y}{K} - \gamma_2 + \delta = \alpha - \gamma_2 + \delta - \frac{\gamma_1}{1 - c_1 - \gamma_1} \left(\frac{Z}{K} + c_2 + \gamma_2\right)$$

This equation has a stable stationary point at $\frac{Z}{K} = \frac{(1 - c_1 - \gamma_1)(\alpha - \gamma_2 + \delta) - \gamma_1(c_2 + \gamma_2)}{\gamma_1}$. Equation (4) now follows by using this expression for Z/K to substitute for K in equation (3) and rearranging.

A Keynesian model does not, however, require demand components that are autonomous in the long run. As an example, retain the investment function (1) but modify the consumption function (2) by removing the component that is ‘long-run autonomous’:

$$C = c_1 Y + c_2 K$$

We now get

$$Y = \frac{1}{1 - c_1 - \gamma_1} (c_2 + \gamma_2) K$$

and (using the investment function (1) and assuming depreciation at the rate δ) the growth rate becomes

$$\hat{Y} = \hat{K} = \frac{\gamma_1 (c_2 + \gamma_2)}{1 - c_1 - \gamma_1} + \gamma_2 - \delta$$

In this model there is a short-run multiplier, and the wealth effect on consumption (the term $c_2 K$) as well as the effect of the capital stock on investment ($\gamma_2 K$) are autonomous in the short run.⁹ But there is no supermultiplier, and long-run growth is not ‘driven’ by some autonomous component of demand, be it a consumption component ($c_2 K$) or an investment component ($\gamma_2 K$).¹⁰ Instead, the growth rates of both output and the capital stock are determined endogenously by the parameters of the consumption and investment functions. In this simple example the economy jumps straight to the steady growth path without any short-run dynamics. The model would still have a steady growth path, but the path would be unstable if we were to add Harrodian dynamics to the investment function; the dynamics could take the form of a dependence of the change either in γ_2 (as in Allain 2016) or in γ_1 (as in Freitas and Serrano 2015) on the utilization rate.

⁹ The consumption function is in line with recent work on SSM. Brochier and Macedo Silva (2019) make autonomous consumption an endogenous variable dependent on rentier wealth. They allow for endogenous changes in Tobin’s q and the ratio of household wealth to the capital stock. This extension, although interesting in other respects, does not affect the joint determination of the growth rates of output, the capital stock and household wealth by the parameters of the model. It complicates their model, however, and they do not provide an explicit analytical expression for the long-run growth rate (in fact their system may have multiple stationary states and their simulations highlight one that is locally stable).

As argued by Oreiro, Silva and dos Santos (2020, p.527, n.18), however, the dynamics of rentier wealth is determined by rentiers’ saving, which depends on the level of economic activity and rentiers’ income. Thus, it is hard to see how spending that is determined by wealth can be autonomous in any meaningful sense, if the analysis is extended beyond the short run.

¹⁰ Brochier and Macedo Silva (2019, p. 427) and Nomaler et al. (2020, p. 6) misleadingly use the term ‘supermultiplier’ to describe the standard short-run multiplier.

In short, the literature has claimed that autonomous demand drives long-run growth and that asymptotically output is determined as the product of a supermultiplier and an autonomous component of demand. For these claims to be meaningful it is not sufficient to identify a component of demand whose current value is independent of current output; the component must also be independent of past levels of output.¹¹

2.2. Endogenous autonomous demand

Output may be linked to some other variable that grows at a constant rate, even if that variable is not itself a component of aggregate demand. This is essentially what happens in models that ‘endogenize autonomous demand’ by introducing a feedback effect from the employment rate to the average propensity to consume, as in Allain (2019, 2020) and Nomaler et al. (2020).

In Allain (2019, 2020) the consumption rate C/Y is given by

$$\frac{C}{Y} = (1 - s)(1 - \pi) + s\alpha\beta(1 - \pi) \frac{1}{e}$$

where e is the employment rate. Thus, the consumption rate depends inversely on the employment rate. Nomaler et al. (2020) employ a dynamic version of this negative feedback. They assume that¹²

$$\begin{aligned} \frac{C}{K} &= (1 - s) \frac{Y}{K} + \zeta \\ \dot{\zeta} &= \mu(\bar{e} - e) \end{aligned}$$

¹¹ Thirlwall (2002), for whom autonomous means to be exogenous to the economic system, exemplifies a strict application of this condition. According to Blecker and Setterfield (2019, pp.386-387) “[M]odern Kaldorian growth theory shares a methodological affinity with the recent work of some Sraffians and Kaleckians (...) however the unique exogenous driver of growth in Kaldorian models is export growth (...) in a model of strictly regional growth. Exports (or their fundamental determinants) can, of course, satisfactorily be taken as exogenous at the level of individual region, without this presupposing anything about the nature of the growth process globally. Kaldorian growth models are not, therefore, identical to other supermultiplier growth models that seek to furnish explanations of global growth and, in doing so, seem to be creating a new species of exogenous growth theory comparable to first-generation neoclassical growth theory (...)”.

¹² Nomaler et al. include separate wealth effects for workers and capitalists (two ζ variables) as well as separate stocks of physical capital and R&D capital. For present purposes, however, these extensions are of no importance. The notation has also been changed to make it consistent with the notation in this paper.

Thus, the autonomous component ζ changes in response to deviations of the employment rate from some neutral rate (\bar{e}),

Models with feedback effects from employment to saving are interesting, but the autonomous-demand terminology is misleading: the models have no ‘supermultiplier’. By definition, steady growth paths always have stable reduced-form relations between endogenous variables, but a reduced-form relation between endogenous variables does not represent a supermultiplier in any meaningful sense. There is a constant proportionality between output and capital, for instance, if the output capital ratio is constant, but that does not imply that the capital stock drives output (or that output drives the capital stock) in a unidirectional way.

There is also an accounting relation between output, labor productivity, the employment rate and the total labor force:

$$Y = \frac{Y}{L} \frac{L}{N} N = mN$$

where $m = \frac{Y}{L} \frac{L}{N}$. Thus, if both labor productivity and the employment rate are constant in the long run, output can be written as the product of a ‘supermultiplier’ and the size of the labor force. From this perspective the growth in the labor force (or the growth in population if the participation rate is constant) drives economic growth. But when did an exogenously given labor supply become a component of aggregate demand?

Aggregate demand affects long run outcomes in models with feedback effects from the employment rate to saving. The influence, however, runs via the endogeneity of the employment rate and the productivity of labor: the value of the multiplier in the relation between Y and N is endogenous. And in models with technical change the multiplier will be growing at a constant rate in steady growth.

The Allain and Nomaler et al. models have strong similarities with the literature that has stressed feedback effects from employment to investment and/or income distribution. To see this, consider a simple linear model in which saving and/or investment depend on the employment rate as well as on the utilization rate and the profit share:

$$\frac{I}{K} = \gamma + \gamma_u u - \gamma_e e; \quad \gamma_e \geq 0 \tag{5}$$

$$\frac{S}{K} = s\pi u + b e; \quad b \geq 0 \tag{6}$$

In the absence of technical change and normalizing labor productivity to one (or, equivalently, measuring L in efficiency units of labor), we have $e = \frac{L}{N} = \frac{YK}{KN} = uk$ where k is the ratio of the capital stock to the total labor force. Thus, the short-run equilibrium value of u in this closed economy without public sector is given by

$$u = \frac{\gamma}{s\pi - \gamma_u + (b + \gamma_e)k}$$

In order to ensure stability of the short-run equilibrium for all values of k , the Keynesian stability condition $s\pi > \gamma_u$ must be met.

If the labor supply grows at the rate n and capital depreciates at the rate δ , we now have

$$\hat{k} = \hat{K} - n = (s\pi + bk) \frac{\gamma}{s\pi - \gamma_u + (b + \gamma_e)k} - (n + \delta) \quad (7)$$

The right-hand side of this equation is decreasing in k , and the equation has a unique and stable stationary point k^* with $0 < k^* < \infty$ if the conditions $\frac{s\pi\gamma}{s\pi - \gamma_u} > n + \delta > \frac{b\gamma}{b + \gamma_e}$ are satisfied. The stationary values of e and k are given by¹³

$$k^* = \frac{\gamma s\pi - (n + \delta)(s\pi - \gamma_u)}{(n + \delta)(b + \gamma_e) - \gamma b}$$

$$e^* = uk^* = \frac{\gamma}{s\pi - \gamma_u + (b + \gamma_e)k^*} k^*$$

Using the Keynesian stability condition and the constraints that ensure a positive and finite stationary solution for k^* , tedious but straightforward calculations show that as long as $b + \gamma_e > 0$, we have

$$\frac{\partial e^*}{\partial \gamma} > 0; \quad \frac{\partial e^*}{\partial s} < 0; \quad \frac{\partial e^*}{\partial \pi} < 0$$

These comparative static results are independent of whether the employment effect runs through investment or saving. Thus, the analysis shows that (i) employment effects on investment and/or saving can align the warranted rate with the natural rate, (ii) the signs of

¹³ Restrictions on the parameter values are needed to ensure that the employment rate takes a meaningful value between zero and one. As a more satisfactory and robust way to ensure meaningful solutions, well-motivated non-linearities in the employment effects can be introduced. For present purposes, however, the simple linear forms will do.

the long-run employment effect of shocks to animal spirits, the share of profits or the saving rate are independent of whether the employment rate produces a negative feedback on investment (as in Flaschel and Skott 2006) or a positive feedback on saving (as in Allain 2019 and Nomaler et al. 2020). For simplicity, the results have been derived using linear investment and saving functions; they apply more generally, however, to nonlinear specifications (see Appendix A).

The analysis can be extended by introducing Harrodian dynamics: the rate of change in the parameter γ in the investment function may depend positively on the utilization rate. Equation (7) describing the dynamics of k still holds, but it is now combined with a second equation

$$\dot{\gamma} = \lambda(u - u_n)$$

A stationary solution to this 2D system requires that $u = u_n$ and $\frac{S}{K} = \frac{I}{K} = n + \delta$. The Harrodian dynamics pins down the utilization rate, and both the investment and saving functions must therefore include another accommodating variable to ensure the existence of stationary solutions with $\frac{S}{K} = n + \delta$ and $\frac{I}{K} = n + \delta$. For investment, the level of γ may serve that purpose, and no additional degrees of freedom are needed. But if the employment rate does not affect saving directly -- as in equation (6) with a positive parameter b -- a degree of freedom must be introduced in some other way.¹⁴

The obvious solution is for the adjustment to happen via the profit share and -- if the model is extended to include a public sector -- via economic policy. The employment rate affects wage bargaining, the general business climate and firms' price setting, and the average saving rate, in turn, is affected by income distribution. The policy effects are straightforward, too. Economic policy reacts both directly to changes in employment and to the inflationary implications of changes in employment. Inspired by Goodwin's formalization of Marx's 'general law of capitalist accumulation', a large literature has pursued these feedback effects,

¹⁴ The simple specification with an exogenously given desired rate of utilization represents a special case of Harrodian instability. The same problem of instability and reconciliation of the warranted and natural growth rates arise if the long-run accumulation rate is highly sensitive to persistent changes in utilization rates (formally, u_n may be an increasing function of g ; Skott 1989). If, say, the steady-growth accumulation rate must fall in the range $\underline{u} \leq u \leq \bar{u}$, then steady growth at the natural rate becomes impossible if the net saving to capital ratio exceeds (falls below) the natural growth rate for all values of the utilization rate in this interval. The simple specification with a unique desired utilization rate corresponds to the shrinking of this interval to a single point.

while adding Keynesian elements that were completely absent in Goodwin's original model.¹⁵ Like Allain and Nomaler et al., these models include feedback effects from employment to the (public plus private) saving rate but mediated by the distribution of income and/or economic policy.

The feedback effects from employment to aggregate demand are central to the 'endogenization of autonomous demand' as well as to the theories in which the employment rate affects investment, income distribution and economic policy. As indicated above for the case without Harrodian dynamics, the similarity extends to the comparative statics and the preservation of 'Keynesian properties' for the long run, supposedly a major achievement of the autonomous-demand model. Notions of wage-led and profit-led growth cease to be well-defined if the distribution of income becomes endogenous (as in some Harrodian models). But the preservation of a paradox of thrift as well as positive employment effects of a boost to animal spirits does not require the particular specification adopted by the autonomous-demand approach.^{16 17}

One can debate the relative significance of the different feedback effects. Conditional on income, does the employment rate exert a strong positive influence on household saving?¹⁸ Or are the effects of labor market conditions on pricing, investment, income distribution and inflation likely to be more important? Are the feedback effects strong enough to solve the Harrodian problems or does a capitalist economy depend on government policy or on interactions with a non-capitalist sector? These are questions that should be examined, a task that is beyond the scope of this paper.¹⁹ It is difficult to see, however, why direct feedback effects from the employment rate to saving should be uniquely Keynesian and categorically different from feedback effects to investment and the distribution of income. It is difficult to see, moreover, why the feedback effects to saving are being cast as representing the operation of a 'supermultiplier' applied to 'endogenous autonomous demand'.

¹⁵ See, e.g., Skott (1989, 2015), Flaschel and Chiarella (2000), Barbosa-Filho and Taylor (2006), von Arnim and Barrales (2015), Thompson (2018).

¹⁶ See, e.g., Skott (1989). In the simple Harrodian case an increase in animal spirits corresponds to a fall in the desired utilization rate.

¹⁷ Picking assumptions to obtain desirable results would not, in any case, be a great scientific methodology.

¹⁸ A saving effect could also come via corporate saving, as suggested by Thompson (2018), if firms adjust both investment and retained earnings in response to high employment rates.

¹⁹ The questions are not new. The possibility that weak feedback effects fail to ensure the existence and stability of a nontrivial steady growth path was noted by, among others, Skott (1989, sections 5.6 and 6.4.3) and Nakatani and Skott (2007). Franke (2018) and Ryoo and Skott (2019) discuss stabilization of a Harrodian economy through policy.

3. Benchmark Models of Autonomous Demand

If autonomous demand is the driver of long-run economic growth, it must be possible to identify components of aggregate demand that can plausibly be treated as exogenous in the long run. This section examines two models that take this route.

3.1. Allain's Formulation

Allain (2015) focuses on government consumption as the autonomous component of demand and to avoid complications from public debt dynamics, he assumes a balanced budget. His investment function adds Harrodian dynamics to a simple Kaleckian short-run specification,

$$\frac{I}{K} = \gamma + \gamma_u(u - u_n) \quad (8)$$

$$\dot{\gamma} = \lambda(u - u_n) \quad (9)$$

where $u = Y/K$. Private saving (S) is taken to be proportional to after-tax profits,

$$\frac{S}{K} = (1 - \tau)s\pi u \quad (10)$$

where τ , s and π denote the tax rate, the saving rate out of aftertax profits and the profit share.

Unlike Allain we include depreciation; I and S are gross investment and gross saving, and the growth rate of the capital stock is given by $\hat{K} = I/K - \delta$. Two reasons motivate this slight modification of the model. The proportional saving rate, first, seems more plausible as a description of the relation between gross income and gross saving, rather than between net income and net saving. Using gross variables, second, empirical calibration yields a higher saving rate which favors the model: it becomes possible to allow a higher value of γ_u without jeopardizing short-run stability, and an increase in the value of γ_u enhances the stabilizing effect of autonomous demand.

Government consumption is predetermined in the short run but grows at a constant rate,

$$\hat{G} = \alpha \quad (11)$$

The ratio of government consumption to capital ($z = G/K$) therefore follows a differential equation,

$$\dot{z} = \hat{G} - \hat{K} = \alpha - g \quad (12)$$

where $g = I/K - \delta$ is the net accumulation rate. All incomes are taxed at the same rate, and the balanced budget assumption implies that

$$G = \tau Y = T \quad (13)$$

Thus, the tax rate satisfies the condition

$$\tau = z/u \quad (14)$$

Short-run equilibrium requires that $(S + T - G)/K = I/K$, and using equations (8)-(14) we have:

$$(1 - \tau)s\pi u = s\pi u - s\pi z = \gamma + \gamma_u(u - u_n)$$

Hence,

$$\begin{aligned} u &= \frac{\gamma - \gamma_u u_n + s\pi z}{s\pi - \gamma_u} \\ &= \frac{1}{s\pi - \gamma_u} \gamma + \frac{s\pi}{s\pi - \gamma_u} z - \frac{\gamma_u u_n}{s\pi - \gamma_u} \end{aligned} \quad (15)$$

$$\begin{aligned} g &= \frac{I}{K} - \delta = \gamma + \gamma_u \left(\frac{\gamma - \gamma_u u_n + s\pi z}{s\pi - \gamma_u} - u_n \right) - \delta \\ &= \frac{s\pi}{s\pi - \gamma_u} \gamma + \frac{s\pi \cdot \gamma_u}{s\pi - \gamma_u} z - \frac{s\pi \cdot \gamma_u u_n}{s\pi - \gamma_u} - \delta \end{aligned} \quad (16)$$

The dynamics of the economy can now be described by a 2D system of differential equations. Substituting (15)-(16) into equations (9) and (12), we have:

$$\dot{\gamma} = \lambda(u - u_n) = \lambda \left(\frac{\gamma - \gamma_u u_n + s\pi z}{s\pi - \gamma_u} - u_n \right) \quad (17)$$

$$\dot{z} = z(\alpha - g) = z \left[\alpha - \frac{s\pi}{s\pi - \gamma_u} \gamma - \frac{s\pi \cdot \gamma_u}{s\pi - \gamma_u} z + \frac{s\pi \cdot \gamma_u u_n}{s\pi - \gamma_u} + \delta \right] \quad (18)$$

Equations (17)-(18) always have a stationary solution (a steady growth path) with $z = 0$ and $\gamma = s\pi u_n$. This stationary solution describes the standard Harrodian warranted growth path in an economy without autonomous demand. The more interesting case arises when the system allows for a second solution with $z > 0$; this happens if $s\pi u_n > \alpha + \delta$.

Assuming that the existence condition is satisfied, the second stationary solution is given by $(\gamma^*, z^*) = \left(\alpha + \delta, u_n - \frac{\alpha + \delta}{s\pi}\right)$. This stationary point is locally stable if the Harrodian adjustment parameter satisfies the condition (see Appendix B),

$$\lambda < s\pi\gamma_u z^* = \gamma_u[s\pi u_n - (\alpha + \delta)] \quad (19)$$

3.2. A Serrano-Freitas version

As in the Allain example, let government consumption be the autonomous component of demand ($Z = G$) and assume that the government budget is balanced,

$$T = G \quad (20)$$

The equilibrium condition for the goods markets can be written

$$S + T - G = I \quad (21)$$

or simply, using equation (20),

$$S = I \quad (22)$$

Assuming a constant private saving rate s out of disposable income, we have

$$S = s(Y - T) \quad (23)$$

Serrano et al. (2019) (SFB), finally, assume that investment is determined by the following equation:

$$I = hY = \nu y^e Y \quad (24)$$

where ν is the capital-output ratio at the desired utilization rate, and y^e is the expected growth rate of demand.²⁰ I and S denote net investment and net saving.

²⁰ Curiously, in the SFB specification of the investment function there is no predetermined investment, even in the short run. Investment adjusts instantaneously to any short-run increase in the level of output. This determination of investment by the value of current output implies that non-capacity generating demand, by construction, becomes the only predetermined variable. Moreover, it denies any influence of uncertainty and animal spirits on investment spending. In fact, Sraffian or Neo-Ricardian Keynesians do not seem to attach much importance to uncertainty, expectations and animal spirits for economic analysis. In the words of Eatwell and Milgate (2011, p. 301), “If, on the other hand, an attempt is made to locate uncertainty and expectations within the class of the persistent, systematic forces characterizing the workings of the economy, then the analysis becomes bereft of any definite result—the behavior of the economy being as arbitrary as the hypothesis made about the formation of expectations”.

Combining equations (20)-(24), the level of output in short-run equilibrium is given by

$$Y = \frac{s}{s-vy^e} G \quad (25)$$

The Keynesian stability condition is satisfied, and the equilibrium solution is positive if $s > vy^e$. If this condition fails to be met, the model becomes economically meaningless.

Unlike most of the literature on autonomous demand, Serrano et al. (2019) cast their model in discrete time, and from the equilibrium condition (25) it follows that

$$y_t = \alpha + \frac{v(y_t^e - y_{t-1}^e)}{s - vy_t^e} (1 + \alpha) \quad (26)$$

where $y_t = (Y_t - Y_{t-1})/Y_t$ and $\alpha = (G_t - G_{t-1})/G_{t-1}$ are the growth rates of output and government consumption. The dynamics of the expected growth rate are given by:

$$y_t^e = y_{t-1}^e + \beta[y_{t-1} - y_{t-1}^e] \quad (27)$$

The parameter β represents the speed of adjustment of the expected growth rate of output towards the actual growth rate; expected growth influences investment, and this adjustment captures the Harrodian dynamics in the model.²¹

The two-dimensional dynamic system (26)-(27) has no economically meaningful stationary solutions if $s < v\alpha$. Focusing on the case with $s > v\alpha$, the system has a unique stationary solution with $y = y^e = \alpha$. As in the Allain model, the instability condition can be expressed as an upper limit on the adjustment speed; β must satisfy the condition

$$\beta < \frac{s-v\alpha}{v[1+\alpha]} = \frac{1}{1+\alpha} \left[\frac{s}{v} - \alpha \right] \quad (28)$$

Considering the differences in specification, this local stability condition is remarkably similar to the stability condition in the Allain model. Allain's investment equation (8) implies that

$$(g + \delta) - \gamma = \gamma_u(u - u_n) \quad (29)$$

Combining (29) and the Harrodian dynamics (equation (9)), we have

²¹ A similar interpretation of the Harrodian dynamics has been suggested by Lavoie (1995). But unlike SFB and seemingly motivated by a desire to simplify the analysis, Lavoie uses the accumulation rate as an approximation for the growth rate of output.

$$\dot{\gamma} = \frac{\lambda}{\gamma u} (g + \delta - \gamma) \quad (30)$$

If $\beta = \lambda/\gamma u$ denotes the sensitivity of the change in γ to deviations of the accumulation rate from its steady growth value, the limit on the stability condition in equation (19) can now be expressed as

$$\beta < s\pi z^* = s\pi u_n - (\alpha + \delta) = \frac{s\pi - \nu\delta}{\nu} - \alpha \quad (31)$$

The last equation in (31) follows from the observation that the value of the output capital ratio at normal utilization is u_n in the Allain model; thus, $u_n = 1/\nu$.

The net saving rate in the Allain specification is $s\pi - \nu\delta$. Thus, comparing (28) and (31), the only difference is the multiplicative term $1/(1 + \alpha)$ in the stability condition for the SFB model. This multiplicative term tightens the stability condition for the SFB model, but the difference is small: the term is close to one.²²

4. Two Policy Regimes

Our ‘SSM regime’ is straightforward: the growth rate of government consumption (α) is set equal to the target rate of growth. Once α has been chosen, no further adjustments are made on the spending side; following Allain, tax rates are adjusted to maintain a balanced budget.

The functional finance approach advocates an active fiscal policy, rather than a passive, Friedmanite rule that keeps the growth rate of government consumption constant. The fiscal parameters are adjusted continuously in response to changes in the state of the economy. We assume a balanced budget at all times but unlike in the SSM approach, the growth rate of government consumption is not kept constant. There is no reason why fiscal policies based on functional finance should maintain a balanced budget; it may be desirable to run deficits in some periods but surpluses if conditions change. The balanced-budget assumption facilitates comparison with the SSM scenarios, however.

Suppose that the economy is initially following a steady growth path with output and government consumption growing at the rate α_0 and utilization at the desired rate. Policy makers now want to raise the growth rate to α_1 . Suppose, moreover, that they wish to implement this increase as fast as possible, but that utilization rates above \bar{u} (where $\bar{u} > u_n$)

²² Franke (2020) discusses the SFB model in greater detail.

will lead to overheating and bottlenecks with adverse effects on inflation (and, in an open economy, on the current account). Given these targets and constraints, functional finance prescribes an expansionary fiscal policy that raises actual utilization rates to the upper limit \bar{u} as quickly as possible. Once $u = \bar{u}$, any further increase must be avoided, and fiscal adjustments now aim to keep utilization at the ‘safe’ rate \bar{u} until the accumulation rate has increased sufficiently, at which point utilization rates can be brought back down to the desired rate u_n .

In the Allain model output is a jump variable, the Harrodian dynamics determine the change in the investment parameter γ as a function of the utilization rate, and the short-run solution for $u = Y/K$ is given by

$$u = \frac{\gamma - \gamma u_n + s\pi z}{s\pi - \gamma u} \quad (32)$$

Changes in the level of G affect the ratio z of government consumption to capital and thereby also the utilization rate u . Implementing the expansion of the modern sector as fast as possible translates into an instantaneous jump in z to get $u = \bar{u}$.

Setting $u = \bar{u}$ and solving for z , we get

$$z = \frac{G}{K} = \bar{u} - \frac{\gamma + \gamma u_n (\bar{u} - u_n)}{s\pi} \quad (33)$$

The utilization rate now exceeds normal utilization; the value of γ will start increasing, and policy makers reduce z gradually as γ increases in order to keep actual utilization at the upper bound ($u = \bar{u}$). When γ has reached the target value for the gross accumulation rate ($\gamma = \alpha + \delta$), the expansionary policy is abandoned. The government spending ratio z is adjusted to the level that is consistent with $u = u_n$ and the target growth rate, that is,

$$z = z^* = u_n - \frac{\alpha + \delta}{s\pi} \quad (34)$$

The length of the adjustment period can be found analytically in this model: the dynamic equation for γ implies that the transition from α_0 to α_1 will take $(\alpha_1 - \alpha_0)/[\lambda(\bar{u} - u_n)]$ periods.

Given the similarities in stability conditions and convergence speeds – and following the advice of an anonymous referee – we present only the simulations for the Allain model. All simulations consider an initial steady-growth path which is disturbed by a permanent

shock to the growth rate of government consumption. The simulations use the Runge-Kutta method for numerical integration of ODEs.²³

The simulations use the parameters in table 1. The parameter α – the growth rate of public spending -- is raised from 0.03 to 0.05. The values of the depreciation rate δ and the average saving rate $s\pi$ have been chosen as empirically plausible benchmarks. High values of the investment parameter γ_u relax the local stability condition in equation (19), but short-run Keynesian stability requires that $\gamma_u < s\pi$; robustness of the Keynesian analysis to shifts in saving parameters or income distribution, moreover, requires that the stability condition must be satisfied by a significant margin. The value of γ_u balances these concerns. Given the values of the other parameters, the condition for local stability of the new steady growth path requires that $\lambda < 0.005$.

Table 1 – Parameters for Allain-SSM simulation

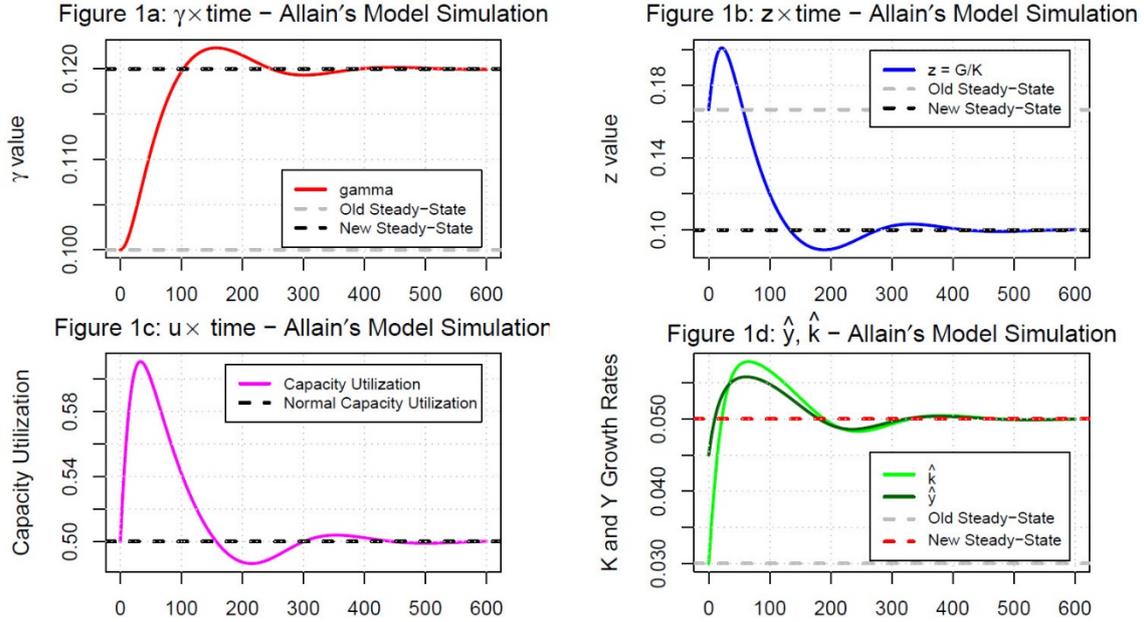
Parameter	Value	Parameter	Value
α_0	0.03	λ	0.0025, 0.0052
α_1	0.05	$s\pi$	0.3
δ	0.07	u_n	0.5
γ_u	0.166		

Figures 1a-1d depict a stable case with $\lambda = 0.0025$; figures 2a-2d illustrate the instability that follows from raising the adjustment speed. The extremely slow rate of convergence is explained by the low value of the adjustment parameter. If the adjustment parameter is interpreted as reflecting adjustments in expected growth, the low value ($\lambda = 0.0025$) implies that the half-life of deviations of expected growth from a constant steady-growth value is about 46 years, that is, if accumulation rates were to increase permanently from 3 percent to 5 percent, it would take 46 years before expected growth rates have

²³ We also analyzed the paths using Euler’s method for numerical integration. No relevant differences appeared among the simulated models.

adjusted from 3 to 4 percent.²⁴ Slow convergence is coupled with a prolonged period – more than 70 years – in which utilization rates exceed the normal rate by more than ten percent.

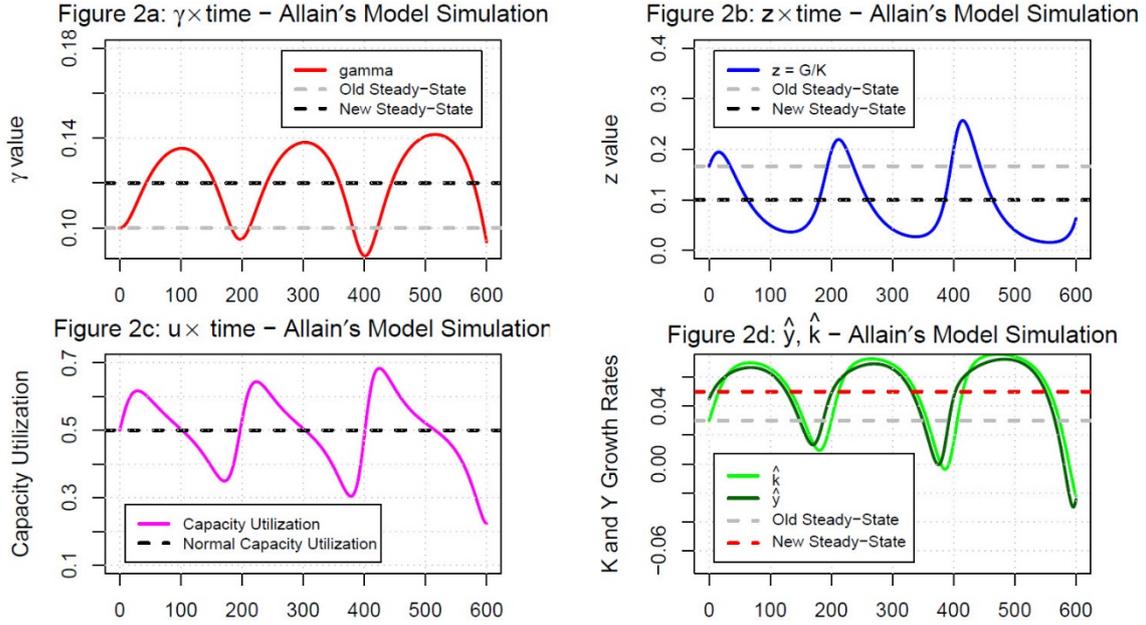
Figures 1a-1d: Allain-SSM trajectories for $\lambda = 0.0025$



The ‘high’ adjustment speed in figures 2a-2d ($\lambda = 0.0052$) exceeds the threshold for stability (which is 0.005), and the steady growth path becomes unstable. The explosive oscillations have a very long period length, however, reflecting the low adjustment speed; the half-life for the adjustment of expectations is about 23 years which is still extremely slow.

²⁴ The solution to the differential equation $\dot{x} = \beta(\bar{x} - x)$ is given by $x = \bar{x} + (x_0 - \bar{x})e^{-\beta t}$ where x_0 is the value of x at $t = 0$. Thus, setting $(x - \bar{x})/(x_0 - \bar{x}) = 1/2$, we have $e^{-\beta t} = 1/2$ or $t = \ln 2/\beta$. This general result can be applied to the thought experiment in the text. The experiment, which concerns the adjustment of expected growth to a hypothetical shift in the actual rate of growth, involves the Harrodian dynamics in equation (30) with $g + \delta$ treated as exogenous. The adjustment speed for expected growth is $\beta = \lambda/\gamma_u$, and the result now follows.

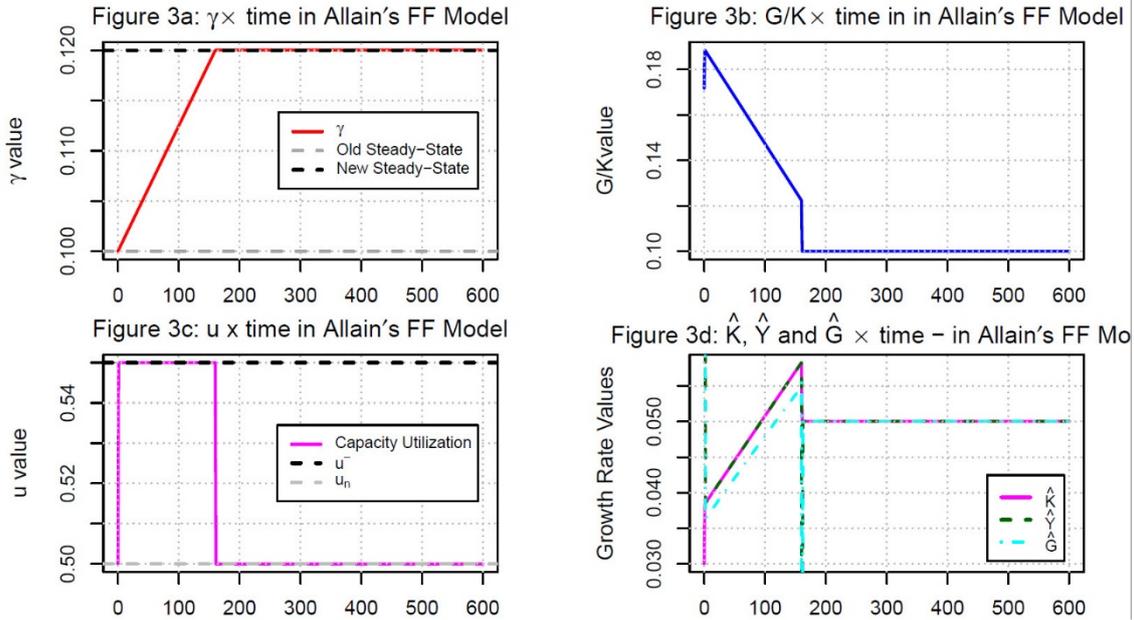
Figures 2a-2d: Allain-SSM trajectories for $\lambda = 0.0052$.



The simulations in figures 3a-3d for the functional finance version of the Allain model use the same parameters as in Allain-SSM simulation in Figures 1a-1d. One new parameter has been added, however: we set the threshold for the output capital ratio to 0.55 ($\bar{u} = 0.55$); that is, increases in utilization rate of more than 10 percent above the normal rate must be avoided to prevent overheating.

The convergence is still slow, but the profile has changed: the utilization rate jumps immediately to the upper limit of the safe range and stays at this upper limit during the transition process. The accumulation rate therefore increases more quickly than in the Allain-SSM specification during the early stage of the transition. The relative speeds are reversed during the later stages as the utilization rates increases above the safe rate in the SSM specification. The SSM trajectory overshoots the new steady growth path, and the times to full convergence have the same order of magnitude in the two cases.

Figures 3a-3d: Allain-FF trajectories for $\lambda = 0.0025$



Figures 4a-4d: Allain-FF trajectories for $\lambda = 0.06$

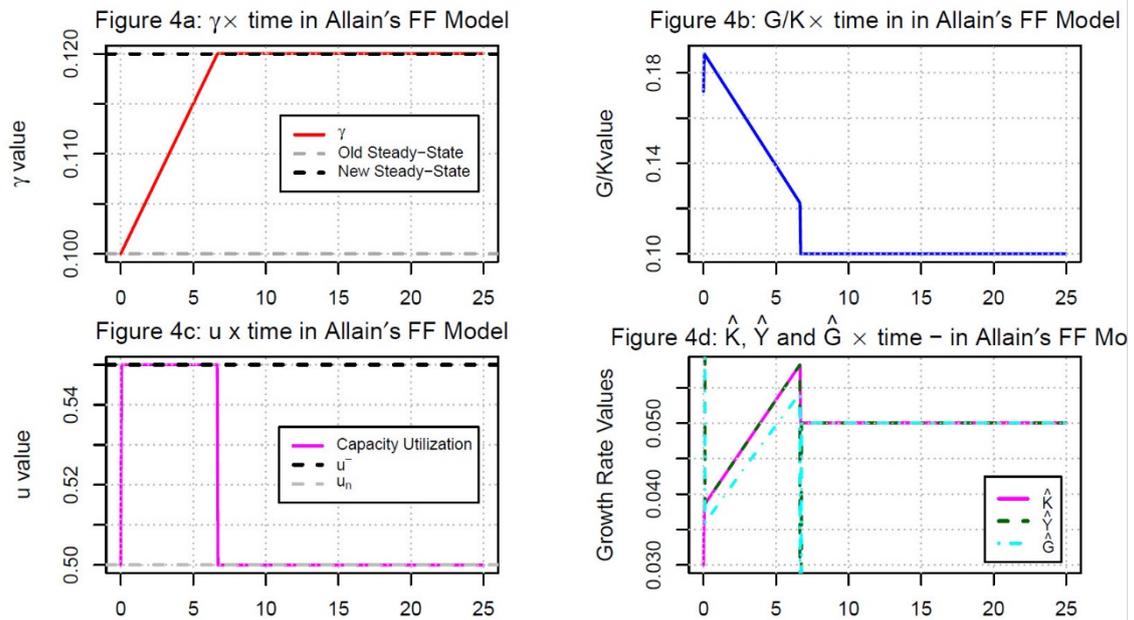


Figure 4a-4d corresponds to figure 2a-2d for the Allain SSM model. Stability conditions do not limit the permissible adjustment speeds in the FF version, however, and the high value of the adjustment speed has been raised to $\lambda = 0.06$. This value implies a

plausible half-life of expectation adjustment of about 2 years, and the time to full convergence is now reduced to less than 7 years.

The simulations illustrate a general point: magnitudes matter. Dynamic models with more than one state variable often contain stabilizing and destabilizing forces. When this happens, the stability of the system cannot be determined without quantitative assessments. If autonomous demand – rather than active policy or some other state dependent feedback effect – is the stabilizing force, the dynamics will be slow, and convergence to the steady growth path will be drawn out as illustrated by the numerical example.²⁵

Autonomous demand could, in principle, be the driver of long-run growth, even if stabilization is achieved through active stabilization policy. But the simulations of our stylized functional finance regime also show, first, that the growth rate of government consumption deviates from its asymptotic value for prolonged periods; functional finance is not just a matter of adding countercyclical, short-run movements in fiscal policy. Second, a policy that relies on adjustments in the share of autonomous demand in total income to produce the desired growth rate raises serious questions if government consumption is seen as the autonomous component. In the numerical examples, the steady-growth share of government consumption falls from 33 percent when the growth rate is 3 percent to 20 percent when the growth rate is 5 percent. Is it economically or politically desirable to determine the size of the public sector in this residual way?

5. Conclusion

The post-Keynesian literature increasingly recognizes the potential significance of Harrodian instability. The SSM approach has addressed the instability issue by emphasizing the stabilizing forces of components of demand that do not create capacity and are ‘autonomous’, that is, independent of current and past movements in output.

²⁵ Citing Lavoie (2017), a referee questioned the use of numerical simulations to evaluate theoretical models and their stability properties. This is a curious position.

Consider a static model and suppose a change in some variable x has both positive and negative effects on another variable y . Suppose, further, that economist A makes a claim that the net effect is negative. If economist B suggests that in fact the negative effect is weak and that the net effect is likely to be positive, then economist A can challenge this assessment. But an argument to the effect that quantitative assessments of the opposing forces are intrinsically meaningless in abstract models would also undermine the initial claim made by economist A. No statements about comparative statics can be made without reference to the relative strength of the opposing effects.

Stability analysis in models with both stabilizing and destabilizing forces resemble this static example.

This approach has been attractive to many researchers because allegedly it preserves a ‘Keynesian position’ in which investment is independent of saving and long-run growth is driven by autonomous demand (e.g. Garegnani 1992, pp.47-48, Serrano and Freitas 2015, p.17). This argument is peculiar. Why would one insist that changes in the rate of saving have no effects on current or future investment? Saving rates influence aggregate demand and the utilization rate, and firms react to changes in capacity utilization. There is nothing ‘un-Keynesian’ about feedback effects from saving behavior to investment²⁶. In fact, in the SSM models autonomous demand allows utilization to converge to the desired rate precisely because the trajectory of autonomous demand generates changes in the average saving rate. These changes endogenize Harrod’s warranted growth and influence investment, thereby reversing the cumulative process by which actual growth rate diverges from the warranted growth rate (Oreiro, Silva and Santos 2020; Skott 2019b).

Clearly, some demand components are autonomous in the short run. This, indeed, is a standard element of all short-run Keynesian models. Fractions of both consumption and investment can plausibly be seen as predetermined (exogenously given) in the short run, and we routinely analyze the short-run effects of shifts in consumer confidence or animal spirits. It is also true that some components of demand can be semi-autonomous in the medium run. Relaxations of credit constraints, for instance, can generate asset bubbles that feed on themselves and influence aggregate demand (Oreiro 2005). But asset bubbles do not continue forever, and they are not exogenous: feedback effects from output are important, both when it comes to sustain the bubbles and for an understanding of why the bubbles burst at some point. These autonomous short-run components and semiautonomous medium-run processes, moreover, are irrelevant for the analysis of the local stability properties of the steady growth path in the SSM models.

Government consumption could play the role of autonomous demand in the long run: within limits, it could be set to grow at an exogenous rate. A sensible policy, however, will adjust fiscal (and monetary) policy so as to achieve the policy makers’ targets. In a mature economy, as Lerner (1943) argued, economic growth with full employment would seem an obvious and fairly uncontroversial target. To be sure, ‘full employment’ may not be precisely defined, even in a mature economy, and in dual economies there is no similar, obvious target

²⁶ It is important to notice that the existence of such feedback effects of savings over investment does not mean that “prior savings” are required to finance investment. As Argued exhaustively by Post-Keynesians like Davidson (1994, chapter 8) investment finance is essentially a demand for money, not for savings. The same argument is made by Bibow (2009, p.30).

for the growth rate. But the principle of functional finance still applies: if a growth target has been decided upon, we can analyze the implications for fiscal and monetary policy. Policy makers still face a double challenge: to adjust the warranted growth rate to the target rate and prevent divergence from the warranted growth path. To best meet this challenge, they must respond to movements in output and, more generally, to economic performance.

The simulations in this paper illustrate the difference between the two approaches to economic policy. The rate of convergence is extremely slow in all scenarios when the Harrodian adjustment parameter is kept within the range that will ensure stability in the SSM cases. This range is very narrow, however, and the implied half-life of adjustments in expectations is unreasonable large. Plausible adjustment speeds generate fast convergence in the FF cases, but divergence in the SSM cases.

Even in the stable cases, the SSM simulations produce prolonged periods with very high utilization rates. Empirically, utilization rates exhibit large cyclical fluctuations, but it would be unprecedented to have utilization rates stay 10-20 percent above normal for a period of more than 50 years; yet this is what happens in the SSM simulations of a two percentage point increase in growth rate of autonomous demand. The FF simulations capped the utilization rate at 10 percent above normal; if 20 percent is safe – in the sense that it does not lead to bottlenecks and inflation -- the cap could be raised, and the convergence would be faster.

SSM proponents could object that the cards have been stacked against the SSM policy: the simulations of functional finance presume an unrealistic ability of policy makers to control and fine tune the economy. This is a fair point. Complete stabilization is illusory and short-run fluctuations cannot be eliminated. But economic policy (aided by endogenous feedback effects from employment to investment, income distribution and saving) can prevent local instability from turning into global divergence, and the idealized trajectories in the simulations may represent a good first approximation if the amplitude of the fluctuations can be kept small. Imperfect policy, moreover, does not affect the main message: policy makers may not be able to fine tune the economy in the precise way suggested by the simulations, but they can do better, surely, than keep constant the growth rate of government consumption and maintain a balanced budget. If an economy is in deep recession, then presumably Keynesian economists would recommend aggressive stimulus, rather than balanced budgets and the continuation of the previous trend in government spending.

Proponents of SSM models could raise a second objection. The emphasis in SSM models on the stabilizing influence of autonomous demand suggests a simple rule for fiscal policy: set the growth rate of government consumption and rely on the long-run convergence of accumulation and economic growth to this growth rate. This is the policy rule that we have simulated and that we criticize in this article. But proponents of SSM have not, to our knowledge, been explicitly advocating this rule. In their paper entitled “A baseline supermultiplier model for the analysis of fiscal policy and government debt” Freitas and Christianes (2020) consider the steady-growth effects of a “fiscal policy using the rate of growth of government expenditures as an instrument” (p. 331). But they also point out that the focus of the paper on fully adjusted equilibria with a constant growth rate of government consumption “limits our ability to evaluate the economic effects and relative merits of different stylized versions of fiscal policy rules (and conventions) adopted by governments in practice” (p. 314). As another example, having considered the stabilizing effects of a constant growth rate of government consumption, Allain (2015, p. 1365) suggests that his “results support the use of discretionary counter-cyclical fiscal policies to maintain activity, employment and economic growth”.

We would be delighted if SSM proponents view the fixed growth rate of government consumption as neither a policy recommendation nor a good first approximation to the policies adopted by governments in practice. But if active policy – perhaps along the lines of functional finance -- is what the SSM proponents recommend, there would seem to be neither a need for autonomous demand to stabilize the economy nor any good candidates for the roles of long-run autonomous demand and driver of economic growth. Instead of a vain search for any such candidates, we can focus on discussing how policy should be designed to meet the challenges we face.

The simple models and simulations in this paper have many limitations. We have completely eschewed open economy complications except for a brief reference to overheating and balance of payments problems. We have also said nothing about monetary policy, and we have restricted the types of fiscal policy under consideration. Sensible fiscal policies may require a non-balanced budget, and we excluded this possibility by assumption.²⁷ The exclusion forced us to treat government consumption and the share of government

²⁷ Freitas and Christianes (2020) and Hein and Woodgate (2020) analyze debt dynamics and economic growth when (wholly or partially) debt-financed government consumption grows at a constant rate; Skott (2016, 2020) consider debt dynamics under functional finance, assuming that taxes are used as the fiscal instrument while the ratio of government consumption to capital is kept constant at whatever level is deemed desirable.

consumption in total income as mere policy instruments to control the accumulation rate. The government consumption ratio G/Y should not, however, be treated as an accommodating variable whose value has no independent interest aside from its effects on aggregate demand. Fast growth may necessitate a squeeze on consumption (this happens in the SSM models through the decline in the share of autonomous demand). But it does not have to be government consumption that is squeezed; in fact, large parts of government spending may be essential for long-term development (as are other policies, including industrial, trade and education policy). Decisions must be made about how many teachers and roads are needed; taxes can then be used to adjust aggregate demand. For the purposes of this paper the restricted policy space and the balanced-budget assumption are harmless. But policy discussions need to consider a much richer menu once we abandon fiscal rules of constant growth rates for government consumption.

Appendix A: Comparative statics with general functional forms

Suppose that investment and saving functions are given by

$$\frac{I}{K} = F(u, k); F_k \leq 0$$

$$\frac{S}{K} = G(u, k); G \geq 0$$

and that $F_k - G_k < 0$. If the Keynesian stability condition is satisfied ($F_u - G_u < 0$), the short-run equilibrium solution for u is an inverse function of k (assuming the existence of an equilibrium with $I = S$),

$$u = u(k)$$

The dynamics of the ratio of the capital stock to the total labor force is given by

$$\hat{k} = G(u(k), k) - n = \phi(k)$$

If this equation has a stable stationary solution, we must have $\phi'(k^*) < 0$.

Now consider shocks to the investment or saving functions. Positive shocks to investment and negative shocks to saving both have the same qualitative effect on the short-run equilibrium: they generate an upward shift in the $u(k)$ function. An upward shift in the $u(k)$ function, in turn, produces an upward shift in the $\phi(k)$ function, and the

stationary solution for k^* increases. With an increase in k^* and an upward shift in the $u(k)$ function, finally, the steady growth value of the employment rate ($e = uk$) must also increase.

Appendix B: Local stability in the Allain model

The local stability of the stationary solution to equation (17)-(18) is determined by the Jacobian matrix evaluated at the stationary state. We have:

$$J(\gamma, z) = \begin{bmatrix} \frac{\lambda}{s\pi - \gamma_u} & \frac{\lambda s\pi}{s\pi - \gamma_u} \\ -z^* \frac{s\pi}{s\pi - \gamma_u} & -z^* \frac{s\pi \gamma_u}{s\pi - \gamma_u} \end{bmatrix}$$

The determinant is unambiguously positive,

$$\det J = z^* \frac{\lambda s\pi}{(s\pi - \gamma_u)} > 0$$

Thus, local stability of the steady growth path is ensured if the trace of Jacobian is negative; formally, if

$$\text{tr } J = \frac{\lambda - s\pi \gamma_u z^*}{s\pi - \gamma_u} < 0 \quad (\text{B1})$$

By assumption, the short-run equilibrium is stable ($s\pi > \gamma_u$). The stability condition (B1) therefore imposes an upper limit on the Harroddian adjustment parameter,

$$\lambda < s\pi \gamma_u z^*$$

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