Using meta-regression to explore moderating effects in surveys of international achievement

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This article demonstrates how meta-analytic techniques, that have typically been used to synthesize findings across numerous studies, can also be applied to examine the reasons why relationships between background characteristics and outcomes may vary across different locations in a single multi-site survey. This application is particularly relevant to the analysis of data from international surveys of student achievement. A brief introduction to the method of meta-regression is provided and the technique is demonstrated in an analysis of the extent to which the relationship between school autonomy and achievement varies depending upon the level of accountability in a country. The results show that the meta-regression approach to analysis is more accurate than combining data across all countries into a single simple model.

Imagine, for example, we have a set of research studies examining the differences in achievement between boys and girls. Some of the studies find that boys’ achievement is higher whereas others find that girls’ achievement is higher. Furthermore, even where two studies agree on the direction of the difference, the size of the reported difference between boys and girls may differ. When faced with conflicting results from different studies (a fairly typical scenario) we want to understand why. This might include looking for any patterns associated with the skills measured in different studies, the age at which measurements were made, or the form of measurement. This attempt to understand the reasons for differences between different studies is a common issue within the wider topic of meta-analysis (Bangert-Drowns and Rudner, 1991).

Although the methodologies presented in this article are applicable more widely, this article particularly focusses on surveys of international achievement such as the Programme for International Student Assessment (PISA) and the Trends in International Mathematics and Science Study (TIMSS). Although the majority of attention on the results from these studies focusses on the international rankings, analyses examining how the relationship between various school and pupil characteristics and achievement varies between countries are also provided. For example Organisation for Economic Co-operation and Development, OECD, (2010a) provides details of gender differences across countries and OECD (2010b) provides details on how the impact of socio-economic background and immigrant status varies across countries. By viewing results from different countries in the same way as we would results from different studies, we can use methods from meta-analysis to conduct exploratory analysis examining why such differences exist. This article provides some background to a particular type of meta-analysis known as meta-regression, and gives an example of how such an approach can be implemented in R (R Core Team, 2013). The example will also illustrate how this approach can lead to very different results from less nuanced approaches.
Similar approaches to those described in this paper have been applied to examine treatment effects in multisite interventions. This methodology, described by Kalaian (2003), aims to understand why the impact of an intervention may vary between different schools, classrooms, or geographic locations. Similarly, Springer et al. (2004) describe meta-analysis applied to the CSAP National Cross-site Evaluation of High Risk Youth Programs. This multi-site survey used the same methodology and data collection instruments to evaluate the effectiveness of 48 different substance abuse prevention programs implemented in different parts of the USA. Analysis focused on identifying underlying reasons to explain the effectiveness of programs in different locations.

The greater the difference between sites, the more we might expect effects to vary between them. As such, using techniques from meta-analysis to examine differences between sites may be particularly pertinent to data from international surveys of student achievement. Having said this, such approaches to analysis of data from international surveys have seldom been applied in practice. Rare examples are provided by Lietz (2006) and Else-Quest et al. (2010) each of whom examined the reasons underlying gender differences across different countries.

**Brief description of meta-regression**

Thorough descriptions of the different methods available for meta-regression are given by Thompson and Higgins (2002), and Higgins and Thompson (2004). According to the authors “in contrast to simple meta-analysis, meta-regression aims to relate the size of effect to one or more characteristics of the studies involved”. There are two different approaches to meta-regression: the fixed effects meta-regression model and the random effects meta-regression model.

If the estimated effect of interest (e.g. the difference in achievement between genders) in the ith study is denoted by \( y_i \) then the fixed effects meta-regression model assumes that:

\[
y_i \sim N(\theta_i, \nu_i) \quad \theta_i = x_i^T \beta
\]

Where the \( \theta_i \) is the (unknown) “true” effect in the ith study, \( \nu_i \) is the variance of the estimated effect from the ith study (that is, the standard error squared), \( x_i \) is a vector of study level covariates (that is, the moderating variables) including a constant term for the intercept and \( \beta \) is the vector of study level covariates (that is, the moderating variables). The fixed effects meta-regression model can be fitted using standard weighted regression where the weights are set to equal \( 1/\nu_i \). However, because the above model assumes that all of the variance in estimated effects can be explained by the moderating variables and the within-study sampling errors, the standard errors need to be adjusted (see Higgins and Thompson, 2004, p. 1665 for further details).

In contrast to the fixed effects model, the random effects model allows for the fact that there may be residual, unexplained variance in true effects across different studies. These residual variances are assumed to follow a normal distribution so that:

\[
y_i \sim N(\theta_i, \nu_i) \quad \theta_i = N(x_i^T \beta, \tau^2)
\]

Where \( \tau^2 \) is the residual variance in true effects. Programs to fit random effects meta-regression models using the method of restricted maximum likelihood (REML) have been written in Stata (Sharp, 1998), SAS (van Houwelingen et al., 2002) and R (Viechtbauer, 2010).

Given the wide variety of differences in context across different countries in an international survey, the assumptions of the fixed effects meta-regression model are unlikely to hold for the data we are interested in. For this reason, it is better to use the random effects meta-regression model to explore differences in effects across countries.

One of the key dangers of applying meta-regression is the temptation to engage in “data dredging”. This is particularly pertinent in the realm of international achievement data as the number of potentially interesting country-level contextual variables far exceeds the number of countries participating in such studies. As a result, there is a danger that excessive data exploration, without initial thought to generate a small number of well-defined hypotheses, could lead to false positives. The best advice to avoid this is “to minimize the number of covariates investigated, to select those justified through scientific rationale and to specify them in advance” (Higgins and Thompson, 2004, p. 1679).

**Example: An analysis of the relationship between school autonomy and reading achievement**

As an example, we examine the relationship between school resource autonomy and reading achievement across OECD countries participating in the...
Programme for International Student Assessment (PISA) in 2009. This relationship was examined within one of the OECD’s reports on the same data (OECD, 2010c). Resource autonomy was measured by the extent to which schools had a say over which teachers they employed, how much they paid individual teachers and how they allocated their budget.

The original analysis was based upon an amended form of Ordinary Least Squares (OLS) regression. Regression models applied to data from international surveys need to account for the fact the data must be weighted (to ensure responses within each country are representative of the population of that country as a whole and that all countries are given equal weight overall), that achievement is defined using multiply imputed plausible values, and that the standard errors of regression need to account for the within-school clustering of pupils in each country (see OECD, 2009, pages 35-45 for further details). Fortunately a number of tools have been provided by the OECD and others (see for example Caro, 2013) that make applying this form of OLS regression relatively straightforward.

The original analysis examined the relationship between school autonomy for resource allocation and achievement across 33 different OECD countries. Of particular interest was the extent to which this relationship interacted with the degree of school accountability, defined as the percentage of students attending schools that posted achievement data publicly. The results of this original analysis are reproduced below with the main coefficients of interest highlighted.

Based on the results in Table 1, in particular the significantly positive interaction between autonomy and accountability, the original analysis was used to suggest that in countries where there is a high degree of accountability in a country (that is, most schools report achievement data publicly), there is a positive relationship between the degree of resource autonomy a school has and the reading achievement of their pupils. Conversely, the significantly negative main effect for autonomy indicates that, where there is a low degree of accountability in a country, greater autonomy will be associated with lower achievement.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>Standard Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>School autonomy for resource allocation</td>
<td>-3.24</td>
<td>1.45</td>
</tr>
<tr>
<td>Interaction with percentage of students in schools that post achievement data publicly (additional 10%)</td>
<td>0.58</td>
<td>0.28</td>
</tr>
<tr>
<td>School autonomy for curriculum and assessment</td>
<td>0.04</td>
<td>0.59</td>
</tr>
<tr>
<td>Private school</td>
<td>-0.48</td>
<td>1.49</td>
</tr>
<tr>
<td>PISA index of economic, social and cultural status of student (ESCS)</td>
<td>17.98</td>
<td>0.26</td>
</tr>
<tr>
<td>PISA index of economic, social and cultural status of student (ESCS squared)</td>
<td>2.06</td>
<td>0.22</td>
</tr>
<tr>
<td>Student is a female</td>
<td>36.23</td>
<td>0.51</td>
</tr>
<tr>
<td>Student’s language at home is the same as the language of assessment</td>
<td>17.02</td>
<td>1.23</td>
</tr>
<tr>
<td>Student without an immigrant background</td>
<td>11.64</td>
<td>1.2</td>
</tr>
<tr>
<td>School average PISA index of economic, social and cultural status</td>
<td>58.13</td>
<td>0.97</td>
</tr>
<tr>
<td>School in a city (100 000 or more people)</td>
<td>-2.36</td>
<td>1.21</td>
</tr>
<tr>
<td>School in a small town or village (15 000 or less people)</td>
<td>2.93</td>
<td>1.14</td>
</tr>
<tr>
<td>School size (100 students)</td>
<td>1.61</td>
<td>0.13</td>
</tr>
<tr>
<td>School size (100 students, squared)</td>
<td>-0.01</td>
<td>0.00</td>
</tr>
</tbody>
</table>

N 267,425

Note: Individual dummy variables were included in the regression to account for the effect of each of the 33 countries. These coefficients were not reported in the OECD report and for brevity are not included here.
Although it is appealing to be able to produce analyses such as those in Table 1 in a single step, it is not without its difficulties. One of the problems of the initial OECD analysis is that it assumes that each of the background variables included in the model have the same effect on students across different countries. This is despite the fact that existing research has shown that many of these effects vary. For example, other results (OECD, 2010a, 2010b) have shown differences in the link between deprivation or gender and attainment across countries. Moreover, the OLS model assumes that the residual variance in achievement (that is, the variance in achievement not accounted for by the variables included in the model) is constant across different countries. These issues could potentially be resolved by including large numbers of interaction variables in the above OLS model and by extending the method to account for non-constant variance. However, even then the analysis would need to assume that there is a single “true” coefficient both for the main effect of school autonomy and for the interaction with accountability that is invariant across countries. Note that attempting to analyze the trans-national data using hierarchical modeling (Osborne, 2000) where we imagine pupils are clustered within schools, which in turn are within countries, would not address these issues as such models also assume that the level of residual variance (that is, the extent of variability between pupils within schools and between schools within countries) is the same across all countries.¹

The above issues can be addressed using meta-regression. This requires a two-step procedure. Firstly, we estimate the relationship between resource autonomy and achievement in each country and the standard errors associated with these estimates. Once these country-level coefficients have been derived we can then apply meta-regression to explore whether there is any significant association with the level of school accountability.

**Step 1: Estimation of country-level coefficients**

The relationship between autonomy and achievement is estimated in each country using regression. In common with the original analysis, these within-country regressions need to account for the way the data is weighted and clustered within schools as well as the multiple imputation of plausible values for achievement. Also, as with the original analysis, in estimating this relationship we take account of the influence of other background variables.

Specifically we account for:

- The socio-economic status of pupils as measured by the index of Economic, Social and Cultural Status (ESCS).² A squared version of this measure is also included to account for the potentially nonlinear relationship between ESCS and achievement.
- Gender.
- Whether the test language is the same as the language the student speaks at home.
- Immigrant status.
- The average socio-economic status of pupils within schools.
- School location (whether in a city, town or small town/village).
- School size. A squared version of school size is also included to account for the potentially non-linear relationship between school size and achievement.

Accounting for these additional variables means that we are essentially calculating the partial regression coefficient of autonomy in each country. As with any calculation of partial regression coefficients, the choice of variables that we control for may have an important influence on both the results and the subsequent interpretation. As such, it is important to examine the sensitivity of results to the inclusion or exclusion of different variables. Indeed, in order to properly understand the relationship between given variable and the outcomes of interest, it is often worth calculating the regression coefficients before, as well as after, accounting for other background variables. As part of considering which variables to account for, it is worth calculating the correlations of each of the variables we

¹ Very recent developments in hierarchical modeling could potentially address such issues. For example, the mixed effects location scale model of Hedeker and Nordgren (2013) allow for the level of residual variance to vary between higher level clusters. However, the development of such methods is at an early stage so applying the method to data from international surveys would not be a straightforward task.

² This index is calculated for each student individually based upon their responses to various questions about parental occupation and education, as well as detailed questions about the possessions they have in their home.
control for with the main variable of interest (in this case autonomy). High correlations may potentially indicate that two variables are measuring the same construct so that controlling for one, whilst examining the effect of the other may not be appropriate.

Within this particular data set, in order to avoid such problems of multicollinearity in the within-country regressions, we have not included controls for whether schools are public or private, or for the level of school autonomy for curriculum and assessment. These are the only variables included in Table 1 that we have not accounted for. However, it should be noted that the same analysis recorded here has been run again including these two extra variables, and that this resulted in no change to the substantive conclusions.

Note that, because we are using a separate regression analysis within each country, our new analysis does not assume a constant relationship between each of the above factors and reading achievement. On the contrary, our analysis now explicitly accounts for the fact that, for example, the difference in attainment between males and females is larger in Finland than in Mexico (OECD, 2010a, p. 57), or that the difference between immigrant and non-immigrant pupils is larger in Italy than in Hungary (OECD, 2010b, p. 70).

Regression analyses based upon data from international studies can be done very simply and efficiently using the intsvy package in R (Caro, 2013) and an example of the code required to run the separate regressions is included in Appendix 1.

The estimated coefficients from these within-country regressions are shown in Table 2. Within each country, these coefficients display the estimated average increase in pupils’ reading scores associated with a change of one standard deviation in the level of a school’s resource autonomy. The standard error of each of these coefficients is also shown.

The countries in table 2 are sorted according to the level of accountability in each country as measured by the percentage of students attending schools that post achievement data publicly (the final column). According to the hypothesis proposed by the OECD, the relationship between resource autonomy and reading achievement should be stronger for countries at the bottom of Table 1 than for those at the top.

To further explore the extent to which these coefficients are associated with the level of school accountability, the resource autonomy coefficients for each country are plotted against the level of accountability in each country in Figure 1. These results are shown again in Figure 2 with three outlying countries (Belgium, Greece and Turkey) excluded from the chart.
formally combine the results across countries and calculate the statistical significance of any relationship, we must make use of meta-regression. This can be easily applied to the results in Table 2 using the R package metafor (Viechtbauer, 2010). The code required to run this analysis is given in Appendix 1.

The results of analysis are given in Table 3 (model 1). Because the application of meta-regression assumes that true coefficients follow a normal distribution across different studies, it is prudent to remove the outlying countries (Belgium, Greece and Turkey). The results with these countries removed are also shown in Table 3 (model 2). Neither of these models show any significant relationship between accountability and the association between autonomy and achievement. Finally, from the three countries in the bottom left hand corner of Figure 2 (Switzerland, Japan and Austria), we might generate the hypothesis that the relationship between autonomy and achievement is more likely to be negative in countries where the level of accountability is very low (below 10%). A test for this hypothesis is included in Table 3 (model 3) which shows a significant association between very low accountability and a negative effect of autonomy, but no significant association between autonomy and accountability beyond this. However, this hypothesis has only been generated after looking at the data and so should perhaps be seen as a case of data dredging rather than testing a genuine hypothesis. As a result, this finding should be treated with extreme caution.

**Discussion**

In studying data from multi-site surveys it is often our aim to understand the differences in results between different locations. In particular, in the case of data from international surveys of student achievement, often we are not only interested in differences in achievement across countries but also in differences in the pattern of achievement. This might include examining why particular subgroups of pupils (e.g. girls) perform particularly well in some countries but not in others. In such cases we might wish to explore the moderating effect of country-level variables on within-country regression coefficients.

**Step 2: Applying meta-regression**

The initial exploration above casts some doubt over the results of the original analysis. However, to more

https://scholarworks.umass.edu/pare/vol19/iss1/3

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We have seen in this article that simply including interaction effects within a single trans-national regression analysis is inadequate as it fails to fully account for:

- Variation in the influence of the explanatory variables across countries.
- Differences in residual variance across countries.
- The fact that differences in the effect of the variable of interest on achievement across countries are not purely due to the impact of whatever moderating variable (or variables) we are examining but may have genuine variation beyond this.

Meta-regression provides a straightforward method to address the above issues. All that is required is simply to break the analysis into two stages: a set of within-country regressions, and a meta-regression examining the effect of country-level moderating variables on whichever coefficient we are most interested in. As is shown in Appendix 1, this approach can be implemented very easily using recently developed tools.

It may be the case that, with further work, the two-stage procedure recommended here could be collapsed into a suitable one-stage analysis. However, as shown by (the brevity of) the code in Appendix 1 this would provide very little advantage in terms of time saving. Furthermore, the two-stage approach recommended here provides analysts with a chance to stop and plot the data (as in Figures 1 and 2) as is strongly recommended by Thompson and Higgins (2002) before applying the meta-regression. This allows a greater opportunity to evaluate the assumptions of the model and identify the influence of individual countries upon findings.

The example we have provided in this article shows that the recommended two-stage approach can lead to different conclusions to the simple method of trans-national regression. There should be no doubt that the conclusions of the meta-regression approach are preferable to those of the original analysis. As can be seen from Figure 2, straightforward analysis of the relationship between resource autonomy and achievement across different countries provided results that were directly contradictory to the results implied by the original overall model. This is unfortunate as there appears to be little point in generating overall analyses of international data if the majority of similar studies conducted within an individual country will give the opposite result. Making use of meta-regression provides a suitable method to ensure that international analyses will provide results that are consistent with results within individual countries.

### References


Citation:

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Appendix

R code to perform analysis of data from PISA detailed in this paper

#DATA FROM OECD COUNTRIES INITIALLY STORED IN DATA FRAME OECD1.  
#THIS DATA FRAME CONTAINS PLAUSIBLE VALUES FOR READING ACHIEVEMENT  
#(PV1READ-PV5READ), FINAL STUDENTS WEIGHTS (W_FSTUWT), REPPLICATE WEIGHTS  
#(WFSTR1-WFSTR80), COUNTRY IDENTIFIERS (CNT),  
#AND COVARIATES DERIVED FOR THE SPECIFIC ANALYSIS INCLUDING THE PERCENTAGE  
#OF STUDENTS IN SCHOOLS THAT POST ACHIEVEMENT DATA PUBLICLY (CNTPOST).  

#ORIGINAL OECD ANALYSIS (SINGLE OLS REGRESSION ACROSS ALL COUNTRIES)

library(intsvy)
pisa.reg.pv(x=c("CNT","RESPRES","INTRESP","RESPCURR","PRIVSCH","ESCS"  
 ,"ESCSSQ","FEMALE","LANG","NONIMMIG","ESCSSCH","SCHINCIT","SCHINSMA"  
 ,"SCHSI10","SCHSISQ"),pvlabel="READ",by="OECD",data=oecd1)

#PRODUCE RESULTS FROM SEPARATE REGRESSION IN EACH COUNTRY
reg1=pisa.reg.pv(x=c("RESPRES","ESCS","ESCSSQ","FEMALE","LANG"  
 ,"NONIMMIG","ESCSSCH","SCHINCIT","SCHINSMA","SCHSI10","SCHSISQ")  
 ,pvlabel="READ",by="CNT",data=oecd1)

#STORE COEFFICIENTS OF INTEREST INTO A SINGLE TABLE (SEE TABLE 2)  
coef=data.frame(COEF=rep(NA,length(reg1)),SE=rep(NA,length(reg1)))  
for (iz in 1:length(reg1)){  
  coef$COEF[iz]=reg1[[iz]]$Estimate[rownames(reg1[[iz]])=="RESPRES"]  
  coef$SE[iz]=reg1[[iz]]$"Std. Error"[rownames(reg1[[iz]])=="RESPRES"]}  
coef$CNT=names(reg1)

#MATCH IN DATA ON SCHOOL ACCOUNTABILITY IN THESE COUNTRIES
postach=unique(oecd1[,c("CNT","CNTPOST")])
coefs=merge(coefs,postach)

#META REGRESSION OF ESTIMATED COEFFICIENTS (EXCLUDING OUTLIERS)  
#ON LEVEL OF SCHOOL ACCOUNTABILITY IN THE COUNTRY
library(metafor)
rma.uni(yi=COEF,sei=SE  
 ,data=coefs$CNT%in%c("BEL","GRC","TUR") ==FALSE,)  
 ,method="ML",knha=TRUE,mods =-CNTPOST)