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Economic growth in dual and mature economies: revisiting the Pasinetti and neo-Pasinetti theorems

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Abstract

This paper (i) examines the role of income distribution in the determination of the average saving rate and the growth process in dual and mature economies, and (ii) revisits the Pasinetti and neo-Pasinetti theorems. The profit share may influence saving because of differences in the saving rates across households (the Pasinetti theorem) or because firms retain part of their earnings (the neo-Pasinetti theorem). The two mechanisms are not mutually exclusive, and the alignment between warranted and natural growth rates in mature economies can happen through feedback effects from employment to the distribution of income.

JEL: E12, E21, O41
Key words: Harrod's problems, income distribution, saving rates, Cambridge equation

1 Introduction

The warranted growth rate, Roy Harrod argued, may differ from the natural rate, and the warranted growth path is likely to be unstable. Many economists thought that these dire predictions about obstacles to full-employment growth were inconsistent with the stylized facts; there must, they argued, be mechanisms that reconcile the warranted and natural rates and prevent cumulative divergence.

Solow's theoretical solution to Harrod's first problem – the reconciliation of warranted and natural growth – allows the capital intensity to adjust appropriately. This argument is unconvincing. The Cambridge capital controversy demonstrated serious aggregation issues and undermined the notion that

*This paper draws on Skott (2022, chapters 4 and 8).

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economies move along a smooth production function as the relative supplies of capital and labor change. The capital controversy and its lessons have been largely forgotten, however, and neoclassical production functions remain a staple of most contemporary growth models. This state of affairs is all the more surprising since other mechanisms may explain how some – but not all – economies experience full-employment growth.

This paper considers the effects of income distribution on the warranted growth rate in models with a Leontief production function. The average saving rate is one of the determinants of the warranted growth rate, and the average saving rate depends on the share of profit. Thus, endogenous changes in distribution may align the warranted with the natural rate.

The dependence of saving on income distribution may arise because of heterogeneity between households; households that receive a large proportion of their income from profits may save a larger proportion of their income than households that receive mainly wage income. Pasinetti (1962) emphasized this link. Alternatively, saving rates out of profits may be high because firms retain some of their earnings, as argued by Kaldor (1996). The two explanations are not mutually exclusive, but I shall consider them separately. The average saving rate can also change for reasons that are unrelated to the distribution of income; I shall return to this point in the conclusion.

Section 2 presents a brief outline of Harrod's argument and three solutions to Harrod's first problem. Section 3 examines the household-based argument for differential saving rates and the Pasinetti theorem. Capitalists and workers have different saving behavior but, unlike in Pasinetti's original model, the saving rates are endogenous. Following Foley et al. (2019, chapter 17), workers are life-cycle savers and leave no bequests, while capitalist dynasties optimize with an infinite horizon. Section 4 discusses a version of Kaldor's 'neo-Pasinetti theorem' which emphasizes the distinction between firms and households. Section 5 concludes.

2 Warranted and natural growth rates

2.1 Harrod's problems

Consider a closed, one-sector economy without a public sector. If the production function is Leontief and λ and σ are the technical coefficients, we have

$$Y = \min\{\lambda L, \sigma K\} \tag{1}$$

Using standard notation, Y, L and K denote output, employment and capital.

There are minor variations in the utilization rate of labor (labor hoarding) over the cycle, and the utilization rate of capital exhibits substantial cyclical fluctuations. Firms, however, will adjust their investment decisions in response to persistent deviations of the utilization rate from the rate that they consider optimal. Thus, as a first approximation we may disregard labor hoarding and assume that capital is utilized at the desired rate in steady growth. Formally,

$$u = u^* \tag{2}$$

$$\frac{Y}{K} = u\sigma \tag{3}$$

$$\frac{Y}{L} = \lambda \tag{4}$$

where u^* is firms' desired rate of utilization of capital. Equation (2) describes the long-run investment function: in this extreme version of a Harrodian model the accumulation rate is perfectly elastic at $u = u^*$. To simplify notation the desired utilization rate is set to one and the technical coefficients λ and σ are taken to be constant. There is no technical change, and the natural growth rate n is equal to the growth rate of the labor force N,

$$\hat{N} = n \tag{5}$$

'Hats' over a variable will be used to denote growth rates throughout this paper; i.e. $\hat{x} = (dx/dt)/x$).

A linear saving function, an equilibrium condition for the goods market and an accounting equation for the evolution of the capital stock complete the Harrodian benchmark model:

$$S = sY \tag{6}$$

$$I = S \tag{7}$$

$$\dot{K} = I - \delta K \tag{8}$$

where s and δ are the average saving rate and the rate of depreciation.

Equations (1)-(8) can be used to derive the warranted growth rate g_w:

$$\hat{Y} = \hat{L} = \hat{K} = s\sigma u^* - \delta = g_w \tag{9}$$

This rate of growth is 'warranted' because of the consistency between expectations and outcomes: if they invest (on average) at the rate $\hat{K} = g_w$, then the investment decisions turn out to have been warranted in the sense that – taking into account the multiplier effects of investment on output – firms on average achieve the desired rate of utilization of capital.

This simple framework gives rise to two observations. If the natural rate of growth n and the values of s and σ are exogenously given, first, there is no reason to expect equality between the warranted and the natural rate. Only by a fluke will it be possible for a pure capitalist economy to follow a steady growth path with full employment. The warranted growth path, second, is likely to be unstable. The actual utilization rate u is determined by the equilibrium condition (7). If investment is predetermined in the short run, we have

$$u = \frac{\hat{K} + \delta}{s} \tag{10}$$

The actual utilization rate in equation (10) is increasing in the accumulation rate and exceeds the desired rate if for some reason $\hat{K} > g_w$. Faced with a shortage of capital, firms will tend to increase investment. A rise in $\hat{K}$, however, aggravates the problem: it causes the rate of utilization to increase further, implying that the warranted path becomes unstable.[1]

The focus in this paper is on Harrod's first problem. I shall assume throughout that the economy is on – or fluctuates around – the warranted path; this outcome is consistent with the local instability of the warranted path (the second problem) if nonlinearities or policy interventions turn local instability into bounded fluctuations.[2]

2.2 Three solutions

Dual economies Economies with employment rates that fluctuate around a fairly high level would come up against labor constraints if aggregate demand were to expand rapidly over periods lasting more than few years. Large-scale immigration could alleviate the labor shortages but would meet political opposition, and it is limited how fast and to what extent new groups can be drawn into the labor market (through changes in the retirement age, for instance). This group of 'mature' economies arguably include countries like the US, Japan and Germany. Economies that could sustain high growth rates for prolonged periods without running into general labor shortages are 'dual'; most developing countries fall into this category.

Harrod's first problem does not arise in developing economies with a small modern sector and large amounts of underemployment. In these economies the modern sector can grow at the warranted rate without running into labor constraints, even if the warranted rate exceeds the natural rate. In fact, it is desirable to have the modern sector grow rapidly, thereby absorbing (open and hidden) unemployment in traditional and informal sectors .

Solow While an elastic supply of labor to the modern sector 'solves' Harrod's first problem for dual economies (or more accurately, implies that the problem does not arise), a full-employment path represents a good approximation to the long-run growth rate in mature economies. The equality between the warranted and natural growth rates in these economies requires that

$$s\sigma u^* = n + \delta \tag{11}$$

[1] A simple formalization specifies a linear adjustment equation for the accumulation rate $\hat{K}$:

$$\begin{aligned}\frac{d\hat{K}}{dt} &= \nu(u - u^*) \\ &= \nu(\frac{\hat{K} + \delta}{s} - u^*)\end{aligned}$$

[2] E.g. Skott (1989, 2015, 2022), Chiarella et al. (2005), von Arnim and Barales (2015), Fazzari et al. (2013), Ryoo and Skott (2017).

The five terms in equation (11) cannot be determined independently; at least one of them must be allowed to adjust.

Robert Solow singled out Harrod's "crucial assumption that production takes place under conditions of fixed proportions" (Solow 1956, p. 65), assuming instead that the technical output-capital ratio σ adjusts.

Equation (11) can be used to determine the steady-growth value of the technical coefficient σ. The existence of a solution is ensured if the range of possible output capital ratios is sufficiently wide, but merely solving for σ does not establish that the economy will converge to this steady growth path. Solow examines this dynamic issue, but the analysis is conducted under very restrictive conditions.

Harrod had set out to examine whether a pure market economy would be likely to converge towards a steady growth path with full employment. Solow, by contrast, addresses a more limited question. He assumes, that "full employment is perpetually maintained" (p. 67) and that there is also "full employment of the available stock of capital" (p. 68). Imposing these assumptions, he then shows that it is possible to construct a logically consistent story in which both labor and capital are fully employed at all times and in which, starting from arbitrary initial values of the labor supply and the stock of capital, the trajectory of the economy converges to a steady growth path with a constant output capital ratio.

Considering its flimsy foundations, the fictional aggregate production function carries a very heavy load in Solow's story. Product and process innovations or shifts in the sectoral composition of output can affect aggregate capital output ratios, and firms may also change techniques in response to changes in factor prices. But changes in capital intensity typically require new and different physical capital goods and – contrary to the assumptions embedded in the neoclassical production function – the costly and slow transition to a new technique following a rise in interest rates may lead to lower capital intensity. Thus, in light of the Cambridge capital controversy and the literature on aggregation, models that rely heavily on the movements along a well-defined neoclassical production function should be avoided.[3] Instead, a Leontief production function represents a simple, neutral starting point in much the same way that linear functions may be preferred as a benchmark specification if there are no good arguments for introducing nonlinearities.[4]

Kaldor/Marx Nicholas Kaldor presented his 'Keynesian' explanation of growth with (near-) employment in Kaldor (1955-56). Leaving problems of the trade

[3] See Cohen and Harcourt (2003) and Felipe and Fisher (2003) for surveys of the capital controversy and the aggregation literature.

[4] Harrod himself recognized that in principle the equalization of warranted and natural growth rates could be accomplished through changes in interest rates and their effects on the choice of technique. He explicitly rejected this possibility, however, suggesting that "the rate of interest and the MARC [the minimum acceptable rate of return on capital] do not often have a big effect on the method chosen" and that an attempt to derive a rate of interest "which brought the warranted growth rate into equality with the natural growth rate ... really makes no sense" (Harrod, Economic Dynamics, Macmillan 1973, pp. 172-3).

cycle "outside the scope of this paper", he assumed that the natural growth rate "governs the growth rate" (p. 97).[5]

With a Leontief production function there are no marginal products to pin down the distribution of income, leaving open the possibility that adjustments in the share of profits can serve to equalize the warranted and natural rates.[6] Formally, if ω denotes the share of wages in income and the saving propensities out of wages and profits are s_w and s_p, the share of wages (ω) must satisfy

$$[s_w\omega + s_p(1-\omega)]\sigma = n + \delta$$

or

$$\omega = \frac{s_p\sigma - (n+\delta)}{(s_p - s_w)\sigma} \tag{12}$$

This solution exists and is economically meaningful if $(1 \geq s_p > \frac{n+\delta}{\sigma} > s_w \geq 0)$.

Kaldor focuses on the steady growth path, offering no discussion of investment decisions and of how the accumulation rate comes to be adjusted to the natural rate. But suppose factor markets are competitive and that the gross rate of return on capital (r) falls to zero if capital is in excess supply, while real wages (w) fall to some minimum if labor is an excess supply. A standard efficiency-wage argument – whether based on nutrition or asymmetric information – suggests that the minimum wage level will be positive.[7]

Formally, let

$$\left.\begin{matrix} w = w_{\min}; \quad 0 \leq w_{\min} < \lambda \\ r = r_{\max} = \sigma(1 - \frac{w_{\min}}{\lambda}) \end{matrix}\right\} \qquad \text{if } N \;>\; \frac{\sigma}{\lambda}K \tag{13}$$

$$\left.\begin{matrix} r = 0 \\ w = w_{\max} = \lambda \end{matrix}\right\} \qquad \text{if } N \;<\; \frac{\sigma}{\lambda}K \tag{14}$$

These distributional assumptions imply that the average saving rate will be s_w if $\lambda N < \sigma K$ and $s_p - s_w w_{\min}/\lambda$ if $\lambda N > \sigma K$. It follows that

$$\hat{k} = \hat{K} - n = \left\{ \begin{matrix} s_w\lambda N - (n+\delta) < s_w\sigma - (n+\delta) & \text{if} \quad \lambda N < \sigma K \\ (s_p - s_w\frac{w_{\min}}{\lambda})\sigma - (n+\delta) & \text{if} \quad \lambda N > K \end{matrix}\right.$$

As long as $s_w\sigma < n+\delta < (s_p - s_w\frac{w_{\min}}{\lambda})\sigma$, the economy will converge to a steady growth path with full employment and $\lambda N = \sigma K$; along this path the wage share is given by (12) and the profit rate is

$$r = \frac{n + \delta - s_w\sigma}{s_p - s_w}$$

[5] Kaldor subsequently changed his interpretation of the stylized facts, suggesting that he had been "wrong in thinking that 'low-earnings' sectors have been eliminated" and that Britain and other developed economies had reached maturity (Kaldor 1978, p. xx).

[6] The analysis can be extended to cases with limited substitutability between 'aggregate capital' and labor.

[7] The lower bound on the profit rate when $\lambda N < \sigma K$ was set to zero to simplify notation. It could be set, analogousely to $w_{\min}$, at a value $r_{\min} \gtreqless 0$.

Kaldor's mature-economy solution has close affinities with another, even more influential contribution. Using different terminology, Karl Marx discussed the relation between the warranted and natural growth rates in chapter 25 of *Capital*. Fast accumulation gradually reduces the 'reserve army of labor'; a decline in the reserve army strengthens workers and at some point the wage share will start rising. As the profit share decreases, however, accumulation falls, and low accumulation means that the reserve army gradually becomes replenished.

Goodwin (1967) formalized Marx's argument. Using the employment rate $e = L/N$ as an inverse indicator of the size of the reserve army of labor, he assumed that the growth rate of the real wage is an increasing function of the employment rate. If productivity grows at a constant rate, this assumption translates into a positive relation between employment and the wage share,

$$\hat{\omega} = f(e); \quad f' > 0 \tag{15}$$

Retaining the differential saving rates out of wages and profits and focusing on the case where $\lambda N > \sigma K = Y$, we have[8]

$$\frac{S}{K} = [s_w\omega + s_p(1-\omega)]\frac{Y}{K} = s(\omega)\sigma; \quad s' < 0 \tag{16}$$

Combining equations (7)-(8) and (16), the movements in the employment rate are given by another differential equation,

$$\begin{aligned} \hat{e} &= \hat{L} - \hat{N} \\ &= \hat{K} - n \\ &= s(\omega)\sigma - (n+\delta) \end{aligned} \tag{17}$$

The 2D system defined by equations (15) and (17) has an economically meaningful stationary solution with $0 < e^* < 1$ and $0 < \omega^* < 1$ if $f(0) < 0 < f(1)$ and $s(0) > (n+\delta) > s(1)$. The first condition stipulates that the wage share must be falling when workers are very weak (if $e \to 0$) but rising when they are strong (if $e \to 1$); the second condition requires that the accumulation rate (the ratio of net saving to capital) be greater than the natural growth rate if the wage share is zero, but smaller than the natural rate if the wage share is one. Assuming the conditions are met, the stationary solution is given by $\omega^* = s^{-1}(\frac{n+\delta}{\sigma})$ and $e^* = f^{-1}(0)$.

It can be shown that starting from any initial point, the Goodwin model produces conservative fluctuations around the stationary solution. As in the Kaldor solution, an endogenous distribution of income aligns the (long-run average of the) warranted rate with the natural rate. Unlike in the Kaldor version, however, the model generates cycles, the balance of power between capital and labor determines the changes in the wage share, and the stationarity condition for the wage share pins down the solution the employment rate e^*; an increase

[8] Goodwin set $s_p = 1$ and $s_w = 0$.

in workers' power and militancy – an upward shift in the $f-$function – reduces the stationary solution for the employment rate.[9]

3 Pasinetti theorems

The Kaldor/Marx alignment of the warranted with the natural growth rate relies on differences between the saving rates out of profits and wages. Pasinetti (1962) pointed out what appeared to be a flaw in this argument. If workers save, Pasinetti argued, they will receive profit income, and a distinction must be made between the profits going to capitalists (who have a high saving rate) and the profits going to workers (who have a low saving rate). The distribution of capital between capitalists and workers now evolves endogenously, and the average saving rate out of profits becomes a weighted average of the two saving rates, with the weights determined by the fraction of capital owned by workers; the higher this fraction, the lower will be the difference between the average saving rates out of profits and wages. This dependence seemingly jeopardized the Kaldor/Marx analysis, but Pasinetti showed that as long as workers' saving rate s_w is relatively low, capitalists' share of total capital stock converges to a strictly positive, stationary solution. Moreover, at the stationary solution the economy will satisfy the 'Cambridge equation': $g + \delta = s_c r$. This 'Pasinetti theorem' has been extended to cases in which the saving rates s_c and s_w are determined endogenously.

In a series of contributions Tom Michl has analyzed long-run growth in models with two classes, workers and capitalists. Both capitalists and workers maximize utility and both may save. But capitalist dynasties optimize over an infinite horizon, while workers engage in life-cycle saving and leave no bequests. The production function is Leontief and the economy may be either dual (with a 'conventional wage' and a perfectly elastic labor supply) or mature (with an accumulation rate that is equal to the growth rate of the labor force in efficiency units). The model is used to analyze social security and public debt issues in Michl (2007, 2013) and by Foley et al. (2019, chapter 17) to examine the Pasinetti theorem.

[9] It is not entirely clear from Marx's own verbal argument whether the level or the change of the share of wages is related to the size of the reserve army:

> If the quantity of unpaid labour supplied by the working class, and accumulated by the capitalist class, increases so rapidly that its conversion into capital requires an extraordinary addition of paid labour, then wages rise, and, all other circumstances remaining equal, the unpaid labour diminishes in proportion. But as soon as this diminution touches the point at which the surplus labour that nourishes capital is no longer supplied in normal quantity, a reaction sets in: a smaller part of revenue is capitalised, accumulation lags, and the movement of rise in wages receives a check. The rise of wages therefore is confined within limits that not only leave intact the foundations of the capitalistic system, but also secure its reproduction on a progressive scale. (Marx 1967 [2001], p. 891)

If equation (15) is replaced by a relation between employment and the *level* of the wage share, the resulting dynamics reduce to a single differential equation, and the economy converges monotonically to a steady growth path with a constant employment rate.

Using a discrete-time setting, Foley et al. assume that both capitalists and workers have logarithmic per-period utility. Workers live for two periods, working in the first period and providing for retirement in the second period by saving a part of their first-period wage income, as in Diamond (1965). The slightly streamlined version in this section is set in continuous time, and the OLG specification is replaced by a general life-cycle specification.

Formally, capitalists's consumption is determined by

$$\begin{aligned} & \max \int e^{-\rho t} \log c^c dt \\ & st. \\ \dot{k}^c &= (r-\delta)k^c - c^c \end{aligned} \tag{18}$$

All capitalists have the same utility function, and the optimization implies that

$$\hat{K}^c = \hat{k}^c = (r-\delta-\rho) \tag{19}$$

$$C^c = \sum \rho k^c = \rho K^c \tag{20}$$

where C^c and K^c denote the capitalists' total consumption and capital, with c^c and k^c as the corresponding values for single capitalist dynasties.

Workers' consumption follows Ando and Modigliani's (1963) specification of an aggregate consumption function derived from aggregating the consumption decisions of life-cycle optimizing households. The expressions for workers' consumption and the growth rate of workers' wealth are[10]

$$C^w = (1-s)wL + bK^w \tag{21}$$

$$\hat{K}^w = s(\sigma - r)\frac{K}{K^w} - (b-(r-\delta)) \tag{22}$$

A continuous-time version of the 2-period OLG version in Foley et al. would have $b = 1 + r - \delta$.

As in the Kaldor model in section 2.2, factor prices are determined by equations (13)-(14). Thus, in a dual economy the growth rate of aggregate capital can be written.

$$\hat{K} = (r_{\max} - \delta - \rho)\frac{K^c}{K} + [s(\sigma - r_{\max})\frac{K}{K^w} - (b - (r_{\max} - \delta))]\frac{K^w}{K} \tag{23}$$

Combining equations (19) and (23), capitalists' share of capital follows that the differential equation

$$\hat{K}^c - \hat{K} = [b - \rho - s(\sigma - r_{\max})] - (b-\rho)\frac{K^c}{K}$$

[10] Ando and Modigliano (1963) estimate coefficients of about 0.6 and 0.06 for $(1-s)$ and b which, they suggest, is in line with what would be expected theoretically. Muellbauer (2010) discusses more recent empitical models that use extended versions of the LCH, including the Federal Reserve's FRB/US model.

This equation has a stable stationary solution with $0 < K^c/K < 1$ if $0 < s(\sigma - r_{\max}) < b - \rho$:

$$\frac{K^c}{K} \to 1 - \frac{s(\sigma - r_{\max})}{b - \rho}$$

At the stationary solution we have

$$\hat{K} = r_{\max} - \delta - \rho \tag{24}$$

Convergence to maturity is assured if the growth rate of the capital stock exceeds the natural rate of growth (if $r_{\max} - \delta - \rho > n$).

Once maturity is reached, continued full employment requires that $\hat{K} = n$, that is,

$$\begin{aligned} \hat{K} &= (r - \delta - \rho)\frac{K^c}{K} + s(\sigma - r) - (b - (r - \delta))\frac{K^w}{K} \\ &= (b - \rho)\frac{K^c}{K} + s\sigma - b - \delta + (1 - s)r = n \end{aligned} \tag{25}$$

Equation (25) determines the profit rate r in a mature economy:

$$r = \frac{n + \delta + b - s\sigma}{1 - s} - \frac{b - \rho}{1 - s}\frac{K^c}{K}$$

Substituting this value of the profit rate into equation (19) and using (25), the dynamics of wealth distribution is given by.

$$\begin{aligned} \hat{K}^c - \hat{K} &= r - \delta - \rho - n \\ &= \frac{sn + s\delta + b - s\sigma - (1 - s)\rho}{1 - s} - \frac{(b - \rho)\frac{K^c}{K}}{1 - s} \end{aligned}$$

Hence, if $0 < s(\sigma - n - \delta - \rho) < b - \rho$, we get convergence to a stationary solution with $0 < K^c/K < 1$:

$$\frac{K^c}{K} \to 1 - \frac{s(\sigma - n - \delta - \rho)}{b - \rho}$$

The associated profit rate at the stationary solution is

$$r = n + \delta + \rho \tag{26}$$

Equations (24) and (26) capture the Pasinetti theorem. For both dual and mature economies there is convergence to a steady growth path with a direct positive relation between the growth rate and the profit rate (assuming that the inequality conditions are met). The relation is completely independent of the parameters that describe the workers' saving behavior; an increase in capitalists' discount rate ρ (an increase in their 'propensity to consume'), by contrast, raises the rate of profit for any given rate of economic growth.

The original Pasinetti formulation assumed exogenous saving rates, and the relationship took the form $s_c(r - \delta) = g$ where s_c is the saving rate out of net profits and g is the growth rate. The saving rates are endogenous in this version, and capitalists' saving rate is an increasing function of the profit rate, $s_c = (1 - \frac{\rho}{r-\delta})$. Using this notation – with s_c as a increasing function of $r - \delta$ rather than a constant parameter – equations (24) and (26) can be written in Pasinetti's original form $s_c(r - \delta) = g$.

The Cambridge equation shows that workers' saving and the endogenous determination of the fraction of capital owned by workers need not sever the link between the functional distribution of income and the average saving rate. Changes in the profit share still allow the equalization of warranted and natural growth rates.

4 The neo-Pasinetti theorem

4.1 Firms and households

By assumption all capitalists maximize the same utility function and the parameters of this function enter directly into the Cambridge equation. In 19th century capitalism capitalists owned and controlled the means of production, while workers were poor and did little saving. Marx could argue, moreover, that all capitalists save and invest because competition compels them to behave in this way. The delineation of capitalists is less obvious today. In a corporate economy ownership has been separated from the running of firms, and competition among firms does not enforce a uniform saving behavior across owners of equity and other financial assets. Bequests have also become important for groups that we may not want to categorize as capitalists.

Jeff Bezos, Elon Musk and Mark Zuckerberg clearly belong to the group of capitalist households but what about the Rockefellers, Gettys and Carnegies that may now live off inherited wealth? What about hedge fund managers or CEOs who may run major corporations but whose wealth is significantly less than that of many households that exert no direct control over the means of production? And should other members of the top 0.01 percent, including rock stars, baseball players and surgeons, who are likely to leave large bequests, be merged with life-cycle saving workers? It may seem churlish to ask concrete questions like this of a model that is designed to analyze properties of capitalist economies at a high level of abstraction. But the model defines two groups of households, and it should be possible to outline general criteria for the delineation of the groups. One group leaves bequests while the other does not, and in a corporate economy this criterion would seem to correlate more with a rich-poor distinction than with a traditional Marxian emphasis on control over the means of production. The aggregation of households into infinite-horizon optimizers and life-cycle savers may still provide a decent first approximation, but a defense of this particular delineation based on Marxian 'character masks' becomes less compelling in a corporate economy.

Samuelson and Modigliani (1966) brought up related issues in their analysis of the Pasinetti theorem when they questioned the "assumption of 'permanent' classes of pure-profit and mixed-income receivers with given and unchanging saving propensities" (p. 297). Kaldor (1966) responded to the critique in the same issue of the Review of Economic Studies. The high saving propensity out of profits, he argued, was "something which attaches to the nature of business income and not the wealth (or other peculiarities) of the individuals who own property" (p. 310). His 'neo-Pasinetti theorem' – which does not require a distinction between worker households and capitalist households – presented the analysis behind this claim.[11]

It may appear that Kaldor's argument can be made quite simply. In a closed economy without public sector, households' flow of disposable income (Y^D) is given by

$$Y^D = Y - R$$

where R is retained earnings. If retained earnings are proportional to profits and households save a constant fraction s of their disposable income, aggregate saving – the sum of household and corporate saving (S^H and S^F) – is given by

$$\begin{aligned} S &= S^H + S^F = sY^D + R \\ &= [s + s_f(1-s)\pi]Y \end{aligned}$$

where π is the profit share and s_f denotes the share of retentions in profits. Thus, if the retention rate is given, it would seem that an increase in the profit share must raise the average saving rate. But the simple proportionality between household saving and household disposable income can be questioned: households may 'pierce the corporate veil' and adjust their own saving rate out of disposable income so as to offset changes in corporate saving.

Retained earnings can be used by the firm to finance investment, pay off debt or buy back shares. These possible uses of retained earnings all raise future profits per share (net of interest payments) relative to what profits would have been otherwise, with greater reliance on internal finance also reducing the riskiness of the firm's shares. The increase in future profits per share and the reduced risk will tend to raise the value of the firm's shares. A household that wants to maintain its consumption despite higher retentions and an unexpected reduction in dividends now has two options.

One option is for the household to take out loans or reduce its holdings of liquid assets in an amount corresponding to the reduction in its dividends. This option, following Modigliani and Miller (1958), will be 'undoing the firm's retention policy': households offset the fall in firms' leverage by increasing their own leverage. The option, however, may be barred by credit constraints. Unlike the firm, the household may not be able to obtain loans, at least not on the same terms as the firm, even if the value of its shares has increased.[12]

[11] The name of the 'theorem' is misleading. Kaldor's analysis also rebutted Pasinetti's original criticism of his saving function by showing there is no logical slip in assuming different saving propensities out of wages and profits.

[12] Assuming 'full rationality' there are two main constraints on the validity of the Modigliani-Miller result: the possibility of firm bankruptcy and the presence of credit constrained households (Stiglitz 1969).

There are other reasons to disregard this option. Sophisticated real-world households diversify their portfolio compositions. They may also adjust the compositions towards less risky assets as they approach retirement age. And they may change the portfolio compositions in response to major – real or imagined – shifts in the riskiness and returns of different types of assets. But one would expect households to focus on the factors that appear most important. Assessments of the riskiness of and likely returns on tech shares, for instance, will have to take into account the regulatory zeal of the EU, Chinese industrial policies, and the likelihood of future pandemics, while climate change and its effects on government interventions and policies are crucial to the prospects of not just the energy sector but a whole slew of industries, from airlines to real estate in Florida. In this context differences in retention policies across firms may influence the relative weights of individual shares, but the influence of the firms' average retention rate on the overall weight of equity in the portfolio is likely to be far down the list of priorities, if it makes an appearance at all.

Empirical evidence supports the insensitivity of portfolio compositions to changes in riskiness. The Swedish government introduced a defined contribution component of its social security system in 2000. Each retirement saver could select among hundreds of mutual funds, allocating her retirement savings between up to five funds. Analyzing detailed data on the choices made by the entire population of 7,315,209 retirement savers in Sweden during the period 2000-2016, Cronqvist et al. (2018) show that 'nudging' had strong effects on the portfolio choices and that the effects of nudging were highly persistent: "the participants seem to have a 'set it and forget it' mindset" (p. 154). Following a change in regulation in 2010, the default fund, which had attracted a large proportion of savers and which had been entirely in equity, switched to having 50 percent leverage. Despite this large, sudden increase in the riskiness of the fund almost no one switched away from the default fund. Cronqvist et al. conclude that (p. 157) "[i]n outer space, an object that has been nudged will keep going in that direction until it is nudged again. Retirement savers appear to resemble such objects."

Overall, the behavioral evidence makes it implausible to assume that households respond to an increase in the average retention rate by taking out loans and/or reducing their holding of bonds and bank deposits. A rejection of this mechanism leaves households with a second option, however: a household may respond to a rise in share prices by selling a fraction of its shares. Thus, if an increase in retained profits causes the share valuation of a company to go up by the same amount, a household owning shares in the company can maintain exactly the same consumption and wealth as if it had received the dividends (rather than the capital gain): the household can 'declare its own dividends', not by adjusting its own borrowing but by selling shares. This is the argument addressed by Kaldor's 'neo-Pasinetti theorem'.

The intuition behind Kaldor's counterargument is simple. The suggestion that share prices will appreciate automatically in line with retained earnings involves a fallacy of composition, Kaldor argued. It may be correct that the share price of a single firm (relative to the general level of share prices) responds positively to an increase in the firm's retained earnings. It is also correct that an individual shareholder can declare her own dividends by selling some of her shares. But households as a group cannot finance consumption by selling shares: there is no one to buy. Households' attempt to compensate for compressed dividends by selling off equity leads to capital losses as equity prices fall. The capital losses temper the desire to consume, and the average saving rate out of income increases as a result of the rise in corporate retentions.

4.2 A formal model

Households in a corporate economy do not save in the form of physical capital. Disregarding housing and other, less important real assets, household wealth is financial wealth. Households may be the ultimate owners of the capital stock, but the ownership of productive capital takes the form of equity.

Equity is not the only financial asset, and even as a first approximation it is essential to include at least two types of financial assets: an asset with a contractual rate of return and an asset, equity, that gives ownership rights to firms but promises no contractual rate of return.[13] Thus, following Skott (1981, 1989) and Skott and Ryoo (2008), assume that household wealth consists of equity and bank deposits (money).[14]

Firms finance investment, dividends and interest payments on external debt by a combination of profits, share issues and new bank loans. The financial constraint is given by

$$I + D + iM^L = \Pi + v\dot{E} + \dot{M}$$

where E and M^L are the number of shares and the external debt (bank loans); v, i, I, D and Π are the price of shares, the interest rate, investment, dividends, and profits; 'dots' over a variable denote time derivatives, i.e. $\dot{x} = dx/dt$). To simplify the exposition, the price of output and the rate of interest i are taken to be constant, and the price output is normalized to one.

If s_f is the retention rate, the financial constraint can be rewritten

$$I = \Pi - D - iM^L + v\dot{E} + \dot{M}^L = s_f\Pi + v\dot{E} + \dot{M}^L$$

For given profits and a given level of investment, firms choose two of the three financial variables. Suppose they set the retention rate s_f $(= 1 - (D + iM^L)/\Pi)$ and the rate of new issues $\dot{E}$. The financial constraint now pins down the required change in bank loans (external finance).

In analogy with firms' financial constraint, households have a budget constraint. They receive wage income W and interest income $(i^M M)$ on their bank

[13]The formal model in the appendix to Kaldor (1966) had equity as the only financial asset. This assumption implies that the household sector can only abstain from consuming all distributed incomes to the extent that firms issue new equity. In Kaldor's setting, furthermore, firms cannot set investment, the retention rate and new equity issues independently.

[14]The introduction of bonds as a second contractual-return asset would add little to the analysis.

deposits (M), while their holdings of equity yield a flow of dividends (D). By assumption firms retain a fraction s_f of their gross profits Π, and the dividends are given by $D = (1-s_f)\Pi - iM^L$. Households' flow of income is either added to bank deposits or spent on consumption and the purchase of new shares. Thus, we can write households' budget constraint as

$$W + (1 - s_f)\Pi - iM^L + i^M M = C + \dot{M} + v\dot{E}^H \tag{27}$$

Neither firms nor households hold cash (by assumption there are only two assets, equity and bank deposits) and if, for simplicity, it is assumed that banks hold no reserves and have neither costs nor profits, we have $i = i^M$ and $M^L = M$.[15] The number of shares owned by households must also be equal to the number of shares issued by firms ($E^H = E$), with the price of shares adjusting to ensure this equilibrium condition. Thus, the budget constraint simplifies to

$$W + (1 - s_f)\Pi = C + \dot{M} + v\dot{E} \tag{28}$$

Using a traditional consumption function with non-property income and wealth as the determinants of household consumption, let[16]

$$C = (1 - s)(1 - \pi)Y + bA \tag{29}$$

where π is the profit share and $A = M + vE$ denotes household wealth. The specification is almost identical to the one in equation (21) but with one crucial difference: households' financial wealth takes the place of the capital stock.

If α_M denotes the share of deposits in household wealth, we have

$$M = \alpha_M A; \qquad vE = (1 - \alpha_M)A \tag{30}$$

The stock of deposits is predetermined, while the endogenous determination of the share price makes it possible to adjust the value of shareholdings instantaneously: the share price is given by $v = \alpha M/E$ and the aggregate financial wealth can be written $A = (1 + \alpha)M$ where $\alpha = (1 - \alpha_M)/\alpha_M = \frac{vE}{M}$.

Using equations (29)-(30) and dividing by Y, the consumption rate can now be written

$$\frac{C}{Y} = (1 - s)(1 - \pi) + b(1 + \alpha)\frac{M}{Y} \tag{31}$$

The short-run consumption rate in equation (31) depends inversely on the profit share and positively on the ratio of equity to deposits (the portfolio parameter α) and the deposit-income ratio.

The changes in households' deposits can be found from the budget constraint (28):

$$\dot{M} = (1 - s_f\pi)Y - v\dot{E} - C$$

[15] The analysis is substantively unchanged if banks make profits ($i^D < i^L$) and pay out these profits as dividends.

[16] The specification of household behavior in equations (28)-(30) endogenizes movements in the stock-flow ratio A/Y. Using an alternative approach, Skott (1981, 1989) specify target stock-flow ratios and derive the implied, endogenous saving decisions.

or

$$\begin{aligned}(\frac{\dot{M}}{Y}) &= \frac{M}{Y}(\widehat{\frac{M}{Y}}) = (1 - s_f\pi) - (\alpha\hat{E} + \hat{Y})\frac{M}{Y} - \frac{C}{Y} \\ &= (1 - s_f\pi) - (1-s)(1-\pi) - [\alpha\hat{E} + \hat{Y} + b(1+\alpha)]\frac{M}{Y} \qquad (32)\end{aligned}$$

The portfolio composition α may change, both endogenously in response to changes in relative returns and as a result of exogenous shocks to 'household sentiment'. But as argued above, the Modigliani-Miller argument for compensating changes in response to shifts in average financial practices by firms has no behavioral support, and for present purposes the value of α may be taken as constant. If output and the number of shares grow at constant rates, and $\alpha\hat{E} + \hat{Y} + b(1+\alpha) > 0$,[17] the differential equation (32) implies that the deposit-income ratio converges to stationary solution:

$$\frac{M}{Y} \rightarrow \frac{(1 - s_f\pi) - (1-s)(1-\pi)}{\alpha\hat{E} + \hat{Y} + b(1+\alpha)} = \frac{s(1-\pi) + (1-s_f)\pi}{\alpha\hat{E} + \hat{Y} + b(1+\alpha)} > 0 \qquad (33)$$

Plugging the stationary value into equation (31), we get

$$\frac{C}{Y} \rightarrow (1-s)(1-\pi) + b(1+\alpha)\frac{s(1-\pi) + (1-s_f)\pi}{\alpha\hat{E} + \hat{Y} + b(1+\alpha)} \qquad (34)$$

Equation (34) implies that the inverse relation between the consumption rate and the profit share carries over to the long run.[18]

[17] The inequality condition holds trivially if $\alpha > 0, \hat{Y} > 0, \hat{E} \geq 0$. The rate of new issues has been negative in the US since 1980s, but the condition is satisfied for plausible values of the variables and parameters. Empirically, the ratio of buybacks to gross investment has been of the order of 0.1 to 0.25 during this period, and with gross investment at about 15-20 percent of gdp, we have $vE \approx -0.03Y$. Output growth at 2-3 percent, a wealth-gdp ratio above 2, and a consumption rate out of wealth of more than 3 percent, now imply that

$$\begin{aligned}\alpha\hat{E} + \hat{Y} + b(1+\alpha) &= \alpha M\frac{vE}{vE}\frac{1}{A}\frac{A}{M} + \hat{Y} + b\frac{A}{M} \\ &\approx -0.03\frac{Y}{A}\frac{A}{M} + \hat{Y} + b\frac{A}{M} \\ &> 0.015\frac{A}{M} + 0.02\end{aligned}$$

[18] Using the benchmark numbers in footnote 13, we have

$$\begin{aligned}\frac{\partial\frac{C}{Y}}{\partial\pi} &= -(1-s) - b(1+\alpha)\frac{(s_f - (1-s))}{\alpha\hat{N} + \hat{Y} + b(1+\alpha)} \\ &= -s_f - (1-s-s_f)\frac{\alpha\hat{N} + \hat{Y}}{\alpha\hat{N} + \hat{Y} + b(1+\alpha)} \\ &\approx -s_f - (1-s-s_f)\frac{-0.015\frac{A}{M} + 0.02}{0.015\frac{A}{M} + 0.02} \\ &< -s_f + (1-s-s_f) \quad \text{for all positive values of } \frac{A}{M} \\ &< 0 \;\; \text{for } s_f > \frac{1-s}{2}\end{aligned}$$

The analysis has other interesting implications. Households' portfolio composition (the value of α) and firms' financial behavior (the values of s_f and $\hat{E}$) influence the consumption rate: the consumption ratio is decreasing in s_f and $\hat{E}$, and decreasing (increasing) in α if $\hat{E} > \hat{Y}$ (is $\hat{E} < \hat{Y}$). Intuitively, households as a group can only spend disposable income on shares to the extent that firms issue new shares. Thus, the fraction of disposable household income that goes into purchasing shares depends on both the rate at which firms expand the number of shares and the valuation of the shares. In the extreme case in which there are no new issues ($\hat{E} = 0$) and households keep all their wealth in shares ($\alpha \to \infty$), consumption must be equal to household disposable income ($C/Y = (1 - s_f\pi)$); attempts by households to buy shares merely lead to prices being bid up until household wealth has reached a level that makes desired consumption equal to disposable income.[19]

4.3 Feedback effects on firms

The analysis leading to (34) focused on household behavior, taking firms' investment and financial decisions as given. This partial analysis ignores potential feedback effects on firms.

Undoubtedly firms react to signals from financial markets as well as from goods and labor markets, and households would be calling the tune if firms always made investment and financial decisions that fully reflected households' preferences, information and expectations. But dynamic feedback effects between households' consumption and portfolio decisions and firms' investment and financial decisions do not automatically solve the coordination problems between households and firms. The feedback effects need not even produce adjustments in the right direction.

Consider a simple scenario in which the share of investment that is financed by retained earnings is subject to a financial norm. Specifically, let the financial valuation of a single firm (the firm-level value of Tobin's q) depend on deviations of the firm's behavior from the norm and on the average value of Tobin's q; firms that deviate from the norm receive a lower valuation than other comparable firms.

Formally, suppose that the valuation ratio for firm i is determined by

$$q_i = f(\frac{s_{fi}\pi_i p_i Y_i}{p_{Ki} I_i} - \mu(z_i), \bar{q}, z_i)$$

where

$$f_1(\frac{s_{fi}\pi_i p_i Y_i}{p_{Ki} I_i} - \mu(z_i), \bar{q}, z_i) \gtreqless 0 \text{ for } \frac{s_{fi}\pi_i p_i Y_i}{p_{Ki} I_i} - \mu(z_i) \lesseqgtr 0$$

Thus, the very weak condition $s_f > 0.5(1-s)$ is sufficient (but not necessary) for $\partial \frac{C}{Y} / \partial \pi < 0$.

[19] Paradoxically, the long-run consumption rate can be *decreasing* in the consumption parameters c and b : this happens if $\alpha\hat{E} + \hat{Y} < 0$. An increase in the propensities to consume reduces the long-run wealth-income ratio, and if the reduction is sufficiently large, the net effect of a rise in a consumption parameter can be negative.

Subscripts i indicate firm-level variables. The average value of Tobin's q is denoted by $\bar{q}$; q_i is the firm-level value of q; z_i represents a vector of firm-level characteristics (size, industry, current profitability, future prospects, etc.) that affect the valuation. The prices of the firm's own output (p_i) and of the capital goods that it uses (p_{Ki}) may deviate from the average price level (which has been normalized to one). The financial norm μ depends on legal and institutional factors – the structure of taxation, for instance, or whether share buybacks are permitted. Path-dependent Minskian views on prudent behavior are also likely to play a significant role, and the norm will almost certainly be contingent on the firm's characteristics (z_i).

Now consider the effects of a change in financial norms. Specifically, assume that for some reason there has been a rise in the average value of μ. Firms respond by raising their retention rates, the average saving rate increases, and aggregate demand declines. On average, consequently, firms experience a fall in their output Y_i or price p_i. If investment is kept unchanged, each firm has an incentive to raise its retention rate further in order to satisfy the norm. But as all firms do so, the fall in aggregate demand is reinforced. And if the fall in aggregate demand leads to a decline in investment, the problem is further exacerbated. In short, the feedback effects may not be stabilizing.

Empirically, beliefs that firms' investment and finance decisions reflect household preferences and that corporate saving is irrelevant for the average saving rate face problems, too. The literature on the aggregate saving effects of corporate retentions is sparse. There is evidence, however, that aggregate saving depends on retained earnings. Poterba (1986, p. 503) finds that

> the most conservative estimates suggest that a one dollar shift in corporate saving induces a 23 cent shift in private saving. For the longer sample period, the implied effects are much larger

In a more recent study, Bebczuk and Cavallo (2016, p, 2281) conclude that "a $1 increase in business saving raises private saving by approximately $0.59". It may be noted as well that the q theory of investment has not been particular successful econometrically (e.g. Blanchard et al. 1993) and that Tobin's q does not hover around one (or some other constant) as one would expect if the distinction between financial assets and physical capital were irrelevant (figure 1).

5 Conclusion

Harrod's first problem, the reconciliation of warranted and natural growth rates, does not arise in dual economies. The two rates are aligned, however, in mature economies, and this alignment calls for explanation.

The warranted rate of growth could in principle be adjusted to the natural rate through changes in the capital intensity, as suggested by Solow. There are reasons to be skeptical of this mechanism, however, and the adjustment can happen in other ways. In order to examine (one of) these ways the technical

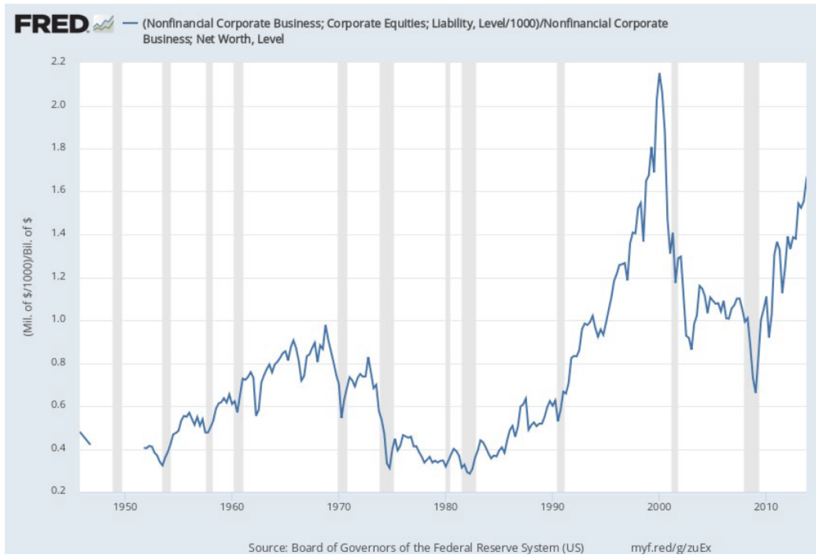

Figure 1: Tobin's q

coefficients of the production function and the natural rate of growth have been taken as exogenously given and constant in this paper.

Empirical evidence as well as strong theoretical arguments support the dependence of the average saving rate on the share of profits in income. The share of profits, in turn, may be influenced by the employment rate which produces a feedback from employment via income distribution to the average saving rate and the warranted growth rate.

High employment rates translate into high (or rising) real wages and downward pressures on the profit share in a Marxian analysis, but although this mechanism seems plausible, it can be questioned. Keynesian models reject a direct determination of real wages in the labor market, which complicates the chain of causation: the influence of employment on income distribution must be mediated by firms' investment and output decisions and/or by an influence of employment rates in saving. High employment is associated with strong workers, weak 'discipline in the factories' (to use Kalecki's term), shortages of workers with appropriate skills, and high search and hiring costs. The business climate suffers; firms' employment, production and investment decisions will be affected, and derived effects on aggregate demand puts downward pressure on the profit share (Skott 1989, Skott and Zipperer 2012). The feedback from the rate of employment to firms' incentives to invest and expand output may be supplemented (or replaced) by a direct influence on saving (as in Allain 2021).[20] Or extending the analysis to include a public sector, a policy of 'functional finance'

[20] Kaldor (1955) eschewed these problems by assuming that capital wll be fully utilized, with investment automatically adjusting to the saving.

can be used to influence the average saving rate and adjust the warranted rate to the natural rate (Skott 2016).

Whichever form it takes, a negative feedback from employment to one of more of the variables that define the warranted rate constitutes the essential element in any story about the adjustment of warranted to natural growth. If the Solow story is rejected and σ is exogenous, this basically leaves the average saving rate as the accommodating variable with income distribution and economic policy as the most important mediating mechanisms.

This conclusion, it should be noted, does not depend on the exogenous utilization rate u^* in equation (9). The exogenous u^* in (9) describes a long-run property of a (simple version of a) Harrodian investment function: it is based on the assumption that utilization rates above u^* would lead to ever-increasing accumulation rates while rates below u^* would lead to decreasing accumulation rates. Thus, the steady-growth investment function is vertical at $u = u^*$, and the warranted rate can be found found by substituting $u = u^*$ into the saving function.

If this (extreme version of the) Harrodian investment is abandoned, the steady-growth value of the utilization rate and the warranted growth rate will depend on the specification of both investment and saving. As an example, let

$$g^d = a + bu \qquad (35)$$

$$\dot{g} = \nu(g^d - g) \qquad (36)$$

where g is the accumulation rate, $g = I/K$. A Harrodian version of (35)-(36) has $b > s\sigma, a < 0$ and $\nu > 0$; a benchmark Kaleckian version sets $\nu = 0, a > 0$ and $b < s\sigma$. The Harrodian and Kaleckian versions differ with respect to the local stability properties of the economy, but both specifications imply that in steady growth we have $a + bu = s\sigma u$. The steady-growth value of utilization ($u^* = a/(s\sigma - b)$) is determined endogenously by the parameters of the saving and investment functions, but the reconciliation of warranted and natural growth rate still requires feedback effects from employment to the variables that define the warranted rate.

The analysis in this paper might seem to be at odds with empirical evidence of increasing profit shares and declining growth rates in the US and many other countries. The analysis does not, however, posit an invariant positive relation between growth and profits. Firms' financial behavior has undergone significant change; retention rates have fallen and high levels of share buybacks have taken the place positive net new issues of equity. These changes unambiguously reduce the average saving rate in a corporate economy. Asset bubbles and shifts in household portfolios towards equity (increases in α) work in the same direction if $\hat{E} < \hat{Y}$, a condition that has been satisfied since the 1980s. Veblenian emulation effects and a relaxation of credit constraints for low-income households also affected the aggregate saving rate. In addition to these effects on private saving, the average saving rate has been reduced by significant fiscal deficits; see Ryoo (2016, 2018) for a detailed analysis of the US case along these lines. In short, the adjustment of warranted to natural growth can take place via changes in the distribution of income – the topic of this paper. But saving rates are also influenced by other factors, some of them exogenous and others, including economic policy, responsive to movements in employment.

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