

University of Massachusetts Amherst

**ScholarWorks@UMass Amherst**

---

Economics Department Working Paper Series

Economics

---

2022

## **Marx after Okishio: Falling Rate of Profit with Constant Rate of Exploitation**

Deepankar Basu

Oscar Orellana

Follow this and additional works at: [https://scholarworks.umass.edu/econ\\_workingpaper](https://scholarworks.umass.edu/econ_workingpaper)

---

# Marx after Okishio: Falling Rate of Profit with Constant Rate of Exploitation

Deepankar Basu\*      Oscar Orellana†

May 17, 2022

## Abstract

Can cost-reducing, technical change lead to a fall in the long run rate of profit if class struggle manages to keep the rate of exploitation constant? In this paper we demonstrate, in a general circulating capital model, that if (a) the technical change is capital-using labor-saving (CU-LS), (b) the real wage bundle can change, and (c) the decline in the unit cost of production is bounded above by the change in the nominal labor cost associated with the new technique of production, then viable technical change can be consistent both with a constant rate of exploitation and a fall in the long run rate of profit. This result vindicates Marx's claim in Volume III of *Capital*, that if the rate of exploitation remains unchanged then technical change in capitalist economies can lead to a fall in the long run rate of profit.

**Keywords:** Okishio theorem, rate of exploitation, uniform rate of profit.

**JEL Codes:** B51.

## 1 Introduction

A large literature in Marxian political economy, starting with Volume III of *Capital*, has analysed the impact of technical change on the rate of profit in capitalist economies. Technical change is a key feature of capitalist economies, and the rate of profit is arguably the

---

\*Department of Economics, University of Massachusetts Amherst, 310 Crotty Hall, 412 N. Pleasant Street, Amherst MA 01002. Email: [dbasu@econs.umass.edu](mailto:dbasu@econs.umass.edu).

†Departamento de Matemática, Universidad Técnica Federico Santa María, Avenida España 1680, Valparaíso-Chile. Email: [oscar.orellana@usm.cl](mailto:oscar.orellana@usm.cl). This author acknowledges financial support provided by ANID Santiago-Chile under the Proyecto Fondecyt Regular número 1181414 and Universidad Técnica Federico Santa María, Valparaíso-Chile.

most important indicator of the health of a capitalist economy - viewed from the perspective of capital. Naturally, then, theoretical and empirical analyses that link the two have been of great interest to Marxist political economy.

In developing the law of the tendential fall in the rate of profit in Volume III of *Capital*, Marx had argued that technical progress in capitalist production, which brings about a rise in the organic composition of capital, would manifest itself as a ‘tendency’ of the rate of profit to fall (Marx, 1993). Starting with Okishio (1961), a large literature has argued that Marx’s result cannot be sustained if capitalists behave in a reasonable manner, that is, they adopt new a technique of production only if it reduces the cost of production at existing prices. In fact, if capitalist producers choose to adopt a new technique of production only if it is cost-reducing at current prices and the real wage rate remains unchanged before and after technical change, then the long run rate of profit in the economy will rise (Okishio, 1961; Bowles, 1981; Roemer, 1981; Dietzenbacher, 1989). This is the crux of the famous Okishio theorem.

Okishio’s result rests on the assumption that the real wage rate does *not* change. But this is an extremely restrictive assumption, given that the analysis is about *long run* prices and profit rates. There is no theoretical or empirical reason to believe that the real wage rate remains constant over the course of technical change, i.e. the adoption of a new technique of production by an innovating capitalist and its subsequent diffusion through the rest of the economy.<sup>1</sup> In fact, technical change interacts with larger social and economic forces, including those relevant to labour market outcomes, and it is not inconceivable that the real wage rate can change - one way or the other - after technical change. Taking a Marxian

---

<sup>1</sup>Even Okishio (2000) admits that the assumption of a constant real wage rate is unrealistic. “The assumption of a constant real wage rate implies either a non-monetary economy or the instantaneous adaptation of the money-wage rate to the prices of consumption goods. Both are unrealistic. A capitalistic economy is a monetary production economy. Labourers receive a money-wage. The money-wage rate and the prices of consumption goods change owing to competition in the consumption goods market and in the labour market. The assumption of a constant real wage rate cannot be maintained.” (Okishio, 2000, pp. 493).

view of the matter suggests that the real wage rate is an outcome of the class struggle, and it is unclear why class struggle would not be able to change the real wage rate over the course of technical change. At the least it seems plausible to argue that, since technical change increases labor productivity, workers will attempt to bargain for some part of the gain of technical change. Hence, it is eminently possible that the real wage rate will *increase* with technical change, rather than remain unchanged. An important finding of the Marxist literature on technical change and distribution, one that is often not appreciated, is that Okishio's result will no longer hold if we allow the real wage to change over the course of technical change (Roemer, 1981; Foley, 1986; Dietzenbacher, 1989; Laibman, 1992; Liang, 2021).

There have been two broad approaches to providing more structure to how the real wage rate might change. The first approach has worked with a constant wage-profit share as a plausible description of how the real wage rate might behave over the course of technical change. An analysis of the effect of technical change on the rate of profit when the profit-wage ratio remains constant was worked out in a 2-commodity model in Roemer (1977, 1981). Two important findings in Roemer (1977) are that, first, we can only define sectoral profit-wage ratios, but not the aggregate profit-wage ratio, without reference to the scale of production, and second, that sectoral profit-wage ratios can remain constant only when the real wage varies across sectors, i.e. we need to assume non-competitive labour markets. The main result in Roemer (1977, 1981) is that the rate of profit falls (or remains unchanged) if there is cost-reducing capital-using labour-saving (CU-LS) technical change in the capital goods (consumer goods) sector. This result has been generalized to the case of an  $n$ -commodity model - without distinguishing between capital and consumer goods industries - in Chen (2019), which shows that when there is cost-reducing CU-LS technical change in any sector with sectoral profit-wage ratios remaining constant, the equilibrium profit rate falls.

The second approach uses a constant rate of exploitation as a description of how the real

wage might vary over the course of technical change.<sup>2</sup> The idea that the rate of exploitation might remain constant before and after technical change goes back to Marx (Marx, 1993). His analysis of the law of the tendential fall worked with the often implicit assumption of a constant rate of exploitation. Laibman (1982, 1992) incorporated this assumption in a two-sector model and analysed the effect of technical change on the rate of profit. The main finding of Laibman (1982) was that it is *possible* for the rate of profit to fall after cost-reducing technical change if the rate of exploitation remains constant. While Basu (2021, Chapter 6) presents the same result in a one sector model, Liang (2021) has generalized Laibman’s result to an  $m$ -sector two department model with fixed capital.

This paper contributes to this literature by extending the results in Laibman (1982), Basu (2021) and Liang (2021). We extend the analysis of Laibman (1982) and Basu (2021) to a general  $n$ -sector circulating capital model of a capitalist economy. Unlike Laibman (1982), we do not distinguish between capital and consumption goods. We extend the analysis in Liang (2021) by allowing for a general change in the real wage bundle. Whereas Liang (2021) only allows proportional changes in the vector of the real wage bundle, we allow for the real wage bundle to change in an arbitrary manner over the course of technical change. In this general setting, we demonstrate that under certain plausible conditions, the long run rate of profit can fall after viable technical change if the rate of exploitation remains constant. One advantage of using the constant rate of exploitation description of real wage behavior is that we do not need to assume non-competitive labor markets, as is needed in Roemer (1981) and Chen (2019).

The intuition for our result is straightforward. When a new technique of production becomes available in a sector, capitalists compare the cost of production associated with the new technique and the old technique using *current* prices and wage rates. Capitalists

---

<sup>2</sup>If we adopt the New Interpretation of Marx’s value theory, the two approaches would be the same. This is because in the *ex post* accounting framework that *is* the New Interpretation, the rate of exploitation is equal to the profit-wage ratio (Foley, 1982; Mohun, 2004).

do not know the direction in which class struggle will proceed and therefore do not take account of possible changes in the nominal or real wage rate - an outcome of class struggle - when arriving at their decision to adopt the new technique of production. Hence, if the technique reduces costs of production at current prices and wage rates, capitalists adopt the new technique of production.

The course of class struggle can, under certain circumstances, lead to an increase in the real wage bundle in such a way that it not only becomes more expensive at current prices but also keeps the rate of exploitation remains unchanged. If technical change is of the capital-using labor-saving (CU-LS) type, the predominant form of technical change in capitalism (Foley et al., 2019), then the labor value of all commodities will (weakly) fall (Roemer (1981, Theorem 4.9)). Hence, a ‘larger’ real wage bundle will still be compatible with a constant rate of exploitation. The ‘larger’ real wage bundle can accommodate relatively higher magnitudes of commodities for which the labor values have fallen relatively more. If these commodities also had relatively high prices in the original situation compared to labor values after technical change, then the monetary cost of the real wage bundle will increase to such an extent that it will lead to a fall in the long run, equilibrium rate of profit.

One important condition that ensures the fall in the equilibrium rate of profit is that the reduction in cost afforded by the new technique of production, evaluated with the original prices and the original real wage bundle, not be too large. In fact, if the cost reduction is bounded above by the change in the nominal labor cost associated with the new technique of production, then the equilibrium rate of profit will fall - squeezed by the rise in the nominal labor cost coming from the new real wage bundle. Since new techniques of production are perturbations of current techniques (Duménil and Lévy, 1995), the assumption of an upper bound on the cost reduction associated with a new technique of production seems reasonable.

Given this bounded nature of cost reduction, the rate of profit falls because capitalists are unable to fully take account of the effects of technical change on the labor market. While

capitalists might be able to control wage movements at the level of their firm, technical change has larger impacts on the labor market that is beyond the control of individual capitalists. It is this inability to fully control wage movements that, under certain plausible configurations of technological change, will lead to a fall in the long run, equilibrium rate of profit. Hence, individually rational capitalist actions can lead to an overall undermining of the interest of the whole capitalist class.

The rest of the paper is organized as follows. In section 2, we describe the basic set up and define viable technical change; in section 3, we show that, under certain conditions, the rate of profit can fall after viable technical change if the rate of exploitation remains constant; in section 4, we present an example of a 3 sector model to illustrate my argument; finally, we conclude the paper in section 5.

## 2 The Set-Up

### 2.1 Initial Configuration

Consider an economy with  $n$  sectors of production, where the technology is given by the non-negative, productive,  $n \times n$  input-output matrix,  $A \geq 0$ , and the  $1 \times n$  vector of direct labor inputs,  $L > 0$ , and the real wage bundle is given by the  $n \times 1$  vector  $b \geq 0$ .<sup>3</sup> Each sector produces one commodity with one technique of production and there is no fixed capital.

The cost of producing one unit of the commodity in sector  $i$  is given by  $p \cdot A_{*i} + wL_i$ , where  $A_{*i}$  denotes the  $i$ -th column of  $A$ ,  $\cdot$  denotes a dot product, and  $w = pb$  is the nominal wage rate. Using the normalization that the nominal wage rate is unity, the  $1 \times n$  vector of long run equilibrium prices (prices of production),  $p$ , and the long run equilibrium (uniform)

---

<sup>3</sup>For vectors and matrices, we will follow the following notation:  $x \geq 0$ , if for  $i = 1, 2, \dots, n$ ,  $x_i \geq 0$  and  $x \neq 0$ ;  $x > 0$ , if for  $i = 1, 2, \dots, n$ ,  $x_i > 0$ .

rate of profit,  $\pi$ , are given by

$$p = (1 + \pi) pM, \text{ and } pb = 1, \tag{1}$$

where  $M = A + bL$ , is the augmented input matrix.

We assume that the input-output matrix,  $A$ , is productive and indecomposable. Then, an application of the Perron-Froebienius theorem shows that  $p > 0$  and  $\pi > 0$  (Dietzenbacher, 1989, pp. 36). For this configuration of technology, the  $1 \times n$  vector of labor values,  $\Lambda$ , is given by

$$\Lambda = L(I - A)^{-1}. \tag{2}$$

Standard results in linear algebra show that, since  $A$  is productive,  $(I - A)^{-1} > 0$  (Pasinetti, 1977, Appendix). Hence, we have  $\Lambda > 0$ .

## 2.2 Viable, CU-LS Technical Change

Suppose there is a cost-reducing (viable) technical change in sector  $i$ , i.e. the cost of producing one unit of output with the new technique of production is lower than with the older technique of production when both are evaluated at current prices and wage rate. Hence,

$$pA_{*i} + L_i < p\bar{A}_{*i} + \bar{L}_i, \tag{3}$$

where  $A_{*i}$  and  $\bar{A}_{*i}$  denote the  $i$ -th columns of the matrices  $A$  and  $\bar{A}$ , respectively,  $L_i$  denotes the  $i$ -th element of  $L$ , and we have used the normalization, once again, that the nominal wage rate is 1. In addition, suppose technical change is capital-using and labor-saving (CU-LS). This means the the amount of material inputs used to produce one unit of the commodity



in sector  $i$  rises, while the amount of direct labor input falls. Hence, for  $j = 1, 2, \dots, n$ ,

$$a_{ji} < \bar{a}_{ji}, \text{ and } L_i > \bar{L}_i, \quad (4)$$

where  $a_{ij}$  and  $\bar{a}_{ij}$  denote the  $(i, j)$ -th elements of  $A$  and  $\bar{A}$ , respectively, and  $L_i$  denotes the  $i$ -th element of  $L$ .

Since the new technique of production reduces unit cost of production, evaluated at current prices, capitalist firms in sector  $i$  will adopt the new technique of production (Okishio, 1961). All other sectors continue using the old technology - because technical change occurs only in sector  $i$ . Hence, the new technology in the economy is captured by the  $n \times n$  input-output matrix,  $\bar{A}$ , and the  $1 \times n$  vector of direct labor inputs,  $\bar{L}$ , where the columns of  $A$  and  $\bar{A}$  are identical other than for column  $i$ , and the elements of  $L$  and  $\bar{L}$  are identical, other than the  $i$ -th element. With the new technology, the  $1 \times n$  vector of labor values,  $\bar{\Lambda}$ , is given by

$$\bar{\Lambda} = \bar{L} (I - \bar{A})^{-1} > 0, \quad (5)$$

where strict inequality follows because  $\bar{A}$  is productive.

### 2.3 Class Struggle and a New Real Wage Bundle

Suppose class struggle, during and after the technical change, leads to the emergence of a new real wage bundle,  $\bar{b} \geq 0$ , such that it is more expensive to purchase at old prices, i.e.

$$p\bar{b} > pb. \quad (6)$$

This just means that workers are able to bargain for and secure a higher nominal wage rate as the process of technical change works itself out over the long run. In addition, this new

real wage bundle satisfies two properties.

**Property 1.** *The new real wage bundle,  $\bar{b}$ , keeps the labor value of the real wage bundle unchanged, i.e.*

$$\Lambda b = \bar{\Lambda} \bar{b}. \quad (7)$$

The rate of exploitation, before technical change, is given by  $e = (1 - \Lambda b)/\Lambda b$ . After technical change and with the new real wage bundle, it is given by  $\bar{e} = (1 - \bar{\Lambda} \bar{b})/\bar{\Lambda} \bar{b}$ . Hence, this property ensures that workers are able to secure a new real wage bundle that keeps the rate of exploitation unchanged even after technical change.

**Property 2.** *The decline in the unit cost of production in sector  $i$  (the sector that witnessed technical change) is bounded above by the change in the nominal labor cost corresponding to the new technique of production,*

$$0 < pA_{*i} + L_i - p\bar{A}_{*i} - \bar{L}_i < \bar{L}_i (p\bar{b} - 1) \quad (8)$$

Since technical change in sector  $i$  is viable, it reduces the unit cost of production at current prices and wage rates, which gives the left hand side of the above inequality. The condition in (8), in addition, puts an upper bound on the decline in the unit cost of production - which gives the right hand side of the above inequality. Note that the unit cost of production in sector  $i$  before technical change is given by  $pA_{*i} + L_i$ ; and, after technical change in that sector, it is given by  $p\bar{A}_{*i} + \bar{L}_i$ . Hence, the left hand side of the above inequality is the decline in the unit cost of production in sector  $i$ . Since  $pb = 1$  was the nominal wage rate in the initial situation and  $p\bar{b}$  is the nominal wage rate with the new real wage bundle, where both are evaluated at the original prices,  $\bar{L}_i(p\bar{b} - 1)$  is the change in the nominal labor cost in sector  $i$  corresponding to the direct labor input requirement associated with the new technique,  $\bar{L}_i$ .

This condition is reasonable because technical change involves the emergence and adoption of new techniques of production that are local perturbations of the existing techniques of production (Duménil and Lévy, 1995). Thus, while the amount of material and labor inputs required by the new technique is different from the old, the changes are not too large. The intuition of the change in cost corresponding to the new technique of production being not ‘too large’ is captured by the above condition for the bound on the cost reduction associated with the new technique of production.

### 3 Main Results

Our main result consists of two theorems. First, we show that if there exists some  $\bar{b} \geq 0$  that satisfies property 1 and 2, then viable technical change keeps the rate of exploitation constant even as the uniform rate of profit falls. Second, we show that under certain plausible conditions there always exists some  $\bar{b} \geq 0$  that satisfies properties 1 and 2. Hence, if class struggle leads to the emergence of such a real wage bundle then viable technical change will be accompanied by a fall in the uniform rate of profit.

**Theorem 1.** *Let  $\bar{p}$  and  $\bar{\pi}$  denote the price of production vector and the uniform rate of profit with the technology  $\bar{A}, \bar{L}$  and a real wage bundle  $\bar{b}$ . If this  $\bar{b}$  satisfies property 1 and 2, then  $\bar{\pi} < \pi$ .*

*Proof.* Since  $\bar{b}$  satisfies property 1, the rate of exploitation remains unchanged. An application of Dietzenbacher (1989, Theorem 5) shows that, since (6) and (8) hold, the uniform rate of profit declines. □

The implication of this result is interesting. It shows that if a real wage bundle satisfying property 1 and 2 exists, then viable technical change can, at the same time, keep the rate of exploitation constant and also lead to a fall in the uniform rate of profit. Hence, this

shows that Marx's claim in Volume III of *Capital* can be sustained under certain conditions. Of course, to complete the argument, we must demonstrate that such a real wage bundle  $\bar{b}$  exists, and that is what we show in the next result.

**Theorem 2.** *Let  $e$  denote the rate of exploitation before technical change, i.e.*

$$e = \frac{1 - \Lambda b}{\Lambda b}, \quad (9)$$

let  $p$  denote the initial price of production vector, and let  $g$  denote the decline in the cost of production in sector  $i$  (the sector which witnessed technical change) as a fraction of the labor cost corresponding to the new technique of production in that sector evaluated at the old wage rate,

$$g = \frac{(pA_{*i} + L_i) - (p\bar{A}_{*i} + \bar{L}_i)}{\bar{L}_i}. \quad (10)$$

Let  $\bar{\Lambda} = [\bar{\lambda}_i]$  denote the vector of labor values after technical change. If, for some  $j = 1, 2, \dots, n$ ,

$$\left( p_j / \bar{\lambda}_j \right) > (1 + e)(1 + g) \quad (11)$$

then there exists some  $\bar{b} \geq 0$  that satisfies property 1 and 2.

*Proof.* Consider the  $n$  dimensional space whose coordinate system is  $(\bar{b}_1, \dots, \bar{b}_n)$ . Any point in this space is a candidate real wage bundle. We will only consider the nonnegative orthant of this space because negative elements in the real wage bundle are not meaningful.

Consider the hyperplane in this space given by the set of points  $P$  defined by

$$P = \{ \bar{b} \geq 0 \mid p \cdot \bar{b} - \alpha = 0 \}, \quad (12)$$

where  $i$  denotes the sector in which technical change occurred, and

$$\alpha = (pA_{*i} + L_i - p\bar{A}_{*i})/\bar{L}_i > 0, \quad (13)$$

where the strict inequality in (13) comes from the fact that the numerator and denominator are both strictly positive. The numerator is strictly positive because the new technique of production is cost-reducing, and hence,  $pA_{*i} + L_i - p\bar{A}_{*i} > pA_{*i} + L_i - p\bar{A}_{*i} - \bar{L}_i > 0$ ; the denominator is positive because  $\bar{L}_i > 0$ . Note that, for  $j = 1, 2, \dots, n$ , the hyperplane  $P$  intersects the coordinate axes at points of the form  $x_j e_j$ , where  $e_j$  is a  $n$ -vector with 1 as the  $j$ -th element and 0 as every other element, and  $x_j = \alpha/p_j > 0$ , where the strict inequality follows from (13) and  $p_j > 0$ . Thus, the hyperplane  $P$  intersects the coordinate axes at strictly positive points. The important point to note is that all points ‘above’ hyperplane,  $P$ , satisfies property 2, i.e. for such real wage bundles, the decline in the unit cost of production in sector  $i$  is bounded above by the change in labor cost corresponding to the new technique of production.

Now consider the hyperplane, in the same  $n$  dimensional space, given by the set of points  $V$  defined by

$$V = \{\bar{b} \geq 0 \mid \bar{\Lambda} \cdot \bar{b} - \beta = 0\}, \quad (14)$$

where

$$\beta = \Lambda b > 0, \quad (15)$$

where the strict inequality in (15) follows because  $\Lambda > 0$  and  $b \geq 0$ . Note that, for  $j = 1, 2, \dots, n$ , this hyperplane intersects the coordinate axes at points of the form  $y_j e_j$  where,  $y_j = \beta/\bar{\lambda}_j > 0$ , where the strict inequality follows from (15) and  $\bar{\lambda}_j > 0$ . Thus, the hyperplane

$V$  also intersects the coordinate axes at strictly positive points. For the hyperplane,  $V$ , the important point to note is that all points on this hyperplane satisfy property 1, i.e. such real wage bundles ensure that the rate of exploitation remains unchanged before and after technical change.

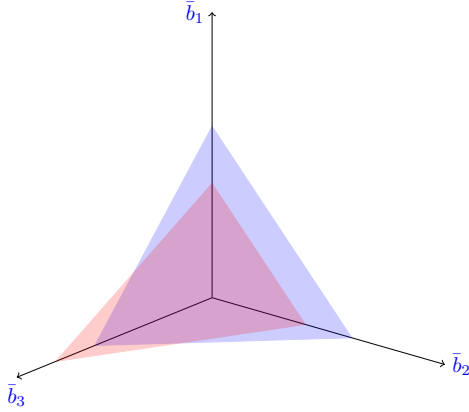
Using the assumption of the theorem, condition (11), we have, for at least one  $j = 1, 2, \dots, n$ ,  $(p_j/\bar{\lambda}_j) > (1+e)(1+g)$ . This shows, after a little algebraic manipulation, that  $\beta/\bar{\lambda}_j > \alpha/p_j$ . This means that some portion of the hyperplane,  $V$ , lies above the hyperplane,  $P$ , in the *positive orthant*, as shown in Figure 1a and 1b for a 3-dimensional setting. This provides us with an infinite number of points  $\bar{b} \geq 0$  for which property 1 and 2 will be satisfied. To show this more formally, we need to demonstrate that points on the hyperplane  $V$  that lie in the positive orthant *and* to the right of the intersection with hyperplane  $P$  are above the hyperplane  $P$ .

Let  $j = k$  for which (11) is satisfied, i.e.  $(p_k/\bar{\lambda}_k) > (1+e)(1+g)$ . Consider the hyperplane given by the set of points  $B_{k-1}$  defined by  $B_{k-1} = \{\bar{b} \geq 0 | \bar{b}_{k-1} = 0\}$ . Let  $x$  denote the point of intersection of  $P$ ,  $V$  and  $B_{k-1}$ . Let  $y$  denote the point where the hyperplane  $V$  intersects the  $b_k$  coordinate axis, and let  $z$  denote the point where the hyperplane  $P$  intersects the  $b_k$  coordinate axis. Let  $u$  denote the unit vector given by  $u = (y - x)/\|y - x\|$ , and let  $v$  denote the unit vector given by  $v = (z - x)/\|z - x\|$ , where  $\|w\|$  denotes the Euclidean norm of the vector  $w$ . The vector  $(y - x)$  lies on the hyperplane  $V$ , and the vector  $(z - x)$  lies on the hyperplane  $P$ .<sup>4</sup> Hence, the angle between  $u$  and  $v$  is the angle between  $p$  and  $\bar{\Lambda}$ , because the vector  $p$  is perpendicular to the hyperplane  $P$  and the vector  $\bar{\Lambda}$  is perpendicular to hyperplane  $V$  (see Figure 1b).<sup>5</sup> Hence, if  $\theta$  denotes the angle between  $u$  and  $v$ , then  $\cos \theta = (p \cdot \bar{\Lambda})/\|p\|\|\bar{\Lambda}\| > 0$ , where the strict inequality comes from the fact that both  $p$  and  $\bar{\Lambda}$  are strictly positive vectors.

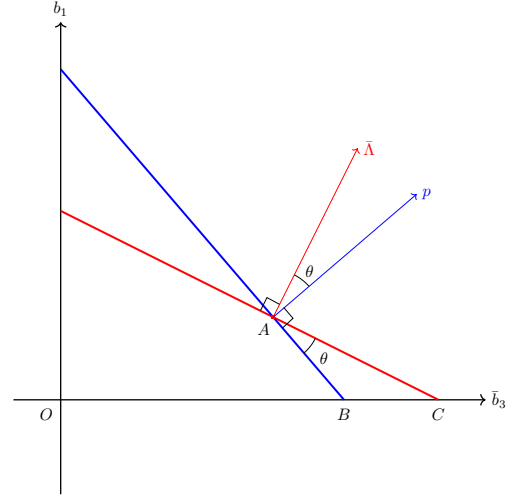
---

<sup>4</sup>In Figure 1b,  $x = OA$ ,  $y = OC$ ,  $z = OB$ . Hence,  $y - x = CA$  and  $z - x = BA$ .

<sup>5</sup>Recall that the  $P$  is given by  $p \cdot \bar{b} = \alpha$ , and  $V$  is given by  $\bar{\Lambda} \cdot \bar{b} = \beta$ . This shows that the vector  $p$  is perpendicular to  $P$  and the vector  $\bar{\Lambda}$  is perpendicular to  $V$ .



(a) The hyperplanes  $P$  (blue, extending outward on the  $\bar{b}_1$  axis) and  $V$  (red, extending outward on the  $\bar{b}_3$  axis).



(b) Projection of the hyperplanes  $P$  (blue) and  $V$  (red) onto the plane defined by  $\bar{b}_2 = 0$ .

**Figure 1:** Intersecting hyperplanes given by equations (12) and (14) for the case  $n = 3$ .

Let  $0 < t < 1$ , and consider a point  $x_0 = x + tu$ , and note that  $x_0$  lies on  $V$ . We will show that  $x_0$  is ‘above’  $P$  by showing that it lies on the other side of  $P$ , compared to the origin. When we plug in the zero vector (origin) in the equation for the hyperplane  $P$ , we get  $-\alpha < 0$ . When we plug in  $x_0$  into the equation for the same hyperplane  $P$ , we get  $p \cdot x + tp \cdot u$ . Since  $x$  lies on  $P$ , we have  $p \cdot x = \alpha > 0$ . Additionally,  $tp \cdot u = t\|p\| \cos \theta > 0$ , because  $\cos \theta > 0$  (as we saw above). Hence,  $p \cdot x + tp \cdot u > 0$ . This shows that the point  $x_0$  is ‘above’ the hyperplane  $P$ , i.e. the origin and  $x_0$  are on *different* sides of the hyperplane  $P$ .  $\square$

*Discussion.* The key condition in Theorem 2 is captured in (11). This instructs us to look at the ratio of the price of production before technical change and the labor value after technical change, sector by sector. If for any sector, this ratio is bounded below by the product  $(1+e)(1+g)$ , then the condition will be satisfied. When technical change is CU-LS, this condition is not restrictive, because  $p > \Lambda \geq \bar{\Lambda}$ , i.e. the vector of prices of production before technical change is strictly larger than the vector of values after technical change.

To see the first inequality,  $p > \Lambda$ , note that, from (1), we have,  $p = (1 + \pi)L[I - (1 + \pi)A]^{-1} = (1 + \pi)L \sum_{j=1}^{\infty} (1 + \pi)^j A^j > (1 + \pi)L \sum_{j=1}^{\infty} A^j = (1 + \pi)L[I - A]^{-1} = (1 + \pi)\Lambda > \Lambda$ , where we have used the facts that  $0 < \pi < R$  (where  $R$  is the maximal rate of profit, i.e. the rate of profit when the real wage bundle is the zero vector) and  $A$  is productive. As long as the real wage bundle,  $b \geq 0$ , has at least one strictly positive element, we have  $0 < \pi < R$ , where  $1 + R$  is the reciprocal of the maximal eigenvalue of  $A$ , and this ensures the validity of the infinite series matrix expansion of  $[I - (1 + \pi)A]^{-1}$ ; and, as long as  $A$  is productive, we have a valid infinite series matrix expansion for  $[I - A]^{-1}$ .<sup>6</sup> The second inequality,  $\Lambda \geq \bar{\Lambda}$ , follows from Roemer (1981, Theorem 4.9) because the technical change under consideration is CU-LS.

Given these inequalities, the condition in (11) is merely stating that the ratio of price of production (before technical change) and the labor value (after technical change) in at least one sector must not only be larger than unity, but be larger than the quantity appearing on the right hand side of (11). The intuition behind this condition is that, since CU-LS technical change reduces the labor value, we can increase the magnitude of commodities in the real wage bundle with relatively low labor value and yet keep the value of the wage bundle unchanged. But, if these commodities had relatively high prices in the original situation compared to their labor values after technical change, then the monetary cost of the real wage bundle increases to such an extent that it leads to a fall in the equilibrium rate of profit.

In the next section, we provide an example of a 3-sector economy where we can find an infinite number of such real wage bundles. But a caveat is necessary at this point. Our argument is not that class struggle will always discover a real wage bundle that satisfies property 1 and 2. Rather, we have demonstrated that such a real wage bundle does exist

---

<sup>6</sup>Note, we have used the notation  $A^0 = I$ , the 0-th power of the matrix  $A$  is the identity matrix. For a discussion of this infinite series matrix expansion, see Pasinetti (1977, Appendix, pp. 266).



and that it is not possible to rule it out without additional restrictions on technical change or class struggle. Hence, it is *possible* that such a real wage bundle will be discovered by class struggle. In that case, the equilibrium rate of profit will fall even when capitalists have adopted cost-reducing techniques of production.

## 4 An Example

Consider the example of a 3-sector economy discussed in Dietzenbacher (1989, pp. 39).<sup>7</sup>

### 4.1 Initial Situation

Let initial technology be given by

$$A = \begin{bmatrix} 0.35 & 0.05 & 0.25 \\ 0.15 & 0.45 & 0.05 \\ 0.15 & 0.15 & 0.35 \end{bmatrix}$$

and

$$L = \begin{bmatrix} 0.15 & 0.15 & 0.15 \end{bmatrix}.$$

Let the initial real wage bundle be given by

$$b = \begin{bmatrix} 1/3 \\ 1/3 \\ 1/3 \end{bmatrix}.$$

For this configuration of technology and real wage bundle, we can calculate the uniform

---

<sup>7</sup>R code to implement this example is given in the Appendix.

rate of profit,  $\pi = 0.25$ , and the price of production vector as

$$p = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix}.$$

The vector of values is given by

$$\Lambda = \begin{bmatrix} 0.4285714 & 0.4285714 & 0.4285714 \end{bmatrix}.$$

## 4.2 CU-LS, Viable Technical Change

A CU-LS, viable technical change takes place in sector 3. The new technology is given by

$$\bar{A} = \begin{bmatrix} 0.35 & 0.05 & 0.27 \\ 0.15 & 0.45 & 0.07 \\ 0.15 & 0.15 & 0.37 \end{bmatrix}$$

and

$$\bar{L} = \begin{bmatrix} 0.15 & 0.15 & 0.08 \end{bmatrix}.$$

Note that the new technology is

- CUS-LS: because the third column of  $\bar{A}$  is, element by element, greater than the third column of  $A$ , and
- viable: because the cost of production in sector 3 falls from 0.80 to 0.79 (using the price vector computed above and using the normalisation that the nominal wage rate is 1).

Hence, a capitalist producer will adopt this new technology. With this new technology, the new vector of value is given by

$$\bar{\Lambda} = \begin{bmatrix} 0.4034091 & 0.4034091 & 0.344697 \end{bmatrix}.$$

### 4.3 Class Struggle and a New Real Wage Bundle

Suppose class struggle leads to the emergence of a new real wage bundle given by

$$\bar{b} = \begin{bmatrix} \bar{b}_1 \\ \bar{b}_2 \\ \bar{b}_3 \end{bmatrix}.$$

We need to ensure that the new real wage vector  $\bar{b} \geq 0$  is more expensive than the original real wage bundle

$$p_1\bar{b}_1 + p_2\bar{b}_2 + p_3\bar{b}_3 > 1 = pb, \quad (16)$$

and that the decline in the unit cost of production is bounded above by the change in the nominal labor cost associated with the new technique of production,

$$pA_{*3} + L_i < p\bar{A}_{*3} + (p_1\bar{b}_1 + p_2\bar{b}_2 + p_3\bar{b}_3)\bar{L}_3, \quad (17)$$

where  $A_{*3}$  and  $\bar{A}_{*3}$  denote the third column of  $A$  and  $\bar{A}$ , respectively, and, finally, that the labor value of the real wage bundle remains unchanged,

$$\bar{\lambda}_1\bar{b}_1 + \bar{\lambda}_2\bar{b}_2 + \bar{\lambda}_3\bar{b}_3 = \Lambda b, \quad (18)$$

which ensures that the rate of exploitation remains constant.

Since the new technique of production reduces the unit cost of production in sector 3, we have  $(pA_{*3} + L_i - p\bar{A}_{*3})/\bar{L}_3 > 1$ . Hence, the above three conditions can be reduced to two conditions:

$$p_1\bar{b}_1 + p_2\bar{b}_2 + p_3\bar{b}_3 > (pA_{*3} + L_i - p\bar{A}_{*3})/\bar{L}_3, \quad (19)$$

$$\bar{\lambda}_1\bar{b}_1 + \bar{\lambda}_2\bar{b}_2 + \bar{\lambda}_3\bar{b}_3 = \Lambda b. \quad (20)$$

Since  $(pA_{*3} + L_i - p\bar{A}_{*3})/\bar{L}_3 = 1.125$ , and using the vector of old price of production, the vector of old and new labor values, we have the following two equations:

$$1 * \bar{b}_1 + 1 * \bar{b}_2 + 1 * \bar{b}_3 > 1.125, \quad (21)$$

$$0.4034\bar{b}_1 + 0.4034\bar{b}_2 + 0.3447\bar{b}_3 = 0.4286. \quad (22)$$

The condition in (21) is satisfied by all points in the positive orthant of the 3-dimensional space with coordinates  $(\bar{b}_1, \bar{b}_2, \bar{b}_3)$  that lie above the hyperplane  $\bar{b}_1 + \bar{b}_2 + \bar{b}_3 = 1.125$ . This hyperplane intersects each of the three axes at 1.125. The condition in (22) is satisfied by all points in 3-dimensional space with coordinates  $(\bar{b}_1, \bar{b}_2, \bar{b}_3)$  that lie on the hyperplane given by (22). This hyperplane intersects the  $\bar{b}_3$  axis at  $1.24 = 0.4286/0.3447$ , which is larger than 1.125. Hence, the hyperplane given by  $\bar{b}_1 + \bar{b}_2 + \bar{b}_3 = 1.125$  and the hyperplane given by  $0.4034\bar{b}_1 + 0.4034\bar{b}_2 + 0.3447\bar{b}_3 = 0.4286$  will intersect in the positive orthant. Thus, there are an infinite number of points that lie on the hyperplane given by (22) and that also satisfy (21).

To choose one *particular* real wage bundle,  $\bar{b} \geq 0$ , that satisfies (21) and (22), let us select some  $\bar{b}_3$  so that  $1.125 < \bar{b}_3 < 1.24$ . To be specific, let us choose  $\bar{b}_3 = 1.15$ . Let us also impose the condition  $\bar{b}_1 = \bar{b}_2$ . Hence, using (22), we get  $\bar{b}_1 = (0.4286 - 1.15 * 0.3447)/0.8068$ .

Hence,  $\bar{b}_1 = 0.0399$ . Hence, the new real wage bundle is given by

$$\bar{b} = \begin{bmatrix} 0.0399 \\ 0.0399 \\ 1.1500 \end{bmatrix}.$$

Using this real wage bundle, we see that  $\Lambda b = \bar{\Lambda} \bar{b} = 0.4285714$  and the new uniform rate of profit is  $\bar{\pi} = 0.2163382 < 0.25 = \pi$ .

## 5 Conclusion

Technical change is a characteristic feature of capitalist economies. Since the profit rate is one of the clearest indicators of the health of a capitalist economy, seen from the perspective of capital, it is of great interest to investigate the effect of technical change on the rate of profit. In Volume III of *Capital*, Marx had argued that technical change will impart a falling tendency to the rate of profit when the rate of exploitation remains constant. In this paper, we have demonstrated that this result *can* obtain in a multisector economy. To be more concrete, we have demonstrated that there are real wage bundles which keep the rate of exploitation constant and lead to a fall in the new equilibrium rate of profit.

The picture of technical change that Marx gave us in the volumes of *Capital* remains extremely relevant. Competitive pressures force capitalists to search for new cost-reducing techniques of production. The innovator capitalist who manages to adopt such a technique is able to make super-normal profits. That creates the incentive for capitalists to constantly look for and adopt, when found, cost-reducing techniques of production. The adoption of the new technique by the innovator disrupts the prevailing equilibrium. This is because the economy is interconnected in complex ways. The output of the innovator capitalist is used as inputs in other industries; the innovator's demand for the output of other industries also

change because of the technical change. When all these changes have played themselves out, a new equilibrium profit rate and a new set of prices of production emerge.

Okishio (1961) had shown that the new equilibrium rate of profit would be higher than the one that prevailed before technical change if the real wage rate remains unchanged. In this paper, we have shown that if the rate of exploitation remains unchanged, which will imply that the real wage rate has to increase, the rate of profit can fall after cost-reducing technical change of the type analysed by Okishio (1961), as long as the reduction in cost is bounded above by the change in the nominal labor cost associated with the new technique of production. The constancy of the rate of exploitation is one way to capture the balance of class forces. Hence, the result in this paper shows that if the balance of class forces manages to keep the division between paid and unpaid labour time unchanged, cost-reducing technical change can lead to a fall in the rate of profit - if the cost reduction from technical change is not too large. In such cases, individually rational decisions by capitalist producers might harm the collective interest of the capitalist class. This is just one pathology of a competitive, capitalist economy.

## References

- Basu, D. (2021). *The Logic of Capital: An Introduction to Marxist Economic Theory*. Cambridge University Press, Cambridge, UK.
- Bowles, S. (1981). Technical change and the profit rate: A simple proof of the okishio theorem. *Cambridge Journal of Economics*, 5:183–186.
- Chen, W. (2019). Technical change, income distribution, and profitability in multisector linear economies. UMass Amherst Economics Working Papers. 273.

- Dietzenbacher, E. (1989). The implications of technical change in a Marxian framework. *Journal of Economics*, 50(1):35–46.
- Duménil, G. and Lévy, D. (1995). A stochastic model of technical change: An application to the us economy (1869–1989). *Metroeconomica*, 46(3):213–245.
- Foley, D. K. (1982). The value of money, the value of labour power and the Marxian transformation problem. *Review of Radical Politics Economics*, 14(2):37–47.
- Foley, D. K. (1986). *Understanding Capital: Marx's Economic Theory*. Harvard University Press, Cambridge, MA.
- Foley, D. K., Michl, T. R., and Tavani, D. (2019). *Growth and Distribution*. Harvard University Press, second edition.
- Laibman, D. (1982). Technical change, the real wage rate and the rate of exploitation: The falling rate of profit reconsidered. *Review of Radical Political Economics*, 14(2):95–105.
- Laibman, D. (1992). *Value, Technical Change and Crisis: Explorations in Marxist Economic Theory*. M. E. Sharpe, Inc., Armonk, NY.
- Liang, J. (2021). The falling rate of profit under constant rate of exploitation: A generalization. *Review of Radical Political Economics*, 53(3):501–510.
- Marx, K. (1993). *Capital: A Critique of Political Economy, Volume III*. Penguin. (First published in 1894).
- Mohun, S. (2004). The labour theory of value as foundation for empirical investigations. *Metroeconomica*, 55(1):65–95.
- Okishio, N. (1961). Technical changes and the rate of profit. *Kobe University Economic Review*, 7:85–99.

Okishio, N. (2000). Competition and production prices. *Cambridge Journal of Economics*, 25:493–501.

Pasinetti, L. L. (1977). *Lectures on the Theory of Production*. Columbia University Press, New York, NY.

Roemer, J. E. (1977). Technical change and the “tendency of the rate of profit to fall”. *Journal of Economic Theory*, 16:403–424.

Roemer, J. E. (1981). *Analytical Foundations of Marxian Economic Theory*. Cambridge University Press.

## Appendix A R Code for the Example

```
> # Input-output data
> A <- matrix(c(0.35,0.05,0.25,
+             0.15, 0.45, 0.05,
+             0.15, 0.15, 0.35),
+           byrow = TRUE, ncol = 3, nrow = 3)
>
> # See A matrix
> A
[,1] [,2] [,3]
[1,] 0.35 0.05 0.25
[2,] 0.15 0.45 0.05
[3,] 0.15 0.15 0.35
>
> # Direct labor input
```



```

> l <- matrix(c(0.15,0.15,0.15),ncol=3)
>
> # See l vector
> l
[,1] [,2] [,3]
[1,] 0.15 0.15 0.15
>
> # Real wage bundle
> b <- matrix(c(1/3,1/3,1/3),ncol = 1)
>
> # See real wage bundle
> b
[,1]
[1,] 0.3333333
[2,] 0.3333333
[3,] 0.3333333
>
> # Augmented input matrix
> M <- A + b%*%l
>
> # See M matrix
> M
[,1] [,2] [,3]
[1,] 0.4 0.1 0.3
[2,] 0.2 0.5 0.1
[3,] 0.2 0.2 0.4

```

```

>
> # Compute uniform rate of profit
> r <- (1/(max(Mod(eigen(M)$values))))-1
>
> # See r
> r
[1] 0.25
>
> # Compute price of production vector
> D <- diag(3) - (1+r)*A
> p <- (1+r)*1%*%solve(D)
>
> # See price of production vector
> p
[,1] [,2] [,3]
[1,] 1 1 1
>
> # Compute vector of labor value
> B <- diag(3) - A
> v <- 1 %*% solve(B)
>
> # See value vector
> v
[,1] [,2] [,3]
[1,] 0.4285714 0.4285714 0.4285714
>

```

```

> # Compute value of real wage bundle
> c2 <- v%*%b
>
> # See value of real wage bundle
> c2
[,1]
[1,] 0.4285714
>
> # ----- CU-LS technical change in sector 3
>
> # New input-output vector
> A_new <- matrix(c(0.35,0.05,0.27,
+                 0.15, 0.45, 0.07,
+                 0.15, 0.15, 0.37),
+                 byrow = TRUE, ncol = 3, nrow = 3)
>
> # See new input-output vector
> A_new
[,1] [,2] [,3]
[1,] 0.35 0.05 0.27
[2,] 0.15 0.45 0.07
[3,] 0.15 0.15 0.37
>
> # New labor input vector
> l_new <- matrix(c(0.15,0.15,0.08),ncol=3)
>

```

```

> # See new labor input vector
> l_new
[,1] [,2] [,3]
[1,] 0.15 0.15 0.08
>
> # Check that the viability condition is satisfied
> # Should be positive
> (p%%A[,3] + l[1,3] - (p%%A_new[,3] + l_new[1,3]))
[,1]
[1,] 0.01
>
> # Compute new vector of labor values
> B_new <- diag(3) - A_new
> v_new <- l_new %% solve(B_new)
>
> # See new vector of labor values
> v_new
[,1]      [,2]      [,3]
[1,] 0.4034091 0.4034091 0.344697
>
>
> # The constant needed to implement the condition for
> # cost reduction to be bounded above
> c1 <- (p%%A[,3] + l[1,3] - p%%A_new[,3])/l_new[1,3]
>
> # See c1

```

```

> c1
[,1]
[1,] 1.125
>
> # Maximum coordinate of constant rate of exploitation hyperplane
> B2 <- max(c2/v_new[1,1],c2/v_new[1,2],c2/v_new[1,3])
>
> # Condition (If B2>B1, the problem is solved!)
> c(B1, B2)
[1] 1.125000 1.243328
>
>
> # ---- Example of a real wage bundle that keeps the
> # rate of exploitation constant by leads to a fall in the
> # uniform rate of profit
>
> # Construct new real wage bundle
> # b3 lies between 1.125 and 1.24, and b1=b2
> b3_new <- 1.15
> b1_new <- (c2 - b3_new*v_new[1,3])/(v_new[1,1]+v_new[1,2])
> b_exm <- matrix(c(b1_new,b1_new,b3_new),ncol = 1)
>
> # See new real wage bundle
> b_exm
[,1]
[1,] 0.03987257

```

```

[2,] 0.03987257
[3,] 1.15000000
>
> # New augmented input matrix
> M_new <- A_new + b_exm %*% l_new
> M_new
[,1]      [,2]      [,3]
[1,] 0.3559809 0.05598089 0.27318981
[2,] 0.1559809 0.45598089 0.07318981
[3,] 0.3225000 0.32250000 0.46200000
>
> # New uniform rate of profit
> r_new <- (1/(max(Mod(eigen(M_new)$values))))-1
> r_new
[1] 0.2163382
>
> # Compare old and new profit rates
> c(r,r_new)
[1] 0.2500000 0.2163382
>
> # Compare values of real wage bundles
> c(v_new %*% b_exm, v%*%b)
[1] 0.4285714 0.4285714

```