1994

SYMMETRY-TESTS AND STANDARD MODEL BACKGROUNDS

BR Holstein
holstein@physics.umass.edu

Follow this and additional works at: https://scholarworks.umass.edu/physics_faculty_pubs

Part of the Physical Sciences and Mathematics Commons

Recommended Citation
Retrieved from https://scholarworks.umass.edu/physics_faculty_pubs/352

This Article is brought to you for free and open access by the Physics at ScholarWorks@UMass Amherst. It has been accepted for inclusion in Physics Department Faculty Publication Series by an authorized administrator of ScholarWorks@UMass Amherst. For more information, please contact scholarworks@library.umass.edu.
Symmetry Tests and Standard Model Backgrounds
Barry R. Holstein
Department of Physics and Astronomy
University of Massachusetts
Amherst, MA 01003

Abstract
Symmetry tests provide an important probe for the structure of elementary particle interactions and for the validity of the standard model. However, it is pointed out that in the interpretation of such experiments one must keep in mind that in many cases apparent "violations" of such tests are actually the result of ordinary effects within the standard model.

God, thou great symmetry,
Who put a biting lust in me
From whence my sorrows spring,
For all the frittered days
That I have spent in shapeless ways
Give me one perfect thing.

Anna Wickham

1 Introduction
The name Ernest Henley to me is inexorably linked with the idea of symmetry tests. Indeed I learned my first information about parity violation in nuclei from his classic work on time reversal and parity[1] and it was reading this paper which eventually led to my interest in and work in this field. Of course, it is not just Prof. Henley who has long been interested in the subject of invariance. Indeed, mankind has from the earliest days been fascinated with the concept of symmetry. The Pythagorans considered the circle and sphere to be the most perfect of two and three dimensional objects respectively because of their obvious radial symmetry. The planets were assumed to move in precise circular orbits and the stars were assumed to be situated in the heavenly spheres. It is during recent decades that symmetry studies in physics have had a rebirth, however. The reason for this has to do with the development of modern physics. As long as physics was focussed on its classical roots of mechanics and electrodynamics, for which the exact laws of motion were known, the use of symmetry methods was, strictly speaking, not necessary. Indeed most books on classical mechanics written during the early part of the present century do not even mention the conservation of angular momentum—they just solve the Kepler problem exactly[2] However, with the advent of particle and nuclear physics, wherein the underlying interactions and equations of motion are complex and unknown, the use
of symmetries in order to flesh out the structure of these fundamental interactions has become commonplace, especially after the discovery of parity violation within the weak interaction in 1957.\[^{[3]}\]

In the interpretation of the results of such tests, however, it must always be kept in mind that apparent violations can be the result of quite pedestrian effects within the standard model rather than bona fide symmetry breaking. A familiar example is the mocking of T-violation by final state strong interactions. Similar effects are expected for other symmetries and below we review the size of these standard model "backgrounds" which can be expected in such tests. Knowledge of the size of such symmetry violation simulations is essential in planning experiments since this represents a fundamental and inescapable upper bound on the sensitivity of these measurements.

## 2 Symmetry Tests in Electroweak Physics

When as particle/nuclear physicists we write down the so-called standard model of electroweak interactions, we simultaneously build in without thinking a significant number of symmetry assumptions, each of which is intrinsic to the standard electroweak model and is subject to experimental test.

Because of space limitations I shall concentrate here on the low-energy and light quark sector of the weak interaction, which then takes the form

$$\mathcal{L}_{wk} = -\frac{G}{\sqrt{2}} J_{\mu}^\dagger J_{\mu}$$

with

$$J_{\mu} = \sum_{e,\mu} \bar{\ell} \gamma_\mu(1 + \gamma_5) \nu_{\ell} + (\bar{d} \gamma_\mu(1 + \gamma_5) U_{KM} \left( \begin{array}{c} u \\ c \end{array} \right) \right)$$  \hspace{1cm} (1)

where $U_{KM}$ is the KM matrix in this sector. The set of processes allowed by this Lagrangian is a large one and includes: a) leptonic: $\mu^- \rightarrow e^- \bar{\nu}_e \nu_\mu$, b) semileptonic: $n, \Lambda \rightarrow p e^- \bar{\nu}_e, \mu^- p \rightarrow n \nu_\mu$, and c) nonleptonic: $np \rightarrow np, \Lambda \rightarrow p\pi^-$ reactions. We begin by considering each symmetry in turn and the means by which it can be tested. Many such tests involve correlation measurements in nuclear beta decay. It is then useful to summarize the set of such correlations, and we do so in the Appendix.

i) V,A Character: The form of the various beta-decay correlations in the case of V,A coupling is given in many sources.\[^{[4]}\] The deviations expected in the presence of S,P or T interactions can be found in the classic work of Jackson, Treiman and Wyld [JTW].\[^{[5]}\] A classic test for the presence of such additional couplings is to look at the longitudinal polarization of the outgoing electron/positron in a beta decay process. An alternative tack in the case of Fermi transitions is to look for a systematic dependence\[^{[6]}\]

$$ft \propto (1 - 2b_F \gamma \frac{m_e}{E})$$  \hspace{1cm} (2)
in the $F_t$ values of such decays. Recently Adelberger has pointed out that the broadening of the proton peak in delayed proton emission accompanying beta decay can also be used in order to provide improved limits on $S,T$ couplings.\textsuperscript{7} Averaging measurements such as these one finds rather strong limits\textsuperscript{8}—$b_F < 0.005$—on the absence of such non-$V,A$ couplings.\textsuperscript{7}

Finally, recent claims have been made that analysis of radiative pion decay experiments—\[ \pi^- \to e^- \bar{\nu}_e \gamma \]—performed at Serpukov has provided evidence for the existence of a nonzero tensor coupling. However, the ”measured” number\textsuperscript{9}—$F_T = -0.0066 \pm 0.0023$—is in contradiction with the upper bound—$F_T = 0.0018 \pm 0.0017$—given by beta decay measurements\textsuperscript{10} and should I believe be discounted.

ii) Left-handedness: In seeking evidence for the possible presence of right-handed weak currents, again the best sensitivity comes from examination of possible deviations of experimental correlation measurements from the values expected within a pure left-handed picture. It is traditional to parameterize the results in terms of:

\[ \sigma = \frac{m_R^2}{m_L^2} \quad \text{and} \quad \zeta = L-R \text{ mixing angle} \quad (3) \]

Figure 1: Experimental limits on right-handed currents.

Again, one of the best present limits arises from electron/positron longitudinal polarization measurements\textsuperscript{12} while another approach involves the use of the beta-polarization coefficient $A$ in superallowed decay as a probe of right handed effects. In the latter case the most stringent limit is obtained in cases where the measured asymmetry is small such as $^{19}$Ne\textsuperscript{13} or neutron beta decay.\textsuperscript{14} A third and powerful probe is provided by the Michel parameters in muon decay. Finally a new approach has been proposed by Quin and Gerard who pointed out the sensitivity of the nuclear-electron spin correlation function to the presence of right-handed currents.\textsuperscript{15} A recent such experiment has been performed using $^{107}$In by Severijns et al.\textsuperscript{16} The present limits on the right-handed parameters $\eta, \zeta$ are summarized in Figure 1.

iii) CVC: There exist two classic tests of CVC within nuclear beta decay. The most familiar (and most precise) is the prediction of identical $F_t$ values for superallowed
$0^+ - 0^+$ decays. Here

$$F_t = \frac{2\pi^3 \ln 2}{G_F^2 m_e^5 |V_{ud}|^2 a_0(0)} \left(1 - \frac{2\alpha}{\pi} \frac{m_Z}{m_N} + \ldots\right)$$ \hspace{1cm} (4)$$

where $a_0(0) = \sqrt{2}$ is the CVC requirement on the Fermi matrix element. Such an analysis has been performed by many groups and all results agree at the 0.5% level. However, as discussed shortly, it is important to know the result even better. The second test involves the comparison of shape factors in mirror $\beta^+, \beta^-$ decays, which are predicted in lowest order for the familiar $A=12$ system in terms of the measured M1 decay width from the 15.11 MeV $^{12}$C excited state to be

$$\frac{dS_\pm}{dE} \approx \pm \frac{4}{3} \frac{b}{A_c m_N} \sim \pm 0.5\% / \text{MeV}$$ \hspace{1cm} (5)$$

The experimental number obtained by Lee, Mo and Wu using an iron free magnetic spectrometer yields values in good agreement with this prediction.

iv) PCAC: In the context of beta decay/muon capture PCAC makes two predictions. One is the Goldberger-Treiman relation

$$F_\pi g_{\pi NN} = m_N g_A$$ \hspace{1cm} (6)$$

relating strong and weak nucleon couplings. The second is that the induced pseudoscalar should have the size

$$r_P = \frac{m_\mu}{2m_N} g_P(q^2 = -0.9 m_\mu^2) = 7.0$$ \hspace{1cm} (7)$$

in a muon capture process.

Unfortunately at the present time the precise validity of both predictions is open to question. In the case of the Goldberger-Treiman relation the problem has primarily to do with the value of the strong $\pi - N$ coupling constant, as will be discussed in the next section. In the case of the induced pseudoscalar the difficulty is that sensitivity is available only in muon capture experiments and even then only at the cost of significant model dependent assumptions. Present results are

$$r_P = \begin{cases} 
7.4 \pm 2.0 & ^3\text{He} \ [21] \\
6.5 \pm 2.4 & p \ [22] \\
9.1 \pm 1.7 & ^{12}\text{C} \ [23]
\end{cases}$$ \hspace{1cm} (8)$$

Each of these results agrees then with the PCAC prediction, but the accuracy is only at the 30% level and could be substantially improved.

v) G-Invariance: Using the quark model one finds that the usual polar-, axial-vector currents satisfy the relations

$$GV_\mu G^{-1} = V_\mu, \quad GA_\mu G^{-1} = -A_\mu$$ \hspace{1cm} (9)$$


Second class currents were defined by Weinberg as being those having opposite signs under the G-parity operation and are not present in the standard model.\cite{24} Their absence has definite implications for weak matrix elements. The most general axial vector matrix element between spin 1/2 systems has the form

\[ < p|A_\mu|n> = \bar{u}(p_2) \left( g_1 \gamma_\mu - ig_2 \frac{\sigma_\mu q^\nu}{2m_N} + \frac{g_3}{2m_N} q_\mu \right) \gamma_5 u(p_1) \]  

(10)

Here for an analog transition such as neutron decay, the absence of second class currents requires \( g_2 = 0 \).\cite{25} In the case of nuclear beta decay, the analog of \( g_2 \) is the tensor form factor \( d \) and the absence of second class currents requires that\cite{25}

i) \( d = 0 \) for an analog transition, e.g. \( ^{19}\text{Ne} \to ^{19}\text{F} \)

ii) \( d_{\beta^+} = d_{\beta^-} \) for mirror decay, e.g. \( ^{12}\text{B} \to ^{12}\text{C} \leftarrow ^{12}\text{N} \)

The tensor term may be measured via correlation experiments, and the best of these measurements involves the alignment correlation in the A=12 mirror system,\cite{26} for which the present experimental number is\cite{27}

\[ d^{II}/b = -0.05 \pm 0.13 \]  

(11)

\textit{i.e.} second class currents are ruled out at the level of 20\% of weak magnetism—not a particularly precise limit.

vi) T-Invariance: Neglecting final state interaction effects, time reversal invariance requires that all amplitudes contributing to a process be relatively real. Thus one looks for T-violation by seeking a phase difference between two or more multipoles with participate in a decay. The most precise experimental work has been done by measuring the D coefficient in beta decay

\[ D^{exp} = \begin{cases} 
(0.7 \pm 6) \times 10^{-4} & ^{19}\text{Ne} \quad [28] \\
-1.1 \pm 1.7 \times 10^{-3} & ^{n} \quad [29] 
\end{cases} \]  

(12)

On the theoretical side a non-zero value of D can arise from a phase difference between leading Fermi and Gamow-Teller terms or between leading and recoil terms.\cite{30} An alternative approach is to measure the R coefficient, which is sensitive to a possible phase difference between V,A and a tensor interaction.\cite{5} Present experimental numbers are consistent with zero but with sizable errors

\[ \text{Im}(G_T + G'_T) = \begin{cases} 
0.136 \pm 0.091 & ^{19}\text{Ne} \quad [31] \\
0.024 \pm 0.027 & ^{8}\text{Li} \quad [32] 
\end{cases} \]  

(13)

However, since such experiments involve measurement of the electron polarization, they will never approach the statistical precision of the corresponding measurements of the D correlation.
3 Standard Model Backgrounds

Two standard model symmetries have no corrections even when all components of the standard model are considered. These are CPT, whose validity follows simply from the tenets demanded of any reasonable quantum field theory, and the other is the left-hand nature of the weak interaction, which is only modified by going outside the standard model. All other symmetries, however, are no longer exact when higher order effects are included.

One of these is not particularly important, however. In the case of the V-A nature of the weak interaction, Higgs boson exchange can introduce effective S,P components into the weak interaction. However, in view of the present mass limits on the Higgs, such effects should be negligible.

i) T-invariance: In the case of T invariance, any real effects arising from higher loop graphs within the standard model are negligible since heavy quarks must be involved. However, as is well-known, T-violation can be simulated by strong and electromagnetic final state interactions, which according to the Fermi-Watson theorem, give rise to different phases for different multipole amplitudes. In the case of the very precise D coefficient measurements, a quick glance at JTW indicates that within the V-A picture there are no such corrections at leading order. However, at the level of recoil this is no longer the case and one finds from one-photon exchange\[33\]

\[
D_{EM}^E = \frac{1}{|a|^2 + |c|^2} (\pm \frac{Z \alpha E^2}{4Mp}) [\delta_{JJ'} \left(\frac{J}{J+1}\right)^{\frac{3}{2}} \text{Rea}^* (b \mp c)(1 + 3 \frac{m^2}{E^2})] - \frac{1}{2 J + 1} \gamma_{JJ'} \text{Rea}^*(c \pm d \mp b)(3 + \frac{m^2}{E^2}) + \cdots
\]

For these superallowed transitions then the leading effect comes from interference between the Gamow-Teller and weak magnetism form factors leading to small but non-negligible values

\[
D_{EM} = \left\{ \begin{array}{l}
2 \times 10^{-4} & \text{for } ^{19}\text{Ne} \\
2 \times 10^{-5} & \text{for } n
\end{array} \right.
\]

In fact such experiments if done to this precision can be turned around—measurement of the final state interaction effect can be used as a probe of weak magnetism even if no bona fide T-violating signal is detected.

For other correlations the electromagnetic effect can arise at leading (non-recoil) order. Thus one finds for the R coefficient \(R_{EM}/A \approx \alpha Z m/p[5]\) which can simulate T violation even at the \(10^{-2}\) level in some cases.

ii) PCAC: A careful look at the derivation of the Goldberger-Treiman relation shows that it should read\[20\]

\[
F_{\pi} g_{\pi NN}(q^2) = M_N g_A(q^2)
\]

so that both the strong and weak couplings should be evaluated at the same value of momentum transfer. Generally what is quoted and used, however, are \(g_A(0)\) and
Table 1: Experimental values of the pion-nucleon coupling constant and the associated Goldberger-Treiman discrepancy.

<table>
<thead>
<tr>
<th></th>
<th>$g^2/4\pi$</th>
<th>$\Delta_\pi$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\pi^\pm$</td>
<td>13.31(27)</td>
<td>0.002</td>
</tr>
<tr>
<td></td>
<td>14.28(18)</td>
<td>0.043</td>
</tr>
<tr>
<td>$\pi^0$</td>
<td>13.55(13)</td>
<td>0.017</td>
</tr>
<tr>
<td></td>
<td>14.52(40)</td>
<td>0.051</td>
</tr>
</tbody>
</table>

$g_{\pi NN}(m_\pi^2)$. Thus a discrepancy is expected for the Goldberger-Treiman relation, and this can be characterized in terms of the quantity

$$\Delta_\pi = 1 - \frac{M_N g_A(0)}{F_\pi g_{\pi NN}(m_\pi^2)}$$

for which one expects $\Delta_\pi \sim m_\pi^2/2m_\pi^2 \sim 0.015$. The experimental size of the discrepancy is at present unclear, due to uncertainty over the size of the pi-nucleon coupling constant. The situation is summarized in Table 1.

iii) G-invariance: If we write the most general axial matrix element between neutron and proton as in Eq. 10 then, since neutron and proton are members of a common isomultiplet, G-invariance requires $g_2 = 0$. Many precise measurements have attempted to check this prediction. However, as discussed above, since this structure function is associated with recoil the experimental limits obtained thereby are relatively weak—$g_2^{\text{exp}} < 0.4$.

Within the standard model, one expects that $g_2$ should be nonvanishing due to both electromagnetic effects and quark mass differences. In particular the latter can be estimated within a relativistic quark model, wherein one finds

$$g_2 = \frac{\int d^3x r(u_u \ell_d - u_d \ell_u)}{\int d^3x (u_u u_d - \frac{1}{3} \ell_u \ell_d)} - \frac{1}{4} \left( \frac{m_n}{m_p} - \frac{m_p}{m_n} \right)$$

For $\Delta S = 0$ processes such as nuclear beta decay one finds $g_2/g_A \sim 10^{-3}$ so that the effect is essentially unmeasurable. However, for $\Delta S = 1$ hyperon decays the strange quark mass is involved and one finds $g_2/g_A \sim 0.3 - 0.4$, which should certainly be detectable. Unfortunately previous analyses have not been precise enough to see this effect. In fact usually $g_2$ is simply set to zero. However, future work involving correlation studies together with rate measurements should be able to resolve this question.

iv) SU(2), SU(3): For the case of SU(2) violation a particularly illuminating example involves $K_{\ell 3}$ decays

$$K^+ \to \pi^0 e^+ \nu_e \quad \text{and} \quad K_L \to \pi^- e^+ \nu_e$$
For the P-wave coupling in such decays, one would have in exact SU(2)

$$f^{K^+\pi^0}(0)/f^{K_L\pi^-}(0) = 1$$

(20)

and one might suspect little change in this result because of the Ademollo Gatto theorem, which seems to assert that any violation of Eq. 20 must be second order in symmetry breaking. However, this is not the case. In fact because of $\eta - \pi^0$ mixing one has

$$1 - |f^{K_L\pi^-}(0)|^2 = \mathcal{O}(\epsilon^2)$$

but

$$1 - \frac{1}{4}|f^{K^+\pi^0}(0)|^2 - \frac{3}{4}|f^{K^+\eta^0}|^2 = \mathcal{O}(\epsilon)$$

(21)

Thus one predicts

$$f^{K^+\pi^0}(0)/f^{K_L\pi^-}(0) \approx 1 + \frac{3}{4} \frac{m_d - m_u}{m_s - \frac{1}{2}(m_d + m_u)} = 1.02$$

(22)

which is in good agreement with the experimental number

$$f^{K^+\pi^0}(0)/f^{K_L\pi^-}(0) = 1.029 \pm 0.010$$

(23)

In the case of SU(3) violation, it is interesting to examine semileptonic hyperon decay—$\Lambda \to p e^- \bar{\nu}_e$, $\Sigma^- \to n e^- \bar{\nu}_e$, etc. Generally such decays are fit via the assumption of SU(3) symmetry

$$< B_b | J_\mu | B_a > = \bar{u}_b (F_V f_{abc} \gamma_\mu + F_A f_{abc} + D_A d_{abc} \gamma_\mu \gamma_5) u_a$$

(24)

and such fits are very good but certainly not perfect. In fact there is good evidence for SU(3) breaking from such fits if one compares the experimental and theoretical predictions in the case of the $\Sigma - \Lambda$ transition. Defining the SU(3) breaking parameter $\rho$ via

$$g^{\Sigma - \Lambda}_A = \sqrt{\frac{2}{3}} \frac{D}{D + F} g^{np}_A$$

(25)

we find $\rho = 0.914 \pm 0.022$ when the experimental value for $g^{\Sigma - \Lambda}_A$ and the fit value for $D, F$ are employed. However, it should be kept in mind that the assumption $g_2 = 0$ was made in performing such fits. In any case it is necessary to understand such effects in both meson and baryon sectors in order to extract believable values of $V_{us}$.

v)CVC: The importance of electromagnetic effects which violate the naive prediction $a(0) = \sqrt{2}$ is critical and possibly lies at the origin of possible KM matrix unitarity violating effects which have been reported by some groups. Thus including radiative and other effects one finds

$$\mathcal{F}t = ft(1 + \Delta_\beta + \delta_r + \frac{\alpha}{\pi} C_{NS})(1 - \delta_c + \Delta_c^2)$$

(26)
where here $\Delta_\beta, \delta_r$ are the usual radiative correction factors, $C_{NS}$ is an axial current correction, $\delta_c$ is a valence nucleon mismatch factor and $\Delta_Z^c$ is a term recently proposed by Wilkinson to account for core nucleon mismatch. Using the form $\Delta_Z^c \sim \gamma Z^{1.8}$ one can achieve a reasonably good fit to the $0^+ - 0^+$ decay ft values and use of this correction brings about no violation of the KM unitarity condition

$$1 - \sum_j |V_{uj}|^2 = \begin{cases} 0.0044(12) & \text{without } \Delta_Z^c \\ 0.0008(12) & \text{with } \Delta_Z^c \end{cases}$$

(27)

However, this simple phenomenological procedure is not substitute for a careful theoretical analysis.

### 4 Conclusion

We have seen that use of the standard model in order to describe the electroweak interactions implicitly assumes the validity of a host of symmetries—CVC, PCAC, G-invariance, etc. Each of these symmetries (except CPT) is subject to experimental verification in low energy leptonic and semileptonic decays. However, in many cases the standard model also provides the ultimate "background" to such tests, in producing non-zero results whose origin is not at all related to the symmetry violation which one is trying to probe.

### Appendix

The standard notation for correlation parameters in nuclear beta decay was given by Jackson, Treiman and Wyld and has the form

$$d\Gamma = \Gamma_0 \left(1 + \frac{p \cdot p'}{E E'_{\nu}} + \frac{m}{E} b_\nu < J > \cdot \left[ \frac{P}{E} A + \frac{P_\nu}{E'_{\nu}} B + \frac{P \times P_\nu}{E E'_{\nu}} D \right] \\
+ \; < \sigma > \cdot \left[ \frac{P}{E} G + \frac{< J > \times P}{E} R \right] + < J, J' > \left[ (\frac{p \cdot p}{E^2} - \frac{p^2}{3 E^2} \delta_{ij}) H \right. \\
+ \left. \left( \frac{p \cdot p_\nu}{E E'_{\nu}} - \frac{P \cdot P_\nu}{3 E E'_{\nu}} \delta_{ij} \right) K \right] \right)$$

(28)

Here the correlation parameters A,B,C, etc. are expressed in terms of the nuclear form factors, which for a general allowed transition can be written in the form

$$< \beta_{p2}|V_{\mu}(0) + A_{\mu}(0)|\alpha_{p1}> = \frac{1}{2M} a P \cdot \ell \delta_{J,J'} \delta_{M,M'} - \frac{i}{4M} \epsilon_{ijk} (J' M' 1 k | J M) \times [2 b \ell q_j + i \epsilon_{ijk} \ell^X (c P^n - d q^n)] + \cdots$$

(29)

where $J,J'$ are the spins of the parent and daughter nuclei respectively, and $M,M'$ represent the initial and final components of nuclear spin along some axis of quantization. The four-vector quantity $\ell^\mu = \bar{u}(p) \gamma^\mu (1 + \gamma_5) v(k)$ is the lepton matrix element.
and a, b, c, d represent reduced matrix elements. Using standard notation

\[ a = g_V M_F, \quad c = g_A M_{GT} \]  

(30)

where \( M_F, M_{GT} \) are the Fermi, Gamow-Teller matrix elements respectively, while \( b \) is the weak-magnetism contribution which, between nuclear analog states would be given by

\[ b = A \left( \frac{J + 1}{J} \right)^{\frac{1}{2}} M_F \mu_V \]  

(31)

where \( A \) is the mass number and \( \mu_V \) is the isovector magnetic moment measured in terms of nuclear magnetons. The coefficient \( d \), the induced tensor, is uniquely correlated with the existence of second class currents if \( \alpha, \beta \) are isotopic analogs. On the other hand, if \( \alpha, \beta \) are not members of a common isotopic multiplet, the existence of \( d \) is not forbidden by G-parity considerations and even receives a contribution from first class currents in the nuclear impulse approximation.

Acknowledgement

This work is supported in part by the National Science Foundation.

References


