1999

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Recommended Citation

CLOSING IN ON Ω_0: THE AMPLITUDE OF MASS FLUCTUATIONS FROM GALAXY CLUSTERS AND THE LYMAN-ALPHA FOREST

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Submitted to ApJ, September 30, 1998

ABSTRACT

We estimate the value of the matter density parameter Ω_0 by combining constraints from the galaxy cluster mass function with Croft et al.'s recent measurement of the mass power spectrum, $P(k)$, from Lyα forest data. The key assumption of the method is that cosmic structure formed by gravitational instability from Gaussian primordial fluctuations. For a specified value of Ω_0, matching the observed cluster mass function then fixes the value of σ_8, the rms amplitude of mass fluctuations in 8 h^{-1}Mpc spheres, and it thus determines the normalization of $P(k)$ at $z = 0$. The value of Ω_0 also determines the ratio of $P(k)$ at $z = 0$ to $P(k)$ at $z = 2.5$, the central redshift of the Lyα forest data; the ratio is different for an open universe ($\Lambda = 0$) or a flat universe. Because the Lyα forest measurement only reaches comoving scales $2\pi/k \sim 15 - 20$ h^{-1}Mpc, the derived value of Ω_0 depends on the value of the power spectrum shape parameter Γ, which determines the relative contribution of larger scale modes to σ_8. Adopting $\Gamma = 0.2$, a value favored by galaxy clustering data, we find $\Omega_0 = 0.46^{+0.12}_{-0.10}$ for an open universe and $\Omega_0 = 0.34^{+0.13}_{-0.09}$ for a flat universe (1σ errors, not including the uncertainty in cluster normalization). Cluster-normalized models with $\Omega_0 = 1$ predict too low an amplitude for $P(k)$ at $z = 2.5$, while models with $\Omega_0 = 0.1$ predict too high an amplitude. The more general best fit parameter combination is $\Omega_0 + 0.2\lambda_0 \approx 0.46 + 1.3(\Gamma - 0.2)$, where $\lambda_0 \equiv \Lambda/3H_0^2$. Analysis of larger, existing samples of QSO spectra could greatly improve the measurement of $P(k)$ from the Lyα forest, allowing a determination of Ω_0 by this method with a precision of $\sim 15\%$, limited mainly by uncertainty in the cluster mass function.

Subject headings: cosmology: observations, cosmology: theory, large-scale structure of universe

1. INTRODUCTION

In theories of structure formation based on gravitational instability and Gaussian initial conditions, the observed mass function of galaxy clusters constrains a combination of the density parameter Ω_0 and the amplitude of mass fluctuations. The physics underlying this constraint is simple: massive clusters can form either by the collapse of large volumes in a low density universe or by the collapse of smaller volumes in a high density universe. To a good approximation, models that reproduce the observed cluster masses have $\sigma_8\Omega_0^{0.5} \approx 0.5$, where σ_8 is the rms mass fluctuation in spheres of radius 8 h^{-1}Mpc and $h \equiv H_0/(100 \ \mathrm{km\,s^{-1}\ Mpc^{-1}})$. White, Efstathiou, & Frenk (1993) were the first to express the cluster normalization constraint in this form and to point out its insensitivity to the shape of the mass power spectrum $P(k)$. Since σ_8 is given by an integral over $P(k)$, an accurate measurement of $P(k)$ could be combined with this constraint to determine Ω_0. Unfortunately, studies of galaxy clustering yield only the galaxy power spectrum, which is related to the mass power spectrum by an unknown (or at best poorly known) "bias factor."

In this paper, we estimate Ω_0 by combining the cluster mass function constraint with Croft et al.'s (1998b, hereafter CWPHK) recent determination of the linear mass power spectrum from Lyα forest data. The argument is slightly more complicated than the one just outlined because the $P(k)$ measurement is at redshift $z = 2.5$ and the observed units are km s^{-1} rather than comoving h^{-1}Mpc. However, the value of Ω_0 determines the linear growth factor, which relates $P(k)$ at $z = 0$ to $P(k)$ at $z = 2.5$, and it determines the relation between comoving h^{-1}Mpc and km s^{-1} at $z = 2.5$. The scalings are different for a flat universe with a cosmological constant ($\lambda_0 \equiv \Lambda/3H_0^2 = 1 - \Omega_0$) and an open, zero-$\Lambda$ universe, so the derived Ω_0 will be different in the two cases.

2. OBSERVATIONAL INPUTS

The $P(k)$ measurement from Lyα forest data is described in detail by CWPHK, who also test for many possible systematic uncertainties. The key feature of this measurement, for our purposes, is the absence of unknown bias factors — the method (introduced and extensively tested on simulations by Croft et al. [1998a]) directly estimates the linear theory mass power spectrum, under the assumption of Gaussian initial conditions. The dominant uncertainty in the measurement at present is the statistical uncertainty resulting from the small size of the data set. Fitting a power law,

$$P(k) = P_p \left(\frac{k}{k_p} \right)^n, \tag{1}$$

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1

to the derived power spectrum, CWPHK find a logarithmic slope $n = -2.25 \pm 0.18$ and a normalization $P_p = 2.21^{+0.90}_{-0.64} \times 10^7 (\text{km s}^{-1})^3$ for the power at the "pivot" wavenumber $k_p = 0.008 (\text{km s}^{-1})^{-1}$, which is chosen so that errors in n and P_p are statistically independent. The measurement probes comoving scales $2 - 12 \ h^{-1}\text{Mpc}$ for $\Omega_0 = 1$, $3 - 16 \ h^{-1}\text{Mpc}$ for $\Omega_0 = 0.5$, $\lambda_0 = 0$, and $4 - 22 \ h^{-1}\text{Mpc}$ for $\Omega_0 = 0.3$, $\lambda_0 = 0.7$.

For the cluster normalization constraint, we adopt the results of Eke, Cole, & Frenk (1996, hereafter ECF):

$$\sigma_8 = (0.52 \pm 0.04)\Omega_0^{-0.46+0.10\Omega_0} \qquad \lambda_0 = 0 \qquad (2)$$
$$\sigma_8 = (0.52 \pm 0.04)\Omega_0^{-0.52+0.13\Omega_0} \qquad \lambda_0 = 1 - \Omega_0.$$

These constraints are derived using the Press-Schechter (1974) formalism, cross-checked against large cosmological N-body simulations. The 8% uncertainty in the normalization includes a combination of statistical uncertainties and potential systematic errors, both observational and theoretical, as discussed by ECF. There have been numerous other determinations of this constraint using different methodologies and different treatments of the observational data, and although they yield slightly different values for the normalization and power law indices, they generally agree with equation (2) to 10% or better (e.g., Cen 1998; Eke, Cole, & Frenk 1998; Pen 1998; Viana & Liddle 1998). This agreement suggests that the uncertainty quoted by ECF is reasonable, although the various analyses share many assumptions and often rely on the same observational data (e.g., Edge et al. 1990; Henry 1997).

3. CONSTRAINTS ON Ω_0

Figure 1 illustrates our method, by comparing the CWPHK measurement of $P(k)$ to the predictions of cluster-normalized models (i.e., models satisfying equation [2]) that have a power spectrum with shape parameter $\Gamma = 0.2$, in the parameterization of Efstathiou, Bond, & White (1992, hereafter EBW). The EBW parameterization is motivated by physical models with scale-invariant primordial fluctuations and cold dark matter, for which $\Gamma \approx \Omega_0 h$ if the baryon fraction is small (see also Peebles 1982; Bardeen et al. 1986).[1] For our purposes, the EBW form serves as a convenient and plausible description of the power spectrum shape that seems in reasonable accord with observations. From top to bottom, the curves in Figure 1 show power spectra for $\Omega_0 = 0.1$, 0.2, 0.3, 0.4, 0.5, 0.6, and 1. The left panel is for $\lambda_0 = 0$, and the right for $\lambda_0 = 1 - \Omega_0$. The uncertainty in the overall normalization is indicated by the error bar on the open circle; at the 1σ level, all points can shift coherently in amplitude with this uncertainty. The (1σ) error bars on the individual solid points are derived from the dispersion among ten subsets of the Lyα forest data; they represent uncertainties in the relative amplitudes of power at different k.

For $\Omega_0 = 1$, the normalization of $P(k)$ is low today, and reduction by the linear growth factor $(1 + 2.5)^2$ between $z = 2.5$ and $z = 0$ puts the predicted $P(k)$ well below the CWPHK measurement. Conversely, for $\Omega_0 = 0.1$, the

$P(k)$ normalization is high at $z = 0$, and scaling back to $z = 2.5$ reduces fluctuations by a smaller factor, yielding a predicted $P(k)$ that is too high. The value of Ω_0 that yields the best fit to the CWPHK data is $\widehat{\Omega}_0 = 0.46$ for $\lambda_0 = 0$ and $\widehat{\Omega}_0 = 0.34$ for $\lambda_0 = 1 - \Omega_0$. The flat model yields a lower Ω_0 because of the larger linear growth factor between $z = 2.5$ and $z = 0$ in a Λ-dominated universe. One could also envision the Ω_0 constraint by evolving the CWPHK $P(k)$ forward in time to $z = 0$: there is one and only one value of Ω_0 for which forward extrapolation yields a combination of σ_8 and Ω_0 in agreement with the cluster constraint (2).

Because the Lyα forest $P(k)$ measurement only reaches comoving scales $\sim 15 - 20 \ h^{-1}\text{Mpc}$, the value of Ω_0 derived from this argument depends on the assumed shape of $P(k)$, which determines the relative contribution of larger scale modes to σ_8. For Figure 1, we adopted the shape parameter $\Gamma = 0.2$, a value favored by a number of studies of large scale galaxy clustering (e.g., Maddox et al. 1990; Baugh & Efstathiou 1993, 1994; Peacock & Dodds 1994; Gaztañaga & Baugh 1998). Figure 2 shows a more general result, confidence intervals for cluster-normalized models in the (Ω_0, Γ) plane. We derive these by computing the amplitude P_p and logarithmic slope n of $P(k)$ at $z = 2.5$ for each (Ω_0, Γ) combination, using equation (2) to fix σ_8, then computing the $\Delta\chi^2$ of these $P(k)$ parameters relative to the best fit power law parameters using the $\Delta\chi^2$ curves shown in figure 15 of CWPHK. These curves (and hence the contours of Figure 2) account for the covariance of the individual $P(k)$ data points and for the contribution of uncertainty in the mean Lyα forest opacity to the uncertainty in the amplitude P_p. Higher Γ implies less large scale power and less contribution to σ_8 from scales beyond those of the CWPHK measurement. Since the cluster mass function determines the combination $\sigma_8 \Omega_0^{0.5}$, we obtain higher values of Γ for higher values of Ω_0. The value of Ω_0 affects the mapping from $h^{-1}\text{Mpc}$ at $z = 0$ (the units of the EBW power spectrum) to km s^{-1} at $z = 2.5$, so it influences both the slope and the amplitude of the predicted $P(k)$ — in terms of Figure 1, changing Ω_0 shifts the $P(k)$ curves both vertically and horizontally. However, because the uncertainty in the slope is fairly large, it is primarily the amplitude P_p that constrains Ω_0, while the slope constrains Γ.

If we consider the Lyα forest $P(k)$ and cluster normalization as our only constraints, then the dashed contour ($\Delta\chi^2 = 2.30$) represents the "1σ" (68% confidence) joint constraint on Ω_0 and Γ. With the statistical uncertainty of the CWPHK measurement, we cannot rule out the combination of high Γ and high Ω_0. However, if we fix the value of Γ based on large scale structure data (thus implicitly assuming that biased galaxy formation does not distort the *shape* of the power spectrum on large scales), then we obtain 1, 2, and 3σ constraints on Ω_0 from the intersection of a vertical line with the solid contours, which represent $\Delta\chi^2 = 1$, 4, and 9. The ridges of minimum $\Delta\chi^2$ are well described by

$$\widehat{\Omega}_0 = 0.46 + 1.3(\Gamma - 0.2) \qquad \lambda_0 = 0, \qquad (3)$$

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[1] The identification of Γ with $\Omega_0 h$ is sensitive to the assumptions of a scale-invariant inflationary spectrum and a pure CDM matter content. For example, a CDM model with a tilted ($n < 1$) inflationary spectrum would be roughly equivalent to an EBW model with $\Gamma < \Omega_0 h$ on the scales that are relevant to our calculation.
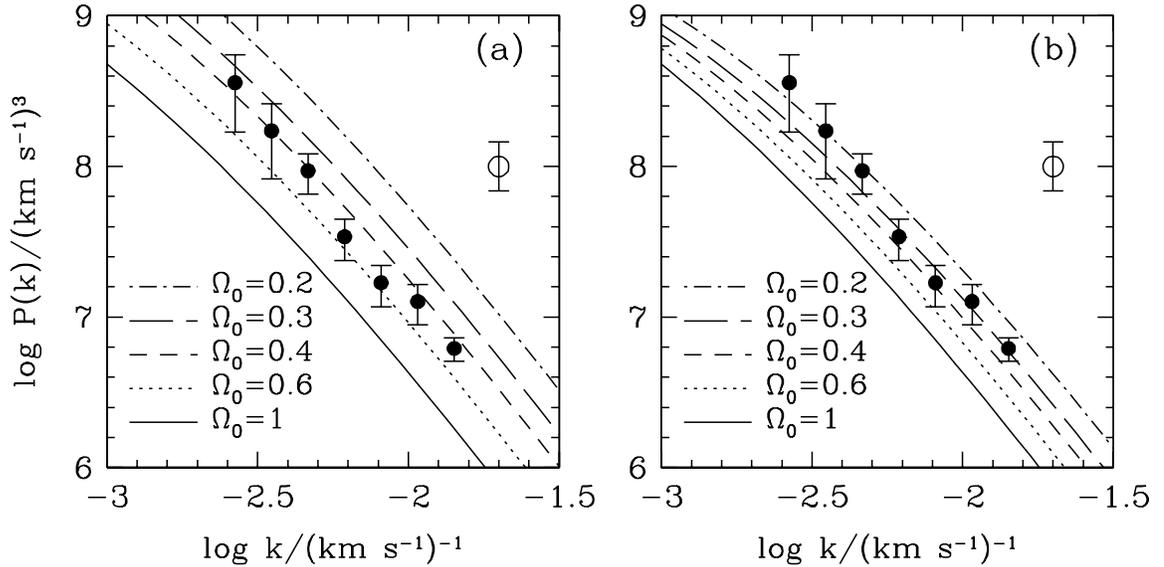
FIG. 1.— Filled circles (with 1σ error bars) show the power spectrum of mass fluctuations at $z = 2.5$, derived from Lyα forest spectra by CWPHK. The error bar on the open circle indicates the normalization uncertainty: at the 1σ level, all points can be shifted coherently up or down by this amount. Curves show $P(k)$ at $z = 2.5$ for cluster-normalized models with a power spectrum shape parameter $\Gamma = 0.2$ and various values of Ω_0, as indicated. Models with high Ω_0 predict a $P(k)$ that is too low to match the Lyα forest results, and models with low Ω_0 predict a $P(k)$ that is too high. (a) Open models, with $\lambda_0 = 0$. (b) Flat models, with $\lambda_0 = 1 - \Omega_0$.
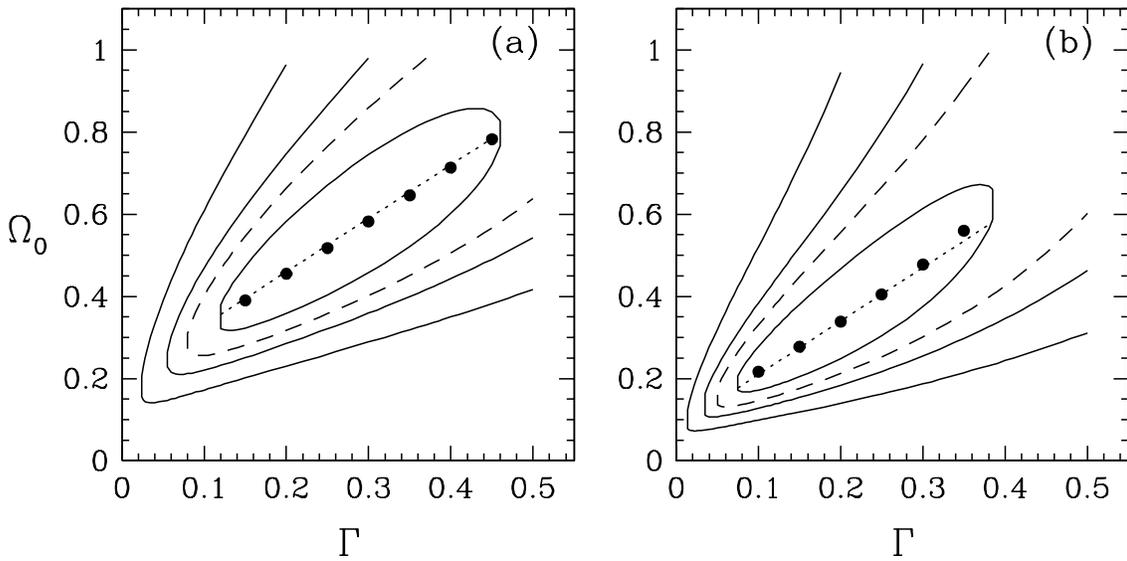

FIG. 2.— Constraints on the parameters of cluster-normalized power spectra, for open models (a) and flat models (b). Filled circles show the value of Ω_0 that gives the best match (minimum χ^2) to the CWPHK power spectrum parameters at each Γ. Dotted lines show the ridge-line equations (3). Solid lines show contours of $\Delta\chi^2 = 1, 4, 9$ in the $\Omega_0 - \Gamma$ plane. For a specified value of Γ, the intersection of a vertical line with the solid contours gives the 1, 2, and 3σ confidence intervals on Ω_0. If one ignores external information about Γ and considers only the Lyα forest data themselves, then the dashed contour at $\Delta\chi^2 = 2.30$ represents the 68% confidence constraint on the parameter values.

$$\widehat{\Omega}_0 = 0.34 + 1.3(\Gamma - 0.2) \qquad \lambda_0 = 1 - \Omega_0.$$

For $\Gamma = 0.2$, the derived values of Ω_0 and corresponding uncertainties are

$$\Omega_0 = 0.46 \,^{+0.12}_{-0.10} \,(1\sigma) \,^{+0.29}_{-0.17} \,(2\sigma) \qquad \lambda_0 = 0, \qquad (4)$$
$$\Omega_0 = 0.34 \,^{+0.13}_{-0.09} \,(1\sigma) \,^{+0.32}_{-0.16} \,(2\sigma) \qquad \lambda_0 = 1 - \Omega_0.$$

The fractional uncertainty in Ω_0 is smaller for open models because the stronger Ω_0-dependence of the fluctuation growth factor in an open universe increases the sensitivity of P_p to Ω_0. A critical density universe is formally ruled out at the 3σ level for $\Gamma \lesssim 0.2$ and at the 2σ level for $\Gamma = 0.3$.

The uncertainties in equation (4) do not include the uncertainty in the cluster normalization itself. For a fixed value of this normalization at $z = 0$, inspection of Figure 1 shows that the value of P_p is approximately proportional to $\Omega_0^{-3/2}$ in open models and to Ω_0^{-1} in flat models over the range $0.2 < \Omega_0 < 1$. The ECF estimate of the cluster normalization uncertainty translates to $\sim 15\%$ uncertainty in σ_8^2, the quantity proportional to P_p, for fixed Ω_0. It therefore contributes $\sim 10\%$ (open) or $\sim 15\%$ (flat) uncertainty to our derived value of Ω_0, to be added in quadrature to the uncertainty listed above. Because the uncertainty in P_p with the present Lyα forest sample is large, this additional uncertainty has little effect on our error bars.

4. ASSUMPTIONS, CAVEATS, AND PROSPECTS

Equation (4) is the principal result of this paper. It rests, however, on a number of assumptions:

1. *Primordial fluctuations are Gaussian, as predicted by inflationary models for their origin.* This assumption is built into the determination of the cluster normalization constraint and into the normalization of the Lyα forest $P(k)$. It is this assumption that allows us to combine results from Lyα forest spectra, which respond mainly to "typical" ($\sim 0 - 2\sigma$) fluctuations in the underlying mass distribution, with results from rich clusters, which form from rare, high fluctuations of the density field. The assumption is supported by studies of galaxy count distributions (e.g., Bouchet et al. 1993; Gaztañaga 1994; Kim & Strauss 1998), the topology of the galaxy distribution in redshift surveys (e.g., Gott et al. 1987; Colley 1997; Canavezes et al. 1998), by other large scale structure statistics (e.g., Weinberg & Cole 1992), and by the statistics of cosmic microwave background (CMB) anisotropies (Colley, Gott, & Park 1996; Kogut et al. 1996). However, current constraints still leave room for some non-Gaussianity of the primordial fluctuations, perhaps enough to affect the Ω_0 value determined by our approach.

2. *The primordial power spectrum has approximately the EBW form with shape parameter $\Gamma \approx 0.2$.* This assumption allows us to to calculate the contribution of large scale modes to σ_8 once $P(k)$ is fixed to match the amplitude determined from the Lyα forest. Changing Γ changes the best fit value of Ω_0 by $\Delta\widehat{\Omega}_0 = 1.3(\Gamma - 0.2)$, and the influence of Γ on the uncertainty in Ω_0 can be read from

Figure 2. A radical departure from the EBW form of $P(k)$, such as truncation at $2\pi/k \sim 20 \ h^{-1}$Mpc, would have a more drastic impact on our conclusions, but it would be clearly inconsistent with galaxy clustering data. Gaztañaga & Baugh (1998, a further examination of the results in Baugh & Efstathiou 1993, 1994) argue that the EBW form of $P(k)$ predicts a turnover that is too broad to match angular clustering in the APM galaxy survey. However, scales near the turnover make little contribution to σ_8, so our derived Ω_0 would not change much if we adopted the Gaztañaga & Baugh (1998) power spectrum instead of a $\Gamma = 0.2$ EBW model.

3. *The cluster mass determinations used to obtain equation (2) are correct.* Cosmological N-body simulations give straightforward predictions of cluster masses for specified cosmological parameters, but cluster masses are not directly observed — they are inferred with aid of assumptions from galaxy motions, X-ray data, or gravitational lensing. The approximate agreement of these different methods (see, e.g., Wu et al. 1998) supports the robustness of these mass determinations, but the agreement is not yet demonstrated with high precision, and the physics of cluster formation could cause a breakdown of the standard assumptions that would systematically affect all three methods in the same direction. Many papers, including ECF, compare model predictions to observed X-ray temperatures instead of inferred masses, but this approach still requires assumptions to translate theoretically predicted masses into cluster gas temperatures. On the whole we regard the cluster normalization constraint as fairly robust, and comparisons between hydrodynamic simulations (e.g., Evrard, Metzler, & Navarro 1996; Bryan & Norman 1998; Pen 1998) and expanded X-ray temperature samples should solidify and refine it over the next few years. However, it is worth noting that analyses of peculiar velocity data imply different normalizations, ranging from $\sigma_8\Omega_0^{0.6} \approx 0.375$ (Willick et al. 1997; Willick & Strauss 1998) to $\sigma_8\Omega_0^{0.6} \approx 0.8$ (Kolatt & Dekel 1997; Zaroubi et al. 1997; Freudling et al. 1998; Zehavi 1998).[2] These normalizations would imply substantially different values of Ω_0, as is evident from Figure 1.

4. *The physical picture of the Lyα forest that underlies Croft et al.'s (1998a) method of $P(k)$ determination is correct.* The essential feature of this picture is that most Lyα forest absorption arises in moderate density fluctuations ($\rho/\bar{\rho} \sim 0.1 - 10$) of the diffuse, photoionized intergalactic medium, leading to a tight relation between local mass overdensity and Lyα optical depth (Croft et al. 1997; Weinberg, Katz, & Hernquist 1998). This picture is derived from hydrodynamic cosmological simulations (Cen et al. 1994; Zhang et al. 1995, 1998; Hernquist et al. 1996; Miralda-Escudé et al. 1996; Wadsley & Bond 1997; Theuns et al. 1998b), and a similar description was developed independently as a semi-analytic model of the forest (Bi 1993; Bi, Ge, & Fang 1995; Bi & Davidsen 1997; Hui, Gnedin, & Zhang 1997). It is empirically supported by the success of the simulations and semi-analytic models in reproducing observed properties of the Lyα forest (see

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[2]The VELMOD studies (Willick et al. 1997; Willick & Strauss 1998) find $\beta_I \equiv \Omega_0^{0.6}/b_I \approx 0.50$, which we translate to $\sigma_8\Omega_0^{0.6} = \sigma_{8I}(\sigma_8/\sigma_{8I})\Omega_0^{0.6} = \sigma_{8I}\beta_I \approx 0.375$ using the measured clustering of IRAS galaxies, which implies $\sigma_{8I} \approx 0.75$ (Fisher et al. 1994; Moore et al. 1994). This translation implicitly assumes that the bias factor b_I affecting the peculiar velocity analysis is the same as the rms fluctuation ratio σ_{8I}/σ_8. The most recent analysis using the POTENT method finds a much higher $\beta_I = 0.89$ (Sigad et al. 1998), which translates to $\sigma_8\Omega_0^{0.6} \approx 0.67$.

the above papers and Davé et al. 1997, 1998; Rauch et al. 1997; Zhang et al. 1997; Bryan et al. 1998; Theuns, Leonard & Efstathiou 1998a). It is also supported by the coherence of Lyα absorption across widely separated lines of sight, which gives direct evidence that the absorbing structures are low density (Bechtold et al. 1994; Dinshaw et al. 1994, 1995; Rauch & Haehnelt 1995; Crotts & Fang 1998). Small scale "cloudlet" structure is ruled out by the nearly perfect correlation of Lyα absorption along lines of sight towards gravitationally lensed QSOs (Smette et al. 1992, 1995; Rauch 1997). The $P(k)$ determination method relies on general properties of this physical picture, not on details of particular simulations or a particular cosmological model.

5. *The CWPHK determination of $P(k)$ is correct.* If assumptions (1) and (4) are correct, then the most likely source of a systematic error larger than the estimated statistical uncertainty would be an error in the adopted value of the mean opacity of the Lyα forest. CWPHK take this value from Press, Rybicki, & Schneider (1993) and incorporate Press et al.'s estimated statistical uncertainty into the $P(k)$ normalization uncertainty. Rauch et al. (1997) find a similar value of the mean opacity from a small sample of Keck HIRES spectra. However, some other determinations (Zuo & Lu 1993; Dobrzycki & Bechtold 1996) yield significantly lower mean opacities, and these would in turn imply higher $P(k)$ normalizations and lower estimates of Ω_0 (see Figure 1). CWPHK examine a number of other potential sources of systematic error and find none that are as large as the statistical uncertainty, and they obtain consistent results from high and low redshift halves of the data sample and from a second independent data set. Nonetheless, since this is the first determination of $P(k)$ from Lyα forest data (except for the illustrative application to a single high resolution spectrum in Croft et al. [1998a]), it should be treated with some caution until it is confirmed. At the 1σ level, the best fit values of P_p and n depend on the selection of the data and the parameter fitting procedure, and the statistical error bars are themselves uncertain because they are estimated by breaking the data into small subsets. For our constraints on Ω_0 it is P_p that matters much more than n, and for this parameter we believe that the CWPHK error estimate is likely to be conservative.

There are good prospects for checking these assumptions and improving the precision of the Ω_0 measurement with existing or easily obtainable data. Statistical properties of high resolution spectra, such as the flux decrement distribution function (Miralda-Escudé et al. 1996, 1997; Rauch et al. 1997), can independently constrain the amplitude of $P(k)$ on these scales, and can test assumptions (1) and (4). Larger samples of moderate resolution spectra can provide new determinations of $P(k)$ with smaller statistical uncertainties. These will test assumption (5), and because a precise determination of n will tightly constrain Γ, they will also test assumption (2), though even with a larger data set we will probably need to extrapolate $P(k)$ to larger scales with an assumed form in order to calculate σ_8. With a sample of ~ 100 moderate resolution spectra, it should be possible to reduce the uncertainty in the amplitude of $P(k)$ well below the uncertainty in the cluster normalization constraint. In this limit, the fractional uncertainty in Ω_0 in flat models is similar to the fractional

uncertainty in σ_8^2 at fixed Ω_0 from cluster normalization, and it is smaller by a factor of $3/2$ in open models (see the discussion at the end of §2).

Our current results clearly favor a low density universe over a critical density universe. However, a conspiracy of small errors could still make our results consistent with $\Omega_0 = 1$, without requiring a drastic violation of any of the above assumptions. For example, if we increase the cluster normalization by 1σ, decrease the CWPHK value of P_p by 1σ, and adopt $\Gamma = 0.3$ instead of $\Gamma = 0.2$, then our best fit Ω_0 for open models rises from 0.46 to 0.84. Analysis of larger QSO samples should make the discrimination between critical density and low density models more decisive in the near future.

Other recent determinations of Ω_0 include the estimate $\Omega_0 \approx 0.2$ from careful analyses of cluster mass-to-light ratios (Carlberg et al. 1996, 1997b) and estimates based on cluster evolution that range from $\Omega_0 \approx 0.3 - 0.5$ (Bahcall, Fan, & Cen 1997; Carlberg et al. 1997a; Henry 1997; Bahcall & Fan 1998; Eke, Cole, & Frenk 1998) to $\Omega_0 \approx 0.7 - 1$ (Blanchard & Bartlett 1998; Reichart et al. 1998; Sadat, Blanchard, & Oukbir 1998; Viana & Liddle 1998). Eke et al. (1998) and Viana & Liddle (1998) provide good accounts of the current systematic uncertainties in the cluster evolution technique. Despite its short history, we believe that the method adopted here will ultimately lead to a more compelling measurement of Ω_0 than either cluster mass-to-light ratios or cluster evolution, because it is independent of complex galaxy formation physics on the one hand and of systematic uncertainties in high redshift cluster masses on the other. A systematic error in cluster mass determinations at $z = 0$ would affect all three methods, in the same direction.

We have not considered models with space curvature *and* non-zero λ_0 in detail. However, for $0 < \lambda_0 < 1 - \Omega_0$ the best fit Ω_0 should lie between that of the open and flat cases illustrated in Figure 2. We have investigated results for open, non-zero λ_0 models with $\Gamma \approx 0.2$ and find that the best fit parameter combinations approximately satisfy $\Omega_0 + 0.2\lambda_0 = 0.46 + 1.3(\Gamma - 0.2)$. As expected, our method is sensitive primarily to Ω_0, because of its direct influence on cluster normalization, and is only weakly sensitive to λ_0. It therefore complements measurements of cosmic acceleration using Type Ia supernovae, which most tightly constrain a combination that is approximately $\Omega_0 - \lambda_0$ (see the constraint diagrams in Kim [1998] and Riess et al. [1998]). It also complements measurements of the angular location of the first acoustic peak in the CMB anisotropy spectrum, which most tightly constrain $\Omega_0 + \lambda_0$ because the peak location is sensitive to space curvature (Kamionkowski, Spergel, & Sugiyama 1994).

Our result strengthens the Type Ia supernova case for a non-zero cosmological constant (Kim 1998; Riess et al. 1998) because it rules out $\lambda_0 = 0$ models with very low Ω_0. It is consistent with current CMB anisotropy data for either a flat or an open universe (see, e.g., Hancock et al. 1998; Lineweaver 1998). Constraints on (Ω_0, λ_0) parameter combinations from all three methods should become substantially more precise in the near future. The combination of the three measurements should yield good, non-degenerate determinations of Ω_0 and λ_0 and hence an empirical test of the theoretical prejudice that favors a flat universe. Alternatively, the three methods may yield in-

consistent results, indicating either that the assumptions underlying at least one of the methods are incorrect or that the combination of pressureless matter and a constant vacuum energy does not adequately describe the energy content of our universe.

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