Axiomatic Marxian Exploitation Theory: a Survey of the Recent Literature

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Abstract

In this paper we review recent developments in axiomatic studies of Marxian exploitation theory. First, given the acute controversy over the formal definition of exploitation during the 1970-1990s, we review the study of the axiomatic framework, which identifies some fundamental properties – technically, domain conditions – that any definition of exploitation should satisfy. Moreover, we provide a survey on the axiomatic studies about the proper measures of exploitation which coherently preserve the basic Marxian perceptions represented by two axioms, **Profit-Exploitation Correspondence Principle** and **Class-Exploitation Correspondence Principle**. Finally, we examine the relevance of the labour theory of value in these axiomatic studies of the proper measures of exploitation.

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Keywords: Axiomatic analysis; Labour Exploitation; Profit-Exploitation Correspondence Principle; Class-Exploitation Correspondence Principle; Labour Theory of Value.

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1 Introduction

Exploitative and dominance relations characterise capitalist societies. Karl Marx (1867, 1894) argued that the conflicting distributional relationship between workers and capitalists is exploitative, and exploitation is a generic and persistent feature of capitalism. According to Marx, though the capitalists-workers relationship is mediated by a contract in labour markets that the worker is free to enter and exit, it is all but fair: workers cannot but spend part of their time working for the capitalist when entering into such a contract, since otherwise they could not procure their necessities because they lack of access to the means of production.

Therefore, just like feudal lords exploit serfs who spend part of their time working for themselves and another part on uncompensated work for the lord, capitalism is characterised by an *unequal exchange of labour* (UE), that is, by exploitative relations involving systematic differences between the amount of labour that individuals contribute to the economy and the amount of labour they receive via their income.

The application of the notion of UE exploitation to capitalism, however, involves a fundamental difficulty: unlike in the feudal system, the division of workers labour into the part for themselves and the part appropriated by capitalists cannot be directly observed. Therefore, in a capitalist economy UE exploitation can be detected, and measured, only via accurate economic analysis. This requires identifying an operational measure of the difference between the labour expended and the labour received by each individual.

In Marx (1867, 1894), such an operational measure was based on the notion of *labour values*: while the amount of labour that workers contribute to the economy is defined as their working time, the amount that they receive is defined as the labour embodied in the consumption bundle that they (can) purchase via their wage revenue.

Following this classical proposal, Okishio (1963) defined the labour value of each commodity as the solution to a system of linear equations in simple production economies with a single technique of production of the Leontief type (i.e. with every production process producing a single output).[1] Based on this definition, Okishio (1963) developed a formal approach of exploitation theory consistent with Marx's own argument (Marx, 1867). Since then, Okishio (1963) has sparked a vast formal literature in mathematical Marxian economics. A number of UE approaches have been proposed that either extend Okishio's to more general production economies, in which both the choice of techniques and the joint production are available, such as Morishima (1974) and Roemer (1982), or are alternative to it, like the New Interpretation à la Duménil (1980)-Foley (1982).

In contrast, Roemer (1982, 1994) has criticised the notion of UE exploitation and proposed instead a *property relation definition of exploitation* (PR exploitation) which,

[1] For a more thorough description, see section 2 below.

in his view, better captures the normative concern of exploitation theory, namely the injustice of the unequal distribution of productive assets, rather than the unequal exchange of labour *per se*.

While PR exploitation theory – and the critique of the UE approach – have gained some traction for a while, recently a number of authors have criticised it, leading to a revival of UE exploitation (Cohen, 1995; Wright, 2000; Vrousalis, 2013). Critics argue that exploitation should be conceptualised as the systematic product of the structure of economic transactions, in which some of the fruits of the labour of the exploited agents is appropriated by the exploiters owing to the asymmetric power relations generated from private ownership.[2]

Despite the debates on alternative definitions of exploitation and the revival of UE theory, the issue of the proper definition of UE exploitation has remained unresolved. A novel, *axiomatic* approach has been recently proposed to address this issue. This paper provides a preliminary survey of this literature, which examines the foundations of UE exploitation in a broad class of economic environments. We will also reexamine the relevance of the labour theory of values in the issue of appropriate definitions of the Marxian notion of exploitation.

2 The Framework

In this section, we lay out a general framework to analyse the notion of exploitation in capitalist economies. An economy comprises a set of N agents, $\mathcal{N} = \{1, .., N\}$, with generic element $\nu \in \mathcal{N}$.

The production possibility set P comprises all production techniques, or *activities*, that can be used to produce n (private) goods. An activity can be written as $\alpha \equiv (-\alpha_l, -\underline{\alpha}, \overline{\alpha}) \in \mathbb{R}_- \times \mathbb{R}^n_- \times \mathbb{R}^n_+$,[3] where α_l and $\underline{\alpha}$ describe the amount of *effective* labour and the vector of produced goods used as inputs in order to produce a vector of outputs $\overline{\alpha}$.[4] Denote the vector of net outputs (outputs minus inputs) arising from α as $\widehat{\alpha} \equiv \overline{\alpha} - \underline{\alpha}$. Assume that technology displays constant returns to scale and $\mathbf{0} = (0, ..., 0) \in P$.[5]

[2]A detailed review of these debates can be found in Yoshihara (2017).

[3]Let $\mathbb{R}$ be the set of real numbers and $\mathbb{R}_+$(*resp.* $\mathbb{R}_-$) the set of non-negative (*resp.* non-positive) real numbers. For all x, $y \in \mathbb{R}^n$, $x \geqq y$ if and only if $x_i \geqq y_i$ $(i = 1, \ldots, n)$; $x \geq y$ if and only if $x \geqq y$ and $x \neq y$; and $x > y$ if and only if $x_i > y_i$ $(i = 1, \ldots, n)$. All vectors are columns, unless otherwise specified.

[4]Observe that while we shall allow for heterogeneous *skills* across agents (see below), we are assuming that only one type of homogeneous labour is used in production in all industries. This is without significant loss of generality in our context. For an axiomatic analysis of the measurement of labour content in economies with heterogeneous labour, see Yoshihara and Veneziani (2023a). For a discussion of UE exploitation in economies with heterogeneous labour see Veneziani and Yoshihara (2017a).

[5]Formally, P is a closed, convex cone such that (i) for any $\alpha \in P$, $\overline{\alpha} \geq \mathbf{0}$ implies $\alpha_l > 0$ (labour is

A production set P is of a *von Neumann type* if there exists a profile (A, B, L) such that P can be represented as follows:

$$P_{(A,B,L)} \equiv \left\{\alpha \equiv (-\alpha_l, -\underline{\alpha}, \overline{\alpha}) \in \mathbb{R}_- \times \mathbb{R}^n_- \times \mathbb{R}^n_+ \mid \exists y \in \mathbb{R}^m_+ : \alpha \leqq (-Ly, -Ay, By)\right\}.$$

where A is an $n \times m$ matrix, whose generic component, $a_{ij} \geqq 0$, represents the amount of commodity i used as an input to operate one unit of the j-th production process; B is an $n \times m$ matrix, whose generic component, $b_{ij} \geqq 0$, represents the amount of commodity i produced as an output by operating one unit of the j-th production process; and $L > \mathbf{0}$ is a $1 \times m$ row vector of direct labour input coefficients. The j-th column of of A (resp. B) is denoted as A_j (resp. B_j). Let I be the $n \times n$ identity matrix. A *Leontief* production set is a special case of a von Neumann production set $P_{(A,B,L)}$ where $m = n$ and $B = I$.[6]

For each agent $\nu \in \mathcal{N}$, let $s^\nu > 0$ be ν's *skill* level. If an agent ν spends an amount of *time* λ^ν in production, then $\Lambda^\nu = s^\nu \lambda^\nu$ is the *effective* labour performed by ν. Let $\bar{l} > 0$ be the maximum amount of time each agent can spend in production. Then, each ν's *effective* labour endowment is $l^\nu = s^\nu \bar{l}$. Let $\omega^\nu \in \mathbb{R}^n_+$ denote ν's endowment of productive assets, and let $\omega = \sum_{\nu \in \mathcal{N}} \omega^\nu$ be the vector of social endowments.

An economy is specified by a list $E = E\langle \mathcal{N}, P, \boldsymbol{\omega}, \boldsymbol{l}\rangle$, where $\boldsymbol{\omega} \equiv (\omega^\nu)_{\nu \in \mathcal{N}}$ and $\boldsymbol{l} \equiv (l^\nu)_{\nu \in \mathcal{N}}$. Denote the set of economies by $\mathcal{E}$ and let

$$\mathcal{E}_{\mathcal{L}} \equiv \left\{E\langle \mathcal{N}, P, \boldsymbol{\omega}, \boldsymbol{l}\rangle \in \mathcal{E} \mid \exists \text{ a Leontief technique } (A, L) \text{ with } P = P_{(A,I,L)}\right\}$$

be the subset of Leontief economies.

An allocation of labour and produced commodities is specified by a vector $\boldsymbol{x} \equiv (\boldsymbol{c}, \boldsymbol{\Lambda}) \in \mathbb{R}^{nN+N}_+$, where $\boldsymbol{c} = (c^\nu)_{\nu \in \mathcal{N}}$ represents an assignment of the n commodities to the N agents and $\boldsymbol{\Lambda} = (\Lambda^\nu)_{\nu \in \mathcal{N}}$ is a profile of (effective) labour supplied by each agent.

An allocation $\boldsymbol{x}$ is called *feasible* for the economy E if and only if the labour performed by each agent does not exceed their endowment ($0 \leqq \Lambda^\nu \leqq l^\nu$ for each agent $\nu \in \mathcal{N}$) and there exists a production activity $\alpha \in P$ that can be activated given the aggregate capital endowment ($\underline{\alpha} \leqq \omega$), aggregate net output is equal to aggregate consumption ($\widehat{\alpha} = \sum_{\nu \in \mathcal{N}} c^\nu$), and aggregate labour used in production is equal to aggregate labour expended ($\alpha_l = \sum_{\nu \in \mathcal{N}} \Lambda^\nu$).

A feasible allocation $\boldsymbol{x}$ for E is called *balanced* if and only if it is realisable in competitive markets in that there exist a vector of commodity prices $p \geq \mathbf{0}$, a (scalar) wage rate $w \geqq 0$, and a (scalar) profit rate $r \geqq 0$ such that the production activity $\alpha \in P$ is the most profitable ($p\overline{\alpha} = (1+r)\, p\underline{\alpha} + w\alpha_l$ and $p\overline{\alpha}' \leqq (1+r)\, p\underline{\alpha}' + w\alpha_l'$ for

indispensable for the production of positive net output); and (ii) for any $c \in \mathbb{R}^n_+$, there exists $\alpha \in P$ such that $\widehat{\alpha} \geqq c$ (any non-negative vector of commodities can be produced as a net output).

[6]For simplicity, we assume A to be productive and indecomposable.

any $\alpha' = (-\alpha'_l, -\underline{\alpha}', \overline{\alpha}') \in P$); and every agent's consumption bundle c^ν is affordable ($pc^\nu = w\Lambda^\nu + rp\omega^\nu$ for all $\nu \in \mathcal{N}$).

Then, a pair of a balanced allocation $\boldsymbol{x}$ and its associated vector (p, w, r) is called an *equilibrium* if and only if no agent would (strictly) prefer any other balanced allocation at the price vector in E. For each economy $E \in \mathcal{E}$, let $\mathcal{S}_E$ be the set of equilibria in E.

In the rest of the paper, we shall write "For any $E \in \mathcal{E}$ and any $(p, w, r; \boldsymbol{x}) \in \mathcal{S}_E$" as a shorthand for "For any economy $E \langle \mathcal{N}, P, \boldsymbol{\omega}, \boldsymbol{l} \rangle \in \mathcal{E}$ and any equilibrium $(p, w, r; \boldsymbol{x}) \in \mathcal{S}_E$".

3 The concept of exploitation in Marxian economics

At the most general level, a *definition of exploitation* is a rule that specifies, in each $E \in \mathcal{E}$ and each $(p, w, r; \boldsymbol{x}) \in \mathcal{S}_E$, a partition of $\mathcal{N}$ into $\mathcal{N}^{ted}$, $\mathcal{N}^{ter}$, and $\mathcal{N}^{non}$, where $\mathcal{N}^{ted}$ is the set of *exploited agents*; $\mathcal{N}^{ter}$ is the set of *exploiters*; and $\mathcal{N}^{non}$ is the set of agents who are *neither exploited nor exploiters*. Different approaches to exploitation theory can be conceived of as different ways of specifying how the equilibria of capitalist economies map into partitions of the set of agents. For example, in the set of capitalist economies with uncoerced transactions and no government interference, a libertarian approach can be conceptualised as stipulating that $\mathcal{N}^{non} = \mathcal{N}$ at all $(p, w, r; \boldsymbol{x}) \in \mathcal{S}_E$.

A number of definitions of exploitation have been proposed in the literature on Marxian economics, and we shall briefly present the main ones in this section.

First, for any commodity bundle $c \in \mathbb{R}^n_+$, let $l.v.\,(c)$ denote the *minimum* amount of (effective) labour necessary to produce c as net output.[7] Morishima (1974) defines the exploitation status of each agent ν by focusing on the bundle actually purchased, c^ν, whose labour content is defined as $l.v.\,(c^\nu)$:[8]

Definition 1. [Morishima (1974)] For any $E \in \mathcal{E}$ and any $(p, w, r; \boldsymbol{x}) \in \mathcal{S}_E$, an agent $\nu \in \mathcal{N}$, who supplies Λ^ν and consumes c^ν, is *exploited* if and only if $\Lambda^\nu > l.v.\,(c^\nu)$ and *an exploiter* if and only if $\Lambda^\nu < l.v.\,(c^\nu)$.

Next, let $P(p, w, r) \equiv \left\{ \alpha \in P \mid \frac{p\widehat{\alpha} - rp\underline{\alpha} - w\alpha_l}{p\underline{\alpha}} = \max_{\alpha' \in P} \frac{p\widehat{\alpha}' - rp\underline{\alpha}' - w\alpha'_l}{p\underline{\alpha}'} \right\}$ be the set of activities that maximise the rate of return on capital at a price vector (p, w, r). Given $(p, w, r; \boldsymbol{x}) \in S_E$, for any commodity bundle $c \in \mathbb{R}^n_+$, let $l.v.\,(c; p, w, r)$ be the *minimum*

[7] Formally, $l.v.\,(c) \equiv \min\{\alpha_l \mid \alpha \in P, \widehat{\alpha} \geqq c\}$. Under mild assumptions on P, $l.v.\,(c)$ is unique, well-defined and positive whenever $c \neq \mathbf{0}$.

[8] For consistency, in Definition 1 exploitation status is defined at an equilibrium allocation. However, Definition 1 can be generalised to hold at any balanced allocation $\boldsymbol{x}$ for E.

amount of (effective) labour necessary to produce c as net output with a profit-rate-maximising activity $\alpha \in P(p, w, r)$.[9] Then:

Definition 2. [Roemer (1982)]: For any $E \in \mathcal{E}$ and any $(p, w, r; \boldsymbol{x}) \in \mathcal{S}_E$, an agent $\nu \in \mathcal{N}$, who supplies Λ^ν and consumes c^ν is *exploited* if and only if $\Lambda^\nu > l.v.\,(c^\nu; p, w, r)$ and *an exploiter* if and only if $\Lambda^\nu < l.v.\,(c^\nu; p, w, r)$.

Finally, given $(p, w, r; \boldsymbol{x}) \in S_E$, let $\alpha^{p,w,r} \in P$ be the *aggregate (profit-rate-maximising) production activity* at this equilibrium and let $\widehat{\alpha}^{p,w,r}$ be the corresponding vector of net output. For any $c \in \mathbb{R}^n_+$ with $pc \leqq p\widehat{\alpha}^{p,w,r}$, let $\tau^c \in [0, 1]$ be defined by $\tau^c \equiv \frac{pc}{p\widehat{\alpha}^{p,w,r}}$: τ^c is the cost of bundle c as a fraction of national income. Then:

Definition 3. [Duménil (1980); Foley (1982)] For any $E \in \mathcal{E}$ and any $(p, w, r; \boldsymbol{x}) \in \mathcal{S}_E$, an agent $\nu \in \mathcal{N}$, who supplies Λ^ν and consumes c^ν is *exploited* if and only if $\Lambda^\nu > \tau^{c^\nu} \alpha_l^{p,w,r}$ and *an exploiter* if and only if $\Lambda^\nu < \tau^{c^\nu} \alpha_l^{p,w,r}$.

All three definitions formalise a notion of exploitation as the unequal exchange of labour and incorporate some key intuitions of Marxian exploitation theory. But they have rather different properties, and implications. Definition 1 identifies exploitation status of each agent prior to and independent of price information, as in the standard Marxian approach, focusing only on production data. However, it is independent of the social relations of production in that it identifies exploitation status based on (possibly counterfactual) activities that need not, and may never be used by profit-maximising capitalists.

In contrast, by requiring that the labour content of the bundle actually bought by each agent should be determined focusing on profit-rate-maximising activities, Definition 2 identifies exploitation status based on production techniques that can be actually used under the capitalistic relations of production.

In Definition 3, social relations are even more prominent since exploitation is directly related to the production and distribution of national income and the distribution of social labour expended in actually used production activities. More precisely, the labour content of aggregate net output, $\widehat{\alpha}^{p,w,r}$, is equal to total social labour, $\alpha_l^{p,w,r}$, and the amount of labour contained in a given bundle c is equal to the fraction τ^c of social labour necessary to produce a fraction of aggregate net output, $\tau^c \widehat{\alpha}^{p,w,r}$.

This admittedly brief review of some of the most prominent definitions of UE exploitation shows that many different approaches can, and actually have been adopted to identify exploitation status in mathematical Marxian economics. These differences reflect different normative intuitions and will lead to different partitions of the set of agents. For example, Definitions 1-2 both focus on agents' actual consumption choices

[9] Formally, $l.v.\,(c; p, w, r) \equiv \min\{\alpha_l \mid \alpha \in P(p, w, r), \widehat{\alpha} \geqq c\}$.

and thereby make exploitation status dependent on idiosyncratic factors, such as individual preferences, whose relevance in Marxian theory is not obvious. Definition 3 abstracts from preferences but at the cost of identifying exploitation status based on a potentially counterfactual consumption bundle.

Two questions then naturally arise. First, can a common core of UEL theory be identified that is common to all of the main approaches and defines the domain of admissible formalisations of Marxian exploitation theory? Second, within this (potentially large) domain is there a way of identifying one, or a small class of definitions that capture some important Marxian intuitions? We survey the literature addressing the two questions in turn.

4 UE exploitation theory: an axiomatic approach

Yoshihara and Veneziani (2018) apply the axiomatic method to identify the domain of admissible UE approaches. "Domain axioms are routinely formulated and analysed in social choice theory and axiomatic bargaining theory. They do not represent full-fledged theories or definitions. Rather, they can be interpreted as meta-properties which usually identify the main object of research (e.g., a social welfare functional, a social welfare ordering, or an allocation mechanism), the space in which such object is analysed (e.g. welfare allocations, or economic environments with certain properties), and some foundational properties defining the set of admissible solutions (e.g. completeness, transitivity, or single-valuedness). Domain axioms thus delineate the basic perimeter of the theoretical exercise." (Yoshihara and Veneziani, 2018, p.383). No such axiom exists in exploitation theory and it is indeed unclear *a priori* what the object and space of the analysis are, and the information relevant to define exploitation has hardly been explicitly and systematically discussed.

The starting point of Yoshihara and Veneziani (2018) is the acknowledgement that there are in principle infinitely many ways of mapping economies and equilibria into partitions of the set $\mathcal{N}$, but only a subset of those will capture the fundamental intuitions of the Marxian theory of exploitation as the unequal exchange of labour. Yoshihara and Veneziani (2018) identify a list of properties that restrict the admissible mappings – i.e. the ways in which agents can be classified into exploiters, exploited, or neither. Their axioms thus identify some normatively relevant restrictions on the information that can be brought to bear in identifying the admissible definitions of exploitation.

The first axiom states the most foundational property, and basic principle, of UE theory, namely the idea that exploitation status depends on a mismatch between labour contributed, in some relevant sense, and labour received, in some relevant sense.

Axiom 1. ***(UE)*** *For any $E \in \mathcal{E}$ and any $(p, w, r; \boldsymbol{x}) \in \mathcal{S}_E$, for each $\nu \in \mathcal{N}$, there exist the upper and lower bounds, $L^\nu_{\min}, L^\nu_{\max} \in \mathbb{R}$, of the labour received by agent ν with $L^\nu_{\min} \leqq L^\nu_{\max}$, such that agent ν is an exploiter if and only if $\Lambda^\nu < L^\nu_{\min}$; and is exploited if and only if $\Lambda^\nu > L^\nu_{\max}$.*

Two observations concerning Axiom 1 should be made. First, the labour expended by an agent is presumed to be measured in terms of effective labour in this paper. This is the main approach in the literature and it corresponds to what Yoshihara and Veneziani (2018) have dubbed the *contribution view*, according to which exploitation theory captures an ethical notion of proportionality between contribution and reward whose philosophical foundations can be traced back to Aristotle in the *Nichomachean Ethics*, and it can be justified in terms of the Kantian categorical imperative according to Roemer (2010).[10]

Second, Axiom 1 stipulates the existence of upper and lower bounds of the labour received by each agent but it does not specify how they are determined. The following axioms impose some restrictions on the way in which the two bounds are determined.

Axiom 2 stipulates that the reference labour amounts $(L^\nu_{\min}, L^\nu_{\max})_{\nu \in \mathcal{N}}$ are associated with a profile of reference commodity bundles, called the *exploitation reference bundles* (ERBs).

Axiom 2. ***(ERBs)*** *For any $E \in \mathcal{E}$ and any $(p, w, r; \boldsymbol{x}) \in \mathcal{S}_E$, for each $\nu \in \mathcal{N}$, there exist $c^\nu_{\min}, c^\nu_{\max} \in \mathbb{R}^n_+$ and a function f^ν such that $L^\nu_{\min} = f^\nu(c^\nu_{\min})$ and $L^\nu_{\max} = f^\nu(c^\nu_{\max})$.*

Axiom 2 embodies what may be interpreted as a materialistic perspective in that the amount of labour received by agents depends on some reference bundles of goods.

For all $\nu \in \mathcal{N}$ and all $\Lambda^\nu \leqq l^\nu$, let $B(\omega^\nu, \Lambda^\nu; p, w, r)$ be the set of consumption bundles that agent ν can purchase at prices (p, w, r), if she supplies Λ^ν units of labour, given her endowment ω^ν.[11] Axiom 3 states that the ERBs are affordable via market exchanges at equilibrium prices.

Axiom 3. ***(Economic Feasibility of ERBs)*** *For any $E \in \mathcal{E}$ and any $(p, w, r; \boldsymbol{x}) \in \mathcal{S}_E$, for each $\nu \in \mathcal{N}$, $c^\nu_{\min}, c^\nu_{\max} \in B(\omega^\nu, \Lambda^\nu; p, w, r)$.*

By Axiom 3 the amount of labour received by agents must be linked to their purchasing power.

Axiom 4 stipulates that the ERBs are also producible as net outputs under the present economy with production techniques

[10] Alternatively, one may adopt the *well-being view* (Yoshihara and Veneziani, 2018) and measure the labour expended by agents in terms of labour *time*. From this viewpoint, UE exploitation captures some inequalities in the distribution of material well-being and free hours, where both of them are key determinants of *individual well-being freedom* (Rawls, 1971; Sen, 1985).

[11] Formally: $B(\omega^\nu, \Lambda^\nu; p, w, r) \equiv \{c^\nu \in \mathbb{R}^n_+ \mid pc^\nu = w\Lambda^\nu + rp\omega^\nu\}$.

Axiom 4. ***(Technological Feasibility of ERBs)*** *For any $E \in \mathcal{E}$ and any $(p, w, r; \boldsymbol{x}) \in \mathcal{S}_E$, for each $\nu \in \mathcal{N}$, there exist α'^{ν}, $\alpha''^{\nu} \in P$ such that $\widehat{\alpha}'^{\nu} \geqq c^{\nu}_{\min}$ and $\widehat{\alpha}''^{\nu} \geqq c^{\nu}_{\max}$.*

Axiom 4 incorporates the Marxian intuition that technological knowledge, and production conditions, are central in the determination of exploitation status.

While Axiom 4 requires the ERBs to be productively feasible, it imposes no constraints on the functional relation between the amount of labour received and the ERBs. Axiom 5 identifies the admissible class of functional relations:

Axiom 5. ***(Reference labour Amounts of ERBs)*** *For any $E \in \mathcal{E}$ and any $(p, w, r; \boldsymbol{x}) \in \mathcal{S}_E$, for each $\nu \in \mathcal{N}$, $L^{\nu}_{\min} = f^{\nu}(c^{\nu}_{\min})$ and $L^{\nu}_{\max} = f^{\nu}(c^{\nu}_{\max})$ hold if and only if there exist $\alpha^{\nu}_{\min}$, $\alpha^{\nu}_{\max} \in P$ such that $\widehat{\alpha}^{\nu}_{\min} \geqq c^{\nu}_{\min}$, $\widehat{\alpha}^{\nu}_{\max} \geqq c^{\nu}_{\max}$, $L^{\nu}_{\min} = \alpha^{\nu}_{l\min}$, and $L^{\nu}_{\max} = \alpha^{\nu}_{l\max}$.*

In other words, the upper and lower bounds of the labour received by each agent are the amounts of labour necessary for the production of the ERBs by some specific choices of production activities under the current production technology.

Axioms 1-5 capture the key intuitions of Marxian UE exploitation theory and rigorously state the intuitions behind Definitions 1-3 – and indeed all of the main definitions in the literature. From this perspective, Axioms 1-5 do not help select a particular approach: they identify the minimum common denominator of *all* UE approaches in the Marxian tradition and thus map the domain of admissible UE definitions. Yoshihara and Veneziani (2018) summarise them into the following domain axiom:

Labour Exploitation (LE): For any $E \in \mathcal{E}$ and any $(p, w, r; \boldsymbol{x}) \in \mathcal{S}_E$, for each $\nu \in \mathcal{N}$, there exist $c^{\nu}_{\min}, c^{\nu}_{\max} \in B(\omega^{\nu}, \Lambda^{\nu}; p, w, r)$ and $\alpha^{\nu}_{\min}$, $\alpha^{\nu}_{\max} \in P$ satisfying $\widehat{\alpha}^{\nu}_{\min} \geqq c^{\nu}_{\min}$, $\widehat{\alpha}^{\nu}_{\max} \geqq c^{\nu}_{\max}$, and $\alpha^{\nu}_{l\min} \leqq \alpha^{\nu}_{l\max}$ such that the following condition holds:

$$\begin{aligned} \nu &\in \mathcal{N}^{ted} \textit{ if and only if } \alpha^{\nu}_{l\max} < \Lambda^{\nu}, \\ \nu &\in \mathcal{N}^{ter} \textit{ if and only if } \alpha^{\nu}_{l\min} > \Lambda^{\nu}. \end{aligned}$$

LE requires that, for each economy and each equilibrium, the exploitation status of every agent ν is determined by the difference between the labour that ν 'contributes' to the economy, and the labour she 'receives'. Whereas the former quantity is given by the (effective) labour Λ^{ν} this agent supplies measured in skill-adjusted labour time, the labour she receives is determined by identifying two affordable and technically feasible bundles, $c^{\nu}_{\min}$, $c^{\nu}_{\max}$, *and* their labour content which is equal to the labour necessary to produce them as net output, $\alpha^{\nu}_{l\min}$, $\alpha^{\nu}_{l\max}$. The amount of labour that ν receives is the (possibly degenerate) interval $[\alpha^{\nu}_{l\min}, \alpha^{\nu}_{l\max}]$, and so, for any $\nu \in \mathcal{N}$, if Λ^{ν} is more (*resp.*, less) than $\alpha^{\nu}_{l\max}$ (*resp.*, $\alpha^{\nu}_{l\min}$) then ν is regarded as 'giving' more (*resp.*, less) labour than ν 'receives' and therefore a member of $\mathcal{N}^{ted}$ (*resp.*, $\mathcal{N}^{ter}$).

Proposition 1. *[Yoshihara and Veneziani (2018)]. A definition of exploitation satisfies* ***LE*** *if and only if it satisfies Axioms 1-5.*

With this characterisation, Yoshihara and Veneziani (2018) show that all of the main definitions proposed in the literature satisfy **LE**, and thus Axioms 1-5, thus confirming the relevance of the latter in identifying the domain of admissible definitions.

Corollary 1. ***[Yoshihara and Veneziani (2018)]*** *Definitions 1-3 satisfy* ***LE****.*

Proof. Definition 1: let $c^{\nu}_{\min} = c^{\nu}_{\max} = c^{\nu}$ and $\alpha^{\nu}_{\min} = \alpha^{\nu}_{\max} = \arg\min\{\alpha_l \mid \alpha \in P, \text{ with } \widehat{\alpha} \geqq c^{\nu}\}$.
Definition 2: let $c^{\nu}_{\min} = c^{\nu}_{\max} = c^{\nu}$ and $\alpha^{\nu}_{\min} = \alpha^{\nu}_{\max} = \arg\min\{\alpha_l \mid \alpha \in P(p,w,r), \text{ with } \widehat{\alpha} \geqq c^{\nu}\}$.
Definition 3: let $c^{\nu}_{\min} = c^{\nu}_{\max} = \tau^{c^{\nu}}\widehat{\alpha}^{p,w,r}$ and $\alpha^{\nu}_{\min} = \alpha^{\nu}_{\max} = \tau^{c^{\nu}}\alpha^{p,w,r}_{l}$. □

In the rest of the paper we shall use the expression "definition of labour exploitation" as a shorthand to identify the subset of definitions that satisfy **LE**.

5 UE Exploitation: two characterisations

While **LE** is far from vacuous or trivial, and a number of approaches – such as libertarianism, or Roemer's property rights definition (Roemer, 1982) – are ruled out, there are in principle many definitions in the relevant domain – many definitions of labour exploitation. "The fundamental question is how to choose among all of the existing and the conceivable definitions. Thus far, the debate has largely been reactive: new definitions have often emerged as the product of a process of adjustment of the theory to various anomalies and counterexamples identified in the literature" (Veneziani and Yoshihara, 2017a, p.1609).

Two important insights of Marxian economics have been central in debates on the appropriate definition of Marxian exploitation: one is the so-called *Fundamental Marxian Theorem* (FMT), which was originally proved by Okishio (1963); the other is known as the *Class-Exploitation Correspondence Principle* (CECP) originally examined by Roemer (1982).

The FMT shows that a capitalist economy is profitable if and only if the (average) rate of exploitation of the working class by the capitalist class is positive, thus proving that the surplus value appropriated in the capitalist production process is the only source of positive profits. The CECP verifies another classical Marxian claim: every member of the capitalist class is an exploiter while every member of the working class is exploited.[12]

[12] In equilibrium, class membership and exploitation status emerge endogenously: the wealthy can rationally choose to belong to the capitalist class among other available options and become an exploiter, while the poor have no other option than being in the working class and are exploited.

While the FMT and the CECP have originally been proved as *results* in the literature, their epistemological status has been that of a *postulate*: alternative definitions have been evaluated in terms of their ability to preserve their validity. Typically, whenever a counterexample has been found showing that, in a given class of economies, or for a certain set of allocations, either the FMT or the CECP did not hold for a given definition of exploitation, an alternative definition has been proposed under which the FMT and/or the CECP would hold. In other words, the validity of each form of exploitation has been tested by the robustness of the equivalence between exploitation and positive profits, and exploitation and class status. However, this process of exploration of the domain of possible definitions by trial-and-error is both inefficient, because it may involve an infinite repetition of counterexamples and new proposals, and relatively uninformative, because each step sheds only *some* light on the properties of different definitions.

Yoshihara (2010) and Veneziani and Yoshihara (2015, 2017a,b) have turned the procedure on its head and formulated the two basic Marxian claims as axioms that should be satisfied by any definition of exploitation. "Rather than proposing another definition, and comparing it with the existing alternatives, we develop an axiomatic framework to analyse what exploitation is, and how it should be measured. The axiomatic method is used to rigorously and explicitly state the normative and positive foundations of the notion of exploitation" (Veneziani and Yoshihara, 2017a, pp.1609-1610).

5.1 Profit-Exploitation Correspondence Principle

Given an economy $E \in \mathcal{E}$, an agent $\nu \in \mathcal{N}$ is *propertyless* if and only if $\omega^\nu = \mathbf{0}$. Moreover, given an equilibrium $(p, w, r; \boldsymbol{x})$ in the economy E, a propertyless agent $\nu \in \mathcal{N}$ is *employed* if and only if $\Lambda^\nu > 0$. Then, following Veneziani and Yoshihara (2015, 2017a), the key intuitions of the FMT can be captured by the following axiom:

Profit-Exploitation Correspondence Principle (PECP): For any $E \in \mathcal{E}$ and any $(p, w, r; \boldsymbol{x}) \in \mathcal{S}_E$:

$$r > 0 \Leftrightarrow \text{every propertyless employed agent is exploited,}$$

in terms of a given definition of exploitation.

The **PECP** states that a given definition of exploitation is appropriate only if it captures the existence of a mechanism by which for any economy and any equilibrium, (part of) the productive fruits of the exploited are transferred to exploiters. In perfectly competitive markets, neglecting the issue of rent, net outputs (the aggregate value added) are distributed into wage income and profit income. Moreover, every party receives an equal

wage per unit of (effective) labour. Therefore, the appropriation of more of the productive fruits by exploiters must be explained as a source of profits.

The correspondence between profits and exploitation is required to hold for a large class of economies which include fixed capital, joint production, and multiple activities but **PECP** *per se* is not very strong. For it imposes no constraints on the exploitation status of the unemployed, and it allows for the possibility of *some* propertyless employees being exploited in an equilibrium with zero profit.[13]

Veneziani and Yoshihara (2015, 2017a) have identified a necessary and sufficient condition for **PECP**:

Proposition 2. *[Veneziani and Yoshihara (2015, 2017a)] For any definition of labour exploitation, the following two statements are equivalent:*
(1) PECP holds;
(2) for any $E \in \mathcal{E}$ and any $(p, w, r; \boldsymbol{x}) \in \mathcal{S}_E$, if $r > 0$, then for any propertyless employed agent $\nu \in \mathcal{N}$, there exists an activity $\alpha_\pi^\nu \in P$ with $\alpha_{\pi l}^\nu = \Lambda^\nu$ such that $\widehat{\alpha}_\pi^\nu \in \mathbb{R}_+^n$, $p\widehat{\alpha}_\pi^\nu > w\Lambda^\nu$, and $(\alpha_{l\pi}^\nu, \underline{\alpha}_\pi^\nu, \overline{\alpha}_\pi^\nu) \geqq \eta^\nu \left(\alpha_{l\,\max}^\nu, \underline{\alpha}_{\max}^\nu, \overline{\alpha}_{\max}^\nu\right)$ for some $\eta^\nu > 1$.

Condition (2) states that if the equilibrium rate of profit is positive, then by spending exactly the same amount of labour, and using the appropriate amount of capital, each propertyless employee $\nu \in \mathcal{N}$ *could* in principle activate a (counterfactual) production process which yields a net revenue higher than ν's earnings.[14] Because $\Lambda^\nu = \alpha_{\pi l}^\nu > \alpha_{l\,\max}^\nu$, agent ν is exploited at this equilibrium, according to the given definition satisfying **LE**.[15]

Proposition 2 provides a demarcation line (condition (2)) by which one can test which of infinitely many potential definitions preserves the essential relation of exploitation and profits in capitalist economies. Thus, if a definition of exploitation satisfying **LE** does not meet condition (2), then it will not satisfy **PECP**, which implies that it is not a proper definition of UE exploitation.

With this characterisation of **PECP**, Veneziani and Yoshihara (2015, 2017a) examine which of the main definitions proposed in the literature passes the test of **PECP**. They show that, among Definitions 1-3, only Definition 3 passes this test.

To see that Definition 3 satisfies **PECP** recall that $\alpha_{\max}^\nu = \tau^{c^\nu} \alpha^{p,w,r}$. Then, let $\alpha_\pi^\nu \equiv \frac{\Lambda^\nu}{\alpha_l^{p,w,r}} \alpha^{p,w,r}$. As $\tau^{c^\nu} = \frac{pc^\nu}{p\widehat{\alpha}^{p,w,r}}$ by definition, $pc^\nu = w\Lambda^\nu$ for any propertyless employed

[13] In this respect, **PECP** is weaker than the FMT, which implies that no propertyless employee is exploited at an equilibrium with zero profit. In contrast, however, while the FMT focuses on the rate of exploitation for the whole working class is positive, **PECP** restricts the exploitation status of *every* propertyless worker, making it a stronger claim than the FMT.

[14] This counterfactual production activity is a proportional expansion of the activity $\alpha_{\max}^\nu$.

[15] In other words, through a reorganisation of the social relations of production, exploited workers could increase their income vis-à-vis the current mode of organisation.

agent ν in equilibrium, and $p\widehat{\alpha}^{p,w,r} - w\alpha_l^{p,w,r} > 0$ holds by $r > 0$, it follows that $\frac{\Lambda^\nu}{\alpha_l^{p,w,r}} > \frac{pc^\nu}{p\widehat{\alpha}^{p,w,r}} = \tau^{c^\nu}$ for any propertyless employed agent ν. Therefore, Definition 3 satisfies condition (2) of Proposition 2.

To see why the other definitions do not pass the test, consider the following example.

Example 1. Let $n = 2$. Assume there are two production processes, and let

$$A = \begin{bmatrix} 1 & 1.5 \\ 1 & 0.5 \end{bmatrix}, \; B = \begin{bmatrix} 2 & 3.5 \\ 2 & 1.5 \end{bmatrix}, \; L = (1, 1).$$

As $B - A = \begin{bmatrix} 1 & 2 \\ 1 & 1 \end{bmatrix}$ and $L = (1, 1)$, process 2 is superior to process 1 in that it produces 2 units of good 1 and one unit of good 2 as net output per unit of labour input, compared with one unit of each commodity.

Let $\omega = \begin{bmatrix} 1.5 \\ 0.5 \end{bmatrix}$ be the social endowment of capital goods. Assume that every agent needs to consume a bundle $c^* = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ in order to supply one unit of labour. Suppose there is one propertyless agent and one capitalist who holds ω and employs the propertyless agent in order to implement profit maximising production activity by investing ω.

It is not difficult to show that the vector $((p^*, w^*, r^*), \alpha^*)$ with $p_1^* = 0.5 = p_2^*, w^* = 1$ and $\alpha^* \equiv (-1, -(1.5, 0.5), (3.5, 1.5))$ is an equilibrium, in which $r^* = 0.5$, only process 2 is operated, and the propertyless agent earns one unit of wage revenue and purchases c^*, while the capitalist earns 0.5 unit of profit revenue and purchases one unit of commodity 1 for her consumption. In this economy,

$$l.v.(c^*) = 1 = l.v.(c^*; p^*, w^*, r^*)$$

holds, which implies that the propertyless employed agent is not exploited in terms of Definitions 1-2, though $r^* > 0$. Therefore, none of Definitions 1-2 satisfies **PECP**. □

A different conclusion is reached, however, if attention is restricted to the subset of Leontief economies, in which the equivalence between positive profits and exploitation of each propertyless employee holds for *any* definition of exploitation satisfying **LE**.

Proposition 3. ***[Veneziani and Yoshihara (2015)]:*** *For any $E \in \mathcal{E}_\mathcal{L}$ and any $(p, w, r; \boldsymbol{x}) \in \mathcal{S}_E$, $r > 0$ if and only if every propertyless employed agent is exploited, for any definition of labour exploitation.*

5.2 Class-Exploitation Correspondence Principle

We now introduce some additional information about each agent's equilibrium actions. Let $(\alpha^\nu, \beta^\nu, \gamma^\nu) \in P \times P \times [0, l^\nu]$ denote a profile of production actions, where $\alpha^\nu \in P$

represents a production activity operated by agent ν using her wealth $p\omega^\nu$ and employing her own labour; $\beta^\nu \in P$ represents a production activity operated by ν using her wealth $p\omega^\nu$ to hire other agents; and γ^ν is the amount of labour that agent ν sells in the labour market. By the definition of equilibrium, $\sum_{\nu\in\mathcal{N}} \alpha^\nu + \sum_{\nu\in\mathcal{N}} \beta^\nu = \alpha^{p,w,r}$; $\alpha_l^\nu + \gamma^\nu = \Lambda^\nu$ for any $\nu \in \mathcal{N}$, which in turn imply $\sum_{\nu\in\mathcal{N}} \beta_l^\nu = \sum_{\nu\in\mathcal{N}} \gamma^\nu$ in equilibrium.[16] Let $\boldsymbol{\xi} \equiv (\alpha^\nu, \beta^\nu, \gamma^\nu)_{\nu\in\mathcal{N}}$. With a slight abuse of notation we shall write an equilibrium as $(p, w, r; \boldsymbol{x}; \boldsymbol{\xi}) \in \mathcal{S}_E$.

In this subsection, without loss of generality, we focus on a special case of economies with $\bar{l} = 1$ and $s^\nu = 1$, for each $\nu \in \mathcal{N}$ and assume that agents only care about consumption. In this case, for any $(p, w, r; \boldsymbol{x}; \boldsymbol{\xi}) \in \mathcal{S}_E$, $\Lambda^\nu = \lambda^\nu = 1$ holds for all $\nu \in \mathcal{N}$. Then, $pc^\nu = rp\omega^\nu + w$ holds for any $\nu \in \mathcal{N}$ in equilibrium.

According to Roemer (1982), classes in a capitalist economy can be defined as follows:

Definition 4. [Roemer (1982, Chapter 5)] For any $E \in \mathcal{E}$ and any $(p, w, r; \boldsymbol{x}; \boldsymbol{\xi}) \in \mathcal{S}_E$, agent ν belongs to:

- a subset C^H of $\mathcal{N}$ if and only if $\alpha^\nu \geq \mathbf{0}, \beta^\nu \geq \mathbf{0}, \gamma^\nu = 0$;
- a subset C^{PB} of $\mathcal{N}$ if and only if $\alpha^\nu \geq \mathbf{0}, \beta^\nu = \mathbf{0}, \gamma^\nu = 0$;
- a subset C^S of $\mathcal{N}$ if and only if $\alpha^\nu \geq \mathbf{0}, \beta^\nu = \mathbf{0}, \gamma^\nu > 0$;
- a subset C^P of $\mathcal{N}$ if and only if $\alpha^\nu = \mathbf{0}, \beta^\nu = \mathbf{0}, \gamma^\nu > 0$.

The set C^H represents the capitalist class, since in equilibrium, its members optimally choose to employ others by investing their own capital; C^{PB} represents the middle class, since its members optimally choose to be self-employed by investing their own capital; C^S represents the class of part-time workers, since its members optimally choose to spend some of their time as self-employed producers, and some of their time as workers employed by others; C^P represents the proletariat, since its members can only choose to work for others.

Roemer (1982) proves that the equilibrium class structure of a capitalist economy is determined by the unequal private ownership of capital assets. To see this, for any $(p, w, r; \boldsymbol{x}; \boldsymbol{\xi}) \in \mathcal{S}_E$, recall that $P(p, w, r)$ is the set of activities which achieve the maximal profit rate r at equilibrium prices. Let us call such activities *efficient*. Moreover, let $\alpha \in P(p, w, r)$ be an *efficient unit-activity* if $\alpha_l = 1$.

Let an efficient activity α be *at least as capital-intensive as* (resp. *more capital-intensive than*) another efficient activity α' if and only if $\frac{p\alpha}{\alpha_l} \geqq \frac{p\alpha'}{\alpha'_l}$ (resp. $\frac{p\alpha}{\alpha_l} > \frac{p\alpha'}{\alpha'_l}$). Equivalently, α' is *no more* (resp. *less*) *capital-intensive than* α. Then, there exist two

[16] To see this, note that in equilibrium $\sum_{\nu\in\mathcal{N}} \alpha_l^\nu + \sum_{\nu\in\mathcal{N}} \beta_l^\nu = \sum_{\nu\in\mathcal{N}} \Lambda^\nu = \sum_{\nu\in\mathcal{N}} \alpha_l^\nu + \sum_{\nu\in\mathcal{N}} \gamma^\nu$.

efficient unit-activities, $\alpha^{\max}$ and $\alpha^{\min}$, that are, respectively, at least as capital-intensive as and no more capital-intensive than every other efficient activity.[17]

Lemma 1 identifies a correspondence between class status and wealth in equilibrium.

Lemma 1. ***(Wealth-Class Correspondence) [Roemer (1982, Chapter 5)]*** *For any $E \in \mathcal{E}$ and any $(p, w, r; \boldsymbol{x}; \boldsymbol{\xi}) \in \mathcal{S}_E$, for every $\nu \in \mathcal{N}$:*
(i) $\nu \in C^H \Leftrightarrow p\underline{\alpha}^{\max} < p\omega^\nu$;
(ii) $\nu \in C^{PB} \Leftrightarrow p\underline{\alpha}^{\min} \leqq p\omega^\nu \leqq p\underline{\alpha}^{\max}$;
(iii) $\nu \in C^S \Leftrightarrow 0 < p\omega^\nu < p\underline{\alpha}^{\min}$;
(iv) $\nu \in C^P \Leftrightarrow p\omega^\nu = 0$.

Members of the capitalist class C^H are the richest agent, as their equilibrium wealth is greater than the upper bound $p\underline{\alpha}^{\max}$ which is the value of the inputs necessary to activate the most capital intensive technique. They are so rich that, even if they activated the most capital intensive technique, worked the whole day as self employed producers, they could not use up all their wealth. In order to optimise, in equilibrium they must hire other agents because $p\underline{\alpha}^{\max} < p\omega^\nu$ implies that to fully activate their assets, more than one unit of labour (the agents' endowment, by assumption) is required. A similar argument can be applied to each agent in the other classes.

Following Yoshihara (2010) and Veneziani and Yoshihara (2017b), the CECP can be formalised as an axiom.

Class-Exploitation Correspondence Principle (CECP): For any $E \in \mathcal{E}$ and any $(p, w, r; \boldsymbol{x}; \boldsymbol{\xi}) \in \mathcal{S}_E$ with $r > 0$, for every $\nu \in \mathcal{N}$:

$$\nu \in C^H \Rightarrow \nu \in \mathcal{N}^{ter};$$
$$\nu \in C^S \cup C^P \Rightarrow \nu \in \mathcal{N}^{ted},$$

in terms of a given definition of exploitation.

The **CECP** states that a correspondence between the class and exploitation structure should exists in every economy and at each equilibrium with a positive profit rate: every agent in the capitalist class must be an exploiter, while every agent in the working class must be exploited.

The next proposition derives a necessary and sufficient condition for **CECP** to hold.

Proposition 4. ***[Yoshihara (2010); Veneziani and Yoshihara (2017b)]*** *For any definition of labour exploitation, the following two statements are equivalent:*
(1) ***CECP*** *holds;*

[17] Formally, $\alpha^{\max} \in \arg\max_{\alpha \in P(p,w,r);\ \alpha_l = 1} p\underline{\alpha}$ and $\alpha^{\min} \in \arg\min_{\alpha \in P(p,w,r);\ \alpha_l = 1} p\underline{\alpha}$.

(2) For any $E \in \mathcal{E}$ and any $(p, w, r; \boldsymbol{x}; \boldsymbol{\xi}) \in \mathcal{S}_E$, there exist at most two bundles $\bar{c}_\pi, \underline{c}_\pi \in \mathbb{R}^n_+$ and associated activities $\alpha^{\bar{c}_\pi}, \alpha^{\underline{c}_\pi} \in P$ such that $\widehat{\alpha}^{\bar{c}_\pi} \geqq \bar{c}_\pi$, $\widehat{\alpha}^{\underline{c}_\pi} \geqq \underline{c}_\pi$, $\alpha_l^{\bar{c}_\pi} = 1 = \alpha_l^{\underline{c}_\pi}$ and $rp\underline{\alpha}^{\max} + w \geqq p\bar{c}_\pi \geqq p\underline{c}_\pi \geqq rp\underline{\alpha}^{\min} + w$. Moreover, for any $\nu \in \mathcal{N}$:

$$\begin{aligned} pc^\nu_{\min} &> p\bar{c}_\pi \Rightarrow \nu \in \mathcal{N}^{ter}; \\ pc^\nu_{\max} &< p\underline{c}_\pi \Rightarrow \nu \in \mathcal{N}^{ted}. \end{aligned}$$

Condition (2) states that there are two commodity bundles $\bar{c}_\pi, \underline{c}_\pi$ which can be produced as net outputs by operating some unit-activities $\alpha^{\bar{c}_\pi}, \alpha^{\underline{c}_\pi}$ and whose value is bounded above by $rp\underline{\alpha}^{\max} + w$ and below by $rp\underline{\alpha}^{\min} + w$. While there may be many bundles with these properties, condition (2) requires that the sets of exploiters and exploited agents should be characterised by choosing at most two such bundles, and an agent ν is an exploiter (resp. exploited) if and only if the cost of her ERB $c^\nu_{\min}$ (resp. $c^\nu_{\max}$) is higher (resp. lower) than the cost of these bundles.[18]

The intuition behind Proposition 4 is simple: noting that agents are assumed to work the same amount of time, and have the same skills, both class and exploitation status are determined by an agent's income, which is in turn determined by wealth. By Lemma 1, there exist at most two cut-off levels of an agent's income/wealth that determine their class. By **LE**, there exist at most two cut-offs which determine their exploitation status. Proposition 4 says that such cut-offs must be linked as specified in condition (2) for the **CECP** to hold.

Yoshihara (2010) and Veneziani and Yoshihara (2017b) use Proposition 4 to examine which of the main definitions proposed in the literature satisfies **CECP**, and prove again that only Definition -3 passes this test.

Example 1 shows that Definitions 1-2 do not satisfy **CECP**, since a propertyless employed agent is shown to be not exploited in terms of either definition. To see that Definition 3 satisfies **CECP**, let $\bar{c}_\pi = \frac{\widehat{\alpha}^{p,w,r}}{N} = \underline{c}_\pi$. It is then immediate to verify that condition (2) of Proposition 4 is satisfied.

However, again, a different conclusion is reached if one focuses on the class of Leontief economies.

Proposition 5. ***[Yoshihara (2010)]*** *Within $\mathcal{E}_\mathcal{L}$, **CECP** holds for any definition of exploitation satisfying **LE**.*

5.3 Is it all about technology?

The results surveyed in this section characterise the definitions of labour exploitation that satisfy two important properties of Marxian exploitation theory – the **PECP** and

[18] In other words, by **LE**, for each $\nu \in \mathcal{N}$, $pc^\nu_{\min} > p\bar{c}_\pi$ if and only if $\alpha^\nu_{l\,\min} > 1$; and $pc^\nu_{\max} < p\underline{c}_\pi$ if and only if $\alpha^\nu_{l\,\max} < 1$.

the **CECP**. Among all of the main definitions proposed in the literature, only Definition 3 satisfy both axioms in general.

However, the difference between the New Interpretation and the other approaches emerges *only when* general production economies with complex production technology are considered. Indeed, as shown by Propositions 3 and 5, every definition of exploitation satisfying **LE** passes the tests of both **PECP** and **CECP** within the class of simple Leontief economies.

To be sure, the validity of a definition of exploitation should not critically hinge on the complexity of production technology, and the exploitation status of agents should not depend on the presence of such features as the existence of fixed capital, multiple techniques, and joint production. Therefore, the failure to satisfy **PECP** and **CECP** casts significant doubts on Definitions 1-2.

Nonetheless, another interpretation may also be possible. That is, the problems of Definitions 1-2 do not arise from the complexity of the production technology. Rather, they may be due to the failure of well-defined individual labour values, as discussed in the next section.

6 Exploitation and the Labour Theory of Value

As mentioned in the Introduction, the classical Marxian literature defined the operational measure of UE by means of the notion of *labour values*. This classical view came into question in the 1970s when Steedman (1975) and Morishima (1973, 1974) showed that standard labour values (and even surplus value) may be negative in economies with joint production, casting serious doubts on the additive model of labour valuation. Morishima (1974) argued that UE exploitation could be defined without relying on individual commodities' labour values. From this perspective, in general economies, the formal definition of labour value of each individual commodity is not only generally impossible, but also unnecessary. Since then, all of the main UE approaches in the literature – including those surveyed in the previous sections – have been independent of individual commodities' labour values.

Recently, however, Yoshihara (2021, 2023) has suggested that the notion of individual labour values may be relevant to identify the appropriate definitions of UE exploitation. Even when a general technology is considered, a definition of exploitation satisfying **LE** satisfies **PECP** whenever the labour values of all goods are well-defined and the amount of labour received by agents is defined by means of such individual labour values.

6.1 Individual Labour Values

In what follows, assume $n = 2$ for the sake of simplicity. Consider a von Neumann production possibility set $P_{(A,B,L)}$. Then, $\left(A^{(ij)}, B^{(ij)}, L^{(ij)}\right) \equiv ((A_i, A_j), (B_i, B_j), (L_i, L_j))$ is called (i,j)-*von Neumann technique* if and only if it is a combination of the technique of the i-th process, (A_i, B_i, L_i), and the technique of the j-th process, (A_j, B_j, L_j), where $i, j = 1, \ldots, m$ with $i \neq j$, such that there exists a strictly positive commodity bundle $c \in \mathbb{R}^2_{++}$ satisfying $\left(B^{(ij)} - A^{(ij)}\right) y \geqq c$ for some vector $y \in \mathbb{R}^2_+$. An (i,j)-von Neumann technique is called *all-productive* if and only if for any non-negative $c \in \mathbb{R}^2_+$, there exists a vector $y \in \mathbb{R}^2_+$ such that $\left(B^{(ij)} - A^{(ij)}\right) y = c$ holds: if a technique is all-productive, then, for each commodity, it is possible to produce a net output consisting of a unit of that commodity alone with a nonnegative intensity vector (Kurz and Salvadori, 1995, pp.238-9). Let $\mathcal{T}_{(A,B,L)} \equiv \left\{\left(A^{(ij)}, B^{(ij)}, L^{(ij)}\right) \mid i, j = 1, \ldots, m \text{ with } i \neq j\right\}$ be the set of all von Neumann techniques derived from $P_{(A,B,L)}$. The *set of all von Neumann techniques is all-productive* if and only if every von Neumann technique available in this set is all-productive.

While it is *mathematically* stringent, all productiveness is arguably reasonable from an *economic* viewpoint. For instance, if all of the 'jointly produced outputs' in every von Neumann technique come from the presence of one-period-depreciated fixed capital goods at the end of each production period, then such techniques can be all-productive, as argued by Yoshihara (2021).

Assuming that $\mathcal{T}_{(A,B,L)}$ is all-productive, Yoshihara (2021) defines labour values of individual commodities for a technique $\left(A^{(ij)}, B^{(ij)}, L^{(ij)}\right) \in \mathcal{T}_{(A,B,L)}$ as follows:

$$v^{(ij)} = L^{(ij)}(B^{(ij)} - A^{(ij)})^{-1} > \mathbf{0}.$$

As $\mathcal{T}_{(A,B,L)}$ is all-productive, $v^{(ij)}$ exists and is well-defined for any (i,j)-von Neumann technique in $\mathcal{T}_{(A,B,L)}$. Therefore, if $\left(A^{(p,w,r)}, B^{(p,w,r)}, L^{(p,w,r)}\right) \in \mathcal{T}_{(A,B,L)}$ is optimally chosen according to the profit-rate maximisation at an equilibrium $(p, w, r; \boldsymbol{x})$, then the vector of labour values associated with this technique $v^{(p,w,r)}$ is unique, well-defined and strictly positive. A formal definition of UE-exploitation can then be proposed based on it.

6.2 PECP in All-Productive von Neumann Economies

Let $\mathcal{E}_{vN} \equiv \left\{E\left\langle \mathcal{N}, P_{(A,B,L)}, \boldsymbol{\omega}, \boldsymbol{l}\right\rangle \in \mathcal{E} \mid \mathcal{T}_{(A,B,L)} \text{ is all-productive}\right\}$ be the set of economies with all-productive von Neumann techniques. In this section, we characterise the set of admissible definitions of exploitation which satisfies **PECP** within $\mathcal{E}_{vN}$ by means of the following axiom which was originally introduced by Yoshihara (2023) in economies with multiple Leontief techniques – a strict subset of $\mathcal{E}_{vN}$.

Labour Value Theory of Exploitation (LVE): Consider a definition of labour exploitation. For any $E \in \mathcal{E}_{vN}$ and any $(p, w, r; \boldsymbol{x}) \in \mathcal{S}_E$, $\alpha^{\nu}_{l\,\max} \leqq v^{(p,w,r)} c^{\nu}_{\max}$ holds for any propertyless employed agent $\nu \in \mathcal{N}$.

LVE requires that any admissible definition of exploitation should measure the labour content received by propertyless agents according to the labour theory of value. That is, the socially necessary labour time for the reproduction of labour power should be evaluated by means of a properly defined vector of labour values.

Given an equilibrium $(p, w, r; \boldsymbol{x}) \in \mathcal{S}_E$, the labour value vector $v^{(p,w,r)}$ based on the competitively chosen technique $\left(A^{(p,w,r)}, B^{(p,w,r)}, L^{(p,w,r)}\right) \in \mathcal{T}_{(A,B,L)}$. In economies with a single Leontief technique, $v^{(p,w,r)}$ reduces to the standard vector of labour values. If, however, there exist multiple Leontief or all-productive von Neumann techniques, the definition of labour values is not obvious and the vector $v^{(p,w,r)}$ would not necessarily be adopted as the formulation of labour values.[19] Given this indeterminacy, **LVE** identifies the upper bound of admissible forms of labour values by means of $v^{(p,w,r)}$.

This upper bound is weak enough to allow for a rather broad class of admissible labour theories of value, in that there are infinitely many possible definitions of labour values – and definitions of labour exploitation – satisfying this constraint. Indeed, all of Definitions 1-3 satisfy **LVE**.

The next result generalises Theorem 2 in Yoshihara (2023), and proves that **PECP** holds within $\mathcal{E}_{vN}$, for any definition of labour exploitation that satisfies **LVE**.[20]

Theorem 1. *Consider a definition of labour exploitation. If this definition of exploitation satisfies **LVE**, then **PECP** holds over $\mathcal{E}_{vN}$.*

As Definitions 1-2 satisfy **LVE** within $\mathcal{E}_{vN}$, Theorem 1 implies that the significant contrast between the New Interpretation definition and the others in terms of **PECP** disappears within $\mathcal{E}_{vN}$ even if the complexity of production technology still prevails. This may suggest that the real reason why Definitions 1-2 fail to preserve **PECP** would be that the desired performance of these as the proper measure of exploitation crucially hinges on the well-defined individual labour values. In other words, the New Interpretation can serve as the proper measure of UE exploitation by coherently preserving a basic Marxian intuition, even outside of $\mathcal{E}_{vN}$, that is in general economies in which individual labour values are not necessarily well-defined.[21]

[19] For instance, Morishima (1974) focuses on "*optimum values*" which are the minimizer of the labour expenditure, as in Definition 1, in contrast with the vector $v^{(p,w,r)}$, which he calls the "actual values".

[20] The demonstration of Theorem 1 can be found in Appendix A.2.

[21] Similar conclusions hold for Definition 5 in Appendix A.1 which also satisfies **LVE**.

7 Concluding Remarks

We have surveyed recent contributions in the axiomatic literature on Marxian exploitation theory, focusing on the issue of the appropriate measurement of the unequal exchange of labour. In closing this paper, it is worth commenting on the other foundational aspect of exploitative relations, namely the *asymmetric power structure in the capitalist-worker relationship* (Wright, 2000; Vrousalis, 2013), which we have not addressed and on which the above results shed little light.

Such a power structure is typically viewed in Marxian literature as originating from capitalist relations of production, and the following famous remark by Weber (1978) well-represents the features of that structure: "The formal right of a worker to enter into any contract whatsoever with any employer whatsoever does not in practice represent for the employment seeker even the slightest freedom in the determination of his own conditions of work, and it does not guarantee him any influence on the process. It rather means, at least primarily, that the more powerful party in the market, i.e., normally the employer, has the possibility to set the terms, to offer the job 'take it or leave it,' and, given the normally more pressing economic need of the worker, to impose his terms upon him." Sociology also considers other dimensions of power, especially for understanding the differential class-positioning of different subgroups of workers.[22]

In contrast, what we have a main interest in here is the power dimension primarily linked to the studies of the mechanism which persistently engenders exploitative relations in capitalist economies. More concretely speaking, it should provide any relevant information about how an equilibrium wage rate in competitive markets is determined in order to extract surplus labour.

This is a non-minor point. First, it has long been shown that market competition *alone* is insufficient to allow for the extraction of surplus labour (Veneziani, 2007, 2013). For one, accumulation tends to make capital abundant, leading profits to disappear in the long run. In the classical Marxian literature, capitalists are assumed to react by introducing *capital-using and labour-saving* innovations, thus replenishing the industrial reserve army and restoring capital scarcity. This is possible because capitalists have effective decision-making powers on their firms' production and investments, while workers are essentially powerless. (This classical intuition has been formalised in a computational framework by

[22]To do so, Bourdieu (1983, 2010), for example, extended the concept of capital to include also *cultural* and *social capital*, which influence economic prospects, just as do their holdings of *economic capital*. Also, the power of normativity is relevant, in that social action generally depends upon consensus to be effective, which operates *in part* independently of the relative economic, cultural and social positions of the actors involved. In this context, the communicative dimension in which such norms are affirmed, contested and ultimately transformed is given compelling treatment, in the work of Tomaskovic-Devey et al (2019).

Cogliano et al (2016, 2019, 2023).)

However, as Okishio (1961) suggested, capitalists introduce cost-reducing innovations in order to appropriate extra-profits, not to increase aggregate unemployment. But then, as shown by Yoshihara and Veneziani (2023b), the decisive power of each capitalist to adopt a new technique cannot *by itself* guarantee the persistence of exploitation, as many cost-reducing innovations adopted by the capitalist class may cause the equilibrium profit rate to fall – for example, when *capital-saving and labour-using* innovations are universally adopted, while others may lead to *indeterminacy of the equilibrium distribution between wages and profits*.

Second, and related, the equilibrium distribution is actually *generically indeterminate*, in that a one-dimensional *continuum of equilibria* generically emerges as the result of perfect competition in capitalist markets. This indeterminacy was originally pointed out by Sraffa (1960) in a simple classical framework and has recently been extended by Mandler (1999) and Yoshihara and Kwak (2023a,b) to the standard Walrasian framework. It implies that, even when both labour and capital are fully employed, market competition *alone* cannot identify a single equilibrium point on the so-called *wage-profit frontier*: either the wage or the profit rate must be determined by some *out-of-market mechanism*.

As positive profits have prevailed in actual capitalist economies even in periods of near-full employment, as in the post-war era, there must be some non-market mechanism associated with the asymmetric power relations between the capitalists and the workers, which keeps wages persistently in check. It would be desirable to develop a general theory to predict the equilibrium distribution without relying on any institutional specification of the non-market mechanism.[23][24] We believe that this is a promising line for future research in exploitation theory taking into account both the distributive and the power dimension of exploitative relations.

[23]Some preliminary steps in this direction have been taken by Cogliano et al (2016) and Yoshihara and Kaneko (2016) within a cooperative bargaining framework in which unequal asset holdings create power asymmetries.

[24]An alternative framework to analyse asymmetries in bargaining power, and their distributional implications, is given by the celebrated contested exchange model (Bowles and Gintis, 1990). In this model, however, power derives from the principal-agent relationship associated with the asymmetric information structure involved in industrial firms, rather than the class structure in the capitalist society associated with unequal private ownership of capital assets. Moreover, the wage-determining mechanism explained by that model must presume imperfect competition in labour markets, that is different from the perspectives of Marxian theory of exploitation which views that exploitation emerges even in perfectly competitive markets, as Roemer (1982) emphasized.

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A Appendix

A.1 An alternative definition of exploitation (Roemer, 1982)

Roemer (1982) also proposed an alternative approach in which agents' exploitation status is independent of their preferences over bundles of produced goods, focusing on the *maximum* and the *minimum* amounts of labour embodied in bundles that they can purchase.

Definition 5. [Roemer (1982)]: For any economy $E \in \mathcal{E}$ and any equilibrium $(p, w, r; \boldsymbol{x}) \in \mathcal{S}_E$, an agent $\nu \in \mathcal{N}$, who supplies Λ^ν and possesses ω^ν at that equilibrium, is *exploited* if and only if $\Lambda^\nu > \max_{c \in B(\omega^\nu, \Lambda^\nu; p,w,r)} l.v.\,(c; p, w, r)$ and *an exploiter* if and only if $\Lambda^\nu < \min_{c \in B(\omega^\nu, \Lambda^\nu; p,w,r)} l.v.\,(c; p, w, r)$.

It is easy to see that Definition 5 satisfies **LE**:

$$\begin{aligned}
c^\nu_{\min} &= \arg \min_{c \in B(\omega^\nu, \Lambda^\nu; p,w,r)} \min \{\alpha_l \mid \alpha \in P(p, w, r), \text{ with } \widehat{\alpha} \geqq c\} \\
c^\nu_{\max} &= \arg \max_{c \in B(\omega^\nu, \Lambda^\nu; p,w,r)} \min \{\alpha_l \mid \alpha \in P(p, w, r), \text{ with } \widehat{\alpha} \geqq c\} \\
\alpha^\nu_i &= \arg\min \{\alpha_l \mid \alpha \in P(p, w, r), \text{ with } \widehat{\alpha} \geqq c^\nu_i\}, \text{ where } i \in \{\min, \max\}.
\end{aligned}$$

Further, noting that $l.v.\,(c^*; p^*, w^*, r^*) \leqq \max_{c:\ p^* c = w^*} l.v.\,(c; p^*, w^*, r^*)$, Example 1 can be used to show that Definition 5 does not satisfy either **PECP** or **CECP**.

A.2 Proof of Theorem 1

The demonstration of Theorem 1 is analogous to that of Theorem 2 in Yoshihara (2023).

Proof. Take any definition of exploitation satisfying **LE** and **LVE**. Take any economy $E = E\left\langle \mathcal{N}, P_{(A,B,L)}, \boldsymbol{\omega}, \boldsymbol{l} \right\rangle \in \mathcal{E}_{vN}$ and let $(p, w, r; \boldsymbol{x}) \in \mathcal{S}_E$ be an equilibrium. Then, for each propertyless employed agent $\nu \in \mathcal{W}$, there exist $c^\nu_{\max} \in \mathbb{R}^2_+$ and $\alpha^\nu_{\max} \in P_{(A,B,L)}$ satisfying $\widehat{\alpha}^\nu_{\max} \geqq c^\nu_{\max}$, such that $pc^\nu_{\max} = w\Lambda^\nu$ holds, and $\nu \in \mathcal{N}^{ted} \Leftrightarrow \alpha^\nu_{l\max} < \Lambda^\nu$. Then, there exist $\left(A^{(\nu)}, B^{(\nu)}, L^{(\nu)}\right) \in \mathcal{T}_{(A,B,L)}$ and $\boldsymbol{y} \geq \mathbf{0}$ such that $\alpha^\nu_{\max} = \left(-L^{(\nu)}\boldsymbol{y}, -A^{(\nu)}\boldsymbol{y}, B^{(\nu)}\boldsymbol{y}\right)$.

Then, at the equilibrium $(p, w, r; \boldsymbol{x}) \in \mathcal{S}_E$, it follows that:

$$\begin{aligned}
pB^{(p,w,r)} &= (1+r)\, pA^{(p,w,r)} + wL^{(p,w,r)} \\
pB^{(ij)} &\leqq (1+r)\, pA^{(ij)} + wL^{(ij)} \text{ for any } \left(A^{(ij)}, B^{(ij)}, L^{(ij)}\right) \in \mathcal{T}_{(A,B,L)},
\end{aligned}$$

where the former equations are equivalent to:

$$p = rpA^{(p,w,r)} \left(B^{(p,w,r)} - A^{(p,w,r)}\right)^{-1} + wv^{(p,w,r)} > \mathbf{0}.$$

Then, by multiplying $c^\nu_{\max}$ from the right in both sides of the above equations, we have:

$$pc^\nu_{\max} = rpA^{(p,w,r)} \left(B^{(p,w,r)} - A^{(p,w,r)}\right)^{-1} c^\nu_{\max} + wv^{(p,w,r)} c^\nu_{\max}.$$

As $pc^{\nu}_{\max} = w\Lambda^{\nu}$ holds, the above equations can be rewritten by:

$$\begin{aligned} w\Lambda^{\nu} &= rpA^{(p,w,r)}\left(B^{(p,w,r)} - A^{(p,w,r)}\right)^{-1} c^{\nu}_{\max} + wv^{(p,w,r)}c^{\nu}_{\max} \\ &\geqq rpA^{(p,w,r)}\left(B^{(p,w,r)} - A^{(p,w,r)}\right)^{-1} c^{\nu}_{\max} + w\alpha^{\nu}_{l\max}, \end{aligned}$$

where the last inequalities follow from **LVE**.

Note that $\left(B^{(p,w,r)} - A^{(p,w,r)}\right)^{-1} \geq \mathbf{0}$ by all-productiveness of $\left(A^{(p,w,r)}, B^{(p,w,r)}, L^{(p,w,r)}\right)$ and $pA^{(p,w,r)} > \mathbf{0}$ by $p > \mathbf{0}$, and thus

$$rpA^{(p,w,r)}\left(B^{(p,w,r)} - A^{(p,w,r)}\right)^{-1} c^{\nu}_{\max} > 0 \Leftrightarrow r > 0.$$

Hence, it follows that

$$r > 0 \Leftrightarrow \Lambda^{\nu} > \alpha^{\nu}_{l\max}$$

for any propertyless employed agent $\nu \in \mathcal{N}$. This implies that **PECP** holds for this definition of exploitation. □