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Driver Dynamics and the Longitudinal Control Model

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DRIVER DYNAMICS AND THE LONGITUDINAL CONTROL MODEL

A Thesis Presented

by

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for all the artists in my life

ABSTRACT

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Driver psychology is one of the most difficult phenomena to model in the realm of traffic flow theory because mathematics often cannot capture the human factors involved with driving a car. Over the past several decades, many models have attempted to model driver aggressiveness with varied results. The recently proposed Longitudinal Control Model (LCM) makes such an attempt, and this paper offers evidence of the LCM's usefulness in modeling road dynamics by analyzing deceleration rates that are commonly associated with various levels of aggression displayed by drivers. The paper is roughly divided into three sections, one outlining the LCM's ability to quantify forces between passive and aggressive drivers on a microscopic level, one describing the LCM's ability to measure aggressiveness of platoons of drivers, and the last explaining the meaning of the model’s derivative. The paper references some attempts to capture driver aggressiveness made by classic car-following models, and endeavors to offer some new ideas in study of driver characteristics and traffic flow theory.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

CHAPTER I

RESEARCH OBJEVTIVES

The objective of this research is to develop a connection between the realm of human factors associated with operating a car and the realm of mathematics. As drivers navigate roads, they steer their vehicle making judgments based on their senses and emotions, which may be classified as reactions, feelings, anxieties, desires, or any other internal factors that cause a driver to initiate a certain maneuver. Two layers of analysis link these human factors with a proposed mathematical model, titled the Longitudinal Control Model. The first layer of analysis classifies these human elements associated with operating cars by using formal terminology within the world of transportation engineering, using the standard terms “desired speed,” and “desired spacing,” as well as “deceleration tolerances.” The second layer of analysis converts these engineering principles into mathematical parameters by assigning each a metric. In this research, desired spacing is measured in meters, (m) desired speed is measured in meters per second (m/s), and deceleration tolerances are measured in meters per second per second (m/s^2).

Using each of these parameters and their respective metrics and building blocks for a robust equation, The Longitudinal Control Model is structured to predict a driver’s response based on desire to increase speed or pass a leading car, preference for a certain amount of space between themselves and a leading car, and tolerance for abrupt or sudden decelerations. Using this model, this research seeks to develop a better understanding of traffic flow dynamics and better predictions of driver reactions. In order to accomplish this goal, this research employs a methodology that explores mathematical relationships between essential human factor parameters that relate to people’s everyday driving experiences. Finally, this research endeavors to capture specific properties of driver behavior based on many types of data, to test safety thresholds.

CHAPTER II

LITERATURE REVIEW

The level at which the Longitudinal Control Model seeks to model human behavior sets both the model and this research apart from some preceding work of the same lineage. While traditional models commonly found in the study of transportation engineering typically use parameters such as perception-reaction time as a human factor while predicting when a driver may need to start braking to avoid an accident.(1) Previous models may simplify driver dynamics for the sole sake of mathematical ease, which sometimes occurs as researchers build computer simulation models (2), or at times if researchers seek to derive more fundamental relationships of engineering principles by eliminating complex factors.(3) Many of the parameters and concepts incorporated in the Longitudinal Control Model are lend themselves to a deeper level of analysis than previous simpler models, stipulating that a driver will react not only to avoid an accident, but also to "comfortably" stop short of an accident.

Under these more realistic conditions, a person operating a car who desired a less abrupt braking maneuver and larger desired spacing, will produce mathematical results in the Longitudinal Control Model that may differ greatly from those produced by a typical perception-reaction time equation. Research conducted at Chung-Ang University in Korea, indicates that driver sensitivities to upstream road conditions are extremely important in modeling accurate real-world phenomena, especially on highways, when vehicles reach very high speeds and must decelerate to avoid accidents with very large margins for error. The desired spacing algorithm developed in part by Peter G. Gipps in 1981(4) allows the Longitudinal Control Model to not only model these deceleration effects not considered in previous equations, but also with more accuracy than some other models using less robust algorithms. Likewise, in non-braking conditions, a car-following pair including a leading and following vehicle may also be modeled with attention to desired spacing. In this scenario, two vehicles may travel at the same speed for a time, before "the leading vehicle decelerates and travels at a speed…and the following vehicle will also start to decelerate at a certain time after the leader started to decelerate." (5) Considering the size of the headway before, during and after a series of accelerations and decelerations, eventually the two cars may reach a “lull”, at which point desired spacing fluctuates very little and the two cars can be considered in a state of equilibrium. In this state, both the leading and following drivers likely feel “comfortable” with the distance between themselves. If no other traffic interferes with these two cars, this condition may persist, which may provide a future direction of mathematical analysis that could be performed using Longitudinal Control Model. In recent decades, many traffic flow theorists have studied this same idea of spacing equilibrium, most notably Rainer Wiedemann, who is credited with the development of the car-following model used in the popular traffic simulation package known as “VISSIM.” Wiedemann’s model belongs to a family of models known as “psychophysical or action-point models. This family of models uses thresholds or action-points where the driver changes his/her driving behavior. Drivers react to changes in spacing or relative speed only when these thresholds are crossed. The thresholds and the regimes they define are usually presented in the relative speed/spacing diagram for a pair of lead and follower vehicles.” (6)

In this context, the Longitudinal Control Model belongs to a family of more advanced equations that are able to model road conditions differently than previous equations, and with more regard for bigger-picture road conditions. While many traffic flow models consider factors such as speed, perception-reaction time, or sight distance, the proposed Longitudinal Control Model includes parameters not commonly addressed such as deceleration tolerance, driver desire for speed and desire to closely follow other cars.

CHAPTER III

MODEL FORMULATION

Over the past 60 years a wide variety of traffic models with varying scopes have found niches in the realm of traffic flow theory. Some models are microscopic and are intended to capture only very specific phenomena or relationships between leading and following car pairs, while others are macroscopic in nature and attempt specifically to measure the large scale effects of incidents like bottlenecks and traffic jams. To date, traffic flow theorists have yet to deem any particular model as the "best" in terms of being both comprehensive and capable of accurately modeling any type of traffic flow regime with elegance. The recently proposed Longitudinal Control Model (LCM), formulated by Ni (2011), is designed to be mathematically elegant on microscopic and macroscopic levels, and effective without requiring a vast amount of parameters. The need for this model stems from its ability to serve as a link between the microscopic and macroscopic realms of traffic flow theory. Very few mainstream traffic flow models are able to capture the dynamics of car-following pairs on a small scale, while also containing parameters such as spacing and deceleration, both of which have strong and direct mathematical relationships to the "bigger-picture" parameters of density and aggression. On a microscopic level, the model accounts for speed, acceleration, deceleration, and perception-reaction time. From a macroscopic perspective, the LCM model accounts for free-flow speed, density, and average response time.

These are the same parameters that will appear in all versions of the LCM throughout this paper. The relationships between these parameters arise from a so-called Unified Field Theory. A diagram of the unified theory resembles a free-body diagram involving a leading and following car. In this diagram, included in Figure 1, the forces exerted on the following car are roadway gravity force, roadway resistance, and a vehicle interaction force. The roadway gravity force equates to a driver's desire for speed, while the roadway resistance force corresponds to deterring factors presented by other drivers or obstacles in the vicinity. The vehicle interaction force defines the physics of the interplay between the leading and following car in terms of speed, spacing, acceleration and deceleration.

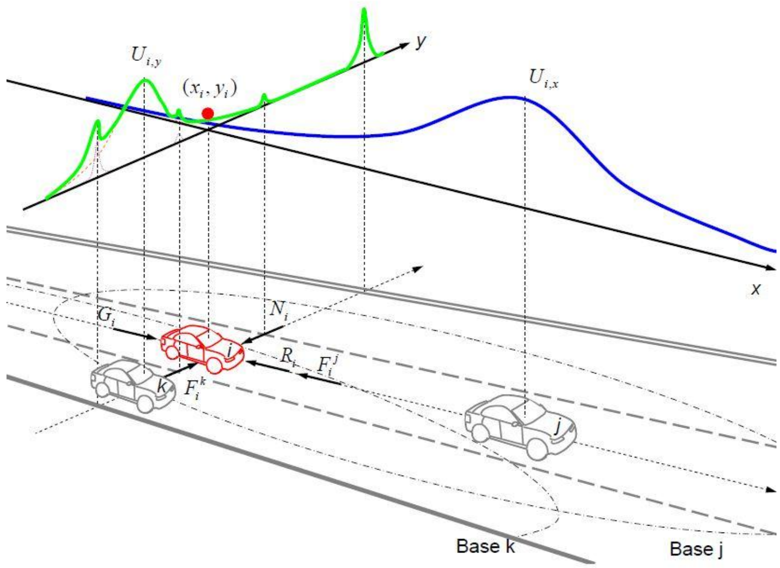

Figure 1 Free Body Diagram used to develop the Longitudinal Control Model

Based on this diagram, the term $\frac{\dot{x}_i(t)}{v_i}$ is derived as a microscopic ratio of actual speed divided by desired speed, designed to measure how content a driver is with their current speed. The term $\frac{s_{ij}(t)}{s_{ij}^*(t)}$ represents another ratio, which corresponds to actual spacing divided by desired spacing, intended to represent how content a driver is with the spacing between themselves and the car they trail. These ratios are included in the microscopic version of the LCM, described further in the next section. Along with these two ratios, the additional factors of A_i, the maximum acceleration capabilities of the vehicle under analysis, as well as τ_i, the perception reaction time of the driver of this vehicle, are used in an equation to generate an estimate for the momentary acceleration that a driver will achieve as they trail another car. The macroscopic version of the model contains similar parameters, which correspond to the same ratios, and appear as density derived from spacing, aggressiveness derived from deceleration tolerance, and perception reaction time, which remains as an aggregated measure from the microscopic model. The final and most reduced version microscopic version of the LCM places speed and spacing on one side of the equation, and a matching acceleration and perception reaction time on the other. The most reduced macroscopic version balances flow with speed, density, aggressiveness and perception reaction time.

CHAPTER IV

AGGRESSIVENSS PARAMETERS OF CAR FOLLOWING MODELS

In the realm of traffic flow theory, the patterns and habits of drivers range from being extremely cautious to extremely aggressive. The microscopic derivation of the Longitudinal Control Model (LCM), which originates from a unified field theory model proposed in 2011(7), uses two parameters to account for this aggressiveness, namely desired spacing and desired speed.

The most simplified case of the microscopic Longitudinal Control Model is found in equation (1):

$$\ddot{x}_i(t+\tau_i) = A_i\left[1 - \frac{\dot{x}_i(t)}{v_i} - e^{1-\frac{s_{ij}(t)}{s_{ij}^*(t)}}\right] \quad (1)$$

In this equation, A_i represents max acceleration from standstill, ($\dot{x}_i$) is speed, v_i is desired speed, τ is perception-reaction time. Desired spacing is one of the most important parameters of the LCM model and is represented as [$s^*_{ij}(t)$]. This equation (1) is the most useful and intuitive way to express of the microscopic LCM.

The [$s^*_{ij}(t)$] desired spacing term that appears in this equation (1) could be defined in many ways, using models that range anywhere from the car-following models of the 1960s, to cutting edge models used in car manufacturing laboratories. One of the strengths of the LCM is that the model allows the user to insert the desired spacing algorithm they deem most appropriate for a particular microscopic environment. In a rural area with low population, the LCM may function better using simpler algorithms, while in heavily populated areas where spacing is at a premium, the model may need more complicated desired spacing algorithms. In this paper, authors choose the Gipps model for desired spacing, which appears in equation (2) below:

$$s_{ij}^*(t) = \left[\frac{(x_i')^2(t)}{2b_i}\right] - \left[\frac{(x_j')^2(t)}{2Bj}\right] + (x_i')(\tau_i) + \ell_j \quad (2)$$

In equation (2) above, *s* represents a mathematical result for desired spacing based on values for x'(t), which represents the velocities of the subscripted cars, τ, the reaction time of the following driver, and ℓ_j , which is the length of the leading car. The two terms (b_i) and (B_j) correspond to deceleration rates maintained by drivers of the (i^{th}) and (j^{th}) cars.

This algorithm, derived by Gipps, is adopted because of the realistic results it provides for desired spacing based on speeds and decelerations that might occur in the field. For example, if an aggressive driver was behind the wheel of the following (i^{th}) car, they would be considered more likely than other drivers to leave very little distance (spacing) between themselves and a leading car. Because an aggressive driver may apply their brakes more stiffly to avoid accidents and, the risks they take are reflected in the model as the equation's deceleration rate (b_i) increases. As [$_{\lim} b_i \rightarrow \infty$], then the spacing [$s_{ij}^{*}(t)$] term in equation (2) necessarily approaches smaller values. A crash occurs if $s_{ij}^{*}(t)<0$.

The following inequality is derived from the Gipps model and presents a scenario in which deceleration parameters predict an accident:

$$\left|\left[\frac{(x_i')^2(t)}{2b_i}\right] - \left[\frac{(x_j')^2(t)}{2Bj}\right]\right| > (x_i')(\tau_i) + \ell_j \qquad (3)$$

Specifically, the left-hand side of (3) presents itself for analysis because the velocity and deceleration terms play the most dynamic role in determining spacing.(8) For example, consider a driver traveling at 115 km/hwho is following another car and harshly applies their brakes to achieve a complete standstill in about two seconds. Converting the speed of 115 km/h to 31.94 m/s, the average deceleration ($\frac{\Delta v}{\Delta t}$) for the following car would be computed as b_i = (31.94 m/s)/(2) = 15.97 m/s^2. In this example, consider the driver of the leading car to be more cautious and let B_j = 9.26 m/s^2, which back-calculates to a scenario in which a driver is traveling at a speed of 100 km/h and is able to come to a complete stop in about three seconds.

Solving the rest of the equation using τ set at 1.36 seconds based on empirical data (9) for highway conditions, and using an average car length of 8 m, the spacing between the two cars would be predicted as 38.50 m.

In this case, no accident occurs because;

$$125.899\,m = \left| \left[\frac{(x_i')^2(t)}{2b_i} \right] - \left[\frac{(x_j')^2(t)}{2Bj} \right] \right| \ngtr (x_i')(\tau_i) + \ell_j = 164.4\,m.$$

For drivers using passenger cars equipped with good tires who are traveling on dry highway, most vehicles can decelerate at a comfortable rate of about 4.6 m/s^2(10), which equates to about $0.47g$.

Figures 2 and 3 below depict pairs of cars traveling at various matching speeds that both decelerate to a standstill. The curved lines represent the absolute value $\left| \left[\frac{(x_i')^2(t)}{2b_i} \right] - \left[\frac{(x_j')^2(t)}{2Bj} \right] \right|$, while the straight lines represent slopes for $[(x_i')(\tau_i) + \ell_j]$. The intersection of the two inequalities marks the point at which $s_{ij}^*(t)$ falls below zero, denoting the speed in (m/s), at which cars with deceleration rates of b_i and B_j are no longer able to avoid crashes according to the Gipps algorithm. *Note: b_i is follower and B_j is leader*

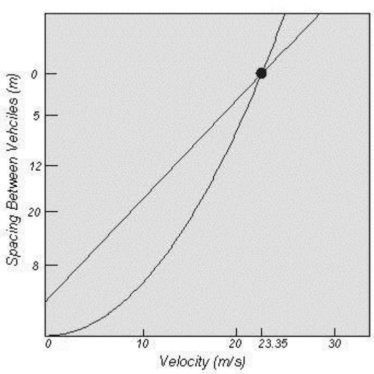

Figure 2 Crash thresholds for B_j=1g, b_i=0.9

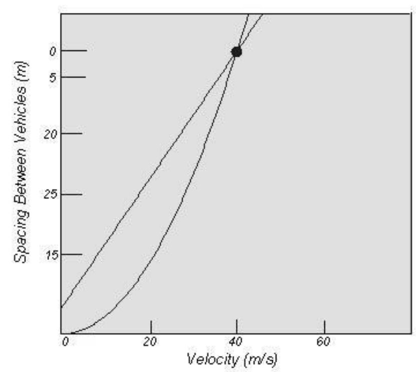

Figure 3 Crash thresholds for B_j=1.4g, b_i=1.3g

Figure 2 depicts a scenario in which a following driver tolerates a deceleration rate at $0.9g$, while the leading driver is more aggressive and decelerates at a rate of $1g$, and a τ of 2.25 seconds is assumed for both drivers.(11) These conditions predict no accidents until a speed of 23.35 m/s (72 km/h or 60 mph). Figure 3 depicts a scenario in which the leading car decelerates more aggressively at a rate of $1.4g$ (13.2 m/s^2), while the follower is slightly less aggressive (b_i =1.3), and the same τ for both drivers is set at 2 seconds to model fairly alert motorists. Under these conditions, spacing does not reach zero until both cars travel at about 40 m/s (80 km/h or 58 mph).

Similar calculations yield results for drivers with a perception reaction time set at "normal" response time for drivers on highways.(12) The Gipps equation indicates that a driver who can decelerate abruptly at $2g$ (19.7 m/s^2), while a leading car decelerates very slowly at $0.3g$ (3.1 m/s^2), stipulates that accidents can be avoided at speeds up to 43 m/s (154 km/h or about 95 mph). If both drivers are very aggressive and can tolerate deceleration rates of $2g$, the Gipps model predicts these drivers can avoid accidents at speeds up to 53.3 m/s (191 km/h or 119 mph).

One of the advantages of the LCM is that it holds intrinsic limits for boundary values of $s_{ij}^*(t)$ that result from various levels of aggressiveness, regardless of whether the Gipps method or any other algorithm may be used to estimate spacing. The limit of $s_{ij}^*(t)$ implies one of two conditions. For $_{\lim}s_{ij}^*(t) \to 0$, then the term $e^{1-\frac{s_{ij}(t)}{s_{ij}^*(t)}}$ approaches zero. In this case, the LCM could be considered as modeling an infinitely aggressive driver who desires no space at all between themselves and the car in from of them, and continuously wants to maintain a relationship of $s_{ij}^*(t) < s_{ij}(t)$. In this imaginary scenario, the "aggressive" version microscopic LCM equation boils down to appear as:

$$\ddot{x}_i(t+\tau_i) = A_i\left[1-\frac{\dot{x}_i(t)}{v_i} - e^{1-\hat{\hat{\infty}}}\right] = A_i\left[1-\frac{\dot{x}_i(t)}{v_i} - 0\right] = A_i - (A_i)\left[\frac{\dot{x}_i(t)}{v_i}\right] \quad (4)$$

This equation (4) is intuitive because when a driver is following or keeping very close spacing between vehicles ($s_{ij}^* < 3\ell_j$), which is defined as tailgating, the acceleration term (A_i) begins to dominate the right hand side of the LCM equation. Making a further assumption that this same aggressive driver also has a very good perception reaction time ($_{\lim}\tau_i \to 0$), then their acceleration increasingly becomes defined solely by $\ddot{x}_i(t) = A_i$. In this case the microscopic LCM successfully models a driver in the i^{th} car who is continuously accelerating as they would from standstill in a never-ending quest to maintain an infinitesimally small amount of space between their fender and the leading car's rear bumper. Furthermore, aggressive drivers often desire a greater speed (v_i) than their actual speed, $(\dot{x}_i)$. As this following driver dramatically increases their speed, and $_{\lim}(\dot{x}_i(t)) \rightarrow v_i > v_i$, then this condition in turn forces the condition of $\left[\frac{\dot{x}_i(t)}{v_i}\right] > 1$. In this case the LCM is able to predict a lane change by the aggressive tailgating driver, otherwise $A_i - (A_i)\left[\frac{\dot{x}_i(t)}{v_i}\right]<0$, and the aggressive driver is forced to slow down.

Conversely, an indiscriminately passive driver who desires a nearly unlimited amount of space between themselves and the car in front of them forces the condition $_{\lim}s_{ij}^*(t) \rightarrow \infty$. By virtue of the fact that in the real world, the inequality $\left[0 < \frac{s_{ij}(t)}{s_{ij}^*(t)}\right]$ must hold otherwise the equation would yield an accident, these boundary conditions force the tautology of $max\left[e^{1-\frac{s_{ij}(t)}{s_{ij}^*(t)}}\right] = e^1$. Allowing this boundary condition to hold, the "passive" version of the LCM equation would appear as:

$$\ddot{x}_i(t+\tau_i) = A_i\left[1 - \frac{\dot{x}_i(t)}{v_i} - e^{1-\widehat{\hat{0}}}\right] = \left[A_i - \left[(A_i)\frac{\dot{x}_i(t)}{v_i}\right] - (A_i)(e)\right] = A_i\left(-1.72 - \left[\frac{\dot{x}_i(t)}{v_i}\right]\right) \quad (5)$$

The predicted value of $\ddot{x}_i(t+\tau_i)$is always negative in this "passive" version of the model unless a driver is traveling in reverse at a negative speed that satisfies$\left[\frac{\dot{x}_i(t)}{v_i}\right] < -1.72$. Within these boundary conditions, the LCM model can be calibrated to the psychology of much more "typical" drivers with more intuitive and realistic results. More specifically;

Aggressive LCM $(\ddot{x}_i) \cong \left[A_i - (A_i)\left[\frac{\dot{x}_i(t)}{v_i}\right]\right]$> **"Typical Driver"** >$\left[A_i\left(-1.72 - \left[\frac{\dot{x}_i(t)}{v_i}\right]\right)\right] \cong$ Passive LCM

The following examples are included as an attempt to validate the LCM's predictive abilities by inputting middle tier AASHTO standards into the microscopic LCM equation, and considering whether the values output match those that could be expected in the real world. According to AASHTO reasonable parameters for the LCM are to set, A_i = 0.4g and $s_{ij}(t) = 56m$, while velocity in each situation is set at a "safe" speed of about 20 m/s below critical speeds for crashes.(13) The general range for velocities(x_i') is between 10m/s (36 km/h or 22 mph) to 50 m/s (180 km/h or 112 mph), which in turn are used to compute the desired spacing, $s_{ij}^*(t)$. If a following car is traveling at 15m/s, and b_i= 2.0$g B_j$= 0.3g, then based on equation (2), the desired spacing $s_{ij}^*(t)$ is predicted using the previous Gipps algorithm as 13.4

meters. However, if the following driver traveling at $25 m/s$ ($55\ mph$) suddenly desires a speed (v_i) of 37 m/s ($80\ mph$), then the LCM model predicts the following acceleration:

$$\ddot{x}_i(t+\tau_i) = A_i\left[1 - \frac{\dot{x}_i(t)}{v_i} - e^{1-\frac{s_{ij}(t)}{s^*_{ij}(t)}}\right] = 3.83\left[1 - \frac{25}{37} - e^{\left(1-\frac{56}{13.4}\right)}\right] = 1.27\ m/s^2. \quad (6)$$

The result in equation (6) is based on middle tier AASHTO standards yields an estimate for acceleration comparable to the typical acceleration rates displayed by motorists on a highway. According to field studies dating tabulated in a 2001 research paper, acceleration rates on highways generally range from 0.15 m/s^2 for tractor-trailer trucks, to 2.27 m/s^2 for "inner-lane drivers."(14) This range was amalgamated from field observations throughout the past 40 years, taken from areas such as London, England, suburban areas of Detroit, Michigan, and roads in New Jersey near New York City, New York.

Even more dramatically, if an aggressive driver is moving at 30 m/s, but unexpectedly desires to slow down to 5 m/s, then the LCM predicts a sharply negative acceleration of -25.68 m/s^2. If a driver was cruising at 20 m/s and desired to incrementally speed up to 22 m/s, the LCM equation predicts a slight positive acceleration of 0.16 m/s^2. One of the drawbacks of the LCM is that it generally tends to predict somewhat small estimates of acceleration, within the range: $\{-15\ m/s^2 < \ddot{x}_i(t+\tau_i) < 5\ m/s^2\}$. However, these predictions make more sense in light of the above research reported in 2001, as well as a 2007 report titled, "Force Model for Single Lane Traffic."(15) Authors of this report argue that a microscopic model based on Van Der Waal's molecule model can be used to mathematically show that in a two vehicle "system" involving a leading (j^{th}) and following (i^{th}) car, that the "conception of keeping a certain distance between two (cars) is similar to that (relationship) in molecule dynamics."

The function the researchers used to model the equilibrium of two drivers is originated from the Lennard-Jones potential model for physical distances between molecules, which somewhat resembles the LCM model:

$acceleration = \frac{c}{s_{ij}(t)}\left[-\left(\frac{d_{Opt}}{s_{ij}(t)}\right)^4 + \left(\frac{d_{Opt}}{s_{ij}(t)}\right)^2\right]$, in which d_{Opt} is an "optimal" distance similar to the LCM's $s^*_{ij}(t)$ term, and c is a coefficient that attempts to measure deceleration aggressiveness.

Based on this force model for single lane traffic derived from the Lennard-Jones model, the report states that; "The max acceleration lies in the line *v*=0 and computation shows that $a_{max} = 3.033\ m/s^2$…which is consistent with the common capability of a vehicle on a city street." (15**)**

The notion that a driver will only accelerate their car until they cannot go any faster in a state of equilibrium, approaching an asymptote, helps put a better perspective on the LCM's projections for accelerations. The two following graphs illustrate the asymptotic effect of acceleration captured by the LCM.

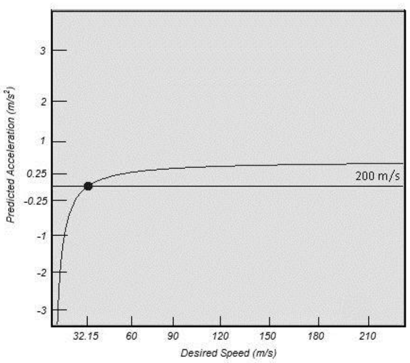

Figure 4 Critical Point for Aggressive Drivers

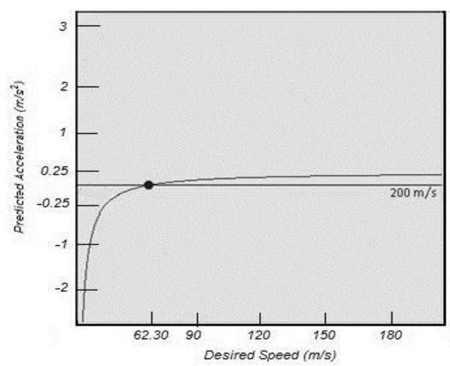

Figure 5 Critical Point for Passive Drivers

The curve in Figure 4 portrays a situation in which an aggressive driver is already traveling at a 30 m/s speed limit and is comfortable with a value of $2g$ for b_i, while desired spacing is set very low, and their desired speed is allowed to vary from 0 to 200 m/s. In this case, the LCM predicts an aggressive driver will initiate a change from deceleration to acceleration ($\ddot{x}_i(t + \tau_i) > 0$) when their desired speed increases by 2.15 m/s above the hypothetical 30 m/s speed limit. In Figure 5, a more passive following driver's desired speed is also allowed to vary from 0 to 200 m/s, while their desired spacing is set using a less aggressive value of $1g$ for b_i. In this case, tolerable spacing gaps are projected to be about twice as large, and the LCM predicts that a more cautious driver will not start to initiate a change from deceleration to acceleration until their desired speed is a much greater 32.2 m/s above the same hypothetical 30 m/s speed limit.

CHAPTER V

MICROSCOPIC AND MACROSCOPIC RELATIONSHIPS

The dynamics of aggressiveness that can be captured in the microscopic version of the LCM model have parallels in the macroscopic version of the LCM model. Under steady-state conditions, vehicles in traffic on a highway behave somewhat uniformly, implying that the speeds of both cars are nearly the same.

Using the same microscopic equation but allowing$[(x_i')^2(t)] = [x'_j(t)] = v^2$, because both cars are moving at the same speed, then: $s_{ij}^*(t) = (v^2)\left[\frac{1}{2}\left(\left(\frac{1}{b_i}\right) - \left(\frac{1}{B_i}\right)\right)\right] + (x_i')(\tau_i) + \ell_j$.

Let $\left[\frac{1}{2}\left(\left(\frac{1}{b_i}\right) - \left(\frac{1}{B_i}\right)\right)\right] = \gamma$, then; $s_{ij}^*(t) = (v^2)\gamma + (x_i')(\tau_i) + \ell_j$.

Plugging in the new spacing term $s_{ij}^*(t)$= $(v^2)\gamma + (x_i')(\tau_i) + \ell_j$into the microscopic LCM yields:

$$\ddot{x}_i(t+\tau_i) = A_i\left[1 - \frac{\dot{x}_i(t)}{v_i} - e^{1-\frac{s_{ij}(t)}{(v^2)\gamma + (x_i')(\tau_i) + \ell_j}}\right].$$

Given that the domain of traffic flow theory defines spacing between (j^{th}) and (i^{th}) cars as (s) = $s_{ij}(t) = \frac{1}{density} = \frac{1}{k}$, then$\left(s_{ij}^*(t) = (v^2)\gamma + (x_i')(\tau_i) + \ell_j = \frac{1}{k^*}\right)$, and the terms in the exponent of the exponential can be inverted to appear as: $\frac{s_{ij}(t)}{(v^2)\gamma + (x_i')(\tau_i) + \ell_j} = \frac{\frac{1}{s_{ij}(t)}}{\frac{1}{(v^2)\gamma + (x_i')(\tau_i) + \ell_j}} = \frac{k^*}{k}$

After some rearranging, the full macroscopic LCM model appears as:

$$v = v_f\left[1 - e^{1-\frac{k^*}{k}}\right] \tag{7}$$

$\left(where\ k^* = \frac{1}{\gamma v^2 + \tau v + \ell}\right)$

In this equation (7), v is traffic space-mean speed, v_f is free-flow speed, k is density, and k^* varies based on the parameters of γ, a macroscopic measure of aggressiveness, and τ, average response time. The same deceleration parameters b_i and B_j from the microscopic version of the LCM still permeate the macroscopic version of the LCM, infused into the model through the single γ parameter. As a result, the macroscopic version of the LCM has just as equally well-defined boundary conditions as the microscopic version.

For $_{\lim}(k) \rightarrow 0$, spacing is allowed to be infinitely large, because $k = \frac{1}{s_{ij}(t)} = 0$, so $_{\lim}s_{ij}(t) = \infty$. In this case, regardless of aggressiveness, $_{\lim}\left(e^{1-\frac{k^*}{k}}\right) = (e^{1-\infty}) = 0$. This equation's physical meaning is that density reaches an arbitrarily low value, and the space-mean speed of traffic simply becomes the free-flow speed of the road defined by $v = v_f[1 - e^{1-\infty}] = v_f[1 - 0] = v_f$. In the opposite case of jam density condition, when $_{\lim}(k) \rightarrow k_j$, spacing approaches zero, because $k = \frac{1}{s_{ij}(t)} \rightarrow k_j$, so $_{\lim}s_{ij}(t) = 0$. Depending on aggressiveness, two cases arise, each one corresponding to diverging limit conditions relating to k^*. The boundary conditions relating to k^* are very similar to those of k, except that they are also constrained by the physical reality that the actual density of a road reaches a saturation point well below an infinite number of cars per space.

When $_{\lim}(k^*) \rightarrow max$, then the equation $s^*_{ij}(t) = (\gamma v^2 + \tau v + \ell)$ approaches very small values. In the case when γ attains a negative value, the value of the term $(\gamma v^2 + \tau v + \ell)$ becomes as low as it possibly can, because γ is associated with the v^2 term. As values of $(\gamma v^2 + \tau v + \ell)$ tend to zero, meaning that drivers are absolutely aggressive, then k^* approaches extremely high values. As the limits of k^*, or "predicted density," approach these higher values, then this boundary condition is also defined partly by empirical density, k, because in the real world a saturation point exists at jam density in which no more cars can fit on a road. As such, when drivers become infinitely aggressive and road conditions approach jam density, then $_{\lim}\frac{1}{\gamma v^2 + \tau v + \ell} \rightarrow 0$, which means $_{\lim}(k^*) = k_j$and $_{\lim}(k) \rightarrow k_j = k_j$, forcing $_{\lim}\left(\frac{k^*}{k}\right) = \left(\frac{k_j}{k_j}\right) = 1$, and $_{\lim}\left(e^{1-\frac{k^*}{k}}\right) = \left(e^{1-\frac{max}{max}}\right) = (e^0) = 1$.

Summarily put, as aggressiveness increases without bounds, so too does density, and the macroscopic LCM predicts a standstill traffic jam, $v = v_f[1 - e^0] = v_f[1 - 1] = 0$.

If $_{\lim}(k^*) \to 0$, then the equation $s_{ij}^*(t) = (\gamma v^2 + \tau v + \ell)$ approaches very large values. In this case γ attains large positive values and aggressiveness decreases. As the term $(\gamma v^2 + \tau v + \ell)$ increases, k^* decreases, and actual density, k, can vary freely mathematically but likely will decrease in the real world. If (k^*) is allowed to reach its limit of zero and density is extremely low, then $_{\lim}(k^*) \cong 0$ and $_{\lim}(k) \cong 0$, and once again, $_{\lim}\left(\frac{k^*}{k}\right) = 1$. The full equation appears as $v = v_f[1 - e^0] = v_f[1 - 1] = 0$. The LCM model in this scenario predicts the opposite extreme boundary condition in which drivers are all non-aggressive to the point that none of them move.

By this logic, the LCM yields the overall result that when platoons of drivers operate their cars aggressively, γ values will be large and negative. Platoons of passive drivers will cause γ values to rise.

Even under the most extreme driver mentalities, the value for the aggressiveness parameter γ in the macroscopic LCM lies within the range: $\{-0.073\ s^2/m < \gamma < 0.54\ s^2/m\}$. The following table lists some field data relating to aggressiveness on roads in Europe and the United States. (16)

Table 1 Macroscopic Field Data

Data Source			**Empirical Parameters**				**Capacity Condition**		
Location	Facility	No.obs.	v_f m/s	*l* m/s	τ s	γ s^2/m	q_m v/h	k_m v/km	v_m km/h
Atlanta	GA400	4787	29.5	4	1.46	-0.038	1883.8	22.0	85.8
Orlando	I-4	288	24.2	8.6	1.09	-0.040	1795.5	22.1	81.4
Germany	Autobahn	3405	43.3	10	1.0	-0.018	2114.1	22.3	95.0
CA/PeMs	Freeway	2576	31	6.3	2.4	-0.060	1124.9	11.0	102.5
Toronto	Hwy 401	286	29.5	12	0.8	-0.026	1945.7	21.8	89.2
Amsterdam	Ring Rd	1199	28.4	7.5	0.82	-0.026	2452.2	27.2	90.3

The data in Table 1 originates from several sources, including some raw data collected over a full year from sensors on Georgia 400 provided for this paper by the Georgia Institute of Technology, a data set extracted via the internet from Caltrans database with public information about Orange County, California, and international data gathered from past research papers. All the data is amalgamated over a one year period and only the most reliable data points are included. The highest free-flow speed is on the Autobahn in Germany, where drivers are allowed faster speeds, accommodating for more drivers with high desired speeds and high tolerance for abrupt decelerations. The Autobahn in Germany is a road which lends itself for analysis because the prevailing mentality of all drivers is to drive as fast as possible, with nearly half of all motorists traveling above 130 km/hr.(17) Statistics relating to this unique highway indicate that the Autobahn is perhaps the best example in the world of a stretch of road on which the largest percentages of platoons of drivers are all acting with the same intent to travel as fast as possible while tolerating quick decelerations. Because so many drivers on the Autobahn act with the same mentality, data relating to the road helps define boundary conditions of the LCM. Because drivers all operate with a similar mindset and a higher level of aggression, then the γ-value shrinks because the difference of $\left[\frac{1}{2}\left(\left(\frac{1}{b_i}\right)-\left(\frac{1}{B_i}\right)\right)\right]$ is much closer to zero, making it the lowest in Table 1. Not surprisingly, the γ-value associated with higher visible behaviors of aggression belongs to a road with a high accident rate, which is again the autobahn.

In California, drivers may have the same psychology, although the free flow speed is much lower, causing a higher γ-value because aggressive drivers are more likely to get stuck behind passive drivers traveling at a lower speed limit, causing a greater discrepancy between drivers, and thus a larger difference between $\left[\frac{1}{2}\left(\left(\frac{1}{b_i}\right)-\left(\frac{1}{B_i}\right)\right)\right]$.

The values for spacing [$s^*_{ij}(t)$] that are produced from the microscopic equation as a function of b_i and B_j, show that faster, more aggressive cars perhaps deserve to be grouped into their own category of study, rather than being aggregated with all other data points from any given study. On a macroscopic level, the LCM is able to model an accentuated point on the flow density curve corresponding to aggression.

Using realistic γ-values that have either been empirically collected or derived from the same b_i and B_j rates that were selected for analysis in the microscopic version of the LCM, the two graphs in Figures 6 and 7 portray the effects of γ on flow, speed, spacing and density.(18) The graphs support the claim that the LCM accurately models driver dynamics on a macroscopic level. The macroscopic LCM is able to reproduce a similar effect as the microscopic LCM because as density ($k = \frac{1}{spacing}$) decreases, then speed increases. More noticeable is the fact that in both models, the diagrams almost perfectly associate slower speeds with less spacing.

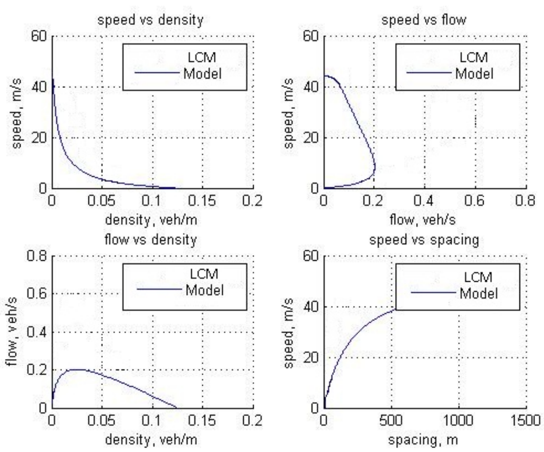

Figure 6 Relationships for Passive Platoons

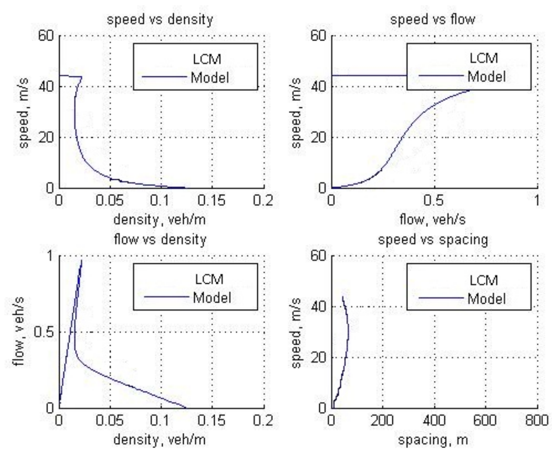

Figure 7 Relationships for Aggressive Platoons

The diagrams in Figure 6 depict a scenario based on deceleration values from an aggressive lead driver and a slow follower, yielding γ = 0.076 s^2/m. The flow-density curve in this scenario is very round because spacing is projected to be very large due to a timid following driver. Density and flow levels in Figure 6 are low and not many cars travel the road per period of time (q). By contrast, Figure 7 portrays a much sharper γ-value of -0.05s^2/m, indicating very aggressive drivers. In this family of graphs, cars travel quickly and follow closely, allowing large amounts of flow, but higher risk for density, as is reflected in the q-k curve. Spacing in Figure 7 is also noticeably smaller in general. Because platoons modeled in Figure 7 are more aggressive, rates for flow *(q)* are able to remain much higher even when speeds increase, since there are no slow drivers to block faster platoons. The two graphs above display a stark contrast in flow-density plots as aggressiveness increases. The difference between the aggressiveness factor of timid and bold drivers becomes more measurable using a wider range of γ values.

The discrepancy between the habits of the extremely aggressive drivers and extremely timid drivers are what can cause a mismatch between models and real world data. Yet while other models may not produce curves that match empirical data that include such a wide variety of dynamics, the LCM is able to capture these phenomena. When a large discrepancy exists between b_i and B_j values due to differing behaviors of aggressive and timid drivers, flow and density curves change dramatically.

Based on this discrepancy, the LCM model could be used to respond to comments by Rahka in a 2009 research paper (19), stating some macroscopic models have trouble modeling realistic conditions. In reference to the work of Del Castillo and Benitez, Rahka wrote that empirical data often times "have probably been grouped together, representing conditions that probably never occurred, because vehicles belonging to a certain speed class could not have traveled together."(20) The family of graphs in Figures 6 and 7 indicate that even when drivers with a different mentality are grouped together, the macroscopic LCM model is able to adapt appropriately. The data is aggregated over thousands of cars, yet the LCM avoids the pitfall of representing "conditions that never occurred." Furthermore, the Longitudinal Control Model can be used to model the mentality of platoons, demonstrated on the following page in Figures 8 through 10.

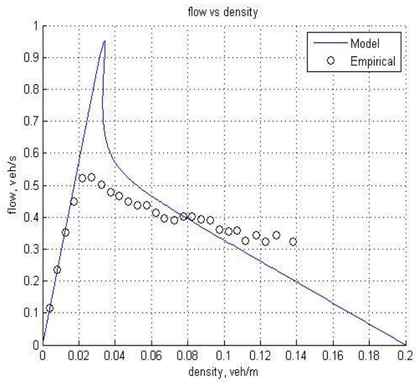

Figure 8 Aggressive Platoons

The curve in Figure 8 has a γ-value of -0.046. The deceleration values that yielded this curve indicate significant discrepancies between driver psychologies, and more specifically that the following driver is aggressive while the leading driver is somewhat passive. The effect of aggressiveness on the macroscopic LCM is a reverse lambda curve, which is exaggerated in Figure 8 for illustrative purposes. The tip of the curve symbolizes a point at which cars start to fill the road and aggressive followers cause spacing to be at a premium. This effect is what causes the sudden amount of heavy traffic and greater density.

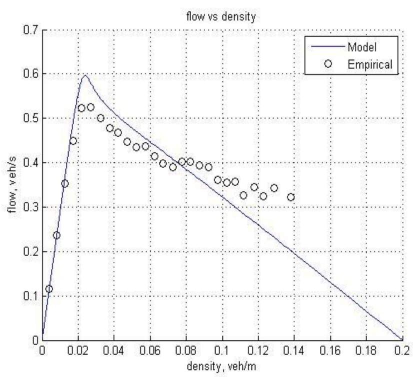

Figure 9 Typical Platoons

The curve in Figure 9 has a γ-value of -0.038. This curve better matches empirical data due to the fact that in reality, discrepancies between driver psychology exist, but are not so exaggerated as in Figure 8 above. Under more uniform conditions in which drivers share a more common psychology, the reverse lambda effect shrinks and the LCM model is able to quantify a small peak that occurs at a density of approximately 0.03 veh/m, where two empirical data points exist. The LCM is able to encompass these two points while still maintaining a well fit line that corresponds to the remaining data points through a density of 0.2 veh/m.

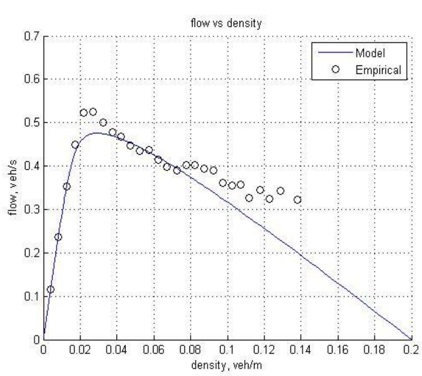

Figure 10 Passive Platoons

The curve in Figure 10 has a γ-value of -0.031, This accounts for the more rounded shape of the curve, and why it appears as almost a perfectly straight descending line after density of 0.04 veh/m. As γ-values get closer to zero, the reverse lambda starts to disappear. If γ is zero, then the curve resembles a triangle, corresponding to less variation among driver psychology and resembles car-following models that use linear algorithms, such as Greenshields..

CHAPTER VI

DERIVATIVES AND WAVE SPEEDS

The derivative of the standard LCM macroscopic equation, $v = v_f \left[1 - e^{1-\frac{k^*}{k}}\right]$ can be found by first rearranging the equation to appear as$k = \frac{1}{\gamma v^2+\tau v+\ell\left[1-\ln\left(1-\frac{v}{v_f}\right)\right]}$, where γ, aggressiveness, τ, average response time, and v_f, free flow speed are defined in the same ways as before. Given that $q = kv$, then $\frac{dq}{dk} = \frac{d(kv)}{dk}$. To prove that$\frac{dv}{dk} = \frac{1}{\frac{dk}{dv}}$, suppose that $v = f(k)$ defines v as a function of k. Differentiate both sides with respect to v, and the result is: $1 = \frac{df(k)}{dv}$.

Applying the chain rule to the right side of equation (11), yields $1 = \frac{df(k)}{dv} = \frac{df(k)}{dk}\frac{dk}{dv} = \frac{dv}{dk}\frac{dk}{dv}$. Therefore, $\frac{dq}{dk} = \frac{d(kv)}{dk} = v + k\frac{1}{\frac{dk}{dv}}$. After substituting the previous equation $k = \frac{1}{\gamma v^2+\tau v+\ell\left[1-\ln\left(1-\frac{v}{v_f}\right)\right]}$ into this chain, the result can be simplified as $(k)\left(\frac{1}{\frac{dk}{dv}}\right) = -\left(\frac{(\gamma v^2+\tau v+\ell)\left[1-\ln(1-\frac{v}{v_f})\right]}{(2\gamma v+\tau)\left[1-\ln(1-\frac{v}{v_f})\right]+(\gamma v^2+\tau v+\ell)\frac{1}{v_f-v}}\right)$. Plugging this result into equation, $\frac{dq}{dk} = v + (k)\left(\frac{1}{\frac{dk}{dv}}\right)$; yields $\frac{dq}{dk} = v - \frac{(\gamma v^2+\tau v+\ell)\left[1-\ln(1-\frac{v}{v_f})\right]}{(2\gamma v+\tau)\left[1-\ln(1-\frac{v}{v_f})\right]+(\gamma v^2+\tau v+\ell)\frac{1}{v_f-v}}$. (8)

Given that spacing, $s = \frac{1}{density} = \frac{1}{k}$, a similar derivative calculation can be made by inserting ds into equation (8), and solving for $\frac{dv}{ds} = \frac{1}{\frac{ds}{dv}}$. The result can also be simplified at jam density, such that when spacing is reduced to the length of a car, ℓ, and $v = 0$, then $\frac{dv}{ds} = \frac{1}{\tau+\frac{\ell}{v_f}}$. (9)

This result in equation (9), which represents the value of the slope of a tangent line to any point on a *q*-*k* graph of a macroscopic LCM equation, is known as kinematic wave speed, or ω_i. The value for

ω_i takes on a clearer meaning under jam density conditions, because $k = k_j$, and $v = 0$. Plugging these conditions into equation (9) zeroes out many of the terms and the derivative simply becomes;

$$\frac{dq}{dk} = \frac{\ell}{\tau + \frac{\ell}{v_f}}. \qquad (10)$$

The nature of many flow-density plots means that the simplified derivative in equation (10) often applies to a significant portion of a typical q-k graph. For example, the slope of the tangent at k_j often continues running in a fairly straight line between jam density and a certain point, defined as(k_m, q_m). In this case, the derivative for $\frac{dq}{dk}$ at jam density roughly holds for a majority of the graph, implying that $\omega_i = \frac{dq}{dk} \approx \frac{q_m - 0}{k_m - k_j}$. Thus, the simplified $\frac{dq}{dk} = \frac{\ell}{\tau + \frac{\ell}{v_f}}$ expression yields a kinematic wave speed, ω_i, that is frequently applicable for measuring wave speed from capacity condition, all the way through jam density.

The connection between driver aggressiveness and the LCM's wave speed term $\left(\frac{\ell}{\tau + \frac{\ell}{v_f}}\right)$, lies within the average response time parameter τ, which in turn is heavily influenced by density. Sudden changes in driver response time before and after a sudden, unexpected deceleration related to aggressiveness therefore play a role in kinematic wave speed.

According to a driver psychology report by Forbes in 1963, when cars are following each other on a densely packed road, drivers become more alert and aggressive, and a sudden change in driver response time is reflected in the length of headways before and after a leading car aggressively slows down.(21) The study states that, "time headways of the experimental platoon were in the 1 to 1.5 second range while following at constant speed before the slow-down, but about twice as large after the experimental and unexpected deceleration by the lead car."

In other words, a sudden or aggressive deceleration or stop by a leading car has a tendency to "wake up" a following driver. The following driver then typically pays more attention to the road and tends to leave more headway between themselves and their leader in case they need to suddenly decelerate again based on actions of the aggressive leader. As volume increases, driver response time to leading vehicles becomes a larger factor and limits headway and flow. If drivers respond not only to the car immediately ahead, but also to other vehicles and conditions on the road upstream, the average response time, measured in seconds, effectively increases by a factor related to the free-flow speed of the road. Appropriately, the Longitudinal Control Model predicts faster wave speeds in situations when drivers focus on sharpening their average response times. If the LCM's τ parameter was cut in half, then $\omega_i = \frac{dq}{dk} = \frac{\ell}{(\frac{\tau}{2})+\frac{\ell}{v_f}}$ increases greatly.

While Forbes developed a predictive equation for wave speed, ω_F, as $\frac{dq}{dk} = \frac{\ell}{\tau}$, the Longitudinal Control Model derives a similar equation, but postulates an extra term such that; $\omega_{LCM} = \left(\frac{\ell}{\tau+(extra\ margin)}\right) = \left(\frac{\ell}{\tau+\frac{\ell}{v_f}}\right)$.

This extra margin of $\frac{\ell}{v_f}$ in the denominator is generally fractions of a second in size and corresponds to the time needed to pass a vehicle, since the distance of a car, ℓ, divided by the free-flow speed of the road, v_f, is roughly equivalent to the time, in seconds, needed for a fast-moving car to pass its leader.

Research by Lighthill (22), states that mean headway or "temporal space," increases or decreases by about a meter for every kilometer per hour of speed. Using this stipulation for headway as a guideline, the LCM model roughly reflects a change in headway by about the same proportion as this extra margin. At low values of concentration, the mean speed, $v = \frac{q}{k}$, has been regarded as a function of q, flow. Speed drops as q increases, and "at high values of concentration, most writers have regarded the mean headway as a function of mean speed…At $v = 0$, the mean headway takes a value only just greater than the average vehicle length." As headway increases, flow decreases, and the LCM model predicts an appropriate decrease in wave speed, or rate of change of flow with respect to density.

For example, a highway that prohibits trucks and has a high free-flow speed of 40 m/s would be filled with small vehicles that may average 6 meters in length. Given these conditions, drivers would be traveling at their own pace either in platoons or individually as stragglers, and may generally have a more lax response time of 2.5 seconds because they are not as concerned with “future” upstream road conditions. The wave speed on this road would be relatively small by LCM standards, and can be calculated as $\omega_i = \frac{\ell}{\tau+\frac{\ell}{v_f}} = \frac{6}{2.5+\frac{6}{40}} = 2.26\, m/s$. This means that the effects of events upstream such as crashes or bottlenecks do not ripple along the distance of the road very quickly and thus may not influence all the road’s drivers.

In the opposite scenario, longer cars, trucks, and heavy vehicles on a highway all make traffic denser more quickly and heighten the awareness of other drivers on the road due to their size. These larger vehicles may have an average length of 12 meters and travel more slowly, creating higher volume on the route. If a highway is used as a frequent passage way for trucks, platoons will be more tightly packed and the free-flow speed of the road may be in the range of 20 m/s. Drivers on this highway would be much more focused because they find themselves surrounded by other cars, leading to a heightened reaction time, which could be as low as 0.7 seconds.(23) Any events upstream like crashes, bottlenecks, or events initiated by an aggressive driver will ripple along the road much faster. The wave speed predicted by the LCM in this case would be: $\omega_i = \frac{\ell}{\tau+\frac{\ell}{v_f}} = \frac{12}{0.7+\frac{12}{20}} = 9.23\, m/s$.

Based on empirical data collected from six highways around the world (Table 1), average response time is generally in the range of about1second, while free-flow speed of a highway is generally on the order of 25 to 30 m/s (55 to 70 mph). Thus, the macroscopic LCM can be expected to predict wave speeds generally in the range of ω_i = {0 m/s, 10 m/s}. In particular, the LCM assigns interstates in the following cities with following wave speeds; Atlanta: 0.45 m/s, Orlando: 2.2 m/s, Germany: 1.9 m/s, California: 0.86 m/s, Toronto: 3.68 m/s, Amsterdam: 1.62 m/s.

CHAPTER VII

CONCLUSIONS AND FUTURE DIRECTIONS

The many traffic flow theory formulations that have been offered throughout the past several decades have stimulated much fundamental thinking on important phases of traffic flow theory. At present, to explain different slow-down or speed-up relationships on highways between two cars, models which are slightly more complicated than traditionally posed may be more useful tools when trying to account for human factors. (24) To this end, the Longitudinal Control Model is designed to include flexibility for spacing algorithms, such as the Gipps equation, which includes deceleration parameters. These deceleration parameters allow the LCM to model driver dynamics on a more refined scale than predecessors formulated by Greenshields, Pipes or Van Aerde. The inclusion of a "desired speed" parameter also sets the LCM apart from other models, because the unfulfilled desire for speed factors into the resulting acceleration that a driver seeks on the road.

As demonstrated in the second section of this paper, the LCM can further incorporate any microscopic spacing algorithms as desired to produce intrinsic and distinct microscopic limits, as well as generate a range of predicted values of acceleration that match the full empirical range of previously cited field data. In order to produce these acceleration estimates, the model's user may employ perception-reactions times defined by AASHTO that correspond to various environments, while only requiring a user to collect field data relating to speed and spacing.

Given that the Longitudinal Control Model produces acceleration estimates that fall within the minimum and maximum empirical ranges that exist on a highway, the model may be used to reliably back-calculate the parameters of speed or spacing. If all parameters are known except for desired spacing, then the LCM could be used as a back-calculation algorithm to benchmark the validity of any desired spacing algorithm. For example, if empirical values for speed, acceleration, desired speed, and perception-reaction time are all evaluated by the Longitudinal Control Model, and the LCM stipulates that a driver's desired spacing from their leader's bumper is *x* meters, this would provide a target value to test the accuracy of pre-existing desired spacing algorithms such as the Gipps, General Motors models, or any other equations that attempt to predict desired spacing. Most of the parameters included in the Longitudinal Control Model are also commonly found in traffic software programs, making the microscopic LCM a suitable candidate to be adopted for use within a computer-based traffic simulation program.

Macroscopically, the model provides similarly useful results because flow-density curves intuitively relate to aggressiveness and spacing, and extremely low conditions of density force low speeds, while extremely high speeds are achievable only at low densities. Vehicle platoons operated by drivers who can generally tolerance quick decelerations are able to maintain much higher flow rates under higher densities, until a delicate equilibrium breaks, resulting in a road condition that quickly cuts the road's flow roughly in half before bringing the platoon to jam density. Conversely, within vehicle platoons operated by drivers who desire a more comfortable deceleration, flow never reaches levels as high as those of "aggressive" platoons, and speeds slowly decrease along with flow and density, leaving increasingly larger gaps in spacing as time passes. The ability for the Longitudinal Control Model to measure and predict road densities based on subtle differences in prevailing driver characteristics relating to deceleration, may make the macroscopic LCM a better tool in calculating the level of service on a highway. While some models may use more linear equations that relate speed and flow to density, the Longitudinal Control Model can be used to predict more dynamic estimates for density, resulting in more realistic ranks for level of service. In terms of measuring wave speed, section four of this paper also indicates that the extra terms in the wave equation may provide better estimates than those created by predecessors such as Forbes.

Unlike some other models, the LCM possess a strong microscopic and macroscopic link, contains no discontinuities, no piecewise components, and offers some solutions for loopholes in flow-density relationships emphasized in previous models. Future work on the model could include developing more links between the LCM and other equilibrium models, more testing of driver psychology, and deeper analysis of acceleration predictions. Further study involving how the average response time parameter τ, is related to aggressiveness, may also provide new insights into better understanding the boundary conditions and data-fitting capabilities of the Longitudinal Control Model.

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