Development of a Cost Minimizing Strategy to Mitigate Bird Mortalities in a Wind Farm

Karamvir Singh
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DEVELOPMENT OF A COST MINIMIZING STRATEGY TO MITIGATE BIRD MORTALITIES IN A WIND FARM

A Thesis Presented

by

KARAMVIR SINGH

Submitted to the Graduate School of the University of Massachusetts Amherst in partial fulfillment of the requirements for the degree of

MASTER OF SCIENCE

May 2012

INDUSTRIAL ENGINEERING AND OPERATIONS RESEARCH
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Karamvir Singh
ABSTRACT

DEVELOPMENT OF A COST MINIMIZING STRATEGY TO MITIGATE BIRD MORTALITIES IN A WIND FARM

MAY 2012

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Wind is the second largest renewable energy source after solar. It is one of the fastest growing sources of electricity in the world and currently 29,440 Megawatts of wind energy is installed in the United States and an additional 5,866 MW is under construction (Office of Energy and Environment Affairs, 2011). For the growth of wind electricity, one of the most prominent environmental concerns relates to the death of birds, bats and other avian species resulting from collision with turbine blades.

This thesis develops a model that provides the optimal strategy of turning the turbines off in a wind farm for certain periods to mitigate bird mortalities. We first create a single turbine optimization model for each hour on each day of a single month. We maximize the expected revenue generation and limit the expected bird mortalities to a certain level to solve for the dates and times for which the turbine should be turned off. The optimization problem is found to be part of common class of problems called Knapsack problems and through experiments we conclude that a linear programming (LP) relaxation of the problem provides
a near-optimal solution. We extend the single-turbine model to a multiple-turbine model applicable to a wind farm. In this case, we solve for the percentage of wind turbines that should be turned off to limit the expected bird mortalities to a certain level. Finally, we carry out an uncertainty analysis and estimate probability distributions over the outcome of optimal strategy of turning the turbine off.

We consider the Cape Wind project as a case study and limit the analysis to only one species of endangered birds called the common loon. We find that in order to save an expected number of 10 such birds in the month of March; we need to turn the turbine off for a total of 23 hours spread over specific dates and times. The average cost per bird was found to be $171.
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CHAPTER 1

INTRODUCTION AND BACKGROUND

1.1 Introduction

This thesis provides a strategy to mitigate bird mortalities in a wind farm caused due to collision with turbine blades. The strategy that is proposed is to turn the turbines off for a certain period. The dates and times for which turning the turbines off is most beneficial is governed by the expected revenue generation and the expected bird mortalities during that period.

The thesis develops a model that estimates the time periods for which turbines should be turned off to save a certain (average) number of target birds in a particular month. The Cape Wind Project has been considered as a case study. Only one species of endangered bird, the common loon, has been considered for study. The model that is developed provides the lowest cost dates and times for which the turbines should be switched off to save a given number of common loons in the month of March.

1.2 Background

Many elements of human society and the environment are sensitive to climate variability and change. Human health, agriculture, natural ecosystems, coastal areas, and heating and cooling requirements are examples of climate-sensitive systems. Global climate change has already had observable effects on the environment. Glaciers have shrunk, ice on rivers and lakes is breaking up earlier, plant and animal ranges have shifted and trees are flowering sooner. Scientists have high confidence that global temperatures will continue to rise for decades to come, largely due to greenhouse gasses produced by human activities. The Intergovernmental Panel on Climate Change (IPCC), which includes more than 1,300 scientists from the United States and other countries, forecasts a temperature rise of 2.5 to 10 degrees Fahrenheit over the next century.
It is critical to develop economically acceptable global technology solutions to counter the uncertainty in energy supply while alleviating the current climatic conditions. Wind energy is a massive power source that is available virtually everywhere in the world. There are no fuel costs, no geo-political risk and no supply import dependency. Wind power is a clean, emissions-free power generation technology. Like all renewable sources it is based on capturing the energy from natural forces and has none of the polluting effects associated with ‘conventional’ fuels.

Not only is wind energy a power generation technology that can deliver the deep cuts in CO$_2$ emissions the world needs to combat the worst effects of climate change, it also provides numerous other environmental benefits. It has a positive effect on air pollution, which is choking cities around the world, by not emitting dangerous air pollutants as other generation technologies do. Wind energy does not produce any toxic waste. And, in addition, wind energy uses virtually no water, which, in an increasingly water-stressed world, is a major environmental consideration.

The growth of the market for wind energy is being driven by a number of factors, including the wider context of energy supply and demand, the rising profile of environmental issues, especially climate change, and the impressive improvements in the technology itself. Over the past ten years, global wind power capacity has continued to grow at an average cumulative rate of over 30%, and 2008 was another record year with more than 27 GW of new installations, bringing the total up to over 120 GW. Wind energy has grown into an important player in the world’s energy markets, with the 2008 market for turbine installations worth about €36.5bn. The wind industry also creates many new jobs: over 400,000 people are now employed in this industry and that number is expected to be in the millions in the near future (Global Wind Energy Council, 2011).

For wind electricity, one of the major environmental concerns relates to the death of birds, bats, and other avian species that can fatally collide with turbine towers, blades, and power lines, an issue termed ‘bird mortality’. Many ecologists, biologists, ornithologists, and environmentalists at large have spoken out against wind power on the grounds that it presents too great a risk to avian wildlife.
Studies have generally noted that onshore and offshore wind turbines present direct and indirect hazards to birds and other avian species. Birds can smash into a turbine blade when they are fixated on perching or hunting and pass through its rotor plane; they can strike support structures; they can hit parts of towers; or they can collide with associated transmission and distribution (T&D) lines. These risks are exacerbated when turbines are placed on ridges and upwind slopes, built close to migration routes, or operated during periods of poor visibility such as fog, rain, and at night. Some species, such as bats, face additional risks from the rapid reduction in air pressure near turbine blades, which can cause internal hemorrhaging through a process known as barotrauma (Baerwald et al., 2008). Indirectly, wind farms can positively and negatively physically alter natural habitats, the quantity and quality of prey, and the availability of nesting sites (Fielding et al., 2006; National Wind Energy Coordinating Committee, 1999).

The rest of the thesis has been organized as follows:

Chapter 2 presents a relevant literature review on the methods that have been adopted to estimate the bird mortalities in a wind farm and the measures previously suggested to mitigate bird mortalities. Chapter 3 uses data to estimate the probability distribution for four random variables – energy generated, electricity price, bird mortality and net revenue on an hourly basis for each day of the month of March. Chapter 4 formulates the optimization problem for both single turbine and multiple turbine systems; and solution methods are discussed. Chapter 5 presents and discusses the results obtained from solving the optimization model. Chapter 6 presents an uncertainty analysis over the results and discusses how policy decisions can be made under uncertainty. Chapter 7 discusses the limitations of the current work and provides a scope for future work. Finally, Chapter 8 concludes the thesis.
CHAPTER 2

LITERATURE REVIEW

In this Chapter we review the literature relevant to the thesis. This includes reviewing studies on bird mortality estimates and previous bird mortality mitigation studies.

2.1 Bird Mortality estimates

In this section, we review two methods widely used to estimate bird mortalities in a wind farm – Counting method and the Collision risk model (CRM) method. The estimation of bird mortalities is particularly significant since it serves a baseline to assess the future mitigation measures and also provides an assessment of the potential impacts of other proposed wind farms.

2.1.1 Counting method

This approach involves counting the bird carcasses within a certain region of the wind farm for a given period of time.

Orloff and Flannery (1992) carried out a study in the Altamont Pass Wind Resource Area, California (APWRA) and concluded that 96% of the carcasses deposited by wind turbines were less than 50 meters from the turbines. Smallwood and Thelander (2008) estimated the bird mortality in the APWRA by searching bird carcasses within 50 meters of 4,074 turbines for periods ranging from 6 months to 4.5 years. Scavenger trials were used to estimate carcasses that are not found due to scavenger removal and searcher error. Such conventional trials generally place $\geq 10$ carcasses at once within small areas already supplying scavengers with carcasses deposited by wind turbines. The mortality rates were then adjusted for scavenging rates to estimate the annual wind turbine caused bird mortalities. The adjusted annual bird mortality rate was found to be 4.7 deaths per MW per year. Smallwood et al. (2010) used novel scavenger removal trials to estimate the scavenger removal rates and searcher detection error. To avoid scavenger swamping, which might bias mortality estimates low, Smallwood et al. placed only 1-5 bird carcasses at a time amongst 52 turbines of the APWRA region. Each carcass was monitored by a motion-activated camera. The mortality
rates were again adjusted to estimate annual wind turbine caused mortalities. The adjusted annual bird mortalities were found to be 7.8 deaths per MW per year. It is noted that there is a significant difference in annual mortalities using novel scavenger removal trials and conventional trials.

Kuvlesky et al. (2007) concluded that the risk of bird death differs according to weather, layout of wind farm, type of wind technology, specific bird migration routes, and topography, along with the particular bird species and number of birds found in the area. The Table 2-1 (Sovacool, 2009) shows the variation in bird mortality per turbine per year for different wind farms:

<table>
<thead>
<tr>
<th>Source</th>
<th>Location</th>
<th>Bird mortality (deaths/turbine/year)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Kunz et al. (2007)</td>
<td>United States</td>
<td>1.3-38.2</td>
</tr>
<tr>
<td>Kuvlesky et al. (2007)</td>
<td>Europe and the United States</td>
<td>0-30</td>
</tr>
<tr>
<td>Winegrad (2004)</td>
<td>United States</td>
<td>1.8-7.5</td>
</tr>
<tr>
<td>Osborn et al. (2000)</td>
<td>United States</td>
<td>1.6</td>
</tr>
<tr>
<td>Lubbers (1988)</td>
<td>Denmark</td>
<td>0.8</td>
</tr>
<tr>
<td>Marsh (2007)</td>
<td>Spain</td>
<td>0.2</td>
</tr>
<tr>
<td>Lowther and Stewart (1998)</td>
<td>United Kingdom</td>
<td>0</td>
</tr>
</tbody>
</table>

| **Table 2-1: Estimates of bird mortality at different wind farms (Sovacool, 2009)** |
| It should be noted that counting method will not be applicable to estimate bird mortality in offshore wind farms since it would not be possible to count the number of dead birds. This is because the birds would sink in the water after collision with wind turbines. |

**2.1.2 Collision Risk Models**

In this section, we review the collision risk models used to estimate bird mortalities in a wind farm.

The model of Band et al. (2005) used data describing the structure and operation of turbines: number of blades, maximum cord width, pitch angle of blades, rotor diameter, rotation speed,
bird size, body length, wingspan, flight speed, flapping and gliding flight to derive a probability of collision. This approach was found to be generally sound mathematically (Chamberlain et al. 2005). Sensitivity analysis showed that key parameters in determining collision risk were bird speed, rotor diameter and rotation speed. Band et al. estimated the probability of collision as the bird passes through the rotors to be in between 0.10 to 0.15. Mortality was estimated by multiplying the collision probability by the number of birds passing through the area at risk height, determined from survey data.

Desholm et al. (2006) examined the estimation and use of avoidance rates in conjunction with Band collision risk model. The avoidance rate was defined as the probability of a bird taking action when encountering a turbine. The mortality rate was calculated by multiplying the collision risk probability with the non-avoidance rate. The bird mortalities were estimated by multiplying the mortality rate with the number of birds passing through the risk height. Painter et al. (1999) estimated an avoidance rate of 0.9962 for gulls and Madders (2004) estimated it at 0.9950 for Golden Eagles. Using these avoidance rates, Desholm et al. concluded that including avoidance rates in the Collision Risk Models can drastically impact the bird mortality rate and the resulting mortality estimation.

The Band model assumed that birds have straight flight path which is parallel to the ground. Holmstrom et al. (2011) improved upon the Band model by accounting for different angles of bird approach. It was demonstrated that the angle of approach between flight path and turbine orientation had a significant effect on the collision probability and resulting mortality estimates. It was found that collision probabilities are higher in case of oblique angle of approach (up to 25% higher at certain angles in comparison to Band model).

It is observed that taking avoidance rate into consideration makes the mortality rate very low. We note that taking a collision risk of 15% (Band et al., 2005) and an avoidance rate of 99% would yield a mortality of just 0.15%.

2.2 Mitigation measures

In this section, we review some bird mortality mitigation strategies that have been previously suggested.
Tucker (1996) developed a mathematical model for collision between birds and propeller-type turbine rotors and identified variables that can be manipulated to reduce the probability of bird collision. The study defined a “safety index” that allows rotors of different sizes and designs to be compared in terms of wind energy converted to electrical energy per bird collision. The collision model accounted for variations in wind speed during the year and showed that for model rotors with simple, one-dimensional blades, the safety index increases in proportion to rotor diameter, and variable speed rotors have higher safety indexes than constant speed rotors. It was found that the safety index can also be increased by enlarging the region near the center of the rotor hub where the blades move slowly enough for birds to avoid them. Painting the blades to make them more visible was also found to increase the safety index.

Erickson et al. (2001) concluded that turbines that are more widely spaced and operated at lower rotor speed (rotations/min) are safer for birds. Hunt (2002) found that larger turbines would be safer for golden eagles. But Orloff and Flannery (1992) and Smallwood and Thelander (2004, 2005) found that turbines with larger rotor-swept areas killed more of some raptor species.

As a part of the mitigation measure, Alameda County in California suggested replacing the old-generation wind turbines with new repowered wind turbines in the Altamont Pass Wind Resource Area (APWRA). Smallwood and Karas (2009) studied the bird mortality rates at Old-generation and Repowered wind turbines in APWRA. It was found that repowered wind turbines reduced bird mortality up to 65% for some birds on power generation basis. The overall adjusted bird mortality fell from 7.8 deaths per MW per year for old turbines to 5.6 deaths per MW per year for new turbines. The main reason for this is that the repowered wind turbines are more efficient and almost double the wind energy generation.

To test one mitigation option aimed at reducing bat fatalities at wind energy facilities, Baerwald et al. (2009) altered the operational parameters of 21 turbines at a site with high bat fatalities in southwestern Alberta, Canada, during the peak fatality period. It is known that more bat fatalities occur in low wind speeds (Fiedler 2004) and that non-moving turbine blades do not kill bats (Arnett 2005). Baerwald et al. examined whether reducing the amount that turbine rotors turn in low wind speeds would reduce bat fatalities. This was done either
by changing the wind-speed trigger at which the turbine rotors were allowed to begin turning or by altering blade angles to reduce rotor speed. The blades were nearly motionless in low wind speeds and this resulted in a significant reduction in bat fatalities (by 60.0%).

We did find any literature that considers the trade-off between expected revenue generated and expected bird mortality. All previous work corresponds to relating power produced with bird mortalities. The mortality rate in terms of number of bird deaths per MW of power produced has been calculated at many different wind sites and is widely cited in literature.
CHAPTER 3

PROBABILITY MODELING

In this chapter, we carry out a case study of the Cape Wind project area site by using historical data of wind speed, electricity price and birds observed at the site. The case study estimates probability distributions of the revenue generated by a turbine and the bird mortalities at the site for each hour of the day for all days of the month of March. We find the probability distribution over hourly revenue by combining the distribution of energy and electricity price using Monte Carlo sampling method.

3.1 Energy

In this section, we derive a probability distribution over the average energy produced on each hour of the day for each day of the month of March by an off-shore wind turbine in the Cape Cod bay area.

3.1.1 Data Analysis

This section gives the method used to analyze the wind speed data and derive the probability distribution of energy from it.

The energy produced in time \( t \) is given by the power produced in time \( t \) multiplied by the time \( t \). Since, we are considering only one hour time intervals the magnitude of energy produced in an hour is equal to the power produced in the same hour. For our analysis, we conclude that the probability distributions of energy will be same as the probability distributions of power. So, we will first estimate the distributions of power and then simply say that the distributions of energy produced look the same. It is, however, noted that the unit of energy will be different from that of power. For energy, the unit is Kilo-Watt hours while for power it is Kilo-Watts.

The wind speed data from a buoy in Boston harbor is collected (National Data Buoy Center, 2011). The anemometer height is 5 meters above the sea level. For our analysis, we assume that this is a good approximation of the wind speed in the Cape Wind project area. The data
contains average wind speed on each hour for 20 years (1984-2003). About 5% of data points are missing due to unavoidable reasons (icing, broken sensors etc.).

The power output of a wind turbine varies with wind speed and every wind turbine has a characteristic power performance curve. With such a curve, it is possible to predict the energy production of a wind turbine without considering the technical details of its various components.

Power curves for existing machines are obtained from the manufacturer. The curves are derived from field tests, using standardized testing methods. We know the power curve of a land based GE wind turbine in the form of tabular data (GE Energy, 2010). We have 48 wind speeds data points and the corresponding power produced at those speeds. The power produced is plotted against the wind speed (see figure 3-1). The cut-in speed or the speed at which the turbine starts to deliver useful power is 3 m/s. The cut-out speed or the maximum speed (usually limited by engineering design and safety constraints) at which the turbine is allowed to deliver power is 25 m/s. The rated power or the maximum power generated by this wind turbine is 1500 KW.

![Power Curve of a GE turbine](image)

**Figure 3-1: Power Curve of a GE turbine**

We are interested in simulating an off-shore wind turbine for the Cape Wind project. So, for our newer off-shore wind turbine, we scale the power produced at each of the 48 wind speed
data points by a factor of 2.4 (since newer wind turbines have higher power rating). The maximum power generated by the off-shore wind turbine is 3600 KW. For simplicity, we assume the cut-in and cut-out speeds to be same for new off-shore turbine. To find the power produced between any two wind speed data points, we use linear interpolation. Thus, we estimate the power produced at all our wind speed data points of 20 years.

The probability mass function (pmf) of power is estimated by plotting histograms of generated power for each hour of the day. For simplicity, we assume no variation in the diurnal wind statistics over the course of the month of March. This means that each day of the month of March is considered same (which may not be true in reality). For each hour we have approximately (20*31) wind speed data points. Here, 20 denote the number of years for which we have the wind speed data and 31 is the number of days in the month of March. Each bar in the histogram represents the fraction of total data points of power that lie within a particular interval. We interpret the histogram as the probability mass function such that each bar in the histogram gives the probability that power produced will lie within a certain range. Thus, we have a discrete probability distribution for the power produced on each hour of the day: for each day we have 24 histograms each corresponding to an hour of the day. We say that the corresponding distributions of energy produced on hourly basis are same as the distributions of power.

The 24 histograms each corresponding to an hour of the day are same for all 31 days of the month of March since each day of the month of March is considered to be same.

3.1.2 Energy histograms

The figure 3-2 shows the histogram of the average energy produced between 8:00 PM and 9:00 PM for all days in March.
Figure 3-2: Histogram of average energy between 8:00 PM and 9:00 PM

The histograms at all other hours of the day show a similar trend. The following observations are made by looking at all the 24 histograms (all histograms not shown here):

- All histograms are bi-modal (2 peaks). This is because the wind speed data cluster around two intervals of wind speeds – very low wind speed (0-3 m/s) and very high wind speeds (>15 m/s). From the power curve (see figure 3-1), we note that these intervals correspond to zero and maximum power respectively. So, the energy histograms (which are actually same as power histograms) have two peaks – each corresponding to zero energy and maximum energy.

- At each hour there is at least 25% probability that no energy will be produced (due to a very low wind speed).

- Towards the late afternoon and evenings (from 2 PM to 7 PM), the probability of zero energy generation is comparatively higher.

- During the night at 10 PM and from 12 AM to 1 AM, the wind speeds are high and the probability of zero energy production is lowest.
The histograms are used to estimate the discrete probability mass function which is then used to find the expected energy at each hour.
To get further insights into the trend of energy generation, the expected energy (or, mean energy) is calculated at each hour and plotted in figure 3-3.

![Figure 3-3: Expected energy on each hour of the day](image)

It is noted that the expected energy is remarkably low between 3:00 PM to 7:00 PM. And, the expected energy is higher on and around mid night.

The Appendix-I gives the MATLAB code for generation of energy histograms.

### 3.2 Price
In this section, we derive a probability distribution over the price of electricity on each hour of the day for each day of the month of March.

#### 3.2.1 Data Analysis
This section gives the method used to analyze price data and derive probability distributions from it.

The location marginal price of electricity is the cost to serve the next MW of load at a specific location, using the lowest production cost of all available generation, while observing all transmission limits. The location marginal price (LMP) of electricity over the last 7 years in the Southeastern Massachusetts zone, known as the SEMASS zone, is
collected (ISO New England, 2011). The electricity price in this zone is a good approximation for the price in Cape Wind project site. So, we have hourly electricity prices since de-regulation (03/2003-04/2011).

For simplicity, we have assumed that the price does not show much variation on weekdays and weekend (or on other holidays). We assume no variation in diurnal price statistics over the course of the month of March. In other words, each day of March is considered the same. The histograms of LMP’s are plotted for each hour of the day to estimate the probability mass function. For each hour, we have approximately \((07*31)\) electricity price data points. Here, 7 denote the number of years for which we have LMP data points and 31 is the number of days in the month of March. Each bar in the histogram represents the probability that electricity price will lie within a particular range. Thus, we have a discrete probability distribution for the electricity price for each hour of the day. All days for the month of March have the same set of 24 price histograms since each day of the month is considered same.

### 3.2.2 Price histograms

The figure 3-4 shows the histogram of electricity price between 8:00 PM and 9:00 PM for all days in March.
Figure 3-4: Histogram of electricity price between 8:00 PM and 9:00 PM

It is noted that all price histograms are not smooth and there are some missing bars in the histograms (meaning that probability of price in the corresponding interval is zero). The figure 3-5 shows the histogram of price between 7:00 AM and 8:00 AM. It is noted that this histogram is not smooth and is more spread at the tails.
Figure 3-5: Histogram of electricity price between 7:00 AM and 8:00 AM

The following observations are made by looking at all the 24 price histograms (*all histograms not shown here)*:

- All histograms are unimodal. The data points are clustered around a single peak. During the night, from 11:00 PM to 5:00 AM, the price data points have a smaller mode. During the day, the mode is higher. This is because the demand is lower during the night and therefore, the price is also low. Hence, data points are clustered around a lower peak.
- Some histograms are more spread at the tails than the others.
- Towards the early morning (between 7:00 AM to 11:00 AM), there is a certain probability that the price will reach extremely high values (up to 23.8 cents per KWh).
- Towards the early evening (between 6:00 PM to 8:00 PM), there is again a probability that price might touch high values. At 7:00 PM, there is a chance that the price might go as high as 28.7 cents per KWh.
To gain further insights into the price trends, the expected value of price (or, mean price) at each hour is calculated and plotted in figure 3-6. The probability mass function is estimated from the histograms and is used in calculation of expected price.

The Appendix-I gives the MATLAB code for generation of price histograms.

![Figure 3-6: Expected price at each hour of the day](image)

The graph shows that very high mean price is reached early morning between 8:00 AM to 12:00 PM. Also, between 7:00 PM to 9:00 PM, price is expected to be comparatively higher. These are the peak periods of demand. It is clear from the graph that expected price is lower during the night than during the day because the demand of electricity falls during the night in the month of March.

It is noted that the price distribution is heavily dependent on the month. A very different pattern of hourly expected price is anticipated for any other month, say July. In July, we expect more demand during the night (and higher price) since it would be warmer and most people would be using air-conditioning.

### 3.3 Revenue
In this section, we derive the probability distribution of the hourly revenue generation for running one turbine for each day of the month of March.
3.3.1 Monte Carlo Sampling

This section gives the Monte Carlo method used estimate the probability distribution of generated revenue.

The value of running a turbine for any hour or the hourly revenue generated by a turbine is given by the relation:

\[ \text{Value of running turbine for any hour} = \text{price} \times \text{energy} \]

In sections 3.1 and 3.2, we have discussed deriving the probability distributions of power and price for each hour of the day. The probability mass function of the value of running the turbine for each hour can be generated by combining these two distributions using the Monte Carlo random sampling method. The random sampling is done for 100,000 times. So, we have 100,000 data points for revenue generated at each hour.

Finally, histograms are plotted to estimate the probability mass function of revenue generated for each hour on each day of the month of March. Each bar in the histogram provides the probability that the revenue lies in a particular interval. Thus, we have a discrete probability distribution for the revenue generation for each hour of the day. All days for the month of March have the same set of 24 revenue histograms.

Once the probability mass function is known, we can calculate the expected revenue for each hour of the day.

3.3.2 Revenue histograms

The figure 3-7 shows the histogram of the value of keeping the turbine running between 8:00 PM and 9:00 PM.
From all histograms it is noted that there is a large probability that no revenue will be generated at a particular hour. This corresponds to the fact that the power histogram had a mode at zero power production. Also, most histograms are not smooth, i.e., we note some bars missing between certain ranges (In Figure 3-7, there is no revenue data point at mean revenue of $350). This is because the price histogram was also not smooth. Some histograms are more spread than others at the tails. For example, the histogram of revenue between 7:00 AM and 8:00 AM (see figure 3-8) is more spread at the tail than the histogram of revenue between 8:00 PM and 9:00 PM (see figure 3-7). We relate this behavior to the fact that the price histogram was more spread between 7:00 AM and 8:00 AM (see figure 3-5) than between 8:00 PM and 9:00 PM (see figure 3-4).
Figure 3-8: Histogram of revenue generated between 7:00 AM and 8:00 AM

The Appendix-I gives the MATLAB code for Monte Carlo simulation used to estimate the probability mass functions.

The maximum energy that can be generated by the turbine is 3600 KWh (from figure 3-2). Also, from the probability mass function of price, the maximum mean price that can be reached between 8:00 PM and 9:00 PM is 0.135 $/KWh (from figure 3-4). So, the maximum limit to the value of running turbine during this time interval is 3600*0.135 = $ 486. This is exactly what the histogram of revenue depicts. We note that the maximum value of revenue generated between 8:00 PM and 9:00 PM lies between $ 475 and $ 525 (see figure 3-7). We can say that the Monte Carlo simulation is giving result consistent with our expectation.

The expected value of keeping one turbine running for each hour of the day has been calculated and tabulated in Appendix-II. To make an easy comparison, we have reproduced the expected energy, expected price and expected revenue graphs all in one page (see figure 3-9).
Figure 3-9: Comparison in trends of expected energy, expected price and expected revenue on each hour of the day
We note that this trend is similar to what we expect by multiplication of the expected values of power and price. At many hours, the expected energy lies between 1.2-1.3 MWh, so the expected revenue graph (which can simply be approximated by multiplying power and price) simply follows the trend of expected price graph during those hours. The expected energy graph shows a big dip between 3:00 PM to 7:00 PM. The low value of expected energy during this period also drags the corresponding values of expected revenue lower. For example: the expected price on the 19th hour lies well above the expected price on the 11th hour. But the expected energy is very low on the 19th hour (less than 1.1 MWh). This fact drags the expected revenue on the 19th hour down and we note that the expected revenue on the 19th hour is on level with the expected revenue on the 11th hour.

By observation, it is noted that the expected value of keeping a turbine running is comparatively low during the night.

### 3.4 Bird Mortality

In this section we estimate the probability distribution of bird mortality for each hour on each day of the month of March.

The US Army Corps of engineers released the draft environmental impact statement (EIS) in November’2004 to study the possible impacts of the Cape Wind offshore wind farm on the environment (Cape Wind, 2011). The EIS comprehensively analyzed the possible effects of Cape Wind project on marine species, water quality, terrestrial ecology, wildlife, protected species etc. The EIS also provides data on the number of birds observed near the Cape wind project site. Both aerial surveys and boat surveys were done to find the birds observed in the study area.

We choose an endangered bird, the Common Loon, for our analysis. The Common Loon is protected by the State and Federal law as a migratory, non-game bird.

It is noted that the EIS provides bird data only on certain dates of each month. So, for the month of March, we do a piecewise linear extrapolation to calculate the number of common loons observed on each day. The figure 3-10 shows the number of loons observed on each day of the month of March.
It is noted that a large number of common loons are observed towards the end of March. This corresponds to the start of the migratory period of the bird.

**Figure 3-10: Number of common loons observed on each day of the month**

We assume a uniform distribution for the number of birds observed during the course of one day. Let $p$ denote the probability of bird collision (a collision will lead to mortality). The probability distribution function of the bird mortalities for any hour is modeled by the Binomial distribution, $B (n, p)$ where $n$ is the number of birds observed during that hour and $p$ is the probability of bird collision. So, the probability of $k$ bird mortalities is given by:

$$P[K = k] = C(n, k)p^k(1 - p)^{n-k}$$

where, $C(n, k) = \frac{n!}{k!(n-k)!}$

Thus the number of expected bird (only the common loon) mortalities for any hour on any day of the month of March is estimated by the relation:

$$Expected\ bird\ mortalities\ in\ any\ hour\ of\ the\ day = n * p = \frac{(Total\ birds\ observed\ during\ the\ day) * p}{24}$$
CHAPTER 4

OPTIMIZATION MODELING

In this chapter, we set up the optimization problems in case of a single turbine and multiple turbine systems. The single turbine problem has been formulated as an integer program and the multiple turbine problem has been formulated as a linear program. Solution methods are discussed to solve these problems in Section 4.2.3.

4.1 Optimization problem
In this section, we define the optimization problem that we model in the later sub-sections.

The optimization problem is to maximize the expected revenue subject to the constraint of limiting the expected bird mortalities to $\varepsilon$. We need to find the expected cost minimizing dates and times for which the turbines should be turned off to limit the mortalities to a certain level.

Here, $\varepsilon$ can lie anywhere between 0 and the total number of expected mortalities for the whole month. It is noted that the total number of expected bird mortalities for the month of March is simply the sum of expected bird mortalities on each day.

4.2 Single Turbine Problem
In this section, we formulate the single turbine optimization problem and provide solution methods to find the optimal solution.

4.2.1 Integer Programming formulation
This section develops an integer programming formulation of the single-turbine optimization problem.

Let $x_{ij} = 1$ if the turbine is switched ON for the $j^{th}$ hour of the $i^{th}$ day

$= 0$ if the turbine is switched OFF for the $j^{th}$ hour of the $i^{th}$ day

where, $i = 1, 2, ..., 31$ and $j = 1, 2, ..., 24.$
Let $R_{ij}$ denote the expected value of keeping one turbine running for $j^{th}$ hour on $i^{th}$ day.

Let $O_{ij}$ denote the number of birds observed on $j^{th}$ hour of $i^{th}$ day.

As mentioned before, the birds observed on any day are assumed to be uniformly distributed.

It may be noted that the expected revenue generated hour-wise is same for all days of the month. Also, the number of birds observed is same for all hours on any day of the month.

Finally, the integer program can be formulated as below:

$$\text{Max } \sum_{i=1}^{31} \sum_{j=1}^{24} x_{ij} R_{ij}$$

subject to: $p \sum_{i=1}^{31} \sum_{j=1}^{24} x_{ij} O_{ij} \leq \varepsilon$

$x_{ij} = \{0,1\}$

Here, $\varepsilon$ denote the number of bird mortalities allowed and $p$ is the probability of bird collision. Clearly, the integer program has $31 \times 24 = 744$ binary variables. The integer program is identified as a 0-1 Knapsack problem. It is an NP complete problem. This means that there exists no polynomial time algorithm which can provide an optimal solution to the problem.

4.2.2 Knapsack Problem comparison

In this section we define the classical 0-1 Knapsack problem and compare it with our optimization problem set up in the previous section.

The 0-1 Knapsack problem is defined as follows: Given a set of items, each with a benefit value and a weight, pack the knapsack with a specific weight carrying capacity such that the benefit value is maximum. Each item can be placed only once and a fraction of any item cannot be placed.

Let each item have a weight $w_i$ and benefit value $b_i$ (all $w_i$, $b_i$ and $W$ are integer values). The weight carrying capacity is $W$. 

Mathematically, the 0-1 Knapsack problem can be expressed as

\[
\text{Maximize} \sum_{i=1}^{n} b_i x_i \\
\text{Subject to:} \sum_{i=1}^{n} w_i x_i \leq W \quad x_i \in \{0,1\}
\]

For our optimization problem modeled in previous section, the benefits \(b_i\) correspond to the expected revenue generated each hour. The weights \(w_i\) correspond to the number of expected bird mortalities each hour. The constraint is on the number of bird mortalities and the maximization is on the expected revenue. In our problem, the expected bird mortalities each hour and the expected hourly revenue are non-integer.

### 4.2.3 Solution Methods

In this section, we give two methods commonly used to provide an approximate optimal solution to the 0-1 Knapsack problem and apply these methods to our optimization problem.

The optimal solution for the integer program (IP) can be found by invoking the IP solver in MATLAB. The solver uses a Branch & Bound algorithm to reach the optimal solution.

#### 4.2.3.1 Greedy Algorithm

In this section, we illustrate how the computationally fast greedy algorithm can be used to provide an approximate optimal solution to our optimization problem.

A greedy algorithm is an algorithm that follows the problem solving heuristic of making the locally optimal choice at each stage with the hope of finding the global optimum. We can make whatever choice seems best at the moment and then solve the sub-problems that arise later. The choice made by a greedy algorithm may depend on choices made so far but not on future choices or all the solutions to the sub-problem. It iteratively makes one greedy choice after another, reducing each given problem into a smaller one. In other words, a greedy algorithm never reconsiders its choices.
The Greedy Algorithm can fail to reach near an optimal solution in certain cases. For example: Consider the problem in figure 4-1. Here the objective is to find the largest sum path.

![Figure 4-1: Illustration of Greedy Algorithm](image)

In this case, the Greedy Algorithm will choose 12 instead of 3 in the second stage and will never reach optimal solution.

The Greedy Approximation Algorithm to solve the Knapsack problem involves sorting the items in decreasing order according to the $\frac{b_i}{w_i}$ values. Here, $b_i$ represents the benefits and $w_i$ represents the respective weights. The item with the largest $\frac{b_i}{w_i}$ value is first inserted into the Knapsack and so on until the limit on maximum weight limit is reached.

Applying the Greedy Algorithm to our optimization problem, we divide the hourly expected revenue by the expected number of bird mortalities for the same hour. We get the expected value per bird (dollars/bird) for each hour of each day and we sort the expected values in ascending order.

In order to save $\mu$ number of birds, the approximate optimal strategy (as per the greedy algorithm) is to shut the turbine off for hours having minimal expected value per bird. So, we start with the hour having minimum expected value per bird, then the hour having second minimum expected value per bird and so on till the target number of saved birds, $\mu$ is achieved.
The algorithm is coded in MATLAB to return the date and hours for which the turbine should be turned off to achieve the target number of saved birds. The Appendix-I gives the code.

4.2.3.2 LP Relaxation
This section illustrates how a Linear Programming (LP) relaxation technique can be used to solve our optimization problem.

The LP relaxation of a 0-1 integer program is the problem that arises by replacing the constraint that each variable must be 0 or 1 by a weaker constraint that each variable belong to the interval [0, 1].

To apply the LP relaxation to our optimization problem, we replace the integrality constraint on \( x_{ij} \) by the constraint: \( 0 < x_{ij} < 1 \).

If the optimal solution to the linear program happens to have all variables either 0 or 1, it will also be an optimal solution to the integer program. For a maximization problem, the relaxed linear program has an objective value greater than or equal to the optimal solution of the original problem.

4.3 Multiple Turbines
In this section, we extend the analysis to a wind farm and develop the optimization problem in case of multiple turbines.

4.3.1 Linear Programming Formulation
This section develops the linear programming formulation of the optimization problem in case of multiple turbines. The problem is formulated to solve for the fraction of turbines that should be turned off to save an average particular number of birds in the month of March.

Let \( x_{ij} \) denote the fraction of turbines turned ON in \( j \)th hour of \( i \)th day.

The linear program can be formulated as below:

\[
\text{Max} \sum_{i=1}^{31} \sum_{j=1}^{24} x_{ij} R_{ij} * N
\]
Here, \( N \) is the number of turbines in the wind farm.

Clearly, the linear program has \( 31 \times 24 = 744 \) variables. It is solved by invoking the LP solver in MATLAB.

\[
\text{subject to: } p \sum_{i=1}^{31} \sum_{j=1}^{24} x_{ij} o_{ij} \leq \varepsilon \\
0 \leq x_{ij} \leq 1
\]
CHAPTER 5

RESULTS AND DISCUSSION

In this chapter, we present and discuss the results obtained from solving single turbine and multiple turbine optimization problems. We also compare the results obtained from solving the single turbine problem with different algorithms.

5.1 Optimal Strategy

This section provides the expected cost minimizing strategy to save a particular average number of birds in the month of March for both single turbine and multiple turbine systems. We arbitrarily assume the probability of bird collision to be 1%.

5.1.1 Single Turbine

In this section, we give the expected cost minimizing (optimal) strategy to save a particular average number of birds in the month of March.

In order to save an average of 10 birds in the month of March, the optimal strategy is to turn the turbine off for certain hours on the last two days of the month. The graphs in figure 5-1 and 5-2 shows the hours for which the turbine should be turned off. Here, hour 1 indicates the time between mid-night and 1:00AM and so on. 0 indicates that the turbine is off and 1 indicates that it is on.

![Graph showing optimal strategy for turning the turbine off on 30th March](image)

Figure 5-1: Optimal strategy of turning the turbine off (on 30th March)
Figure 5-2: Optimal strategy of turning the turbine off (on 31st March)

So, by turning the turbine off for 23 hours in the month of March, we can save an average of 10 endangered birds. The expected lost revenue due to shutting off the turbine is $1,715.

The optimal strategy (which is to turn the turbine off for specific hours during the last two days of the month) is driven by various factors. One is that the number of birds observed increase towards the end of the month (See figure 3-10). The numbers of birds observed are actually highest on the last two days of the month and therefore, the expected bird mortalities are also highest during these days. Since the number of birds is uniformly observed during the day, the specific hours of the day for which the turbine should be turned off are governed by the corresponding expected revenue generation. We can see that the hours for which the turbine is turned off mainly correspond to the dips in the expected revenue graph (See figure 3-9). It is noted that the hours do not strictly correspond to dips in expected price and expected power graph (See figure 3-3 and figure 3-6). For example, the 6th hour of the day has a lower expected price than 16th and 17th hour but it does not come in the solution set of hours for which the turbine should be turned off on 30th March. The 16th and 17th hours figure in the optimal solution since they correspond to lower expected revenue as compared to the 6th hour.

5.1.2 Multiple Turbines

In this section, we give the expected cost minimizing (optimal) strategy to save a particular average number of birds in a wind farm for the month of March. The strategy will give the percentage of turbines to be turned off and the corresponding dates and times.
In order to save an average of 10 birds, the optimal strategy of turning a fraction of turbine is given by figure 5-3 and 5-4. We need to turn the turbines off for certain hours on the last two days of the month of March. We note we get a 0-1 kind of solution for all hours except one.

Figure 5-3: Optimal strategy of turning the turbines off for a wind farm
(on 30th March)

Figure 5-4: Optimal strategy of turning the turbines off for a wind farm
(on 31st March)
It is noted that the Linear Program gives an optimal solution only when the number of turbines in the wind farm is very large. Otherwise, the LP gives a near-optimal solution. The following scenario will make this clear:

Let us say that the number of turbines in the wind farm is 100. We solve for the percentage of turbines that should be turned off on each hour of the day for all days of March. Now, the number after the decimal has to be rounded off to zero so as to get the actual number of turbines to be turned off (as an example, we can say that since it is not possible to turn off 36.8 % of turbines in a 100 turbine wind farm, we should turn off 37 % of the turbines as an approximation).

We notice that as the number of turbines in the wind farm becomes larger, the number after the decimal in the percentage value starts to make more sense. For example, it is actually possible to turn off 36.8 % turbines in a 1000 turbine wind farm. So, we conclude that LP formulation for a multiple turbine provides a near optimal solution in most cases.

**5.2 Solution Strategy Comparisons**

In this section we compare the results obtained using the IP solver, Greedy Algorithm and LP relaxation for a single turbine optimization problem.

In order to save 10 birds, the LP relaxation gives an integral solution except for one particular hour (which is 24\textsuperscript{th} hour of 30\textsuperscript{th} March). Since a fractional solution does not make sense for a single turbine problem, we round off the fraction and consider the corresponding hour as an hour for which the turbine should be turned off. All three methods give the same dates and hours for which the turbine should be turned off to save a target of average 10 birds. However, in certain cases (e.g., when the target expected number of saved birds is 20 or 25), the three methods do give different results. The table 5-1 compares the lost revenue due to shutting the turbine for variable numbers of birds saved.
We note that both the LP relaxation (after rounding after fractions) and the greedy algorithm provide an exactly similar solution in all instances.

It is calculated that if the number of birds to be saved is changed to any value, the LP relaxation and the greedy algorithm gives the lost revenue within 2% of what is obtained using the IP solver (see table 5-2). The difference between the lost revenue obtained using IP solver and LP/greedy is never more than $85.

It is noted that the IP Solver is not able to provide any solution for a running time of 10 hours in case the target number of saved birds is 45 and 50. If the analysis is extended to the whole year, the number of binary variables in the optimization problem would be $31 \times 24 \times 12 = 8928$. We anticipate that in such a case, it will be computationally very hard to reach the optimal solution. On the other hand, both LP relaxation and greedy heuristic provide solutions in polynomial time (the average time complexity of a LP solved using Simplex method is polynomial).

Taking the computational savings and closeness to the optimal solution into consideration, we deduce that both LP relaxation and greedy heuristic provides a good approximate solution for our optimization model.

**Table 5-1: Comparison of lost revenue obtained using different solution methods**

We note that both the LP relaxation (after rounding after fractions) and the greedy algorithm provide an exactly similar solution in all instances.

It is calculated that if the number of birds to be saved is changed to any value, the LP relaxation and the greedy algorithm gives the lost revenue within 2% of what is obtained using the IP solver (see table 5-2). The difference between the lost revenue obtained using IP solver and LP/greedy is never more than $85.

It is noted that the IP Solver is not able to provide any solution for a running time of 10 hours in case the target number of saved birds is 45 and 50. If the analysis is extended to the whole year, the number of binary variables in the optimization problem would be $31 \times 24 \times 12 = 8928$. We anticipate that in such a case, it will be computationally very hard to reach the optimal solution. On the other hand, both LP relaxation and greedy heuristic provide solutions in polynomial time (the average time complexity of a LP solved using Simplex method is polynomial).

Taking the computational savings and closeness to the optimal solution into consideration, we deduce that both LP relaxation and greedy heuristic provides a good approximate solution for our optimization model.
5.3 Cost Analysis

In this section, we provide an analysis on the revenue lost due to turning a turbine off for a variable number of saved birds for the month of March. We also derive the marginal cost and study its implications on policy making. All analysis has been done for the Single turbine problem solved using the greedy heuristic. We first report the statistics and then analyze the trends graphically.

The table 5-2 reports the cost, the revenue generated and the marginal cost for a variable number of birds saved. The Marginal cost is defined as the cost to save the last bird. For example, if the target is to save 10 birds in the month of March, then the Marginal cost is the cost to save the 10\textsuperscript{th} bird. It is calculated by finding the total revenue lost when 9 birds are saved and subtracting it from the revenue lost when 10 birds are saved (greedy heuristic used in each case).

<table>
<thead>
<tr>
<th>Target saved birds</th>
<th>Lost revenue (in $)</th>
<th>Revenue generated (in $)</th>
<th>Marginal Cost (in $)</th>
<th>Avg. cost per bird (in $)</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>1,715</td>
<td>63,419</td>
<td>188.16</td>
<td>171.50</td>
</tr>
<tr>
<td>15</td>
<td>2,733</td>
<td>62,401</td>
<td>218.11</td>
<td>182.20</td>
</tr>
<tr>
<td>20</td>
<td>3,910</td>
<td>61,224</td>
<td>236.81</td>
<td>195.50</td>
</tr>
<tr>
<td>25</td>
<td>5,219</td>
<td>59,915</td>
<td>268.16</td>
<td>208.76</td>
</tr>
<tr>
<td>30</td>
<td>6,674</td>
<td>58,460</td>
<td>315.67</td>
<td>222.47</td>
</tr>
<tr>
<td>35</td>
<td>8,306</td>
<td>56,828</td>
<td>368.35</td>
<td>237.31</td>
</tr>
<tr>
<td>40</td>
<td>10,599</td>
<td>54,535</td>
<td>512.80</td>
<td>264.98</td>
</tr>
<tr>
<td>45</td>
<td>14,222</td>
<td>50,912</td>
<td>924.13</td>
<td>316.04</td>
</tr>
<tr>
<td>50</td>
<td>19,145</td>
<td>45,989</td>
<td>1035.87</td>
<td>382.90</td>
</tr>
</tbody>
</table>

Table 5-2: Cost analysis for a variable number of birds saved

The graph between saved birds and lost revenue is plotted in figure 5-5. It is found to be convex.
Figure 5-5: Revenue lost vs. Birds saved

From the convex nature of the above graph, we expect the graph between revenue generated vs. birds saved to be concave. The graph in figure 5-6 is therefore consistent with our expectation.

Figure 5-6: Revenue generated vs. Birds saved

The graph in figure 5-7 shows the average cost per bird vs. the number of birds saved. The graph is convex. This is consistent with the convexity of the graph between total revenue lost vs. birds saved.
Since the graph between average cost per bird and birds saved is convex, it will be interesting to see how the marginal cost compares with the average cost.

It is observed that marginal cost compares somewhat with the average cost per bird when the target number of birds to be saved in the month is small (say, between 10 and 20). When the target number of birds to be saved is large (say, 40 or 50), the marginal cost is significantly larger than the average cost per bird. It is noted that there is a significant jump in the marginal cost when we change the target number of saved birds from 40 to 45. For a target number of saved birds greater than 40, we are actually targeting to save more than 50% of the expected bird mortalities in the month. In order to save 40 birds, the solution set of hours correspond to dates at the end of the month where bird observations are high. But, when the...
target is to save 45 birds, we need to turn the turbine off for certain hours on days earlier in the month when bird observations are much lower. So, when the solution set reaches periods when expected mortalities are not very high, it becomes very costly to save each bird since the turbine might have to turned off for many hours to save that one extra bird. This behaviour is depicted in the marginal cost graph where we see a big jump change the target number of saved birds from 40 to 45.

The marginal cost plays a significant role in policy making when the goal is welfare maximization. Let us assume that we can quantify the benefits from saving endangered birds in numerical figure and that the curve of marginal benefit is known. In such a case, we can find the optimal policy that would be give the number of expected birds that should be saved for welfare maximization. For welfare maximization, we can say,

\[ \text{Marginal Cost} = \text{Marginal Benefit} \]

So, the point of intersection of the marginal cost and marginal benefit curve will give the optimal policy that should be implemented in order to maximize social welfare.
CHAPTER 6

UNCERTAINTY ANALYSIS

In this chapter, we provide the probability distribution over the outcome of optimal strategy of turning the turbine off. The uncertainty analysis is done only for Single turbine system.

6.1 Birds saved

In this section, we derive the probability distribution over the number of birds saved using the optimal strategy of turning the turbine off in the month of March. The probability distribution is found for a given number of expected saved birds.

As discussed in section 3.4, the number of bird mortalities in any hour is a binomial distribution \( B(n, p) \) where \( n \) is the number of birds observed in the corresponding hour and \( p \) is the probability of collision. In order to find the probability distribution over the number of birds saved, we add binomial random variables (corresponding to bird mortalities) for the hours for which the turbine is turned off.

We use the following result to estimate the overall probability distribution: Let \( X_1, X_2, \ldots, X_k \) be independent binomial random variables where \( X_i \) has a Binomial, \( B(n_i, p) \), distribution for \( i = 1, 2, \ldots, k \). Then \( X_1 + X_2 + \cdots + X_k \) has a Binomial \( B(n_1 + n_2 + \cdots + n_k, p) \) distribution.

The graph in figure 6-1 gives the probability distribution over the number of birds saved in the month of March using the optimal strategy for a single turbine system. The probability distributions are plotted for different values of expected birds saved. In the graph, \( \mu \) denotes the expected saved birds.
We note that as the expected number of birds saved becomes higher, the probability plot becomes more spread at the tails. Let $X_1, X_2, X_3$ and $X_4$ denote the discrete random variable of the number of birds saved when the expected number of birds saved is 10, 20, 30 and 40 respectively.

To study the dispersion of probability distribution of each random variable, we calculate the coefficient of variation. The coefficient of variation (CV) is defined as the ratio of standard deviation, $\sigma$ to the mean, $\mu$.

$$CV = \frac{\sigma}{\mu}$$

The standard deviation of a discrete random variable is given by:

$$\sigma = \sqrt{\sum_{i=1}^{N} p_i(x_i - \mu)^2}$$
The CVs of $X_1, X_2, X_3$ and $X_4$ are found to be $0.31, 0.22, 0.18$ and $0.15$ respectively. It is noted that all random variables have $CV < 1$ and hence we conclude that all distributions are of low variance. The coefficient of variation gives a measure of riskiness of the random variables.

On riskiness, the four random variables compare as follows: $X_1 > X_2 > X_3 > X_4$. The riskiness can have a significant impact on policy making. For instance, a policy maker choosing between two policies – one which saves an expected number of 10 birds and the other which saves an expected number of 20 birds must take into consideration that the former is a riskier policy.

In case the expected number of birds saved is 10, there is a 6.6% chance that the only 5 or fewer birds are saved. Clearly, the average cost per bird doubles if exactly 5 birds were saved.

### 6.2 Lost Revenue

In this section, we derive the probability distribution over the lost revenue using the optimal strategy of turning the turbine off in the month of March. The probability distribution is found for a given number of expected saved birds.

Let $Z$ denote the random variable of the revenue lost due to turning the turbine off to save an expected number of 10 birds. Let $S$ denote the set of all the hours and dates for which the turbine is turned off. Then $Z$ can be expressed as a summation of random variables of revenue generated over the set $S$.

$$Z = \sum_{i,j \in S} Z_{ij}$$

Here, $Z_{ij}$ denotes a random variable representing revenue generated in the $j^{th}$ hour of $i^{th}$ day.

The overall probability distribution of lost revenue (or, the random variable $Z$) is estimated by combining the distributions of revenue generated over the hours for which the turbine is turned off using Monte Carlo random sampling method. The sampling is done 100,000 times.
So, we have 100,000 data points of lost revenue. The Appendix-I gives the MATLAB code for Monte Carlo simulation.

Histograms are plotted to estimate the probability mass function of revenue lost in the month of March. Each bar in the histogram provides the probability that the lost revenue lies in a particular interval. Thus, we have a discrete probability distribution for the revenue lost in the month of March due to shutting the turbine off to save an expected number of 10 birds.

The figure 6-2 represents the histogram of the revenue lost in the month of March. Each interval of the histogram is 200 units.

![Histogram of the lost revenue for the month of March](image)

**Figure 6-2: Histogram of the lost revenue for the month of March**

The lost revenue has the peak probability of lying in the range $[1500, 1700]$ dollars. This is comparable to the expected lost revenue which was found be $1,715$ in previous sections.

**6.3 Marginal Cost**

In this section, we discuss the method used to derive probability distribution over the marginal cost of saving the 10\textsuperscript{th} bird. The results are also presented and analyzed.
We find the strategy (i.e., the dates and hours) of turning the wind turbine off in order to save an expected 10 birds. We again run the optimization model (discussed in section 4.2.1) and find the strategy of turning the turbine in order to save an expected 9 birds. Please note that both runs are done using the greedy heuristic. Let $S_1$ and $S_2$ denote the set of hours for which the turbine should be turned off to save an average of 10 and 9 birds respectively. Let $Z$ denote the random variable representing the marginal cost of saving the $10^{th}$ bird and $Z_{ij}$ denote the random variable representing the revenue generated in the $j^{th}$ hour of the $i^{th}$ day. The random variable $Z$ can be expressed as follows:

$$Z = \sum_{i,j \in S_1} Z_{ij} - \sum_{i,j \in S_2} Z_{ij} \quad \text{... eq. (1)}$$

The overall probability distribution of marginal cost (or, the random variable $Z$) is estimated by using eq. (1) with a Monte Carlo random sampling method. The sampling is done 100,000 times. So, we have 100,000 data points of marginal cost. We plot histograms to estimate the probability mass function. The Appendix-I gives the MATLAB code for Monte Carlo simulation.

The figure 6-3 represents the histogram of marginal cost of saving the $10^{th}$ bird. Each interval of histogram is 100 units.
Using the probability mass function, the expected marginal cost is calculated to be $150. We note that it is very close to the marginal cost calculated using the greedy heuristic in table 5-2.

We see how the probability of saving more than 10 birds varies if we change the policy from saving an expected 10 birds to saving an expected 9 birds. Let $W_1$ and $W_2$ denote the random variables of the number of birds saved when the expected number of saved birds is 10 and 9 respectively.

\[
P(W_1 > 10) = 1 - P(W_1 \leq 10) = 1 - 0.583 = 0.416
\]

\[
P(W_2 > 10) = 1 - P(W_2 \leq 10) = 1 - 0.706 = 0.294
\]

We can say that to increase the probability of saving >10 birds by 12.2%, we need to pay a cost whose distribution is given by figure 6-3.
CHAPTER 7

FUTURE WORK

In this chapter, we compile all the assumptions that have been made in this thesis and analyze the limitations of the current work. We also discuss the scope of future work which can be done by relaxing different assumptions.

For the month of March, we have to estimate $31 \times 24$ (where, 31 denotes the number of days and 24 denotes the number of hours in day) probability distributions for both power and price. To keep the analysis simple, we have assumed each day of the month to be same and that all days of the month have the same set of 24 probability distributions for price and energy. For future work, we suggest estimating the actual probability distributions corresponding to each hour for all days and repeat the analysis to solve for the time periods for which the turbines should be turned off. It is anticipated that the analysis would be more cumbersome, time consuming and subject to data limitations (since we will have fewer data points for each point we are estimating).

We have arbitrarily assumed the probability of bird collision to be 1%. We recommend calculating the probability of bird collision using some collision risk model (e.g. Band Model) and also including the effect of avoidance rates for better estimation. We have also ignored the effect of the angle of approach between flight path and turbine orientation. Radar studies that provide flight speeds and directions can be carried out and an approach angle dependent model can be used estimate probability of collision.

We have assumed a constant probability for bird collision. The probability of collision is a function of the time of the day and is also dependent on the location of wind turbine. There is not much literature that would quantify the variation of bird collision either with time of the day or with the location of wind turbine. As future work, we suggest to carry field experiments in a wind farm using motion cameras that would note the time of bird death and location of a dead bird. Then, a mathematical analysis can be done to co-relate the probability of bird collision with the time of day and geometric location of a wind turbine.
Due to the lack of data on the number of bird mortalities in a wind farm on hourly basis, we have assumed a uniform distribution of bird mortalities over the day. Previous research has shown that more birds are killed (as they fly lower) when it is overcast and there is large cloud cover. So, more birds are killed on particular days and few are killed on other days. We propose carrying field experiments in future and counting the number of dead birds on hourly basis to get an idea about the variance.

Previous research has shown that with time some birds alter their migratory patterns and deviate from the path having the wind farm. The probability of bird collision in a wind farm becomes smaller with time. In this thesis, we have not taken into account this behavior of migratory birds. For future work, it would be a good idea to count the number of dead birds in a particular wind farm on monthly basis over a large time period (say, 6-8 years) and then estimate the factor by which the probability of bird collisions is diminishing over time.

We have not taken altitude of bird migration into account. We have simply multiplied the number of birds that are observed in the project site by the probability of bird collision to estimate the number of bird mortalities. The birds that fly at a higher altitude than the maximum turbine height might actually have nil probability of collision. In future work, we recommend advanced studies that would provide the altitude of bird migration. Only the birds flying below the turbine height will be considered in danger of collision.

We have limited data on the number of birds that are observed on different dates. The data that is available gives only the number of birds observed on certain specific dates. We have assumed that a linear interpolation provides a good estimate of the number of birds observed on dates for which we do not have data. In future, it would be useful to conduct more surveys to collect data about the number of birds observed on different days.

In future, we propose to explore the idea of slowing down the wind turbines for certain hours rather than completely shutting them down to mitigate bird mortalities. The turbines can be slowed down either by changing the wind speed trigger or by altering the blade angles. It would be interesting to study the effect on mortality rates.

In this thesis, we have used a linear programming formulation to find the optimal strategy in case of multiple turbines. A LP formulation actually provides a near-optimal solution only if
the number of turbines in the wind farm is sufficiently large (say, 100 or more). In case of a wind farm with small number of turbines, we will need to formulate an integer program where a binary variable will denote the on/off state of each turbine for each hour. Also, we have assumed that each turbine of the wind farm produce same energy. In reality, a wind farm loses energy due to wake effects and the energy produced by some turbines is slightly lower than others.
CHAPTER 8

CONCLUSIONS

The issue of bird mortality and electricity generation through wind turbines is a complex one. We make two major conclusions based on the analysis carried out in this thesis.

First, we conclude that a far more detailed, rigorous, and sophisticated analysis is called for to take into account the complexities involving bird mortalities in a wind farm. The shortcomings of this preliminary analysis are discussed in the Chapter 7. In fact, to develop a robust model to mitigate bird mortalities, we would need an exhaustive data on the migratory habits of birds, flying altitudes on different days and mortalities on hourly basis in a wind farm. Second, we can say that turning the turbines off for specific periods provides an effective strategy to mitigate bird mortalities in a wind farm. The uncertainty analysis indicates that there is an almost 42% chance that more than 10 birds will be saved if we turn the turbines off with the optimal strategy of saving an average of 10 birds in the month of March.

While the rudimentary numbers provided in the thesis are intended to provoke further research and discussion, they nonetheless emphasize the importance of detailed data collection and represent a method to develop a model to limit bird mortalities.
% This code plots the power histograms and estimates the pmf of power

W = load('wind_data.txt');  % load the wind data
W = W(:,[1 2 3 4 8]);      % only keep the 1,2,3,4,and 8th columns (year,month, day, hour, wind speed)

ind_3 = find(W(:,2)==3);  % find data points that are in march
W_3 = W(ind_3,:);         % only the date and wind speed data from march

hr = [0:23]';            % a vector of hours 0-23
L1 = length(hr);         % length of the hours vector

u = [0:.01:ceil(max(W_3(:,5)))];  % a wind speed vector from 0 to the max wind speed in the data set (rounded up) in steps of 0.01 m/s.
L2 = length(u);

% load power curve
Rating = 3.6;  % in MW
load ge_15_sl  % load the power curve data from file
clear power_curve PC P_pdf CF
power_curve(:,1)=ge_15_sl(:,1);  % first column is wind speed
power_curve(:,2)=ge_15_sl(:,2)*Rating/1.5;  % scale up to rating of 3.6 MW. second column is power
power_curve = [power_curve; [25.01 0]];  % cut out
power_curve = [power_curve; [1000 0]];  % for interpolation

for j=1:L1;  % loop through each of the 24 hours
  ind_j = find(W_3(:,4)==hr(j));  % find all data in march for this particular hour
  eval(strcat('U_',num2str(hr(j)),'=W_3(ind_j,5);'));  % create a wind speed vector for this particular hour
  eval(strcat('P_',num2str(hr(j)),'=interp1(power_curve(:,1),power_curve(:,2),W_3(ind_j,5));'));  % interpolate to find the vector of power outputs of a single turbine for each hour
end

% Plot histograms
h_p = [0:400:3600]';

for j=1:24
  Hp(:,j) = eval(strcat('hist(P_',num2str(hr(j)),',h_p);'));

  figure
  bar(h_p,Hp(:,j)/sum(Hp(:,j)));
  power_pmf(:,j) = Hp(:,j)/sum(Hp(:,j));
end
% This code plots the price histograms and estimates the price pmf %

P = load('price_data.txt');
P = P(:,[1,2,9]); % Keep only the month, hour and LMP%

ind_3 = find(P(:,1)==3); % Find index of data points in March

P_3 = P(ind_3,:);
hr = [1:24]';
L = length(hr);

for j=1:L
    ind = find(P_3(:,2)==hr(j));
    eval(strcat('Q_',num2str(hr(j)),'=P_3(ind,3);')); % Create a price vector for each hour%
end

% Plot the price histogram%

h_p = [0:15:300]';
for i=1:24
    Hp(:,i) = eval(strcat('hist(Q_',num2str(hr(i)),';','h_p');'));

    figure
    bar(h_p,Hp(:,i)/sum(Hp(:,i)));
    pmf(:,i)= Hp(:,i)/sum(Hp(:,i));
end
Monte Carlo simulation to estimate the pmf of the value of running one turbine (revenue) for each hour of the day

% Monte Carlo simulation to estimate the pmf of the value of running one turbine (revenue) for each hour of the day

K = xlsread('price_pmf'); % read the pmf of price from excel file
L = xlsread('power_pmff_new'); % read the pmf of power from excel file

% Generate the cdf of price and power
for x = 1:24
    for w = 1:21
        if w == 1
            K_cdf(1,x)=K(1,x);
        else
            K_cdf(w,x)=K_cdf(w-1,x)+K(w,x);
        end
    end
    for y = 1:10
        if y==1
            L_cdf(1,x)=L(1,x);
        else
            L_cdf(y,x)=L_cdf(y-1,x)+L(y,x);
        end
    end
end

K_cdf=xlsread('K_cdf_new'); % Remove the extra 1s from the cdf of price and insert a row with all elements zero using excel. Load the new file.
h = [9 10 11 11 11 14 12 13 10 10 9 11 8 9 12 20 12 10 10 9 9]'; % Length of each column of the cdf of price
b(1,24)=0;
L_cdf=[b;L_cdf]; % Insert a row of value zero in the beginning

for i=1:24 % loop to calculate revenue hour-wise
    for m=1:100000
        r(m,i)=rand(1);
        for f=1:h(i,1) % Length of each column is picked from h vector
            if K_cdf(f,i)<=r(m,i)&&K_cdf(f+1,i)>r(m,i)
                p(m,i)=(15)*(f-1);
            end
        end
        for g=1:10 % 10 is the length of each column of power
            if L_cdf(g,i)<=r(m,i)&&L_cdf(g+1,i)>r(m,i)
                e(m,i)=(400)*(g-1);
            end
        end
        a(m,i)=(p(m,i)*e(m,i))/1000;
    end
end
hr = [1:24]';  % a vector of hours 1-24
L1 = length(hr);  % length of the hours vector

for n=1:24
    eval(strcat('V_',num2str(hr(n)),'=a(:,n);'));
end

h_p = [0:50:1200]';

for n=1:24
    Hp(:,n) = eval(strcat('hist(V_',num2str(hr(n)),'[:,h_p]);'));  % generate a separate revenue vector for each hour
    figure
    bar(h_p,Hp(:,n)/sum(Hp(:,n)));
    pmf_revenue(:,n)= Hp(:,n)/sum(Hp(:,n));
end
% This code provides the strategy for turning the turbine off using Greedy Algorithm

G = xlsread('dollar per bird');
G = sortrows(G,5); % Sorts the dollar per bird in ascending order

% Strategy to save 10 birds in the month of march
H=zeros(744,1);C=zeros(744,1);
H(1,1)=G(1,3);C(1,1)=G(1,4);

for i=1:743
    if(H(i,1)<=10)
        H(i+1,1)=H(i,1)+G(i+1,3);
        C(i+1,1)=C(i,1)+G(i+1,4);
    else
        H(i+1,1)=H(i,1);
        C(i+1,1)=C(i,1);
    end
end

H=unique(H); C=unique(C);
len=length(H);

B = sum(G);
Actual_expected_mortalities = B(1,3)
expected_saved_birds_with_strategy = H(len,1)
revenue_lost=C(len,1)

% Loop to return the date and hour for which the turbine should be switched off

for k=1:len
    Date.Hour(k,1) = G(k,1);
    Date.Hour(k,2) = G(k,2);
end

Date.Hour;
% Monte Carlo simulation to estimate the overall distribution of lost revenue. expected 10 birds saved

R = xlsread('revenue_pmf'); % read the pmf of revenue from excel file

% Generate the cdf of revenue
for x = 1:24
    for w = 1:25
        if w == 1
            R_cdf(1,x)=R(1,x);
        else
            R_cdf(w,x)=R_cdf(w-1,x)+R(w,x);
        end
    end
end

R_cdf=xlsread('R_cdf_new'); % Remove the extra 1s from the cdf of price. Load the new file.
b(1,24)=0;
R_cdf=[b;R_cdf]; % Insert a row of value zero in the beginning
R_cdf=R_cdf(:,[1 2 3 4 5 6 14 15 16 17 18 22 23 24]); % Keep cdf of only those hours for which turbine will be turned off.
R_cdf=R_cdf(:,[1 2 3 4 5 6 7 8 9 10 11 12 13 14 1 2 3 4 5 9 10 13 14]); % Hours are repeated on on different days

% Find length of revenue cdf columns
h = zeros(1,23); c(1,23)=0; % h stores the length of cdf columns. c is a counter.
for p=1:23
    for q=1:23
        if R_cdf(q,p)==1
            h(1,p)=c(1,p)+1;
        else
            c(1,p)=c(1,p)+1;
        end
    end
end
h=h';

% Find overall distribution of the sum of random variables
a(100000,1)=0;
for i=1:23
    for m=1:100000
        r(m,i)=rand(1);
        for f=1:h(i,1) % Length of each column is picked from h vector
            if R_cdf(f,i)<=r(m,i)&amp;R_cdf(f+1,i)>r(m,i)
                rev(m,i)=(50)*(f-1);
            end
        end
        a(m,1)=a(m,1)+rev(m,i); % adding random variables
    end
end

h_p = [0:200:4500];
Hp(:,1)=hist(a,h_p);
figure
bar(h_p,Hp(:,1)/sum(Hp(:,1))); % find the pmf of revenue lost when 10 birds were saved

pmf_rev_lost(:,1)= Hp(:,1)/sum(Hp(:,1));

pr=sum(pmf_rev_lost(1:13)); % probability of losing less than 2500 dollars

prob=1-pr; % probability of losing more than 2500 dollars
% Linear Programming formulation

f1 = xlsread('dollar per bird');
f2 = f1(:,4);
f = -f2'; % convert to a minimization problem
A = f1(:,3)';
b = [32.467];
lb = zeros(744,1);
ub = ones(744,1);
[x,fval] = linprog(f,A,b,[],lb,ub);
f1_new = f1(:,[1,2]); % Keep only the date and hour columns
f1_final = [f1_new x]; % Join the date and hour columns with the corresponding fraction of turbines to be kept ON during that time

Total_rev_no_strategy = 65134;
revenue_lost_discard = Total_rev_no_strategy-(fval); % Discarded value of lost revenue

% Round the LP reported solution appropriately
for i=1:744
    if x(i,1)<0.01
        x1(i,1)=round(x(i,1));
    elseif x(i,1)>0.99
        x1(i,1)=round(x(i,1));
    else
        x1(i,1)=floor(x(i,1));
    end
end

f1_round=[f1_new x1];
ind=find(x1(:,1)==0);
lost_revenue=sum(f2(ind,1))
saved_birds=sum(f1(ind,3))
% Integer Programming formulation

f1 =xlsread('dollar per bird');
f2 = f1(:,4);
f = -f2';
A =f1(:,3)';
b = [72.467];

options=optimset('MaxTime',72000);
[x,fval] = bintprog(f,A,b,[],[],[],options);
f1_new = f1(:,[1,2]); % Keep only the date and hour columns
f1_final = [f1_new x];
lost_revenue = sum(f1(:,4))-(fval)
ind=find(x==0);
expected_birds_saved = sum(f1(ind,3))

Date_Hour(:,1)=f1(ind,1);
Date_Hour(:,2)=f1(ind,2);
% Monte Carlo simulation to estimate the distribution of marginal cost

R = xlsread('revenue_pmf'); % read the pmf of revenue from excel file

% Generate the cdf of revenue
for x = 1:24
    for w = 1:25
        if w == 1
            R_cdf(1,x)=R(1,x);
        else
            R_cdf(w,x)=R_cdf(w-1,x)+R(w,x);
        end
    end
end

R_cdf=xlsread('R_cdf_new'); % Remove the extra 1s from the cdf of price. Load the new file.
b(1,24)=0;
R_cdf=[b;R_cdf]; % Insert a row of value zero in the beginning
R_cdf=R_cdf(:,[23 24]); % keep only distinct hours

h = [11 11]'; % length of cdf columns

% Find overall distribution of marginal cost
a(100000,1)=0;
for i=1:2
    for m=1:100000
        r(m,i)=rand(1);
        for f=1:h(i,1) % Length of each column is picked from h vector
            if R_cdf(f,i)<=r(m,i)&&R_cdf(f+1,i)>r(m,i)
                rev(m,i)=(50)*(f-1);
            end
        end
        a(m,1)=a(m,1)+rev(m,i); % adding random variables
    end
end

h_p = [0:50:900]';
Hp(:,1)=hist(a,h_p);
figure
bar(h_p,Hp(:,1)/sum(Hp(:,1)));

pmf_marginal_rev(:,1)= Hp(:,1)/sum(Hp(:,1));
## APPENDIX II

### EXPECTED VALUE OF KEEPING ONE TURBINE RUNNING

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<th>Expected value hour-wise (in $)</th>
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REFERENCES


