Norms for Bayesians

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NORMS FOR BAYESIANS

A Dissertation Presented

by

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NORMS FOR BAYESIANS

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ABSTRACT

NORMS FOR BAYESIANS

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Bayesian epistemology provides formal norms that govern our degrees of belief both at a time and over time. It tells us that our degrees of belief ought to obey the probability axioms. It tells us that, when we get evidence, we ought to revise our beliefs in accordance with our conditional probabilities. Bayesianism can be made compatible with the norms for evidence that traditional epistemology has to offer us. But the question of whether Bayesianism itself implies some norm for evidence has never been addressed. This dissertation considers this question and develops two new Bayesian updating rules as a response to it. These rules refine the requirements that Bayesianism already provides us with by supplementing these requirements with two formal accounts of evidence.

In chapter two, I set some of the groundwork for the dissertation by comparing the problem of how evidence, and the experience that gives rise to it, constrain an update to two problems in the Bayesian literature that have similar structures, but that are better understood. In chapters three and four, I look at some places where the literature has founndered on the lack of an account of evidence. I consider two discussions that illustrate how the lack of a constraint on how experience gives rise to an update causes us to see
problems where there aren't any and to overlook problems where they do indeed exist. In chapter five, I develop a new account of the structure of Bayesian justification, which I call Bayesian coherentism. Bayesian coherentism is an updating norm that is motivated, both by the problems of the previous chapters and by the desire to provide a unified account of the structure from which updates on certain and uncertain evidence proceed. Finally, in chapter six, I develop a different norm that is guided by considerations similar to those that motivate Bayesian coherentism, but that is also compatible with the recently popular idea that there are no norms of diachronic rationality.
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CHAPTER 1

NORMS FOR BAYESIANS

Traditional epistemology gives us norms for turning true belief into knowledge. Some say that in order to constitute knowledge a true belief must meet certain constraints that are internal to the agent. Others claim that in order to constitute knowledge a true belief must be hooked up to the world in the right sort of way. While still others hold views about knowledge that jettison these norms for belief, the aim of epistemology, for many, is to provide them.

But, on the other hand, epistemology cannot be exhausted by these sorts of norms. For in addition to the fact that we believe certain things, it also seems true that we believe things to varying degrees. For instance, I am more sure that the sun will rise tomorrow than that my committee will pass my dissertation. In order to do justice to this sort of fact, we need the notion of a degree of belief, or ‘credence’. If we are to incorporate the notion of a credence into our epistemology, then we will need norms that govern these as well.

This dissertation is about one system of norms that governs our credences called Bayesian epistemology. There are aspects of Bayesian epistemology that are clear and intuitive. Bayesian epistemology gives us norms that tell us how our credences ought to hang together at a time: it tells us that our credences should obey the probability axioms. Bayesian epistemology also gives us norms that tell us how our credences ought to evolve over time when we get evidence: it tells us that our credences should evolve in accordance with our conditional probabilities. The norms that regulate what our credences ought to look like between the times we get evidence, and after we get evidence, are well defined. But what happens before we get evidence is the opposite of this: a black box. Though
Bayesianism can be made compatible with the accounts of evidence that traditional epistemology has to offer, the question of whether Bayesianism itself implies some norm for evidence has never been addressed.

In some way or other, each of the papers in this dissertation addresses this question: each of the papers in this dissertation is about Bayesianism’s black box. This dissertation looks at where the literature has foundered on the lack of an account of evidence. It uses these discussions to advance two new Bayesian updating rules—refinements of the requirements that Bayesianism already provides us with—that are motivated by these mistakes. While each of the discussions in this dissertation leaves us with a different result, then, together they provide a unified picture of part of the structure of epistemic justification for Bayesian epistemology. They provide a picture of how an agent’s credences ought to evolve over time. This is a dissertation about the structure of diachronic norms for Bayesians.

The products of this dissertation are, at least in part, the result of a focus on a Bayesian updating rule that rarely receives the attention it deserves. Most discussions about Bayesianism focus on Bayesian conditioning, the rule that tells us how to revise our beliefs whenever we get evidence of which we are certain. They do this at the cost of ignoring Jeffrey conditioning, the Bayesian updating rule that tells us how to revise our beliefs whenever we get evidence that we hold with any degree of confidence. On first reflection, it might seem odd to focus on the less general case. But on further reflection, this move makes sense. Most discussions that talk about Bayesian updating aren’t discussions about the formal features of the framework. When using Bayesianism as a tool to address other philosophical concerns, as we so often do, it makes sense to focus on the most idealized version of it.

But this isn’t a dissertation that will use Bayesianism as a tool or as a backdrop. This is a dissertation about Bayesian epistemology. Therefore, it’s a dissertation about Jeffrey
conditioning. Again, it seems reasonable that this should be so. A framework that acknowledges, as Bayesianism does, that belief comes in degrees, should not withhold this assumption from our evidence. As Jeffrey (1992, p.11) himself writes, opinion, no matter how it comes to us, should be probabilities “all the way down, to the roots”.

1.1 Methodology

Bayesians have long assumed that an agent’s subjective probabilities—the cornerstones of Bayesian epistemology—can be understood in terms of her preferences. This naturally gives rise to the idea that we can justify Bayesian norms by showing that conforming to such norms has practical advantages. Dutch book arguments dramatize this idea by illustrating that an agent that fails to satisfy Bayesianism’s norms have credence functions that sanction a set of bets that would result in a sure loss of money, or utility. But, as many have noted, these arguments make strong assumptions—too strong, many think, to justify norms as weak as Bayesianism. Though these arguments offer interesting analyses of the philosophical concepts that Bayesianism relies upon, they are unsatisfying justifications for Bayesianism itself. Recent arguments that attempt to show that conforming to Bayesian norms renders the agent more liable to end up with a set of beliefs that she deems to be more accurate face similar worries.

A different way of justifying Bayesian norms is by appeal to the method of reflective equilibrium advanced by Goodman (1955) in the process of defending precisely the sort of logic that Bayesian epistemology assumes. As he writes (p. 64): “the process of justification is the delicate one of making mutual adjustments between rules and accepted inferences; and in the agreement achieved lies the only justification needed for either”. Since the aim of this dissertation is to provide the account of evidence that yields the least flawed, most coherent interpretation of Bayesianism’s norm of diachronic coherence, consistent with Bayesianism’s core commitments and our considered judgments
about what these commitments entail, it can’t help but assume this methodology. This is a dissertation about how Bayesians ought to interpret their own norms.

1.2 Dissertation Summary

Summing up, then, what this dissertation undertakes to provide is the account of evidence that yields the best interpretation of Bayesianism’s diachronic norm, when we take seriously the motivation for Jeffrey conditioning. As I’ve mentioned already, each paper in this dissertation is intended to be read and understood on its own. But, for the interested reader, here’s a sketch of how these discussions hang together.

In chapter two, I argue that we get the clearest view of Bayesian epistemology from the perspective of Jeffrey conditioning. First, in generalizing the regular framework’s account of evidence, we’re able to better focus on its essential features. This prompts us to ask: what is the best way of understanding the role of evidence on the Bayesian framework? Second, the sort of empiricism that motivates Jeffrey conditioning underlines the fact that all updates begin in experience. This prompts us to wonder: what is the best way of understanding the role of experience on the Bayesian framework? Chapter two won’t resolve either of these problems. Instead, it provides some of the groundwork for this task by comparing these problems to two different problems in the Bayesian literature with similar structures. First, I consider the question of whether, and how, evidence is normative for us on the Bayesian framework. While some hold that evidence is just whatever partition of propositions an agent’s probabilistic belief transition is conditional on, most assume that evidence is a constraint on how the agent revises her beliefs. With this in mind, I compare the prospects of providing a constraint on how evidence gives rise to an update with those of providing a constraint on the probability functions that are vindicated by representation theorem arguments for Probabilism. Though both are the problem of showing how evidence and probabilities, respectively, can be held to guide an agent’s behavior, there’s
a difference between them that makes it possible to provide an account of the former, but not of the latter.

Second, I consider some of the problems that arise when we assume that all updates are justified by experience. I compare the prospects of providing a constraint on how experience gives rise to an evidence partition with those of giving the correct dynamics for *de se* beliefs. While, again, these problems are similar, in virtue of dealing with non-propositional content, I argue that there is an important difference between the two that makes it possible to resolve the latter, but not the former. I conclude that there is reason to think that “the experience problem” is intractable in a way that “the evidence problem” is not.

Chapters three and four illustrate how the lack of a constraint on how experience gives rise to an update causes us to see problems where there aren’t any and to overlook problems where they do indeed exist. It’s argued in Weisberg (2015) that Bayesian updating and undermining defeat are in tension. In chapter three, I show that there is no tension between Bayesian updating and undermining defeat. Attending to the role that higher-order beliefs play in the updating story allows us to provide an account of undermining defeat that is compatible with Bayesianism. The temptation to overlook this solution, I suggest, arises from the experience problem. It’s because we have no normative account of how experience gives rise to an update that we find Weisberg’s argument so plausible.

In chapter four, I argue that the experience problem is also the source of a genuine worry. Lange (2000) famously argues that although Jeffrey conditioning is formally non-commutative over updates, the framework isn’t defective in virtue of this feature, since it isn’t non-commutative over experiential information. In chapter four, I argue that the updating framework Lange’s argument assumes is stronger than Jeffrey conditioning. Moreover, I show that once we account for the normative properties that this new framework must have, we see that the account of experience we would need to get Jeffrey condi-
tioning off the hook—one that takes experiences to be individuated independently of the commutative property—gives rise to a problem that’s just as serious as the one this account of experience enables Jeffrey conditioning to avoid. Just as chapter two considers and rejects the possibility that evidence is simply whatever partition an update is rigid with respect to, chapter four considers and rejects the possibility that identical experiences are simply those experiences that figure in commutative relations.

In the last two chapters of this work, I develop two updating rules motivated by the problems of the previous chapters. These updating rules provide modified versions of Jeffrey conditioning by supplementing it with different constraints on evidence.

In chapter five, I argue that the best way to interpret the constraint on evidence that Bayesian updating imposes is as the constraint that our updates minimize the degree to which they fail to commute. I develop a degree-theoretic measure of this diachronic inconsistency, or diachronic incoherence. The account of Bayesian updating that results is attractive for two reasons. First, it’s a natural response to the problem raised in the previous chapter. Second, unlike the traditional foundationalist interpretation of Bayesian epistemology, it yields the result that updates on certain and uncertain evidence proceed from a common normative framework. Since my commutative norm is to updates what the norm of evidential consistency from traditional formulations of coherentism is to beliefs, it looks as though the best way of understanding the structure of Bayesian updating is as a form of coherentism.

In chapter six, I provide a synchronic surrogate of Bayesian coherentism. The updating rule I develop is consistent with the recently popular idea that there are no norms of diachronic rationality. What makes this rule a surrogate of Bayesian coherentism is that it is motivated by the desire to eliminate a kind of inconsistency that is the mirror image of the inconsistency that makes Jeffrey conditioning non-commutative. The very feature that entails that updating sequentially sometimes leaves us with non-commutative
updates also entails that updating on a bundle of evidence we’ve accumulated over time will sometimes lead to a violation of Probabilism over an interval. The updating norm I develop appeals to higher-order beliefs to resolve the inconsistency yielded by these violations.
CHAPTER 2

JEFFREY CONDITIONING: FROM REQUANTIFICATION TO DYNAMICS

ABSTRACT. Jeffrey conditioning is a rule that tells us how to revise our beliefs whenever we get evidence that we hold with any degree of confidence. Despite the simplicity of this rule, the way that we get evidence on the framework remains largely unexplored. In this paper, I provide some of the groundwork necessary for considering this part of the Bayesian story.

Ever since Richard Jeffrey’s inaugural description of Jeffrey conditioning in The Logic of Decision (1965), this rule for updating on uncertain evidence has been assumed simply to be a more permissive, or liberal, version of Bayesian conditioning.¹ In one sense, this is clearly true. The aim of Jeffrey conditioning is to make the traditional Bayesian framework applicable to a broader range of epistemic situations by generalizing its account of evidence. On the traditional Bayesian framework, an agent updates her beliefs by Bayesian conditioning, a rule that tells her how to revise her beliefs whenever she gets evidence that she holds with certainty. Jeffrey conditioning is a rule that tells the agent how to revise her beliefs whenever she gets evidence that she holds with any degree of confidence. On this understanding of things, Jeffrey conditioning is a mere footnote to the Bayesian program, in the sense that we know all there is to know about it once we’ve considered regular Bayesian conditioning.

This paper considers the formal features of Bayesian updating with a very different assumption in mind. Not only does an analysis of the Bayesian framework seem to call for an analysis of it in its most general form, but we get the clearest view of

¹Strictly speaking, the first discussion of this updating rule occurs in Jeffrey’s 1957 doctoral dissertation.
Bayesian epistemology from the perspective of Jeffrey conditioning. First, in generalizing the regular framework’s account of evidence, we’re better able to focus on its essential features. This prompts us to ask: what is the best way of understanding the role of evidence on the Bayesian framework? Second, the sort of empiricism that motivates Jeffrey conditioning underlines the fact that all updates begin in experience. This prompts us to wonder: what is the best way of understanding the role of experience on the Bayesian framework?

This discussion won’t provide an answer to these questions. Instead, it will provide some of the groundwork for this task. The aim of this paper is to gain a better understanding of the structure of Jeffrey conditioning by comparing the “evidence problem” and the “experience problem” to two problems in the Bayesian literature with similar structures, which are better understood.

First, I consider the question of whether, and how, evidence is normative for us on the Bayesian framework. While some hold that evidence is just whatever partition of propositions an agent’s probabilistic belief transition is conditional on, most assume that evidence is a constraint on how the agent revises her beliefs. With this in mind, I compare the prospects of providing a constraint on how evidence gives rise to an update with those of providing a constraint on the probability functions that are vindicated by representation theorem arguments for Probabilism. Though both are the problem of showing how evidence and probabilities, respectively, can be held to guide an agent’s behavior (Zynda 2006), there is a subtle difference between these problems that makes it possible to resolve the former, but not the latter.

Second, I consider some of the problems that arise when we assume that all updates are justified by experience. I compare the prospects of providing a constraint on how experience gives rise to an evidence partition with those of giving the correct dynamics for de se beliefs. While, again, these problems are similar, in virtue of deal-
ing with non-propositional content (Schwarz ms.), I argue that there is an important
difference between the two that makes it possible to resolve the latter, but not the
former. I conclude that there is reason to think the experience problem is intractable
in a way that the evidence problem is not.

2.1 Assumptions

In what follows, I’ll make a few standard assumptions. I’ll assume that a Bayesian
agent’s belief state can be represented as an algebra, \( \mathcal{A} \) — a set of propositions closed
under disjunction and negation—over \( \Omega \). I’ll assume that an agent has a degree of
confidence, or credence, in each of these propositions, and that these are represented
as an assignment of real numbers to those propositions. Therefore, I’ll assume that
credences are precise, rather than imprecise, since nothing important turns on this
question.\(^2\)

Finally, I’ll assume that an agent’s credence function satisfies the probability ax-
ioms, \( p : \mathcal{A} \rightarrow \mathbb{R} \):

1. \( \forall A, p(A) \geq 0 \).
2. \( p(\top) = 1 \).
3. \( \forall A, B, \text{if } A \land B = \bot \text{ then } p(A) + p(B) = p(A \lor B) \).

Together, these constraints provide an account of subjective probability.

2.2 From Requantification to Kinematics

2.2.1 Jeffrey Conditioning as Rigidity

Jeffrey conditioning is sometimes simply described as “updating subjective probabil-
ities”. But, to most, the idea of updating by Jeffrey conditioning carries with it com-

\(^2\)For the alternative approach, see e.g. Levi (1974, 1980, 1985), Joyce (2005, 2010), Weatherson (2008), Sturgeon (2008), and Moss (2014).
mitments beyond those that the probabilities it trades in are subjective and changeable. To many, it carries with it the assumption that these probabilities change by means of a particular mechanism.

To see this, we might note some other procedures for revising subjective probabilities that have been discussed in the literature. These include, among others, (1) complete reassessment; (2) retrospective conditioning and; (3) exchangeability. Complete reassessment is the most general of these rules. As its name suggests, it involves re-quantifying over $p$ in order to generate $p'$—“presumably by whatever technique was used to quantify the original distribution”. Retrospective conditioning and exchangeability are, along with Jeffrey conditioning, special routes to the re-quantification of the approach of complete reassessment: “each is valid and useful under different assumptions”.

We can get a better handle on the idea that Jeffrey conditioning is but one of many methods for revising subjective probabilities by considering how it differs from one of these other methods—for instance, from exchangeability. An exchangeable sequence of events is a sequence of events, such that future samples behave like earlier ones. Therefore, the exchangeability of a sequence of events justifies us in assigning future events the same probabilities that we’ve assigned to earlier ones. For example, say that a set of experiments has led us to conclude that there’s a sixty percent chance of rain in Montreal during the summer months, and that this leads us to adopt a credence of $0.6$ in this event. Now, suppose that we take a different future event to take place under the same conditions—for instance, the event of a rainfall in Montreal the following year. If these events are similar enough, it may be that they are exchangeable: it may be that we are justified in assigning to the second event the same proba-

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3For a discussion of these methods, see Diaconis and Zabell (1982, p. 822).
bility that we’ve assigned to the first.

While exchangeability yields is a simple and intuitive procedure for revising our subjective probabilities, it is of limited use. This is because, in most cases, the standard of “similar enough” will not be met. In most cases, there will be some difference—say, in the barometric conditions in Montreal in 2018—that makes us unjustified in assigning these events the same probabilities. In reality, there will be very few exchangeable events.

Nevertheless, our example gets at something more general, which is that the occurrence of some events are probabilistically relevant to the occurrence of others. This idea might still hold in cases where we aren’t justified in thinking that the probabilities of these events are perfectly symmetrical. One way of capturing this idea is to say that the occurrence of some event, like a rainfall in Montreal during a certain period of time, provides us with evidence that we can use to infer the probability of a second event. The main feature that distinguishes Jeffrey conditioning from exchangeability, then, is that the technique used to re quantify over p assumes, not the exchangeability of a set of events, but the “availability” of an evidence partition.6

What it means for an evidence partition to be available is a question that we will come back to shortly. For now, we can say that, just as the exchangeability of a set of events justifies our assigning some event the same probability that we’ve assigned to a different one, the availability of an evidence partition justifies our conditioning our probability of some event on a different event. The availability of an evidence partition justifies our transitioning from one probability function to another in a way that is ‘rigid’ on this event—this piece of evidence. Hence the name by which Jeffrey conditioning—or probability kinematics—was first introduced:

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6Diaconis and Zabell (1982, p. 823)
In Physics, Dynamics is a contrary of Kinematics as well as of Statics: it is the first contrariety that I had in mind when I called Chapter 11 of *The Logic of Decision*, ‘Probability Kinematics’. Take a see-saw, with fulcrum 2/3 of the way toward your end. If you push your end down two feet, the other end will go up three. That is kinematics: You talk about the propagation of motions throughout a system in terms of such constraints as rigidity and manner of linkage.\(^7\)

A Bayesian agent’s evidence is then the mechanism that constrains the propagation of her probabilities over time. Where we begin with a probability distribution, \(p\), and where experience directly changes an agent’s credences along the partition \(\{B_1, \ldots, B_n\}\), such that \(p(B_i) > 0\), Rigidity says that our prior credence distribution, \(p\), conditional on an evidence proposition, \(B_i\), stands in the following relation to our posterior credence distribution, \(p'\):

\[
\text{RIGIDITY: } \forall A, \forall B_i, p(A \mid B_i) = p'(A \mid B_i). \tag{8}
\]

Jeffrey conditioning, understood as Rigidity, says that updating our subjective probabilities means updating in a way that preserves those conditional probabilities that are conditional on our evidence. This is the method that distinguishes it from exchangeability, as well as from other methods that might be used to requantify a probability distribution.

Interestingly, it’s been shown that the Rigidity condition is satisfied by anyone who already endorses Probabilism.\(^9\) There is some partition that is sufficiently fine-grained to represent any probabilistic belief transition as an update by Rigidity, or by conditional probability. Given this, a natural question to ask is whether Rigidity is a restriction—whether it implies that evidence “propagates” or guides the agent’s probabilities—or whether it is merely a way of describing the process that the agent carries out in requantifying her probability distribution.

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\(^7\)Jeffrey (1970, p. 172)

\(^8\)Where \(p(A \mid B) = \frac{p(A \cap B)}{p(B)}\).

\(^9\)For the proof of this, see Diaconis and Zabell (1982, p. 825).
We can get a firmer grip on what this question is asking by comparing it with a well-known argument from the Bayesian literature that raises a similar concern. Representation theorems say that if preferences satisfy certain constraints, there is a representation of those preferences, in terms of subjective probabilities and desirabilities. More specifically, if preferences satisfy certain constraints, there is a unique probability function and a utility function that is unique up to linear transformation that ranks items exactly as these preferences do.\(^{10}\) Representation theorems are held to provide an argument for Probabilism. Here's how the argument goes. In order for the representation theorem to go through, preferences must meet certain conditions. While some of these conditions are assumed for technical convenience, others—like that preferences be asymmetric and transitive—are clearly rational constraints. Given this, representation theorems are supposed to provide a justification for both decision theory and for Probabilism. For one _ought_ to meet these constraints on preferences. And if one _does_ meet these constraints, then one can be represented as an expected utility maximizer and, therefore, as someone who satisfies Probabilism.

However, as some have noted, it's far from clear that representation theorems are capable of establishing an argument for Probabilism. The problem is the inference from the idea that these theorems vindicate probability functions to the idea that they vindicate functions that can be interpreted as representing the doxastic state of an agent. If preferences are defined in terms of probabilities and utilities, then it becomes less plausible to understand these probabilities and utilities as _determining_ our preferences. On this picture of things, it doesn't look like I deliberate about the probabilities of a set of options and then come to form my preferences on the basis of this deliberation. My preferences aren't revealed, or restricted, by my probabilities

\(^{10}\)There are many representation theorems that fall within expected utility theory, the most notable of which is Savage (1954).
and desirabilities. Instead, they are “dragged along” (Zynda 2006, p. 979) by these things. This undermines the idea that the probability function and utility functions that fall out of these theorems represent the beliefs and desires that are characteristic of agency. Therefore, it undermines the idea that representation theorems vindicate Probabilism by vindicating a logic of decision.\(^{11}\)

A similar point can be made about the relation between Rigidity and an agent’s probabilistic belief transition. If our evidence is just something that can be reverse-engineered out of any probabilistic belief transition, then there’s no sense in which our evidence determines or guides the propagation of an agent’s probabilities, as Jeffrey’s description suggests. Instead, our evidence is dragged along by these probabilities. This undermines the idea that our evidence constrains our posterior probability distribution. Therefore, it undermines the idea that Jeffrey conditioning represents the norm of diachronic rationality that many assume it to be. Before moving on, then, it’s important to consider whether, and in what sense, Rigidity is normative for us.

2.2.2 Jeffrey Conditioning as an Account of the Value of Updating

One way of getting a handle on how Rigidity is normative for us is by comparing it with a rule for updating subjective probabilities that also tells us to update on our evidence, but that isn’t Jeffrey conditioning. A recent example is the updating rule that Leitgeb and Pettigrew (2010) defend as a surrogate for Jeffrey conditioning as part of their accuracy-based vindication of the Bayesian framework. The authors introduce a rule for updating on uncertain evidence that they insist is not Jeffrey conditioning. Call this rule LP-updating.

Both Jeffrey conditioning and LP-updating impose additional requirements on

\(^{11}\)See Zynda (2006, p. 979) for this point (cf. Meacham and Weisberg 2011, §4), as well as for a discussion of how Jeffrey’s work as a whole negotiates the border between the descriptive and the normative.
an agent’s probabilistic belief transition. We’ve seen that one way of describing the constraint imposed by Jeffrey conditioning is as the requirement that our conditional probabilities remain the same over time. But there’s another, formally equivalent, way of describing this constraint—one that affords us a nice way of comparing it with LP-updating. The Rigidity condition is equivalent to what Diaconis and Zabell call ‘mechanical updating’, which is the constraint that a probabilistic belief transition minimize the distance between the propositions in the agent’s prior and posterior probability distribution, relative to a certain class of metrics. Rigidity and mechanical updating are two ways of describing the requirement that we multiply each element of the agent’s evidence partition by the smallest constant required to get us from her prior distribution to her posterior distribution.

Interestingly, LP-updating can also be described as a norm that requires that we minimize the distance between two probability distributions. However, it assumes a different measure of distance. While Rigidity, qua mechanical updating, takes minimizing the difference between an agent’s prior and posterior credence distribution to involve multiplying the former by the smallest constant required to get us to the latter, LP-updating takes minimizing the difference between an agent’s prior and posterior credence distribution to involve adding to the former the smallest constant required to get us to the latter.

For our purposes, what’s interesting aren’t the details of these accounts, but the fact that the formal difference between LP-updating and Rigidity corresponds to a difference in the values that these norms represent. Leitgeb and Pettigrew provide a compelling argument for the claim that the class of measures, according to which the distance between credence functions is minimized by adding a constant to each

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12See Diaconis and Zabell (1982, p. 828). Though, as the authors note, there exist other reasonable distance measures, according to which Jeffrey conditioning does not minimize distance.
world in our evidence partition, have properties that make them best suited to represent measures of inaccuracy. By contrast, Diaconis and Zabell assume that multiplying each world in the agent’s evidence partition by the smallest possible constant represents revising one’s beliefs in the most conservative way possible. Therefore, the comparison with LP-updating illustrates that, not only is Rigidity a non-trivial description of how an agent transitions from one probability distribution to another, but it is a value laden description of this transition. While LP-updating describes a way of transitioning from one probability distribution to another in a way that is accuracy-promoting, Rigidity describes a way of transitioning from one probability distribution to another in a way that promotes conservativism. Even if Rigidity were just a way of describing a probabilistic belief revision, then, this description would not be entirely non-normative, for these descriptions assume an account of what makes a belief revision valuable.

### 2.2.3 Jeffrey Conditioning as an Account of Updating

Despite all this, the idea that Jeffrey conditioning is a mere description—even a value laden one—is not the picture of it that most have in mind. While some have held the view that Jeffrey conditioning, qua Rigidity, is just a way of describing how an agent’s probability distribution evolves over time—including Jeffrey himself at certain points—most tend to think of Jeffrey conditioning as a *constraint* on how the agent ought to revise her beliefs, in light of her evidence. Most hold that the dif-

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13The authors are aware of the intuitive costs of abandoning rigidity (See Leitgeb and Pettigrew (2010, fn. 14). For a discussion that dramatizes these costs, see Levinshtein (2014)). They acknowledge that there may very well be cases where we would want to appeal to a rigid updating rule. But they argue that rigidity is not the be-all-and-end-all of normative constraints. They claim that, at the very least, a constraint that minimizes expected inaccuracy seems like one that might be useful in certain contexts.

14For the view that Jeffrey conditioning is a mere description of how the agent transitions from one probability distribution to another, see Jeffrey (1983). See also Wagner (2013) and Bradley (2005) for this view.
ference between Jeffrey conditioning and LP-updating is a difference in the prescriptions that they issue.

How do we go from thinking of Jeffrey conditioning as a description of evidence, or the value of evidence, to thinking of it as a normative constraint? We do this by imposing a constraint on evidence that isn’t entailed by Rigidity. Earlier we noted that the following existential claim has been shown to hold:

**FORMAL DIACHRONIC COHERENCE FOR BAYESIANS**

\[ \forall p, \forall p', \exists S(\forall B_i \in S), \forall A, p(A \mid B_i) = p'(A \mid B_i) \].

We can get a constraint on evidence by stipulating that only *some* partitions that the rest of an agent’s beliefs are conditional on are evidence partitions: only some ways that an agent might transition from one probability function to another are the result of the agent *having evidence*. Where \( E = \{ B_1, ..., B_n \} \) are the elements of the set of beliefs that meet the criteria for being evidence, and where \( p(B_i) > 0 \), we can formulate this as the constraint that the agent’s prior credence distribution \( p \), conditional on each \( B_i \) stand in the following relation to her posterior credence distribution, \( p' \), by means of the obligatory operator, \( O \):

**NORMATIVE DIACHRONIC COHERENCE FOR BAYESIANS:**

\[ \forall t, \forall t', \forall p, \forall p', \forall B_i \in E, \forall A, O(p(A \mid B_i) = p'(A \mid B_i) \].

Though the claim that Jeffrey conditioning should be thought of as something along the lines of **NORMATIVE DIACHRONIC COHERENCE FOR BAYESIANS** is widely accepted, the fact that we are able to get to **NORMATIVE DIACHRONIC COHERENCE FOR BAYESIANS** from **FORMAL DIACHRONIC COHERENCE FOR BAYESIANS** isn’t trivial. The comparison with representation theorem arguments for Probabilism will again help put things into perspective. Remember we said earlier that the problem of understanding Jeffrey conditioning as a substantive diachronic norm parallels the
problem of understanding representation theorems as arguments for Probabilism. The general worry, recall, was that it looked like those very things that were supposed to be guiding an agent (evidence, in the first case; a probability function in the second case) were being reverse-engineered out of those things they were supposed to be leading the agent towards (a posterior credence distribution in the first case; preferences in the second case). Therefore, we might have thought that we can save representation theorem arguments for Probabilism in the way that we’ve just saved Jeffrey conditioning—by imposing an additional requirement on that which is supposed to be guiding the agent. But we can’t. Representation theorem arguments for Probabilism face the following dilemma:

Horn 1: The probabilities entailed by an agent’s preferences don’t need to meet some different constraint that make them beliefs, in which case they don’t represent the belief functions of agents.

Horn 2: The probabilities entailed by an agent’s preferences do need to meet some different constraint that make them beliefs, in which case not all agents with coherent preferences can be represented by these probability functions.

The first horn of the dilemma holds in virtue of what we said in the previous section: we need representation theorems to vindicate probability functions qua beliefs if they are to vindicate Probabilism, via vindicating decision theory.

The second horn is a bit trickier, but it looks to be true as well. Say we require that the probabilities that get extracted from an agent’s preferences must have some different characteristic that make them beliefs for the representation theorem argument to go through. Say, for instance, that they must supervene on the mental states of the agent in question. In this case, we get a slightly different version of the original
objection. In this case, it will be possible for the probability function in question not to represent the actual agent, not because we fail to require of all probability functions that they have some additional feature that make them a belief function, but because the agent in question might fail to have this feature. The probability function implied by the agent’s preferences might not represent the agent’s actual belief state. The agent in question might fail to have the particular probabilistic belief state that the representation theorem entails. Or her belief state might fail to be probabilistic at all. Both these possibilities lead proponents of representation theorem arguments back to the original problem of not being able to ensure that the probability functions that get extracted from the agent’s preferences make the actual agent an expected utility maximizer.

This same problem does not arise for NORMATIVE DIACHRONIC COHERENCE FOR BAYESIANS. The analogue of the previous dilemma is as follows:

Horn 1: The partition that a belief transition is rigid with respect to needn’t meet some different constraint that makes it evidence, in which case it can’t vindicate an agent’s posterior credence distribution.

Horn 2: The partition that a belief transition is rigid with respect to needs to meet some different constraint that makes it evidence, in which case not all agents with probabilistic credence distributions will have their posterior credence distributions vindicated.

Again, the first horn of the dilemma holds given what we said above. The only way to get a normative account of Jeffrey conditioning is to cut a difference between Rigidity and some other updating rule like LP-updating. And the only way to do this is to identify the evidence partition independently of the way an agent revises her beliefs.
But the second horn does not pose a problem for Normative Diachronic Coherence for Bayesians. Though it may be that the partition that an agent’s belief revision is conditional on is not one that meets whatever constraints on evidence are in place, this is unproblematic for Normative Diachronic Coherence for Bayesians. In such a case, we simply have a norm that the agent has failed to satisfy. Since we want the norm in question to be non-trivial, this possibility is one that we should welcome.

In short, then, the difference between the cases we’ve been considering is that, in the one case, what we are trying to provide is a norm for an agent, while, in the other case, what we are trying to provide is a norm for a norm. In illustrating the possibility of violating a norm for an agent in the former case, we simply show that the norm in question is one that can be violated—as one would want and expect. In illustrating the possibility of violating a norm for a norm in the latter case, we sacrifice our justification for Probabilism. That representation theorems don’t justify Probabilism, via justifying decision theory, is an intractable problem then. On the other hand, the task of providing a normative account of Jeffrey conditioning isn’t intractable; it’s just incomplete.

2.3 From Kinematics to Dynamics

2.3.1 The Input Problem

Can we conclude that there is no problem with understanding Jeffrey conditioning as a substantive diachronic norm? Not quite yet. Part of what motivates a move away from regular Bayesian conditioning and towards Jeffrey conditioning is the thought that experience rarely leaves us with propositional information. The motivation for thinking that we ought to adopt a more permissive framework, then, arises from the
acknowledgment that all updates begin in experience. In the passage cited earlier, Jeffrey specifies the sense of kinematics he intends when describing his updating rule as ‘probability kinematics’. Here he continues:

[Kinematics] is the physics of position and time, in terms of which you can talk about velocity and acceleration, but not about force and mass. When you talk about forces—causes of accelerations—you are in the realms of dynamics.

To give a dynamical account of an agent’s update is to give an account of what initiates it. Since all updates begin in experience, a dynamical account of Bayesian updating is one that we might think of as beginning there as well.

We concluded at the end of the last section that some constraint must be placed on the evidence partition the agent gets, though we never identified that constraint. One option is to say that the constraint we would want is one that tells us what piece of evidence the agent has, given the experience she has had. If this is the way we want to go, then we will need an account of experience: we will need a rule that tells us how particular experiences justify particular weighted evidence partitions—or, equivalently, how particular experiences justify particular updates. The problem of providing such an account is sometimes referred to as ‘the input problem’ for Jeffrey conditioning. Alternatively, Bradley (2005) has called this the problem of ‘domesticating the inputs’.

In this second part of the paper, then, I want to consider whether we can do for experience what we’ve just done for evidence: I want to consider whether there is any impediment to thinking that the Bayesian framework has the resources to provide a

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15See, for instance, Jeffrey (1965, 1983), and Diaconis and Zabell (1982, p. 823).
16Jeffrey (1970, p.172)
rule that tells us how experiences constrain updates. The strategy will again be to consider some of the issues that would seem to arise for such a rule, and then to go on and compare its general prospects with a nearby problem in the literature that has received more careful treatment.

2.3.2 The Normalizing Problem and the Scale Problem

There are two main commitments that Jeffrey conditioning maintains with respect to evidence. The first is that evidence is a weighted partition of propositions. We’ve seen this already. The second commitment that Jeffrey conditioning maintains is that evidence is what results when some experience changes the agent’s credences along some partition directly, and so, non-inferentially.\(^{18}\) Providing an account of how experience justifies a Bayesian update would presumably require abandoning the second commitment, since such an account would need to maintain that there is an inference that takes us from an experience to an evidence partition. In this section, however, I want to explore a more fundamental reason for thinking that experience could not change the agent’s credences along some partition of propositions non-inferentially, in the course of justifying an update. I want to suggest that, not only is there good reason to think that we could not devise a relation that tells us what is the correct weighted evidence partition to adopt directly from experience, but that there is good reason to think that we could not get any weighted evidence partition at all directly from experience.

We might begin by noticing a difference between cases where we get certain evidence and cases where we get uncertain evidence. When we get evidence of which we are certain, our evidence seems to be represented in a way that comes close to mirroring the content of our experience. When I have the experience as of a green

\(^{18}\)See again Jeffrey (1965) for both of these commitments.
dress, the evidence that corresponds to this is either the set of worlds where the dress is green (where we take a worlds-based approach to the objects of credences), or some sentence whose structure mirrors my experience (where we take a sentence-based approach to the objects of credences). Both these types of objects seem “close” enough to my experience to make the transition from experience to evidence unproblematic.\textsuperscript{19} But where my evidence is uncertain—I get only a quick glimpse of the dress, so that my credence that it is green is only .7—and we assume that my evidence is a partition, the evidence that my experience gives rise to is partly comprised of the proposition that the dress is not green, to which I have assigned a credence of .3. Intuitively, however, my experience as of a green dress doesn’t have the same logical structure as what it gives rise to. My experience as of a green dress, uncertain though it may be, seems not even partly made up of the experience as of not a green dress. In general, it seems strange to say that you can have an experience that is both as of x and also as of not x. This seems false to the way that we understand experience.

We can give this problem a probabilistic gloss—one that focuses on the weights we assign our evidence partition, rather than the partition itself—and parse it into two distinct concerns. The first concern is that, in order for the impact of my experience to be representable as a weighted partition, it needs to be the case that the weights my experience gives rise to, which represents something like the clarity or intensity of my experience, sum to one. And it’s not obvious how this constraint could be encoded in phenomenal experience. But if this constraint is not encoded in experience, then I must apply this constraint to my experience in order to generate an evidence partition, contra the assumption that experience directly, and so non-inferentially, changes my credences over some partition of propositions. Call this the normalizing problem.

\textsuperscript{19} Though some would probably contest that this inference is unproblematic.
The second worry is that, in order for the impact of my experience to meaningfully assign a value to a member of my evidence partition, we must assume some scale. In order for the values I assign to some proposition to be meaningful, there needs to be an answer to the question of how much more the value I assign to one proposition is to the value that I assign to another. Again, it’s not obvious how this constraint could be encoded in phenomenal experience. And again if this constraint is not encoded in phenomenal experience, I must draw an inference from a scale, contra the assumption that experience directly, and so non-inferentially, changes my credences over a partition of propositions. Call this the scale problem.

Both the normalizing problem and the scale problem challenge Bayesian orthodoxy then. Bayesian orthodoxy tells us that experience changes our credences over a partition of propositions directly. Both problems point to the fact that it seems impossible that experience actually does do this. Just as some have held that experience never yields knowledge without concepts (cf. Sellars (1956)), it also looks like experience never yields a probability distribution without at least a scale and a normalizing principle.

Do either the normalizing problem or the scale problem have a solution? Since the normalizing problem is a problem about making sure that some set of values sum to one, there might be some middle-ground between the suggestion that experience never changes our credence in a partition non-inferentially and the suggestion that it never changes our credence in anything at all non-inferentially. It may be that experience changes our credence non-inferentially in some of the propositions that are part of our evidence partition. It may be that we can infer from these changes the rest of the members of our partition. Here’s one way we might do this.\(^\text{20}\) We might

\(^{20}\)The following solution is adapted from a proposal in Diaconis and Zabell (1982, p. 827), though the authors do not use this proposal to address either the normalizing problem or the scale problem. Instead they appeal to this proposal to handle cases of simultaneous updates, i.e., cases where our
assume that we get from experience to an evidence partition in two steps. The first step is that experience changes the agent’s credence in one or more propositions. For instance, our agent in the dress case might have some experience that justifies our assigning the proposition that the dress is green a value of $p(G) = .7$ and the proposition that the dress is violet a value of $p(V) = .2$. The second step is an independence constraint that we use to generate the set of propositions that partition the agent’s doxastic state. That is, we assume that $p(VG) = p(V)p(G)$. Where we impose this constraint, and assume Probabilism, this will entail that $p(VG) = .14$, $p(VG) = .06$, $p(\neg VG) = .56$, $p(\neg VG) = .24$. Using this method allows us to claim that experience does affect a non-inferential change in the agent’s doxastic state, while nonetheless letting us hold onto the idea that the input to the updating process is a partition.

Of course, this two-step approach does nothing to resolve the scale problem. We cannot posit an extra step after the agent has changed her credence in some proposition since, in order to change her credence in some proposition in the first place, the agent needs to draw an inference from a scale. If phenomenal experience does not itself encode such a scale, then contra the way that Jeffrey conditioning is typically formulated, we must conclude that experience never directly changes our credences in anything at all.\footnote{There is further reason to think that experience never gives rise to an evidence partition (or evidence proposition) non-inferentially, which I explore in chapter six.}

### 2.3.3 Experience and De Se Belief

Before closing, I want to step away from the particular worries raised in the previous section and consider what the general prospects of resolving the input problem might be. Some have noted a similarity between the problem of giving an account of how experience justifies an update and the problem of giving an account of the dy-
namics of de se, or self-locating, belief.\textsuperscript{22} There are a couple of things these problems have in common. First, both can be cast as the problem of how to incorporate non-propositional content into the Bayesian framework.\textsuperscript{23} And, in a certain way, both can be cast as the problem of how to incorporate self-locational content into the Bayesian framework. De se beliefs provide us with information about where we are spatio-temporally located in a world, while experiences provide us with information about what the world is like from our idiosyncratically internal point of view. Given these similarities, one might reason that the fact that we’ve been able to construct accounts of de se updating that are more or less successful means that there should be a solution to the input problem. In this last section, I show why this comparison, and this reasoning, is too quick.

As Titelbaum (2012) notes, most discussions of de se updating appeal to one of a few general frameworks. One of these frameworks is what Titelbaum calls a ‘stable base scheme’. This method supplements regular Bayesian conditioning with a rule that tells us how to deal with centered content, once we have updated by Bayesian conditioning in the usual way. For instance, Meacham (2008)’s version of this scheme proceeds in three steps. First, we consider the distribution over uncentered worlds only and assign worlds that are incompatible with our strongest evidence a credence of zero. Second, we renormalize over our remaining uncentered worlds. Finally, we renormalize our credences in the remaining centered worlds at each uncentered

\textsuperscript{22}See, for instance, Schwarz (ms.). It’s unclear, however, that Schwarz has in mind the same interpretation of the input problem than I am assuming. As we will see in chapter four, there are two interpretations of this problem, one that takes it to be a normative problem, and one that takes it to be a descriptive problem.

\textsuperscript{23}This description might be thought to be contentious for certain ways of understanding de se belief, since they are derivatively propositional on Lewis’s view, in virtue of being properties. For the purposes of this discussion, let us count Lewisian centered worlds as being non-propositional. For a more direct and useful comparison of the input problem with the problem of accommodating de se content, see again Schwarz (ms.).
world. So, for example, if some of the worlds where \( \text{today is Sunday} \) are centered on the proposition that \( \text{the sun is shining on Sunday} \), getting as evidence that the sun is shining on Sunday will eliminate all my credence in the worlds where it’s not the case that the sun is shining on Sunday. Meacham’s updating rule will then tell me to redistribute my credence over the centered worlds where today is Sunday in a way that preserves the proportional relations between them.

A second scheme for \textit{de se} updating integrates centered and uncentered content more directly. Kim (2009)’s ‘shifting scheme’ does this by means of a primitive “at” operator that maps centered propositions to uncentered propositions. The main idea behind Kim’s framework is the common one that placing our indexical information within a context can turn it into a regular proposition. On Kim’s framework, if WY is the centered proposition expressed by \( \text{the Blue Jays won their game yesterday} \), then \( \text{WY at May 12th} \) is the uncentered proposition that the Blue Jays won their game on May 11th. This means that where all I learn is that the time has passed from May 11th to May 12th, my credence in the proposition that the Blue Jays won their game yesterday should equal the credence I had the day before that the Blue Jays would win their game yesterday at May 12th: \( P_{\text{May12th}(WY)} = P_{\text{May11th}(WY \text{ at May 12th})} \). And where I learn something stronger than this—say, that the Blue Jays’s best pitcher was scheduled to play yesterday (BPY)—my credence on May 12th that the Blue Jays won their game yesterday should be just what it would have been on May 11th had I learned on May 11th that the Blue Jays would win their game yesterday at May 12th, conditional on their best player pitching yesterday at May 12th: \( P_{\text{May12th}(WY)} = P_{\text{May11th}(WY \text{ at May 12th} \mid \text{BPY at May 12})} \). Instead of updating in two steps, then, Kim’s updating rule lets us shift indexicals in a way that naturally preserves the results that we normally get when we condition on our evidence.

\footnote{See also Halpern (2004) for a similar proposal.}
Despite the differences in their frameworks, Meacham and Kim appeal to the same general strategy: both build into the standard conditioning framework a mechanism that coordinates our centered and uncentered content so that we always have information about one when we have information about the other. More specifically, both Meacham and Kim build into the standard conditioning framework a mechanism that grounds centered content in uncentered content, so that we always have information about the former sort of content when we have information about the latter sort of content. What about cases where we don’t have information about the latter sort of content? These are cases where both of these frameworks break down. Meacham’s updating rule can’t accommodate cases where we haven’t gotten any uncentered evidence. Without any uncentered evidence, we cannot redistribute our credences in our centered worlds in the evidence partition, in a way that preserves the proportional relations between them. Kim’s updating rule can’t accommodate cases where we don’t have uncentered information about what context we are in. Without information about the context, Kim’s rule can’t turn our centered information into uncentered information.

These blindspots are instructive. What they show us are that the dynamics of de se belief are grounded in regular, old propositional evidence. And this shows us how the problem of providing the correct dynamics of de se belief is relevantly different from the input problem. For what a solution to the input problem is looking to provide an account of is precisely the opposite: it is looking to provide an account of how we get regular, old propositional evidence from something non-propositional, rather than vice versa. It’s not the nature of experience as non-propositional, then, that makes the input problem a problem, but, rather, the role that we are looking for it to fill. The input problem is that experience is all blindspot.

Where does that leave us? Nothing we’ve said here means there’s no possible way
of giving a normative account of how experience figures into the updating process. But just as the comparison with representation theorem arguments for Probabilism shows us that the evidence problem is not so serious, the appeal to the literature on the dynamics of \textit{de se} belief leads us to conclude the opposite about the experience problem. The comparison with representation theorem arguments for Probabilism shows us that one can impose a constraint on the agent’s evidence partition unproblematically. The comparison with \textit{de se} updating suggests that the constraint that gets imposed most likely cannot be one that depends upon us resolving the input problem.
CHAPTER 3

BAYESIANISM AND THE UNDERMINING PROBLEM

Abstract. Jonathan Weisberg has argued that Bayesianism’s rigid updating rules make Bayesian updating incompatible with undermining defeat. In this paper, I argue that when we attend to the higher-order beliefs we must ascribe to agents in the kinds of cases Weisberg considers, the problem he raises disappears. Once we acknowledge the importance of higher-order beliefs to cases of undermining defeat, we are led to a different understanding of how these cases arise. And on this different understanding of things, the rigid nature of Bayesianism’s updating rules is no obstacle to its accommodating undermining defeat.

The tension between holism and foundationalism is nothing new to traditional epistemology. However, only recently has it been noticed that a similar tension appears to exist between holism and formal epistemologies, like Ranking Theory, Dempster-Shafer Theory and Bayesianism. These accounts are foundational in the sense that many of the “updates” they sanction are anchored in a set of prior commitments that aren’t themselves subject to this process of revision. For instance, the Bayesian assumes that all our beliefs ought to depend upon the evidence we get, along with our prior conditional probabilities. And many of these prior conditional probabilities are assumed to be unchangeable. So, though I may come to change my degree of belief that it will rain tomorrow after listening to the weather forecast, my belief about the degree of correlation between these two events—that which determines how much my belief in the weather ought to change after getting this evidence—can’t be revised in this same sort of way. This is the sense in which Bayesianism might be
described as being foundational.¹

Jonathan Weisberg has argued that, due to this trait, the Bayesian framework mishandles the phenomenon of perceptual undermining defeat.² He argues that Ranking Theory and Demspter-Shafer Theory have similar traits that result in similar failures. Since Weisberg’s discussion focuses on the Bayesian case, I’ll only consider his argument against Bayesianism in this paper.³ I’ll argue that Bayesian updating and Bayesian undermining actually are compatible, once we conceive of undermining in the appropriate way.

In §1, I give a bit of background on the Bayesian framework. In §2, I outline the account of perceptual undermining Weisberg assumes, and his argument that it is incompatible with Bayesianism. In §3, I consider Weisberg’s rebuttal of the idea that we can appeal to higher-order beliefs to resolve the dilemma he describes. In §4, I outline a way of deploying higher-order beliefs, which escapes Weisberg’s rebuttal, and which indeed resolves the dilemma. Finally, in §5, I explain what makes us so prone to overlook the higher-order beliefs solution by considering the more general question of whether Bayesian epistemology is compatible with confirmation holism.

3.1 Background

Standard Bayesianism assumes that a rational agent’s degrees of belief, or credences, are well-defined over the space of possibilities, Ω. It further assumes that, for every possibility she entertains, the agent has in it an exact, real-numbered degree of belief.

¹In chapter five, I consider a different understanding of what foundationalism might mean in the Bayesian context. While I don’t endorse the description of foundationalism that Weisberg assumes in this work, nothing in his argument or mine turns on this label.

²Note that what goes by the name “undermining defeat” in Weisberg’s discussion is more commonly referred to as ‘undercutting defeat’. For the canonical account of undercutting defeat, see Pollock (1986, 2008).

³The worry he raises is an extension of the argument from Weisberg (2009). The bulk of my discussion will focus on Weisberg (2015), where Weisberg attempts to sharpen this worry. However, we’ll return to consider some of the more general issues raised in Weisberg (2009) in §4.
In addition, most Bayesians endorse two norms. First, they maintain that an agent ought to satisfy the following synchronic norm:

**Probabilism:** An agent’s degrees of belief ought to obey the probability axioms.

Second, they hold that an agent’s credences ought to satisfy a diachronic constraint. It’s this second constraint that is the target of Weisberg’s argument. It tells us how we ought to update our beliefs when we learn some new piece of evidence:

**Conditionalization:** When you get B as evidence, your new degree of belief in A, for any A, should be \( p'(A) = p(A|B) \), where A and B are propositions.\(^4\)

Since our evidence always gets a credence of one on this updating framework, a commitment that those who endorse it take on is that we are always certain of our evidence. But, to many, it seems as though we often get what one might call “uncertain” evidence: for example, the slight smell of cinnamon that makes us think that there may be an apple pie baking in the oven, or a glimpse of color, which makes us increase our credence in the proposition that the sock in the drawer is red, without making us sure of it. To accommodate this type of evidence, Richard Jeffrey (1965) proposed an alternative to Conditionalization that generalizes this model. Jeffrey suggests that our evidence takes the form of a weighted partition of propositions, where a partition is a set of mutually exclusive and exhaustive propositions. Formally, on this picture of things, our evidence is an n-tuple of propositions \( \langle B_1, \ldots, B_n \rangle \) that partitions \( \Omega \), and that are assigned an n-tuple of credences (which, assuming Probabilism, will sum to one). These weighted partitions are the inputs to Jeffrey Conditionalization, in the way that propositions are the inputs to Conditionalization:

\(^4\)And where \( p(A|B) = \frac{p(A \land B)}{p(B)} \).
Jeffrey Conditionalization: If your degrees of belief are given by the probability function $p$, and (i) experience directly affects your credences over the partition $\{B_i\}$ from $p(B_i)$ to $p'(B_i)$, but (ii) experience does not affect any other credences, then your new credences should be given by $p'(A) = \sum_i p(A | B_i) p'(B_i)$.$^5$

So, if my quick glimpse leaves me uncertain about whether the sock in the drawer is red, this more liberal approach to updating will tell me to hedge my bets: it will tell me to assign some credence to the proposition that the sock is red, and some credence to the proposition that it isn’t. The partition comprised of these two propositions will weight the impact that updating by my conditional probabilities has on my credence function. If I’m almost sure I’ll put on socks that day, conditional on the fact that my socks are red, a high credence in the latter proposition will mean that I should be pretty sure of putting on socks that day. And a low credence in this same proposition will mean that I should be pretty sure that I will go barefoot.

3.2 Weisberg’s Argument

Weisberg’s diagnosis of where Bayesianism goes wrong appeals to a formal property shared by both Conditionalization and Jeffrey Conditionalization— a property called ‘rigidity’. An updating rule is rigid just in case it preserves those conditional probabilities that are conditional on the elements of the evidence partition. Relative to Jeffrey Conditionalization, rigidity can be defined in the following way:

---

$^5$The correct philosophical interpretation of Jeffrey Conditionalization is a complicated matter, made all the more complicated by the difference in how Jeffrey understood his own updating rule, and how others have appropriated it (and made more complicated still by the fact that Jeffrey’s own views about his updating rule were only clarified over time (cf. Jeffrey (1965) and Jeffrey (1983))). I follow Weisberg, and others, in understanding Jeffrey Conditionalization as a substantive diachronic updating norm. See Weisberg (2015), 3-4.

$^6$It was Jeffrey himself who first coined this term. See Jeffrey (1965).

$^7$To avoid this cumbersome expression, in what follows, I will simply use the term “conditional probabilities” to denote those conditional probabilities that are conditional on the elements of the evidence partition. Where I don’t have this more restricted usage in mind, context should make that clear.
JEFFREY CONDITIONALIZATION IS RIGID: If \( p \) is a probability function and \( p' \) comes from \( p \) by Jeffrey Conditionalization on the partition \( \{B_i\} \), then \( p'(A | B_i) = p(A | B_i) \), for every \( A \) and \( B_i \) (Weisberg 2015, 4).

As Weisberg notes, in many cases, rigidity seems like a nice feature for an updating rule to have. Getting evidence that confirms some proposition—say, that it will rain in a few hours—seems to leave our credence in all the conditional relations involving this belief unaffected. So, for example, one belief that it seems should not be affected by my belief that it’s going to rain is the conditional belief that if it rains, I will not go biking. No increase in my belief that it will rain ought to affect my conditional credence in that proposition. Most evidence, we might think then, doesn’t change our conditional probabilities. Instead, most evidence that lessens our credence in some proposition does so by “rebutting” it. And rebutting defeaters are encoded in our conditional probabilities.

For example, if my sister hates red socks, my credence in the proposition that the sock in her drawer is red, conditional on this information, is probably low. If it is, Bayesianism issues a clear recommendation: it tells me that when I get this rebutting evidence—that my sister hates red socks—I should lower my credence in the proposition that the sock in her drawer is red, in the way prescribed by my conditional probability. Bayesianism handles these cases of rebutting defeat with ease.

But sometimes the evidence we get is evidence that the relations of conditional support we were relying on were mistakenly held. Sometimes our evidence actually seems to change our conditional probabilities themselves: it seems to change how I ought to change my credence conditional on my receiving the evidence that my sister hates red socks. Weisberg’s claim is that Bayesianism founders on these types of cases.

To illustrate with his own example, the proposition that the lighting in the room makes all socks look red (F) ought to be probabilistically independent of the propo-
sition that the sock is red \( E \) if my credence in the latter proposition is due entirely to the testimony of my friend (who, let’s suppose, did not come to hold this opinion by viewing the sock). In this case, \( F \) is not probabilistically relevant to \( E \) at all: i.e., 
\[
p_0(E \mid F) = p_0(E).
\]

Notice, however, what happens after I have an appearance that increases my credence in \( E \) (let us denote this perceptual experience by \( \zeta \)). Once I have received this evidence, \( F \) does seem to become probabilistically relevant to \( E \): after all, part of my reason for believing that \( E \) is true is now based on the way things look to me. Thus, after \( \zeta \), it must be the case that \( p_1(E \mid F) < p_1(E) \): after my credence in \( E \) becomes partly based on my visual experience—after I’ve increased my credence in \( E \) in response to \( \zeta \)—the belief that this experience has been distorted by tricky lighting ought to move me to lower my credence in \( E \). It would appear, then, that \( F \) is probabilistically independent of \( E \) before my perceptual experience, and negatively relevant to \( E \) afterwards. Thus, we get the following set of relations:

\[
\begin{align*}
p_0(E \mid F) &= p_0(E) \\
p_1(E \mid F) &< p_1(E)
\end{align*}
\]

The problem is that Jeffrey Conditionalization preserves the rigidity of conditional probabilities on \( \{E_i\} \): when the only evidence I have received is evidence that changes the value along this partition—evidence that leads me to assign different values to \( E \) and \( \overline{E} \)—the conditional probability of \( E \) on \( F \) must remain, after \( \zeta \), the same as it was before \( \zeta \):

**RIGIDITY PRESERVES INDEPENDENCE**: If the transition from \( p \) to \( p' \) is rigid on the partition \( \{E_i\} \), and \( p(E_i \mid F) = p'(E_i) \) for every \( E_i \), then \( p'(E_i \mid F) = p'(E_i) \) for
every $E_i$.\(^8\)

If this is right, the pair of credence assignments depicted above will be impossible for the Bayesian agent to hold, when the transition from $p$ to $p'$ comes from updating on the partition $\{E_i\}$. We can name the schema these relations instantiate the Naive Account of perceptual undermining:

**The Naive Account:**

$$p_0(X \mid Y) = p_0(X)$$
$$p_1(X \mid Y) < p_1(X)$$

Since Bayesianism is partly comprised of Jeffrey Conditionalization, and since rigidity is a formal property of Jeffrey Conditionalization, the Bayesian framework will be unable to capture how our beliefs ought to change in cases where our conditional probabilities should intuitively change.\(^9\) Given that we seem to have no reason to doubt our intuitions in cases like the previous one, the fault must lie with the framework. Where $X$ is the evidence proposition and $Y$ is the undermining defeater, Bayesians remain unable to satisfy the following constraint.

**The Undermining Constraint:**

If one has the intuition that $X$ and $Y$ are negatively correlated, then one’s epistemological framework ought to capture this.

The inability of Bayesians to satisfy this norm is *The Undermining Problem.*

---

\(^8\)For the proof of this, see Weisberg (2015), 5, fn. 4.

\(^9\)Here, again, what we mean to say is that the Bayesian framework will be unable to introduce the needed dependence between the elements of the evidence partitions being conditionalized on. Given this, the obvious solution might seem to be to say that, in cases like the preceding one, the agent updates on partitions that are finer-grained than $\{E, \overline{E}\}$ and $\{F, \overline{F}\}$, respectively: for example, $\{EF, \overline{EF}, \overline{EF}, \overline{EF}\}$. Weisberg considers this solution and rejects it on the grounds that it is objectionably ad hoc. (See Weisberg 2015, 21-24.) Nonetheless, for an argument that appeals to this strategy, see Wagner (2013).
3.3 An Alternative to the Naive Account

3.3.1 The Challenge

It’s important to be clear about the burden that’s assumed by someone who wants to challenge the Undermining Problem. Weisberg takes The Naive Account to be a counterexample to the claim that the Bayesian formalism can accommodate all forms of rational belief revision. His claim is not merely that we can contrive of a set of relations that violates rigidity. Any substantive theory will admit of violations. What makes a violation of some theory a counterexample to that theory is that it is a case we feel pressure to accommodate.

Compare with a different theory. What makes Utilitarianism a substantive moral theory is that, for any given agent, at any given time, there will be some options that don’t maximize utility. But not all of these options are counterexamples to Utilitarianism. Cases where we fail to privilege the welfare of our nearest and dearest are counterexamples to Utilitarianism: they are cases of intuitively morally permissible behavior that don’t maximize utility.

Insofar as we assume that The Naive Account is the way we must understand undermining, and insofar as we take undermining to be rational, we have reason to think that Bayesianism has failed us. But while The Naive Account may be one way of accounting for our intuition about how the agent ought to rationally revise after having gotten $\zeta$ and $F$ in the red sock case, there may yet be better ways of accounting for this intuition. For unlike the nearest and dearest objection, the description of the phenomenon that triggers our intuitions in such cases is, plausibly, part of what’s up for grabs.\footnote{To appreciate the difference here, consider that there are a number of ways the Utilitarian might try to deflect the appeal to cases where we privilege our nearest and dearest: she might bite the bullet, or argue that cases of privileging one’s loved ones will always maximize utility, etc. One thing she could not credibly do is challenge the description of the phenomenon itself, i.e., of privileging one’s loved ones.} With this in mind, in what follows, I’ll propose and defend a different
formal description of undermining defeat. This will establish that while the relations depicted by the Naive Account are inconsistent with the Bayesian formalism, this is no mark against the Bayesian formalism. Since they no longer correspond to the intuitively rational phenomenon of undermining defeat, we no longer have reason to want to accommodate them.

3.3.2 The Higher-Order Beliefs Approach

Earlier I conceded, on behalf of the Bayesian, that we have no reason to doubt our intuitions in the red sock case. This is what led us to impugn Bayesianism. This concession may have been a bit too quick however. Notice how our intuitions change according to how we fill out the agent’s evidence set. If I am the agent and I already know I have been given a drug that makes me immune to tricky lighting, it seems I should not lower my credence in the proposition that the sock is red when I learn the lighting is tricky. I should instead keep the credences I had at t₀. If I have the belief that I have been given a drug that makes me immune to tricky lighting, and in addition I have the belief that I have a particularly high tolerance for this drug, so that it might have been ineffective, this will again change how I ought to respond to learning that the lighting is tricky. Clearly, the agent’s other beliefs will play a critical role in determining what response is rational in cases like the one we’ve been considering. We might be tempted to think, then, that what accounts for the weird result in the previous case is that it’s been underdescribed. Taking a more careful inventory of the agent’s commitments will perhaps reveal that there really is no problem for Bayesianism after all.

Weisberg is aware that his argument invites this kind of response. He briefly considers the possibility that someone might try to fill out the agent’s evidence set in a way
that makes the Undermining Problem disappear. In particular, he considers the possibility that someone might try to block his objection by arguing that the metacognitive—or higher-order—proposition \textit{my credence at }t'\text{ that the sock is red is based on its having appeared red at }t\textit{ (AB) can be used to introduce a negative correlation between E and F after we update over }\{E_i\}\textit{ in response to }\zeta\textit{ (Weisberg 2015, 20).}^{11}\text{ Here’s how this response would go:}

After the first update in response to the sock appearance, there is an update in response to that update, on the proposition \(AB = \text{My credence at }t'\text{ that the sock is red is based on its having appeared red at }t\). That is, I make a metacognitive observation that prompts an update in between the updates on E and on F. And that intermediate update is where the negative correlation between E and F is introduced. Thus, when I finally do learn F, it will reduce my credence in E. Unless, of course, I do not make the metacognitive observation that my credence in E is appearance-based. Then there is no intermediate update and learning F does not affect my credence in E, as is appropriate (Weisberg 2015, 20).^{12}

While Weisberg seems to concede the importance of higher-order beliefs to the undermining story, he doesn’t think this strategy gets the Bayesian off the hook:

This solution has prima facie appeal because it acknowledges what seems plausible: that perceptual beliefs are often accompanied by metacognitive information about their sources, and that the rational response to subsequent underminers should depend on what meta-data we have about a belief’s sources. Nevertheless, it fails for a fairly simple reason. The metacognitive observation that my credence in E is appearance-based cannot introduce the desired negative correlation. To see this, notice that at the outset, before I look at the sock, the conjunction \textit{The lighting is deceptive }[F] \land \textit{My credence at }t'\text{ that the sock is red will be based on its appearing red at }t\textit{ [AB]} is probabilistically independent of the proposition \textit{The sock is red }[E].\text{ The fact that in a moment my credence in E will be high based on misleading appearances has no bearing on what the actual color of the sock is. In other words, at the outset E is independent

\footnote{Note that I have replaced }E'\text{ with }AB\text{ in this passage, as well as in the passages that follow.}

\footnote{For another recent defense of this idea in a non-Bayesian context, see Sturgeon (2012).}
of $F \land AB$, i.e. $p(E | F \land AB) = p(E)$. Because Rigidity will Preserve this Independence, learning $AB$ in addition to $F$ cannot lower the probability of $E$. Thus the intermediate, metacognitive update does not provide an opportunity to introduce the negative correlation between $E$ and $F$ that we are after (Weisberg 2015, 20).

I agree with Weisberg that the proposal he considers does not succeed. But I don’t think this means that higher-order beliefs don’t provide the resources to resolve the Undermining Problem. I think that a different way of deploying higher-order beliefs, along with a better understanding of the role of higher-order beliefs, can indeed make Bayesian updating compatible with undermining defeat.

### 3.4 A Solution to the Undermining Problem

#### 3.4.1 The Modified Higher-Order Beliefs Approach

Consider the relations that represent the higher-order beliefs approach described in the passage above, relations that Bayesianism still can’t accommodate:

**The Higher-Order Beliefs Approach**

$$p_0(E | F \land HO) < p_0(E)$$

$$p_1(E | F \land HO) < p_1(E)$$

Since what these relations imply is that $E$ is undermined by the complex undermining defeater, $F \land HO$, it shouldn’t come as a surprise that they don’t resolve the Undermining Problem. They merely re-instantiate the Naive Account.\(^{13}\) Happily,

\(^{13}\)In other words, $F \land HO$ slots into $Y$ in the schema from §2:

$$
p_0(X | Y) = p_0(X)$$

$$p_1(X | Y) < p_1(X)$$
though, these relations aren’t the only option for incorporating higher-order beliefs into the undermining story. Indeed, one might think that the more obvious way to go about doing this is to distinguish these beliefs formally from the undermining defeater by placing them on both sides of the relation:

**The Modified Higher-Order Beliefs Approach**

\[
p_0(E | F \land HO) < p_0(E | HO) \\
p_1(E | F \land HO) < p_1(E | HO)
\]

This seems to better capture what a proponent of the higher-order beliefs strategy should be after. Our higher-order beliefs aren’t part of the underminer for E. Instead, they are the background beliefs, or assumptions, that we hold constant in determining whether F undermines E.

Nevertheless, it might seem as though these relations don’t resolve the Undermining Problem either. Consider again Weisberg’s objection, which was that knowing at the outset that one’s experience will be appearance-based (AB) does nothing to confirm the evidence proposition E. My proposal seems to founder on this fact as well. If we take \( F \land AB \) to be a proposition that defeats the experience that supports E of its full evidential force without providing any extra reason for or against E, The Modified Higher-Order Beliefs Approach yields, \( p_0(E | F \land AB) = p_0(E) \). However, assuming that AB neither supports, nor deprives E of support, also leaves us with, \( p_0(E | AB) = p_0(E) \). By substitution, we get, \( p_0(E | F \land AB) = p_0(E | AB) \). This contradicts the set of relations I have proposed.

Despite this, I think The Modified Higher-Order Beliefs Approach is still our solution to the Undermining Problem. I think that what the previous objection shows is just that we have ascribed to the agent the wrong higher-order belief. It shows
that, insofar as we think the agent ought to revise after having gotten F as evidence, we must be making some assumptions about the case that the agent is not making in reflecting upon her own beliefs. In order to see this, it will be useful to go back and briefly consider what made the Undermining Problem seem so persuasive to begin with.

3.4.2 The Role of Higher-Order Beliefs

Consider again the original red sock case, that which was our model for the The Naive Account. I want to suggest that the intuition that rigidity has been violated in this case trades on the fact that we are forced to view it from two different perspectives. From the point of view of the reader, we know that F is an underminer for E, since we are told after the case has been described that we’ve been presented with a case of undermining defeat. We have the intuition that learning that F should lower the agent’s credence in E because we are told we ought to have this intuition. We are led to make the sorts of assumptions that our priors require to secure a negative correlation between E and F: for example, that the agent’s experience was sourced by a reliable visual faculty.

By contrast, nothing about the information that has actually been ascribed to the agent suggests that we ought to regard her as being in a case of undermining defeat. All that we know about her is that she has had some experience that has triggered an update on E, and has then been told that the lighting is red tinted—two facts that aren’t correlated in any obvious way.

Appealing in turn to these distinct epistemic standpoints enables Weisberg to make two observations about the red sock case that seem reasonable, and that, when taken together, imply that the Bayesian formalism is defective. The first observation is that Bayesianism ought to deliver the result that we get a negative correlation between
E and F at t₁. Since the case has been stipulated to be a case of undermining, our intuitions are informed by whatever implicit assumptions secure this. And, so, although we have no way of accounting for these assumptions in these relations, we assume the negative correlation between E and F that these assumptions would entail:

**The Implicit Perspective:**

\[ P_{IP}(E|F) < P_{IP}(E) \]

The second observation, triggered by the information that is explicitly assigned to the agent, is that Bayesianism doesn’t deliver the result that we get a negative correlation between E and F, since it’s been stipulated that the agent’s evidence set only contains E and F:

**The Explicit Perspective:**

\[ P_{EP}(E|F) = P_{EP}(E) \]

It’s in assuming that the agent has all of the implicit information that vindicates our perspective, while failing to assign to her the evidence that this perspective entails that we get the appearance of a violation of rigidity:

**The Undermining Problem**

\[ P_{EP}(E|F) = P_{EP}(E) \]
\[ P_{IP}(E|F) < P_{IP}(E) \]

How do these observations support The Modified Higher-Order Beliefs Approach? The way I’ve just diagnosed the Undermining Problem suggests that
it arises because, before we were provided with a plausible story about the role that higher-order beliefs play in cases of undermining defeat, it wasn’t possible to explicitly assign to the agent evidence that could vindicate the implicit perspective. This is what led us to be pulled by two incompatible sets of intuitions. But conceding the importance of higher-order beliefs changes things. Conceding the importance of higher-order beliefs gives us a way of explicitly assigning to the agent those assumptions we were implicitly making about her first-order beliefs, since this is precisely what higher-order beliefs are designed to do: they are designed to represent the agent’s perspective on her first-order beliefs. Since the higher-order perspective we assign to the agent must reflect our intuitions about the agent’s first-order beliefs in the case in question, and since these intuitions must be sufficient to secure the negative correlation between E and F that the framework supposedly can’t capture in cases of undermining defeat, the higher-order perspective must secure the negative correlation between E and F that the framework supposedly can’t capture in cases of undermining defeat. This means that the appeal to higher-order beliefs can’t help but make the Undermining Problem disappear.

With the role of higher-order beliefs made clear, we are now in a position to see how the objection we were left with at the end of the last section can be handled. Recall this was the worry that knowing at the outset that one’s experience will be appearance-based (AB) does nothing to confirm the evidence proposition E since, for all we know, this appearance might be misleading, in which case it would not establish a negative correlation between E and F at the outset. However, given what we’ve just said, it should be clear that there are two possibilities in the red sock case, each of which The Modified Higher-Order Beliefs Approach can accommodate. The first possibility is that the agent’s higher-order evidence does establish a negative correlation between E and F, from the point of view of her priors. If the agent has priors
like most of us, this higher-order evidence might be the belief that the experience she has had is appearance-based and proceeds from a reliable process (RAB):

\[
\begin{align*}
 p_0(E \mid F \land \text{RAB}) &< p_0(E \mid \text{RAB}) \\
 p_1(E \mid F \land \text{RAB}) &< p_1(E \mid \text{RAB})
\end{align*}
\]

But it doesn’t matter which particular belief is needed to secure these relations. The bottom line is that, in order for our intuitions to be tracking what is rational from the point of view of the agent, we must have assigned to the agent \textit{whatever assumptions were informing our intuitions about the case}, in the form of a higher-order belief—whether this is RAB or something else. Since there will always be some fact of the matter about what beliefs are needed to establish a negative correlation between E and F, relative to our priors, a tension between our intuitions and the verdict that the case at hand delivers will be impossible.\footnote{Perhaps one might object that a regress threatens here. One might worry that the higher-order belief that secures a negative correlation between E and F will need to be made more and more specific in order to establish a negative correlation between E and F. In other words, just as we have assumed that RAB is required to block some possibility that AB leaves open, we will need something even stronger than RAB to block some possibility that if leaves open, etc. But this objection is misguided. A regress occurs whenever the justifier for one thing itself requires justification, which itself requires justification, etc. The argument I’ve advanced does not have this structure. Instead what it says is that either you’ve made some higher-order update or you haven’t. If you have made that update, and that update establishes a negative correlation between the object proposition and the defeater, then we get a case of undermining defeat. And if you haven’t made that update, then we don’t. Since justification is going to be a matter of coherence with the agent’s priors, no regress threatens.}

Of course, the second possibility is that the agent’s higher-order evidence \textit{does not} establish a negative correlation between E and F, from the point of view of her priors. If the agent has priors like most of us, this higher-order evidence may be the belief that the experience she has had is appearance-based (AB):
\[ p_0(E \mid F \land AB) = p_0(E \mid AB) \]
\[ p_1(E \mid F \land AB) = p_1(E \mid AB) \]

Or this higher-order evidence may be the belief that the experience is appearance-based and misleading (MAB):

\[ p_0(E \mid F \land MAB) = p_0(E \mid MAB) \]
\[ p_1(E \mid F \land MAB) = p_1(E \mid MAB) \]

But, again, regardless of which particular beliefs are needed to secure these relations, the bottom line is that, in order for our intuitions to be tracking what is rational from the point of view of the agent, we must have assigned to the agent whatever assumptions were informing our intuitions about the case, in the form of a higher-order belief—whether this is AB or MAB or something else. This means that a tension between our intuitions and the verdict that the case at hand delivers will be impossible, for if these higher-order beliefs do not establish a negative correlation between E and F, from the point of view of the agent’s priors, this means that they do not establish a negative correlation between E and F, from the point of view of our priors. This means that we don’t have the intuition that the agent is in a case of undermining defeat.

In short, once we acknowledge the role that higher-order beliefs play in cases of undermining defeat, we acknowledge that they can’t help but rationalize the agent’s first-order beliefs by representing our implicit assumptions about the case at hand. And once we acknowledge this, we can’t help but make the Undermining Problem disappear.
3.5 A Deeper Error Theory for the Undermining Problem

Weisberg’s claim is that cases of perceptual undermining are cases where certain conditional probabilities change. I’ve argued that this way of understanding undermining defeat is called into question once we acknowledge the importance of higher-order beliefs to these cases. Acknowledging not only the importance of higher-order beliefs, but also their role, means that the verdicts rendered in such cases will necessarily track our intuitions about these cases, thereby dissolving the Undermining Problem.

It’s possible, however, that our discussion has overlooked a more fundamental concern. In the introduction, I noted that Weisberg thinks the Undermining Problem is really a problem about the incompatibility of Bayesianism and holism. Weisberg (2009) suggests that the rigidity of the Bayesian’s priors represents the unrevisability of certain beliefs, assumed to exist by foundationalists about epistemic justification, and denied by holists about epistemic justification (Weisberg 2009, 17; Weisberg 2015, fn. 6). In this last section, I want to briefly consider the relation between Bayesianism and holism. While this will uncover the sense in which Bayesianism might be held not to accommodate holism, I’ll argue that this doesn’t explain the genesis of the Undermining Problem. Instead, it leaves us with an error theory for it.

In order to understand the relation between Bayesianism and holism, we need to get a better grip on what holism commits us to. In Weisberg (2009), where the Undermining Problem is first introduced, Weisberg suggests that the tendency for some beliefs to be revisable—i.e., to have a defeater—is a consequence of the fact that such beliefs depend for their justification on our background beliefs. He provides us with the following passage:

HOLISM. For any experience and any [empirical] proposition, there is a “de-
feater” proposition, such that your degree of belief in the first proposition, upon having the experience, should depend on your degree of belief in the defeater proposition (Weisberg 2009, 5).

As stated, this description can be given a reading where the empirical proposition being referred to is any proposition the agent has some credence in, and a reading where the proposition being referred to is a proposition in the agent’s evidence partition. On the former reading, holism commits us to something like the following idea:

Holism (general) = df. The way you rationally revise your empirical beliefs will depend upon your background beliefs.

Bayesianism, of course, is not only compatible with this version of holism; it entails it. Jeffrey Conditionalization requires, after all, that the agent’s posterior credences be a function of her prior credences, i.e. of her background beliefs.

It’s perhaps the second reading of the passage that Weisberg has in mind, since in Weisberg (2009), we get the following definition:

Holism (Weisberg) = df. The evidential import of an experience is always sensitive to background assumptions (Weisberg 2009, 1).

Bayesianism is compatible with this version of holism as well. Since Jeffrey Conditionalization is compatible with the agent having gotten any evidence partition, it’s clearly compatible with any manner in which this evidence partition has been informed: it’s compatible with the agent’s evidence having been informed by her background beliefs, and it’s compatible with the agent’s evidence not having been informed by her background beliefs. Thus, Bayesianism entails Holism (general) and is compatible with Holism (Weisberg) even though Bayesianism is rigid.

It would seem, then, that rigidity isn’t a symptom of a more general anti-holistic feature of the Bayesian framework. Rather, rigidity represents an innocuous way that
part of the framework is fixed that does not preclude the agent’s empirical beliefs from being informed by her background beliefs. Having already established, coming into this section, that undermining defeat is compatible with rigidity, it’s clearly also the case that undermining defeat is compatible with holism.\textsuperscript{15} Rigidity, undermining and these versions of holism are mutually compatible.

Can we conclude, then, that Bayesianism does indeed accommodate holism? Not quite. In order for an updating rule to accommodate holism, it’s perhaps not enough that it be merely consistent with the idea that the evidential import of an experience is always sensitive to background assumptions. Plausibly, to accommodate holism, an updating rule needs to regulate how this happens: it must give us some sort of guidance as to how our background beliefs inform our updates, via our evidence:

\[ \text{HOLISM (GUIDANCE)} =_{df} \text{The evidential import of an experience is always a function of background assumptions (Weisberg 2009, 1).} \]

Insofar as there is a legitimate worry about Bayesianism being unable to accommodate holism, I think it must be the worry that it does not accommodate HOLISM (GUIDANCE). Some remarks Weisberg makes in the introduction of Weisberg (2015) suggest that he thinks the Bayesian’s inability to accommodate HOLISM (GUIDANCE) is related to the Undermining Problem:

The worry that Bayesianism runs afoul of perceptual undermining originates with David Christensen (1992). Christensen argues that Bayesian update rules at best treat the interaction between perception and background belief as a black box. My response to a reddish glimpse of a sock should depend on what I think about the reliability of my vision, but Bayesianism does not model or regulate this interaction. If I suspect my

\textsuperscript{15}Since undermining defeat is simply an instance of Jeffrey Conditionalization, and Jeffrey Conditionalization entails HOLISM (GENERAL) and is compatible with HOLISM (WEISBERG), undermining defeat entails HOLISM (GENERAL) and is compatible with HOLISM (WEISBERG).
vision is unreliable, the Bayesian can recommend that I Jeffrey Conditionize on the proposition that The sock is red with a middling probability instead of a high one. But the choice of a middling input instead of a high one is not something the Bayesian formalism explains or prescribes.

In my (2009), I took Christensen’s argument a step further. Jeffrey Conditionalization doesn’t just fail to regulate perceptual undermining, it bungles it. Consider the case where I first glimpse the sock, then learn the lighting is deceptive. Surprisingly, Jeffrey Conditionalization makes my discovery about the deceptive lighting ineffectual, failing to undermine my belief that the sock is red. The reason is that Jeffrey Conditionalization is “rigid” (§1.1), and thus independence preserving. (Weisberg 2015, 1-2).

Weisberg, following Christensen, is certainly right that Jeffrey Conditionalization doesn’t accommodate HOLISM (GUIDANCE). And he’s right to point out that both are problems about perceptual undermining. Nevertheless, it’s important to recognize that HOLISM (GUIDANCE) is orthogonal to the Undermining Problem. Christensen’s worry is that Jeffrey Conditionalization doesn’t provide a rule for how background beliefs inform our evidence partition before we update. The Undermining Problem is the worry that a formal property of Jeffrey Conditionalization blocks us from accommodating undermining defeaters, after we know which evidence partition we ought to use. Since the Undermining Problem begins where Christensen’s worry ends, it can’t be the case that the latter gives rise to the former. It can’t be that the Undermining Problem is a problem about Bayesianism being unable to accommodate HOLISM (GUIDANCE).

What I think we get from Christensen’s worry isn’t the source of the Undermining Problem, but an error theory for it. The lack of regulation, when it comes to ascribing to the agent the evidence she must have in some particular case, provides an explanation for why we mistakenly fail to ascribe to the agent the higher-order beliefs that dissolve the Undermining Problem in the sorts of cases Weisberg considers. If we had
a complete, two-step updating rule—one that took us from an experience and set of background beliefs to a weighted evidence partition, and from a weighted evidence partition and set of background beliefs to an update—it would be impossible to do this, since part of what this rule would determine is what the agent’s evidence is. The fact that we don’t have such a rule is what allows us to assume that the agent occupies our perspective—the implicit perspective—without also ascribing to her the higher-order evidence that this perspective entails.
Chapter 4

Commutativity, Normativity and Holism: Lange Revisited

Abstract. Lange (2000) famously argues that although Jeffrey Conditionalization is non-commutative over evidence, it’s not defective in virtue of this feature. Since reversing the order of the evidence in a sequence of updates that don’t commute does not reverse the order of the experiences that underwrite these revisions, the conditions required to generate commutativity failure at the level of experience will fail to hold in cases where we get commutativity failure at the level of evidence. If our interest in commutativity is, fundamentally, an interest in the order-invariance of information, an updating sequence that does not violate such a principle at the more fundamental level of experiential information should not be deemed defective. This paper claims that Lange’s argument fails as a general defense of Jeffrey Conditionalization. The account of experience we would need to get Jeffrey Conditionalization off the hook gives rise to problems that are just as serious as the one this account enables Jeffrey Conditionalization to avoid.

The non-commutativity of Jeffrey Conditionalization is typically regarded as a mark against it. The reason isn’t difficult to grasp. Consistency seems to require that identical pieces of evidence be treated the same, no matter the order in which they are received. A non-commutative updating rule appears to flout this requirement.

Lange (2000) famously argues that although Jeffrey Conditionalization is non-commutative over weighted evidence partitions, the Jeffrey framework isn’t defective in virtue of this feature. This is because reversing the order of the evidence in a sequence of non-commutative updates does not reverse the order of the experi-
ences that underwrite these revisions. If our interest in commutativity is, fundamentally, an interest in the commutativity of information, an updating sequence that does not violate such a principle at the more fundamental level of *experiential* information should not be deemed defective.

This paper contends that Jeffrey Conditionalization cannot be saved by Lange’s argument. The account of experience we would need to get Jeffrey Conditionalization off the hook gives rise to problems that are just as serious as the one it enables Jeffrey Conditionalization to avoid.

At the heart of this discussion is the question of what exactly it means to be *defective* in virtue of being non-commutative. In many contexts, it makes no sense to talk about some operation being defective because it is non-commutative. There’s no sense in which multiplication and addition are better operations than division and subtraction because the former are commutative, whereas the latter are not. The fact that Jeffrey Conditionalization is typically assumed to be defective in virtue of not commuting updates, then, means that, unlike in these other cases, it fails to commute something that it *ought* to commute. The main question this paper addresses is whether it can compensate for this failing by regulating something else—something more fundamental. The main contribution this paper makes, then, is to spell out what Lange’s argument teaches us about the normative structure of Jeffrey Conditionalization. In the end, we’ll see that we cannot escape the conclusion that this structure is defective.

Here’s how the discussion will go. In §1, I outline Lange’s argument. In §2 and §3, I consider the account of experience that Lange’s argument assumes. I argue that the account of experience we would need to defend Jeffrey Conditionalization against the charge that it is defective in virtue of being non-commutative is a stronger account of experience than the one Jeffrey Conditionalization entails.
Therefore, the norm that isn’t defective in virtue of being non-commutative is actually a stronger norm than Jeffrey Conditionalization. In §4, I show that this stronger norm faces a different problem: it requires that experience rationalize a belief revision. The lack of some principled way of giving content to such a norm undermines Lange’s argument in a different way. Finally, in §§5, I consider how the literature on this problem supports these results. I suggest that this literature also offers us a different way of understanding what makes it so tempting to buy into Lange’s argument to begin with.

4.1 Lange’s Argument

Standard Bayesianism assumes that an agent’s degrees of confidence, or credences, in those propositions she entertains can be represented as an assignment of real numbers to those propositions. It further assumes that the following synchronic norm governs this assignment:

**Probabilism:** An agent’s credences ought to obey the probability axioms.

More importantly for our purposes, most Bayesians also take an agent’s credences to satisfy a diachronic constraint. This constraint tells us how we ought to update our beliefs when we are presented with some new evidence:

**Bayesian Conditioning:** When you acquire new evidence B, your credences in A for any A should be \( p'(A) = p(A | B) \).\(^1\)

Since our evidence always ought to get a credence of one on this updating framework, a commitment that those who endorse it take on is that we should always be

\(^1\)Where \( p(A | B) = \frac{p(A \land B)}{p(B)} \).
certain of our evidence. But, to many, it seems as though the evidence we get from our sensory experience is often uncertain. Indeed, to some it seems as though the experiences that underwrite belief changes almost never leave us certain of anything at all.

To accommodate this intuitive idea, Jeffrey (1965) proposed an alternative to Conditionalization that generalizes this model. Jeffrey suggests that our evidence takes the form of a weighted partition of propositions: an n-tuple of propositions \( \langle B_1, ..., B_n \rangle \) that partitions the agent’s credal state, and that are assigned an n-tuple of credences:

**JEFFREY CONDITIONALIZATION:** When experience directly changes your credences over a partition \( \{B_i\} \) from \( p(B_i) \) to \( p'(B_i) \), your new degree of belief in \( A \), for any \( A \), should be \( p'(A) = \sum_i p(A \mid B_i) p'(B_i) \).

It’s easy to see that, in many cases, the order in which two Jeffrey updates happen will determine the credence distribution one ends up with. Consider the case where I see a raven and this leads to me directly changing my credence in the proposition that *the raven is black* to .9, i.e., \( p(RB) = .9 \), and then after a second glance, I come to directly change my credence in this proposition to .7. After I update on these two pieces of evidence, I am left with a credence of .7 in RB. But had I made these two updates in reverse order, I would have been left with a credence of .9 in RB. Therefore, the order in which these two pieces of evidence are received makes a difference to my post-observational credence distribution. Jeffrey Conditionalization is non-commutative over evidence partitions.

In some cases, this feature of the framework seems exactly right: the probative value of my most recent evidence seems to swamp the value of the evidence I’ve gotten before. If I glance an object from afar, and then glance it again at a much
closer distance, the information I get from the second glance should make my first glance irrelevant.

But in just as many cases, this seems like the wrong result. Two consecutive glances taken from the very same vantage point shouldn’t necessarily lead us to discount the deliverances of the first.

An assumption made by those who criticize Jeffrey Conditionalization for being non-commutative is that the elements for which the commutative property ought to hold are updates—or, equivalently, weighted evidence partitions. In his paper, Lange calls into question these criticisms by pointing out that updates that don’t commute in this way will be underwritten by different experiences, in the original case and its permutation. If experiences are different in two sequences of updates where the order of the weighted evidence partitions have been reversed, then the conditions required to generate commutativity failure at the level of experience will fail to hold in cases where we get commutativity failure at the level of weighted evidence partitions. Therefore, there’s a more fundamental type of information that isn’t non-commutative under the framework: experiential information.

Why think that experiences will be different in cases where evidence partitions don’t commute? Lange thinks that the answer to this question lies with the agent’s background beliefs. It lies especially with one kind of background belief: the agent’s prior opinion about her evidence. To begin to see this, consider the following passage from Skyrms (1986, p.197), which Lange cites in his paper:

Suppose I see a bird at twilight which I clearly identify as a raven. Because the light is not so good, the probability I can assign to him being black on the basis of that observation is only 0.8. Suppose further that I hold the theory that all ravens are black and that this theory is buttressed by massive numbers of previous observations. In such a situation the final probability I assign to the statement that the raven
is black will be higher than the observational probability, and quite properly so. Otherwise I could disconfirm lots of theories just by running around at night.

Skyrms’s point is that the agent’s background beliefs will be relevant to the probative value of an experience. If we remember that all the ravens we’ve seen in the past have been black, and we know that the lighting conditions aren’t that great, a clearly identified raven whose color we cannot rule out as being brown should not result in a low credence in RB, even though the same appearance might have led us to assign a low credence to this proposition, had we believed that it was daytime, or had we not expected to see a black raven.

Things get interesting when we consider the relevance of a particular kind of background belief: the agent’s prior opinions about her evidence. It’s easy to see why such beliefs are important. If my prior credence in the proposition that the raven I will see a moment from now will be black is pretty low, an experience that’s not so clear or intense will result in my still having a low credence in this proposition. By contrast, if my prior credence in the proposition that the raven will be black had been higher, this same mildly informative experience would have resulted in my having a relatively higher credence in this same proposition. Therefore, along with my experiences, my prior opinion about my evidence will determine the weights that I will assign to the members of the evidence partition. The same experience in the presence of different prior opinions will yield different posterior credence distributions.

This same reasoning suggests something else. It suggests that the same posterior credence distribution gotten from different prior opinions will imply that the agent has had different experiences. That is, like Lange, we might reason as follows:

For an experience at twilight to have lowered our confidence in \( \epsilon \) from 0.99 to 0.8, the bird must have not looked much the way a black bird
would be expected to look at twilight, whereas for an experience at twilight to have raised our confidence in $e$ from 0.75 to 0.8, the bird must have looked about the way that any dusky colored object would be expected to look under those conditions. Plainly, these are different experiences.

We can walk through things more slowly to see how this reasoning is supposed to lead to the conclusion that the Jeffrey framework isn’t defective in virtue of being non-commutative. First, consider that since any update involves the agent changing her credence in the evidence proposition, a non-commutative update will entail that the agent’s priors are different in the original sequence and its permutation ($p \neq q'$ and $p \neq q$):

$p \quad r$
$\bar{\xi}_1 \quad \bar{\xi}_2$
$t_0 \quad t_1 \quad t_2$

$p \quad q' \quad r'$
$\bar{\xi}_3 \quad \bar{\xi}_4$
$t_0 \quad t_1 \quad t_2$

Figure 1: Different Priors, Same Posteriors = Different Experiences

Second, notice that in order for an update to be non-commutative, we must have had the same evidence in the original sequence and its permutation. Therefore, it must be the case that the agent’s posterior credence distribution after the first update in the first sequence is identical to her posterior credence distribution after the second update in the second sequence, and vice versa ($q = r'$ and $q' = r$).

By Lange’s reasoning in the previous passage, this means that the experiences that trigger the two updates in question must have been different in the original sequence and its permutation ($\bar{\xi}_1 \neq \bar{\xi}_4$ and $\bar{\xi}_2 \neq \bar{\xi}_3$), in order to “offset” the difference in prior opinions to yield the same posterior credence distribution. And this
gets us Lange’s conclusion. Since the conditions required to get a violation of experience commutativity (two experiences whose orders have been reversed) fail to hold in cases where we have reversed the order of the evidence partitions, the non-commutativity of evidence partitions does not entail the non-commutativity of experiences. Since the framework isn’t non-commutative over a more fundamental set of elements, we ought to conclude that it is not defective.\(^2\)

Let’s consider a final example. Imagine that a whiff of pie leaves you with a credence of .3 in the proposition that the pie in the oven is rhubarb (R) at \(t_1\) and then, a moment later, another whiff leaves you with a credence of .7 in the same proposition at \(t_2\). Now imagine that you’d gotten the previous evidence in reverse order:

\[
\begin{align*}
\text{at } t_0 & \quad \text{at } t_1 & \quad \text{at } t_2 \\
p(R) = .1 & \quad q(R) = .3 & \quad r(R) = .7
\end{align*}
\]

\[
\begin{align*}
\text{at } t_0 & \quad \text{at } t_1 & \quad \text{at } t_2 \\
p(R) = .1 & \quad q'(R) = .7 & \quad r'(R) = .3
\end{align*}
\]

Figure 2: Smell of Rhubarb Pie

Here, again, we have a case where the fact that weighted evidence partitions don’t commute under the framework doesn’t seem like a defect of the framework since the experiences underlying these partitions in the original case and its permutation aren’t likely to be identical. The fact that we have raised our credence slightly

\(^2\)Of course, one might ask: what about those non-commutative updates that don’t involve a change in the agent’s prior opinions about her evidence? Though Lange does not address these sorts of cases in his paper, plausibly he believed that appealing to the normative import of the agent’s background beliefs could help us here too. (See Wagner (2002) for an argument for this. For a more detailed discussion of the conditions under which an update will fail to commute, see Wagner (2003) and Diaconis and Zabell (1982), §3.3.) In §4, it will be shown that, to whatever extent invoking the normative import of an agent’s background beliefs can help us make sense of a sequence of updates, this move will be open to the general objection that I raise for Lange’s argument.
at $t_1$ from $p(R) = .1$ to $q(R) = .3$ in the first sequence, in response to the evidence
gotten in between $t_0$ and $t_1$, suggests that the experience we’ve had is something
like a decisive whiff of rhubarb pie. By contrast, where we decrease our credence
from $q'(R) = .7$ to $r'(R) = .3$ in the second sequence, in response to having gotten
the same weighted evidence partition in between $t_1$ and $t_2$, that must have been
because we had a “disconfirming” experience—maybe a whiff of lemon (which is
not an ingredient in rhubarb pie). A similar story could be told about the second
update in the first sequence, and the first update in the second sequence.

Though weighted evidence partitions don’t commute under Jeffrey Conditional-
alization, then, that’s altogether appropriate, since they don’t supervene on the same
experiences in the original case and its permutation. If $p \neq q'$ and $p \neq q$, but $q = r'$
and $q' = r$, then the experience that triggered the first update in the first sequence
must have been different than the experience that triggered the second update in
the second sequence, and vice versa. Therefore the experiences underlying these
sequences aren’t the same. Therefore, there’s no reason we would want it to be the
case that $r = r'$.

Although Jeffrey Conditionalization isn’t commutative over weighted evidence
partitions, then, examples like this one suggest that it’s not defective in virtue of this
feature of it.

4.2 A Partial Account of Experience

The crucial move in Lange’s argument is his claim that Jeffrey Conditionalization
is not defective because it doesn’t fail to commute experiences. A natural question
to ask is whether this defensive move commits Lange to some positive thesis about
commutativity. I think that it must. In particular, I think that it must commit Lange
to the thesis that experiences ought to commute, and that they do commute. Without this assumption, his reasoning that Jeffrey Conditionalization is not defective because it doesn’t fail to commute experiences makes no sense. It amounts to the claim that Jeffrey Conditionalization isn’t defective because it doesn’t fail to commute experiences, but that this isn’t in any way a good thing. Therefore, Lange’s argument seems to be committed to the following principle:

LANGE’S ASSUMPTION: The elements that ought to commute under Jeffrey Conditionalization are experiences and Jeffrey Conditionalization is commutative over experiences.

We can take a step back and appreciate that, regardless of what Lange’s actual intentions may have been, LANGE’S ASSUMPTION is what is needed for his argument to succeed. The question Lange sets out to address in his paper—the question of whether Jeffrey Conditionalization is defective in virtue of being non-commutative—assumes that the question of how our evidence behaves does not fall beyond the purview of a theory of rational belief revision. An updating framework is not defective insofar as it regulates this evidence, either directly by commuting it, or indirectly by regulating the experiences that give rise to it. Since the Jeffrey framework does not regulate evidence directly, it needs to vindicate LANGE’S ASSUMPTION in order to be deemed non-defective.

Does the framework vindicate LANGE’S ASSUMPTION? Let’s focus on the second conjunct: that Jeffrey Conditionalization is commutative over experiences. Some remarks Lange makes near the end of the paper about the account of experience his argument assumes suggest how he hopes to secure this part of the assumption. Recall that Lange’s conclusion is that experiences must have been different in two sequences of updates that feature different priors, but identical posterior cre-
dence distributions. This implies that, had both prior and posterior credence distributions been the same, the experiences involved could have also been inferred to have been the same. We get something like this idea near the end of the discussion:

Consider two agents who undergo sensory experiences, where neither agent is left with a full belief in some proposition that captures all that the agent learned from her sensory experience. I am inclined to suggest that the two agents are undergoing the same sensory experience exactly when it is the case that had the two agents begun with the same prior probability distribution, then they would as a result of their actual sensory experiences have imposed exactly the same constraints on that distribution, and this agreement would have resulted no matter what the two agents’ common prior probability distribution had been (2000, p. 401).

This passage makes it sound as though Lange is putting forth necessary and sufficient conditions for experience identity. In the next section, we will consider more carefully whether or not this passage should indeed be read in this way. For now, we can say that, at the very least, this passage says that if two experiences are identical, then they will have the same impact on an agent’s credence distribution.

We can formulate this idea more precisely and say that, for experiences, \(\xi\) and \(\xi^*\), and credence functions, \(p(\cdot)\) and \(q(\cdot)\) defined over every proposition, \(X\), in some \(\sigma\)-algebra, \(\mathcal{A}\) (and where \(p_\xi(\cdot)\) comes from \(p(\cdot)\) by updating on the weighted evidence partition induced by \(\xi\), and \(q_{\xi^*}(\cdot)\) comes from \(q(\cdot)\) by updating on the weighted evidence partition induced by \(\xi^*\)), the following holds:

**An Account of the Impact of an Experience:**

\[
\forall p, q \forall X \in \mathcal{A} [\forall \xi, \xi^* (\xi = \xi^*) \rightarrow (p = q \rightarrow (p_\xi(X) = q_{\xi^*}(X)))].
\]

While there are a number of ways the subjunctive component of this constraint might be read, one interpretation of it seems natural. The impact, or degree of
change, that some experience induces in an agent’s credence distribution is often
identified with the Bayes factor of that update. Formally, two updates have the
same Bayes factors just in case the ratios of the new-to-old odds of the elements
of each of the evidence partitions, taken pairwise, are identical. 3 Substituting the
requirement that two updates have the same Bayes factors \( \mathcal{B}_F_{p_p\xi} = \mathcal{B}_F_{q_q\xi^*} \)
for the subjunctive constraint from An Account of the Impact of an Experience
leaves us with:

**An Account of the Impact of an Experience (BF):**

\[
\forall p, q [\forall \xi, \xi^*(\xi = \xi^*) \rightarrow (\mathcal{B}_F_{p_p\xi} = \mathcal{B}_F_{q_q\xi^*})].
\]

Finally, this formulation of the passage from Lange’s discussion gets us the con-
nection to experience commutativity we are after. This is because a series of results
confirm that two updates will commute just in case they yield the same Bayes fac-
tors in the original case and its permutation.4 Since An Account of the Impact
of an Experience (BF) says that a necessary condition for experience identity
is a property that is necessary and sufficient for experiences to commute, it vind-
icates the second conjunct of Lange’s Assumption. Therefore, An Account of
the Impact of an Experience (BF) is a necessary condition for the account of

---

3

\[
\frac{p_\xi(E_{i_1})/p_\xi(E_{i_2})}{p(E_{i_1})/p(E_{i_2})} = \frac{q_\xi^*(E_{i_1})/q_\xi^*(E_{i_2})}{q(E_{i_1})/q(E_{i_2})},
\]

for all \( i_1, i_2 \).

4See Field (1978) for the left-to-right half of the biconditional. See Wagner (2002) for a theo-
rem that establishes both directions of the biconditional for commutativity on updates. Wagner’s
theorem also “generalizes out” Field’s nominalism in establishing the result that Bayes factor iden-
tities individuate commutative updates involving countable evidence partitions as well.
4.3 A Complete Account of Experience

Is An Account of the Impact of an Experience (BF) also a sufficient condition for the account of experience Lange’s argument relies upon? Although Lange certainly makes it sound as though he is putting forth necessary and sufficient conditions in the passage that points us towards this account, at the very end of the paper, he says the following:

Of course, a fuller account of the sameness of sensory experiences would be very welcome. But I cannot offer one at present (2000, p. 402).

How do we reconcile these two seemingly inconsistent passages? In a way, this will be the question that guides the rest of this discussion. Plausibly, Lange makes the latter statement because he believed that a necessary condition for experience identity is some sort of qualitative constraint—one that we might think of as distinguishing experiences with different phenomenal characters. In this section, I argue that not only is such a constraint plausible, but that some such constraint on experience identity looks like it’s required to secure the conclusion that Jeffrey Conditionalization is not defective. This means that the norm that Lange’s argument defends as not defective is actually a stronger norm than Jeffrey Conditionalization, for it must include a constraint that Jeffrey Conditionalization does not entail.

To begin to see this, recall again the following assumption that Lange’s argument relies upon:

Lange’s Assumption: The elements that ought to commute under Jeffrey Conditionalization are experiences and Jeffrey Conditionalization is commutative over experiences.
Let’s now focus on the first conjunct: that the elements that ought to commute under Jeffrey Conditionalization are experiences. Lange is careful in his discussion to distinguish the claim that: (1) Jeffrey Conditionalization is *formally* non-commutative over weighted evidence partitions, from the claim that (2) Jeffrey Conditionalization is *defective* in virtue of being non-commutative over weighted evidence partitions (2000, pp. 393, 397). The first is a mathematical claim; the second is a normative claim. And, of course, it is the second claim that Lange’s argument targets.

We’ve said already that the second claim is an unusual one, since it assumes that commuting evidence is a good thing, whereas the commutative property is most often regarded as an arational feature of operations. In order for experience commutativity to save Jeffrey Conditionalization from being defective, then, we must assume that, unlike arithmetic operands, the identity conditions for experience make experience commutativity also a good thing.

The most obvious thing to say here is that the reason why experiences ought to commute is that they are phenomenally indistinguishable. In other words, what saves Jeffrey Conditionalization from being defective is that it treats phenomenally indistinguishable experiences the same. But maybe this is too quick. Maybe we don’t actually need to identify experiences with anything phenomenal or qualitative to do justice to the idea that treating experiences consistently makes the Jeffrey framework non-defective. Maybe there’s something about the degree of an impact on a credence distribution that makes this the thing that ought to be treated consistently, and so that ought to commute.

There are several reasons to reject this suggestion. First, it does not seem to be what Lange had in mind; never once in the examples he provides throughout the paper does he talk about the agent’s doxastic behavior. Instead, he talks about ‘sensory
experiences’—or sometimes just ‘experiences’—being different in two sequences of updates where the weighted evidence partitions have been reversed (2000, pp. 394-95, 397). Sometimes he talks about particular sensory experiences, like “the way that any dusky colored object would be expected to look under those conditions” (2000, p. 398). And, of course, there is the passage we were left with in the last section, where Lange acknowledges that he has failed to offer a complete account of experience, in providing an account of how identical experiences impact an agent’s credence distribution.

Second, regardless of Lange’s intentions again, this is clearly not what he should have had in mind. To whatever extent we think that there is something intuitively good about belief revisions commuting, it’s difficult to imagine that this is not because we are assuming that the experiences that ground these revisions are phenomenally identical—i.e., that the feeling that induces these changes is the same. As Bradley (2005, p. 6) puts it, “it does not follow from the fact that your Bayes factors represent what you have gleaned from observation, that they have the kind of objectivity which obligates others to modify their beliefs using them as constraints on their posteriors.”

To see Bradley’s point more clearly, consider the lack of rhetorical force Lange’s argument would have if it did not include a description of the experiences that prompted the belief revisions he describes. To claim that Jeffrey Conditionalization is not defective because reversing the order of the weighted evidence partitions involved in two updates does not entail that the magnitudes of these revisions are identical comes pretty close to claiming that Jeffrey Conditionalization is not defective in virtue of being non-commutative because its updates don’t commute. It comes pretty close to being just a description of the fact that Jeffrey Conditionalization is non-commutative, rather than a defense of it.
These considerations suggest that Lange’s Assumption—that Jeffrey Conditionalization is not defective in virtue of commuting experiences—requires that we think of experiences as individuated by something more than merely Jeffrey Conditionalization’s commutative property. Plausibly, this “something more” is an experience’s phenomenal character. But then this means that experience commutativity won’t be trivially preserved by the Jeffrey framework. If we want to ensure that experiences, individuated in a way that would make Jeffrey Conditionalization not defective in virtue of commuting such experiences (as required by the first conjunct of Lange’s Assumption) actually do commute (as required by the second conjunct of Lange’s Assumption), we need an additional norm to secure that experiences with the same phenomenal characters actually commute under the framework. The norm that isn’t defective in virtue of being non-commutative isn’t Jeffrey Conditionalization, then, but an updating rule comprised of two constraints. The first constraint is Jeffrey Conditionalization. The second constraint secures Lange’s Assumption, by ensuring that phenomenally indistinguishable experiences yield updates with the same Bayes factors, and so commute:

**Experience-Commuting Jeffrey Conditionalization (ECJC):**

1. When an experience, \(\xi\), directly changes your credences over a partition \(\{E_i\}\) from \(p(E_i)\) to \(p'(E_i)\), your new degree of belief in \(A\), for any \(A\), should be

\[
p'(A) = \sum_i p(A|E_i)p'(E_i).
\]

---

5Strictly speaking, the structure of my objection does not require that experience identity be individuated by phenomenal character. To get the conclusion that the norm that isn’t defective in virtue of being non-commutative is a stronger norm than Jeffrey Conditionalization it suffices that (1) experiences be partly individuated by some feature that makes commutativity on experiences plausibly a norm, and that (2) this feature is not entailed by Jeffrey Conditionalization. While this could be any number of things, an experience’s phenomenal character seems like the most natural candidate.
2. For any two experiences, $\zeta$ and $\zeta^*$, that are identical with respect to phenomenal character, and for any credence distributions, $p(\cdot)$, $q(\cdot)$, the following should hold: $BF(p, p_\zeta) = BF(q, q_\zeta^*)$.\(^6\)

Lange’s argument entails that the norm that isn’t defective in virtue of being non-commutative over weighted evidence partitions is ECJC. Jeffrey Conditionalization alone does not commute the elements that ought to commute. But ECJC is able to do this.

### 4.4 Commutativity, Normativity and Holism

We’ve just seen that the Jeffrey framework is too weak to vindicate the claim that it is not defective in virtue of being non-commutative. However, even if one is persuaded by the argument, perhaps it does not seriously undermine the possibility of defending the Jeffrey framework. Perhaps it simply highlights that the framework needs to be supplemented with the second condition of ECJC. In this section, I argue that this optimistic thought is mistaken. While ECJC is a coherent schema, giving content to its second condition in a way that would turn it into a well-defined rule looks like it might be impossible to do. This means that ECJC does not, in general, fare any better than regular Jeffrey Conditionalization. It’s just defective in a different way.

To begin to see the problem, consider Jack and Jill, who have the same priors and who have had the same phenomenal experience in response to a whiff of rhubarb pie. Suppose that, in response to this experience, Jack updates in a way that leaves his credence in the proposition that *the pie in the oven is rhubarb* at $p(R) = .7$.

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\(^6\)Where, again, $p_\zeta(\cdot)$ is the function that comes from $p(\cdot)$ by updating when $\zeta$ directly changes the agent’s credences along the partition $\{E_i\}$, and $q_\zeta^*(\cdot)$ is the function that comes from $q(\cdot)$ by updating when $\zeta^*$ directly changes the agent’s credences along the same partition, $\{E_i\}$
Suppose that Jill updates in a way that leaves hers at $p(R) = .8$. Recall that ECJC says that phenomenally indistinguishable experiences will be represented by the same Bayes factors. Since an agent’s priors and the Bayes factor of her update—both of which Jack and Jill share—determine a unique posterior credence distribution, this means that ECJC would have to say that either Jack or Jill has gotten something wrong. But there’s nothing we can appeal to in order to justify giving content to ECJC in a way that would render the verdict that either Jack or Jill is wrong. The problem is that, though the appeal to intuition Lange makes in his paper convinces us that ECJC should map experiences to Bayes factors, it does not tell us anything about how particular experiences should be mapped to particular Bayes factors, in the way that we would need to give content to a norm like ECJC. Without such guidance, any way of giving content to ECJC threatens to make its prescriptions arbitrary. And this undermines the rationality of its prescriptions. For if phenomenal experiences are mapped to Bayes factors arbitrarily, it’s difficult to see why they ought to commute. It’s difficult to see why we should care about regulating them at all.\(^7\)

The Jack and Jill case is just the tip of the iceberg. Even if particular experiences could be mapped to particular Bayes factors, as many have noted, we get weird results when we fail to acknowledge that background beliefs other than the agent’s prior opinions about her evidence seem to be relevant to the probative value of her

\(^7\)One might object here that what generates the problem is the assumption that whatever relation maps phenomenal experiences to Bayes factors must make it the case that two agents have the same ECJC function. But maybe this assumption is unwarranted. Maybe different agents have different ECJC functions, so that these functions are consistent with Jack and Jill both rationally ending up with different posterior credence distributions after having had the same experience.

But the interpersonal feature of the example is not an essential feature of the worry. To see this, we might imagine that two different time-slices of Jill update to two different credence distributions, in response to the same experience and against the same background beliefs. In this case, we face a parallel dilemma: either the first time-slice of Jill has gotten something wrong, or the second one has. Which one?
update. Famously, Garber (1980) argues that mapping phenomenal experiences to Bayes factors yields the wrong result in cases where we have the same phenomenal experience over and over again. To modify Garber’s example, say that I keep having the same blurry visual experience of a black raven. If each blurry experience yields the same Bayes factor, then having it enough times will lead me to hold the proposition that the raven is black with something close to certainty, which seems absurd.

More recently, Weisberg (2009) has taken essentially the same objection in a different direction. Rather than arguing that we get bad results when we assume that the impact of phenomenally identical experiences is additive, he notices that we get bad results when we assume that the impact of phenomenally identical experiences commute. To illustrate with his own example, say that I update on the proposition that the lighting is red tinted, and then I update on the proposition that the sock is red. If the phenomenal experiences underlying these updates are the same, then they should commute. But this means that my background belief about the tricky lighting cannot influence the probative value of my experience of the red sock, in the way that it should if some reasonable version of confirmation holism is correct. For whether it should influence the probative value of my experience of a red sock depends upon whether it comes before or after the experience, contra the assumption that the order of these experiences does not make a difference.8

Wagner (2002) offers a suggestion that overcomes these worries. Wagner maintains that it is considered experiences—experiences in the light of all relevant background beliefs—that should be mapped to Bayes factors. If we assume this, then

8It’s unclear how committed Weisberg himself is to this argument, since it is not the main argument of the paper, but, instead is used as support for the claim that the rigid nature of Bayesian updating rules makes it incompatible with undermining defeat.
Garber’s objection is no worry. Since the considered experiences will be different in each case where we have the same mildly informative phenomenal experience over and over again—since something like the memory of our previous experience must play a role—what this will yield are updates whose Bayes factors decrease over time. In this way, we avoid the result that continually updating on the same phenomenal experience will lead us to be almost certain of some proposition.

In a similar way, in the case that Weisberg describes, the fact that the agent’s considered experiences will be different in the original case and its permutation—since the agent’s belief about the tricky lighting will be a relevant background belief when it comes before the experience of the red sock, but not after—means that the Bayes factors of the updates in the original case and its permutation won’t be identical. In this way, we avoid the result that having the same phenomenal experiences in reverse order should lead us to ignore those background beliefs that seem relevant.

But while Wagner’s proposal helps with these problems, it comes at a cost. Mapping considered experiences to Bayes factors seems even less feasible than mapping simple phenomenal experiences to Bayes factors. Any rule that does the former would have to encode information about which background beliefs are relevant to the probative value of an experience—alone and in combination with other background beliefs—and of how relevant these background beliefs will be. Such a rule would make the question of whether it is Jack or Jill who updates in a rational way a matter of all of the relevant background beliefs that they hold.910 It’s even less clear

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9For a discussion of the difficulties involved in providing this sort of account, see Christensen (1992).
10Whether or not this idea requires that we reject ECJC depends upon whether one thinks that background beliefs actually affect the phenomenal character of some experience. If they do, then ECJC stands intact: these arguments show that we could not have possibly had the same phenomenal experience in the presence of different background beliefs. If background beliefs don’t actually affect the phenomenal character of some experience, then ECJC is wrong, and we need a rule that says that Bayes factors should be mapped to pairs of background beliefs and phenomenal experiences. But whatever we think about this, the main point stands: either of these things will be a
than before that such a rule could be formulated in a non-arbitrary way. Again, the inability to formulate this kind of rule in a non-arbitrary way would undermine the rationality of its prescriptions: it would undermine the idea that considered experiences ought to commute. It would undermine Lange’s argument by undermining the first conjunct of Lange’s Assumption.

We can again take a step back and consider what all this tells us about the structure of the Jeffrey framework—or the more adequate version of it that ECJC depicts. Recall we said earlier that the question Lange addresses in his paper entails that how our evidence behaves does not fall beyond the purview of a theory of rational belief revision. We said that this means that the Jeffrey framework will be defective, insofar as it fails to regulate weighted evidence partitions or, alternatively, that which gives rise to them. The Jeffrey framework does not regulate evidence partitions—it does not commute them. And we have just seen the difficulties involved in giving content to a norm that would regulate, by commuting, those experiences (or considered experiences) that give rise to these partitions. Therefore, it looks like the Jeffrey framework cannot do either of the things that would save it from being defective.11

4.5 Two Kinds of Accounts of Experience

I’ve just argued that the account of experience that Lange needs to get the Jeffrey framework off the hook for not commuting evidence looks like it gives rise to problems that are just as serious as the one that it enables this framework to avoid. Before

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11Note that an interesting upshot of our discussion is that Bayesian conditioning and Jeffrey Conditionalization appear to be two different norms, despite the fact that the latter generalizes the mathematical features of the former. For Bayesian conditioning is able to do one of the things that makes a Bayesian updating rule non-defective: it is able to commute evidence. More will be said about this seemingly strange result in the following chapter.
closing, I want to consider some of the history surrounding the attempt to provide a norm, like ECJC, that assumes such an account of experience. This history perhaps provides some insight into what made Lange’s argument seem so plausible. It also confirms that the argument does not succeed as a general defense of the Jeffrey framework.

The first worries about a norm like ECJC come from Carnap’s 1957 correspondence with Jeffrey (reprinted in Jeffrey 1975), where he tells Jeffrey that he himself had attempted to formulate such a rule. His correspondence expresses two worries about its prospects. We have assumed so far that experience must be represented as some sort of magnitude. But this assumption wasn’t always taken for granted. Carnap’s first worry arose as a result of assuming that experience ought to be represented, not as a Bayes factor, but as a probability attached to some experientially-affected sentence which indicates “the subjective certainty of the sentence on the basis of the observational experience” (1975, p. 42). Carnap’s concern was that representing experience in this way made it impossible to represent the agent’s evidence as a function of these experiences and her prior opinions about her evidence.

To see the difficulty, suppose that the degree of support Jill’s experience of rhubarb pie lends to R is 0.9. Suppose also that her probability for R before the experience is 0.6. What should Jill’s post-experience probability for R be, given these two facts? To assume that it should be 0.9 would be to ignore her priors. To assume that it should be 0.6 would be to ignore her most recent experience. Intuitively, the correct post-experience probability for R should be a function of both of the previous probabilities. But there’s no obvious way of defining this function (cf. Christensen (1992), 550-51).

We avoid this worry by taking the contribution of experience to an update to be representable as the Bayes factor of that update, as we have done throughout this
discussion. Since Bayes factors are functions of an agent’s prior opinions and her evidence, they make an agent’s experience a function of her prior opinions and her evidence. This, in turn, makes an agent’s evidence a function of her prior opinions and experience.

Since this first problem that Carnap identifies stems from the assumption that experience ought to be represented, not as a magnitude like a Bayes factor, but as a probability, call it the representation problem.

Carnap’s second concern was the concern we considered in the previous section. Even if we were to represent experience as a Bayes factor, so that we could represent the Jeffrey framework as representing the agent’s evidence as a function of her experiences and prior expectations—and so that we could also represent the agent’s experiences as commuting—we still would not have a rule for how to map a particular experience (or considered experience) to a particular Bayes factor. The sort of rule Carnap thought we would need is one that would govern how features like “the clarity of the observation (or the feeling of certainty connected with it or something similar)” (p. 44) determine the latter. As he complained to Jeffrey, the latter rule was no part of Jeffrey Conditionalization:

You emphasize correctly that your [posterior probability distribution] is behavioristically determinable. But this concerns only the factual question of the actual belief of [the agent] in [her evidence]. But [the agent] desires to have a rule which tells him what is the rational degree of belief (44, emphasis in original).

In short, Carnap thought we needed a rule that tells us whether we ought to update like Jack or like Jill. Call this the normative problem.

Carnap’s frustration that he could not adequately address the representation
problem and the normative problem ultimately led him to abandon the project of crafting a rule like ECJC.\footnote{See Jeffrey (1975), 41. For other expressions of this skepticism, see Christensen (1992) and Schwarz (ms.).} The first discussion that actually addresses both the representation problem and the normative problem is Field (1978). Field proposes a reparametrized version of Jeffrey Conditionalization, one that, like ECJC, assumes that an account of Bayesian updating ought to say something about the experience that gives rise to an update.\footnote{Field maintains we ought to generalize Jeffrey Conditionalization from the form $p' = \omega(p, E)$, into the form $p' = \omega(p, E, \alpha)$, where $E$ is the observation sentence that is directly affected by experience, and $\alpha$ is the degree to which the sentence is affected, rather than the degree of certainty that we are left with in it after the experience (1978, p. 361). Field argues that this input parameter, $\alpha$, ought to take the following form:

$$\alpha = \text{def} \left( \frac{1}{2} \log \left( \frac{p'/p}{(1-p)/p} \right) \right)$$

Where we ignore the $1/2 \log$ (which merely serves to make Field’s quantity a scale-invariant relevance measure), and we assume for simplicity that the evidence partition is $\{E, \overline{E}\}$, we get:

$$\alpha = \frac{p'(E)/p'(\overline{E})}{p(E)/p(\overline{E})}$$

The first discussion that actually addresses both the representation problem and the normative problem is Field (1978). Field proposes a reparametrized version of Jeffrey Conditionalization, one that, like ECJC, assumes that an account of Bayesian updating ought to say something about the experience that gives rise to an update. Field explicitly distinguishes his approach from Carnap’s attempt at such an account based upon the responses he offers to the representation problem and the normative problem (pp. 363-64). First, Field’s updating rule entails that experience’s contribution to an update is the Bayes factor of that update. Therefore, Field’s account avoids the representation problem. Field’s response to the normative problem is equally decisive. He claims that it does not ask a legitimate question:

Carnap’s criticism of Jeffrey is that there are cases where a person ought

if he is rational to come to attach a high probability $q$ to a directly affected sentence $E$, but that nothing in Jeffrey’s constraints requires him
to do so. I do not think that this way of putting the problem makes the

problem very persuasive. In any case, it did not persuade Jeffrey (p.
364, emphasis in original).

Having identified experience with a Bayes factor rather than a probability, the normative problem for Field becomes that there are cases where a person ought, if he is rational, come to attach a certain value to the Bayes factor of the update on the basis of an experience, but that nothing in Jeffrey’s constraints requires him to do this. While Field acknowledges the normative problem, then, he does not provide an answer to it, since he believed that the task of providing an account of how experience figures into the updating process should be conceived of as a purely descriptive one, as: “the problem of giving a complete psychological theory for a Bayesian agent” (p. 364). On this interpretation of what an account of experience aims to provide, it does not matter that the phenomenal experience of rhubarb pie changes the agent’s credence in the proposition that a Democrat will win the next election. What’s important for Field is that we have a way of representing how this happens that has experiences commute under the framework.

Where does all this leave us? Neither Carnap, nor Field provides an answer to the normative problem. Carnap believed that such an answer was impossible to give. Field believed that such an answer did not belong as part of a theory of rational belief revision. He believed that while what happens after the agent gets evidence might be a rational process, what happens before she gets evidence is not.

It’s perhaps in siding with Field, and against Carnap, that we were initially led to find Lange’s argument so plausible. However, this paper has argued that the very question Lange addresses in his paper—the question of whether Jeffrey Conditionalization is not defective in virtue of commuting the elements that it ought to commute—precludes us from siding with Field. It assumes that the task of providing an account of how experience figures into the updating process cannot be
conceived of as a purely descriptive one, but that it must be conceived of as a normative one. It requires that we think of experiences as mapped to Bayes factors in some sort of rational way. Given the difficulties involved in doing this—that led Carnap to abandon the project of crafting a norm like ECJC—I think we can’t escape the conclusion that the structure of Jeffrey Conditionalization is defective.
CHAPTER 5

BAYESIAN COHERENTISM

ABSTRACT. This paper considers a problem for Bayesian epistemology and goes on to propose a solution to it. On the traditional Bayesian framework, an agent updates her beliefs by Bayesian conditioning, a rule that tells her how to revise her beliefs whenever she gets evidence that she holds with certainty. In order to extend the framework to a wider range of cases, Richard Jeffrey (1965) proposed a more liberal version of this rule that has Bayesian conditioning as a special case. Jeffrey conditioning is a rule that tells the agent how to revise her beliefs whenever she gets evidence that she holds with any degree of confidence. The problem? While Bayesian conditioning has a foundationalist structure, this foundationalism disappears once we move to Jeffrey conditioning. If Bayesian conditioning is a special case of Jeffrey conditioning then they should have the same normative structure. The solution? To reinterpret Bayesian updating as a form of coherentism.

Foundationalism and coherentism are competing views about the structure of epistemic justification. It’s surprising then that they co-exist on the Bayesian framework. The explanation: Bayesianism is committed to norms that govern our degrees of belief—our credences—in propositions that stand in particular logical relations to each other at each time. It’s also committed to norms that govern how these credences change over time in response to new evidence. Traditional Bayesian epistemology is coherentist with respect to the first set of norms. It’s foundationalist with respect to the second. It has two strands of justification running through it.
This paper considers a problem for Bayesianism’s second strand of justification, and goes on to propose a solution to it. On the traditional Bayesian framework, an agent updates her beliefs by Bayesian conditioning, a rule that tells her how to revise her beliefs, whenever she gets evidence that she holds with certainty. In order to extend the framework to a wider range of cases, Richard Jeffrey (1965) proposed a more liberal version of this rule. Jeffrey conditioning is a rule that tells the agent how to revise her beliefs, whenever she gets evidence that she holds with any degree of confidence. Jeffrey claimed that his rule has Bayesian conditioning as a special case. This claim is now a truism of Bayesian epistemology.

The problem? While Bayesian conditioning has a foundationalist structure, this foundationalism disappears once we move to Jeffrey conditioning. But if Bayesian conditioning is a special case of Jeffrey conditioning, then these two updating rules should have the same normative structure. We are then left with the following inconsistent triad: (1) If one norm is a special case of another, then they should have the same normative structure, (2) Bayesian conditioning is a special case of Jeffrey conditioning, (3) Bayesian conditioning and Jeffrey conditioning have different normative structures.

I will argue for an interpretation of the Bayesian framework that resolves this inconsistency by rejecting (3). I’ll reject (3) by arguing that both regular Bayesian updates and Jeffrey updates proceed from frameworks with a coherentist structure.¹ My strategy will be to appeal to what has long been deemed to be a defect of Jeffrey conditioning: the fact that its updates aren’t guaranteed to commute. To say that Jeffrey updates aren’t guaranteed to commute is to say that an agent’s credences after a sequence of updates will sometimes be determined by the order in which this evidence has been received. This feature of Jeffrey updates is a defect because the order in which some

¹In certain places, I will speak loosely and refer to updates on uncertain evidence as ’Jeffrey updates,’ or updates by Jeffrey conditioning. Strictly speaking, this is not correct, of course, since updates on certain evidence are also Jeffrey updates. However, in some contexts, it would be awkward to talk in any other way.
evidence has been received seems irrelevant to the impact it ought to have. While the fact that the Jeffrey framework can’t guarantee that its updates will commute is standardly taken to show that the framework fails to satisfy an important desideratum for an updating rule, in this paper, I propose that we take the commutative property to play a more fundamental role. I propose that we take the commutative norm that Bayesianism is committed to to be one that grounds particular updates. In other words: some set of updates will be justified to the extent that they commute. Since the sort of consistency this norm encodes is to updates what the norm of evidential consistency from traditional formulations of coherentism is to beliefs, it looks as though the best way of understanding the structure of Bayesian updating is as a form of coherentism.

Here’s how we’ll get to this conclusion. In §1, I give some background. In §2, I describe the sense in which regular Bayesian updating has a foundationalist structure. In §3, I explain why adopting Jeffrey conditioning entails abandoning this foundationalism. In §4, I propose a constraint that looks like a version of coherentism about updating and argue that it more clearly supports the truism that Bayesian conditioning is a special case of Jeffrey conditioning. In §5, I give this constraint a formal backbone. Finally, in §6, I revisit the motivation for this constraint.

5.1 Some Background

5.1.1 Diachronic Coherence for Bayesians

I’ve suggested that it’s possible to ask whether Bayesian updating is a form of foundationalism or a form of coherentism. Let’s begin by getting clear on exactly what this question means.

Standard Bayesianism assumes that an agent’s credences in the propositions she entertains can be represented as an assignment of real numbers to those propositions. It further assumes that two norms of coherence govern this assignment. First, Bayesian-
ism is committed to the constraint that, at each time, the agent’s credences be a probability function. To say that a Bayesian agent is synchronically coherent, then, is to say that she conforms to Probabilism: 1) she assigns every proposition her credence function is defined over a non-negative value, 2) she assigns a credence of one to any tautology and, 3) for any mutually exclusive propositions, A and B, that her credence function is defined over, \( cr(A) + cr(B) = cr(A \lor B) \).

Second, Bayesianism is committed to the constraint that the agent’s beliefs evolve over time in accordance with her conditional probabilities. If my credence in the proposition that I will play baseball tomorrow is .3, and my credence in the proposition that I will play baseball tomorrow conditional on the proposition that it will not rain is .7, then when I learn that it will not rain—when I get this as evidence—my credence that I will play baseball tomorrow should shoot up from .3 to .7. My current credence in any proposition \( (p'(A)) \) should always be my prior credence in that proposition, conditional on the evidence that I’ve gotten \( (p(A|E)) \).\(^2\)

To say that a Bayesian agent is diachronically coherent, then, is to say that her conditional probabilities guide her belief revisions. One way of capturing this idea is to require that the agent’s current probabilities be determined by her conditional probabilities, in the way that we’ve just described. A different, though equivalent, way of capturing this idea is to require that the values of the conditional probabilities that yield the agent’s current probabilities be the same before and after the update. We can think of the agent’s conditional probabilities as arrows that proceed from her evidence and that guide the propagation of the rest of her probabilities. To serve this guiding function, they must remain fixed.\(^3\)

As it happens, every probabilistic belief transition has a set of arrows. For any prob-

\(^2\)Where \( p(A|B) = \frac{p(A \land B)}{p(B)} \).

\(^3\)The arrow analogy is borrowed from Weisberg (2015). This guiding feature of our conditional probabilities is often referred to as ‘rigidity’ (see Jeffrey (1965)).
ablistic belief transition, there will be some information—like learning that it will not rain—that each of my beliefs are conditional on in the same way before and after this transition. More formally, there will always be a partition (a set of mutually exclusive and exhaustive propositions, like \{RAIN, \overline{RAIN}\}) that is sufficiently fine-grained to represent this transition as an update that is conditional on that partition.\(^4\)

**Descriptive Diachronic Coherence for Bayesians:** There is a sufficient partition for every probabilistic belief transition.

Or, equivalently, where \(S=\{B_1, ..., B_n\}\) is a set of beliefs that form a partition, and where an agent has an experience that causes her to revise her beliefs, the transition between the agent’s prior probability distribution, \(p\), and posterior probability distribution, \(p'\), at \(t\) and \(t'\), respectively, can be formulated in a way that underlines that there will always be some conditional probability that remains the same before and after the update, so that it can be understood to guide her belief revision:

**Descriptive Diachronic Coherence for Bayesians:**

\[
\forall p, \forall p', \exists S (\forall B_i \in S), \forall A (p(A|B_i) = p'(A|B_i)), \text{ if defined.}
\]

Since **Descriptive Diachronic Coherence for Bayesians** is no stronger than Probabilism, one might suspect that it will be too weak to capture any interesting notion of diachronic coherence. To see that this is indeed the case, consider the very simple agent who only has beliefs about whether or not she will play baseball tomorrow. Her credences are only defined over the partition \{PLAY, \overline{PLAY}\}. Suppose these credences are \(p(\text{PLAY})=.4\) and \(p(\overline{\text{PLAY}})=.6\). Suppose further that the agent revises her beliefs to \(p(\text{PLAY})=.7\) and \(p(\overline{\text{PLAY}})=.3\). In this case, there is a partition that is

\(^4\)For the proof of this, see Diaconis and Zabell (1982, p. 824). As Diaconis and Zabell note, there will be cases where our conditional probabilities are undefined for some partition—namely, where we assign a member of our partition a credence of zero. However, their result still holds for all updates if we take a sufficient partition to be a partition that is sufficient to represent a probabilistic belief transition as an update that is conditional on every proposition in this partition, for which a conditional probability is defined.
sufficient for the update: \( \{ \text{PLAY, PLAY} \} \). Therefore, this belief transition satisfies DESCRIPTIVE DIACHRONIC COHERENCE FOR BAYESIANS. Intuitively, however, this case doesn’t look much like an agent responding to her evidence. For we tend to think that updating in accordance with one’s evidence happens when we come to change our belief in some proposition, on the basis of some different information. It happens, as in the example above, when our views about whether we will play baseball tomorrow change in response to listening to the weather forecast and learning about the chance of rain. The lesson is that some belief transitions that satisfy DESCRIPTIVE DIACHRONIC COHERENCE FOR BAYESIANS don’t look much like an agent being diachronically coherent at all, if we take such coherence to involve the agent getting evidence. Instead, what they look like is an agent swapping one set of probabilities for another.

We can remedy this by strengthening our account of diachronic coherence. We can do this by stipulating that it is only when the agent conditions her beliefs on partitions that meet some additional constraint for being evidence that she is diachronically coherent:

**Normative Diachronic Coherence for Bayesians:** A probabilistic belief transition ought to be such that:

(a) there is a sufficient partition, \( \{ B_i \} \), for the transition, and

(b) \( \{ B_i \} \) satisfies the conditions for being evidence.

Or, equivalently, where \( E = \{ B_1, \ldots, B_n \} \) meets the criteria for being evidence, the following prescribes the relation between an agent’s prior credence distribution \( p \) and her posterior credence distribution, \( p' \), at \( t \) and \( t' \), respectively, by means of the obligatory operator, \( O \):

**Normative Diachronic Coherence for Bayesians:**
\[
\forall p, \forall p', \forall B_i \in E, \forall A, O(p(A|B_i)) = p'(A|B_i), \text{ if defined.}
\]
What makes this formulation normative is that it is stronger than Probabilism: an agent might transition from one probability function to another in a way that violates it. What makes this formulation a norm of coherence is that it is defined over a set of credence functions. Finally, what makes this norm of coherence diachronic is that these credence functions are indexed to different times.

Before moving on, I want to mention one last way of understanding diachronic coherence for Bayesians: a middle path between descriptive and normative diachronic coherence. Instead of overcoming the weaknesses of the former by restricting the conditions under which some partition is evidence, we might simply take for granted the existence of an evidence partition, and ask about what follows from it. In other words, we might take the Bayesian agent’s diachronic obligations to consist in how she ought to proceed, assuming that she has a certain piece of evidence. On this picture of things, the evidence partition is not normatively determined, but causally determined: it is “an internal or psychological condition that must be checked or accepted at each stage.”

This understanding of diachronic coherence looks like a more modest way of getting us what we are after. By stipulating that some partition of propositions constitutes the agent’s evidence, it avoids the worry that it is too weak to capture any interesting notion of diachronic coherence. But since this understanding of diachronic coherence does not require an agent’s evidence to satisfy any additional constraints, it is also no stronger than Descriptive Diachronic Coherence for Bayesians. Moreover, it makes sense of the way that people tend to talk about Bayesian updating. Some might even call this the default view of diachronic coherence for Bayesians.

I want to defer saying anything more about the default view for the moment. It will become clear a bit later on why this account of diachronic coherence cannot be used to unify Bayesian updates in the way that we are looking to do.

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5.1.2 A Formal, Deflationary Account of Evidence

For now, then, let us assume that the aim of this paper will require a normative account of diachronic coherence. And this will require that we adopt an account of evidence. There are a couple of ways that we might go about this. The most familiar of these ways is to appeal to a substantive account of evidence: for instance, to the requirement that evidence be what one knows, or be related to what one has internal access to, or be formed by a reliable process, etc. What makes such accounts substantive ones is that Bayesians, qua Bayesians, aren’t committed to the normativity of knowledge, or of access, or of reliability, etc. A substantive account of evidence, then, is a constraint on evidence formulated in terms of a property that is not already part of the Bayesian formalism.

This paper will take a different approach by defending a formal account of evidence: a constraint on evidence that is formulated in terms of some feature of the Bayesian formalism. A Bayesian formalist about evidence will hold that it is in virtue of the sufficient partition of an update being assigned certain values, or weights, that the agent can be said to have evidence—whether or not these weighted partitions are further justified by the sorts of substantive considerations that we have just mentioned.\(^6\) Exactly what it will look like for a formal constraint on evidence to be satisfied will become clearer

\(^6\)The distinction between formal and substantive norms roughly tracks the distinction between ‘thin’ normative concepts like consistency—concepts that anyone, regardless of their other normative commitments would have to concede is a prima facie, good-making feature—and the sorts of thick normative concepts that are capable of distinguishing normative views (for the canonical account of the distinction between thin and thick normative concepts in the moral domain, see Williams (1985), especially pp. 140-142, 150-152.).

Perhaps an easier way of understanding what makes knowledge and reliability substantive, rather than formal, is that they can be made sense of out of context: they can be defined independently of the other epistemic commitments one happens to hold. Formal norms are different in this regard. Take, for instance, the formal norm of consistency. The way that a Bayesian treats evidence consistently will differ from the way that a defender of Dempster-Shafer theory treats evidence consistently. While the Bayesian will spell out her notion of consistency by means of probability functions, a Dempster-Shafer theorist, who trades in belief functions (or mass functions), will cash out her notion of consistency in terms of these. Unlike knowledge or reliability, the norm of consistency is so thin that it isn’t complete, absent a framework that gives it content.
in just a little bit. For now, notice that the appeal to a formal account of evidence leaves us able to understand how it is possible to ask whether Bayesian updating is a form of foundationalism or a form of coherentism. Since what we will be after is a formal, or structural, account of evidence, and since foundationalism and coherentism are both structural norms, they will both be candidates for such an account.

The reason I defend a formal account of evidence in this paper is because I think it’s of interest to consider how much normativity can be defined of in terms of the commitments Bayesians already hold. It’s worth emphasizing that a formal account of evidence will be a deflationary one. My proposal takes seriously the idea that there is nothing more to being a constraint on evidence than being a constraint on the sufficient partition of a belief transition. Insofar as we are tempted to talk of weighted partitions as "being evidence", then, it is because the weights these partitions get assigned are what determine the extent to which our constraint on evidence gets satisfied.\textsuperscript{7} This deflationary picture of evidence will allow us to develop an account of normative diachronic coherence with the following features:

1. Agents aren’t diachronically coherent, full-stop. Instead, they are diachronically coherent to varying degrees.

2. The constraint that determines an agent’s degree of diachronic coherence isn’t defined over weighted partitions. Instead, it’s defined over sets of weighted partitions.\textsuperscript{8}

3. The sets of weighted partitions that our constraint is defined over isn’t assigned to an agent at a single time. Instead, it’s assigned to temporally extended sequences of the agent.

\textsuperscript{7}Nevertheless, I will continue to talk in this way.

\textsuperscript{8}A consequence of this is that \textsc{Normative Diachronic Coherence for Bayesians} is more perspicuously formulated in terms of a pair of evidence partitions, instead of just one. How this can be done will become clearer in §4.
We’ve said that an agent’s standing as synchronically coherent will depend upon how her credence functions are related to each other. In defining diachronic coherence over sets of weighted sufficient partitions that the agent has at different times, my account entails that an agent’s standing as diachronically coherent will depend upon how her updates are related to each other. My account, then, leaves us with an interpretation of Bayesian epistemology that is coherentist, with respect to both strands of justification that run through it.

5.1.3 An Assumption

Finally, an assumption. Epistemic theories can be given one of two interpretations. On the one hand, we might think that what any such theory provides is guidance for how a rational agent ought to act. On the other hand, we might think that what any such theory provides is a way of evaluating an agent’s actions, whether or not we would want to say that an agent ought to have done what she did. Bayesian epistemology, when understood in the first way, has received its fare share of criticism. This is because the sorts of idealizing assumptions that we need to get the framework off the ground require of ordinary agents that they perform operations that are computationally intractable. Here’s Harman (1988, p. 25-26) on this:

One can use conditionalization to get a new probability for P only if one has already assigned a prior probability not only to E but to P∧E. If one is to be prepared for various possible conditionalizations, then for every proposition P one wants to update, one must already have assigned probabilities to various conjunctions of P together with their denials. Unhappily, this leads to combinatorial explosion, since the number of such conjunctions is an exponential function of the number of possibly relevant
evidence propositions.\footnote{See also Kornblith (1992), 910, for a similar version of the objection.}

And Earman (1992, p. 56):

‘Ought’ is commonly taken to imply ‘can’, but actual inductive agents can’t, since they lack the logical and computational powers required to meet the Bayesian norms. The response that Bayesian norms should be regarded as goals toward which we should strive even if we always fall short is idle puffery unless it is specified how we can take steps to bring us closer to the goals.

In light of these sorts of criticisms, I will assume that Bayesianism is best understood as a set of evaluative norms, rather than as a set of action-guiding norms.\footnote{Defenders of evaluative norms in general include Feldman (2001) and Wolterstorff (2010). (Note that I will continue to use the word “norm” to refer to evaluative standards, even though most put the normative and the evaluative at odds with each other. I do so mainly for ease of exposition. But also because it seems intuitive (at least to me) that there might be norms for states of affairs, in addition to norms for agents. For discussion of this point, see, for instance, Chrisman (2008).)} This means that although Bayesian epistemology sets certain standards, there are no obligations issued by the theory. Just as we can say that cars are good, insofar as the brakes work, and bad insofar as they don’t, without imposing any obligations on anyone to do anything, we can say that updates are good or bad, in virtue of certain features of them, without imposing any obligations on anyone to do anything.

The way that we’ve set things up in this section already points us in the direction of conceiving of Bayesian epistemology as an evaluative theory. The natural question to ask on the action-guiding approach is: given what I take my evidence to be, how should I update? By contrast, the natural question to ask on the evaluative approach is: does my update have the right features? By defining evidence \textit{in terms} of the update that it triggers, as we have done above, we set ourselves up to pursue the second of these questions. An agent’s update will be good insofar as the formal constraint on evidence
defended in this paper is satisfied, and bad insofar as it isn’t. However, this does not obligate the agent to update in any particular way.

5.2 Bayesian Conditioning: Foundationalism about Updating

We’ve said that, without a constraint on evidence, the sort of diachronic coherence that Bayesian updating involves amounts to no more than Probabilism. But where one of the members of the sufficient partition of an update is a certainty, such an update does include a constraint on evidence: it includes a foundationalist constraint on evidence.\textsuperscript{11} To see this, we will need to get clear on what foundationalism amounts to when applied to updates. And in order to do this, we will need to get clear on what foundationalism amounts to when applied to beliefs.

Traditional foundationalism about epistemic justification says that the ultimate source of the justification of all our beliefs is some privileged set of cognitive states that is the locus of this justification, but that can’t be the target of it. It’s the conjunction of the claims: (1) that some cognitive states are basic, in the sense of their being justified not in virtue of their relations to other cognitive states and, (2) that all non-basic states are justified in virtue of some relation that they bear to basic states. In addition, classical foundationalism assumes (3) that the distinguishing mark of basic states is their infallibility.\textsuperscript{12} Given this, one obvious candidate for an agent’s basic state on the Bayesian framework is her evidence. We can formulate a constraint that captures this idea by focusing, once again, on the relation that Bayesian updating secures between an agent’s conditional probabilities and the credence function they direct her to adopt:

\textsuperscript{11}Those who have explicitly taken standard Bayesian conditioning to instantiate a foundationalist structure include Christensen (1992), Skyrms (1997), Bradley (2005), and Weisberg (2009), among others.

\textsuperscript{12}By contrast, many recent, non-classical foundationalist accounts, like Goldman’s (1988) reliabilism, Plantinga’s (1991) proper basicality, Pryor’s (2000) dogmatism and Huemer’s (2001) phenomenal conservativism defend some form of fallible foundationalism. That is, they maintain that the property that makes beliefs basic is something other than their infallibility.
Bayesian conditioning: If the strongest evidence you get raises your
credence in B to one, then your new degree of belief in A, for any A, should be

\[ p'(A) = p(A \mid B), \]

where A and B are propositions.

Where we assume the sort of Cartesian foundationalism that identifies infallibility
with certainty, Bayesian conditioning satisfies (3) by requiring that some proposition
in the agent’s evidence partition receive a value of one—by requiring that it be a propo-
sition of which she is certain. This formulation clearly satisfies (2) as well: the values
we assign the rest of our beliefs depend upon our evidence. What about (1)? While
the agent uses her evidence proposition to infer the credences she holds in other propo-
sitions, the evidence proposition itself cannot receive this sort of support. This is be-
cause propositions that receive a credence of one cannot have their values changed by
Bayesian conditioning at some later time.\(^\text{13}\) Therefore, once a belief becomes a basic
state—once it becomes evidence—it is no longer able to receive the same sort of in-
ferential support that it offers. Perhaps most importantly of all then, (1) is satisfied as
well.

Foundationalism, then, is a structure that applies just as easily to updates as it does
to beliefs. Traditional foundationalism makes justification a function of whether some
belief \(P\) is in the set of beliefs justified by an agent’s basic state \(S\):

**Traditional foundationalism:** \(f_S : P \rightarrow \{0, 1\}\)

\[(1 \text{ if } p \in S, \text{ and } 0 \text{ otherwise})\]

By contrast, where we take an update \((UP)\) to be a probabilistic belief transition,
Bayesian foundationalism says that **normative diachronic coherence for bayesians**
is satisfied when \((UP)\) is in the set of updates justified by the agent’s evidence \((E)\),

\(^{13}\text{That certainties stay certainties is simply a mathematical feature of the formalism.}\)
where the constraint on evidence is the foundationalist constraint described by Bayesian conditioning:

**Bayesian foundationalism:** $f_E: \mathcal{U} \rightarrow \{0, 1\}$

(1 if $\mathcal{U} \in \mathcal{E}$, and 0 otherwise)

There are a few things to notice about Bayesian foundationalism. First, unlike traditional foundationalism, Bayesian foundationalism governs an update. It tells us what our beliefs ought to look like in the future, rather than whether they are justified at any given moment. Second, Bayesian foundationalism governs the values we assign these beliefs. Finally, and most importantly, the constraint that Bayesian foundationalism imposes is a merely formal one. While it requires the agent’s probability function to be encoded with certain values before and after an update, there is no further norm that underwrites the assignment of these values. These values constitute a form of foundationalism, regardless of whether or not they are justified by some further substantive consideration.14

5.3 Jeffrey Conditioning: Foundationalism Undermined

Most take the fundamental idea behind Jeffrey conditioning to be the thought that, as Jeffrey (1983, p. 171) himself put it: “it is rarely or never that there is a proposition for which the direct effect of an observation is to change the observer’s degree of belief in that proposition to one.” Most of the time we have an experience that changes our credence in some proposition, without making us sure of it. We get a quick glimpse of color on the floor that makes us think that the sock might be red. But maybe it’s really

14One might object that we defined a formal constraint on evidence in §1.2, not as a constraint on the agent’s entire credence function but, rather, as a constraint on the sufficient partition of her update. But, of course, since the values of a sufficient partition entail values for the credence distribution it is sufficient for, these amount to the same constraint.
brown. Or maybe it’s purple.

In order to capture this more realistic class of cases, we need a rule that tells us how we ought to revise our beliefs whenever we get this sort of uncertain evidence. Jeffrey (1965) introduces a rule that does just this by allowing our evidence to assign values other than zero and one to the members of our partition:

**JEFFREY CONDITIONING:** If experience directly changes your credences over a partition \( \{B_i\} \) from \( p(B_i) \) to \( p'(B_i) \), then your new degree of belief in \( A \), for any \( A \), should be \( p'(A) = \sum p(A | B_i) p'(B_i) \).

It’s clear from this formulation of it that Jeffrey conditioning has Bayesian conditioning as a special case. Both updating rules say that we should revise our beliefs in accordance with the conditional probability that our evidence determines. Assuming our evidence to be a partition allows us to accommodate the uncertainty of some pieces of evidence by, for instance, allowing us to assign probabilities other than zero and one to the possibility that the sock is red, and to the possibility that it is brown, and to the possibility that it is purple—which, together, will sum to one. Assuming our evidence to be a partition also allows us to accommodate the certainty of some pieces of evidence by, for instance, allowing us to assign probability one to the possibility that the sock is red and probability zero to the possibility that it isn’t.

But although Bayesian conditioning is a special case of Jeffrey conditioning, it is a degenerate case of it. This is because Jeffrey conditioning lacks the foundationalist constraint that governs Bayesian conditioning. There are a couple of ways of understanding how this structure is lacking. First, assume that we take the agent’s basic state to be the evidence partition that she updates on. Since the propositions in this partition can receive a value of less than one—less than complete certainty—it doesn’t include an infallible belief. More importantly, since the propositions in this evidence partition can receive a value of less than one, they are able to have their values changed by means
of the same sort of inferential support that they offer by a future update. Therefore, the beliefs that comprise these evidence partitions violate the first and third conditions of foundationalism identified above.

Here’s a different way of understanding how Jeffrey conditioning fails to be a form of foundationalism.15 Earlier we said that if there is no constraint on the evidence partition that generates a particular belief transition—if it can generate any belief state that is consistent with Probabilism, by receiving any set of values consistent with Probabilism—then it fails to satisfy Normative Diachronic Coherence for Bayesians. Given this, it’s tempting to think that if we want to satisfy Normative Diachronic Coherence for Bayesians, we should take the values that a partition gets assigned to be constrained by the experience that gives rise to it. That is, we should take this experience, rather than our evidence, to be our basic state. However, the Bayesian formalism does not regulate how experience gives rise to an update. Since experiences lack the inferential relation to updates that a basic state bears to non-basic states, they aren’t better candidates for the role we are looking to fill. On this understanding of things, our updating rule violates the second condition of foundationalism identified above.16

Therefore, Jeffrey conditioning is strictly weaker than Bayesian conditioning: the latter includes a formal constraint on evidence that the former lacks. These considerations also show us why the default view of Bayesian diachronic coherence considered earlier can’t help us. Recall this is the view that takes an agent’s diachronic obligations to follow from an evidence partition that we have assumed the agent to have gotten. Since the default view is not committed to foundationalism, it is strictly weaker than regular Bayesian conditioning. Therefore, it will be too weak to serve as an account that can unify Bayesian updates.

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15The following line of argument has a steady, if diffused, presence in the literature on Jeffrey conditioning. There are references to it as early as Carnap (1957) (reprinted in Jeffrey (1975)).

16Here, again, we note that to impose a constraint on an evidence partition just is to impose a constraint on an agent’s credence distribution.
How do we unify Bayesian updates then? Since Bayesian conditioning entails a constraint that makes it stronger than Jeffrey conditioning, putting these two updating rules on a par will require making Jeffrey conditioning stronger. But we can’t make Jeffrey conditioning stronger by making it a form of foundationalism. Putting these two updating rules on a par, then, will require reinterpreting the normative consequences of the formal property that makes Bayesian conditioning so strong. It will require finding a norm capable of governing all Bayesian updates. The rest of this discussion will propose and defend a norm that does just this.

5.4 A Solution: Coherentism about Updating

We’ve just seen that Jeffrey conditioning lacks the formal constraint on evidence that makes regular Bayesian conditioning a form of foundationalism. In this section, I’ll argue that we can get a unified account of Bayesian updating—one that allows us to claim that updates on certain and uncertain evidence proceed from frameworks with the same normative structure—by reconceiving of Bayesian updating as a form of coherentism.

The fundamental difference between regular Bayesian conditioning and Jeffrey conditioning has always been assumed to be that the latter generalizes the certainty of evidence. A second notable difference in these frameworks is that only the regular framework is commutative over evidence partitions: only the regular framework makes the order in which we get evidence irrelevant to the credence distribution we end up with at each and every time that we update. This is a significant mark against the Jeffrey framework. Consistency seems to require that identical pieces of information be treated the same, no matter the order in which they are received. And Jeffrey updates aren’t guaranteed to be consistent in this way.

While much discussed in the literature, the non-commutativity of Jeffrey condi-
tioning has never been assumed to be a defining feature of it, in the way that the uncertainty of evidence has been so understood. Instead, it has been assumed to be an unfortunate, but non-essential defect of the Jeffrey framework. This suggests an intriguing possibility: why not take the fundamental norm that governs all Bayesian updates, including Jeffrey updates, to be that they minimize the defect of failing to commute. Why not take the norm for evidence that governs all updates to be, not that these updates be grounded in a certainty, but that they be minimally non-commutative. This would mean understanding the formal norm for evidence that governs updates to be the requirement that the values these updates yield be as insensitive as possible to the order in which these updates were made. It would mean requiring that all updates be consistent in this way.

Whether or not this way of grounding Bayesian updates is a reasonable move to make depends upon whether we think that minimizing the extent to which updates fail to commute is a norm that Bayesians ought to be interested in. Given that so much has been made of the commutative property in the Bayesian literature, it’s clear that it is a norm that Bayesians ought to be interested in. Moreover, I think we can give this norm an interesting gloss. I think that a norm that requires that we minimize the extent to which Bayesian updates fail to commute makes the Bayesian framework look like a form of coherentism. To get to the conclusion that Bayesianism is a form of coherentism about updating, it will again be useful to consider what this structure of justification looks like when it is applied to beliefs.

17See Domotor (1980) and Doring (1999). For the classic rebuttal of the charge that Jeffrey conditioning is defective in virtue of being non-commutative, see Lange (2000). Lange claims that though the Jeffrey framework isn’t commutative over evidence, this does not entail that it isn’t commutative over the experiences that underwrite belief revisions. Therefore, it isn’t non-commutative in a way that makes it defective.

In other work, I argue that Lange’s argument does not target Jeffrey conditioning, but a more sophisticated updating rule. Moreover, I argue that it is not even an adequate defense of this updating rule. Therefore, there is reason to think that the Jeffrey framework is indeed defective, in virtue of not being commutative over evidence. Or, as I will argue in §6, it is defective, in virtue of not being commutative over evidence, provided that there are no other relevant normative considerations in play.
Like traditional foundationalism, traditional coherentism assumes that the locus of justification is a set of beliefs. It assumes that some set of beliefs is justified exactly when its component beliefs cohere, or fit correctly, with one another. On many coherentist accounts, probabilistic coherence, logical coherence, and evidential coherence are each measures that contribute to a belief set’s coherence.\(^{18}\)

Logical coherence and probabilistic coherence will both be preserved over time by Bayesianism’s synchronic constraints: they will be preserved no matter how we understand the structure of diachronic coherence for Bayesians. The interesting question, then, is what an account of evidential coherence will amount to in a Bayesian setting. It’s well-understood what evidential coherence amounts to in a static setting. It is a measure of the degree to which some proposition confirms each other belief in the set to which it belongs. It is a measure of the degree to which every proposition in a set is evidence for every other proposition in the set. If I hold the belief that it will rain in a few hours \((P_1)\), and also the belief that the owner of the shop down the street just put out her umbrella stand \((P_2)\), then the belief that the baseball game will be rained out this afternoon \((P_3)\), if it increases the proportion and strength of the inferential connections between the beliefs in this set, increases the evidential coherence of this set of beliefs.\(^{19}\) Traditional coherentism makes justification a function of the degree to which some set of propositions, \(P_1, P_2, \ldots, P_n\), cohere:

**Traditional Evidential Coherence:** \(f: \{P_1, P_2, \ldots, P_n\} \rightarrow \mathbb{R}^+\)

We can triangulate on an account of Bayesian coherentism from the descriptions

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\(^{18}\)There are, of course, many different kinds of coherentist accounts of justification, both historically and contemporaneously. And many earlier coherentists did not endorse all three of these constraints. Ewing (1934), for instance, takes coherence to be a matter of logical coherence alone, while Lewis (1946) takes coherence to be a matter of probabilistic coherence alone. Notably, Bonjour (1985) takes coherence to be a matter of logical and probabilistic coherence, as well as a number of other requirements that might be held to fall under the heading of evidential coherence (see pp. 97–99 for the details).

\(^{19}\)Contemporary coherentist accounts tend to spell out the notion of an inferential connection probabilistically. For instance, some have said that, in the previous example, what accounts for the increased coherence provided by \(P_3\) is that \(p(P_1 | P_2) < p(P_1 | P_2 \land P_3)\) and \(p(P_2 | P_1) < p(P_2 | P_1 \land P_3)\).
of Bayesian foundationalism and traditional coherentism we already have. From *traditional coherentism*, we borrow the idea that the locus of justification is a consistent set. From *Bayesian foundationalism*, we borrow the idea that the target of this justification are the values triggered by updates, rather than the contents of beliefs. What we are interested in isn’t whether the contents of some set of beliefs are consistent at a time, but whether the values of some set of updates are consistent over time. The ideal of justification for the Bayesian coherentist is a consistent set of updates, UP, where, again, consistency comes in degrees:

**Bayesian Evidential Coherence**: \( f: \{UP_1, UP_2, ..., UP_n\} \rightarrow \mathbb{R}^+ \)

Therefore, where we take a set of updates to be a set of probabilistic belief transitions, the Bayesian coherentist will say that *normative diachronic coherence for Bayesians* is satisfied to the extent that one’s updates are coherent, or consistent. The underlying requirement is that the evidence partitions implied by a set of updates be treated consistently. Since the most obvious way for an updating framework to treat consistently the evidence partitions implied by a set of updates is to require that they yield the same values whenever we get them, this version of *normative diachronic coherence for Bayesians* looks like the requirement that updates commute.

If all this is right, then an alternative to understanding Bayesian updating as a form of foundationalism is to understand it as a form of coherentism. I’ll go on to say more in the next section about what Bayesian evidential coherence amounts to. In particular, I’ll offer a proposal for how this sort of coherence can be represented as a gradable property. But, before we do that, it will be useful to get a feel for where we are right now. We can state Bayesian foundationalism and Bayesian coherentism in a way that illustrates that each gives us a different version of *normative diachronic coherence for Bayesians*:

**Bayesian foundationalism**: A probabilisitic belief transition will be
such that:

(a) There is a sufficient partition, \( \{E_i\} \), for the transition.

(b) It is diachronically coherent iff some \( E_i \) is held with certainty.

**Bayesian Coherentism:** A pair of probabilistic belief transitions will be such that:

(a) There are sufficient partitions, \( \{E_i\} \), \( \{F_j\} \), for each of these transitions, and

(b) They are diachronically coherent to the extent that they maximize evidential coherence (or minimize evidence incoherence), in a sense that will be made more precise in the following section.

As I alluded to earlier, an interesting feature of Bayesian coherentism is that, unlike either Bayesian conditioning (i.e., Bayesian foundationalism) or Jeffrey conditioning, it is undefined for a single update. Therefore, it is not entailed by either Jeffrey conditioning or Bayesian conditioning. Notice, however, that while this makes Bayesian coherentism an amendment to the traditional Bayesian framework, it is not an amendment that requires this framework to take on any additional substantive commitments. No matter what other commitments one maintains, inconsistency will always be a *prima facie* defect. This explains the importance that Bayesians, and formal epistemologists in general, have placed on the commutative property. In effect, what Bayesian coherentism represents is just a different way of articulating a commitment that Bayesianism, as well as every other normative theory, already holds.

The final piece of the puzzle is to see how adopting Bayesian coherentism helps us with the problem of being able to say that both regular updates and Jeffrey updates proceed from frameworks with the same normative structure. For, at first glance, it looks
as though this problem persists. It looks as though Jeffrey conditioning bears the same relation to Bayesian coherentism that it bears to Bayesian foundationalism. Jeffrey updates fail, in general, to be updates on basic states. But they also fail, in general, to be updates that commute. So, is appealing to a commutative norm really any different than appealing to a norm that makes justified belief revision a matter of maintaining a certain relation with some basic state?

I think there is a relevant difference between these two sorts of appeals. What makes Bayesian foundationalism problematic is that adopting it would mean having to say that every Jeffrey update, qua Jeffrey update, is incapable of making the agent diachronically coherent. Trivially, updates on uncertain evidence aren’t capable of being updates on certain evidence. And updates on certain evidence are the only updates that have foundationalist properties.

But Bayesian coherentism would not have this same feature. This is because both updates on certain and uncertain evidence are capable of commuting. If we are looking for a norm to unify these two types of updates, then, a norm that makes diachronic coherence a matter of updates commuting is capable of fulfilling this function. The fact that updates on uncertain evidence, qua updates on uncertain evidence, are capable of satisfying the norm to commute, suggests that the best interpretation of why some Jeffrey updates fail to commute is that they have failed to conform to Bayesian coherentism. By contrast, the fact that updates on uncertain evidence, qua updates on uncertain evidence, aren’t capable of satisfying the norm to be an update on a basic state, entails that the foundationalist norm that we would need to render this verdict just isn’t there.20

In short, the fact that updates on uncertain evidence can’t conform to a norm formulated in terms of a basic state entails that such updates aren’t governed by Bayesian

20This follows from standard deontic logic, which says that a norm can’t require X if X is logically impossible. Thanks to Chris Meacham for helping me to clarify this point.
foundationalism. It entails that there is no such norm. By contrast, the fact that Jeffrey updates are capable of conforming to Bayesian coherently suggests that they are
governed by Bayesian coherently. It suggests that Bayesian coherently is a norm
for such updates. And I think we can say something even stronger than this. I think
we can say that, not only are all updates on uncertain evidence capable of satisfying
Bayesian coherently, but that all updates on uncertain evidence do satisfy Bayesian
coherently—to some extent. I’ve already suggested that coherence is most plausibly
interpreted as a gradable property. Identifying commutativity with coherence, then,
makes it natural to want to give commutativity a degree-theoretic interpretation, as I
do in the following section. This will enable us to say that all Bayesian updates are di-
achronically coherent, to a degree.

5.5 Bayesian Evidential Incoherence

In the last section, I proposed a way of grounding Bayesian updates that would allow us
to say that updates on certain and uncertain evidence proceed from the same normative
structure. This proposal rests in the intuitive idea that we can assess the incoherence
of sets of updates based upon the extent to which they instantiate what has long been
deemed to be a bad-making feature of the Bayesian formalism. If one wants to reject the
proposal, then one must either deny that (1) commutativity is an important feature for
an updating rule to guarantee, or that (2) the fact that commutativity is an important
feature for an updating rule to guarantee does not mean that it is an important feature
for individual sequences of updates to have. Absent an argument for at least one of
these claims, I assume that we have good reason to proceed with the question of how
a norm that draws on this intuitive idea might be developed.21

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21 Can we reject one of these two claims? I’ve suggested that the first claim seems unimpeachable: the
formal epistemology literature seems to care a lot about the commutativity of formal updating rules.
What about (2)? Perhaps one might want to argue that the kind of defect the non-commutativity
of updates represents is not a defect of particular updates, but is a sort of inconsistency that inheres in
As I’ve noted already, many Jeffrey updates will commute. Therefore, the simplest way of developing the proposal of the previous section is to say that some sequence of updates satisfies **Normative Diachronic Coherence for Bayesians** iff it commutes, and fails to satisfy **Normative Diachronic Coherence for Bayesians** iff it fails to commute. We can spell out this idea in formal terms by appealing to a property that is both necessary and sufficient for commutativity. This is the property of Jeffrey independence.\(^{22}\)

**Jeffrey Independence (JI):** Let \( P \) be a probability function. And let \( P_\mathcal{E} \) and \( P_\mathcal{F} \) be the probability functions that result from updating \( P \) on the partitions \( \mathcal{E} = \{ E_i \} \) and \( \mathcal{F} = \{ F_j \} \), respectively. The partitions \( \mathcal{E} \) and \( \mathcal{F} \) are Jeffrey independent with respect to \( \{ p_i \} \) and \( \{ q_j \} \) if \( P_\mathcal{E}(F_j) = P(F_j) \) and \( P_\mathcal{F}(E_i) = P(E_i) \) holds for all \( i \) and \( j \).

Thus, Jeffrey independence says that Jeffrey updating on \( \mathcal{E} \) with probabilities \( p_i \) does not change the probabilities on \( \mathcal{F} \) and similarly with \( \mathcal{E} \) and \( \mathcal{F} \) interchanged.

The most straightforward way of unifying certain and uncertain updates under a commutative norm, then, is to require that a sequence of updates be Jeffrey independent. However, if we are looking to mimic the concept of evidential coherence—or evidential incoherence—that we are borrowing from traditional epistemology, we will want a degree-theoretic account of this. How do we get a degree-theoretic account of evidential incoherence? If we identify complete diachronic coherence with Jeffrey independence, then an obvious approach to partial diachronic coherence is to quantify the framework in general. However, it’s difficult to imagine what it would mean for the framework in general to be defective, in a way that doesn’t accrue to particular updates. I think, then, that we are safe in proceeding.

\(^{22}\)The term ‘Jeffrey independence’ was coined by Diaconis and Zabell (1982).
the degree of a violation of Jeffrey independence for a sequence of updates. In the appendix, I develop and defend a measure that does just this. Here’s what this measure ends up looking like:

**Evidential Incoherence (EI):** Let $P$ be a probability function, and let $P_C$ be the probability function that results from updating $P$ on $E$ with probabilities $p_j$. Finally, let $r_{ij} = \frac{P(E_i | F_j)}{P(E_j)P(F_j)}$, $r_{ij}' = \frac{P_C(E_i | F_j)}{P_C(E_j)P_C(F_j)}$, if defined, and 1 otherwise.\(^{23}\)

A sequence of updates over the partitions, $\{E_i\}$, $\{F_j\}$, is coherent to the extent that it minimizes $\sum_j (|1-\sum_i r_{ij}p_i| + \sum_i (|1-\sum_i r_{ij}'q_j|)$.

It’s an interesting question how EI might best be put to use in a norm. Since the aim of this paper is merely to establish Bayesian coherentism as an alternative to Bayesian foundationalism, I won’t consider that question here. Perhaps we would want our norm to govern only pairs of updates that are sequential—that happen one after the other. Or maybe we would want our norm to govern larger sets of updates taken pairwise. But however we choose to go, it’s clear that the kernel of the norm we would want is represented by the description of minimizing incoherence that EI encodes. This description allows us to sharpen our formulation of Bayesian coherentism in the following way:

**Bayesian Coherentism (Revised):** A pair of probabilistic belief transitions will be such that:

\(^{23}\)One might worry about the ‘if defined’ clause in this formulation, which is meant to deal with those cases where we update on a certainty. It might be objected that this ad hoc fix undermines the aim of this paper, which is to unify updates on certain and uncertain evidence under the same norm. But requiring this clause in cases where an update on certain evidence makes this measure undefined is innocuous. However we choose to iron out the details our evaluative norm, it will always be the case that updates on certain evidence are consistent, in virtue of commuting. It is this intuitive notion of consistency that binds certain and uncertain updates, rather than the perhaps inelegant way that we are forced to give this notion formal content.
(a) There are sufficient partitions, \( \{E_i\} \), \( \{F_j\} \), for each of these updates, and

(b) Their degree of diachronic incoherence is determined by \( EI \).

It’s a common idea that there are degrees of probabilistic incoherence. This discussion introduces the idea that there are also degrees of diachronic incoherence that aren’t reducible to the latter by defending a degree-theoretic account of **Normative Diachronic Coherence for Bayesians** that isn’t reducible to Probabilism—that isn’t reducible to **Descriptive Diachronic Coherence for Bayesians**. On the account that I’ve called Bayesian coherentism, perfect normative diachronic coherence will be the special case where the agent’s updates commute.

### 5.6 Final Thoughts

Say I am told there’s a thirty percent chance of rain tomorrow by Jack. And then I am told there’s a seventy percent chance of rain tomorrow by Jill. Given certain plausible assumptions, if these experiences cause me to revise my beliefs, they may very well yield two updates that don’t commute. If I update twice on the proposition that it will rain tomorrow, my final credence that it will rain tomorrow will be .7. But had I gotten Jack and Jill’s testimony in reverse order, my final credence that it will rain tomorrow would have been .3. Therefore, Bayesian coherentism will say that these updates are not perfectly diachronically coherent.

But now suppose that Jill is reliable, when it comes to matters of the weather, whereas Jack isn’t. If my updates are the same as before—with the evidence that Jill’s testimony gives rise to swamping the evidence that Jack’s testimony gives rise to—Bayesian coherentism will again tell me that I’m not perfectly diachronically coherent. But is this still the right result?
The account developed in the previous two sections says that the formal constraint that guides all Bayesian updates is that they be made in a way that makes the order in which evidence has been received irrelevant to the goodness of the update in question. But clearly there are cases where it makes a lot of sense to privilege a later update over an earlier one: namely, where we have some substantive reason for doing so. I’ve emphasized throughout the difference between formal and substantive reasons for evidence. I’ve emphasized that my account is an account of the former. Nevertheless, it’s important to have an idea of how my formal norm can be made consistent with the existence of substantive reasons. While I am again going to defer providing a much more worked out story than the one we already have on the table, I think we can at least say that the considerations that guide our instincts in the Jack and Jill case indicate that the second condition of Bayesian Coherentism (Revised) is best understood as a prima facie constraint. Contra the way we have formulated Bayesian coherentism so far, then, the constraint on diachronic coherence encoded in EI should hold only in those cases where there are no substantive reasons for either of the updates that EI is defined over. This makes Bayesian coherentism consistent with the idea that, in some cases, the best possible credence distribution is incoherent by the lights of EI. This might be because we have substantive reason to favor one piece of evidence over another, as in the case where we have reason to favor the evidence given to us by Jill over the evidence given to us by Jack. Or, it might be because we have substantive reason to take both these updates seriously. In the latter case, being diachronically coherent will be a more complicated matter. The bottom line, however, is that it is only where there is no reason to do either of these things that EI kicks in to tell us something about the goodness of the belief revisions in question. Let us then revise Bayesian Coherentism one last time:

Bayesian Coherentism (final): Where there is no substantive reason for either of a pair of probabilistic belief transitions, this pair of probabilis-
tic belief transitions will be such that:

(a) There are sufficient partitions, \( \{E_i\} \), \( \{F_j\} \), for each of these updates, and

(b) The degree of diachronic incoherence of these updates is determined by EI.

Does revising our norm in this way make it objectionably weak? I think the answer to this question is that it makes Bayesian coherentism exactly as strong as we would want it to be, given the purpose for which it has been contrived. Recall our objective has been to get a norm capable of unifying Bayesian updates by replacing the foundationalist’s constraint. But the foundationalist’s constraint is itself remarkably weak. To really appreciate its weakness, consider an agent who, after being knocked over the head, directly changes her credence in some proposition to one, and then updates in accordance with her conditional probabilities. Such an update conforms to Bayesian foundationalism. Nevertheless, such an update clearly still goes wrong in an important way. The way in which it goes wrong has to do with the lack of substantive reason the agent has to revise her beliefs in the first place. The fact that Bayesian coherentism is also weak, in virtue of the updates it governs also lacking these reasons, is no objection to it then. Quite the opposite: it’s exactly what we would expect of a formal norm for evidence.

I want to conclude by re-considering the motivation for this discussion. We have been assuming throughout that Jeffrey conditioning and regular Bayesian conditioning ought to be brought together; that updates on certain and uncertain evidence ought to proceed from frameworks with the same normative structure. But maybe they shouldn’t. Maybe one can provide a principled explanation for why they don’t. For instance,
maybe like Field (1978, p. 365) claims when discussing his own updating rule, we
would want to say that, unlike Jeffrey conditioning, Bayesian conditioning is too much
of an idealization to ever be of any use:

I suspect that the fact that [Bayesian conditioning] is not a special case of
[Field conditioning] is no loss—I suspect that [Bayesian conditioning]
should be regarded as an oversimplification that can’t ever really arise—
but if you want to allow it, you can allow change to occur by [Bayesian
conditioning] as well as by [Field conditioning].

Or maybe, like Lange (2000, p. 397), we would want to hold that the conditions
under which updates on uncertain evidence happen differ in relevant ways from those
under which updates on certain evidence happen:

Whether the stimulus we receive succeeds in pushing our confidence in
e to a given level in the open interval (0, 1) depends on our prior opin-
ions. This does not arise in cases to which Bayesian conditioning applies,
since then we would presumably have come away from our experience
with \( p' = 1 \) whatever our prior level of confidence in e had been.

Lange does not elaborate on why he thinks only updates on uncertain evidence
are sensitive to an agent’s prior opinions. At one point, he notes that cases where we
update on evidence to which we have assigned a credence of one are cases where our
background beliefs fail to function as “extended sense organs”, in the way that they do
when we assign our evidence any other value.\(^{24}\)

Despite the weird imagery, this does not seem like a crazy suggestion. For starters,
it does seem as though some propositions, though they might be triggered by expe-
rience, are not justified by experience. When I change my credence in the proposition

\(^{24}\)Lange (2000, p. 400).
that a difficult math proof is correct, though this change may be accompanied by certain sensory experiences that are brought about by introspection, these experiences do not seem to be what justify these revisions, in the way that my belief that the sky is blue is justified by an experience with a certain phenomenal character. If this is the case—and if it is also the case that updates on certain evidence are exactly those that aren’t justified by experience—then the agent’s prior expectations (her background beliefs) won’t have a hand in determining what her experience justifies, since, in these cases, the agent’s experience does not justify anything at all.

More generally, the previous passage raises a possibility that we have not yet considered, which is that the type of content to which we happen to be justified in assigning a value of one might differ in some relevant way from the type of content to which we happen to be justified in assigning a lesser value. If there is indeed this difference in content between our certain and uncertain evidence, a unified account of the normative structure that this evidence partakes in may be inappropriate. For while it may be implausible that there is a sharp cut-off between certain and uncertain evidence, there may very well be a sharp cut-off between different types of propositions this evidence corresponds to. If so, then there may, after all, be reason to think that updates on certain and uncertain evidence ought to proceed from frameworks with different normative structures.

Of course, those who tell this sort of story owe us an account of why we might be justified in assigning only some particular class of propositions a credence of one. This might be a considerable task. Or it might not be. A modest proposal along these

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25See Cassell (ms. a) for further discussion.
26Of course, this will depend upon how they are different. In the end, such a difference might very well turn out to be irrelevant as well.
27It’s true that we have principles that regulate content (think Lewis’s (1980) Principal Principle and Van Fraassen’s (1984) Rational Reflection). However, these principles are still formal, or ‘syntactic’, principles, in the sense that their prescriptions are based upon the formal relations that certain beliefs hold to certain other beliefs. Or, put another way, these principles hold for all beliefs, irregardless of their content. The Principal Principle maintains that you ought to calibrate your beliefs about certain
lines would be to appeal to the principle of Continuing Regularity. This principle says that we should assign probability one only to logical truths and zero only to contradictions (or, to necessary and impossible propositions, respectively). While not universally endorsed, this principle is believed by many Bayesians to be quite plausible. And since there is more or less agreement about which propositions are necessary and impossible, we would easily be able to identify the sorts of propositions to which we are justified in assigning a credence of one.

Maybe, then, there’s some argument from Continuing Regularity to the conclusion that certain and uncertain evidence ought to proceed from frameworks with different normative structures. It would be interesting if there were. I think it’s of interest to consider all the possible ways that the commitments underlying Bayesianism can be articulated. One such way, which we have been considering here, is suggested by what appears as a footnote in nearly every paper on Bayesian epistemology. This is the assumption that Bayesian conditioning and Jeffrey conditioning are perfect parallels, with respect to their formal structures. Since Bayesian updating is a normative theory, I have argued that it makes some sense to ask what it would mean for these updating rules to also be perfect parallels, with respect to their normative structures. This paper has tried to answer this question. Maybe there’s not much going for the answer that’s been provided besides its connection to the truisms that appears in all of these footnotes. But I do think it’s interesting—and, also, surprising—to discover what this apparent truisms ends up committing us to.

Propositions to your beliefs about the objective chances of those propositions, no matter what those propositions happen to be. Rational Reflection says that you ought to calibrate your beliefs about certain propositions to your beliefs about your future beliefs about those propositions, no matter what those propositions happen to be. In both cases, the constraint holds merely in virtue of the contents of two sorts of beliefs being identical. And identity is a formal relation.

By contrast, if we were looking for a principle to govern which beliefs ought to be assigned a value of one, this principle would have to target this content directly. It could not make use of a formal property, like identity.
CHAPTER 6

HYPOTHETICAL PRIORS AND DIACHRONIC RATIONALITY

Abstract. In this final chapter, I consider whether the ideal that guides Bayesian coherentism can be made compatible with recent sceptical challenges to diachronic rationality. I offer an account of updating that is friendlier to this form of scepticism and that is guided by considerations similar to those that motivate Bayesian coherentism.

Recently, norms of diachronic rationality have come under attack by those who claim that there are no diachronic norms. In this chapter, I offer an account of Bayesian updating that is friendlier to this form of scepticism. The account that I propose is guided by considerations similar to those that motivate Bayesian coherentism. Recall that Bayesian coherentism includes a formal evidential norm designed to address the inconsistency of non-commutative updates. But the non-commutativity problem for Jeffrey conditioning has a problem that is its mirror image. The problem is the following. In cases where we assume that the agent can accumulate evidence before she updates, the evidence the agent accumulates will violate a norm similar to Probabilism (in a sense that will be made clearer in the following sections) in those cases where an agent’s updates would have failed to commute, had she updated on this evidence sequentially. Just as Bayesian coherentism includes a norm for evidence designed to address the inconsistency of non-commutative updates, the account proposed in this paper includes a norm for evidence designed to address the inconsistency yielded by these sorts of violations.
There are two ways the results of this paper are of interest then. First, they alert us to a type of inconsistency—one that the literature has largely ignored—that arises when we assume that we can accumulate over time the information that we update on. Second, they show that we can develop a norm equipped to treat this inconsistency, in a way that is compatible with the idea that there are no norms of diachronic rationality.

6.1 Skepticism about Diachronic Rationality

In this section, I spell out the argument for the claim that there are no norms of diachronic rationality, as well as its consequences. Ultimately, what I will want to show is that the account developed in this paper can be made consistent with this claim.

A rough gloss on what accounts of diachronic rationality all have in common is that they are committed to the idea that what attitudes you ought to have at a time directly depend upon what attitudes you have at other times. Skepticism about diachronic rationality is the negation of this claim:

**Skepticism about Diachronic Rationality**: The question of what attitudes you ought to have at a time does not directly depend upon what attitudes you have at other times.

There are a couple of reasons for thinking that skepticism about diachronic rationality might be true, so that there are no diachronic norms. Some have noticed that diachronic norms are inconsistent with internalism (Meacham 2010). Therefore, the attractiveness of internalism should compel one to reject these norms. Others have claimed that the relation one bears to one’s past or future self is similar to the relation one bears to other persons. Therefore, just as one should not feel bound by the commitments of some other person, one should not feel bound by the commitments of prior instances of oneself (Hedden 2015). It’s important to notice that the appeal to internalism and the appeal to personal identity are usually taken to be two ways of appealing
to the same general consideration. The reason we hesitate to take seriously other temporal instances of ourselves is that they are bound by different commitments than our present self, if we take seriously a weak version of internalism and take our epistemic commitments to supervene on our current mental states.¹

Some who have raised these concerns about diachronic rationality have also proposed replacing norms of Bayesian updating with synchronic surrogates: norms that are similar in spirit, but that entail no commitment to diachronic rationality. For instance, the synchronic surrogate outlined in Meacham (2010) says that our current credences should be a function, not of our prior credences, but of our beliefs about our prior credences. More specifically, Meacham’s account requires our current credences to be the weighted average of what we believe our previous credence function recommends about how we ought to revise our beliefs, in light of our current evidence. If we assume an internalist constraint on evidence, Meacham’s account entails that an agent’s current credences will be a function only of her current mental states. Therefore, Meacham’s amendment to the traditional Bayesian formalism leaves us with a norm that governs the agent at each time, rather than over time.

Hedden (2015) also develops a synchronic version of Bayesian updating, albeit one that works a little differently. Rather than appealing to our beliefs about our credence functions at earlier times, Hedden’s account appeals to a uniquely rational credence function. He holds that, at every moment, we are rationally required to condition this rational credence function on our current evidence, which he takes to be those propositions that supervene upon our current mental states. Here, too, then, we get a norm that can be satisfied at every moment, given a weak internalist constraint on evidence.

Call the view implied by both Meacham and Hedden’s accounts—that there are only synchronic norms for Bayesian updating—the synchronic view.²

¹In §6.3, I will suggest that the appeal to internalism and the appeal to personal identity really aren’t tracking the same general consideration.
²I say ‘implied’ here because, strictly speaking, only Hedden offers positive reasons for thinking that
One way of understanding how the synchronic view can get away with jettisoning a coherence constraint is that it trades it in for a stronger constraint elsewhere. Hedden’s account gives up diachronic coherence, at the cost of a uniquely rational prior probability function. Meacham’s account gives up diachronic coherence by handing the work that it does over to the agent’s higher-order beliefs about her prior probability function. There’s a tradeoff, then, between coherence and the amount of information we need to encode in the agent’s other commitments. The less we need of the one, the more we need of the other (Cf. Hedden 2015, 26).

A more rigorous way of understanding this tradeoff structure is to see that diachronic coherence falls out of these synchronic surrogates in certain special cases. Take Hedden’s synchronic version of Jeffrey conditioning:

**JC—Synchronic (Hedden):** When your credences over a partition \( \{E_i\} \) are \( p'(E_i) \), then your degree of belief in \( A \), for any \( A \), should be

\[
p'(A) = \sum_i p^*(A | E_i)p'(E_i),
\]

where \( t' \) is the time at which one has gotten evidence, and \( p^* \) is the uniquely rational prior probability function.\(^3\)

We’ve said that what does all of the work for JC—Synchronic (Hedden) is the constraint that we condition on a rational credence function. But this assumption means that this updating rule will entail Jeffrey conditioning in the special case where we haven’t lost any evidence. For if we assume that our evidence grows monotonically, then our uniquely rational credence function is equivalent to our earlier credence function conditioned on our earlier evidence. Therefore when we condition our uniquely rational credence function on our total evidence, what we are in effect doing is conditioning the credence function that arises from our earlier credence function and earlier evidence on

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\(^3\)While Hedden does not provide a formulation for synchronic Jeffrey conditioning, he writes (fn. 30) that synchronic conditioning can be generalized to synchronic Jeffrey conditioning, presumably in the way I’ve just done.
our current evidence. This is just the requirement prescribed by regular Jeffrey conditioning.

A similar point can be made about the synchronic surrogate developed by Meacham. Meacham’s surrogate can be formulated in the following way:

JC—Synchronic (Meacham): If your credences over a partition \( \{E_i\} \) are 
\[ p'(E_i) \] then your degree of belief in \( A \), for any \( A \), should be 
\[ p'(A) = \sum_{ij} p'( \{ p = p_j \} ) \cdot p_j(A | E_{ij}) p'(E_i), \]
where \( t' \) is the time at which one has gotten evidence.\(^4\)

What does all this work for JC—Synchronic (Meacham) is the constraint that our current credences be a function of our current credences about our prior credence function. But this assumption will entail that where we know what our prior credences were (i.e., \( p'( \{ p = p_j \} ) = 1 \)), our current credences about our prior credence function just is our prior credence function. Therefore, in the case where we know our prior credence function, our current credences will be a function of our prior credences and our evidence. This is just the requirement prescribed by regular Jeffrey conditioning. Therefore, where the agent’s priors encode higher-order knowledge, JC—Synchronic (Meacham) is equivalent to a requirement of diachronic coherence.

In short, then, Meacham and Hedden’s accounts get us synchronic versions of Bayesian updating by appealing to an idiosyncratic account of the agent’s prior function and an internalist account of evidence.\(^5\) Is the view that there are no diachronic norms compatible only with internalism? Hedden thinks no. He maintains that skepticism about diachronic rationality is compatible with other, non-internalist accounts of evidence, like, for instance, Williamson (2000)’s E=K theory of evidence: the view that your evidence consists of all and only the propositions that you know.\(^6\) As Hedden notes, though causal

\(^4\)Here too I’ve modified Meacham’s original formulation to accommodate cases of uncertain evidence.

\(^5\)Following Hedden and others, I will continue to refer to the function that we condition our evidence on as a prior function, even in cases where we are assuming that the function we condition our evidence on is not one that is temporally prior to our evidence.

\(^6\)More precisely, Hedden maintains, against orthodoxy, that Williamson’s view counts as an internalist account of evidence, for his purposes.
facts about the past might determine whether some belief is knowledge (as opposed to true belief), knowledge is still a mental state on this view. And, so, even on this view, one’s evidence can supervene upon one’s present mental states. Accordingly, what one ought to believe might still supervene upon one’s present mental states (2015, p. 29).

While orthodox internalism may be especially friendly to the synchronic view, then, in light of the seriousness with which it takes the agent’s perspective, the synchronic view does not discriminate against most accounts of evidence.

But the synchronic view does discriminate against the account of evidence developed in the previous chapter. Since this account makes evidence a function of our updates at different times, it makes the rationality of our attitudes a function of our priors at different times, contra the assumption made by both Meacham and Hedden that ordinary priors cannot figure in a synchronic account of Bayesian rationality.

Bayesian coherentism then includes a norm for evidence that is inconsistent with the synchronic view. In what follows, I will propose a norm for evidence that is motivated by considerations similar to those that motivate Bayesian coherentism, but that, when coupled with either Meacham or Hedden’s account of priors, is compatible with the synchronic view, in much the way that Williamson’s account of evidence is compatible with the synchronic view. My norm for evidence will, then, provide us with a synchronic surrogate of Bayesian coherentism. Before we begin to develop the account, however, we will need to backtrack a little and consider more carefully the formal structure of the priors that we will be assuming.

6.2 Hypothetical Priors

Normally we talk about the agent’s prior probability function in pretty general terms, as the credences she has before she gets any evidence. But there are a number of ways of filling out this idea. On the most literal way of doing this, an agent’s priors are the
credences she has before she gets any evidence whatsoever. They are the commitments that she has right at the point that she pops into existence, and before she has gotten any information about the world. They are her initial prior function.

The initial priors account has been criticized on the grounds that a function that encodes the agent’s commitments when she pops into existence seems implausible. Simply put, it’s hard to make sense of how an agent could have any commitments at all at this time. There’s a different understanding of priors that gets around this worry. This account says that the function an agent is required to condition her evidence on isn’t an actual set of priors that the agent has had at some point in her existence, but is instead a hypothetical prior function. A hypothetical prior function is any function that bears the right relation to a subject’s credences and the evidence that she’s gotten over time:

**THE HYPOTHETICAL PRIORS ACCOUNT:**

If $E_t$ is a conjunction of the agent’s total evidence at $t$, then her hypothetical prior function, $cr_H$, is the function such that:

$$cr_t(\cdot) = cr_H(\cdot | E_t).$$

One might worry that appealing to hypothetical priors threatens to trivialize Bayesian updating. The worry is a familiar one from chapter two, where we saw that any probabilistic belief transition can be represented as an update that is rigid on some evidence partition. The hypothetical priors account looks like it lets us make a similar claim: it lets us say that, granted that there is some way of making sense of the conjunction of an agent’s evidence, any probabilistic belief transition can be represented as an update from some set of priors. (In the following section, we will come to see that it’s not trivial that

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7Or what is sometimes referred to as a *ur*-prior function. For some historical discussions of these functions, see Carnap (1950) and Levi (1980). For some more recent discussions, see Meacham (2016b) and Titelbaum (ms.).

8Meacham (2016a, 2016b) distinguishes between between the commitment that one's evidence be
an agent’s evidence can be conjoined. For now, I assume there is some way of making sense of this idea.)

But just as we saw in chapter two that we could overcome this worry about triviality by placing normative constraints on evidence, in a way that turns Bayesian updating into a non-trivial norm, so too can we place normative constraints on our priors, in a way that accomplishes the same. These constraints might encode a number of things. On the one hand, our priors might encode normative commitments, like Lewis (1980)’s Principal Principle and van Fraassen (1984)’s Rational Reflection, that place constraints on our other priors. So, for example, my priors will constrain my belief that the coin will come up heads in a moment by my belief that the chance of the coin coming up heads in a moment is .5. On the other hand, our priors might encode the evidential standards, or background beliefs, against which our evidence is interpreted, and which makes sense of two agents who accumulate the exact same evidence, but who come to have different credence distributions as a result. To modify an example from Titelbaum (ms.), consider two agents, one of whom is naturally trusting and one of whom is naturally skeptical—both of whom have received the same evidence and updated to different posterior credence distributions. We can assume that both agents are rational by assuming that they have begun with hypothetical prior functions that encode different evidential standards.

If we assume an account of hypothetical priors that encodes the previous kinds of normative constraints, we can offer a non-trivial account of diachronic coherence from a different angle, by appealing to a normative hypothetical credence function:

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9For discussions of evidential standards, see Meacham (2016b), Schoenfield (2016) and Titelbaum (ms.).
HP-COHERENCE: Let $E_t$ be a conjunction of the agent’s total evidence at $t$. An agent is hp-coherent iff there is a normative hypothetical credence function $cr_H$ such that:

$$cr_t(\cdot) = cr_H(\cdot | E_t)$$

Finally, we might think that we can generalize the account of coherence described above, in a way that accommodates uncertain evidence:

HP-COHERENCE (JC): An agent is hp-coherent iff there is a normative hypothetical credence function $cr_H$ such that:

$$cr_t(\cdot) = \sum_i cr_H(\cdot | E_i)cr_t(E_i)$$

6.3 The Problem: Interval Incoherence

In the last section, I outlined a couple of roles that priors are traditionally held to occupy. It’s an open question what other work priors can be called upon to do. In the following sections, I will propose that the hypothetical priors account can be used to help us formulate a synchronic surrogate of Bayesian coherentsim. The formal structure of the hypothetical priors account gives rise to a problem for evidence that is similar to the non-commutativity problem for Jeffrey conditioning. Since this problem entails a gap between the evidence the agent gets and the evidence the agent updates on, imposing a rational constraint on her priors can actually be used to constitute the latter: it can be used to constitute evidence.

To begin to see this, notice again that we’ve formulated HP-Coherence (JC) in a way that assumes that we are able to make sense of the conjunction of the agent’s evidence over some period of time. However, there’s an obstacle to understanding the agent’s total evidence as the conjunction of her evidence over a given interval. Consider the case
where I directly change my credence in some proposition twice, in turn: I have an experience at one time that prompts me to directly change my credence in the proposition that the raven is black (B) to .2 and my credence in the proposition that it’s not the case that the raven is black (¬B) to .8. And then I have an experience at a later time that prompts me to directly change my credence in the proposition that the raven is black to .3 and my credence in the proposition that it’s not the case that the raven is black to .7. HP-Coherence (JC) assumes that there is a probabilistically weighted partition that represents the conjunction of my evidence over these two times. But it’s not clear how we would conjoin the evidence I have gathered in the case just described. We seem to have a set of values that makes this impossible, since each evidence proposition has been assigned two different values over the interval in question.

It’s important to be clear that this problem does not stem from the normative account of priors we left off with at the end of the last section. Even if we assume that there aren’t any constraints at all on the agent’s priors, the previous problem entails that it will not be the case that, for any set of probabilistic belief transitions, there will be a set of priors that can represent these revisions as Bayesian updates. It entails that there might not be a coherent way of formulating the notion of a hypothetical prior. The problem we are grappling with now is more fundamental than the question of how an agent’s priors are constrained, or the question of which evidential standards she ought to adopt.

The values that we have in the case described above then makes the agent incoherent, in a certain sort of way. I think there’s a way of interpreting this sort of incoherence as something familiar. Plausibly, one way of characterizing what goes wrong in cases like the previous one is that Probabilism is violated “over an interval”. Trivially, this happens whenever Probabilism is violated at a particular time. But I want to suggest that this also happens, in cases like the previous one, where the following modified version of Normality is violated:
Evidential Normality:

∀ p, p', ∀A, p(A) + p'(-A) = 1, where p, p' are credence functions the agent has in between two updates, and where A is a proposition the agent has gotten as evidence at both t and t'.

Evidential Normality imposes an additional constraint on an agent’s beliefs in between the times of two updates by requiring, not just that the propositions in the agent’s algebra conform to Probabilism at a time, but that they conform to something in the spirit of Probabilism when we mix and match propositions that we’ve gotten as evidence at different times. Evidential Normality is satisfied whenever the values we’ve assigned the propositions that form a tautology sum to one, even if they are taken from different evidence partitions, provided that these evidence partitions have not been updated on. This means that violations of Evidential Normality occur anytime we change our credence twice in the same proposition in between updates. The raven example from above illustrates this. If my credence in the proposition that the raven is black is .2 at t, and my credence in the proposition that the raven isn’t black is .7 at t', then p(B) + p'(-B) = .2 + .7 = .9 ≠ 1.

In a way, violations of Evidential Normality are the mirror image of non-commutative updates—they are identical in form, but with the structure reversed. These defects both result from assigning a proposition that is a member of two different evidence partitions two different values or, equivalently, getting some proposition as evidence twice. But each arises within a different way of structuring Bayesian updating. The non-commutativity problem for Jeffrey conditioning arises when we assume that we update our beliefs at each and every time we get new evidence. By contrast, violations of Evidential Normal-

\[^10\text{Of course, as we saw in the previous chapter, these cases are only a proper subset of those that violate Jeffrey independence, and so they are only a proper subset of those cases that make the agent’s updates fail to commute on the sequential model of conditioning. Nevertheless, insofar as violations of Evidential Normality are those violations of Jeffrey independence that the framework makes possible, I take there to be some connection between the defects described above.} \]
ity arise when we accumulate evidence and condition on it all at once.

Here’s a more contentious way of spelling out the difference between the two types of inconsistencies just described. The non-commutativity problem for Jeffrey conditioning is a problem about diachronic coherence—it’s a problem that arises when we update. By contrast, insofar as violations of Evidential Normality arise in between updates, I think we can say that such violations represent a problem about synchronic coherence. This might seem like an odd suggestion, for two reasons. First, this constraint is defined over credence functions the agent holds at different times. Therefore, it does not seem like it could be a synchronic constraint. Second, unlike violations of Probabilism, violations of Evidential Normality don’t really seem like a problem. There doesn’t seem to be anything especially bad about the agent moving from one probability function to another, in a way that violates Evidential Normality.

There are two things to notice here however. First, notice that, from the perspective that we are concerned with—the perspective of updating—violations of Probabilism and violations of Evidential Normality are equally bad. Both types of violations in our evidence ensure that we don’t have the sort of object that can be updated on, in virtue of ensuring that the product of what the agent has gotten in between two updates doesn’t yield a probabilistically weighted partition.

Second, from the perspective of updating, Evidential Normality isn’t actually defined over different times, in the relevant sense. To see this, we might return to the discussion of the previous section and notice that we can distinguish between two interpretations of skepticism about diachronic rationality. The first interpretation says that skepticism about diachronic rationality is the idea that one should not be beholden to one’s commitments at different times because one’s commitment at the present time ought to take priority. The second interpretation says that skepticism about diachronic rationality is the idea that one should not be beholden to one’s commitments at differ-
ent times because time is irrelevant to what one’s commitments happen to be. The first interpretation is committed to the idea that there is something inherently special about our present time-slice that isn’t explained by the agent’s commitments. The second interpretation suggests that what makes some time-slice special is *precisely* the agent’s commitments.\footnote{In light of this, we might think that ‘skepticism’ about diachronic rationality is a bit of a misnomer. Instead, it seems more appropriate to call the account we are after a reductive account of diachronic rationality.}

It is the second version of skepticism about diachronic rationality that strikes me as the more plausible one. There’s nothing special about time—it’s just the receptacle for our normative commitments.\footnote{Admittedly, the first way of thinking about skepticism about diachronic rationality is more compelling for internalists who are antecedently motivated by the idea that rationality ought to be a matter of the way that things seem to one at the present moment. However, the second way of thinking about diachronic rationality is the more ecumenical one: it is compatible with any account of evidence.} But if this is the case, then, for normative purposes, times should be individuated, not by natural instances, but by how temporally extended the evidence the agent updates on turns out to be. Since the hypothetical priors account assumes that the agent updates on her total evidence, rather than the evidence she has at any particular time, Evidential Normality’s constraint will be a synchronic constraint, since Evidential Normality’s constraint isn’t defined over those times that individuate updates, but is instead defined *within* those times. (Of course, in order to say all this, we need to be able to say how it is that the conjunction of an agent’s evidence over some period of time could be the evidence that she updates on, even though it violates Evidential Normality, and so could not be the sort of weighted partition that could serve as an input to the updating process. In the next section, a solution to this problem will be proposed.)

Violations of Evidential Normality, then, result from a mismatch between the different ways that Probabilism and Bayesian updating individuate times. It’s the possibility that our commitments, relative to the norm of Probabilism, might change without our commitments, relative to the norm of updating, changing that makes such violations
possible. Or, in other words, it’s the possibility of an agent’s commitments, relative to
the norm of Probabilism, changing without the agent having updated on evidence that
makes such violations possible.

Arguably, violations of Evidential Normality pose an even more serious challenge for
the Bayesian framework than the non-commutativity problem for Jeffrey conditioning.
The sort of inconsistency that arises from a non-commutative set of updates is at least
compatible with Bayesian updating—it’s compatible with the agent being coherent, to a
degree. But a set of evidence partitions that violates Evidential Normality cannot serve
as the input to the Bayesian updating process. An agent cannot update at all on such
evidence. Therefore, this second type of inconsistency needs to be, not only minimized,
but resolved.

6.4 The Solution: Higher-Order Beliefs as an Account of Evidence

We’ve just seen that the hypothetical priors account of updating does not entail a con-
dition that is necessary for evidence on the Bayesian framework: it does not entail that
an agent’s total evidence conforms to Evidential Normality. In this section, I will argue
for a constraint whose main aim is precisely to secure this formal necessary condition
for an account of evidence. Insofar as this is the case, I will again be defending a formal
account of evidence. Since the account of evidence I will defend is one that mitigates the
effects of violations of Evidential Normality, it will be one that depends upon the agent’s
attitudes at earlier times. However, we’ve just seen that the times that are relevant, for
normative purposes, will be individuated by updates. Therefore, just as Williamson’s
account makes what evidence the agent has depend upon facts at earlier times, while
being compatible with the synchronic view, so too is my account compatible with the
synchronic view. For, on my account, as on Williamson’s, the attitudes we ought to hold
at any given time will depend upon the evidence we have at that time.
Earlier we noted that both Meacham and Hedden’s arguments assume a weak, internalist constraint on evidence, a constraint that tells us that whether or not we have evidence, and what evidence we have, is determined by the way that things seem to us. But there’s an ambiguity here when we talk about ‘having evidence’. Evidence might denote one of two things: (1) that which the internalist constraint is being applied to, or 2) that which the internalist constraint generates. In other words, we might think of evidence as the mental states that our weighted evidence partition supervenes upon, or we might think of evidence as this weighted partition itself. Normally, we are sloppy and call both of these things ‘evidence’. From here on, let us be more careful and call (1) ‘evidence’, and (2) ‘evidence*’.

Like Meacham and Hedden’s account, my account will assume that we can understand evidence* to be a weighted partition. But instead of understanding the agent’s evidence to be whatever mental states the agent happens to have at a time, I take the agent’s evidence to be the set of weighted partitions that she accumulates over an interval. This means that, though an agent might directly change her credence along the partition \( \{ E_i \} \) twice, neither of these changes constitute evidence*. Neither of these changes justifies an update. It’s only when a normative constraint gets imposed upon a set of weighted partitions that the weighted partition that results is evidence* the agent is able to update on.\(^\text{13}\)

What is the normative constraint that might reasonably make a set of weighted partitions evidence*? I want to suggest that it is a constraint that Bayesians are, in large part, already committed to. We’ve already noted that expert functions like Rational Reflection and the Principal Principle have an important role to play in an account of Bayesian rationality. With this in mind, I want to suggest extending the use of expert functions in

\(^{13}\)In other words, when one gets some proposition as evidence, there are, strictly speaking, two steps that account for the value that the agent maintains in this proposition, though the second step is redundant. First, the agent gets this proposition as evidence. Second, the agent conditions this proposition on itself, thereby maintaining in it the same value as before. Without a normative constraint on evidence, it is only the first step that changes the agent’s credence in this proposition.
a different direction. I want to propose that beliefs about how relatively reliable we were at the times at which our evidence was received should constrain the agent’s beliefs before she updates—whether we take reliability to be a matter of how closely our beliefs align with the objective chances, with our future opinions, or with some other expert.

To see how this constraint will naturally resolve violations of Evidential Normality, consider again our agent who assigns a credence of .6 to the proposition that the raven is black and a credence of .4 to the proposition that the raven isn’t black, and then, a moment later, changes her mind and raises her credence in these propositions to .8 and .2, respectively. In this case, the agent violates Evidential Normality. But notice that weighting these values by how relatively reliable the agent takes herself to have been at the times at which these revisions were made has the potential to resolve the inconsistency that these revisions involve. Perhaps the agent believes that at $t_2$ she was much more reliable than she was at $t_1$. In this case, her higher-order beliefs will weight how the evidence she has gotten at these different times should bear on her evidence*. Or perhaps the agent believes that she was three times as reliable at $t_1$ as she was at $t_2$. In this case, the weighted evidence partition the agent updates on will be the following:

$$p(B) = (.6)(.75) + (.8)(.25) = .65$$

$$p(\overline{B}) = (.4)(.75) + (.2)(.25) = .35$$

More generally, the normative constraint on an agent’s evidence that higher-order beliefs impose will be one that weights our evidence by how reliable we take this evidence to be, relative to how reliable we take our competing evidence to be—or, equivalently, relative to how reliable we take ourselves to have been at each of the times that some different evidence was gathered:
HO-CONSTRAINT: Given any evidence, \( p(E_i) \), that the agent gets at \( t_i \), the agent’s evidence* \( p_n(E^*) \), will be determined in the following way:

\[
p_n(E^*) = \sum_{i=1}^{n} p(E_i) R_i, \quad R_i = \frac{p_n(\text{REL}(E_i))}{\sum_{j=1}^{n} p_n(\text{REL}(E_j))}
\]

where \( p(E_i) \) is the agent’s credence in \( E \) at \( t_i \), and where \( p_n(\text{REL}(E_i)) \) represents how reliable the agent believes herself, at \( t_n \), to have been at \( t_i \), and where \( \sum_{j=1}^{n} p_n(\text{REL}(E_j)) \) represents the sum of how reliable the agent takes herself, at \( t_n \), to have been at each of the different times in question.\(^{14}\)

We can unpack \( R_i \) a little bit more. We’ve just said that what it does is to weight the agent’s evidence, \( p(E_i) \), by how reliable she believes this evidence to be, relative to how reliable she believes herself to have been at the other times at which she’s gotten evidence. Where the agent updates on some proposition more than once, this constraint will have the effect of weighting competing pieces of evidence by weighting the agent’s

\(^{14}\)In the case described above, then, the HO-Constraint can be unpacked in the following way:

\[
p_H(B^*) = \sum_{i=1}^{n} p(B_i) R_i
\]

\[
p_H(B^*) = p(B_{i_1}) R_{i_1} + p(B_{i_2}) R_{i_2}
\]

\[
p_H(B^*) = p(B_{i_1})(.75)/(.75 + .25) + p(B_{i_2})(.25)/(.75 + .25)
\]

\[
p_H(B^*) = (.6)(.75)/(.75 + .25) + (.8)(.25)/(.75 + .25)
\]

\[
p_H(B^*) = .65
\]

\[
p_H(\overline{B}^*) = \sum_{i=1}^{n} p(\overline{B}_i) R_i
\]

\[
p_H(\overline{B}^*) = p(\overline{B}_{i_1}) R_{i_1} + p(\overline{B}_{i_2}) R_{i_2}
\]

\[
p_H(\overline{B}^*) = p(\overline{B}_{i_1})(.75)/(.75 + .25) + p(\overline{B}_{i_2})(.25)/(.75 + .25)
\]

\[
p_H(\overline{B}^*) = (.4)(.75)/(.75 + .25) + (.6)(.25)/(.75 + .25)
\]

\[
p_H(\overline{B}^*) = .35
\]
first-order perspective on her evidence by her second-order perspective about how reliably this evidence was formed, relative to those other times at which the evidence proposition was received, in a way that will yield a probabilistically weighted partition. And in cases where the agent doesn’t update on the same proposition more than once, but instead updates on a different proposition each time, so that there is no need to adjudicate between conflicting pieces of evidence, the HO-Constraint will have no effect at all, since the same piece of evidence will be weighted by one.

There are several concerns one might have about the HO-Constraint, and so several adjustments one might want to make to it. One concern one might have about the HO-Constraint is that it goes silent in cases where the agent does not have the requisite higher-order beliefs, or where there are gaps in these higher-order beliefs. Suppose the agent has beliefs about how reliable she was at t₁ and t₃, but has no opinion at all about how reliable she was at t₂ and t₄. How should things work out in a case like this?

One option is to say that where one has no higher-order beliefs about one’s first-order beliefs, these first-order beliefs should not contribute to one’s evidence* at all. After all, if beliefs about how reliable we are are essential to our normative constraint on evidence then, if these beliefs are lacking, there’s a sense in which our normative constraint on evidence is lacking as well.

But maybe this is too severe. Maybe we would want an account that assumes that experience has some prima facie import. In this case, a more reasonable option for treating cases where an agent has gappy higher-order beliefs is to appeal to a principle of indifference and say that an agent’s credences ought to be uniformly spread among those times about which she has no opinion. If she has opinions about how reliable she was at t₁ and t₃ but not at t₂ and t₄, then whatever credence remains after assigning the appropriate amount of credence to t₁ and t₃ should be spread evenly between times t₂ and t₄. If she has no opinion at all about how reliable she was between t₁ and t₄, then her credence
should be spread evenly over all of these times:

**HO-CONSTRAINT (REVISED):** Given any evidence, \( p(E_t) \), that the agent gets at \( t_i \), the agent’s evidence*, \( p_n(E^*) \), will be determined in the following way:

\[
p_n(E^*) = \sum_{i=1}^{n} p(E_i)R_i, \quad R_i = \begin{cases} 
\frac{p_n(\text{REL}(E_i))}{\sum_{j=1}^{n} p_n(\text{REL}(E_j))} (n_d/n), & \text{if defined}, \\
1/n, & \text{otherwise}
\end{cases}
\]

where \( n_d \) is the number of intervals between \( t_1 \) and \( t_n \) for which \( p_n(\text{REL}(E_i)) \) is defined, and where \( p_n(\text{REL}(E_i)) \) represents how reliable the agent believes herself, at \( t_n \), to have been at \( t_i \), and where \( \sum_{j=1}^{n} p_n(\text{REL}(E_j)) \) represents the sum of how reliable the agent takes herself, at \( t_n \), to have been at each of the different times in question.

Finally, as before, we can derive an account of coherence from a hypothetical function that includes those higher-order beliefs that deliver our evidence*, \( p_n(E^*) \):

**GENERALIZED HP-COHERENCE:** An agent is hp-coherent if there is a credence function \( p_H \) such that, for any \( p(E_i) \) the agent gets at \( t_i \),

1. \( p_n(E^*) = \sum_{i=1}^{n} p(E_i)R_i, \quad R_i = \begin{cases} 
\frac{p_n(\text{REL}(E_i))}{\sum_{j=1}^{n} p_n(\text{REL}(E_j))} (n_d/n), & \text{if defined}, \\
1/n, & \text{otherwise}
\end{cases}
\)

2. \( p_n(\cdot) = \sum_k p_H(\cdot | E_k^*)p_n(E_k^*) \).

where \( n_d \) is the number of intervals between \( t_1 \) and \( t_n \) for which \( p_n(\text{REL}(E_i)) \) is defined, and where \( p_n(\text{REL}(E_i)) \) represents how reliable the agent believes herself, at \( t_n \), to have been at \( t_i \), and where \( \sum_{j=1}^{n} p_n(\text{REL}(E_j)) \) represents the sum of how reliable the agent takes herself, at \( t_n \), to have been at each of the different times in question.

Notice that the way the HO-Constraint (Revised) gets us a normative account of evidence mimics the way that a typical internalist account of evidence does this. On
Meacham and Hedden’s accounts, an agent’s ‘evidence’ are her mental states, and her ‘evidence*’ is the weighted evidence partition that supervenes on these mental states, according to the internalist constraint. On my account, an agent’s ‘evidence’ are the weighted evidence partitions she has gotten over time, and her ‘evidence*’ is the weighted evidence partition that supervenes on this set of weighted evidence partitions, according to the HO-Constraint (Revised). The HO-Constraint (Revised) reproduces the structure of the account of evidence that the synchronic view assumes.

One might object that there is an important difference between the internalist’s account of evidence and mine, which is that only on my account is the process of getting evidence temporally extended. However, when we consider more carefully the process that yields evidence on the internalist’s account of things, we come to see that it is a mistake to think that it is not a temporally extended process, in the very same way that it is on my account, and on Williamson’s E=K account. There will be some point in time when some collection of mental states, or experiences, that I’ve had makes it seem like there is a red sock in front of me. At this point in time, the internalist’s constraint kicks in and justifies my getting evidence*. But the experience as of a red sock, or the set of experiences that comprise it, takes place over some, albeit probably short, interval of time. We tend to idealize away this fact about experiences being temporally extended when we talk about updating because, usually, nothing turns on it. However, in the present context, attending to it allows us to see that the account I have proposed has exactly the same structure as the traditional internalist’s account. On my account, we have some particular set of mental states over a period of time. At a certain point—the point at which the higher-order belief constraint gets imposed—these mental states justify my getting evidence*.15

15Notice that adopting the HO-Constraint (Revised), in addition to providing a solution to the problem described in the beginning of this paper, enables the framework to avoid many of the worries explored in chapter two. In a way, violations of Evidential Normality are analogous to the general problem of how something like experience—something that doesn’t have the structure required to slot into an inferential
Priors have long been held to serve two important roles. First, they tell us how we ought to update. Second, they tell us how we ought to adjust our other beliefs before we update. In this section, I have suggested that the second role we ascribe to priors can be extended in a way that allows them to do still more work for us. Priors, in conjunction with the evidence the agent has gotten, provide us with the agent’s evidence*. 

6.5 A Solution for the Skeptics

Earlier we considered two synchronic surrogates for Jeffrey conditioning. We saw that Meacham’s and Hedden’s updating rules work by giving us a different interpretation of the function that we condition our evidence on. On Meacham’s view, this function is the weighted average of our beliefs about our prior function. On Hedden’s view, this is a uniquely rational prior function. We also noted earlier that, since the norm for evidence that Bayesian coherentism includes makes coherence a function of the agent’s updates at different times, appealing to the sorts of functions that Meacham and Hedden propose won’t, by itself, get us an account of Bayesian updating that avoids diachronic commitments.

To this end, I have developed an account of evidence that, when coupled with the functions that Meacham and Hedden propose, *does* get us an account that avoids diachronic commitments.16 In addressing violations of Evidential Normality—the synchronic analogue of violations of commutativity—the account of evidence developed in this paper can be understood to be a surrogate for Bayesian coherentism. Like Bayesian coherentism, it is an account of evidence motivated by a form of incoherence that arises

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16Note that because we have adopted an account of evidence that is stronger than the one that Meacham and Hedden’s accounts assume, we might not, in the end, need to adopt accounts of priors that are as strong as theirs. But the point is that their priors, along with my account of evidence, are sufficient for a synchronic account of Bayesian updating.
from the formal properties of Bayesian evidence. While Bayesian coherentism tells us to minimize this incoherence, the account developed in this paper provides us with the means for resolving it.
APPENDIX A

A MEASURE OF BAYESIAN EVIDENTIAL INCOHERENCE

There are different formal measures that might be used to give content to the notion of Bayesian evidential incoherence. In this appendix, I outline one such measure, which draws upon the seminal work of Diaconis and Zabell (1982) on the formal properties of Jeffrey conditioning.¹ This measure follows naturally from a property that is both necessary and sufficient for commutative updates (p.825):

JEFFREY INDEPENDENCE: Let P be a probability function. And let P_\mathcal{E} and P_\mathcal{F} be the probability functions that result from updating P on the partitions \mathcal{E}={E_i} and \mathcal{F}={F_j}, respectively. The partitions \mathcal{E} and \mathcal{F} are Jeffrey independent with respect to \{p_i\} and \{q_i\} if P_\mathcal{E}(F_j)=P(F_j) and P_\mathcal{F}(E_i)=P(E_i) holds for all i and j.

Thus, Jeffrey independence says that Jeffrey updating on \mathcal{E} with probabilities \(p_i\) does not change the probabilities on \mathcal{F} and similarly with \mathcal{E} and \mathcal{F} interchanged.

It should be clear how the lack of Jeffrey independence undermines the commutativity of updates. Since Jeffrey updating fixes the probabilities on the evidence partition, the values for \mathcal{F} that we get when we update on it first will be the same values for \mathcal{F} that we get when we update on it second. If \(P_\mathcal{E}(F_j)\neq P(F_j)\), then where we get \mathcal{F} first and \mathcal{E} second (where \(P_\mathcal{E}=P_\mathcal{F}\)), it will change the values along \mathcal{F} from those that we got

¹The following draws heavily from §3.3 of this discussion.
from updating on $\mathcal{F}$ first. Therefore, these values will differ from those that we would have gotten from updating on $\mathcal{F}$ second. Our final probabilities for the propositions in $\mathcal{F}$ will be different, then, depending upon whether we update on $\mathcal{F}$ first or second. Similarly, since Jeffrey updating fixes the probabilities on the evidence partition, the values for $\mathcal{E}$ that we get when we update on it first will be the same as the values for $\mathcal{E}$ that we get when we update on it second. If $P_{\mathcal{F}}(E_i) \neq P(E_i)$, then where we get $\mathcal{E}$ first and $\mathcal{F}$ second (where $P_{\mathcal{F}} = P_{\mathcal{E} \cap \mathcal{F}}$), it will change the values along $\mathcal{E}$ from those that we got from updating on $\mathcal{E}$ first. Therefore, these values will differ from those that we would have gotten from updating on $\mathcal{E}$ second. Our final probabilities for the propositions in $\mathcal{E}$ will be different, then, depending upon whether we update on $\mathcal{E}$ first or second.

Since, as the authors show, Jeffrey independence is necessary and sufficient for commutativity (pp. 825-26), if we are looking to assess the degree to which some set of updates fails to commute, understanding what it would mean for this standard to fail to be met to varying degrees looks like the place to start. We can begin by distinguishing Jeffrey independence from a different, stronger type of independence (p. 825):

P-INDEPENDENCE: Two partitions $\mathcal{E} = \{E_i\}$, and $\mathcal{F} = \{F_j\}$, such that $P(E_i) > 0$, $P(F_j) > 0$ for all $i$ and $j$, are P-independent if $P(E_i | F_j) = P(E_i)$ and $P(F_j | E_i) = P(F_j)$ for all $i, j$.

Thus, P-independence says that conditioning on $\mathcal{F}$ does not change the probabilities on $\mathcal{E}$ and similarly with $\mathcal{E}$ and $\mathcal{F}$ interchanged.

As should be clear, P-independence entails Jeffrey independence: it is Jeffrey independence, for all $p_i, q_j$. The reason that Jeffrey independence, rather than P-independence, is sufficient for commutativity is that, given two partitions whose members aren’t all P-independent of each other, it is possible to update on these partitions and assign them values that perfectly offset these relations of dependence, in a way that secures the com-
mutativity of updates. Therefore, the dependence that gives rise to non-commutativity—that which we are looking to capture, and to make gradable—will be a function, both of the relations of p-dependence that precede an update, and of the weighted evidence partition that gets updated on.

To begin to get a handle on this function, notice that since Jeffrey independence says that $P_E(F_j) = P(F_j)$, for all j, and $P_{\mathcal{E}}(E_i) = P(E_i)$, for all i, it will hold where $\frac{P_E(F_j)}{P(F_j)} = 1$, for all j, and $\frac{P_{\mathcal{E}}(E_i)}{P(E_i)} = 1$, for all i. Plausibly, then, the degree of a violation of Jeffrey independence will be a function of the amount by which each of these diverges from 1. To formulate a measure that can account for this in a perspicuous way, we can note, with Diaconis and Zabell, that $\frac{P_E(F_j)}{P(F_j)} = \sum_i p_i r_{ij}$ where,

$$r_{ij} = \frac{P(E_i F_j)}{P(E_i) P(F_j)}.$$

Given this, the degree of a violation of Jeffrey independence will correspond to the sums of the amount by which $\sum_i p_i r_{ij}$ diverges from 1, for all j, and the amount by which $\sum_j q_j r_{ij}$ diverges from 1, for all i. These observations point us towards the following measure of incoherence:

**EVIDENTIAL INCOHERENCE (EI):** Let $P$ be a probability function and let $P_\mathcal{E}$ be the probability function that results from updating $P$ on $\mathcal{E}$ with probabilities $p_i$. Further, let $r_{ij} = \frac{P(E_i F_j)}{P(E_i) P(F_j)}$, $r_{ij}' = \frac{P_E(E_i F_j)}{P_E(E_i) P_E(F_j)}$, if defined, and 1 otherwise.

A sequence of updates over the partitions, $\{E_i\}$, $\{F_j\}$, is coherent to the extent that it minimizes $\sum_j (|1 - \sum_i r_{ij} p_i|) + \sum_i (|1 - \sum_j r_{ij}' q_j|)$.

We can run through a couple of examples to see how EI will work.

(1) Consider the following initial credence distribution $P(E_i F_j)$:
\[
\begin{array}{c|ccc}
 & F_1 & F_2 & F_3 \\
\hline E_1 & .25 & .125 & .125 & .5 \\
\hline E_2 & .125 & 0 & .125 & .25 \\
\hline E_3 & .125 & .125 & 0 & .25 \\
\hline .5 & .25 & .25 &
\end{array}
\]

Now assume that the agent gets as evidence \( p(E_1) = .5 \), \( p(E_2) = .2 \), \( p(E_3) = .3 \) and \( p(F_1) = .2 \), \( p(F_2) = .4 \), \( p(F_3) = .4 \), in turn. We are left with the following after we update on \( \{E_i\} \), with values \( \mathbf{p}_i \) (left), and on \( \{F_j\} \) with values \( \mathbf{q}_j \) (right):²

\[
\begin{array}{c|ccc}
 & F_1 & F_2 & F_3 \\
\hline E_1 & .25 & .125 & .125 & .5 \\
\hline E_2 & .1 & 0 & .1 & .2 \\
\hline E_3 & .15 & .15 & 0 & .3 \\
\hline .5 & .275 & .225 &
\end{array}
\quad
\begin{array}{c|ccc}
 & F_1 & F_2 & F_3 \\
\hline E_1 & .1 & .182 & .22 & .502 \\
\hline E_2 & .04 & 0 & .18 & .22 \\
\hline E_3 & .06 & .218 & 0 & .278 \\
\hline .2 & .4 & .4 &
\end{array}
\]

Before the first update, \((r_{ij})\) represents the relations of dependence that hold between \( \{E_i\} \) and \( \{F_j\} \). Before the second update, \((r_{ij}')\) represents the relations of dependence that hold between \( \{E_i\} \) and \( \{F_j\} \):

\[
(r_{ij}) = \begin{pmatrix}
1 & 1 & 1 \\
1 & 0 & 2 \\
1 & 2 & 0
\end{pmatrix}, \quad (r_{ij}') = \begin{pmatrix}
1 & .91 & 1.11 \\
1 & 0 & 2.22 \\
1 & 1.82 & 0
\end{pmatrix}
\]

²Some of the values in the second chart are approximations. However, since they are not inputs into our measure, this makes no difference.
Multiplying each column (i.e., each member of \( \{ F_j \} \)) of \((r_{ij})\) by \(p_i\), and taking the amount by which the value of each column diverges from one, and multiplying each row (i.e., each member of \( \{ E_i \} \)) of \((r_{ij}')\) by \(q_j\), and taking the amount by which the value of each row diverges from one yields:

\[
\sum_i (|1 - \sum_j r_{ij} p_i|) + \sum_j (|1 - \sum_i r_{ij}' q_j|) \approx 0.37
\]

This is a measure of the evidential incoherence of the update.

(2) Now assume that, given the same initial credence distribution, the agent gets as evidence \(p(E_1) = .6\), \(p(E_2) = .2\), \(p(E_3) = .2\), and \(p(F_1) = .4\), \(p(F_2) = .3\), \(p(F_3) = .3\), in turn. We are left with the following after we update on \(\{ E_i \}\), with values \(p_i\) (left), and on \(\{ F_j \}\) with values \(q_j\) (right):

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<tr>
<td>(E_1)</td>
<td>.24</td>
<td>.18</td>
<td>.18 .6</td>
</tr>
<tr>
<td>(E_2)</td>
<td>.08</td>
<td>0</td>
<td>.12 .2</td>
</tr>
<tr>
<td>(E_3)</td>
<td>.08</td>
<td>.12</td>
<td>0 .2</td>
</tr>
<tr>
<td></td>
<td>.4</td>
<td>.3</td>
<td>.3</td>
</tr>
</tbody>
</table>

Before the first update, \((r_{ij})\) represents the relations of dependence that hold between \(\{ E_i \}\) and \(\{ F_j \}\). Before the second update, \((r_{ij}')\) represents the relations of dependence that hold between \(\{ E_i \}\) and \(\{ F_j \}\):

\[
(r_{ij}) = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 0 & 2 \\ 1 & 2 & 0 \end{pmatrix} \quad (r_{ij}') = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 0 & 2 \\ 1 & 2 & 0 \end{pmatrix}
\]

Multiplying each column (i.e., each member of \(\{ F_j \} \)) of \((r_{ij})\) by \(p_i\), and taking the amount by which the value of each column diverges from one, and multiplying each row
(i.e., each member of \( \{E_i\} \)) of \( (r_{ij}^r) \) by \( q_j \), and taking the amount by which the value of each row diverges from one yields:

\[
\sum_j (|1 - \sum_i r_{ij}^r p_i|) + \sum_i (|1 - \sum_j r_{ij}^r q_j|) = 0
\]

This is a measure of the evidential incoherence of the update. This update exhibits perfect evidential coherence.
APPENDIX B

BAYESIAN COHERENTISM IN ACTION

The real payoff of Bayesian coherentism is a theoretical one, rather than a practical one: it allows us to claim that both updates on certain and uncertain evidence proceed from the same norm. Unlike many tweaks to the traditional Bayesian formalism, Bayesian coherentism isn’t problem driven: it’s not a remedy to instances where the framework yields unintuitive or unclear verdicts. It’s not going to solve the Sleeping Beauty problem.

Nevertheless, it’s a constraint on adequacy of any norm that it function in an intuitive way. To show that Bayesian coherentism meets this constraint, we can walk through some examples to get a sense of how its verdicts compare with those rendered by Bayesian foundationalism (i.e., Bayesian conditioning) and Jeffrey conditioning in four types of cases: 1) cases where we update on uncertain evidence and where Bayesian coherentism and Jeffrey conditioning yield different verdicts, 2) cases where we update on uncertain evidence and where Bayesian coherentism and Jeffrey conditioning yield the same verdict, 3) cases where we update on certain evidence and where Bayesian coherentism and Bayesian foundationalism yield the same verdict, and 4) cases where Bayesian coherentism yields the same verdicts as both Bayesian conditioning and Jeffrey conditioning, regardless of what the agent’s evidence consists in.

Consider an agent with a credence function defined over the propositions:

\[ E_1: \text{it's going to rain before noon today} \]
\[ E_2: \text{it's going to rain after noon today} \]
\[ E_3: \text{it's not going to rain today} \]
F₁: the barometric pressure is low

F₂: the barometric pressure is high

F₃: the barometric pressure is within medium range

Assume that the agent has the following initial credence distribution P(EᵢFⱼ):

<table>
<thead>
<tr>
<th></th>
<th>F₁</th>
<th>F₂</th>
<th>F₃</th>
</tr>
</thead>
<tbody>
<tr>
<td>E₁</td>
<td>.25</td>
<td>.125</td>
<td>.125</td>
</tr>
<tr>
<td>E₂</td>
<td>.125</td>
<td>0</td>
<td>.125</td>
</tr>
<tr>
<td>E₃</td>
<td>.125</td>
<td>.125</td>
<td>0</td>
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<tr>
<td></td>
<td>.5</td>
<td>.25</td>
<td>.25</td>
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</tbody>
</table>

(1) Now suppose that the agent gets as evidence \( p(E₁) = .2, p(E₂) = .4, p(E₃) = .4 \) and \( p(F₁) = .5, p(F₂) = .2, p(F₃) = .3 \), in turn. Since this is a case where the agent has gotten uncertain evidence, we can either appeal to Bayesian coherism, or we can appeal to Jeffrey conditioning—whether conceived of as Descriptive Diachronic Coherence for Bayesians, or as by the default view.

Suppose that we appeal to Jeffrey conditioning. By the lights of Descriptive Diachronic Coherence for Bayesians, the agent is perfectly diachronically coherent. Her posterior credence distribution reflects that she has updated in accordance with her conditional probabilities, since she has updated in accordance with some conditional probabilities. Recall that we know this simply because her posterior belief state is a probability function. Moreover, if these conditional probabilities are those that are conditional on some partition of propositions the agent has, as a matter of fact, changed her credences in directly, then she is perfectly coherent by the lights of the default view of Jeffrey conditioning.

By contrast, suppose that we appeal to Bayesian coherism. Recall that this norm judges an update, not only according to whether it reflects that the agent has updated in
accordance with some conditional probability, but according to whether the agent’s evidence minimizes evidential incoherence, as encoded in EI. Therefore, if it turns out to be the case that the value of her update has been a function of the order of her evidence, her update is less than perfectly diachronically coherent. As it turns out, the value of her update is a function of the order of her evidence. Therefore, the agent is diachronically incoherent to the extent prescribed by Bayesian coheretism, via EI (See example (1) in Appendix A for the details).

(2) Suppose that, given the same initial credence function, the agent gets as evidence $p(E_1) = .6, p(E_2) = .2, p(E_3) = .2$ and $p(F_1) = .4, p(F_2) = .3, p(F_3) = .3$, in turn. Again, given that this is a sequence of updates on uncertain evidence, we can judge this set of updates, either by the lights of Jeffrey conditioning, or by the lights of Bayesian coheretism. In this case, both standards render the same verdict. For, in this case, the value of the agent’s update is not a function of the order of her evidence. Therefore, in this case, EI entails that the agent is perfectly diachronically coherent (See example (2) in Appendix A for the details). So does the standard set by Jeffrey conditioning, for the reasons given in (1).

(3) Suppose instead that the agent gets as evidence $p(E_1) = 1, p(E_2) = 0, p(E_3) = 0$ and $p(F_1) = 1, p(F_2) = 0, p(F_3) = 0$, in turn. Since what we have here is an update on certain evidence, we can either appeal to Jeffrey conditioning, Bayesian foundationalism or Bayesian coheretism. In this case, it’s easy to see that Jeffrey independence will be satisfied, so that Bayesian coheretism entails that these updates commute. Therefore, the agent will be perfectly diachronically coherent by the lights of Bayesian coheretism. The agent will also be (perfectly) diachronically coherent by the lights of Bayesian foundationalism, but for a different reason: because the agent is certain of her evidence. Given a suitable description of the case, the agent will be diachronically coherent by the lights of Jeffrey conditioning, conceived of as by the default view. Of course, she will satisfy DESCRIPTIVE DIACHRONIC COHERENCE FOR BAYESIANS as well.

(4) Finally, consider an agent with the following initial credence distribution $P^x(E_i F_j)$:
<table>
<thead>
<tr>
<th></th>
<th>$E_1$</th>
<th>$E_2$</th>
<th>$E_3$</th>
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</thead>
<tbody>
<tr>
<td>$F_1$</td>
<td>.25</td>
<td>.25</td>
<td>.25</td>
</tr>
<tr>
<td>$F_2$</td>
<td>.25</td>
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<td>.25</td>
</tr>
<tr>
<td>$F_3$</td>
<td>.25</td>
<td>.25</td>
<td>.25</td>
</tr>
<tr>
<td></td>
<td>.5</td>
<td>.5</td>
<td>.5</td>
</tr>
</tbody>
</table>

Since each of the propositions in $\{E_i\}$ is probabilistically independent of each of the propositions in $\{F_j\}$, we know that \textit{whatever} the input distribution, it will be the case that the agent’s evidence is Jeffrey independent, and so commutes. Again, then, the agent’s update will exhibit diachronic coherence, by the lights of Bayesian coherentism. And if $E_i=1$, $F_j=1$, for some $i$, $j$, then her update will exhibit diachronic coherence, by the lights of Bayesian foundationalism as well. Finally, given a suitable description of the case, the agent will be diachronically coherent by the lights of Jeffrey conditioning, conceived of as by the default view. Again, nearly trivially, the standard set by \textit{Descriptive Diachronic Coherence for Bayesians} will be satisfied as well.
BIBLIOGRAPHY


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