Children's Understanding of Compositionality of Complex Numerals

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CHILDREN’S UNDERSTANDING OF THE COMPOSITIONALITY OF COMPLEX NUMERALS

A Thesis Presented

by

JIHYUN HWANG

Submitted to the Graduate School of the
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ABSTRACT

CHILDREN’S UNDERSTANDING OF COMPOSITIONALITY OF COMPLEX NUMERALS

FEBRUARY 2021

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Counting is the first formal exposure for children to learn numerals, which are constructed with a set of syntactic rules. Young children undergo many stages of rote-memorization of the sequence and eventually count through 100. What core knowledge is necessary to expand their number knowledge to higher numbers? The compositionality of numerals is a key to understanding the natural number system as in learning languages. Higher numbers (e.g., two hundred five) are constructed with the lexical items such as earlier numbers (e.g., one to nine) and multipliers. If children develop their understanding of the compositionality of numerals, they might comprehend complex numerals far beyond their count list. In a novel task, the Number Word Comparison task, we tested whether children’s skill to compare the ones (e.g., five versus eight) can extend to complex numerals (e.g., two hundred five versus two hundred eight). Sixty-eight preschoolers completed three tasks, which measured counting fluency, number word comparison skills, and their cardinal principle knowledge. Children who were capable of comparing the ones performed above chance on average in comparing complex numerals. The performance in comparing complex numerals was strongly associated with their counting fluency. Based on these empirical results, we discuss a linguistic account of number acquisition in early childhood, proposing a link between learning the syntax of numerals and understanding the meaning behind them.
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CHAPTER 1

INTRODUCTION

Children's experience with number starts from reciting number words in the count list (Fuson, 1988, Gelman and Gallistel, 1978, Wynn, 1990, Wynn, 1992, Mix, 1999, Mix et al., 1996). Children often younger than two years of age start to rote memorize the counting sequence from one to ten as a meaningless string, after which they learn to recite higher numbers while exploring the relational meaning between them (Fuson, 1988). By around the age of five, children associate the verbal numerals with the cardinal meaning of each, as they become the so-called Cardinal Principle knower (Wynn, 1990, Wynn, 1992; hereafter, CP knower), referring to the principle that the last number word counted indicates the cardinal value of the set. While the literature has well documented these important developmental milestones of knowing the meaning of small numbers, little is known about how children expand their knowledge of these relatively small numbers to larger numbers (e.g., beyond ten and twenty).

One main difference between small and large numbers is that small numbers are represented by simplex numerals that are made up of one lexical item (e.g., two, seven, ten) while larger numbers are represented by complex numerals that are compositional following the base-10 system in most languages. For instance, sixty-five is a combined numerical phrase between sixty and five. Thus, we begin with the premise that understanding larger numbers requires the understanding of the structure of complex numerals that represent them. This study investigates children’s knowledge about this structure of complex numerals. Do children understand the compositionality of complex numerals? The following sections provide a brief overview of the literature that build up our core questions.

THE GAP ON HOW CHILDREN PRODUCTIVELY LEARN HIGHER NUMBERS

The underlying mechanism of early number acquisition has been proposed by Carey’s bootstrapping theory (Carey, 2004, Carey, 2007). In the bootstrapping theory, until the age of three or
four, infants can hold each of the objects to distinguish the numbers in their memory. Children can discriminate the first three or four items at the preverbal stages (Starkey, Spelke, & Gelman, 1990, Wynn, 1992, Wynn, 1996). Children learn the meaning of ‘two’ from natural language quantifiers such as singularity, duality, triality, quantifiers for more numbers (e.g., one, two, three, some, many, etc) and from the parallel individuation of each items (Carey, 2004). In contrast to these preverbal stages, children use the successive relation of numbers to learn the meaning of the word ‘five’. The first few numbers below ‘five’ play a role as place holders. Through the bootstrapping process, ‘five’ means one apart from four which is one apart from three, and so on. This inductive reasoning of numbers (i.e., $S(n) = n + 1$, where $S$ is the successor function and $n$ is natural numbers) enables children to acquire the meaning of the number. Children understand the bootstrapping process that what the next number comes after the preceding number, under the premise where children understand the semantic induction based on the successor function.

This bootstrapping theory explains that children come to understand the semantic induction based on the few examples of consecutive natural numbers. Moreover, children come to realize this process after they integrate this semantic induction with the fact that the count list corresponds to meaning of each number (Carey, 2004). However, it’s hard to directly ask if children have this semantic induction in mind. Aside from the debates on whether or not the bootstrapping theory is rational, it is unlikely that this semantic induction operates over all numbers in children’s developing number concept. The bootstrapping theory can work over small numbers without knowing the structure of verbal numerals or the symbolic meaning of Arabic numerals. However, it is uncertain if the non-linguistic apparatus can work in the path of early number acquisition (see p190 in Hurford, 1987, also see Núñez, 2017 for more recent debates). For instance, numerous empirical studies now demonstrate that acquiring the cardinal principle does not indicate the acquisition of the inductive reasoning in higher numbers represented by complex numerals (Cheung, Rubenson, and Barner, 2017; Spaepen et al, 2018). Moreover, Guerrero and colleagues (2020) have recently demonstrated that having such an inductive reasoning for higher numbers does not indicate children’s knowledge about the structure of complex numerals.
The abstract idea of natural numbers is represented in the form of verbal numerals. Hurford (1975, 1987) conjoined the linguistic principles and the psychological bases of numerals during number acquisition. He believed that numbers are expressed in a unique numeral system, which have been used beyond the ordinary language to express higher numbers, zero, irrational numbers. Therefore, the structure of natural numbers was devised in a grammatically foreseen way, expressed by linguistic categories such as *Number, Phrase, and Multiplier* (Hurford, 1975; Hurford, 2007). Hurford proposed a set of phrase structural rules that denoted how numerals are structured, in which 1) *Digit* (i.e., numerals from one to nine) is projected to *Number*, 2) *Phrase* consists of either *M* (i.e., multiplier; e.g., ten, hundred, thousand) or a sequence *Number M*, and 3) *Number* consists of *Phrase* or another sequence *Phrase Number*. Complex numerals are constructed following this syntactical rule, henceforth referred to as numerical syntax in the rest of this paper (see the example in Figure 1).

Hurford raised a possibility that, with repetitive experience, children come to understand the arithmetic operations instantiated by the numerical syntax (Hurford, 1987). With multiplicative merge between *Number* and *Multiplier*, children learn how decade names (e.g. twenty is a composite of *Number* [two] and *Multiplier* [ten]) are constructed. With additive merge between *Phrase* (e.g., twenty which is the combination of *Number* and *Multiplier*: *[Number [Digit two] [M ty]]*) and *Number* (e.g. five), children learn how to segment or generate complex numerals in inter decades (e.g., twenty five is a composite of *Phrase* [twenty] and *Number* [five]). Although children may not acquire an explicit understanding of the proposed syntactic categories (i.e., *Number, Phrase, and Multiplier*), it is plausible to hypothesize that early experiences with spoken numerals build knowledge about arithmetic operations behind the grammatical structure of numerals. Such an idea, however, had not been empirically tested.

**Children’s acquisition of generative rules in abstract counting**

Although children’s understanding of numerical syntax, from the perspective of Hurford’s theory, has not been explicitly tested, developmental studies on abstract counting has provided
important insights into how children may acquire generative rules to produce complex numerals (Siegler and Robinson, 1982, Song and Ginsberg, 1988, Gould, 2017). In these studies, young children are simply asked to count from one until they make a mistake (or until they reach a specified number), and researchers examine the patterns of children’s errors to infer their knowledge about the rules underlying complex numerals. Siegler and Robinson (1982) suggested the three groups model that explains the developmental progress of children’s counting skills: 1-20 group, 20-99 group, and beyond 100 group. In their account, children memorize the local sequence one to nine and the rest of the words as lexical items. After 20, children generate numbers by picking up these specific items coming from their 1-20 group knowledge. In other words, they learn to re-use the counting sequence in the earlier range in larger numbers. In later numbers, the hurdles that stopped children were earlier in the regular numeral system (e.g., 99 as in the Korean number system, see Song and Ginsberg, 1988) than in the less transparent numeral system (e.g., 109 in English number system).

Gould (2017), in a more recent study, has proposed that preschoolers who initially develop item based number concepts gradually transition into more systematic ideas of the syntactic rules for producing numerals. Children tend to make errors or “stop” counting after counting towards the last number in a decade (e.g., 29, 39, 49, and 109). These stopping points indicate children’s difficulties in understanding generative rules of numbers (see also Song and Ginsberg, 1988). These error patterns signal that children may be rote memorizing some of the earlier decade transitions (counting from 29 to 30, from 39 to 40, and from 49 to 50) but that after repeated exposure to the syntactic rules for generating the decade names children learn to produce the decade names from those syntactic rules at higher decades. Also, the rest points (e.g., 10, 20, and 100) at which children tend to take a longer pause reflect that children realize the concatenative rule combining the local sequence from one to nine and multipliers.

**THE PRESENT STUDY**

This study investigates children’s knowledge about the syntactic structure of complex numerals. As mentioned in Sections 1.2 and 1.3, a linguistic theory on numerals provide a theoretical basis for
hypothesizing how children acquire the meaning of complex numerals, and empirical studies on abstract counting demonstrate children’s implicit understanding of the generative rules for counting sequences. However, it remains unclear when and whether children understand that complex numerals are syntactically organized.

We begin with the premise that a complete understanding of complex numerals comes with the comprehension of arithmetic operations implicated in the syntactic structure (e.g., Guerrero et al., 2020), which may be derived from repeated exposure to generative rules for counting sequences (e.g., Gould, 2017). We then hypothesize that an earlier developmental milestone is the somewhat simpler understanding that complex numerals are compositional. Thus, we questioned, do children understand that, for instance, the number [[two hundred] seven] is composed of two numbers, [two hundred] and [seven], and when, during the developmental time period, do children understand it?

In order to address this question, we devised a novel task named the number word comparison task. In this task, children compared two numbers represented in the form of [[Digit1 hundred] Digit2] where only Digit2 differed between the two numbers (e.g., Which is more: two hundred seven or two hundred three?). If young children, without the complete knowledge of high numbers, nevertheless understand that complex numerals are compositional, they should be able to compare those high numbers beyond their count list. With additional tasks assessing children’s counting fluency and cardinal principle knowledge, we questioned the conditions under which children understand the compositionality of complex numerals. Counting fluency is defined as children’s ability to successfully pass the decade boundaries (e.g., 39, 49, 109, etc) while counting the number sequence. Cardinal principle knowledge (hear after, CP knowledge) refers to the knowledge that the last word in the counting sequence means the total number of entities in a given set.
CHAPTER 2

METHODS

PARTICIPANTS
Sixty-eight monolingual Korean-speaking children (33 boys and 35 girls) aged 3 years 5 months to 6 years 5 months (mean age = 5 years) were initially recruited from three different local public preschools in Korea. Three children were excluded from the analysis for the following reasons: Two children (1 boy and 1 girl) who show an atypical psychological or linguistic development and one boy/girl failed to follow the instructions on one of the tasks.

PROCEDURES
Participants were tested interactively one-on-one with an experimenter in a private room. We tested all the children on the following three tasks: the Interval Counting task (iCount), the Number-Word Comparison task (NWC), and the Give-a-Number task (Give-N). All the children completed the iCount task first, but the order of the other two tasks were alternated to suit the children’s level of moment-to-moment engagement at the time of the study. The administration of one task did not depend on the administration of another.

INTERVAL COUNTING TASK
The aim of this task was to examine the extent to which children understand the regularities in the count list (Siegler & Robinson, 1982; Miller & Stigler, 1987; Song & Ginsberg, 1988; Gould, 2017). Many previous studies have achieved this goal by asking children to count from the number one to the highest number that they can possibly count. In various pilot tests following the same procedure, we realized that such a task makes many children extremely frustrated or bored. Thus, as in one of our previous studies (Guerrero et al., unpublished), we targeted children’s understanding of the transition of decades and hundreds in the count list by asking them to selectively count certain intervals that contain numbers that are known to elicit counting errors in young children (e.g., 10, 20, 29, 39, 49, 100, and 109; see Song & Ginsberg, 1988; Gould, 2017).
As our primary objective was to assess children’s understanding of the regularities in complex numerals, we focused on the regular numeral system in Korean. In the regular system, a systematic combination of a digit (1 to 9) and the multiplier ten allows one to express all the numbers between 1 and 99. For example, the number 16 is expressed as sip-yuk (literal translation would be ten-six) and the number 60 is expressed as yuk-sip (six-ten). Thus, the combinatorial rule for constructing a decade word is transparent in the regular numeral system. This is not true in the irregular numeral system, in which each decade word is close to a unique lexical item. Nevertheless, we also assessed children’s counting in the irregular system for the completeness of the study. We counterbalanced the order of the regular and the irregular system. In this task, the children were asked to rote-count the following intervals (i.e., without referring to objects) in that order: 1-11, 16-32, 37-51, 95-111, and 285-311. Children were asked to count all the interval sets regardless of whether or not they made any errors. The experimenter introduced the task by saying “We will play a counting game.” The first interval was then tested by asking, “Do you know the number, [one, two, three]? Now let’s start with [one ].. ?” where [one, two, three] were replaced with appropriate Korean numerals, [il, i, sam]. The experimenter rose the tone of voice to refer the question.

When the child counted all the numbers in a given interval without an error, he/she proceeded to the next interval. When the child made an error in a given interval, the highest number (N) that the child counted up to before the error was first recorded. In that case, we asked “What number comes after N?” without giving any hint about the next number after N. If the child did not know the next number, we stopped the counting activity for that interval and proceeded to the next interval. If the child knew the next number and resumed counting the numbers, we let him or her continue until the second error, if there was any. We prodded the child the same way at the second error. At the third error, if it existed, we stopped the interval and proceeded to the next interval by saying for example “Okay, then let’s now count from [ten-six] (which is a literal translation of the word sixteen in Korean).” Unlike in

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1 The inclusion of the multiplier for 100 in this combination allows the representation of numbers up to 9,999.
the case of the first interval (1-11) where the first three numbers were introduced by the experimenter, only the very first number was introduced in the rest of the intervals.

In order to define a measurable index of a child’s counting fluency and understanding of the regularity in the counting list, the number of successful decade transitions (e.g., counting from 39 to 40) was counted within each number interval and summed over all the intervals. The points of decade transition were 9, 19, 29, 39, 49, 99, 109, 289, 299, and 309. Therefore, the highest possible score was 10.

**NUMBER WORD COMPARISON (NWC) TASK**

The primary aim of this Number Word Comparison task (hereafter, NWC task) was to assess children’s knowledge about the combinatorial nature of complex numerals. Unlike many Indo-European languages, the expression of complex numerals is highly regular in the regular Korean numeral system (e.g., the number 65 is represented as six ten five), making it ideal for this task. In this task, children were presented with an iPad showing picture of two animals on each side of the screen without any verbal numerals. They were then told by the experimenter that each of the two animals has some number of objects or has earned a number score, and were asked to choose the animal that has more.

In the baseline *ones* condition, children compared two numbers between 1 to 9. In two other (i.e., *tens* and *hundreds*) conditions, complex numerals were introduced but the only difference between the two numbers was in the ones value. Single lexical items (i.e., Digit) in the baseline condition do not include any merge operation between phrases, whereas, each of the tens and the hundreds in the complex numeral conditions does include two levels of merge operations, which is based on Hurford’s phrase structural rule (i.e., [merge 2 [merge 1 Digit M] Digit]). In the *tens* condition, children were asked to compare two different numbers with the identical tens value (e.g., six ten five vs. six ten eight; see figure 2). In the *hundreds* condition, children were asked to compare two different numbers with the identical hundreds value and without the tens value (e.g., two hundred five vs. two hundred eight; see also figure 2).
If a child understands that a complex numeral is compositional (e.g., [[six ten] eight] and [[two hundred] eight]), the child will be able to parse the elements in the structure, which results in performing well in the tens and the hundreds conditions (assuming that he/she performs well in the baseline ones condition). On the other hand, if a child understands a complex numeral as a single entity, the child will not perform well in the complex numeral conditions (even when the child performs well in the baseline condition). We adjusted the alpha level (.05) based on Bonferroni correction method for controlling the family-wise type I error rates. The corrected alpha level was .0167 for comparing the average score of each condition (i.e., ones, tens, and hundreds) of NWC task performance with the 50% of chance level.

**GIVE-A-NUMBER (GIVE-N) TASK**

The Give-a-Number task (hereafter the Give-N task) was used to determine whether or not the children were cardinal principle (CP) knowers, that is whether they understood the cardinal meaning of a given number word and were capable of producing the cardinality by counting objects (Wynn, 1990). We adopted a simpler procedure following Davidson et al. (2012) because we did not intend to categorize children based on their subset knowledge. After presenting 12 plastic bears, the experimenter said to the child, “Now, we will play the bear game. When I ask for N bears, you will count N bears and give them to me.” A total of four trials were performed, each asking for 7, 8, 7, and 8 bears to a child. On each trial, the experimenter asked, “Can you give me N bears?” After the child’s response, the experimenter asked “Are those N bears? Are you sure? Can you count them?” If the child confirmed that the answer is correct, for example by saying “Yes, these are N bears,” regardless of its correctness, the experimenter moved to the next trial by saying “Thank you, then let’s try another number.” If the child found an error at the first try, the experimenter gave a child a chance to add or remove the bears, after which the experimenter asked for the confirmation again before moving on to the next trial. If a child failed at any of four trials, we marked the child as a non-CP knower.
CHAPTER 3

RESULTS

INTERVAL COUNTING TASK

Children’s understanding of the regularities in the count list was assessed using the interval counting task. Our primary focus was to assess whether children are able to produce the right decade names after each number that ends with 9. Given that English speaking children stopped counting at numbers ending with 9 rather than ending with 0 (see figure 2 in Siegler and Robinson, 1982), we defined the \textit{decade transition} (DT) score by the total number of correct decade transitions made by the child. The number of times that the child successfully counted 10, 20, 30, 40, 50, 100, 110, 290, 300, and 310 (which are within the intervals used in this study) was summed up, making the maximum DT score of 10.

Previous literature on children’s abstract counting have characterized the developmental pattern that children use abstract counting to construct the cardinal meaning of numbers, specifically for very small numbers or very large numbers which are hard to process conceptually (Fuson, 1992). Therefore, we divided the group into non-CP knowers and CP knowers based on their Cardinal Principle knowledge, since knowing the cardinal meaning of numbers is closely linked to the transition from abstract counting to counting on the objects (Fuson, 1992). Around 40\% of non-CP knowers who failed at counting from 1 to 9 could be simply due to non-CP’s lack of experiences in understanding the relation between abstract counting and cardinality given that their mean age has not yet been 5 (see p128 in Fuson, 1992).

Overall, CP knowers showed better counting skills than non-CP knowers (Fig. 3). Some decades are hurdles for children before they master the grammatical rules to generate numbers (Gould, 2017). The syntactic structure of the decade names is transparent (e.g., forty is spoken as four ten in
Korean regular numeral system). However, CP knowers who knew the decade names in the regular Korean numeral system still struggled at 109, which is the first decade transition with three digits. It is highly unlikely that CP knowers who counted up to 109 memorized the sequence all the way to the point. Therefore, CP knowers should have understood the fact that the decade names cycle repeat with the hundreds so that they can count numbers after 109 to much larger numbers. This suggests that effective counting of large numbers has to be supported by the knowledge that the familiar numbers are hierarchically embedded in the hundreds likewise in the decades in addition to CP knowledge.

Since Korean number system is transparent, the results that 3 CP knowers succeed at passing 289, 299, and 309 suggests that those children may have acquired the grammatical rule (Fig 3). The CP knowers made more successful DTs overall. Given that the age difference between non-CP knowers (Mage = 4;6) and CP knowers (Mage = 5;5) is around 1 year, children become confident in counting from the age of five and a half. Age would be one of the major factors to influence children’s understanding of counting at different ages (Fuson, 1983, Siegler and Rogen, 1982, Song & Ginsberg, 1988, Gould, 2017), which will be controlled later in our regression analysis, where the DT scores regress on the number word comparison scores.

**NUMBER WORD COMPARISON (NWC) TASK**

The NWC task was designed to assess children’s understanding of the compositionality of complex numerals. The baseline measure of this task involved comparing two single-digits (the ones condition), and the subsequent tens and hundreds conditions, which involved comparing two complex numerals where only the digit in the ones position differed between them. One sample t-test showed that children performed above chance in all three conditions (Meanones: 75.09, t(64) = 8.20, p = .000; Meantens: 61.26, t(64) = 3.63, p = .000; Meanhundreds: 63.52, t(64) = 4.59, p = .000, see Fig. 4). Given that the decade transition score ranged from 0 to 10, each for 9 to 309, the number word pairs in the complex numerals conditions (i.e., the tens and the hundreds) were certainly beyond most of the children’s counting range. In fact, all but three children were not able to count beyond 289. These results indicate that children were not likely to understand the meaning of those high numbers used in the tens and the
hundreds conditions. It is worth noting that the overall performance of the tens and the hundreds conditions across children may have been underestimated because we do not expect children who unreliably solve the ones condition to solve the complex numeral conditions.

Thus, we re-analyzed the data treating the ones condition as a baseline. Specifically, we first categorized children into those who got 5 or 6 correct out of 6 total trials in the ones condition ($p = .109$ in a binomial test) versus those who did not. According to this criterion, 28 children passed the ones condition ($M_{age} = 5.45; SD = 0.71$) and 37 children failed the ones condition ($M_{age} = 4.46; SD = 0.57$). On average, those who failed the ones condition performed near chance in the baseline ones condition, $t(27) = .22, p = .826$, and did not perform above chance in the tens condition, $t(27) = -1.29, p = .207$, or in the hundreds condition, $t(27) = -1.06, p = .299$), which is expected, given their lack of knowledge for comparing two numbers in the ones condition.

Those who passed the ones condition performed above chance in both the complex numeral conditions, $t(36) = 5.80, p < .001$ in tens $t(36) = 8.70, p < .001$ in hundreds (Figure 5). However, their performances in complex numerals were significantly lower than their baseline performance in the ones condition in a post hoc pairwise t-tests, between baseline vs tens, $t(36) = 5.64, p < .001$, and between baseline vs hundreds, $t(36) = 5.84, p < .001$. Their performance in the two complex numeral conditions did not differ significantly, $p = 1.0$. These results indicate that children, even those who can reliably compare single digits, on average have difficulties understanding that complex numerals are compositional. Given that most of our preschoolers have not yet mastered abstract counting, which makes them practice the grammatical understanding of numerals, these results are expected according to our main hypothesis that understanding the compositional structure of complex numerals is a key to comprehend large numbers.

We questioned what determines children’s understanding of the compositionality of complex numerals. One promising candidate is children’s understanding of the grammatical regularity of counting sequence (Gould, 2017, Siegler and Robinson, 1982; Fuson, 1982). We tested this hypothesis by assessing children’s counting skills using the Interval Counting task. Children’s, particularly the
one-passers’, performance in the tens and the hundreds conditions in the NWC task was, in two separate models, regressed on their counting skill quantified by the DT score (Figure 6). DT score significantly predicted the performance both in the tens condition \((b = 3.088, p = .0270)\) and in the hundreds condition \((b = 3.213, p = .008)\), while controlling for age. The variable age was a significant predictor for predicting the NWC performances in the tens \((b = 11.379, p = 0.039)\), but not in the hundreds \((b = -1.283, p = 0.779)\). One-passers \((N = 37)\) were composed of 4 non-CP knowers and 33 CP knowers. All of them counted a first decade transition cycle (i.e., one to eleven) except for 4 non-CP knowers who verbally counted up to 9. One CP knower only verbally counted up to 3 and failed at counting to later ranges as well, indicating that this CP knower might lack of the attention at the moment. Therefore, we did not exclude this one-pass CP knower to see how the different levels of counting skills can predict the performances in our model and this one-passer is expected to understand the cardinal meaning of the numbers in the range of the target Digit.

When would children become confident in comparing the numbers? First, one-passers \((M_{age} = 5;5)\) are around 1 year older than non- one-passers \((M_{age} = 4;5)\). Second, ten-passers \((M_{age} = 5;8)\) are also around 1 year older than non- ten-passers \((M_{age} = 4;9)\). Third, hundred-passers \((M_{age} = 5;5)\) are around a half- year older than non- ten-passers \((M_{age} = 5)\). Overall, children start to reliably compare the numbers at least by age 5 and a half. We ran post hoc chi-square tests to further expand our question and to see which decade boundaries are needed for CP knowers to get a reliable NWC performance. Since children’s counting fluency is a key factor to predict children’s comprehension in complex numerals, their performances at decade transition should be informative for confirming the stage where children pass the complex numeral conditions. We ran a chi-square test between whether children pass each DT and whether children pass in each complex numeral condition (i.e., equal to or more 83% correct in either the tens or the hundreds conditions) to address this question. We only consider the CP knowers and then categorize them respectably into the tens- or hundreds- passers in this analysis. To correct the family-wise error rate, we corrected the p-value with Bonferroni method.
The chi-square value was highest in the association between 50 passers and tens passers ($\chi^2 = 14.18, p < 0.005$), meaning that successful counting from 49 to 50 is the most critical point for succeeding the tens condition in the NWC task (Figure 7). The association between passing the tens and passing the decade transition at 20, 30, 40 and 100 showed a relatively lower chi-square and a significant association ($\chi^2 = 8.42, 8.42, 9.59$ and 9.9, respectively; $p < 0.05$). In contrast to this, there was not a significant association between passing each decade and passing the hundreds condition. If when we used a less robust familywise error rate correction with alpha level of .05, we still find 40 and 100 are similar at the critical point where DT is associated with children passing the hundreds condition. These findings so far suggest three novel implications: 1) Children overall understand that complex numerals beyond their counting range are compositional, 2) This knowledge about compositionality is predicted by their counting skills which reflect their knowledge about the syntactic regularities in number words and 3) Children might come to know the grammatical structure of complex numerals, especially the tens, when they begin to count beyond 49.

**Give-a-Number (Give-N) task**

We measured Cardinal Principle knowledge as another factor that influences children’s NWC performance in complex numerals. CP knowers who better understand the cardinal meaning of numerals than non-CP knowers are likely to catch the relevant Digit pairs from the whole phrase. This between-subject variable (i.e., the children’s CP knowledge) served as a factor in explaining some of the effects in subsequent analyses. Out of a total of 65 children, 37 were CP knowers and 28 were non-CP knowers. CP knowers ranged between 3.2 and 6.25 years in age with the median of 5.25 years. Non-CP knowers ranged between 3.58 and 6.24 years with the median of 4.63 years. Only 4 of these 28 non-CP knowers were one-passers who got 5 out of 6 trials and knew the local sequence from one to nine. We will manifest these non-CP knowers’ performances in the later section.

We examined whether the performances in the baseline and the complex numeral conditions depend on children’s Cardinal Principle knowledge. A two-way ANOVA with CP knowledge (non-CP knower vs. CP knower) x Condition (ones vs. tens vs. hundreds) revealed a significant main effect of
Condition as expected, $F(2, 189) = 9.202, p < .001$, and a significant main effect of CP knowledge, $F(1, 189) = 105.217, p < .001$. The interaction was not significant, $F(2,189) = .749, p = .474$. Non-CP knowers, on average, did not perform above chance in all the conditions (Mean$_{ones} = 56, t(27) = 1.686, p = .103$, Mean$_{tens} = 47.04, t(27) = - .919, p = .366, Mean_{hundreds} = 46.89, t(27) = -.853, p = .400$), whereas, CP knowers did well above chance in all the conditions (Mean$_{ones} = 89.54, p = .000$; Mean$_{tens} = 72.03, p = .000$; Mean$_{hundreds} = 76.11, p = .000$). Post hoc pairwise t-tests revealed that CP children scored significantly higher in the baseline ones than in the tens and in the hundreds (both $ps < .05$). No significant difference was found in the performances between the tens and the hundreds ($p = 1.000$).

CP children showed that children’s performance was robustly worse with more merge operations without the difference between the tens and the hundreds. This led us to develop the future study to see an effect of the different type of level of merge operations in complex numerals.

Non-CP children showed no significant difference in their scores across conditions (ones vs tens: $p = .22$, ones vs hundreds: $p = .20$, tens vs hundreds: $p = 1.000$). Non-CP knowers did not even get the trials in the baseline and the complex numeral conditions better than chance. Their scores in the baseline ones were not correlated with the Interval Counting task, $r = .29, p = .142$. Since it is very likely that non-CP knowers chose a bigger numeral by guessing, it could be either by lack of knowing the cardinal meaning of numerals or lack of the mapping ability, which associates the corresponding numerical value with the verbal numeral.

Notably, among all the children who counted up to 9, there were 36 CP knowers and 17 non-CP knowers (Fig. 8). These 17 non-CP knowers’ knowledge of local sequence can be a potential factor that distinguishes them from the rest of the non-CP knowers or from CP knowers, since the local sequence is in the same range with the baseline condition. First, if non-CP local sequence knowers’ performance in the baseline is over the chance, the local sequence knowledge is at least necessary for comparing the ones. The non-CP knowers who counted up to 9 performed above chance in the ones condition, (M: 60.82%) $p = .028$, although the mean score in these non-CP local sequence knowers was not different from the mean score in the rest of the non-CP knowers who failed to count up to 9, (M:
Furthermore, by looking into the non-CP local sequence knowers’ performance in the baseline ones, there were only four children who passed the ones with 83% of success. Therefore, the local sequence knowledge did not distinguish the developmental stage among the non-CP knowers in terms of their baseline comparison performances.

Second, we checked if CP knowledge still makes a difference in the baseline condition when both groups know the local sequence from 1 to 9. Non-CP local sequence knowers’ performances were significantly lower compared to the CPs’ performances (M: 60.82 vs M: 89.72, p<.001). It was evident that CP knowledge makes the major difference. Non-CP local sequence knowers’ performances were not even above chance in the complex numeral conditions (M_{tens}: 46.06; M_{hundreds}: 44), therefore, there was not any effect of local sequence (i.e., 1 to 9) knowledge in complex numeral conditions. We further examined non-CP local sequence knowers’ performance between the baseline and the complex numerals to see a similarly developmental pattern like CP knowers. These non-CPs’ performance in the baseline was not significantly better than the complex numeral conditions (p_{tens} = .386; p_{hundreds} = .221) unlike the CPs. The major difference between these non-CP local sequence knowers and CP knowers was whether or not they acquire the CP knowledge. To sum up, we can be sure that the effect of CP knowledge experimentally worked well.
CHAPTER 4

GENERAL DISCUSSION

THE AIM OF THE STUDY

A core, but scarcely studied, question is how children understand higher verbal numerals on the basis of their knowledge of small numbers (e.g., abstract counting, CP knowledge). Children’s understanding of natural numbers gradually develops as a function of age while learning verbal numerals in abstract counting, ordinal relations between numbers, cardinality principle knowledge and semantic induction, which have been studied mostly in the domain of simplex numerals from one to nine (Carey, 2004; Gelman & Gallistel, 1978; Fuson, 1992; Fuson & Hall, 1983; Siegler & Robinson, 1982; Wynn, 1992). Children must extend aforementioned knowledge of numbers to higher numbers so that they acquire higher numbers. Children should gain a minimal grammatical understanding, since young children are likely to extract the syntactic principles of verbal numerals from their incidental experiences in the written and spoken numerals. Though children would not be explicitly taught about the linguistic labels, they come to understand the grammatical role of each lexical item. A few pieces of evidence have been examined. Young children understand the first few numerals one to three based on the singular and plural distinction (i.e., understanding the number word one to three before the mastery of semantic induction, see Carey, 2004, and also Sarnecka et al., 2007). Children’s understanding of grammaticality in numerals has developed long before they master the count list. Children better recalled the higher numerals when the structure of those numerals were legal rather than illegal (Barrouillet, Thevenot, and Fayol, 2010).

Compositionality of numerals is a foundational knowledge that leads a child to acquire the grammatical understanding of numbers. In the current study, we addressed this theory by following the logic of the previous literature. Children’s grammatical knowledge from incidental exposure to numerals would facilitate the process of acquiring the compositionality of larger numbers. Children
should understand that numbers are not a single string, as they come to know the relations between earlier numbers. The number sequence is no more a whole thing to rote-memorize, rather, a detachable chain of each numeral (Fuson, 1982). Children recite much larger numerals without rehearsal after acquiring the number sequence one through one hundred. Children would see each numeral as a lexical item at first glance, however, they come to know sixty-five is merged between sixty and five. This relationship between numerals is formally labeled with grammatical categories, as *Number* is a merged lexical item between *Phrase* (e.g., [[Number [Digit six]] [M -ty]]) and *Number* (e.g., [Number [Digit five]]) (Hurford, 1975). Only after children gain this knowledge that larger numerals are compositional, they should comprehend the meaning of large numbers based on syntactic structure of numerals. In producing numbers, children generate large numerals based on the grammatical rule, not on the memorized count sequence. This grammatical understanding becomes necessary to generate larger numbers, and then a key to understand the number system. Therefore, we addressed the hypothesis that children might understand the compositionality of numerals, which makes the comprehension of large numbers based on their knowledge of earlier numbers. We examined whether young preschoolers show their understanding of the compositionality of numerals by experimenting children to solve the verbal numeral comparison tasks. This theoretically based hypothesis broadens the base of this study by considering the grammatical ideas of comprehension with complex numerals.

**THE SUMMARY OF THE FINDINGS**

We questioned the extent to which children have grammatical ideas—albeit implicit—of how *Digit* and *Multiplier* form large numbers (i.e., compositionality of numbers). Specifically, we hypothesized that children with limited knowledge in large numbers should nevertheless understand that the complex numerals representing those large numbers are compositional. In our novel task, the Number Word Comparison task, children who were able to compare verbal numerals in the baseline ones condition were, on average, able to compare numbers expressed on complex numerals as expected. This suggests that children understand that large numbers beyond their count list are not novel lexical items and are rather compositional.
Also, we examined children’s counting fluency which tells us how many times children successfully passed the decade transition boundaries (e.g., 39) in the interval Counting task. Aligned with the previous literature, only less than 30% of the children were able to pass 39 and 49. 109 was still an obvious hurdle to clear. Noticeably, 3 children who were able to pass 109 counted successfully up to the last number 311. Counting numbers from different starting points (i.e., our Interval Counting task) requires a broader understanding of the grammatical rule to compose numerals. One can debate the strategies that children used, either rote-memorizing the sequence or understanding the grammatical rule. Since the preschoolers in our study have never learned to count beyond 20 in their curriculum from the preschool, it is highly unlikely that our participants rote-memorized numbers after 109. Such a progress through the decade boundaries should depend on children’s likely implicit understanding of the grammatical regularity in the structure of the complex numerals (Gould, 2017, Cheung et al., 2017). Then, we wanted to see whether children need to understand the regularities in the count sequence to perform well in the complex numeral conditions which are far beyond their count list. We filtered out children who performed with less than 83% of success and then ran the regression analysis only with the children who passed the baseline ones. Children’s decade transition score predicted their performance in those complex numeral conditions. This indicates that children with better counting fluency performed well in comparing tens and hundreds words since their understanding of the regularity in the count list helps to understand the compositionality of complex numerals. The most cumbersome was the decade boundary 49, that is associated with whether children pass the tens condition in the NWC task. If children know counting from 49 to 50, they are likely to perform well at comparing the tens. Therefore, these findings provide evidence that understanding syntactic regularities in the count list supports children in understanding the grammatical structure of much larger numbers.

Finally, the result examining children based on their CP knowledge confirmed our prediction that CP knowledge might be a strong indicator of children’s comprehension skill with complex numerals. Cardinal principle knowledge has been identified as a key factor of early number concepts (Carey, 2009) and as a predicting factor for preschooler’s later addition arithmetic competency and, likewise, other symbolic number knowledge (Chen and Li, 2014, Fazio et al., 2014, Geary et al., 2018,
Schneider et al., 2017). Non-CP knowers performed poorly in the baseline ones and merely solved the questions in the complex numeral conditions. As opposed to non-CP knowers, CP knowers performed better than chance in the complex numeral conditions as well as the baseline ones condition. This result is contextually consistent with Lipton and Spelke (2005)’s finding that CP knowers at the age of 5 already acquire the logic of the association between large numerals and exact cardinal values. When presented with the numerosity cards (e.g., the card printed with 120 rectangles) or the real objects (e.g., bears, balls, etc), children applied large numerals to the specific correct value outside of their counting range. Since the researchers asked “Is this N objects?”, they might not need to accurately estimate the exact number of cardinal values without counting. However, this case is same in our NWC task, in that we provide two spoken numerals. Our CP knowers should have used this logic - each numeral represents unique cardinal values when comparing two large numerals that they are not familiar with. Overall, we believed that substantiating these ideas with empirical findings upholds the importance of grammatical understanding in verbal numerals.

**DEVELOPMENTAL ASPECT OF THE STUDY**

We started this developmental question in the compositionality of numerals with the grammatical point of view. Previous literatures have built up similar evidence that children have minimal understanding of the grammatical structure of numerals (Fuson, 1988, Hurford, 1987, Miller and Stigler, 1987, Siegler and Robinson, 1982). Children’s grammatical understanding to produce the correct number sequence has been observed using the local sequence one to nine and the decade names below 100. And then, children should understand the grammatical rule that combines *Digit Multiplier* and *Digit* to construct the structure of tens words and the decades structure repeat over 100. Recent studies about the grammatical understanding of numbers focused mainly on children’s abstract counting (Gould, 2017) or the place value system in Arabic numerals (Mix et al, 2014) rather than the comprehension of verbal numerals which is early foundation for later numerical thinking.

Number word comprehension is the foundational process for understanding the numerical meaning of ones and grasping the structure of numerals- the conjoined form of each *Digit* with
Multiplier. This line of work revealing children’s comprehension of spoken larger numerals is developmentally meaningful in that there was no work on whether children can compare large numbers regardless of their familiarity to the numbers. Children’s use of their grammatical knowledge to compare the large numbers implies the involvement of three skills: 1) knowing the cardinal meaning of the lexical items one to nine, 2) understanding the syntactical merge between Digit and Multiplier, and 3) parsing the targeted Digit (e.g., two) from Number (e.g., seventy two) composed of Phrase and Digit. In our data, children around 5 years old compared large numerals in the spoken form, where each numeral contains the syntactic structure with the tens and with the hundreds. Passing the decade boundary at 50 was the verge point associated with whether children pass the tens condition. Thus, the overall pattern for number word acquisition of complex numerals and their grammatical understanding of it emerges in the early preschool age before they fully acquire the productive skill to generate those numbers. By 6 years of age, different children understood more complex structure with two multipliers (e.g., two hundred thousand in our unpublished data). The grammatical understanding of numbers gradually extends to increasingly complicated structures of large numbers.

This current finding shed light on the relative grammatical difficulty of the comprehension and the production of larger numerals for young children. Children with CP knowledge comprehended much larger numerals ranging between 300 and 1000 in the Number Word Comparison task, whereas only 3 of them produced numbers up to 311 successfully in the interval Counting task. This apparent difficulty of producing numbers should not be neglected for studying the origin of children’s understanding of large numbers. The comprehension of much larger numbers could have been more difficult than children’s experience in learning to combine digit and multiplier in decade, since our NWC task includes understanding the structure of unfamiliar complex numerals and comparing the value of each. Even CP knowers were not able to easily conjoin the decade names into hundreds level when verbally counting the sequence. This result provides the comparative evidence for the early grammatical comprehension of the numbers that is predicted by their counting fluency (i.e., the production) but is not restricted to it.
CARDINAL PRINCIPLE KNOWLEDGE AND VERBAL NUMERAL COMPREHENSION

CP knowers excelled in the complex numeral conditions of the NCW task, whereas, non-CP knowers performed poorly in both the baseline and the complex numeral conditions. CP knower might have used either the numerical meaning of the spoken numerals or the later-greater principle which gives a hint that later number in the count list means larger magnitude (Le Corre and Carey, 2007). In fact, a previous study showed that only CP knowers who were able to map magnitudes onto numerals used the later greater rule (Le Corre and Carey, 2007, Sarnecka and Carey, 2008). In alignment with this previous research, our non-CP knowers who might not have yet acquired the mapping between the numerals and the magnitudes couldn’t use this knowledge even in the baseline, where they had already known the local sequence 1 to 9. The fact that non-CP knowers cannot use this analogy regardless of their understanding of local sequence is supported by Fuson’s work (1983). As Fuson’s model suggests, knowing the local sequence is a cornerstone when comparing the meaning of numerals based on their understanding of the either ordinal or cardinal relation between numerals. However, thinking of numbers as separate items rather than a single list is necessary for comparing numbers likewise mastering the CP knowledge and mapping between numerals and magnitudes (Fuson, 1983). Non-CP knowers with the successful local sequence can still lack this number item idea. The non-CP knowers’ complex numeral condition performances confirmed that children’s early number concept is influenced by their abstract counting skill, and that CP knowledge is necessary for understanding complex numerals.

Non-CP knowers mostly did not even compare the ones which are obvious in that non-CP knowers are lack of the cardinal principle knowledge necessary to compare the numerical value (Carey, 2009) and of the later greater principle to use analogy that later numeral in the counting sequence means the greater value than prior ones (Le Corre and Carey, 2007). Non-CP knowers were less likely to realize either the cardinal meaning of each numeral or understanding the association between the number word and the corresponding magnitude. We noted that non-CP knowers counting one to nine showed that they knew how to compare the ones at least better than chance as opposed to non-CP
knowers who were not able to count one to nine. Non-CP knowers with successful local sequence, however, were not at the developmentally different stage compared with the rest of the non-CP knowers and the CPs in number comparison. Knowing the local sequence facilitates non-CP’s understanding on numerical value comparison, which may not be sufficient to know comparing all the ones. These findings shed light on the linguistic apparatus in children’s number word learning, which is rooted in the previous models such as Siegler and Robinson (1982)’s three groups account based on children’s counting skills (0-20, 20-99, after 100) and Fuson (1992)’s developmental stages in the context of sequence, ordinal and cardinal relations. Aligned with this literature, children gain some-albeit limited comprehension for comparing the ones without the CP knowledge, which must depend on the production of the number sequence.

LIMITATIONS

The results were clear that children have foundational grammatical understanding of the structure of complex numerals. However, there are a few limitations to further examine in our study. Socio-Economic Status (hereafter, SES) might be one of the strongest factors that can be associated with children’s numerical thinking. Children’s cognitive development, which are rooted in the environmental input from SES, can be accounted for their different performances in the number talk, vocabulary size, academic achievement or IQ score. The relationship between SES and cognitive development from infancy to childhood has been heavily reported (Bradley and Corwyn, 2002). Low SES has an impact on parent-child verbal communication and cognitive input in the parent-child dyads (Hackman, Farah, and Meaney, 2010). In the home environment, children acquire preverbal and verbal competencies of numbers before entering the elementary school. During the critical periods of learning numbers, low SES diminishes learning opportunities and social experiences which can bring all the primary exposures for children to gain early number competencies (Jordan and Levine, 2009). In parent-child interactions, middle-class mothers engaged children with higher level of number activities than low-class mothers did (Saxe et al., 1987). Four different levels of number activities were: 1) doing number play or saying numbers without any numerical operations (e.g., pushing the elevator buttons),
Middle-class children, rather than the low-class, participated in number activities in a wider range of complexity levels. This reciprocal goal seeking interaction in number play between mother and child influences on children’s mathematical achievements. In nursery school, where there are teacher-child interactions, the pattern was similar in that the level of SES expels children’s mathematical knowledge following their familial background even over time (Kilbanoff et al., 2006). Children from high SES families showed better performance than middle and low SES families, which, again, demonstrated the better performance in children from middle SES families over low SES families. Their mathematical knowledge difference stemming from SES didn’t change when measuring their knowledge again with 7 months gap from the beginning of the semester to the end of the semester.

In alignment with this measure of SES, preschoolers’ vocabulary knowledge can be a strong predictor of our preschoolers’ ability understanding the structure of complex numerals. The production and the comprehension of vocabulary is strongly associated with learning early number word knowledge (LeFevre et al., 2010, Negen and Sarnecka, 2012). However, the results are mixed, given that vocabulary knowledge measured with Peabody Picture Vocabulary Test (PPVT) was not a significant predictor on mathematical skills in preschoolers aged from 1 to 3 years (Verdine, Irwin, Golinkoff, & Hirsh-Pasek, 2014). Therefore, the influence of children’s linguistic ability on comprehending numerals can be varied by their age during early childhood.

For suggesting the further examination, we do not know the extent to which our CP knowers, who were measured by the Give-N task, are different from one passers who got correct 5 or 6 out of 6 trials in the baseline ones (which was measured by our novel task, the NWC task). CP knowers and one passers did not show the salient difference in performing the complex numeral conditions of the NWC task. Few CP knowers and one passers showed the stable performances in the complex numeral conditions, meaning that most of their performances in the complex numeral conditions were disrupted by the complex syntax of large numerals. However, there can be meaningful influence on the
comprehension of complex numerals, which caused by the different level of CP knowledge. There can be some ways to study this group difference between CP knowers and one passers.

First, we only distinguished the group in a dichotomous way between non-CP knowers and CP knowers, which does not lead us to examine the specific level of our CP knowers who were tested with a simpler version of the Give-N task. If the level of CP knowledge is relevant to comprehending the structure of complex numerals, CP knowers who are older at higher level of CP knowledge could have performed more reliably. The classic version of Give-N task measures which level of CP knowledge is acquired in children (Sarnecka and Carey, 2008, Wynn, 1992b). The original version of the Give-N task classifies children’s CP knowledge levels between being a subset knower and being a CP knower. Whereas, subset knowers come to understand the meaning of each numeral in a stepwise way (e.g., learning each numeral serially), CP knowers suddenly acquire the meaning of each numerals from 5 to the highest number that they can count. We could have tested the children’s CP level more in depth following the original version of the Give-N task to see if a specific level of being a subset knower (i.e., distinguishing numerals 1 to 4) or a CP knower (i.e., distinguishing numerals from 5 to their highest count) is associated with their performances in the complex numeral conditions of the NCW task. If we had sampled older children with CP knowledge, we also could have tested the individual differences of CP knowers’ performance of comprehending the complex numeral which can be rooted in their CP level. Dissecting the difference depending on CP knowers’ knowledge levels can give us more distinct information to study the developmental difference between CP knowers and one passers.

Second, if our CP knowers have had the later-greater principle to use the analogy that later numerals in the sequence means the greater magnitude, they would have excelled in the complex numeral conditions easily over one passers. Prior literature showed that only CP knowers who were able to map magnitudes onto numerals understand the later-greater rule when they gauge the numerosity of the dot cards. This developmental stage of understanding the later greater rule comes late after they come to understand the association between each numerals and magnitude (Le Corre and Carey, 2007, Sarnecka and Carey, 2008). As our mean age of the sample was 5 years old, most of our CP knowers
might have become a CP knower recently before participating in our study. It is highly likely that our CP knowers didn’t acquire the later-greater principle rule yet. The older CP knowers who are more proficient in counting on the objects would have showed the developmentally different performances as opposed to one passers.

Lastly, one might argue the sole effect of children’s memory capacities over children’s understanding of numerical syntax when comparing two verbal numerals in the NWC task. Children’s performances can be disrupted in the complex numeral conditions by memory interference, which can stem from children’s memory span or lack of attention. How can we be sure that children’s understanding of the structure in verbal numerals, rather than memory interference, was the reason for worse performance with complex numerals? First, as we tested children’s working memory by asking what verbal numerals children heard, children had to hold both of the exact numerical phrase (e.g., six-ten five and six-ten eight) in their memory before deciding which number is greater than the other. Further, we can directly measure children’s digit span to rule out the possibility that children who are capable of comparing complex numerals compare numerals with their memory capacities while holding two verbal numerals in mind. In our follow up studies, when the number of the words in each phrases is the same between the complex numeral phrase (e.g., two hundred five chairs vs. seven hundred five chairs) and the adjectival phrase (e.g., two tiny yellow chairs vs. seven tiny yellow chairs), which has same syntactic structure with the numerical phrase condition, children’s performance when comparing numbers was only disrupted with the complex numeral condition. When we compared children’s performances in the baseline ones (e.g., two chairs vs, seven chairs) and the complex numeral condition (e.g., two hundred five chairs vs. seven hundred five chairs), there was not a difference in performance. It was evident that children’s memory was not interfered by the number of the words, rather by the structure of numerical phrase.

CONCLUSION
This study is the first step to examine the importance of grammatical knowledge in children’s developing number comprehension. In contrast to attention towards how children learn smaller numbers, there has been the lack of studies which examined the mechanism involved in how children expand their knowledge of small numbers to understand much large numbers. Our study suggests that the empirical evidence of studying children’s understanding of complex numerals should lead us to how they eventually acquire the natural number system. By the age of 6, children can count numbers from one to one hundred without formal education. What would be the core knowledge that children gain when they are counting the number sequence? Becoming a proficient counter provides children with the possibility that they are more likely to comprehend larger numerals even beyond their count list. Importantly, we found that children should acquire the grammatical understanding of numbers that higher numbers (e.g., sixty five) are constructed with earlier numbers (e.g., six and five) and multipliers (-ty) that they rote-memorized in order to compare large numerals in our Number Word Comparison task. Furthermore, passing the decade boundary at 50 was strongly associated with whether children passed the tens condition in the Number Word Comparison task. Our results are consistent with previous findings that children with better counting fluency acquire other numerical concepts well (Carey, 2009, Cheung et al., 2017, Sarnecka & Carey, 2008, Gelman & Gallistel, 1978). Our findings indicate that producing large numerals in the count sequence might share the linguistic mechanism of comprehending large numerals based on numerical syntax. To date, our study validates the importance of children’s understanding of the grammatical compositionality when understanding large numerals.
## APPENDIX A

### TABLES

Table 1. Number comparison task stimuli

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<td></td>
<td>three hundred</td>
<td>seven</td>
<td>three hundred</td>
</tr>
<tr>
<td></td>
<td>seven hundred</td>
<td>six</td>
<td>seven hundred</td>
</tr>
</tbody>
</table>
Table 2. Demographic information was presented by children’s cardinal principle knowledge.

<table>
<thead>
<tr>
<th>Group</th>
<th>Age</th>
<th>N</th>
</tr>
</thead>
<tbody>
<tr>
<td>non-CP knower</td>
<td>3;5 – 6;3</td>
<td>28</td>
</tr>
<tr>
<td>CP knower</td>
<td>3;7 – 6;5</td>
<td>37</td>
</tr>
<tr>
<td>Total</td>
<td>3;5 – 6;5</td>
<td>65</td>
</tr>
</tbody>
</table>

Table 3. Demographics of children who failed (getting 4 or less correct; non-one-passer) and who passed (getting 5 or 6 correct; one passer) the ones condition were presented.

<table>
<thead>
<tr>
<th>Age*</th>
</tr>
</thead>
</table>

<table>
<thead>
<tr>
<th>Group</th>
<th>N</th>
<th>Mean (SD)</th>
<th>Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>Children who failed the baseline ones condition</td>
<td>28</td>
<td>4;6 (.6)</td>
<td>3;4-5;5</td>
</tr>
<tr>
<td>Children who passed the baseline ones condition</td>
<td>37</td>
<td>5;5 (.7)</td>
<td>3;6-6;4</td>
</tr>
<tr>
<td>total</td>
<td>65</td>
<td>5;0 (.8)</td>
<td>3;4-6;4</td>
</tr>
</tbody>
</table>

*Ages are in years and months (years; months)
Appendix B

Figures

Figure 1. The example of the tree structure is represented to illustrate on how complex numerals are constructed.

Figure 2. The examples of the syntactic structure of verbal numeral stimuli were presented with their grammatical categories labeled: (a) Baseline condition where only one verbal numeral was spoken, (b) complex numeral condition with tens where Multiplier -ty was applied to the first Digit and then the target Digit was combined, and (c) complex numeral condition with hundreds where Multiplier hundred was applied to the first Digit and the target Digit was combined. The stimuli were spoken to children without any visualization.
Figure 3. The bar represents that the number of children who correctly passed at each of the decades in the interval counting task. The bars at “Non” represent the number of children who did not even succeed to pass the first decade, ten.

Figure 4. The bar represents that the percentage of the number of children who correctly passed at each of the decades in the interval counting task. The bars at “Non” represents the number of children who didn’t even succeed to pass the first decade, ten. The percentage was calculated by dividing the number of children who passed the corresponding decade transitions with the total number of each group, non-CP knowers, CP knowers, or total group accordingly.
Figure 5. Children’s performances in the Number Word Comparison task were grouped by their Cardinal Principle knowledge.

Figure 6. The children’s performances in the Number Word Comparison task were grouped by whether they passed the baseline one condition.
Figure 7. Regression analysis with One-passer’s performances with the tens in the Number Word Comparison task regressing on the Decade Transition score.

Figure 8. Chi-square values were presented for each association between successful decade transition and successfully passing the tens condition in the Number Word Comparison task.
Figure 9. Distribution of the number of children and their highest number counted in the local sequence one to nine in the interval Counting task. Although the median of CPs and non-CP knowers were same as 9, 11 out of 28 non-CP children did not count numbers up to nine perfectly.
REFERENCES


