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Physical Model of the Feeding Strike of the Mantis Shrimp

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PHYSICAL MODEL OF THE FEEDING STRIKE OF THE MANTIS SHRIMP

A Thesis Presented

by

SUZANNE M. COX

Submitted to the Graduate School of the
University of Massachusetts Amherst in partial fulfillment
of the requirements for the degree of

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Mechanical and Industrial Engineering
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ABSTRACT

PHYSICAL MODEL OF THE FEEDING STRIKE OF THE MANTIS SHRIMP

SEPTEMBER 2012

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A physical model was built to study the properties of the feeding strike of the mantis shrimp that are responsible for drag reduction and cavitation control. The model had three goals: 1) The model was to be outfitted with a method to collect kinematic, force and cavitation data. 2) The velocity and acceleration profile of the model were to be predicted with a mathematical model of the mechanism. 3) The model was to match as many drag and cavitation sensitive properties of the mantis shrimp strike as feasible and have a means to control the rest. The first iteration of the model met the first goal but not the second or third. It matched the strike in maximum velocity, appendage size and shape and environmental temperature and salinity but did not control acceleration profile, water quality or pressure. Data collected with high-speed video of strikes of the model and Gonodactylus smithii showed the model to cavitate at speeds at which no cavitation was seen in animal strikes. The model was redesigned to be driven by the stored elastic energy in the deflection of a beam spring. The redesigned model reached the animals maximum accelerations but not velocities. Environmental variation was found to not substantially contribute to the variation in cavitation onset velocity between the model and animal experiments.
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44. Predicted and experimental tangential velocity and acceleration profiles for a beam width at two different beam heights. Dashed line are predictions made without mathematical model corrections. Gray lines are the experimental data. The dotted white line is the corrected predicted profile. Black area shows the area of predicted uncertainty.

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46: Kinematic measurements from digitized high-speed video and accelerometer data. Data for two strikes are shown in each plot. Accelerometer data are jagged. Digitized data are smooth. For each strike, the velocity data measured with the accelerometer and found through digitization of high-speed video are shown with the same line type.

47: Kinematic measurements from digitized high-speed video and accelerometer data. Velocity measurements from centripetal accelerometer readings are shown by the solid line. The hollow triangles mark the velocity profile determined from digitized high-speed video of the strike. The natural oscillation of the model is depicted by the curve marked with hollow circles.
Stomatopods, commonly known as mantis shrimp, appear to operate at the physical limits of animal motion in water. *Odontodactylus scyllarus* has been shown to strike prey with a blow that reaches 23 m/s while accelerating up to 104 km/s² (Patek et. al. 2004). Moving at these speeds, the animal meets two significant forces. Drag becomes formidable. A sphere moving at these speeds experiences a drag force 50 times its own weight\(^1\). Even more challenging, fast motions can cause water to vaporize, or cavitate. At great speeds, the local water pressure can drop below that of the water vapor, forming cavitation bubbles that, upon collapse, emit shock waves so powerful they erode holes in metal.

Cavitation has long plagued engineers. Material damage and loss of efficiency have led engineers to try to minimize cavitation in hydro pumps, spillways and on propellers for nearly a century (as reviewed in Brennen 1995 [34, 94, 96]; Rayleigh 1917; Parsons and Cook 1919). Mantis shrimp have not only managed to operate under these extreme conditions, but they may even have evolved to take advantage of them. Despite moving at such large speeds and accelerations, mantis shrimp avoid cavitation except upon impact with their prey when the force of the collapse of the cavitation bubble may be used to help them open shells (pers. comm. Patek; Patek et al. 2004; Patek and Caldwell 2005).

\(^1\) Estimated with Drag/Weight = Cd \( \rho \) \( U^2 \) \( A \) / 2mg where Cd = 1, \( \rho = 1000 \) kg/m\(^3\), \( U = 23 \) m/s, Area(A) = 4e-6 m\(^2\) (per. measurements) Mass(m) = 1e-4 kg (per. measurements), \( g = 9.8 \) m/s\(^2\).
Cavitation during the mantis shrimp strike is generally limited to when it would be advantageous to the animal and is rarely seen when it would only damage its own exoskeleton. This suggests that this animal may have evolved to control cavitation.

The goal of this study is the development and analysis of a simple mechanical system designed to emulate the rotational strike of the mantis shrimp. This model will be a tool with which to study drag reduction and cavitation control under the rotating and accelerating conditions found in the shrimp strike. The mechanical model has three goals. 1) The model should match the animal strike in as many properties relevant to drag and cavitation as feasible. The model should have a means to control any properties that are not feasible to match but are shown to be relevant. 2) The mechanism driving it should be simple enough to allow for a mathematical description of the model dynamics. And, 3) it should be outfitted with a method of kinematic and cavitation data collection.

The description of the design, building and analysis of a physical model of the strike of the mantis shrimp is divided into two sections. In Chapters 1 through 5, the design and testing of the physical model are discussed in light of how completely the model captures the fluid dynamics of the mantis shrimp strike. What is known about the strike of the mantis shrimp is reviewed section 1 of Chapter 1. Chapters 2 and 3 focus on the fluid dynamic properties relevant to drag and cavitation that are important to take into consideration when trying to emulate the stomatopod's strike. Chapter 4 presents the design of the first iteration of the model and how it incorporated the constraints highlighted in Chapters 2 and 3. Chapter 5 compares kinematic and cavitation data
collected from the model and one species of mantis shrimp. The model is found to cavitate at lower velocities than the animals. Chapters 6 through 8 focus on fine tuning the physical model in order to provide a platform for more sensitive future experiments. A model redesign is presented in Chapter 6. The model is redesigned to change the driving mechanism to one that can be mathematically described and to incorporate an accelerometer for more accurate data collection. A mathematical model of the redesign is developed in Chapter 7. Kinematics of the redesigned physical model are analyzed in Chapter 8 and compared to the results predicted by the mathematical model. Chapter 9 presents the accelerometer data. Chapter 10 presents a study evaluating the influence of environmental variation on the cavitation variation seen between the animals and the model. A discussion of this work is found in Chapter 11.

1.1 The Mantis Shrimp

Mantis shrimp are small crustaceans in Order Stomatopoda that live on the ocean floor in burrows either found or self-constructed (Ahyong 2001). They defend themselves and attack prey by either spearing with the sharp point on the end of their open appendage or by using their closed appendage (see white arrow in Figure 1) like a hammer, rotating it quickly away from their body and smashing their target (Figure 2) (Caldwell and Dingle 1976). Previous work had found *O. scyllarus* to smash their prey at speeds up to 23 m/s (Patek et al. 2004). During work to determine the mechanism by which these accelerations are produced and the resultant force to which prey are subjected, Patek et al. (2004) discovered that mantis shrimp strike with a force thousands of times their body weight. They also saw that mantis shrimp form a cavitation bubble
upon impact with their prey that exerts a second impact upon collapse, an impact of similar or larger force than the initial strike. Despite extreme speeds, cavitation was rarely seen in these animals during any other portion of the strike except impact (pers. comm. S. Patek). Cavitation is not present, despite very fast speeds, when it would only be damaging to the animal's own exoskeleton and yet is seen during impact when it could aid in opening the shells of prey. This suggests that these animals may have evolved to control cavitation.

It has been shown that the strike of the mantis shrimp cannot be accounted for by muscle contraction alone (Burrows 1969; Patek et al. 2004). Muscles have a functional power limit where power is defined as

\[
P = (F \times U) + \Delta t * \omega(i+1) + \omega(i)
\]

where \(P\) is power, \(F\) is force, and \(U\) is velocity. Muscles that can contract quickly produce less force than those that contract slowly (Alexander 1983 [24]). To overcome

Figure 1: *Odontodactylus scyllarus*. Hammer appendage indicated with white arrow. (Patek et al. 2007, Figure 1)
this limitation, animals have developed elastic structures to store the slowly produced energy and release it quickly (Alexander 1983; Alexander and Bennet-Clark 1977; Gronenberg 1996). Such mechanisms can produce movements faster than any muscle while still imparting a large force. Humans have adopted the same strategy in the design of bows, catapults and sling shots. By slowly storing up potential energy in a spring or in the bend of a bow and releasing it with the flick of a trigger, objects can be hurtled much farther and faster than with muscle power alone.

Most motions in crustaceans are generated by alternating contractions of two opposing muscles. One is contracted to move the appendage away from the body, the other to bring it closer (Burrows 1969; Patek et al. 2007). The mantis shrimp, in contrast, in preparation for a strike pulls the appendage snug to its body and then continues to contract both of the opposing muscles, causing no additional motion in the appendage. Instead, the contraction deforms the exoskeleton. One of the two contracted muscles is then released (m1 Figure 3), and the appendage linearly slides forward and then accelerates radially in a strike. The mantis shrimp is thought to store elastic energy in the deformation of two particularly calcified sections of its exoskeleton, the ventral

Figure 2: *Gonodactylus smithii* striking. The hammer like appendage starts snug against the animal's body and then rotates away to strike a wooden stick. Two milliseconds pass from first to last frame. (photos by author)
bars (Figure 3) (Patek et al. 2007). Mechanical testing of the exoskeleton deformation has shown the exoskeleton to have a linear Hookean spring constant (Zack et al. 2009).

Patek et al. (2007) developed, and McHenry et al. (2012) further refined, a mathematical model of this strike as a four bar linkage based on digital analysis of the relative motion of the exoskeleton segments and the relationship between the maximum force the muscle could exert and the strike’s impact force. A simplified understanding of the mechanism is modeled in Figure 3. The ventral bar acts as a tape spring which is deflected in preparation for a strike. During release, the meral-V (mv) returns to its pre-deformed state, widening the gap between it and the merus (M). In doing so, it slides
along the appendage exerting a moment upon the carpus which rotates with the dactyl (D) and propodus (P) in a rapid acceleration.

1.2 Summary

The strike of the mantis shrimp has been shown to move at speeds that are among the fastest observed for any aquatic animal. In doing so, the mantis shrimp navigates the complex fluid phenomena of cavitation and large drag forces. Studying the mechanical, material and kinematic properties of the strike provides us with insights into novel drag reduction and cavitation controlling techniques. This thesis presents the work of designing, building and evaluating a physical model designed to emulate the strike. With this physical model, we will study the properties of the mantis shrimp strike that enable drag reduction and cavitation control. While the physical model of the strike will not replicate the complex biological mechanism that drives this motion, it will be driven by the basic principle of elastic energy storage known to power the animal strike. With this foundation, we look to the literature on drag and cavitation to inform the details of the model design.
The first step in developing a model is to determine the relevant physical parameters. In this chapter we will look to the literature to compile a list of physical properties relevant to drag. Chapter 3 will follow the same approach while focusing on the properties relevant to modeling and measuring cavitation.

This chapter will focus on the properties that affect drag in open flows. The major contributions to drag for the mantis shrimp are skin friction drag, associated with viscous shear in the boundary layer of the fluid around the body, and pressure drag, the result of pressure differences normal to the surface due to boundary layer separation (Blevins 1984). Because the appendage is accelerating, there may also be added mass (Wilcox 2007) and Basset history force components to drag (Soo 1967). Each of these elements will be discussed in turn.

2.1 Skin Friction Drag

Drag is the result of momentum transfer from the surrounding fluid to a body. The velocity of the fluid at the surface matches the velocity of the body on most surfaces. This is known as the no-slip boundary condition. If the velocity of the body varies from the free stream velocity, the transition between the two occurs in a thin boundary layer near the body’s surface. This shear layer exerts a force tangent to fluid flow, decreasing
the difference in velocity between the body and surrounding fluid. This skin drag increases with the length of the boundary layer, the viscosity of the liquid and, at slow speeds, the difference between the free stream and body velocity (Blevins 1984 [351]).

Recent work has called into question the no-slip boundary condition on some solid surfaces. Some materials allow for slip and thus greatly reduce skin drag. It has been shown that ultra-hydrophobicity allows for some slippage (Muralidhar et al. 2011; Daniello et al. 2009; Min 2004). Surface roughness, once thought only to increase drag, has also been shown to alter boundary layer flow and result in lower drag for some patterns (Priezjev 2011; Vinogradova 2011; Bandyopadhyay, 1986).

2.2 Form Drag

The resultant pressure variation across a body's surface due to velocity gradients on many shapes will result in detachment of the boundary layer from the surface forming a low pressure turbulent wake behind the body. Pressure drag results from the difference between the pressure exerted on the body in the attached boundary layer and in the low pressure wake. Pressure drag is a net force in line with the fluid flow (Figure 4).

For any given body, the relationship between drag, geometry and velocity is captured in its most simple form by the non-dimensional coefficient of drag, \( C_d \).

\[
\alpha(i+1) = \frac{\text{Torque}(i+1)}{I_{tot}}
\]  

(2)
where \( D \) is the drag force, \( A \) is the frontal area, \( \rho \) the free stream fluid density and \( U \) is the free stream velocity. In practice, this is a broad simplification. The drag coefficient is related to free stream velocity by a much more complex relationship as will be detailed in the remainder of the chapter.

### 2.3 Drag in Uniform Flow: greater detail

Newton first explored drag on a sphere in a viscous fluid in 1710 (Soo 1967 [15]). His experiments at high velocities gave a drag force:

\[
F = 0.22 \pi a^2 \bar{p} U^2
\]  

(3)
where $a$ is the radius of the sphere. Stokes, in 1850 (Soo 1967 [17]), suggested that at very low velocities, the inertial effects of moving the water out of the way of the body are so small that they can be neglected from the Navier-Stokes equation. This results in an approximation for drag in a symmetric flow about a sphere proportional to velocity rather than velocity squared:

$$F = 6\pi \mu a U$$  \hspace{1cm} (4)

where $\mu$ is the mean fluid viscosity. Both of these approximations can be compared with experimental data in Figure 5, which shows the complexity of the actual relationship. The non-dimensional Reynolds number, $R_E$, is useful in quantifying these phenomenon.

$$R_E = \frac{\rho U L}{\mu}$$  \hspace{1cm} (5)

Here $L$ is a characteristic length of the body and $\mu$ is the fluid viscosity. Stokes' drag is considered accurate for Reynolds numbers under 1. Newton's formulation roughly holds from $300 < R_E < 2 \times 10^5$, but it is drastically inaccurate for accelerating or turbulent flows or for shapes other than spheres.
A closer look at the flow patterns around a sphere at different Reynolds numbers sheds some light on some of the flow factors that alter the drag coefficient (Figure 6). Stokes drag is a fair approximation for laminar flow (Reynolds numbers below 5), and thus represents skin drag over the entire surface. At Reynolds numbers between 5 and 40, a pair of symmetric vortices trail the sphere. Comparing this to the experimental data in Figure 5, we see Stokes drag underestimates drag coefficient in this range. This suggests that the separated boundary layer adds some pressure or form drag. Yet Newton's approximation doesn't fully capture drag behavior until the range where the vortex street downstream from the sphere is separated and producing a low pressure wake. Before the wake becomes turbulent, from $40 < R_e < 300$, the oscillating vortices

Figure 5: A collection of experimental data from several sources for coefficient of drag for flow past a sphere (unless otherwise noted) vs. Reynolds number. (Soo 1967, Figure 2.1)
give rise to some form drag but the trailing oscillating wake pressure is not as low as a fully turbulent one.

Figure 5 also suggests other factors affecting drag. Measurements in turbulent or accelerating flows or for non-spherical bodies vary drastically from Stokes' or Newton's
approximations. Figure 7 illustrates drag variation across Reynolds numbers for different body shapes. The coefficient of drag, $C_d$, decreases as the bodies become rounded across the front, the thickness to length ratio decreases, the trailing edge becomes more pointed, and the cross sectional area decreases. Each shape alteration decreases the size of the trailing wake, and thus the pressure drag until the shape is a flat plate which has only skin drag.

![Figure 7: Drag Coefficient vs. Reynolds number for spheres of different surface roughness. Drag crisis region is at lower Reynolds numbers for rougher surfaces, $\epsilon$, and the effect is lessened. (Blevins 1984, Figure 10-23)](image)

Note the drop in drag coefficient for rounded bodies for Reynolds numbers between $10^5$ and $10^6$. This drop in $C_d$, referred to as the 'drag crisis' is illustrated in Figure 6 for $3 \times 10^5 > R_E > 3 \times 10^6$. This region corresponds to a period of transition to a
fully turbulent boundary layer. Before this transition, the boundary layer can be seen to separate less than 90 degrees from the stagnation point on the upstream section of the body, subjecting more than half of the body to high drag turbulent wake. As the boundary layer becomes turbulent it remains attached to the body further downstream, decreasing the width of the wake and thus the proportion of the body exposed to low pressure, hence decreasing the form drag. During this transition, the vortex street is lost.
to turbulence, producing a small window without oscillatory pressure forces. This transition can be seen to happen only in bodies without sharp corners, and is observed at lower Reynolds numbers and with less impact for greater surface roughness, as is seen in Figure 7.

It is interesting to note that the cross sectional profile of the mantis shrimp's appendage is a streamlined shape with a thickness approximately of 0.4 its diameter and operating in the $10^5$ Reynolds numbers range\(^1\) placing it within the drag crisis for that shape.

![Figure 9: Cross section of propodus and dactyl for G. smithii from a Micro-Computed Tomography scan.](image)

2.4 Hydrofoil Pressure Distribution.

Given the similarity between the cross section of the appendage and that of a hydrofoil, an understanding of what is known about the pressure distribution across the surface of a hydrofoil will be useful.

Velocity increases from zero at the stagnation point to a maximum just forward of the maximum width of the hydrofoil (Figure 10). Minimum pressure can be seen at

\[ \text{Re} = \frac{\rho U L}{\mu} \]

\(^1\) Re = $\rho U L/\mu$ where $\rho=1027 \text{ kg/m}^3$, $U = 29 \text{ m/s}$, $L = 10e-3 \text{ m}$ (from personal measurements), $\mu = 1.88e-3 \text{ kg/(m/s)}$
maximum velocity. As the stream lines come back together the velocity decreases and the pressure increases (Blevins 1984 [351]).

2.5 Summary of Parameters that Affect Drag in Uniform Flow

The above discussion of drag focused almost primarily on drag in flows with a constant velocity in both direction and magnitude. And while the mantis shrimp strike is both accelerating and rotational, the factors that alter drag in transient flow may also be relevant to accelerating rotational flow. In summary the properties of bodies and flow that affect drag thus far discussed are:

- surface properties: roughness, hydrophobicity, and surface pattern
- flow velocity
- cross-sectional area
- roundness of body's upstream edge
- pointedness of trailing edge
- length to thickness ratio
- turbulence in free stream
2.6 Drag in Non-uniform Flow

2.6.1 Added Mass

An accelerating body has additional forces acting on it besides skin and form drag. When a body accelerates in a fluid, it displaces some of the fluid. As it accelerates, some of this fluid is dragged along with it. The acceleration of the surrounding liquid requires additional force which retards the motion, as if the body were more massive. The associated drag term is thus called 'added mass'. The force needed to accelerate a body in a fluid is

\[ F = (m + m_h) \frac{dU}{dt} \]  

(6)

where \( m \) is the mass of the body and \( m_h \) is the added mass defined in terms of the relative kinetic energy of the fluid by

\[ \frac{1}{2} m_h U \cdot U \equiv \iiint V \frac{1}{2} \rho U_{rel} \cdot U_{rel} dV \]  

(7)

where \( U_{rel} \) is the fluid velocity relative to the object and the integration is performed for the entire volume of the surrounding object. For a cylinder this reduces to

\[ F = 2\pi \rho R^2 \frac{dU}{dt} \]  

(8)
which is simply the mass of the water displaced by the volume of the cylinder multiplied by the acceleration of the body (Wilcox 2007 [474]).

Complex geometries are not so easily analytically solved. Numerical methods have been proposed by Sahin et al. (1993), Lin and Laio (2011) and Ghassemi and Vari (2011), among others, for calculations using panel, boundary element and the fast multiple boundary element methods, respectively. Experimental techniques most often include measurement of the slope of the acceleration profile for a body under a constantly accelerating force (Chan and Kang 2011; Bilo and Nachtigall 1980).

2.6.2 Basset History Term

The surrounding water accelerated by a body moving through a liquid has some inertia of its own. This fluid can in turn exert force upon the body during large accelerations. For instance, when moving through a fluid, if a body is suddenly stopped the surrounding fluid moving under its own inertia runs into the body, exerting force upon it. This component of drag is termed the Basset history term because it takes into account the sum of the result of previous accelerations with the integral over time of the acceleration. Formulated for the resultant force on a sphere (Soo 1967 [32]), the Basset history term has the form:

\[
F_b = 6 \pi a_p^2 \sqrt{\pi \rho_F \mu_F} \times \int_0^t \frac{(\frac{d}{dt})(U_F - U_p)}{\sqrt{t-\tau}} \, d\tau
\]  

(9)
where $F_B$ is force, $U_f$ is the fluid velocity and $U_p$ is the particle velocity over the interval from zero to time $t$.

This relationship has only been experimentally verified for flows with Reynolds numbers up to 62. For Reynolds numbers between 8000 and 16,000 Tsuji and Tanaka (1990) show that drag increases in non-rotational accelerating flow and decreases in decelerating flow. Odar's 1968 experimental data for drag on a sphere along a circular path for Reynolds numbers from 6 to 185 shows added mass and that the effects of the history term increase drag in relation to linear motion. It is not clear how these two effects combine or should be applied to organic forms at the Reynolds numbers we are trying to model.

2.6.3 Summary of Drag in Non-uniform Flow

The drag forces on a body in accelerating and rotating flow are not quantified for non-spherical bodies. Yet, the experimental evidence on spheres rotating or accelerating suggests under similar conditions drag may be altered by:

- path of motion
- cross section area
- acceleration profile.

2.7 Summary

Combining the conclusions from what is known about drag in uniform and non-uniform flows, we can arrive at a list of parameters that may affect drag in the rotating, accelerating conditions of the physical model.

The properties of the flow that may alter drag are
- path of motion
- acceleration profile.
- flow velocity
- turbulence in free stream

The properties of the body in the flow that may affect drag are

- cross sectional area
- roundness of body's upstream edge
- pointedness of trailing edge
- length to thickness ratio
- surface properties: roughness, hydrophobicity, and surface pattern
Chapter 2 established the physical and kinematic properties relevant to drag. The goal of this chapter is a similar treatment of cavitation and its measurement.

Cavitation is the development of bubbles in a liquid, somewhat akin to boiling. Both cavitation and boiling occur when the local pressure equals the water's vapor pressure resulting in a change of phase from liquid to vapor. Cavitation and boiling are distinguished only by the path taken. In boiling, the vapor pressure is increased to equal the water pressure by raising the temperature of the water. In cavitation, vapor bubbles are formed when the local water pressure drops below the vapor pressure (Brennen 1995).

A classical phase change diagram can be useful in understanding the relationship between boiling and cavitation (Figure 11). The curve from the triple point, T, to the critical point C, separates the liquid and vapor phases. Starting at a point in the liquid phase, A, one could move horizontally, increasing the temperature and keeping the pressure constant, as seen in boiling, to cross the curve to the vapor domain, or move vertically, keeping the temperature constant while decreasing the pressure, as in cavitation. Either path will result in the local pressure equaling the vapor pressure at the intersection of the curve and will result in bubble formation once it is crossed.
There is much debate about the molecular explanation of this change of phase.

Some have suggested that the decrease in local pressure below the vapor pressure of the liquid causes a strain on the hydrogen bonds between molecules (Caupin and Herbert 2006; Levitas et al. 2011). When the stress between molecules exceeds a threshold, a propagating crack is formed, which is widened into a vapor bubble by the pressure differential. Others (Brennen 1995 [22; 31; 37]; Franc and Michel 2004 [Chpt 2]) have focused on the expansion, at lowered pressure, of microscopic voids that are always present in the liquid. These views can be combined by considering microscopic nuclei to

Figure 11: Phase Change Diagram: Boiling occurs moving from the liquid state at point, A, to vapor by an increase of temperature, moving horizontally. Cavitation can occur with little to no temperature change because of a decrease in pressure.
be either the point of weakness at which the cracks form, or the source of the voids which grow into cavitation bubbles. Although still debated, these two views are macroscopically identical, and the differences are relevant mostly when one is concerned with the sources of variation in experiments studying cavitation onset.

3.1 Types of Cavitation

Cavitation is categorized by either its cause or form, but categories are vague, overlap. Moreover, different types of cavitation are often seen together, making categorization difficult. Cavitation can take the form of transient isolated bubbles that appear in areas of low pressure, then expand, and persist until reaching an area of increased pressure, whereupon they collapse. Cavitation also forms attached sheets or cavities along low pressure areas of surfaces, as is often seen along hydrofoils or propeller blades. Attached cavitation can be fully developed, leaving a large vapor cavity, or merely result in a cloud of small bubbles, or some combination thereof.

Cavitation can be seen in the low pressure centers of vortices, the result of rough surfaces or of shear flow, as seen in jets or wakes (as reviewed by Brennen 1995 [Chpt. 7]). Figure 12 illustrates the many forms of cavitation that can occur simultaneously or on the same body at different velocities. Flow is from the left. Here we see bubble cavitation in (a) with some cloud cavitation seen behind the sphere in (b). Cavitation in the vortices can be seen in (c). The cloud cavitation transitions to fully detached sheet cavitation at (h).
Figure 12: Cavitation around a sphere in uniform flow at different cavitation numbers. (Figure and text from Brandner et al. 2010)
3.2 Conditions Favorable to Cavitation

Cavitation is formed in areas of lower pressure. Cavitation is often seen along bodies in steady flow with geometries of that lead to a large acceleration, such as around corners or in areas of constriction. High shear due to surface roughness can produce small vortices that cavitate at their center. Pressure fluctuations due to the vibrations or impacts often produce cavitation (as reviewed by Franc and Michel 2004 [1; 5; 123]).

3.3 Effects of Cavitation

Cavitation has been the topic of much research among hydraulic engineers because it causes loss of efficiency in pumps, hydro-turbines and propulsion systems (as reviewed by Tropea et al. 2007 [960]). Additionally, cavitation can cause erosion that may range from minimal pitting after many years (Figure 13) to catastrophic failure within hours (Figure 14). Cavitation bubbles collapse dramatically, forming high amplitude shock waves reaching several thousand bars (Frank and Michel 2004) and producing heat which can be $10^4$ times the ambient temperature, eroding holes in any nearby substance (Brennen 1995 [84]). Such shock waves produce a detectable sound signature and often a very brief flash of light (Brennen 1995 [3.9]; Tropea et al. 2007 [985]).

Nonetheless, the power of cavitation has not been overlooked. In fact, it has even been harnessed by several technologies. Cavitation is used in milk homogenization, crystallization, microbial disinfection, ultrasonic cleaning, the release of enzymes, and the breaking up of kidney stones (as reviewed by Tropea et al. 2007 [960]; Gogate and Kabadi 2009).
3.3.1 Measurement of Cavitation Bubble Collapse Forces

Since it is difficult to measure impact forces, cavitation damage is often experimentally quantified by measuring material damage rather than impact force. The collapse of the cavitation bubble, although powerful, is very brief and localized. Thus measurements that require devices be capable of sampling at very high frequencies and positioned at precisely the correct location. Arndt et al. (1997) and Soyama and Kumano (2002) have used a piezoelectric polymer, polyvinylidene fluoride (PVDF) to measure bubble collapse pressure impulses. PVDF is a thin film capable of a time resolution between 10 and 40 ns that, when placed on the surface of a cavitating body, can capture

Figure 13: Photograph of typical cavitation damage on the blade of a mixed flow pump (Figure and text from Brennen 1995, Figure 3.11)
impact forces along its entirety, eliminating the need to place a small sensor precisely where cavitation bubble collapse will occur.

Force sensors can be used to study the pressure wave resulting from the collapse of the cavitation bubble when the location of cavitation is precisely known. In work on impact forces of cavitation, Patek and Caldwell (2005) developed a technique using a piezoelectric force sensor with upper frequency limit of 75 kHz as a target for mantis shrimp strikes. Since the cavitation bubble forms between the appendage and the force sensor, the location of bubble collapse was controlled.

### 3.3.2 Quantification of Cavitation Inception

Figure 14: Cavitation erosion of cement in spillway after only 4 hours of use. For scale, people are circled. (Tropea et al. 2007, Figure 15-1)
In an attempt to quantify the degree of cavitation occurring for a given shape and given conditions cavitation index, $\sigma$, a non-dimensional parameter, was defined as

$$\sigma = \frac{(p_0 - p_c)}{\frac{1}{2} \rho U_0^2}$$  \hspace{1cm} (10)$$

where $p_0$ and $U_0$ are characteristic pressure and velocity, $\rho$ is the density, and $p_c$ is a critical pressure. Large cavitation numbers usually correspond to no cavitation, either in cases of a high characteristic pressure or low flow velocity. As $\sigma$ decreases, cavitation is more likely (Figure 12). Since cavitation theoretically occurs when the minimum local pressure equals the vapor pressure, a value for cavitation inception, $\sigma_i$, can be defined as

$$\sigma_i = \frac{(p_0 - p_v)}{\frac{1}{2} \rho U_0^2}$$  \hspace{1cm} (11)$$

where $p_v$ is the vapor pressure of the liquid. The cavitation inception index can be thought of as a performance boundary, such that cavitation is most likely to occur for $\sigma$ less than $\sigma_i$. Note that the cavitation number and the inception index are both independent of scale. But it should be emphasized that, as with most things in cavitation, the experimental data vary widely from these approximations.

### 3.4 Cavitation Inception

#### 3.4.1 Factors that Affect Inception
Although it is well understood that cavitation bubbles form in areas of lower pressure, since experiments on cavitation began, theoretical predictions of the precise conditions under which cavitation will occur have been thwarted by wild variations in the experimental data for cavitation inception, even on the same body. (Figure 15). While theory suggests cavitation will occur when local pressure drops below vapor pressure, experimental evidence rarely matches this approximation closely. Early work on cavitation in small tubes showed that the tensile strength of water can vary widely, sustaining pressures well below that of the vapor pressure, when experiments were

Figure 15: Cavitation inception index vs. flow velocity for the same body in several testing facilities (Tropea et al. 2007, Figure 15-28)
performed with very clean water and newly washed tubes (Berthelot 1850; Henderson 1980). Cavitation can also occur at pressures above the vapor pressure of the liquid, most often in flows where small bubbles are already present (Holl 1960). Despite extensive research into the conditions that alter cavitation inception, there is still no universally agreed upon set of properties with quantifiable known effects. However, the list of factors that contribute is finite.

3.4.2 Vapor Pressure

As the vapor pressure drops, cavitation onset is seen at lower velocities and higher cavitation numbers. Two major factors contribute to variation in vapor pressure of a liquid. Vapor pressure increases with increased temperature and with an increase in the presence of solvents. Thus salinity and water temperature are known factors in cavitation onset conditions.

3.4.3 Body Geometry

In a flow, the body's geometry, in part, determines the distribution of pressure across its surface. As with drag, shapes that cause sudden changes in velocity will lower cavitation numbers. Thus, cavities, voids, sharp corners or projections will increase cavitation.

3.4.4 Surface Properties
Surface roughness and hydrophobicity each play a role in drag production by altering the boundary layer velocity. Changes in velocity affect localized pressure and thus sensitivity to cavitation. In general, properties that decrease drag increase cavitation.

### 3.4.5 Water Quality

In numerous experiments the impact of water quality has been shown to be the most difficult to quantify of those properties known to be relevant to cavitation inception. Since cavitation bubbles are formed at microscopic nucleation sites, water with more gas bubbles cavitates at lower velocities. This was determined to be the cause of the variation seen in Figure 15. Other particulates can also act as nucleation sites at which bubbles form, and can lower tensile strength of the water at the gas-bubble interface, decreasing the pressure needed to both start and expand a bubble. Particulates can also act to increase surface tension by increasing the local shear (Brennen 1995 [33; 134; 147]).

Measurements of gas content in a liquid are most often inferred from levels of dissolved oxygen in solution. Unfortunately, while dissolved oxygen can approximate dissolved gas content, it is both difficult to measure accurately and has not been shown to have a quantifiable relationship with gas content (as reviewed by Tropea et al. 2007).

Several other techniques have been employed to measure nucleation sites. One approach introduced acoustic vibrations into the liquid to induce bubble oscillations. The frequency of the resultant sound was indicative of bubble size and quantity. Others compared the speed of sound propagation, since water with many bubbles propagates sound at a different rate than water with few. But both approaches tend to overestimate
the presence of small nuclei and underestimate the gas content of liquids with large nuclei (as reviewed by Tropea et al. 2007).

Optical techniques have also had their proponents, and have ranged from the brute force method of simply photographing a highly illuminated plane through the liquid to count the bubbles, to the more sensitive technique of measuring the amount of light scattered through a liquid. Both techniques, as with the acoustic and dissolved gas measurements, have their limitations and inaccuracies (as reviewed by Tropea et al. 2007 [980-982]).

3.5 Cavitation Inception Detection

Predictions of cavitation onset are often imprecise due to the localized effects of water impurities, and the variation in pressure differences needed to expand nuclei of different sizes found within any water sample. A clear definition of cavitation inception often further confuses the situation. Techniques used for cavitation detection have usually fallen into one of two categories, the visual and the auditory. Most often cavitation is defined by the visual appearance of bubbles due to a decrease in pressure, but it has also been defined as define it by the presence of an acoustic signature often detectable long before bubbles are visible. Visualization of cavitation bubbles is often carried out in a water tunnel where the pressure is decreased or the velocity increased until cavitation is seen. Bubbles are detected by eye, laser interference, or light scattering. Each method can be susceptible to variations due to subjectivity. Auditory detection requires small hydrophones to be placed near the location of cavitation inception and with this method it
can be difficult to account for timing differences caused by the speed of sound when cavitation is spread across a large surface.

However, like the conditions that affect cavitation inception, each method of detection is susceptible to differences in measurement due to subjectivity and these differences lead to discrepancies between data collection at different facilities. For these reasons, at this point there is no clear way to distinguish between variation in detection thresholds or methods, and environmental conditions (as reviewed by Tropea et al. 2007 [986]).

3.6 Cavitation Scaling

Since cavitation inception is dependent on the number and size of nuclei, it is dependent on scale. In the same water conditions, the same nuclei density and size will be proportionally more abundant for a larger scale model and result in larger cavitation bubbles (as reviewed in Tropea et al. 2007 [986-987]. Experimental evidence of this can be seen in Figure 16.

3.7 Summary of Cavitation-sensitive Properties

In experiments cavitation inception is sensitive to:

• Body geometry
• Body surface roughness
• Body material properties (hydrophobicity, elasticity)
• Velocity
• Acceleration (instantaneous and historical)
• Water Pressure
• Temperature
• Water Quality
• Scale
3.8 Summary of Drag or Cavitation-sensitive Properties

Combining the lists of properties that may alter drag or cavitation we find many overlaps. In the list below of properties known to alter drag are marked with an asterisk. Cavitation sensitive properties are marked with a "˟".

- Body geometry˟
- Body surface roughness˟
- Body material properties (hydrophobicity, elasticity)˟
- Velocity˟
- Acceleration (instantaneous and history)˟
- Environmental conditions˟
- Scale˟
CHAPTER 4

INITIAL MODEL DESIGN

This chapter presents the design of the model of the stomatopod's strike. The design criteria are first articulated. The design solution is then presented with an explanation of how each criterion is satisfied.

4.1 Model Design Criteria

4.1.1 Type of Model

The model of the mantis shrimp strike will be used to isolate the properties relevant to drag reduction and cavitation control. It is essential that the model fully capture the fluid dynamics of the strike. Modeling these complex body-fluid interactions lies just outside the boundaries of what is currently computationally efficient for a mathematical model. While CFD simulations can successfully model cavitation around bodies (Senocak and Shyy 2001), accurate simulation of cavitation resulting from the impact of two bodies is beyond this scope of this work. A physical model, in contrast, is not limited by current knowledge of the underlying physics and our ability to mathematically represent it. Thus, like many other explorations of the interaction between fluid and moving biological forms (Koehl 1996; Dickinson et al. 1999; as reviewed by Alexander 2003), a physical model was used for our studies.
The difficulty of altering and adding to a physical model adds constraints to the model design. The design must not only take into account each of the drag and cavitation-sensitive properties discussed in the last two chapters, but must also incorporate experimental flexibility. Forethought must be used to design variability into the structure so as to allow different properties to be tested. Therefore, a focus of the design will be the model variability.

4.1.2 Structural Requirements

The design criteria for the model fall into two categories. First, the study of drag and cavitation requires the model to meet some structural requirements. Second, the model must match or keep consistent drag and cavitation sensitive properties. We begin with structural requirements.

Each phenomenon the model was designed to study has distinct structural requirements. Drag measurements are most easily analyzed by combining the power of the physical model with a mathematical model. The model's motion is the result of the interaction of several forces; the driving force, the fluid dynamic drag forces, the inertial force and any body forces present. If we can measure the motion of the model and account for all but the fluid forces in a mathematical model of the mechanism, by subtraction we can infer the drag forces acting during the model strike. Thus, it is important that the model be designed to be simple enough to allow a mathematical analysis of the driving mechanism.

Cavitation studies can be distinguished by the conditions under which they occur. The mantis shrimp studied here cavitates almost universally upon impact and rarely
during the rapid acceleration of the forward motion of the strike. Thus, we can break the study of cavitation control in mantis shrimp down into two phases:

- cavitation during forward rotational motion
- impact cavitation.

Study of each type of cavitation require a purely experimental technique and will rely on kinematic, cavitation onset detection and cavitation bubble collapse force measurements. Kinematic and cavitation onset data will be collected with high-speed video and do not impose any structural requirements on the model. The model must be allowed to move differently to study each type of cavitation. Cavitation during forward motion requires the model to rotate freely. Impact cavitation requires that the model rotate and then strike an impact force sensor at a consistent angle and distance. An additional removable structure to support the force sensor is needed for the study of impact cavitation. The model will be designed with this variability.

4.1.3 Physical Property Matching

We have looked to the literature on drag and cavitation to determine which strike properties were important to try to match on the model. The model was designed to match most, but not all, of the relevant properties. The model was designed to match the animal's:

- Maximum velocity
- Maximum acceleration
- Rotational motion and scale
- Appendage scale and shape
- Environmental conditions of temperature, salinity and initial water quality
One set of properties that was not matched but is known to alter drag and cavitation are the surface and material properties of the animal appendage. The mantis shrimp striking appendage is materially quite complex (Weaver et al. 2012). The dactyl club is composed of three distinct regions. A hard outermost region made of layers, rotated much like plywood, covers an inner region of layered chitin fibers. The chitin fibers both rotate between layers and coil helically within layers. These two regions are encompassed by a thickened striated region of parallel chitin fibers. Rather than match the appendage material properties, we simply aim to keep them consistent across trials with the model. To change these properties across trials, we can vary the appendages used. To keep them consistent, we use the same appendage. Thus, although we do not match the surface and material properties of the animal appendage, we can still study their effects.

**4.2 Overview of a Preliminary Design**

The physical model uses the same basic principle of power amplification through stored elastic energy known to power the stomatopod's strike. The spring is compressed slowly and then the stored energy is released quickly with the flip of a latch. A very large spring can reliably and repeatably produce large accelerations in salt water with little damage.

In this design, a latch is pulled up by 100 lb. braided fishing line attached through the center of the shank of a hand crank and through a hole drilled in the end of the latch. Pivoting around a shaft, the latch pushes down on the cam of the appendage assembly with a 4:1 mechanical advantage. The cam in turn compresses the spring. The latch
rotates as it is pulled up, moving the point of contact between it and the cam down the cam surface until, at the point of release, the cam slips by the latch. No longer held in check by the latch, the spring releases, pushes on the cam to rotate the appendage, which then strikes the force sensor.

The entire mechanism is held between two plates which constrain the motion to one plane and house the axles around which the cam and latch rotate. The mechanism can be raised or lowered in relation to a frame which surrounds and houses a 20 gallon
glass aquarium tank. Forces of impact and cavitation bubble collapse are measured with a one axis force sensor that is fixed to a platform that can be raised, lowered and rotated to allow impact at various portions of the strike while maintaining impact perpendicular to the sensor. The impact platform structure can be removed for full appendage 180 degree rotation without impact, for forward cavitation studies.

4.3 Design Specifics

4.3.1 Scale

The scale of the appendage (between 1 and 3 cm) and the aim to minimize the total mass and drag of the cam and the appendage were the major limiting factors on the size of the mechanism. Minimizing the size of the cam limited the working distance of the spring, necessitating a very large spring constant to produce the required accelerations. The stainless steel rod on which the cam rotates experiences very large forces in this design, and keeping its length to a minimum was required to avoid failure. This constrained the distance between the plates holding the mechanism and provided a limitation on spring width.

4.3.2 Springs

The spring of the exoskeleton had been shown to approximate a linear spring. Compression springs were available in the size and strength necessary to power the model's motion. Three different sizes were used to give a variety of spring constants: 2.54 cm x 2.54 cm 5.5 mPa, 2.54 cm x 19. cm 3.69 mPa and 2.54 cm x 1.43 cm 1.79 mPa. Springs were held in place with a pressed fit cylinder through their center that was tapped
and held fast to the model assembly with a screw. Multiple aluminum shims (0.156 cm and 0.317 cm) were placed under the spring to increase the displacement up to an additional 1.27 cm. The entire spring assembly was dipped in enamel paint to minimize corrosion.

4.3.3 Ball

To maintain the point of contact in the vertical plane as the spring releases, a stainless steel ball just slightly larger than the spring's inner diameter was epoxied to the top of the spring.

4.3.4 Cam

A cam is required to maintain contact between the spring and appendage assembly and serves as a base into which various appendages can be attached. When the spring is compressed, the distance from the point of contact between the spring and the appendage assembly is longer than when relaxed. Thus the shape of the cam alters the lever arm of the moment of rotation through the motion. Although there was no attempt in this preliminary version to match the acceleration profile of the animal, variation in the cam's shape allows variation in the acceleration profile.

4.3.5 Axles

The cam and latch rotate on stainless steel rods, 0.317 cm x 3.175 cm and 0.635 x 3.175 cm respectively, that puncture the vertical plates supporting the mechanism. The cam's axle rotates freely in graphite bearings suitable for underwater use. The cam and
latch are held in the same plane with spacer cylinders which maintain them in the center between the vertical plates.

### 4.3.6 Force Sensor Platform

A force sensor is screwed down onto a 2.54 x 2.54 cm plate milled flat within 2.54e-4 cm that is bolted to a 2.54 x 7.62 x 5.08 cm aluminum beam. This beam is attached at two points, one fixed and the other able to slide in an arched slot in the vertical plates, for angle rotation. The platform angle is fixed with the tightening of a bolt. This structure is bolted to a 2.54 x 2.54 x 15.24 cm aluminum bar that is attached to the model structure with a sliding rail allowing the impact platform to be raised and lowered easily.

### 4.3.7 Force Sensor

The force sensor is Model 200B02 from PCB Piezotronics (NY, USA) with a force range of 444.8 N, upper frequency limit of 75 kHz. The stainless steel force sensor has a 12.7 mm diameter load surface and a stiffness of 1.9 μm⁻¹.

### 4.3.8 Material Choices

In an attempt to minimize erosion through electrolysis between dissimilar metals, where choices were available and structurally sufficient, all submerged parts were machined from aluminum. Springs with the required spring constant were only readily available in steel and required coating in enamel paint. The axles on which the cam
rotates required the strength of stainless steel. Bearings to minimize friction in axle rotation are non-corroding graphite.

### 4.3.9 Latch

The latch is 0.635 x 5.08 x 1.27 cm machined aluminum.

### 4.3.10 Appendage

The appendages are attached with a pin and set screw for interchangeability. Two different types of appendages were used; a 0.635 x 2.54 cm aluminum cylinder and casts of animal appendages. Actual appendages were fragile once off the live animal and tended to break when run on the model. Cast appendages range from 1.905 to 3.175 long and can be cast from a variety of materials. For this experiment, only one casting material was used.

### 4.3.11 Appendage Casts

Molds of five appendages from three individual peacock mantis shrimp, one of the largest smashers, were made using Polydimethylsiloxane (PDMS), a silicone widely used for mold making and capable of the surface fidelity necessary to replicate nanocircuits (Dang et al. 2012). Appendages were suspended in liquid PDMS mixed at a 20:1 ratio on a wire during curing. The mold was sliced when hardened for appendage removal. After resealing, casts were made with UV curing 2-P, a strong clear polymer with low enough viscosity to capture surface detail of the mold. Screws or pins were embedded in the molds for attachment onto the model. Molds are reusable and allow
multiples of any given shape to be made in a variety of materials to compare material and surface properties.

4.3.12 Data Collection Technique

The system is outfitted with high-speed video kinematic and cavitation data collection. This method was used in earlier mantis shrimp kinematic and cavitation studies (Patek et al. 2004; Patek and Caldwell 2005; Patek et al. 2007). Cavitation studies have also been conducted with high-speed video imaging used to detect cavitation onset and categorization (as reviewed by Tropea 2007).

4.3.13 Environment

Given the sensitivity to environmental conditions in cavitation inception experiments, particular attention was paid to environmental conditions. Taking into account that these animals operate under ever changing conditions, the first iteration of the model simply matched the range of animal water conditions, allowing variables to vary with the stomatopod's environment. Animals were kept in water with a salinity of 34±2 ppt and a temperature of 26±2 °C. Tanks were open to the air, aerated with bubbles and kept clean with bacteria filters. The model was also open to ambient air. Salinity was kept between 32 at 34±2 ppt and temperature constant at 26± 2°C and monitored. With no organic life in the model tanks, bacterial filters were not thought necessary. The water was made with RO filtered water (Millipore Progard 2, QGuardel) and marine salt.

4.4 Variability
In summary, it is possible to vary the velocity, acceleration, acceleration profile, impact rotational angle, appendage size, shape and material and environmental conditions of the model by:

**Maximum Velocity:** Variation of the spring displacement

**Maximum Acceleration:** Variation of the spring constant with different springs

**Acceleration Profile:** Variation of the cam profile

**Impact Rotational Angle:** Rotation and height adjustment of impact platform

**Appendage:**

- **Shape:** Casts from various appendages vs cylinder
- **Size:** Casts from various appendages
- **Material Properties:** Casts in different materials
- **Surface Properties:** Casts in different materials

**Environmental Conditions:**

- **Vapor Pressure:** Variation in temperature or salinity
- **Water quality:** Variation in initial water filtration

### 4.5 Summary

The focus of the study of the feeding strike of the mantis shrimp is the interaction between the biological form and the fluid through which it moves. A physical model was chosen to capture the details of this complex dynamic. The model was designed to meet the structural requirements needed to study each phenomenon and match the relevant physical properties.
Experiments related to three different types of phenomena each necessitated different structural requirements. First, drag experiments required the model to be simple enough to mathematically describe. The motion of the motion was driven by the same principle of elastic energy storage known to power the strike of the mantis shrimp. Under most conditions, the dynamics of an unloading spring rotating a rigid body are simple to analyze. Second, drag and forward cavitation experiments required the model to match the rotational path of the animal motion. The model incorporated a removable adjustable platform to hold the force sensor for impact measurements.

The model design also matched or controlled several cavitation and drag sensitive properties. The model's environment matched the animals' in temperature, salinity and initial water filtration. The model's appendage matched the animals' in shape and size. Material and surface properties were controlled. The springs driving the model were adjusted to match the peak velocity and acceleration observed in animal strikes\(^1\).

The model was designed to incorporate variability to allow the testing of the influence of several properties. The model can vary the maximum velocity and acceleration of the appendage, the acceleration profile, the rotational impact angle, the appendage shape, size, material and surface properties, and the vapor pressure and water quality.

\(^1\) Details of the kinematic measurements can be found in Chapter 5.
CHAPTER 5

MODEL ANALYSIS

In this chapter, we present the analysis of the physical model. The model was evaluated according to two standards: 1) Does the model meet the structural requirements necessary to use it to study drag and cavitation? 2) Does the model match the animal for cavitation and drag sensitive properties? The latter question was evaluated in two ways. Kinematic and cavitation onset data were collected from model and animal strikes. Kinematic measurements were used to evaluate whether the model was able to match the animal kinematic range. Cavitation onset experiments were used to evaluate the properties chosen to match between the model and the animals. If the cavitation onset velocity was comparable between the model and animals, the choice of properties was sufficient.

5.1 Structural Requirements

The study of drag, forward and impact cavitation each placed different structural requirements on the model. Impact cavitation required a removable structure to hold the impact force sensor. Drag and forward cavitation studies required that the model appendage rotate freely. Drag studies also required that the model be simple enough to mathematically describe. The first two structural requirements were easily met by the model design (Figure 17). The last required a mathematical analysis of model dynamics.
The mathematical description of the model dynamics was composed of 1) the forces driving the model, 2) the geometry of how the force is applied to the appendage assembly, and 3) the moment of inertia of the appendage assembly. The velocity profile of the model's strike would be predicted from these three components.

A description of the dynamics of decompression of the spring driving the model's motion proved beyond the scope of this work. While a dynamic description of spring unloading is generally straightforward, the relative mass of the spring and appendage assembly used here makes it more complex. The mass of the spring required to produce the desired accelerations is two to four times more massive than the appendage assembly. Dynamic behavior of compression springs is easily analytically described for springs with a load less than 1/4 the spring mass. But with a more massive load, the mass of the spring plays a major factor in spring dynamics, resulting in a surge wave through the spring, with very complex behavior (Maier 1959). As a result, one of the three pieces necessary for a mathematical description of the model mechanism is beyond the scope of this work.

To summarize, the model met the structural requirements for forward and impact cavitation studies. Due to the complex unloading of massive springs, the mathematical description of the model for use in drag studies is too complex in light of the particular the compression springs used here, in the first design of the physical model.

5.2 Matching Drag or Cavitation Sensitive Properties

The model was designed to match the animal strikes in
All but maximum velocity and acceleration have been shown, thus far, to match between the model and animal strikes. The model design fixed the motion as rotational. Model appendages were cast from molds taken from mantis shrimp appendages; the appendage shape and scale are matched. The environmental conditions of temperature, salinity and initial water quality were monitored and held consistent with the animal's conditions. To determine if the model matched the remaining physical properties, maximum velocity and acceleration, kinematic data was collected of the model strike.

To improve the comparison between the model and kinematics of the mantis shrimp, higher resolution kinematic data were collected from the *Gonodactylus smithii* species of mantis shrimp. The earlier kinematic studies of mantis shrimp were collected with high-speed video at a frame rate of 5000 fps (Patek et al. 2004). With the rapid rotational portion of the mantis shrimp strike lasting only on average 2.7 ms (Patek et al. 2004), at this frame rate only 10 to 15 data points were available over the course of a strike. Curve fitting data from so few points can lead to large errors. To improve accuracy of kinematic comparisons, high-speed video were collected of *G. smithii* strikes at 30,000 fps improving the data resolution six fold.

Earlier kinematic studies were made with *Odontodactylus scyllarus* mantis shrimp. Here we used *G. smithii* instead, for two reasons. First, *G. smithii* were available in the Patek lab where these studies were performed, whereas *O. scyllarus* were not.
Second, at the time kinematic studies had not yet been published on any other species of smashing mantis shrimp. Kinematic data on the strike of *G. smithii* would add to scientific knowledge. It should be noted that *G. smithii* average a total length of 62 mm (male, 53 mm female) (Ahyong 2001 [73]) while *O. scyllarus* average almost twice that (m: 112 mm, f: 98 mm) (Ahyong 2007 [80]). The appendage of *G. smithii* is correspondingly smaller than that of *O. scyllarus*.

5.3 Kinematic Experimental Setup

Kinematic data from both live animal strikes and the physical model were collected with a Photron APX-RS high-speed video camera, which films up to 250,000 frames per second, under 1200 W lighting. Videos were filmed between 20,000 and 35,000 fps with a shutter speed between 1/30,000 and 1/50,000 seconds and a resolution between 128x128 and 256x384 pixels depending on the size of the image captured and the available light (Figure 18). Video was captured with Photron FastCam Viewer software. Calibration images were shot of a ruler along with each strike in order to calibrate for distance.

5.4 Model Kinematic Data Collection

Preliminary animal kinematic data collected by Patek et al. (2007) offered a range of velocities and accelerations at which to run the model. Model strikes were captured on video for two different springs, at five different displacements, for four different cast appendages and with the cylinder for maximum velocities from 2 to 45 m/s and accelerations from $5 \times 10^2$ m/s$^2$ to $9 \times 10^4$ m/s$^2$. 

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Animal data were collected from *G. smithii* by swaddling and holding their hind quarters steady within the focal length of the high-speed camera while enticing them to strike with a wiggling stick or snail shell (Figure 19). Holding them produced strikes that were most often perpendicular to the line of sight of the video camera. Data were collected from 12 individuals, 5 of which supplied at least 10 digitizable strikes.

5.5.1 Data Analysis
5.5.2 Calculation of Velocity and Acceleration

Video was digitized in Matlab, extracting x and y coordinates of a designated landmark in each frame. Calibration data were extracted from calibration photos and x and y coordinates were scaled and then individually fit to 8th order polynomials for x(t) and y(t) for a given frame rate with a least squares method. Velocity was calculated from the evaluation of the polynomials according to:

\[ V = \frac{d}{dt} \sqrt{x^2 + y^2} \]  

(12)

Figure 19: Animal kinematic data collection setup: Animals were prodded to strike a stick or snail shell while held on axis to camera. 10 strikes from five different animals were collected at between 20,000 and 36,000 fps. (photos by author)
These velocity data were then fit by least squares to an 8th order polynomial. The derivative was taken to find acceleration. Order of polynomial fit was determined by increasing the polynomial order until the change in residuals fell below a maximum threshold.

5.5.3 Off Axis Correction

Animal strikes were not necessarily perpendicular to the video frame, potentially leading to decreased measured velocity. To correct for this, landmark measurements of each individual's appendage were compared between videos and calibration photos (resolution of 4288 x 2848 px). If the difference in length of the landmark measurements from the video and the photos was greater than the margin of digitizing error, the distances measured from the video were scaled proportionally to the ratio of the photo length to the video length.

5.5.4 Data Filtering

Data were filtered to compare similar portions of the animal and model strike. Typical velocity and acceleration profiles for the model and an animal strike are shown in Figure 20. In the velocity profile of the model (Figure 20 A) the velocity starts at zero, increases to a maximum (gray portion of Figure 20 A) and then slowly decreases (striped portion of Figure 20 A). These two sections correspond to different periods of model interactions. As the velocity increases (gray A), the model is accelerating the appendage. The moment it reaches a maximum, the model loses contact with the appendage. During
the striped portion of Figure 20 A, the appendage is rotating through the water and is no longer accelerating and losing speed to drag forces.

The period of rapid rotation in the animal can also be identified from the velocity and acceleration profile. The animal strike starts with a linear slide of the appendage (Patek at al 2007) along the merus which is followed by a rapid rotation. In Figure 20 C and D, the animal strike can be seen to start slowly and then rapidly accelerate. The beginning portion of the strike, with slowly increasing velocity (Sections of C highlighted with wavy lines), corresponds to the lateral slide of the appendage along the merus. The acceleration almost returns to zero as the animal transitions to a rotational motion. The gray portion of D isolates the period of rapid rotation in the animal strike. During this period, the energy stored in the deformation of the mantis shrimps' exoskeleton is used to accelerate the appendage. The striped portion of the animal strike, as in the model graphs, represents the non-elastically driven motion.

Analysis of model and animal data compared the portions of the velocity and acceleration profiles that corresponded to the rapidly rotating period of the strike. In Figure 20 the solid gray portion of the model velocity and acceleration profiles (A and B) were compared with the solid gray portions of the animal velocity and acceleration profiles (C and D).

5.6 Kinematic Results

The range of maximum animal and model velocities can be seen in Figure 21. The kinematic range of *G. smithii* is compared to the kinematic range of the model below.
Animal Kinematic Range

Model Kinematic Range

Velocity

- 5 to 29 m/s (mean 15)
- 2 to 45 m/s

Acceleration

- 2.6 to 15 \( \times 10^{-4} \) m/s\(^2\) (mean 7)
- 5 to 9 \( \times 10^{-4} \) m/s\(^2\)

For any one model setting, there was a 14% standard deviation in measured velocity across three runs and a 23% STDV across accelerations. Standard deviation was

Figure 20: Typical model and animal velocity and acceleration profiles. Solid gray portions correspond to the spring driven accelerating portion of strikes and were used for comparisons of maximum velocity, acceleration and jerk. The stripped portions of the strikes correspond to non-elasticity driven motion. Sections identified with wavy lines of graphs C and D correspond to the sliding portion of the animal strike preceding rapid rotation and not present in the purely rotational motion of the model. Spurious peaks due to curve fitting in A and B are circled and pointed to by S were not included in comparison data.
calculated for five different model settings and then averaged. Digitization error was 1.87% of velocity and 6.63% of acceleration. Digitization error was determined from the maximum variation in velocity or acceleration of 10 digitizations of one strike.

5.6.1 Kinematic Conclusions

The model can span the full range of animal velocities. The model's acceleration range is smaller than that of *G. smithii*. *G. smithii* reached a maximum linear acceleration of $15 \times 10^4$ m/s$^2$ and the model's maximum acceleration was only $9 \times 10^4$ m/s$^2$. The model matched the animal maximum velocities but not maximum accelerations.

5.7 Cavitation Inception Comparison

Cavitation onset velocity was used to compare the animal's and the model's propensity to cavitate. This test evaluated the sufficiency of the model to match the animal in cavitation and drag-sensitive properties and was to serve as a baseline for future studies. While cavitation and drag are sensitive to the same physical properties, cavitation is sensitive to an additional few (see Chapter 2 section 7). Being the more sensitive phenomena, cavitation can be used as a standard against which to measure similarity in both properties.

If the model matched the animals in all drag and cavitation sensitive properties, at the same speeds the cavitation produced on the model would be identical to the cavitation produced on the animals. The model could then be used to isolate the physical properties of the strike of the mantis shrimp to which cavitation and drag are most sensitive. With the model's built in ability to vary cavitation and drag sensitive properties,
one property could be varied at a time, and the effect on cavitation observed. It could be inferred that properties that lower the cavitation onset velocity of the model maybe responsible for cavitation control and/or drag reduction in the animal strike.

If the model did not have the same cavitation onset velocity as the animals, it could be concluded that at least one physical property to which cavitation is sensitive significantly varies between the model and the animal strikes. Again, the model could be used to isolate the property or properties responsible for the variation. In this case, properties that raise the cavitation onset velocity of the model, decreasing the tendency for the model to cavitate, could be among the properties that play a role in cavitation control or drag reduction in the animal strike.

Figure 21: Maximum velocities for strikes with observed cavitation for both animals and the model with cast appendages: Animal data compiled from 12 individuals with from 1 to 10 strikes each, evenly spread throughout distribution. The two cases of non-impact animal cavitation were from different individuals, each of which contributed at least 5 strikes to the data set. Neither cavitating animal strike was indisputably due to forward motion alone. The cavitating animal strike in the 30 m/s column may have been caused by the opening of dactyl during strike. It is unclear whether the cavitating animal strike in 20 m/s column is due to glancing impact with the target.
The model can be used to study drag and cavitation in rotating and accelerating flows whether or not it matches the animal's cavitation onset velocity. In order to use the model to study these phenomena, a baseline of model and animal cavitation must be measured. The cavitation onset experiments presented here are intended to serve that purpose.

5.7.1 Cavitation Inception Methods

Strikes were filmed of the model and *G. smithii* with high-speed video as was described in section 2 of this chapter. Cavitation inception was detected visually. The cavitation inception velocity for any strike was defined as the maximum velocity calculated for a strike where cavitation was visually detectable. The cavitation onset velocity of an animal or apparatus was defined as the velocity at which 50% of the strikes cavitated. To keep the shape consistent between model and animal appendages, model

Figure 22: Cavitation as seen on appendages run on the model. From left to right. Clear cast of *O. scyllarus* appendage. Cylinder showing sheet, vortex and cloud cavitation and actual animal appendage of non smasching mantis shrimp displaying cloud cavitation (photos by author)
appendages were cast from another smasher mantis shrimp species, *Odontodactylus scyllarus*, as was described in section 11.11. Appendages cast from *O. scyllarus* are larger than the focal species, *G. smithii*. *G. smithii* appendages were too small to be practical. One observer was used throughout the experiments to minimize subjective variation on cavitation presence. Kinematic measurements were made as described in section 2.4 of this chapter.

5.7.2 Cavitation Inception Results

During animal strikes, cavitation was observed during forward acceleration without any impact twice in 49 filmed strikes. (0% of strikes below 15 m/s cavitated. 9% of strikes between 15 and 20 m/s cavitated. 0% of strikes between 20 and 25 m/s cavitated. 50% of strikes between 25 and 30 m/s cavitated) (Figure 21). Animal peak velocities ranged from 5 to 28 m/s and had a cavitation onset velocity of 27.5 m/s.

 Strikes from 51 model trials were used. No cavitation was seen for strikes with a peak velocity below 15 m/s. Of strikes with peak velocities between 15 and 20 m/s, 31% cavitated. Of strikes with peak velocities between 20 and 25 m/s 92% cavitated. Strikes over 25 m/s cavitated 100% of the time. Model maximum velocities ranged from 3 to 30 m/s and had a cavitation onset velocity of 20 m/s.

5.7.3 Discussion of Cavitation Inception Experiment

Neither of the cavitating animal strikes were clear cases of open flow cavitation. In one strike the animal's appendages appeared to have struck each other. During the other, the propodus of the striking appendage separated from the dactyl. Either of these
events could have caused the cavitation seen during the two animal strikes. Only two animal strikes were measured to reach peak velocities over 25 m/s (4%). A 50% cavitation rate for *G. smithii* strikes at 27 m/s may, therefore, be an overestimate. Therefore, the conclusion of a cavitation onset velocity of 27 m/s is conservative.

Even with a conservative interpretation of the animal cavitation onset velocity, the model cavitates at lower velocities. The cavitation onset velocity of the model was 20 m/s in comparison to the animals' 27 m/s. The model, therefore, does not match the cavitation onset velocity of *G. smithii*. From this we conclude that there is at least one cavitation sensitive property that varies between the model and the animals.

There are many possible explanations for a discrepancy in cavitation inception between animals and the model. Each may be the key to the cavitation control of the mantis shrimp. The properties known to vary between the model and animal strikes are water quality, acceleration profile, appendage size, and surface and material properties.

Given the extreme sensitivity to water quality seen in cavitation research, environmental effects cannot be overemphasized. The water in which the animal experiments were performed is regularly aerated and filtered with micro-organisms. The model's water is neither filtered nor aerated. The net result of this variation is unclear. While the animal water is filtered, it also houses live animals that produce waste. The filter itself is composed of millions of micro-organisms through which the water flows. The micro-organisms in total may decrease contaminants or they may enter the water stream and act as nucleation sites themselves. While, it is known that the water quality is
not consistent between the animal and model environments, it is not simple to predict the effect on cavitation inception.

It is known that drag at high accelerations and low Reynolds numbers is dominated by the Basset history term. The Basset history term is sensitive to the accumulation of the effects of the rate of change of acceleration, jerk, throughout the strike. Keeping all else the same, matching the maximum acceleration of the animal strikes may not be enough to replicate the patterns of fluid flow around the appendage. Cavitation and drag may be sensitive to the jerk of the strike. The jerk of the strike of the model has not been matched to the jerk of the animal strikes.

Water pressure was not controlled for in these experiments. The animal and model tanks were open to the atmosphere allowing the water pressure to vary with weather changes. Barometric pressure changes between animal and model trial may contribute to the variation in cavitation onset velocity observed.

Appendages cast for use on the model were 1.5 to 3 times the size of appendages of the animals from which we collected kinematic data. Given what is known from experimental work about cavitation onset scaling in uniform flow, larger appendages will cavitate at lower velocities, all else being held equal. This is consistent with our data and suggests exploration of size as a cause of cavitation onset variation.

Exoskeleton surface properties vary from that of the cast appendages. Surface roughness variation could be explored and quantified. Mold making may lose surface roughness detail or, through voids or bubbles in casts, add roughness. Hydrophobicity
differs between the casting material used and appendages and, given its effects on surface
drag, could play a large part in cavitation onset.

The stiffness of the animal appendage also may vary from the model appendage.
Stiffness may correlate with appendage vibration during the strike. The forces acting on
the appendage during the strike are not only parallel to the motion. Any forces off axis of
the motion could result in appendage vibrations. Vibrations alone can cause the
formation of cavitation (as reviewed by Tropea 2007).

To summarize, the model was shown to cavitate at lower speeds than the mantis
shrimp. The possible causes of lower cavitation onset velocity on the model may be due
to a difference between the model and the animal trials in

- Environmental conditions
- Acceleration profile
- Appendage size, surface or material properties

5.8 Exploration of Acceleration Profile

The correlation between maximum jerk and cavitation can be distilled from
further analysis of the cavitation onset data. The maximum jerk for each model strike
was calculated from the acceleration profile. If the rate of change of acceleration altered
the flow around the appendage in a way relevant to cavitation production, we would
expect a correlation between maximum jerk and the presence of cavitation during a
strike.
A histogram of maximum jerks for cavitating and non-cavitating model strikes is presented in Figure 23 A. For comparison, a histogram of maximum model acceleration for the same set of strikes is presented in Figure 23 B. All model strikes were run with appendages cast from mantis shrimp.

The histogram of maximum acceleration shows a clear relationship between maximum acceleration and the presence of cavitation. As maximum acceleration increases, the likelihood of cavitation increases. This is the same pattern seen between cavitation and peak strike velocity in Figure 21. There is a maximum velocity and acceleration past which all strikes cavitate and a minimum below which no cavitation is seen. No such pattern is evident in the maximum jerk histogram. Strikes with the lowest maximum jerks cavitated and some strikes with the highest maximum jerks did not cavitate. Cavitation is scattered throughout the jerk histogram with little trend evident.
The data show a clear relationship between cavitation, maximum velocity and maximum acceleration. As peak velocity and acceleration increase, the probability of cavitation increases. Maximum jerk does not correlate with cavitation. From this we conclude that the rate of change of acceleration is not a significant factor in the variation seen in cavitation onset velocity between the model and the animal strikes. We must look to other differences to explain the model's lower cavitation onset velocity.

5.9 Summary of Physical Model Analysis

The analysis of the model was done in three pieces. The structural requirements were evaluated. Then, the kinematic range of the model was compared to the range of the G. smithii species of mantis shrimp. And, lastly, the cavitation onset velocity of the model was compared with the animal strikes.

The model met the structural requirements of two of the three areas to be studied. Requirements for forward and impact cavitation studies were built into the model design. Drag studies rely on a mathematical analysis of the mechanism driving the model. Although the model was kept simple enough to mathematically describe, the mathematical analysis proved more difficult than expected. The relative mass of the spring to the rotated appendage made the resulting spring unloading dynamics too complex to model in this study. Thus, with the present design, drag studies cannot be conducted as planned.

Kinematic data were collected from the model and strikes of Gonodactylus smithii. The data showed the model was able to span the velocity range of G. smithii. The model could not reach the full acceleration range of the animals. The maximum
acceleration measured for the model was $9 \times 10^4 \text{m/s}^2$ while the animal was measured accelerating at $15 \times 10^4 \text{m/s}^2$.

Cavitation onset experiments were intended to evaluate whether the model matched enough cavitation sensitive properties to capture the animals' cavitation behavior. Results showed the model to have a lower cavitation onset velocity than *G. smithii*. From this, it was concluded that the model differs from the animal appendage or strike in at least one cavitation sensitive property. Relevant properties thought to vary between the model and the animals were water quality, appendage size, material and surface properties and jerk. Further analysis of cavitation onset data showed jerk not to correlate with the presence of cavitation.

The first model of the feeding strike of the mantis shrimp almost meets two of the three goals set out for it. Meeting the first goal, if we include the use of the high speed video, the model is outfitted for kinematic, force and cavitation measurement and detection. Towards the second goal, the model matches the animals' strike in path of motion, range of maximum velocities, appendage shape and approximate size, and the environmental properties of temperature, salinity and initial water filtration. The model does not reach the maximum animal accelerations. The present design of the model does not meet the third goal; the motion of the model is not simple enough to mathematically analyze. To overcome these limitations, the model was redesigned. Chapter 6 presents the model redesign.
CHAPTER 6

REDESIGN OF THE PHYSICAL MODEL

The first design for a physical model of the feeding strike of the mantis shrimp met many of the design requirements set out for it. It was outfitted for kinematic, force and cavitation data collection. Structurally it could accommodate experiments in forward and impact cavitation. It matched the strike of the mantis shrimp in rotational motion and maximum velocity. The model appendage was the same shape and approximate size of the animal appendage.

However, model did not meet all of the design goals. The mathematical description of the model motion was too complex for this study. The model was unable to reach the maximum accelerations seen in the animal kinematic study. The model was redesigned to overcome these limitations.

Practical considerations also were factored into the redesign. Because digitization of high-speed video is a time consuming and imprecise method of data collection, an accelerometer was incorporated into the redesign to improve the accuracy of kinematic measurements and to decrease the time required to analyze data.

6.1 Improvements
Figure 24: In essence, the redesigned model consists of an appendage assembly, the cam and appendage, which is rotated by the unloading of a deflected beam spring. The cam acts as an attachment point for the appendage and rotates rigidly with it. The accelerometer is embedded radially in the cam, measuring centripetal acceleration. \( F_b \) is the force of the beam spring in the horizontal direction.

The goals of the redesign of the model were to: 1) expand the range of accelerations of the model to reach the maximum velocities observed in mantis shrimp; 2) complete a mathematical analysis of the driving mechanism of the model and the resulting kinematics; and 3) to improve the accuracy and decrease the time needed for kinematic analysis through the incorporation of an accelerometer. To achieve these three goals, two fundamental improvements were implemented. First, we changed the spring
used to power the motion from a compression spring to a beam spring. The unloading of a cantilevered beam spring is a simple calculation when dimensioned within the constraints of linear beam theory. Thus, mathematical analysis of the predicted kinematics of the model was a more straightforward problem when a beam spring was employed. The beam spring also helped to circumvent the space limitations of the previous model and allowed larger springs to produce higher accelerations. The second improvement was the incorporation of an accelerometer into a section of the model that rotates with the appendage but is not changed with appendage replacement. This would allow kinematic data to be collected instantaneously without the error inherent in digitization of high-speed video.

The redesign shares many basic principles with the original design. In both, the motion is driven by the release of stored elastic energy. In the original design, a compression spring stored the energy while the redesign model used a beam spring. Both designs rely on a latch to allow the appendage assembly to deflect the spring and then almost instantaneously release (see Figures 24 and 25). The latch is attached to a hand crank with a thin filament and pivots on an axle of 1/8” stainless steel. Rotation of the hand crank winds the filament and rotates the latch. As the latch rotates clockwise, it slides along the top of the cam, causing the cam to rotate counter-clockwise. The cam intersects with the beam spring, deflecting it as it continues to rotate. At the angle of release, the latch slides past the end of the cam (Figure 25). The cam, now free to rotate, is accelerated by the unloading of the beam spring.

6.2 Design Challenges
Each design improvement came with its own challenges. The mounting of the beam spring did not allow for the variation of the beam spring's deflection and thus its maximum velocity. To solve this, a linear motion guide was added to allow for fine adjustment of beam deflection. Mounting the accelerometer also posed difficulties. Even the smallest available accelerometer would increase the width of the cam by five times if placed to measure tangential acceleration. The cam rotates with the appendage and
enlarging it would drastically increase the force needed to rotate the appendage at the desired rate. Instead, with only a 50% increase in the width and length of the cam, the accelerometer was mounted axially, measuring radial centripetal acceleration (Figure 24).

6.3 Summary

The model redesign loses none of the variability achieved with the original design and overcomes its limitations. The accelerometer should allow data collection without time-consuming digitization. The use of the beam rather than compression spring expands the model's possible range of velocities and accelerations and is simple enough to mathematically analyze. Thus, the redesigned physical model should meet all of the design requirements.

The building of the next iteration of the physical model involved three steps. First, the model was designed on paper to include all of the necessary improvements. This was the focus of this chapter. Rough calculations were used to approximate the dimensions of the mechanism to achieve the required velocity and accelerations. Second, a mathematical model was developed based on the proposed design which predicted the kinematic behavior of the physical model given the particulars of the proposed design. This allowed fine-tuning of the model's dimensions before building the physical model. The physical model was then built and its kinematics compared to those predicted by the mathematical model. This allowed an evaluation of the importance of the simplifications made in the mathematical model. Chapter 7 will discuss the details of the mathematical analysis. Chapter 8 compares the predicted and experimental kinematic data of the model redesign.
This chapter presents a mathematical model of the mechanics of the redesigned physical model. The mathematical model served two purposes. The mathematical model was used to fine-tune geometry of the redesigned model to ensure that model would produce the desired kinematic output. Second, predictions using the mathematical model were to serve as a baseline in drag experiments of the model kinematics in the absence of fluid.

The mathematical model predicts the velocity and acceleration profiles of the rotating model appendage. The model calculates the instantaneous acceleration from the quotient of the moment of inertia of the appendage and the instantaneous torque. Torque is a function of the angle and distance at which the force of the beam spring is applied. This chapter details the derivation of the predicted redesigned model kinematics.

7.1 Overview of Analysis

The mathematical analysis should be tailored to predict the motion of the model in terms of variables that can easily be measured while the model is in use. The model can be run with different appendages and beam springs which vary in length and cross-section. Once the appendage is attached and the beam spring is secured, the model can be adjusted to raise and lower the beam spring by thousandths of an inch in order to
finely adjust maximum velocity by controlling deflection. Thus, for any appendage and beam spring configuration, we would produce an expected acceleration and velocity profile in terms of the beam spring height.

In order to predict the model's motion, we started with the fundamental equation describing the driving forces and the resulting motion. The deflected beam spring exerts a force on the cam causing the appendage to rotate. How quickly it rotates is a function of the torque applied and the appendage's moment of inertia. The equation for torque is

\[ T = I \alpha \]  

where, \( T \) is torque and \( I \) is the moment of inertia of the appendage and \( \alpha \) is the angular acceleration. Solving for angular acceleration,

\[ \alpha = \frac{T}{I} \]  

we have the two quantities that we need to predict the model's motion; torque and the appendage's moment of inertia.

The moment of inertia is a constant and is therefore a straightforward calculation based on the geometry of the model. Torque is more complex. Torque is dependent on the magnitude of the beam spring's force and the angle and distance at which it is applied. Each of these components change as the appendage rotates. An equation for torque during appendage rotation will be found by writing each as a function of appendage angle. In order to determine the acceleration profile that results from a force that changes
with appendage angle, we need to introduce time. By discretizing equation 14 in time and approximating the torque during each time step, we can solve for the motion of the appendage as a function of time.

### 7.2 Moment of Inertia

Calculation of the moment of inertia of the rotating appendage will have to take into account the moment of inertia of the cam. The force of the beam spring is actually exerted on the cam, which rotates with the appendage as one body. We will call this body the “appendage assembly”. The moment of inertia of any complex body is the linear combination of the moments of its individual parts. The moments of the cam and the appendage, thus, will be calculated individually and summed.
Neither the cam nor the appendage rotates around its center of mass. For any body, the moment of inertia about a point other than its center of mass is given by the parallel axis theorem.

\[ I_z = I_{cm} + mr^2 \]  

(15)

where \( I_{cm} \) is the moment about the center of mass, \( m \) is the object's mass and \( r \) is the perpendicular distance from the center of mass to the point of rotation.

Thus in order to calculate the moment of inertia of the cam or the appendage we need the moment about its center of mass, the objects' mass, and the distance from its center of mass to the point of rotation. For a rectangular solid like the cam, the moment about its center of mass is

\[ I_{cam} = m_{cam} \frac{(h^2 + (a + b)^2)}{12} \]  

(16)

where \( h \) is the height, \( a+b \) is the rectangle's length (Figure 27), and \( m_{cam} \) is the mass of the cam (Beer and Johnston, 1984 [947]). The moment of the cam about the origin will then be

\[ I_{cam} = m_{cam} \left( h^2 + \frac{(a+b)^2}{12} \right) + m_{cam} \left( a - \frac{(a+b)}{2} \right)^2 \]  

(17)

since the center of mass along the x axis would be found at \((a+b)/2\), the cam length divided by two.
Calculation of the moment of inertia of the appendage follows similar steps. For the sake of simplicity, the appendage used here will be a cylindrical appendage. The moment of a cylinder about it center of mass is

\[ I_c = \frac{m_c L^2}{12} \]  \hspace{1cm} (18)

where \( m_c \) is the mass of the cylinder, \( r \) is the radius and \( L \) is the cylinder length (Beer and Johnston, 1984 [947]). Using equations 15 and 18, the moment of inertia of the cylindrical appendage about the origin is

Figure 27: Appendage assembly and beam spring geometry. The origin is at the center of rotation of the appendage assembly. When the appendage assembly is rotated to touch the beam, the point \( f \) on the assembly will touch the point \( e \) on the beam. The coordinates of the point \( e \) are \((X_b, Y_b)\). The length of the line from the origin to \( e \) and \( f \) is \( R \). The distance from the origin to \( f \) perpendicular to the long axis of the cam is \( d \). The height of the cam is \( h \). The horizontal distance from the end of the cam to the origin is \( a \). The distance from the origin to the end of the cam at which the cylinder attaches is \( b \). The cross sectional radius of the cylinder is \( R_c \) and its length is \( c \). The beam spring's length is \( L \) and width is \( w \). The center of mass of the cam is at the horizontal position given by \((a+b)/2\).
\[ I_c = m_c \left( \frac{c^2}{12} \right) + m_c \left( \frac{c + b}{2} \right)^2 \]  

because, as is denoted in Figure 27, \( R_c \) is the cylinder radius, \( c \) is its length and \( b \) is the distance from the end of the cam to the axis of rotation. Combining equations 17 and 19, the total moment of inertia of the appendage assembly for a cylindrical appendage is

\[ I_{tot} = m_{cam} \left[ h^2 + \left( \frac{a+b}{2} \right)^2 + \left( a - \frac{a+b}{2} \right)^2 \right] + m_c \left( \frac{c^2}{12} + \left( \frac{c}{2} + b \right)^2 \right) \]  

(20)

7.3 Torque

The torque applied by the beam spring on the cam assembly will determine the acceleration profile of the appendage and will change as the appendage rotates. Torque is given by the magnitude of the force and the distance and angle at which the force is applied (Figure 28).

\[ T = \frac{F_b R_p}{\cos(\theta)} \]  

(21)

Here \( T \) is torque. \( F_b \) is the force exerted by the beam. The perpendicular distance at which the force is applied is designated by \( R_p \), and \( \theta \) is the applied angle. The force on the appendage is proportional to the deflection of the beam. As the beam spring unloads, the applied torque will rotate the appendage, deflecting the beam spring less, while changing the distance and angle at which the force is applied. Thus, torque will vary with appendage angle.
Since the angle and distance at which the torque is applied changes as the appendage rotates, the force exerted on the beam spring as it unloads will be written as a function of appendage angle. It is complex to solve the equation for the motion of a beam spring with an applied end-point load that is a function of appendage angle. In order to avoid this non-linear differential equation, we can simplify this to a discretized statics problem if the time it takes to accelerate the appendage is much slower than the natural frequency of the beam. At any instant, the force exerted by the beam spring can be calculated as if the beam spring were not in motion. In order to ensure that this is a valid simplification, the model will be designed to use beams with unloading periods that

Figure 28: Torque is a function of the force, $F_b$, and the angle, $\theta$, and distance, $R_p$, at which it is applied.
are at minimum of 1/10 the time needed to rotate the appendage at the maximum velocity through the angles the beam spring will be applying force. The natural frequency of the beam spring should be high enough that the beam's motion will be controlled by the time it takes to rotate the appendage.

### 7.3.1 Natural frequency versus Acceleration Time

#### 7.3.1.1 Beam Spring Natural Frequency

In order to ensure that the beam spring is not limited by its natural frequency, we must calculate both a) the natural frequency of the cantilevered beam spring and b) the time it takes to rotate the appendage assembly through the desired angles. The natural frequency of a cantilevered beam is a solved problem with natural frequencies equal to

\[ \omega_n = (Lk_n)^2 \sqrt{\frac{EI}{m_b L^3}} \]  

(22)

where E is the beam's elastic modulus, I is the cross sectional moment of inertia of the beam, \( \omega \) is angular frequency in radians per second, \( m_b \) is the mass of the beam and L is its length. Values for \( k_n L \) corresponding to the different modes are given in Table 1 (Maymon 2008).

<table>
<thead>
<tr>
<th>n</th>
<th>k_nL</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.87510</td>
</tr>
<tr>
<td>2</td>
<td>4.69409</td>
</tr>
<tr>
<td>3</td>
<td>7.85478</td>
</tr>
<tr>
<td>4</td>
<td>10.99554</td>
</tr>
<tr>
<td>5</td>
<td>14.13717</td>
</tr>
<tr>
<td>6</td>
<td>17.27876</td>
</tr>
</tbody>
</table>

Table 1: Normalized eigenvalues for a uniform cantilever beam
Since we are trying to ensure that the minimum natural frequency of the beam spring is above a threshold, we can use the smallest value for $k_nL$, 1.87510.

In order to apply equation 22 to our beam, we need the beam's cross sectional moment of inertia. For a square beam spring the moment is

$$I_b = \frac{w^4}{12}$$

(23)

where $w$ is the beam spring width (Beer and Johnston, 1984 [947]). The mass of the beam, $m_b$, can be rewritten

$$m_b = \rho L w^2$$

(24)

where $\rho$ is the density of the material composing the beam, $L$ is the length and $w^2$ is the cross sectional area. Linear beam theory imposes a minimum length to width ratio of 10:1. We will impose a constraint on the length such that,

$$L = 11w$$

(25)

Substituting equations 25, 24, 23 and the lowest value for $k_nL$ into equation 22, and using the values for a stainless steel square beam spring of $E = 1.9305 \text{e}^{11} \text{ Pa}$ and $\rho = 7944 \text{ kg/m}^3$, we find the that first mode of the natural frequency of a cantilevered square beam spring with a length 11 times its width is,
Given a natural frequency, $\omega_1$, the time for the beam spring to unload will be $\frac{1}{4}$ the period of the motion, or $1/(4\omega_1)$. Thus, for cantilevered square stainless steel beam spring of any width with no external load, the time to move from fully deflected to non-deflected due to its own internal stiffness and mass is

$$T_{unload} = 0.0060w$$  \hspace{1cm} (27)

This is the time it would take the beam spring to oscillate at its natural frequency from a position of maximum deflection to no deflection. This corresponds to the motion the beam spring will travel as it accelerates the appendage assembly. We want this time to be much smaller than the time needed to accelerate the appendage, discussed in the next section, to ensure that we can ignore the dynamics of the beam spring when calculating the acceleration profile of the appendage.

### 7.3.2 Approximate Time to Accelerate Appendage

The time required to rotate the appendage through the angles the beam spring will be applying a force must be at least ten times the time required to unload the beam. The rotation time will be approximated by the time needed to rotate the appendage tip at the maximum velocity mantis shrimp have been observed striking, 30 m/s. This time will be a conservative estimate. Thus, this simplification will further decrease the likelihood that the motion will be limited by the beam spring dynamics.
To determine the angles through which the appendage is rotated as it is accelerated, we need to know both the angle at which it is released and the angle at which the beam spring stops accelerating the appendage. The geometry fixes the latch release angle, which we will call $\theta_R$. The beam spring will no longer accelerate the appendage when it returns to its non-deflected position. The angle at which the beam spring stops accelerating the appendage, $\theta_3$ (see Figure 29), can be calculated from the geometry of the model and will be a function of beam spring height, $Y_b$. The beam spring height is

![Figure 29: Geometry of the appendage assembly and the beam spring when the appendage assembly is rotated to just touch the non-deflected beam. The point of contact between the beam spring and the appendage assembly, e, has x and y coordinates, $X_b$ and $Y_b$ respectively. The angle of rotation of the appendage assembly, $\theta_3$ can be found from the difference between $\theta_1$ and $\theta_2$. $R$ is the line from the origin to the point e. $\theta_1$ is the angle the line R makes with the horizontal. $\theta_2$ is the angle the line R makes with the long axis of the cam.](image)
the distance from the end of the beam, point e, to the origin. We will assume that the beam spring will stop accelerating the appendage at the angle where the assembly is rotated to just touch the beam. This angle, \( \theta_3 \), will be found by calculating the difference between \( \theta_1 \) and \( \theta_2 \), where \( \theta_1 \) is the angle the line R makes with the horizontal, and \( \theta_2 \) is the angle R makes with the long axis of the cam (see Figure 29). R is the line from the origin to the point of contact, e. Its length is given by

\[
R = \sqrt{Y_b^2 + X_b^2}
\]  

(28)

where \( Y_b \) is the vertical distance from e to the origin, and \( X_b \) is the horizontal distance. These values can also be used to calculate the angle, \( \theta_1 \), the angle that line R makes with the horizontal according to

\[
\theta_1 = \tan^{-1}\left(\frac{X_b}{Y_b}\right)
\]  

(29)

The angle, \( \theta_2 \), between the line R and the long axis of the appendage assembly is equal to

\[
\theta_2 = \sin^{-1}\left(\frac{d}{R}\right)
\]  

(30)

where \( d \) is defined as in Figure 29. The angle at which the beam spring stops accelerating the appendage, \( \theta_3 \), will then be the difference between \( \theta_1 \) and \( \theta_2 \).

\[
\theta_3 = \tan^{-1}\left(\frac{Y_b}{X_b}\right) - \sin^{-1}\left(\frac{d}{\sqrt{Y_b^2 + X_b^2}}\right)
\]  

(31)
where \( X_b \) and \( d \) are constants. Thus, for any beam spring height, \( Y_b \), the appendage assembly will be accelerated from the angle at which the latch releases through \( \theta_a \) degrees where

\[
\theta_a = \theta_R - \text{atan}\left( \frac{Y_b}{X_b} \right) - \text{asin}\left( \frac{d}{\sqrt{Y_b^2 + X_b^2}} \right)
\]  

(32)

The time needed to rotate through \( \theta_a \) degrees will vary with the velocity of the appendage assembly. We are interested in approximating the time needed to move the tip of the appendage at a tangential velocity of 30 m/s. This will depend on the distance from the tip of the appendage to the point of rotation. That distance is given by the sum of \( b \) and \( c \) (Figure 27), where \( b \) is the distance from the point of rotation to the beginning of the appendage, and \( c \) is the length of the appendage. The linear distance the tip of the appendage travels as it rotates through \( \theta_a \) degrees is equal to

\[
D = 2 \pi (b + c) \times \frac{\theta_a}{360} m
\]

(33)

Moving at 30 m/s, this will take

\[
t = \frac{D}{30 \frac{m}{s}}
\]

(34)
seconds. Substituting equations 32 and 33 into equation 34, we get the time to move the appendage at 30 m/s through the angles during which the beam spring will be exerting a force.

\[
\begin{align*}
    t_{\text{driven}} &= \frac{(2\pi(b + c)) \cdot (\Theta_r - \text{atan}\left(\frac{Y_b}{X_b}\right) - \text{asin}\left(\frac{d}{\sqrt{Y_b^2 + X_b^2}}\right))}{30 \text{ m/s} \cdot 360} \\
\end{align*}
\]

(35)

The proposed geometry allows \(Y_b\) to vary from 8 mm to 13 mm, \(X_b\) is 3 mm, \(b\) is 14 mm and \(c\) is 24 mm giving a range of rotation times from 2.1 to 2.3 milliseconds. For 6 mm, 8mm and 10mm square beams, from equation 27, the natural frequency gives unloading times of 36.2, 48.3 and 60.5 microseconds, respectively. The unloading times are shorter than the time to accelerate the appendage by a factor of 100. Thus, we can be sure that the dynamics of the unloading of the beam spring are dominated by the acceleration of the appendage and not by the natural frequency of the beam. This will allow us to approximate the motion of the appendage using fairly simple static beam equations.
7.3.3 Torque as a Function of Appendage Angle

To determine the velocity profile of the appendage as it is accelerated by the unloading beam, we need to describe how the force changes as the appendage rotates and determine the motion that results. This will be done in two steps. Here in section 4, the
changing magnitude of the torque will be described as a function of the appendage angle. In section 5, we will combine torque, the moment of inertia of the appendage assembly, and the angles over which the torque is applied, to determine the kinematics of the appendage by discretizing the problem in time.

7.3.4 Force as a Function of Appendage Angle

Calculation of the torque as a function of appendage angle requires solving for the magnitude of the force on the cam, $F_p$, and the perpendicular distance from the point of rotation at which the force is applied, $R_p$ (Figure 30). Each of these will be written as a function of the angle of the appendage from the horizontal. From static beam equations we know that the force needed to produce a deflection for a cantilevered beam is

$$F_b = \frac{3EI_b x}{L^3}$$

(36)

where $x$ is the deflection perpendicular to the beam's axis, $F_b$ is the force in the same direction, $E$ is the elastic modulus, $I_b$ is the moment of inertia of the beam spring as given by equation 23 and $L$ is the beam spring length (Case and Chilver 1971 13.19 pg 218).

In equation 36, the force of the beam spring was written in terms of beam spring deflection. We need to determine the deflection in terms of rotation angle. The deflection of the beam spring will be the difference between the $x$ value of the point e when non-deflected, $X_b$, and when deflected, $X_t$ (Figure 30).

The calculation of the $x$ value of the point e for a given deflection is involved. In order to find $X_t$, we will model the rotation of the surface of the cam that comes into
contact with the beam spring as the line $\overline{TC}$ (see Figure 31). The height of the beam spring changes infinitesimally during the small deflections used here. We will assume that it is constant. As the appendage assembly rotates, the point of contact it makes with the beam spring will always be along the horizontal line $y = Y_b$. The line $\overline{TC}$ rotated through angle $\theta$, shall be designed as $\overline{TC_r}$. Thus, the point of contact between the appendage assembly and the beam spring will lie at the intersection of the lines $\overline{TC_r}$ and $y = Y_b$. The $x$ value at the point of contact, $X_t$, for any appendage assembly rotation angle can be found by solving at $Y_b$ the equation of the line $\overline{TC}$, rotated about the origin.

Rotation of the line $\overline{TC}$ about the origin can be done by rotating each of the points T and C individually about the origin and calculating the equation of the line that goes through the two rotated points. The line $\overline{TC}$, when horizontal, can be described by the equation $Y = d$ for $-a < x < b$. Thus, the points T and C have the coordinates $(-a, d)$ and $(b, d)$ respectively. We rotate these points about the origin according to
where \( x \) and \( y \) are the coordinates of each point when the line is horizontal, \( \theta \) is the new angle of the center line of the appendage assembly measured from the horizontal, and \( X \) and \( Y \) are the rotated point coordinates.

The equation of the new line, \( \overline{TC}_r \), can then be found to be

\[
y = mx + b
\]  

where

Figure 31: Rotation of the line along the top of the cam. The line, \( \overline{TC} \) lines along the top of the cam when horizontal and goes through the points \( T(-a,d) \) and \( C(b,d) \). \( \overline{TC}_r \) is the line \( \overline{TC} \) rotated about the origin through the angle \( \theta \). \( \overline{TC}_r \) is found by rotating the points \( T \) and \( C \) about the origin and calculating the line which goes through the rotated points. The point, \( e \), lies along the horizontal line \( y = Y_b \). The \( x \) coordinate of the point \( e \) will lie at the intersection of the lines \( y = Y_b \) and \( \overline{TC}_r \).
\[ m = \frac{Y_T - Y_C}{X_T - X_C} \]  \hspace{1cm} (39)

and

\[ b = Y_T - \frac{Y_T - Y_C}{X_T - X_C} X_T \]  \hspace{1cm} (40)

Here the rotated x and y values for point \( T \) are designated \( X_T \) and \( Y_T \) and the rotated x and y coordinates for point \( C \) are \( X_C \) and \( Y_C \). This line intersects the point \( e \) at the y value equal to the height of the beam, \( Y_b \). Thus, we can use the equation of the line to find the x coordinate of the point of contact between the beam spring and cam, \( X_t \), at any given angle by finding the x value corresponding to a y value of \( Y_b \). Solving equation 38 for \( x \) and substituting \( Y_b \) for \( y \) we get

\[ X_t = \frac{Y_b - b}{m} \]  \hspace{1cm} (41)

Substituting equations 40 and 39 into equation 41, using the proposed values for the coordinates of \( T \) and \( C \) (Figure 31), and simplifying, \( X_t \) can be written

\[ X_t = (Y_b + asin\theta - dcos\theta \tan\theta(acos\theta - dsin\theta)) \cot\theta \]  \hspace{1cm} (42)

Equation 42 gives us an expression for the x coordinate of the point of contact between the beam spring and the appendage assembly for any rotation angle.

To recap, in this section we are finding an expression for the force acting tangentially to the motion of the appendage as a function of rotation angle. From the coordinates of the point of contact between the beam spring and the cam, we can
calculate the deflection of the beam. The force of the beam spring in the x direction can be found from the deflection. With the appendage rotation angle and the force in the x direction, we can calculate the force that the beam spring exerts on the appendage assembly tangential to the assembly's motion. This force is one of three pieces needed to determine torque as a function of rotation angle, which is in turn necessary to find the acceleration profile of the appendage.

We have found the x coordinate, $X_t$, of the point of contact between the beam spring and the cam. The deflection of the beam spring can be determined from $X_t$. The non-deflected beam spring makes contact with the appendage assembly at x equal to $X_b$. The deflection of the beam, $x$, will simply be the difference between $X_b$ and $X_t$ (Figure 30)

$$x = X_b - X_t$$  \hspace{1cm} (43)

Substituting equation 42 into equation 43, the deflection of the beam spring as a function of rotation angle can be written

$$x = X_b - (Y_b + asin\theta - dcos\theta - tan(-acos\theta - dsin\theta))cot\theta$$  \hspace{1cm} (44)

Thus, the force of the beam spring acting perpendicular to the axis of the beam spring can be written in terms of the rotation angle of the appendage assembly (from equations 44 and 36).
Equation 45 describes the beams force in the x direction. The component of the force applied by the beam spring tangential to the motion of the appendage is dependent only on \( F_b \) and the appendage angle. From equation 21, we have the component of the force acting tangential to the motion equal to \( F_b / \cos(\theta) \). From Figure 30 we can see that \( \theta_4 \) is simply the complement of \( \theta \).

\[
\theta = 90^\circ - \theta_4
\]  

(46)

Thus, the force can be written in terms of the appendage assembly rotation angle

\[
F_p = \frac{F_b}{\sin(\theta)}
\]  

(47)

Substituting equation 47 into equation 45, we write the force of the beam spring acting tangential to the motion of the appendage assembly as

\[
F_p = \frac{3EI_b (X_b - (Y_b + \sin \theta - \cos \theta - \tan(\theta)(-\cos \theta - \sin \theta)) \cot \theta)}{\sin \theta L^3}
\]  

(48)

7.3.5 Radius of Applied Torque as Function of Angle

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The calculation of torque as a function of rotation angle requires expressions, as functions of rotation angle, for both the force acting tangential to the assembly's motion and the distance from the point of rotation at which the force is applied. In section 3.1 we found the equation for that force (equation 48). Here we will find an expression for the distance at which the force is applied.

The distance at which the beam spring applies a moment on the appendage assembly will vary with appendage angle. Once the x and y coordinates, $X_t$ and $Y_b$ of the point e are known, they can be used to calculate the direct distance from the origin to the point e, $R_d$ (Figure 30).

$$R_d = \sqrt{X_t^2 + Y_b^2} \quad (49)$$

The perpendicular radius at which the force is applied can then be determined according to

$$R_p = \sqrt{R_d - d^2} \quad (50)$$

which can be re-written substituting equation 49 into equation 50.

$$R_p(\theta) = \sqrt{X_t^2 + Y_b^2 - d^2} \quad (51)$$

Here $X_t$ is given by equation 30 and $Y_b$ and d are constants from our geometry. Thus, $R_p$ is written as a function of only $\theta$.

7.3.6 Final Equation for Torque as a Function of Rotation Angle
The torque exerted on the appendage assembly by the beam spring is given by equation 13 as simply the product of $F_b/cos(\theta)$ and $R_p$. Substituting equations 51, 47 and 45 into equation 13, we can write torque as a function of the appendage assembly rotation angle.

$$T(\theta) = \sqrt{X_t^2 + Y_b^2 - d^2} \frac{3EI_b}{3EI_b} \left( X_b - (Y_b + a \sin \theta - a \cos \theta - \tan \theta(\cos \theta - \sin \theta)) \theta \right) \frac{cot \theta}{\sin \theta L^3}$$ (52)

### 7.4 Appendage Motion as a Function of Time

Thus far we have determined the moment of inertia of the assembly and the applied torque as a function of rotation angle. We have determined that the natural frequency of the beam spring is high enough that the beam spring dynamics do not influence the motion of the assembly. If the torque were constant, this would mean that we could find the acceleration by simply dividing the torque by the assembly's moment of inertia. With a constant acceleration, we could take the integral of acceleration over our time domain to find an equation of velocity as a function of time.

But the torque here changes with rotation angle. Acceleration, then, is not constant and analytically solving for acceleration and velocity becomes very complex. A numerical approach will allow us to find a solution by approximating the acceleration as constant during very small time steps. In general terms, we determine the rotation angle of the assembly, and from equation 52, find the corresponding torque. Instantaneous acceleration is found using equations 14 and 20. We approximate a linearly varying acceleration by taking the average between the present acceleration and that value during the last time step. Assuming an average value over the time step, the change in velocity is
calculated from the derivative of the acceleration. We follow a similar process to approximate the change in angle during the time step, assuming a constant velocity. The new angle is then used to find the torque at the next time step using equation 52. If the time step is small enough, this method should provide us with a reasonable approximation of the velocity and acceleration profiles as a function of time.

To solve for velocity and acceleration of the appendage assembly as a function of time, we must first define initial conditions. When the latch releases and the acceleration of the assembly begins, the angular velocity, \( \omega \), is zero. The initial angle is the angle of release, \( \theta_R \) degrees. Angular acceleration, \( \alpha \), is found from equations 52, 20 and 14.

Finding the angular velocity at the next time step requires approximating the derivative of angular acceleration during this time step. There are many ways to do this. Choosing among the first order approximations, the backward difference approximation leads to the least difficulties. Since the angular acceleration varies with rotation angle, \( \alpha_{i+1} \) is dependent on \( \theta_{i+1} \). Forward difference approximations would result in a dependency circle with \( \theta_{i+1} \) calculated from \( \omega_{i+1} \) which in turn would be calculated from \( \alpha_{i+1} \). A backwards difference approximation for angular velocity avoids this and will also be used to approximate the change in angle for consistency. Here we use a backwards trapezoidal rule\(^1\). Thus, angular velocity is approximated

\[
\omega(i + 1) = \omega(i) + \frac{\alpha(i-1) + \alpha(i)}{2} \times \Delta t
\]

\( (53) \)

\(^1\) A second order Adams-Bashforth formula could also be used.
For all time steps after the first, we similarly use the present and last values of angular velocity to calculate the angle at the next time step.

\[ \theta(i+1) = \theta(i) + \Delta t \times \frac{\omega(i-1) + \omega(i)}{2} \]  

(54)

The angular acceleration is determined at this new angle from

\[ \alpha(i+1) = \frac{\text{Torque}(i+1)}{I_{\text{tot}}} \]  

(55)

where the torque is determined from the appendage rotational angle as in equation 52 and the assembly's moment of inertia is given in equation 20. This new angular acceleration is used to iterate the next time step until the angle is reached at which the beam spring no longer touches the appendage assembly. The time step used was 1 ns. Strikes take between 1 and 2 ms, depending on the velocity. This results in one to two million time steps during the period of acceleration.

The angular velocity and acceleration for the first time step must be treated differently since we have no values for angular acceleration and velocity at time minus one. Thus, we assume a constant angular acceleration for the first time step such that \( \alpha_1 \) equals \( \alpha_0 \). This gives a value for angular velocity at the second time step. The approximation of the rotation angle at the next time step is made with this value using a forward difference approximation rather than assuming no angular rotation as would result from the backward approximation.
Stepping through time, we can thus find the angular velocity and acceleration of the appendage assembly between the angle of release and the last angle that the beam spring accelerates the appendage.

### 7.5 Plots and Discussion

The above calculations result in predicted acceleration profiles for any beam spring width and height. Maximum velocity versus beam spring height is plotted for each beam spring width in Figure 32. We can see that the model predicts that the physical model will be able achieve the maximum velocity and accelerations observed in the animal strikes of at least 30 m/s and 1e5 m/s\(^2\).

Looking at general trends made apparent from these graphs, as we would expect, the maximum velocity achievable by each beam spring is proportional to its cross-sectional width. It requires more force to deflect a thicker beam spring by the same amount as a thinner one. When released, the thicker beam spring will accelerate the appendage with more force, resulting in a larger maximum velocity. Thus, we see in Figure 32 that the maximum velocities and accelerations are produced by the largest 10 mm beam, depicted in darkest gray.

Additionally, for any given beam, a lesser beam spring height results in a larger angle over which the beam spring accelerates the appendage (see Figure 33). The appendage assembly needs to rotate less to make contact with the beam spring at a lower beam spring height. Since the beam spring is always released at the same angle, this leads to a greater deflection and larger maximum velocities as can be seen in Figure 32.

In these ways, the predictions made by the mathematical model are as we would expect.
One surprising point about these predicted profiles should be commented on. As can be seen in Figure 32 A, is that the greater the beam spring height, the greater the maximum acceleration. At a height of 9 mm, an 8 mm beam spring has a maximum acceleration of $2.5e4 \text{ m/s}^2$ while at a height of 6 mm the maximum acceleration is only $2e4 \text{ m/s}^2$. This is unexpected because a $Y_b$ of 9 mm corresponds to less beam spring...

Figure 32: Velocity (A) and acceleration (B) profile ranges for each beam spring width. The velocity and acceleration profiles for each beam at beam heights of 6 and 10 mm show the kinematic range predicted for each beam.
deflection that a height of 6 mm (see Figure 33). From this factor alone we would expect the maximum force exerted by the beam spring on the assembly to be greatest at the lowest height. The acceleration at a height of 9 mm drops off much faster and acts for less time than for a height of 6mm, thus producing a smaller maximum velocity, as is expected. But, acceleration being greatest for beam spring heights that are larger seems counter-intuitive. A closer look at the factors that go into determining angular acceleration explains this result.
From equation 14, we recall that acceleration is proportional to the torque divided by the moment of inertia of the assembly. The moment is a constant. Torque, however, is a function of the force of the beam, the angle and the distance from the axis of rotation at which it is applied (equation 52). The angle of release is constant. But, as the height of the beam spring increases, both the deflection of the beam spring and radius at which the force is applied change. Torque is linearly dependent on both deflection and applied radius, Rp (equations 21 and 48). While a decrease in deflection results in a smaller applied force, the corresponding increase in radius will always dominate the combined effect (see Figure 34). Thus, counter-intuitively, as the beam spring height increases, decreasing the deflection, the maximum acceleration increases.

The velocity and acceleration profiles predicted by the mathematical model behave as would be expected. The acceleration starts at a maximum and decreases as the deflection of the beam spring is unloaded. The corresponding velocity starts at zero and increases as the acceleration is positive, reaching a maximum at the end of the beam's influence. Decreasing the height of the beam spring increases the maximum velocity but, counter intuitively, decreases the maximum acceleration. Practically, then, to increase the velocity, the beam spring should be lowered. To increase the acceleration, the beam should be raised. To increase the velocity and acceleration, a larger beam spring should be used at a greater height.
Two corrections were made to the mathematical model to make it a more accurate model of the actual physical system as built. First, as with most prototypes, the final working physical model had to be altered slightly from the proposed design to make it fully functional. This resulted in slight variations between the dimensions of the physical geometry and the values originally used in the mathematical model of the structure. Each of these values was an input of the mathematical model and easily redefined.

Measurements were taken from the physical model and updated in the mathematical
analysis. The final values of each of the geometric parameters are compared with the original values in Table 3.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Designed Value</th>
<th>Measured Value when Built</th>
</tr>
</thead>
<tbody>
<tr>
<td>Yb</td>
<td>8 to 13 mm</td>
<td>6 to 10 ± 0.5 mm</td>
</tr>
<tr>
<td>Xb</td>
<td>3 mm</td>
<td>3.0 ± 0.5 mm</td>
</tr>
<tr>
<td>Rc</td>
<td>2.42 mm</td>
<td>2.4 ± 0.1 mm</td>
</tr>
<tr>
<td>$\Theta_r$</td>
<td>76 degrees</td>
<td>76 ± 5 degrees</td>
</tr>
<tr>
<td>a</td>
<td>24 mm</td>
<td>24 ±0.1 mm</td>
</tr>
<tr>
<td>b</td>
<td>14 mm</td>
<td>10.6 ± 0.1 mm</td>
</tr>
<tr>
<td>c</td>
<td>24 mm</td>
<td>24 ±0.1 mm</td>
</tr>
<tr>
<td>d</td>
<td>5 mm</td>
<td>3.4± 0.1 mm</td>
</tr>
<tr>
<td>Cam width</td>
<td>9.5 mm</td>
<td>9.5 ±0.1 mm</td>
</tr>
<tr>
<td>Cam height</td>
<td>12.7 mm</td>
<td>12.7 ± 0.1 mm</td>
</tr>
</tbody>
</table>

Table 2: Final values and proposed values for geometry of the model. Variation from the proposed design is highlighted in bold.

Second, the structure supporting the beam spring was modeled as perfectly rigid. In fact, each bolt hole allowed some movement and each structural member deflected slightly. Instead of the loading being taken up entirely by the deflection of the beam spring, some of it was distributed throughout the structure. This resulted in the beam spring itself deflecting less and returning less energy than was predicted. In a complex structure, modeling this behavior is quite difficult.

In order to quantify this effect, the entire mechanism was put into a material testing machine (Instron ElectroPuls E3000, Instron Corp. Norwood, Ma). The force-deflection curve was measured of the structure loaded to the deflection that is ordinarily
seen during operation. These measurements were done for three different beam springs (Figure 36). Each force deflection curve was incorporated into the mathematical model.

Figure 35 shows that the predicted force for any deflection was much higher than the model actually produced. The maximum force produced by the mechanism was only one third of the predicted force. The variation between beam springs of different cross sectional widths was also much lower than is expected. Each of these trends was consistent with each beam spring deflecting less than was predicted. This could be accounted for by a rotation, rather than deflection, of the beam spring and/or the deflection of other less stiff members of the structure.

Measurements taken from high-speed video of the beam during a strike confirmed movement at both ends of the beam spring. The location of the top and bottom of the beam spring were digitized using methods described in section 2 of Chapter 5. While the bottom of the beam spring deflected the amount predicted by the model within the margin of error of measurement (0.3 mm), the top of the beam spring did not. Deflection of 1.5 ± 0.3 mm was observed where the model predicted none. This decreased the total beam deflection by 20% and the resultant force by up to 500 N.\(^1\)

\(^1\) For a 10 mm beam with a force/deflection slope of 321 N/mm calculated from Figure 36
7.7 Conclusion

The method used to predict the kinematics of the physical model involved, fundamentally, two steps. Angular acceleration was found from the quotient of the moment of inertia of the appendage assembly and the torque applied to it. The moment of inertia was a straightforward calculation of the linear combination of the moments of

Figure 35: Force versus Deflection: The predicted force for each beam is shown in the three top lines marked with hollow markers. The measured force for each beam is given by the lower three lines, denoted with filled in markers.
the cam and the appendage each rotated about a point distant from their center of mass. Finding the applied torque required a few more steps.

The torque applied to the assembly was the product of the force produced by the deflection of the beam spring and the angle and distance at which it was applied to the assembly. Each of those three contributors changed as the assembly rotated and had to be written in as functions of assembly angle. An equation for torque as a function of rotation angle was thus found.

To find the resultant motion of the assembly when accelerated by this rotationally varying torque required discretizing the equation in time. For any rotation angle, the corresponding torque was used to find the instantaneous angular acceleration. Approximating the derivative of this with the trapezoidal rule allowed us to find the angular velocity at the next time step. A similar process was used to find the next angular position. This was then used to find the next value for torque to start the process over for the next time step. In this way we found the acceleration and velocity profiles for any beam spring height and width. Energy was conserved in this analysis. The final kinetic energy was within 0.0022 % of the stored elastic energy in the beam when run with a time step of 1 nanosecond. The model of the was improved by incorporating measured force-deflection curves for each beam width and using the values of the geometry as actually built.

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1 The stored elastic energy was found by numerically integrating the force times the deflection from zero to the maximum deflection. The kinetic energy is 1/2 Iω², where I is the moment of inertia of the appendage assembly and ω is the angular velocity.
CHAPTER 8

EVALUATION OF MATHEMATICAL MODEL

In this chapter we evaluate the mathematical model. Predictions made with the updated mathematical model are compared to the velocities and accelerations produced by redesigned physical model. The mathematical model was evaluated according to two standards: validity and accuracy. First we compare the experimental to the predicted model kinematics to validate the model. Second, the predicted and experimental values are compared to determine if the mathematical model is accurate enough to use as a baseline for drag studies.

8.1 Comparison of Experimental and Theoretical Kinematic Data.

The predictions made with the corrected mathematical model were compared with experimental data. Kinematic measurements were collected with high-speed video of the redesigned model run in air. Data was analyzed with methods described in section 2 of Chapter 5. Accelerometer measurements were not used, as will be discussed in section 3 of this chapter.

Since the model was designed to produce a range of velocities and accelerations, many comparisons between mathematical predictions and experimental data can be
made. Results for one beam at two different heights are shown in Figure 36\(^1\) and were selected because they are typical. Velocity profiles are shown with a white background. Acceleration profiles are on light gray.

Each plot in Figure 36 compares three sets of kinematic data. Experimentally collected kinematic measurements are shown as solid gray lines. The dashed lines are mathematical predictions made with the uncorrected mathematical model. The predictions for the mathematical model, corrected as discussed in the last section, are shown with a dotted line centered in the black shaded area. The black swath represents the uncertainty of the mathematical prediction\(^2\).

Three general trends can be seen from the four graphs in Figure 36\(^3\). As would be expected from relationship between the predicted force-deflection curves and those measured, the corrected model produced maximum velocities and accelerations.

\[\text{\textsuperscript{1} For the sake of brevity, similar plots for the remaining beam/height combinations are found in Appendix B.}\]

\[\text{\textsuperscript{2} Each measurement was taken off the model three times. The variation among the measurements was taken to be the uncertainty. The model was run with every combination of high and low values for each measurement to determine the range of uncertainty. For instance, the angle at which the latch releases the cam was measured to be 76 degrees with a ±5 degree uncertainty and the beam height could be determined within 0.5 mm. The mathematical model was run with a release angle of 71 and 81 degrees. Each of these settings were run with two different values for the beam height, and so on for each measurement. In this way, each possible combination was compared to every other possible combination. The blackened region depicts the range of predicted values for each beam height and width within the measurement uncertainty.}\]

\[\text{\textsuperscript{3} Plots of other beam widths and heights can be found in Appendix A.}\]
approximately one half the magnitude of the idealized model. Lower force at any given deflection resulted in a lower acceleration and, thus, a corresponding lower final velocity.

Second, the experimental acceleration profiles rarely started at a maximum and decrease as is predicted. Most started at zero, quickly reached a maximum often far above the predicted value and oscillated with damped amplitude to a minimum. This trend resulted in a corresponding early hump in the velocity profiles as can be seen in both beam height velocity plots. Despite these variations, the final velocities most often lie within or slightly below the predicted maximum velocity range.

The third point concerns the variation in experimental data. While the acceleration profiles often oscillated two to three times around the predicted profile during a strike, the magnitude and timing of the oscillations were very inconsistent. This was most apparent in Figure 37, which summarizes the predicted and experimental data for each beam height and width. The maximum predicted accelerations underestimate the experimental values by as much as 830%. The frequency of the oscillations also vary. For a 8mm beam height (Figure 36 D), for instance, the maximum acceleration can be found within the first 1/10 of the strike but can be seen as late as half way through the strike. Thus, the experimental data were not very consistent.

Let us examine Figure 36 and 37 in terms of the two criteria for evaluation of the mathematical model discussed in the introduction to this chapter. First, did the experimental evidence validate the mathematical model? And second, were the predictions made by the mathematical model accurate enough to use as a baseline against which to compare future drag experiments in water?
Figure 36: Predicted and experimental tangential velocity and acceleration profiles for a beam width at two different beam heights. Dashed line are predictions made without mathematical model corrections. Gray lines are the experimental data. The dotted white line is the corrected predicted profile. Black area shows the area of predicted uncertainty.
The experimental data showed that while the mathematical model represented the dominant trend in the model's motion, it did not capture many details. While each of the experimental plots oscillated with a higher frequency, they each approximately oscillated around the predicted profile or slightly below. Figure 36 D is a clear example of this. Each of the experimental acceleration profiles could be decomposed into a low frequency signal that would fall within the black uncertainty range of the predicted profile and a higher frequency oscillation. The mathematical model captures the dominant trend in the acceleration but fails to model the higher frequency oscillation.

The variation between the predicted and experimental values can be accounted for by two simplifications of the mathematical model. The first was unexpected. The second was known. First, the higher oscillation suggests that more than just the beam was storing and releasing elastic energy. Some other structural element was being deflected and the resultant damped oscillation can be seen superimposed on the predicted motion. Second, the trend of the experimental data to fall below the predicted values can be accounted for by the fact that the mathematical model did not include any sort of friction. Any frictional forces would strip energy from the system resulting in lower final velocities.

Thus, the model matched the general trends observed in the experiments, but was not quantitatively predictive. The mathematical model predicted the length of the strike within 1 ms. The magnitude of the experimental maximum velocity fell within the mathematical margin of uncertainty 60% of the time (Figure 37). The remaining values fell below predictions as would be expected for a model that does not incorporate
friction. The mean maximum velocity was at most 56% lower than predicted with a standard deviation of no more than 2.5 m/s. The acceleration range was wider, but the variation appears to be due to the release of stored elastic energy in some member the mathematical model assumed rigid. The mean maximum experimental accelerations varied from those predicted by as much as 230% with a maximum standard deviation of $3.8 \times 10^4$ m/s$^2$.

Figure 37: Maximum experimental and predicted tangential velocities (A) and accelerations (B) for each beam and beam height.
While the mathematical model predicted the dominant motion of the physical model, it did not capture the details in at least two important and problematic ways. First, for any physical model setting, the higher frequency oscillations were irregular. If the higher frequency oscillation were consistent, we could determine the frequency and changing amplitude and incorporate it into the mathematical model. But the great variation seen for any one physical model setting suggests that the mathematical model missed more in its simplifications than just the influence of an additional elastic element. The cause of the variation in magnitude and frequency of the higher harmonic is unknown and appears quite complex. The second issue is similar. Friction was neglected in this mathematical model. But the experimental data offer few insights into a simple modification that would account for it. Some experimental runs showed great dissipation of energy while many others showed very little.

Complex forces left out of the mathematical model's analysis resulted in inconsistent experimental data for any physical model setting. The combination of the complexity of the unaccounted for forces at play in the mechanism and the resultant variability they caused make the mathematical model insufficient, at present, to serve as a base line for physical model kinematics in air.

In summary, the mathematical model captured the dominant forces at work in the model mechanism and can be said to have been validated by in-air experiments. Yet, unaccounted forces cause unpredictable variation for any physical model setting. This variation makes the analysis insufficient to use to compare in-air and in-water kinematic measurements to infer the effects of drag.
8.2 Kinematic Range of the Redesigned Model

Figure 37 shows the maximum velocities and accelerations measured with the redesign model. The redesigned model range is summarized and compared with the animals' and 1st model's range below:

<table>
<thead>
<tr>
<th></th>
<th>Animal Range</th>
<th>1st Model Range</th>
<th>Redesigned Model Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>Velocity</td>
<td>5 to 29 m/s</td>
<td>2 to 45 m/s</td>
<td>6 to 22 m/s</td>
</tr>
<tr>
<td>Acceleration</td>
<td>2.6 to 15 x10^4 m/s^2</td>
<td>5 to 9 x10^4 m/s^2</td>
<td>0.5 to 10.6 x10^4 m/s^2</td>
</tr>
</tbody>
</table>

The redesigned model had a smaller velocity range than both the original model design and the animals. While the maximum acceleration of the redesign slightly improved upon the first model, it still did not match the maximum accelerations measured in animal strikes.

The mathematical model predicted a velocity and acceleration range for the redesigned model far in excess of the animal range. Yet, as we saw in section 1 of this chapter, the actual force the model exerted on the appendage assembly for a given deflection was much lower than predicted, resulting in lower peak velocities and accelerations.

The corrected mathematical model of the physical model was able to predict the final velocity of experimental strikes within 34%\(^1\). Therefore, the mathematical model could inform slight design changes to compensate for the structural deflections neglected

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1 This is the mean value of the percent error for all model configurations. Maximum percent error 131%. Minimum 0.9%. Mean percent error of predicted maximum acceleration was 65%.
in our predictions. While the predictions of mathematical model are insufficient to serve as a basis for drag studies, they can be used to fine tune the model redesign to expand its kinematic range.

8.3 Kinematic Patterns

The corrected kinematic patterns do not agree with the patterns predicted by the idealized model. It was shown in section 5.1 of Chapter 7 that the maximum velocity of the model was predicted to be produced by the largest beam spring at the lowest beam height. The largest acceleration were predicted also for the largest beam spring at the lowest beam height.

Inspection of Figure 37 shows a similar trend for maximum predicted acceleration and a slightly different trend for maximum predicted velocity as was predicted by the non-corrected mathematical model. From Figure 37 B, we can see that, as predicted with the idealized mathematical model, the greatest acceleration was expected for the largest beam set at the greatest beam height. This is the same counter-intuitive trend predicted and explained earlier. However, the difference in predicted maximum acceleration between the beams is much lower with the corrected mathematical model, as we would expect. The force measured at any given deflection was not significantly different for the two largest beams.

It is interesting to note how the experimental data compare to this prediction. The greatest experimentally measured accelerations were produced by the smallest beam at the lowest beam height, with a trend of decreased magnitude with increased beam width. This is precisely the opposite of the theoretical prediction. This discrepancy may be
explained by the fact that the mathematical model does not capture the higher frequency oscillations responsible for peaks in maximum acceleration.

The predicted maximum velocities of the corrected model do not follow the exact trend predicted by the idealized mathematical model. The idealized model predicted the greatest velocity would be achieved with the largest beam spring set to the highest beam height (Figure 32). In contrast, the corrected mathematical model predicts the greatest velocities to be produced by the smallest beam (Figure 37). It is not clear whether the experimental data support this prediction.

8.4 Summary

In this chapter, we evaluated the mathematical analysis. The experimental kinematic data collected with the redesigned physical model run in air were compared to the predictions made by the mathematical model Two criteria were used for evaluation of the mathematical model. First, the soundness of the mathematical model was validated by the experimental velocity falling within the region of mathematical uncertainty of the model 60% of the time. The variation between the model and the experimental values were explained by factors left out of the mathematical model. The average velocity falling, at most, 56% below the predicted value is consistent with the model neglecting friction. The acceleration profiles appear to have been influenced by the excitation and damping of another spring element. This resulted in the maximum acceleration values reaching as much as 230% of the predicted values. Yet, the dominant trends in the experimental data were predicted by the mathematical model. Thus, we conclude the mathematical model is somewhat helpful, but not very accurate.
The second criteria of evaluation required more accurate predictions by the mathematical model. In order to use the mathematical predictions in drag experiments as a baseline of the model's kinematics in the absence of fluid forces, the variation between the experimental data of the model collected in air and the predictions must be consistent. It was found that the experimental data varied in ways that were not consistent. This was most apparent in the experimental acceleration data which showed a standard deviation of over three times the range of uncertainty of the mathematical model. The variation could be explained as the contribution of the release of stored elastic energy in some other structural member besides the beam spring resulting in a higher frequency oscillation on top of the predicted acceleration profile. Yet, the variation in the magnitude and frequency of this second component is inconsistent. This suggests that the forces at work here are complex and do not offer a straightforward way to be incorporated into the mathematical model. Thus, while the mathematical model is useful insofar as it captures the dominant forces at work in the mechanism, it is not sufficient to use for drag related experiments.
In this chapter, we present one experiment that can serve as an example of how the physical model can be used to isolate cavitation sensitive properties. Much can be learned from the fact that the model does not match the animals’ cavitation onset velocity. By systematically exploring the effect on cavitation of each property we know to vary between the model and the animal trials, we can isolate the property or properties of mantis shrimp that reduce the production of cavitation in open flows. It is already known that the environment was one of several properties that were not consistent between the animal and the model in our earlier tests. Temperature, salinity and initial water filtration were held constant across animal and model tests. Pressure and water quality were not. In the experiment discussed here, we attempt to isolate the influence on cavitation of variation in water pressure and quality between the animal and model environments.

Throughout the literature on experimental cavitation studies, variation in environmental conditions confounded results and confused researchers (Berthelot 1850; Henderson 1980; as reviewed by Tropea et al. 2007). Figure 15 in Chapter 3 shows the variation in critical cavitation index of the same hydrofoil measured at different testing facilities across which environmental conditions were not precisely matched. The results vary widely.
We will break the environmental conditions known to influence cavitation into two categories: those that are easy to measure and control and those that are difficult. In this study, we hold constant those properties that are easy to control and allow the others to vary. Cavitation is sensitive to water vapor pressure, water pressure and water quality. Vapor pressure is easy to measure and control. Water pressure and quality are both difficult with the present setup.

The vapor pressure of water is altered with a change in temperature or the quantity of solvents. An increase in temperature increases the vapor pressure and makes it easier to cavitate (Brennen 1995). An increase in solvents lowers the vapor pressure and makes it more difficult to cavitate (as reviewed by Tropea et al. 2007). The dominant solvent in our experiments was marine salt. Both temperature and salinity were held constant across the animal and model tests. Thus, they should not contribute to cavitation variation.

Water pressure is primarily a function of the ambient air pressure. Air pressure varies locally and decreases with altitude\(^1\). The model and animal tanks are in two different rooms approximately 50 yards apart on the same floor of a building. While pressure variation between testing facilities at different locations may be a significant factor, in the same building the pressure variation is minimal. For instance, the local air pressure over Massachusetts for July 22, 2012 varied by less than 0.2\(^%\)\(^2\). Over the month of June 2012 in Amherst Ma the daily air pressure varied no more than 1.4\(^%\) from the

\(^1\) The pressure at any height above sea level is given by \( p = p_0 \times \exp(1.125e^{-4}h) \) where \( p_0 \) is the pressure at sea level and \( h \) is the height above sea level in meters. (Lerner 1996 [494])

\(^2\) (http://www.usairnet.com/weather/maps/current/ma/).
mean value of 29.92 in mg (wunderground.com). In contrast, the change in barometric pressure due to a change in altitude can be much larger. The minor difference in altitude between Amherst (312 ft) and Boston, Ma (6 ft), for instance, results in a 1.1% change in air pressure. If we tried these experiments at University of Michigan (elevation 876 ft) they would be performed with 2.1% lower air pressure than in Amherst. The University of Colorado at Boulder (elevation 5381 ft) has a 17.6% lower air pressure than in Amherst. Thus, while air pressure does vary and could cause water pressure variation across testing facilities, the variation in one location and altitude is minimal.\(^1\)

Yet, since we do not know the relationship between the magnitude of pressure variation and visible changes in cavitation production under these conditions, the minor variations present in our experiments may have significant effects. Precise water pressure control has been deemed important enough to warrant the construction of cavitation tunnels for accurate experimental cavitation work (as reviewed by Tropea et al. 2007). Cavitation tunnels are large recirculating chambers in which the pressure, temperature and flow rate of the liquid can be carefully controlled. Our laboratory set up cannot control water pressure.

Water quality is the most difficult to quantify of those environmental conditions to which cavitation is sensitive. The presence of larger quantities of nucleation sites is hypothesized to correlate with an increase in cavitation production (Brennen 1995). However, the exact nature of nucleation sites is unknown. Some suggest that dissolved air bubbles are expanded at lowered pressure. A greater number of bubbles to expand

\(^1\) (Elevation data from the USGS Geographic Names Information System geonames.USGS.gov)
lead to more cavitation (Brennen 1995 [33; 134; 147]). Others suggest that impurities in the water may weaken the bonds between water molecules and allow cracks to form in the water that then expand into bubbles at lowered pressure (Brennen 1995).

Unfortunately, no technique free of major limitations has been developed to measure of either cause of impurity, and most of the proposed methods are complex and expensive (as reviewed by Tropea et al. 2007 [986]).

Water quality maintenance methods varied between the water in which the animals and model tests were performed. For simplicity, we will refer to the animal environment as 'the aquarium' and the model environment as 'the laboratory'. Both aquarium and laboratory water are made from water filtered by reverse osmosis (RO). The aquarium water supports several animals and is recirculated, aerated, and filtered continuously. The laboratory water has no biological waste and is not aerated, or filtered. The constant aeration of the aquarium water and the presence of biological waste would suggest greater nucleation site concentration in the aquarium water/ resulting in a lower cavitation number and greater cavitation in the aquarium (Brennen 1995). While, this is opposite the trend observed in cavitation onset experiments, where more cavitation was seen in the laboratory, our inability to measure nucleation site density leaves open the possibility that unknown factors contribute to water quality variation.

We know water quality and pressure are important to cavitation inception, however, with our current set up, we have little ability to control either one. We also don't know the extent to which the present variation in either property affects cavitation development. Before investing in a costly and involved system that could detect and
control nucleation sites or water pressure, we chose to test the combined effect of all of the environmental variation between the animal and the model environments that we cannot presently control. This method evaluated the combined effects of several properties at once. Because the literature suggests that each property alters cavitation number in the same direction (no property is likely to cancel out the effect of another), then experimental results of combined tests can be informative. If environmental variation does not alter cavitation inception, most likely none of those properties are dominant factors in the cavitation variation we are trying to explain. If an effect is observed, further experiments can be designed to determine which environmental property is significant. In this way we can either eliminate environmental causes as a

Figure 38: Forward cavitation comparison between the animal strikes and the model. The animal strike (A) was performed in the aquarium. The right hand image shows two overlaid images from the same strike. In (B) a cavitation cloud covers the trailing edge of the appendage. The model's appendage with no cavitation, for comparison, can be seen in (C). Both the animal and model strikes depicted here reached peak velocities of 27 m/s.
contributor to cavitation variation or know we need to develop more sensitive experiments to study environmental variation further.

Figure 39 shows the difference in cavitation observed in animal and model studies. In cavitation onset experiments described in section 3.1 of Chapter 5, almost no forward cavitation was seen during the animal strikes\footnote{One out of over 100 filmed strikes showed evidence of forward cavitation.}, as is seen in (A) of Figure 38. At (B), we see the cavitation formed on the model's appendage when rotated at the same tangential velocity in the laboratory. A clear cavitation cloud encases the trailing edge of the model appendage. For comparison, the appendage with no cavitation is shown at (C). This is the variation in cavitation production we tried to explain with these environmental variation experiments.

There are many differences between the animals' and the model's appendage and strike that could be responsible for cavitation variation. The model's and animal's appendage shape, surface and material properties vary. In order to eliminate all variation except the environmental conditions, we ran the model mounted with an identical appendage in both environments. If the differences seen in Figure 38 were primarily due to environmental differences, we would expect similar variation in cavitation clouds between the model run in the aquarium and in the laboratory.

The methods used in this study are primarily qualitative. We compare the development and size of cavitation clouds as seen in images from high-speed video of model strikes. Qualitative results can be informative in this context for two reasons. First, the magnitude of the difference in cavitation between the animals and the model
was so great that qualitative studies should be sensitive enough to pick out a wide range of variation. Second, the goal of this study was to determine whether the differences between the model and animal environments should be a focus of future cavitation reduction studies. The goal was not the development of a precise quantitative relationship between cavitation and environmental variation. Experiments designed to produce quantitative results are complex and require sophisticated equipment. This experiment was preliminary and intended to determine whether the investment needed to develop more sensitive experiments is warranted. As such, precision in the results are not needed.

Results can be categorized as falling into one of three broad categories. Either we will see 1) more cavitation in the laboratory than in the aquarium 2) an equal amount of cavitation in both environments, or 3) less cavitation in the laboratory than in the aquarium. Results in either the first or the third categories would warrant further investigation of the role of environmental variation. We hypothesize we will see more cavitation in the laboratory than in the aquarium. Results that fall into the first category would suggest that environmental differences are a primary cause of cavitation variation. Results from the third category would suggest that environmental variation is one of several causes. Both of these results justify an investment in more sensitive experiments to determine the individual effects of water quality and water pressure on cavitation production.

9.1 Methods
Twenty five trials were run of the redesigned model with a cylindrical 1/4” appendage. Thirteen trials were performed in the aquarium, where the appendage had an average speed of 12.04 m/s (maximum speed of 16.2 m/s, minimum speed of 8.3 m/s). Twelve trials run were in the laboratory and averaged 12.53 m/s (maximum speed of 17.7 m/s, a minimum speed of 9.3 m/s). Kinematic data were collected with high-speed video as described in section 2 of Chapter 5. Cavitation was evaluated by the size and development of the maximum cavitation cloud formed during a given strike. All tests were performed in 20 gallon glass aquarium tanks open to the atmosphere. Tests were performed on the same day to minimize atmospheric pressure variation. The animals were each housed in a 20 gallon tank as part of an integrated recirculating system that monitors and controls several environmental properties. Temperature was kept at $26 \pm 2^\circ$ C. The salinity was held at $34 \pm 2$ ppt. In laboratory experiments here and in the earlier trials, the temperature and salinity were held within the same range as in the aquarium. Salinity was monitored with a refractometer. Temperature was measured with a thermometer. Both were rechecked before each trial. Both the aquarium and laboratory salt water were made by adding marine salt to RO filtered water.

The major difference between the aquarium and laboratory water was the filtration and aeration. Laboratory salt water was neither aerated nor filtered once in the tank. It was mixed from water filtered that day and poured into the tank. Aquarium water was constantly aerated and filtered by micro-organisms.

9.2 Results
Figure 39 shows images from the model run at three different velocities in both environments. Images performed in the laboratory can be seen on the right. Aquarium results are shown for three corresponding speeds on the left. The variation in cavitation is minor in comparison to that seen in Figure 38. At appendage speeds of 9.3 m/s (images (A) and (B)), the cavitation cloud was similar between the aquarium and laboratory tests,
although small variations can be seen\(^1\). The cavitation cloud has begun to form at the tip of the appendage in (A) while none was seen in (B). The cloud was more tightly formed in (A) than the wispy cloud in (B). The cloud trailing the cylinder in (C) was much more developed than in (D), but the maximum velocity of the strike in (C) was 0.1 m/s faster. While this difference in velocity is within the digitizing margin of error (1.2\%), it may be a reason to suggest that they do not form a perfect comparison. The cavitation clouds seen in (E) and (F), again, vary slightly, with the cavitation cloud more developed in the aquarium trial, (E). A clear cavitation cloud can be seen at the tip of the appendage in E whereas none is seen in (F). The cavitation trailing the cylinder is also denser in the aquarium's cavitation cloud, in (E), than the cloud formed in the laboratory, in (F).

### 9.3 Discussion

Earlier tests showed that the animals produced little or no cavitation during the forward motion of their strike, while the model almost always cavitated at peak velocities over 15 m/s. We hypothesized that environmental variation was at least partly responsible for this discrepancy in cavitation production. This would imply that the laboratory water had a much lower cavitation number than the aquarium water; That is, it was easier to cavitate in the laboratory.

Our variation in cavitation may be explained by the variation in properties of the earlier animal and model trials. Water pressure and quality were not held constant across animal and model tests. Although the model appendages were the same shape as the animal appendages, they had different material, surface and mechanical properties. Any

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\(^1\) Figure 12 shows images of the development of cavitation with decreasing cavitation number.
of these variations could contribute to cavitation reduction. In order to isolate the environmental effects, we ran the model with the same appendage in both environments.

Size and development of cavitation clouds were compared between trials that reached the same peak velocities (±0.05 m/s). Peak velocity was shown in experiments in Chapter 5 to correlate with cavitation inception. Earlier experiments compared cavitation onset velocity. Size of cavitation cloud was used here for comparison because the 1/4” cylinder appendage used in these experiments cavitated at all speeds. The cylinder was used for comparison rather than model appendages because of temporary difficulties attaching appendages to the redesigned model. Therefore, instead of comparing the speed at which cavitation is first seen, here we compare the size of the cavitation cloud between trials. This assumes that a system that is more prone to cavitation would result in a larger cavitation cloud at the same speed (Keller 2001).

If our hypothesis were correct and the laboratory water was more conducive to cavitation, we would expect the trials run in the laboratory to show a much larger and more developed cavitation cloud than those run in the aquarium. The results from the environmental tests shown in Figure 39 do not support this hypothesis. There is very little difference between the size and development of cavitation clouds across environments. The extent to which this result varies from the results of the earlier experiments is striking. In the animal and model studies, the animals generally produced no forward cavitation while the model appendage was engulfed in a cavitation bubble. The gap between cavitation production in the aquarium and the laboratory is very wide. In the present experiments, the gap is barely visible. Additionally, closer inspection of the
variation in cavitation clouds shows that the little discrepancy that can be detected between the aquarium and laboratory trials is in the opposite direction from what our hypothesis predicts. The hypothesis suggests that the cavitation clouds formed during trials in the laboratory will be larger than those formed in the aquarium. But, if any consistent variation could be concluded from the images in Figure 39, it would be that the cavitation clouds formed in the aquarium were larger.

However, any conclusions to the effect that the aquarium water produced more cavitation are certainly disputable. The visible differences in cavitation clouds could be due to different lighting conditions or resolution. The methods of quantification of cavitation cloud size are not discriminating enough to confidently draw conclusions from the minor differences between the aquarium and laboratory results.

As mentioned at the outset, the goal of this study is not the development of a quantitative relationship between environmental conditions and cavitation production. The more modest goal was to determine whether the results justified more sensitive experiments into environmental variation. We only needed to determine whether environmental variation alone resulted in a difference in cavitation production in the aquarium and the laboratory. We found no significant difference in cavitation production.

The results from this experiment suggests that as long as the temperature, salinity and ambient air pressure are hold constant between the aquarium and laboratory, the resulting cavitation is comparable. We do not, therefore, need to invest in complex nucleation site or pressure monitoring and controlling devices in order to infer from
results made in the laboratory to those made in the animal's aquariums. Other factors such as appendage vibration or surface or material properties should be the focus of future work to determine the cause of cavitation variation.
The physical model of the feeding strike of the mantis shrimp is part of a bigger project to study the interaction between evolution and physics. Stomatopods accelerate their appendages through the water at speeds that are among the fastest measured for any aquatic animal. This suggests that mantis shrimp may be operating at the physical limits of what is possible for animal motion in water.

The physical model was designed to study two of the fluid dynamic phenomena that may limit how fast animals can move. Drag forces resisting motion increase with speed. The energy needed to overcome drag forces at the speeds the mantis shrimp strike would be formidable without some method of drag reduction. Cavitation may also impose an upper limit on the speed of animal motion in water. Any shape will cavitate if moved fast enough through water. The erosion caused by cavitation could prove deadly for any fast moving animal if cavitation were not controlled.

One fundamental question, then, drove the building of this model: Have the mantis shrimp evolved to decrease drag and/or control cavitation, and, if so, how? Knowledge of which properties alter drag and cavitation in these environments could be used together with information of how these properties are distributed through the
phylogeny to come to a deeper understanding of the role physical forces play in evolution.

The desire to answer this question set the parameters by which the physical model was designed. The model was built as a tool to be used to explore the strike of the mantis shrimp to determine which physical parameters decrease drag, decrease cavitation during free rotational motion and promote cavitation upon impact. To that end, the mechanical model had three goals:

1) The model was to be outfitted with a method to collect kinematic, force and cavitation data.
2) The velocity and acceleration profile of the model were to be predicted with a mathematical model of the mechanism.
3) The model was to match as many drag and cavitation sensitive properties of the mantis shrimp strike as feasible and have a means to control the rest.

The first goal was necessary to study each phenomena of interest and was met. The model was outfitted with a removable force sensor for impact cavitation data collection. High-speed video served to collect both kinematic and cavitation data for use in drag and cavitation studies. The second goal was to predict the model kinematics with a mathematical model. While the model captured the dominate forces driving the motion, inconsistent kinematic output of the model made the predictions inaccurate. The presence of an additional high frequency oscillation suggested that another elastic element was releasing stored energy during the strike. The most likely candidate for the additional spring is the axle on which the appendage assembly rotates. As the model is loaded for a strike, both the latch and the beam exert forces on the appendage assembly on either side of the axle. In the present design, the axle is a thin stainless steel rod. Doubling the diameter of the axle will decrease the axle's resistance to
bending by 16 times. If the variability in the model kinematic output is the result of the release of stored elastic energy in the bent axle, this change to the model design may bring the model kinematics more in line with the mathematical predictions.

The third goal of the model was to match the animals' strike in as many drag and cavitation sensitive properties as possible and have a means to control the rest. To that end, the redesigned model matched the animals in

- Rotational motion
- Appendage shape and approximate scale
- Environmental qualities of temperature, salinity and initial water quality

The redesign model could achieve some, but not all, of the animal peak velocities and peak accelerations. The variation in appendage material and surface properties were controlled with changeable appendages cast in different materials.

The properties known to alter drag and/or cavitation that were not matched or controlled were

- Water pressure
- Water quality
- Acceleration profile

The influence on cavitation of the properties not matched by the model properties was evaluated. Cavitation has been shown to be sensitive to water pressure and quality. The literature indicates that maximum acceleration and jerk may alter both drag and cavitation. Yet, analysis of the experimental data collected in cavitation onset velocity experiments showed no correlation between maximum jerk and cavitation. This suggests that in rotating accelerating flows, the jerk is not a relevant physical property to cavitation production. Additionally, experiments on the influence of the difference in
water pressure and water quality on cavitation showed that the environmental differences
did not influence cavitation. Thus, while the model experiments may differ from the
animal experiments in water quality, pressure and acceleration profile, the variations here
have been shown to not alter cavitation production.

Thus, the physical model met two of its goals. The model is outfitted for
kinematic, force and cavitation data collection. The present design does not satisfactory
meet the second goal. While a mathematical model was developed that predicts the
dominant kinematic behavior of the physical model, it is not accurate enough to serve as
a baseline for drag experiments. The third goal was satisfied. The model matched the
animals in many physical properties relevant to drag and cavitation, and is capable of
holding the the rest constant. Thus, the model can be used to study the properties of the
mantis shrimp that control cavitation.
Figure 40: Predicted and experimental tangential velocity and acceleration profiles for a 6 mm beam at a 10 mm height. Dashed line are predictions made without mathematical model corrections. Gray lines are the experimental data. The dotted white line is the corrected predicted profile. Black area shows the area of predicted uncertainty.
Figure 41: Predicted and experimental tangential velocity and acceleration profiles for a beam width at two different beam heights. Dashed line are predictions made without mathematical model corrections. Gray lines are the experimental data. The dotted white line is the corrected predicted profile. Black area shows the area of predicted uncertainty.
Figure 42: Predicted and experimental tangential velocity and acceleration profiles for a beam width at two different beam heights. Dashed line are predictions made without mathematical model corrections. Gray lines are the experimental data. The dotted white line is the corrected predicted profile. Black area shows the area of predicted
Figure 43: Predicted and experimental tangential velocity and acceleration profiles for two beam widths at two different beam heights. Dashed line are predictions made without mathematical model corrections. Gray lines are the experimental data. The dotted white line is the corrected predicted profile. Black area shows the area of predicted uncertainty.
Figure 44: Predicted and experimental tangential velocity and acceleration profiles for a beam width at two different beam heights. Dashed line are predictions made without mathematical model corrections. Gray lines are the experimental data. The dotted white line is the corrected predicted profile. Black area shows the area of predicted uncertainty.
Figure 45: Kinematic measurements from digitized high-speed video and accelerometer data. Data for two strikes are shown in each plot. Accelerometer data is jagged. Digitized data are smooth. For each strike, the velocity data measured with the accelerometer and found through digitization of high-speed video are shown with the same line type.
Figure 46: Kinematic measurements from digitized high-speed video and accelerometer data. Data for two strikes are shown in each plot. Accelerometer data are jagged. Digitized data are smooth. For each strike, the velocity data measured with the accelerometer and found through digitization of high-speed video are shown with the same line type.
Here we present the results of incorporating an accelerometer into the physical model. The accelerometer mounting and testing are described. The device output is presented and discussed.

1 Mounting and Testing

The accelerometer was mounted radially (Figure 25) in a pocket in the cam and encased in a rigid marine epoxy. Mounting the accelerometer in this orientation measured centripetal rather than tangential acceleration. Centripetal acceleration is proportional to tangential velocity and can be found from the equation

\[ A_c = \frac{V_t^2}{R} \]  \hspace{1cm} (56)

where \( V_t \) is the tangential velocity, \( A_c \) is the centripetal acceleration and \( R \) is the distance from the center of rotation to the center of the accelerometer. With this formula, the output of the accelerometer was converted to instantaneous velocity. The accelerometer was tested by mounting it to a test and measurement exciter (Brüel & Kjaer shaker, model 4808) driven by an amplified generated signal ranging from 100 Hz to 10 kHz. The exciter oscillated the accelerometer at a frequency proportional to the input. Under
these conditions, the accelerometer produced a clear signal corresponding to the driving oscillation.

2 Accelerometer Data

A typical signal generated during a model strike is shown in Figure 36\(^1\). The plot shows the same region depicted in the velocity and acceleration profiles in Figure 36 and corresponds to the portion of the strike during which the beam spring is accelerating the appendage. In Figure 37, the accelerometer data are shown by the jagged solid line.

![Figure 36](image)

Figure 36: Typical signal generated during a model strike.

![Figure 37](image)

Figure 47: Kinematic measurements from digitized high-speed video and accelerometer data. Velocity measurements from centripetal accelerometer readings are shown by the solid line. The hollow triangles mark the velocity profile determined from digitized high-speed video of the strike. The natural oscillation of the model is depicted by the curve marked with hollow circles.

\(^1\) All the data are available in Appendix B
Velocity measured through digitization of high-speed video of this strike is shown by the line marked with triangles.

Two features of the accelerometer data should be noted. The digitized velocity shows a smooth upward flow to a maximum of 18 m/s. This is consistent with the last data point corresponding to the moment the beam spring stops accelerating the appendage. At this moment, the appendage is moving at its maximum velocity. The accelerometer data, in contrast, reached a similar maximum acceleration in the middle of the strike and then falls back to zero. Additionally, the accelerometer data resolved a higher frequency oscillation with a period at most half the period of the strike. While the digitized velocity profile has 100 times fewer data points than the accelerometer data, at least 50 points are available and suggested no such oscillation.

Three factors are hypothesized to be responsible for the accelerometer data complexity. First, while the accelerometer was mounted radially and was most sensitive to radial accelerations, it had, at most, a 5% sensitivity to accelerations perpendicular to this motion. While the radial accelerations were on the order of \(100 \text{ m/s}^2\), the tangential accelerations were 100 times larger (see Figure 36 and 37). As seen in Figure 36, the tangential acceleration often oscillated at a much higher frequency than the velocity. The higher oscillation of the acceleration data may be the contribution of 5% of the tangential acceleration. Yet, subtracting 5% of the tangential acceleration, as calculated from high-speed video of the strike, still did not result in a usable signal. Other factors played a significant role.

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1 For velocities of 20 m/s and with a radius of 3.5 cm, we expect centripetal acceleration of 571 m/s\(^2\) using equation 56. Tangential values range from \(10^3\) to \(10^4\) m/s\(^2\).
The smallest accelerometers, like the one used here, were designed to measure very high frequency oscillations. As such, they were not designed to be sensitive to unchanging or slowly changing accelerations regardless of the magnitude (personal communication with manufacturer). As the rate of change of the velocity drops in the latter half of the strike, the accelerometer loses sensitivity and the output floats back to zero. Thus, while the accelerometer was sensitive to the magnitude of the range of accelerations expected from the redesigned model and could respond quickly enough to operate at this time scale, the accelerometer was insensitive to DC accelerations. This results in kinematic data collected from the accelerometer showing the velocity to be zero at the point where the digitized video suggests the velocity of the model was at a maximum.

The third possible confounding factor in the accelerometer data complexity was, in fact, a consequence of the second. Since the accelerometer was very sensitive to high frequency oscillations, it may have picked up on vibrations traveling through the cam as a result of the release of the latch. These vibrations may show up as the very high frequency noise on top of the other signals or it may contribute to the larger oscillations. Thus, the accelerometer's sensitivity to only high frequency oscillations may have made it unresponsive to the accelerations resulting from the motion in which we are interested and, instead, pick up on reverberations in which we have little interest.

As with any accelerometer, signal processing is required to filter out inconsequential data and isolate the relevant information. Here, the magnitude of the

\[1 \text{ Data sheet for Endevco model 22 accelerometer}\]
noise appears to dominate the signal. Filters that eliminate the larger oscillations alter the timing and magnitude of the signal to an unacceptable degree.

There was one hypothesis for the additional peaks in the accelerometer data that was ruled out. The additional peaks could have been caused by oscillations of the driving mechanism. While the natural frequency of the beam spring itself was calculated to be much higher than what is seen in the accelerometer data, when mounted on the model, the entire structure could have a much lower natural frequency. And, in fact, high-speed videos of the beam spring after it stopped pushing the cam showed an oscillation much slower than would be expected of the beam spring itself. Yet, when this oscillation was digitized, it was found to be no more than half of the mysterious frequency seen in the accelerometer data. The measured model natural oscillation is plotted in Figure 47 with hollow circles as markers. It was concluded that the large oscillations in the accelerometer data were not the result of oscillations in the model structure.

In summary, it proved difficult to extract the centripetal acceleration that the accelerometer was intended to measure. While the accelerometer was much more resolved than the digitized data, it was, in fact, too sensitive to accelerations in which we are not interested and not sensitive enough to the ones in which we are. Thus, the accelerometer data was not used in the kinematic analysis of the redesigned model.

3 Summary

The accelerometer was intended to improve the accuracy of kinematic measurements and decrease analysis time. The accelerometer did not meet these
expectations. Two facts explain the accelerometer's most serious limitations. Mounted radially, the accelerometer is 100% sensitive to centripetal acceleration and 5% sensitive to tangential acceleration. The tangential acceleration is at least 100 times larger than the centripetal acceleration and may be contributing a large component to the accelerometer's data. This could potentially be filtered out with more complex signal processing.

The second factor complicating the accelerometer data is more difficult to overcome. The accelerometers available that are small enough to not drastically alter the scale of the model are not sensitive to slowly changing accelerations. The rate of change of the accelerations during the model strike fall right at the edge of the accelerometer's sensitivity. The model's largest accelerations are picked up by the accelerometer, but as the jerk decreases, the output floats back to zero. This behavior is very difficult to compensate for. The combination of these two effects has made the accelerometer less accurate than time consuming, but consistent, digitization of high-speed video. Thus, the accelerometer was not used for kinematic data collection.


