2017

Non-Euclidean Shells: A Study of Growth-Induced Fabrication and Mechanical Multi-Stability

Nakul Bende

Follow this and additional works at: https://scholarworks.umass.edu/dissertations_2

Recommended Citation
https://scholarworks.umass.edu/dissertations_2/1023

This Open Access Dissertation is brought to you for free and open access by the Dissertations and Theses at ScholarWorks@UMass Amherst. It has been accepted for inclusion in Doctoral Dissertations by an authorized administrator of ScholarWorks@UMass Amherst. For more information, please contact scholarworks@library.umass.edu.
NON-EUCLIDEAN SHELLS:
A STUDY OF GROWTH-INDUCED
FABRICATION AND MECHANICAL
MULTI-STABILITY

A Dissertation Presented
By
NAKUL PRABHAKAR BENDE

Submitted to the Graduate School of the
University of Massachusetts Amherst
in partial fulfillment of the requirements for the degree of

DOCTOR OF PHILOSOPHY
September 2017

Polymer Science and Engineering
NON-EUCLIDEAN SHELLS: A STUDY OF GROWTH-INDUCED FABRICATION AND MECHANICAL MULTI-STABILITY

A Dissertation Presented
By
NAKUL PRABHAKAR BENDE

Approved as to style and content by:

________________________________________
Ryan C. Hayward, Chair

________________________________________
Alfred J. Crosby, Member

________________________________________
Christian D. Santangelo, Member

________________________________________
E. Bryan Coughlin, Department Head
Polymer Science and Engineering
DEDICATION

Dedicated to my grandfather *baba*, whose curiosity and respect for education took us on an amazing journey.

*Baba*, I will always remember and cherish the trust you put in all of us.
ACKNOWLEDGEMENTS

This thesis would not have been possible without the personal and professional support of some of the most amazing people I have met in my life.

To begin with, I would like to offer my sincerest thanks to my advisor, Prof. Ryan Hayward, for allowing me to be a part of an incredible research group, and for always providing me with encouraging guidance on both a scientific and a professional front. His curiosity and zeal for a thorough understanding of the problem at hand will forever provide me with inspiration and much-needed guidance. Ryan: thanks for giving me the playground to grow, learn and most importantly, fail. I would also like to thank my two committee members, Profs. Alfred Crosby and Christian Santangelo for providing me with helpful feedback and making me think hard about my scientific conclusions. Al always encouraged me to think and formulate my answers right, and at the same time pushed me to ask the right questions. Throughout my PhD, Chris helped me understand the intricacies with geometry and mechanics and inspired me with his fascinating approach for tackling a problem. Additionally, I am extremely grateful for all my previous academic mentors throughout my undergraduate studies, Profs. Yuvraj Negi (IIT Roorkee), Doris Vollmer (Max Planck Mainz) and Sadhan Jana (Univ. of Akron), for preparing me for a career in science.

I will like to thank my funding sources for work under this thesis dissertation: Emerging Frontiers in Research and Innovation (EFRI) Origami Design for the Integration of Self-assembling Systems for Engineering Innovation (ODISSEI) (1240441), and Material Research Science and Engineering Center (MRSEC DMR-0820506) on Polymer at the University of Massachusetts Amherst. I would also like to thank all the central instrumentation facilities in UMass Amherst, and the wonderful staff that manages them.

This acknowledgement would be incomplete without mentioning the department of Polymer Science and Engineering, which provided me a unique platform for developing
personal, scientific and professional skills. Above all, I would like to thank the greatest asset that makes this department a truly amazing place to work at: the people. I never imagined that I would make so many friends across the professional backgrounds, ages and nationalities in one place. Lisa Groth, Maria Farrington, Sophie Shu, Linda Chatfield, Jessica Skrocki, Andre Mulek, Alyssa Krystek, Greg Debowksi, Jennifer Green and Jim were always there to kindly help me with whatever was on my plate throughout my PhD, and deserve a sincere mention. I would like to thank all the people involved in my outreach team, and all the volunteers who ever helped me for this great cause.

It was a honor and a pleasure to work with my collaborators: Arthur Evans, Marcelo Dias, James Hanna, Jesse Silverberg, Itai Cohen, Tom Hull and Robert Lang. Prof. Michael Dickey and Dr. Tim White were very helpful in discussions regarding life in science and I am grateful for their kind guidance and encouragement. Working with all of my seven undergraduates: Sarah Innes-Gold, Thao Do Vy Le, Luis Marin, Nivedita Sharma, Jordon Kornfeld, Sarah Selden and Autumn Phaneuf, was as much as a learning experience for me as for them, and I wish them all the best in their careers. Last and certainly not the least, I would like to thank members of the Hayward group, the most hardworking and dedicated people I have worked with. Scott, Felicia, Cheol, Maria, Dayong, Jinhye, Anesia, Rachel, Kyle, Daniel, Adam, Ying, Tetsu, Di, Qi, Hyunki, Alexa, Carolyn, Minjung, David, Weiguo, Hyeongjun, Myunghwan, Bin, Laju, Kyu, Junhee and Seog-Jin: please accept my humble gratitude for all the wonderful time I spent with all of you.

The class of 2011 is the best class PSE has ever seen, and time will only further demonstrate that. I feel lucky to be a part of this family, and truly a family away from home it was for me. I will always cherish the time I spent with all of you during classes, cumes, Will Smith Wednesdays, playing hookie to go tubing, job hunts, defenses and farewells - all of it has a special place in my memories from grad school. I also am grateful for my roommates: Mike, Joel, Nihal, Tetsu and Marco were great to hang out and de-compress with. Other
friends from Amherst: Ishaan, Jignesh, Rohit, Ranadeep, Sagun, Anish, Celina, Greta, Kim and Carmen were always fun and encouraging to be with. My childhood friends from India: Bipin, Saurabh, Anupam, Arpit, Maa, Amit, Praths, Eesha and Sunny were always ready to hear about my random problems and stories throughout the grad-school, for which I can not thank them enough. It was always comforting to talk to my classmates from IIT Roorkee: Prateek, Ankit, Hari, Farhan, Hareesh, Abhinav and Pranav; knowing that they too share and understand the challenges I faced.

All of this could not have been possible without my family. Mummy, Papa: thanks for everything. They made sacrifices after sacrifices for my education, never squashing the curious child in me. It is their unconditional love and trust in me that helped me through all the high and lows of grad school. Talking to them and Manas, my brother always brought unexplainable calmness in me. My in-laws Bill and Patricia treated me as their son for the last three years, once again giving me the warmth of being in a family miles away from home.

And to end with, I can not thank my wife Liz enough, but still will take this chance to. Through highs and lows, acceptances and rejections, tremendous happiness and stress, you were the one standing right next to me encouraging me with love and support like I never knew to be possible. It will take a lifetime to thank you for this, and that is exactly what I intend to do.
ABSTRACT

NON-EUCLIDEAN SHELLS: A STUDY OF GROWTH-INDUCED FABRICATION AND MECHANICAL MULTI-STABILITY

SEPTEMBER 2017

NAKUL PRABHAKAR BENDE
B.TECH., INDIAN INSTITUTE OF TECHNOLOGY ROORKEE
M.TECH., INDIAN INSTITUTE OF TECHNOLOGY ROORKEE
M.S., UNIVERSITY OF MASSACHUSETTS AMHERST
PH.D., UNIVERSITY OF MASSACHUSETTS AMHERST
DIRECTED BY PROF. RYAN C. HAYWARD

Non-Euclidean shells are ubiquitous in both natural and man-made systems, yet fabrication at smaller length-scales (nanometer to micrometer) and the underlying mechanical behavior of these geometries is not well understood. In this dissertation, we develop a framework for improvements in fabrication and control over deformation pathways of non-Euclidean elastic shells.

For programming a target non-Euclidean geometry, we study the non-uniform growth-induced buckling in flat sheets. To deepen our understanding of this powerful mechanism, we present an experimental verification of its mathematical equivalence with a mechanism involving topological defects. We establish a framework for correlating topological defect-induced buckling, realized through a simple cut-and-glue construction, with growth-induced buckling realized through non-uniform growth of patterned 2D hydrogel sheets. Validating the obtained mathematical results, we demonstrate fabrication of a cylindrical
and conical dipole geometry through both mechanisms under consideration. In addition, upon applying a similar treatment on tetrahedron geometry we realize the limitations of growth-induced buckling mechanism for programming 3D geometry, and find that optimizing for pattern resolution and swelling range can lead to a higher fidelity in target geometries.

Next, we turn towards the interplay between geometry and mechanics in non-Euclidean shells, to harness multi-stability between different geometric configurations. Under this theme, we study deformation of arbitrarily curved surfaces along pre-defined creases (curves with local weakening) finding that the continuity of this deformation can be predicted through a simple consideration of the curvature of the crease. We establish that simple geometric control over the crease curvature can be used to program on-demand snap-through instabilities between bi-stable states of developable, elliptic and hyperbolic surfaces. Using experiments, FEA and theoretical analysis of bending and stretching energies involved while indenting a hemispherical shell, we establish the geometric phase space in which the isometric state of a creased sphere is stable.

Finally, we extend this notion by considering the ability to program multiple stable states in a tiled conical geometry, commonly found in bendable drinking straws (bendy straws) and other similar structures. These corrugated structures exhibit stability in axial (resulting in change in length), non-axial (change in ‘bent’ angle) and azimuthal direction (change in azimuthal angle) imparting a desirable high-degree of freedom in possible configurations. By analyzing the stability behavior of elastic double conical frusta shells, we study the necessary conditions for programming multi-stability, and find that axial stability depends on geometrical parameters. Interestingly, we conclude that a stable non-axial state requires a combination of appropriate geometry and presence of a pre-stress in azimuthal direction. The controlled multi-stability opens pathways toward a truly re-configurable shape-programmable system, useful for manipulators and actuators in soft-robotics.
# TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>ACKNOWLEDGEMENTS</td>
<td>v</td>
</tr>
<tr>
<td>ABSTRACT</td>
<td>viii</td>
</tr>
<tr>
<td>LIST OF TABLES</td>
<td>xiii</td>
</tr>
<tr>
<td>LIST OF FIGURES</td>
<td>xiv</td>
</tr>
<tr>
<td>CHAPTER</td>
<td></td>
</tr>
<tr>
<td>1. INTRODUCTION AND DISSERTATION OUTLINE</td>
<td>1</td>
</tr>
<tr>
<td>1.1 Shape-programming as an alternate route for fabrication of non-Euclidean geometry</td>
<td>2</td>
</tr>
<tr>
<td>1.2 Fabrication of non-Euclidean geometries by non-uniform growth in flat sheets</td>
<td>3</td>
</tr>
<tr>
<td>1.3 Programming stability in non-Euclidean elastic shells</td>
<td>7</td>
</tr>
<tr>
<td>2. EXPERIMENTAL VALIDATION FOR EQUIVALENCE OF NON-UNIFORM GROWTH AND TOPOLOGICAL DEFECTS FOR SHAPING OF HYDROGEL SHEETS</td>
<td>9</td>
</tr>
<tr>
<td>2.1 Introduction</td>
<td>9</td>
</tr>
<tr>
<td>2.2 Theory: mapping between non-uniform growth and topological defects</td>
<td>10</td>
</tr>
</tbody>
</table>
2.3 Materials and technique ........................................ 12
2.4 Experimental observations for shaping of hydrogel sheets using non-uniform growth induced buckling ........................................ 14
2.5 Limitations and Conclusions ....................................... 17

3. GEOMETRIC CONTROL FOR SNAP-THROUGH TRANSITIONS OF CREASED ELASTIC SHELLS ........................................ 19

3.1 Introduction ..................................................... 19
3.2 Geometrical mechanics of folding a shell ....................... 20
3.3 General design principle for snap-through instabilities ........... 23
3.4 Fabricating creased, non-Euclidean elastic shells .................. 24
3.5 Studying the geometrically programmed snap-through instability in curved elastic shells ........................................ 25
  3.5.1 Negative Gaussian curvature surface, helicoid .................. 25
  3.5.2 Zero Gaussian curvature surface, cylinder ....................... 27
  3.5.3 Positive Gaussian curvature surface, hemisphere ............... 29
  3.5.4 Energetics of a deformed spherical shell ......................... 31
3.6 Time-scale for snap-through transition between isometric states .... 33
3.7 Conclusion .................................................. 34
3.8 Future work ............................................... 35

4. MULTI-STABILITY IN ELASTIC CONICAL FRUSTA: A STUDY ON RE-CONFIGURABILITY OF A DRINKING STRAW ...................... 37

4.1 Introduction .................................................. 37
  4.1.1 Limitations of shape-programmable systems .................... 37
  4.1.2 Re-configurability of a drinking straw ......................... 38
4.2 Conical frusta geometry ....................................... 39
4.3 Fabrication of re-configurable conical frusta (RCF) .......................... 40

4.4 Stability of elastomer RCF geometry .............................................. 41

4.5 Effect of residual stress present in commercial conical frusta on non-axial stability .......................................................... 42

4.5.1 Possible origins of residual stress in commercial samples ............ 44

4.6 Fabricating RCF geometry with built-in stress ................................. 44

4.7 Non-axial stability in pre-stressed RCF ........................................... 45

4.8 Fundamental mechanism behind stress-induced stability in RCF geometry ........................................................................... 48

4.8.1 Experimental details for characterization of RCF geometry upon deformation ................................................................. 49

4.8.2 Analyzing stretching and bending during axial deformation: ....... 50

4.8.3 Analyzing stretching and bending during non-axial deformation ................................................................. 52

4.9 Conclusions ................................................................................. 54

4.10 Future work .............................................................................. 55

5. SUMMARY AND OUTLOOK ................................................................. 57

APPENDIX: PROTOCOL FOR GRAYSCALE PROJECTION LITHOGRAPHY ................................................................. 60

BIBLIOGRAPHY ............................................................................... 73
# LIST OF TABLES

<table>
<thead>
<tr>
<th>Table</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.1 Recipe for photo-crosslinkable, thermally responsive PolyNipam copolymer</td>
<td>12</td>
</tr>
<tr>
<td>3.1 Observed time-scales for snap-through transitions for different geometries</td>
<td>34</td>
</tr>
</tbody>
</table>
## LIST OF FIGURES

<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.1</td>
<td>Non-Euclidean geometries in natural and architectural examples</td>
<td>1</td>
</tr>
<tr>
<td>1.2</td>
<td>Applications of shape-programmable systems</td>
<td>3</td>
</tr>
<tr>
<td>2.1</td>
<td>Topological defect-induced buckling for obtaining a cone using a simple cut-and-glue method</td>
<td>9</td>
</tr>
<tr>
<td>2.2</td>
<td>Equivalent domains for growth-induced and topological defect-induced buckling into a spherical geometry</td>
<td>11</td>
</tr>
<tr>
<td>2.3</td>
<td>Halftone patterning for defining a growth metric in 2D hydrogel sheets</td>
<td>13</td>
</tr>
<tr>
<td>2.4</td>
<td>Equivalent domains for a cylindrical geometry</td>
<td>15</td>
</tr>
<tr>
<td>2.5</td>
<td>Equivalent domains for a dipole</td>
<td>15</td>
</tr>
<tr>
<td>2.6</td>
<td>Equivalent domains for a tetrahedron</td>
<td>17</td>
</tr>
<tr>
<td>3.1</td>
<td>Creasing a shell by thinning locally along a curve to form a ‘trench’ with the local coordinate system ( {s, v} ) indicated</td>
<td>21</td>
</tr>
<tr>
<td>3.2</td>
<td>Isometric states of a curved surface</td>
<td>21</td>
</tr>
<tr>
<td>3.3</td>
<td>Schematics for prototypical geometries</td>
<td>23</td>
</tr>
<tr>
<td>3.4</td>
<td>Fabrication of Euclidean and non-Euclidean geometries with creased region along controlled curves</td>
<td>24</td>
</tr>
<tr>
<td>3.5</td>
<td>Continuity of deformation of a helicoid ( (K &lt; 0) ) geometry</td>
<td>26</td>
</tr>
<tr>
<td>3.6</td>
<td>Load response for deformation of a helicoid geometry with creases</td>
<td>27</td>
</tr>
<tr>
<td>3.7</td>
<td>Rigid boundary constrained for a cylindrical geometry</td>
<td>27</td>
</tr>
</tbody>
</table>
3.8 Load response for deformation of a cylindrical surfaces ($\kappa = 0$) ........ 28
3.9 Creased spherical boundary in natural and folded states ............... 29
3.10 Load response for deformation of a spherical surfaces ($\kappa > 0$) ........ 30
3.11 Stability of creased spherical shells ........................................ 30
3.12 Deformation profile for a un-creased and creased spherical shell ....... 31
3.13 Origins of a stable isometric state in creased spherical shell - comparing bending and stretching energy contributions ................................. 32
3.14 Energy landscape for indentation of a spherical shell for various values of $\alpha$ 33
4.1 Re-configurability through multi-stability ...................................... 38
4.2 RCF geometry .................................................................................. 39
4.3 Design and fabrication of 3D printed molds ...................................... 40
4.4 Phase diagram for stability of elastic double conical frusta, with fixed $t/R = 0.017$, $t_c = 0.25$ mm, $\alpha_2 = \alpha_1 - 10^\circ$ and varying $h_1, \alpha_1$ ................. 41
4.5 Load-displacement curves for RCF samples with $\alpha_1 = 45^\circ$ .............. 42
4.6 Effect of residual stress ...................................................................... 43
4.7 Closing the topology of a relaxed unit to isolate effect of stress .............. 43
4.8 Programming a built-in stress for exploring non-axial stability in RCF geometry ....................................................................................... 45
4.9 Glued sample ($r = R$) for inducing stress, with the seam region shown in inset ................................................................. 45
4.10 Load response of samples with similar geometry ($h_1 = 6$ mm, $\alpha_1 = 45^\circ$, $\alpha_2 = 35^\circ$) but varying starting $\psi$, hence encoded with increasing amount of $\sigma_p h_i$ .............................................................. 46
4.11 The effect of studied pre-stress in conical frusta is clearly evident through the induced stability under non-axial deformation, for $h_1 > 5$ mm), originally absent in the un-stressed phase diagram ........................................ 47
4.12 Front and top view of stressed RCF \((h_1 = 10\, mm, \alpha_1 = 45^\circ, \psi = 60^\circ)\) demonstrate re-configurability to stable states corresponding to multiple azimuthal positions \(\phi = 0^\circ, 90^\circ\) and \(180^\circ\) ........................................... 48

4.13 Harness for CT scanning ........................................... 50

4.14 Distribution of Gaussian \((\kappa)\) and Mean \((H)\) curvature during axial deformation of RCF with \(h_1 = 10\, mm, \alpha_1 = 45^\circ\) and \(\psi = 0^\circ\) .......................... 51

4.15 In-situ curvature measurement ................................. 53

A1. Grayscale lithography for programming shape in hydrogel sheets ....... 60
A2. Possible inquiries to the DMD device ............................ 67
A3. Modes for resetting the DMD device ............................ 68
A4. Returned values and their meaning for error reporting ............... 72
CHAPTER 1

INTRODUCTION AND DISSERTATION OUTLINE

The distinction between Euclidean and non-Euclidean geometry can be made using the fifth postulate of geometry, the parallel postulate as defined by Euclid [1]. In short, given a line and a point not on it, zero or infinite non intersecting parallel lines to the given line can be drawn through the point, for elliptical or hyperbolic geometry respectively. Non-Euclidean geometries are ubiquitous around us, evident in natural examples such as pollen grains and red blood cells [2–4], ancient architecture such as the Stupas (Sanchi, India, 3rd century BCE) and the Pantheon (Rome, Italy, 118-128 AD), and engineered micro-structures [5–7]. These curved geometries offer notable unique properties compared to their flat counterparts, such as the controlled folding of pollen grains upon dehydration [3] or the enhanced rigidity commonly seen in architectural domes or egg shells [8].

The curvature in these geometries also presents a unique challenge in fabrication and mechanical analysis of the shells. In this chapter, we briefly introduce the strategies established in this dissertation work for improvements in fabrication and control over deformation pathways for elastic shells.

Figure 1.1: Non-Euclidean geometries in natural and architectural examples: (A) Pollen grain, Lilium auratum (Public domain image sourced from remf.dartmouth.edu). (B) Red blood cell (Public domain image sourced from The National Cancer Institute at Frederick). (C) Spherical dome of the main stupa, Sanchi, India, built in 3rd century BC (Image reproduced here under creative commons license, ©Aditya Maurya, 2012). (D) Hyperboloid water tower designed by Vladimir Shukhov, Nizhny Novgorod, Russia, built in 1896 (Public domain image sourced from Wikimedia commons).
1.1. Shape-programming as an alternate route for fabrication of non-Euclidean geometry

Due to their curved nature, fabrication of non-Euclidian geometries requires additional considerations compared to their Euclidian counterparts. With advances in industrial manufacturing, traditional subtractive or additive methods which are inherently 2D in nature, can approximate these curved geometries with sufficient resolution. Wherever applicable, molding based fabrication methods have also been used to produce geometries with precise control over local curvatures. Yet, limitations arise for adapting these commonly used fabrication techniques across various material systems and different length-scales, especially for micron- or nano-scale fabrication. Leveraging the significant advances made for fabrication at these smaller length-scales through lithographic processes, there is recent interest in the development of techniques for transformation of a processed 2D sheet into a 3D geometry with controlled curvature [9–11].

In general, these ‘shape-programmable’ systems can transform their geometry to suit a desired application, in turn providing a responsive handle on switching and fine tuning properties such as effective packing [12, 13], optical [6, 7, 14] and acoustic characteristics [15, 16] or mechanical behavior [17, 18]. The impact of these broad range of efforts can be seen in applications ranging from aerospace [13, 19–21], architectural [22] and medical [23–25] fields, along with numerous studies on the fundamental physics behind deployable structures [12, 26, 27] (see Fig. 1.2).
As discussed in following sections, we introduce two major pathways for encoding this shape-transformation in programmable systems:

(i) **developing mechanisms for patterning non-uniform, in-plane growth in flat sheets,**

and

(ii) **developing pathways to program multiple stable states in an elastic shells by controlling local stability during deformation.**

### 1.2. Fabrication of non-Euclidean geometries by non-uniform growth in flat sheets

Fabrication of 3D (Euclidean or non-Euclidean) geometries starting form a 2D flat sheet of material is a desirable capability, especially provided the advances in high repro-
ducibility, resolution and ease of 2D manufacturing techniques such as printing or lithography. Several natural examples demonstrate similar shape-programming for finely-tuned tasks such as bending of sunflower stems [28], coiling of cucumber tendrils [29], seed dispersion in chiral Bauhinia seed pods [30] and shaping of leaves, flowers and intestinal villi [31, 32] with exceptional complexity and beauty. In general, this shape transformation arises from incompatibility of local curvatures in the target state compared to the initially flat state, causing the resulting strains to relax by buckling the original geometry in a compatible configuration.

This geometric incompatibility usually arises from external constraints on geometry or differential strains in the body. For example, an un-constrained flat beam can buckle out-of-plane due to non-uniform growth along the thin direction. This is the general working principle to program mean curvature ($H$) for a final Euclidean geometry, a method commonly used in thermostats. The geometry and mechanics behind this phenomenon is well understood in beam-buckling literature [33]. On the other hand, programming non-Euclidean geometry starting form a flat sheet necessitates a non-uniform growth along the in-plane direction, and requires a deeper analysis of geometry and mechanics.

To understand the general framework behind programming non-Euclidean geometry in 2D sheets, we consider how local strains in a body under non-uniform growth affect the resulting geometry. Following Sharon and co-workers [9], we define any arbitrary geometry in terms of the distances between neighboring points on its surface as,

$$dl^2 = g_{ij}dx^i dy^j,$$

where $g$ is a 2D metric tensor such that,

$$g = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}.$$  \hspace{1cm} (1.1)

Any in-plane growth applied this geometry defined by Eq 1.1 results in a metric tensor $g'$,
defined according to the local change in distance between points due to this growth. This ‘target’ metric ($g''$) tensor and the resulting strain tensor ($\epsilon$) upon growth can simply be written as,

$$g''_{ij} = \begin{pmatrix} \Omega_{ij} & 0 \\ 0 & \Omega_{ij} \end{pmatrix} \text{ (where } \Omega_{ij} \text{ is the local areal growth)} , \text{ and}$$

$$\epsilon \sim (g' - g). \quad (1.3)$$

To understand the role of this metric-induced shape transformation to non-Euclidean geometry, we invoke a thought experiment. Imagine a circular disc which undergoes in-plane growth in the azimuthal direction by a factor of $\alpha(r)$. The initial and target metric tensors written in polar coordinate system are,

$$g = \begin{pmatrix} 1 & 0 \\ 0 & r^2 \end{pmatrix} \rightarrow g' = \begin{pmatrix} 1 & 0 \\ 0 & \alpha(r)^2 r^2 \end{pmatrix}. \quad (1.4)$$

According to the new metric defined in Eq. 1.4 the disc perimeter grows from $2\pi R$ to $2\pi \alpha(R) R$, presenting an incompatibility with maintaining a flat configuration for any finite growth factor $\alpha(r)$. Hence, this disc must buckle out-of plane to accommodate the extra material into a 3D non-Euclidean configuration.

Under this formulation, the final geometry realized by $g'$ due to this growth can be obtained by using Gauss’ *Thorema Egregium*, which addresses the relationship between metric and Gaussian curvature $\mathcal{K}$ for a surface [9]. Specifically, for a metric tensor,
\[ g = \begin{pmatrix} 1 & 0 \\ 0 & \omega(x)^2 \end{pmatrix}, \] the Gaussian curvature \( \mathcal{K} \) simply takes the form of, \hspace{1cm} (1.5)

\[ \mathcal{K} = -\frac{1}{\omega} \frac{\delta^2 \omega}{\delta x^2}. \] \hspace{1cm} (1.6)

Hence, encoding a local in-plane growth in 2D sheets can provide a powerful mechanism for programming 3D geometries with control over local Gaussian curvature.

Based on these principles for programming Euclidean or non-Euclidean geometry in flat sheets, significant advancements have been made in the field of shape-programmable systems. As discussed earlier, several studies have focused on developing specific material systems in conjugation with fabrication techniques, for stimuli-responsive control of the spatial distribution of strains. Typical mechanisms explored for generating these strains are differential swelling in hydrogels [11, 30, 31, 34–46], spatially-controlled heating in materials [47–53], changing mesogen orientation in liquid crystal elastomers [54–57] or pneumatic actuation of elastomers [25, 58, 59].

Through this dissertation work, we aim to further develop this mechanism for encoding complex geometries using non-uniform growth, by discussing two studies based on theoretical and experimental considerations. In Chapter 2, we provide experimental verification for a proposed mathematical equivalence between this mechanism with another pathway based on topological defects, to program non-Euclidean geometries. The established mapping can be used to correlate a simple approach of ‘cut-and-glue’ construction with the growth-induced buckling, providing an easier route to visualizing the buckled geometries and calculating the desired growth metric [41].
1.3. Programming stability in non-Euclidean elastic shells

Encoding a non-uniform growth for programming target geometries holds promise for the fabrication of materials with well-defined 3D shapes using well-established 2D fabrication techniques. However, the scarcity of practical applications to date indicate limitations to realizing the full potential of this approach. Specifically, there are considerable difficulties involved with encoding, driving and tuning the shape-transformation which restrict the library of material and fabrication protocols that can be utilized. Furthermore, a majority of systems employing non-uniform growth to pattern 3D geometries are plagued by highly specific material systems, length-scale, test conditions (such as environmental requirements) and associated fabrication pathways.

In contrast, applications leveraging shape-transformation due to an instability between locally stable states are common in both natural and man-made examples [7, 29, 60–64]. For example, Venus flytraps (*Dionaea muscipula*) use this mechanism to generate a snapping motion to close their leaves [60], hummingbirds (*Aves: Trochilidae*) twist and rotate their curved beaks to catch insect prey [63], and bacteria (*V. alginolyticus*) exploit a buckling instability of their flagella for re-orientation while swimming [64]. All of these examples harness the otherwise ‘catastrophic’ snap-through instability to switch between desired states which are locally stable, demonstrating a powerful mechanism for shape-programming. The local stability during deformation of thin elastic shells arises due to the inherent coupling between bending and stretching energies during deformation of a shell [65, 66]. This coupling ensures that any deformation in a thin shell (thickness $t$) requires both the costlier stretching (energy, $E_S \sim t$) and bending ($E_B \sim t^3$), driving the system towards deformation pathways that minimize $E_S$. As a result, *isometric* (or without any stretching) states become energetically favorable, opening pathways to encode shape-transformation. The purely geometric nature of these principles allow for a more universal approach to shape-programming, enabling adoption to a wider range of material system
and length-scales. In notable examples, harnessing this instability driven mode to program a shape-transformation between locally stable states has been realized using curvature inversion in a 1D Euler’s beam [18, 54, 67] or 2D shell [7, 68].

In this dissertation work, we discuss theoretical principles from differential geometry and mechanics for developing simple design rules for programming local stability in non-Euclidean shells. In Chapter 3, we discuss the general design rules for harnessing a snap-through instability between bi-stable, isometric states, for three prototypical geometries: cylindrical (Euclidean), spherical and helicoid (non-Euclidean). Next, we consider further improvements to shape-programmable systems by addressing the limitation in the number of geometries that can be programmed in most practical examples. Under this theme in Chapter 4, moving beyond the ability to program bi-stability, we discuss the fundamental conditions necessary to impart multi-stability to an elastic shell.
CHAPTER 2

EXPERIMENTAL VALIDATION FOR EQUIVALENCE OF NON-UNIFORM GROWTH AND TOPOLOGICAL DEFECTS FOR SHAPING OF HYDROGEL SHEETS †

2.1. Introduction

The underlying concept of growth-induced buckling of 2D sheets has been exploited for shape-programming complex 3D geometries, both in natural and man-made examples. Yet, another well known shaping-mechanism exists with similar ability to prescribe Gaussian curvature, involving topological defect-induced buckling. To demonstrate this mechanism we provide a simple tabletop experiment in Fig. 2.1; simply by removing a wedge, and gluing along the newly cut edges, we can transform a flat sheet of paper into a 3D buckled configuration with singular Gaussian curvature at the conical tip [69, 70].

This relationship between topological defects and geometry has far-reaching implications, from faceting of viral capsids [71] and fullerenes [72], shape-transitions in protein-coated cell membranes [73], buckling in graphene [74] and the art-form of kirigami. The effect on global curvature arising from topological defects has also been used in architecture [75] and other engineered structures [76, 77].

† The following work has been reproduced in part with permission from Royal Society of Chemistry, as published on June 24th, 2014, in Soft Matter, Vol. 10, Issue 34, Page 6382-6386. ©RSC, 2014
Although there has been significant attention towards studying the geometrical implications of topological defects, an in-depth analysis of a mathematical mapping between the two mechanisms under discussion had not been established before this work. Such a mapping between growth-induced and topological defect-induced buckling can be of great advantage: allowing visualization of the final geometry for a specified metric with basic ‘cut-and-glue’ experiments, and inversely presenting a pathway for calculation of the growth-metric required for a specified 3D geometry.

In this chapter, we provide an experimental proof for validation of one such mapping. Under this framework, we correlate the growth-metric to be patterned for a specific 3D geometry, with its topological construction using established mathematical mapping. We limit ourselves to geometries with $\mathcal{K} = 0$ everywhere except at singularities, and consider the mapping for a cylinder, conical dipole and tetrahedron geometry. In addition to serving as a proof-of-concept, these experiments are also used to establish trade-offs which affect the required resolution of the target metric, and understand the practically realistic limits of shape-programming with non-uniform growth.

2.2. Theory: mapping between non-uniform growth and topological defects

The origins of mapping between growth-induced and topological defect-induced buckling is far from just being a mathematical equivalence: it is actually an equivalence between minima of the elastic energy derived from both mechanisms. To understand this equivalence, we consider the general form of growth metric as:

$$ds^2 = \Omega(x, y)(dx^2 + dy^2),$$

(2.1)

where $\Omega(x, y)$ denotes a local areal growth factor [78]. The resulting equilibrium geometry is then solved for using a covariant set of equations (Eq. 1.6), as discussed in Chapter 1.
This covariance implies that we can write an equivalent metric in another coordinate system \{u, v\} as

\[ ds^2 = \tilde{\Omega}(u, v)[du^2 + dv^2], \quad (2.2) \]

without altering the equilibrium geometry. Hence, given a specified geometry, two flat domains can be reached by ‘un-growing’ by either \(1/\Omega(x, y)\) or \(1/\tilde{\Omega}(u, v)\) (see Fig. 2.2). Converting to complex coordinates \((z = x + iy\) and \(w = u + iv\)), the metric in the domain of complex \(w\)–plane can be expressed as \(ds^2 = 2\Omega(w, \overline{w})dwd\overline{w}\). For mapping this on the domain of complex \(z\)–plane, an analytical function \(g(z)\) can be defined such that \(w = g(z)\). Hence, the metric in the domain of \(z\)–plane becomes

\[ ds^2 = 2\Omega[g(z), g(z)]|\partial g(z)|^2 dzd\overline{z}, \quad \text{where} \quad \partial \equiv (\partial_x - i\partial_y)/2. \quad (2.3) \]

This constitutes as first result of the proposed mapping: \textit{a domain in the} \(z\)–plane \textit{with isotropic growth} \(\tilde{\Omega}(z, \overline{z})\) \textit{will result in the same equilibrium geometry as the domain} \(g(z)\) \textit{in the} \(w\)–plane \textit{with isotropic growth} \(\Omega[w, \overline{w}]\).

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{equivalent_domains.png}
\caption{Equivalent domains for growth-induced and topological defect-induced buckling into a spherical geometry: Two domains, both of which can be mapped to the same sphere with different growth patterns. The mapping between the two is \(w = g(z) = z/(1 - z)\).}
\end{figure}

Next, using Gauss’s \textit{theorema egregium} and solving for \(g(z)\) by setting \(\Omega(w, \overline{w}) = 1\), we obtain \(\partial g(z) = e^{h(z)/2} \prod_i(z - z_i)^{-K_i/(2\epsilon)},\) where \(h(z)\) is an arbitrary analytical function.

\[ \text{\dag} \quad \text{Detailed mathematical proof available in the original publication [41]} \]
This mapping \( g(z) \) then implies that a non-uniform growth function \( \tilde{\Omega}(z, \bar{z}) \) (in \( z\)-plane) has the same equilibrium geometry as that obtained from a domain with no growth, given that some of the edges are identified (in \( w\)-plane).

In the following discussion, we validate this mapping by applying it to the topological construction and corresponding growth function for a specific geometry. We confirm the buckled geometry for this growth pattern using controlled swelling of hydrogel sheets and by performing spring-bead numerical simulations (in \( z\)-plane); meanwhile comparing this to the geometry realized by the topological construction (in \( w\)-plane).

2.3. Materials and technique

For defining a growth-metric in 2D sheets, we use halftone lithography [38] to define a local crosslinking density to a polymer network. We use a photo-crosslinkable, thermally-responsive copolymer, poly(N-isopropyl acrylamide)-co-(acrylic acid)-co-(benzophenone acrylamide)-co-(fluorescein acrylate) (PolyNipam), synthesized using simple free-radical random copolymerization according to the following recipe:

<table>
<thead>
<tr>
<th>Monomer (mole %)</th>
<th>Functionality provided</th>
</tr>
</thead>
<tbody>
<tr>
<td>N-isopropyl acrylamide (87)</td>
<td>thermal responsiveness (LCST at ( T = 32^\circ C ))</td>
</tr>
<tr>
<td>benzophenone acrylamide (7)</td>
<td>photo-crosslinking (upon UV exposure [79])</td>
</tr>
<tr>
<td>acrylic acid (5)</td>
<td>control on swelling of final hydrogel in water</td>
</tr>
<tr>
<td>fluorescein acrylate (1)</td>
<td>fluorescence signal for confocal imaging</td>
</tr>
</tbody>
</table>

Table 2.1: Recipe for photo-crosslinkable, thermally responsive PolyNipam copolymer

For casting films of \( \sim 10 \ \mu m \) thickness (and lateral dimensions of \( \sim 300 - 500 \ \mu m \)) with minimal stress, we slowly evaporate solvent by dropcasting a 1% solution (90 – 120 \( \mu l \)) in chloroform on silicon wafer substrate (1 \( cm^2 \)) enclosed in a small chamber with elevated vapor pressure of solvent. The resulting films are then patterned using UV exposure.
according to halftone lithography [38, 41]. In halftone lithography, the areal fraction of highly crosslinked ‘dots’ ($\phi_{\text{low}}, \Omega_{\text{low}} = 2.5$) embedded in lower crosslinked matrix ($\Omega_{\text{high}} = 7.5$) is controlled by arranging dots of diameter $d$ arranged in a regular hexagonal lattice with constant $a$; in turn defining an effective local crosslinking density or growth (see Fig. 2.3A). Similar to halftone printing, this discrete pattern relies on elasticity of film to produce a smooth curved surface in buckled configuration.

![Image](image-url)

Figure 2.3: Halftone patterning for defining a growth metric in 2D hydrogel sheets: (A) The basic process of halftone patterning, by using two masks to define regions with high crosslinking density ($\Omega_{\text{low}}$) embedded in a low crosslinked matrix ($\Omega_{\text{high}}$). (B) Areal growth factor ($\Omega$) varying with UV dose and the areal fraction of highly crosslinked regions ($\phi_{\text{low}}$).

According to the resulting calibration curves (Fig. 2.3B), target metrics were discretized into a pattern of dots on a hexagonal lattice using MATLAB (Mathworks Inc., USA), and converted to a CAD (AutoCAD, Autodesk, USA) format for printing (Front Range Photomasks, USA) as a chrome on glass mask with a resolution of 3 $\mu$m. Regions lying outside the allowed range of swelling of PolyNipam polymer were truncated to extrema, hence setting an ‘iso-swelling’ boundary for the patterned gel. In cases where the pattern boundary does not coincide with the $\Omega_{\text{low}}$ or $\Omega_{\text{high}}$ limit, an arbitrary iso-swelling boundary was chosen. For each geometry, two photo-masks were then used to define regions with different degrees of crosslinking using an inverted optical microscope (Axio-vert 200, Carl Zeiss AG,
Germany) and a fluorescence excitation lamp (X-Cite120 Q, Excelitas Technologies Corp., USA) as a UV source. The patterned film was next developed in an ethanol:water mixture (2:1 by volume) and released by swelling in phosphate buffered saline (PBS) solution (containing 1 mM NaCl and 1 mM phosphate buffer, pH 7.2) (Fig. 2.3A). The final shape of the swelled gel film was then characterized using a laser scanning confocal fluorescence microscope (LSM510 Meta, Carl Zeiss AG, Germany) followed by 3D reconstruction using ImageJ.

2.4. Experimental observations for shaping of hydrogel sheets using non-uniform growth induced buckling

We first apply the proposed mapping to a simple geometry, a cylinder. In the domain without any growth, a cylinder may be constructed by taking a strip of paper of length $2\pi R$, and identifying the edges to obtain a cylinder of radius $R$. Additionally, a cylinder can also be realized by imparting a non-uniform growth metric $\Omega(r)$ to a disc of radius $R$ as,

$$\Omega(r) = c \left( \frac{r}{R} \right)^{-2} \tag{2.4}$$

In Fig. 2.4 we demonstrate the equivalence between these mechanisms through a paper model of the derived topological construction, and photo-patterning a hydrogel sheet with the derived growth metric (Eq. 2.4). Corroborating the obtained cylindrical geometry, we also present the equilibrium shape obtained by numerically minimizing a bead-spring model upon growth.

Next, we extend the mapping to a dipole geometry. A dipole has two oppositely charged $\mathcal{K}$ singularities with two tips of conical region pointing in opposite directions, as represented by the growth function,

$$\Omega(r) = c \left( \frac{r + d_2}{r - d_1} \right)^{\beta}, \tag{2.5}$$
indicating a conical singularity of deficit angle $\beta \pi$ at distance $d_1$ from origin, and another with deficit angle $-\beta \pi$ at a distance $d_2$ [80]. Moreover, bounding the growth function within $\Omega_{min} \leq \Omega \leq \Omega_{max}$ results in excised regions around the singularities as seen in Fig. 2.5.

This growth function $\Omega(r)$ is used to map onto a domain with no growth, and the necessary topological construction is obtained, resulting in a paper model with the desired geometry (see Fig. 2.5). Similarly, the growth pattern generates a predicted buckled geometry in both simulations and hydrogel experiment.

Finally, we consider the inverse problem of optimizing a growth pattern required for a tetrahedron geometry using our mapping and the topological construction. The generic form of a growth function required for this geometry is given by,

$$\Omega(z, \bar{z}) = \Omega_0 |e^{h(z)}|^2 |z^3 - R^3|^{-1} \text{ (where } h(z) \text{ is an analytical function)} \tag{2.6}$$
representing three $\mathcal{K}$ singularities on corners of an equilateral triangle. For an optimum growth function, the excise areas should be minimized. In addition, to impart a uniform growth at a far away point from these singularities ($z \gg R$), we use ‘virtual’ singularities of opposite charge that lie outside the boundary of the sample (at $D > R$). Hence, fulfilling these conditions we fix the form of $h(z)$ and growth function as,

$$|e^{h(z)}|^2 = |z^3 - D^3|^{-1}, \text{ and}$$

$$\Omega(z, \bar{z}) = \Omega_0 \left( \frac{z^3 - D^3}{z^3 - R^3} \right).$$

noting that an increasingly uniform growth function away from the singularities can be achieved as $D \to R$, along with smaller excised regions. The corresponding topological construction relies on folding an equilateral triangle by identifying a length $L$ around the excised cores (Fig. 2.6), equivalent to the distance between real and virtual singularities in buckled state for the domain with growth. From the exact solution for $L$ (obtained using Eq. 2.8, [41]), it can be inferred that the length $L$ decreases as $D \to R$.

The antagonistic effect of these conditions across the two domains has a striking effect on the final buckled geometry. **We can optimize the growth-induced patterning by making boundaries more uniform and minimizing the excised area, but at the expense of shortening the length to be identified.** As a result, experiments in both domains exhibit a systematic deviation from a closed tetrahedron geometry as shown in Fig. 2.6, demonstrating the first tradeoff in shaping of hydrogel sheets. One way to close this geometry is to use a higher range of growth ($\Omega_{\max} - \Omega_{\min}$), allowing more length to be identified.

In addition to the discussed limitation, another practical hurdle arises due to the finite resolution of lithography process used for photo-patterning. The use of virtual singularities demands the patterned growth to go through the entire range: from $\Omega_{\text{low}}$ to $\Omega_{\text{high}}$ over a small distance between real and virtual singularities respectively. This again presents a contradiction, **the closer these singularities are placed for an optimal metric, higher the required resolution of patterning.**
Figure 2.6: Equivalent domains for a tetrahedron: Topological construction for a tetrahedron with singularities removed (red edges of length $L$ to be identified). The corresponding growth pattern and domain with $D/R = 1.75$. The holes and boundary are chosen so that, $\Omega_{\text{high}}/\Omega_{\text{low}} = 3$. Results of numerical minimization of for a tetrahedron of thickness $0.04R$ (inset), topological construction of poly(vinyl siloxane) elastomer sheet with thickness $\approx 0.03R$ (scale bar: 10 mm) † and the hydrogel experiment all exhibit a striking similarity regardless of deviation from the intended equilibrium geometry.

Hence, through a comparison between two mechanisms studied here we obtain a unique perspective on limitations in metric patterning, suggesting the importance of balance between desired geometry, resolution of the patterning and range of growth available.

2.5. Limitations and Conclusions

In conclusion, we have experimentally verified the theoretical work by Santangelo on an explicit mapping between two mechanisms of metric patterning: non-uniform growth and topological defects. This mapping opens a pathway for one to visualize and optimize final geometry to be patterned using non-uniform growth, by employing a simple cut-and-glue construction. More importantly, the discussion allows us to realize the limitations and

† The model for topological construction of tetrahedron geometry is fabricated using an elastomer sheet instead of paper to effectively model the elastic hydrogel material. Further details on construction can be found elsewhere [41]
tradeoffs in growth-induced patterning and provides insight towards optimizing several elements for shape-programming: the growth function, material system and fabrication techniques.

Our discussion for the proposed mapping is limited to geometries with $\mathcal{K} \neq 0$ only in singularities. Despite this, the mapping signifies an important development towards furthering the understanding of different mechanisms for shape-programming.
CHAPTER 3

GEOMETRIC CONTROL FOR SNAP-THROUGH TRANSITIONS OF CREASED ELASTIC SHELLS ‡

3.1. Introduction

Despite the ubiquity of curved shells both in artificial and natural environments, the mechanics behind their deformation is not well understood. In recent years, the plethora of geometrically induced properties that these shells offer has attracted both fundamental and applied studies. Owing to their curvature, there exists a coupling between bending and stretching in these shells resulting in geometrically induced rigidity [8, 68, 81] which enhances the structural stability of such structures. This coupling alters geometry and mechanical behavior of these shells, as seen in both naturally occurring scenarios such as intestinal villi and pollen grains [3, 32], and man-made structures such as programmable meta-materials [82–85]. Particularly in shells with multi-stable isometric states, this geometrically induced rigidity presents a high energy barrier towards the transition between them, often leading to high forces and acceleration associated with stretching mediated ‘snap-through’ transition [7, 54, 60–63, 67, 86, 87]. Though usually avoided in structural design due to the resulting loss in load-bearing ability, examples of utilizing these otherwise ‘catastrophic’ instabilities in shells are prevalent in natural phenomena across an impressive range of applications, mechanisms and length-scale [60–64]. This abundance of bio-inspiration has resulted in recent interest towards controlling snap-through instability for incorporation in artificial systems. Notable efforts include engineered micro lenses

‡ The following work has been reproduced in part with permission from National Academy of Sciences, as published on September 8th, 2015, in Proceedings of National Academy of Sciences, Vol. 112, Issue 36, Page 11175-11180. Copyright: PNAS, 2015
which rapidly switch from convex to concave shapes in order to tune their optical properties [7], and utilizing the well known instability of a 1D beam under compression for a rapid actuator [54, 67]. Despite this ability to engineer bi-stability and snapping transitions in a variety of systems by using curvature inversion, pre-stress or material anisotropy [88–94], there does not exist a general geometric design rule for creating a snap-through between stable states of arbitrary shells. Contrary to this, there exist a robust framework for understanding deformation of a flat sheet, evident in both the art-form of origami and recent works leveraging its underlying design principles [12, 95, 96]. Deforming the sheet to access low energy states can be easily realized in origami, by simply weakening the structure locally to minimize stretching resulting in a well-guided deformation path.

Inspired by these ideas from origami and the need for establishing design rules to program snap-through instabilities, we consider addressing the deformation of a curved surfaces along specific curves (‘creases’). In this chapter, we use concepts from differential geometry and mechanics and establish specific rules to be followed for designing creases that allow for a robust control on presence of bi-stable states, and the continuity of deformation between them can be realized.

3.2. Geometrical mechanics of folding a shell

Folding curved surfaces poses a non-trivial problem, as the continuum mechanics of a shell under these deformations is far from fully understood. The concept has been realized on rare occasions in art-form [96–98], but systematic studies have been impeded by both experimental and theoretical difficulties.

To begin, we consider an arbitrary curved surface as shown in Fig. 3.1. A ‘crease’ on this surface along which it is to be deformed is defined as a narrow region of local weakening, realized experimentally through thinning the material.
Though locally this crease allows an hinge-like bending behavior, presence of a finite global curvature induces large deformations in the shell and hinge itself. To proceed, we assume this deformation to be approximately isometric (no stretching, or in-plane strain) for a sufficiently thin shell.

We parametrize the crease on this arbitrary surface using the arc length $s$ and tangent vector $t(s)$, resulting in the space curvature to take form $\kappa \equiv |dt(s)/ds|$. The surface tangent (simultaneously orthogonal to the crease tangent $\hat{t}$ $\hat{u}$ and surface normal $\hat{n}$ can also be defined for the surface under consideration as shown in Fig. 3.2. Finally, by using the angle $\psi$ between Frenet normal $\hat{N}_F$ (normal to crease, in the osculating plane) and surface tangent $\hat{u}$ we can define geodesic curvature, $\kappa_g = \kappa \cos \psi$, and the normal curvature $\kappa_N = \kappa \sin \psi$. These quantities are the respective projections of the space curvature onto the tangent ($\hat{u}^\pm$) and normal ($\hat{n}^\pm$) of the shell.

The angle $\psi$ can be used as an handle for quantifying deformation, expressed as,

$$\psi = \pm \cos^{-1} \left( \sqrt{1 - (\kappa_N/\kappa)^2} \right). \quad (3.1)$$

The uniqueness of solutions for $\psi$ is the first result of this treatment: for a curve with $\kappa_N$
there are two solutions $+\psi$ and $-\psi$, except when $\kappa_N$ vanishes and $\psi = 0$.

As discussed previously, in the case of finite thickness shells, the transition between these two isometric states ($\pm \psi$) is characterized by a region with high stretching energy and that therefore provides an energetic barrier. To further clarify the origin of this non-linear behavior, let us consider the mean curvature ($H$) and bending energy density ($E_B$) of a shell undergoing deformation from $\psi \rightarrow -\psi$. The mean curvature near the hinge as calculated form the first and second fundamental form of surface, can be written as

$$H = \frac{1}{2} \left( \kappa_N + \frac{\kappa + (\partial_\psi \psi + \tau)^2}{\kappa_N} \right), \quad (3.2)$$

where $\tau$ is the torsion of the fold, which measures the rate that the osculating plane twists around the hinge and, hence, the non-planarity of the hinge [99]. According to Eq. 3.2, the mean curvature diverges as angle $\psi$ changes sign during deformation, leading to a divergence in bending energy density $E_B = \frac{b}{2} (H - H_0)^2$ for a deformed shell ($H_0$ is background curvature of the shell). This infinite energy barrier between two isometric states is overcome by favoring stretching in a finite thickness shell, exhibited by the presence of a snap-through instability (further details and mathematical proofs in [100]).

The discussed formulation also points to a flaw in one of our assumptions: finite thickness shells do indeed undergo stretching near the hinge to overcome the infinite bending energy barrier, similar to the already studied examples of stress localization in deformed shells [68, 101–106]. Another special case arises for creases with vanishing $\kappa_N$, in which case there exist no bi-stable states and the shell can be folded with monotonically increasing bending energy barrier.
3.3. General design principle for snap-through instabilities

Based on this purely geometrical treatment, we propose a length-scale and material independent design rule for programming and controlling the continuity of deformation between bi-stable states on arbitrarily curves surfaces: *deforming a curved surface along a crease with non-zero $\kappa_N$ induces a discontinuous snap-through instability between isometric states due to the infinite bending energy barrier between them. On the other hand, if the surface is deformed along a crease with $\kappa_N = 0$, a continuous hinge-like deformation is expected.*

Applicable to any arbitrary surfaces, the unique features offered by all three prototypical curved geometries {spherical, cylindrical and elliptical} such as effect of Gaussian curvature $\mathcal{K}$ and presence of $\kappa_N = 0$ curves further justify a careful consideration to each of them. Of specific interest is the different cases related to $\kappa_N = 0$ curves that exist on various geometries. On a Euclidean geometry ($\mathcal{K} = 0$) such as a cylinder, there exist only one family of such curves along the axes, necessarily straight in space [99] (depicted as white lines, Fig. 3.3a). For a shell with positive $\mathcal{K}$, these curves do not exist altogether so that all creases shall generate a snap-through instability (Fig. 3.3b). Finally, and most interestingly, a surface with $\mathcal{K} < 0$ poses a further interesting case - having two directions at each point along which $\kappa_N = 0$, which can exist as both planar or non-planar curves [107] (depicted as white lines, Fig. 3.3c).

Figure 3.3: Schematics for prototypical geometries: cylindrical ($\mathcal{K} = 0$), spherical ($\mathcal{K} > 0$) and helicoid ($\mathcal{K} < 0$) with special curves with $\kappa_N = 0$ denoted as white curves.
3.4. Fabricating creased, non-Euclidean elastic shells

To test our hypothesis as described in the previous section, we lay down a framework to fabricate elastic shells with regions of different thickness to realize creases as follows. Owing to their Euclidean nature, cylindrical shells can be fabricated starting with a flat sheet by using conventional 2D techniques. Here, we demonstrate the same by fabricating cylinders with controlled crease regions using a commercial laser cutter (Zing Epilog 16) to score poly(ethylene terephthalate) sheets (Grafix Dura-Lar®, 120 μm thick, Y ~ 5 GPa). The shape of this plane curve is set to be sinusoidal, such that when the sheet is wrapped to form a cylinder the resulting space curve is the intersection between a plane at an oblique angle (θ), and the amplitude can be scaled to obtain various combinations of d, θ as seen in Fig. 3.4.

![Figure 3.4: Fabrication of Euclidean and non-Euclidean geometries with creased region along controlled curves: (a) A flat sheet scored with calculated curve to realize a planar κ_N ≠ 0 crease in cylindrical form as shown (b, c) 3d printed molds with embossed ridge to realize crease on non-Euclidean (K ≠ 0) geometries.](image)

We mold elastic or thermoplastic polymers in 3D printed molds to impart controlled non-Euclidean geometries with creased regions. We prepare two-part negative molds by 3D printing using a commercial 3D printer (uDimensions, Stratys Inc.), consisting of embossed features to generate final shell with thinned profile (creases). Hemispherical shells were fabricated using a curable two part elastomer poly(vinyl siloxane) (Zhermack SpA Elite Double 32, elastic modulus (Y) = 1.36 MPa). Prior to filling the mold, the 1.2:1 base:catalyst ratio...
mixture was degassed to remove bubbles that may otherwise serve as defects. The helicoid samples were fabricated using poly(caprolactone) (Monomer-Polymer & Dajac Labs, 1258, Y = 353 MPa), by melting polymer in the mold at 70°C, and allowing it to cool. The final shells were 1 mm in thickness, and creased regions had a rectangular cross-section 0.75 mm deep and 1 mm wide along the appropriate curve. Only samples without structural defects were included for further testing.

3.5. **Studying the geometrically programmed snap-through instability in curved elastic shells**

In this section, we explore the snapping behavior and lack thereof in geometries with varying global Gaussian curvature (\(K\)) and crease parameters (\(\kappa_N\)). For appropriate cases, we also study the load response for deformation of shells along these creases by using a custom load displacement setup. By combining a linear translation stage (T-LSM 100, Zaber Technologies Inc., Canada) and a load cell (RPG-10, Loadstar Sensors Inc., USA), along with 3D printed indenters and grips that apply highly specific boundary conditions, we achieve the ability to record strain-controlled (typically 5 mm/min) response using an in-house algorithm in MATLAB.

**3.5.1. Negative Gaussian curvature surface, helicoid**

As previously discussed, a helicoid geometry is of great interest since at any given point, there exist two family of orthogonal curves with zero \(\kappa_N\) [99]. Of these, the planar curves can be thought of as along the generating lines, while the other non-planar \((\tau \neq 0)\) family of curves is locally orthogonal to them. To test our hypothesis, we fabricate plastic helicoids with creases along three kinds of curve: (i) a \(\kappa_N \neq 0\) curve generated by slicing the helicoid with a plane, (ii) the first \(\kappa_N = 0\) curve along the generating lines, and (iii) the second \(\kappa_N = 0\) along the helical curve orthogonal to ruling lines (Fig. 3.5a, b, and c respectively).
Figure 3.5: Continuity of deformation of a helicoid ($\kappa < 0$) geometry: (a) Five frames at equal time interval depict the discontinuous snap-through deformation of a helicoid possessing a crease with $\kappa_N \neq 0$ from initial folded state (0) to the final folded state (+4). Frame 3 falls mid-snap, and is blurred. For both (b) planar (along generating line), and (c) non-planar (or helical) $\kappa_N = 0$ creases, a composite image is used to depict torsion along either sides of the folded state (0) of the crease. As predicted, a continuous deformation characteristic of a hinge is observed for both of these cases and frames at equal time intervals on either sides (1, 2 on one side, and i, ii on another) depict the same.

Supporting our hypothesis and design principle, we observe a discontinuous transition between isometric states around the $\kappa_N \neq 0$ curve evident of snap-through transition, as shown in a time-lapse composite image in Fig. 3.5a. Further, deforming the sample along a planar crease with $\kappa_N = 0$ exhibits a smooth hinge-like motion, intuitive and as predicted by our hypothesis (Fig. 3.5b). Lastly, bolstering the design rule we observe a similar continuous deformation of this curved geometry for a non-planar crease with $\kappa_N = 0$, which although is less obvious is predicted by our hypothesis (Fig. 3.5c).

Recording load-response of a helicoid geometry itself poses a challenge given the non-trivial path a point indenter should travel for maintaining contact throughout the deformation of shell. Given the linear nature of our actuator, and the importance to avoid stick-slip nature that may arise by a contact based indenter, we attach a carbon fiber tow between load cell and helicoid samples under deformation. As shown in Fig 3.6a we observe a monotonically increasing force response as expected for the hinge like behavior of samples with $\kappa = 0$. On the other hand, deforming the helicoid along a crease with $\kappa = 0$
is characterized by a snap-through instability, resulting in a ‘slack’ in the tether and hence a drop in load values (see insets, Fig 3.6b). Combined, these qualitative experiments on a helicoid geometry validate our design rule for a negative Gaussian curvature geometry.

3.5.2. Zero Gaussian curvature surface, cylinder

A cylindrical geometry, as a singly curved surface, also provides a unique case for studying deformation of curves surfaces. The load response and localizations upon indentation has been well established for thin, elastic and un-creased cylinders [68, 104, 108], and can be recovered in our experiments using a un-creased cylinder.

To study the effect of inclusion of \( \kappa \neq 0 \) creases, we fabricate cylindrical shells as described earlier, and use a point indenter to deform the shell along these creases. Remarkably, despite the prediction from our design rule, we observe a continuous global bending instead of a snap-through instability for a free cylinder. This indeed results from the singly-curved nature of cylindrical geometry, making it possible to bend without stretching in one direction due to absence of a coupling between the two modes. Preventing this global bending can easily be achieved by constraining the boundaries to be fixed as a cylindrical shell as shown in Fig. 3.7.

Figure 3.6: Load response for deformation of a helicoid geometry with creases: \( \kappa_N = 0 \) (a) and \( \kappa_N = 0 \) (b) measured with help of a carbon fiber ‘tether’ attached to a load cell. The continuity of deformation can be seen as ‘smooth’ hinge-like for (a) and discontinuous, ‘snap-through transition’ as predicted by our proposed hypothesis.

Figure 3.7
Two samples, both with $\kappa \neq 0$ creases parametrized by lower values of \( \{d, \theta\} = \{2.8 \text{ mm}, 6.6^\circ\} \) and higher values of \( \{d, \theta\} = \{11.4 \text{ mm}, 12.8^\circ\} \) are indented on the apex of cylinder with a point indenter, and the load response is recorded. As shown in Fig. 3.8, the sample with ‘smaller’ crease has a lower effective stiffness for small displacement, but recovers to that of an un-creased shell for higher displacements (blue curve) without any indication of a snap-through instability. On the other hand, for ‘larger’ crease the load response follows a similar trend to that of a un-creased shell till an intermediate anti-symmetric mode (marker 2), finally snapping to an as-predicted mirror symmetric isometry (marker 3, green curve). Numerical simulations using finite element analysis (performed using ABAQUS, Dassault Systemes) shown in Fig. 3.8 show consistency with the above described behavior for both samples along with the observed anti-symmetric mode.

![Figure 3.8: Load response for deformation of a cylindrical surfaces (\( \mathcal{K} = 0 \)): Force-displacement curves for un-creased (red), crease with $\theta = 6.6^\circ$, $d = 2.8$mm (blue) exhibiting mono-stable behavior. In contrast, a creased cylinder with $\theta = 12.8^\circ$, $d = 11.4$mm (green) indicates an anti-symmetric behavior leading to a bi-stable snapped state. Snapshots from experiments and FEA simulations show different stages of deformation. A contact-slip profile is seen at higher displacements for these samples.](image)

The lack of stability of an isometric state despite a $\kappa_N \neq 0$ crease opens an interesting consideration, that of a competition of bending energy vs. stretching energy for finite thickness shells. We consider the deformation of shell in two parts: a region of mirror symmetric isometry that contains only bending, and a localized ridge that concentrates most of the stretching energy [68, 104, 108, 109]. Thinning the shell in creased regions lowers stretching energy cost for a localized ridge, and may stabilize any resulting isometric states.
only if the associated energy gain is lower than bending energy cost of the inverted region. Specifically, we propose that for lower values of \( \{d, \theta\} \) the bending energy cost of the mirror isometry is large enough that the energy gain from creasing is insufficient to result in bi-stability, whereas larger \( \{d, \theta\} \) values lower the cost of the ridge sufficiently to induce bi-stability.

This competition is of a great importance for considering presence of stability in the bi-stable state, and shall be discussed further for spherical geometry due to ease in analyzing the deformation analytically.

3.5.3. Positive Gaussian curvature surface, hemisphere

Among curved surfaces, spherical geometries have been most commonly studied given their ubiquity in both natural [110] and architectural occurrences. Recent studies have focused on localizations on sphere upon indenting, which are result of localized distribution of stretching energy to maximize regions with isometry [68, 105, 111]. Moreover, a spherical shell naturally requires stretching upon any deformation avoiding any possibility of pure bending modes as present in cylindrical counterparts [65, 66, 68]. To systematically establish a analytically tractable solution for this fully axi-symmetric system, we fabricate elastic shells \((R_s = 35 \text{ mm})\) as described with \(\kappa_N \neq 0\) creases obtained by intersecting shells with a horizontal plane, and define the normalized crease radius as \(\alpha = R_i/R_s\) (Fig. 3.9).

Upon indenting hemispherical shells, we observe a monotonically increasing load response for an un-creased \((\alpha = 0)\) sphere (Fig. 3.10, red curve), similar to literature studies [68, 105, 111]. For a crease with \(\alpha = 0.5\) we observe an overall weaker response compared to the un-creased samples, devoid of any bi-stability (blue curve). Finally, for a creased sample with \(\alpha = 0.6\) we observe a intermediate non-axisymmetric snap (marker 2), followed by a snap-through instability to the isometric state (3, green curve).
We fabricate hemispherical shells with varying Föppl-von Kármán number $\gamma$ and normalized crease radius $\alpha$ to construct a phase-like behavior for understanding the stability of isometric state for creased samples (see Fig. 3.11). Using both experiments (■) and FEA simulations (+) we observe a clear demarcation between a region with stable isometric state (■, + and shaded green region) and hence bi-stability, and that exhibiting mono-stability (■, + and shaded red region).

Figure 3.10: Load response for deformation of a spherical surfaces ($K > 0$): Spherical shells with crease radius $\alpha = R_c/R_s = 0$ (red), 0.5 (blue), and 0.6 (green), and over all radius $R_s = 35$ mm are tested for presence of stable isometric state. For the smaller value of $\alpha$, no stable isometric state is observed indicating mono-stability, while for larger value of $\alpha$ a non-axisymmetric deformation, followed by a stability in isometric state is observed.

Figure 3.11: Phase diagram for stability behavior of creased spherical shells over a phase space in $\alpha$ and $1/\sqrt{\gamma}$. Stability behavior in experiments is characterized as bi-stable (■), mono-stable (■) or temporarily-stable (■, stable only for a few seconds). FEA (points solved denoted with +) provides regions of mono-stability (red shading) and bi-stability (green shading). Each experimental data point was analyzed for at-least 3 shells of appropriate parameters.
3.5.4. Energetics of a deformed spherical shell

We extend our treatment presented for cylinders to spheres by considering well known isometric deformations of a thin spherical shell by Pogorelov put forward in his seminar works [109, 112]. For displacements larger than thickness but smaller than the crease size, the deformation is characterized by an inverted bulge of radius $r$ and bounded by a ridge of size $\ell \sim \sqrt{tR_s}$ (Fig. 3.12). The energy for this state is comprised of the bending energy contribution from the inverted bulge ($E_B$) while the ‘Pogorelov ridge’, acting as an elastic boundary layer, contains all the stretching energy ($E_P$) [102, 113]. For a thin hemispherical shell of radius $R_s$ the individual energy contributions to total energy $E_T$ can be written as,

$$E_T \approx \frac{Yt^3}{12(1-\nu^2)} \left( \frac{r}{R_s} \right)^2 + Yr^3 \left( \frac{t}{R_s} \right)^{5/2},$$

or upon simplification,

$$\frac{E_T}{B} \approx \left( \frac{r}{R_s} \right)^2 + \gamma^{1/4} \left( \frac{r}{R_s} \right)^3,$$  \hspace{1cm} (3.3)

where $B$ is bending rigidity of the shell, $Y$ is the Young’s modulus of material, $\gamma$ is the Föppl-von Kármán number $\gamma \equiv \hat{Y}R_s^2/B$ and $\hat{Y}$ is stretching modulus of the material. For a thin shell $\hat{Y} = Yt$, and $B = Yt^3/12(1 - \nu^2)$, where $\nu$ is Poisson’s ratio, such that $\gamma \sim (R_s \sqrt{12(1 - \nu^2)}/t)^2$. The Föppl-von Kármán number characterizes the important balance between bending and stretching energies, and is useful to be defined even for not-essentially thin shell geometries such as viruses and polymerized membranes [71].

For a creased shell, we assume that the deformation of shell will maintain this same energetics, with a corrected thickness as a function of bulge radius $r$ (or the indentation depth $h$). This correction allows us to realize the thinned region of crease as $t(r = R_s) = e^\epsilon t$, where $\epsilon = 0.75$, effecting the stretching contained in ‘Pogorelov ridge’ to drop drastically due to a thickness change. To visualize this schematically, we plot $E_B$, $E_P$ and $E_T$ for
hemispherical shell with various $\alpha$ values in Fig. 3.13. As predicted, both the stretching and bending energy raise monotonically in absence of a crease region ($\alpha = 0$ case). For small values of $\alpha$ ($= 0.2$), a local minimum in Pogorelov energy around $r = R_t$ demonstrates the effect of thinning, only to be overcome by the monotonically increasing bending energy and hence, failing to result in a local minimum in total energy. Interestingly, for larger values of $\alpha$ ($= 0.3$) the energy gain by this thinning in $E_P$ can be seen to overcome $E_B$, successfully inducing a local minimum in $E_T$ and hence bi-stability.

Figure 3.13: Origins of a stable isometric state in creased spherical shell - comparing bending and stretching energy contributions: For an un-creased shell, the energy of an indented shell is composed of the bending energy $E_B$ (black dotted) and the Pogorelov ridge $E_P$ (colored dotted). For a creased shell $E_P$ takes a substantial dip localized at $r \sim R_t$, but the total energy $E_T$ (colored solid) only has a local minimum if the crease is large enough. In these schematics the function $t(r)$ is chosen to mimic the profile provided by experimental molds.

Thus, we propose that **the stability of creased shells in isometric state is governed by competition between bending energy of the inverted shell and stretching energy contained in the creased region.** The discussed treatment successfully explains the stability regions we observe in our phase-like construction, for any fixed $R_s$ a creased shell attains stability for $R_t$ larger that some threshold $R_t$ (Fig. 3.11). This is further bolstered by presence of temporarily stable samples (for times on the order of seconds) near the phase boundary of two regions (denoted by ■), demonstrating an agreement between experiments and simulations.

Finally, we again employ FEA to understand this critical value of $\alpha$ for attaining a stable isometric state. We report the total energy for axisymmetric solutions with $\gamma = 10^4$
(corresponding to $R_s = 35$ mm), as a function of the indenter displacement ($h$) for samples with varying normalized crease radius ($\alpha$). As seen in Fig. 3.14 we find a bifurcation in stability for samples with larger $\alpha$ as the energy curves develop a well-defined local minimum (solid) and maximum (dashed), causing the shell to snap into a stable isometric state.

![Energy landscape for indentation of a spherical shell for various values of $\alpha$.](image)

Figure 3.14: Energy landscape for indentation of a spherical shell for various values of $\alpha$: Numerically calculated energy landscape for a creased shell with $\gamma \approx 10^4$ for a variety of $\alpha$. The Pogorelov solution is recovered for $\alpha = 0$ (red plot), while for small values of $\alpha$ the energy gain from crease is insufficient to create a local minimum. However, above a critical $\alpha$, local minima (solid green) and maxima (dashed green) bifurcate to generate a region of stability.

3.6. **Time-scale for snap-through transition between isometric states**

Though a full analysis of deformation dynamics during a snap-through transition is of great interest, it is beyond this proof-of-concept study. To demonstrate the possible applicability of snap-through instabilities as rapid actuators, we simply measure the time of instable deformation in various geometries under consideration by obtaining high speed videos. We define the time-scale for snap-through instabilities as time taken to reach the final isometric state, starting at a time-point at which the indenter leaves contact with the shell.
<table>
<thead>
<tr>
<th>Geometry, time</th>
<th>Characteristic length-scale</th>
<th>Material, Young’s modulus (E)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Helicoid (42 ms)</td>
<td>Length of generator = 60 mm</td>
<td>Poly(caprolactone) (E = 353 MPa)</td>
</tr>
<tr>
<td>Cylinder (12 ms)</td>
<td>R = 25 mm, h = 100 mm</td>
<td>Poly(ethylene terephtalate) (E = 5 GPa)</td>
</tr>
<tr>
<td>Sphere (93 ms)</td>
<td>Base radius = 35.52 mm</td>
<td>Poly(vinyl siloxane) (E = 1.3 MPa)</td>
</tr>
</tbody>
</table>

Table 3.1: Observed time-scales for snap-through transitions for different geometries

These values provide an idea of achievable time-scales and a detailed tracking of shell geometry is needed to understand the true dynamics especially given the observed breaking in symmetry of deformed regions.

3.7. Conclusion

Controlling the nature of deformation in curved shells in terms of both speed and continuity represents a major step towards shape-programmable materials with actuation capabilities. Using structural inhomogeneity as an handle, we propose a geometric design principle for programming snap-through instabilities in elastic shells. As discussed, this snap-through behavior is driven by stretching of the material, resulting in the speed of transition to be scaled as speed of sound, which is the upper physical limit for any non-destructive deformation. Interestingly, the discussed mechanism is inertially mediated, preventing common limitations in natural examples such as poro-elasticity or hydraulic dampening [114]. In conclusion, this work lays foundation to be build upon the growing field of non-Euclidean origami, suitable for applications in deployable structures.
3.8. Future work

The fundamental design rule established by this study for programming snap-through instability opens pathways to guide stress, and hence controlled deformation of elastic shells with all the prototypical curvatures. The exciting possibilities offered by this capability provides a robust base to further develop and apply the concept in many points of interest.

For the field of deployable structures, this ability to control deformation dynamics (continuous vs. dis-continuous) in a shell can be pivotal for moving beyond typical hinge-based active geometries. Most of the current state of the art in deployable structures use locally flat panels similar to origami, to fold the global geometry into a smaller package. The ability to maintain a curved geometry while a controlled folding takes place is highly desirable for applications especially in aerospace and architecture. A thorough study for understanding the overall deformation of shell along various creases will be of interest for such an application. The following avenues can be general points of interest to understand deformation dynamics of elastic shells using high-speed imaging:

(i) Further investigating time-scale of snap-through instability can be of interest to establish the predicted inertial nature for this deformation. This can be studied in an ordered fashion by comparing dynamics with properties of a elastic wave for samples at different length-scales.

(ii) Tracking of deformation in elastic shells using strain markers can also be used to validate our assumptions of stress-distribution in undeformed regions, which is of importance in deployable structures.

(iii) This tracking will also be useful in understanding the higher order modes for cylindrical and spherical geometries observed in our experiments.
Similar principles applied in this study can also be useful for optimizing soft-actuators, which are of recent interest in field of micro- and soft-robotics. Finally, extending the phenomena of geometrically controlled stability behavior, we strive to program not just two, but multiple geometries as discussed in Chapter 4.
CHAPTER 4

MULTI-STABILITY IN ELASTIC CONICAL FRUSTA:
A STUDY ON RE-CONFIGURABILITY OF A DRINKING STRAW

4.1. Introduction

Our work on snap-through instabilities enabled robust pathways to program bi-stability in an elastic shell, by simply using geometrical design. Extending this framework further, we consider an ability to program not just two, but many stable states in an elastic shell. This highly desired re-configurability is crucial for practical applications of shape-programmable systems in soft-robotic actuators and deployable structures.

4.1.1. Limitations of shape-programmable systems

In addition to advances and limitations for shape-programmable systems discussed in Chapter 1, most systems are limited to switching geometry between only two pre-encoded configurations, sometimes irreversibly, hindering their feasibility in wider applications. Encoding more than two configurations has been realized in literature, but often requires composites with multiple stimuli-responsive material systems [115, 116] or complicated actuation cycles [47, 52, 117–119], again impeding the adaptation to out-of-laboratory applications. Hence, there is a clear need for developing pathways that enable re-configuration of a system to multiple states, possibly without pre-programming to unlock on-the-fly re-configurable systems. We predict that such an ability will allow exploitation of shape-programmable systems to their true potential for application in high degree of freedom actuators for soft robotics and deployable structures.
4.1.2. Re-configurability of a drinking straw

Curiously, surpassing the need of a complicated shape-programmable mechanism for achieving high degree of re-configurability is a simple and ubiquitous object from our day-to-day life: a bendable drinking straw. Based on corrugated straws invented by Joseph Friedman in 1937 [120], these tubular structures typically consist of tiled conical frusta possessing an intriguing ability to shape-transform into an infinite number of stable configurations [121, 122]. The unsophisticated yet powerful re-configurability enables these initially straight tubes to approximate any space curve within practical reasoning (Fig. 4.1A). This unique feature is manifested through an ability of each of the constitutive units to exist in multiple stable states: either through a complete (and mirror symmetric) collapse along the axial direction to change its length, or a partial collapse (asymmetric) in non-axial direction to change its angle. Furthermore, the non-axial stability of each of these units is degenerate in the azimuthal direction, allowing continuous re-configuration to stable states for changing torsion angle (see Fig. 4.1B). The combination of a high degree of freedom, with the tiled nature of these active units enables the overall structure to possess high degree of re-configurability without the need for pre-programming desired geometries.

Inspired by this remarkably high degree of freedom in a simple structure, we inves-
tigate the pathways to reproduce this multi-stability and attempt to understand the fundamental mechanics behind this specific geometry.

4.2. Conical frusta geometry

Based on our study involving the presence of stability in nearly isometric configurations for elastic shells, we hypothesize that stability of isometric states in a conical frusta geometry also depends on the competition between stretching \( E_S \) and bending energy \( E_B \) contributions during deformation (Chapter 3, [100]). To design a framework to harness this geometric effect, we establish a canonical unit geometry for re-configurable structures based on commercial samples of drinking straw and other re-configurable structures like transport ducts and collapsible camping utensils [120, 123, 124].

The re-configurable conical frusta (abbreviated as RCF) consist of two conical frusta inverted with respect to each other as shown in Fig. 4.2.

In general, we design an ‘upper’ frustum to have base radius \( R \), height \( h_1 \) and slant angle \( \alpha_1 \) (corresponding to an aperture angle of \( \pi - 2\alpha_1 \)). To avoid self contact upon inversion along the axis (lower inverted shell depicted by dashed lines), the slant angle of ‘lower’ frustum \( (\alpha_2) \) is less than \( \alpha_1 \) by \( \Delta \alpha \). The base radius of lower frustum is kept equal to \( R \), which fixes its height as \( h_2 = f(R, \alpha_1, \Delta \alpha) \). The shell also deviates from its constant thickness \( t \) near the two bases, where a thinned region (thickness \( t_c \)) similar to a ‘crease’ acts in a hinge-like fashion to guide inversion along this boundary. Finally, considering the practical difficulty involved with studying this six parameter phase space \((t, R, h_1, \alpha_1, \Delta \alpha \text{ and } t_c)\), we start by
fixing \( t/R (= 0.017) \) and \( \Delta \alpha (= 10^\circ) \), similar to that of a commercially sourced ‘toy’ model, \textit{Pop Toobs} (Slinky Inc., USA).

### 4.3. Fabrication of re-configurable conical frusta (RCF)

To fabricate elastic shells with well-defined geometries we use a two-part curable silicone elastomer poly(vinyl siloxane) or PVS (Zhermack Inc., Italy) for resin cure molding at room temperature. For designing appropriate molds to impart the necessary geometrical parameters for RCF, we develop a parametric CAD algorithm (\textit{Grasshopper plug-in, Rhino, Robert McNeel Inc., USA}) to obtain four-part negative molds for any given inputs \((t, R, h_1, a_1, \Delta \alpha \text{ and } t_c, \text{ see Fig. 4.3A})\). These 3D CAD models are then printed with poly(acrylonitrile-butadiene-styrene) using a commercial 3D printer (\textit{Dimension uPrint SE Plus, Stratasys, USA}) as shown in Fig. 4.3B.

![Figure 4.3: Design and fabrication of 3D printed molds: (A) CAD model, with indicated assembly for 4-part molds. (B) 3D printed negative molds used for fabricating RCF with a room temperature resin-cure molding of poly(vinyl siloxane) elastomer.](image)

To cast the RCF geometry, we first apply a mold release agent (\textit{MR311, Sprayon, Canada}) to our 3D printed molds. Then, we mix a 1:1 (prepolymer:crosslinker) ratio of the PVS formulation, followed by immediate degassing under vacuum for 8-10 min to remove any air bubbles. Next, we pour in the degassed mixture, assemble the molds as shown in Fig. 4.3A, and let the elastomer crosslink at room temperature and pressure for 30 min. Finally, we open the molds to remove the cured RCF geometry. Any samples with bubbles
or other defects are discarded due to the sensitive nature of defects in this study.

4.4. Stability of elastomer RCF geometry

To establish the multi-stability, and characterize the load response of the fabricated RCF, we use a similar custom load-displacement setup for indenting samples as our previous study. By combining a linear translation stage, load cell, two orthogonal cameras (*HD Pro Webcam C920, Logitech Corp., USA*) and customizable 3D printed grips we impart controlled axial and non-axial deformation.

To start, we analyze a RCF fabricated using parameters scaled to *Pop Toobs*, as \( t/R = 0.017, \alpha_1 = 45^\circ, \Delta \alpha = 10^\circ \) and \( h_1 = 6 \) mm. Interestingly, replicating these four geometrical parameters results in a mono-stable sample, lacking the axial or non-axial stable states observed in commercial samples (coordinate * on Fig. 4.4). To systematically investigate this deviation from expected results, we fabricate additional RCF by fixing \( t, t_c, \Delta \alpha \) and \( R \) while varying frustum height \( h_1 \) and slant angle \( \alpha_1 \).

![Phase diagram for stability of elastic double conical frusta](image)

Figure 4.4: Phase diagram for stability of elastic double conical frusta, with fixed \( t/R = 0.017, t_c = 0.25 \) mm, \( \alpha_2 = \alpha_1 - 10^\circ \) and varying \( h_1, \alpha_1 \). The lack of any stable states in either axial or non-axial direction (○) is observed for all the sample with \( h_1 \leq 6 \) mm in the indicated phase space. Axially stable states are recorded for samples with higher \( h_1 \) (▮), as visualized in the bottom panel. Notice the complete absence of non-axial stability.
Among the fabricated RCF, we observe mono-stability for all the samples with $h_1 \leq 6$ mm (denoted by ○). The corresponding load response (see Fig. 4.5, red curve) for this sample indicates an initial linear regime, followed by a region of non-linear deformation manifested through buckling around central regions of the shell (discussed later), and finally a lack of stability when the lower conical frustum inverts. On other hand, samples with higher $h_1 = 7.5$ mm (blue curve) and $10$ mm (green curve) deform through a similar initial linear regime followed by a non-linear buckling, but snap to a stable final state (denoted by □, Fig. 4.4). This axial stability results from a stable inversion of the lower frustum along its base, and is similar in nature to that in commercial samples. As all the samples fail to reproduce non-axial stability in RCF, it is clear that a hypothesis based on a simple geometric effect is incorrect.

4.5. Effect of residual stress present in commercial conical frusta on non-axial stability

To address this lack of non-axial stable states, we delve deeper in the fundamental mechanism of re-configurability in the drinking straw. While measuring dimensional parameters of commercial samples (initial radius $r = R_o$) for establishing geometry of RCF, we found that samples cut open along the axial direction consistently ‘opened’ to adopt a larger radius ($r = R_n$) of curvature. Although expected as an unintentional byproduct of
thermoplastic processing, this residual stress is surprisingly consistent for elastomer based commercial products \textit{(X-Series camping gear, Sea to Summit, USA [125])}, and indeed has the propensity to render the samples devoid of their original re-configurable nature.

Figure 4.6: Effect of residual stress: \textbf{(A)} Starting at \( r = R_o \), the sample is first cut along a direction parallel to the axis, which relaxes stress by opening to \( r = R_n \) and renders the sample devoid of any multi-stability. This stability is restored by gluing back the relaxed sample to \( r = R_o \). \textbf{(B)} Load response to axial deformation of multiple RCF geometries in original stressed state (green curve) and opened relaxed state (red curve). Notice the force jumps at (i) and (ii) due to axial collapse of individual RCF geometries.

To demonstrate, we present the stability behavior of ‘as-is’ samples with radius \( R_o \), and that of samples cut along a straight line parallel to the axis: which relaxes the samples to their natural radius \( r = R_n \) (see Fig. 4.6A). The stressed sample possesses axial stability as indicated by jumps in force value associated with axial collapse of individual RCF (Fig. 4.6B, green curve). Upon relaxation, the sample demonstrates a smooth accordion like behavior lacking any snap-through modes (Fig. 4.6B, red curve). Further bolstering the role of residual stress, a relaxed sample regains all the stable modes.
when glued back to its original radius $R_o$ (see Fig. 4.6A). Finally, to rule out any effect from topology on the stability of relaxed samples, we glue an appropriate piece from another sample to close off the gap in a relaxed RCF, at its natural radius $R_n$ (Fig. 4.7). This too fails to re-instate the stability to relaxed sample in axial or non-axial direction, indicating the presence of residual stress as a necessary condition for stability of non-axial states.

4.5.1. Possible origins of residual stress in commercial samples

To understand the importance this stress and its origin in commercial samples, we take a closer look at how these structures are fabricated. A thorough survey of patent literature reveals that the commonly used fabrication method is based on the one proposed by Joseph Friedman in his original patent for drinking straws in 1937 [120]. Adapted accordingly, these straws are fabricated on ‘straw-bending’ machines which apply the active corrugated geometry (canonical geometry discussed in section 4.2) to a initially straight-walled tubes. These tubes are held on an inner mandrel, typically at elevated temperatures (above the glass transition temperature of the polymer used) and pressed against a rotating mold with the desired pattern to be embossed on the tubes. Finally, the straws are removed from the mandrel and compressed axially for storage and transportation.

Keeping the described fabrication process in mind, we infer at-least two potential origins for induction of residual stress. While embossing the tubes, the rotating mandrel will impart shear stress which can result in a residual stress. In addition, long term storage in axially closed state will also result in introduction of stress due to plastic deformation of structures fabricated in open state [121].

4.6. Fabricating RCF geometry with built-in stress

To investigate the possibility of a pre-stressed state as a necessary condition for stability in non-axial direction, we reiterate the resin-molding protocol for fabrication of RCF geometries. To systematically encode controlled amount of stress, we redesign the
negative molds so as to exclude a wedge of arc angle $\psi$. Upon closing this shell molded at a natural base radii of $R_n(\psi) = R/(1 - \psi/2\pi)$ to an ‘under-curved’ radius $R$, we expect to program a built-in pre-stress in azimuthal direction $\sigma_\phi$, with a magnitude that can be varied by controlling wedge angle $\psi$ (see Fig. 4.8).

![Figure 4.8: Programming a built-in stress for exploring non-axial stability in RCF geometry. The modified molds with an extra ‘wedge’ of arc angle $\psi$ are shown along with the molded RCF geometry.](image)

To systematically study the effect of residual stress on multi-stability of elastic RCF, we fabricate samples with gradually increasing values of starting radii $R_n$ by altering the corresponding angles of excluded wedge ($\psi = 30^\circ, 45^\circ, 60^\circ, 75^\circ, 90^\circ$). After gluing, the samples attain similar final glued-geometry ($h_1/R$, $t/R$ and $\alpha_1$), except a minor dimensional variation along the axis ($\sim 0.1 \times h_1$) due to Poisson’s effect (see Fig. 4.9).

![Figure 4.9: Glued sample ($r = R$) for inducing stress, with the seam region shown in inset.](image)

4.7. **Non-axial stability in pre-stressed RCF**

With an ability to control amount of pre-stress, $\sigma_\phi$ programmed in RCF geometry, we next focus on characterizing the stability behavior in non-axial direction. For measuring the load-response, imparting a controlled non-axial deformation to RCF geometry poses a considerable challenge, as the point of contact travels in a curvilinear path. To work around this, we design a custom grip and indenter setup, consisting of a linearly displaced (along
Z axis) point indenter for imparting a controlled non-axial tilt (angle $\theta$), at the same time allowing the indenter to freely move laterally by designing a smooth recessed rail in the grip (setup schematic in Fig. 4.10).

Figure 4.10: Load response of samples with similar geometry ($h_1 = 6$ mm, $\alpha_1 = 45^\circ$, $\alpha_2 = 35^\circ$) but varying starting $\psi$, hence encoded with increasing amount of $\sigma_p h_i$: A stable non-axial ‘bent’ state is observed for $\psi \geq 60^\circ$. Shaded regions envelop max/ min values around the mean for 2 runs on 2 samples each.

As seen in Fig. 4.10, RCF geometries with all values of $\psi$ initially have a linear response to displacements ($\Delta Z \sim t$), as expected for thin elastic shells with similar geometries, subsequently reaching a peak force value around $\Delta Z = 2$ mm. Further non-axial deformation reveals an interesting effect of increasing built-in stress, in an agreement with our hypothesis. Samples with $\psi = 0^\circ, 30^\circ$ and $45^\circ$ exhibit mono-stability under non-axial deformation; characterized by a load response that decays until $\Delta Z \sim 6$ mm followed by a stronger response on further displacement. Contrary to this, samples with $\psi = 60^\circ$ and $90^\circ$ clearly demonstrate non-axial stability, as noticed by the force dropping to a null value, indicating loss of contact with the indenter. Notably, for samples with mono-stability under axial deformation (Fig. 4.4), we do not observe a induced stability similar to the non-axial direction.
Figure 4.11: The effect of studied pre-stress in conical frusta is clearly evident through the induced stability under non-axial deformation (👈, for $h_1 > 5$ mm), originally absent in the un-stressed phase diagram.

Extending this result, we revisit the stability of RCF in previously explored parameter space (Fig. 4.4), now with a programmed built-in stress $\sigma_\phi$ corresponding to $\psi = 75^\circ$. For lower values of $h_1$ (3 mm and 5 mm) we see no change in axial or non-axial stability due to the addition of pre-stress (see Fig. 4.11). For all the other tested values of $h_1$ (≥ 6 mm) and $\alpha_1$ (= 30°, 45°), we observe the intended stability in non-axial direction (denoted by 👈). Additionally, we note the continuous re-configurability of RCF by rotating the sample in azimuthal direction ($\phi$) while in a non-axially stable configuration. As shown in Fig. 4.12, one such sample demonstrates stability in multiple configurations ($\phi = 0^\circ$, 90° and 180°).
Figure 4.12: Front and top view of stressed RCF ($h_1 = 10\text{mm}$, $a_1 = 45^\circ$, $\psi = 60^\circ$) demonstrate re-configurability to stable states corresponding to multiple azimuthal positions $\phi = 0^\circ$, $90^\circ$ and $180^\circ$. The glued seam is positioned at $\phi = 90^\circ$.

Based on the discussed observations, we propose an **empirical design rule** for programming multi-stability:

(i) **Axial stability of RCF geometry** can be programmed by using a sufficiently high conical frustum ($h_1$).

(ii) **Non-axial stability of RCF geometry** requires a combination of appropriate geometry and sufficient built-in stress ($\sigma_{\phi i}$). Interestingly, the axial stability does not change upon introduction of this stress, though the non-uniformity introduced by the glued seam-region is a possible issue in our experiments. In addition, the non-axial stability is also characterized by a **continuous stability in azimuthal direction** providing a high degree of freedom to these structures.

4.8. **Fundamental mechanism behind stress-induced stability in RCF geometry**

Our empirical findings point to a successful framework for fabricating a truly re-configurable system inspired by commercial drinking straws, but a thorough consideration of the mechanics and geometry is required for a complete understanding of the mechanisms for programming axial, non-axial and azimuthal stability to RCF geometry. This is of a
considerable difficulty, as the deformation of a conical frustum is not fully understood, and only a few experimental studies of axial indentation in metallic conical frustum commonly used in projectile tips and shock absorption exist [126–129]. The lack of literature and difficulties to analyze deformation of a non-axisymmetric shells analytically turns our approach towards alternate methods for understanding the mechanics behind this deformation.

We draw parallels with our discussion on stability of isometric deformation in spherical shells by considering the competition between bending and stretching energies (Chapter 3, [100]), and characterize the distribution of bending and stretching energies of axial and non-axial deformation in RCF geometry.

4.8.1. Experimental details for characterization of RCF geometry upon deformation

To measure the distribution of stretching energy (related to change in local Gaussian curvature) and bending energy (related to change in mean curvature) we obtain 3D geometry of deformed RCF using an in-situ X-Ray computed tomography (X-Ray CT) setup. To fix and deform RCF between CT scans, we prepare separate 3D printed harnesses for controlled axial and non-axial deformation. As shown in Fig. 4.13A, end grips constraint the sample according to the appropriate boundary conditions, and the upper grip has capability to slide on vertical pegs along $Z$ direction. This harness ensures a controlled axial deformation ($\Delta Z$) with reasonable step size (0.1 – 1 mm) possible between subsequent scans. For a control over non-axial deformation, an upper grip similar to that used in load-displacement experiments was designed to be compatible with linear displacement of a turn-screw based indenter. As shown in Fig. 4.13B controlled linear displacement by turning this screw ($\Delta Z$) is converted to the tilt angle $\theta$. 
A Perkin Elmer IVIS Spectrum CT, Perkin Elmer imager is used to generate X-Ray Computer Tomography (CT) scans by exposing samples at a 23 mGv dose through a Cu (120 mils) filter. The obtained scans (voxel size 150 μm) are thresholded accordingly in ImageJ for generating a 3D image stack for desired geometry. To avoid possible interference during curvature measurement, the resulting outer and inner surfaces are separately identified using MATLAB, Mathworks Inc., USA and a 3D point cloud depicting the inner surface is exported to Meshlab software package [130]. Therein, following a reduction of dense point clouds [131] surface curvature is measured using algebraic point set surface method [132].

4.8.2. Analyzing stretching and bending during axial deformation:

We begin by analyzing the 3D geometries representing axial deformation of RCF geometry. We obtain several scans of a RCF geometry ($h_1 = 10$ mm, $\alpha_1 = 45^\circ$ and $\psi = 0^\circ$) by deforming the sample gradually (step size < 1 mm) between each scan. We plot the Gaussian curvature ($\mathcal{K}$) on a 3D mesh as seen from the bottom view in Fig. 4.14A at different indentation depth $\Delta Z$. In ground state ($\Delta Z = 0$), we observe as expected $\mathcal{K} = 0$ everywhere, except near the central region (where $\mathcal{K} < 0$ is expected on inner surface of the fabricated geometry, Fig. 4.2). A break in axi-symmetry of the deformed region, mentioned previously in load-displacement experiments, is clear from distribution of $\mathcal{K}$ upon further indentation. As shown in Fig. 4.14A this buckling appears to be periodic at
ΔZ = 2.9 mm, characterized by a periodicity m = 5. In the next few steps of indentation at ΔZ = 4, 8.9 and 10.9 mm, we consistently note two ‘crest’ regions merging into one to break the periodicity of this buckled region from m = 5, to m = 4, 3 and 2. Finally, on further deformation to ΔZ = 10.9 mm, we observe a stable, axi-symmetric inversion of lower frustum.

Figure 4.14: Distribution of Gaussian (\( \kappa \)) and mean (\( H \)) curvature during axial deformation of RCF with \( h_1 = 10 \) mm, \( \alpha_1 = 45^\circ \) and \( \psi = 0^\circ \): (A) Measured Gaussian curvature (\( \kappa \)) from X-Ray computed tomography data, for steps in axial indentation depth ΔZ. After an initial axisymmetric deformation regime, we observe periodic buckling of the central region upon axial indentation characterized with an initial periodicity \( m = 5 \) at ΔZ = 2.9 mm, gradually depleting due to merging ‘crest’ regions at ΔZ = 7.9 mm (\( m = 4 \)), 8.9 mm (\( m = 3 \)) and 10.8 mm (\( m = 2 \)), before regaining an axi-symmetry in the isometric state at ΔZ = 10.9 mm. As clearly seen in color-maps, a majority of Gaussian curvature is concentrated in the central buckled region. (B) Measured mean curvature \( H \) of isometric states at ΔZ = 0 and 10.9 mm.

Throughout deformation, most of the conical regions remain free of \( \kappa \), indicating the concentration of stretching energy in the buckled region. Additionally, the negligible change in magnitude of \( \kappa \) between two isometric states (ΔZ = 0 and 10.9 mm) indicates isometry, further bolstered by the inversion of sign for mean curvature expected for mirror inversion (Fig. 4.14B).

Drawing an analogy to our previous study, we hypothesize that this axial stability results from sufficient relaxation of the stretching energy concentrated in buckled regions.
compared to the opposing bending energy to invert the lower frustum. For samples with lower frustum height $h_1$, this energetic barrier posed by bending energy must be larger than the gain by relaxing stretching energy, essentially rendering the isometric state unstable. In other words, if the stretching energy contained in buckled regions right before inversion is $E_s(\Delta)$ and bending energy for inverting lower conical frusta is $E_B^i$, the following condition,

$$E_s(\Delta x) > E_B^i$$

makes switching to an stretching-free axially inverted state energetically favorable.

### 4.8.3. Analyzing stretching and bending during non-axial deformation

Next, we consider the non-axial deformation of RCF geometry ($h_1 = 6$ mm, $\alpha_1 = 45^\circ$) with ($\psi = 60^\circ$) and without ($\psi = 0^\circ$) a built-in stress. Each sample is loaded in the previously discussed harness in step size of $\Delta \theta < 2^\circ$, and the obtained curvature data is plotted as seen from below in Fig. 4.15.

Following a similar trend as axial deformation, an initial ground state for both samples exhibits the expected $\mathcal{K} = 0$ in conical regions. For the unstressed sample (top panels), a buckled regime with higher concentration of $\mathcal{K}$ (‘crests’ marked with black arrows) is seen centered azimuthally around the point of contact with indenter (red arrow). Tilting the sample further increases this concentration of $\mathcal{K}$ in buckled regions, until an unstable state with a partially-inverted lower frustum is reached ($\theta = 10.5^\circ$).

Interestingly, while comparing these results at similar values of tilt angle $\theta$ for a stressed RCF, we observe a buckled region with a higher mode and concentration of $\mathcal{K}$. As discussed earlier, this stressed sample has a stable non-axial state which we observe when indented to $\theta = 10.5^\circ$. 
Figure 4.15: In-situ curvature measurement: (A) Gaussian curvature during non-axial loading for similar tilt angles $\theta$ in unstressed ($\psi = 0^\circ$, upper panel) and stressed ($\psi = 60^\circ$, lower panel, glued seam position indicated) samples with $h_1 = 6$ mm, $\alpha_1, \alpha_2 = 45^\circ, 35^\circ$ (top view). Buckled regions (crests marked with black arrows) appear symmetrically around the indentation point (red arrow). The amplitude and magnitude of $\kappa$ both increase with $\theta$ before collapsing into an unstable and stable isometric state for $\psi = 0$ and $60^\circ$ respectively. (B) Side view of indented RCF at $\theta = 5^\circ$, with a higher buckling mode clearly visible for the sample with built-in stress (right).

The lack of mechanical model prevents establishing a clear distinction between critical strain ($\psi_c$, purely geometric) or stress ($\sigma_{\psi c}$, material dependent) as the necessary condition for non-axial stability. Though we find a deviation in under-curvature between fabricated and commercial samples pointing to the case of critical stress, a direct comparison without further study should be avoided due to variation in stress states encoded. Based on these empirical observations, we expect that a sufficient stress $\sigma_{\psi}$ affects the stability behavior by increasing the stretching energy contribution, $E_S(\theta)$ compared to the bending energy required for partial inversion of this shell in ‘bent’ state $E_B(\theta_{\text{invert}})$. We realize that to prove this hypothesis an exact comparison of bending vs. stretching energy must be made with calculated energies for deformed geometries, but acknowledge that such a measure-
ment requires tracking of material points which is beyond the resolution of both our X-Ray CT scans and post-processing analysis.

4.9. Conclusions

Unlocking the true potential of shape-programmable systems clearly demands further advancement to the current state-of-the-art, which often poses a challenge to adaptability in practical systems and limits the number of shapes to be encoded. To develop a canonical model capable of on-the-fly switching between multiple shapes for a truly reconfigurable system, we draw inspiration from the ubiquitous drinking straw and other commercial products. By considering fundamental rationale behind multi-stability of the canonical geometry (termed here as RCF), we aim to establish design rules for ease in incorporating re-configurability in a wider range of application.

According to our observations, ensuring the axial stability of RCF geometry required stability of the isometric state obtained by inverting the lower conical frustum. Generally, this involves designing a conical frusta with sufficient height $h_1$ for other fixed parameters. Programming non-axial stability on the other hand, requires not only the right geometry but also presence of a pre-stress. As observed, this pre-stress has the propensity to increase stretching energy in deformed geometries, making a stable partially inverted state with lower stretching energetically favorable.

In conclusion, we propose an empirically derived design rule for programming reconfigurability in conical frusta like geometries. Our treatment allows us to control the stability in the axial, non-axial and azimuthal direction, enabling applications in high degree of freedom manipulators. Further understanding of the stability and deformation mechanics is needed for theoretical proof of the proposed explanations, and remains of an important future interest.
4.10. **Future work**

Following this study, many exciting avenues remain unexplored given the infancy of shape-programmable systems and lack of understanding in shell mechanics.

With a general increase in interest towards articulated, high degree of freedom actuators [25, 58] for applications in micro-robotics, surgical instruments and soft-robotics, we predict that the proposed multi-stable elastic shells discussed here will effectively fulfill the purpose. The RCF geometry can be tiled to fabricate an articulated structure for providing a robust way to program multiple possible configurations, and more importantly without the need to pre-program all the desired states. To achieve this, there needs to be an effective actuation mechanism for providing the necessary deformation to the structure. Though it is not clear which specific pathway is suitable for driving this capability, we propose one possible actuating mechanisms which can be effectively incorporated into the fabrication and geometry under consideration.

Pneumatic actuators have been studied as a cheap and effective actuation mechanism for high DOF actuators. For example, the so called ‘McKibben air muscles’ effectively convert the radial contraction or expansion of a pneumatically activated core to an overall increase or decrease in transverse direction respectively [133, 134]. In a RCF geometry, three such actuators can be distributed along the azimuthal direction, attached to the base of conical frusta. Using all of the actuators at the same time results in an axial deformation, while any one of the three actuators can be used for non-axial deformation. For accessing the continuous re-configuration in the azimuthal direction, a control system for actuating more than one channels will have to be established based on the geometrical parameters for RCF. For a high DOF structure with tiled RCF, we predict that use of zones can be reasonably implemented for accessing multiple configurations with an optimized number of channels. Lastly, commenting on the power efficiency of this structure, we note that presence of multi-
stability in RCF based actuators allows for a null or lower power draw from the actuating mechanism once the geometry switch is complete, increasing power efficiency crucial for applications such as micro-robotics. This is in contrast with the present mechanisms which essentially ‘hold’ the structure in the desired position by applying a continuous stress [25, 58].

The fundamental questions regarding the effects of pre-stress upon deformation pathway are a great challenge and need a serious effort to achieve a true understanding of mechanics behind the observed phenomena. For this, perhaps using a simpler geometry such as a cylindrical surface with varying amount of under-curvature or pre-stress encoded will make this problem more tractable both by analytics and finite element numerical simulations.
SUMMARY AND OUTLOOK

Shape-programming serves as an alternate route for fabrication of complicated 3D geometries, and has been successfully used for a wide range of material systems and length-scales. In this dissertation, we study the fundamental principles that enable the ability to transform the shape of system by considering geometry and mechanics of non-Euclidean shells.

In Chapter 2, we aim to establish a framework for optimization of growth-induced buckling in 2D sheets as a pathway to obtain controlled 3D non-Euclidean geometries. Under this theme, we experimentally demonstrate how target geometries obtained by this complex mechanism can be visualized using a simple cut-and-glue construction of macro-scale models. This technique relies on equivalence between growth-induced buckling and topological defect-induced buckling, and establishes the simple yet impactful ability to visualize and optimize the final geometry. The results in this study offer a unique perspective into limitations involved in shape-programming with growth-induced mechanisms. Through our analysis of programming a tetrahedron geometry with both the discussed routes, we gain an important insight into the balance between the available range of growth and pattern resolution, and how they affect the fidelity of final geometry to be patterned.

Further improving on the state-of-the-art, we provide fundamental design principles for a robust control over shape and deformation pathways of non-Euclidean shells. Inspired by natural systems which harness snap-through instabilities to switch between locally stable states, we discuss the design rules for programming these instabilities in chapter 3. By considering the deformation of a shell along weakened curves, or ‘creases’ we establish that the presence of local stability and continuity of this deformation can be controlled using the
curvature of the crease. We find this geometric design rule is applicable to multiple material systems, promising a highly adaptable mechanism for shape-programming. Additionally, this study provides greater control over the final geometry of the system, compared to the natural counterparts and previous studies where a simple mirror symmetry of the overall shell structure is utilized. In conclusion, our results lay the foundation for programming on-demand instabilities between isometric states in non-Euclidean shells.

Extending this mechanism to program bi-stability, we propose a solution to one of the major challenges involved with shape-programmable systems by aiming to program multiple locally stable states. For this, we consider the origins of re-configurability of a day-to-day object, a bendable drinking straw (bendy straw). A bendy straw offers infinite possible configurations through presence of multi-stability in axial, non-axial and azimuthal directions. We find that reproducing this multi-stability in a similar structure requires both the appropriate conical geometry and a built in azimuthal pre-stress in the structure. These results on programming locally stable states by introducing a pre-stress in structure will open an important discussion on the fundamental mechanics behind this behavior.

Shape-programming through both the pathways considered here is a relatively new field of study, and important questions remain unanswered. Hence, the work summarized in this dissertation can be a useful basis for further studies. To begin, a deeper understanding of growth-induced mechanism can be achieved by applying similar mechanisms to a wider range of material systems and length-scales. For example, the growth-induced shaping mechanisms discussed in section 1.2 in principle can be applied to pattern cm-scale textile sheets with non-Euclidean geometry. In addition to the commercial importance, these studies can also be used to realize fundamental limitations regarding the effect of sheet thickness, pattern resolution by systematically varying these factors and analyzing the deviations in target geometry.

Shape-programming through local stability in non-Euclidean shells also presents
new questions that need to be addressed. In our analysis for snap-through instability in spherical and cylindrical geometries, we notice anti-symmetric transition states which need further investigation. These higher order modes are not considered in the proposed energetics of spherical shell deformation and warrant careful consideration. Additionally, in the current study we consider the deformation of shells along curves generated by intersecting the surface with a plane. Establishing the stability of a shell deformed along curves which do not lie in a plane can also be of interest and needs further attention.

Finally, our studies on programming multi-stability in conical frusta geometry presents a template upon which further studies can be designed. The effect of pre-stress in shell deformation is not a well understood phenomenon, and can be carefully considered in simpler geometries such as cylinders. Further studies call for an improvement on fabrication processes including the incorporation of a controlled built-in stress. One possibility to fabricate ‘seam-less’ RCF is to use elastomer composites with varying crosslinking density through thickness, which can build up an internal stress due to migration of free polymer chains [135]. This can be achieved both using molding multiple layers or compatible 3D printing setups, and will need well designed studies for optimization. Finally, a finite element analysis for deformation of RCF can be useful for a more quantitative comparison between bending and stretching energy contributions in all the discussed cases of axial and non-axial deformation.
APPENDIX:

PROTOCOL FOR GRAYSCALE PROJECTION LITHOGRAPHY

In this appendix, we present a protocol for grayscale patterning, developed as an improvement to the discussed halftone patterning for defining growth in a 2D hydrogel sheet [44]. In grayscale lithography, rather than relying on discrete levels of growth ($\Omega_{\text{low}}$ and $\Omega_{\text{high}}$) we define an arbitrarily high number of growth levels ($L$) by using projection lithography system capable of exposing multiple masks without a need for alignment step between subsequent exposures (Fig. A1. (A) and (B)). The smoother gradient between different levels of growth leads to higher fidelity for prescribed target geometries, as shown in Fig. A1. (c) for a spherical metric patterned with varying $L$.

Figure A1. : Grayscale lithography for programming shape in hydrogel sheets: (A) Rather than patterning with just two levels of $\Omega$ as in halftone lithography, grayscale lithography uses an arbitrarily high number of levels ($L$) in swelling range. (B) The projection lithography setup with a digital micro-mirror device for grayscale lithography. (C) An example metric of spherical cap prescribed at different number of levels $L$. Images reproduced and modified with permissions from the royal society of chemistry, ©2016, [44].

The protocol for grayscale lithography is divided into two parts, and discussed in following
sections:

- A MATLAB based algorithm for calculating the sequence of masks required for patternning an arbitrarily high number of crosslinking levels.

- A control protocol for displaying these masks on a DMD device.

MATLAB code for calculating masks for gray-scale lithography

The following code divides the swelling range into $L \left(2^{bits}\right)$ intervals, calculates the mask and exposure time corresponding to each of this interval according to the supplied swelling function.

```matlab
clear all

Define basic parameters for calculation of swelling factors

bit = 8; \% Number of bits to be patterned. The final number of masks will be
   L = 2^{bits} - 1 (all black mask not calculated)

n = 768; \% Resolution of DMD (rows)

m = 1024; \% Resolution of DMD (columns)

of_x = 512; \% x offset with respect to center

of_y = 384; \% y offset with respect to center

R_out = 350; \% Outer radius of the disc to be patterned

\% Pre-allocating matrices for speed

R = zeros(n,m); swelling = zeros(n,m); dose = zeros(n,m); exposure = zeros(n,m);

\% Calculate the calibration curve for areal swelling vs. UV dose (Sample
\% values indicated in the matrix curve, please change accordingly)

curve(:, 1) = [0.642857143 1.028571429 1.285714286 3.857142857 6.428571429]
```
12.85714286 18 24.42857143 32.14285714]; % UV dose in J/cm^2
curve(:, 2) = [8.46954 7.67333 7.0393 4.62146 3.50338 2.54864 2.22988 2.1887
2.0883]; % Areal swelling ratio
curve(:, 3) = [0.45495 0.43239 0.40519 0.28931 0.29281 0.16585 0.09461
0.14156 0.10272]; % Standard deviation in areal swelling
curve_fit = fit(curve(:, 2), curve(:, 1), 'smoothingspline'); % Fit the
calibration curve, this is later used to calculate dose for given areal
swelling factor
max_swelling = max(curve(:, 2)) min_swelling = min(curve(:, 2))
swelling_range = max_swelling - min_swelling % max, min and the range =
max-min

% Dividing the overall swelling range in L equal intervals
L = 2^bit - 1; % Number of levels to be patterned
delta_bit = (max_swelling-min_swelling)/L; % (max-min)/L
bit_bins = [min_swelling:delta_bit:max_swelling];

% Define a rectangular coordinate system, each coordinate depicting a micro-
mirror on DMD device
i = 1; j = 1;
for i = 1:n
    for j = 1:m
        x(i, j) = j;
        y(i, j) = i;
        R(i, j) = ((j-of_x).^2 + (i-of_y).^2)^0.5;
    end
end

% Calculate areal swelling ratio at each coordinate. Replace the swelling
function with desired analytical function in terms of coordinate system \{x, y\} or R\{x, y\}

\[ i = 1; j = 1; \]
\[ \text{for } i = 1:n \]
\[ \quad \text{for } j = 1:m \]
\[ \quad \quad \text{if } R(i, j) \leq R_{\text{out}} \& R(i, j) \geq R_{\text{in}} \% \text{ Restrict calculation within a specified area} \]
\[ \quad \quad \quad \text{swelling}(i, j) = \text{insert analytical function here}; \]
\[ \quad \quad \quad \%\text{swelling}(i, j) = \frac{\text{max_swelling}}{((1 + (R(i, j)/R_{\text{out}})^2)^2)}; \% \]
\[ \quad \quad \quad \text{Swelling function for a spherical cap} \]
\[ \quad \quad \text{else swelling}(i, j) = 0; \]
\[ \quad \text{end} \]
\[ \text{end} \]
\[ \text{end} \]

\% Write the lithography masks corresponding to each interval in form of a multi-dimensional array of size \(n, m, L\)

frame = zeros(768, 1024, L);
\[ i = 1; j = 1; \]
\[ \text{for } b=1:L \]
\[ \quad \text{for } i = 1:n \]
\[ \quad \quad \text{for } j = 1:m \]
\[ \quad \quad \quad \text{if swelling}(i, j) \leq \text{bit_bins}(1, b+1) \&\& \text{swelling}(i, j) > \text{bit_bins}(1, 1) \]
\[ \quad \quad \quad \quad \text{image}(i, j) = 1; \% \text{Write the pixel } \{x, y\} \text{ with value 1, if it falls in the appropriate swelling interval} \]
\[ \quad \quad \quad \text{else image}(i, j) = 0; \]
\[ \quad \text{end} \]
\[ \quad \text{end} \]
\[ \text{end} \]
% Calculate the UV dose needed (in J/cm^2) at each point \{x, y\},
corresponding to the calculated swelling at \{x, y\}

intensity = 46.15*10^-3; % Measured intensity of the UV source in W/cm^2

for i = 1:L
    dose_bins(1, i) = curve_fit(bit_bins(1, i));
    exposure_bins(1, i) = dose_bins(1, i)/intensity;
end

exposure = zeros(1, L);
exposure(1, L) = exposure_bins(1, L);
for j = (L-1):-1:1
    exposure(1, j) = exposure_bins(1, j) - exposure_bins(1, j+1);
end
exposure = exposure';

total_exposure = sum(exposure)

save('frame.mat','frame'); % Save the masks in an array, to be used later
save('exposure.mat','exposure'); % Save exposure times for each mask, to be used later

Output:
frame: array of dimension (rows*columns*L) representing the L masks for an effective grayscale photolithography.
exposure: scalar variable with dimension (1, L) representing exposure times for each of the L masks.

**MATLAB code for controlling DMD setup for gray-scale lithography** †

The following functions describe a MATLAB code, for controlling DLP based micro-mirror devices using a ViALUX ALP controller. This set of functions are freely available in an online repository at https://github.com/nakulbende/ALP41_API_Matlab, and the following may be used as documentation.

**Notes:**

(i) This code requires the function library (X64/ X86) provided by *ViALUX GmbH, Germany*, accompanied with the controller as a part of drivers and documentation.

(ii) This API was developed with grateful aid from Dr. Martin Vogel (Max Planck Institute of Biophysics) [136] and VIALUX.

(iii) Get appropriate ALP drivers at http://www.vialux.de/transfer/alp-4.2/ALP42_install.exe OR http://www.vialux.de/transfer/alp-4.1/ALP41_install.exe

The following elements from the required code loads library, connects the DMD device and load images to be displayed for photo-lithography.

**Load libraries in MATLAB** (api_library)

[return_lib] = api_library(dll_name, dll_header)

**Input:**

dll_name: name of the .dll file, without file extension

---

† This code is maintained and published online in an open source repository: https://github.com/nakulbende/ALP41_API_Matlab
dll_header: name of the header file, without file extension

**Output:**

return_lib = 'Library is loaded'; or 'Error: Library was not loaded' (Opens a list of functions available in library in a separate window)

**Connect/ Allocate a DMD device (api_allocate)**

Connects the device to MATLAB, and generates a device handle which will be used as an address for subsequent operations.

```
[return_allocate, hdevice] = api_allocate(dll_name)
```

**Input:**

dll_name: name of the .dll file, without file extensions

**Output:**

hdevice: device handle generated by api_allocate function

**Inquire device/ controller parameters (api_inquire)**

Sends a query to the device, and stores the value in an out pointer. Available query types can be found in Fig. A2.

```
[return_inquiry, return_query] = api_inquire(dll_name, hdevice, query)
```

**Input:**

dll_name: Loaded control library

hdevice: Device handle generated by allocate function

query: Common query types in Fig. A2.
<table>
<thead>
<tr>
<th>Control/query type</th>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
</table>
| ALIB_DLL_TIMEOUT      | 0     | Set or query the multi threading timeout (AlibDllControl, AlibDllInquire)  
Parameter type: unsigned long  
Values: 0: no timeout; ALIB_INFINITE |
| ALIB_DLL_VERSION      | 1     | DLL version information (AlibDllInquire)  
Parameter type: struct ALIB_VERSION |
| ALIB_DEV_HALT         | 0     | Hall or resume the device (AlibDevControl, AlibDevInquire)  
Parameter type: unsigned long  
Values: 0: resume, 1: halt |
| ALIB_DEV_DRIVER_VER   | 1     | Device driver version information (AlibDevInquire)  
Parameter type: struct ALIB_VERSION |
| ALIB_DEV_FIRMWARE_DATE| 2     | Version information of the USB controller firmware (AlibDevInquire)  
Parameter type: struct ALIB_DATE |
| ALIB_DEV_CONFIG_DATE  | 3     | Version information of the application FPGA configuration (AlibDevInquire)  
Parameter type: struct ALIB_DATE |
| ALIB_DEV_SERIAL       | 4     | Serial number of the ALP (AlibDevInquire)  
Parameter type: unsigned long |
| ALIB_DEV_DMSTYPE      | 5     | Configure ALP basic to use another DMD type.  
ALP-4 basic devices: inquire the DMD type after AlibDevAlloca (AlibDevControl, AlibDevInquire).  
Parameter type: unsigned long  
Values: ALIB_DMSTYPE_XGA (default for ALP-3 basic),  
ALIB_DMSTYPE_MGA, ALIB_DMSTYPE_MGA_PLUS,  
ALIB_DMSTYPE_1080P_695A,  
ALIB_DMSTYPE_1080P_755A,  
ALIB_DMSTYPE_1080P_755A,  
ALIB_DMSTYPE_1080P_DISCONNECT (emulate 1800p) |
| ALIB_DEV_VERSION      | 6     | Read the ALP hardware version (AlibDevInquire)  
Parameter type: unsigned long |
| ALIB_DEV_DDC_VERSION  | 7     | Read the DDC chipset version (AlibDevInquire)  
Parameter type: unsigned long |
| ALIB_DEV_SWITCHES     | 8     | Read the DIP switch status (AlibDevInquire)  
Parameter type: unsigned long (bit map meaning) |
| ALIB_DEV_DDC_SIGNALS  | 9     | Adjust and inquire miscellaneous DDC signals: complement data, enable watchdog timer, DMD power down/power boxed,  
adjust reset groups (AlibDevControl, AlibDevInquire)  
Parameter type: unsigned long (bit map meaning) |

Figure A2. : Possible inquiries to the DMD device

Output:

**return_queryptr**: C style pointer with the readout from device/controlled about the specific query.

**Reset the DMD device (api_reset)**

Executes the essential reset step for DMD device before loading an image file.

```
return_reset = api_reset(dll_name, hdevice, reset_mode, reset_address)
```
Input:

dll_name: Loaded control library
hdevice: Device handle generated by allocate function
reset_mode: DMD blocks to be reset,

- 1: Single
- 2: Pair
- 3. Quad
- 4. Global

reset_address: address of block to be reset (0 for global)

<table>
<thead>
<tr>
<th>ResetType</th>
<th>ResetAddr</th>
<th>Addressed blocks</th>
</tr>
</thead>
<tbody>
<tr>
<td>ALPB_RESET_SINGLE</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>...</td>
<td>ResetAddr</td>
</tr>
<tr>
<td></td>
<td>16</td>
<td>15</td>
</tr>
<tr>
<td>ALPB_RESET_PAIR</td>
<td>0</td>
<td>0, 1</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>1, 2</td>
</tr>
<tr>
<td></td>
<td>...</td>
<td>ResetAddr/2, Rese</td>
</tr>
<tr>
<td></td>
<td>7</td>
<td>14, 15</td>
</tr>
<tr>
<td>ALPB_RESET_QUAD</td>
<td>0</td>
<td>0–3</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>4–7</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>8–11</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>12–15</td>
</tr>
<tr>
<td>ALPB_RESET_GLOBAL</td>
<td>0</td>
<td>0–15</td>
</tr>
</tbody>
</table>

Figure A3. : Modes for resetting DMD device

Output:

return_reset: Return for success/ error reporting
**Clear the DMD mirrors** (*api_clear*)

The clear operation sets the memory content of whole reset blocks to logic 0, and should be executed in the order, reset → clear → load.

\[
[\text{return}\_\text{clear}] = \text{api}\_\text{clear}(\text{dll}\_\text{name}, \text{hdevice}, \text{first}\_\text{block}, \text{last}\_\text{block})
\]

**Input:**

dll\_name: Loaded control library

hdevice: Device handle generated by allocate function

first\_block: First block to be cleared (0)

last\_block: Last block to be cleared (15)

**Output:**

return\_clear: Return for success/ error reporting

**Load an image file to the DMD device**

Should always be performed in order, reset → clear → load.

\[
[\text{return}\_\text{load}] = \text{api}\_\text{load}(\text{dll}\_\text{name}, \text{hdevice}, \text{image}, \text{first}\_\text{row}, \text{last}\_\text{row})
\]

**Input:**

dll\_name: Loaded control library

hdevice: Device handle generated by allocate function

image: Binary image matrix with appropriate dimensions (rows*columns = 768*1024).

Note that C style structures are transpose of that in MATLAB.

first\_row: First row to be loaded (0)

last\_row: Last row to be loaded (767)

**Output:**

return\_load: Return for success/ error reporting
Run the reset → clear → load order in a loop for grayscale lithography

These set of commands demonstrate a typical loop for setting up the DMD device to display a series of mask, necessary for a gray-scale lithography. This algorithm uses the frame and exposure variables depicting the lithography masks and corresponding exposure times, generated using the discussed code in last section.

```matlab
black = zeros(768, 1024); % Define a black image and load it on mirrors
$return_load$ = api_load(dll_name, hdevice, black, first_row, last_row);
return_check($return_load$)
$return_reset$ = api_reset(dll_name, hdevice, reset_mode, reset_address); % Reset once
return_check($return_reset$)
$return_clear$ = api_clear(dll_name, hdevice, first_block, last_block); % Clear once
return_check($return_clear$)

disp('exposure is going to start...')

pause(5) %Pause for 5 seconds, start the light source at this point

loop_time = zeros(1, size(frame, 3));

for b = 1:size(frame, 3)
    m1 = sprintf('Starting exposure %0.0f for %0.0f seconds', b, exposure(b, 1))
    image = frame(:, :, b); % Load a specific image
tic;
$return_load$ = api_load(dll_name, hdevice, image, first_row, last_row);
```

70
% Display the mask on mirrors
[return_reset] = api_reset(dll_name, hdevice, reset_mode, reset_address);
% Ready a reset command, saves time when you actually change the mask
pause(exposure(b, 1)) % Exposure time for specific mask
[return_clear] = api_clear(dll_name, hdevice, first_block, last_block); % Clear the mask, showing a blank between this and next loop execution
loop_time(b) = toc; % Use this to rectify exposure times
end

[return_reset] = api_reset(dll_name, hdevice, reset_mode, reset_address);
[return_clear] = api_clear(dll_name, hdevice, first_block, last_block);
disp('Exposure finished')

Input:
frame: array of dimension (rows*columns*L) representing the L masks for an effective grayscale photolithography.
exposure: scalar variable with dimension (1, L) representing exposure times for each of the L masks.

Free DMD device after use (api_free)
Initiates the shutdown protocol by returning mirrors to a floating position.

[return_free] = api_free(dll_name, hdevice)

Input:
dll_name: Loaded control library
hdevice: Device handle generated by allocate function
Output:
return_free: Return for success/ error reporting

**Error checking (return_check)**

An error checking utility for troubleshooting using the generated return variable.

[out_signal] = return_check(return_var)

Input:

return_var: Return variable from above discussed functions

Output:

out_signal: Verbose meaning of the return signal:

<table>
<thead>
<tr>
<th>Return code</th>
<th>Value</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>ALFB_SUCCESS</td>
<td>0</td>
<td>The function succeeded. Output data is valid.</td>
</tr>
<tr>
<td>ALFB_SUCCEED_PARTIAL</td>
<td>1</td>
<td>The function succeeded. However, output has been truncated.</td>
</tr>
<tr>
<td>ALFB_ERR_NOT_FOUND</td>
<td>8000 0000h</td>
<td>No free ALP basic device with the specified serial number was found.</td>
</tr>
<tr>
<td>ALFB_ERR_DUPLICATE</td>
<td>8000 0002h</td>
<td>The ALP is already in use.</td>
</tr>
<tr>
<td>ALFB_ERR_INIT</td>
<td>8000 0003h</td>
<td>ALP device initialization failed.</td>
</tr>
<tr>
<td>ALFB_ERR_RESET</td>
<td>8000 0004h</td>
<td>ALP device initialization failed. Toggle reset switch and try again.</td>
</tr>
<tr>
<td>ALFB_ERR_INVALID</td>
<td>8000 0005h</td>
<td>The device handle is invalid.</td>
</tr>
<tr>
<td>ALFB_ERR_DISCONNECT</td>
<td>8000 0006h</td>
<td>The device has been disconnected. Despite use AlpDevFree to destroy the handle!</td>
</tr>
<tr>
<td>ALFB_ERR_CONNECTION</td>
<td>8000 0007h</td>
<td>A connection error occurred, but the device has already been re-connected. Re-allocate by calling AlpDevFree and AlpDevAlloc.</td>
</tr>
<tr>
<td>ALFB_ERR_MT</td>
<td>8000 0008h</td>
<td>Multi threading: Another concurrently executed function denies this call.</td>
</tr>
<tr>
<td>ALFB_ERR_HALT</td>
<td>8000 0009h</td>
<td>The device has been halted. Resume it using AlpDevControl.</td>
</tr>
<tr>
<td>ALFB_ERR_MEM</td>
<td>8000 000Ah</td>
<td>The required memory could not be accessed.</td>
</tr>
<tr>
<td>ALFB_ERR_MEM_I</td>
<td>8000 000Bh</td>
<td>Insufficient memory situation occurred while creating internal objects.</td>
</tr>
<tr>
<td>ALFB_ERR_PARAM</td>
<td>8000 000Ch</td>
<td>An argument has an invalid value.</td>
</tr>
<tr>
<td>ALFB_ERR_DONGLE</td>
<td>8000 000Dh</td>
<td>The USB dongle is missing or defective.</td>
</tr>
</tbody>
</table>

Figure A4. : Returned values and their meaning for error reporting
BIBLIOGRAPHY


