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Search for Deconfined Criticality: SU(2) Déjà Vu

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Monte Carlo simulations of the SU(2)-symmetric deconfined critical point action reveal strong violations of scale invariance for the deconfinement transition. We find compelling evidence that the generic runaway renormalization flow of the gauge coupling is to a weak first order transition, similar to the case of U(1)×U(1) symmetry. Our results imply that recent numeric studies of the Néel antiferromagnet to valence bond solid quantum phase transition in SU(2)-symmetric models were not accurate enough in determining the nature of the transition.

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Within the standard Ginzburg-Landau-Wilson description of critical phenomena a direct transition between states which break different symmetries is expected to be of first-order. The existence of a generic line of deconfined critical points (DCP) proposed in Refs. [1, 2, 3] — an exotic second-order phase transition between two competing orders — remains one of the most intriguing and controversial topics in the modern theory of phase transitions. In particular, the DCP theory makes the prediction that certain types of superfluid to solid and the Néel antiferromagnet to valence bond solid (VBS) quantum phase transitions in 2D lattice systems can be continuous. Remarkably, the new criticality is in the same universality class as a 3D system of $N = 2$ identical complex-valued classical fields coupled to a gauge vector field (referred to as the DCP action below). This makes the DCP theory relevant also for the superfluid to normal liquid transition in symmetric two-component superconductors [4].

An intrinsic difficulty in understanding properties of the $N$-component DCP action is its runaway renormalization flow to strong coupling at large scales and the absence of perturbative fixed points for realistic $N$ [3, 4]. One may only speculate that the value of $N$ must be of little importance since the possibility of the continuous transition for $N = 1$ is guaranteed by the exact duality mapping to the SU(2)-symmetric NCCP [14]. For our simulations we consider the lattice version of the SU(2)-symmetric NCCP [2, 3] and map it onto the two-component $J$-current model. The DCP action for two spinon fields $z_a$, $a = 1, 2$ on a three-dimensional simple cubic lattice is defined as

$$S = - \sum_{<ij>,a} t(z^*_a z_{aj} e^{i A_{<ij>}^a} + c.c) + \frac{1}{8g} \sum_{\Box} (\nabla \times A)^2 \ ; \sum_a |z_a|^2 = 1 \ , \ (1)$$

where $<ij>$ runs over nearest neighbor pair of sites $i, j$, the gauge field $A_{<ij>}^a$ is defined on the bonds, and $\nabla \times A$ is a short-hand notation for the lattice curl-operator. The mapping to the $J$-current model starts from the parti-
tion function $Z = \int Dz Dz^* DA \exp(-S)$ and a Taylor expansion of the exponentials $\exp\{t z_{a,i}^* z_{a,j} e^{i A_{<ij>}}\}$ and $\exp\{t z_{a,j}^* z_{a,i} e^{-i A_{<ij>}}\}$ on all bonds. One can then perform an explicit Gaussian integration over $A_{<ij>}$, $z_{ai}$ and arrive at a formulation in terms of integer non-negative bond currents $J_{i,\mu}^{(a)}$. We use $\mu = \pm 1, \pm 2, \pm 3$ to label the directions of bonds going out of a given site the corresponding unit vectors are denoted by $\hat{\mu}$. These $J$-currents obey the conservation laws:

$$\sum_\mu I_{i,\mu}^{(a)} = 0, \quad \text{with} \quad I_{i,\mu}^{(a)} = J_{i,\mu}^{(a)} - J_{i+\hat{\mu},-\mu}^{(a)}. \quad (2)$$

The final expression for the partition function reads

$$Z = \sum_{\{J\}} Q_{\text{site}} Q_{\text{bond}} \exp(-H_J), \quad (3)$$

where

$$H_J = \frac{g}{2} \sum_{i,j; a,b \mu \nu \delta \lambda = 1,2,3} I_{i,\mu}^{(a)} V_{ij} I_{j,\nu}^{(b)} \quad (4)$$

$$Q_{\text{site}} = \prod_i \frac{\mathcal{N}_i^{(1)} \mathcal{N}_i^{(2)}}{1 + \mathcal{N}_i^{(1)} + \mathcal{N}_i^{(2)}}!, \quad \mathcal{N}_i^{(a)} = \frac{1}{2} \sum_\mu J_{i,\mu}^{(a)}$$

$$Q_{\text{bond}} = \prod_{i,a,\mu} \frac{t_{i,a}^{(a)} J_{i,\mu}^{(a)}!}{J_{i,\mu}^{(a)}!}. \quad$$

The long-range interaction $V_{ij}$ depends on the distance $r_{ij}$ between the sites $i$ and $j$. Its Fourier transform is given by $V_{q} = 1/\sum_{\mu=1,2,3} \sin^2(q\mu/2)$ and implies an asymptotic behavior $V \sim 1/r_{ij}$ at large distances.

This formulation allows efficient Monte Carlo simulations using a worm algorithm for the two-component system $\text{SU}(2)$. For the flowgram analysis we measure the mean square fluctuations of the winding numbers $\langle W_{2,a}^2 \rangle \equiv \langle W_{a>2}^2 \rangle$ of the conserved currents $I_{i,\mu}^{(a)}$ or, equivalently, $\rho_\pm = \sum_\mu \langle (W_{1,\mu} \pm W_{2,\mu})^2 \rangle / L \equiv \langle W_{2}^2 \rangle / L$. In particular, we focused on the gauge invariant superfluid stiffness, $\rho_-$ measuring the response to a twist of the phase of the product $z_1^* z_2$.

Similar to the $\text{U}(1) \times \text{U}(1)$ case $\text{SU}(2)$, the NCCP$^1$ model features three phases, Fig. 4 characterized by the following order parameters:

VBS: an insulator with $\langle z_{ai} \rangle = 0$ and, accordingly, $\langle \rho_+ \rangle = \langle \rho_- \rangle = 0$.

2SF: two-component superfluid (2SF) with $\langle z_{ai} \rangle \neq 0$, $\langle \rho_+ \rangle \neq 0$ and $\langle \rho_- \rangle \neq 0$.

SFS: supesolid (a paired phase $\text{SU}(2)$) with $\langle z_{ai} \rangle = 0$, $\langle z_1^* z_2 \rangle \neq 0$, $\rho_+ = 0$ and $\rho_- \neq 0$.

The point $g = 0$ and $t \approx 0.468$ features a continuous transition in the $O(4)$ universality class. The relevant part of the phase diagram is the region of small $g$ close to this $O(4)$ point, far away from the bicritical point $g_{bc} \approx 2.0$. Where SFS phase intervenes between the VBS and 2SF phases. The corresponding direct VBS-2SF transition has been proposed to be a deconfined critical line (DCP line) $\text{SU}(2)$.

The key idea of the flowgram method $\text{SU}(2)$ is to demonstrate that the universal large-scale behavior at $g \to 0$ is identical to that at some finite coupling $g = g_{\text{coll}}$ where the nature of the transition can be easily revealed. The procedure is as follows:

(i) Introduce a definition of the critical point for a finite-size system of linear size $L$ consistent with the thermodynamic limit and insensitive to the order of the transition. In our model we used the same definition as in Ref. $\text{SU}(2)$. Specifically, for any given $g$ and $L$ we adjusted $t$ so that the ratio of statistical weights of configurations with and without windings was equal to 7.5.

(ii) At the transition point, calculate a quantity $R(L, g)$ that is supposed to be scale-invariant for a continuous phase transition in question, vanish in one of the phases and diverge in the other. Here we consider $R(L, g) = \langle W_{2}^2 \rangle$.

(iii) Perform a data collapse for flowgrams of $R(L, g)$, by rescaling the linear system size, $L \to C(g)L$, where $C(g)$ is a smooth and monotonically increasing function of the coupling constant $g$. In the present case we have $C(g \to 0) \propto g^2$.

A collapse of the rescaled flows within an interval $g \in [0, g_{\text{coll}}]$ implies that the type of the transition within the interval remains the same, and thus can be inferred by dealing with the $g = g_{\text{coll}}$ point only. Since the $g \to 0$ limit implies large spatial scales, and, therefore, model-
FIG. 2: (Color online) Flowgrams for the short-range model. The lower horizontal line features the O(4) universality scaling behavior, so that for \( g < g_c \approx 0.95 \) all flows are attracted to this line. The upper horizontal line is the tricritical separatrix (marked as TP). Above it, flows diverge due to the first-order transition detected by the bi-modal distribution of energy.

FIG. 3: (Color online) A typical flowgram of the gauge invariant superfluid stiffness in the NCCP\(^1\) model. The inset shows a fan of diverging flows for \( 0.125 < g < 1.4 \). The yellow line is a fit representing the master curve. The horizontal axis is the scale reduced variable \( C(g)L \) with \( C(g) = \frac{(\exp(bg) - 1)}{(\exp(bg_1) - 1)} \), \( b = 2.28 \pm 0.02 \) and \( g_1 = 1.3 \). Error bars are shown for all data points.

FIG. 4: (Color online) Data collapse for the NCCP\(^1\) flows. The yellow line is a fit representing the master curve. The horizontal axis is the scale reduced variable \( C(g)L \) with \( C(g) = \frac{(\exp(bg) - 1)}{(\exp(bg_1) - 1)} \), \( b = 2.28 \pm 0.02 \) and \( g_1 = 1.3 \). Error bars are shown for all data points.

independent runaway renormalization flow pattern, the conclusions are universal.

To have a reference comparison, we first simulated a short-range analog of the NCCP\(^1\) model \( 4 \) with \( V_{ij} = g \delta_{ij}. \) The short-range model has a similar phase diagram, but with a second order phase transition for small \( g \) and a first order one at large \( g. \) Figure 2 clearly shows that the corresponding flowgram cannot be collapsed on a single master curve by rescaling the length (shifting the lines horizontally in logarithmical scale), and the separatrix at the tricritical point (TP) at \( g \approx 0.95 \) is clearly visible.

Contrary to the short range model we find no such separatrix for the DCP action. As shown in Fig. 3 the flows feature a fan of lines diverging with the system size and with the slope increasing with \( g \) without any sign of a TP separatrix.

One can notice that the NCCP\(^1\) flows exhibit a slope change, see Fig. 3 (also observed in Ref. \( 12 \) for the \( J-Q \)-model) that might be interpreted as a sign of the evolution towards a scale invariant behavior \( \langle W^2 \rangle = \text{const}. \) This behavior all but rules out the existence of the \( J-Q \)-model (see Fig. 4). The recasting function \( C(g) \) exhibits a linear behavior \( C(g) \propto g \) at small \( g \) consistent with the runaway flow in the lowest-order renormalization group analysis \( 3. \) This behavior all but rules out the existence of the TP on the VBS-2SF line.

Though our conclusions directly contradict claims made in Refs. \( 14, 15, 16 \), the primary data are in agreement. A data collapse of the flowgram presented in the lower panel of Fig. 13 in Ref. \( 16 \) shows the same qualitative behavior as our Fig. 4 \( 22 \). We are also consistent with the conclusion reached in Ref. \( 17 \) that the slope change is an intermediate scale phenomenon and the Néel antiferromagnet to VBS transition in the \( J-Q \)-model violates the scale invariance hypothesis as observed by the divergent flow of \( \langle W^2 \rangle \).

The flow collapse within an interval \( g \in [0, g_{\text{coll}}] \) does
not yet imply a first-order transition. What appears to be a diverging behavior in Fig. 3 might be just a reconstruction of the flow from the O(4)-universality (at \(g = 0\)) to a novel DCP-universality at strong coupling. To complete the proof, we have to determine the nature of the transition for \(g = g_{\text{coll}}\). In this parameter range the standard technique of detecting discontinuous transitions by the bi-modal energy distribution becomes feasible. As shown in Fig. 4 a clear bi-modal distribution develops at \(g = 1.65\) which is below the bicritical point \(g_{bc}\) and within the data collapse interval \([0, g_{\text{coll}}]\).

This leaves us with the clear conclusion that the whole phase transition line for small \(g\) features a generic weak first-order transition identical to the one observed in the U(1)\(\times\)U(1) case. Driven by long-range interactions, this behavior develops on length scales \(\propto 1/g \to \infty\) for small \(g\) and thus is universal. It cannot be affected by microscopic variations of the NCCP\(^2\) model suggested in Ref. 16 to suppress the paired (molecular) phase.

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[19] See Ref. 4 for discussions of 2d as well as 3d field induced paired phases in two-component superconductors.
[20] The interaction constant \(K\) in Ref. 16 is defined as \(K = 1/(4g)\).
[21] A flow collapse is meaningful even when the collapsing lines \(R(L)\) are relatively short and reminiscent of straight lines: a straight line is described by two independent parameters, while the rescaling procedure has only one degree of freedom of shifting the line horizontally in logarithmic scale. The master curve may significantly deviate from a straight line and prove indispensable for understanding the global character of the flow and difficulties with the finite-size scaling in specific models.