October 2018

USING STRATEGIC DISCOURSE FOR BUILDING UNDERSTANDING IN ELEMENTARY MATHEMATICS: WHAT DO TEACHERS AND STUDENTS THINK?

Mary Coakley
University of Massachusetts Amherst

Follow this and additional works at: https://scholarworks.umass.edu/dissertations_2

Part of the Curriculum and Instruction Commons, Curriculum and Social Inquiry Commons, Educational Assessment, Evaluation, and Research Commons, and the Educational Methods Commons

Recommended Citation
https://doi.org/10.7275/11935836.0 https://scholarworks.umass.edu/dissertations_2/1333

This Open Access Dissertation is brought to you for free and open access by the Dissertations and Theses at ScholarWorks@UMass Amherst. It has been accepted for inclusion in Doctoral Dissertations by an authorized administrator of ScholarWorks@UMass Amherst. For more information, please contact scholarworks@library.umass.edu.
USING STRATEGIC DISCOURSE FOR BUILDING UNDERSTANDING IN ELEMENTARY MATHEMATICS: WHAT DO TEACHERS AND STUDENTS THINK?

A Dissertation Presented

By

MARY MCGEE COAKLEY

Approved as to style and content by:

_____________________________________
Kathleen S. Davis, Chair

_____________________________________
Linda Griffin, Member

_____________________________________
Eric Sommers, Member

_____________________________________
Jennifer Randall
Associate Dean of Academic Affairs
College of Education
DEDICATION

This dissertation is dedicated to my amazing husband, Daniel Coakley, and my greatest blessings Connor Coakley, Erin Coakley and Maura Coakley.

I would also like to dedicate this dissertation to my parents Bernard G. McGee and Mary T. McGee who taught me to work hard and to believe in myself. This day would not have been possible without them.
ACKNOWLEDGEMENTS

Writing this dissertation has been a long personal and professional journey for me. I have thought about this day for many years and so writing this acknowledgment is long overdue. I would like to thank the people who have supported this endeavor throughout these years. This dissertation would not have been possible without the extended collaboration among many who contributed to my work and research since I began my study of mathematical discourse in 1993.

First, I would like to acknowledge my dissertation committee chair, Dr. Kathleen Davis for her continuous support and dedication to this project. Her encouragement and commitment went far and above her responsibilities as she became my mentor and coach throughout the entire process.

I would also like to acknowledge my other committee members for their contributions and participation. First, I am extremely thankful to Dr. Linda Griffin, for all of her valuable feedback and expertise. Also, I would like to thank Dr. Eric Sommers for his contributions including the mathematical expertise he brought to this project.

This dissertation could not have been possible without the encouragement of my amazing husband, Dan. Thank you for embracing my dream. It would never have been possible without you by my side. Your unwavering support throughout these years has been truly remarkable. I look forward to our next phase together. I love you!

Thank you to my three wonderful children, who in their own ways have added so much love and joy to my life. Connor, thank you for your technical savvy and jumping in to help me even when you had your own commitments. Erin, thank you boundless
generosity and for helping to make the tough times so much easier. Maura, thank you for understanding the commitment I made to this work, and for appreciating all of the fun times we spent together. I love you all. I am so very proud of all of you!

Also, I would like to thank all of my family members and wonderful friends who supported me in pursuit of this dream. Taking an interest in my research, as well as, offering kind words to encourage me to pushing forward was appreciated.

I would also like to acknowledge principals and curriculum directors who have shared this important work with me. Thank you to the teachers and their students who opened up their classrooms and minds to worked beside me to make successful attempt to include discourse with their students.
ABSTRACT

USING STRATEGIC DISCOURSE FOR BUILDING UNDERSTANDING IN ELEMENTARY MATHEMATICS: WHAT DO TEACHERS AND STUDENTS THINK?

SEPTEMBER 2018

MARY MCGEE COAKLEY, B.S. FLORIDA SOUTHERN COLLEGE M.Ed., LONG ISLAND UNIVERSITY BROOKLYN CAGS, UNIVERSITY OF SOUTHERN CONNECTICUT NEW HAVEN Ed.D, UNIVERSITY OF MASSACHUSETTES AMHERST

Directed by: Professor Kathleen Davis

The mathematics reform movement has not had a significant or lasting impact on the practice of teachers and learning of students throughout the country (Boylan, 2010, Kazemi & Stipek, 2001). Students are not developing the types of skills critical thinking skills needed to solve problems in mathematics. Research suggests a need for structural changes that include providing opportunities for students to develop more autonomy and authority in the mathematics classroom (Cuban, 2013). To meet these challenges, teachers and students must make significant changes to implement instruction that fulfills this demand. This expectation has left teachers struggling to determine essential changes and how to implement them. Although educators have begun to use discourse practices as a means for advancing understanding, how and why they do so is unclear.

The purpose of this descriptive case study is to identify the discourse practices used by two elementary teachers and their students as they solve problems together in
mathematics. A description of the dynamic interactions occurring among members in the classroom community of practice will be included (Lave and Wenger, 1991). Furthermore, the aim of the study is to describe specific discourse strategies that are used by teachers and students to support the building of understanding involving the mathematics concepts and skills being studied. Moreover, specific discourse strategies will be described, detailing the level of cognitive complexity of these methods. The study will include a focused investigation the on-the-spot decision making of classroom teachers and their students as they engage with one another to identifying strategies and articulate solutions with one another. The results will inform policy makers and educators by providing greater insight about the discourse strategies used to effectively facilitate student discussions while learning mathematics in a community of practice.
# TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>ACKNOWLEDGEMENTS</td>
<td>v</td>
</tr>
<tr>
<td>ABSTRACT</td>
<td>vi</td>
</tr>
<tr>
<td>LIST OF TABLES</td>
<td>xvii</td>
</tr>
<tr>
<td>LIST OF FIGURES</td>
<td>xix</td>
</tr>
<tr>
<td>CHAPTER 1. INTRODUCTION</td>
<td></td>
</tr>
<tr>
<td>A Vision for Elementary Mathematics</td>
<td>1</td>
</tr>
<tr>
<td>Statement of the Problem</td>
<td>4</td>
</tr>
<tr>
<td>Barrier 1: Understanding the Necessary Changes Needed</td>
<td>5</td>
</tr>
<tr>
<td>Barrier 2: Time and Involvement During Implementation</td>
<td>5</td>
</tr>
<tr>
<td>Barrier 3: Communicating the Changes Needed</td>
<td>6</td>
</tr>
<tr>
<td>Purpose of the Study</td>
<td>8</td>
</tr>
<tr>
<td>Guiding Research Questions</td>
<td>14</td>
</tr>
<tr>
<td>Scope of Study</td>
<td>15</td>
</tr>
<tr>
<td>Significance of the Study</td>
<td>15</td>
</tr>
<tr>
<td>Assumptions of the Study</td>
<td>17</td>
</tr>
<tr>
<td>Limitations of the Study</td>
<td>19</td>
</tr>
<tr>
<td>Establishing Trust</td>
<td>20</td>
</tr>
<tr>
<td>Organization of the Dissertation</td>
<td>22</td>
</tr>
<tr>
<td>2. LITERATURE REVIEW</td>
<td>24</td>
</tr>
<tr>
<td>Elementary Mathematics Reform Movement</td>
<td>25</td>
</tr>
<tr>
<td>Traditionalists</td>
<td>25</td>
</tr>
<tr>
<td>Progressives</td>
<td>26</td>
</tr>
<tr>
<td>Historical Perspective</td>
<td>27</td>
</tr>
<tr>
<td>Role of Assessment</td>
<td>28</td>
</tr>
<tr>
<td>Back to Basics Movement</td>
<td>29</td>
</tr>
<tr>
<td>Common Core Standards</td>
<td>30</td>
</tr>
<tr>
<td>Situated Learning in a Community of Practice</td>
<td>32</td>
</tr>
<tr>
<td>Conceptual Framework</td>
<td>34</td>
</tr>
<tr>
<td>Situated Learning: Example One</td>
<td>39</td>
</tr>
<tr>
<td>Example Two</td>
<td>41</td>
</tr>
<tr>
<td>Example Three</td>
<td>42</td>
</tr>
<tr>
<td>Reform-Based Practices in Elementary Mathematics</td>
<td>44</td>
</tr>
</tbody>
</table>
Mrs. Washington-Classroom A .................................................................153
Ms. Littleton .........................................................................................156

Observation Setting ..............................................................................158
Lakeview Elementary ...........................................................................158

Sources of Data and Collection Strategies ...........................................158
Observational Data ............................................................................162

Classroom Observations .....................................................................162
Videotaped Classroom Observations ..................................................164

Interviews .............................................................................................166
Individual Teacher Interviews .............................................................167
Teachers Focus Group Interview .........................................................167
Individual Student Interviews .............................................................169
Students Focus Group Interview ..........................................................169

Documents And Artifacts .....................................................................170
Field Notes and Memos .......................................................................170
Curriculum Mapping Documents ........................................................171
Teacher Designed Tasks .......................................................................171

Data Analysis Procedures ....................................................................172

Phase 1 of the Data Analysis .................................................................174
Classroom Talk ...................................................................................181
Dialogic Talk ........................................................................................183
Exploratory Talk ..................................................................................185
Mrs. Washington ..................................................................................187
Ms. Littleton ..........................................................................................189

Phase 2 Data Analysis Cycle .................................................................190
Exploratory Talk ..................................................................................192

Phase 3 of Data Analysis ......................................................................194
Gaining Entry and Informed Consent ...................................................195
<table>
<thead>
<tr>
<th>Summary</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>4. MRS. WASHINGTON AND STUDENTS</td>
<td>197</td>
</tr>
<tr>
<td>Strategic Discourse</td>
<td>199</td>
</tr>
<tr>
<td>Implementation of Strategic Discourse</td>
<td>206</td>
</tr>
<tr>
<td>Engage In Joint Reasoning</td>
<td>207</td>
</tr>
<tr>
<td>Classroom Example 1-Event 1</td>
<td>208</td>
</tr>
<tr>
<td>Classroom Example 2-Event 2</td>
<td>224</td>
</tr>
<tr>
<td>Summary</td>
<td>229</td>
</tr>
<tr>
<td>Everyone Invited To Contribute (GR1)</td>
<td>231</td>
</tr>
<tr>
<td>Classroom Example 1-Event 3</td>
<td>234</td>
</tr>
<tr>
<td>Classroom Example 2-Event 4</td>
<td>250</td>
</tr>
<tr>
<td>Summary</td>
<td>251</td>
</tr>
<tr>
<td>Multiple Solutions Are Encouraged (GR7)</td>
<td>252</td>
</tr>
<tr>
<td>Classroom Example 1-Event 5</td>
<td>254</td>
</tr>
<tr>
<td>Classroom Example 2-Event 2</td>
<td>265</td>
</tr>
<tr>
<td>Summary</td>
<td>269</td>
</tr>
<tr>
<td>Reform-Based Practices For Learning Mathematics With Understanding</td>
<td>271</td>
</tr>
<tr>
<td>Problem Posing</td>
<td>273</td>
</tr>
<tr>
<td>Well Designed Problem</td>
<td>275</td>
</tr>
<tr>
<td>Well Designed Tasks</td>
<td>278</td>
</tr>
<tr>
<td>Enriches Concepts and Skills</td>
<td>279</td>
</tr>
<tr>
<td>Provides Structure for Discussion</td>
<td>283</td>
</tr>
<tr>
<td>Active Learning With Authenticity</td>
<td>284</td>
</tr>
<tr>
<td>Engages In Learning</td>
<td>284</td>
</tr>
<tr>
<td>Connects To Real Life</td>
<td>288</td>
</tr>
<tr>
<td>Honors Mathematics As A Discipline</td>
<td>291</td>
</tr>
<tr>
<td>Learning Through Interaction</td>
<td>294</td>
</tr>
<tr>
<td>Learning Is Socially Constructed</td>
<td>294</td>
</tr>
<tr>
<td>Contributes To The Learning of Others</td>
<td>299</td>
</tr>
</tbody>
</table>
Knowledge Is Made Public (GR4) .......................................................... 483
Participants Offer Knowledge About Mathematics (GR4A) .................. 483
Strategies Are Explained in Words, Pictures And/Or Numbers (GR4B).... 485
Contributions Are Restated (GR4C) ....................................................... 486
Reasoning Is Visible In The Talk (GR5) ............................................... 487
Justifications/Rationales Are Provided To Explain Thinking (GR5A)....... 487
Engage in Joint Reasoning (GR6) ......................................................... 488
Ideas And Solutions Are Discussed With Others (GR6A)............... 489
Questions Are Posed to the Community to Direct Thinking (GR6B)..... 491
Questions Are Posed To Encourage Exchange Of Ideas (GR6C) ......... 492
Community Members Ask Questions to Understand Thinking (GR6D).... 493
Tasks Are Assigned To Initiate Working Together Find Solutions (GR6F).... 494
Assistance Offered To Work Through Process/Scaffold Learning (GR6G).... 494
Multiple Solutions Are Encouraged (GR7) ....... 496
Many Ways Of Solving Problems/Thinking Are Encouraged (GR7A) .... 496
Many Ways Of Solving Problems/Thinking Is Shared (GR7B) .............. 498

Ms. Littleton and Students ..................................................................... 499

Knowledge is Made Public (GR4) .......................................................... 500
Participants Offer Knowledge About Mathematics (GR4A) .................. 501
Strategies Are Explained In Words, Pictures And/Or Numbers (GR4B).... 502
Contributions Are Restated (GR4C) ....................................................... 503
Reasoning Is Visible In The Talk (GR5) ............................................... 505
Justifications/Rationales Are Provided To Explain Thinking (GR5A)....... 505
Steps In Solutions are Explained (GR5B) ............................................. 506
Engaging In Joint Reasoning (GR6) ....................................................... 507
Ideas And Solutions Are Discussed With Others (GR6A)............... 508
Questions Are Posed to the Community to Direct Thinking (GR6B)..... 511
Questions Are Posed To Encourage Exchange Of Ideas (GR6C) ......... 513
Community Members Ask Questions to Understand Thinking (GR6D).... 514
Multiple Solutions Encouraged (GR7) ....... 516
Many Ways of Solving Problems Encouraged (GR7A) .................... 517
Many Ways Of Solving Problems/Thinking Is Shared (GR7B) .............. 518

Summary of Theme 2: Developing Mathematical Knowledge.................. 519

Theme 3: Strengthening Critical Thinking ............................................. 523

Mrs. Washington and Students ............................................................. 524

Ideas Extended Together (GR9) ............................................................ 525

Ms. Littleton and Students ................................................................. 527

Partners Engage Critically With Each Other (GR11) ......................... 528
Summary of Theme 3: Strengthening Critical Thinking ........................................529
Research Question #2 ..........................................................................................530

Successful Strategic Discourse Practices Identified By Teachers ......................530

- All Are Valued And Capable Members (GR3D) .............................................531
- Ideas Are Discussed With Others (GR6A) .......................................................532
- Questions Are Posed To Direct Thinking (GR6B) ..........................................534
- Questioning To Exchange Ideas (GR6C) Understand Thinking (GR6D) ....534
- Assistance Is Offered To Try To Help Work Through The Process (GR6G) ..536
- Many Ways Of Problem Solving Are Encouraged (GR7A) .........................537
- All Have Opportunities To Question Others Ideas (GR11A) .......................539

Additional Discourse Strategies Identified By Teachers ......................................541

- Thinking Is Highlighted To Spotlight Different Ways Of Thinking (GR6E) ....541
- Listening To Understand Other Ways Of Thinking (GR10C) .......................542

Additional Strategies Identified By Students .......................................................542

- Steps In Solutions Are Explained (GR5B) ......................................................542
- Listening To Understand Other Ways Of Thinking (GR10C) .......................543

Summary ................................................................................................................543

Research Question #3 ..........................................................................................545

Mrs. Washington and Students ..............................................................................547
Mrs. Littleton and Students ...................................................................................549

Reform-Based Practices For Learning Mathematics With Understanding ..........552

- Problem Posing ...............................................................................................552
- Active Learning with Authenticity ....................................................................561
- Learning Through Interaction ..........................................................................571

Shifting Authority Toward Shared Authority: Littleton ........................................587
Shifting Authority Toward Shared Authority: Washington ...............................591
Summary ................................................................................................................594

Research Question #4 ..........................................................................................596

Teachers Perspective Of Impact of Discourse on Students’ Understanding ........596
Impact of Strategic Discourse on Their Own Understanding .............................597

Summary ................................................................................................................599
7. DISCUSSION........................................................................................................601

Restatement of Hypothesis......................................................................................602
Reformed-Based Practices ......................................................................................605

Problem Posing........................................................................................................606
Active Learning with Authenticity ........................................................................608
Learning Through Interaction..............................................................................612

Strategic Discourse ...............................................................................................619

Encouraging Student Participation ......................................................................625
Developing Mathematical Knowledge ..................................................................627
Strengthening Critical Thinking .........................................................................634

Intentional Strategic Discourse ............................................................................636
Sharing Mathematical Authority .........................................................................641
Students and Teachers Inform Practice ...............................................................649
Limitations ............................................................................................................653
Implications ..........................................................................................................655
Conclusions ...........................................................................................................658

APPENDICES .......................................................................................................660

APPENDIX A: ASSENT FORM ..............................................................................661
APPENDIX B: CAREGIVER INFORMED CONSENT ...........................................662
APPENDIX C: RESEARCH PROJECT CONSENT FORM .......................................663
APPENDIX D: PARTICIPANT INFORMED CONSENT .........................................664
APPENDIX E: FOCUS GROUP INTERVIEW STUDENT PROTOCOL ....................665
APPENDIX F: FOCUS GROUP INTERVIEW TEACHER PROTOCOL .....................666
APPENDIX G: INDIVIDUAL TEACHER INTERVIEW PROTOCOL .....................667
APPENDIX H: INDIVIDUAL STUDENT INTERVIEW PROTOCOL .......................668
APPENDIX I: STUDENT SURVEY/QUESTIONNAIRE PROTOCOL .......................669
APPENDIX J: LESSON OBSERVATION PROTOCOL: PHASE 1 .........................670
APPENDIX K: TEACHER DESIGNED TASKS .......................................................671
APPENDIX L: OBSERVATION PROTOCOL: PHASE 2 .......................................674
APPENDIX M: OBSERVATION PROTOCOL: PHASE 3 .......................................676
APPENDIX N: ANALYTICAL MEMO .................................................................678
APPENDIX O: OBSERVATION PROTOCOL: REFORM-BASED METHODS ........679
APPENDIX P: COMPARING CASE TO DISCOURSE TALK MODELS ................680
APPENDIX Q: LEARNING MATH WITH UNDERSTANDING IN COP ..................682

REFERENCES ......................................................................................................683
# LIST OF TABLES

<table>
<thead>
<tr>
<th>Table</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Results of the Boaler (1998) Study</td>
<td>65</td>
</tr>
<tr>
<td>2. Extending student thinking framework</td>
<td>95</td>
</tr>
<tr>
<td>3. Models for Using Mathematical Discourse with Students</td>
<td>108</td>
</tr>
<tr>
<td>4. Models for Using Mathematical Discourse with Students</td>
<td>109</td>
</tr>
<tr>
<td>5. Models for Using Mathematical Discourse with Students</td>
<td>110</td>
</tr>
<tr>
<td>6. Data Collection Sources Case A</td>
<td>159</td>
</tr>
<tr>
<td>7. Data Collection Sources Case B</td>
<td>161</td>
</tr>
<tr>
<td>8. Comparing Case A to Discourse Among Talk Models</td>
<td>175</td>
</tr>
<tr>
<td>9. Comparing Case A to Discourse Among Talk Models</td>
<td>177</td>
</tr>
<tr>
<td>10. Teacher Use of Ground Rule Elements Case A</td>
<td>201</td>
</tr>
<tr>
<td>11. Student Use of Ground Rule Elements Case A</td>
<td>204</td>
</tr>
<tr>
<td>12. Student Survey Questionnaire/Results Case A</td>
<td>223</td>
</tr>
<tr>
<td>14. Teacher Use of Ground Rule Elements Case B</td>
<td>310</td>
</tr>
<tr>
<td>15. Student Use of Ground Rule Elements Case B</td>
<td>313</td>
</tr>
<tr>
<td>16. Student Survey/Questionnaire Results Case B</td>
<td>381</td>
</tr>
<tr>
<td>17. Implementation of Reform Practices for Teaching Mathematics Case B</td>
<td>419</td>
</tr>
<tr>
<td>18. Ground Rules Frequency Comparison: Case 1 and Case 2</td>
<td>452</td>
</tr>
<tr>
<td>19. Most Frequently Used Strategic Discourse Elements by Teacher</td>
<td>458</td>
</tr>
<tr>
<td>20. Most Frequently Used Strategic Discourse Elements by Student</td>
<td>459</td>
</tr>
</tbody>
</table>
# LIST OF FIGURES

<table>
<thead>
<tr>
<th>Figure</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Chips placed on the overhead projector (McClain, &amp; Cobb, 2001, p. 249). The bottom portion of the chart is the notation suggested by the students.</td>
<td>53</td>
</tr>
<tr>
<td>2. The crate used by students to fill pumpkins (McClain, &amp; Cobb, 2001, p. 254)</td>
<td>54</td>
</tr>
<tr>
<td>3. Sample homework questions (Sherin, p. 222)</td>
<td>116</td>
</tr>
<tr>
<td>4. Spiraling Approach to Data Analysis</td>
<td>173</td>
</tr>
<tr>
<td>5. Implementation of Ground Rules in Case A</td>
<td>199</td>
</tr>
<tr>
<td>6. Implementation of Ground Rules in Case B</td>
<td>308</td>
</tr>
<tr>
<td>7. The Big G Graphic Organizer</td>
<td>324</td>
</tr>
<tr>
<td>8. Strategic Discourse Strategies by Theme</td>
<td>456</td>
</tr>
<tr>
<td>9. Learning with Understanding in a Community of Practice</td>
<td>605</td>
</tr>
<tr>
<td>10. Most Frequently Used Discourse Among Teachers by Theme</td>
<td>624</td>
</tr>
<tr>
<td>11. Most Frequently Used Discourse Among Students by Theme</td>
<td>625</td>
</tr>
<tr>
<td>12. Complexity of Discourse Elements According to Theme</td>
<td>636</td>
</tr>
</tbody>
</table>
CHAPTER 1
INTRODUCTION

A Vision for Elementary Mathematics

Mathematics reform has been on the education agenda for over two and a half decades with the goal of addressing the many of the barriers impacting learning (Fulllan, 2001; Lortie, 1975; NCTM, 1989; Oakes, Quartz, Ryan & Lipton, 2000; Ravitch, 2010; Tyack & Cuban, 1995). The movement includes a de-emphasis on the acquisition of arithmetic skills and an emphasis on developing conceptual thinking and understanding. The National Council for Teachers of Mathematics created a vision that included many recommendations for wide sweeping changes in school mathematics (2000). This vision proposes students understand the mathematics that they learn in school. It also calls for an increase in the types of experiences that would help students learn mathematical concepts instead of manipulating numbers and focusing on identifying correct answers. The vision required students, along with their teacher, to think and reason more about strategies and solutions and to be able to communicate this thinking aloud and in writing. Teachers are being asked to view students as mathematicians responsible for making and refining conjectures and engaging in conversations to confirm or disprove those conjectures. The shifts embedded in the vision required educators to think more deeply about their roles, their student roles, and how it all connects to instruction and learning. Although the vision captivated enormous energy, the profound instructional shifts needed were vast and complex. Many wondered, is this possible?

Educational reformers rallied around the ideals supported in the vision. Many built strong arguments for the need to transform classrooms through the use of the
practices included in the vision (Boaler, 2002, Chapin & O’Connor, 2007; Kazemi & Franke, 2004; Lampert & Cobb, 2003; McClain & Cobb, 2001; Mueller & Maher, 2008; Washaw & Anthony, 2008). This vision also resonated with educators and the movement began to build momentum. The vision also mirrored the Common Core’s recommendations for teachers to provide students with experiences to learn how to share their own thinking, develop reasonable arguments, justify conclusions, and make sense of the reasoning of others (Chapin & O’Connor, 2007; Sherin, 2002; McClain & Cobb, 2001). All of these initiatives pushed the reform agenda to the national forefront with what seemed to be a major commitment by all in the educational arena. In addition, identifying ways to assist students to gain experiences in developing skills needed for success in college and career were added to this reform agenda.

However, difficulties with implementation emerged. It became apparent that implementation was difficult and practices were superficial at best (Ball, Hill & Bass, 2005; Boylan, 2010; Millet, Brown & Askew, 2004; Cuban, 2013). Researchers discovered that although the content of math had changed somewhat, how students learn remained very similar to the way our grandparents did many years ago (Cuban, 2013). In fact, many of the educators who believed that they were implementing reform practices were not using methods reflective of the changes envisioned by reformers (Hiebert & Morris, 2012; Millet, Brown & Askew, 2004). A lack of pedagogical content knowledge and expertise around implementing practices among educators were limiting progress (Kennedy, 2004, Putnam & Borko, 2000). In the face of these challenges many reverted back to using more traditional methods (Cuban, 2013; Kennedy, 2004). Was this the result of a hasty implementation?
Also problematic to the reform effort was the constant and conflicting agendas from many constituents including community members, parents, policy makers, administrator, and reformers (Cuban, 2013). These agendas included a focus on: preparing for high stakes testing, revising standards, raising international performance ratings, and or reinstating back to the basics methodology. This revolving door of initiatives and adoption of new methods has seriously detracted educators from making a commitment to change. Policy makers and society as a whole, support quick and sweeping innovations but often are not willing to demonstrate patience and trust as educators work over time to merge the initiatives with existing practice. Accepting that change takes time and careful planning is a necessary first step for lasting school reform.

As a former District Mathematics Curriculum Coordinator and current Assistant Principal, I have had many opportunities to observe teachers when handed new resources or mandates such as, textbooks, state frameworks and Common Core standards. More support is necessary for teachers to make curriculum modifications, pedagogical changes, and assessment revisions (Kober & Rentmer, 2011). Sometimes resources come with manuals, but often curriculum documents are packaged as a list of isolated standards for teachers to use as their grade level curriculum. Teachers are left to interpret these resources and standards on their own or with their colleagues. They are not informed about ways to adapt their teaching methods or merge these standards with their existing curriculum. Often this results in teachers piecing together a mathematics program that is simply a series of disconnected lessons across the course of a year and may or may not look different from classroom to classroom. This job, typically done in isolation,
becomes overwhelming leaving teachers to question themselves and most often revert back to what they know and have always done.

The task of implementing the standards that dictate changes in pedagogy requires teachers to redesign the structure and culture of learning (Cuban, 2013). Most teachers use a top-down approach to instruction. A majority of what students do in the classroom involves whole group instruction, teacher directed tasks and independent work. Students often are not essential to the activity, nor do they have authority in lesson execution or decision-making about the route of their own learning. It is taken for granted that teachers have the skills, especially the pedagogical content knowledge, to make these changes. Even those who embrace the student-centered approach do not have the skills to write curriculum or plan instruction correlating to reform ideals (Kennedy, 2004). Additionally, reform practices compete with other pressures, time and techniques that teachers juggle during the school day. The lack of attention to these issues has held students back from learning mathematics with understanding.

**Statement of the Problem**

Clearly mathematics reform in still in a state of flux, as a process for leading students toward mathematical understanding still seems to be elusive. Reformers, policy makers and teachers do not comprehend the deep and systemic changes necessary to achieve lasting change in teaching and learning (Cuban, 2013; Kennedy, 2004). Teachers and students are given little support, time or voice while implementing the changes. In addition, not enough specific recommendations have been made so that teachers are knowledgeable of what it is they should be doing (Cochran-Smith & Fries, 2005; Putnam & Borko, 2000). These barriers form challenges that have resulted in years of superficial
Barrier 1: Understanding the Necessary Changes Needed

The first barrier for change is the lack of understanding among reformers and policy makers about the types of systemic changes necessary to bring about reform in classrooms (Cuban, 2013). This lack of knowing includes understanding the complexity of the classroom and the process involved in transforming educators’ thinking about learning and instruction called for in the vision outlined previously. A limited concept surrounding the complexity involved in transitioning teachers and students from a traditional learning environment to one that is designed around reformed learning practices has been harmful to the process. This limitation has made the things that need to be changed unclear and contributed to a superficial understanding about how students learn most effectively. The transformational change in schooling that is needed requires attending to the countless small and large shifts of everyday practices and customs. Methodology and structure requires reconstruction, as will teacher and student roles and interactions (Boylan, 2010). Teachers will need to move away from a traditional framework toward one that encourages students to increase their position, responsibility and engagement in learning. Students acting as passive receptacles where information is dumped must be a thing of the past.

Barrier 2: Time and Involvement During Implementation

Teachers require time and support to implement the changes, acquire new skills and transition their instruction. A different type of knowledge is needed to teach mathematics so that students can understand and explain concepts. Effective teachers
need content knowledge and “math knowledge for teaching” (Hill, Rowan & Ball, 2005, p. 373). There is so much uncertainty about how to structure the learning to increase thinking and discourse while assisting students to understand the mathematics they study (Ding, Li, Piccolo, & Kulm, 2007; Webb, Nemer, & Ing, 2006; Yackel & Hanna, 2003). Many teachers only experience with mathematics has been through traditional application (Kirchner, 2002).

Changes in instruction must occur in small steps and includes opportunities to discuss these experiences with other teachers and professionals. Incorporating all of the skills required to engage students in logical thinking, critical reasoning, and communicating ideas takes practice. Learning to understand student thinking is critical for assessing student growth and for planning instruction. Educators need time to study this thinking and structure appropriate ways of engaging with others in the classroom. Likewise time must be allocated to helping students adapt to the transformative changes required (Corbett & Wilson, 1995). Most of the changes will be completely different from what students are accustomed to doing in their classrooms.

**Barrier 3: Communicating the Changes Needed**

The third barrier is the lack of articulation about the complexity of the reforms proposed by reformers and policy makers. Although research demonstrating the benefits of many reform practices in mathematics education is widely available, a systematic process for using these methods with students has been elusive to teachers. The intent of the changes and the specific things that must be changed has not been clearly defined (Cuban, 2013). Teachers lack the authority, knowledge, experience and time, making it extremely difficult for them to identify and execute the quality experiences their students
need and to commit to doing so long term (Kennedy, 2004). Even when teachers understand and utilize reforms, this understanding does not ensure that they know how to implement them in their own classroom. More than just the national and state frameworks they need specific process to help all teachers with this process so that all students benefit (Davis & Simmt, 2003; Martin, Towers & Pirie, 2006; Warfield, Wood & Lehman, 2005). Initiating change through top-down mandates leaving teachers to wonder on their own how to execute an implementation out of the decision-making has been detrimental to mathematics reform.

In addition, teachers and students have typically not been included in the decision-making, planning or revision of the changes. The organization of the school hierarchy places them at the bottom, which promotes the idea that they are not organizationally or socially valued (Shed & Bacharach, 1991). Yet, teachers are making the daily decisions regarding what students learn, how they engage in learning, and when and how they are evaluated (Corbett & Wilson, 1995; Oakes & Lipton, 1990). This has fostered a disconnection between educational policy outside of the classroom and the reality of inside the classroom. These accepted regularities continue to be accepted by society as an effective process for educating our youth (Cuban, 1993; Lortie, 1975; Sarason, 1990). In fact, as graduates of the teacher-dominated education system, we have been socialized to accept the power structure that endures in the classroom as “right, natural, and proper,” and therefore, it does not seem appropriate to question its existence (Sarason, 1990, p. 89). According to researchers, certain regularities have endured because they have become part of the culturally held beliefs associated with the conception of school and the practices that are embedded into it (Tyack & Cuban, 1995).
The regularities associated with the management and hierarchy of schools makes it difficult to transform how students learn in the classroom.

Understanding how teachers implemented a participation structure around strategic discourse practices in a community of practice will help other educators establish their own learning environment that supports understanding. Additionally examining the specific types of discourse practices the teachers implemented helped to shed light on the methods used to engage students in conversations about the mathematics studied in their own classrooms. Moreover, considering the perspectives that teachers and students have about the discourse practices assist in developing an understanding and identifying mathematical teaching practices that are most effective for learning mathematics with understanding.

**Purpose of the Study**

The purpose of this descriptive case study was to describe the discourse practices used by teachers and students as they solved problems together in a community of practice. Teachers need a more detailed vision for transforming the existing structure of their mathematics instruction to include studying math in new and authentic ways. The process for engaging students with one another to build understanding in a community of practice while solving problems with real life connections that I propose here is grounded in situated learning theory. Using this process to guide the instruction in a classroom community to build mathematical understanding with their students is essential to making lasting change.

The theoretical framework that orientates this research study is grounded in the assumption that learning is social and situated. The situated learning theory developed by
Lave and Wenger (1991) is based on the view that educational practice resides naturally in the interaction among students within a participant framework. According to Lave and Wenger, learning is a function of the activity, content and culture in which it occurs. Optimum learning, in this view, does not occur in the individual’s mind or actions, as traditionally believed, but instead through the ongoing pursuit of joint meaning making.

The vehicle for learning in the situated learning theory is the “community of practice” [CoP]. This includes a process referred to as “legitimate peripheral participation” [LLP] (1991, p. 29). Within LLP, participants build knowledge while submerged in a task as they make connections with the other members of the community while developing understanding.

Members of the communities studied by Rogoff and Lave were not just focused on getting tasks done, but are on relating to each other as they attempted to “resolve inevitable conflicts in ways that maintain the relationship” (1984, p. 10). In successful communities of practice members are dependent on positive relationships and productivity among members. Critical to the success of the community is the establishment of a foundation for participation based on mutual respect among members. All must actively engage as equal contributors while making sense of the concepts studied.

Research pertaining to this view is focused on participation as a way of understanding interaction and how this can lead to improvement in pedagogical practice and student learning (Fuller, 2007; Handley, Sturdy, Fitcham & Clarke, 2006; Hendricks, 2001). The concept of learning as a social activity allows for the understanding of the moment to moment interactions among students and how these interactions impact
learning and the learners (Boylan, 2010). Situating the interactions among students offers a context for students to engage in authentic learning that is easily transferred to knowledge needed in their lives (Fuller, 2007; Handley, Sturdy, Fitcham & Clarke, 2006; Hendricks, 2001). Learning is optimized when students become partners with other students and their teachers. The transformational changes to student and teacher roles and expectations are critical for enabling student to engage freely to make conjectures, ask questions, and challenge ideas with others in the community.

However, there has been criticism of using the situated learning theory as a lens to study learning in school settings. A lack of a historical perspective, a defined model and/or concepts explaining learning are among the reasons been cited as reasons preventing the viability of this perspective on learning in schools (Engeström, 2007; Hughes, 2007).

According to some (Boylan, 2010) the situated learning theory was originally developed in informal environments and therefore cannot be applied to the more formal environments such as school. Traditionally learning is too restrictive for a situated perspective. Students learn a predetermined set of skills taught to them by their teacher with very little interaction (Boylan & Povey, 2009). Traditional learning, especially in mathematics, exposes students to decontextualized learning objectives which is not an appropriate environment for a situated learning perspective (Boylan, 2010). However, classrooms that are reformed based are conducive to this perspective.

Additionally, critics (Adler, 1998, Lemke, 1997) suggest that learning in classrooms cannot not be viewed using an apprenticeship model because a community does not exist in the classroom. Learning in schools is structured with a teacher working with a large group of students. This structure does not allow students to have tailored
learning experiences to expand their knowledge. Classrooms where students are learning from other experts and acting as experts to contribute to the learning in the community is in line with this perspective.

Additionally, critics question the use of the situated lens because students are not learning to become master teachers, they are simply learning to master a skill that they may or may not use outside the classroom (Adler, 1998, Lemke, 1997). This thinking is attributed to the idea that students do not have the same prospect of coming to share the identical social position and relation to practice as the teacher nor will they eventually engage in the identical practice as their teacher. However, building knowledge year after year will eventually lead to mastery as students move through the grades.

Unfortunately, there are two areas lacking attention in the Lave and Wenger research, these involve peer learning and relationships among participants. Although they recognize the importance of peer to peer or near peer relationships their lack of explanation of these relationships leaves this phenomena unexplained in their research. Additionally, Lave and Wenger acknowledge the existence of power relationships among participants but they offer very little interpretations about them and their effect on learning (Linehan & McCarthy, 2000).

Even with these limitations, there is tremendous potential for applying the situated learning perspective to the classroom. Researchers in the educational field have offered insights for applying this perspective to learning where Lave and Wenger have not (Boylan, 2010; Carpenter, Franke, & Levi, 2003; Cobb, 2000). Boylan (2010) suggests a more flexible conceptualization of participation as central to learning, especially as it applies to learning in school. It is through the patterns of interaction and discourse
created in the classroom that students develop ways of thinking and learning with and from one another.

Cobb and colleagues provide an example of the theoretical and practical contributions of a situated perspective during a study of classroom practices (2000). In this way, situated learning allowed these researchers to abandon the false dichotomy of individual cognition versus participation in social context. In their teaching experiment designed to facilitate and investigate students’ mathematical development within the social context of a third grade classroom, these researchers documented both the development of individual students’ place value conceptions and the evolution of the communal mathematical practices in which they participated. They found that the relationship between practices used by groups of students and individual students was interchangeable and automatic. That is, students contributed to the development of practices within the classroom community; these practices, in turn, constitute the immediate context for their own learning. Others claim the importance of discussion and argumentation to learning mathematics with understanding (Carpenter, Franke, & Levi, 2003). Expressing and justifying ideas to oneself and others to prove that they are true is valuable in advancing understanding.

The sociocultural perspective associated with situated learning is important for framing this research because it describes learning as it occurs through participation among the cultural practices in classrooms. The settings in this research are reformed classroom environments where students do have greater opportunities for getting involved in their own learning. Furthermore, the students will be engaged in opportunities to investigate, plan and discuss their mathematical solutions with other students and
teachers. They will be encouraged to think, plan and discuss their ideas in a variety of ways while solving problem together. Students will learn from and with their teacher and classmates. Although the link between the situated learning perspective and schooling may not be automatic, this study will demonstrate how learning in a community of practice is a natural process that can thrive in any classroom.

In my years as a math teacher, curriculum coordinator, and professional developer, I have come to know that students make the most progress when they have significant involvement in their own learning. My understanding of how students come to know is based on the idea that individuals are active builders of knowledge rather than passive receptacles waiting to be filled (O’Connor, 1998). I also believe understanding increases when students are engaged in the learning process through experiences that encourages them to discuss and solve problems with others. Furthermore, I believe that students are most successful when they engage in activities and discussions about the mathematics that they study in the process of building understanding. Knowing how to manipulate number along with a conceptual understanding of the meaning and ideas behind the number is critical to effective mathematics learning (Boaler, 2002; Cohen, Raudenberg & Ball, 2003; Lampert, 1990; McClain & Cobb, 2001). In a classroom community designed around understanding, students are empowered to think. Having a greater emphasis on understanding, students develop a self-directed questioning disposition and become more inquisitive. They have more opportunities to develop decision-making skills that help in solving problems. Students do not wait for someone else to tell them what to do and how to do it. They ask questions, identify important information and make generalizations for themselves. Learning situated in this way,
allows teachers to know their students and understand their thinking more fully. Teachers become more informed instructors and students are more informed learners. Teachers instinctively identify student needs, recognize typical barriers and misconceptions and provide more experiences for building understanding. However, student and teacher participation, as described, does not just happen. It is dependent on the establishment and cultivation of specific ways of participating and with a lot of work and development to hone these ways so that they become habits of mind for all in the community.

**Guiding Research Questions**

The analysis of videotaped classroom events within this descriptive case study were collected to describe the discourse practices teachers and their students used as they engaged with one another while solving problems in a community of practice. The research questions guiding this study are as follow:

1. Which types of strategic discourse do teachers use to guide mathematical thinking?
2. Which types of strategic discourse are students using as they engage with peers throughout the problem-solving process?
3. According to the teacher, which types of strategic discourse are most successful? Are these similar or different from the types of strategic discourse identified by students?
4. In what ways do teachers come to understand and implement reform-based practices?
5. In what ways do teachers believe that student understanding was improved was by the use of strategic discourse used in the classroom? In what ways do
students believe that student understanding was improved by the use of strategic discourse in the classroom?

**Scope of Study**

The aim of this research study is to examine how two teachers have worked to overcome these barriers and are implemented mathematics reform practices within their instructional mathematics programs. Also, this study will investigate the current discourse practices to used by teachers to engage students in mathematical discussions to build understanding of mathematical concepts. This study will explore how teachers support students as they use and adapt to standards based curriculum built on the conceptual development of content and skills through problem solving by making connections to real world problems and interacting with others. Aspects of their methodology, the involvement of students in the learning process and planning of instruction will also be investigated. Insights emerging from the perspective of teachers and their students will also be explored.

**Significance of the Study**

The goal is to move away from students simply stating their own solution and work toward building deeper meaning with others (Yackel & Hanna, 2003). The mathematics reform movement has not had a significant or lasting impact on the practice of teachers and the learning of students throughout the country (Boylan, 2010, Kazemi & Stipek, 2001; O’Connor, 1998). Students are not developing the types of skills needed to solve a variety of problems in mathematics. To meet these challenges, the two teachers in this study teachers have made changes to utilize instruction that fulfills this demand and the requirements of the Common Core Standards and Massachusetts State Standards.
They have implemented structural changes that include providing opportunities for students to develop more autonomy and authority in the mathematics classroom (Cuban, 2013). Studying these teachers will shed more light on why the teachers have implemented changes to support their students as they engage in discourse. This will provide other teachers with more guidance to help transform learning that is consistent with reform ideals.

My research on teachers and students use of strategic discourse is innovative because it brings together an analysis of existing models of discourse (see Table 2.1) with those used by two practicing educators and their students. Linking the emerging theory to existing literature enhances the internal validity, generalizability and theoretical level of theory building in case research (Yin, 2009). Furthermore the proposed study will describe the dynamic interactions that occur between participants in the quest for mathematical understanding. Of particular interest is the process that is used by teachers and their students to initiate, sustain, and extend student interactions (Franke, Turrou & Webb, 2015; Cengiz, Kline, & Grant, 2011) This will include the discourse choices and rationale from classroom teachers and their students as they communicate with one another while solving problems together in the classroom. This is significant because of the value it places on the contributions and voice of the participants in the research process and in the school improvement process (Feagin, Orum & Sjoberg, 1991).

Additionally, peer learning has not been interpreted as significant to learning in the mathematics classroom (Boylan, 2010). However, peer learning is essential for building meaning as students work together to solve problems together (Yackel & Hanna, 2003). This study will help to fill the existing gaps in the research describing the
interactions between students as they learn together in the classroom (Davis & Smith, 2003; Martin, Towers, & Pirie, 2006). Due to the critical nature of the types of interactions when using discourse in the classroom, it seems to be a necessary component to provide this information to teachers. Focusing the research on the participants can help gain a more ground up understanding of the interactions and changes in the participants as it relates to practice and those who are engaged in the process (Boylan, 2010).

I believe this research will be helpful in identifying practices for engaging and extending students in strategic discourse with their own ideas and the ideas of others to build understanding. This research will assist teachers in first identifying the benefits of these practices and then intentionally selecting those that help students attain the intended outcomes of each lesson. I am particularly committed to sharing the results of my analysis with teachers, especially throughout my own district, in the hopes that my work will not just be a fulfillment of dissertation requirements, but will provide them with information with which they can better guide their instruction.

**Assumptions of the Study**

The main assumption is that mathematics teaching and learning is socially constructed (Lave & Wenger, 1991) and this approach supports a focus on developing understanding among students (Kazemi & Franke, 2004). Moreover, learning in a community of practice does not have a prescribed path to achievement in mathematics. This is different from the cognitive and psychological learning practices that dominate learning in our classrooms. Instruction in this way “requires the participation of people inventing and adapting customs and traditions, who learn from their efforts to develop the principals and practices for themselves” (Rogoff & Lave, 1984, p. 10). A nonlinear path
to mathematical understanding has infinite possibilities, unlike the lockstep thinking encouraged by more traditional learning, but it will pose challenges for teachers who attempt to implement one in the classroom. These challenges will include using in-the-moment decision making to select practices that challenge students to engage with ideas shared by the community. Furthermore, teachers will also need to provide opportunities for students to move beyond developing mathematical thinking toward strengthening critical thinking.

It is also assumed that teachers in this study acknowledge the importance of using the reformed practices to improve learning in their mathematics classroom (Boaler, 2002; Fuller, 2007; Handley, Sturdy, Fitcham & Clarke, 2006, Hughes, 2007; McCrone, 2005; Walshaw & Anthony, 2008). These teachers have made an effort to use four fundamental reform-based practices for building mathematical understanding problem that include problem posing, making real life connections, learning through interaction and strategic discourse. They have also cultivated an environment that promotes the sharing of ideas and mathematical authority among all members of the classroom community.

Finally, the two teachers have a strong knowledge of subject content and pedagogy and therefore it is expected that they will experience more success in developing effective experiences for improving student understanding than those without this expertise (Millet, Brown, & Askew, 2004; Gee & Clinton, 2000; Sherin, 2001).

The genre of the descriptive case study requires the acceptance of the idea that reality is conditional to the individual’s perception of reality and the changing nature of it (Merriam, 1998). To assist with defining this reality, students and teachers were given the opportunity to communicate their ideas about discourse and shed light on the
phenomenon through their participation in interviews and student survey questionnaires. It is assumed that the participants answered the questions posed for these purposes truthfully. Attention to strengthening validity of methods throughout the selection and communication of methodology is critical to effective qualitative research. To strengthen the quality and validity of the methodology conducted all instruments will be tested prior to their use in the study to increase the validity and is measuring the desired.

**Limitations of the Study**

Case studies are particularly useful in studying a process or innovation (Merriam, 1998). However, limitations do exist when applying this approach to research. One limitation of this study includes the small sample size making it hard to have representativeness or to offer results in generalizable terms. Although, generalizations are difficult to justify through the use of qualitative data, presenting data in a way that uses the case to teach us about other cases can help transform the data from the specific to the general (Guba & Lincoln, 1987).

Another limitation to this study is my role as an administrator in the same district as the participants and setting. Though not a direct supervisor of the teachers in this study, my role may have an effect on the teachers’ level of participation. This power relationship must be shared so if that any hesitation on the part of teachers to share their ideas arises, it can be acknowledged. These participants may feel more motivated to work harder to please the researcher than without this relationship. Supportive, not evaluative feedback will be offered to assist in building a non-threatening dynamic between participants and the researcher (Rossman & Rallis, 2003). Allowing the participants to share their stories empowers them to participate in ways that will enact change and
encouraging more contributions (Creswell, 2007). Furthermore, as a former math teacher and curriculum director, I must be reflexive in my interpretations (Rossman & Rallis, 2003). Throughout the entire collection of data and analysis, I will need to stay true to the data that has been collected. I cannot let my own biases as an administrator or curriculum leader cloud the authenticity of the data. Although it is “not possible to be completely free of bias,” using the participant validation technique will assist in authenticating the data and decreasing bias (Strauss & Corbin, 1998, p. 97).

**Establishing Trust**

As I review, analyze and synthesize the multi facets of all the data collected in this study, I will commit to making my decisions and biases transparent. As a mathematics educator for sixteen years, I have been focused on assisting students with developing students understanding through the co-construction of ideas an solution planning. I have been studying how teachers helps students to do this in classrooms through communication of knowledge, ideas and strategies. Throughout this investigation, I plan to take every measure to establish a high level of credibility by using procedures to establish trustworthiness during the data analysis process. The research project will include the following procedures:

1. **Triangulation:** A variety of data sources and methods of data collection will be used to allow for triangulation. All data sources will be analyzed to identify similar information. The videotapes will be viewed with students and teachers to investigate common interpretations and conflicting interpretations of data. This way the validity of a participant’s input can be cross-checked with their own responses or the responses of others. An inquiry into the rival conclusions will be
conducted (Miles & Huberman, 1994). This inquiry will involve reviewing the
interviews, observations and surveys.

2. Participant Validation: Findings will be shared with participants to clarify,
elaborate or correct information in the data (Rossman and Rallis, 2003). This will
also make the project more transparent and allowing the participants access to the
information gathered. The use of “member checking” will further assist in raising
credibility of the findings (Cresswell, 2007).

3. Clarifying Researcher Biases: Full disclosure about my biases relating to the data
will be made visible during the collection and results stages of the project
(Rossman & Rallis, 2003).

4. Audit Trail: Describing in detail data collection, categories development and
decision making during the inquiry allow for an audit trail (Merriam, 1998).
Reliability in research refers to consistency in results. Test-retest methods will be
used to increase the likelihood that the surveys will yield consistent results.
Reliability will be increased by using questions that have similar questions but
with slightly different phrasing. Correlation among items will increase
consistency and therefore raise the reliability factor.

5. Establishing Validity: Validity refers to the accuracy of a measurement. Although
this is often subjective to the researcher. To decrease this subjectivity, the surveys
used in this project will be pre-tested to assess the validity. This process will be
recorded in field notes. Providing detailed information about how the data is
collected and how decisions are made will increase the reliability (Merriam,
1998).
Organization of the Dissertation

The proposed research study will investigate discourse practices among teachers and students while in a community of practice focusing on four fundamental reform-based practices for building mathematical understanding. This practice for building understanding includes problem posing, making connections to real world problems, learning through interaction and strategic discourse. Refer to the attached diagram, see Appendix A for a graphic representation of this vision for learning. It is essential to this vision for learning mathematics that students and teachers communicate their thinking with and among members of the community of practice while they work and learn together to develop understanding. This vision is dependent on the productive and strategic discourse used by teachers and their students to engage with one another while using the other reformed based practices. This is the piece that ties the others together.

Although talk between students and teachers has increased in many classrooms, strategic discourse that is planned and purposeful is rare. Strategic discourse maps out an intentional instructional path for studying particular ideas and concepts during discussion among teachers and students in math class. Teachers who use strategize their discourse encourage students to engage in certain distinct ways including; discussing their own thinking, questioning themselves and others, seek clarification and offer justifications while learning in the classroom community.

In order for this vision for mathematics to thrive, an environment that values shared thinking and communication of ideas and expertise should be cultivated. Problem posing, making connections to real world problems, learning through interaction and strategic discourse are essential for learning mathematics with understanding, as I will
demonstrate in the next section.

Unlike traditional practices, using strategic discourse practices to guide the instructional program in a community of practice does not follow a script but involves being able to plan and facilitate a series of important moments that unfold concurrently. The uncertainty of this can be difficult terrain for teachers to navigate. Utilizing the four reformed practices as outlined below can be used to lead teachers more easily through the unchartered waters.

The remaining chapters will be divided into four chapters. The second chapter contained a literature review of the research on elementary mathematics reform, situated learning in a community of practice, reform-based instructional practices, strategic discourse and professional learning. The third chapter presents a framework used to ground the data collection methods and analysis used in this study. Chapters four and five present the results of each of the two case studies. Chapter six is a cross-case analysis. The final chapter contains a discussion of key findings and conclusion about the implications and recommendations of this analysis.
CHAPTER 2
LITERATURE REVIEW

This chapter provides a review of the literature that identifies some of the contributing factors that the impact of the traditional notions of schooling have on the teaching practices of teachers as they relate to the implementation of strategic discourse while solving problems with their students. Additionally an analytic overview of the historical and contemporary thinking regarding reform-based instruction as they relate to learning mathematics with meaning and understanding in a community of practice. The first section focuses on mathematics reform and the barriers for implementing reform-based practices. Section two looks at the viability of applying the situated learning theory as a viable option for teaching and learning mathematics for understanding in a community of practice. The third section will present four tenets of mathematics reform and the need for improving mathematics teaching and learning. The three reform practices problem posing, active learning with authenticity and learning through interaction will be introduced. The review of the literature addressing strategic discourse practices, including key ideas and the structural changes needed to build a partnership among teachers and students will follow. Additionally, the literature presented in this chapter will also include the discussion of factors contributing to the lack of authority among and between students and teachers. This discussion includes the lack of influence teachers have in deciding the types of methods that are beneficial to learning, as well as the lack of authority students have maintained in the learning environment. Here the argument will be made that studies are needed to help know and understand the types of
strategic discourse used to engage students with one another and their ideas while learning mathematics in a community of learning built on respect and shared authority.

**Elementary Mathematics Reform Movement**

**Traditionalists**

On the one side, the traditionalists criticize the reform methodology for being fuzzy, hands-on, fun math, which lacks skill development and rigor (Askey, 2001; Cheney, 1997; Hirsch, 1996; Jennings, 1997). Most are outraged by the decrease of computational skills proposed by reformers and the National Council of Teachers of Mathematics [NCTM] (Klein, 2002; Loveless, 1997; Loveless & Coughlan, 2004). Cheney (1997) condemns the reformed learning practices for focusing less on the correct answers and more on “having a good rationale for a wrong one” (p. A22). Loveless and Coughlan (2004) argue that students must master number operations before having the opportunity to learn higher-level mathematics, which is not the case in the reformed classrooms. Further adding to the firestorm, mathematicians and policy makers have tried to block reform-based textbook programs aligned with the NCTM standards as a demonstration of their opposition to the movement (Klein, 2002). Many believe that instituting federal policy could be the path to success (Cobb & Jackson, 2011). Also, parents, such as those affiliated with the Mathematically Correct organization, are vocal about their distrust in the changes, especially those concerning the decrease of technical skills recommended by the NCTM standards document (Loveless, 1997). Furthermore, mathematicians have condemned the NCTM for failing to gather input from those working in the field or other successful international mathematics programs as reasons
for the organization’s so-called “botched” attempt to revolutionize mathematics education (Askey, 2001).

**Progressives**

On the other side of the debate, progressives argue against a mathematics education program that is built on teacher-dominated practices, including the “drill and kill” methodology associated with the “back to basics” movement (O’Brien, 1999, Schoenfeld, 1985; Stigler & Hiebert, 1999). Also, progressive reformers criticize traditional mathematics grounded in behaviorist theory because they believe that students spend entirely too much of their time “learning terms and practicing procedures” and too little time actually thinking (Stigler & Hiebert, 1999, p. 39). In fact, this practice has been dubbed “parrot math” because of the emphasis it has on the non-thinking transfer of information from teacher to student that occurs while learning mathematics in the classroom (O’Brien, 1999).

Instead, reformers believe that students should be allowed to discover how and why mathematical procedures are applied and to understand how and when to use the skills they have learned (Lampert, 1990, Stigler & Hiebert, 1999). They disagree with conservatives about the need for students to master number operations before experiencing authentic mathematical problem-solving activities (Ball, 1993; Schoenfeld, 1992). They argue that using methods that encourage students to design, defend, and discuss their ideas while learning mathematics have significant benefits over traditional methods in strengthening their knowledge of mathematics (Boaler, 2002; Cobb, Boufi, McClain, & Whitenack, 1997; Lampert, 1990; McCrone, 2005). Furthermore, reformers suggest that using problem-solving activities that require an increased level of
participation and communication helps students learn while providing teachers with more effective opportunities to evaluate students’ knowledge than the traditional assessments (Cobb, Stephan, McClain, & Gravemeijer, 2001; Lampert, 1990; McClain & Cobb, 2001). A more thorough description of this research will be presented later in this document.

**Historical Perspective**

The reform movement took a big leap forward when the NCTM, backed by reformers and the National Science Foundation, published and presented their standards documents as the agenda for improving mathematics education (2000). This long-awaited and highly praised vision for school mathematics recommended a shift in content from learning skills before solving problems to developing skills while engaged in problem solving, a shift in teaching from disseminating information to stimulating student thinking, and a shift in assessment from end-of-unit tests to diagnosing students’ strengths and weaknesses (Leinwand & Burrill, 2001; NCTM, 2000). This document advised teachers to increase experiences to raise their students’ level of mathematical understanding in an environment that was less restrictive and more interactive (Ball, 1993; Boaler, 1999; Cobb et al., 2001; Hiebert & Wearne, 1993; Lampert, 1990). The progressive mathematics educational community believed that they were moving in a new and positive direction. However, by the turn of the century, the movement was being jeopardized because of faltering national test scores and the traditionalist backlash against textbook programs and new progressive state math standards and programs (Hirsch, 1996; Klein, 2002; Loveless & Coughlan, 2004).
Role of Assessment

The standardized test scores in mathematics among our students have generated much negative attention throughout the country. In 2003, U.S. performance was reported as below average for both low and high skill levels for item difficulty in comparison with 12 other countries participating in the Trends in International Mathematics and Science Study [TIMMS] and Program of International Student Assessment [PISA] tests (Ginsburg, Cooke, Leinwand, Noel, & Pollock, 2005). These deficiencies have been attributed to a lack of curriculum coherence and an overabundance of topics being covered by teachers during a school year (Schmidt, McKnight, & Raizen, 1997). This problem was referred to as the “splintered vision” because of the tendency of American mathematics programs to emphasize out-of-context, disconnected, low-level thinking methods (Schmidt et al., 1997). Also, the lack of a uniform curriculum, limited instruction time in particular topics, and weaker teacher content knowledge were all believed to have contributed to the poor outcomes (Ginsburg et al., 2005). In addition, the results of the National Assessment of Educational Progress [NAEP] were not encouraging either. Most notably, the NAEP report called the “Nation’s Report Card” reported that only 27% of American eighth graders could correctly shade one-third of a rectangle and 45% were unable to solve a word problem that required dividing fractions. Even though our students’ scores have risen steadily during recent years on the main math test, the long-term trend NAEP indicates weaknesses in our students’ computational skills. The results are a major concern and require increased attention to make the proper improvements to strengthen our students’ understanding of mathematics.
Back to Basics Movement

Complicating this issue is the movement by states such as California to denounce their reformed mathematics frameworks and revert back to a more traditional, content-specific set of standards (Loveless & Diperna, 2000). This decision was the result of the intense pressure from the media, parent organizations, and a newly emerging conservative political climate determined to raise test scores in mathematics (Cheney, 1997; Lambdin & Walcott, 2007; Loveless, 1997).

Even those who were initially supportive of the federal legislation [NCLB] now disagree with the promising legislation because it failed to raise academic standards and led teachers to water down mathematics curriculum (Ravitch, 2000). This occurred because the original goals set by the policy were unachievable for students in the states and districts that were targeted by the legislation. Additionally, the schools identified as failing did not have the resources or pedagogical training needed to help close their students’ tremendous gaps in learning.

Common Core Standards

The National Governor’s Association Center for Best Practices (NGA Center) and the Council of Chief State School Officers (CCSSO) launched another standards movement entitled the Common Core Standards Initiative (2010). These standards were developed to improve the quality of the existing standards found in many states across the country. According to the governors, the standards provide a more focused and consistent framework for preparing students for college and the workforce. They define what students should understand and be able to do within their K-12 educational experience. The standards focus on procedural skill, as well as, conceptual
understanding. Related standards are grouped into clusters that are grouped into mathematical domains. The Common Core Standards (2010) includes eight learning principles are included in the mathematics portion of the standards to insure that quality experiences are available to all students:

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning. (pp. 6-8)

The standards do not state specific curriculum or teaching methods. The details of how these practices are connected to the mathematics content are left up to the individual states and districts. According to the authors of the Common Core Standards Initiative, the learning opportunities should continue to vary across schools and educators should make every effort to meet the needs of individual students in their own classrooms.

The standards initially received accolades throughout the educational community. According to researchers, the standards offer well-written and deliberate goals that include the learning progressions that identify central mathematical ideas that should develop among students over time (Cobb & Jackson, 2011).
Additionally, a group of prominent educators, researchers, policymakers, and scholars supported by the Albert Shanker Institute detailed the advantages of a national curriculum in a manifesto they created entitled called A Call for Common Content (2011). Most importantly, the authors praised the standards for encouraging states to create more equitable environments then currently exist throughout our nation. They believe that a national adoption of the standards would significantly improve the learning situation for students because high expectations and high quality materials would be instituted across the board. Furthermore, the manifesto promoted the standards as a resource that would assist teachers with gaining a more sophisticated knowledge of how children should progress mathematically at each grade level. According to the scholars, this would decrease the learning gaps and curricular repetition that occur nationwide. A nationalized curriculum would also allow teachers to share expertise and resources across states. Furthermore the implementation of standards would allow teachers and parents feel more confident that their students were getting the experiences they needed to excel in college and beyond during their public school education.

However, not all of the reaction to the national standards has been positive. Shortly after this manifesto was released a group of educational reformers and professors represented a wide variety of political and educational views created their own manifesto called “Closing the Door on Innovation” to voice their opposition of a nationalized curriculum (2011). In their manifesto, they expressed a skepticism about the over involvement of the government in public education and questioned the need for nationalized standards. Unlike the authors of A Call for Common Content (2011), they worry about the government getting too involved in the process, which might eventually
lead to making policy decisions and deals that compromise student learning. They also anticipate teachers being expected to use the materials only offered by the government, which could lead to a greater focus on rote memorization, superficial coverage and ultimately a lack of innovation. The group also opposes a nationalized curriculum citing that research has not established a link between increased academic performance and national standards. Additionally, one curricular approach for each subject or for particular students had not even been identified which according to them cast serious doubts about whether or not the effort is meaningful.

Both sides agree that the implementation will be time laden and complex. First, assessments aligned with the standards needed to be created. Next, informing teachers about what students will be expected to have learned so that they can assist them to do so. According to the Common Core State Standards Initiative, formal assessments will be aligned with the the benchmarks outlined in the standards. Responsibility for making decisions about which assessments they use will be given to the states. Some states planned to create their own assessments and others planned to use a universal assessment system. In the meantime, states needed to learn the standards given because they are not aligned with the existing state standards (Porter et al., 2011). Also, professional development would be needed to support the necessary curriculum modifications, pedagogical changes, and assessment revisions (Kober & Rentmer, 2011). All of these changes will require additional time and financial support from states.

**Situated Learning in a Community of Practice**

Clearly the school mathematics reform movement is still in a state of flux, as a path for leading mathematics toward competence and understanding still seems to be
elusive. Frustrating for both sides of the debate is the limited reliable and empirical evidence to support the use of either traditional or progressive learning practices to strengthen students’ mathematics understanding and achievement (Kohn, 1999; Loveless, 1997; O’Brien, 1999). So the question is, can the educational community stop the finger pointing and listen to one another? This is possible if they retreat from taking sides and commit to identifying the best practices drawn from both ideologies so that educators and their students can move forward under a more collective and effective leadership.

Surprisingly, a group of well-respected researchers from both sides have already begun this process. These researchers support connecting the knowledge of basic skills [traditional] with in-depth conceptual understanding [reformed] as a means to developing an effective mathematics program for students (Askey, 2001; Battista, 1999; Gamoran, 2001; Ravitch, 2010; Wu, 1999).

One positive outcome of the bitter debate is that it has pressed reformers to strengthen their arguments for developing new methodologies by gathering data to support the changes they view as necessary to identifying a more effective program for learning mathematics with meaning and understanding in all classrooms throughout our country (Gamoran, 2001; McCrone, 2005). More details on these undertakings appear later in this document.

As a final point, although some support of the attempt to unite the traditional movement with the reform movement has emerged, much more work still needs to be done to determine which aspects of these ideologies are successful and to what extent and why.
Conceptual Framework

One particular body of educational research that has demonstrated positive results for improving mathematics learning involves a social approach to learning. Specifically, situated learning theory has been promoted as ideal for transforming how students learn and interact while studying mathematics in the classroom (Boaler, 1999; Cobb et al., 2001; Greeno, 1998). Situated learning supports developing knowledge through a combination of environmental, social, and psychological elements as a more complete method for achieving the intended learning outcomes for students. More specifically, the situated learning theory identifies understanding as being constructed within a “community of practice” by students learning skills in context alongside their teacher and peers (Lave & Wenger, 1991). The foremost characteristic of the situated learning theory is that learning takes place in the same context in which it is applied. Learning is more sophisticated than simply acquiring skills or completing daily tasks as individual learners, occurring through a process in which students interact with one another to strengthen thinking skills.

This differs from traditional learning environments where student learning is individual, abstract, and out of context. Students do not merely observe the teacher modeling a process that they will imitate independently, but practice the skills with support from others with more expertise in the community (Lave & Wenger, 1991).

For these reasons, the situated learning lens may be a more viable method for helping students to develop the thinking skills they need to achieve greater understanding and mastery in mathematics within their existing classroom settings (Boaler, 1999; Cobb et al., 2001; Greeno, 1998). It may also fill an existing void for educators by helping
them to gain a better sense of why and how learning through classroom communities will enable them to help their students find deeper meaning and understanding in mathematics.

The situated learning theory developed by Lave and Wenger (1991) is based on the view that educational practice resides naturally in the interaction among students within a participant framework. Optimum learning, in this view, does not occur in the individual’s mind or actions, as traditionally believed, but in a process where students pursue meaning together. The vehicle for learning in the situated learning theory is the “community of practice” [CoP]. Within the CoP, learners or “newcomers” work alongside expert community members to study the skills and learn the culture of their joint practice (1991, p. 29). This learning process is referred to as “legitimate peripheral participation” [LLP] (1991, p. 29). According to LLP, participants deepen their knowledge as they become submerged in the task and develop connections with the other members of the community. Lave and Wenger studied these habits, interactions, and learning practices in five apprenticeship groups: midwives in the Yucatan, Vai and Gola tailors, naval quartermasters, meat cutters, and members of Alcoholics Anonymous. They concluded that all groups developed the skills they needed and learned more effectively because they had been part of a community of practice. The outcome of their research as described below demonstrates how developing knowledge within a community can result in powerful and robust learning among all participants.

One of Lave’s investigations (1991) included the study of 250 masters and apprentices at Tailors’ Alley in Vai and Gola, Liberia, between 1973 and 1978. The apprentices averaged five years experience working alongside masters, journeymen and
other apprentices. They were learning the process of creating inexpensive men’s trousers to be sold at market. The tailors engaged in making a variety of garments including dresses, short trousers, shirts, prayer gowns to demonstrate the complex set of ordered tasks essential to becoming a master tailor. The apprentices began with skills like sewing by hand, cutting material, using a treadle sewing machine and pressing clothes.

The skills the apprentices learned did not mirror the sequential steps for producing a garment. Apprentices actually began the process by examining the final product. This allowed them to have knowledge of the expected outcome to guide their learning by observing how the individual components of the garment came together. This fostered a higher level of engagement in the process. Apprentices became less peripheral when they determined their own readiness for completing each of the components to making a particular garment. Next, they spent time practicing the same component until it was finished quickly and competently. Finally, the apprentice created the garment entirely on his own.

The apprentices’ progress is evaluated based on the completion of the entire garment rather than on the individual components. Their knowledge was measured according to “meaning, understanding, and learning” as “relative to actual contexts, not to self-contained structures” (Lave & Wenger, 1991, p. 15). Even though individual growth was observed among the participants, Lave and Wenger were much more interested in the group’s dramatic transformation as the members developed and became full members in the community. Evaluating participant knowledge through observation and final product without a formal assessment was successful for three reasons. First, the expectation was that all learners could and would master the task. Of all of the tailor’s
apprentices observed, 85% became masters. The remaining 15% of the participants did not become masters because of other extraneous reasons not attributed to the learning process used in the study. Secondly, learning occurred because participants spent quality time learning what it was that they needed to learn and then practiced until they were able to master the tasks. Also, the multilevel curriculum was a set of landmarks instead of individual procedures, which enabled individual learners to move through the tasks at their own rate. Finally, participants demonstrated a high level of motivation for mastering their skills. According to the researchers, apprentices are aware that once they achieved mastery, they are able to enter a field with experience and competency.

Although the link between apprenticeship training and conventional schooling may not be automatic, apprenticeship learning is a natural process that can thrive in any classroom. The common understanding of the master-apprenticeship relationship usually involves apprentices learning from their masters, but the opportunities for learning in Tailors’ Alley had benefits for both groups. Masters and apprentices worked side by side and contributed to one another. The successful sociocultural practices that emerged grew out of the natural and unplanned interactions between the members of the community as they worked together to reach common goals. Lave and Wenger (1991) describe the relationship among the members of the community while learning:

For to shift as we have from the notion of an individual learner to learner to the concept of legitimate peripheral participation in communities of practice is precisely to decenter analysis of learning. To take a decentered view of master-apprentice relations leads to an understanding that mastery resides not in the master but in the organization of the community of practice of which the master is part. (p. .94)

Lave and Wenger’s research demonstrates the potential of solving authentic problems with students. The tailors first observed the different ways to complete the task
with the assistance of their peers and a master. Gradually they developed the ability to complete the task on their own. The authenticity of the tasks and the support they received from the community created a productive learning environment. Teachers can provide similar experiences for students to solve authentic tasks that allow them to make connections with the mathematics they use in their own lives (Boaler, 1999; Cobb et al., 2001; Greeno, 1997).

This idea becomes more transparent in Lave’s (1988) study of housewives’ understanding of mathematics. The participants [housewives] in the study were unable to successfully compute mathematically in their classroom, yet they were able to do the same calculations successfully while in a more authentic situation [supermarket]. This research demonstrates the potential for helping students understand difficult concepts and procedures through making connections to real-life situations. Although the limitations of schooling can make it difficult to offer students the opportunity to perform tasks in actual contexts [supermarkets], teachers can use simulated experiences and make direct connections for students as often as possible.

Many of the skills used by the participants in these studies are similar to the skills our students are expected to master in schools. These learning experiences require participants to apply higher-order thinking skills, gain experiences with ill-defined and authentic tasks, and develop inferential reasoning, metacognitive and communication skills (Lave & Wenger, 1991). These experiences are supported by current research in mathematics education as avenues for learning mathematics with meaning and understanding (Cobb et al., 2001; Lampert, 1990; McClain & Cobb, 2001). Furthermore, learning in the community of practice described previously does not have a prescribed
path to excelling in mathematics. This is different from the cognitive and psychological learning practices that dominate our classroom learning. Although this nonlinear path to mathematical understanding has infinite possibilities, unlike the lockstep thinking encouraged by traditional education, it will pose challenges to teachers who attempt to implement it in their classrooms.

In addition, the members of the communities studied by Lave and Wenger (1991) are not just focused on getting tasks done, but they also concentrated on relating to each other as people and attempting to “resolve inevitable conflicts in ways that maintain the relationship” (Rogoff & Lave, 1984, p. 10). The bond that developed between the members as a result of working together with a common purpose was deep and meaningful and contributed to the success of the community. Establishing a similar working relationship would be essential to using these practices in mathematics classrooms.

As mentioned earlier, the situated learning theory may be a viable solution for improving the learning of elementary mathematics. Some research highlights the strengths of the elements of the situated learning theory in the classroom (Boaler, 1999; Cobb et al., 2001; Greeno, 1997; Lave & Wenger, 1991).

**Situated Learning: Example One**

The first example is a three-year longitudinal case study involving nine 11-year-old mathematics students at two British schools (Boaler, 1999). The 200 students from Amber Hill used a traditional textbook-driven transmission method, while the other 110 students from Phoenix Park used an open, project-based methodology. Both schools were well matched in that they had similar academic and socioeconomic profiles;
however, their methodologies and environments were entirely different. Phoenix Park students learned to choose, adapt, and apply methods to various problem-solving situations. Similar to the behaviors of the participants in Lave and Wenger’s (1991) study, these students interacted with multiple resources, developed their own ideas, and discussed solutions with other members of the learning community. They were encouraged to identify their own learning paths and then seek out the support of fellow classmates to test and improve their solutions. According to Boaler, the students succeeded because the methods learned in school were similar to those that were used in the real world. The students from Amber Hill were provided with opportunities that were teacher directed, individualistic, procedural, and abstract. This resulted in an inability of the students to think mathematically in some situations and to apply their knowledge in a variety of ways. These deficiencies were likely due to the lack of experience with tasks that required formulating solutions, changing and adapting methods, and discussing content with other students, like those engaged in by Phoenix Park students (Boaler, 2000). However, the students at Amber Hill were not inferior to the students at Phoenix Park because they also learned a great deal of math. The Amber Hill students acquired the skills expected of them by their teachers. Unfortunately, however, the learning tasks that they were expected to complete did not provide them with an opportunity to learn mathematics with understanding. The mathematics they experienced included interpreting textbook cues and engaging in computational practice. This distinction is made to demonstrate the influence that the learning environment has on what and how students learn and what it means to be successful in mathematics. This has serious implications for the future of mathematics education because students can be held
accountable only for meeting the expectations set for them by their teachers and the environment in which they work. Unfortunately, in some classrooms, these expectations do not provide students with adequate opportunities to improve their mathematical development and can have a negative impact on their achievement and attitudes.

**Example Two**

The second contribution is an example of how learning can be enhanced by providing students with the opportunity to co-develop the mathematical practices they use while working collectively to solve problems. The outcomes from this project mirror the characteristics of the situated learning theory. First, learning was measured by the skills that were developed by the group, rather than in the mastery of individual students (Lave & Wenger, 1991). Also, as students developed an effective system to guide their study, they showed a commitment to the pursuit of common goals and worked to find meaning together. The bond that developed between the members as a result of working together with a common purpose was deep and meaningful and contributed to the success of the community (Rogoff & Lave, 1984).

In this study, the first-grade students and their teacher developed sociomathematical norms to assist them in building their understanding during a multi-week study of measurement (Cobb et al., 2001). The mathematical norms or practices included determining between “different mathematical solutions, insightful mathematical solutions, efficient mathematical solutions and acceptable mathematical explanation” (p. 124). The mathematical norms were developed and modified continuously as the students’ mathematical knowledge and communication abilities improved during the project. Developing these practices is significant because the students began the project
with weak communicative abilities, especially when being asked to rely on their own judgment early in the project. Their skills improved as they worked together through the unit. By the middle of the project, students were able to use conceptual explanation rather than the calculational reasoning that was characteristic of their earlier work. They also provided backings to their warrants to justify their thinking and to add to the developing understanding of their peers. In addition, the first graders revised their thinking through reflection and reasoning after listening to and talking with peers. Having the norms in place enabled students to practice these skills and eventually communicate more appropriately as they increased their expertise and independence as learners. The researchers viewed the students’ involvement in the project as an evolving microculture that continually regenerated, rather than as a set of predetermined steps that students moved through as they reasoned and communicated about mathematics.

Building a successful community for learning similar to that used by Lave and Wenger (1991) is very sophisticated and challenging to both teachers and students. It requires flexibility and a deep grasp of the mathematics being taught that goes beyond the topics, methods, and procedures usually found within traditional school mathematics (Cobb et al., 2001). Cobb et al., viewed this study as an important example of how the emergent practice of communal classroom practices enhances learning.

**Example Three**

The third example is a middle school mathematics learning experience, which is part of the Middle School Mathematics Application [MMAP] Project (Greeno, 1997). The MMAP Project is deeply rooted in context and allows students to engage in activities supported by authentic software simulations in four domains: architecture, population
biology, cryptography, and cartography. The activities are designed to mirror the work and experiences of professionals during their own commercial practices. For example, the architecture component requires students to create a living and working space for a research team spending two years in Antarctica.

Many of the practices that students engaged in during this project also reflected the practices originating in the situated learning theory. First, students were given the freedom to create their own parameters for the amount of insulation in their walls, windows, and roofs, while remaining within the budget provided. They used quantitative reasoning to calculate the proportions, ratios, and rates essential to making the necessary logical decisions. Also, as part of the project, students were expected to confer with other students while making decisions to complete their tasks. Their progress was measured based on their achievement as a group and not as individuals. The students thrived because they used the practices of discourse and inquiry with one another to construct meaning and solve the tasks appropriately. Also, the students who participated in the MMAP Project went beyond simply acquiring skills and manipulating symbols to understanding the conceptual meaning of the ideas embedded in the tasks. They furthered their knowledge and the knowledge of their classmates by engaging in the mathematical practices of formulating questions, hypothesizing ideas, making conjectures, and sharing evidence while solving the tasks together.

The three examples present information about the ways that situated learning theory can be used to support effective mathematics instruction. Most important, the key elements of the situated learning theory: problem posing, active and authentic learning and group learning prove to be potential effective strategies that can be used to reform
teaching and learning. These reform-based practices are linked to both historical and contemporary philosophy and research. However, there is much yet to be known about impact these practices have on individual student understanding of specific math concepts and skills.

**Reform-Based Practices in Elementary Mathematics**

Traditional learning was about behaving, not itself a behaving. (Kilpatrick, 1951)

Four historical educational reformers, Dewey (1899), Friere (1992), Kilpatrick (1951), and Vygotsky (1978) spoke out against traditional practices and initiated the movement for change in American schooling. They each developed nontraditional theories concerning how children learn most effectively with meaning and understanding. Embedded in these theories are four learning practices including: problem solving through problem posing, making connections to real-life experiences through authentic activities, group learning through interaction and strategic discourse. The ideas sustaining these methods are directly connected to contemporary research that supports their use in the classroom today. Moreover, these methods are key to implementing the situated perspective to teaching and learning. Although the four methods are described separately, it should be understood that all are interdependent and equally vital to a community of practice.

Next, the first reform-based practice Problem Posing included in the vision for mathematics as identified previously will be discussed. This practice contains effective methods for learning mathematics with understanding in a community of practice.
Problem Posing

By tradition, problem solving has been taught after all the necessary computational skills have been mastered or as the final unit in the curriculum. But historical reformers advocated against this unnecessary practice of waiting and instead advocated using problem solving throughout the curriculum as a means for teaching the technical skills in a more meaningful way (Dewey, 1899; Kilpatrick, 1951). First, Dewey encouraged educators to engage students in more intellectual work, rather than emphasizing skill-driven methodology (Wirth, 1966). He believed that teachers could successfully help students’ increase their level of understanding by encouraging them to develop a questioning attitude through the study of real life problems (Dewey, 1899). He hoped that this approach would make developing skills less mechanical and more interactive. Students would spend less time reciting to their teacher and more time communicating with each other about their experiences which would be more stimulating.

Likewise, William H. Kilpatrick (1951) creator of the “project method,” emphasized the importance of using problem solving during the learning of mathematics and not as the culmination of the yearly study. This innovative learning concept divided the curriculum into multidisciplinary units or projects for students to complete alone or in small groups throughout the year. His innovative learning concepts, which were student selected, can be found in child-centered classrooms throughout the country. For example, Kilpatrick’s methods have led to students having more experiences in developing as self directed learners by identifying and planning their own topics of study and involvement in self directed learning experiences.
Also, Friere (1992) encouraged teachers to use problem-posing education to stimulate “true reflection and action upon reality” as a means of achieving a more authentic experience through “inquiry and creative transformation” (p. 65). He wanted students to take the necessary time to think about what they had learned and how they could use their learning to understand the world better.

All three reformers viewed the learning of mathematics education as improved when it contained more stimulating experiences than just learning rules and manipulating numbers. Moreover, Dewey (1902), Kilpatrick (1951), and Friere (1992) valued spending quality time working on fewer, more thought-provoking problems in greater depth as a more effective format for learning. Solving problems instead of a series of isolated tasks aligned with the situated learning theory. In this manner, students are encouraged to focus on tasks and develop mathematical skills while engaged in problem solving together. Learning in this view is more typically nonlinear and fluid, requiring students to draw upon the knowledge they have and to seek out resources to find answers when needed.

Recently researchers have reported the benefits of using problem posing as a method for helping students study mathematics in a way that requires them to think and make connections with ideas and concepts (Cobb et al., 2001; Hmelo-Silver, 2004; Lampert, 1990; McClain & Cobb, 2001; Yackel & Cobb, 1996).

**Contemporary Connections to Problem Posing**

Engaging students in the reflection process before, during and after the problem posing process provides teachers with the opportunity to identify gaps and to evaluate how students have transferred their knowledge to new situations (Cobb et al., 2001;
Hmelo-Silver, 2004; Lampert, 1990). Lampert (1990) used problem posing along with discussion to develop mathematical understanding. Her work with students demonstrated the emphasis she placed on developing mathematical knowledge during the process of solving problems together in the classroom. In the sample conversation below, Lampert in her role as teacher-researcher begins the session by introducing a new problem to her students (1986). This is followed by a small group session where students generate ideas and possible solutions. The culminating activity included a full class discussion where strategies, ideas, and misconceptions are shared. The following example is taken from a problem posing session conducted with students to describe the types of discourse she uses daily with her students. The reflective comments offered by Lampert, as found in the article, are written in italics and have been included to help interpret each section of dialogue (pp. 322-325).

Teacher: Can anyone give me a story that could go with this multiplication expression…12 X 4?

Jessica: There were 12 jars, and each had 4 butterflies in it.

Teacher: And if I did this multiplication and found the answer, what would I know about those jars and butterflies?

Jessica: You’d know you had that many butterflies altogether.

_Jessica has constructed a way of giving meaning to the operation 12 X 4. The next step in the lesson is to illustrate Jessica’s story and to construct a legitimate procedure for counting large numbers of objects arranged in groups by taking the groups apart and putting them together. The procedure is constructed as a joint endeavor by the teacher and students, drawing on actions that make intuitive sense to the students._

Teacher: Okay, here are the jars. The stars in them will stand for butterflies. Now, it will be easier for us to count how many butterflies there are altogether.
if we think of the jars in groups. And as usual, the mathematicians’ favorite number for thinking in groups is (draws a loop around 10 jars).

Sally: 10

Here I am using the language of “loops” to represent the principle of decomposition. The picture shows that the total number of jars and butterflies stays the same, but for the purpose of counting the contents, they can be decomposed into a group of 10 and a group of 2.

Teacher: Each of these 10 jars has 4 butterflies in it, so how many butterflies are inside this circle?

John: It’s 4 X 10.

Teacher: I add 10 jars and 2 jars and I get 12 jars. Each jar has 4 butterflies in it (points to the two 4s in 4 X 10 and 4 X 2). So how many butterflies are there altogether?

Chorus: 48.

In the final step, the two groups of jars were “recomposed,” and the distributive law was illustrated by adding together the total number of butterflies in each group of jars. Even though we have arrived at the “answer” at this point, I continue with the lesson, analyzing the procedure we used and verbalizing its structure.

Teacher: I added the 10 and the 2 to get 12 jars. Should I also add the 4 and the 4 to get 8 butterflies?

Shawn: No. There are just 4 butterflies in each jar. That will never change.

This is usually the glitch for children in what mathematicians call the “distributive law”; as a principle, it is obviously warranted when it is attached to quantities in stories like this one. But when students see only (4 X 10) + (4 X 2), it is hard to explain why the answer is not obtained by doing 8 X 20. The next part of the lesson is intended to get at the idea of finding another grouping to illustrate that the principle of
decomposition and the distributive law are legitimate to use on groups other that those determined by place value (pp. 322-325).

Teacher: Suppose I erase my circle and go back to looking at the 12 jars again altogether. Is there any other way I could group them to make it easier for us to count all of the butterflies?

Jean: You could do 6 and 6.

Teacher: Now, how many do I have in this group?

Steve: 24

Teacher: How did you figure that out?

Steve: 8 and 8 and 8. (He put the 6 jars together, 6 into 3 pairs, intuitively finding a grouping that made the figuring easier for him.)

Teacher: That’s 3 X 8. It’s also 6 X 4. Now, how many are in this group?

Jean: 24. It’s the same. They both have 6 jars.

Teacher: And now how many are there altogether?

Patty: 24 and 24 is 48.

Teacher: Do we get the same number of butterflies as before? Why?

Patty: Yeah, because we have the same number of jars, and they still have 4 butterflies in each.

In her last statement, Patty uses her intuitive understanding of the story context to make a statement about what is mathematically legal procedure. I asked several other children to explain in their own words why there was the same total number of butterflies each time. It was clear from watching and listening to them that some children were surprised that it came out the same, which was a cue to do lots more of these different kinds of groupings.
The discourse that accompanies problem posing requires students to provide strategies and offer a rationale about the problems they solved. Students did not simply report what they did to solve the problem. As evidenced in the dialogue above, students are encouraged to talk about their responses and determine whether or not they are viable. A priority is placed on assisting students to make connections between symbols and operations and their effect on quantities. Also, Lampert demonstrates the appropriate types of talk she wants her students to use and scaffolds their responses to help make the discussion flow. She uses a type of discourse technique called “stepping in and stepping out.” “Stepping in and stepping out” discourse is so effective because it allows the teacher to interact with students in different ways, depending on the topic and the flow of the discussion. At times the teacher is part of the discussion, asking questions and engaging students, and at other times he or she is listening and observing. The technique is used to redirect learning, spotlight thinking, or discuss how the collaborative process is working.

Although most of the problem-posing literature is part of a larger body of research within the medical field, there is valuable research involving the benefits of using the problem-posing approach with elementary school students (Barrows, 2000; Hmelo-Silver & Lin, 2000). Research shows that problem posing can help students establish a system for interacting with others and increase their opportunities to share thinking and contribute to the collective understanding of the learning community (Cobb et al., 2001; Hmelo-Silver, 2002; McClain & Cobb, 2001).

Providing a forum for communicating about the problems they solve together in the classroom can be beneficial to students. One group of second graders from a large
suburban school district demonstrated their ability to do so by developing a set of sociomathematical norms along with their teacher and researchers, which guided their interaction and study during one school year (McClain & Cobb, 2001). As students began to talk about their thinking and interpret their reasoning to their teacher and classmates, certain behaviors or techniques used by the teacher and students contributed to the process. These behaviors and techniques, which eventually became known as sociomathematical norms, provided a format to be used to guide all discussions (McClain & Cobb, 2001; Yackel & Cobb, 1996). Over time, the sociomathematical norms helped govern a collected acceptance of what counted as a different, sophisticated, and efficient mathematical solution and were agreed upon by the entire group of students and their teacher. The sociomathematical norms listed below are those that were developed and agreed upon by the students and their teachers as a system for discussing mathematical ideas and solutions:

1. Students are expected to explain their thinking and justify their reasoning mathematically by offering a rationale about the decisions they made while solving the problem.

2. As students discuss their solutions, the teacher helps to explain the student’s thinking as a way to clarify the individual’s thinking for other students. This includes assisting students to rework an invalid explanation so as to avoid embarrassment.

3. Every student is expected to listen to and attempt to understand their classmates’ explanations.
4. To further reinforce the mathematical skills and ideas within each student’s solution, the teacher draws attention to the symbols and notations by writing them on the board for all to view. In some cases the teacher will help the student create the corresponding notation for their explanation if one was not presented.

5. Students are required to indicate nonunderstanding of solutions and to pose clarifying questions to the student explaining the problem.

6. If students disagree with another students’ explanation they must justify why they did not accept the solution as valid. (McClain & Cobb, 2001)

The purpose of the 12-week instructional unit was to develop numerical relationships that would become automatic and “ready-at-hand” thinking strategies for students to draw on as needed while solving problems. During this project, students were studying patterning specifically by using mental partitioning and recomposing of objects to develop more flexible thinking (McClain & Cobb, 2001). To do this, the students were shown an arrangement of chips on the overhead projector for three or four seconds. The teacher asked the students to determine how many chips they saw and to explain how they were arranged. The goal of the lesson was to help students develop the ability to reason about collections without counting them (see Figure 1). The top portion of the chart is the combination of chips placed on the overhead projector.
Five chips arranged on the overhead

\begin{align*}
1+4 &= 5 \\
3+2 &= 5
\end{align*}

Chips with notation

Figure 1: Chips placed on the overhead projector (McClain, & Cobb, 2001, p. 249). The bottom portion of the chart is the notation suggested by the students.

Throughout the unit, the second-grade students demonstrated their ability to reason about the patterns, their solutions, and the solutions of their peers. The teacher was vital to the success of the process by either redescribing a student’s contribution or by assisting them with making notations to help clarify an individual student’s methods so that other students could understand the reasoning more clearly. This action taken by the teacher helped students to develop the higher-order thinking skills they needed for discussing the similarities and differences within the solutions presented. This was significant because these young students took on the responsibility of judging whether or not a solution was different and the ways in which it was different, which increased their
autonomy as learners. This is the crate students were shown and asked to fill with ten pumpkins (a) and the second is an example of one student’s solution (b).

One example of the second-grade students’ discussions is pulled from a dialogue involving pumpkins in a crate. The teacher placed a ten-frame with five chips arranged in rows of three and two on the overhead projector and asked students to determine how many pumpkins it would take to fill the entire crate (see Figure 2).

![Figure 2: The crate used by students to fill pumpkins (McClain, & Cobb, 2001, p. 254).](image)

The discussion of their individual strategies and rationale is below. The first student to respond was Kitty. She explained that she “saw five as a group of two and a group of three” and continued by saying that it would take “five more to fill the crate since five plus five is ten” (p. 255). The dialogue continues below with Dan:

Dan: Um, the way I saw was, I saw four things and another one and I know… Okay, five plus five makes ten…

Teacher: Okay [notates as shown in Figure 2].

Dan: I had the same theory as Kitty, but I did it in a different way. (p. 255)
This dialogue demonstrates Dan’s ability to reason through the similarities and differences that exist between the two solutions. Along with Kitty, he thought that it would take five more pumpkins to fill the crate, but then decided that the combination of four and one would also work. In addition, the dialogue demonstrates the sociomathematical norms established in the learning community. Dan offered a different solution and explaining his rationale to support his thinking for the group. This research provides evidence that students, even young students, can engage in productive discussions while solving problems in the mathematics classroom.

Problem posing also gives students the opportunity to develop their mathematical ideas by talking about their understanding of concepts and solutions in small groups and with the whole classroom community (Cobb et al., 2001). This thinking-aloud process contributes to their understanding and the understanding of their classmates. The sample dialogue is part of an investigation involving 16 first graders and their teacher as they discuss and solve a set of problems during a unit on linear measurement. All of the problems were posed through a narrative about a king who measured items in his kingdom by pacing heel to toe. Each lesson was planned so that every new problem was an extension of the last. The goal was to assess “whether or not the activity of measuring by iterating a tool along the physical extend of an object would come to signify the accumulation of distance for the first graders” (p. 132). The problems in the unit were designed by the researchers to help these young students engage in conceptual discussions about the methods they were using to measure objects in their environment.

A section of the dialogue from the study has been chosen to illustrate how students approached a problem and participated in a discussion about the length of a
white board hanging in their classroom. At the beginning of the exchange, the researcher helps a first-grade student [Megan] to make a symbolic notation by marking “10” after the first iteration and “20” after the second as Megan explained her reasoning. This notation was made to scaffold the learning and make it easier for the student and the group to reflect on and interpret the explanation.

Researcher: Where’s the 20? What does it mean?

Megan: 20 means 20 food cans.

Researcher: That means 20 food cans. How much space would that be? Can somebody show me how much space 20 cans would take up there?

Mitch: About that long [indicates the space between 10 and 20].

Nancy: No [indicates the space between 10 and 20]. [This is because Mitch indicated that he has changed his mind.]

Researcher: Oh, so it’s the whole 20.

Megan: [continues to measure with the Smurfbar, as the teacher marks each iteration] 21, 22,…30: 40: 60: 61, 62, 63, 64. (p. 141)

The researchers used a whole-class discussion format to discuss the students’ solutions so that they could ensure that students were able to justify their thinking, which was not occurring during their small-group problem-solving experiences. Encouraging students to explain and justify their thinking during the problem-solving process allows educators to uncover weaknesses and helps students to develop reasoning about the topic. The dialogue above demonstrates Nancy’s successful understanding of measuring as the accumulation of distance while using the “smurfbar” tool (p. 138). Helping Nancy to work through her thinking helped increase her understanding and that of her classmates. The smurfbar tool mentioned in the dialogue was part of a narrative that the teacher invented to show students how the Smurfs measured with everyday objects. In the
narrative, Smurfs placed empty food cans end to end to measure the objects. Likewise, the students solved the same problems using Unifix cubes to represent the cans.

This episode also demonstrated that measuring as the accumulation of distance was not taken as shared for every student in the class. The group viewed Nancy’s explanation as legitimate, while Mitch’s was treated as illegitimate. Even though his thinking was incorrect, Mitch makes a valuable contribution to the discussion by directing attention to an ineffective way of reasoning that others may have also considered. The teacher in this example was able to dissect the thinking of her students during the problem-posing session and redirect their understanding. Mitch’s statements and the fact that he had changed his mind after hearing the reasoning of his peers was an excellent example of how learning evolves when using this method correctly.

The students in both of these examples required some support from their teachers for productive dialogue to occur. Again, the process of negotiating with members of the community to find meaning in the thinking of one another is an important ingredient when learning in the situated perspective (Lave & Wenger, 1991). However, older and more established students have experienced greater success with independently assessing thinking and behaviors (Hmelo-Silver & Barrows, 2002).

Elementary students are young and inexperienced. They have not had years of developing skills as successful independent learners. Although the students did show marked improvement in explaining and justifying mathematical solutions, they needed their instructor to scaffold the process so that they could make the transition from dependence to independence during the communication process (Cobb et al., 2001; Lampert, 1986; McClain & Cobb, 2001).
The support that the students needed raises some questions concerning the reasons why students experience difficulty when asked to explain and evaluate their mathematical thinking independently and collectively. Are elementary school students too young to engage in the process on their own? Are there certain aspects of the problem-posing process that are too difficult for some learners? What types of methods can teachers use to establish and reach success in using the process? Are there methods that teachers should put in place to help students develop their skills? Finding answers to questions such as these can provide important information for those who are interested in using this method with students. The elementary students in the two examples worked together to meet common goals, one of the tenets of the situated learning theory. Like the apprentices and masters, students were able to work together with their teachers and peers to build relationships as a means of pursuing meaning (Lave & Wenger, 1991). This is in contrast to the long-standing learning culture that encourages the individualized and competitive pursuit of knowledge in school.

Next, the second reform-based practice Active Learning with Authenticity included in the vision for mathematics as identified previously will be discussed. This practice contains effective methods for learning mathematics with understanding in a community of practice.

**Active Learning with Authenticity**

Both historical and contemporary reformers agree that students learn to solve problems more effectively by engaging in activities that require them to make connections with learning mathematics in school and the mathematics that they use in their lives (Boaler, 1999; Bracha, Zemira, & Arami, 2002; Dewey, 1899; Vygotsky,
1978; Weiss, Herbst, & Chen, 2009). Vygotsky and Dewey valued authentic learning that connected life knowledge with school knowledge because it helped students to view school learning as meaningful and useful in their everyday lives. According to Dewey (1899), schools were wasteful and failed to make any significant difference in the lives of their students because they were organized around bits and pieces approach of presenting irrelevant and disjointed skills. Vygotsky (1997) criticized schools for being disconnected with the outside world. Instead, students would benefit more from actively engaging in authentic activities that required a reaching out of the mind and that would result in greater initiative and independence than students in more traditional classrooms (Dewey, 1902; Vygotsky, 1997). This thinking inspired Dewey (1938), to develop his inquiry theory, which was contrary to other theories of his time and is still relevant in the contemporary reform-based pedagogy today. The inquiry theory focused on the students developing content knowledge during the problem solving process, rather than focusing on learning skills separately. In addition, as part of the inquiry-based study, students formulated their own hypotheses and judged the validity of their solutions based on the strength of the hypotheses, not because someone else had said that it was true or false. If the students’ ideas were not successful, they reflected upon the hypotheses with the help of the community to present a more effective hypothesis. He believed that learning in this way was more authentic and that the skills that students developed in these situations would be similar to the skills they needed to be successful citizens. Dewey’s thinking was revolutionary because in the early 1900s, learning was generally scripted and students were not given the freedom to experience formulating hypotheses as a method for solving problems. Furthermore, both Vygotsky and Dewey advocated students
developing conceptual ideas in the process of thinking as a more favorable way for students to reconstruct, extend, and enrich their learning experiences, as opposed to simply learning and following procedures that were provided to them by a teacher.

Clearly, Dewey was not a proponent of a skill-driven curriculum, but he did not advocate for the removal of skills practice in the curriculum. He opposed having to make the decision between one or the other and understood the necessity of both aspects of learning in a student’s education. Instead, he advocated for improving skill development to complement conceptual learning (Dewey, 1940). However, the vision that Dewey advocated did not come without controversy. Ironically, the controversy around this dichotomy still remains in the educational community today.

An additional controversy surrounding his methods was that educators criticized his inquiry methods by calling them “lofty” and arguing that they were too challenging to implement (Wirth, 1966). This difficulty has been attributed to Dewey’s refusal to define a pedagogical formula for educators to follow. Dewey defended his actions by explaining the need for creative professionals to develop their own action plans that could be designed to meet the needs and specifications of individual classrooms. This would allow pedagogy to be enacted with intelligence and flexibility, rather than with a mandate. Dewey envisioned instruction as a fluid and evolving process that could not be created for teachers. He believed that giving teachers the power to design their own interesting and dynamic process according to their curriculum goals and the needs of their students would be more ideal. This is a clear example of theory being introduced without a direct connection to practice.
Contemporary Connections to Active Learning with Authenticity

Unfortunately, many students across the country are not invited to actively participate in the learning process as envisioned by Dewey and Vygotsky (Hiebert, Gallimore, Garnier, Givven, Hollingsworth, 2003). However, mathematics educational reformers are working hard to change this practice (Ball & Bass, 2000; Boaler, 2002; Lampert, 1990; Weiss et al., 2009). They believe that increasing the authenticity in learning will get students more involved and therefore improve learning.

Authentic mathematics has been defined in the literature in many different ways (Weiss et al., 2009). Four of those conceptions have implications for connecting learning to students and their lives (Ball & Bass, 2000; Boaler, 1999; Lampert, 1990). First, mathematical activities should be “rooted” in real-world contexts and not “decontextualized” or learned in a “context that is contrived or artificial” (Weiss et al., 2009, p. 276). Students need to become more aware of the different ways that real people use mathematics while doing authentic activities such as building houses, carpeting rooms, and cooking. Researchers have found that students benefit more from being able to make greater connections with mathematics in school and in their lives by engaging in activities that are more useful and dynamic (Boaler, 1999).

Second, students benefit from learning mathematics with authenticity by studying mathematics in ways that are reflective of working mathematicians (Weiss et al., 2009). Specifically, learning experience characteristic of this conception include “revisiting assumptions, reformulating definitions, changing hypotheses” and using guessing, experimentation, and plausible reasoning (p. 277). Students benefit from authentic
experiences and engage in these practices successfully when they are given time to learn how while working together in a community of learners (Lampert, 1990).

The third conception identifies authentic mathematical activity as taking place within the activity of students as they discuss mathematics (Ball & Bass, 2000). In this sense, authenticity is found in the ideas and solutions generated by the students, even the ideas that are not developed or identical to those traditionally accepted in the discipline (Weiss et al., 2009). Also, authenticity is identified within the students’ interpretations and descriptions of the mathematics they are studying.

Finally, learning mathematics with authenticity means that students come to understand mathematics in ways that mirror the structure and content of mathematics as a discipline (Weiss et al., 2009). This conception is the thread that runs through the other three as teachers work to remain true to the discipline of mathematics as they teach with authenticity. This conception occurs when teachers connect the ideas that are presented either by themselves or their students with the theories, concepts, and notations found in the discipline of mathematics. Also the conception includes developing an increased use of accurate terminology and information among students. In addition, teachers must find opportunities to unearth the mysteries of mathematics by making connections among ideas with current, past and future experiences for students. Additionally, it is important to constantly honor the dynamic beauty and richness along with the complexity of mathematics as much as possible during the study of mathematics in the classroom.

**Authenticity: Rooted in Real-World Math**

Although students spend much time learning mathematics in school, they often seem to be unable to apply this knowledge to situations outside of the classroom.
Researchers attribute this deficiency to a heavy focus on learning mathematics that is out of context and lacks the development of thinking (Boaler, 1998; Schoenfeld, 1985). Still others blame the way students perceive using mathematics in school as different from how they use it in real-world situations (Boaler, 1998). For researchers, the problem lies in how students learn mathematics. According to Boaler, students who learned using an authentic, open, process-oriented environment developed a greater understanding of, and facility in, their mathematical knowledge. Again, we use the data from the two schools in Boaler’s study to demonstrate the importance of increasing authenticity during the study of mathematics.

Mathematics instruction during the first year at Amber Hill required students to fill out workbooks independently and speak only to their teacher when they needed help to independently proceed with an exercise. In year 2, the students learned by using a textbook-driven mathematics approach. The students’ experiences did not require them to discuss rules and methods or compare solutions, so they did not. Also, students did not demonstrate a desire to think about what they were learning. Lacking both of these experiences has a lot to do with their conception of mathematics as tedious and boring. These views are evident in the statements given by two students during an interview conducted during the investigation:

Neil: In maths there’s a certain formula to get to, say from A to B, and there’s no other way to get to it, or maybe there is, but you’ve got to remember the formula; you’ve got to remember it.

Louise: In maths you have to remember; in other subjects you can think about it. But in exams the questions don’t really give you clues on how to do them. (p. 46)
The lack of authenticity caused Amber Hill students to experience difficulty in transferring their knowledge and to struggle when asked to apply the procedures they learned in context during formal and informal assessments. In addition, they were unable to make connections to what they were doing in the classroom and the demands of their lives outside of school. A student [Gary] explains facing this challenge:

It’s different, and like the way it’s they’re like-not the same. It doesn’t like tell you it, the story, the question; it’s not the same as in the books, the way the teacher works it out. (p. 56)

Conversely, the students at the Phoenix Park School were encouraged to take responsibility for their own learning and become independent thinkers. They were encouraged to engage in learning methods that were project and problem-based. Unlike students from Amber Hill, these students had a positive perception of learning mathematics and viewed their learning tasks as requiring active and flexible thought. In addition, the students did not see a real difference between the math they used in school and the math they used outside. Through participation in the instructional tasks, students developed the ability to adapt their knowledge of concepts and apply them to new and unique situations. This is evident in one of the student’s [Lindsey’s] statements during an interview:

Well, if you find a rule or a method, you try and adapt it to other things. When we found this rule that worked with circles, we started to work out the percentages and then adapted it, so we just took it further and [used] different steps and tried to adapt it to new situations. (p. 58)

By participating in the types of activities offered to them at Phoenix Park, students were able to score significantly higher on all of the assessments that they were given. First, 88% of the Phoenix Park students passed the national General Certificate of Secondary Education (GCSE) exam, which was a higher percentage than the national
average, even though they had not been able to study all of the material included on the test. Only 71% of the Amber Hill students passed the GCSE exam. These results are significant because both schools performed below the national average on the national exam before the investigation began. Furthermore, the Amber Hill students did not score higher than Phoenix Hill students on the traditional closed questions, including the seven short written tests given in the study.

Table 1: Results of the Boaler (1998) Study

<table>
<thead>
<tr>
<th>Amber Hill Study Description</th>
<th>Phoenix Park Study Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Year One-Used workbooks exclusively</td>
<td>Required to take responsibility in learning</td>
</tr>
<tr>
<td>Year Two-Used textbook-driven approach</td>
<td>Encouraged to be independent learners</td>
</tr>
<tr>
<td>Discussion of methods or solutions</td>
<td>Used Project and problem based approach</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Results</th>
<th>Results</th>
</tr>
</thead>
<tbody>
<tr>
<td>Students did not demonstrate a desire to think about what they were learning</td>
<td>Students had a positive perception of mathematics</td>
</tr>
<tr>
<td>Viewed math as tedious and boring flexible</td>
<td>Viewed math as requiring active and thought</td>
</tr>
<tr>
<td>Students were unable to transfer knowledge</td>
<td>Students were able to adapt their knowledge</td>
</tr>
<tr>
<td>No connections between school mathematics and real life by students</td>
<td>Students did not see a difference between mathematics and real life</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Score high on all assessments</th>
<th>Scored higher on the applied tasks</th>
</tr>
</thead>
<tbody>
<tr>
<td>71% of students passed the GCSE exam after scoring below the national average prior to the study</td>
<td>88% passed GCSE exam (higher than national average) after scoring below the national average prior to the study</td>
</tr>
<tr>
<td>Both schools scored similarly on the GCSE exam traditional test items</td>
<td>Scored higher on the applied tasks</td>
</tr>
</tbody>
</table>

Phoenix Park students also performed better on the applied tasks developed by the researcher. Students at Phoenix Park achieved a 61% on the carpet assessment, compared with 31% of the Amber Hill students. This authentic assessment provided students with a scale plan of an apartment that showed external walls and windows, and the task included creating an apartment. Also, students were asked to find the
approximate cost of carpeting for the entire apartment. Thirty-four percent of Amber Hill students ignored the word “approximate” in the description and went on to work out the exact area of the floor space. When asked afterward to explain themselves, they said that they would not have done so in a real-life situation, but because they were in math class, they needed to show their math and work with as much accuracy as possible. This example demonstrates that their sense making was determined by the context of the math class and the “constraints and affordances” to which they had become attuned (p. 9).

Again, the researcher found it unfortunate that becoming effective in mathematics at Amber Hill did not require students to develop mathematical reasoning. Students were exposed only to learning experiences that were useful for answering textbook questions. This lack of authenticity contributed to their apparent difficulty with applying their knowledge to other situations. Table 1 reveals the list of the results from this study.

Nevertheless, not all of the Phoenix Park students’ experiences were positive. Some students from Phoenix Park had difficulty adapting to the open, self-directed format for learning, which resulted in a lower time on task for some students. Also, one-fifth of the students, mostly boys, did not like the approach, especially the freedom they were given to complete their assignments. The students were often disruptive. According to the researcher, the students felt more comfortable working in textbooks and being told what to do by their teachers. A few of the girls were very forthcoming when asked to discuss the issue:

Well, I don’t think they were stupid or anything. They just didn’t want to do the work; they didn’t want to find things out for themselves. They would have preferred it from the book – they needed to know straightaway sort of thing.

Hilary: They just couldn’t be bothered, really, to find anything out. (p. 51)
Authenticity: Investigating Math as Mathematicians

Lampert (1990) provided an authentic environment for her students to study mathematics by requiring them to investigate mathematics as mathematicians do. For instance, students learned to formulate mathematical arguments, make conjectures, and respond to input presented by other members of the classroom community. A process similar to Lakatos’s zigzag path is used to guide the conversation during their mathematical discussions. This is the same process used by mathematicians in which a hypothesis is presented, discussed, and revisited by multiple problem solvers in a joint effort to find a solution or deeper understanding. Studying mathematics in this way requires students to take a more prominent role in the classroom because the process requires their full participation. They participate by proposing their own discoveries, reflecting on the ideas presented, and deliberating about the solutions with other students and their teacher during mathematical discussions.

This process is demonstrated in the following classroom episode that focuses on the mathematical patterns involving fifth graders during an exponent unit. The students were asked to offer their hypotheses publicly and reflect on their ideas before either supporting or rejecting the assumptions, rather than simply presenting answers and moving on to the next question. The sample dialogue follows a set of introductory lessons in which students and their teacher/researcher discuss the patterns they have found within the topic. The lesson objective required students to predict or identify the last digit of the answer when squaring exponents without actually doing the calculations. Lampert assists students to develop an understanding of the “predictive power in
quantitative order” (p. 45). The following discussion occurred after the students were asked to formulate a hypothesis to the question, “What about 7 to the fifth power?”

Arthur: I think it’s going to be a 1 again.

Sarah: I think it’s 9.

Soo Wo: I think it’s going to be 7.

Sam: It is 7.

T writes: 7 to the fifth = 1? 9? 7?

Teacher: Arthur, why do you think it’s 1?

Arthur: Because 7 to the fourth ends in 1 then it times 1 again.

Gar: The answer to 7 to the fourth is 2,401. You multiply that by 7 to get the answer, so it’s 7 X 1.

Teacher: Why 9, Sarah?

Gar: Maybe they think it goes 9, 1, 9, 1, 9, 1.

Molly: I know it’s 7, ‘cause 7…

Abdul: Because 7 to the fourth ends in 1, so if you times it by 7, it’ll end in 7.

Martha: I think it’s 7. No, I think it’s 8.

Sam: I don’t think it’s 8 because it’s odd number times odd number and that’s always an odd number.

Carl: It’s 7 because it’s like saying 49 X 49 X 7.

Arthur: I still think it’s 1 because you do 7 X 7 to get 49 and then for 7 to the fourth you do 49 X 49 and for 7 to the fifth, I think you’ll do 7 to the fourth times itself and that will end in 1.

Teacher: What is 49 squared?

Soo Wo: 2,401.

Teacher: Arthur’s theory is that 7 to the fifth should be 2,401 X 2,401 and since there’s a 1 here and a 1 here…
Soo Wo: It’s 2, 401 X 7.

Gar: I have a proof that it won’t be a 9. It can’t be 9, 1, 9, 1 because 7 to the third ends in a 3.

Martha: I think it goes 1, 7, 9, 1, 7, 9, 1, 7, 9.

Teacher: What about 7 to the third ending in 3? The last number ends in …9 X 7 is 63?

Martha: Oh….

Carl: Abdul’s thing isn’t wrong, ‘cause it works. He said times the last digit by 7 and the last digit is 9, so the last one will be 3. It’s 1, 7, 9, 3, 1, 7, 9, 3.

Arthur: I want to revise my thinking. It would be 7 X 7 X 7 X 7 X 7. I was thinking it would be 7 X 7 X 7 X 7 X 7 X 7 X 7 X 7 X 7. (pp. 50-51)

In arguing whether the last digit in 7 to the fifth power was 1 or 7, the students were deciding which one was the correct solution and how finding the correct answer would impact their further work with exponents. The authentic process of negotiating truth exposed student thinking and strengthened their understanding by surfacing both learning and misunderstandings. As the students offered ideas, Lampert wrote each one on the board for all to reflect and comment. As apparent in the dialogue above, students who disagreed with a proof were required to justify themselves as if they were actually mathematicians arguing for their point of view. In addition, Lampert did not resolve any of the students’ assertions or their arguments. She allowed students to take the responsibility for identifying the appropriate solution together and guided them through this process only when necessary. All assumptions remained on the board until the community confirmed a mutually agreed-upon proof as the solution.

As demonstrated in the dialogue, students acted like mathematicians and successfully and respectfully examine and refute the mathematical assumptions and
strategies offered by others. They also demonstrated their ability to assess and revise their own assumptions throughout the process. It is important for students to be courageous and modest because this process will expose weakness or underdeveloped thinking. For instance, Gar demonstrated his “intellectual modesty” when he disagreed with Sarah’s assumption about 7 to the fourth ending in 9 by trying to understand her way of thinking. He did not tell her that she was wrong. Also, a similar interaction occurred in a dialogue between Martha and Sam regarding whether or not the last digit could be 8.

In addition, students need to feel free to speak up when they realize that there are flaws in their thinking. For example, during the dialogue above, Arthur said, “I want to revise my thinking,” a phrase that the members of the community use when they want to change their minds about an assertion made earlier in the discussion (p. 52). Students need to stretch themselves to develop these new expectations so that they can eventually “end up with better ideas” (p. 53). In fact, Lampert found that as her expectations for students increased, the level of student interest in the work rose as well.

**Authenticity: Found in the Ideas of Students**

The third conception defines authenticity in the ideas and solutions that emerge from students as they encounter mathematics (Weiss et al., 2009). In this view, teachers provide opportunities for students to explore mathematical questions and discuss the ideas that surface as they reason through problems or concepts. This process provides teachers and students with opportunities to focus on the authentic student-driven ideas to develop greater understanding in mathematics. An example of this process is found in an episode extracted from a research project involving third graders studying odd and even
numbers in a suburban school (Ball & Bass, 2000). The teacher/researcher’s comments are presented in italics:

Sean: Six can be even and it can be odd. Three two’s to make that and two three’s make six.

Cassandra: I disagree with Sean when he says that six can be an odd number. I think six can’t be an odd number because... look. (she gets up and comes up to the board)... Six can’t be an odd number because this is (she points to the number line, starting with zero) even, odd, even, odd, even, odd, even, how can it be an odd number because (starting with zero again) that’s odd, even, odd, even, odd, even, odd. Because zero’s not an odd number.

Sean: There can be three of something to make six, and three of something is like odd, like see, um, you can make two, four, six. Three twos to make that and two threes make six.

Keith: That doesn’t mean that six is odd.

Tembe: Prove it to us.

Ball, sensing the need to clarify terms, asks what the working definition of an even number is.

Jeannie: It is um, if you have a number that you can split up evenly without um, having, having um, split one in a half, then um, it’s an even number.

Sean agrees that 6 is even.

Sean: It can fit the definition of odd, too.

Tembe: Prove it to us. Prove it to us.

Sean walks slowly up to the board, where he draws six circles and places lines between each pair of circles.

O O  |  O O  |  O O
Sean: Well, see, there’s two, (he draws) number two over here, put that there. Put this here. There’s two, two, and two. And that would make six.

Mei: I think I know what he is saying… I think what he is saying is that it is that you have three groups of two. And three is an odd number so six can be an odd number and an even number.

*Ball looks at Sean and asks whether that is what he is claiming. He nods. Mei becomes excited and says she disagrees. She asks Ball whether she can go to the board.*

Mei: It’s not according to how many groups it is. Let’s say that I have (pauses) Let’s see. If you call six an odd number, why don’t (pause) let’s see (pause) let’s see – ten. One, two… (draws circles on board) and here are ten circles. And then you would split them, let’s say I wanted to split, split them, split them by twos… (she draws)…

OO  OO  OO  OO  OO

One, two, three, four, five – then why do you not call ten, a like – a, an odd number and an even number, or why don’t you call other numbers an odd number and an even number?

Sean: I didn’t think of it that way. Thank you for bringing it up, so I say it’s ten can be an odd and an even.

Mei: If he goes on like that… maybe it will turn out that all numbers will be odd and even and that wouldn’t make sense… It won’t make sense that all numbers should be odd and even, because if all numbers were odd and even, we wouldn’t be even having this discussion!

Ofala: *(after being prompted for a definition of odd numbers, she explains)* Even numbers have two in them, and also odd numbers have two in them, except they have one left.

II II  II II  I

Ofala: Well, an odd number is something that has one number left over.

Ball: After you do what?
Ofala: After you circle the twos.

Ball guides the students through some experimentation with other numbers they know to be odd, testing Ofala’s definition.

Riba: Twenty doesn’t work. There’s ten groups.
Sean: I’m not saying that twenty can be an odd number… I’m saying twenty-two can be an odd number.

When Ball asks whether there is a pattern to the ones that Sean is calling odd, several children chorus yes.

Sean: Every four numbers, like um. There’s one starting out like that, and it can be an odd number, and then four more…The fourth one would be another odd number…because you can split it in odd groups…odd number of groups.

Riba: Four can’t, six can, eight can’t, ten can, can, can’t, can, can’t (marking off alternating even number with a pointer).

Sean: Two isn’t, two isn’t, because you can’t make it an odd number…There’s only one group there.

Riba: (and several other students) One’s an odd!

Sean: That’s what I’m trying to say. (pp. 215-217)

Sean’s discovery of something interesting about the number six becomes the topic of intense thinking for the group. He claimed that when six is divided into groups of two, the result is three, and therefore six must be an odd number. Initially, this argument is strongly denied by his classmates because his use of the word odd did not fit with their understanding of odd. As the students debated his idea, it occurs to the teacher/researcher that the students have more than one definition of odd and even. Therefore, choosing to use Sean’s novel idea to reason through and clarify the group’s uncertainty was an effective use of time. After the students’ understanding of the definitions is clarified, the group moves on to discussing Sean’s idea in more depth. Also, once the class determined that the number of groups of two that a number could be divided by was not relevant to
the definition of odd, they were able to consider Sean’s reasoning. In addition, dissecting the topic assisted the teacher in helping students understand the definition more clearly. Moreover, this research emphasized the importance of defining and using correct mathematical terms and definitions during conversations about mathematics.

This idea relates to the fourth conception of authenticity that identifies mathematics as an intellectual discipline (Weiss et al., 2009). The teacher in the study required students to use correct terminology and notation to ensure that the discussion remained true to the mathematics being studied. Also, as a result of participating in this mathematical study, all students improved their knowledge and were able to place even and odd numbers correctly on the Venn diagram when completing their assessment. Furthermore, this excerpt provides an example of a well-conducted discussion wherein one teacher took a student’s emerging idea and created a valuable mathematical conversation and worked with her students to find understanding from an idea that was posing difficulty for them.

The definitions of authenticity provided by Weiss et al. (2009) provide an excellent framework for understanding the impact that authenticity has in students’ learning experiences. Not only did the research on authenticity provide evidence of improved skills in transferring learning to new situations, formulating solutions, and engaging in mathematical conversations, but also helped students to view mathematics as enriching, useful, and meaningful (Ball & Bass, 2000; Boaler, 1998, Lampert, 1990). These conceptions of authentic thinking are in line with the situated practices valued by Lave and Wenger (1991). Similarly, they viewed learning as embedded in the natural and relevant settings and applications that would typically occur when using that situation.
or knowledge. In fact, it was thought that learning should include engaging in the authentic process of an expert and not include merely acquiring a body of abstract knowledge to transport and reapply (Lave & Wenger, 1991). Together with Weiss, these researchers provide options for considering a new conception of what it means to learn with meaning and understanding.

Next, the third reform-based practice Learning Through Interaction included in the vision for mathematics as identified previously will be discussed. This practice contains effective methods for learning mathematics with understanding in a community of practice.

**Learning Through Interaction**

Successful interaction is very important, but not automatically used by teachers and students in the process of group learning. Students must have opportunities to engage with others while during the learning process.

Dewey initiated a restructuring of learning that would lead to more stimulating interactions between teachers and students so that students could gain more experience living and studying together (Wirth, 1966). His social learning theory was based on the presumption that true learning occurred only when students interacted with others and the world around them. This was revolutionary because at the time students sat still and listening as teachers imparted knowledge, and then after being asked, students would return it back to teachers. According to Dewey, the ultimate goal of education was to merge the psychological and social aspects of learning as a means for providing more complete and meaningful experiences for students. In this sense, interaction meant discussing the topics and sharing ways that the information connected to the students’
thinking and lives. Convincing his peers and the public of his ideas to concomitantly
develop both psychological and social skills became a major thrust in Dewey’s campaign
to improve learning.

Similarly, Kilpatrick (1951) agreed that successful learning was dependent on the
thriving interaction among students and teachers. Kilpatrick’s project method
encouraged students to become more involved in learning and experience firsthand how
mathematical processes could be discussed and applied to many different situations.
However, it was Vygotsky who seems to have had the most influence on the social
aspects of classroom learning. The sociocultural theory that he developed described his
thinking that language was both a “cultural tool” and a “psychological tool” (Vygotsky,
1978, p. 98). Language was used as a cultural tool to locate and share knowledge with
society and as a psychological tool to learn how to process thought. He described the
development of a child’s mind as occurring both individually and socially through a
process of developmental events that the student gained by interacting with people,
things, and the world. This interaction allowed students to develop a strong level of
cognition while learning (Lave & Wenger, 1991; Vygotsky, 1978).

One significant concept within his sociocultural theory that has applications to the
classroom is the “zone of proximal development” (Vygotsky, 1978, p. 90). The zone of
proximal development [ZPD] was based on the thinking that children are more capable of
achieving at a higher level by interacting with others. Research shows the successful use
of using ZPD by teachers to support and guide students as they work through a skill or
concepts above their independent learning level (Gillies, 2003; Rojas-Drummond &
Mercer, 2004). Using ZPD allows teachers to identify the zone between a child’s
independent functioning [lower mental functioning] and their academic frustration level (Vygotsky, 1978). Afterwards, teachers provide experiences for students to strengthening their skills at a level [higher mental functioning] than would be possible without their guidance and support. This learning theory is also reflected in the legitimate peripheral participation (LPP) technique used within the situated learning theory used by Lave and Wenger (1991). Newcomers in the tailor community relied on other, more experienced community members to learn by observing and practicing the skills they needed to master their craft. They were able to understand the more sophisticated concepts and tasks required by the trade of tailoring then they would have been able to do alone. With this assistance, the newcomers gradually developed their skills and needed less and less support from old-timers to completed tasks more independently.

**Contemporary Connections to Research on Interaction**

The traditional teacher-to-student communication sequence known as “Initiation-Response-Evaluation” (I-R-E) has been an industry standard in K-12 schools for years (Mehan, 1979). Essentially I-R-E involves the teacher asking a question, a student responding, and the teacher evaluating whether or not the solution provided is correct. However, the goals associated with interaction using the I-R-E format are very different for students and teachers learning in a community of practice. First and foremost, a community of practice encouraged a more interactive dialogues among students and their teacher. These interactions are similar to Lampert’s research on her own teaching which has been instrumental in demonstrating a paradigm shift that has occurred in classrooms promoting an increase of interactions among and between students and teachers.
As discussed earlier, there are numerous benefits for students working in classrooms that embrace learning within a community of practice (Cobb et al., 2001; Lampert, 1990; Lave & Wenger, 1991; McClain & Cobb, 2001). These communities are built on the philosophy that students can achieve more by learning with and from the other members of the community (Lave & Wenger, 1991; Vygotsky, 1978). Contemporary reformers have been influenced by this notion and have investigated and designed interventions to increase these types of opportunities to learn while engaging in conversations (Cengiz, Kline, & Grant, 2011; Chapin & O’Connor, 2007; Franke, Turrou, & Webb, 2015; Lampert, 1990; Sherin, 2002). A discussion of other important models for classroom talk will be described later in this chapter.

Research has demonstrated a link between students using interactive learning methods with students and increased learning outcomes. Two projects from this research describe the benefits of the utilization of interaction structures to support students and teachers as they engage with one another in the classroom (Gillies, 2003; Mercer, 2008; Rojas-Drummond & Mercer, 2004). This is followed by two studies that support the need to move students beyond the initial engagement toward interacting more deeply with the mathematical ideas of others (Cengiz et al., 2011; Franke et al., 2015)

**Exploratory Talk**

The first research study describes a successful intervention called “Exploratory Talk” used by teachers to encourage and guide student interactions while working together to improve understanding (Rojas-Drummond & Mercer, 2004, p. 102).

This study involved 700 students between the ages of 6 and 13 years old in the United Kingdom and Mexico (Rojas-Drummond & Mercer, 2004). Both the Mexican
and United Kingdom components of the study set out to determine whether improving interaction could result in raising group and individual problem-solving abilities. The intervention was a series of lessons designed to establish a method for using Exploratory Talk, as opposed to Cummulative Talk, as a tool for interacting in the learning community (Mercer, 2008, p. 96). Exploratory Talk is defined as “a joint, coordinated form of co-reasoning, in which speakers share relevant knowledge, challenge ideas, evaluate evidence, consider options, and try to reach agreement in an equitable manner” (p. 96). On the other hand, teachers using Cummulative Talk encouraged students to offer solutions, after which they evaluated ideas with students simply agreeing with the teacher.

Researchers encouraged the use of Exploratory Talk because “it embodies a valuable kind of ‘co-reasoning,’ with speakers following ground rules which help them [students] share knowledge, evaluate evidence and consider options in a reasonable and equitable way” (Rojas-Drummond & Mercer, 2004, p. 99). According to the researchers, the intervention was conducted to help students develop a conception of how they should talk and think effectively together.

The Mexican component involved 84 participants, including six teachers, from two parallel state primary schools with students between the ages of 10 and 12 years old. They were considered parallel schools because of their equivalent socioeconomic levels. The participants were assigned in equal numbers to either an experimental or a control group during a five-month period. Sessions from both the target and control classes in Mexico and the United Kingdom were videotaped. Likewise, all students were given parallel forms of individual and group versions of Raven’s Progressive Matrices Test.
These evaluations were used to assess the students’ dialogue and curriculum knowledge.

The intervention developed by researchers included a set of 10 “Talk Lessons” used to guide teachers dialogue with students. The first five lessons contained experiences for establishing “ground rules” to set the stage for using Exploratory Talk with students (p. 106). The remaining lessons contained curriculum-related experiences and were delivered using both teacher-directed and group-driven activities. Students were expected to apply the methods that they learned earlier during the remaining lessons.

One example taken from this study provides a format for the types of dialogue that are characteristic of those used in the project. The section piece of dialogue is from a group of sixth graders working on a test item during the pre-intervention stage of the study.

This example provides a format for the types of dialogue that are characteristic of those used in the project.

**Dialogue Before the Talk Lesson Intervention**

Luis: (Laughs)

Georgina: Let’s see, number three (points to the booklet).

Luis: Number three.

Georgina: Three?

Mauro: Two.

Luis: It’s three.

Georgina: Number three? Number three.

Luis: Three...yes, three. Yes, we are thinking correctly.

Mauro: Number three.

[Georgina writes down option number 3 on the answer sheet, which is incorrect] (p. 109).
The second dialogue is from the same group of students’ post-test problem-solving session. Both problems were in the area of geometry and were similar in difficulty level.

Dialogue After the Talk Lesson Intervention

Georgina: Here they remove the dots and this, this cross (points at a drawing).

Mauro: No but wait, it does not fit.

Luis: No, wait.

Mauro: No.

Georgina: Let’s look at the sequence. Here it has like this, they remove the cross and the dots. Here they are not there, any more, here (points)?

Luis: And here they remove only the dots.

Georgina: Yes, the dots. And this part, only the star.

Mauro: It would be this one, look at it (points).

Luis: Which they have removed.

Mauro: It would be this one, because, look, it goes like this (points).

Georgina: But how, if it doesn’t have dots?

Luis: It doesn’t have dots. It would remain just the cross.

Georgina: Yea, for what they have removed!

[Georgina writes down option number 1 on the answer sheet, which is correct]. (p. 109)

The students in the pre-intervention session engaged in a form of Cumulative Talk and did not justify their thinking or question the other group members. The dialogue that occurred after the intervention highlighted the students engaging in negotiation and reasoning. They also made more arguments and presented different perspectives while
working on the problem. Additionally, this group of students solved the problem correctly the second time using the dialogic skills they learned during the intervention.

The analysis included reviewing the types of questions, instructions, and encouragement used by teachers. In addition, researchers gathered information about the ways peer interactions contributed to improving academic results, both collectively and individually. Pre and post test data also informed academic growth among students. Data from the pretest indicated that both groups used Cumulative Talk, which the authors described as simply agreeing without debate or justification (Rojas-Drummond & Mercer, 2004). However, the posttest indicated that the groups who had received the Talk Lessons intervention used significantly more Exploratory Talk than the groups who did not. The effective use of the Talk Lessons intervention also led to a significant increase in both group and individual scores on standardized tests. Teachers used question-and-answer dialogue to monitor knowledge acquisition to monitor understanding and help students to make sense of problems while treating learning as a social process. During this experience, the students were encouraged to make explicit their own thought processes to others. They were required to share how and why they solved their problems aloud. To assist with this process, teachers also used “why” questions to help students to reflect on their own thinking and to further explain their reasoning to their peers. Secondly, teachers provided students with experiences to develop and share a variety of strategies for solving problems. The meaning and purpose of concepts and skills were communicated to students to assist them with making sense of the content covered. Finally, teachers treated learning as a social and communicative process. They encouraged the exchange of ideas between students. Students were also
expected to support the learning of their peers. Taking a more active and vocal role in the education process was required as part of the program.

The United Kingdom component of the study involved primary school students ranging from ages 8 through 13 in two neighboring schools. The two schools were matched with students of the same age that were not using the intervention. Similar results were achieved. Students in the intervention “target” classes used much more exploratory talk than students in the “control” classes, who had not received the intervention. The target group solved problems on the standardized assessments more successfully and made significantly greater gains on math and science tests than did those in control classes. These results were made possible through many lessons wherein students successfully engaged in thinking and reasoning together to solve problems.

The sample dialogue included below is an example of a teacher-led discussion that followed a small-group activity in which children were asked to determine if a set of statements about the solar system were either true or false.

Teacher: Keighley, would you read out number 9 for us?

Keighley: [reads] “The moon changes shape because it is in the shadow of the earth.”

Teacher: Right, now what does your group think about that?

Keighley: True.

Teacher: What, why do you think that?

Keighley: Hmm, because it’s when earth is dark then, hmm, not quite sure but we think it was true.

Teacher: Right, people with hands up [to Keighley]. Who would you want to contribute? (p. 97)
The questions used by the teacher in the dialogue are similar to the methods the teachers used in the initial Talk Lessons. In the sequence above, the teacher guides the discussion in a way that gives students the chance to elaborate on their ideas so that they can share their thinking and further their understanding of the concept. In addition, engaging in dialogue helps teachers collect information about how students are processing the material being covered in the lesson. By demonstrating these questions, students are provided with the types of questions that the teacher would be using with them during future mathematical discussions. The questions used by the teacher also served as a model for the types of questions they would be expected to use when discussing solutions with other students independently. Furthermore, the preparation each of the target groups received enabled them to successfully apply the skills they learned with support from their teacher. The ground rules used by the teacher in the above Exploratory Talk dialogues guided each discussion (Mercer & Littleton, 2007). The ground rules helped to initiate the kind of talk that encouraged co-reasoning to help share knowledge, evaluate evidence and consider options reasonable and equitable ways. The ground rules, developed organically to govern conversation with each class encouraged students to share relevant information, justify assertions, offer opinions, ask for suggestions and reasons, attentively listen to others, discuss alternatives before making decisions, and accept and respond to challenges. This body of work provides a clearer picture of the ways productive talk can guide the development of children’s understanding. The following list is a set of ground rules developed by these authors to support teachers using Exploratory Talk:

1. Everyone participates and engages with the topic through talk
2. Tentative ideas are treated with respect
3. The ideas offered for joint consideration can and should be challenged
4. Ideas and challenges have to be justified with reasons
5. Alternative ideas or understandings are offered
6. Everyone’s ideas are asked for and considered
7. The group tries to negotiate a shared agreement
8. Listen to other peoples because it helps you understand the way they think
9. Take part! Your ideas are as valuable as everyone else
10. Treat people fairly-don’t interrupt them, don’t put them down
11. Criticize the idea, not the person (Mercer & Dawes, 2010, p. 23)

Similar to the way the apprentices in Lave and Wenger’s (1991) study the students in Drummond and Mercer (2004) were supported initially in their learning by observing experts and gradually coming out of the periphery and learn for themselves. The questions that are used by the teacher, in the discussion above, demonstrate the types of questions that teachers use to model the interactions with the goal of students using the questions with peers. After receiving this support, students should be able to accomplish similar task independently with success in the future.

Another commonality is the importance placed on the participants’ working together to develop ideas as a group, instead of simply practicing skills independently. Rojas-Drummond and Mercer believed that teaching students about the ways of talking productively together by developing specific strategies for thinking collectively was the reason for the success they achieved during this project (2004). Unlike Lave and Wenger (1991), Mercer described specific procedures or ground rules to govern participation
with students in classrooms (Mercer, 2008). This rare research clearly establishes the benefits of using a process for “shared conception of relevant knowledge and of how they should talk and think together effectively” to help students engage in successful discourse (Rojas-Drummond & Mercer, 2004, p. 103).

A second study highlighting the benefits for students using procedures for interacting and working with their peers prior to using them independently follows (Gillies, 2003). The key to the success of both interventions was the guidance students and teachers received before and while using the interventions. This intervention is described in the following section.

**Dialogic Talk**

The second research study describes an additional interaction structure that also contributed to the literature on dialogic or prescribed talk involved a project that encouraged students to use procedures for working cooperatively with others to solve problems in small groups. Gillies (2003) discovered students whose experiences included interacting with other students to complete tasks gave more detailed and explanatory help to peers, asked more comprehensive questions, and achieved higher learning outcomes. These findings were generated from five field-based intervention studies that investigated cooperative learning methods with first through eighth graders. Comparison groups of the same aged peers were part of all five of the studies. Each study had between 168 and 220 students and lasted from 9 to 12 weeks. Groups were arranged in mixed-ability and gender-balanced groups of three to four students. All groups were videotaped in the final stage of problem solving during each unit, and the videotapes were coded.
Each teacher who participated received training in using small-group cooperative learning practices with students. The teachers then established structured, cooperative learning in their own classrooms. These experiences were guided by the key elements of cooperative learning described by Johnson and Johnson (1990):

1. Task interdependence, which was established in the groups so that each member had to contribute to the group task.
2. Individual accountability was established so that all members understood they were required to report on their own contributions.
3. Students actively promoted each other’s learning.
4. Students were trained in the interpersonal and small-group skills needed to facilitate group work. (Gillies, 2003, p. 40)

The students in the target group were given time to learn and develop how to interact with other students before they were expected to engage in cooperative learning activities. These skills included: actively listening to other students; providing constructive feedback including suggestions and ideas; encouraging everyone to contribute to the activity; sharing the workload and resources fairly; trying to understand other student’s perspectives; and monitoring and evaluating the group’s progress (p. 40). The students in the control group used the same structures only they had not been apprised of the structures for cooperative learning before they were expected to use them while working with peers.

For example, the primary students in the target groups practice using the structures through role-playing to understand how they would be used in groups. Additionally, the older students discussed the structures and even developed their own
guidelines for conducting group discussions. These activities helped students learn how to promote each other’s learning, accept responsibility and seek help from other members of the group. These structures helped guide the students’ interaction during group work and provided direction for discussing and presenting problems, evaluating findings, and challenging other students’ perspectives. Listed below are two examples of the dialogue students used that supported the interaction structure provided to them:

Student 1: Look at this [pointing to information]. Maybe we could see if we can find out more on this [information needed]. It looks like it could tell us more about it [problem they are trying to solve].

Student 2: That’s just what we want cause it’s got that pointy part [pointing to picture] that’s like the one we want. Don’t you think it’s like what we need [pointing to significant aspects of the picture]? (p. 460)

The results of the analysis found that the students who were provided with the structures and given time to practice using them were more prepared to interact and optimized more on their work with classmates during the collaborative activities. In addition, these students made contributions to the learning of their peers by acting as mediators when they explained ideas, focused attention on mutual interests, and encouraged their group to pursue new directions. They exhibited more cooperative and less non-cooperative behaviors than the students in the control groups. Moreover, students in all five of the studies who received the interaction structure used more content-related talk, gave more assistance to one other, and achieved higher learning outcomes than their peers who did not receive the intervention. Furthermore, all students in the target groups, regardless of their academic levels at the beginning of the project, demonstrated the ability to contribute ideas and knowledge and even learned to appropriate their ideas as needed during the interactions with their classmates.
The participants in this study shared similar behaviors with the participants demonstrated in the Lave and Wenger’s study (1991). Both groups of participants in Gillies (2003) were encouraged to interact and contribute to the learning among the other members in the respective communities. Specifically, the students from the target groups in this study increased the amount of unsolicited explanations they offered to help clarify the information that was shared during cooperative learning. This may be a result of students being more in tune with the needs of their fellow group members from working so closely with one another.

The third research study developed by Franke, Turrou, & Webb, (2015) is a project that included talk to include studies engage students with others through mathematical discussions to promote understanding

**Engaging Students with the Ideas of Others**

Franke, Turrou, & Webb, (2015) found that using talk moves were useful to teachers as they work to figure out how to support student communication and the rigor called for in the Common Core Standards for Mathematical Practices (Kober, & Rentmer, 2011). Hiebert and Grouws (2007) describe this type of classroom discourse as engaging students in a “productive struggle” by providing deeper explanations and analyses that requiring more “intellectual work” from students (p. 390).

This study included videotaped whole class and small-group discussions from 12 pre-K through sixth-grade elementary school classrooms, analyzing engagement patterns as teachers supported interactions with talk moves (Franke et al., 2015). Researchers informally observed in the schools weekly for six months. Formal data collection involved multiple days of observation, two to three observations over two weeks,
approximately for one hour each during mathematics lessons. One day of observation was selected per teacher for analyses. These observations were selected based on whether or not they represented the teacher’s usual practices concerning participation and interactions. Talk move invitations used by teachers were observed across classroom contexts and mathematical topics. The school was selected as a site for this study because inquiry methods were the focus of the instruction and were used throughout. However, the authors note that there was variability in how teachers enacted these goals and practices within the school. The study included descriptions of students and teacher interactions described using vignettes from actual classrooms. Professional development or support during the project for teachers to elicit students’ mathematical thinking or engage students in each other’s ideas was not included in this project.

As part of the study, Franke et al., (2015) described how they examined the support teachers provided to students as they engaged with other students’ while engaged in problem-solving in mathematics. The results highlighted the ways teachers used in-the-moment invitations and support moves with students while they discussed problems. The invitation moves were the initial asking of students to engage with others. The support moves followed the invitation to help the student engage with the mathematical ideas of others. Originally, the support moves developed as a response to challenges teachers faced while trying to sustain engagement with students. The challenges encountered by students included being unable to: engage with a peer’s ideas; provide little or no detail about others’ thinking; or not address the mathematical ideas underlying a strategy shared by a classmate. According to the authors, teachers used talk moves after an initial talk move was used that did not result in a high level of engagement. Using the follow-up talk
moves assisted teachers in inviting students to participate, and encouraging deeper engagement with the mathematical ideas presented by their classmates. For example, the invitation moves fell into the six categories listed below:

1. Explain someone else’s solution
2. Discuss differences in solutions
3. Make a suggestion about another student’s idea
4. Connect one student’s idea to another student’s idea
5. Create a solution together with other students
6. Use a solution that was offered by another student (p. 133)

There were no initial moves that did not fit into the categories above. These moves occurred across classrooms.

The vignettes from the research project demonstrated how teachers listened to students’ explanations asked questions and used moves to initiate more detail in their explanations. Three “teacher support moves” identified as probing, scaffolding and positioning were used to help students interact more with the ideas of peers (p. 135). Probing was used to “press students to engage further by questioning or revoicing in a way related to the mathematics and in service of supporting engagement in the other’s mathematical idea” (p. 136). Scaffolding was used to link a representation, context, or idea already discussed including providing some information or clarify the ideas on the table. Finally, teachers used positioning to discuss and interact with the students in ways that acknowledge the connection the student made to the mathematical idea that has been shared publicly. Using positioning was significant because this move allowed teachers to move beyond using probing and scaffolding to sustain engagement than they had noted.
during an earlier research study conducted by the authors (Webb, Franke, Ing, Wong, Fernandez, Shin, & Turrou, 2014). These researchers stressed this effective use of positioning by teachers arguing that following-up with students about their own ideas and ideas of other students had the potential for extending mathematical thinking during mathematical discourse with elementary students. The talk moves utilized by teachers were not rated based on their effectiveness but presented as examples of ways teachers could further engage students with ideas offered during mathematical conversations. Additionally, the authors acknowledged that teachers carried out moves for different purposes but they only included when teachers used the moves to engage with each other’s ideas in more specific ways.

The results of this study indicated the talk moves were not used consistently across classrooms, nor were they used consistently during similar interactions. Also, the type of teacher invitation did not determine the level at which students engaged in another’s idea. Moreover, researchers observed each type of teacher invitation led to a range of engagement levels. Some engagements included referencing ideas, adding to ideas or agreeing or disagreeing. Researchers indicated three reasons why students did not respond to the initial teacher invitation in a detailed way were noted. First, students did not know how to respond to the invitation, and the invitation from teachers to engage was not always enough for students to know how to engage. Secondly, the student provided few, if any, details about the other student’s idea. Finally, students engaged but did not discuss the mathematical idea surfaced by the other student. It was also determined that teachers used two different types of talk moves with students. These moves were used in different ways. Sometimes teachers used the three support moves
individually or in combination with the others. Moreover, teachers did not use the same series of support moves even when responding to students during the challenges mentioned above. Therefore, the authors concluded that teacher moves were not observed to be a set of planned strategies that teachers applied repeatedly. Instead, they described the moves as “a repertoire of pedagogical moves that teachers drew upon in-the-moment” (p. 143). Furthermore, the moves were described as a developing a set of norms that shaped the ways teachers and students interacted.

Franke et al., (2015) surfaced needs of students while engaging in productive conversations about mathematics. As a result, teachers developed moves to initiate and support students to discuss their own ideas and the ideas of others more effectively. The participants also shared similar behaviors with the participants demonstrated in the Lave and Wenger’s study (1991). Students needed some support from their teachers for productive conversations to occur. This finding was similar to the process of negotiating with members of the community to find meaning in the thinking of others which was crucial to learning in the situated perspective (Lave & Wenger, 1991). Specifically, teachers used invitation and support moves to focus on the mathematical details within student explanations to share those ideas with other students.

The final part of this section includes a description of the research study developed by Cengiz, Kline & Grant (2011). The study pushes mathematical talk beyond Engaging Students with the Ideas of Others (Franke et al., 2015) toward including more opportunities for Extending Mathematical Thinking.
Extending Mathematical Thinking

The study involved six teacher’s classes in first through fourth grades (Cengiz, Kline & Grant, 2011). Teachers in these classrooms had been using a National Science Foundation resources between eight and 12 years before the study began. Teachers also received extensive professional development regarding the mathematics they taught and how to implement these materials with students about the mathematics they were teaching and on implementing using the materials. Researchers acknowledged that these experiences resulted in a highly developed mathematical knowledge for teaching (MKT) of the teachers involved. One episode was selected and coded for each teacher. Two episodes were coded for two of the teachers because additional extending episodes occurred within the same lesson.

The three extensions that were coded are listed below:

1. Encouraging mathematical reflection
2. Going beyond initial solution methods
3. Encouraging mathematical reasoning

Next, lessons were coded for the type of instructional actions by the teachers. All lessons and interviews were videotaped and transcribed. Data was also collected to measure the participants knowledge of content. Furthermore, the link between the participants content knowledge and instructional actions were examined.

The authors defined two categories of talk moves in the framework Extending Student Thinking. The first category extending episodes was described as helping students move beyond initial mathematical observations to developing an understanding of a mathematical phenomenon. The extensions occurred when teachers posed questions
or shared an observation to focus students. A list of extending episodes are listed in the framework, see Table 2. Teachers extended the conversation beyond the initial solution by engaging students in reflection or reasoning about the mathematical idea. The second category included instructional actions. Several instructional actions were used during each of the extending episodes. The instructional actions are also listed in Table 2.

Table 2: Extending student thinking framework

<table>
<thead>
<tr>
<th>Extending Episodes</th>
<th>Instructional Actions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Encouraging reflection</td>
<td>Eliciting actions:</td>
</tr>
<tr>
<td>Encouraging students to understand, compare, and generalize mathematical concepts/claims</td>
<td>Inviting students to share methods</td>
</tr>
<tr>
<td>Encouraging students to consider and discuss interrelationships among concepts</td>
<td>Supporting actions:</td>
</tr>
<tr>
<td>Using multiple solutions to promote reflection</td>
<td>Suggesting an interpretation of a claim/observation</td>
</tr>
<tr>
<td>Encouraging students to consider the reasonableness/validity of a claim</td>
<td>Reminding students of goal of the discussion, the problem, or other information</td>
</tr>
<tr>
<td>Going beyond initial solution methods</td>
<td>Repeating a claim</td>
</tr>
<tr>
<td>Pushing individual students to try alternative solution methods for one problem situation</td>
<td>Recording student thinking</td>
</tr>
<tr>
<td>Promoting use of more efficient solution methods for all students</td>
<td>Introducing different representations/contexts</td>
</tr>
<tr>
<td>Encouraging mathematical reasoning</td>
<td>Extending actions:</td>
</tr>
<tr>
<td>Encouraging students to offer a justification for their solutions/claims</td>
<td>Inviting students to:</td>
</tr>
<tr>
<td>Encouraging students to engage with each others justifications</td>
<td>Evaluate a claim or an observation</td>
</tr>
<tr>
<td>Instructional Actions</td>
<td>Provide reasoning for a claim Compare different methods</td>
</tr>
<tr>
<td><em>Eliciting actions:</em></td>
<td>Use same method for new problems Provide counter speculation for a claim</td>
</tr>
<tr>
<td><em>Supporting actions:</em></td>
<td></td>
</tr>
<tr>
<td><em>Extending actions:</em></td>
<td></td>
</tr>
</tbody>
</table>

95
The researchers examined three instructional actions to extend mathematical conversations which included: eliciting, supporting, and extending. Eliciting actions required students to express their existing thinking about the mathematics discussed. Supporting actions involve more teacher telling. However, according to the authors supporting actions were significant in extending episodes. Students needed assistance to focus their mathematical reflection and reasoning on important mathematical concepts. Extending actions encouraged students to make connections among ideas and solution methods and to move beyond their existing mathematical knowledge. All of the extending episodes observed in this study were from three main categories in the extending student thinking framework. These extending episodes included: encouraging mathematical reflection, going beyond initial solution methods, and encouraging mathematical reasoning.

The results indicated that eliciting actions took place in all six classrooms. The most frequently used extending action was to evaluate claims or observations. Another commonly used extending action used by teachers was posing questions to invite students to share their reasoning after making a claim. Furthermore, comparing different solution methods was also commonly used. Two of the least frequently used extending actions were inviting students to solve a new problem by applying a solution method along with providing a counter speculation.

Teachers also utilized five types of supporting instructional actions during extending episodes. First teachers shared their own interpretations about a students’ claims. They also reminded students about information related to problems or ideas during discussions. Thirdly, they repeated claims or prompted students to repeat claims.
Recording student thinking on the board was another instructional action used. Another instructional action that supported the extending episodes was recording student thinking on the board. Finally, teachers introduced representations or contexts that were familiar to students but this occurred infrequently. Researchers conclude that a variety of instructional actions are more effective in extending students thinking.

Even though the data sample was limited, this study provided a useful vision of the how teachers extend thinking during discourse. Similar to other researchers studying mathematical discourse who agree that having a list of instructional actions as potential steps, a recipe does not exist (Franke et al., 2015; Mercer, 2007). In fact, they found that in some cases individual moves were effective and in other cases ineffective. This suggests a more nuanced framework is appropriate. The researchers attribute teachers’ knowledge about mathematics and pedagogy greatly influences the way they teach mathematics. According to researchers, teachers in this study executed the extended episodes because of their ability to recognize the potential in the conversations. Other ways teachers created opportunities to extend thinking was focusing on listening to students, as well as establishing clear goals about the mathematical ideas and concepts to pursue.

According to Rogoff and Lave, a community of practice does not mean “applying a recipe of techniques to a new collection of people….it requires the participation of people inventing and adapting customs and traditions, who learn from their efforts to develop the principles and practices for themselves” (1984, p. 10). Members of the community are not just physically gathering in a group, but exchanging ideas, helping one another to further their knowledge and the expertise of the group (Lave & Wenger,
1991). Using a combinations of ground rules and other conventions in-the-moment to
guide and extend conversations to develop critical thinking so that participants build
understanding is the focus of the studies outlined above (Cenzin et al., 2011; Franke et
al., 2015; Gillies, 2003; Mercer, 2000).

The complexities described in the studies included in this section suggest that
facilitating discussions with students is a challenging undertaking that requires a great
deal of support for teachers who attempt implementations. However, the results presented
in this section have important implications for learning in schools. First, these studies
provide a clear example of the possible benefits of establishing and practicing a format
for interacting and talking can assist students in using it independently with success.
However, because many factors were likely to have contributed to the effectiveness of
these students’ classroom experiences, it is difficult to make a definite link from the
interaction among teachers and peers in the learning community and greater
understanding. On the other hand, these studies do demonstrate that those students who
were expected to use a high level of productive interactions during group work were able
to apply their skills both collectively and individually. Even though this research is not
specific to learning mathematics, it is exceptionally valuable because it highlights the
importance of preparing students for using productive methods of interaction to make the
most of their experiences (Gillies, 2003). Furthermore, two additional studies expand the
literature on using mathematical discourse to include assisting students to sustain talk
beyond the initial engagement toward interacting more fully with the mathematical ideas
of others (Cengiz et al., 2011; Franke et al., 2015).
The latter two studies described in this section demonstrate ways teachers can extend learning and strengthen students critical thinking abilities (Franke et al., 2015; Cengiz, Kline, & Grant, 2011). However, more research demonstrating the importance of attending to details in students’ explanations and providing vignettes of teachers using in-the-moment decisions that focus on the details in students’ explanations and connecting those to mathematics are necessary (Franke et al., 2015). Furthermore, research demonstrating the effectiveness of specific talk moves and building a repertoire of moves that teachers can use in-the-moment to engage students with ideas of others is also needed.

Next, a description of the fourth reform-based practice named Strategic Discourse which was included in the vision for mathematics as identified previously. This practice includes effective methods for learning mathematics with understanding in a community of practice.

**Strategic Discourse**

Social science researchers define “discourse” as an institutionalized way of thinking expressed through language. For instance, Foucault (1972) describes “discourse” as a system for exchanging attitudes, ideas, actions, interests, and concerns in the process of developing knowledge. More recently, discourse has been referred to as purposeful talk in mathematics where there are genuine contributions and interaction (Pirie & Schwarzenberger, 1988). In line with recommendations from the research reviewed in the previous section, students must have countless experiences including “ongoing, authentic exposure to talk and ways of being” to develop effective discourse practices (Gee, 1991, p. 169). Developing an “identity kit” made up of ways of thinking,
acting, and speaking also helps students to increase their mathematical power (Gee, 1991).

Interaction is crucial in a community using discourse to communicate ideas for the purpose of developing understanding. Discourse is made up of several elements referred to as socio-mathematical norms (Yackel & Cobb, 1996). These norms guide the shared standards for practice, explanation and justification for engaging in mathematics learning within a community. They focus on what counts as mathematically different, sophisticated, efficient and elegant. The socio-mathematical also include appropriate and acceptable ways of engaging with one another and are specific to the mathematical aspects of students’ activity. These norms include students being willing to participate in discussion with teachers and peers. By participating, students articulate their thinking and debate solutions. They explain why they disagree with solutions and they share this feedback aloud. Students are required to compare their thinking to others, make conjectures, restate solutions and commit to participating until a problem has been solved. Students agree to listen to others and revise their thinking with the help of others (Hiebert, et al., 1997; Kosko, 2012). While students contribute with these in mind, they are required to use the mathematical language and expressions to get their ideas across (Askey, 1999). They also compare their own mathematics strategies and thinking with conventional mathematics. Using norms to guide the effective interactions during mathematics with a classroom of learners is helpful to facilitating an effective process.

The discourse that is the focus of this study is “strategic discourse.” “Strategic discourse” promotes a highly defined path for engaging students in discourse to advance understanding of mathematical concepts and ideas. The goal of strategic discourse is
developing shared understandings, new insights and a deeper analysis of mathematics on the part of all members in the community (Manouchehri, 2007). This form of discourse focuses on building reasoning skills needed for students to autonomously plan solutions rather than simply applying a set of routines. Teachers using strategic discourse map a path or charting the key mathematical ideas, mathematical norms, and mathematical vocabulary they intend to intentionally visit during the discussion. Strategic discourse also involves using teacher moves, as defined in this chapter, to facilitate talk that keeps the discussion moving in the direction of understanding (Martino & Maher, 1999).

Clearly, there are many things to consider when charting the path in strategic discourse. Teachers use strategic discourse to steer students by assisting them to make connections to past experiences and future concepts. Strategic discourse can also be used to target a lack of understanding or the weak development of ideas (NCTM, 2000). Knowledge of the curriculum and essential questions will assist in planning before the discourse experiences and while engaged in mathematical discussion. According to NCTM, “effective mathematics teaching requires understanding what students know and need to learn and then challenging and supporting them to learn it well” (2000, p. 16). Even more important is having inclinations about where students typically have trouble or stumble is crucial for planning effective discourse. Teachers who develop these insights are able to quickly enact a plan, on the spot, to scaffold learning for students during discourse. Productive scaffolding is not just leading students to the answer or telling them how to arrive at a solution in a different way. It is the tailored support provided to students to help them arriving at a solution on their own. This support is given by teachers and peers.
Along with laying the path, teachers need to use questioning to skillfully decipher understanding and direct strategic discourse. The use of timely questions should be connected to thinking, strengthening justification, connecting ideas and explaining strategies which are all essential in strategic discourse (Franke et al., 2015; Martino & Maher, 1999). Skillful questioning can provide essential information about students developing knowledge that may otherwise be inaccessible to teachers. Using questions that encourage participation in the discussion and not those that limit the interaction is crucial to strategic discourse. These and other successful talk moves and models (see Table 3) used by teachers during discourse provide additional information in helping teachers conduct successful discussions with students in classrooms (Forman, 2003, Huffered-Ackles, Fusion & Sherin, 2004; O’Connor & Michaels, 1996; Rojas-Drummond & Mercer, 2004; Sherin, 2002; Wood, 1998).

There are many considerations for teachers to consider while charting the path for strategic discourse. Teachers must attend to the quality of the evidence offered, the similarities and differences between explanations, the mathematical ideas embedded, missing components in solutions, and students’ history within the discourse community (Choppin, 2007).

Success takes work and comes with experience listening to students explain their thinking and developing fluency with procedures, new ideas and techniques. Discourse is a major investment in time for teachers, time they will tell you that they do not have. However, not taking the time to organize a path for discourse as described here could have serious consequences. Teachers who facilitate discourse without a plan may lose sight of the important aspects and may get lost in the weeds or become disheartened and
avoid using it altogether. This would not be the first time a practice has gone by the wayside (Kazemi & Stipek, 2001).

**Shifting Authority**

Strategic discourse requires a great deal of planning on the part of the teacher. However, it is not meant to be entirely teacher directed. In the traditional classroom, the key source of mathematical ideas has always been the teacher and the mathematical talk was most often something that students received, not interacted with. Strategic discourse, in a community of practice, on the other hand, requires a more academically and socially equalized environment. Lave and Wenger acknowledged the importance of this equalization in the community of practice but did not address how to structure it in their literature (1991). Shifting authority away from the teacher to community is difficult to institute due to the structure of power existing in school environments (Liston, & Zeichner, 1990; Lortie, 1975). A resistance among teachers, administrators and the public to altering this hierarchy has perpetuated the status quo for far too long. Obstacles exist at every level of the organizational hierarchy governing schools that make change difficult (Shed & Bacharach, 1991). At the top, the district-level administration is charged with overseeing the direction of the schools. They determine the district policies and curriculum under the guidance of the board of education and communicate the expectations and initiatives to faculty (Lortie, 1975). Furthermore, these policies change often and can mean that school districts move back and forth between traditional and reform methodology based on the progressive and conservative shifts that shape the political climate. Principals receive the information and pass along the changes to their teachers. Employees are expected to support the district-level initiatives and to work to
ensure that these changes are instituted in their respective buildings (Shed & Bacharach, 1991). These directives are often ill defined, which means the principal and teachers interpret the policies and curricula themselves and then communicate it to teachers and they implement them with students.

Teachers, along with their students, have always been on the last rung of the ladder. They work in environments whose formal decision making authority is given to those outside of the classroom, some of whom are not even educators (Lorti, 1975; Oakes & Lipton, 1990; Ravitch, 2010). Teachers and students maintain the smallest voices in the organization (Corbett & Wilson, 1995). This placement at the bottom in the hierarchy perpetuates the culture that they are not organizationally or socially valued (Shed & Bacharach, 1991). As part of this power dominated culture, teachers assume the role of director, providing knowledge and algorithms and assigning menial tasks. Similarly, students accept information and direction from their teachers without question (Cuban, 1993). On the rare occasion when students speak, they raise their hands and wait to be called on. These interactions are usually one-word responses.

Teachers who are graduates of the teacher-dominated education system have also been socialized to accept the power structure that endures in the classroom as “right, natural, and proper, “and therefore does not seem appropriate to question its existence (Sarason, 1990, p. 89).

The pedagogy and behaviors resulting from this culture are a pervasive and deep-seated misunderstanding of mathematics as a discipline (Walls, 2007).

This hierarchy has also had negative effects on the learner. The regularities have created passive and dependent students; they have not been encouraged to actively
engage in planning and learning, be self-directed, proactive or collaborative (Roth & Bowen, 1996; Tyack & Cuban, 1995). If there is to be any change in the enculturation embedded in schooling, it will require the teacher to initiate and sustain the changes; specifically, how students learn not just to what they learn (Cuban, 2013). Utilizing strategic discourse practices can help facilitate the increase in students’ participation and improve their learning experiences (Corbett & Wilson, 1995; Fullan, 2001). The first step is creating environments free of hierarchies that encourage students working with other students and their teacher as partners in the learning process.

**Shared Authority in Strategic Discourse**

Teachers can struggle with roles and allowing students to have instructional control (Hoek & Gravemeijer, 2011). However, the success of strategic discourse is dependent on the level of engagement and the willingness of the teacher to share authority with her students (Webel, 2010). Strategic discourse occurs when students and teachers are partners in learning and powerful sources of mathematical ideas and thinking. They act as major contributors to the discussion and are valid and worthy members of the learning community. Validating students position sends a message to students that they are important to the process. Including students more also encourages greater levels of engagement leading to heightened responsibility regarding their own thinking and evaluating their ideas and others (Huffered-Ackles, Fuson & Sherin, 2004). This shift also results in an increased desire to ask questions, share their knowledge and help others. Finding ways to acknowledge students as competent thinkers also establishes them as strong sources of mathematical ideas (Choppin, 2007). Spotlighting their ideas during discourse will assist others in understanding and establish their authority and
identity. This encourages “the result is often an explanation that is better than one students could produce.” (Choppin, 2007, p. 307).

To clarify, the major direction of strategic discourse is provided by the teacher initially but the goal if for it to become distributed among all participants, as they share ownership of the standards of practice in the learning community. Teachers need to be willing and able to let the mathematical conversation take direction from all participants. Students must be able to navigate the direction of the discourse and feel that their ideas are worthy of time and consideration. Students will see right through a teacher who acts like they are sharing authority but still directs the path with too much rigidity as insincere. This approach to strategic discourse magnifies the complexity and significance of the teacher’s real-time decision-making where interactions, even for the most organized teacher, are unpredictable (Choppin, 2007). Even though the teacher has a plan in mind, she needs to be able to make instantaneous decisions to follow a trajectory from students that may take them from their path for a few minutes, while remaining mindful of returning to the intentional path she started on.

**Models for Implementing Mathematical Discourse**

Recently, a focus on discourse within mathematics education literature has led to an increase in both the quality and quantity of talk in our classrooms (Boaler, 2002; Cengiz et al., 2011; Franke et al., 2015; Kline, K., & Grant, 2011; McCrone, 2005; Walshaw & Anthony, 2008). This literature base also recommends that teachers move away from providing students with the mathematical formulas and procedures for calculating answers, toward allowing students to design strategies and solutions generated by their own conception of mathematics, testing these solutions and identifying
what they need to find out to solve the problem correctly (Ball & Bass, 2000; Lampert, 1990; McClain & Cobb, 2001). Additionally, this research praises the benefits of students offering justifications, asking more questions and evaluating ideas presented so as to build on shared ideas, as well as, considering how others think and solve problems (Ball & Bass, 2000; Cobb et al., 2001). Information detailing the aspects of several prescriptive talk models that have been developed and studied by researchers are included, see Tables 3-5 (Lampert, 1990; O’Connor & Michaels, 1996; Sherin, 2002; Wood, 1998).

Many recommend the use of a structure to help guide teachers and students in interacting more critically with one another to build higher order thinking (Cenzin et al., 2011; Franke et al., 2015; Mercer, 2008; Hiebert & Grouws, 2007). Furthermore, effective teachers guide the development of student’s understanding and treat learning as a social, communicative process (Rojas-Drummond & Mercer, 2004). Although the studies present strong resources for implementing strategic discourse in the classroom, the use of these models for engaging in talk are rarely used in classrooms without researcher support (Mercer & Dawes, 2010).
Table 3: Models for Using Mathematical Discourse with Students

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Definition</td>
<td>Students come together in a joint, coordinated form of <strong>co-reasoning</strong></td>
<td>Three step process of <strong>generating</strong> ideas, <strong>comparing</strong> the ideas and discussing an idea in depth.</td>
<td><strong>Five Step</strong> process includes revoicing, revisiting, analyzing, adding and wait time to increase exchange of ideas</td>
<td>Spontaneous and <strong>unscripted</strong> discussion between teacher and students</td>
</tr>
<tr>
<td>Interactions</td>
<td>Students and teachers <strong>exchange</strong> questions and ideas, offering only relevant knowledge</td>
<td>Students generate the ideas and teacher facilitates the comparing and discussing of select ideas</td>
<td>Students <strong>provided</strong> with steps to contribute in discussions</td>
<td><strong>Student initiates</strong> the discussion by offering ideas to be discussed</td>
</tr>
<tr>
<td>Questioning</td>
<td>Students and teacher ask <strong>question</strong> to investigate thinking that is shared</td>
<td>Teacher <strong>probing</strong> students to insure <strong>in-depth</strong> thinking about ideas</td>
<td>Multiple strategies to help students contribute and remain <strong>highly involved</strong></td>
<td>Teacher selects the idea based on those <strong>generated</strong> by students in the discussion</td>
</tr>
<tr>
<td>Presenting Solutions</td>
<td><strong>Solutions</strong> are presented and <strong>justifications</strong> are evaluated by the community</td>
<td><strong>Process</strong> along with content is stressed</td>
<td>Students learn to explain <strong>without prompting</strong></td>
<td>Solutions are student generated</td>
</tr>
<tr>
<td>Listening</td>
<td>Students and teachers listen to <strong>analyze</strong>, evaluate and extend ideas together</td>
<td>Students highly involved in the process of <strong>analyzing</strong> logical and illogical ideas</td>
<td>Students are skilled at listening to <strong>repeat, summarize and analyze</strong> the reasoning Presented</td>
<td>Students actively listen showing a <strong>high level of engagement</strong></td>
</tr>
<tr>
<td>Assessment</td>
<td>Students and teacher <strong>negotiate</strong> the validity of ideas and solutions together</td>
<td>Students determine <strong>viability</strong> of ideas and solutions</td>
<td><strong>Learning gaps and misconception</strong> s are evident in the talk</td>
<td>Focus on determining what students know and do not understand from the concepts they discuss</td>
</tr>
</tbody>
</table>
Table 4: Models for Using Mathematical Discourse with Students

<table>
<thead>
<tr>
<th>Title</th>
<th>Definition</th>
<th>Interactions</th>
<th>Questioning</th>
<th>Presenting Solutions</th>
<th>Listening</th>
</tr>
</thead>
<tbody>
<tr>
<td>I-R-E Mehane (1979)</td>
<td>A three part sequence of talk including initiation, response and evaluation.</td>
<td>Students wait to be called on before offering an idea</td>
<td>Teachers ask the questions</td>
<td>Answers are collected by T and determined to be correct or incorrect without discussion</td>
<td>Focus is on listening to the teacher</td>
</tr>
<tr>
<td>Classroom Talk Lampert (1990)</td>
<td>Collective discussion among students and teachers about problems presented</td>
<td>Students and teacher make conjectures and converse about thinking.</td>
<td>Students and teachers ask questions to understanding thinking of others</td>
<td>Solutions are justified and determined viable by the group</td>
<td>Students listen to analyze and interpret ideas and differing ideas are offered</td>
</tr>
<tr>
<td>Cumulative Talk Rojas-Drummond &amp; Mercer (2004)</td>
<td>Initiations are accepted without discussion or little amendments</td>
<td>Students offer ideas at any time without debate or justification</td>
<td>Questions are open-ended and not cued to specific responses</td>
<td>Students offer solutions to problems</td>
<td>Students are encouraged to listen to ideas of others</td>
</tr>
<tr>
<td>Dialogic Teaching Gillies (2014)</td>
<td>Interacting with others to complete tasks and build on ideas</td>
<td>Students and teacher actively engage to share ideas and consider alternate perspectives</td>
<td>Questions are encouraged by all</td>
<td>All contribute and assist in building ideas of others</td>
<td>Actively listening, constructive feedback is encouraged</td>
</tr>
</tbody>
</table>
Table 5: Models for Using Mathematical Discourse with Students

<table>
<thead>
<tr>
<th>Title</th>
<th>Franke, Turrou, &amp; Webb (2015)</th>
<th>Cengiz, Kline, &amp; Grant, (2011)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Definition</td>
<td>Students come together in a joint, coordinated form of <strong>co-reasoning</strong></td>
<td>Three step process of <strong>generating</strong> ideas, <strong>comparing</strong> the ideas and discussing an idea in depth.</td>
</tr>
<tr>
<td>Interactions</td>
<td>Students and teachers <strong>exchange</strong> questions and ideas, offering only relevant knowledge</td>
<td>Students generate the ideas and teacher facilitates the comparing and discussing of select ideas</td>
</tr>
<tr>
<td>Questioning</td>
<td>Students and teacher ask <strong>question</strong> to investigate thinking that is shared</td>
<td>Teacher <strong>probing</strong> students to insure <strong>in-depth</strong> thinking about ideas</td>
</tr>
<tr>
<td>Presenting Solutions</td>
<td><strong>Solutions</strong> are presented and <strong>justifications</strong> are evaluated by the community</td>
<td><strong>Process</strong> along with content is stressed</td>
</tr>
<tr>
<td>Listening</td>
<td>Students and teachers listen to <strong>analyze</strong>, evaluate and extend ideas together</td>
<td>Students highly involved in the process of <strong>analyzing</strong> logical and illogical ideas</td>
</tr>
<tr>
<td>Assessment</td>
<td>Students and teacher <strong>negotiate</strong> the validity of ideas and solutions together</td>
<td>Students determine <strong>viability</strong> of ideas and solutions</td>
</tr>
</tbody>
</table>

The following discourse models add to the research outlined earlier and provide exceptional tools for implementing mathematical discourse to compliment the curriculum and state frameworks teachers are already using to teach mathematics in schools across the country (Chapin & O’Connor, 2007; Sherin, 2002; Wood 1998). All three models were selected because of the potential they hold for redefining the nature of learning mathematics in the classroom. In addition, all of the models support the idea that students achieve greater mathematical understanding while tackling authentic problems with others, rather than alone (O’Connor & Michaels, 1996; Sherin, 2002; Wood, 1998). Each of the model will be presented through a sample dialogue as a format for
demonstrating the ways that discourse can be implemented with success in conventional classrooms.

**Focusing**

The first model was developed as an alternative to the conventional classroom interaction format wherein learning is mostly “predictable,” and students are instructed about “how to speak and about what to speak” (Wood, 1998, p. 169). One of these conventional methods is the “funnel pattern” (Bauersfeld, 1980), which is a process wherein the teacher asks a question and continues to collect answers from students until the intended solution is shared. This process communicates to a student that identifying and using the one method to solve each problem is most important in learning mathematics.

To improve this interaction format, Wood (1998) developed a process called “focusing,” which encourages students to formulate solutions and discuss their strategies with classmates before determining the best one to use. What makes this model unique is the way that each discussion is formulated around students’ ideas, not one that the teacher has planned in advance. One of the students’ ideas or methods is selected based on the potential it has in reinforcing or advancing a line of thinking for the group of students in the class. Using focusing during discourse requires an increased level of cognition and interaction from both students and teachers because of the spontaneous, unscripted, and student-driven nature of the process. A sample dialogue from Wood’s research has been chosen to demonstrate how focusing can be used in the classroom in this way. The conversation occurred after the second-grade students shared their self-generated methods for solving two-digit subtraction problems with regrouping. The students in this
class are encouraged to invent their own ways of solving problems. They have not been taught the standard algorithm. The problem students are solving is $66 - 28 = \text{____}$. The session begins with John explaining his solution to the class. The comments offered by Wood, written in italics, are included after the dialogue to clarify the students’ thinking and teacher’s actions.

John: We put the 66 under the 28. Then we took off the 6 and the 8 and, if you take that away, 20 plus 60, it’s 40. And if you put the 6 back on and the 8, we have 46. Then we take away…we still have to take away that 8. Then you take away that 6, now you have 40 back and you still have to take away 2.

Elisabeth: But, but why did you take the 6 and 8 off?

John: It was more easier. (p. 172).

The teacher realizes that John’s solution is new for him and that it is not one that has been given previously in the class.

Teacher: OK, could you write down beside it what you did? Maybe that would help us see it. Instead of 66 minus 28, what did you do?

John: 60 take away 20 equals. (He writes $60 - 20$ vertically and looks at the teacher.)

Teacher: Would you write what you get? (He writes $60 - 20$ vertically and looks at the teacher.)

Teacher: Would you write what you get? (He writes 40 under $60 - 20$.) OK, what did you do next? (pp. 172-174).

Having asked John to explain his thinking by using symbol notation, she enables him to clarify his thinking to the students.

John: Then we put the 6 back on. (He writes $+6$ next to the 40.) Then it equaled 46. And you still have to take away 8 so you have 40 back. And …um…if you take away 2, you have to take away 2 more, so we got 38.

Teacher: (to the class) Make sense? (pause) Do you understand what he said about his part? (coming to the front of the class to use the overhead
He said, I have, let’s put 46 up here. (She writes 46 on the top.) That’s what he has, and then he said ‘I’ve got to go back to 40.’ Okay, why did you go back to 40? (pp. 172-174).

At this point, the teacher decides to step in and summarize the aspects of the solution that she feels most of the children understand. Then, intending to draw attention to the point at which she feels that John’s reasoning has been difficult for others to follow, she asks, “Why did you go back to 40?” She rephrases John’s original statement, “So you have 40 back,” as an action he has performed in an attempt to focus attention on the discriminating aspect of his solution strategy so he will provide a rationale for the rest of the class.

John: Cause we took away that 6, ‘cause you have to take away 8. And you still have to take away 2 more.

Teacher: You understand how he did that?

Class: Yeah.

Teacher: (long pause) Very interesting way to do that. Thank you. (pp. 172-174)

This innovative method, which increases the level of investment and attention during the dialogue because it was student generated, created an opportunity for students to reflect on their thinking and on the reasoning of their peers. It also presents students with the opportunity to initiate ideas and have their contributions validated. The teacher in this example does not try to guide John in a way that funnels his thinking, but gives him control of his explanation while focusing his words and notation so others can follow his mathematical ideas. In addition, students are not directed to use John’s method, but are required to study his solution and follow his “reasoning” by trying to make “sense of it” (p. 175). This process demonstrates the value of working to find meaning in the
different ways others interpret and solve mathematical problems. Also, the most obvious idea may not always be the one that generates the greatest opportunity for a discussion or for expanding students’ understanding.

**Filtering**

The second discourse model originated from the work of Sherin (2002) during the Fostering a Community of Teachers as Learners Project. This model is a system for organizing classroom discourse designed around a technique called “filtering” (p. 203). Filtering requires students to reflect on an idea more deeply than what is typically required when discussing solutions in traditional mathematics classrooms. It is typically used during the final stage of mathematical discourse. The teacher in Sherin’s study uses a discussion framework that has three components: idea generation, comparison and evaluation, and filtering. In the example provided, the teacher Mr. Louis uses the following three questions, which correspond to the three components of the model to guide the discourse:

1. What do you think?
2. Why? or Can you explain that?
3. What do other people think? (p. 58)

Most significant to the effectiveness of this model is the way that students are expected to consider one idea in light of another during their discussions. This helps students to limit repeating the same idea and keeps the level of awareness and involvement high in the conversation. Mr. Louis and his students also discuss whether or not they agree or disagree with each idea generated by the students. During the final component or filtering of the discussion, ideas are discussed in more depth and detail.
than at the initial stages of the discussion. During this part of the conversation, the teacher, Mr. Louis, demonstrates his choice to introduce new mathematical content or complex ideas or to extend the concepts that have been introduced. The following discussion and researcher insights have been chosen to enlighten the reader about both the complex nature of this mode and the potential it has for expanding students’ mathematical thought. The comments offered by Sherin as found in the article are written in italics to provide background to the classroom situations.

*The Slingshot lesson took place during a unit on functions. An important goal of the unit was for students to explore the relationship between changing quantities.* Students were to first collect some data, they would then graph the data, and finally they would write an equation to represent the relationship involved. These students were not in a pre-algebra or algebra class, and the goal of the unit was not the standard \( y = mx + b \) material. Instead, the unit was intended to give students experience in exploring data, interpreting graphs, and writing simple linear equations. Small groups of students were given an apparatus that resembled a slingshot. The apparatus, which consisted of a rubber band strung between two nails, rested on the floor. Using the rubber band, students were to measure the distance that a small ball made out of tinfoil traveled along the floor after being released from the slingshot. The groups were to begin by pulling the rubber band back one centimeter and letting the ball go. They would then repeat the experiment for two and three centimeters. Students were encouraged to take more than one measurement for each of the three distances and to average their results. (pp. 222-225)
The discussion session begins with the analysis of two questions that the students completed for homework the night before (see Figure 3). The purpose of the assignment was to encourage students to reflect on the day’s lesson and to discuss their ideas during the next day.

<table>
<thead>
<tr>
<th>Patrice's group did the slingshot experiment today in class. Her group found that the relationship between the STRETCH OF THE BAND and the DISTANCE OF THE SHOT could be approximated by the equation:</th>
</tr>
</thead>
<tbody>
<tr>
<td>Y = 120 • X</td>
</tr>
<tr>
<td>1. Explain in words what this equation means. (The 120 is centimeters.)</td>
</tr>
<tr>
<td>2. What does the X represent?</td>
</tr>
<tr>
<td>What does the Y represent?</td>
</tr>
<tr>
<td>What does the 120 mean?</td>
</tr>
</tbody>
</table>

Figure 3: Sample homework questions (Sherin, p. 222).

The reflective comments offered by Sherin, as found in the article, are written in italics and have been included to help interpret each section of dialogue.

*Patrice, who said that y corresponded to the distance that the ball traveled and that x corresponded to the amount that the rubber band was stretched and that for every centimeter that you pulled the rubber band back, the ball traveled another 120 centimeters. The class agreed with her equation. Then Mr. Louis asked. “Was it pretty accurate to say that it’s about 120 centimeters?”*

*In response, students introduced a number of factors that they believed would affect whether or not the ball traveled 120 centimeters.*

Mr. Louis: What do you think?

Jeff: Depends on what floor it is.

Mr. Louis: Okay, depends upon what floor it was. Why do you say that?

Jeff: The more, the less, the less friction, the further it goes.
Mr. Louis: Okay, what do other people think? (pp. 221-225)

The students recognized that their data did not demonstrate the constant increase suggested by Patrice’s equation. Thus, they suggested other factors as possible reasons for some variation within each group’s data. Additional variables mentioned included human error and the fact that the balls did not always travel in a straight line. After a few minutes, Ben joined the conversation, raising an issue that was somewhat different from the types of comments made up to this point. Ben explained that while the factors that students had named already would account for some of the variation the groups encountered in collecting their data, there might also be another issue in play. Specifically, Ben wondered if the increase in distance might not actually be constant. Another student, Robert, then explained that if this were the case, graphing the data would produce a curve rather than a line.

Ben: I also think it depends like on how far you pull it back.

Mr. Louis: What do you mean?

Ben: Like if you pull it back to the 1 centimeter, and you do that like three times, like it might be 120 centimeters. But then the first time that you pull it back it, say the second one, it might be farther than 120 centimeters. It might just keep going at a steady rate, but . . . it might be larger than 120 centimeters apart.

Mr. Louis: Does anyone understand what Ben is saying because I don’t quite exactly understand . . .

Robert: I think he means that the graph might not be linear. If you make a graph out of it, it might not go at a constant rate.

Mr. Louis: Is that what you’re saying?

Ben: Yeah.

Mr. Louis: What do other people think about that?
Jeff: The change between 0 cm and 1 cm will be less than the change between 1 cm and 2 cm. (221-225)

In contrast, Sam argued that the variation was due to human error and was not because the difference in the distance traveled was increasing. At this point, David [Mr. Louis] highlighted these two issues for the class:

So I hear people saying two things. One group of people [is] saying that you pull back a certain amount, and then it will go that much farther each cm you pull it back. So each time it goes 120 centimeters farther . . . the same amount farther each time. I hear another group of people saying that possibly, the further you pull it back each time, it goes a little farther. So if you pull it back the first time it goes 120, and you pull it back the second time, or 2 cm back, it might go 140. So it goes a little farther each time you pull it back. So what do you guys think about that idea? (pp. 221-225)

To respond to the teacher, students began to look at their data to see which pattern fit most accurately. Soon he suggested that the class pursue this issue, using the graphing calculator. Mr. Louis introduced the notion of a “scatter plot” as a graph whose values do not make a perfectly straight line. With the students’ help, he entered one group’s data into a graphing calculator that worked with the overhead projector. Mr. Louis selected the scatter plot function so that the data was now displayed in view of the entire class. The students discussed how to visually estimate which one line would most accurately represent the data. In addition, they used the graphing calculator to determine a “line of best fit.” In this way, the class was beginning to deal with different ways to interpret the complex set of data that had been collected. In fact, students began to offer a number of different ideas about why the notion of scatter plot was useful for them, and how they could determine whether one of their own estimates was a line of best fit. After students raised several ideas concerning why the distance might not consistently be 120 centimeters, there is evidence that the class shifts into the comparison
and evaluation structure. In particular, Ben’s comments indicate that he has classified the ideas raised thus far as being about physical factors that affect data collection. In contrast, Ben had a different kind of argument to make. Following Ben’s comment, the teacher encouraged a line of best fit. For a time then, Mr. Louis has taken control of the content of the conversation, has narrowed the space of ideas being raised and discussed. Open-ended discourse is not closed off completely; in fact, he often asks for student input to explain the ideas he is presenting. However, this part of the discussion resembles teacher-directed discourse more than that which occurred earlier. The class then uses this filtering by the teacher to redirect their attention and return again to idea generation. Specifically, they began to discuss what a “line of best fit” would look like (e.g., “There must be the same number of data points above and below the line.” “Should some data points pass through the line?”). It is important to note that considering how to interpret a scatter plot and how to determine the features of a line of best fit constitute significant mathematical content for these students. In the past, they had explored data intended to represent linear functions more precisely—the difference between data values was often consistently the same. Here the students were dealing with a very different set of data, and they needed a new set of mathematical tools to do so. The combination of the graphing calculator with the notion, not of a line that fit perfectly, but rather of a line of best fit, had the potential to help them explore these issues productively (pp. 221-225).

This lesson exhibits how filtering can assist teachers in balancing the process and content in mathematical discourse. As demonstrated in the dialogue and teacher reflections above, filtering or narrowing of ideas allows educators to focus on specific content that they identify as mathematically significant and suitable for a productive
discussion at that moment in time. Although the teacher at the center of this study had success with giving attention to process and content, he did experience some struggles with trying to provide equal attention to both. However, he was able to resolve this conflict by moving back and forth between process and content and dedicated the appropriate attention necessary for each lesson. According to Sherin, teachers should establish the norms they use for discussing mathematics first before moving on to the content. Investing the necessary time to establishing solid discourse practices at the beginning will lessen the amount of time needed to restore the process later as teachers try to shift the focus to content. In addition, the section of the example that explains how and why Mr. Louis decides to extend the lesson is invaluable because it helps teachers see exactly how this transition can be made with students.

**Academically Productive Talk**

The third model is “academically productive talk,” developed as part of Project Challenge (Chapin & O’Connor, 2007, p. 113). Project Challenge involved 400 students in grades 4-7 from a low-income school district located in the Northeast. The teachers using academically productive talk in the classroom made a “powerful impact on students’ comprehension of concepts, on their understanding of computational procedures, and on their ability to reason about mathematical problems” (p. 115). Researchers attributed the positive outcomes to the five “talk moves” that the teachers used to help guide students during the implementation of academically productive talk. The following section provides a description of these moves, including samples of actual classroom dialogues that occurred within the project.
The first move, “revoicing,” is used by all participants in the classroom community (O’Connor & Michaels, 1996, p. 119). It occurs when someone restates a previous speaker’s words followed by a question to verify whether the statement is correct. The goal of the first talk move is to clarify thinking and to improve mathematical understanding. According to this research, this form of revisiting ideas allowed more time for students to catch details, process them, and clarify their ideas than in more traditional formats.

The teacher in the following example has just given her third graders a series of numbers and asked them to identify whether they were even or odd. The first student [Philipe] chooses to discuss the number 24. His rationale is not completely clear.

Philipe: Well, if we could use three, then it could go into that, but three is off. So then if it was…but…three is even. I mean odd. So if it’s odd, then it’s not even.

Ms. Davies: OK, let me see if I understand. So you’re saying that twenty-four is an odd number?

Philipe: Yeah. Because three goes into it, because twenty-four divided by three is eight. (pp. 12-16)

Ms. Davies provides Philipe a chance to clarify his thinking. In addition, in using the revoicing talk move with Philipe, the teacher can uncover where the misunderstanding originated. This talk move also allows the teacher to gain a greater understanding of Philipe’s thinking because now she is sure that he is having difficulty with the concept.

The second talk move, “repeating,” encourages students to repeat a classmate’s reasoning, which is initiated by the teacher to help the learning community make sense of another student’s thinking (Chapin & O’Connor, 2007, p. 121). Students using this talk
move were able to increase their understanding during their classroom discussions. The researchers found that as students gained experience in using this talk move, the more they were able to produce elaborate and clear explanations when presenting their ideas.

In the classroom example below, Ms. Davies used this move to continue the classroom conversation.

Ms. D: Can anyone repeat what Philippe just said in his or her own words? Miranda?

Miranda: Um, I think I can. I think he said that twenty-four is odd, because it can be divided by three.

Ms. D: Is that right, Philippe? Is that what you said?

Philippe: Yes. (pp. 12-16)

The third talk move, “analyzing,” involves asking students to apply their own reasoning to the reasoning of someone else. The goal of this talk move is to “elicit” students’ reasoning and to develop their ability to analyze the ideas that their classmates present (p. 122). According to researchers, teachers were able to identify learning gaps and misconceptions in students’ understanding by structuring the discussions in this manner. According to the authors, students who used this move regularly explained their thinking without being asked. In addition, this talk move increased the teacher’s ability to design instruction to meet the needs of the students. The interaction is described below.

Ms. D: Miranda, do you agree or disagree with what Philippe said?

Miranda: Well, I sort of…like, I disagree?

Ms. D: Can you tell us why you disagree with what he said? What’s your reasoning?

Miranda: Because I thought that we said yesterday that you could divide even numbers by two, and I think you can divide twenty-four by two. And it is twelve. So isn’t that even? (pp. 12-16)
The fourth talk move, “adding,” prompts students to offer additional contributions to the discussion. In this move, teachers request that students add to, or comment about, what another student has said so that they can dive deeper into the concepts. Also, this talk move is used to encourage participation and give those who have not yet participated in the discussion a chance to contribute. Below is an example dialogue used in one of the classrooms.

Ms. D: So we have two different ideas here about the number twenty-four. Philippe, you’re saying that twenty-four is odd because you can divide it by three?

Philippe: Uh-huh.

Ms. D: And Miranda, you’re saying that it’s even because you can divide it by two? Is that correct?

Miranda: Yes.

Ms. D: OK, so what about other people? Who would like to add to the discussion? Do you agree with Miranda’s or Philippe’s ideas? Tell us what you think, or add another comment or insight. (pp. 12-16)

The fifth and final talk move, “using wait time,” is essential for encouraging student participation. Wait time occurs when a teacher asks a question and allows time for a student to give his or her response. This allows students time to organize their thoughts and feel less pressure to share before they are ready (Chapin, O’Connor, & Anderson, 2003). Instituting wait time helps to raise the participation level, which also increases the benefits of the discourse process.

There is significant data from this project that demonstrate that students improved their understanding of mathematics (Chapin et al., 2003; O’Connor & Michaels, 1996). Students who engaged in “academically productive talk” for 1, 2, or 3 years performed
significantly better on standardized testing than students who had not used this model (Chapin et al., 2003). This is important because before participating in the project, only 4% of the students in the first cohort were rated “superior” or “very superior” in their mathematical ability, 23% scored “above average,” and the remaining 73% were “average” or “below average,” according to the TOMA-2 (Test of Mathematical Abilities, Second Edition). After the second year of using the talk moves in Project Challenge, 41% of the students tested in the “superior” or “very superior” range, 36% scored “above average,” and 23% were “average,” with no students falling in the “below average” category. In addition, the students scored better than 91% of their peers nationally. This research provides the empirical evidence to demonstrate the advantages of using strategic discourse techniques during the study of mathematics as a means for building understanding with students (O’Connor & Michaels, 1996).

**Key Ideas Embedded in the Discourse Models**

The techniques used in each of the three models can assist teachers in understanding how and why using discourse to solve problems benefits student learning (Chapin & O’Connor, 2007; Sherin, 2002; Wood, 1998). The models are excellent resources that can be used to guide the implementation in classrooms. Teachers can select from the methods within the models to gain the experience they need until they can identify which will be most beneficial for their students. With that in mind, teachers must use three key ideas to implement effective discourse practices in their classrooms: (1) shared mathematical authority (2) using productive and consistent talk, and (3) appreciating the value of listening to others. Although these ideas are embedded in the techniques within the discourse models, they need added attention due to their
significance to the process. Also, additional educational research connected to strengthening the case for using the idea will be shared, as applicable.

**Shared Mathematical Authority**

The reform-based learning practices presented in this document have encouraged methods that require teachers to move away from seeing teaching as identifying students’ deficiencies and more toward finding potential in their contributions. According to Lampert (1986), it is the educators role to ‘bring students’ ideas about how to solve and analyze problem into the public forum of the classroom, to referee arguments about whether those ideas were reasonable, and to sanction students’ intuitive use of mathematical principles as legitimate” (p. 339). What is most significant about these learning methods is the view that students are viewed as “sense-makers rather than re-memberers or forgetters” (p. 340). The impact these changes have on raising students level of authority during discussions is noteworthy. Engaging students in discourse practices helps facilitate students’ thinking and build mathematical flexibility and power (Gee 1991; Lampert, 1990; NCTM 2000). “Mathematical power” is defined by the NCTM (1989) as “an individual’s ability to explore, conjecture and reason logically, as well as the ability to use a variety of mathematical methods effectively to solve non-routine problems” (p. 5). Research supports using discourse as an effective mechanism to help students achieve mathematical power (Lampert, 1990).

**Using Productive and Consistent Talk**

As described previously, research has stated that structured mathematical talk is beneficial to the students’ learning process (Rojas-Drummond & Mercer, 2004; Cenzin et al., 2011; Franke et al., 2015). However, this process does not just happen because
students have been exposed to a discourse model. Teachers must be diligent about promoting productive talk and providing ample time to practice using it in the classroom with students. This can be extremely challenging for teachers. For example, the teacher [David] in the Sherin (2002) study struggled with the constant challenge of keeping the focus of conversations on mathematical content while using class discussions. In addition, he expressed concerns about students adhering to the discourse process.

Periodically, students would listen to peers’ solutions without providing useful feedback even though the expectation was that students should provide reasoning to support their ideas and assess whether other students had stated their case well, they did not always do it. When this occurred, David reminded them of the expectations, and they were able to assume their responsibilities, but the process did require constant maintenance. Overall, however, David was impressed by his students’ achievement as a result of using the discourse model. Observations like the one below were documented in a journal he kept throughout the study:

There are several interesting things happening here. First, the [discourse] norms are hard at work. Students are building on each other’s knowledge and work. . . . The second . . . is the mathematics. I never would have expected to discuss [the mathematics] in such detail and depth. (p. 21)

The discourse models were developed to help educators increase the mathematical talk in classrooms and bring about deeper understanding of the mathematical concepts students are studying. Academically Productive Talk (Chapin & O’Connor, 2007) provided teachers with a system for initiating productive talk with the goal of developing habits that could be used by students independently. The teachers using this model used the talk moves and made the most out of their discussions, which proved to assist
students in developing a greater understanding of mathematics. The project’s success was attributed to the students’ abilities to express themselves clearly.

However, issues developed during the study. At times, some of the participants refused to talk or were ridiculed by classmates when sharing their ideas or asking questions. There were also instances in which certain students dominated the conversation and left little chance for others to participate. These issues remind reformers of the importance of building and sustaining a supportive, respectful, and equitable environment that supports the success of using mathematical talk with students when building mathematical understanding.

In Wood’s (1998) discourse model, students were required to be proactive by offering their ideas for presentation to the group. They could not sit back and wait for their teacher to plan the discussion or encourage their involvement because the discussion was planned around their contributions to the conversation. In addition, when students did present ideas, they were given the time and freedom to explain their thinking to the group. Their ideas were not just offered but also investigated. For instance, when a student [John] shared a novel idea, the teacher in the study allowed him to explain what appeared to be a confusing method for his classmates to follow. If fact, she even encouraged him to create a written representation of his thinking so that his strategy could become more transparent to his peers. This student and teacher interaction demonstrates the value that students and their ideas bring to learning mathematics. Successful interaction can also assist in encouraging students to participate even more fully in future discussions.
Although using productive and consistent talk with students can be beneficial, it also presents some challenges for both teachers and students (Chapin et al., 2007; Sherin, 2002). For example, researchers identified young students who participated in interesting discussions and offered remarkable insights, but were not always able or willing to support their ideas (Carpenter, Franke, & Levi, 2003). In addition, students are not always able or forthcoming when asked to help encourage fellow students to reach deeper levels of understanding or achieve more academically (McClain & Cobb, 2001).

This does not mean that teachers should give up. Many positive outcomes have emerged from research on this topic, for example Khisty and Chval (2002) have identified that teachers who model the discourse practices and mathematical terminologies that they want students to use have more success than teachers who do not. They reported that fifth-grade bilingual students who experienced vocabulary-rich mathematical conversations with the support of their teacher were able to use more sophisticated math talk and successfully explained the meanings behind their calculations than other bilingual students who did not receive the intervention. Also, the teacher in this group did most of the talking early in the year to model the discourse behaviors that students would emulate later in the year. These students reported that this helped them to understand the expectations and to make an easier transition to using discourse on their own.

Conversely, the students from the other second-grade bilingual classroom who did not receive the intervention were unable to engage in effective discourse practices because their teacher did not use discourse strategies or sophisticated mathematics vocabulary. Although these students worked productively and solved problem-solving
tasks correctly together, they did not engage in talk about the problem or the problem-solving process. Furthermore, only a few of the students from this group were able to discuss the mathematics when asked to explain their solutions (Khisty & Chval, 2002). Although these students were younger and probably less knowledgeable, the researchers concluded that the deficiency had more to do with the absence of talk than with their age.

Clearly the experiences and deficiencies between students who engage in practices and those who do not are noteworthy. Not implementing mathematical talk that adds to the learning of mathematics in the classroom is detrimental to students because it restricts their ability to learn mathematics with meaning and understanding (Khisty & Chval, 2002; Ross, 1995). In fact, Ross (1995) found that in classrooms where mathematical discussions were not used, the students avoided asking questions for clarification. These students reportedly assumed that they were correct even when they were not because their thinking was never shared or monitored. This lack of discussion caused conceptual uncertainty and misunderstandings that could have been remedied by receiving quality feedback from teachers and peers. The research detailed has implications for future research because it brings to the surface the factors that can impede the process of using productive talk successfully with students (Chapin et al., 2007; Khisty & Chval, 2002; Ross, 1995; Sherin, 2002). Being aware of the benefits and the impediments can help educators make the important decisions about how to initiate and use talk effectively with students for a more successful implementation.

**Valuing Listening to Others**

All three of the discourse models were developed to advance thinking by building on students’ ideas and opinions as they wrestle with mathematical ideas in a
learning community. However, this goal cannot be actualized without having teachers who really listen to what students are saying and then seize on the opportunity to interact with their thinking (Cenzin et. al, 2011). Listening and capitalizing on what is heard are two actions that can help teachers get a better understanding of what students know and what they do not know so that they can develop a plan to improve learning more effectively.

Listening to students permitted the teacher [Ms. Davies] in Chapin and O’Connor’s (2007) academically productive talk model to be more aware of the difficulty a student was experiencing. This occurred when a student [Philippe] was having trouble understanding the concept of even and odd numbers. By using the “revoicing” talk move, Ms. Davies was able to gain a better sense of Philippe’s thinking, pinpoint the problem, and quickly help him clarify his understanding. In this case, listening was instrumental in helping this teacher determine a student’s misunderstanding, as well as designing instruction to meet her students’ needs more appropriately.

Listening is also key to the success of Wood’s (1998) focusing model because the model requires the teacher to select one of the students’ ideas that have emerged during a mathematical discussion with the community. Listening for and selecting the right idea that is beneficial to the academic growth of the entire group are crucial to this technique. After the teacher identifies something valuable in a student’s thinking, the teacher focuses the group’s attention on the idea and highlights the student’s reasoning in a way that peers can follow. With the help of the student who originated the idea, the teacher elaborates on his or her thinking and assists the student in selecting the notation that will further clarify his or her idea. The success of this model is contingent on the ability of
the teacher to listen, choose the proper idea, and lead students in a direction that will be meaningful to all.

Although listening to students is not groundbreaking, it is often overlooked. Teachers have too many students and not enough time to allow every student to share his or her ideas. However, listening to the different ideas that students have about how and what they are learning is beneficial and worth the investment of time (Chapin & O’Connor, 2007; Sherin, 2002; Wood, 1998). Students have a different view of the classroom and can add valuable input about learning and life in the classroom. For example, in Sherin’s (2002) study, students shared their frustrations with their teacher [Mr. Louis] regarding the data collection component of the Slingshot unit they were studying. In addition, Mr. Louis asked students to provide feedback about how working in groups was helping their learning and solving of tasks. Both types of feedback provided information about how students were handling the activities in which they were engaged. In addition, Mr. Louis was more informed about the needs of his students and acted to improve his practice based on the input he received.

As teachers increase their focus on listening, students will begin to perceive its value and use it with purpose for their own needs. Furthermore, honoring student contributions by providing them with a forum to speak and by modifying practices based on their input demonstrates the value they bring to the learning community (Yackel & Cobb, 1996).

Recently, there has been a push by mathematics education reformers and researchers to encourage teachers to delve into the minds of their students to advance their mathematical thinking (Hiebert & Grouws, 2007). In addition to the types of
listening found within the models, additional research exhibits the advantages of changing classroom practice from an environment consumed by teacher talk to an environment where learning is constructed by students and teachers listening and reacting to others with purpose (Martino & Maher, 1999; White, 2003). First, teachers must aspire to get all students participating in discourse activities. Even students who have been put at a disadvantage by virtue of their race, economics, or academic ability can find success using this process (Martino & Maher, 1999; White, 2003). Research has revealed that students who are supported as they talk and listen while discussing mathematical solutions in a community learn to use and value these skills as habits of meaningful talk (White, 2003). Second, teachers who actively listen for the purpose of extending students’ thinking and understanding have a greater effect on developing meaning with their students than those who do not (Martino & Maher, 1999).

According to White (2003), students who received encouragement from their teachers to engage in discourse first by listening to the ideas and solutions of others and then by offering their own perspective developed as learners. The participants were from two third-grade classrooms in a large, urban school district in the Washington, DC area. The students were similar in their diverse ethnic and socioeconomic backgrounds, but heterogeneous in their academic abilities. Both of the teachers participating in the project successfully promoted mathematical learning through discourse, which led to the success of their students. Students were expected to solve problems in their own way and to explain their answers and solution strategies to the group. They were also expected to ask questions of one another before deciding on a correct solution. As a result of the discourse patterns used in this study, students were more fluent and able to express their
ideas more easily. Furthermore, the students’ conversations reflected more mathematical ideas as they learned to build upon their familiarity with the discourse process and knowledge of mathematics. By placing a premium on active listening, teachers demonstrated the value of listening and of their students’ contributions to furthering the learning in their classrooms. Furthermore, the teachers and their students benefited from being exposed to a variety of different ways to think about solving problems because students shared so many different perspectives about both the problem and the solution strategies. These achievements are significant because the student population involved in this study had not typically been encouraged to use discourse while learning mathematics. This research dismisses the thinking that some students need to be told what and how to think and solve problems to be successful.

Also, the teachers in this study reported being more informed about their students’ thinking after listening and watching how they solved and discussed the problems they were working on in class (White, 2003). Still, to realize the full benefit of discourse, teachers cannot just monitor students’ contributions for logical and accurate solutions. Teachers must also actively listen to students to assess their understanding, interpret their thinking, and plan their next move (Martino & Maher, 1999; White, 2003).

These were the actions taken by teachers who participated in a longitudinal study of 150 third-, fourth-, and fifth-grade New Jersey students from three diverse school districts (Martino & Maher, 1999). All teachers involved received training regarding how to work through the mathematics themselves and how to analyze what students should be able to know and do before leading discussions with students. This allowed
them to skillfully listen, interpret student ideas, and seize on opportunities to stimulate further thought.

According to the results of the study, the teachers achieved success in furthering students’ ideas by requiring them to justify their solutions and trying to convince their peers of their reasoning (Martino & Maher, 1999). Using active listening allowed teachers to develop greater meaning with their students by extending both their thinking skills and understanding. For example, the teachers in the study extended students’ thinking by helping them to re-examine solutions, requiring them to justify them, and asking how they had determined that they had identified all the possible solutions. The teachers also helped students to make connections between past learning and new learning. This was achieved by asking students to reflect on past problems to determine whether the new strategy could be a more effective solution to the original problem.

In addition, this research demonstrates that teachers who listen to students and then use timely questions to support students’ reinvention and extension of ideas have a positive impact on their mathematical thinking and understanding (Martino & Maher, 1999). Asking quality questions to draw out necessary information from students can be used to direct the entire group’s attention to an individual’s line of thinking (Lampert, 1990). Lampert used questions to insure that students were actively listening and encouraged her students to ask questions and offer feedback themselves.

The quality questioning used in the Martino and Maher (1999) study played a critical role because it allowed the teachers to challenge students to examine their solutions in more depth than they were able to do on their own. Furthermore, this research showed that students valued the skills that they were learning as part of their
conversations with their teachers about mathematical solutions (White, 2003). Providing students with experience in listening to other students and expecting them to ask questions of one another, helps them to use these skills while solving problems independently. Yet, to attain the benefits of listening, teachers must develop the skills they need to support students during the process. If teachers have limited understanding of mathematical content or pedagogy, they will be unable to help students clarify and extend their understanding or even identify the weaknesses in their thinking (Sherin, 2002).

**Professional Learning**

Using discourse with students is a challenging and complex process. Knowing the mathematics content can assist teachers to make more informed decisions when directing the discourse (Wood, 1998, Cenzin et. al, 2011). Second, handling the spontaneity of discourse and effectively manage the course of the entire discussion while teaching the lesson is also advantageous (Cenzin et al., 2011; Franke et al., 2015; Mercer, 2000;). Third, teachers need a way to encourage students to participate fully in the process (O’Connor & Michaels, 1996; Sherin, 2002; Wood, 1998). As difficult as the process can be, teachers who use discourse to teach mathematics reveal that their own teaching had taken on a “new depth and interest” (Chapin et al., 2003, p. 113). In addition, allowing students to share their creative ideas can help make the discussions more interesting and engaging for all. However, there are a few things that teachers must know and do to make using discourse effective for improving instruction.

First, Wood (1998) explains how using talk productively requires an increased level of cognition and interaction from students and teachers because it is spontaneous,
unscripted, and student driven. Teachers need the mathematical knowledge to support their students’ learning through discourse practices. In addition, when teachers draw attention to novel ideas, they must know and be able to communicate the value of the idea to students. Having well-developed mathematical knowledge will assist teachers in selecting the appropriate idea that will contribute to the collective knowledge of the class and support the curriculum (Cenzin et al., 2011). This is especially true for teachers who are using focusing during the final stage in their discussions (Sherin, 2002). For example, David chose to introduce new mathematical content to assist students in determining a solution by presenting an idea that would extend their thinking during the final part of the Slingshot lesson. Making this decision would not be possible if he had not been knowledgeable of the mathematics he was covering with his students.

Another strategy that can be used to encourage more student participation in the discourse is “wait time” (O’Connor & Michaels, 1996). This talk move requires teachers to provide time for all students to process the questions or statement presented by members of the community so that they increase their opportunity to contribute to the dialogue. Establishing and encouraging an environment that includes all students and supports discourse constitutes a complex topic that requires a commitment by teachers, students, and administrators. This topic will also be emphasized in Part II of this document.

**Increasing Capacity and Skill**

Guided discourse practices are complex, and teachers need time to understand the benefits and support to make the changes in their pedagogy (Cobb et al., 1997; Lampert, 1990; Hmelo-Silver, 2004). Second, teachers can plan more effectively when they have
the mathematical knowledge they need to support their teaching, especially while using mathematical conversations (Cenzin et al., 2011).

This knowledge of mathematics is apparent in Lampert’s (1990) comments that illuminate her thought process during the point where she determines the next stage of her work with students during a multiplication unit:

In the final step, the two groups of jars were “recomposed,” and the distributive law was illustrated by adding together the total number of butterflies in each group of jars….This is usually the glitch for children in what mathematicians call the “distributive law”; as a principle, it is obviously warranted when it is attached to quantities in stories like this one. But when students see only (4 X 10) + (4 X 2), it is hard to explain why the answer is not obtained by doing 8X20. The next part of the lesson is intended to get at the idea of finding another grouping to illustrate that the principle of decomposition and the distributive law are legitimate to use on groups other that those determined by place value. (p. 323)

Unfortunately, the overall confidence in the teachers’ ability to successfully manage the complexities of productive discourse is low (Askey, 2001; Ding, Li, Piccolo, & Kulm, 2007). Specifically, research has uncovered how teachers are not skilled in guiding students or scaffolding while teaching mathematics (Ding et al., 2007). These results are part of a multi-case study of six sixth-grade teachers using the Middle School Mathematics Project with their students in Delaware and Texas. The five-year longitudinal study examined the use of teacher interventions and their impact on student learning. Researchers analyzed six videotaped lessons of teachers as they taught the same content, using cooperative learning structures and the identical Connected Mathematics Project textbook. In addition to the weaknesses mentioned above, teachers also had difficulty in leading students to identify multiple approaches for solving problems or elaborating on the other students’ ideas. In addition, researchers found that some teachers had a tendency toward guiding students to identify just one solution.
According to the authors, these teachers were less effective in improving students’ thinking skills than teachers who assisted their students in finding multiple solutions to problems (Ding et al., 2007).

Other researchers raised their concerns about the difficulty that teachers have in facilitating discourse and assessing students, especially young students (Cohen, Lotan, Abraham, Scarloss, & Schultz, 2002). However, researchers found that when teachers used specific task-aligned evaluation criteria and provided it to their students before the unit of study, favorable results were achieved. This study involved 163 sixth-grade students and five teachers from a linguistically, ethnically, and racially diverse population in California’s Central Valley during 1998-1999. All teachers involved in the study were highly trained in the instructional strategies used during the project. Three of the teachers used activities to practice interacting and using the evaluation criteria before their use in the final unit. In the other two classrooms, students used general activities designed to improve the quality of their class discussions. Data from group products and presentations were also collected. Audios of the students’ conversations were analyzed. In addition, essays detailing the activities from the unit and their connection to the objectives were collected from all students. According to the data set presented in the study, students who were given the evaluation criteria had a higher rate of evaluating their work and achieved higher scores on their group product than the groups without access to the criteria. In addition, students were observed as being more engaged in the process of solving their tasks effectively because they knew how they would be judged. The increase in the productivity of the students was cited as the reason why students achieved the high scores on their essays.
This research also identified specific benefits of using criteria for assessing discourse-based activities. First, group products provided an excellent tool for assessing students’ understanding of the fraction concept studied in this research. Secondly, using an evaluation criterion that is specific to the unit provided the teachers with a more successful method for conducting the formative assessments. This meant that they were able to evaluate the objectives for the group work and the project and were able to offer specific feedback to students during presentations. Teachers who work without feedback tend to make only general comments and not ones that help with a specific task. However, the authors acknowledge that students and teachers must practice using the criteria to achieve similar results in their classrooms.

Although challenges exist, developing a discourse community in one’s classroom can be a powerful source of professional development for educators (Fennema et al., 1996; Hmelo-Silver, 2004; Hufferd-Ackles, Fuson, & Sherin, 2004). It is not just the students who learn, but the teacher as well. In fact, teachers have been quoted as rethinking their understanding of mathematics and pedagogical strategies after listening to students share novel ideas during conversations (Hmelo-Silver, 2004; Sherin, 2002). Also, teachers have improved their practice by gaining experiences using new methods while participating in research projects (Hmelo-Silver, 2004). These experiences include learning to assess students’ prior learning and new knowledge, building strategies, and developing inferential thinking. The success of the project was attributed to the teachers’ understanding of the valuable objectives and the support they received from the researchers as they implemented the new methods.
**Conclusion**

Learning mathematics in school remains focused on the independent acquisition of isolated skills, with little or no interaction (Hiebert et al., 2003). Even as many call for widespread improvements in, and modifications to, school mathematics, things seem to remain the same. Also, two issues have added to this lack of progress: First, mathematics education in general is being criticized and pressured to make quick changes due to the flat scores on many national standardized tests (Ravitch, 2010). Second, the ongoing debate between reformers about how students learn mathematics effectively and what they learn has been a distraction and an obstacle for the educational leadership. This has left many have to wonder about the future of mathematics education.

Yet, there is reason to be optimistic. The body of promising research grounded in the situated learning theory presented in this document offers an approach with the potential for transforming mathematics learning. This research supports students’ building meaning along with others in a community of practice as a more viable method for learning mathematics in school (Lave & Wenger, 1991). The positive results of using this method with elementary students in actual classrooms have been provided to help educate teachers about the benefits this method has on student learning. First, students learning in a community of practice strengthened their communication skills and mathematical understanding by developing their own system for discussing the concepts they were studying (Cobb et al., 2001). Second, students demonstrated their ability to choose, adapt, and apply methods to various problem-solving situations while solving problems together in a more open-ended classroom (Boaler, 1999). Third, students were observed making their own decisions, reasoning and analyzing information successfully,
while interacting with resources and other students to complete group tasks (Boaler, 1999; Greeno, 1997). All of these experiences have improved students’ mathematical understanding and thinking skills.

Although others have discussed making similar changes to teaching and learning in the mathematics classroom, this document provides a much more comprehensive format. Specifically, the information includes an explanation of the theory [situated learning], the process [community of practice], and the methods supported by both contemporary and historical research that are necessary for implementing the method. As noted earlier, three methods were presented because they were important to establishing and sustaining a community of practice in the classroom. The first method, problem posing, provides teachers with information about the benefits of offering students opportunities to formulate, present, and evaluate ideas while working with others to improve math skills and understanding (McClain & Cobb, 2001; Cobb et al., 2001). The second method recommends increasing active learning through authentic experiences as a way to help students make connections between the mathematics they learn in school and the mathematics they need for living (Ball & Bass, 2000; Boaler, 1999; Lampert, 1990). Conceptions of authenticity were detailed to guide teachers in using authentic activities so that students could experience mathematics as real people use mathematics (Weiss et al., 2009). Cases of students engaging in dynamic and useful activities assisted in demonstrating how these methods increase the students’ achievement and ability to apply and adapt new methods. Also, the research provided a connection between an increase in authentic tasks with the improvement of students’ positive view of mathematics in school (Boaler, 1999).
Finally, the third learning method, learning through interaction, explains the importance of developing collaborative methods and practicing those with students as part of the successful implementation of a community of practice. Interacting with teachers and other students was reported as having a positive impact on students’ behavior and achievement. However, students need support in learning the process and time to practice the skills before using them independently to realize the full benefit (Gillies, 2003; Rojas-Drummond & Mercer, 2004).

Along with the methods, four discourse models were included to assist teachers in acquiring a deeper grasp of how to use mathematical talk to maintain a successful community of practice. The models are described and accompanied by classroom examples to provide a snapshot of how using productive discourse can help guide teachers’ interactions with students. The first model, stepping in and stepping out, accentuates using discussions for initiating students’ thinking, as opposed to telling students what and how to think (Lampert, 1990). The second model, focusing, encourages students to generate solutions, and then one idea is selected to concentrate the group’s attention for the purpose of strengthening thinking and validating contributions (Wood, 1998). The third model, filtering, assists students in learning to reflect deeply on a specific idea (Sherin, 2002). Most significantly, the students learn to compare one idea with many others presented by the group. They work through each of the solutions presented until they can agree on the one that is most suitable for solving the problem. Finally, the fourth model, academically productive talk, helps students to increase their comprehension of concepts, understanding of computational procedures, and reasoning skills by using five talk moves (O’Connor & Michaels, 1996). Each move is designed to
increase the amount of talk used, provide feedback from the community, and increase students’ understanding of the topic. All four of the discourse models were selected for this document to demonstrate specific techniques that teachers could use to examine thinking and to advance students’ understanding.

In addition, three main ideas were extracted from the models to emphasize their importance to the process of implementing a community of practice. The three ideas are using productive and consistent talk, listening and capitalizing on what is heard, and increasing teacher capacity and skills. To begin the process of initiating a community of practice, teachers must get kids to use talk that is productive, meaning that it furthers their learning and the learning of the entire group. Using this type of talk takes effort and time, but as noted throughout the document, the improvement regarding the students’ evaluation, reasoning, and communication skills are worth it (Sherin, 2002). In fact, even when teachers reported having difficulty in using the process with their students, they agreed that students who used talk in math were much better off than students who did not (Khisty & Chval, 2002; Ross, 1995). To improve students’ understanding, all members of the community must listen and capitalize on what they hear. Unfortunately, in our current rush-to-finish mentality, we have overlooked the need to stop and listen to one another. However, listening to other members of the community during problem solving allows teachers and students to be able to support understanding (Martino & Maher, 1999; White, 2003). Furthermore, to keep the community thriving, teachers must have skill and capacity. The research presented previously illustrates that this is an area of concern because many teachers do not have strong mathematical backgrounds and have not been prepared to manage discourse (Askey, 2001; Ding et al., 2007). However,
learning to use the discourse process has reportedly been a positive experience for teachers and students as both groups reported gaining skills (Cohen et al., 2002). Therefore, teachers must be open to attaining these skills because they need a strong knowledge of mathematics to make more informed decisions about how and what to cover (Lampert, 1990). They also need to be able to use their knowledge as they handle the spontaneity required of them during discourse (Sherin, 2002, Wood, 1998).

In addition to the explanation of the methods, models, and strategies, additional research in the field of education was included to further solidify the importance of these ideas to sustaining a community of practice. This is significant because transforming theory into practice is a complex process. First, teachers need to understand the theory, but this must also include being informed about the value it brings to improving their pedagogy and student learning. For instance, in the situated theory, it is important that teachers recognize the value of providing students with more opportunities to participate and communicate while learning mathematics. Unfortunately, in most cases, teachers are typically handed a theory and required to design their own methods to support it in their classrooms. This can also be problematic for administrators because this type of implementation breeds inconsistencies or avoidance.

The research provided in this document demonstrates the potential for using a CoP in their mathematics classrooms. Implementing the methods suggested in this document is a complex process that requires making real changes in the classroom. However, teachers who use these strategies will help students to develop the thinking skills they need to achieve greater understanding and mastery in mathematics (Chapin & O’Connor, 2007; Martino & Maher, 1999; White, 2003). Educators can use the
information provided in this document as a guide for using talk productively to investigate the mathematics they study with their students in a community of practice. After gaining sufficient experience with discourse models, they can develop a specific plan that meets the needs of their students. However, most critical to making these changes is the need to shift the emphasis from individual student learning to group learning. This process will require deep and systemic reform. Restructuring the classroom to include giving students a more predominant role by encouraging their participation and valuing their ideas is fundamental. This kind of change also requires that the traditional teacher and student relationships be modified for the community of practice to thrive. More information about the need for teachers and students to work together as partners to improve the teaching and learning of mathematics will be presented in Part II of this document. In particular, Part II will draw attention to the power structure and power relationships that exist between members of the educational community and the limits they have placed on students, teachers, and the school environment.
CHAPTER 3

DESIGN AND METHODOLOGY

Introduction

The goal of this chapter is to describe the research methodology used in this study, along with the description of the data collection and analysis procedures. The purpose of this research study was to investigate the strategic discourse practices used by two elementary school teachers and their students during problem-solving in mathematics. Additionally, the methods also provided an opportunity to learn about the usefulness of discourse strategies from the perspectives of the participants. This topic interests me because of on my own experience as a classroom teacher working to help students develop solutions and articulate their thinking about the problems during mathematical conversations. During this time, I saw first-hand the success and struggle students had in effectively communicating their mathematical ideas. I worked to identify ways to help them succeed. Additionally, my experiences as a doctoral student, Assistant Principal, and Mathematics Curriculum coordinator has afforded me the opportunity to use and share my knowledge about the implementation of discourse practices with teachers and students. During these experiences, it became clear to me that students and teachers engaging in mathematical conversations were using in-the-moment decision making. Even though the interactions had some meaning, the untapped potential existed. Therefore, I set out to find out more about the discourse practices teachers were using and how they could structure their conversations in a way that would be more productive. I continue to be committed to learning how students and teachers can structure the
discourse in their classrooms to build an understanding of mathematics among their students.

This study provided the opportunity to investigate the use of discourse practices between teachers and students in the natural setting of the elementary mathematics classroom. Additionally, the commonly used discourse practices implemented by teachers and their students were examined. Also noted are the discourse practices that were beneficial in building mathematical thinking and understanding in mathematics, according to the perspective of the participants. Examining the practices that allowed students to share their mathematical authority in a learning partnership with teachers was an additional focus of this study.

Qualitative research is a broad field of inquiry but includes a few common features resulting in narrative, descriptive accounts of settings or practice (Rossman & Rallis, 2003). Most typically qualitative research contains an inductive approach where patterns and themes evolve and constantly change throughout the process of data collection and analysis (Cresswell, 2003; Rossman & Rallis, 1998). In this study, qualitative methods were used to collect information within two math classrooms to gain a more thorough understanding of how strategic discourse was used. Frequency tables were used to present the occurrence of each of the strategic discourse elements used by teachers and students during classroom events (Erickson, 1998). This assisted in determining which of the talk models were most closely aligned with the discourse happening in each of the two classrooms. According to Yin (2003), “A research design is the logic that links the data to be collected, to the initial questions of study” (p. 19). The research design used in this study is the descriptive case study. The descriptive case study
will provide the approach needed to generate an informative account of the phenomena in this investigation.

**Rationale for Case Study**

A case study allows an “empirical inquiry that investigates a contemporary phenomenon within its real-life context… and in which multiple sources of evidence are used” (Yin, 1994, p. 23). Using the descriptive case study research approach to drive the data collection and analysis aligns very closely with the purpose of this study. First, a focus of the study is to provide a description of the ways discourse was used in the classroom to gain a sense of the phenomenon. Therefore, a descriptive case study is most suitable for this study due to the complexity involved in understanding the back and forth nature occurring during the daily discourse among key informants in the classroom (Rossman and Rallis, 2003). Secondly, identifying and explaining the types of discourse practices used by two elementary teachers and their students as they solve problems together in mathematics is critical to the study design. The descriptive case study method will help to determine how each discourse practice is used in these two classrooms (Yin, 1994). Next, critical to the study is shedding light on the perceived effects of the discourse strategies and how they may have impacted the students’ mathematical understanding from the perspective of the key informants. Exploring their perspectives will inform this study even further. Moreover, investigating perspectives of participants will help to uncover how they make sense of their world and experiences. According to Merriam, the reality is in the participant's ever-changing individual interpretation of the phenomenon (2009). Reality in this case, will also include the researcher’s interpretation
of the participants’ reality. Furthermore, data will reveal the ways in which teachers and students share mathematical authority while they study mathematics together.

There are many types of case studies with various components (Mariano, 1993; Stake, 1995; Yin, 1994,). According to Yin (1994), the case study design must have five components: the research question(s), its propositions, its unit(s) of analysis, a determination of how the data are linked to the propositions and criteria to interpret the findings. Propositions are the logical relationships among concepts. A proposition explains how concepts are connected. Theories come from building relationships between propositions. This format guides the outline of each case and the cross-case analysis. Defining the unit of analysis will assist in clarifying information, and with replication and efforts at case comparison. This format, along with the variety of data sources collected and analyzed, will allow this researcher to gain a deeper understanding of each case and then allow for the development of insights among the cases (Stake, 1995). This descriptive case study was written using an inductive investigative strategy with myself, the researcher, as the primary instrument in the data collection and analysis process (Merriam, 2002).

Case studies are also typically bounded by time or place in order to inform a problem (Cresswell, 2007). This study occurred during the Spring of the 2014-2015 school year. All observations took place in the classrooms during their regularly scheduled math block. The choice to study the second semester of school was purposeful because teachers had already established and practices their routines and expectations and were able to focus almost completely on instruction. This time of the year was conducive to collecting the valuable data that Cresswell described as necessary (2007). Also,
teachers have a thorough understanding of their students’ strengths and weaknesses and can build on these more easily during this point in the year. This helped them when reflecting on the progress students were making toward understanding. Finally, this time frame is more suited to the study because students have had ample practice collaborating with their teacher and peers during various academic activities, yielding results that are more focused on developing discourse and growth of understanding, as opposed to practicing how to work well together. Finally, this time frame was purposely chosen based on the fact that the teachers had finished standardized testing and did not feel as much pressure to cover all of the curriculum necessary for mastery of the test. Not feeling this pressure allowed teachers to provide multiple opportunities for the researcher to observe students and teachers using mathematical discourse while engaged in problem solving.

Cresswell also defines a case study as, “a good approach when the researcher has identifiable cases with boundaries and seeks to provide an in-depth understanding of the cases or a comparison of several cases” (2007 p. 74). This comparative case study will include a detailed description of each case along with an analysis of the two cases. The discourse practices will be compared to those found among the research based talk models connected to this study (Chapin & O’Connor, 2007; Rojas-Drummond & Mercer, 2004; Sherin; 2007). Furthermore, the talk models, provided a framework for identifying and classifying the types of strategic discourse that was used. Each case also included critical information about the teacher and three students selected from each classroom.

In summary, the descriptive case study approach supported an investigation that described and explained the interactions of two elementary teachers and their students as
they engage in strategic discourse practices in mathematics (Bromley, 1990). The inductive exploration of context and processes assisted in developing a description of the phenomenon that included observations of the daily use of discourse and the insights that developed (Merriam, 1998; Rossman & Rallis, 2003; Sykes, 1995; Yin, 1994). Furthermore, the case study approach was most appropriate to this study because it helped to illuminate the phenomena (Stake, 1995).

In qualitative research method, there are multiple realities or multiple interpretations, not just one conception of reality or one interpretation (Buba and Lincoln, 1994). Including the key informants, perceptions helped to provide a more natural collection of data that emphasized the subjects’ point of view (Bogdan & Biklen, 2007). Including the perspectives of teachers and students provided the opportunity to for the reader to look at the participants’ reality.

Furthermore, examining the discourse practices among teachers and their students also provided a means for investigating the opportunities teachers provided for students to use reformed based methodology and share mathematical authority in the classroom.

Methods

Participants

The participants in this study represent a purposeful sample (Merriam, 1998). The aim of purposeful sampling is to identify participants who have specific characteristics, knowledge, or direct experience relevant to the phenomenon of interest (Pope & Mays, 1995). With the purpose of this study being to examine the types of discourse teachers were using in the math classroom, it was critical that the teachers selected were using mathematical discussions to support students learning in mathematics. Another criterion
was that the teachers selected needed to have been using reform-based mathematical practices with students effectively in their classrooms.

The participants were chosen upon recommendation from their building administrators. This came about after the researcher met to discuss the possibility with the building administrators. Both teachers were recommended as teachers who met both criteria defined above.

Two teachers were recruited to participate, one third grade teacher and one fourth grade teacher working in the same elementary school. Choosing two settings within the same school allowed for a more focused comparison, eliminating certain factors that could potentially complicate the findings. If the participants were not from the same school, complications among practice could have followed. Complications such as varying district initiatives, professional development opportunities, and student population might have impacted the results. Additionally, both teachers face similar challenges regarding the complexity of teaching and assessing students using a newly adopted standards-based environment. Also, their district was undergoing the revision and implementation of a revised math curriculum heavily guided by state assessment directives and the Common Core Curriculum and Massachusetts State Frameworks. Teachers in their district have spent the last year mapping out the curriculum and the current year working on implement the changes and creating assessments. Both teachers dealt with the added pressure of the possible changes to the state assessments as a result of PARCC. The PARCC test was created to align more closely with the content, and objectives within the K-12 Common Core Standards. If it was determined that they
needed to take the PARCC test then additional content and skill would need to be mastered.

Three students from each of the two classes were randomly selected by the teacher, with the researcher’s help, to represent the students in their class. Teachers sorted their students into three levels below grade level, on grade level, and above grade level. Then a random sample of one student from each group was selected. This process was used to ensure that all levels of mathematical expertise were represented. Selecting one student from each level, allowed the researcher to gain greater insight about how academic levels of students may have impacted how they developed mathematically as a result of using the strategic discourse, as well as, their perceptions about using them. The following section includes a description of each participant including their strengths and weaknesses with the mathematical concepts and strategies for engaging in discourse and problem-solving.

**Mrs. Washington-Classroom A**

Mrs. Washington is a female teacher at Lakeview Elementary. She currently works as a co-teacher, partnering with a special education teacher who is in her classroom for math and English language arts every day. Mrs. Washington is a self-proclaimed non-traditional teacher in her late forties. She has been teaching in elementary schools in Massachusetts, for over 15 years. Mrs. Washington has a bachelor of science degree in Child and Adult Psychology and a master’s degree in Elementary Education. Mrs. Washington has taught third grade for thirteen years, along with two years teaching special education. Administration highly recommended Mrs. Washington as someone who approaches mathematics using methods consistent with reform ideology. She views
the process of studying mathematics as social and typically engages students in mathematical conversations so that they can articulate their thinking. Her years of experience, creativity, and commitment to using reform practices makes her a strong addition to this study. During the time of this study, she was collaborating with colleagues to complete common mathematics assessments and district District Determined Measures (DDM) as part of phase two of the curriculum development process. She has dedicated time to learning more about how to integrate problem-based learning into her instruction. Her future plans include concentrating on curriculum, perhaps in a leadership role in the district. Mrs. Washington’s commitment to identifying ways to reach her students and her connections with students make her a strong addition to the study.

Mrs. Washington’s Students-Classroom A

There are twenty students in Mrs. Washington’s classroom, eight females, and twelve males. Eight students have individualized education plans and receive special education services. One regular education student receives support from the math specialists for remediation purposes. Two students receive free/reduced lunch. One student is currently on a 504 plan and requires accommodations to progress in the classroom. Sixteen of the students are White, two are Asian, and one student is of Hispanic descent.

Gagan-Classroom A. Gagan is a third-grade boy, at Lakeview Elementary, who is performing above grade level in mathematics. Although his teacher described him as being reluctant to share his thinking at the beginning of the school year, he is quite competent in doing so during the study. Gagan also appears to enjoy studying
mathematics. Gagan talks about mathematics with teachers and students freely. He offers his ideas and solutions during group settings, even justifying without prompting. He listens carefully to others and shares his ideas without judgment. Gagan is always willing to help build on the ideas of others. He is thought of by his teacher and classmates as a strong mathematical thinker with a good sense of number.

**Benjamin-Classroom A.** Benjamin is a third-grade boy, at Lakeview Elementary school, who is performing at grade level in mathematics. He too enjoys learning and sharing ideas in mathematics. Benjamin is very confident in his mathematical abilities. He prefers to put his math on paper because he likes to see what he has done before the discussion. Although Benjamin perceives the sharing of ideas as interesting, he does not feel as if he gets to share enough. He feels that others do not always listen to him when he wants to share his ideas. Benjamin likes to be the one who knows the correct answer. He does not feel comfortable when his answer is wrong. His teacher sees Benjamin as being able to figure out the calculations in math, but also as having trouble finding the right words to explain his answers.

**Jaylissa-Classroom A.** Jaylissa is a third-grade girl, at Lakeview Elementary, who performs slightly below grade level in mathematics. Jaylissa always has a positive attitude about math and learning in her classroom. She appears to connect well with her teacher. Mrs. Washington describes Jaylissa as being inconsistent in her knowledge of mathematics. She will often invite Jaylissa to share her ideas both in front of the group and independently. She is especially pleased when Jaylissa is on the right track and can make connections between ideas. Jaylissa enjoys hearing how others think about problems because it helps her to understand. When Jaylissa struggles with an idea, she is
always willing to work with her teacher and other students to find a solution to the problem.

**Ms. Littleton**

Ms. Littleton is in her mid-twenties. She has been teaching fourth grade for just over four years at Lakeview Elementary School. She has a bachelor of arts degree in Mathematics and a master’s degree in Elementary Education. Ms. Little has also taken several classes in special education. Administration highly recommended Mrs. Washington as someone who approaches mathematics using methods consistent with reform ideology. She is a skilled teacher who creatively designs hands-on activities to address her curriculum that engage her students in mathematical conversations. Ms. Littleton is also a very hard worker, who has participated in a several curriculum committees throughout the district. She has gained experience in curriculum design and development through her participation in the district mathematics curriculum committee. This committee was responsible for curriculum review, designing new curriculum and mapping the curriculum with a vertical and horizontal strength. At the time of this study, she was collaborating with colleagues to complete common mathematics assessments and district District Determined Measures (DDM) as part of phase two of the curriculum development process. Ms. Littleton’s commitment to the continuous improvement of instruction and her skills as a teacher and leader make her a strong addition to the study. Her plans include expanding her leadership skills and eventually pursuing a career in administration.
Mrs. Littleton’s Students—Classroom B

There are twenty-five students in Ms. Littleton’s classroom, fifteen females, and ten males. One student has an individualized education plan and receives special education services. Four students receive free/reduced lunch. Twenty-one of the students are White, and four are of Asian descent.

Arthur—Classroom B. Arthur is a fourth-grade boy, at Lakeview Elementary, who is performing above grade level. He is a strong and confident math student. He is quick to understand the mathematics and willing to share his understanding of the math being discussed and often shares his solutions. Arthur is social and articulate. He enjoys working with others and appreciates listening to others explain the different ways they have solved a problem.

Jadiah—Classroom B. Jadiah is a fourth-grade boy, at Lakeview Elementary, who is performing at grade level. Jadiah prefers sharing his ideas with others because it helps him to keep track of his thinking. He enjoys collaborating with others while completing assignments. He especially likes collaborating when he gets the answer correct. Jadiah is articulate and thoughtful when explaining his solutions and ideas. Moreover, he is always willing to help others understand especially when they “get off track” or do not come up with a correct solution.

Evelyn—Classroom B. Evelyn is a fourth-grade girl, at Lakeview Elementary, performs below grade level in mathematics. She is shy and extremely quiet. During group work, it is rare that she speaks up. Often her participation in group involves agreeing with her classmates about how they have solved the problem. Sharing ideas is preferable to Evelyn while she works in groups. She did not like sharing her ideas in
front of the entire class. Evelyn prefers to write her solutions down on paper. Furthermore she is more apt to share her ideas when she thinks her answer is correct.

**Observation Setting**

**Lakeview Elementary**

Lakeview Elementary School is located in a suburban school district about 40 miles southwest of a large New England City. The District enrollment by ethnicity/race for 2014-2015 is African American 1%, Asian 11%, Hispanic 4%, White 81%, Multi-Race, non-Hispanic 3%. Within this population, 15% of the students are first language not English or English language learners. Additionally, 15% of the students receive special education services.

Five-hundred-sixty students are enrolled in grades two through sixth at Lakeview Elementary School. The school enrollment by ethnicity/race for 2014-2015 is African American 1%, Hispanic less than 1%, White 82 %, Asian 1%. Twenty-two students in this population are English language learners, and sixty-nine receive special education services. Seventy-five students receive free or reduced lunch. Twenty-eight students receive Title 1 reading services from a reading teacher.

**Sources of Data and Collection Strategies**

This descriptive case study includes multiple qualitative data methods for collecting and analyzing data used to provide the data needed to create a thorough analysis and rich description (Cresswell, 2007; Merriam, 1998). First data were collected in field notes during classroom observations, during the first two weeks of the study. Then data were collected during additional classroom observations in one third grade and one fourth grade classroom using two video cameras and a digital recorder. Audio-taped
individual student and teacher interviews were included in the data sources. Videotaped Video-taped teacher and student focus interviews that were also included. Table 6 shows the types of data collected and the duration of each source from Case A.

Table 6: Data Collection Sources Case A

<table>
<thead>
<tr>
<th>Date</th>
<th>Title</th>
<th>Description</th>
<th>Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>Video/Audio</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>April 30</td>
<td>Event 1</td>
<td>Measurement: How Tall Is?</td>
<td>18M 3S</td>
</tr>
<tr>
<td>May 20</td>
<td>Event 2</td>
<td>Graphing: Data Collection</td>
<td>16M 45S</td>
</tr>
<tr>
<td>May 20</td>
<td>Event 3</td>
<td>Cupcake Challenge</td>
<td>20M 46 S</td>
</tr>
<tr>
<td>May 27</td>
<td>Event 4</td>
<td>Array Museum Project</td>
<td>25M 5S</td>
</tr>
<tr>
<td>June 3</td>
<td>Event 5</td>
<td>Let’s Plan It Out Project</td>
<td>19M 17S</td>
</tr>
<tr>
<td>June 3</td>
<td>Event 6</td>
<td>Let’s Plan It Out Project</td>
<td>12M 24S</td>
</tr>
<tr>
<td>June 10</td>
<td>Event 7</td>
<td>Class Telephone Data Projects</td>
<td>9M 11S</td>
</tr>
<tr>
<td>June 10</td>
<td>Event 8</td>
<td>Small Group Survey Project</td>
<td>4M 33 S</td>
</tr>
<tr>
<td></td>
<td>1-8</td>
<td>Total Minutes</td>
<td>126M 4S</td>
</tr>
<tr>
<td>Teacher Interviews</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>June 17</td>
<td>1</td>
<td>Individual Teacher</td>
<td>32M 49S</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>Group Focus Teacher</td>
<td>21M 2S</td>
</tr>
<tr>
<td><strong>Student Interviews</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>-------------------------------</td>
<td>-------</td>
<td>------------------------</td>
<td>-------</td>
</tr>
<tr>
<td>June 17</td>
<td>1</td>
<td>Group Focus Student</td>
<td>28M 40S</td>
</tr>
<tr>
<td>June 24</td>
<td>A</td>
<td>Individual Student 1</td>
<td>9M 14S</td>
</tr>
<tr>
<td>June 24</td>
<td>A</td>
<td>Individual Student 2</td>
<td>8M 16S</td>
</tr>
<tr>
<td>June 24</td>
<td>A</td>
<td>Individual Student 3</td>
<td>7M 21S</td>
</tr>
<tr>
<td><strong>Questionnaire</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>June 9</td>
<td>1</td>
<td>Student completed Survey Pilot</td>
<td>20M</td>
</tr>
<tr>
<td>June 17</td>
<td>2</td>
<td>Class A Student Completed</td>
<td>10M</td>
</tr>
<tr>
<td><strong>ARTIFACTS</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Field notes</td>
<td>A</td>
<td>Observations, surveys and interviews</td>
<td></td>
</tr>
<tr>
<td>Researcher Memos</td>
<td>A</td>
<td>Analysis Notes</td>
<td></td>
</tr>
<tr>
<td>Project</td>
<td>A</td>
<td>Let’s Plan It Out</td>
<td></td>
</tr>
<tr>
<td>Project</td>
<td>A</td>
<td>Array Museum</td>
<td></td>
</tr>
<tr>
<td>Project</td>
<td>A</td>
<td>What’s For Lunch</td>
<td></td>
</tr>
</tbody>
</table>
Table 7: Data Collection Sources Case B

<table>
<thead>
<tr>
<th>Video/Audio</th>
<th>Title</th>
<th>Description</th>
<th>Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>May 12</td>
<td>Event 1</td>
<td>Measuring Angles</td>
<td>13M 41 S</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Right Angles in A Circle</td>
<td>6M 32 S</td>
</tr>
<tr>
<td>May 15</td>
<td>Event 2</td>
<td>Measuring Angles</td>
<td>10M 33S</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Parallelogram</td>
<td>1M 35S</td>
</tr>
<tr>
<td>June 5</td>
<td>Event 3</td>
<td>Capacity</td>
<td>10M 28S</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Water Balloons</td>
<td>14 M 0S</td>
</tr>
<tr>
<td>June 9</td>
<td>Event 4</td>
<td>Capacity/Water Balloon</td>
<td>3M 34S</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Presentation 1 Chevron/Lina</td>
<td>2M 12S</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Presentation 2 Arthur/Daniel</td>
<td>3M 46S</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Presentation 3 Evelyn/Winnie/Ty</td>
<td>3M 43S</td>
</tr>
<tr>
<td>June 10</td>
<td>Event 5</td>
<td>Teacher Led Activity</td>
<td>20 M 38 S</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Wrapping A Present</td>
<td></td>
</tr>
<tr>
<td>June 10</td>
<td>Event 6</td>
<td>Wrapping Present</td>
<td>3M 9S</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Group Chetan/ Margaret Part 1</td>
<td>21M 23S</td>
</tr>
<tr>
<td>June 10</td>
<td>Event 7</td>
<td>Bow End</td>
<td>14 M 4 S</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Group Discussion A</td>
<td></td>
</tr>
<tr>
<td></td>
<td>1-7</td>
<td>Total Minutes</td>
<td>126M 18S</td>
</tr>
</tbody>
</table>

**Teacher Interviews**

<table>
<thead>
<tr>
<th>Date</th>
<th>Event</th>
<th>Title</th>
<th>Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>April 27</td>
<td>1</td>
<td>Individual Teacher Interview</td>
<td>26M 26S</td>
</tr>
<tr>
<td>June 27</td>
<td>2</td>
<td>Group Focus Teacher</td>
<td>21M 2S</td>
</tr>
</tbody>
</table>

**Student Interviews**

<table>
<thead>
<tr>
<th>Date</th>
<th>Event</th>
<th>Title</th>
<th>Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>June 17</td>
<td>B</td>
<td>Focus Interview Part 1</td>
<td>1M 15 S</td>
</tr>
<tr>
<td>June 17</td>
<td>B</td>
<td>Focus Interview Part 2</td>
<td>2M 45 S</td>
</tr>
<tr>
<td>June 24</td>
<td>B</td>
<td>Individual Student 1</td>
<td>3M 49 S</td>
</tr>
</tbody>
</table>
Additional data sources included: analytical memos, student survey questionnaires, and other artifacts including the tasks designed by teachers. The decision to include these sources was their potential for creating a complete description of the participants and their involvement and experiences using strategic discourse with others. Including multiple sources of data as part of the methodology of this study to will strengthen the data, findings, and generalizations (Yin, 2003). The following sections will include a description of the various data sources and methodology.

**Observational Data**

**Classroom Observations**

Classroom Observations were a significant part of this investigation. Through observations, the researcher was able to gain an understanding of the key informant's
ways of seeing and the existing culture in a naturalistic setting. In the qualitative research setting, there are most typically two types of observers. Observers that act as participant observers and identified as non-participant observers. Participant observers become part of “the lives of the people being studied with the maintenance of a professional distance that allows adequate observation and recording of data” (Fetterman, 1998, pp. 34-35).

Secondly, non-participant observers maintain a limited type of interaction with the participants. More flexible relationships between the researcher and the informants are becoming more acceptable (Friedenberg, 1998). I was introduced into the setting by teachers as someone who was interested in learning more about how students talked about math in schools. Although my primary role as the researcher in this study was the non-participant observer, there were times that I identified as more of a participant (Cresswell, 2013). For example, while I was casually observing before videotaping began, I sat in the back of the classroom while the teacher presented the lesson, and then I circulated during small group work to interact with students questions about their work. During the videotaping of students during their interactions with their teachers and other students, I purposefully maintained distance from the activity, to blend into the background and have less effect on naturally occurring language and the interactions that occurred between key informants. I also avoided talking with students and teachers during classroom observations for the videotaped/audio taped lessons and spoke with teachers only after the class was over. Yet, during interviews, I was the one conducting both individual and group focus interviews and so I had a central role in the discussion. In traditional research, maintaining objectivity and neutrality are paramount in reducing bias and subjectivity. However, Dewalt & Dewalt (2002) suggest that participant
observation can be a way to increase the validity of the research because a greater role in
the observations can help researchers develop a better understanding of the context and
phenomenon.

**Videotaped Classroom Observations**

Videotaped observations were an essential source of data in this study (Erickson,
2007). Multiple observations were recorded on video and during visits to each classroom.
Transcriptions of the videotaped lessons were completed both by the researcher and
through professional transcription. It was sometimes difficult for the professional
transcriber to interpret students conversations when more that one student was speaking.
Knowing and recognizing the students’ voices made it easier for the researcher to
transcribe those events. Videotaping classroom visits provided an opportunity for the
data to be accessed and reviewed several times and for many purposes throughout the
project. Videotaping participants in their setting allowed for viewing the non-verbal
interactions and participation levels among participants that might not be captured
otherwise through the collection of field notes. Creating transcripts of the observation
data helped in identifying elements of talk models that existed within the discourse in
each of the two classrooms. It also allowed for the development of themes and patterns
around the use of reformed math practices, shared authority and the guiding research
questions framing this research (Engle, Conant & Greeno, 2007).

Data reduction strategies were used to reduce the amount of data involved in this
analysis and to focus the inquiry on particular instances, or events, that are in line with
the purpose of the study (Miles & Huberman, 1994). Several events or observed lessons
were part of the data set. These activities included events where students are engaged in
strategic discourse during problem-solving with their teachers or with other students. The 126-minute data set for analysis included eight events from classroom A. The data set for Classroom B included seven events with a total of 129 Minutes were included in the analysis. The video data included lessons from both small group and large group activities involving the exchange of discourse among participants. Therefore, video involving students working independently or on tasks that do not include engaging in discourse were not included. One other video has been excluded because the verbal exchanges between the teacher and student that focus more on giving directions. The purposeful selection of events will help reduce data and to guide the investigation and result in a more targeted analysis.

Two video cameras, one JVC and the other a Sony camera, were used to videotape during classroom visits. Both audio and video were taken for all observations and some interviews to insure that data was recorded and not interrupted due to power failure, equipment malfunction or human error. There was only one instance when the camera’s battery was dead and additional equipment was required. This occurred during the student group focus interview. Unfortunately, some of that conversation was lost. What could be salvaged was recorded on Ms. Littleton’s cell phone and then emailed to the researcher. As the facilitator conducting the focus interview, this glitch was unavoidable because I was unaware that the camera was off. The implication of this was that the data recorded from this interview was limited.

An Olympus audio voice recorder was used to pick up the students’ voices when background noise was elevated, and noise made it difficult for the video camera to pick
up all of the voices. The digital videos were converted from the camera to a Dell computer using Windows and a digital video conversion application.

Some of the video and audio data was sent out for transcription. A professional transcription service was utilized to transcribe the interview data. Transcripts were transcribed by the researcher when they were too difficult for a professional transcriber to understand. The transcripts were read over and over again to become familiar with the discourse occurring between and among teachers and students. The video and audio were reviewed both before the transcription were complete and afterward several times. Notes and codes were written in the margins or included in memos.

**Interviews**

In-depth interviewing is a strategy used in qualitative research to allow a researcher to elicit the world view of the key informants and the way they make sense of their world (Merriam, 1998; Rossman & Rallis, 2003). Interviewing participants helps to collect information about things that are unobservable (Patton, 2002). Interviews allow for conversations about the participants' background, opinions and ideas, along with, the study of practices in their setting, as well as, their individualized interpretations about them (Denzin & Lincoln, 2007). The in-depth interview enabled the “participants perspective on the phenomena of interest” to “unfold as the participant views it and not as the researcher views it” (Rossman & Rallis, 2003, p. 181). All interviews were guided by protocols using the research questions designed for this study. The interview questions drew from the events from the observations and classroom experiences. All interviews took place on the same day. Additionally, the intent was to develop questions that resulted in authentic responses from the key informants about their experiences and not
simply lead them to a particular response. Attaining perceptions can reduce bias because 
the researcher is not trying to infer what the participant is thinking, but including what 
they are thinking in the study. The information gathered from the individual and focus 
terviews enhances the rich description of the students, teachers, setting and practice 
making for a more comprehensive description of the cases (Rossman & Rallis, 2003).

**Individual Teacher Interviews**

The Semi-structured interviews occurred at the beginning of the project to collect 
data about the teachers' use of reform methodology including discourse practices. The 
terview will include questions about each teachers understanding of mathematical 
reform practices and the importance of discourse (See Appendix G). Using less structured 
interviews allow the conversation to be guided in part by the participant and avenues that 
are important to them (Merriam, 1998). The purpose of these interviews is to gather 
information about each teacher’s use and understanding of reform-based practices, and 
their use of strategic discourse. These interviews were audiotaped and lasted between 25 
and 33 minutes.

**Teachers Focus Group Interview**

A focus group interview, a semi-structured interview, lasted approximately 21 
minutes, was conducted with the two teachers after the researcher has visited both 
classrooms more than twice. Professional transcription services were used to translate the 
focus group interviews. Selected examples in the video/transcript data where strategic 
discourse occurred were identified to focus the teachers discussion (Engle, Conant & 
Greeno, 2007). A coding schemes was used to mine the data from video to select the
events that framed both of the teacher and student interviews (Angelilo, Rogoff, & Chavajay, 2007).

Questions to engage participants in the discussion were guided by the protocol (See Appendix F). During the teacher group focus interview, video clips of particular instances in some events were presented to guide the discussion. These clips highlighted places within certain events or observations where teachers’ used strategic discourse and how students responded to these methods. Clips also demonstrated the ways student used or could have used discourse as they contributed to the understanding of others. The interview encouraged both teachers to reflect on the video clips to surface their interpretations about the types of discourse used. In selecting the events, I was mindful of the complexity of choosing even the smallest clips. Research has found that clips that are too long are difficult for participants to watch without becoming overloaded and “zone out” (Erickson, 2007, p. 146). Targeting shorter clips with a focus helped the key informants make sense of the forest even when they are among the trees. The participants selected their lunch break to engage in the focus interview; this meant that we had less than 25 minutes to meet. Focusing the interview was critical for collecting useful data.

A follow-up letter was sent to the teachers to gather additional information about strategic discourse patterns that emerged in the two classrooms with the purpose of getting more information. A written request, and not the second face to face focus interview, was requested due to the time of year and the time constraints that teachers have on them during the end of the school year and the beginning of summer. Teachers were given the option of meeting with the researcher to discuss the questions instead of
sending them in writing. Unfortunately neither teachers participated in any of these options.

**Individual Student Interviews**

Student interviews were used to collect individual perspectives and opinions about the use of the strategic discourse used by their teachers and peers (See Appendix H). The student interviews were conducted with all sixth of the randomly selected students. These interviews were audiotaped and last approximately 3 to 12 minutes in duration. All interviews were professionally transcribed.

**Students Focus Group Interview**

A focus group interview was conducted to gather information from students about their perceptions of the strategic discourse practices they used. A focus group interview protocol was used to guide this discussion (See Appendix E). Again being aware that complexity of even the smallest clips could result in students being overloaded and “zone out”, the discussion was focused (Erickson, 2007, p. 146). Although I was mindful of this issue and chose small tidbits of data, I did need to adapt the interview to just asking the questions and not sharing the video until the end. The reason for this was that students were much more focused on seeing themselves and others in the video and not on the specific content. After the questions were asked and a discussion followed, I showed them all of the clips.

In the adapted focus group interview, students were encouraged to share their perspectives, attitudes, and beliefs about strategic discourse, shared thinking and the relationship to understanding. The purpose of the interview was to engage students in a collective discussion and gather more open and honest information in a relaxed group
setting. This casual interview encouraged students to build on the thoughts and contributions of others more than possible in a one on one interview (Cresswell, 2007). The interview was videotaped and transcribed. The focus group interview with Mrs. Washington’s class of students lasted approximately 32 minutes and occurred near the end of the project. The focus group interview with Ms. Littleton’s students lasted five minutes and also occurred near the end of the project. The small data set collecte here was a result of faulty equipment. Similarly to the teacher focus interview, students were reminded of certain instances where they had engaged in discourse for them to reflect on the discourse that they engaged in with other students and their teachers.

Data from all interviews were analyzed for the purpose of revealing patterns, themes, consistencies, and inconsistencies. Interview data was also viewed to identify where students responded to specific ways that teachers and students were using strategic discourse in the classroom. The data gathered from all interviews allowed for a comparison of students’ and teachers’ perspectives to be triangulated with the information gathered in the individual interviews and the survey/questionnaire.

**Documents And Artifacts**

**Field Notes and Memos**

Field notes assist in gathering a “written record of…perception” in the field (Rossman & Rallis, 2003, p. 195), turning what is seen and heard into a systematic record of impressions, insights, and hypothesis. Field notes about observations were recorded in a field journal and then transcribed in a word document by the researcher. Field notes
were taken during mathematics classes and while reviewing videotapes of lessons. Notes from these observations were transferred into reflections. These reflections informed making concerning research decisions, survey design and focus group interview questions. Observing in the classrooms before beginning to take analysis on specific The data gathered from this source was also used to triangulate the data found in the individual interviews of the six students. This survey was tested with a fourth-grade group of students from another school within the same district, before using it with students. This process helped to ensure that the questions were clear to students. Only a minor word change was made to the questionnaire after the pilot was given.

**Curriculum Mapping Documents**

The curriculum mapping documents guide the instructional decisions of the two teachers. The Mathematics Curriculum Maps are similar to an online planbook. This source helped to inform the researcher about concepts and skills contained in the third and fourth-grade mathematics curriculum throughout the district. These materials also helped the researcher understand the instructional sequence used by both teachers.

**Teacher Designed Tasks**

A sample of teacher designed tasks were collected to describe the types of tasks that teachers used (See Appendix K). This collection of tasks provided a “written record” of the types of problems teachers designed, and the tasks students were expected to complete (Rossman & Rallis, 2003, p.198). This source assisted in the process of triangulating the data.
Data Analysis Procedures

Data analysis is the process used to inspect the data gathered to explain the data and draw conclusions and make interpretations. With the purpose of this inquiry focused on the type of talk used in math classrooms it seems appropriate that discourse analysis is part of the methodology. All conversations between teachers and students, as well as, between students and students will be reviewed using the process of discourse analysis. Analysis of discourse includes observations of “talk and texts” along with “language in use” (Brown & Yule, 1983; Wetherell, Taylor & Yates, 2001). This is suited to the non-linear process needed to analyze the strategic discourse among teachers and students as they engage in the back and forth dialogue necessary to discussing mathematics during problem-solving activity. Protocols were developed and used to guide this process. The data analysis procedures will also include a plan similar to Cresswell’s spiral approach for data analysis (2007). Data analysis within this approach spiraling continually to allows for repeated cycles of analysis to strengthen the results with each cycle. The analytical methodology follows distinct phases. These phases include data collection; data management; reading and writing memo’s; describing and classifying and interpreting; and representing and visualizing (See Figure 4: Spiraling Approach to Data Analysis).
Data analysis techniques guided all stages of the analysis and continued throughout all phases of the study (Cresswell, 2007). Data were organized in separate notebooks and in different folders on a hard drive to increase researcher organization and efficiency. Entries in the field journal used to create memos that communicated the researcher’s connections, thoughts, and questions that were uncovered in the reading and reviewing of the several data sources. After the data reduction process was finished findings were summarized using matrices and tables to illustrate the relationships.
between the discourse used in the classrooms and those evident in the research models (Miles and Huberman, 1984; Rossman and Rallis, 2003). This was followed by a system to code the data included categories developed by the researcher. Then each case was summarized including interpretations and generalizations. Finally, a cross-case analysis was written to describe similarities among the cases.

The strategic discourse implemented in the two classrooms is authentic to the classroom and the participants utilizing it. An observation protocol was developed and used to focus the collection of data on the elements of talk within the discourse models (See Appendix J). Additional information about the data analysis techniques are explained in the following section.

**Phase 1 of the Data Analysis**

First, key informants were observed in the classroom settings during daily lessons to observe mathematical conversations during class. The purpose of these early visits was to verify that the classes in the study were using methods in line with reformed mathematics instruction, including strategic discourse. The field journal includes a record of these early visits.

Videotaping began during the third visit to each classroom. A deductive approach to identifying codes, themes, and patterns was used to guide the analysis with a focus on the research questions. Although “the traditional approach in social sciences is to allow the codes to emerge during the data analysis,” predefined codes are helpful in addressing “larger theoretical perspectives in research” (Cresswell, 2009, p. 187). The development of the observational protocol was guided by Spradley's work (1980). This protocol focused the inquiry by identifying the elements matching each of category within the talk.
models introduced earlier (See Appendix I). One table was completed for each of the two settings. Each of the events were coded using the protocol. The results are located in two separate frequency tables (See Table 8 and Table 9).

Table 8: Comparing Case A to Discourse Among Talk Models

<table>
<thead>
<tr>
<th>ELEMENT</th>
<th>CLASSROOM TALK</th>
<th>EXPLORATORY TALK</th>
</tr>
</thead>
<tbody>
<tr>
<td>CODE</td>
<td>C A VJ C+ G</td>
<td>C Q SJ A N</td>
</tr>
<tr>
<td>Event 1</td>
<td>64 29 11 19 5 9</td>
<td>Event 1</td>
</tr>
<tr>
<td>Event 2</td>
<td>75 56 41 22 9 4</td>
<td></td>
</tr>
<tr>
<td>Event 3</td>
<td>90 41 18 7 2 0</td>
<td>Event 3</td>
</tr>
<tr>
<td>Event 4</td>
<td>103 44 19 20 11 8</td>
<td>Event 4</td>
</tr>
<tr>
<td>Event 5</td>
<td>80 39 22 27 13 9</td>
<td></td>
</tr>
<tr>
<td>Event 6</td>
<td>10 3 3 2 2 0</td>
<td>Event 6</td>
</tr>
<tr>
<td>Event 7</td>
<td>62 40 1 7 0 0</td>
<td></td>
</tr>
<tr>
<td>Event 8</td>
<td>43 22 3 10 1 0</td>
<td>Event 8</td>
</tr>
<tr>
<td>TOTAL</td>
<td>527 287 118 114 43 30</td>
<td>TOTAL</td>
</tr>
<tr>
<td>Event</td>
<td>Lines</td>
<td>Engaging</td>
</tr>
<tr>
<td>-------</td>
<td>-------</td>
<td>----------</td>
</tr>
<tr>
<td>Event 1.</td>
<td>64</td>
<td>29</td>
</tr>
<tr>
<td>Event 2.</td>
<td>75</td>
<td>56</td>
</tr>
<tr>
<td>Event 3.</td>
<td>90</td>
<td>41</td>
</tr>
<tr>
<td>Event 4.</td>
<td>103</td>
<td>44</td>
</tr>
<tr>
<td>Event 5.</td>
<td>80</td>
<td>39/</td>
</tr>
<tr>
<td>Event 6.</td>
<td>10</td>
<td>3</td>
</tr>
<tr>
<td>Event 7.</td>
<td>62</td>
<td>40</td>
</tr>
<tr>
<td>Event 8.</td>
<td>43</td>
<td>22</td>
</tr>
<tr>
<td><strong>TOTAL</strong></td>
<td>527</td>
<td>287</td>
</tr>
</tbody>
</table>

**DIALOGIC TALK**

<table>
<thead>
<tr>
<th>Element</th>
<th>Lines</th>
<th>Engaging</th>
<th>Questioning</th>
<th>Assisting</th>
<th>Feedback</th>
<th>Collaboration</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>CODE</strong></td>
<td><strong>E</strong></td>
<td><strong>Q</strong></td>
<td><strong>AS</strong></td>
<td><strong>F</strong></td>
<td><strong>CT</strong></td>
<td></td>
</tr>
<tr>
<td>Event 1.</td>
<td>64</td>
<td>24</td>
<td>24</td>
<td>10</td>
<td>5</td>
<td>6</td>
</tr>
<tr>
<td>Event 2.</td>
<td>75</td>
<td>21</td>
<td>28</td>
<td>28</td>
<td>12</td>
<td>7</td>
</tr>
<tr>
<td>Event 3.</td>
<td>90</td>
<td>57</td>
<td>41</td>
<td>12</td>
<td>3</td>
<td>7</td>
</tr>
<tr>
<td>Event 4.</td>
<td>103</td>
<td>51</td>
<td>28</td>
<td>20</td>
<td>6</td>
<td>4</td>
</tr>
<tr>
<td>Event 5.</td>
<td>80</td>
<td>43</td>
<td>23</td>
<td>23</td>
<td>4</td>
<td>6</td>
</tr>
<tr>
<td>Event 6.</td>
<td>10</td>
<td>5</td>
<td>1</td>
<td>5</td>
<td>2</td>
<td>0</td>
</tr>
<tr>
<td>Event 7.</td>
<td>62</td>
<td>38</td>
<td>20</td>
<td>4</td>
<td>3</td>
<td>0</td>
</tr>
<tr>
<td>Event 8.</td>
<td>43</td>
<td>17</td>
<td>14</td>
<td>2</td>
<td>5</td>
<td>0</td>
</tr>
<tr>
<td><strong>TOTAL</strong></td>
<td>527</td>
<td>242</td>
<td>179</td>
<td>104</td>
<td>94</td>
<td>30</td>
</tr>
</tbody>
</table>
Each element was coded on the transcripts and then color-coded with the colors assigned to each of the three models. Video data and transcripts were reviewed several times and analytical memos and notes written in the margins each time it was reviewed.

Table 9: Comparing Case B to Discourse Among Talk Models

<table>
<thead>
<tr>
<th>CLASSROOM TALK</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Element</td>
<td>Lines</td>
<td>Conversing</td>
<td>Analyzing</td>
<td>Viability</td>
<td>Conjecturing</td>
<td>Generalizing</td>
</tr>
<tr>
<td>CODE</td>
<td>C</td>
<td>A</td>
<td>VJ</td>
<td>C+</td>
<td>G</td>
<td></td>
</tr>
<tr>
<td>Event 1.</td>
<td>62</td>
<td>54</td>
<td>11</td>
<td>17</td>
<td>10</td>
<td>2</td>
</tr>
<tr>
<td>Event 2.</td>
<td>110</td>
<td>87</td>
<td>24</td>
<td>35</td>
<td>6</td>
<td>2</td>
</tr>
<tr>
<td>Event 3.</td>
<td>196</td>
<td>156</td>
<td>36</td>
<td>34</td>
<td>7</td>
<td>6</td>
</tr>
<tr>
<td>Event 4.</td>
<td>78</td>
<td>60</td>
<td>13</td>
<td>30</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>Event 5.</td>
<td>96</td>
<td>62</td>
<td>1</td>
<td>19</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>Event 6.</td>
<td>95</td>
<td>68</td>
<td>10</td>
<td>12</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Event 7.</td>
<td>112</td>
<td>94</td>
<td>20</td>
<td>33</td>
<td>3</td>
<td>6</td>
</tr>
<tr>
<td>TOTAL</td>
<td>749</td>
<td>581</td>
<td>115</td>
<td>180</td>
<td>27</td>
<td>17</td>
</tr>
</tbody>
</table>

| DIALOGIC |          |          |          |          |          |          |
| Element    | Lines    | Engaging | Questioning | Assisting | Feedback | Collaboration |
| CODE       | E         | Q        | AS        | F        | CT       |
| Event 1.   | 62       | 55       | 13       | 5        | 8        | 3        |
| Event 2.   | 110      | 88       | 22       | 7        | 9        | 2        |
Several more observations were conducted, each recorded by video and audio.

Each of the additional classroom visits provided an opportunity to deepen the understanding of the discourse experiences in these math classrooms and how the
discourse was used strategically by the teachers and students in this natural setting. Qualitative methods were used to complete this phase of the data analysis process. Although observations and field notes helped identify the strategic discourse being implemented in the classroom, assessing the frequency of the elements within each event was necessary for determining which model corresponded to the strategic discourse used by teachers and students in the classrooms (Erickson, 1985, Miles & Huberman, 1984).

“Data reduction” was used to pinpoint the interactions where students and teachers were using the specific elements of the discourse practices (Miles & Huberman, 1994, p. 10). Data was reduced to instances where teachers were using the defined element with students during a discussion. Each of the events in the analysis was selected because they contained strategic discourse that had been guided by the teacher. For this research, a discussion occurred when a teacher or student is facilitating a conversation about: a mathematical topic, concept or strategy with the entire class or small group. A total of eight events fit the criteria for analysis in Classroom A. A total of seven events fit the criteria for analysis in Classroom B.

The transcripts from the events were coded to identify elements of the three talk models were being used and the level of frequently of each element. Reading and re-reading the transcripts several times also helped recognize the type of discourse included in the discussions among teachers and students. After several cycles of reading, noting and reflecting on the elements, codes were completed. An observation protocol was used to collect data about the occurrence of the implementation of the elements within the three models. Frequency tables were used to represent the number of times the elements were used (Erickson, 1985). See Observation Protocol-Three Models for the results of
this analysis.

The following is a description of the discourse used by both teachers and their students and the connections to the three talk models as it occurred in this study. A preliminary study to compare the discourse models to the discourse practices observed in each class was conducted. The purpose of this activity was to determine if commonalities existed among the models and the discourse practices in the two classrooms. Classroom Talk, Cumulative Talk, Dialogic Teaching, Exploratory Talk, Filtering, Academically Productive Talk and Focusing were all compared to the discourse used to discuss problems in mathematics (Chapin & O’Connor, 2007; Gillies, 2014; Mercer, 2008; Lampert, 1990; Rojas-Drummond & Mercer, 2004; Sherin, 2002; Wood, 1998). Two of the talk models could be eliminated easily, leaving five of the talk models that had elements that were similar to the strategic discourse used in the classrooms. Cumulative Talk, Filtering, and Focusing were eliminated from the analysis early on because they did not have common characteristics to the discourse practices used by either Mrs. Washington or Ms. Littleton’s classroom. These models used specific types of talk moves to initiate particular responses from students. Cumulative Talk was not present because justifying answers and discussing ideas and solutions were not encouraged in this talk model. Additionally, a process for analyzing logical and illogical ideas to determine the viability of solutions did not occur to the level expected in the Filtering model, so it too was eliminated. Finally, focusing was not present either because this discourse model was completely student-driven, and that was not the case in the two classrooms in this study. Mrs. Washington developed the course of the discourse in her classroom and students engaged based on the topic and ideas generated by their teacher. Although slightly less
teacher directed, Ms. Littleton facilitated the content of mathematical discussions, as well.

In the end, both Mrs. Washington’s and Ms. Littleton’s classrooms were aligned with three of the research-based talk models identified in the research in mathematics education. These models included Exploratory Talk, Classroom Talk and Dialogic Talk (Gillies, 2014, Lampert, 1990, Mercer, 2004). The next section contains an investigation to determine which of the three models is most closely aligned to the discourse in each of the classrooms in the study.

**Classroom Talk**

Students were encouraged to share their ideas with one another in the classrooms. Many lessons include a discussion about a problem, this was characteristic of the Classroom Talk model. Teachers and students using Classroom Talk engage in collective discussions about mathematical problems. Mrs. Washington and Ms. Littleton provided opportunities for students to converse by openly discussing and exchanging ideas during the observations or events in this data set. Almost half of the 527 lines coded for Mrs. Washington and her students include the element of Conversing. In Ms. Littleton’s class, three quarters of the 749 statements involved students and teachers Conversing with one another as they discussed mathematics.

According to Lampert, this discourse model encouraged students to think and make conjectures about the mathematical ideas and problems that they are discussing. Making conjectures by drawing a conclusion or making connections to other ideas was a part of the discourse in both classrooms. Forty-three Of the 527 lines of dialogue include conjecturing. Some of the statements were ideas that Mrs. Washington repeated to make
explicit the thinking for others to hear. Mrs. Washington provided assistance and
direction to engage in conjecture. Of the 749 lines of dialogue in Ms. Littleton’s class, 27
were conjectures. Students made more than half of this and did so after being prompted
only five times. Of the Mrs. Littleton, like Lampert stepped in and stepped out to allow
students ideas to emerge naturally.

Also, Mrs. Washington and Ms. Littleton engaged their students in discussions
that included analyzing, evaluating and extending ideas together. Mrs. Washington
engaged students in analysis during 118 instances, and Ms. Littleton engaged students on
115 occasions. Almost half of the analysis students engaged in was offered analysis
independently and without prompting. The remainder of the analysis followed a question
by Ms. Littleton.

Both teachers also encouraged students to share their ideas about the solutions
they presented by requiring justifications. Overall students provided more justifications
than viability when explaining their thinking. Of the 114 times that students in Mrs.
Washington’s class were coded most were justifications, only 18 statements were used by
the teacher to determine the viability of a solution or idea. Mrs. Littleton’s students
provided verifications or justifications 115 times over the course of the eight events. A
little less than half of the time, students in Ms. Littleton’s classroom offered these without
prompting. During these times, Mrs. Littleton would simply ask if students had
something to add or if they had a question for a classmate about their thinking. More than
half of the time, students would respond to a question from their teacher about the
problem they were working on. On a few occasions, when asked, students provided
justifications to classmates. Students determined viability 19 times over the course of the
seven events. They did this in small and large groups, usually to explain their rationale for their solution or to verify the accuracy of their answer.

Additionally, both teachers provided opportunities for students to “Turn and Talk” and discuss their thinking with another student through a partner share. Turn and Talk was an opportunity to ensure that all students processed their thinking along with the support of another student. Turn, and talks occurred seven times during the eight-classroom observations or events in the data set from Mrs. Washington’s classroom. Ms. Littleton used “Turn and Talk” once during the seven events in the data set from her classroom.

Finally, Mrs. Washington and Ms. Littleton provided opportunities for students to offer generalizations about the math that they were studying. Generalizations were made 30 times in Mrs. Washington’s class and 17 times in Ms. Littleton’s class. Students making generalizations in Ms. Littleton’s class made those without being prompted to do so eight times. The other generalizations occurred when the teacher repeated a generalization or extended an idea.

**Dialogic Talk**

Dialogic Talk (Gillies, 2004) encourages students to engage, question, assist, provide feedback and engage in collaboration skill building. Both teachers provided multiple opportunities for students to engage in discussion. Mrs. Washington’s students engaged almost half of the time over the course of the eight events. Ms. Littleton’s students engaged three-quarters of the time during the seven events. The use of questioning existed in both classrooms. Mrs. Washington maintained a high level of
frequency concerning the use of questioning to guide the study of math and problem-solving. Of the 527 statements coded, 179 individual statements were recorded as questions. Ms. Littleton also utilized questioning to guide the discourse in her classroom, of the 749 statements coded, 152 were recorded as questions.

Mrs. Washington’s assisted her students while they made sense of the mathematics that they were studying. Assistance was provided approximately during one-fifth of the discourse among participants. Ms. Littleton also assisted to bridge or solidify thinking. Assistance was provided twenty-five times to students. In both classrooms, students were assisted by their teacher, and in a few instances, students demonstrated their ability to do this independently by providing help to other students, as needed.

Students and teachers engaged in feedback both about their ideas and contributions. Both teachers and their students provided feedback about ideas and effort 94 times during the eight events in the study. Feedback from both teachers most typically included positive comments about a well thought out idea. Rarely feedback was provided to offer constructive instruction on how to improve the individual student’s ideas or the ideas of others as intended by the Dialogic Talk model. More feedback was provided by students in Ms. Littleton’s class than by students in Mrs. Washington’s classroom.

Additionally, a significant element in the Dialogic Talk model is the use of ongoing, direct instruction relating to collaboration. However, this support was seldom provided in both classrooms. Overall, Mrs. Washington did provide input about their interactions during 30 of the 520 statements, and Ms. Littleton guided students eight times among 749 statements. On one occasion, during the May 27 classroom observation,
event 4, students in Mrs. Washington’s class were working on a contract to guide their team work. During this math class, they discussed ways that they could collaborate effectively. Additionally, students in Ms. Littleton utilized a routine that they had come to embed in their practice where they would ask two questions and offer one comment each time a student presented their project work.

**Exploratory Talk**

Exploratory Talk encourages a joint, coordinated form of co-reasoning. Both teachers engaged students in discourse that included co-reasoning consistently during this study. The first element, co-reasoning was directed by Mrs. Washington which aligned with the expectations of the Exploratory Talk Model. Ms. Littleton engaged students in co-reasoning with much less direction. The students in Mrs. Washington engaged with students over half of the time during discussions. Statements were coded on 277 instances when co-reasoning occurred. Ms. Littleton engaged with students more than three-quarters of the time during discussions. Statements were coded on 581 instances when co-reasoning occurred.

Mrs. Washington used questions to engage students and elicit thinking, as in Dialogic Talk. She encouraged thinking through her frequent use of questions, another element of Exploratory Talk. Of the 520 statements coded, 179 were questions. Mrs. Washington typically asked students structured questions to direct or prompt thinking or to check in on student thinking about a particular idea or concept. This is how she strategically kept the discourse moving. Of the 749 statements coded, 152 were questions. However, Ms. Littleton asked more questions that were open-ended which
allow students contributions to be more student directed. Although most of the questions
were initiated by the teachers, students asked questions as part of the discourse. In Ms.
Littleton’s classroom, students asked questions of their teacher and fellow students.

Students were encouraged to given opportunities to provide solutions and
justifications. Students were encouraged to provide solutions and justifications with the
support of the teacher. When students did not provide justifications, they were
encouraged to do so by Mrs. Washington. Ms. Littleton’s students offered solutions and
justifications as a means of communicating their ideas. The teacher provided very little
prompting. Students provided solutions with justifications 180 of the 749 statements
during the seven events.

The fourth element negotiating another element occurred during eight percent of the
discourse. On these occasions, Mrs. Washington presented conflicting ideas to
stimulate discussion so that students could practice negotiating ideas with one another.
She did this in a controlled and safe way that typically involved her presenting the
conflicting idea, which was the inaccurate solution. She did this to encourage students to
argue against it and to test their thinking. It was also done to have a bit of fun too!

Similar to the analysis element in Classroom Talk above, this element in
Exploratory Talk includes reviewing ideas are evaluated and extended together. Both
teachers led students through analyzing ideas to determine if their ideas were logical, as
well as, or original to others proposed by their peers. Mrs. Washington and her students
engaged in analysis 118 times, while Ms. Littleton analyzed contributions, 115 times.
However, Mrs. Washington managed this practice, while almost half of the analysis in
Ms. Littleton’s classroom students offered analysis independently and without prompting.
Mrs. Washington

This analysis determined that the discourse strategies used by Mrs. Washington to engage her students in conversations during problem-solving were most similar to the elements of Exploratory Talk. The development of the conclusion occurred after reviewing the data summarized in the frequency table, along with reviewing the video transcripts and field notes from the eight-classroom events several times over. Mrs. Washington directed the discourse skillfully as she led students to co-reason about mathematical ideas while solving problems together in the classroom. The discourse used by Mrs. Washington and her students was evident in the high number of instances among all of the elements in Exploratory Talk. Mrs. Washington incorporated all five of the major elements throughout the mathematical conversations that she conducted with students. Furthermore, Mrs. Washington structured the discourse by creating opportunities for students to join in co-reasoning while solving problems. She also utilized questions to prompt students to discuss ways that they solved problems and to justify their ideas. She also encouraged students to analyze solutions and at times, initiated ways for students to negotiate ideas together as a class.

Dialogic Talk was eliminated because the training of collaboration and engagement was not the main focus of the discourse used by Mrs. Washington. Additionally, feedback was not a focus area either. Of the two elements only approximately one-fifth of the statements coded included feedback. Similarly, another significant element in the Dialogic Talk Model has directed assistance to develop collaboration skills. However, only 30 instances were noted when a direction toward developing collaboration skills occurred during the eight events. However, Mrs.
Washington did comment in her interview that she did spend more time during the beginning of the year helping students to develop the social interaction skills needed for group problem-solving. Unfortunately, this was not part of the observations collected by the researcher.

Classroom Talk and Exploratory Talk share similar characteristic within three of the elements. These elements include conversing and co-reasoning, viability/justification and solutions/justifications, and analysis. In these elements, there was a similar number of instances where the discourse was used. However, a consistent utilization of this element did not result among Mrs. Washington and her students. Mrs. Washington and her students utilized the remaining Classroom Talk elements conjecturing and generalizing 43 and 30 times consecutively. On the other hand, the Exploratory Talk elements were used more frequently with questioning occurring 179 negotiating occurring 53 times. Additionally, the role of the teacher in the Classroom Talk model is one of a facilitator, moving in and out of the discussion when students require guidance and to have mathematical ideas elicited for them. Mrs. Washington, on the other hand, directed the course of the discourse with a firmer lead. This discourse style is much more in line with the Exploratory Talk Model, with the teacher providing much more support to assist students in mapping out the path of the discourse. Therefore, the Classroom Talk model was not the one most closely aligned with the discourse used in Mrs. Washington’s classroom.
This analysis determined that the discourse strategies used by Ms. Littleton to engage her students in conversations during problem-solving were also most similar to the elements of Exploratory Talk. Data was reviewed several times over. All data sources informed the conclusion including the frequency table, the video transcripts and field notes from the eight-classroom events. Ms. Littleton facilitated the discourse skillfully as she led students in discussion mathematical problems together in class. The discourse elements from the Exploratory Talk Model used by Ms. Littleton and her students were evident in the high number of instances among three of the five elements. Ms. Littleton encouraged students to co-reason as they shared their thinking when discussing problems together. She used questioning to encourage students to share their ideas and to analyze ideas. Students were also asked questions on their own. Ms. Littleton encouraged students to review their ideas and the ideas of others as a means of negotiating accurate ways of determining solutions.

Dialogic Talk did not closely align to Ms. Littleton's discourse because of the low number of elements used by Ms. Littleton and her students over the course of the seven events included in this study. Although Ms. Littleton utilized questioning to guide the discourse with students, four of the other elements were infrequently used. For instance, direct instruction to strengthen collaboration skills, which was an element of Dialogic Talk, was offered only on eight occasions. Also, feedback was used by Ms. Littleton and her students during less than one-fifth of all the discourse. Most of the feedback was simply an acknowledgment of a contribution made by students. Students did offer feedback to other students after they presented explanations as a group. Assisting
students in building knowledge was another key element of Dialogic Talk. Ms. Littleton and her students used this element occasionally with only 25 instances of the 749 discourse statements over the seven events.

As mentioned previously, Classroom Talk and Exploratory Talk share similar characteristic within three of the elements in the model. These elements include conversing and co-reasoning, viability/justification and solutions/justifications, and analysis. A similar amount of instances included these elements. However, Ms. Littleton and her students did not use the remaining two elements. Ms. Littleton and her students utilized the remaining Classroom Talk elements conjecturing and generalizing 27 and 17 times consecutively. On the other hand, the Exploratory Talk elements were used more frequently with questioning occurring 152 times and negotiating occurring 26 times. Ms. Littleton acts more like a facilitator, which is the similar role of the teacher in the Classroom Talk model. Often her questions are much more open-ended allowing students room to share their thinking with little direction. However, Ms. Littleton does ask questions to facilitate the discourse to probe or clarify thinking. This allows students more direction to steer the path of the discourse. Therefore, this discourse style is much more in line with the Exploratory Talk Model, because of the higher frequency of the elements in the discourse model. The next section will further examine the discourse in both classrooms and how it relates to the Exploratory Talk Model.

**Phase 2 Data Analysis Cycle**

This second phase of the data analysis is the deeper investigation of the specific discourse practices used in each classroom by teachers and students. Although there are some comparisons to the broad categories of codes used in Phase 1, the codes will be
expanded to include more in-depth and specific elements associated with each element or ground rule within Exploratory Talk. This cycle also includes a greater opportunity for the researcher to use an inductive approach to identify how and why teachers and students used certain elements. It also allowed for an inquiry into the ways discourse effected the perspectives of the key informants. Steps in the data analysis process were applied again to discover patterns, themes, and categories similarly to the earlier phases.

A new Lesson Observation Protocol: Phase 2 was used to guide the coding process (See, Appendix L). Transcripts were coded and highlighted to note the instances where teachers and students used each element within the ground rules listed in the protocol. This was a more inductive approach because of the inclusion of a discussion that went beyond the elements of the discourse to include how and why the talk may have occurred. Additionally, the ground rules were influenced by the ground rules developed in Exploratory Talk, but were written based on the researchers experience in the classroom observing the norms and practices that teachers and students had developed as a result of the many mathematical conversations they had engaged in during the year (Cobb et al., 2001).

Although frequency tables were used to determine the instances when particular elements of strategic discourse occurred using transcripts, these methods did not provide information about what the purpose or effectiveness in each classroom (Erickson, 1998). Therefore, additional qualitative data sources were reviewed through the spiraling approach to assist in describing, classifying and interpreting the phenomena of strategic discourse in the results chapter. These sources included transcripts of individual interviews; teacher focus interview and student interviews; along with the student
questionnaires. The additional data used here was helpful in exploring and describing teacher and student perspectives about their use of the strategic discourse practices and the impact on the learning environment. It also allowed for the discourse analysis data from the classroom observations to be triangulated with the group and individual interviews and questionnaires.

After determining that the strategic discourse used by Mrs. Washington and Ms. Littleton and their students closely aligned with the Exploratory Talk model, a more focused analysis of the classroom events occurred. In this phase of the discourse analysis data, 527 statements were reviewed from the eight events in Mrs. Washington’s classroom and coded according to the 14 ground rules developed with the guidance of the Exploratory Talk model for both teachers. Additionally, 749 statements were reviewed from the seven events in Ms. Littleton’s classroom and also coded according to the 14 ground rules.

**Exploratory Talk**

In the case of Mrs. Washington, the discourse analysis included 19 more statements than Phase 1 of the discourse analysis. These added statements occurred after listening to the audio and videotaped data. I decided to include additional statements and review the audio and video again after I noticed that the Ground Rule 1 *Everyone Invited to Contribute* numbers seemed low. The numbers seemed low because each time I observed or reviewed the videotape, my impression was that this Ground rule occurred frequently.

Additionally, the fourteen ground rules were broken down into sub-topics to help in identifying specific strategic discourse practices used by each teacher while solving
problems in both mathematics classrooms. Data was added to a frequency chart to signify the number of times each element of the ground rules occurred. Coding was verified several times using the observation protocol and to ensure accuracy. After several cycles of reading, noting and reflecting about the elements or in this case, ground rules were coded completed.

The Exploratory Talk model has fourteen ground rules developed to guide mathematical discussions. The first three ground rules relate to encouraging students to participate. *Everyone Invited To Contribute (GR1), Contributions and Opinions Treated Respectfully (GR2), and Atmosphere of Trust is Present (GR3)* help to build a culture that supports a discourse centered classroom. The elements in these ground rules set the stage for an environment that is built on respect and participation among all. The next five ground rules focused on developing mathematical knowledge. *Knowledge is Made Public (GR4), Reasoning is Visible in the Talk (GR5), Engage in Joint Reasoning (GR6), Multiple Solutions are Encouraged (GR7), and Contributions are Built on Prior Proposals (GR8)* encourage students to interact while developing understanding. The remaining ground rules strengthen critical thinking. *Ideas Extended Together (GR9), Listening Actively to Engage (GR10), Partners Engage Critically with Each Other (G11), Opinions are Considered Before Decisions are Made (GR12), Ideas May be Challenged with Counter Strategy (GR13) and Seek Agreement for Joint Decisions (GR14)* require students to think critically. Students synthesize contributions to determine the viability of ideas and determine effective solutions.

Supporting elements were created for each ground rule to further explain the discourse occurring in the two classrooms in this study. Codes (GR1) through (GR14)
were assigned to assist the reader in noting instances where the ground rules have occurred. Supporting elements within the each of the fourteen ground rules will be coded using letters beginning with A and ranging through G. The discourse practices used in this classroom are comprehensive and the many elements support the overall discourse existing in this classroom. The discussion of the implementation of the ground rules will include the three ground rules that make up the majority of the discourse in both classrooms.

The specific data regarding the use of the ground rules in each case will be presented in the next two chapters as part of the description of each case. Along with the data will be an analysis of the results will be reported. Chapter 4 will include an analysis of these results as part of the discussion involving the discourse practices of Mrs. Washington’s and her students. Chapter 5 will include an analysis of the discourse used by Ms. Littleton and her students.

**Phase 3 of Data Analysis**

The third phase of the data analysis is the cross-case analysis between the key informants of the study. This phase includes a thick and rich description to explain the phenomena detailed in this study as related to both cases. The qualitative methods collected for this study assisted in painting a picture of the phenomena including the discourse practices and the environment present while teachers and students engaged with one another during problem-solving in mathematics. It also includes a summation of the commonalities found in the individual case descriptions of the phenomena, the people involved and the activity observed (Cresswell, 2009). A summary of the perspectives including the importance of practices for engaging students in mathematical discussion,
as well as, the impact on understanding, as a result of the strategic discourse used in each classroom. Of particular interest to this researcher was examining the ways teachers shifted authority to students to encourage the flow of discourse about mathematics as a part of their use of reformed based mathematical practices.

According to Cresswell, studies can use a combination of predefined and emerging categories to support research (2009). In this Phases 2 and 3 of the data analysis also included emerging patterns and categories which were not derived from literature, such as seen previously in Phase 1. Instead, the transcripts gathered for all interviews were coded with categories that were not predefined but emergent. An additional protocol was used to guide the data collection process for this step, as well (See Appendix M). These emergent categories directly connected to the research question about teachers and students’ perspectives about the use and success of discourse practices they are using in math class. This process allowed for a more inductive approach to data analysis, and the discovery of meanings of the key informants, as opposed to that of the researcher (Mays & Pope, 1996). This phase of the data analysis was intended to understand the meanings people bring to the setting (Denzin and Lincoln 1994). The purpose was not to predict the outcome but to understand the characteristics of the situation and the meanings people have constructed (Cresswell, 2009; Patton, 2002).

**Gaining Entry and Informed Consent**

I am an administrator in the district where the school in the study resides. I am currently an assistant principal at the sister school to this elementary school. During the 2013-2014 school year, I formed a relationship with one of the participants while working as a Math Curriculum Committee Co-chair during mathematics curriculum
development. I formed a relationship with the other participant while fulfilling my role as a mathematics implementation facilitator for grades K-3 in my district. This role was one of the responsibilities of my position as assistant principal in the district due to my interest, knowledge, and experience in improving mathematics education for students. As part of this experience, I met with third-grade teachers throughout the district to discuss the implementation of a newly selected math resource. These meetings were open discussions about the struggles and gains regarding the new resource and mathematics instruction in general. Selecting a familiar site provides an opportunity for me to research in a setting where I know the community and the challenges facing teachers and students as they grapple with implementing a new curriculum built on the common core standards and state framework. This location provides a fantastic opportunity how two teachers utilize discourse with students to teach mathematics with understanding. Selecting another site to fit these variables would be difficult.

First, I felt that it was important to inform the students about the project and their role in it. Along with presenting my plan, I provided an assent from to communicate the research study process and solicit their participation (See Appendix A). Parents also received detailed information about the study goals, timeline and their participation in it. I also used an informed consent document to seek the parent permission regarding the participation of their student (See Appendix B). Parents had the opportunity to opt in or out on the different experiences (See Appendix C). This decision was made because I wanted to ensure the highest level of participation possible. In fact, some parents did allow students to participate in parts of the study. Two students were not provided permission to be videotaped. These requests were honored. Teachers were informed of
the expectations and agreements involved in the participation in the study. Each
completed an informed consent (See Appendix D). All participant/guardians in this study
were required to complete an informed consent as a requirement to participate.

**Summary**

According to Rossman and Rallis (2003), the data gathering techniques provide a
structure for weaving the tapestry into a unique expression as observed by the researcher.
It is important in case study methodology to explain what the aim of the case study will
be and what it will not be (Yin, 1994). The purpose of this research study was to
investigate the strategic discourse strategies used by two elementary school teachers and
their students while problem-solving in mathematics and to learn more about their
perspective about the discourse. The Case Study methodology suited this inquiry and
assisted in framing which data was necessary for creating a descriptive story to explain
the phenomena. Throughout the process, data was reviewed several times to increase
familiarity with the dialogue or discourse, and to ensure that codes were accurate. Using
the discourse analysis to transcribe the talk was a critical first step in the data analysis
process. In the spirit of Cresswell’s cyclical process, these transcripts were read and re-
read before, during and after assigning codes to align the discourse with models of talk.

The interpretation of all three phases of the data analysis was also represented in a
diagram to further interpret the emerging ideas and themes. Alternate explanations were
also revealed to provide a complete view of the data (Merriam, 1998). The data analysis
approach that defined this study was critical to the process of the systematic collecting
the evidence and format for interacting with the data helped create the path to concluding.
The explanation of the data compiled by the researcher utilized qualitative, deductive and
inductive methods and has strengthened the data collection process.

The completeness of the process and assisted in developing thorough results. Techniques to help to represent the data in the form of charts, diagrams and results have assisted this researcher in communicating important information drawn from the data. This was necessary as part of the final step in the Spiraling Approach. The data analysis in this study includes a written account of the findings. These conclusions, interpretations and findings will be shared.

The next chapter provides an analysis of the ground rules and refrom-based practices and how they were used as part of the strategic discourse in Ms. Washington’s classroom while they engaged in mathematical discussions.
CHAPTER 4

MRS. WASHINGTON AND STUDENTS

Strategic Discourse

Mrs. Washington and her students used the discourse practices of Exploratory Talk during the course of the eight events using questioning to engage students in shared thinking while building an understanding of a variety of mathematical skills and concepts. As will be explained in the following sections, Mrs. Washington facilitated the talk by guiding her students through several ground rules, from this model. By analyzing the discourse used by Mrs. Washington and her students, a structure of these discussions emerged revealing certain patterns of engagement among the community members. Table 1 shows the frequency of the discourse elements as related to the fourteen ground rules of Exploratory Talk used by Mrs. Washington and her students.

![Discourse Strategies in Case A](chart.png)

Figure 5: Implementation of Ground Rules in Case A

Discourse elements are written in italics to indicate their correspondence to the established ground rules. The results of the examination of the discourse practices in each classroom will be analyzed using examples from within the dialogue that occurred during
events. The analysis will be followed by the identification of the most highly used strategic discourse elements outlined below.

Three strategic discourse practices composed the majority of the discourse over the course of the eight events. These included Engaging In Joint Reasoning (GR6), Everyone Invited To Contribute (GR1), and Multiple Solutions Are Encouraged (GR7). Additionally, within each ground rule are supporting elements included in the discussion to demonstrate the complex nature of the discourse and the strategies Mrs. Washington uses to facilitate each mathematical conversation. The discourse practices used in this classroom are comprehensive, and the many elements included in the ground rules support the overall discourse existing in this classroom.

Mrs. Washington used the element Engaged In Joint Reasoning (GR6) to encourage students to share ideas and solutions as they learned mathematics with others while engaged in discussions about mathematics. This ground rule was fundamental to Mrs. Washington’s practice, as it was evidenced during the course of the eight events, see Table 10. Together she and her students utilized all of the elements within sixth Ground Rule to encourage students to discuss ideas and solutions with others, see Table 11. During these discussions Mrs. Washington uses this ground rule 271 times, equaling half of the discourse strategies used by the teacher during the study. Students Engaged In Joint Reasoning (GR6) 168 times, equaling one-third of the discourse used. Mrs. Washington discussed ideas and solutions with others on 76 occasions and students did 147 times (GR6A). Although students participated in the joint reasoning, by sharing their ideas and solutions, it was rare that they asked questions or initiated any element in the process on their own. Questions were posed to the community to direct thinking (GR6B)
and posed to encourage the exchange of ideas (GR6C). Mrs. Washington posed questions to the community to direct thinking on 49 occasions, while students did not pose questions to the community (GR6A). Mrs. Washington asked questions to encourage the exchange of ideas 40 times as did her students on four occasions (GR6C). Mrs. Washington asked 35 questions to understand thinking, and her students asked seven (GR6D). Contributions were highlighted by spotlighting different ways of thinking eight times by the teacher, while students used this element seven times (GR6E). Tasks were assigned or discussed to initiate students working together to find solutions by the teacher 40 times (GR6F). Assistance is offered to help work through the process by students three times and by Mrs. Washington 23 times (GR6G).

Table 10: Teacher Use of Ground Rule Elements Case A

<table>
<thead>
<tr>
<th>EVENTS</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>TOTALS</th>
</tr>
</thead>
<tbody>
<tr>
<td>GR1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>A</td>
<td>18</td>
<td>14</td>
<td>30</td>
<td>3</td>
<td>14</td>
<td>1</td>
<td>4</td>
<td>12</td>
<td>96</td>
</tr>
<tr>
<td>B</td>
<td>11</td>
<td>14</td>
<td>15</td>
<td>13</td>
<td>14</td>
<td>1</td>
<td>4</td>
<td>0</td>
<td>72</td>
</tr>
<tr>
<td></td>
<td>29</td>
<td>28</td>
<td>45</td>
<td>16</td>
<td>28</td>
<td>2</td>
<td>8</td>
<td>12</td>
<td>168</td>
</tr>
<tr>
<td>GR2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>A</td>
<td>2</td>
<td>3</td>
<td>2</td>
<td>3</td>
<td>5</td>
<td>0</td>
<td>1</td>
<td>2</td>
<td>18</td>
</tr>
<tr>
<td>B</td>
<td>3</td>
<td>1</td>
<td>0</td>
<td>3</td>
<td>2</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>10</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>4</td>
<td>2</td>
<td>6</td>
<td>7</td>
<td>0</td>
<td>2</td>
<td>2</td>
<td>28</td>
</tr>
<tr>
<td>GR3</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>A</td>
<td>5</td>
<td>1</td>
<td>1</td>
<td>6</td>
<td>7</td>
<td>2</td>
<td>2</td>
<td>3</td>
<td>27</td>
</tr>
<tr>
<td>B</td>
<td>2</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>C</td>
<td>2</td>
<td>4</td>
<td>2</td>
<td>2</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>12</td>
</tr>
<tr>
<td>D</td>
<td>3</td>
<td>2</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>11</td>
</tr>
<tr>
<td>E</td>
<td>0</td>
<td>0</td>
<td>3</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>12</td>
<td>7</td>
<td>6</td>
<td>8</td>
<td>9</td>
<td>2</td>
<td>3</td>
<td>10</td>
<td>56</td>
</tr>
<tr>
<td>GR4</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>A</td>
<td>10</td>
<td>3</td>
<td>0</td>
<td>2</td>
<td>4</td>
<td>2</td>
<td>0</td>
<td>0</td>
<td>21</td>
</tr>
<tr>
<td>B</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>C</td>
<td>8</td>
<td>2</td>
<td>6</td>
<td>4</td>
<td>9</td>
<td>2</td>
<td>0</td>
<td>0</td>
<td>31</td>
</tr>
<tr>
<td>D</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>2</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td>GR5</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>A</td>
<td>2</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>2</td>
<td>0</td>
<td>0</td>
<td>3</td>
<td>8</td>
</tr>
<tr>
<td>B</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>GR6</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>A</td>
<td>4</td>
<td>2</td>
<td>18</td>
<td>10</td>
<td>16</td>
<td>1</td>
<td>18</td>
<td>7</td>
<td>76</td>
</tr>
<tr>
<td>B</td>
<td>5</td>
<td>9</td>
<td>4</td>
<td>9</td>
<td>10</td>
<td>1</td>
<td>10</td>
<td>1</td>
<td>49</td>
</tr>
<tr>
<td>C</td>
<td>10</td>
<td>4</td>
<td>7</td>
<td>6</td>
<td>3</td>
<td>0</td>
<td>4</td>
<td>6</td>
<td>40</td>
</tr>
<tr>
<td>D</td>
<td>5</td>
<td>1</td>
<td>5</td>
<td>5</td>
<td>10</td>
<td>0</td>
<td>5</td>
<td>4</td>
<td>35</td>
</tr>
<tr>
<td>E</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>3</td>
<td>3</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>8</td>
</tr>
<tr>
<td>F</td>
<td>0</td>
<td>0</td>
<td>18</td>
<td>3</td>
<td>4</td>
<td>0</td>
<td>14</td>
<td>1</td>
<td>40</td>
</tr>
<tr>
<td>G</td>
<td>1</td>
<td>2</td>
<td>16</td>
<td>0</td>
<td>4</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>23</td>
</tr>
<tr>
<td>GR7</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>A</td>
<td>12</td>
<td>14</td>
<td>5</td>
<td>6</td>
<td>19</td>
<td>0</td>
<td>0</td>
<td>6</td>
<td>62</td>
</tr>
<tr>
<td>B</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>3</td>
<td>2</td>
<td>0</td>
<td>5</td>
<td>0</td>
<td>13</td>
</tr>
<tr>
<td>GR8</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>A</td>
<td>3</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>5</td>
</tr>
<tr>
<td>B</td>
<td>3</td>
<td>3</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>8</td>
</tr>
<tr>
<td>C</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>2</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>4</td>
</tr>
<tr>
<td>D</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>2</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>5</td>
</tr>
<tr>
<td>GR9</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>A</td>
<td>2</td>
<td>3</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>12</td>
</tr>
<tr>
<td>B</td>
<td>4</td>
<td>9</td>
<td>4</td>
<td>8</td>
<td>4</td>
<td>1</td>
<td>0</td>
<td>7</td>
<td>37</td>
</tr>
<tr>
<td>GR10</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>A</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>B</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>C</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>D</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>GR11</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>A</td>
<td>1</td>
<td>3</td>
<td>0</td>
<td>4</td>
<td>5</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>13</td>
</tr>
<tr>
<td>B</td>
<td>0</td>
<td>2</td>
<td>0</td>
<td>2</td>
<td>4</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>8</td>
</tr>
<tr>
<td>C</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>5</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>5</td>
</tr>
<tr>
<td>D</td>
<td>2</td>
<td>5</td>
<td>2</td>
<td>0</td>
<td>4</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>13</td>
</tr>
<tr>
<td>E</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>GR12</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>A</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

202
The second most frequently implemented ground rule utilized by Mrs. Washington and her students was *Everyone Invited To Contribute (GR1)*. Mrs. Washington utilized *Everyone Invited To Contribute (GR1)* as a means of opening up the discussion to everyone in the class. She used this talk move 168 times, close to a third of all the discourse used during the course of the eight events. Her students utilized the Ground Rule *Everyone Invited to Contribute* 110 times throughout the study (GR1). Mrs. Washington *encouraged students to contribute without being singled out* 96 times (GR1A). Her casual acceptance of input, ideas and solutions encouraged students to jump into the discussion very freely. On 72 occasions, Mrs. Washington *chose students strategically to contribute* in the conversation (GR1B). She engaged specific students as a follow up to a contribution they offered when initially joining the conversation. Sometimes additional questions were asked of the same student. Other times other students were invited to participate in the discussion. Students took the opportunity to *contribute without being singled* (GR1A) out 103 times as seen in They also *strategically chose students to assist or contribute* (GR1B) on seven occasions.

The final most frequently implemented ground rule was *Multiple Solutions Are Encouraged (GR7)*. Mrs. Washington used *Multiple Solutions Are Encouraged* as she
prompted students 75 times to *share many ways of solving problems* and provide different ways of thinking about a concept (*GR7*). Mrs. Washington encouraged students to solve problems in many ways on 62 occasions during the course of the eight events (*GR7A*). Mrs. Washington also prompted students to *share many ways of solving problems* (*GR7B*). She invited students to *share their many ways of solving problems or thinking* (*GR7A*) 62 times and *shared* her own solutions (*GR7B*) 13 times over the course of the eight events.

Students utilized the ground rule *Multiple Solutions Are Encouraged* (*GR7*) as they *shared many ways of solving problems* on 78 occasions (*GR7*). Students invited other students to share their *many ways of solving problems or thinking* (*GR7A*) on one occasion. Students also prompted other students shared *many ways of solving problems* on 77 occasions (*GR7B*).

Table 11: Student Use of Ground Rule Elements Case A

<table>
<thead>
<tr>
<th>EVENTS</th>
<th>OBSERVATION PROTOCOL: PHASE 3</th>
<th>WASHINGTON’S STUDENTS</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>GR1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>A</td>
<td>14</td>
<td>8</td>
</tr>
<tr>
<td>B</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>15</td>
<td>9</td>
</tr>
<tr>
<td>GR2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>A</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>B</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>GR3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>A</td>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>B</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>C</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>D</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>E</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>6</td>
</tr>
<tr>
<td>GR4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>A</td>
<td>13</td>
<td>12</td>
</tr>
<tr>
<td></td>
<td>A</td>
<td>B</td>
</tr>
<tr>
<td>---</td>
<td>-----</td>
<td>-----</td>
</tr>
<tr>
<td>B</td>
<td>7</td>
<td>0</td>
</tr>
<tr>
<td>C</td>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>D</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>20</td>
<td>14</td>
</tr>
<tr>
<td><strong>GR5</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>A</td>
<td>12</td>
<td>18</td>
</tr>
<tr>
<td>B</td>
<td>3</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>15</td>
<td>18</td>
</tr>
<tr>
<td><strong>GR6</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>A</td>
<td>11</td>
<td>14</td>
</tr>
<tr>
<td>B</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>C</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>D</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>E</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>F</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>G</td>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>13</td>
<td>20</td>
</tr>
<tr>
<td><strong>GR7</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>A</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>B</td>
<td>12</td>
<td>14</td>
</tr>
<tr>
<td></td>
<td>13</td>
<td>14</td>
</tr>
<tr>
<td><strong>GR8</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>A</td>
<td>1</td>
<td>6</td>
</tr>
<tr>
<td>B</td>
<td>0</td>
<td>5</td>
</tr>
<tr>
<td>C</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>D</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>11</td>
</tr>
<tr>
<td><strong>GR9</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>A</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>B</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td><strong>GR10</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>A</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>B</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>C</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>D</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td><strong>GR11</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>A</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>B</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>C</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>D</td>
<td>0</td>
<td>6</td>
</tr>
<tr>
<td>E</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>9</td>
</tr>
<tr>
<td><strong>GR12</strong></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Students willingly shared the different ways that they solved problems with both peers and their teacher (GR7B). They did so 77 times during the eight events. Although students shared when prompted by their teacher, they also jumped into the conversation to share their thoughts without waiting to be called on or prompted. Only on one occasion did a student encourage others to share many ways of solving problems (GR7B).

**Implementation of Strategic Discourse**

Overall, learning mathematics in Mrs. Washington’s classroom meant all engaged in conversations with others in the community (Case A Transcript, p. 1-18). Mrs. Washington opened up lessons by either gathering students’ thoughts about the previous lesson, asking about a prior lesson taken place during the year or introducing a new concept. Each lesson most often began with a question to encourage shared thinking and sense-making. The questions were initially open-ended and allowed room for students to contribute a wide range of ideas about the topic. Once the question was asked, students joined the conversation to share an idea or strategy. After a student shared a solution or idea, Mrs. Washington would engage that student in a more focused discussion about their ideas. These interactions included restating or recording the contribution, asking a clarifying question, or prompting students to extend the idea. After the problem was
posed students responded by explaining strategies and providing rationales. Multiple ways of thinking and solving problems were encouraged. Contributions were restated, as well as, repeated to be sure that all students had an opportunity to hear them. Each discussion was followed up with a task that students completed either individually, in pairs or small groups.

Below is the in-depth investigation of the discourse practices of the discussions occurring among Mrs. Washington and her students. The events were chosen to provide a clear picture of the ways the ground rules were utilized in those conversations. As mentioned previously, the most consistently implemented strategic discourse strategies were related to three of the 14 ground rules. These included: Engage Students In Joint Reasoning (GR6), Everyone Invited To Contribute (GR1) and Multiple Solutions Are Encouraged (GR7).

Examining the dialogue from the conversations among Mrs. Washington and her students demonstrated the complex and non-linear process for integrating the strategies used during the learning of mathematics. Other ground rules may be referenced during this analysis to unveil the various discourse strategies that were used by Mrs. Washington and her students as they solved problems in mathematics.

Engage In Joint Reasoning (GR6)

This section includes an examination of the discourse practices or ground rules used by Mrs. Washington and her students during conversations in their third-grade classroom from Event 1 (Classroom Observation 1, April 30). I begin with the most commonly used ground rule used by Mrs. Washington and her students called Engage Students In Joint Reasoning (GR6). Mrs. Washington engaged Students in Joint
Reasoning (GR6) using a process of sharing ideas, listening to others and processing ideas to find meaning in the ideas.

Over the course of the eight events, Mrs. Washington utilized all seven of the different elements within this Ground Rule (GR6) as evident in Table 1. Students also utilize five of the six elements as evident in Table 2. The conversations have been examined to demonstrate the ways that Mrs. Washington and her students utilize the elements of Engaged Students In Joint Reasoning (GR6). Again, these elements include ideas and solutions are discussed with others (GR6A), questions are posed to the community to direct thinking (GR6B), questions are posed to encourage exchange of ideas (GR6C), community members ask questions to try to understanding thinking (GR6D), thinking is highlighted to spotlight different ways of thinking (GR6E), tasks are assigned to initiate working together to find solutions (GR6F), and assistance is offered to help work through the process or scaffold learning (GR6G).

**Classroom Example 1-Event 1**

The first example of the Engage in Joint Reasoning comes from the mathematical discourse that was part of Event #1. The focus of the lesson in Event 1 was linear measurement, more specifically understanding the purpose of utilizing a system of standard measurement. The essential question and enduring understandings are written with the end outcome in mind to guide her path toward understanding. These are the questions written and used by Mrs. Washington (April 30, 2015) for this lesson:

**Essential Question**

How are standard measurements used to measure real-life objects?

**Enduring Understanding**
Why are standard forms of measurement useful?

This lesson began with students gathered in meeting formation with Mrs. Washington at the front of the room. Meeting formation is one of the classroom routines requiring students to gather at the front of the room, seated nearest the whiteboard. On this day, Mrs. Washington began the lesson by engaging students in a ‘turn and talk’ with a peer. ‘Turn and talk’ is a technique Mrs. Washington used to provide all students an opportunity to engage with others and the ideas in a one to one situation. During ‘turn and talks’, students turn to a classmate(s) seated near them and share their thoughts, followed by listening to the thoughts of their partner(s). Students usually gather in groups of two to three students, each taking a turn to speak, while the others listen. Students always responded positively to the request to talk with peers (Case A Transcript, Event 1, p1). There is no hesitation, nor resistance to the activity by students. Mrs. Washington used the ‘turn and talk’ strategy nine times during the course of the first five events.

Mrs. Washington begins the discussion by engaging students with one another and the ideas using the question, “Why do we use 12 inches equals 1 foot” (Case A Transcripts, Event 1, p. 1). After students discuss their thoughts with peers, Mrs. Washington restates the question, and the group turns their attention back to the teacher. Next is the dialogue that occurred among Mrs. Washington and her students.

1. **T:** We have to all think about how we use 12 inches equals 1 foot. Why do you think everyone does that?

2. **T:** Turn to someone and talk about why we use 12 inches equals 1 foot.

3. **S:** [Talk amongst themselves about the topic and then the teacher reviews the thinking]

4. **T:** Why do we use the same rules? Molly? Why?
5. Molly: Because you have to measure.

6. T: Because you have to measure, you’re right!

7. T: Why else do we have to use the same measure?

8. Jaylissa: Because if you had to do 100 feet than it would take you three days to even get to like…

9. T: …It would take you forever if you didn’t have that ruler.

10. T: Why does everyone use the same rule. [12 inches equal 1 foot?]

11. A: Because if you said 5 equals something people would say, what?

12. T: Remember when we were measuring things with paper clips and blocks?

13. T: What happened?

14. N: They were all different because of all of the sizes (measurements), all different because the things were different.

15. T: All of the measurements were different.

16. T: We didn’t know what each other were talking about, we had no clue.

17. T: Someone said the piece of paper was 7 paper clips and I’m like mine was 12. Guess what happens? We got confused?

18. T: So, the reason we have the measuring rules is so that everyone is talking about the same unit of measure.

Students had been introduced to ways standard measurements were used to measure real-life objects earlier in the school year (Atlas Rubicon, 2018). She wants students to resurface earlier learning they had acquired about standard measurement. In Turn 12, Mrs. Washington provides a connection to this idea by linking a prior lesson with this lesson (GR8C) to help students recall the experiences they had when they were learning about this concept earlier in the year. Immediately, a Nina joins the conversation without being invited (GR1A). She says, “All the measurements were different.” Mrs.
Washington also probes student thinking to prompt students to explain the reasoning behind their answers.

In Turn 12, Mrs. Washington reminds students of a prior learning experience by restating the contribution (GR4C) to help students make a connection with a past experience and allows them to strengthen their understanding by merging the new understanding with the old. She also repeats answers to make sure that everyone has an opportunity to hear them. Then again in Turn 17, Mrs. Washington brings up the lesson again to remind students of the confusion that they had experienced when they did not use a standard system for measurement.

In Turn 18, after students struggle to make the connection, Mrs. Washington wraps up this phase of the lesson by restating (GR4C) the understanding that has just emerged and makes this knowledge public (GR4A). She does this using the statement, “So the reason we have the measuring rules is so that everyone is talking about the same unit of measure.” This statement was used to wrap up the review portion and move onto the part of the lesson where this understanding would be applied.

Mrs. Washington used prior learning to remind students of a skill they had learned earlier in the year. She did not scaffold to bridge the two students’ learning (GR6G), but instead, she moved on by posing a new question to direct student thinking (GR6B). This question leads to students’ understanding of why the use of a standardized system of measurement is necessary (Event 1, p. 1).

During Event 1, Mrs. Washington recognized the impact of having students offer their knowledge about mathematics (Gr4A) in a specific order as beneficial to building their knowledge of the concepts. Mrs. Washington selected students to share their
thinking by *explaining* and *justifying* for different reasons. According to Mrs.
Washington, some students need to speak so that they do not lose what they discussed
during the ‘*turn and talk*’. Although the student had not mastered the depth of
understanding to be able to answer the question, he wanted to contribute, and therefore
Mrs. Washington responds, see Turn 4. She rephrases the question, “Why else do we
have to use the same measure? However, the class still the class did not provide the depth
of reasoning that she was anticipating.

She discussed her decision-making process during the discourse in the teacher
focus interview. She describes how she layers her discussions for struggling students.
Mrs. Washington said, “I have many different levels of kids that are able to express
themselves and depending on how I want the kids to understand, I actually take a path of
who I ask” (Focus Interview, p.6).

Earlier in the focus interview, she describes the complexity involved in this
planning. She said,

I have a lot of kids who have to be cued. So, for me, it’s also cueing for those
students who may have missed what my question was. So, when you *turn and talk*
to somebody and they look at them like, oh what are we talking about and the
other student is like, oh, we’re supposed to …oh, okay, we’re supposed to do this.
So those are cueing mechanisms for about four of my kids (Focus Interview, p.3).

In the next section of the discourse, see below, Mrs. Washington moves on to the
part of the lesson focused on converting inches to feet. In Turn 19, she begins with telling
students how tall she is, followed by an explanation about how they could convert the 61
inches into feet (Event 1, p 1).

19. T: When I take my shoes off. I am 61 inches tall, this is 61 inches tall (points
to measure on board). I am really tall, aren’t I?

19. Class: NO! YEAH! No, you are not!
21. [Gagan stands up, and all students laugh, he is the same size as the teacher. She brings tall students up to compare the measurement.]

22. T: Sit down! [teachers says jokingly]

23. All kids laugh.

24. T: My doctor does not tell me how many inches tall that I am. He tells me that I am a certain amount of feet tall. Every 12 inches that is hiding in this 61, I can say that I am one foot tall. When I get to 12, I am 1 foot. When I can to 24, I can say, I am two feet tall.

25. T: If I get to.. what comes after that?

26. C: 36

27. T: 36

28. T: I can say I am…?

29. C: Three feet tall.

30. T: Let’s figure it out how many feet tall I am?

31. T: Don’t say as tall as Gagan, because that is not a standard unit. It is a comparison, but it is not a standard measurement. We have to talk about inches and feet.

32. T: Talk with someone near you-Turn and talk about how tall I am?

33. Kids turn and talk about how they figured out the height of their teacher in feet.

33. T: How tall am I and how did you figure it out?

34. T: Come stand near me. I like Jaylissa; she is my friend! (She is smaller than the teacher) . Everyone laughs with her. I cannot get near Will!

35. T: I am 61 inches, but if I wanted to say how many feet I am tall and how do you figure it out?

36. A: You are 5 feet and one inch.

37. T: How did you figure that out?

38. A: I know that 12X5 =60
39. T: Wait! Wait! So your strategy was multiplication? [said with excitement!!]

40. N: Hey (this was her strategy too).

41. A: 12X5 =60 then 60 plus 1 is 61 inches.

42. T: This is what was in Aaron’s brain. [Teacher writes strategy on the board]

43. T: Who did it in a different way?

44. A: I added 12, five times on my fingers, and I got to 60, as Aaron did, and I added one and I knew it was five feet because I had my five fingers up.

45. T: Sh sh sh… Is that how your brain was working? 12 inches plus inches, plus 12 inches equals one foot…that is what I heard you say, is that a good representation. [writes the strategy on the board]

46. T: Does anyone have a different strategy?

47. T: Are these strategies similar? (Aaron and Amy)

48. T: Kevin?

49. K: Yes, because multiplication is just repeated addition.

50. T: Were there any other brains working here, Dana?

51. D: Instead of multiplying I divided.

52. Class: What? Dana, what did you do?

53. T: Get up here, I have no idea, what you are talking about?

54. D: You are 61, I divided 61 by 12.

55. T: Write that out for me.

56. D: It has a remainder which is 5 feet and 1 inch.

57. T: So the remainder is the left-over or 1?

58. D: Yes.
Above in turn 25, the exchange where Mrs. Washington modeled this by leading students to figure out how many times 12 could fit into the total number of inches and invites students to provide input by asking, “What comes after that?” Aaron responds with the number 36 but does not connect that Mrs. Washington is looking for the answer of three feet. After hearing from students, Mrs. Washington again redirects the conversation by posing a question to direct thinking and initiate more conversation (GR6B). Although in-the-moment, framing questions can be difficult, Mrs. Washington uses purposeful questioning and successfully enacts real-time decision making to do so.

When asked to reflect on her use of purposeful questioning she said,

I was the kid who said I don’t get this and everybody around me gets this and I don’t so I am going to fake it. Its for those kids that I need to ask these questions. I think sometimes my experience comes through and I look at the kids and they’ll tell you that I was in sixth grade and every night I cried about math. I’m determined because you need to know that you are able to do it and any effort is great because we can build off of it. (Individual Interview, p.8)

Mrs. Washington also reflected about how using questions provided her with a direction, she said, “I know what I have to get accomplished and the questions are good for me because it also gets people to start thinking (Teacher Interview, p. 2).

Mrs. Washington is also aware of the importance of planning a productive discussion with a guiding or essential question. When asked to reflect on using questions to keep students engaged in the conversations she replied,

It’s a learned thing because it is interesting, my students will ask questions when they are reading somebody’s writing, so it just started happening, so I’m fidgeting around with how do I get them to do that in math, so we have a common language about how to phrase questions when you’re talking to the because they are so little. (Focus Interview, p.2)

Having a plan to ask questions is key to guiding the mathematical discourse and keeping it on track, even though it does not always go exactly as planned. When asked to
comment on this topic she reflected back to the area and perimeter lesson on the Area and Perimeter Lesson (June 3, 2015). She said,

I wanted them to understand that there are purposeful reasons for finding area. I kept changing my question and enduring understandings. I changed it from a who to a what to a why, and then they were like oh. And I kept changing my first word because each time I put something out there, it changed the focus of what I wanted. So, I wanted them to understand why it’s important to know the area of the space. I wanted them to understand that the area on the inside, not the outside because I still have kids that don’t know this. So I was trying to focus that lesson for the lower part of my class with the hope that the higher kids would jump in and help. (Focus Interview, p.5)

Students acknowledged that Mrs. Washington asked many questions for several reasons. Robert said it was a way for the teacher “to give students a challenge” (Student Group Focus Interview, p. 6). Bristol felt that the questions were used “to learn from our mistakes” (Student Group Focus Interview, pg.6). According to Charles, Mrs. Washington wants “to tell us, to tell us what we know because she wants us to get much better” (Student Group Focus Interview, p.6). During the focus interview, students also acknowledged that their teacher used questions to find out what they knew. Aiden said, “She wants to know if we actually know what she’s actually talking about” (Focus Interview, p.6). Kevin contributed, “She makes your brain work” (Focus Interview, p. 6). Aaron added, “she can’t just read us the answer because when you are a teacher, that’s not doing her job” (Focus interview, p.8). Furthermore, students shared that they thought Mrs. Washington asked questions “to review something that they had talked about with her previously” (Focus Interview, p. 2).

Again in the classroom dialogue above, Mrs. Washington redirected students’ thinking the lesson around when she posed a problem and asked students to ‘turn and talk’ to discuss how tall their teacher is and how they would figure this out, see Turn 30.
This strategy was used to pose a question to direct thinking (GR6B). Framing the question in this way emphasized finding a solution as equally important to identifying and explaining the process used.

After reasoning jointly with peers, students shared out to the entire class. In turn 36, the first student share said, “You are five feet and one inch” (Case A Transcript, Event 1, p. 1-2). When the student did not explain, Mrs. Washington prompted her for one. Mrs. Washington asked questions to try to understand the thinking (GR6D). The student responded with the justification, “I know that 12X5=60.” Mrs. Washington acknowledges an important part of her solution and draws the group in by saying, “Wait, wait, wait, Your strategy was multiplication?” The statement is said with a surprised tone as if she cannot believe the words that she has just heard. This grabs everyone’s attention. Spotlighting different ways of thinking (GR6E) in this way puts a positive emphasis the share of ideas because of the excitement Mrs. Washington brings to their contribution. Next, the student then repeats her step by step calculation and adds the one-inch remaining which is the total number of inches in Mrs. Washington’s height. She repeats contributions to continue to provide students with opportunities to offer ideas and solutions to be discussed with others (GR6A). Then Mrs. Washington clarifies the student’s strategy by writing it on the board so that all students could follow her rationale. Taking the time to record this answer sends the message that it is important and worth noting.

As the lesson continues with Turn 43, Mrs. Washington prompts students to share different strategies to sustain the joint reasoning. A student explains her application of the repeated addition strategy by counting the twelves until she reaches 60 inches or 5 feet.
On her own, she compares her solution with the solution of her peer. Afterwards, Mrs. Washington repeats the student’s solution and also writes it on the whiteboard for all to read. This draws student’s attention to the statement.

Then, students are encouraged to share more strategies and when no one joins in on the conversation, Mrs. Washington, thinking quickly, moves on with a question in Turn 47, “Are these strategies similar?” Realizing the similarity, Mrs. Washington asks the question to determine if students have noticed the same thing. A third student generalizes with the statement “multiplication is just repeated addition.”

In Turn 51, a fourth student shares her strategy, and the class responds with excitement to the connection she has made. Other students immediately ask questions to try to understand her thinking (GR6 D). This is significant because this occurred very few times over the course of the eight events. According to the transcripts, students were very excited and interested in her connection to division (Case A Transcripts, Event 1, p. 1-3). They want to know more about her thinking. Mrs. Washington again acts quickly and enthusiastically to spotlight this different way of thinking (GR6E). She invites the student up to the front of the meeting area to discuss her solutions with the class (GR6A). Speaking directly to the class, the student says, “It has a remainder which is 5 feet and 1 inch.” Mrs. Washington asked a question to draw attention to the vocabulary word “remainder” and the student’s use of division as a strategy to solve the problem. Mrs. Washington probed this student’s thinking by asking her, “So the remainder is the leftover or 1?”

This was an instance where Mrs. Washington could encourage students to ask questions of their peers to understand the thinking of other students (GR6D). Students
could have also been invited to provide feedback to the student regarding her contribution.

Tasks and student groupings are made with her students and what their strengths and in mind, according to the video-taped data. Discussions are differentiated through the levels of questions asked of different students. This is transparent and acceptable to all of the students. When asked to reflect on why Mrs. Washington tries to figure out what students know about math, Gagan said, “She usually says everyone gets what they need. She is learning what you need for math” (Individual Interview, p. 2).

Even though it seemed like students had exhausted offering different strategies towards the end of the dialogue in Event 1, this was not the case at all. In the final section of the discussion below, Mrs. Washington resumes her focus on joint reasoning by collecting three more solutions qualified as different than those shared previously. Beginning in Turn 59, Mrs. Washington strategically asks Nathaniel to share his thinking. Mrs. Washington encourages him to participate by giving him room to contribute whatever is on his mind (Class A Transcript, Event 1, p.1).

59. T: Nathaniel, what were you thinking?

60. N: I was thinking 48 was 12 equals 60 4 feet.

61. T: How did you know that?

62. T: Can I ask you a question? How did you decide to start with 48?

63. N: stumbling…had difficulty explaining.

64. T: {Repeated}. You knew that 48 equals 4 feet. Is that what you said? How did you know that?

66. T: So you kind of did this one (pointing to Elena’s) but you stopped at a certain point and then just added 12?

67. N: Yes.

68. T: So I was just wondering why you started at 48. It was just interesting.

69. T: Will?

70. Will: So I was thinking that you could do 2 X12 =24 and 24 plus 1 yard = 60 inches and then you just add one inch.

71. T: Wow! GR6E What does one yard equal?

72. T: What does one yard equal…people? Charles?

73. T: So 24 inches plus 36 =60 and 1 inch.

74. T: Wow! You’ve got to make your brain think. It is the only way to make it work.

75. T: So I am 5 feet 1 inch. Was there one way to figure this out?

76. Class: No!

77. G: As long as you get the right answer, you don’t have to do it the same way.

78. T: As long as you have a reasonable answer…

79. T: Nathaniel started with 48 plus 12, Aaron knows his facts, says I know my facts people. Amy likes to count off the 12 inches. Then we have division, Why does division work? Oh boy!

80. V: You start with 61 and divide it to get your answer.

81. T: (repeats) You start with 61 and divide it to get your answer.

82. T: Kevin?

83. K: Because multiplication and division are related.

84. T: You are on fire today!

85. T: Another way, what, what did you have for breakfast…what ever it is you have to have it again!
86. W: 72-11=61.

87. T: I need to stop here, there is too much brain activity! It is time to go.

Again, Mrs. Washington keeps student engagement in joint reasoning active by posing questions (GR6C). In Turn 61, she asks the student how he knew that 48 equaled four feet and why he started there. The questions she selects are chosen to support the student and encourage him to remain conversing. This student was very soft-spoken and timid. She questions him using a gentle tone as if she is doing all that she can to keep the dialogue going.

In turn 66, above, Mrs. Washington compared the student’s solution to the procedures that another student used previously in the lesson. This provides legitimacy to his thinking even though he struggled to articulate why he chose to start at 48. Most likely he knew his 12 times table by memory up to 12X4 and then added on by 12’s until he reached 60 inches.

Finally, Mrs. Washington revisits the strategies shared with a recap of the solutions, along with again posing a question to support understanding, “Why does division work?” In Turn 80 above, the student explains the process of dividing 61 to get your answer. Knowing that this statement does not explain the process in its entirety, Mrs. Washington repeats this student’s explanation. Then she calls on another student; the same student makes another generalization by saying “Because multiplication and division are related and builds on the idea that division can be used to solve this problem.” The dialogue ends without Mrs. Washington completing the process of finding Mrs. Washington’s height of 5 feet one inch by dividing her height, 61 inches by 12 using division.
Throughout this event, Mrs. Washington expected her students to explain or justify their contributions during their conversations. If students did not automatically explain their thinking, she prompted them using why or how questions. Mrs. Washington reflected on how she wanted students not only to provide an answer but also to explain what they are thinking during her interview. She said,

In math, you [student] have to come up with an answer, and you have to be able to tell me why, how did you [student] get that, what are you thinking...there is so much value in that process, but it’s always [been for others] about getting to the right answer. (Teacher Individual Interview, p.3)

When asked about why they thought Mrs. Washington asked students to explain why they chose to solve a problem in a certain way, Jaylissa said, “she doesn’t want us not to explain how and why we did that. I think she asks us why we did it because we probably thought of the idea or showed something on the board” (Individual Interview, p. 2).

Gagan responded to the same question. He said, “She is trying to see if we know what she is talking about. She does that because she knows what you did and she wants to know if you know what you did. (Individual Interview, p. 2). Likewise, both Jaylissa also reported that when students explain their thinking it helped the teacher to understand what they are thinking (Student Individual Interview, p. 1).

During his interview, Ben pointed out the connection between students sharing and learning from others. He said, “So the class can think, Oh, I should have done it that way” (Individual Interview, p. 3). Ben shared that listening to his fellow students helped him learn “because you know what’s going on” (Individual Interview, p. 3). However, Ben did not agree that explaining his thinking to peers was helpful.
Additionally, according to the survey/questionnaire most of the class liked to share their thinking aloud, see Table 12. Additionally, students thought explaining their thinking to others as helpful. Moreover, students thought explaining their thinking to others was helpful to their peers. Although students have positive perceptions of this process, they also find that the process of explaining their thinking as challenging.

Table 12: Student Survey/Questionnaire Results Case A

<table>
<thead>
<tr>
<th>Question</th>
<th>Yes</th>
<th>No</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Talking about math helps me to understand more clearly.</td>
<td>17</td>
<td>2</td>
</tr>
<tr>
<td>2. I am able to understand another student's thinking when they explain how they solved a problem.</td>
<td>15</td>
<td>4</td>
</tr>
<tr>
<td>3. I understand math more when I talk with others students.</td>
<td>15</td>
<td>4</td>
</tr>
<tr>
<td>4. I understand math better when I talk with my teacher.</td>
<td>15</td>
<td>4</td>
</tr>
<tr>
<td>5. I ask my teacher a lot of questions when I am learning math.</td>
<td>6</td>
<td>12</td>
</tr>
<tr>
<td>6. It is helpful to me when I am asked to explain my thinking</td>
<td>15</td>
<td>3</td>
</tr>
<tr>
<td>7. It is helpful to others students when I explain my thinking</td>
<td>15</td>
<td>3</td>
</tr>
<tr>
<td>8. Listening to how other students explain how they solved a math problem is helpful to me.</td>
<td>15</td>
<td>4</td>
</tr>
<tr>
<td>9. I like to share my thinking aloud.</td>
<td>10</td>
<td>9</td>
</tr>
<tr>
<td>10. I am uncomfortable when I have to share my thinking to the class.</td>
<td>6</td>
<td>12</td>
</tr>
<tr>
<td>11. I prefer to solve problems on my own and not with others.</td>
<td>9</td>
<td>10</td>
</tr>
<tr>
<td>12. Sharing my thinking helps me to figure out if I am on the right track.</td>
<td>15</td>
<td>4</td>
</tr>
<tr>
<td>13. I ask questions so that I can figure out what other people are thinking.</td>
<td>9</td>
<td>9</td>
</tr>
<tr>
<td>14. I ask questions of my classmates to help them find a logical solution.</td>
<td>12</td>
<td>5</td>
</tr>
</tbody>
</table>
Explaining my mathematical thinking is challenging.  
Comparing my answers with other students helps to see if my thinking is correct.  
I prefer writing my answers down, not talking about them.  
I have trouble understanding how other students solve a problem when they explain their solution to the class.  
I like to learn from others.  
Solving problems are easier when I work with other students.  
I have trouble explaining how I solved a problem aloud in math.  
I have trouble explaining my thinking about how I solved a problem in writing.  
I do not like math.  
If my answer is not correct, I can find my mistake by talking with others.  
I help my classmates when they are having trouble while solving math problems.

### Classroom Example 2-Event 2

The second example of Engage In Joint Reasoning comes from the mathematical discourse that was part of the Data and Measurement lesson (Classroom Observation, May, 20).

The lesson began with Mrs. Washington and her students engaged in conversation about the purpose of a tally chart. This was connected to the morning activity where students responded to a question by adding a tally mark to a chart about summer reading.

1. T: Would that be different?
2. A: It would be different but it would still be a tally chart.
3. T: Ooh! How could it be different and similar at the same time?
4. T: Somebody help us out…[Pause]
5. T: Aaron call on a friend to help.

6. A: Can I like draw so she begins drawing on the board?

7. T: Sure, if you need to.

8. T: So, my question to you is could I have gathered my data a different way than a tally chart?

9. T: How is what Amy drew different from what I drew?

10. M: It isn’t, she is just using a different symbol.

11. T: She just said it isn’t, it is just using a different symbol.

12. T: Thanks for the next question, what is she talking about?

13. T: Turn and talk to someone.

14. T: What do we think over here? What are we thinking?

15. A: Martha said that they are pretty much the same they are just using a different symbol.

16. T: Martha who would you like to continue the conversation?

Mrs. Washington asked, “Could I have collected my data in a different way?” Aaron responded, “You could have used a pictograph.” In Turn 1 below, Mrs. Washington engages students in joint reasoning (GR6) by asking, “Would that be different? Additionally, students are provided with opportunities to engage critically with one another when Mrs. Washington encourages them to question the ideas of others (GR11).

In Turn 3, Mrs. Washington builds on Aaron’s prior contribution. She also questions to further students thinking about how a tally chart and a pictograph can be similar and different at the same time. Students do not immediately answer, and Mrs. Washington invites everyone to contribute (GR1) asking “Somebody help us out?” Then to encourage more students to join in the discussion and share their reasoning, she asks
Aaron to call on someone to continue the conversation (GR6G). Having students call on a
is a technique for providing assistance to the student by scaffolding the idea in a way that
is inclusionary.

Next, Mrs. Washington rephrases the question followed by a new question, “How
is what Amy drew different from what I drew?” Martha offers her reasoning when she
says in Turn 10, “It isn’t, she is just using a different symbol.” This challenges the idea
that the two are different. Mrs. Washington repeats Martha’s response and in Turn 12,
asks, “What is she talking about?” This statement draws attention to what was just said
and encourages students to think about and question the ideas of others.

Then students are directed to engage in joint reasoning by turning and talking
with a peer about how the tally chart and pictogram are the same. Again, Mrs.
Washington emphasizes the importance of engaging in joint reasoning (GR6) when she
asks them to report out on what they talked about with peers. In Turn 15, Amy confirms
Martha’s thinking. Martha is invited to keep the conversation going by asking another
student to join in the conversation. This is also a strategy used by Mrs. Washington to
encourage multiple solutions from students (GR7).

Martha calls on Jaylissa and in Turn 17 below, she engages in joint reasoning by
stating her reasoning when she tries to justify that the tally chart is different from the
pictograph (GR6). Jaylissa also spotlights Martha’s thinking by referring to her ideas in
her explanation (GR6E). Mrs. Washington attempts to make sense of Jaylissa’s response
by confirming that the, “smile would equal one vote.”

In Turn 19, Mrs. Washington follows up with a clarifying question because it is
not clear in Jaylissa’s response whether or not she is agreeing with Martha. Disagreeing
is encouraged when finding viable solutions (GR11B). Mrs. Washington uses these questions to prompt students to engage in a disagreement with their ideas.

Again, Mrs. Washington collects ideas from students to continue engaging them in joint discussions. The discussion continues below and in Turn 21, Nathaniel compares tally charts and pictographs noting the similarity among them. More solutions are accepted when Gagan joins the conversation.

17. J: It is different because it is a pictograph, cuz at the bottom of the smiley face equals one vote, but also you could do the same thing as the tally chart, and the same thing as Martha said because it’s different because you are collecting data from each vote.

18. T: So, what you are saying the symbol of the smile would equal one vote.

19. T: Are you agreeing or disagreeing with L?


21. T: Nathaniel, what do you have for me?

22. N: It is like the same thing, but the only thing is that they are using smiley faces and not tally marks.

23. T: So, are you agreeing with L or disagreeing with L?


25. G: They are both the same thing because they are gathering information and they’re like are getting the same amount of stuff or votes but in a different way.

26. G: It would be hard to do it that way because the symbol means more than just one.

27. T: Gagan brings up a really good point when we have been using the pictograph. The symbol has been used for more than one vote.

28. T: Could I have asked you to use the key of two smiley faces equals one vote when I am trying to get data from all of you?

29. T: Martha what were you chatting about?
30. Martha: A tally chart is probably easier.

31. T: Kevin what do you want to share?

32. K: You could go around and ask people their favorite flower and then put a $\frac{1}{2}$ symbol.

33. T: Kevin, I like your thought.

34. T: Calls on a student, he does not answer she says, “She took your idea?”

35. T: If I said to you, and you picked up on this quickly, it was a smiley face to one vote it would be a yes or no question, If I said to you a smiley face to one vote than yes, it would be a pictograph.

36. T: What do we add to a pictograph to make them a little different from a tally chart Will: A key.

37. T: So, the purpose is to show the same data but with different symbols?

38. G: You don’t have to use a tally chart before you use a tally chart but it can be more efficient because if you could double check your work in the rough draft before you do you final draft and if you don’t do something before you would be messed up. T: So, you are thinking about the process of getting the information together it may be a good thing to have a tally chart so when you start to make the symbol for the pictograph?

39. G: It will be more efficient, you can check your work.

40. T: It will be more efficient, and you can check your work. Ooh, I like that, I never thought of that idea.

41. T: So, we are going to move on. We need a shake down.

42. T: Wait! Amy has a question for Gagan! Amy go ahead.

43. A: Are you saying that collecting the data in a tally chart is like the first draft and putting your data into a pictograph is like making a final copy?

44. T: So yes, he is editing and revising his data.

45. G: Yes.

The lesson continues with Mrs. Washington builds on the conversation by asking what students must add to the pictograph when using this tool to represent data. In Turn
38, Gagan extends the understanding by interpreting the use of the tally chart as the initial stage of creating a pictogram. He also justifies his thinking by connecting it to the efficiency of using a tally chart as a rough draft. Then as the lesson closes, another student enters the conversation in Turn 43, by asking a question to help her to understand Gagan’s thinking (GR6D). Gagan engages by acknowledging her question with “Yes.”

**Summary**

Mrs. Washington uses specific strategic discourse practices to engage all of her students in interactive discussions with her about mathematical problems. In Event 1 and 4 she utilizes a multifaceted process that is played out using a moment to moment process of aligning conversations with content. Many elements of strategic discourse are embedded as she steered the discourse to initiate conversation and shared thinking centered around problem-solving. *Engaging In Joint Reasoning (GR6)* is a significant portion of her work with students. The description of the discourse in event one and two illustrate the ways Mrs. Washington navigates the discourse to encourage students to engage in joint reasoning together. Follow up questions are used to clarify, investigate or extend thinking. Mrs. Washington draws her students in by posing questions to initiate the process and then keeps them engage using many other elements of strategic discourse to sustain discussions. Students listen and learn from others. Mrs. Washington demonstrates how students are expected to actively listen, explain their answers and justify their thinking during their conversations about mathematics. She also encouraged students to explain their thinking or solutions, if they do not do so independently by inviting them into the conversation or by asking questions. Moreover, Mrs. Washington
skillfully changes the direction of the conversation if she senses the path is going in a
direction she does not feel to be productive.

Although Mrs. Washington built the classroom discourse on using questioning to
engage students in joint reasoning, it was rare that students directed questions to direct
thinking or exchange ideas toward students or their teacher (GR6 B,C). When students
did ask questions of their teacher it was to gather information about the task that had been
assigned on that day. Students asked questions of their teacher very few times during the
eight events and rarely asked a peer a question.

This lack of questioning among students was also evident in students’ perceptions
of their use of strategic discourse because when asked if they asked questions of their
teacher while talking about math, only six students reported doing so, see Table 3.
According to the same data, both Gagan and Ben reported asking their teacher questions,
and Jaylissa reported that she did not ask questions. On the other hand, when asked if
they asked questions of classmates to help their peers find logical solutions twice as many
students reported yes than no. All three of the students who are part of the case study
answered yes to this question on the questionnaire.

When asked to reflect on her students use of questioning, Mrs. Washington
commented that it was a difficult process that student learned with practice and maturity.
She also reflected on strategies that she admitted could help students when she said,

I think it’s learned, at least for my little ones, it’s a learned thing because it’s
interesting, we use, my students will ask questions when they’re reading
somebody’s writing so this just started happening. So, I am fidgeting around with
how do I get that into math. So, we have common language in how to phrase
questions in writing, so I need to come up with how do you phrase questions
when you’re talking about math. (Focus Interview, p.2)
The next section explains how Mrs. Washington and her students used the Ground Rule *Everyone Invited to Contribute (GR1)* during her discourse practices. This ground rule was the second most commonly used strategic discourse used by students and their teacher.

**Everyone Invited To Contribute (GR1)**

This section includes an examination of the discourse practices or ground rules used by Mrs. Washington and her students during conversations in their third-grade classroom from Event 3 (Classroom Observation 3, May 20). The goal of each discourse session using this ground rule was to get students involved in the conversation. Mrs. Washington strategically phrased her questions to encourage *Everyone Invited To Contribute (GR1)*. The questions invited ideas to be shared so that students could build on their knowledge and the knowledge of others. During her individual interview, I asked Mrs. Washington why she thought it was important for students to talk to others, she answered,

I think math is a social content area, a lot of people don’t see it that way. If we need to be able to solve world problems then we need to be able to do it in math class. You have to be able to talk and communicate. (Teacher Interview, p.4)

Students were also asked about talking in math, Gagan said, “I think it’s pretty interesting because we learn new stuff and when Mrs. Washington calls on us or ask questions we can learn more” (Student Interview, p. 1). Jaylissa said, “I think it’s good because I think that other people will hear what the ideas are, and if I have another idea, I explain it to them” (Student Interview, p.1). Ben said, “So you learn from others” (Student Interview, p.3).
Additionally, when questioned about talking about math, many of the students agreed that they liked learning from others (See Table 12). Students also identified listening to how others solved math problems to be helpful to them, excluding Ben. Most also agreed that it was helpful to them when asked to explain their thinking. Ben did not find it helpful, however. Most identified sharing their thinking as helpful to others. Only half of the students liked sharing their thinking aloud, including the three individuals interviewed. Six of the students reported being uncomfortable when having to share their thinking and ten saw it as challenging. Interestingly, Gagan and eight other students preferred learning math on their own, and Gagan and ten others prefer writing their answers down on paper.

Over the course of the eight events, Mrs. Washington and her students utilized both elements within this Ground Rule Everyone Invited To Contribute (GR1), see Table 1 and 2. Using the dialogue in Events 3 and 4, the implementation of the elements will be examined. The two elements of this ground rule include *everyone is encouraged to contribute without being singled out* (GR1A) and *students are chosen strategically by the teacher/student to contribute* (GR1B). The first element encourages students to join the conversation on their own when they have something to contribute (GR1A). The second element is used when *selectively choosing students to participate* (GR1B). Mrs. Washington usually *chose students to contribute* directly to after they offered a contribution to follow up with them about their thinking (GR1B). She also *strategically selected* students because she knew that they were more likely to share during that particular moment in time. Mrs. Washington also *strategically called on students directly* when she was checking to see that they were listening and understanding (GR1B).
Finally, she *strategically called on students to contribute* because they looked as if they were struggling and needed the assistance (GR1B).

The strategic discourse element associated with *everyone is encouraged to contribute without being singled out* (GR1A) is the ‘turn and talk’ technique. Mrs. Washington used ‘turn and talk’ to increase contributions and to share their ideas. During the *turn and talk* students also *strategically chose* other students to share their *contributions* and listen to the contributions of others (GR1B).

During the individual interview, Mrs. Washington stressed the importance of requiring students to share their ideas because they can “learn from each other” (Teacher Interview, p. 3). She also remarked, “If I can get them to work together maybe something that I said is told to them by a friend in a different way” (Teacher Interview, p. 4). When asked how she manages to get students to interact during these discussions she said, “I spend a lot of time on what do you (students) know, how can you use it and what you see that you can get out of it” (Teacher Interview, p. 1).

During the individual student interview with Jaylissa, she reflected about why it was important to talk and share ideas in math by responding, “Everyone has a good chance to see what you are explaining, and maybe get other people a chance to do their explaining” (Individual Interview, p. 2). Similarly, Jaylissa reported that explaining her thinking was not challenging, see Table 12.

More than half of the students, including Gagan and Ben, reported having to explain their thinking as challenging. During the individual student interview with Ben he also reflected about why his teacher felt that it was important of for him to share ideas in math when he said, “Because well, I am really good at math, like I said before”
(Individual Interview, p. 1). Then when asked about the importance of explaining why he solved a problem in a certain way, he thought some more and said, “Because if you are doing stuff on a project you would know why he [Ben] did it and the they would know what to do for their project and make it good” (Individual Interview, p. 3). Likewise, Gagan felt that talking about math was helpful to him. He reflected, “We can learn from people’s mistakes because if someone did something wrong, she’ll tell us what they did wrong and then fix it. How you learn is by someone doing something wrong you know you can do right next time (Individual Interview, p. 1).”

Overall, the class agreed with sharing their thinking as being helpful in determining if their thinking was correct, according to the questionnaire results, see Table 3. Only four students disagreed with the benefits of shared thinking. Ben was one of four students who disagreed. Most students also agreed that sharing their thinking was helpful in identifying their own mistakes when they talked with others. All three of the students interviewed felt the same way.

When asked to comment about how engaging students in discussion came about in her teaching, Mrs. Washington said,

I was the kid who said I don’t get this and everybody around me gets this and I don’t so I am going to fake it. It’s for those kids that I need to ask the questions. I think sometimes my experiences come through and I look at the kids and they’ll tell you that I was in sixth grade and every night I cried about math. I am determined because you (students) need to know that you are able to do it and any effort is great because we can build off of it. (Teacher Interview, p. 7)

**Classroom Example 1-Event 3**

The first example of Everyone Invited to Contribute comes from the mathematical discourse that was part of Event 3. The focus of this lesson was a project based task using
data about favorite cupcakes (Case A Transcripts, Event 3, p. 4-6). Specifically, students were charged with creating surveys, collecting data and then designing and displaying results on a graph. They were also responsible for creating questions, surveying groups and representing their data in a graph format. After the initial introduction, students were divided into three groups. The essential question and enduring understandings are written with the end outcome in mind to guide the path to student understanding. These are the questions written and used by Mrs. Washington (May 20, 2015). These were written on the whiteboard located in the classroom.

   Essential Question: Can students create a graph to represent the data they collected?

   Enduring Understanding: How can graphs can be used to display different types of information in a variety of different ways?

This lesson began with students gathered in the front of the room to listen to the explanation of the Cupcake Project assignment. Mrs. Washington begins with a question, “How many people like cupcakes?” to build excitement and engage students. Students begin talking excitedly. The dialogue that was part of this event is located below.

1. T: I have a very serious question, how many people like cupcakes?
2. [Excited small talk occurs about ice cream, cake, icing and such]
3. T: Our next project is called “Cupcake Wars.”
4. T: How many people are enjoying doing projects? Vanessa is saying it depends on if I get stuck.
5. T: Here’s what is going to happen?
6. T: You have been hired by your parents to make cupcakes for your parents’ bakery.
7. Group: Ooooooh!

8. T: They only want to make favorite cupcake flavors so that they will have the best bakery in town. They want to make what everyone wants, the best cupcakes.

9. T: I have two separate projects. Why is that?

10. Group: Everyone gets what they need!

11. T: So, the projects are a bit different depending on the team that you are assigned to. The first team is going to just gather information about what the favorite cupcakes are. You are going to survey 20 people.

12. T: They are going to collect data, so what do they have to do first?


14. T: Tally chart about what?

15 J: Like tally chart about like what kind of cupcakes?

16. T: Why are you asking about cupcakes? What is your job, what do you have to find out? Aaron?

17. A: You are going to collect data about our class’ favorite cupcakes.

18. T: So that?

19. A: So that your parents can have the best cupcakes in town.

20. T: Then you are going to use that data to make a bar graph. Then you get to choose one other graph that you want to choose as a way to show your data, pictograph…line plot it is up to you.

20. J: Don’t we need a question?

22. T: Oh Wait…we were just about to the end of the project and guess what Jessie just said to me? What does our question have to be?

23. T: Your parents have hired you to find the best flavors to sell in their bake shop. What is the question going to be?

24. T: Turn and talk to your neighbor?
25  S: [Turned to a neighbor to share what they thought their question would be.]
26.  T: Cameron…what is your question?
27.  C: What is your favorite flavor?
28.  T: Cameron says that his question is going to be what is your favorite flavor?
29.  T: Anyone have a different question?
30.   A: Which cupcake do you like the most?
31.   T: Would that get you the same data?
32.   Class: Yes!
33.   T: What is the main idea of their questions, Cameron and Aaron. Will?
34.  C: What are peoples’ favorite cupcake?
35.  T: Are they both asking the same questions and will they get the same data to help their parents?
36.  C: Yes!
37.  T: Okay, good!
38.  T: Now you will see some things that are very familiar to you, does that look familiar? Yes?

40. The teacher provided this group with graphic organizers to scaffold their work. She explains that these included a graph to be filled in, graph paper and a rubric.
41.  T: Now that we have practiced using rubrics. I added a rubric, and I want to practice and show you and to let you know what is expected of you.

In Turn 4, Mrs. Washington asks students for feedback about the use of projects. This casual interchange is very typical to the manner in which she speaks to students.

These interchanges provide evidence of the Mrs. Washington attempts to equalize the relationship between her and her students. In Turn 7, students yell out in excitement and
giving Mrs. Washington direct feedback about their positive reaction to the assignment. Mrs. Washington is pleased by the response and is not bothered by the calling out.

The lesson continues when Mrs. Washington explains that students will be responsible for also designing an original flavored cupcake, before dividing the group up into three smaller groups to complete the project. In Turn 9, Mrs. Washington announces that she has created two projects and then asks why that is. This is announced in a matter of fact manner that is received well by students. Students answer in unison, “Everyone gets what they need!” These interactions demonstrated that students have come to understand that different students learn at different paces and Mrs. Washington takes that into consideration when assigning the work. When asked to reflect on this during an interview Shub said,

It means like as long as someone has a problem with that type of work, like I have a problem with multiplication sometimes, so she gives us touch points. I mean it like helps me on how to do it more easier and makes me feel better that I can do it, that’s what I mean. Let’s say someone gets multiplication a lot and already knows it. Mrs. Booth sometimes gives us on homework that someone else doesn’t have which means that you get what you need to be successful. (Focus Interview, p.8)

Mrs. Washington continues the lesson by looking at the task directions to review the components of the task. Even though sections of this lesson are teacher directed, Mrs. Washington does invite everyone to contribute by questioning them about the task, instead of simple providing directions. These questions are used to explore the aspects of the task and to ensure that students are involved in the process. For example, in Turn 12, Mrs. Washington asks, “So what do they have to do first” (Case A Transcript, Event 3, p. 4)? The student answers the question and Mrs. Washington probes her response, looking for a rationale or justification This is followed by a few questions to check to see if the
student(s) are understanding the purpose of the assignment. This task has been assigned to required students to work together to complete it (GR6F). Then in Turn 20, students are given the opportunity to select their own graph to display their data. Here mathematical authority is strengthened because students are provided with some decision making concerning the type of graph they can use to display their data.

Just before Mrs. Washington is set to release the students to work in groups, a student blurts out, “Don’t we need a question?” She stops the class from moving, to address the student’s so that they can address her question. “What is the question going to be?” This question is used to understand students’ thinking and determine whether or not students are ready to work on the task.

Students spend the next few minutes contributing ideas without being selected (GR1A) during a ‘turn and talk.’ After a few minutes, students shared their ideas publicly with the small group. Mrs. Washington encourages students to present a variety of question ideas for their survey to encourage multiple solutions (GR7).

In Turn 31, Mrs. Washington asks, “Does anyone have a different question?” The students had shared the decision making and decided to use certain questions to guide their gathering of the data (GR3E). This was not predetermined by the teacher. Instead of listing all questions, Mrs. Washington directs the thinking toward a more critical look at the possible questions that could be used. Noticing wording differences in the questions, Mrs. Washington asked students to compare them so that each question was clear to students. Mrs. Washington wanted to clarify that students were using the types of question that would accurately achieve the intended results.
In order to raise the thinking to a more sophisticated level, students could have been prompted to justify why the questions were useful and have students debate why one question was more effective or less effective than another (GR13B). Here is another example of when the situation is prime for extension, but not utilized.

During the group focus interview with students, they reflected on their experiences with using the ‘turn and talk’ technique. Martha explains the process by saying, “Everyone talks, everyone is sharing” (Student Focus Interview, p. 3). When asked why it was important that everyone had an opportunity to contribute Nina said, “I think it’s because she calls on somebody there’s only a portion of the class that usually are the ones that raise their hand. When it’s ‘turn and talk’ everyone has a turn to participate” (Focus Interview, p. 4).

During the part of the dialogue when Mrs. Washington asked students to think about the different questions they proposed, she asked *specific students to contribute* using follow up questions to ensure that they were following the *along* (GR1B). In Turn 33, she emphasizes the similarities in the questions by asking, “What is the main idea” (Case A Transcript, Event 3, p. 5)? This focus on the format of questions and efforts taken to rework some of them speaks to Mrs. Washington’s understanding of the difficulty of the language of mathematics. This is one example of how she asks questions in different formats to be sure all learners have access to the conversation. Notice here she is using a question that students may have encountered before and requires them to use it her. In Turn 35, she asks these questions again to *compare* and *further* students’ the thinking.
In turn 36 another student jumps in and answers the question (GR1A). An additional question, “Are they both asking the same questions and will they get the same data to help their parents?” was asked. These questions require a higher level of thinking for students. In Turn 36, a student jumped in shouting, “Yes” (GR1A)! Mrs. Washington is ready to move on. Groups 2 and 3 are directed to work at their desks.

Below, a similar conversation transpires when the first small group, group one, meets with Mrs. Washington (Event 3, p. 5). The teacher talked about the plan for organizing data and the process for completing their project. This group is asked to complete an extension of the project. In Turn 42, students freely ask clarifying questions to gain an understanding of Mrs. Washington’s thinking about this extension (GR1A).

Mrs. Washington is open to their questions and input. This group is more mathematically proficient, based on the assignment and the level of independence given to them, than the other groups and therefore are given even more leeway in solution design and execution of the task. Although there are parameters provided, students are expected to make their own choices. In Turn 50, Mrs. Washington asks, “Did you notice what was not included in your packet?” She had not included the graph they needed to complete, she asks this question so that students would exchange ideas about the different graphs and then select the graph to best represented the data that they collected. In Turn, 61, after students asked about particular graphs they hoped to include, she simply let them know that they could explore those options, if they were reasonable when ready.

This section of the dialogue is just one example of Mrs. Washington’s relationship with students and her commitment to treating each respectfully, as well as, to provide an environment built on trust. In this statement, “You get to choose how you
show your data,” she promotes the idea that students’ contributions are valuable, and students are capable members (GR3D). By allowing them the opportunity to pursue these ideas, she demonstrates that they are able to share the decision making in her classroom.

42. G: So do you get to choose the icing?

43. A: Do we write the flavors here?

44. T: Moving on…..she says let’s go back to the top of the sheet…

45. T: So you are going to collect the data using a tally chart. [She draws their attention to question 2].

46. T: You will create a graph of your collected data….one graph will have the information of the neighborhoods favorite flavors and the other one will have the information about the neighborhoods willingness to try the new flavor you came up with. So you are going to show two sets of data.

47. T: Be sure you use appropriate graphs for the data you are collecting. You must be sure to choose graphs that will represent the data you collected. You get to choose how you show your data.

48. T: I am getting two graphs from you, you choose how you show that data.

49. T: Lastly, you are going to write a summary to your parents explaining what you found.

50. T: Did you notice what was not included in your packet?

51. T: Graphing materials will be ready for you as you need them.

52. T: Any questions about what you are doing?

53. B: Can we do other graphs that we have not learned about but that we know about, like pie charts?

54. T: We can talk about pie charts if there is a group that is interested about pie charts. We can talk about that yes.

55. N: Can we do any kind of graphs or are there certain graphs?

56. T: You want to choose a graph that best represents your data.

57. G: Can it be a graph that we have not learned about yet?
58. T: That question was just asked. I said yes…what are you thinking about?

59. G: A line graph.

60. T: Why would a line graph work?

61. T: You know what? After you collect the data we can talk about the different graphs and charts mentioned and discuss the purposes of all those charts you are talking about.

62. T: Here is your rubric so you know what I am looking for.

63. T: This is so that you will understand the elements that I am looking for…and this is how your project will be evaluated. Any questions before you get started?

64. T: Have Fun!

Not only is there a positive relationship between the teacher and her students but there is also one that exists among her students. As evident in the classroom observations, students listen to one another, take turns talking and do not criticize others. Students have a positive attitude about math and about solving problems with classmates (See Student Questionnaire Results). Most students liked math and many find solving problems easier while working with other students, excluding Jaylissa. All students reported their willingness to help classmates when experiencing trouble solving problems. Reflecting upon this during his interview, Gagan said, “If you don’t think that’s right you can say, “Maybe you can try again or that’s not really right, I don’t think it is because you use too much math or you added wrong, something like that.” (Individual Interview, p.1).

Half of the class preferred solving problems with others. Gagan was one of the nine students reporting a preference for solving problems on their own and not with others. Yet, a contradiction was noted when he indicated that solving problems was easier with other students, along with eleven others. Gagan also noted that he liked to learn from others, again with eleven other students. Gagan prefers to work alone because he
moves faster especially on tasks. According to him, larger groups do not allow you to get much done (Individual Interview, p.4).

The productivity within smaller groups, according to Gagan, depends on the people in those groups. Jaylissa preferred working with a partner, Ben did not have a preference (Individual Interview, p.3, p.3). Also, most students agree that explaining their thinking is helpful to others, including Gagan and Jaylissa. In his interview, Ben surfaced the fact that he did not get the opportunity to share his ideas often (Individual Interview, p.1). He also did not view his contributions as being important to others when he shared in a group setting saying, “nobody really listens to my ideas” (Individual Interview, p. 2). When asked why he thought that he said, “I don’t know, because they got better ideas” (Individual Interview, p. 2).

The data also shows evidence that students need support with explaining their thinking. Six of the sixteen students reporting experienced having trouble explaining their thinking aloud and seven of the nineteen reported having difficulty explaining their thinking in writing. Jaylissa and Gagan did not have trouble explaining their thinking aloud, Ben did not respond to either question. Six of seventeen students, excluding the three students interviewed, reported having trouble understanding how others solved a problem. Clearly, there are a variety of perceptions about the methods being used in the classroom.

Furthermore, half of the students reported being uncomfortable sharing their thinking aloud. According to Jaylissa she was comfortable sharing because she knew that she “could listen and share,” but she was uncomfortable at times because she thought that if she got, “the answer wrong sometimes and show copy the other person and you would
think I was going to think of that also” (Individual Interview, p. 4). Although Gagan reported being comfortable talking, he preferred writing. When asked to reflect on this he said, “I think I like writing it more because if I go and read my writing, I find my mistake and I can fix it right away, but when talking, I just keep on talking, you can fix it but it won’t be as much fluent. (Individual Interview, p. 3). During a similar conversation Ben mentioned, “I am uncomfortable when people know more than me” (Individual Interview, p.4). When asked if he liked being the one who knows the answer, Ben replied, “Yes” (Individual Interview, p.4).

The level of support provided by Mrs. Washington continues to be necessary due to the amount of students that required support with explaining their mathematical thinking and participating in the mathematical conversations as evident in the student questionnaire data.

Next Mrs. Washington meets with the other two groups to monitor their progress (Event 3, p. 6). She again provides an opportunity to for students to engage by asking questions or seeking clarification. The questions are framed in a way that requires students to make their own decisions. It is clear that Mrs. Washington wants students to be responsible for their own progress, she tries not to micromanage their work. She facilitates the project with both groups. She begins with group two. The students in group two are less independent than group one.

65. T: Did you get all of your data? Did anyone run into any problems and how did they solve it?

66. S: I had to go to Mr. Adams because there were not enough people in our room to collect enough data.

67. T: Did you get all of your data?
68. S: Yes.

69. T: Ok, did anyone else have any problems and how did you solve it?

70. T: No, no more problems.

Next, Mrs. Washington checks in with Group one. Below is the dialogue exchanged during this meeting.

71. T: The group that I met with…up here while everyone was doing the tallying and writing their question. You have a step 2 in your process, right?

72. T: You also have to gather another set of data, right?

73. Students: Yeah!

74. T: I am going to invite those people to go back and start getting that ready to do all that.

The final group meeting is with group three. They require a greater amount of teacher coordination of their work and therefore the discourse between the students and their teacher is different. In the discussion below, Mrs. Washington directs students in the process of completing the data task (Event 3, p.6). This group was not as forthcoming with responding to the questions as the previous groups. Mrs. Washington provided extended wait time to allow students to engage, but had to answer her own question to get the conversation moving. This demonstrated the students need this assistance and that she provided it, when necessary.

75. T: Alright, Cameron, Molly, Robert, Shub, I need you back?

76. T: What is the next on your to do list?

77. Next thing that you need to do, according to the directions on the front?

78. T: What is the next thing for you to do?

79. T: You need to make a bar graph.
80. T: What do you need to think about when you are going to make that bar graph?

81. T: Turn and talk.

82. T: Say Nathaniel, what do you think?

Above, in Turn 81, Mrs. Washington initiated a turn and talk to enable students to exchange what they knew with peers, since they did not share in front of the whole small group. The students did share their ideas with peers about what they needed to make a bar graph. Washington casually went to a student to assist by listening privately to his thinking to insure that he was on the right track. Then below in Turn 83, Mrs. Washington redirected their attention to her, and asked the original question again in the same format (Event 3, p.6). She breaks apart the tasks by guiding students to tell her the steps in the process, instead of just telling students the steps. This gives them more authority over the process by modeling the skills needed to develop the autonomy of making these decisions on their own.

83. T: What do you need to think about when you are going to make that bar graph?

84. T: Alright, so, what are some of the things that you are going to think about when you put this graph together?

85. T: Robert, what is the first thing?

86. R: A title.

87. T: What do you need to think about when you are going to make that bar graph?

88. T: You are going to need a title, Cameron?

89. C: A scale.

90. T: What is he talking about when he says scale?
91. A Student shouts out…No idea!

92. T: Someone says “No idea, I like it!

Above, in line 89, a student introduces a new vocabulary word and Mrs. Washington strategically halts the conversation by asking the question that students fail to ask. She asks, “What is he talking about when he says scale?” A student in line 91, admitting to not knowing responds with, “No idea!” (Event 3, p. 6). when he does not know the this is a way to admit what they do not know, which can be rare. This is evidence that first, this student knows that he can join the conversation freely and secondly, he is comfortable acknowledging when not knowing something. This demonstrates a level of trust between the teacher and the student. Mrs. Washington responds with, “No idea, I like it!” This statement shows students that this contribution is highly acceptable in this classroom.

When reflecting about how to encourage students’ engagement in mathematical conversation, Mrs. Washington stated, “there are a lot of kids that don’t realize or they know something is different but they don’t know what to ask, so I put myself in that position of asking” (Focus Interview, p. 1). She also acknowledged that pushing students into the conversation, when they were not ready could be problematic. She said,

I have to get their responses when it’s a safe conversation for them. So, in other words they had a chance to hear other people and they know that they are right, I would not call on them first to start a conversation because they would be like, what if I’m wrong? So they need to hear a few more people first and then I can see if in their face and then I call on them, so we’re building confidence that way. (Focus Interview, p.7)

Thinking on her feet and enacting real time decision making, she does not miss a beat and moves the conversation on by initiating another question. Below, the question brings students back to prior learning experience and reminds them of what they learned previously in social studies that could be applied here. After heads nod in understanding,
in Turn 94, she asks a question to further their thinking (GR9B). The question, “What was that scale on the map and how can that help us think about a scale on a graph?” directs students toward how they apply the way they used the scale yesterday to the task today. A student shares his understanding of a scale and explains this knowledge in terms that the group can use. She is cognoscente that students need this interpretation to scaffold this learning in a way that will help them apply it to their work. This is done respectfully through a question that still honors the student’s contribution but furthers understanding.

93. T: We had a scale on the map yesterday. Do you remember that?

94. T: What was that scale on the map and how can that help us think about a scale on a graph?

95. A: You need to use a scale to collect your information. Also, the scale is how much?

96. T: So what I hear you say is, the scale is going to show us how much or how many roads we go and so the scale is going to show us how many votes we got in our data and are going to show?

The conversation continues below, In Turn 97, she provides students with an additional opportunity to share multiple solutions about what students should consider when creating a graph. Next she directs students to think about how the scale on the graph can be structured to measure the cupcake votes. Finally, a student offers knowledge, without being invited, about an additional solution that students should think about including on their graph (GR1A). Mrs. Washington praises the relevant idea, asks for any additional items, and then sends students off to work on their project.

97. T: What else do we have to think about? Anything?

98. T: Aaron says that the scale on a map says one inch equals ten mile but the scale on the bar graph is one vote.
99. T: So on the map one inch equaled 10 miles, what does one square represent on the bar graph if you put your scale is one square equals one vote?

100. A: One vote.

101. T: Anything we need to think about, Cameron?

102: C: You have to think about the names of the choices along the bottom.

103. C: You have to put the names of the things on the bottom

104: T: Good that goes on the bottom. Anything else?

105. T: That is going to have to be enough, everyone in Mrs. Booth’s class…AND STOP AND STOP!

**Classroom Example 2-Event 4**

The second example of *Everyone Invited to Contribute* (GR1) comes from the mathematical discourse that was part of Event #4. As mentioned above, students did join mathematical conversation but did not strategically choose teachers or students to contribute. However, on one occasion during Event 4, a student directed his contribution to another student during an observation. The question Mrs. Washington asked concerned how you could use an array to help you calculate a large multiplication problem. A student originally suggested counting each of the dots in the top row. The teacher called on the next student who suggested counting by 5’s. Will directed his comment to Amy and Mrs. Washington and said, “You could probably count by 15’s.” Amy replied back to both, “Yeah, that’s what I was thinking” (Event 4, p.8). Below is the conversation that occurred.

1. D: You could probably count the top, top row.

2. T: Why?

3. T: I don’t want to count every single one, someone help me out, by what number? Amy? Amy: By 5’s.
4. W: You could probably count by 15’s.

5. A: Yeah, that’s what I was thinking.

6. T: Checked that counting by 5 would work and said, “It works!”

   Also in Turn 6, the student Amy employs higher level thinking when she agrees with the other student’s thinking (GR14B) and verifies that his thinking is also viable (GR11E). Throughout the course of the eight events students agreed with another student’s thinking four times, and verified another student’s solution publicly twice.

**Summary**

Learning through interaction has created a forum to openly share what they think, listen to how others think, consider input and then make informed decisions about how to solve problems either individually or in groups. This community of learners are learning and growing with the support of each other.

The discourse in Mrs. Washington classroom included both elements in Ground Rule 1- Everyone Invited To Contribute over the course of the eight classroom events. Mrs. Washington expected students to be part of the conversation. They were consistently invited into the conversation. As evident in the data from Table 11 students did jump into the conversation often as a response to open ended questions (GR1A). Mrs. Washington also invited students into the conversation with follow up questions that were more specific and focused on the statements students made when joining the conversations (GR1B). These follow up questions were focused on either helping students articulate their thinking or to decipher their knowledge. In either case Mrs. Washington was focused on building understanding of the ideas discussed.
Students consistently joined the conversation (GR1A) during the course of the eight events as evident in Table 11. However, they did not often choose other members to contribute to the conversation (GR1B). Students did invite other classmates to continue the conversation or when they were stuck, if invited to do so by their teacher. Students should be encouraged to take more initiative in inviting others into the conversation.

The next section explains how Mrs. Washington and her students used the ground rule: Multiple Solutions Are Encouraged (GR7) during her discourse practices. The discourse practices used by Mrs. Washington and her students utilized the elements of this strategic discourse. It was the third most commonly used ground rule in the study.

**Multiple Solutions Are Encouraged (GR7)**

This section includes an examination of the discourse practices or ground rules used by Mrs. Washington and her students during their mathematical conversations while solving problems. This practices associated with this discourse practice or ground rule encouraged students to listen to the ways that others solve problems so that they could apply these strategies independently. The goal of each lesson was to explore the many ways a problem could be solved. The discourse also included exchanging the knowledge students had on a given topic or concept. The mathematical conversation began with Mrs. Washington asking a question or posing a problem for students to discuss. This was followed by others students sharing their knowledge and then a discussion about how these understandings could help them in solving the problem.

Mrs. Washington used tasks and projects to encourage different ways of solving real world problem and to meet the needs of a variety of learners in the classroom. Projects were successful because the students were given room to plan out their own
solutions. For example, students worked on multiple projects including; Let’s Plan It Out, Array Museum, and What’s For Lunch. Mrs. Washington planned discussions with the task from the project in mind, usually as a means for setting the stage and brainstorming ideas they might need. She wanted students to be able to make connections with the mathematics that they were studying. This was evident in her response when reflecting about the use of non traditional, reform-based teaching practices. She said,

> I’ve always been one of those people who says, “how can I get the kids to experience what they need to learn and not just learn what they need to learn because I think the more you can make I personal and the more you can experience something, the more it’s going to stay with you and it’s going to have meaning for you. So, it will stick a little bit more. I’m one of those learners.” (Individual Interview, p. 2)

Students also made the connection between learning in the classroom and using math outside of school. In reflecting about how learning to use authentic skill in school could help him he said,

> Yes, because you never know, my dad he does landscaping stuff and you are going to need to know how to do all the calculating like when you get older you will probably pay the bills and you need to figure out how much money you need to give. (Individual Interview, p.4)

Likewise, in the Focus interview Aaron said learning how to solve real world problems, helped him because he could, “notice them around the world and whenever I feel like oh look there’s an array and I could teach my little brother what they are too” (Focus Interview, p. 9).

> Over the course of the eight events, Mrs. Washington utilizes both elements within this Ground Rule (GR7) as evident in Table 10. She encouraged students to share *multiple solutions* to problems (GR7A). She would do this by asking a question to initiate the discussion and then use follow up questions to give all students ample opportunities
to provide a solution or add to the conversation. She also shared multiple solutions of her own to direct thinking or keep the conversation flowing (GR7B). Students also utilize elements of this ground rule as evident in Table 11. Rarely did students encourage the use of multiple solutions (GR7A), but they often shared multiple solutions and ways of solving problems (GR7B). An in depth analysis of the ways multiple solutions were encouraged will be highlighted through an analysis of the discourse among Mrs. Washington and her students during Event 5.

**Classroom Example 1-Event 5**

The first example of Multiple Solutions Are Encouraged (GR7) comes from the mathematical discourse that was part of Event 5 (Case A Transcripts, p.9). The first part of the lesson was a review of the relationship between area and perimeter and to have students reflect upon why this is important. This was necessary before students could proceed to working on the project where they would be designing a playground with specifications defined in a project based learning task. The essential question and enduring understandings guide the path to student understanding. These are the questions written and used by Mrs. Washington.

**Essential Question:** When is it important to know the area and the perimeter of a space?

**Enduring Understanding:** How do student apply their knowledge of area and perimeter to real life applications?

This investigation focuses on the classroom from Event 5 (Classroom Observation 5, June, 5, 2015). This lesson began with students gathered in the front of the room, near the whiteboard. Mrs. Washington informs students about a project that they will be
completing to use their knowledge of area and perimeter. Before they begin the task, Mrs. Washington directs their attention to the essential question and then asks students to turn and talk with a neighbor about when it is important to know the area and perimeter of a space. They discuss their ideas amongst themselves and she asks, “Does anyone remember what area and perimeter is?” This question helps refresh students’ prior knowledge and assesses the ability to draw understanding from the past in order to apply the concept to this lesson. Earlier that day, students had walked the perimeter of the classroom as a warm up to the math lesson. Spiraling back to lessons or ideas that students had learned previously in the year was something that Mrs. Washington described doing during her individual interview. She spoke of spiraling as a means to fit in all of her curricular objectives, as well as, helping students build on the complex ideas that were difficult in the past. According to Mrs. Washington she was able to tie in a wide range of skills using projects. She said, “That’s the kind of spiraling that I’m talking about, I’m not saying that we recreate and reteach it but putting it into everyday conversations with them” (Teacher Interview, p. 3).

In Turn 1 below, Mrs. Washington asks a student to describe what the activity looked like to her. After the students described, “We were walking around the outline of the classroom,” Mrs. Washington repeats the statement in a question format. In turn 5, another students adds, “You were walking around the area.” She directs students to stand in the area to determine if they had made the connection that the entire space inside the perimeter of the classroom constituted the area of the room. Students pop up and walk to where they think they are in the area and stand still. This is an example of Mrs. Washington’s commitment to having students experience their learning. During the focus
interview she reflected about how authentic learning helps students to make connections. She said, “We can teach a concept all we want but if it does not have personal meaning and they haven’t been able to have the opportunity to put organization (make sense of it) in their head, it’s not going to stick” (Focus Interview, p.7).

In turn 7, she questions their answers to encourage multiple solutions (GR7A) and to investigate understanding. She pushes them further when arguing in Turn 8, “It can’t be both!” Mrs. Washington is trying to get students to voice disagreement and debate this idea and challenge them to offer an opposing idea. This motivates students to jump into the conversation and respond by advocating for their way of thinking. In Turn, 10, Mrs. Washington once more tries to determine if the thinking is solid by instigating a debate. However, students do not budge.

Later in Turn 12, Mrs. Washington again encourages students to share multiple solutions for defining area by explaining it as if they were explaining the concept to second graders (GR7A). After the student offers an idea, she asks her to encourage someone else in to join the conversation and add to the solution during Turn 13. The classmate shares her solution. This process is repeated once more to encourage another solution.

1. T: Amy, what did it look like to you?
2. A: We were walking around the outline of the classroom.
3. T: The outline.
4. T: As close as you could get you were walking around the walls, you were walking around the outline and the outside of the classroom?
5. Aaron: You were walking around the area.
6. T: Can you go put yourselves in the area of the classroom?
7. T: Whooa! I have these people just stood up. Where is the area?

8. T: It can’t be both!

9. Class: It is everywhere here, except the outline! [students calling out at once]

10. T: In math, you only have one answer, when you add you only get one answer. When you subtract you cannot have two answers in math? C’mon.

11. Class is moving around saying yes, yes, it is here and here! [calling out]

12. T: If we had a second-grade friend come in and I said that they (students) were in the area, they would be so confused, how could we explain it to them?

13. T: How would you explain it, although you are all in different spaces, how would you explain that you are all in the area? Amy?

14. A: It is the inside of the box.

15. T: So, it is the inside of the box?

16. T: Can you call on someone else to continue?

17. A: Cameron.

18. T: Cameron she called on you! Cameron are you standing in the area?

19. C: No.

20. T: Do you know what we did this morning?

21. C: We walked the perimeter.

22. K: Area [called out].

23. T: Kevin just helped you, it starts with an “A”.

24. C: Area.

25. T: Yes, you are standing in the area, Call on another friend.

26. C: Andrew

27. A: If you walked the perimeter earlier, then if it’s not the outline, it is the area.
28. T: So, if it is not the outline of the room, that’s good so on the inside of that shape, it is the inside? Excellent.

29. T: Come back and meet me in the front.

Mrs. Washington prompts Cameron with a question to which he responds, “No.” Then Mrs. Washington scaffolds her prompt to encourage engagement, “Kevin just helped you, it starts with an A.” With help from another student he comes up with the answer. Then in Turn 25 below, Mrs. Washington invites multiple solutions by asking the same student, Cameron to call on a friend to carry on the discussion (GR7A). After several students contributed solutions, Andrew summarized the understanding. In Turn 27, he justified that the entire inside of a shape was the area and the perimeter was the outline of the shape. Mrs. Washington repeats his contribution before moving forward.

The lesson continues when Mrs. Washington asks students to gather again in the front of the classroom. In Turn 30 below, the essential question is repeated, “Why is it important to know the area and the perimeter of a space?” Gagan offers a justification. Mrs. Washington, trying to get more students involved in the discussion, invites Gagan to call on a friend to continue. Aaron answers, but does not pinpoint what Mrs. Washington is trying to get students to understand. So below in Turn 35 she states, “It is important to know the area and perimeter of a space so that you know where you are in the space.” This statement directs students thinking. Mrs. Washington continues on this path by encouraging more solutions from students by saying, “Is that fair to say?”

Next, Mrs. Washington encourages a discussion with Dana about how knowing the perimeter of a yard helps when planning to install a fence occurred. She prompted Dana to extend her thinking in Turn 40 by asking, “Would I need to know that for another reason? Does it help me know something else?” Dana answers and Mrs.
Washington asks the class, “Why would that help me, the sides all of the way around how
would as the person who was going to buy the fence?” She asks the class to encourage
them to engage by sharing multiple solutions and ideas and even to further thinking
(GR7A). This question is directed at Dana, but it is understood that anyone is encouraged
to join in. Amy joins the discussion and says, “It is going to help you know how much
fence to buy.”

In Turn 44, Mrs. Washington strategically repeats Dana’s contribution in a way
that confirms her understanding but helps others to solidify their thinking. Again this is
another instance where she asks the questions, when she knows that some students will
not because they do know know what questions to ask (Focus Interview, p. 1). Her
decision making process is based on the idea that “working with kids on IEP’s” keeps her
focused on coming “back to what do I want them to know and then how do I present it to
them” (Focus Interview, p.1).

She also acknowledges that students do not ask questions during mathematical
discussions. When asked to reflect on this Mrs. Washington said,

A lot of kids in math its I want to get it done. I want to move on, while writing is
more reflective. A lot of kids in math, it’s just I’m going to get it done. Whereas,
in writing they start to understand it is more of a process but in math it is very
systematic. (Focus Interview, p.2)

30. T: So, let’s get back to my question, why is it important to know the area and
perimeter of a space? C?

31. G: Because you need it for construction.

32. T: Call on a friend to continue….

33. G: Aaron.

34. A: So, um, it is like Kevin’s idea but different. Also, people might think that you
are still on the perimeter because you are close (to the outline).
35. T: It is important to know the area and perimeter of a space so that you know where you are in the space.

36. Is that fair to say?

37. K: Yes.

38. T: Ok, Dana?

39. D: Because you need to know the perimeter of your yard so that you know where to build the fence and where you are not going to put it up.

40. T: Would I need to know that for another reason, does it help me know something else? GR6B 9B

41. Dana: It helps me know the length and the width is so that you know the space of the outline.

42. T: Why would that help me, the sides all of the way around how would as the person who was going to buy the fence?

43. A: It is going to help you know how much fence to buy.

44. T: Oh, it is going to help me know how much fencing to buy.

The lesson in Event 5 continues and Aaron asks to share his thinking about area and perimeter by drawing a diagram on the board. He draws a shape that is five feet long by seven feet wide. She explains the process for finding the perimeter (7 + 7 + 5 + 5 = 24). He continues with detailing how he multiplied the two sides to find the area (7X5=35). Knowing that a distinction had been made about how to calculate area differently than perimeter Mrs. Washington, shouted. In Turn 45 below she hollers, “Wait! Hold on!” followed by a question, “Does anyone know what my problem is? Mrs. Washington, uses this statement for multiple purposes. She is hoping to further their thinking by getting many students to talk about the relationship between the process for figuring out area and perimeter (GR7A). She wants students to decipher that the sums of both are not always the same, a misunderstanding that many third graders hold. She also is
encouraging student to debate the ideas. Periodically Mrs. Washington will raise her voice for effect, she raises her voice to grab every student’s attention and to ensure that everyone stops to reflect. Then in Turn 49, she asks students to turn and talk encouraging everyone to contribute a solution. When asked why it was so important that students share solutions with one another, she said,

So, kind of back to my expectations, it is that I might be coming to ask you so you better pay attention to what your partner says because I’m going to come to you or I may ask a question, Mary said this and I don’t know what she is talking about. I think you two need some more information about that. (Focus Interview, p.4)

45. T: Does anyone know what my problem is?

46. T: We use the same numbers but we did not have the same answer. I have 25 and 34.

47. T: If you are using the same numbers you should have the same answer. You should have the same answer. The product and the sum should be the same.

48. Class: No, you shouldn’t! [trying to convince her that her thinking was wrong]

49. T: Talk with someone near you about why the product and sum are not the same.

50. Students turned to a peer and discussed why the product and the sum are not the same.

The amount of times students are encouraged to offer solutions demonstrates how Mrs. Washington values encouraging students to share solutions while solving problems in mathematics (GR7). Students also see the value in engaging in mathematical conversations. According to the results of the questionnaire, most students find talking about math helps them to understand the math they are learning more clearly. Many say that they understand math more when talking with their teacher and students. Jaylissa was the only one of the three students interviewed, who reported not understanding math
more when she talked with other students. Similarly, Ben reported not understanding math more when he talked about it with his teacher.

In Turn 51 below, Mrs. Washington spotlights something Shub had said to a peer during the turn and talk. After Shub did not agree with this response, Mrs. Washington asked the class to reflect on the thinking, again. She directed them to consider why the answers were different, specifically to engage in joint reasoning about, “His product did not equal his sum.” Jessie explains the difference between calculating the area and perimeter. In Turn 56, Mrs. Washington pushes students to think more about the relationship of area and perimeter when she asks, “But multiplication is just added addition, why wouldn’t it be the same?” She tries to encourage students to engage by offering and comparing solutions by questioning what has already been discussed. Again, she uses questions to facilitate a debate or discussion to uncover their understanding.

Questioning, again is used to encourage more solutions by engaging more students to join the conversation (GR7A). For example, after Will explains his thinking, in Turn 58, Mrs. Washington jumps in with another prompt to get students to reflect on what has been said by asking, “Ben do you agree with Will? She is also trying to investigate if students can interpret what their peers are saying, verify one another’s thinking or provide insights about viability.

51. T: Alright Shub said you (Aaron) had different numbers because you counted it differently, is that what you did? Yes or no?

52. A: No.

53. T: Why did he get two different answers? His product did not equal his sum?

54. T: Jessie?

55. J: Because you are multiplying the area and adding the perimeter.
56. T: But multiplication is just added addition, why wouldn’t it be the same? Will?

57. W: Because it is 5X7 you start with 7 and then you did times you wouldn’t be counting by 1’s you would be counting by 5’s so 7, 14 and so on.

58. T: Ben do you agree with Will?

59. B: Yes, totally, because if you are repeating, if you are repeating you are counting for a bigger number than just one. If you do just one, you just add 1 time you will get a lower answer and if you add many times you will get a higher answer and usually lower and higher answers are different. (This was a bit of talking around the idea).

60. G: Usually when you are doing multiplication you’re the first number you’re are repeating that number as many times as the second number.

61. T: Let me ask you a question, you told me, let me make a statement. You told me the perimeter was around the shape and the area was inside. We went around the perimeter today and then you sat inside the area?

62. T: Thinking about that and her product and her sum, why are they different?

63. G: For the perimeter, you are just adding and not doing multiplication. Cuz….

64. T: Why am I not doing multiplication?

65. T: He is onto it, he is getting it! Dana help him out.

66. D: Because why you are not doing multiplication you are finding the space of the outline, not all the inside.

67. T: Yes!

68. T: She said, can I interpret what you are saying? She is saying make sure I am right, because you know I get a little crazy sometimes. She is saying is it would not be the same because she is finding how long it is around the object and when you are doing the area you are doing what?

As evidenced in Turn 59 below, students experience challenges in organizing and articulate their understanding in a meaningful way that others can understand. In Turn 60, Gagan has more success describing the process used in multiplication, but does not connect his response to the question about why the area and perimeter would not
be the same in the example provided by Aaron. Assessing that students were not making the connection, Mrs. Washington clarifies what is known. She clarifies the question to pinpoint their thinking about the differences in calculating perimeter and area. In Turn 62 when she asks, “Thinking about that and his product and her sum, why are they different?”

This was a productive move because the next student offers a contribution that was focused on the question. Mrs. Washington, continues by encouraging multiple solutions by asking another student to offer a solution to the question (GR7B). Then in Turn 67, she explains the student’s thinking and clarifies her contribution to draw the students attention to the critical elements.

In the focus interviews, Mrs. Washington agrees talking and learning in the way we do in the classroom has positively impacting learning in the classroom and consistently encourages her students to join the mathematical conversations (Focus Interview, p.1). She also acknowledges that students need constant support throughout the process. In event 5, she often needed to ask direct questions again to uncover the mathematical idea. Students answered her questions but were not forthcoming in asking questions on their own. When asked to reflect on how she would engage students in the conversations she said,

It is interesting because my students will ask questions when they are reading someone’s writing. I am fidgeting around with how to get that into math. We have a common language in how to phrase questions in writing so I need to come up with how you phrase questions when you are in math because they are so little. (Focus Interview, p.2)

In addition, many of the students reflected that sharing their thinking helped them to figure out if they were on the right track, see Table 11. Many students also agreed that
comparing their answers with other students assisted them in noticing if their thinking was correct. Furthermore, if they were not correct, many perceived talking with others would help them find their mistake. All of these results included the three of the students who were interviewed individually.

**Classroom Example 2-Event 2**

The second example of Multiple Solutions Are Encouraged comes from the mathematical discourse that was part of the lesson on April 30 (Case A Transcript, Event 2, p. 2-4). Next is the part of the discussion where Mrs. Washington engages with her students using a ‘turn and talk’. During the middle of the lesson, in Turn 32, Mrs. Washington encourages multiple solutions about how tall she is by directing students to *turn and talk* about their thinking with a peer. Students turn to one or two peers and discuss their solutions. After a few minutes, Mrs. Washington poses the question, “How tall am I?” to encourage an exchange of ideas. She continues by asking, “How did you figure it out?”

32. T: Talk with someone near you-Turn and talk about how tall I am?

33. Kids turn and talk about how they figured out the height of their teacher in feet.

33. T: How tall am I and how did you figure it out?

34. T: Come stand near me. I like Jaylissa, she is my friend! (She is smaller than the teacher) everyone laughs with her. I cannot get near Will!

35. T: I am 61 inches, but if I wanted to say how many feet I am tall and how do you figure it out?

36. Aaron: You are 5 feet and one inch.

37. T: How did you figure that out?

38. Aaron: I know that 12X5 =60
39. T: Wait! Wait! So your strategy was multiplication? (said with excitement!!)

40. Nina: Hey (this was her strategy too).

41. Aaron: 12X5 =60 then 60 plus 1 is 61 inches.

42. T: This is what was in Aaron’s brain. (TEACHER WRITES STRATEGY ON BOARD)

43. T: Who did it in a different way?

44. Amy: I added 12, five times on my fingers….and I got to 60, as Aaron did, and I added one and I knew it was five feet because I had my five fingers up.

45. T: Sh sh sh… Is that how your brain was working?…12 inches plus inches, plus 12 inches equals one foot…that is what I heard you say, is that a good representation.

(WRITES STRATEGY ON BOARD)

46. T: Does anyone have a different strategy?

47. T: Are these strategies similar? [Aaron and Amy]

48. T: Kevin?

49. Kevin: Yes, because multiplication is just repeated addition.

The first person to share is Aaron. In Turn 37, Mrs. Washington asks Aaron to encourage engagement and to understand his thinking. She draws attention to Aarons thinking by spotlighting his ideas when he mentions using multiplication as a strategy to solve the problem. She also represents his thinking on the whiteboard for all students to follow. Mrs. Washington continues the discussion by encouraging more students to share different solutions. Amy shares and then Mrs. Washington spotlights her thinking, as well, by recording her solution on the board too. In Turn 46, Mrs. Washington encourages students to engage in joint reasoning again. This time she poses a question to
direct the communities to think by comparing the two strategies. Kevin joins the conversation and explains the connection between addition and multiplication.

Once again, Mrs. Washington encourages more students to engage in joint reasoning. In Turn 51 below, Dana joins in to explain her use of division to solve the problem. Mrs. Washington draws the communities attention to her solution by spotlighting the idea of using division as a strategy to solve this problem. She also asks her questions to try to understand her thinking so that others are able to follow her solution. Furthermore, Dana explains her thinking with others by recording her strategy on the whiteboard. In Turn 57, Mrs. Washington asks a clarifying question to reinforce the communities understanding of Dana’s thinking.

50. T: Were there any other brains working here, Dana?

51. Dana: Instead of multiplying I divided.

52. Class: What? Dana, what did you do?

53. T: Get up here, I have no idea, what you are talking about?

54. Dana: You are 61, I divided 61 by 12.

55. T: Write that out for me.

56. Dana: It has a remainder which is 5 feet and 1 inch.

57. T: So the remainder is the left over or 1?

58. Dana: Yes.

Mrs. Washington encourages more students to engage in joint reasoning by inviting Nathaniel to contribute. This open ended question allows him to share his own ideas with others (GR6A). He explains his solution. Mrs. Washington again asks a clarifying question, “How did you know that?” followed by “Can I ask you a question?”
Nathaniel has some difficulty responded so Mrs. Washington helps him to work through the process by scaffolding his ideas by explains his thinking in her own words (GR6G). In Turn 65, Nathaniel explains his thinking. Then in Turn 68, she spotlights his thinking for all to notice.

59. T: Nathaniel, what were you thinking?

60. Nathaniel: I was thinking 48 was 12 equals 60 4 feet.

61. T: How did you know that?

62. T: Can I ask you a question? How did you decide to start with 48?

63. Nathaniel: stumbling…had difficulty explaining.

64. T: [Repeated] You knew that 48 equals 4 feet. Is that what you said? How did you know that?


66. T: So, you kind of did this one (pointing to Amy’s) but you stopped at a certain point and then just added 12?

67. Nathaniel: Yes.

68. T: So I was just wondering why you started at 48. It was just interesting.

69. T: Will?

70. Will: So, I was thinking that you could do 2 X12 =24 and 24 plus 1 yard = 60 inches and then you just add one inch.

71. T: Wow! What does one yard equal?


73. T: So, 24 inches plus 36 =60 and 1 inch.

74. T: Wow! You’ve got to make your brain think. It is the only way to make it work.
In Turn 69 below, Will also engaged in this joint reasoning by sharing his strategy. He frames his strategy by connecting it to one yard. Mrs. Washington encourages this type of exchange to emphasize effort. She asked Will, “What does one yard equal?” She spotlights his ideas by providing him with praise regarding his hard work. To close the lesson and encourage joint reasoning, Mrs. Washington asks, “Was there one way to figure this out?” Students respond by shouting, “No!” Gagan responds, “As long as you get the right answer, you don’t have to do it the same way.”

**Summary**

The mathematical conversations within Event 5 and 2 were strong examples of the ways Mrs. Washington *encouraged students to contribute multiple solutions* continuously throughout the lesson (GR7A). Students were encouraged to share their thinking, extend their thinking, make connections to prior learning and reflect on the thinking of others. Mrs. Washington even rephrased questions to dig holes in their knowledge to assess whether or not they understood the concept. Even when students came to a shared their own understanding, Mrs. Washington continued to press for widespread understanding among the class. Students experienced *sharing multiple solutions* to various different questions (GR7B). Many were willing to present their solutions and to listen as other were sharing.

However much of the strategic discourse was led by Mrs. Washington. Some of the questions were difficult for students, leading Mrs. Washington to again needing to use “real time decision making” and redirect or refocus students so that they were working toward the path that she had intended. She planted many seeds. At times, when students took the discussion into a territory that Mrs. Washington had not planned, she skillfully...
used questions to redirect the conversation back to the original objective that she had planned.

Compared to the other seven events, students had the most experience being prompted to engage in debates during event 5 (GR11B, GR13B,C). Mrs. Washington encouraged students to deliberate the sum and product of perimeter and areas or problems having multiple solutions. Engaging students in debate requires skills similar to those found in the elements of Ground Rules 9-12, which requiring higher order thinking. If these elements were more widespread during this event and throughout the course of the eight events, the discourse would have been more productive. Finding ways to encourage students to critically think about, and evaluate ideas further increases their flexibility for choosing a path for determining solutions.

In addition, students were asked if they agreed with an idea presented by their teacher or a classmate. One or two students would respond. Providing greater opportunities for all students to share why they agreed with the statement or even utilizing the turn and talk strategy would increase participation. This element can be found among Seeking Agreement for Joint Decisions (GR14) which is a higher level thinking skill. Modeling elements requiring higher order thinking more routinely, and requiring students to contribute more often would also be beneficial.

Thirdly, students are rarely asked to offer their opinions about effectiveness of strategies (GR12). They simply verify correct solutions from incorrect solutions. Analyzing the similarities and differences and effectiveness of a solution can help support students as they attempt to apply a similar solution to a problem in the future (GR11D,E).
Overall, the discourse driven by Mrs. Washington has been effective in getting students to think about mathematics and share their understanding. The overall positive attitude among students as evidenced in the interviews and questionnaires about working with others while problem solving in mathematics supports the process used by Mrs. Washington in this third grade classroom.

**Reform-Based Practices For Learning Mathematics With Understanding**

As the analysis below will show, Mrs. Washington fulfilled her professional responsibility by implementing the Massachusetts State and Common Core Standards in mathematics, as dictated by her school district. Each of the reformed-based practices was embedded into the mathematics lessons and conversations she facilitated with students in a community of practice (See Table 4). Ms. Littleton also used the curriculum mapping tool Atlas to guide her unit planning and made daily decisions about how her instruction was carried out (Rubicon Atlas, 2018). The Atlas tool warehoused the scope and sequence, content, skills, essential questions and enduring understanding developed by teachers in the district using the Common Core and Massachusetts state standards in math, English, social studies, and science (Massachusetts Department of Elementary and Secondary Education 2009; National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010). Common Assessments were also included in the Atlas tool. Teachers developed these assessments in grade level teams and entered them into the on-line planning tool curriculum map to use as a reference. The map has not yet contained lesson plans teachers used to implement the standards. Teachers in the district were aware of the Common Core Standards for School Mathematics document
but have received little guidance from state and local administration about how to implement them into their mathematics instruction.

During the individual interview, Mrs. Washington described her mathematics instruction. She said,

Because I work in a co-taught [special education students with a special education teacher] classroom, students need the skills taught to them in a different way. I learned quickly that when I teach I have to spend a lot of time thinking about the words that we are using, how words communicate a concept to them and then how I am going to develop an opportunity to take what we talked about and play around with it. (Individual Teacher Interview, p. 1)

Mrs. Washington also spoke about how she has learned that some students struggle because they do not know whether or not they know something yet, “they don’t want to ask” (Individual Interview, p.2). In this case, she said that she puts herself in the position of asking because they will not. When asked how she has come to understand what students need, she said,

Prior to working at Lakeview Elementary, I was a special education teacher. I think that experience of understanding an alternative way of how they think and I’m trying to break down barriers for them as learners so that they can get to the content. This is something that I learned along the way. (Individual Teacher Interview, p. 1)

In addition to implementing the reform practices, Mrs. Washington instituted the structural changes that included providing opportunities for students to develop more autonomy and shared authority while learning mathematics. Along with the examination of the practices as implemented by Ms. Littleton are her perspectives, collected during interviews, about how she came to know and use reform-based practices. Perspectives about these methods from students will also be included.
Problem Posing

Problem Posing is the second reformed based instructional practice described in Chapter 1 of this document. Mrs. Washington used problem posing to provide students with opportunities to engage in mathematics using conversations to explore the mathematics. The components of Problem Posing included a well-defined problem or
task, enriches concepts and skills and provides a structure for discussion. The components are embedded in the process, and therefore all three are utilized concomitantly during each of the eight events. The components of Problem Posing were implemented during all of the seven events in this study, as noted in Table 4.

For the purpose of this study, a well-defined task is a project-based problem that required students to connect their learning to a real-life situation. A task can be complex and usually completed over the span of one or two class periods. Similarly, a well-defined problem challenged students to think beyond the skill. A well-defined problem also presented an opportunity for students to solve problems in more than one way. Both problems and tasks provided a level of complexity that allowed for rich discussions among participants. As will be discussed below, each problem and task helped students convey their thinking to others and to exercise their knowledge of strategies used to find solutions. Additionally, the well-defined tasks Mrs. Washington used also required students to build upon the skills learned earlier in the year. Samples of these tasks can be found later in this document (See Appendix K).

As revealed in Table 4, Mrs. Washington used either a well-defined problem or task during all events. Problems were posed during events one, two, four and five to engage students in thinking about the ideas and concepts embedded in the tasks. Tasks required students to complete a project using the skills reviewed in the problems. Students engaged in tasks during four events including the third, sixth, seventh and eighth events.
During the individual interview, Mrs. Washington spoke about the need for students to have opportunities to make decisions and problem solve. She describes an antiquated conception of problem-solving that still exists among her fellow teachers.

Kids don’t have problem solving skills. We have a category called problem solving which cracks me up. Problem-solving is different from what I’m talking about and even going to think problem-solving criteria needs to change. I’m one of those people. I think that it needs to change. (Individual Interview, p.2)

Mrs. Washington was referring to the traditional practice of teaching problem solving as one unit and then moving on. Instead, she uses projects throughout the year to “have students talk with people and try to engage in a conversation and listen” (Individual Interview, p. 8). She also mentioned that she used projects more this year because the students are more controlled than they have been in the past. In years past students tried to “take over the classroom” (Individual Interview, p.7). However, utilizing contracts to support students as they work together has been helpful to the process.

**Well Designed Problem**

Mrs. Washington used problem posing to initiate discussions among students during events one and two. The mathematical discourse facilitated during this lesson focused on students sharing and explaining different ways of determining an accurate solution. During Event 1, Mrs. Washington posed the problem, “My doctor does not tell me how many inches tall I am. He tells me that I am 61 inches. He tells me that I am a certain amount of feet tall. How tall am I” (Case A Transcript, Event 1, p.1)? While students present their ideas, Mrs. Washington interacts with each student using additional questions to prompt explanations and exchanges among her students. Throughout the process, students jump into the conversation to add their ideas.
During Event 2, Mrs. Washington also poses a problem to engage students in a discussion about the different ways to represent data during the second event. Mrs. Washington posed the question, “Could I have collected my data in a different way” (Case A Transcript, Event 2, p. 3)? The discourse she facilitated during this lesson assisted students in thinking about applying skills before having to collect and represent their own data during the following day. Tasks selected by Mrs. Washington were linked to the district curriculum (Rubicon Atlas, 2018). For example, the standard corresponding to this task is referenced in the Atlas Curriculum Map for Grade 3. Below is a sample of the information from the curriculum map linking the assignment to the Massachusetts State Framework.

Measurement/Data: Represent and interpret data.

3. Draw a scaled picture graph and a scaled bar graph to represent a data set with several categories. Solve one- and two-step “how many more” and “how many less” problems using information presented in scaled bar graphs. (Rubicon Atlas, 2018)

Essential Questions:

- How does the type of data collected influence the type of graph to use?
- What parts of graphs are important to understanding the data?
- How can I use graphs to more efficiently display and interpret information? (Atlas Rubicon, 2018).

Mrs. Washington engaged her students in a conversation using the essential questions gathered from the Atlas mapping tool (see above). This discussion helped guide students toward a future task that required students to select the type of graph that would best represent their own data in both Event 3 (Case A Transcript, p. 4-6) and Event 7 (Case A Transcript, p. 11-12).
During Event 4, Mrs. Washington began the lesson by posing the question, “What is an array” (Case A Transcript, Event 4, p. 7)? The discourse that occurred during this event helped to review the concept of an array and the ways arrays could be applied to measurement. Students are provided with opportunities to share the meaning of an array. While they reviewed arrays, students linked using arrays to multiply and using arrays to find the area. Students had developed some knowledge of arrays because they were part of the curriculum experienced earlier in the year (Atlas Rubicon Multiplication Unit, 2018). The purpose of this lesson was to set the stage for a playground/garden design task that would be completed during the next few class sessions (See Appendix K). Students would need to be able to take their knowledge of arrays and apply it to the new task.

Mrs. Washington explained using a spiraling curriculum, circling back to prior learning to strengthen the skill and then extending this knowledge by completing engaging in conversations or problem-solving tasks. She purposefully exposed students to skills all year long and not just during units of practice (Individual Interview, p.18).

In Event 5, Mrs. Washington also reviewed the activity students had engaged in earlier in the day. During this activity, students were asked to demonstrate their understanding of perimeter by walking around the perimeter of the classroom (Case A Transcript, p. 9). The discourse that occurred during this event helped to solidify the differences between area and perimeter. The purpose of this authentic activity was to make a real-life connection with the topic by experiencing what area and perimeters looked and felt like to students. Additional information on this lesson will be included in the section describing Mrs. Washington’s implementation of Active Learning With Authenticity below.
Well Designed Tasks

During the remaining events, as described in the following sections, students were assigned well-defined tasks. During Event 3, students completed a task that required them to collect and represent data about their favorite cupcakes (Event 3, p. 5). A more challenging task was provided to some students during this event. The more challenging task required groups to survey classmates about favorite flavors and even design a new flavor of cupcake. The intention of this new flavor was to help their parent’s bakery become the best one in town!

In Event 6, students were working on the playground/garden design task (See Appendix K). The task required them to build a playground or garden with four rectangular shapes, three square shapes, two combined shapes and one polygon. Students were asked to record their dimension in feet and yards. A brochure was created once students had completed their design and labels. The final design required students to illustrate their work. Additionally, students recorded the area and perimeter of all rectangular and square shapes. While they worked, students shared their ideas and insights with peers at their table groups. They also shared thinking with their teacher and peers. Many of the students agreed that if their answers were incorrect, they could identify a mistake by talking with others (see Table 3). For example, while students were talking about their designs, one approached Mrs. Washington to share her work. Mrs. Washington asked, “Will it fit?!”

During Event 7, students completed a data measurement project that allowed them to identify a question, collect the necessary data and choose a graph to represent the results (p. 12).
Many chose topics on their own including favorite television shows and restaurants. Students in the first group were assigned to collect their data and put the data into an existing graph. The more challenging tasks required the second group of students to collect data and pick a pre-made graph to represent their data. The most difficult task required the third group of students to survey their data and set up a graph of their choice. Students were engaged in these activities during the observation. Mrs. Washington rotated the groups of students to meet with her at the front of the room while others worked at their seats.

Finally, in Event 8, students chose tasks from a variety of options. The choices provided a variety of options that extended the learning from the year. Some tasks were more challenging than others. One task required students to design a mall using predetermined geometric shapes, area and perimeter (Case A Transcript, Event 8, p. 13). The most difficult task required students to compare three cell phone data plans to determine which was the most cost-effective.

**Enriches Concepts and Skills**

The second component within Problem Posing is enriched concepts and skills, as will be shared below. Problem Posing stimulated students to think beyond the grade level curriculum (Rubicon Atlas, 2018). As evident in the observations, Mrs. Washington did not directly teach the content and skills needed to complete the problems, these concepts had been introduced earlier in the year. Instead, she posed questions, described above during the events above, to investigate students thinking and then filled in the knowledge by guiding students to discuss areas where gaps emerged. She even repeated these questions and requested that many students answer them in order to be sure students
did not miss the information shared (Walking The Perimeter Lesson, Event 5, p. 9-10)

The tasks required students to apply the content and skills in different ways to strengthen and enrich their prior learning while connecting the skills to real life (Cupcake Challenge Lesson, Event 3, p. 4-6). These tasks allowed students to shift their study away from practicing algorithms and calculations and toward reasoning about situations and applying methods they had learned earlier (Event 4, p. 6-8). Samples of some of these tasks can be found later in this document (See Appendix K). The focused of each task was to strengthen and extend the grade level concepts and skills except for Event 4 and Event 5 where students were reviewing concepts and skills in preparation for tasks (Atlas Rubicon, 2018)

Most of the tasks were completed by small groups of students. Each group of students was given slightly different assignments based on their academic achievement level. These projects enriched the concept and skill at that particular group’s level. For example, during the Cupcake Challenge Task in Event 3, one group surveyed the class and then determined a “new” flavor that they predicted would make their parent’s bakery the best in town (Event 3, p. 5). This first group was also responsible for designing their own graphs. The second group surveyed the class to determine which of their chosen cupcakes was the favorite. A graph was provided for students to fill in on their own. The final group created a graph of favorite flavored cupcakes, only they were given the flavors and the bar graph to fill in.

Some of Mrs. Washington’s students had significant challenges with the math curriculum. This was reflective of the data from the survey/questionnaire. According to responses gathered, over half of the students reported struggling with math. Similarly,
many students agreed that they have trouble understanding how other students solve a problem when they explain their solution to the class. Six students are uncomfortable when they are asked to share their thinking with the class. Furthermore, all 17 students in this class reported not liking math.

Another example of a problem that was used by Mrs. Washington to encouraged students to enrich concepts and skills occurred during Event 1. Mrs. Washington posed the question, “My doctor does not tell me how many inches tall I am. He tells me that I am 61 inches. He tells me that I am a certain amount of feet tall. How tall am I” (Case A Transcript, Event 1, p.1)?

24. T: My doctor does not tell me how many inches tall that I am. He tells me that I am a certain amount of feet tall. Every 12 inches that is hiding in this 61, I can say that am one foot tall. When I get to 12, I am 1 foot. When I can to 24, I can say…I am two feet tall.

25. T: If I get to.. what comes after that?

26. C: 36

27. T: 36

28. T: I can say I am…?

29. C: Three feet tall.

30. T: Let’s figure it out how many feet tall I am?

31. T: Don’t say as tall as Gagan, because that is not a standard unit. It is a comparison, but it is not a standard measurement. We have to talk about inches and feet.

32. T: Talk with someone near you-Turn and talk about how tall I am?

33. Students turn and talk about how they figured out the height of their teacher in feet.

33. T: How tall am I and how did you figure it out?
34. T: Come stand near me. I like Jaylissa; she is my friend! (She is smaller than the teacher). Everyone laughs with her. I cannot get near Will!

35. T: I am 61 inches, but if I wanted to say how many feet I am tall and how do you figure it out?

36. A: You are 5 feet and one inch.

37. T: How did you figure that out?

38. A: I know that $12 \times 5 = 60$

39. T: Wait! Wait! So, your strategy was multiplication? [said with excitement!!]

40. N: Hey! (this was her strategy too).

41. A: $12 \times 5 = 60$ then $60 + 1$ is 61 inches.

42. T: This is what was in Aaron’s brain. [TEACHER WRITES STRATEGY ON BOARD $12 \times 5 = 60 + 1 = 61$ inches]

43. T: Who did it a different way?

Students know have gain experience measuring objects (Rubicon Atlas, 2018). She extended this learning by asking students to convert her height in inches to her height in inches and feet. After presenting the problem, Mrs. Washington provides an opportunity for the students to think by discussing the problem with a partner, using ‘turn and talk’ before sharing their ideas with the class. While students present their ideas, Mrs. Washington interacts with each student using additional questions to prompt a more thorough explanation. Throughout the process, students jump into the conversation to add their ideas. Mrs. Washington always invites additional students to join the conversation. In this lesson, she keeps the dialogue moving by asking, “Who did it a different way?” to keep the dialogue going (Case A Transcript, Event 1, p. 1).
Some of the tasks were also completed independently. During Event 6, students enriched their understanding of their area and perimeters by designing a playground that required them to choose and identify the area and perimeter of shapes (Plan It Out Lesson, Event 6, p. 11). In Event 7, students also extended their thinking by collecting data, creating questions to go along with their data and designing their graphs instead of receiving a graph and being asked to answer questions about it (Case A Transcript, p. 12).

**Provides Structure for Discussion**

The third component of Problem Posing is *provides structure for discussion*, as shown below. The problem-posing approach helped Mrs. Washington establish a structure for her students to interact with peers while moving toward understanding in the learning community. Mrs. Washington developed a practice by using specific techniques for discussing mathematics with students in her classroom. For the purpose of this study, the practices or structures she used have been defined as elements within the fourteen ground rules, as summarized earlier in this chapter. These ground rules defined the structure that became accepted as what it meant for Mrs. Washington and her students to learn mathematics together in this classroom (Cobb et al. 2001). Using these ground rules, Mrs. Washington was able to encourage students to interact with others and increase their opportunities for shared thinking (Cobb et al., 2001; Hmelo-Silver, 2002; McClain & Cobb, 2001). This structure for *engaging in discussion* was implemented during all eight events in this study, as noted in Table 4.

As described earlier in this chapter, Mrs. Washington and her students implemented certain elements of strategic discourse very frequently within the ground
rules, see Tables 1 and 2. These elements demonstrated that Mrs. Washington emphasized certain elements during her discussions with students. Mrs. Washington engaged students in *joint reasoning* (*GR6*) by posing problems for them to discuss on 271 occasions. She also opened the discussions to *invite everyone to contribute* (*GR1*) on 168 occasions. She encouraged students as they publically *shared their knowledge* (*GR4*) on 66 occasions, and offered *multiple solutions* (*GR7*) on 78 occasions. Additionally, Mrs. Washington created an *atmosphere of trust* (*GR2*) on 57 occasions during events one through eight.

### Active Learning With Authenticity

The third reformed based instructional practice is Active Learning With Authenticity. Mrs. Washington provided authentic experiences for students to learn real-world mathematics by solving real-life tasks that linked school mathematics with real-world mathematics. The components of Active Learning With Authenticity included: *engages in learning, making real-life connections, and honors mathematics as a discipline* mathematics (Weiss et al., 2009). Students were engaged in Active Learning With Authenticity using various discourse elements during each of the observations, as noted in Table 1 and 2. The components of Active Learning With Authenticity were implemented during most of the eight events in this study, as noted in Table 4.

#### Engages In Learning

The first component of Active Learning With Authenticity is *engages in learning*. As noted previously in this chapter, Mrs. Washington consistently engaged students in the learning by providing tasks that encouraged students to increase their involvement in learning, and develop thinking skills. All of the problems and tasks
implemented by Mrs. Washington engaged students in learning because they had a connection to real life (See Case A Transcript, p. 1-18). Additionally, students were observed as being engaged during lesson observations by the researcher.

One example from the data set included a lesson that engaged students authentically in learning while they reviewed area and perimeter during Event 5 (Case A Transcript, p. 9-10). During this activity, students were asked to demonstrate their understanding of perimeter by walking around the perimeter of the classroom. The discourse that occurred during this event helped to solidify the differences between area and perimeter. Students would need this knowledge while completing the Let’s Plan It Out, playground/garden task that was assigned later in the week, see Appendix K.

1. T: Tell me what that looked like when we were walking around the perimeter of the classroom.

2. S1: You were walking around the walls

3. T: To the best that you could, because I had stuff. You were walking along the walls what did it look like to you.

4. S2: We were walking around the outline of the classroom

5. T: The outline, you were walking around the outline and the outside of the classroom.

6. S3: You were walking around the area

7. T: Can you go put yourselves in the area of the classroom?

8. Students moved to several spots in the classroom

9. T: How would you explain it, although you are all in different spaces, how would you explain that you are all in the area?

10. S4: It is inside the box.

11. S5: If you walked the perimeter earlier, then if it’s not the outline, it is the area.
12. T: So, it is not the outline of the room, that’s good.

The discourse that occurred during this event included a conversation that encouraged students to visualize what the activity looked like to students. Next, Mrs. Washington asked her students to stand in the area. Students moved about the room and then stopped in one spot. Some look around confused because many chose different spots throughout the room. Next, Mrs. Washington asked, “How would you explain it, although you are all in different spaces, how would you explain that you are all in the area” (Case A Transcript, Event 5, p. 9)? Someone yelled out, “It is everywhere, except the outline!” To check for understanding, Mrs. Washington asked individuals, “Are you standing in the area?”

After a brief discussion involving students sharing their thinking about the difference between area and perimeter, Mrs. Washington poses the question, “Why is it important to know the area and perimeter of the space?” Mrs. Washington wanted students to know that “the area was not on the outside” of a shape (Focus Interview, p. 5). This was a critical idea that students would need to know to complete the upcoming playground/garden task that she had planned for them.

Additionally, authentic learning occurred within the activity of students as they discussed mathematics with one another (Weiss et al., 2009). Authenticity was evident in the students’ contributions during this mathematical discourse, even if students were not always completely accurate.

As described earlier in this chapter, Mrs. Washington engaged students using elements to encourage them to share ideas and solutions (GR6A) on 76 occasions, see Table 1. She also used questions to encourage the exchanging ideas (GR6C) on 40
occasions and to understand the thinking of others (GR6D) on 49 occasions. Mrs. Washington also engaged students in learning when spotlighting original ideas (GR6E) on eight occasions. She prompted students to share many ways of solving problems (GR7A) on 62 occasions and used questions to further their thinking (GR9B) on 37 occasions.

Also evident in the data, Mrs. Washington’s students often shared their ideas with their teacher and peers, see Table 2. They shared ideas and solutions with others (GR6A) on 147 occasions. They also demonstrated a willingness to exploring different ways of solving problems. Mrs. Washington encouraged students to explain their thinking. Students offered multiple ways of solving problems (GR7A) on 62 occasions. Students provided justifications and rationales (GR5A) to support their thinking and solutions on 69 occasions. They were also provided with opportunities to engage in twelve ‘turn and talk’ experiences to discuss their thinking with peers (GR9A). They also advocated for their way of thinking (GR11C) on 11 occasions, as well. Students gave and were given assistance to work through the process when they stumbled or answered incorrectly (GR6G).

During the individual interview with Mrs. Washington, authentic learning was discussed. She commented that having special education students made her realize that students needed the skills taught more authentically. She said,

I actually spend a lot of time thinking about the words that we're using, how are the words communicating a concept to them and how am I going to develop an opportunity to then take what we talked about and play with it. Let's take what they know and let's start applying it to everything that is going on. (Individual Teacher Interview, p.1)
When Mrs. Washington moved from a special education teacher to a general education teacher, she noticed that students knew how to do the calculations in mathematics but did not understand why they were doing it. That was when she started teaching and thinking differently about her instruction. She wanted to get students "to experience what they needed to learn and not just learn what they need to learn." This was evident in the lesson plan cited above (Individual Teacher Interview, p. 18).

During the individual interview, Mrs. Washington also conveyed teaching more about the process; not just answers in mathematics (p. 18). She mentioned spending a lot of time talking about the process with students. She also described being more comfortable with the experimental side of mathematics, especially acting it out, like in the perimeter and area lesson during Event 5 now that she has taught the content for five years (p. 18). Mrs. Washington shared the importance of providing hands-on experiences in her instruction. She felt strongly that students should be able to figure things out.

**Connects To Real Life**

The second component of Active Learning With Authenticity is making real-life connections. Authentic activities required students to make connections between how the math they used in school could be used outside of the classroom to solve real-world problems. As shown below, Mrs. Washington engaged students in completing tasks that could be applied in the world around them. Applying their learning to new situations extended their thinking beyond mastery of skills toward developing understanding.

All lessons in Mrs. Washington's events included connections to situations where the mathematics discussed could be applied in real life (Case A Transcripts, p. 1-18). For
example, Mrs. Washington engaged students in a discussion about the meaning of an array before exploring the ways arrays could be applied to finding the area and perimeter of shapes or objects found in their environment.

During Event 4, Mrs. Washington began the discussion by posing the question, “What is an array” (A Transcript, Event 4, p. 7)? The discourse that occurred during this event helped to review the concept of an array, which is listed under skills in the curriculum map (Atlas Rubicon, 2018). Students would also explore using arrays in measurement to identify area and perimeter, also noted in the curriculum map.

Furthermore, Mrs. Washington encouraged students to make a real-life connection when identifying the perimeter and area of their classroom by walking around the outline of the room (Event 5, p. 9-11). This discussion also helped set the stage for a task that would be completed during the next few class sessions.

1. T: I am going to ask you to work on the portion that is the Array Project.

2. T: What is an array? Turn and talk to someone.

3. Students talk with others.

4. T: So, my question to you was, what is an array?

5. S: So, you use a symbol for example dots, and you solve a multiplication question like 7 times 5.

6. T: So, if you used a symbol like dots to solve 7 times 5 you would put the dots 5 down and 7 across?

7. S: Yes.

8. S2: An array can help you find the area.

9. T: Oh! An array can help you find the area, is she right?

10. T: I want someone else!
11. S: When you measure the distance, the area you can use an array because in the area you use multiplication, you would use 4X3 with 4 across and 3 going down and 4 and so you did 4 times 3 you would get 12.

12. T: Beautiful.

After discussing their ideas with a partner, during a ‘turn and talk’, students shared their thinking with the class. Students are provided with opportunities to share the meaning of an array. While they review arrays students link using arrays to multiply and using arrays to find the area. Students should have some knowledge of arrays because they are part of the curriculum experienced earlier in the year (Atlas Rubicon Multiplication Unit, 2018). The upcoming playground task would extend the students’ knowledge because students were asked to use arrays to represent the area and perimeter of a certain object included in their playground design.

Then during Event 6, students were observed working on the playground/garden design task (Case A Transcript, p.10-11) The task required students to build a playground or garden with four rectangular shapes, three square shapes, two combined shapes and one polygon (See Appendix K). Students were asked to record their dimension in feet and yards. A brochure was created once students had completed their design and labels. The final design required students to illustrate their work. Additionally, students recorded the area and perimeter of all rectangular and square shapes. While they worked, students shared their ideas and insights with peers at their table groups. They also shared thinking with their teacher and peers.

Using a real-life experience helped students to solidify the meaning area and perimeter. According to Mrs. Washington, she preferred when students experienced the mathematics. She wanted students to spend a lot of time using “hands-on” learning in
mathematics (Individual Interview, p. 1). When asked to reflect some more on this idea this during her interviews, she said,

I’ve always been one of those people who says how can I get the kids to experience what they need to learn and not just learn what they need to learn because I think the more you can make it personal and the more you can experience something the more it’s going to stay with you and it’s going to have meaning for you. (Individual Interview, p. 2)

Then students designed a playground using their knowledge of area and perimeter and identified arrays within their environment (Event 6, p. 11). Next, they completed an open-ended design task incorporating geometric shapes (Event, 7 p. 12) followed by a task where students were given an opportunity to pick from several options (Event 8, p. 13). One group designed a mall, and another determined the best deal on a cell phone plan.

**Honors Mathematics As A Discipline**

The third component of Active Learning With Authenticity is *honors mathematics as a discipline*. Learning mathematics with authenticity means that students come to understand mathematics in ways that mirror the structure and content of mathematics as a discipline. Mrs. Washington *honored mathematics as a discipline* by making connections ideas with the theories, concepts, and notations found in the discipline of mathematics. She also used and emphasized accurate terminology and content when students shared ideas and solutions.

Additionally, Mrs. Washington guided her students to do the same. Students were required to explain their thinking by making connections among their thinking and mathematical content. They were also required to provide explanations that included representations of their thinking using standard mathematical notation, often using
numerical expressions. Several lessons encouraged the honoring of mathematics among Mrs. Washington and her students.

During event one, Mrs. Washington *honored mathematics as a discipline* when she explained the purpose and use of standard measurement. She began by exploring why everyone uses the standard 12 inches equals one foot (Event 1, p. 1) In event two she asked students “What is the purpose of a tally chart” (Event 2, p. 2-3)? Afterwards students discussed the differences between the various elements in a tally chart versus a pictograph, emphasizing why one was more purposeful with the given set of data.

Similarly, Mrs. Washington and her students discussed the bar graph and the several elements included in this type of graph (Event 3, p. 5).

During event four, Mrs. Washington and her students discussed the purpose of an array and how it could be applied to solving two-digit multiplication problems (A Transcript, Event 4, p. 6-8). Mrs. Washington began the lesson by posing the question, “What is an array” The discourse that occurred during this event helped to review the concept of an array and the ways arrays could be applied to measurement. The dialogue below is part of the discussion during Event 4.

1. T: I am going to ask you to work on the portion that is the Array Project.

2. T: What is an array? Turn and talk to someone.

3. Students talk with others.

4. T: So, my question to you was, what is an array?

5. S: So, you use a symbol for example dots and you solve a multiplication question like 7 times 5.

6. T: So, if you used a symbol like dots to solve 7 times 5 you would put the dots 5 down and 7 across?
7. S: Yes.

8. S2: An array can help you find the area.

9. T: Oh! An array can help you find the area, is she right?

10. T: I want someone else!

11. S: When you measure the distance, the area you can use an array because in the area you use multiplication, you would use 4X3 with 4 across and 3 going down and 4 and so you did 4 times 3 you would get 12.

12. T: Beautiful.

After discussing their ideas with a partner, during a ‘turn and talk’, students shared their thinking with the class. Students are provided with opportunities to share the meaning of an array. While they reviewed arrays students between using arrays to multiply and using arrays to find the area. Students had acquired some knowledge of arrays because they are part of the curriculum experienced earlier in the year (Atlas Rubicon Multiplication Unit, 2018).

During this lesson students honored mathematics as a discipline by when representing the number sentences that matched their calculations on the whiteboard. This also occurred during discussion explaining the decomposition strategy used to break apart a large problem using two smaller problems to decrease the complexity (Event 4, p.7).

Moreover, Mrs. Washington honored mathematics as a discipline by helping students to analyze how using “the same numbers” to find the area and perimeter resulted in different answers (Event 5, p. 10). She also honored the content of the discipline by emphasizing the that addition was the accurate operation when finding perimeter, whereas multiplication was needed to identify the area of a shape.
Learning Through Interaction

Learning Through Interaction is the fourth reformed-based practices for building understanding in mathematics. This practice was built on the idea that mathematics is a *socially constructed* endeavor. As discussed below, Mrs. Washington used *socially constructed learning* to encourage students to learn alongside others in the community. The two components of Learning Through Interaction included *learning is socially constructed* and *contributes to the learning of others*. The components of Learning Through Interaction were implemented during all of the eight events in this study, (Case A Transcripts, p. 1-18).

When asked to reflect on the ways she encourages students to talk about mathematics, Mrs. Washington said, "I think math is a social content. If we need to be able to solve world problems, then we should be able to do it in math class. When we do that I think you have to be able to talk and communicate" (Teacher Individual Interview, p. 2).

The problems and tasks used by Mrs. Washington throughout the course of the eight events were used to develop problem-solving skills and social skills. She wanted students to learn how to interact first because the tasks required them to "talk with other people" (Individual Interview, p. 8). The more advanced students, according to Mrs. Washington, may not need to practice all of the skills but they need" to learn how to agree with an explanation to explain their thinking."

**Learning Is Socially Constructed**

The first component of Learning Through Interaction is *learning is socially constructed*. As shown below, Mrs. Washington designed her lessons to include
discussions that enabled students to share their thinking about the topics they studied. This *social construction of learning* was cultivated using problems to incite interactions among Mrs. Washington and her students (Case A Transcripts, 1-18). The mathematical conversations that students participated in allowed students to develop their thinking and understanding by listening and talking with one another. Students also worked with other students to complete tasks during Events three, six, seven and eight in both small and large groups. These tasks provided another mechanism for the *social construction of knowledge* because they required students to plan and design solutions with the help of their teacher and peers.

Mrs. Washington emphasized the social construction of learning while facilitating conversations during mathematical discussions. She typically posed a question that involved a specific skill or concept with students, as described earlier, and then spent several minutes discussing what students knew about it. She listened to their contributions and then posed additional questions to dissect what it was that they knew about the topic. Many students would have opportunities to make their knowledge public. Other students would listen. Mrs. Washington would step in with questions to uncover additional thinking. She would also step in to steer the conversation toward an idea that she wanted students to consider.

According to Mrs. Washington, providing experiences for students to talk about math impacted their understanding (Teacher Focus Interview, p. 7). While discussing this during the focus interview, she stated: “I believe that students make their own meaning and even though we teach them we can teach a concept all we want but if it doesn’t have personal meaning it not going to stick.”
Students also identified the value in using mathematical talk to assist with building understanding. According to the results of the questionnaire, most students find talking about math helps them to understand the math they are learning more clearly, see Table 3. Many say that they understand math more when talking with their teacher and other students. Jaylissa was the only one of the three students interviewed, who reported not understanding math more when she talked with other students. Similarly, Ben reported not understanding math more when he talked about it with his teacher. Additionally, students also felt that listening to others explain their solutions was helpful. Ben was one of four students who did not find listening to others as helpful. Also, many of the students reflected that sharing their thinking helped them to figure out if they were on the right track. Moreover, if they were not correct, many perceived talking with others would help them find their mistake. All of these results included the three of the students who were interviewed individually. Many students also agreed that comparing their answers with other students assisted them in noticing if their thinking was correct.

An example of this occurred during Event 2, Mrs. Washington facilitated a discussion among students while engaged in a discussion about a tally chart the students had completed earlier that morning (Case A Transcript, p. 2-4). Often in these discussions, Mrs. Washington encouraged students to think out loud about skills and concepts as a way of reviewing a topic. In this case, students would be applying skills discussing data collection before having to collect and represent their own data during an upcoming task. The following is the discussion that occurred during Event 1.

1. T: Could I have collected my data another way?

2. Turn and talk to someone near you.
3. Students turned and talked with a partner about the question.

4. T: So, my question was, could I collect my data another way.

5. S: You could have used a pictograph.

6. T: How could I have used a pictograph to collect my data?

7. S: So, you could say 1 smiley face equal one vote so than you would have wrote yes or not, and then you would have us draw a smiley face if we do.

8. T: Ok, if I had you draw the smiley face in the yes or no column is that still a tally chart and I or am I having you just put a different symbol in there?

9. S: Sort of, just the big difference is that you are using a picture instead of tallies.

10. T: Would that be different?

11. S: It would be different, but it would still be a tally chart.

12. T: Ooh! How can it be different and similar at the same time? Somebody help us!

13. T: Student, please call on a friend to help!

14. T: Somebody help us out!

15. S2: It isn’t, she is just using a different symbol.

16. T: Thanks for the next question, what is she talking about?

17. T: Turn and talk to someone.

18. T: S2 just said it is pretty much the same just using a different symbol.

19. S3: It is different because it is a pictograph cuz at the bottom of the smiley face equals one vote, but you could do the same thing as the tally chart.

20. T: So, are you agreeing or disagreeing with S2?


22. T: When did we make pictographs, during the time we collected data or after the time we collected data on our last project?
The discussion was focused on a class graph completed by students earlier in the day. Each student was asked to put a tally mark in the yes column if they went to the library in the summer, or in the no column if they did not go to the library in the summer. To begin the discussion, students were prompted to engage in a ‘turn and talk’ to discuss their ideas with one other person. Students talked with one another about their thinking (p. 2). Students began sharing their ideas, suggesting a type of graph to use. A discussion among students, facilitated by their teacher about the similarities and differences among pictographs and tally charts ensued. The teacher’s questions prompted specific explanations about what students knew about a pictograph. She explained what she heard the student contribute. Then she investigated their thinking when asking, “Would that be different” (p. 3)? Students join the conversation and try to explain. Mrs. Washington prompts other students to help one another out. She does not explain the difference but waits for students to attempt this. In Turn 17, she engages them in another ‘turn and talk.’ Then after a minute or two of partners exchanging their thinking, Mrs. Washington asks, “So what do we think over here?” Mrs. Washington provides opportunities for other students to try to clarify the difference. Then she shifts the conversation a bit toward the critical aspect of tally charts being a tool that students use for collecting data posing another question. Then in Turn 22, she says, “So let me ask another question, when did we make pictographs, during the time we collected data or after the time we collected data on our last project?” One student answers, “Before.” Mrs. Washington does not respond to his incorrect answer. Instead, she looks to a classmate and asks, “Can you help him out?” Gagan steps in and corrects him saying, “After.” Another student confirms, “After, because we collected our data and then after you collect it you can actually do it.”
Mrs. Washington continued to ask students to share during this conversation to confirm that they understood that the tally chart was used in the data collection process.

Overall Mrs. Washington implemented several elements to support the social construction of learning, as evident in Table 4.1 and 4.2. The problem or task was presented, and students shared their knowledge (GR4A) on 43 occasions, along with the strategies that they utilized to solve the problem (GR4B) on 16 occasions. Mrs. Washington restated contributions to help clarify ideas (GR4C) so that students could understand one another (GR4C) on 31. Additionally, students discussed solutions and agreed on decisions (GR14B) on four occasions, with the help of their teacher. Students also spent time reflecting on and comparing solutions (GR11D) on eight occasions. Likewise, their teacher guided the reflection and comparing on 11 occasions. Both Mrs. Washington and her students assisted in building on the ideas of other (GR8B) on eight occasions. Moreover, she always encouraged and invited contributions (GR1A and GR1B), yet ultimately, she guided the construction of ideas.

An example of Mrs. Washington guiding students in a different direction occurred during Event 2 (Case A Transcripts, p. 3) while students were trying to discuss the difference between a tally chart and a pictograph. Students went back and forth trying to articulate the difference, and they struggled. Mrs. Washington stepped in with a question to redirect the conversation.

**Contributes To The Learning of Others**

The second component of Learning Through Interaction is contributes to the learning of others. As shown below, Mrs. Washington initiated opportunities for students to join her in supporting one another during the learning of mathematics. Tables
1 and 2 reveal students offered their knowledge of mathematics and *shared their solutions using words, pictures, and numbers (GR4B)* on sixteen occasions. For example, during Event 4, Ben came up to explain the meaning of an array and to draw the image on the whiteboard (p. 7-8). Additionally, in Event 6, a student came up to share her thinking to offer some advice to her classmates from making a similar mistake. She said,

“If you are making your things (geometric shapes) super tiny, I added like my 4 yards, 4 yards, and 3 yards and realized that I only used 11 of my 40 yards for my sides. So, I am like…Wow! That is not very much. I can make those way bigger! (Event 7, p. 11)

As explained earlier in this chapter, Mrs. Washington provided opportunities for students to learn from others by inviting them to share their thinking so that others could learn from them as they explained different ways of thinking. A couple of the elements that Mrs. Washington encouraged students offered their knowledge about mathematics. Students *Offered Their Knowledge About Mathematics (GR4A)* on 43 occasions. They also *offered justifications and rationales (GR5A)* on 69 occasions, and *many solutions for problems (GR7B)* on 77 occasions. All of these elements allowed students to demonstrate their knowledge to others in the community. The way Mrs. Washington guided discussions allowed student to contribute in these ways with her support and the support of classmates. All were helpful in providing ideas and support, as noted below when others struggled with the concepts.

Mrs. Washington invited students to help their classmates get out of a situation when they were not sure of the answer. She prompted these exchanges by suggesting that students call on a friend to help continue the conversation. Again, during Event 1, students were discussing why a tally chart would be a better choice for graphing the class data during Event 1. One student contributed his thinking and said, “You can do the
pictograph, but it would be hard because it would take a lot more thinking” (Case A Transcript, Event 1, p.1). The second student clarified saying, “It would be too hard to do it that way because the symbol means more than just one.” Mrs. Washington confirmed the thinking and said, “He brings up a good point when we have been using the pictograph. The symbol has been used for more than one vote.”

Additionally, during the same lesson, Gagan steps in to help a classmate during the discussion about whether or not the tally chart was used before or after the pictogram in Event 2. When the student answers incorrectly, Mrs. Washington does not respond to his incorrect answer. Instead, she looks to a classmate and asks, “Can you help him out?” Gagan steps in and corrects him saying, “After.” An additional student confirmed it, “Because we collected our data and then after you collect it you can actually do it” (Case A Transcript, Event 2, p. 3).

Moreover, during Event 5, Mrs. Washington asked a student if he was “standing in the area?” He replied, “No” (Case A Transcript, p.10). He was standing in the area, and Mrs. Washington asked another student to help him out. The students said it begins with “An A.” Afterwards, Mrs. Washington prompted, “So she just helped you, it begins with a [area].” A third student provided additional support to her classmate, “People might think it is the area because you are close to the outline (perimeter).”

Again, for strategic discourse to be truly effective, it cannot be entirely teacher directed. As will be described below, Mrs. Washington set the course for student and teacher roles to evolve so that students could become more autonomous while they studied mathematics together.
Shifting Authority Toward Shared Authority

Mrs. Washington has made use of methods that are reforming how her students learn, not just what they learn. As described above, Mrs. Washington has instituted the reform-based practices and several strategic discourse elements to successfully guide her instruction as she teaches mathematics with understanding. Additionally, and most remarkably, Mrs. Washington has established an equitable environment that encourages a partnership among her students and herself. This equalized environment has established a heightened level of mathematical authority among students. Providing students more experience sharing their knowledge has provided greater expertise in communicating ideas and confidence in mathematics. (interviews liking math). Encouraging students to share thinking and support to their classmates has communicated to them that they are valuable contributors to developing the understanding of mathematics. Students have developed a greater responsibility evaluating their thinking and the thinking of others (Huffered-Ackles, Fuson & Sherin, 2004). These experiences all have promoted greater mathematical authority.

Strategic discourse is built on the idea that both students and teachers are partners and powerful sources of mathematical ideas and thinking (Gee, 1991; Lampert, 1990). The untraditional relationship among Mrs. Washington and her students was much more equalized than in traditional classrooms. As examined earlier in this chapter, there were several instances where the interactions were casual. Students jumped into conversations and did not wait to be called on. They freely joined conversations and jokes went back and forth among teacher and students.
For example, when students were discussing the height of their teacher, she joked quite a lot about her short stature. When she asked, “When I take my shoes off, I am 61 inches tall. I am really tall aren’t I” (Case A Transcripts, Event 1, p.1). Students immediately chimed in and laughed while yelling, “No! You are not!” Mrs. Washington smiled and continued the conversation.

Mrs. Washington consistently encouraged her students to participate by sharing their mathematical expertise and ideas about many different topics and problems during the course of the study. Encouraging participation helped to demonstrate the value and worth of their ideas in the community, in the discourse and treated each as valuable and worthy members of the learning community. The supportive environment thrived as Mrs. Washington provided students with multiple opportunities to demonstrate their knowledge. She also provided them with the mathematical authority to share their thinking and engage in discussions about the topics they studied.

Students demonstrated an increase mathematical authority in the statements they made when sharing their knowledge. For example, a student stated, “I know that 5 X 12 equals 60” (Case A Transcripts, Event 1, p. 1). Also during the array project, Mrs. Washington shared a strategy she had for calculating a double-digit number using arrays. The problem was 15 times 5. She said, “If you have a mathematical equation that is too hard for you to solve, I could do this whole array or could do a break it into two smaller ones. What would be the most logical?” A student speaks up with a suggestion, she says; “You are decomposing.” Mrs. Washington asked and another student, “How many multiplication problems am I doing?” Without hesitation, the student replies, “You are doing two multiplication problems, 15 times 5 and 5 times 5.”
In both situations, students step up and contribute. These are typical ways of participating for students. They do not hesitate, and never answer attempt to provide an answer.

Also, Mrs. Washington typically asked students to tell her what they thought they should do and does not just tell them. For example, when students were reviewing the directions for the survey task in Event 3, Mrs. Washington asked, “What do you think that you need to think about when you are going to make a bar graph” (Case A Transcripts, p. 6). Students respond by listing the several components of the bar graph. She finishes with, “Anything else we need?” The last students says, “You have to put the names of the things on the bottom.” She responds, “Good, that toes on the bottom.” She allows all of these contributions from students to help guide the work of their peers.

Additionally, as detailed in the previous section, students were also provided with the authority to provide help to their classmates, as needed. Reaching out to students in this way promoted students as valuable contributors to the mathematical knowledge among their peers.

Most students found that solving problems was easier when they worked with other students according to the Survey/Questionnaire Results (see Table 3). More than half reported that they asked questions of their classmates to help them find a logical solution. However, only four agreed that they helped their classmates when they are having trouble while solving math problems. Also, when asked about solving problems together with classmates, more than half of the students preferred to solve problems on their own and not with others. Moreover, only six students reportedly liked to learn from others.
Cultivating their mathematical authority has taken time. Mrs. Washington has modeled helping students rethink their strategies, as well as, encouraged her students to offer and get help from their peers. Mrs. Washington also found ways to acknowledge students as competent thinkers when she brought attention to their ideas.

Although students were highly involved in classroom discussion, Mrs. Washington remained very much in control of the conversations. She guided the discussions, interjected her knowledge and interpreted what her students shared. Although students have not been actively engaged in planning, they have had several opportunities to collaborate with their teacher and peers. Moreover, students were given many opportunities to work on tasks independently, some of the assignments were highly teacher directed. The tasks in Event 7 and Event 8 were more open-ended and allowed students more room to complete the assignments independently.

**Summary**

Mrs. Washington implemented all four reformed-based practices including Problem Posing, Active Learning with Authenticity, Learning Through Interaction, and Strategic Discourse. Teachers implemented these practices along with the components consistently throughout the study.

The components of the first reform-based practice Problem Posing included *well-designed problem or task enriches the concept or skill and provides structure for discussion.* The components are embedded in the process and therefore all three are utilized concomitantly during each of the eight events. Mrs. Washington utilized *well-designed problems or tasks* during all events to generate
and structure mathematical discussions she facilitated with students as they solved problems together. All problems enriched the concepts of the topic addressed.

The components of the second reform-based practice Active Learning With Authenticity included engages in learning, connects to real life, and honors mathematics as a discipline were also implemented by both teachers. Mrs. Washington engaged students in learning while connecting learning tasks to real-life problems during all events.

Additionally, Mrs. Washington honored mathematics as a discipline by pointing out the mathematics that mirrored the structure and content of mathematics as a discipline throughout the study. Students were asked to explain their thinking, connected strategies to the discipline, and used logical numeric formats. Mrs. Washington honored the mathematics as a discipline during all events, except for events seven and eight.

Mrs. Washington also utilized the components of the third reformed based practice Learning Through Interaction were utilized by both teachers during each event to support the building of understanding among participants. The first component learning is socially constructed was encouraged as students; interacted with other students to complete tasks in small groups and large groups, provided help to peers, asked questions, and interacted with the thinking of others.

Contributes to the learning of others is the second component of Learning Through Interaction. To help contribute to the learning of others, students; asked questions, compared ideas, shared multiple solutions and identified effective solutions. Mrs. Washington initiated opportunities for students to contribute to the learning of others when she requested that students ask other students to help continue conversations.
She also encouraged students to student interactions to generate more solutions, or to help a student get out of a situation where they were not sure of the answer. Both students and teachers agreed that talking about mathematics with others was beneficial to the understanding of mathematics.

Furthermore, Mrs. Washington has established an environment that encourages equalized working relationships and a greater sharing of authority amongst her and her students. Students provided their own solutions for solving problems and shared different ways of thinking about and solving a variety of problems. Encouraging students to communicate their mathematical expertise and provide support and guidance to peers has perpetuated a sharing of authority among all participants. The existence of a shared authority which resulted in a partnership among Mrs. Washington and her students would not have been possible without the existence of the interactive and supportive environment established by Mrs. Washington.

Although students were implementing the discourse elements at a high level, Mrs. Washington remained very much in control of the conversations, see Table 11. Mrs. Washington still maintained responsibility for guiding the discussions, interjecting her knowledge, interpreting student contributions and redirecting conversations.

The next chapter provides an analysis of the ground rules and refrom-based practices and how they were used as part of the strategic discourse in Ms. Littleton’s classroom while they engaged in mathematical discussions.
CHAPTER 5

MS. LITTLETON AND STUDENTS

Strategic Discourse

Ms. Littleton and her students used the discourse practices of Exploratory Talk during the course of the seven events using problems to engage students in shared thinking while building understanding of a variety of mathematical skills and concepts. As will be explained in the following sections, Ms. Littleton facilitated the talk by guiding her students through several ground rules, from this model. By analyzing the discourse Ms. Littleton and her students used, a structure of these discussions emerged, revealing certain patterns of engagement among the community members. Tables 14 and 15 show the frequency of the discourse elements as related to the fourteen ground rules of Exploratory Talk used by Ms. Littleton and her students. Throughout the document, discourse elements are written in italics to indicate their correspondence to the established ground rules. The results of the examination of discourse practices in each classroom will be analyzed using examples.

![Figure 6: Implementation of Ground Rules in Case B](image-url)
from within the dialogue that occurred during events. The analysis will be follow the identification of the most highly used strategic discourse elements outlined below.

Four ground rules composed the majority of the strategic discourse of students and teachers over the course of the eight events. These included Engaging In Joint Reasoning (GR6); Knowledge Is Made Public (GR4); Atmosphere of Trust Is Present (GR3) and Everyone Invited To Contribute (GR1). Within each ground rule are the supporting elements that will be described later demonstrating the complex nature of the discourse Ms. Littleton used each as a tool to facilitate mathematical conversations. The discourse practices used in this classroom are comprehensive in nature and the variety of elements included in the ground rules support the overall discourse existing in this classroom.

Ms. Littleton used the ground rule Engaged In Joint Reasoning (GR6) to encourage students to share ideas and solutions as they learn with others while engaged in discussions about mathematics. During these discussions Ms. Littleton used this ground rule 401 times, which is about half of the discourse in the study. Students Engaged In Joint Reasoning (GR6) 273 times, equaling just over one third of the discourse. Engaging In Joint Reasoning (GR6) was fundamental to Ms. Littleton’s practice, as evidenced during the course of the eight events (See Table 14 and Table 15). Together she and her students utilized all of the elements within this ground rule as they discussed the mathematics that they were studying.
Table 14: Teacher Use of Ground Rule Elements Case B

<table>
<thead>
<tr>
<th>EVENTS</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>TOTALS</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>GR1</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>A</td>
<td>6</td>
<td>7</td>
<td>5</td>
<td>2</td>
<td>3</td>
<td>0</td>
<td>12</td>
<td>35</td>
</tr>
<tr>
<td>B</td>
<td>14</td>
<td>26</td>
<td>36</td>
<td>4</td>
<td>14</td>
<td>0</td>
<td>35</td>
<td>129</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>164</td>
</tr>
<tr>
<td><strong>GR2</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>A</td>
<td>1</td>
<td>1</td>
<td>20</td>
<td>0</td>
<td>6</td>
<td>2</td>
<td>12</td>
<td>42</td>
</tr>
<tr>
<td>B</td>
<td>5</td>
<td>3</td>
<td>3</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>8</td>
<td>20</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>62</td>
</tr>
<tr>
<td><strong>GR3</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>A</td>
<td>3</td>
<td>3</td>
<td>5</td>
<td>0</td>
<td>6</td>
<td>1</td>
<td>2</td>
<td>20</td>
</tr>
<tr>
<td>B</td>
<td>3</td>
<td>2</td>
<td>21</td>
<td>2</td>
<td>8</td>
<td>1</td>
<td>11</td>
<td>48</td>
</tr>
<tr>
<td>C</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>3</td>
<td>6</td>
</tr>
<tr>
<td>D</td>
<td>5</td>
<td>5</td>
<td>0</td>
<td>1</td>
<td>2</td>
<td>0</td>
<td>0</td>
<td>13</td>
</tr>
<tr>
<td>E</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>89</td>
</tr>
<tr>
<td><strong>GR4</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>A</td>
<td>2</td>
<td>7</td>
<td>12</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>21</td>
</tr>
<tr>
<td>B</td>
<td>0</td>
<td>5</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>5</td>
</tr>
<tr>
<td>C</td>
<td>5</td>
<td>2</td>
<td>11</td>
<td>0</td>
<td>3</td>
<td>0</td>
<td>2</td>
<td>23</td>
</tr>
<tr>
<td>D</td>
<td>1</td>
<td>3</td>
<td>3</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>5</td>
<td>13</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>62</td>
</tr>
<tr>
<td><strong>GR5</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>A</td>
<td>0</td>
<td>4</td>
<td>2</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>3</td>
<td>9</td>
</tr>
<tr>
<td>B</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>11</td>
</tr>
<tr>
<td><strong>GR6</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>A</td>
<td>20</td>
<td>37</td>
<td>49</td>
<td>8</td>
<td>25</td>
<td>17</td>
<td>46</td>
<td>202</td>
</tr>
<tr>
<td>B</td>
<td>1</td>
<td>11</td>
<td>24</td>
<td>1</td>
<td>6</td>
<td>2</td>
<td>7</td>
<td>52</td>
</tr>
<tr>
<td>C</td>
<td>6</td>
<td>5</td>
<td>8</td>
<td>4</td>
<td>4</td>
<td>2</td>
<td>18</td>
<td>47</td>
</tr>
<tr>
<td>D</td>
<td>8</td>
<td>11</td>
<td>15</td>
<td>1</td>
<td>9</td>
<td>10</td>
<td>19</td>
<td>73</td>
</tr>
<tr>
<td>E</td>
<td>0</td>
<td>0</td>
<td>3</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>3</td>
</tr>
<tr>
<td>F</td>
<td>3</td>
<td>1</td>
<td>2</td>
<td>0</td>
<td>2</td>
<td>0</td>
<td>0</td>
<td>8</td>
</tr>
<tr>
<td>G</td>
<td>1</td>
<td>4</td>
<td>8</td>
<td>1</td>
<td>2</td>
<td>0</td>
<td>0</td>
<td>16</td>
</tr>
<tr>
<td></td>
<td>39</td>
<td>69</td>
<td>109</td>
<td>15</td>
<td>48</td>
<td>31</td>
<td>90</td>
<td>401</td>
</tr>
<tr>
<td><strong>GR7</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>A</td>
<td>9</td>
<td>5</td>
<td>23</td>
<td>0</td>
<td>7</td>
<td>0</td>
<td>19</td>
<td>63</td>
</tr>
<tr>
<td>B</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>66</td>
</tr>
<tr>
<td><strong>GR8</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>A</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>2</td>
</tr>
</tbody>
</table>
Ms. Littleton encouraged students to use the element *discuss ideas and solutions* on 202 occasions *(GR6A)*. The students responded by *sharing their ideas and solutions* on 230 occasions while solving problems in mathematics *(GR6A)*. Ms. Littleton plans instruction around tasks that encourage students to *share their ideas and solutions*
As tasks are discussed and completed, she actively engages students in joint reasoning to filter the information they share.

Questioning was a significant part of the discourse. She carefully steps in to ask questions to the class or individual students to direct thinking (GR6B). Ms. Littleton used the ground rule posed questions to direct thinking in a specific direction twice (GR6B). Students also directed the thinking on their own six times (GR6B). Additionally, questions were posed by Ms. Littleton to encourage the exchange of ideas on 46 occasions and on 18 occasions by students (GR6C). Sometimes she repeats the same question to continue the exchange or asks a new question to guide students in a different direction. Questions are also used by Ms. Littleton to help understand students thinking either about the steps in a solution or about their knowledge of the mathematics (GR6D). Students asked 21 questions to understand thinking and Ms. Littleton asked 76 (GR6D).

Contributions were highlighted by spotlighting different ways of thinking three times by the teacher, while students drew attention to an idea through spotlighting once (GR6E). This element is used to emphasize the thinking of students for the purpose of drawing everyone’s attention to a strategy or solution (GR6E). Ms. Littleton assigned problems or tasks to initiate students working together during five of the seven lessons and spoke of these tasks 16 times (GR6F). The majority of the work assigned by Ms. Littleton involves tasks requiring students to work together to identify solutions together (GR6F). Ms. Littleton supports students as they engage in joint reasoning by providing assistance when students need the support. For example, she will talk students through a process or draw their attention to important information needed to make a connection needed to move forward (GR6F).
The remaining lessons included students sharing problems or discussing projects. Lastly, assistance was offered using the final element in this ground rule, assistance provided to help a student work through the process from a student on one occasion and from Ms. Littleton on 16 occasions (GR6G).

Table 15: Student Use of Ground Rule Elements Case B

<table>
<thead>
<tr>
<th>EVENTS</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>TOTALS</th>
</tr>
</thead>
<tbody>
<tr>
<td>GR1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>A</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>2</td>
<td>0</td>
<td>4</td>
</tr>
<tr>
<td>B</td>
<td>0</td>
<td>3</td>
<td>3</td>
<td>11</td>
<td>0</td>
<td>0</td>
<td>4</td>
<td>21</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>25</td>
</tr>
<tr>
<td>GR2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>A</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>3</td>
<td>2</td>
<td>0</td>
<td>7</td>
</tr>
<tr>
<td>B</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>5</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>7</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>14</td>
</tr>
<tr>
<td>GR3</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>A</td>
<td>0</td>
<td>8</td>
<td>16</td>
<td>1</td>
<td>4</td>
<td>16</td>
<td>3</td>
<td>48</td>
</tr>
<tr>
<td>B</td>
<td>1</td>
<td>4</td>
<td>1</td>
<td>7</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>15</td>
</tr>
<tr>
<td>C</td>
<td>5</td>
<td>1</td>
<td>3</td>
<td>6</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>18</td>
</tr>
<tr>
<td>D</td>
<td>0</td>
<td>2</td>
<td>3</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>6</td>
</tr>
<tr>
<td>E</td>
<td>0</td>
<td>0</td>
<td>5</td>
<td>0</td>
<td>1</td>
<td>13</td>
<td>0</td>
<td>19</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>106</td>
</tr>
<tr>
<td>GR4</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>A</td>
<td>9</td>
<td>13</td>
<td>45</td>
<td>21</td>
<td>16</td>
<td>9</td>
<td>0</td>
<td>113</td>
</tr>
<tr>
<td>B</td>
<td>6</td>
<td>7</td>
<td>5</td>
<td>22</td>
<td>3</td>
<td>3</td>
<td>0</td>
<td>46</td>
</tr>
<tr>
<td>C</td>
<td>2</td>
<td>0</td>
<td>5</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>2</td>
<td>11</td>
</tr>
<tr>
<td>D</td>
<td>0</td>
<td>0</td>
<td>3</td>
<td>2</td>
<td>0</td>
<td>0</td>
<td>5</td>
<td>10</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>180</td>
</tr>
<tr>
<td>GR5</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>A</td>
<td>9</td>
<td>5</td>
<td>8</td>
<td>21</td>
<td>11</td>
<td>6</td>
<td>20</td>
<td>80</td>
</tr>
<tr>
<td>B</td>
<td>7</td>
<td>6</td>
<td>9</td>
<td>15</td>
<td>2</td>
<td>4</td>
<td>6</td>
<td>49</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>129</td>
</tr>
<tr>
<td>GR6</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>A</td>
<td>17</td>
<td>28</td>
<td>47</td>
<td>32</td>
<td>16</td>
<td>49</td>
<td>41</td>
<td>230</td>
</tr>
<tr>
<td>B</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>C</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>9</td>
<td>0</td>
<td>7</td>
<td>1</td>
<td>18</td>
</tr>
<tr>
<td>D</td>
<td>3</td>
<td>3</td>
<td>2</td>
<td>7</td>
<td>1</td>
<td>4</td>
<td>1</td>
<td>21</td>
</tr>
<tr>
<td>E</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>F</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td>G</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**GR7**

<p>| | | | | | | | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>1</td>
<td>0</td>
<td>6</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>4</td>
<td></td>
<td>11</td>
</tr>
<tr>
<td>B</td>
<td>9</td>
<td>6</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td></td>
<td>26</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>37</td>
</tr>
</tbody>
</table>

**GR8**

<p>| | | | | | | | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td></td>
<td>1</td>
</tr>
<tr>
<td>B</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td></td>
<td>0</td>
</tr>
<tr>
<td>C</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td></td>
<td>1</td>
</tr>
<tr>
<td>D</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td></td>
<td>1</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>3</td>
</tr>
</tbody>
</table>

**GR9**

<p>| | | | | | | | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>2</td>
<td>0</td>
<td></td>
<td>2</td>
</tr>
<tr>
<td>B</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td></td>
<td></td>
<td>1</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>3</td>
</tr>
</tbody>
</table>

**GR10**

<p>| | | | | | | | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td></td>
<td>0</td>
</tr>
<tr>
<td>B</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td></td>
<td>0</td>
</tr>
<tr>
<td>C</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td></td>
<td>0</td>
</tr>
<tr>
<td>D</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td></td>
<td>0</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0</td>
</tr>
</tbody>
</table>

**GR11**

<p>| | | | | | | | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>3</td>
<td>0</td>
<td>7</td>
<td>2</td>
<td>0</td>
<td>2</td>
<td>0</td>
<td>14</td>
<td></td>
<td></td>
</tr>
<tr>
<td>B</td>
<td>0</td>
<td>1</td>
<td>7</td>
<td>0</td>
<td>0</td>
<td>3</td>
<td>0</td>
<td>11</td>
<td></td>
<td></td>
</tr>
<tr>
<td>C</td>
<td>0</td>
<td>1</td>
<td>5</td>
<td>0</td>
<td>1</td>
<td>5</td>
<td>0</td>
<td>12</td>
<td></td>
<td></td>
</tr>
<tr>
<td>D</td>
<td>2</td>
<td>2</td>
<td>3</td>
<td>0</td>
<td>0</td>
<td>2</td>
<td>0</td>
<td>9</td>
<td></td>
<td></td>
</tr>
<tr>
<td>E</td>
<td>0</td>
<td>2</td>
<td>4</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>6</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>52</td>
</tr>
</tbody>
</table>

**GR12**

<p>| | | | | | | | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>0</td>
<td>0</td>
<td>2</td>
<td>3</td>
<td>3</td>
<td>6</td>
<td>0</td>
<td></td>
<td>14</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>14</td>
</tr>
</tbody>
</table>

**GR13**

<p>| | | | | | | | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td></td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>B</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td></td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>C</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td></td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>2</td>
</tr>
</tbody>
</table>

**GR14**

<p>| | | | | | | | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td></td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>B</td>
<td>0</td>
<td>0</td>
<td>2</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td></td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>C</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td></td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>D</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td></td>
<td>0</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>3</td>
</tr>
</tbody>
</table>
The second most frequently implemented ground rule utilized by Ms. Littleton and her students was *Knowledge Is Made Public (GR4)* as a means of communicating their knowledge and ideas about the concepts they studied, see Table 14. She uses this talk move 62 times over the course of the seven events. Ms. Littleton encouraged students to use the elements within the ground rule *Knowledge Is Made Public (GR4)* and they responded by utilizing the 180 times see Table 15. She strategically makes connections to students’ explanations, lessons or concepts from earlier in the year to help students make connection with and among the concepts being studied (*GR4D*).

Ms. Littleton strategically provides opportunities for students to share what they know mathematically (*GR4A*). She begins her lessons with open-ended questions to encourage all students to join the conversation. Then she asks more directed questions to guide individual thinking, group thinking or to redirect to a new idea.

Ms. Littleton encourages students to explain their strategies in ways that make sense to them (*GR4B*). Ms. Littleton made her own *knowledge public* 21 times, while she explained mathematics for students using words, pictures and/or numbers on five occasions during the *events (GR4A,B)*. She also *made her knowledge public* as she caught errors in students reasoning and then guided their thinking toward viable solutions (*GR4A*).

She listens as students share and often restates their thinking to make their knowledge accessible to their classmates (*GR4C*). While engaged in discussions about problem solving strategies and solutions, *contributions were restated* by Ms. Littleton 23 times (*GR4C*). She *restated contributions* to either reinforce effective ideas or to interpret students thinking. Similarly, Ms. Littleton used *referred to previous lessons, concepts or*
contributions 13 times during the course of the seven events (GR4D). Ms. Littleton also provided opportunities for students to reflect on previous concepts and contributions of students (GR4D).

Students utilized this talk moves more often than their teacher. Students communicated their knowledge publicly (GR4) on 180 occasions, a quarter of all the discourse. Students also offered their knowledge and shared their thoughts about concepts and solutions much more often using this element 113 times (GR4A). They explained their strategies using words, pictures and/or numbers on 46 occasions throughout the seven events (GR4B). The element restated contributions while making connections with their own and the ideas of others was utilized by students on 11 occasions (GR4C). They restated a contribution, most typically to inform the group of a repeated answer (GR4C). This also included students drawing attention to repeats or inaccuracies stated by peers. Finally, students also used the element (GR6D) to refer to previously contributions while discussing their own thinking 10 times. Students referred to other concepts help them while explaining their thinking. They also referred to another student’s answer to correct a mistake the student had made (GR4D).

The third most frequently implemented ground rule was Atmosphere of Trust Is Present (GR3). Ms. Littleton and her students utilized this ground rule while engaging in joint problem solving during the course of the seven events. Ms. Littleton utilized this ground rule a total of 89 times during the seven events. Students used this ground rule 106 times to discuss their thinking in small and large groups.

It is important to note that the implementation of the elements within this ground rule indicated an environment existed that encouraged students to engage in a partnership
with their teacher and peers. This environment was perpetuated by both Ms. Littleton and her students. Students utilized the ground rules that assisted in building an atmosphere of trust and acceptance, as well. This was evident in their acceptance of ideas and the willingness to listen to one another. Furthermore, the elements within this ground rule supported the building and sharing of mathematical authority. The dialogue in the classroom examples have been examined to demonstrate the specific ways that Ms. Littleton and her students utilize the elements of Atmosphere Of Trust Is Present (GR3) to cultivate what it meant to work together in a community of practice.

The first element in this ground rule *casual interchanges demonstrating equalized relationships* was used 20 times by Ms. Littleton and 48 times by students (GR3A). Students utilized this element while speaking on the behalf of others, seeking clarification or acknowledge mistakes. They also casually asked questions and offer suggestions both in small group and large group settings (GR3A). Ms. Littleton was involved in *casual exchanges* when she explained her rationale for activities. This also included her interactions with students during transition times to settle students or warn them that the end of class was near.

Ms. Littleton also implemented the element *offers praise and encouragement* to students on 48 occasions after they made positive contributions during discussions (GR3B). Ms. Littleton often *praises* students’ contributions and provides positive feedback (GR3B). Students also *offered praise and encouragement* to their peers 15 times (GR3B). Most often praise was provided when students presented their ideas, however once praise was provided to the teacher.
At times, Ms. Littleton and her students *asked questions openly and freely* as part of their discourse (*GR3C*). The teacher and her students ask questions *openly and freely* when someone was unclear or misguided without hesitation (*GR3C*). Students did so 18 times and Ms. Littleton used this element on six occasions. The casual interchanges usually occurred when Ms. Littleton sought clarification from students about their thinking or actions. Additionally, students openly asked their teacher questions to seek information about directions she had provided. They also questioned their peers when they had misspoken or eliminated a portion of their answer.

The next element pertains to the aspects of the classroom that supports an environment where *all are valued and capable members* of the community (*GR3D*). This element was evident in the discourse on 13 occasions by Ms. Littleton and six times by students. It occurred when Ms. Littleton invited students to offer their own insights, questions and suggestions to their peers and teacher (*GR3D*). It was evident that students were valued as capable members of the community as they shared in the decision-making responsibility during the course of the seven events. For instance, on two occasions. This contributes to promoting the environment where all were *valued and capable* (*GR3D*).

Ms. Littleton involved students in *shared decision making* when she asked students how they would proceed with completing a task (*GR3E*). Students are also given opportunities to offer input or suggestions, especially in *making decisions* about how to solve to plan solutions (*GR3E*). Students also engaged in *shared decision making* while working with peers 19 times during events three and six (*GR3E*).

The final most frequently used ground rule was *Everyone Invited to Contribute* (*GRI*). Ms. Littleton and her students utilized this ground rule to invite one another to
engaged in discourse about the problems and ideas they discussed during events, see Table 14 and 15. Mrs. Littleton planned activities to include opportunities for students to talk with others about the mathematics they studied. During large and small group discussions, Mrs. Littleton used *everyone is encouraged to contribute without being singled out* (GR1A) on 35 occasions when asking questions directed at the class or during small group discussion with students. Students used this element on four occasions. Mrs. Littleton used the second element *students are chosen strategically by the teacher/student to contribute* (GR1B) more frequently, using it 129 times. During discussions, she often called on students who had their hand raised to answer a question or directed a question at a student who was already explaining a solution or idea. Students used the element *students are chosen strategically by the teacher/student to contribute* (GR1B) on 21 occasions when prompted by their teacher to ask students for questions.

An examination of the implementation of the most frequently used ground rules in this case is described in the following section.

**Implementation of Strategic Discourse**

Overall, learning mathematics in Ms. Littleton’s classroom meant that students built their understanding by engaging in problem solving with peers and then talking about their learning experiences as a group. In general, the format of the lessons included a discussion about a problem or idea with a follow up problem solving task in small groups or pairs. Three of the seven events involved students completing tasks (See Appendix K). The other events involved students working on problems with small groups or engaging in mathematical discussions.
After the initial discussion during each event, students were directed to complete a related assignment in small groups. After the group work, students reconvened as a class to discuss the work they completed in small groups. Throughout the small and large discussions, Ms. Littleton used open ended questions to encourage students to share their thinking. Once the idea or solution was shared by a student, Ms. Littleton interacted with the student either to investigate their thinking or to seek clarification. Ms. Littleton would often ask students to reflect on why they chose a particular path to solve a problem. Sometimes students would be invited to ask a peer if they had a question to encourage the exchange of ideas among them. She encouraged many students to join in the discussion by always asking the group to contribute different ways of solving the problem.

Below is the in-depth investigation of the discourse practices of the discussions occurring among Ms. Littleton and her students. The events were chosen to provide a clear picture of the discourse utilized. As mentioned previously, the most consistently implemented strategic discourse strategies were related to four of the 14 ground rules. These included: Engage Students In Joint Reasoning (GR6), Knowledge Is Made Public (GR4), Atmosphere of Trust is Present (GR3) and Everyone Invited to Contribute (GR1). Examining the dialogue from the conversations among Ms. Littleton and her students demonstrates the complex and non-linear process for integrating the crucial strategies used in the context during the learning of mathematics. Other ground rules may be referenced during this analysis to unveil additional insights gathered during the investigation.
Engage In Joint Reasoning (GR6)

This section includes an investigation of the discourse practices or ground rules used by Ms. Littleton and her students during conversations in their fourth-grade classroom from Event 3 (Classroom Observation, June 5). This analysis examines the most commonly used ground rule used by Ms. Littleton and her students called Engages Students In Joint Reasoning (GR6). Ms. Littleton Engaged Students in Joint Reasoning (GR6) through initiating a discussion about a concept, engaging students with one another to solve problems related to the concept and then coming together to sharing solutions, and to find meaning in the ideas discussed.

Again, Mrs. Washington utilized all seven of the different elements within this Ground rule (GR6) as evident in Table 10. Students also utilize the elements as evident in Table 11. The conversations have been examined to provide specific ways that Ms. Littleton and her students utilize these elements. These elements include ideas and solutions are discussed with others (GR6A), questions are posed to the community to direct thinking (GR6B), questions are posed to encourage exchange of ideas (GR6C), community members ask questions to try to understanding thinking (GR6D), thinking is highlighted to spotlight different ways of thinking (GR6E), tasks are assigned to initiate working together to find solutions (GR6E), and assistance is offered to help work through the process or scaffold learning (GR6F).

Classroom Example 1-Event 3

The first example of the Engage in Joint Reasoning comes from the mathematical discourse that was part of Event 3. The focus of the lesson in was liquid capacity, more specifically solving problems requiring students to calculate liquid measurements.
(Classroom Observation, June 5, p.7-13). The essential question and enduring understanding are written with the end outcome in mind, guiding the path toward understanding of measurement. The essential question and enduring understanding guide Ms. Littleton’s lesson.

Essential Question: Why does “what” we measure influence “how” we measure?

Enduring Understanding: Objects can be measured using different tools and units based on what we are measuring.

Upon entering the room during the lesson, the students are working in groups on a problem-solving activity involving capacity. After a few minutes, Ms. Littleton gathers students’ attention to her by announcing, “Waiting to see that we are all ready to listen.” Students quiet down quickly demonstrating their readiness to begin the next part of the discussion. During the next several minutes, students and their teacher engage in joint reasoning as students offer answers and Ms. Littleton interacts with them about their ideas (GR6A). Ms. Littleton reflects about why talking with teachers and peers is important in mathematics, she said,

I think it’s important because it helps them to develop their thinking. And students really learn from each other and like all the time I’ll hear some student talk about something, and you get the "Ah ha moment" they’re exploring things together and making their own conclusions rather than me telling them this is the formula or we’re going to use for the area. They then find more ownership and while they’re learning, and more understanding of it. (Focus Interview, p.4)

As she facilitates the discussion, Ms. Littleton encourages students to offer multiple solutions by continuing to collect answers. Students respond by sharing many solutions, listing several units of measurement. As students name a unit, Ms. Littleton confirms their response when the contribution is accurate. She praises their contributions immediately. Ms. Littleton restates contributions to reinforce a student’s idea or to ensure
that others are hearing what each student has said. Students offer their feedback when someone repeats a unit that has previously been offered.

1. T: Ty?
2. T: Cups.
3. T: Yes.
4. T: Caleb?
5. C: Fluid ounces.
7. T: When we measure capacity, we can only measure in fluid ounces. We can’t measure in just ounces. Fluid ounces we are talking about capacity.
8. T: Edna?
10. T: Grams actually measures the weight of something.
11. T: Margaret?
14. S: We already said pints.
15. S: I said pints.
16. T: Sometimes we are going to repeat, but we are going to try not to.
17. T: Manny?
19. S: Cups we said, cups.
20. T: Jadiah?
22. T: Good.

23. M: I said that.

Below in Turn 24, Ms. Littleton highlights one of the group’s thinking by spotlighting a strategy that they used to organize units of liquid measurement (GR6E). She asks the group, “Could someone tell me why you used that Big G.” The Big G is a graphic organizer that students use to organize the units of liquid measurement as they relate to a gallon (See Figure 6). Edna begins by stating the labels used in the graphic but does not include an interpretation. That prompts Ms. Littleton to probe her thinking by asking, “So why did you draw this?” The question is used to both understand what the student is thinking, as well as, to exchange the thinking to make the thinking explicit to other students (GR6 C,D). In turn 22, Edna explains the graphic,

G is the gallon and the four Q’s inside the G are four quarts. So draw the big giant G and four Q’s and draw two P’s. Inside of each P stands for pints. Then draw two C’s inside of the P’s and that stands for two cups and that’s it. (Event 3, p.8)

Figure 7: The Big G Graphic Organizer
Jadiah joins the conversation to *offer his knowledge of mathematics* by interpreting the diagram. In turn 30, he refers to the graphic organizer (Figure 6) to explain the units for measuring capacity. He says, “Because four quarts the four G’s each q stands for one quart and four quarts equal one gallon and two pints equal 1 quart and 4 cups equals one quart” (p.8). Jadiah also links prior lessons by conjecturing that the diagram uses multiplication when he says, “I kind of noticed something. It kind of times’.” The group had used this representation earlier in the year to assist them in organizing equivalent liquid measurement units from teaspoons to gallons.

Then, Ms. Littleton engages Jadiah in *joint reasoning* by trying to extend his thinking about the idea using the question, “What do you mean it times’?” In his explanation, “Because in the quarts it has 2 pints and in the 2 pints it has 4 cups. So it’s like 2 and then it is ½ and then 4,” Jadiah rationalizes the steps of his strategy using numbers to explain his solution. She validates his contribution praising Jadiah for his contribution.

Students explain themselves *openly* and freely in this classroom. According to the Students Questionnaire Results 15 students said that they liked to share their thinking aloud. Additionally, eighteen students agree that sharing their thinking helped them to figure out if they were solving the problem correctly. Chetan, one of the students who participated in the interviews, and two others disagreed with this statement. Six students, preferred learning on their own. The three students interviewed in this study did not prefer to learn on their own.

Evelyn, who described herself as “not a good talker” reported that she did not like to share her thinking aloud, as did five of her classmates. During her interview,
Evelyn described sharing thinking in math was helpful. She said, “Well, some people need help with answers and it is good to share answers with others. It is good to help them.” (Individual Interview, p.1). When asked to reflect on a time when she and partner tried to decide on an answer after they had different answers, Evelyn said,

Well you could give a math problem and if you really know how to do it and they are kind of not too good at the strategy or something you can show how to do it, that maybe your answer is right, you can show your answer is right. (Individual Interview, p.2)

Moreover, Evelyn and 13 others reported having trouble explaining how they solved problems in math. Seven students including Chetan expressed that explaining their thinking was challenging. Three students shared that they experienced trouble understanding how other students solved a problem when they explain it to the class.

Yet, all but one student agreed that explaining their thinking was helpful to others. Additionally, when asked to reflect on how helping other students when they did not understand, Chetan said, “I show them how I did my work and they start to understand it a lot more and if they don’t understand it I still talk to them, and see how they are doing and end up knowing what I am saying.” (Individual Interview, p.2)

The measurement discussion during Event 3 continues below (See lines 24-65).

24. T: Could someone tell me why you used that Big G?
25. E: G stands for gallon. Can I explain what the rest is?
26. T: Yes.
27. E: So G is the gallon and the four Q’s inside the G are four Quarts. So, draw the big giant G and four Q’s and draw two P’s inside of each P stands for pints. Then draw two C’s inside of the P’s and that stands for two cups and that’s it.
28. E: Well you could draw more but that would probably take forever.
29. T: So why did you draw this?

30. J: [Jumps in] Because four quarts the four G’s each q stands for one quart and four quarts equal one gallon and two pints equal 1 quart and 4 cups equals one quart.


32. T: What do you mean it times’?

33. J: Because in the quarts it has 2 pints and in the 2 pints it has 4 cups. So, it’s like 2 and then it is $\frac{1}{2}$ and then 4.

34. T: Awesome, so you are recognizing a relationship between different units of measure.

35. T: Awesome.

36. T: If we had four quarts, what can we say four quarts is equal to?

37. T: Lina?

38. L: Um.

39. T: So, we have one quart two quarts three quarts and four quarts. Taps onto the quarts drawn on the board. So, four quarts is equal to what? Then points to each on the board.

40. L: 8 pints.

41. T: Writes on board. 4 quarts equals 8 pints. Good. How did you see that?

42. L: Because 2 pints are in each quart.

43. T: Awesome, who can find another unit that is equal to four quarts? Ella.

44. E: A gallon.

45. T: Awesome!

46. T: How did you know that?

47. J: Because inside of the G there are four q’s and they represent 4 quarts, so there are four quarts in a gallon.

48. T: Excellent.
49. T: Is there another unit we can find for four quarts?

50. T: Zandra?


52. T: 16 cups. How did you figure that out?

53. Z: Pint is two cups and two quarts is 8 cups. 2, 4, 6, 8…

54. M: Shouldn’t it be 32? Shouldn’t it be two pints in each quart and she only did one pint in each quart.

55. T: I don’t know, what do you think?

56. T: So why do you think Zandra thinks it should be 16 and Margaret thinks it should be 32? What do you think friends?

57. T: Devon?

58. D: I agree with Zandra because there in a so…in like in a quart there is 4 cups so 16 cups because 4 times 4 is 16 cups.

59. T: Does that make sense Margaret?

60. M: Um hm.

61. T: Can you tell me why Margaret?

62. M: Because 4 times 4 is sixteen.

63. T: Where did the four come from?

64. M: 4 comes from the 4 pints and the 4 quarts.

65. T: There are 4 cups in each quart and we have 4 quarts.

Ms. Littleton encourages engaging in joint reasoning by asking students questions to direct their thinking toward using the representation highlighted by their classmates (GR6B). She begins with a question that requires students to use the diagram introduced by the students. In Turn 36, referring to the contribution she asks, “If we had four quarts, what can we say four quarts is equal to?” When Lina hesitates to answer, Ms. Littleton
scaffolds the information to help her process it. While drawing on the board she says, “So, we have one quart two quarts three quarts and four quarts, so four quarts is equal to what? answers, “8 pints.” Ms. Littleton asks, “How did you see that?” This question was used to understand the student’s thinking, and she had understood the concept (GR6D). Lina explains her thinking, “Because 2 pints are in each quart.”

Ms. Littleton again engages students in joint reasoning by inviting students exchange ideas by finding another unit equal to four quarts (GR6A). Later in Turn 43, Ms. Littleton initiates another exchange of ideas asks, “Who can find another unit that is equal to four quarts?” (GR6C).

Mathematical talk has a prominent position in the discourse in this classroom as evident in the discussion, planning and interviews involving participants. During the individual interview, Ms. Littleton reflected about why talking in math was important when she said,

I think it's important because it helps them to develop their thinking. And the students really learn from each other. And like all the time I’ll hear some student talk about something, and you get the “ah ha moment” that they're exploring things together and making their own conclusions rather than me telling them okay, this is the formula, we're going to use the area. They then find more ownership and while they’re learning and more understanding of it. (Individual Interview, p.4)

Above in Turn 54, Margaret engages the group in joint reasoning as she directs thinking when asking, “Shouldn’t it be 16? Shouldn’t it be two pints in each quart and she only did one pint?” (GR6B). Zandra explained two quarts was eight cups instead of 16. Margaret was engaging the group in a critical look at the solution by comparing Zandra’s thinking with the model. Ms. Littleton did not answer this question, but reflects it back to the student and asked, “I don’t know, what do you think?” Next, she attempts to engage
the class in joint reasoning to further their thinking by comparing the two solutions. Ms. Littleton encourages all to students to join the conversation by asking, “So why do you think Zandra thinks it should be 16 and Margaret thinks it should be 32? What do you think friends?”

In Turn 57, Devon joined the discussion and offers her justification, “I agree with Zandra because there is like in a quart there is 4 cups so 16 cups because 4 times 4 is 16 cups.” Keeping the exchange of ideas going, Ms. Littleton asked Margaret, “Does that make sense?” When Margaret confirms this, Ms. Littleton follows up with a question to direct her thinking and to understand if Margaret was clear in her thinking (GR6B,D). In Turn 62, Margaret answers, “because 4 times 4 is sixteen.” Ms. Littleton asks, “Where did the four come from?” After Margaret says, “4 comes from the 4 pints and the 4 quarts.” Ms. Littleton assists by adding to her thinking (GR6E) and says, “4 comes from the 4 pints and the four quarts.” Margaret is praised for her efforts.

This is an example where Ms. Littleton stays with a student to ensure that she [Margaret] really understands the conversion of the units. When asked to reflect on how talking about problems helps students understand the mathematics Ms. Littleton said,

The more they discuss in the classroom, the deeper their understanding is. I could see it like through exit tickets and formative assessments and I would just teach it. And because sometimes I do. I don’t always do math talk and from year to year, I change things too. But especially like area and perimeter, I think either that or the year before I did a whole lesson about that them discovering the formulas on their own rather than me telling them what it was. And their understanding was so much stronger because of that. Because they had ownership, I think really understood why it is length times width without me telling them it was length times width. It's not just me telling them okay, you need to memorize this. (Focus Interview, p.9-10)

This dialogue also demonstrates the ways Ms. Littleton includes students in the discussion to help students see the math the way others do. When asked to reflect on how
students talking with their peers differs from students talking with teachers she said, “Sometimes they can just--students can put it in kid friendly language. It's just helps them to understand it better where I might try using the mathematical terms they are not quite ready for.” (Focus Interview, p.5)

According to the Student Questionnaire Results, more than half of the students agreed to understanding math more when they talked with other students. Chetan also reflects on this during the individual interview. He said,

Sometimes I get it wrong and the other person gets it right, and through talking I get it a lot more, and then I do what they’re trying to say on a piece of paper, and then I come out with an answer, and I got it right. They help me understand the problem more. (Individual interview, p.1)

The discussion continues with a transition to the next part of the lesson (See Lines 66-89).

66. T: Something that we are going to work with today is the relationship between liters and milliliters. Does anyone remember the relationship of liters and milliliters?

67. T: Kathryn?

68. K: A liter has 1000 milliliters inside that liter.

69. T: So one liter equals 1000 milliliters?

70. T: So today when we do our problem solving, we are going to need these units of measure. We are going to be working with a partner today on a couple different problem-solving questions. These questions all have to do with water balloons and the amount of liquid inside of water balloons.

71. T: You will receive a paper with four questions on it. You will be assigned a question to start with that one question. The expectation is that you will create a poster showing how you solved that question, so that you could teach someone else how to solve the question.

72. T: So, what do you think that we should put on our poster?

73. T: Nicholas?
74. N: All of your work.

75. T: Awesome, all of your work, Elenor what do you think we should put on this poster?

76. E: Explain how you got your answer and show your work. You also wrote balloon and not balloon.
77. T: Thank you I will fix that. Jordon?

78. J: Label.

79. T: Label, definitely label your work.

80. T: Would it be helpful to show two different ways to solve a problem?

81. T: So, challenge yourself. See if there is a second way that you can solve the problem.

82. T: Anything else we should put on this poster?

83. E: Write your answer neat.

84. T: Xavier could you repeat what Elenor said?

85. X: I don’t know what Elenor said.

86. T: Oh man, who can tell me what Elenor said? Zandra?


88. T: Neat handwriting, awesome!

89. T: You need a title an answer, write the question, show all of your work and write your answer in a complete sentence.

Ms. Littleton linked prior learning with this lesson (GR8C) and directed thinking (GR6B) using the question, “Does anyone remember what the relationship of liters and milliliters?” Kathryn makes her knowledge public by explaining, “A liter has 1000 milliliters inside that liter.”
Next, Ms. Littleton provides a description of the task she wants students to complete during the remainders of class, as a way to *initiate their working together* (GR6F). She says,

So today when we do our problem solving, we are going to need these units of measure. We are going to be working with a partner today on a couple different problem solving questions. These questions all have to do with water balloons and the amount of liquid inside of water balloons. (Event 3, p.10)

She provides more specifics about the task. Then in Turn 72, she invites all students to engage in a joint discussion by asking, “What do you think that we should put on our poster?” This allowed student to share ideas and *engage in joint decision making* instead of simply being provided with the instructions (GR14A). Drawing on prior knowledge from past assignments and lessons, students share their ideas. Ms. Littleton then asked, “Would it be helpful to show two different ways to solve a problem?” as a way to *direct the students’ thinking* and *discuss* her own ideas (GR6A,B). Then Ms. Littleton *encourages* students to contribute additional *multiple solutions* by asking, “Anything else we should put on this poster?” Students provide additional suggestions. Finally, Ms. Littleton summarizes the expectations in Turn 89 when she says, “You need a title, an answer, write the question, show all of your work, and write your answer in a complete sentence.

During the individual interview, Chetan was asked why Ms. Littleton might have asked the students to share different ways to solve the same problem. Chetan said,

Because she wants you to know a different way, because if you don’t know two ways. I would rather know two ways than one. It will help me understand the problem a lot more, and it gives me a lot more ways to do it. If you do it one way and it’s right, you do it another way, you can easily do a different way too. (Individual Interview, p.3)
Also, when Evelyn was asked to comment on why she thought Ms. Powers has students come up with two solutions to a problem she said, “So that you can figure out how to do it in a different way so you know you can understand it better.” (Individual Interview, p.3)

Also during the Focus Interview, Elenor shared why she thought her teacher asked for two solutions to a problem. She said, “It is a good use of brain power instead of thinking of one way the challenge was to think of two ways (Student Group Focus Interview p. 1). Manny added, “It’s like a good way to help you like understand a problem in many ways, you think that there is just on way but here are millions of ways to solve the problem.” Chevron also said, “I think it’s a good idea because you have to think about it and try a different strategy.” Finally, Jadiaih explained,

She wants to see…sure there are a lot of ways to do it but maybe you have thought one way and she wants to see which way pushed you to do it. The other time you had to pick how you would do it. You had to pick one that was challenging for you. She wants to see what you prefer, how you do math. (Focus Interview, p.2)

When asked to reflect on her request for students to demonstrate two ways of solve a problem during her individual interview, Ms. Littleton mentioned that her students needed support to do so. She added,

When I was asking them for two different ways to solve a problem, somebody, one of the kids said well, Nicholas did that one so I’m not sure how he did it, so we had a discussion about that. They’re working together, this is a group project. You should both understand both strategies that we are doing. So, then they spent a lot of time teaching each other. (p.4)

Although students understood the meaning of finding different methods for solving the same problem, it was not always easy for them. Winnie reflected, “It was
kind of harder because we found our first way and we really like spent a lot of time explaining our parts to each other. We didn’t really get to understand it [the second method] like the first one” (p.1). Nicholas also shared his difficulty saying, “It kind of makes me feel a little awkward when she asks me to do it a second way because I am so used to doing it one way and it gets tough for me to do it another way.”

**Classroom Example 2-Event 7**

The second example of Engage In Joint Reasoning (GR6) comes from the mathematical discourse that was part of the lesson in Event 7 (Classroom Observation, June 10). This observation involved a conversation facilitated by Ms. Littleton about the student’s experiences completing the bow task (Event 7, p. 22-24). Ms. Littleton begins the conversation by asking the class “What went really well?” This open-ended question encouraged all to contribute (GR1A) to the discussion. Students share their thoughts relating to their experiences working as a team. Ms. Littleton continues to ask questions to engage students (GR6C) in order to keep the conversation flowing among students.

1. T: What went really well?
2. M: We were working together.
3. T: Awesome. Elenor?
4. E: Well, our group we had to keep focused so we could...get things done, we did this thing where we measured all of the box, so we can get an exactly accurate bow.
5. T: Wow! Nicholas.
6. N: We started working together better. We started to use teamwork a lot more.
7. T: Okay. Did you not start off that way?
8. N: Uh-uh. We started off pretty rough.
9. T: And what helped you to start doing teamwork?

10. N: When I started to pitch in with my group. It started to help us a little more.

After a few students finished sharing, Ms. Littleton flipped the discussion to, “What were some of the challenges you faced?” In the discussion below, students share ideas about the challenges they faced with the task (GR6.A).

11. T: What were some of the challenges that you faced? Zandra?

12. Z: Like we had to--like we had to wrap the ribbon around it and put a bow since like we had like a lot of bow. So, we needed to get the ribbon down so we had to like tape it down. And we had to get the measurement then, like we had to tape it down and then like people…

13. T: So, was your challenge measuring out the amount that you needed?

14. Z: Yeah. And it was like only 60, we have to do it multiple times because we have like two 60s.

15. T: What was only the 60?

16. Z: The measurement like the ruler that came up, so we had to do it multiple times since we had 138 inches. So, we had to do like 60 and 60 and 18 inches.

17. T: Awesome. How did you figure that out?

18. Z: How’d you figure that out?

19. T: How'd you figure out that you needed to do 60 twice?

20. Z: Because 60 + 60 = 120, and then, you had 18 + 120 = 138 and then, you needed an estimate with 138 inches.

21. T: Awesome. What are some of the other challenges you face?

22. T: What's your challenge?

23. C: Well, all three of us, we're measuring the box and all three of us got something different. So, we had to figure out who was right.

24. T: And how did you figure that out? What did you do?
To further the dialogue among students, Ms. Littleton asks probing questions to both understand thinking and encourage the exchange of ideas among her students (GR6C,D). In Turn 12 for instance, Zandra shares the challenges that her group experienced and how they finally managed to wrap it around the box. Ms. Littleton follows up with a question to clarify her explanation, “So was your challenge measuring out the amount that you needed?” Ms. Littleton prompts Zandra to make the steps the group had taken more explicit to the rest of the community. In Turn 14, Zandra explains, “It was like only 60, we have to do it multiple times because we have like two 60’s.” Ms. Littleton next question attempts to move this student toward the specific mathematical ideas involved in finding the solution to this problem. Ms. Littleton asked, “How’d you figure out that you needed to do 60 twice?” In Turn 20, Zandra justifies her group’s thinking by explaining the steps taken to determine the length of the ribbon. They had estimated the sides of the box and then combined the estimates to arrive at 138 inches. Zandra does not communicate whether or not the group’s process connected the attributes of the three-dimensional figure with a strategy for finding their solution. Ms. Littleton praises Zandra for her contribution. She does not ask any follow up questions. Continuing the conversation, Ms. Littleton asks, “What are some of the other challenges you faced?” This question continues the discussion of ideas (GR6A). One group shared how their group members each came up with different answers. Another shared their challenges with focus. Ms. Littleton facilitated these ideas using questions to understand the struggles students experienced. These questions were used to engaged students in a discussion and to investigate how they managed to overcome the challenges on their own. Students did not ask questions of others during these exchanges.
The dialogue continues below with a question Ms. Littleton uses to direct thinking (GR6B). She asked, “So after doing this once, now you have experience and now you have a different size box. What would you do differently? This open-ended question prompted a conversation about how they might use what they had learned today when solving similar problems. This open-ended question, invites all to participate because the question could be answered in many ways (GR6A).

25. T: So, after doing this once, now you have experience and now, you have a different size box. What would you do differently?

26. E: Well, estimate and then add 10 extra inches.

27. T: What was the 10 inches for?

28. E: Just in case it was like a bit small.

29. T: Okay. So, you're giving yourselves an extra ribbon to work with so that way you don’t end up with something too short?

30. T: Awesome. Winnie?

31. W: I think have the same thing like wrapping the tape measure around the box. But you might need to wrap it a lot less or a lot more.

32. T: Uh-hmm.

33. W: And then, just add like 5 to 8 inches for like for the bow.


35. E: We would use a different size ribbon but use to the same thing. So, like do the same thing that was did but since the box is a little bit different size so you have to use a different piece.

36. T: Can you tell us what you did? What is the same thing?

37. E: Well, we wrapped the tape measure around and then, we added on like 3 to 5 inches, Devon and I did rock paper scissor to see whether to have on 2 feet or 3 feet, and we measured 3 feet and it actually worked.

38. T: Okay. So, you just took a random guess. You don’t have any estimates that you used?
39. S: Well, we like …

40. A: We wrapped the ribbon around and estimated and I said two and Devon said 3 and Devon won [Rock Paper Scissors] so we did 3 feet.

41. E: And just for the bow we just added on extra for the bow.

42. T: Okay. So, extra was for the bow?

43. T: Awesome, Elenor.

44. E: So I'd see if the box was big or small and then, I’d estimate to see if it would be more than a hundred inches or less than a hundred inches.

45. T: Okay. So, you would use a hundred inches as your benchmark, Jostos?

46. Z: Yeah.

47. T: Okay. And how you would—why did you pick 100 inches?

48. Z: Because my box was a hundred and thirty-eight inches, so you know, a hundred it could go over 100 and it could go under 100.

49. T: Awesome. Jadiah?

50. J: We want to use a regular ruler. We were only using a yard stick and tape measure.

51. T: Okay. Why are you choosing those two tools?

52. J: Because we're using a yard stick or a tape measure instead of—because like ours was very small and there are much smaller boxes in that.

53. T: Okay.

54. J: So, it was still pretty big like there were still a lot of inches to say for what it looks like. It's actually it looks smaller than it is. So, if we have one a little bigger like that [points to demo box] we would use a yard stick or something.

55. T: Okay. So, I want you think about this question for a second? What if we weren’t going to use a bow, but we needed to tie the ribbon around the width and around the length, and it needed to meet exactly where it crosses over. Would that be more difficult to figure out or less difficult to figure out? Why?

56. T: I want you to turn and talk.
Ella answered first explaining that she would add 10 inches “just in case it was a bit small.” Winnie shared that she would use the tape measure to wrap around the box to get an estimate again and then add the five to eight inches extra for the bow. Edna concluded that a different size ribbon would be needed because the box was different. Encouraging students to discuss ideas (GR6A) provided all a chance to reflect on the lesson as a community. Inviting students to share anything from their experiences also invited more students into the conversation because almost anything they could add would be acceptable (GRA1).

It was not always apparent what the students meant when they attempted to communicate their ideas. Ms. Littleton used questions to understand thinking, often to seek clarification (GR6D). In Turn 26 for example, Ella said that she would estimate and add ten extra inches, but did not explain why. Ms. Littleton attempted to understand her thinking by asking “What was the 10 inches for?” Ella said, “Just in case it was a bit small.” Again, Ms. Littleton probed to surface Ella’s thinking. She asked, “So, you're giving yourselves an extra ribbon to work with so that way you don’t end up with something too short?”

Ms. Littleton also asked questions to direct students thinking toward something in a student’s contribution. In Turn 51 for example, she asked, “Why are you choosing those two tools?” She wanted students to explain more explicitly how these tools may have been part of their decision making while solving the task.

57. T: After doing this once, now you have experience and now, you have a different sizes box. What would you do differently?

58. E: Well, estimate and then add 10 extra inches.

59. T: What was the 10 inches for?
60. E: Just in case it was like a bit small.

61. T: Okay. So, you're giving yourselves an extra ribbon to work with so that way you don’t end up with something too short?

62. T: Turn and Talk.

When asked what would help her utilize math talk in the classroom to make it more productive and lead to greater understanding, Ms. Littleton said,

You asked before like what do I ask them to like probe and, like that’s just through my own experience, my own knowledge. Like I don’t have a list of questions that I feel like I should be saying. It's just like in-the-moment what I think would help. And I don’t know if there's professional development that would be more effective for me to be asking more specific questions or more like topic related questions during those math talks like that may help student's understanding on that specific topic. Because, I mean, I teach that way because I feel like it's the right way to do it but I don’t really, like no one told me that that’s what I need to do. (Focus Interview, p.9)

During the focus group interview, students were asked why their teacher asked questions such as, “what would you have done differently if you did it again?” Daniel responded, “She asks questions to help people understand better.” (Focus Interview, p.1-2). Additionally, students were asked why Ms. Littleton wanted to know about their “challenges or what went really well.” Kate responded, “Well, I think she asked questions to help us understand.” Ella added, “She might ask you what went well because then you could use the strategy next time.” Josh also explained, “She’s trying to help you find another solution to help you solve the problem.

Ms. Littleton also used questions to extend the student’s thinking. She asked Edna, “Can you tell us what you did? What is the same?” This allowed her to highlight something the student said that others could learn from. In Turn 45, Edna responded to the question by explaining how her group figured out their solution. Edna said, “Well, we
wrapped the tape measure around and then, we added on like 3 to 5 inches, Devon and I did rock paper scissor to see whether to have on 2 feet or 3 feet, and we measured 3 feet and it actually worked.” Another group member to join the conversation to add clarity to the explanation after Ms. Littleton asked, “So you just took a random guess?” Arthur explained that they had estimated two different lengths and then decided that Devon’s three feet would be used because she won the Rock, Paper, Scissors game. Students did not explain why the three feet estimate was the closest and how Devon’s way of estimating could have been repeated when solving problems in the future.

Next, Ms. Littleton directed students thinking by asking a more specific question, “What if we weren’t going to use a bow but we needed to tie the ribbon around the width and around the length, and it needed to meet exactly where it crossed over? She immediately follows that question with another, “Would that be more difficult to figure out or less difficult to figure out? Why?” Then in Turn 62, she directs students to “turn and talk” with a peer about the question she asked. After a few minutes, she invites all to share their thinking.

63. T: Let me see or show your hands if you think it will be less difficult. Raise your hand?

64. T: Less difficult? Raise your hand.
65. S: [Approximately 12 students raised their hands]

66. T: Hands down. Do you think will be more difficult, raise your hand?
67. S: [6 students raise their hands]

68. T: All right.

69. T: Hands down. Why do you think it would be less difficult? Margaret?

70. M: Well, when we had the bow, we have to add an extra, an extra number to it. And if we didn’t, we wouldn’t have to add that extra number.
Okay. Let's hear from this side from one more. Why you think it would be more difficult? Devon?

Because like you have to, if it's exactly, you have to get it like the exact measurements and you can't like estimate or anything like that. So, it would be really hard to get it exactly.

Good. So, if there was no bow, if we were just had it measure it, so it was lining up exactly, this bow gives us flexibility, doesn't it?

So it allows us to make an over estimate. We will just make the bow a little bigger. Could we? I saw some groups doing that. I saw that some groups had a very tiny bow like this one. But they still were able to wrap their present, right?

Yes.

And then, I saw that some groups had a larger bow.

Yay! That’s ours.

We go! go group 6.

This one, if they measured two or three inches shorter. Would they have still be able to wrap their box?

No.

And what case it is in real life? Might you have to do something like this. Maybe not wrapping a box, but working with material and going to make an estimate.

Well, this one is a box. Maybe when we wrapping our Christmas present for somebody.

Okay. Excellent, Daniel?

This is good for camping like you need to know like the estimate of like, how much wood for your fire going to need and so you can wrap it up so don’t—like the log don’t roll or run away and you will have wood for you fire. You have to like, you know, you estimate around kind of like tie all the logs together.

Okay. So, tying the logs together with some material?
87. D: Yeah.

88. T: Would it be better to do an over estimate or under estimate there?

89. D: Probably over so you can get like it tighter.

90. T: Good. Because if you have an underestimate you might end up with not enough to tie up your logs. Jadiah?

91. T: It might be good because like if a ribbon on your teddy bear broke so you could be able to measure the like the neck or something. So, you could get it again or another one.

Next the conversation in this lesson continues with students reflecting about the difficulty of the activity. In Turn 65, Ms. Littleton asks students to respond as a group regarding the difficulty of estimating for a group response about whether or not it would be easier to get an exact measurement, not including the bow, before requesting that individuals discuss their ideas (GR6A).

Then Ms. Littleton says, “Show your hands if you think it will be less difficult. Raise your hand?” Twelve students raise their hand. Then she does the same when asking, “Do you think will be more difficult, raise your hand?” Six students raise their hand. Then in Turn 69, she asks the question, “Why do you think that it would be less difficult?” She selected Margaret to share. Margaret explains, “Well, when we had the bow, an extra number to it. And if we didn’t, we wouldn’t have to add that extra number.” This student had not quite made the connection, so Ms. Littleton turned and selected the group seated next to her to share.

Ms. Littleton used the strategy of “Turn and Talk” with her fourth graders during this lesson to initiate and exchange of ideas among students. Although this technique was
only used once during the several lessons observed by the researcher, Ms. Littleton described the strategy as very important. She said,

I think it is awesome thing to use because students get the chance to talk with each other before they present it in front of the whole group and it also let’s them iron out some of those questions that they have about the topic discussed. (Individual Interview, p.4)

In Turn 74, Margaret said, “Well, when we had the bow, we have to add an extra, an extra number to it. And if we didn’t, we wouldn’t have to add that extra number.” Ms. Littleton adds, “So if there was no bow, if we just had to get the exact measurement and you can’t estimate or anything like that, it would be really hard to get it exactly.” Margaret was trying to make the point was that estimating provided more flexibility. Ms. Littleton clarified Margaret’s thinking and said, “So if there was no bow, if we were just had it measure it, so it was lining up exactly, this bow gives us flexibility, doesn’t it?” (Event 7, p. 26) The whole class agreed.

During this mathematical discussion, Ms. Littleton also highlighted the thinking of two groups that she worked with earlier by mentioning how one bow was small and one was large but they were still able to wrap their presents, they just had different size bows (GR6E). In Turn 84, she reinforced this thinking by asking, “If they [referring to the group with the smallest bow] measured two or three inches shorter, would they have still been able to wrap their box?” The students agreed that they would not be able to wrap their box, demonstrating understanding of this idea.

As the conversation continues, Ms. Littleton engage students in problem solving by reiterating the difference between an exact estimate and an over estimate to help students conceptualize why finding the exact measurement is more difficult than
estimating measurements (GR6G). She does this by asking students to connect their thinking to another real-life situation.

Next, in Turn 86, Ms. Littleton asks, “And what case it is in real life? Might you have to do something like this. Maybe not wrapping a box, but working with material and going to make an estimate?” Leo connects the wrapping of this box to wrapping Christmas presents. Daniel connects it to estimating the amount of wood and the string that you might need for a campfire while camping. Ms. Littleton interacts with Daniel asking, “Would it be better to do an overestimate or underestimate there? Daniel responds with, “Probably over so you can get like it tighter.” Ms. Littleton praises and clarifies his contribution for the class by saying, “Because if you have an underestimate you might end up with not enough to tie up your logs. She encourages multiple solutions to her question.

92. T: What if you were a carpenter and you were laying the carpet down in a house?

93. T: Okay. Devon said she's going to say something about this, what do you think about that, Devon?

94. D: Like laying a carpet?

95. T: Yeah.

96. D: Like you would need to measure the stuff to make sure that you got it right.

97. T: So what would happen if you did an underestimate?

98. D: The underneath it would be showing and people would be mad.


100. T: Awesome.

101. D: And if it was over they would like trip over it.
102. T: So when you are working with estimates, it is important to think about what you are working with. Would it be better in a case to do an underestimate or overestimate? So, in that situations, might be better for an over. So, in the situation that we better for an under.

103. T: Excellent job today. It the time for us to clean up and have some social studies, so please leave your papers in the box.

Afterwards, Ms. Littleton directs students thinking again when asking a more pointed question, “What if you were a carpenter and you were laying the carpet down in a house?” Devon begins, “Like you would need to measure the stuff to make sure that you got it right.” Ms. Littleton assists her in articulating her words more clearly and asks, “So what would happen if you did an underestimate?” Devon responds, “The underneath would be showing and the people would be mad.” Ms. Littleton praises her contribution with, “Awesome.” Devon continues, “And if it was over they would like trip over it.”

When asked to reflect on helping students to make connections to mathematics and their own life, she said, “I ask them to make connections to their own life. I wish I could take a video on Jadiah because he makes these connections even like on his shirts. It’s just crazy the connection is a big thing rather than just telling them.” (Individual Interview, p. 8)

Ms. Littleton wraps up the lesson with a general statement about why it is important to think about what you are working with when estimating. She also praises the class for their good work during the mathematics class on that day before transitioning them to social studies.

Summary

Ms. Littleton uses specific strategic discourse practices to engage all of her students in discussions with her and classmates about mathematical problems. During
Events three and seven Ms. Littleton engages students in problem-solving situations either in a large group or small groups and then has them discuss their solutions. She encouraged multiple students to participate in the conversations while she or their classmates ask questions or exchange ideas. Although the path of the discussion is facilitated by Ms. Littleton, students consistently generate their ideas. Both engage with one another by asking questions or offering feedback. Many elements of strategic discourse are embedded in these conversations, centered around problem solving.

*Engaging In Joint Reasoning (GR6)* is a significant portion of her work with students, with students discussing ideas and solutions with others (*GR6A*) In these events, Ms. Littleton begins with open-ended questions to invite all students to contribute then follows up with more specific questions to highlight their thinking and unlock their understanding. Ms. Littleton draws her students in by engaging them in solving problems or tasks together. Afterwards she poses questions to initiate conversation and then keeps them engaged using other elements of strategic discourse to encourage ideas and solutions. In event three, she uses questions to prompt students to explain themselves to scaffold learning in a way that leads them to understanding. Also in event three she presses students to think about the reasonableness of their strategies when selecting the most effective operation to apply to solving problem.

In event seven, Ms. Littleton encourages students to learn from one another. They exchange ideas by sharing their knowledge and experiences about working and solving problems as a group. Students include explanations and justifications when sharing their thinking. If a student does not explain their thinking independently, Ms. Littleton encourages them to do so with follow up questions. However, Ms. Littleton does not
always follow up a student’s response with a question. She chooses when to follow the path raised by students and when not to.

Ms. Littleton also skillfully changes the direction of the conversation. This occurs when she has a specific path in mind that she wants students to get to but has realized they cannot on their own. For instance, in event seven, when she wanted students to apply the learning from the lesson to real life applications where you would need to use a more exact estimate. She had to assist them in making these connections by prompting them to share examples of when they estimating might be used in the world. She engaged students in this discussion before closing the lesson.

Ms. Littleton encourages students to engage in conversations very often without critiquing what is said. At times, however, she did engage critically with students on some occasions by correcting their thinking. These interactions provide evidence that it is acceptable for teachers and students, in this classroom, to question one another and to disagree. For example, in Event 3 when a student offers fluid ounces rather than simply ounces as the unit of measure, she stepped in to correct him. Likewise, in the same event when a student offered grams she added “Grams actually measures the weight of something.”

Ms. Littleton encourages students to ask questions while engaging in joint reasoning within mathematical conversations. However, the amount of questions from students compared to their teacher is much lower, see Table 14 and 15. Students did use questions four times to direct thinking over the course of the seven events (GR6B). These usually involved questions seeking clarification or to draw attention to a mistake. During event three, a student asks her teacher a question to exchange ideas during the discussion.
about the Big G diagram (GR6C). Students also asked other questions to exchange ideas with other students and their teacher five times during the other six events. In events one and seven, students asked questions of others to gain an understanding about how they arrived at an answer or to seek clarification about a question asked by their teacher (GR6D). Assistance was given by a student to another student while solving problems during a mathematical discussion twice during lesson two (GR6G). The infrequent use of questioning among students was also evident in the students’ perceptions when only six students reported about asking questions of their teacher on the student survey questionnaire (See Appendix I).

The next section explains how Ms. Littleton used the ground rule: *Knowledge Is Made Public (GR4)* during her discourse practices. This ground rule was the second most commonly used strategic discourse used by students and their teacher.

**Knowledge is Made Public (GR4)**

This section includes an investigation of the discourse practices or ground rules used by Ms. Littleton and her students during conversations in their fourth-grade classroom from Event 1(Classroom Observation, May 12). Ms. Littleton plans lessons to encourage students to think mathematically and share mathematical ideas related to the concepts that they are studying. She used the elements within the ground rule *Knowledge Is Made Public* to provide students with the opportunity to share this knowledge (GR4). Ms. Littleton’s questions invited ideas to be shared and then studied so that the community can interact while strengthening their understanding.

Over the course of the seven events, Ms. Littleton utilizes all four elements within this ground rule (GR4) as evident in Table 14. Students also utilize elements of this
ground rule as evident in Table 15. Using the conversations from Event 1, the ways Ms. Littleton and her students implement the elements of *Knowledge Is Made Public (GR4)* will be examined. The four elements within this ground rule include *participants offer their knowledge about mathematics (GR4A)* strategies are explained in words, pictures and numbers (GR4B) *contributions are restated (GR4C)* and referrals are made to previous lessons, concepts, or contributions (GR4D).

**Classroom Example 1-Event 1**

The first example of Knowledge Is Made Public comes from the mathematical discourse that was part of Event 1 (Classroom Observation, May 12). The focus of the lesson is angle measurement, more specifically understanding the equivalence of fractional parts within a circle. The essential question and enduring understandings are written with the end outcome in mind to guide her path toward understanding. These are the questions used by Ms. Littleton guiding this lesson:

**Essential Question**

Why is it important to know the fractional part of a circle?

**Enduring Understanding**

Angle measurements can be thought of as a measure of rotation in a circle.

This lesson began with students working on a problem-solving question in small groups (Event 1, p. 1-3). Students usually work in groups of two to four students, each responsible for completing the team’s task. After about 10 minutes, Ms. Littleton gathers students’ attention to the front of the classroom where she is standing near the whiteboard. Students always respond positively to working in groups to complete their assignments. Groups are most typically assigned by the teacher.
Ms. Littleton begins the discussion (Event 1, p.1-24) with the statement encouraging multiple solutions to the problem, “We are sharing out because we’re going to try to share different thinking, try not to share the same thinking.” In Turn 2, Chetan starts to explains his understanding that there are four right angles in a circle (GR4A). He casually asks his teacher if he can represent his thinking on the whiteboard. As he draws his thinking in pictures and numbers (GR4B) he explains, “I think it’s four because there are four right angles in there.”

Next, Ms. Littleton invites students to engage in joint reasoning by asking if anyone has questions for Chetan. When no one answers she thanks Chetan for his contribution and calls on another student to share.

Then Elenor joins the conversation to make her knowledge public (GR4A) by saying, “I know that there’s 90 degrees in one part of the circle.” Ms. Littleton restates her thinking (GR4C) through a question, “You know that 90 degrees is what?” to reinforce this line of thinking. Elenor continues to explain her thinking. She says,

I know that 90 degrees is one-fourth of the 360-degree circle so I divided 90 by four, 360 degrees by 90 and I got 4. I also made a circle and divided it into four pieces starting with 90 here, 90 here, 90 here, and 90 here. And there's 4 pieces. (Event 1 p.1-2)

Ms. Littleton praises Edna and then asks students to consider how she connected her thinking to a prior concept they learned earlier in the year (GR4D). Ms. Littleton calls on Jostos and he suggests fractions. Ms. Littleton restates the contribution again (GR4C). She then refers to the prior contributions (GR4D) she observed while students were working on this problem earlier and adds, “I saw some other people that also connected their thinking with fractions.”
To encourage students to offer multiple solutions, she invites Edna to share (GR7A). Edna extends the conversation by making her knowledge of the relationship between 180 degrees and one half of a circle which is made up of two 90 degree angles public. In Turn 17, Again, Ms. Littleton helps to make Edna’s thinking explicit for all by asking a question to the clarifying her contribution, “So four right angles?” Edna confirms her thinking.

After Edna contributes, Chevron shares her knowledge with the group. Ms. Littleton tries to break apart the steps by asking, “So first you tried three?” She continued, “And then you realized that wasn’t enough so you added one more?” Afterwards, Chevron confirmed that was what she had said. Students responded to questions that Ms. Littleton asked the class willingly engaging with peers about their thinking. For example, Jostos responded in Turn 13 when Ms. Littleton asked, “Elenor connected her thinking with what?” He also responded in Turn 36, after Ms. Littleton asked “Why did Devon use subtraction to try to figure out the fraction of the circle?”

When asked to reflect on why Ms. Littleton asked students if they had any questions for the others as they explained their solution, Arthur said,

If I have a fifteen, ten times and they don’t know how I got it, because I did it a whole different way from them, they would say how did you get your answer and I would say I multiplied ten times fifteen and got me answer and maybe they have a whole different way, or the same way. (Individual Interview, p.1).

Chetan said, “because some people might be struggling, and they might have gotten it wrong, or some people might disagree with me, or do agree with me.”

(Individual Interview, p.3)

1. T: Chetan?
2. C: Ok, so how many right angles would be equal to a full circle. If you put four right angles in a circle, can I show it on the board?

3. T: Yes.

4. C: Like (drew a circle divided into 4 parts.)

5. C: Okay, hold on. Okay. one, two, three, four. Yeah, I think it's four because there's four right angles in there. There's one, two, three and four. [pointed to all 4 angles]

6. T: Does anyone have any questions for Chetan?

7. T: Okay. Thank you. Elenor?

8. E: So I know that there's 90 degrees in one part of a 360-degree circle.

9. T: You know that 90 degrees is what?

10. E: I know that 90 degrees is one-fourth of the 360-degree circle so I divided 90 by four, 360 degrees by 90 and I got 4. I also made a circle and divided it into four pieces starting with 90 here, 90 here, 90 here, and 90 here. And there's 4 pieces.

11. T: Ok, nice job Elenor, so Elenor connected her thinking with what? What else did we learn this year?

12. T: Jostos?


14. T: Fractions, yes, you can use fractions. I saw some other people that also connected their thinking with fractions.

15. T: Edna?

16. E: I know a full circle equals 360 degrees. And 180 is one-half of 360 so 90 is half of 180 and then so if you add 90 four times it equal 360.

17. T: So four right angles?

18. E: Yes.

19. T: Awesome. Any questions for Edna? Okay. We're going to have time for one more.

20. T: Chevron.
21. C: I knew that a whole circle was 360 degrees so I added 90 four times. Well, I added it 90, 90, 90, and I added them up and then got 270 and then I added 90 again and got 360.

22. T: Okay. So first you tried three?

23. C: Uh hum.

24. T: And you realized that wasn’t enough so you added one more, excellent. Any questions for Chevron?

After Chevron shares, Ms. Littleton provides another task for students. Below, she directs them to solve another problem with their group. They are asked to find the angle measurement for three-fourths of the circle and show your thinking. Students are given a few minutes to solve before Ms. Littleton tells them that they will have time for three people to share. Jadiah is first to share, in Turn 28. He makes his knowledge public by describing in words how he used his knowledge of three times nine to figure out three times 90 as his first way of solving the problem (*GR4A,B*). He offers an additional strategy and explains using two times 90 equals 180 and adds a third 90 to get 270 degrees. Ms. Littleton thanks Jadiah and calls on Devon.

25. T: So here’s a question. Find the angle measurement for three-fourths of a circle and show your thinking.

26. T: We will have time for three people to share your thinking.

27. T: Jadiah would you like to share?

28. J: I figured it out, 4 x 90 equals 360 so then I did that so then 3 X9 =27 and 3X90=270, that was my first way of doing it. Then 2x90=180 and 180 plus 90 = 270.

29. T: Does anyone have any questions for Jadiah?

30. T: Thank you for sharing Jadiah!
Devon makes her *knowledge public* by explicitly *explaining* how the angle measurement related to the fractional parts of the circle (*GR4A,B,D*). Her explanation in Turn 32, is similar to Jadiah’s in that she added three sets of 90 degrees but builds on her thinking by making the direct correlation to the fractional parts. Then in Turn 33, Devon shared her knowledge about the second strategy she used to solve the problem. This time she explained in numbers and words how she used subtraction to figure out the fraction of the circle (*GR4A,B,*). She said,

The full circle is 360 degrees so we are only leaving out ¼ so I subtracted the ¼ which is 90 degrees from the full circle which is 360 degrees and I did 360 degrees minus 90 degrees and got 270 degrees. (Event 1, p.2)

Ms. Littleton asks the class to reflect on Devon’s use of the subtraction strategy. Jostos begins by explaining, “Well, since the denominator is fourths then she just did fourths and then took away.” When Ms. Littleton probes him to explain more, Jostos responds, “I don’t really know.” She encourages him to continue and he says, “Well, if the denominator is fourths then you could do all of the circle and take away only one fourth.” Ms. Littleton asks Jostos, “Well, if the denominator is fourths then you could do all of the circle and take away only one fourth?” Jostos agrees by nodding.

During the focus interview teachers were ask if they observed things that students do to make their interactions more productive. Ms. Littleton said,

Yes, I would say that there's some students who have stronger discussion skills than others, and that really listen to what someone is saying and take that into consideration. On the other hand, there's some students who feel like they're right and it's hard for them to listen to someone else giving a suggestion and why they might be wrong. And that’s not just the math piece of it, but that just taking a feedback. So there is another whole skill set in there too. (Focus Interview, p.7)

However, when asked if there were certain strategies that were used to increase talk, she Ms. Littleton agrees and says, “I mean my answer would be yes, but I can’t
think of what strategies” (Focus Interview, p. 6). She thought for a second and then she said, “I would say as students are discussing, to be walking around listening and maybe laying a question that might help them think a certain way or to maybe question what someone else is saying.”

31. T: Devon?

32. D: Um, so, my first way is what I wrote, I know that 180 degrees is \( \frac{1}{2} \), so that is 2/4 and then we had 1/4 left and that is 90 so I did 180 degrees plus 90 degrees equals 270 degrees, so ¾ of the circle is 270 degrees.

33. D: And then my other way, was um the full circle is 360 degrees so we are only leaving out ¼ so I subtracted the ¼ which is 90 degrees from the full circle which is 360 degrees and I did 360 degrees minus 90 degrees and got 270 degrees.

34. T: Devon just shared two different ways, one that was using subtraction why did Devon use subtraction to try to figure out the fraction of the circle?

35. T: Jostos?

36. J: Well, since the denominator is fourths then she just did fourths and then took away.

37. T: What do you mean?

38. M: I don’t really know.

40. T: Think about it because you used the same strategy.

41. J: Well, if the denominator is fourths then you could do all of the circle and take away only one fourth.

42. T: If the denominator is fourths then you could take away the one that is missing?

43. M: Nodds.

44. T: Awesome.

The lesson continues below when Ms. Littleton asks for one more volunteer to share a different way of thinking. Edna casually asks to come to the board to offer her
knowledge by drawing her explanation (GR4A,B). In Turn 48, she explains how she used addition to determine that three quarters of the circle was equal to 270 degrees. After she shared her thinking, Ms. Littleton asks if anyone had any questions for Edna. When no one answers she explained the next part of the lesson to the student. After explaining the task, students are given time to ask questions about the specific elements of the work. Students openly and freely asked questions before getting started.

During the Individual Interview, Ms. Littleton was asked to reflect on if she thought about how sharing was structured. She said,

No. I haven’t. Not for the most part I would say. If there isn't really structure, but if the student who I know doesn’t normally make connections or doesn’t normally share out and their hand is up, I definitely try to call on them to give them that opportunity. But for the most right I don’t really structure it. (Individual Interview, p.6)

Similarly, when asked to share whether or not she had an idea in mind for what she wanted students to discuss. She said,

Yes, which is interesting because a lot of times I have it in my mind planned out and laid, but it's leveled like make that connection here, and were going to need this connection first. But I just trying to live like that so we can still get to the same endpoint. (Individual Interview, p.6)

Ms. Littleton explains that she thinks about the end in mind, and plans her discussions in her head to finish there. She also describes wanted students to connect with certain ideas or concepts on the way.

Students often answered questions asked by their teacher. These questions usually included those directed at specific aspects of a problem they were working on either in a class discussion or during group work. They were usually specifically directed toward an idea. For example, question such as, “Does anyone remember what the relationship liters and milliliters?” resulted in the student exchanging their ideas (Event 3, p. 10). Students
freely shared their thinking when asked open-ended questions such as, “Edna would you like to share your thinking?” When asked to reflect on her students use of questioning, Ms. Littleton commented,

The thing that stands out to me is that some student are not confident in their math skills to begin with. I feel like they don’t get as much out of the math talk. Yeah, I think those are more listeners and they don’t have that confidence or that skillset to be sharing or to be questioning what someone else is doing. They’re going to listen and say oh that must be right. That’s what they're saying. So, it's right. (Focus Interview, p.7)

However, students rarely responded to open-ended questions that prompted them to comment about what a classmate had just contributed. For example, a student shared his solution to the question, how many right angles would be equal to a full circle? After he finished explaining his solution, Ms. Littleton asked the three questions listed below:

- Does anyone have any questions for Chetan? (Event 1, p.1)
- Does anyone have any questions for Jadiah? (Event 1, p.2)
- Does anyone have any questions for Edna? (Event 1, p.3)
- Does anyone have questions for Paris about how she solved the problem? (Event 2 p.5)

None of these questions were answered. The lack of response by the class was most likely because they had not been educated to participate in this way. If they had been expected to ask at least one question the habit of responding may have increased their likelihood to respond. Perhaps using the two questions, one comment protocol as in Event 4, may have resulted in more productive discussions after students were prompted using the questions above.

During her interview, Ms. Littleton acknowledged that students needed more support to be able to exchange ideas with one another. She reflected:
I think they should be more direct modeling, sometimes teachers just take students work for what it is. Yes, you add it correctly, yes you applied it correctly but they don’t necessarily stop to say why did you put it this way? So I think the more teachers do it, then you can model it for students to do it. (Individual Interview, p.2)

When asked to reflect on why Ms. Littleton questions students while circulating amongst groups, Evelyn said, “To help you understand what you are doing.” (Individual Interview, p.4)

Students asked questions of other students on a few occasions. Once during Event 3, Margaret caught a mistake and asked, “Shouldn’t it be 16?” “Shouldn’t it be two pints in each quart and she only did one pint in each quart?” (Event, 3 p.9). Another time when Edna was impressed by Caleb’s conversion of units of measurement she asked, “How do you know that?” (Event 3, p. 9).

During an individual interview with Chetan, he provided examples of the types of questions students asked one another. He said, “They [students] ask how did you show your work, and maybe ask can you explain it again or something?” (Individual Interview, p.4). Also, all students who completed the student questionnaire, except for Evelyn, agreed they asked questions to figure out what other people were thinking. Furthermore, the same nineteen students agreed that asking questions of classmates helps them to find logical solutions, see Table 16.

**Classroom Example 2-Event 5**

The second example of Knowledge Is Made Public (GR4) comes from the mathematical discourse during Event 3 (Classroom Observation, June, 5, p. 7-13). This lesson was also part of the Engage In Joint Reasoning discussion in the first section above.
The dialogue below comes from the conversations between Ms. Littleton and her students while they contributed units that could be used in measuring capacity. A discussion about the relationship between cups, pints, quarts and gallons followed. The final segment involves students working on solving capacity problems in groups of two.

While students share their knowledge, Ms. Littleton helps to navigate the discussion. As students list several units, Ms. Littleton praises their contributions when a new and accurate unit is shared (GR4A). She acknowledges the mathematics that is shared by restating it and offering words of praise.

1. T: Waiting to see that we are all ready to listen.
2. T: Edna can you give me one unit that you can use to measure capacity?
3. T: We are going to try not to repeat.
4. E: Liters.
5. T: Awesome!
6. T: Caleb?
7. C: Milliliter.
8. T: Awesome!
9. T: Margaret?
11. T: Great!
12. T: Winnie?
14. T: Awesome!
15. T: Chevron?
17. T: Good.
18. T: Edna?
20. T: Good.
21. T: Jadiah?
23. T: Nice.
24. T: Chetan?
25. C: Ounces.
26. T: Good, try not to repeat.
27. T: Devon?
29. T: Teaspoons we already said it.
30. T: Manny?
31. M: Did we say ounces?
32. T: [Nods yes.]
33. T: [Looks at Kate]
34. K: Gallons. GR4A
35. T: You said gallons, good.
36. T: Ty?
37. T: Cups.
38. T: Yes.
39. T: Caleb?
40. C: Fluid ounces.

41. T: Fluid ounces.

42. T: When we measure capacity, we can only measure in fluid ounces. We can’t measure in just ounces. Fluid ounces we are talking about capacity.

43. T: Edna?

44. E: Grams.

45. T: Grams actually measures the weight of something.

It is clear that Ms. Littleton and her students are aware of the mathematical knowledge being shared. Ms. Littleton listens critically to students as they share their knowledge publicly. Reminders are given not to repeat. When students do repeat, they are corrected by students. Ms. Littleton is focused on hearing what students are saying. She steps in to share her knowledge and to reinforce mathematical thinking (GR4A).

For example, in Turn 40, Caleb offers “fluid ounces” rather than simply ounces. Ms. Littleton brings everyone’s attention to this detail by saying, “When we measure capacity, we can only measure in fluid ounces. We can’t measure in just ounces. Fluid ounces we are talking about capacity.” She also steps in to correct thinking, as well. During these times, she does not work through but steps in to correct the mathematical thinking. In Turn 44 for instance, Edna suggests, “Grams” and Ms. Littleton corrects her by saying “Grams actually measures the weight of something.” Later while students are working in small groups in the hall, Ms. Littleton questions Chevron’s thinking when she decides to multiply instead of divide to find out how many balloons that she can fill.

Ms. Littleton acknowledges sometimes directing student thinking toward the right answer during the focus interview. She said, “That is one thing I don’t want them to do is keep going on the wrong track.” (Focus Interview, p. 6). She also mentions not choosing
to spiral back to prior learning but instead corrects students by reminding them of what they should already know so that they can focus on the current problem. More details regarding this exchange will come later in the chapter.

Students show evidence of knowing how to critique while discussing ideas with peers. In Turn 53, for example, Manny offers, “Cups” and student quickly responded by saying, “Cups, we said cups.” Also, in Turn 56, Jadiahsays “Quarts” and student responds by saying, “I said quarts.” These are two examples demonstrating that students are listening and processing what is being said, they are not just accepting, but filtering all responses.

46. T: Evelyn: Pints
47. T: Evelyn: Pints
49. M: We already said pints.
50. E: I said pints.
51. T: Sometimes we are going to repeat, but we are going to try not to.
52. T: Manny?
53. M: Cups.
54. S: Cups we said, cups.
55. T: Jadiahs?
57. T: Good.
58. S: I said that.
During the next part of the discussion below, Ms. Littleton draws attention to a strategy used by one of the groups when deciding on the units used in measuring capacity. Ms. Littleton invites the group to explain the words included in their model (GR4B). In Turn 61, Edna explains “The Big G.”

59. T: Over at these two table groups, I see a G written on their table tops.
60. T: Could someone tell me why you used that Big G?
61. E: G stands for gallon. Can I explain what the rest is?
62. T: Yes.
63. E: So G is the gallon and the four Q’s inside the G are four Quarts. So, draw the big giant G and four Q’s and draw two P’s inside of each P stands for pints. Then draw two C’s inside of the P’s and that stands for two cups and that’s it.
64. E: Well you could draw more but that would probably take forever.
65. T: So why did you draw this?
66. J: [Jumps in] Because four quarts the four G’s each q stands for one quart and four quarts equal one gallon and two pints equal 1 quart and 4 cups equals one quart.
67. J: I kind of noticed something. It kind of times’.
68. T: What do you mean it times’?
69. J: Because in the quarts it has 2 pints and in the 2 pints it has 4 cups. So, it’s like 2 and then it is ½ and then 4.
70. T: Awesome, so you are recognizing a relationship between different units of measure.

Ms. Littleton clarifies student thinking above in Turn 70. Then she closes this part of the discussion by making the shared knowledge public through a generalizing statement about what students should take from the diagram. She says, “So you are recognizing a relationship between different units of measure.”
The lesson continues when Ms. Littleton asks other students to use the diagram to solve some equivalent measurement questions below. In Turn 72, she invites all students to offer their knowledge (GR1A, GR4A) by asking, “If we had four quarts, what can we say four quarts is equal to?” When Lina hesitates, Ms. Littleton supports her thinking by referring back to the diagram written on the board by students. She rephrases her question and asks, “Four quarts is equal to what?” Lina says, “8 pints.” Ms. Littleton, clarifies the knowledge that has been made public by students and writes the representation on the whiteboard. She points out the equivalency among the units, and asks Lina, “How do you see that?” Lina responds, “Because 2 pints are in each quart.”

The conversation continues in Turn 80, with Ella, Jadiah and Zandra providing more units and explanations about how they arrived at their solutions. After Zandra shares, “16 cups.” Margaret joins the discussion, freely questioning Zandra’s thinking and asks, “Shouldn’t it be 32, shouldn’t it be two pints in each quart and she only did one pint in each quart?” Ms. Littleton asks Margaret, “I don’t know, what do you think?” Then Ms. Littleton turns and asks the entire class, “What do you think friends?”

It has come up that students do not have the same knowledge about the equivalencies and this needs to be addressed. Ms. Littleton facilitates this part of the conversation by asking Margaret and then the class to consider one solution in light of the other to find validity. This also allows all students the opportunity to publicly share their knowledge with a justification of their thinking about the number of pints in a quart.
Devon joins the conversation and states her agreement with Zandra and makes her knowledge about the topic (GR4A) and says, “I agree with Zandra because there in a so...in like in a quart there is 4 cups so 16 cups because 4 times 4 is 16 cups.” Ms. Littleton then asks Margaret if Devon’s explanation makes sense. After Margaret agrees, she encourages her to tell why. After Margaret says “4 times 4 is sixteen” Ms. Littleton still pushes to make sure Margaret understands completely. She asks, “Where did the 4 come from?” After Margaret answers, Ms. Littleton strengthens the explanation in Turn 101, by adding “There are 4 cups in each quart and 4 quarts”.

71. T: Awesome.
72. T: If we had four quarts, what can we say four quarts is equal to?
73. T: Lina?
74. L: Um. GR6A
75. T: So, we have one quart two quarts three quarts and four quarts. Taps onto the quarts drawn on the board. So, four quarts is equal to what? Then points to each on the board.
76. L: 8 pints.
77. T: Writes on board. 4 quarts equals 8 pints. Good. How did you see that?
78. C: Because 2 pints are in each quart.
79. T: Awesome, who can you find another unit that is equal to four quarts? Ella.
80. E: A gallon.
81. T: Awesome!
82. T: How did you know that?
83. J: Because inside of the G there are four q’s and they represent 4 quarts, so there are four quarts in a gallon.
84. T: Excellent.
85. T: Is there another unit we can find for four quarts?

86. T: Zandra?

87. Z: 16 cups.

88. T: 16 cups. How did you figure that out?

89. Z: Pint is two cups and two quarts is 8 cups. 2, 4, 6, 8….

90. M: Shouldn’t it be 32, shouldn’t it be two pints in each quart and she only did one pint in each quart?

91. T: I don’t know, what do you think?

92. T: So why do you think Zandra thinks it should be 16 and Margaret thinks it should be 32. What do you think friends?

93. T: Devon?

94. D: I agree with Zandra because there in a so…in like in a quart there is 4 cups so 16 cups because 4 times 4 is 16 cups.

95. T: Does that make sense Margaret?

96. M: Um hm.

97. T: Can you tell me why Margaret?

98. M: Because 4 times 4 is sixteen.

99. T: Where did the four come from?

100. M: 4 comes from the 4 pints and the 4 quarts.

101. T: There are 4 cups in each quart and we have 4 quarts.

The next section of the dialogue, included below, includes a conversation with Chevron and Lina’s group as they work in the hallway to complete the capacity problem. Beginning in Turn 102, these students engage in a discussion that requires negotiating a reasonable path for solving the problem. The problem asks, “The package of Mel’s water balloons says that it holds 300 milliliters of water. How many
baloons can he fill if he has two liters of water?” Unfortunately, when they solved the problem individually, Lina and Chevron had two different answers. Ms. Littleton joins Chevron and Lina during their discussion and they immediately bring the issue of having two answers to her attention. Ms. Littleton listens critically as the students explain their issue. She begins to question the route Chevron took in solving the problem.

102. C: What we can do is do a multiplication problem to get the division problem?

103. L: What? No, no, no wait!

104. C: I got this, I know what I am doing.

105. L: But I know what I am doing.

106. L: If you do um a 300.

107. C: Yeah.

108. L: [300] Times what equals 3000 or close to 3000.


110. L: Hold on, you can just do…1 I guess.

111. C: 2X3 is 6, 0x2=0, 0X0=0, 6,000

112. L: 0 – 3 is…

113. L: 7.

114. C: 6X0, 3X0, 3X0, 3X0 and 6X3 is 18.

115. C: Hey Lina, I’m done.

116. C: 1, 2, 3, 4, 5, 6, that’s not right.

117. C: 18.
118. A: Look Lina, if you do 20,000 divided by 300 you get 6,000. 6,000 times 300 is 18.

119. C: Should we ask if we can get two different answers?

120. L: No. That’s not right (pointing to her work).

121. C: Do you just want to go with my answer?

122. L and C: Yeah.

123. L: I don’t think 6 and 200 is a correct answer.

124. L: I think mine would be more of a correct answer.

125. C: So do you want to write it in a marker or colored pencil?

126. C: So I didn’t get 18? [asks the Teacher]

127. C: Yeah.

128. L: But she got 18.

129. L: And I got 200 and I don’t think that was right.

130. T: Ok.

In Turn 131 she asks, “So you did division here [points to Lina’s paper], and what did you do Chevron?” After Chevron told the teacher that she had used multiplication as a strategy, Ms. Littleton questions Chevron’s thinking (GR6D). She says, “We talked about why division was the best strategy, why did you do multiplication?” Ms. Littleton asks this question to critically engage with Chevron and to help her to see why using the operation division is more appropriate to this problem. Chevron begins to provide her justification by explaining her steps involving multiplying 2000 by 300 (GR4 A,B).

In Turn 135, Ms. Littleton directs the students thinking (GR6B) with a question, “Why did you multiply?” Chevron to use multiplication by saying,
We were trying to figure out how many balloons we were trying to fill with 2000 liters of water, right? Since we have 2000 liters of waters and we are trying to get water to each balloon, should we be multiplying or dividing?

131. T: So, you did division here [points to paper], and what did you do Chevron?

132. C: Multiplication.

133. T: So, if Lina did division, why did you do multiplication. We talked about why division was the best strategy, why did you do multiplication?

134. C: Um I did multiplication, instead of division, because well I did 2000 x 300 and that’s our numbers that we had and I got 6000, so 18. Then I…

135. T: Why did you choose to multiply? We were trying to figure out how many balloons we were trying to fill with 2000 liters of water, right. Since we have 2000 liters of waters and we are trying to get water to each balloon, should we be multiplying or dividing?

136. C: Dividing.

Above in Turn 136, Chevron answers, “Dividing.” Ms. Littleton restates the contribution, “dividing.” Then she directs Chevron and Lina to look at using more viable and efficient process for solving this problem. She begins to help the students break the problem down in Turn 137.

137. T: Dividing, so let’s go back and look at our division that we did here.

138. C: What did Lina get?

139. C: 6 and 200.

140. T: What’s wrong with that answer?

141. C: Lina said that we should go with my answer because it would be more realistic.

142. T: More realistic?

143. L: I didn’t say that because I don’t think that you can do 6 and remainder 200 because 200 is bigger than 6.
144. T: It is but you are looking at what you are dividing by and you are dividing by 300, you are looking at groups of 300?

145. T: So if you are looking at groups of 300 and you are leftover with 200?

And then following that, Ms. Littleton assists the student to work through the problem and figure out how to use the information needed to solve the problem. She engages the students in joint reasoning with the question “What did Lina get?” Chevron says, “6 and 200.” Ms. Littleton questions, “What is wrong with that answer?” This requires the students to understand exactly what the questions is asking. They initially are confused with the extra 200 milliliters. Chevron advocates (GR4C) for her thinking and says that Lina thought that her answer sounded more “realistic.” Lina justifies this thinking by explaining that she did not think that having an answer of six with a remainder of 200 made sense because the 200 was bigger than the 6 (GR4D). In Turn 144, Ms. Littleton clarifies Lina’s thinking by making her knowledge of division known (GR4A) when she says, “It is [bigger than 6] but you are looking at what you are dividing by and you are dividing by 300, you are looking at groups of 300?” Ms. Littleton continues sharing her knowledge to confirm that division is the logical operation to use for this problem. She adds, “So if you are looking at groups of 300 and you are left over with 200.

This discussion continues below with Turn 146. Ms. Littleton confirms that Lina’s solution is correct and praises her thinking, “So you are absolutely right with what you did. Your answer is absolutely right.” Then she assists her in understanding why her answer was correct., “You have 200 left over at the end, can you make another group of 300 with it?” Lina replies, “No.” Ms. Littleton confirms with, “No, so it’s okay.”
Before moving to the next small group discussion, Ms. Littleton asks the students a follow up question to reinforce the correct thinking, as well as, asking students as a check for understanding (GR4A). In Turn 151 Ms. Littleton says, “Can you fill up a balloon with left over two hundred?” She also further investigates their understanding of the mathematics involved by probing, “Why is this answer right?” Lina responds, “Because um, well all of this is correct and you add it up and 6 and you have a remainder of 200. You can’t really do anything with the remainder.”

146. T: So you are absolutely right with what you did. Your answer is absolutely right.

147. T: Now we need to understand why our answer is right. So, you have 2000 liters of water and you want to see how many groups of 300 you can make with that.

148. T: You have 200 left over at the end, can you make another group of 300 with it.

149. L: No.

150. T: No, so it’s okay.

151. T: Can you fill up a balloon with left over two hundred?

152. S: [Both girls nod no]

153. T: No, can’t fill it up so it stays as your left overs.

154. T: So tell me again, why is this answer right?

155. L: Because um, well all of this is correct, and you add it up and 6 and you have a remainder of 200. You can’t really do anything with the remainder so.

Ms. Littleton transitions students to the next task. She provides students with an opportunity to think about what their next steps by asking, “What should we do now girls?” In Turn 156, Ms. Littleton moves on by asking students about their next steps. Chevron answers, “Find another way to figure it out.” That was the challenge part of the
lesson described earlier. Ms. Littleton encourages the students to get their thinking on the poster first and then work on the second way to solve the problem.

156.  T:  So what should we do now girls?

157.  A:  Find another way to figure it out.

158.  T:  If you want to start on your poster to start explaining this way first, that is fine. You can get all of your thinking down and then come back and figure it out a second way.

159.  T:  I don’t want you to forget your thinking before you get it onto paper.

160.  T:  Okay?

161.  A:  Okay.

Before moving on she checks in again by asking, “Why are we are dividing and not multiplying.” Chevron responds by saying, “You are trying to figure out how many balloons to fill?” In Turn 164, Ms. Littleton digs deeper and asks, “How many balloons that you can fill, with what?” Chevron answers, “From the water that you have.” Knowing that students struggles earlier about the decision to use multiplication or division in this problem, Ms. Littleton also asks, “Will you end up with a higher or lower number? When Chevron responded, “Lower,” Ms. Littleton praised her and moved on.

162.  T:  Tell me again why we are dividing and not multiplying?

163.  A:  You are trying to figure out how many balloons to fill?

164.  T:  How many balloons that you can fill, with what?

165.  A:  From the water that you have.

166.  T:  Awesome.

167.  T:  Will you end up with a higher or lower number?
The exchange with Chevron was a good example of noticing when a student is off track and assisting in getting them back on track. Helping students navigate through some clouded thinking toward greater clarity is a very challenging aspect of navigating mathematical discourse. Ms. Littleton reflects about this during the individual interview. She said,

Peeling back the layers to identify the gaps. Like I feel like I’m strongest having a student's question what they are doing, helping them try to explain what they're doing. But for those students who aren’t coming in at a fourth-grade level, and peel back where is this gap because it's going to affect everything that we're trying to learn at this point.  (Focus Interview, p.3).

It is evident in the previous dialogue with Chevron above, that she needed to step back to understand why division was the more appropriate operation to use when figuring out how many balloons could be filled. Ms. Littleton successfully achieved this goal. Engaging in this group conversation also reinforced the success of the division strategy used by Lina.

During her interview Ms. Littleton was asked to reflect on how moments where students go wrong and how this can assist everyone in the learning process. She responded,

I would say it happens, not all the time but it definitely happens in the classroom. And when it does I'll bring the whole class back just like analyze why are we thinking this way and is it just the right way to be thinking for this problem, for the skill. So, either you do it in the group or in the whole group. And then, have the students share out their thinking about it, so they can analyze it together and find where the mistakes are. Because they still think that piece is important, like it's okay, we made a mistake. We just seem to understand why it's a mistake (Focus Interview, p.5).
The discussions and negotiations that students have with one another are also very complex. Students must be able to discuss when their answers are not the same as a partner. A certain atmosphere that allows students to have developed mathematical power and authority must exist in classrooms for these types of conversations to occur. An additional discussion about mathematical authority will be included later in this document.

**Summary**

The discourse in Ms. Littleton’s classroom includes all four elements of the ground rule Knowledge Is Made Public (GR4) over the course of the seven classroom events. The mathematical conversations within Event 3 and 4 are examples of the ways Ms. Littleton encouraged students to *publicly share their knowledge of the concepts and strategies they designed while together in mathematics (GR4)*. Ms. Littleton also utilized problem solving to involve students in the studying and discussing of mathematical concepts in the fourth-grade curriculum. She expects students to be part of the conversation either while working in small groups or sharing during whole class discussions. As a result, her students shared their knowledge freely and openly.

As evident in the data in Table 14, students were consistently invited share their *knowledge publicly (GR4A)*. Ms. Littleton listened to the contributions to be sure that students were sharing reasonable and accurate information. She *restated their thinking throughout (GR4C)*. She reinforced the ideas students brought to light, and confirmed accurate thinking. When a student was not quite able to articulate their reasoning, Ms. Littleton supported them by *restating their contribution* so that others could understand,
and to draw out the knowledge that she wanted them to take from it (GR4A,C). Ms. Littleton was selective about the ideas that she followed up on. She did not ask every student to share their ideas about the solutions they developed.

Students were also given opportunities to offer their knowledge to other students to help them build on their understanding (GR4A). They consistently shared their knowledge (GR4A) over the course of the seven events as evident in Table 1. Students shared verbally and in writing using pictures, numbers and words (GR4B). In this lesson students wrote their solutions on the whiteboard and shared the diagram that they used to help them with equivalence (GR4B). Additionally, when a student asked Ms. Littleton if her thinking was accurate, she invited students to help by explaining their thinking about the same problem.

Both Ms. Littleton and her students listened critically to students as they share their knowledge with one another. They do not simply accept all contributions. If someone is not thinking accurately then Ms. Littleton and her students speak up to discuss these discrepancies.

The next section explains how Ms. Littleton and her students used the ground rule: Atmosphere Of Trust Is Present (GR3) during her discourse practices. The discourse practices used by Ms. Littleton and her students utilized the elements of this strategic discourse. It was the third most commonly used ground rule in the study.

**Atmosphere of Trust Is Present (GR3)**

This section includes an examination of the discourse practices or ground rules used by Ms. Littleton and her students during conversations in their fourth-grade classroom that was part of Event 4 (Classroom Observation, June, 9). The elements
within the Ground rule Atmosphere of Trust Is Present occur between and among students and their teacher as they solve problems together in mathematics class.

Over the course of the seven events, Ms. Littleton and her students utilized all five of the elements within this ground rule. She utilized the element *engage in casual interchanges demonstrating equalized relationships (GR3A), praise and encouragement (GR3B), questions were asked openly and freely (GR3C), all were valued and capable members (GR3D) and decision-making responsibility is shared (GR3E).*

**Classroom Example 1-Event 4**

The lesson includes students presenting their work as the culminating activity from the capacity problems. As part of the task, students were asked to represent their data in a presentation format to present to their classmates. Each partner team created a poster to spotlight their thinking. Four of the partner teams shared during this observation. The essential question and enduring understandings guided Ms. Littleton’s path to student understanding. These are the questions guided this lesson during Event 4 (Classroom Observation, June, 9).

**Essential Question:** Why does "what" we measure influence "how" we measure?

**Enduring Understanding:** The same measurement can be represented in different units. The larger the unit the smaller the number you obtain as you measure. Units can be converted in order add, subtract, and compare measurements more easily.

Prior to the lesson, students had discussed expectations for what to include on the poster. Ms. Littleton *invited students to contribute (GR1A) ideas about what should be included. While presenting, all were expected read their question, explain their thinking,
and receive questions and comments. They were also expected to include a title; written question; and a numerical representation of their process for solving their problem, and their answer written in a complete sentence. Some groups included a second way to solve the problem, that was an optional challenge for those who could complete it.

This investigation focuses on the classroom dialogue occurring in Event 4 (Classroom Observation, June, 3, 13-17). The lesson began with students gathered in the front of the room with students seated on the floor waiting for student presentations. Student pairs were given autonomy to present their poster and to engage with the class by inviting peers to offer comments and ask questions on their own. This provided students with an opportunity to engage in shared decision making (GR3E). This also demonstrated that students were valuable members of the classroom community, capable of making decisions about how to solve the problem and plan their own presentations (GR3D,E). Students and their teacher exhibited an equalized relationship while engaging with one another to provide feedback and ask questions to exchange ideas during presentations (GR3A). Ms. Littleton was part of the audience and allowed students to have authority through managing control of the exchanges. Students spoke about their solutions and then called on students to ask questions or comment about their work. This lesson also gave authority to the audience members because they were responsible for offering a comment and asking two overall questions. This partnership resulted in successful interactions among members of the community.

Students were expected to be respectful to their classmates and demonstrated their ability to do this by listening and not talking while the students were presenting. When asked why listening was important, Ms. Littleton said, “Because they're still learning
from other students as they're hearing them discuss.” (Individual Interview, p. 6). Both the presenters and the students in the audience were a capable and valuable members of the lesson (GR3D). They were charged with actively listening to the presentations and then offering questions and providing feedback to the presenters. This was an example of a pre-planned participation structure where students were provided with a clear format for joining the discussion.

During the interview, when asked to reflect on the structured discussions that she uses that require students to offer two questions and a comment. She said, “They get to prepare for it [the interaction with the rest of the class] in addition to just what they say, what they would always know that might be a question I should comment on.” (Teacher Focus Interview, p.3)

Getting all students to participate at the same level is difficult. Using a structure around how students are expected to engage provides an opportunity for more students to participate. This is something that Ms. Littleton referenced during her interview. As noted earlier, when asked what the issues were in getting students to talk she said, “I can’t necessarily get every kid to talk. They are more of a listener which I feel is important. I think it’s also a confidence piece for them” (Focus Interview, p. 5).

Ms. Littleton also mentioned this issue during her individual interview.

They have the idea that if they share something that is wrong, they think it is the end of the world. To help support them. She also explained “I really have those students who really do share, I’ll say well I started this way and I always point out to let students know it is okay to share that you started out the wrong way or maybe you got stuck and you didn’t’ find the right answer but shared their thinking so that we can help them get to the right spot. (Individual Interview, p.7).

Additionally, Evelyn mentioned not liking to speak in front of the class during hwe individual interview, Evelyn said, “I don’t like sharing in front of the class because I
don’t like to talk in front of people that much” (Individual Interview, p.2). However, when asked if it was ever okay to share Evelyn said, “If I know my answer is right, it kind of helps me a little bit more.” Evelyn also shared how she felt when her answer was not right she said, “kind of weird” (Individual Interview, p.2). In addition, when asked if she preferred talking out loud about math, or writing it down? She commented, “Writing it down…I am not a good talker” (Individual Interview, p.4).

Fifteen of the 21 students found that it was helpful to them to share their thinking, according to the results of the student survey/questionnaire, see Table 16. Students also reported that sharing their thinking helped them to figure out if they were on the right track. However, a little less than half of the students indicated that it was helpful to them when asked to explain their thinking. Three students indicated being uncomfortable sharing their thinking with the class. Furthermore, seven students found explaining their mathematical thinking challenging. In fact, six students reported having trouble explaining how they solved a problem aloud in math. These results were surprising given the observed interactions during classroom events. Most surprising was that 11 students did not like math.

During the focus interview, teachers also reflected on the difficulties that some students have with explaining their thinking. Ms. Littleton reflected about the ways even skilled students struggled, she said,

Yeah. I definitely see that. And those were also the kids that if I ask them to explain something, they struggle with it. They can do it. They can show me on paper, but they can’t show me the words or tell me the words.” (Focus Interview, p.8)

Table 16: Student Survey/Questionnaire Results Case B

<table>
<thead>
<tr>
<th>STUDENT SURVEY/QUESTIONNAIRE PROTOCOL</th>
</tr>
</thead>
<tbody>
<tr>
<td>Littleton’s Students</td>
</tr>
</tbody>
</table>

381
The classroom example supporting the use of Atmosphere of Trust Is Present (GR3) begins with following discussion between Chevron and Lina during their presentation. All comments offered by students to the presenters were positive, usually including praise regarding the work they had done, which supports the existence of a supportive environment in this classroom. Most questions were thoughtful and specific to
each group’s presentation. During the presentations, Ms. Littleton joins the conversation, at times to ask a question or to clarify information or understanding.

The first group, Chevron and Lina begin by reading their question, “The package of Mel’s water balloons says that it holds 300 milliliters of water. How many balloons can he fill if he has two liters of water?” Lina decides to start with, “We figured the 2,000 came from the 2 liters which equals 2,000 millimeters then it said that, the packet said that, it can hold 300 millimeters of water.” Chevron adds, “We used subtraction.” They go on to explain their strategies in words while referring to their numeral representation as they outline their solution. They described taking away 300 [the amount of water that each balloon held] starting from 2000 and ending with 200 milliliters. Then the students in the audience offered praise and encouragement by clapping for the group after the presentation was complete (GR3B).

\[
\begin{array}{c}
300 & 2000 \\
\underline{-300} & 1 \\
1700 \\
\underline{-300} \\
1400 \\
\underline{-300} \\
1100 \\
\underline{-300} \\
800 \\
\underline{-300} \\
500 \\
\end{array}
\]
1. C: The package of Mel’s water balloons says that it holds 300 milliliters of water. How many balloons can he fill if he has two liters of water?

2. L: Um, then we did, so we figured the 2,000 came from the 2 liters which equals 2,000 millimeters then it said that the packet said that it can hold 300 millimeters of water.

3. C: So we used subtraction.

4. C: Then I did 2000-300 =1700 then we did minus 300 then I got and then we got 1, 400 and then we minused 300 and then we got 1100 and we minused it by 300 and we got 800 then we minused it by 300 and we got 500, then we minused it by 300 again and got 200 as our remainder 200.

5. L: Then we added the numbers on the right and we got six. So, we got our answer as six.

6. C: We did this because it was the easiest way because we can you can do subtraction multiplication and division. We also had to find out how many balloons he could have with the water.

7. L: And that got our answer 6 remainder 200.


9. S: [Students clapped]

10. T: What was the question asking you?

11. C: How many balloons can he fill if he has 2 liters of water? GR4A

12. T: So what is the answer to that question?


14. T: How many balloons can he fill?

15. C: Six.

16. T: Go ahead and write your answer in a complete sentence.

17. T: Does anyone have questions or comments?
18. L: Elenor?

19. E: Did you say millimeters or milliliters?

20. L: Millimeters.


22. C: Edna?

23. E: What was your second way?

24. C: We did not have enough time to do it a second way.

25. C: Chetan?

26. C: I think you did a really, really good job you showed with colors and details.

In Turn 10, Ms. Littleton asked, “What was the question asking you?” Chevron answered, “How many balloons can he fill if he has 2 liters of water?” Ms. Littleton then asked, “So what is the answer to that question?” After Chevron said, “Six remainder 200,” she pressed on. Both girls then answered, “Six.” Ms. Littleton assists the students to think about the reasonableness of their answer and to come to the understanding that Mel could only fill six even though there was 200 milliliters remaining.

Ms. Littleton steps in here to make help the students to articulate their solutions accurately. When they provided the answer 6 and 200, this could have been interpreted as 206 if not clarified. This complex process requires a great deal of skill on the part of the facilitator. Ms. Littleton successfully handles this situation by questioning the girls to pulls out key information to clarify their answer. But it is not always this easy. Ms. Littleton reflected about how this can be a struggle due to her lack of experience knowing where some of the gaps may be when it is clear that they do not understand a process or concept. She said,
Peeling back the layers to identify the gaps. Like I feel like I’m strongest having a student’s question what they are doing, helping them try to explain what they’re doing. But for those students who aren’t coming in at a fourth-grade level, and peel back where is this gap because it’s going to affect everything that we're trying to learn at this point. (Focus Interview, p.3)

The dialogue continues while the Chevron and Lina completed their answer, Ms. Littleton steps in to support this group with receiving questions and comments from the other students. Ms. Littleton asked, "Does anyone have any questions or comments?" She is communicating that all are capable and valued in this interchange (GR3D). Thinking critically about the group’s presentation, Elenor openly and freely questioned the accuracy of the unit they used to label their answer (GR3C). She asked, “Did you say millimeters or milliliters?” When Lina replied, “Millimeters,” Ms. Littleton stepped in to correct her by saying, “Milliliters.”

Edna was chosen to ask the second question. In turn, she asked if the group had come up with a second way to solve the problem. The group told them that they did not have enough time to complete the second way. Chetan was selected to offer a comment. He kindly offered, “I think you did a really, really good job. You showed with colors and details.”

Next, Evelyn, Ty and Winnie’s group presented their ideas. Winnie presented the problem and the first strategy the group used to solve the problem. Winnie read, “Beverly has 1.5 liters to fill 6 water balloons and each balloon holds .35 liters of water. She explained the strategies with numbers (GR4B) by providing a description of the steps they used to describe the guess and check strategy with estimation. First, they added .35 six times, four times and three times to determine the closest number of balloons that they
can fill with 1.5 liters. Then Evelyn shared how they used multiplication and multiplied .35 four times once they knew that was the closest.

\[
\begin{array}{ccc}
.35 & .35 & .35 \\
.35 & .35 & .35 \\
.35 & .35 & .35 \\
.35 & .35 & 1.05 \\
.35 & 1.40 & \text{too low} \\
.35 & \text{closest} \\
2.10 & \text{too high}
\end{array}
\]

Winnie began reading the problem from the poster. She said, “So we did question 2 and the information is Beverly has to 1.5 liters to fill 6 water balloons and each balloon holds .35 liters of water.” She continued by explaining the steps in the solution that her group developed. They had tried multiplying .35 times three, four, and six. Winnie said, “For our work we did .35 six times and we got 2.10 and it was above what we could do [too much]. Then we tried adding .35 four times and there we got 1.40 liter and we thought that was the closest that we could get. Then the last one we filled 3 balloons and that was way too low.”

Evelyn took over and said, “The second way we did 35 hundredths times four and we got and we got 1 and 40 hundredths.”

Next, Ty shared how they also used division to find the answer a third way. She did get a bit confused while reading from the chart. Initially she said that she divided 1.40
by .35 but realized that the original problem, not their estimate, required them to divide
1.50 by .35. She explained that their answer was four.

Winnie restated this information and said, “Our answer for both of them was
Beverly could fill four out of six balloons. “All three students referred to the
representation of their thinking, illustrated on their poster, during their presentation. This
was followed by T’s explanation, “Then we got .35 hundredths divided by 1.40 no wait
but you could do .35 divided by 1.50 that is easier and we got 4.” Winnie completed the
presentation by offering, “Our answer for both of them was Beverly could fill 4 out of the
six balloons.”

The presenters decided who would ask questions or offer comments (GR3E).
Again questions were asked openly and freely by the audience (GR3C). All students had
the opportunity to contribute as capable and valued members of the classroom
community (GR3D). Zandra went first, she asked, “Did you get confused at any of the
parts?” Winnie explained that she had trouble remembering how they came up with the
four when Ms. Littleton came over to discuss their solution. Then in Turn 31, Ms.
Littleton asked, “How did you come up with the four?” Winnie answered, “We divided
1.50 divided by .35 and that’s how we got the four.”

They based this calculation on the estimate that they determined earlier.

27. W: Zandra?

28. Z: Did you get confused at any of the parts?

29. W: Yes, when Ms. Littleton came over and asked how did you come up with
the four and I didn’t even know how we came up with the four.

30. T: How did you come up with the four?

31. W: We divided 1.50 divided by .35 and that’s how we got the four?
Ms. Powers did not step in to ask any clarifying questions that probably could have been more helpful in surfacing that the students were working using an estimate and not an exact calculation. Students listening did not ask any questions either.

Evelyn called on the second student to contribute. Elenor provided feedback and said, “You guys seem to know what you are doing and I trust you.” Then Winnie chose Arthur. He asked, “Which way did they like the best?” Both Ty and Winnie responded by saying that they liked the addition best. Evelyn did not share. Winnie wrapped up the presentation and the next group came up to present.

32. E: [Called on Elenor to ask a question or comment]
33. E: You guys seem to know what you are doing and I trust you.
34. W: Arthur?
35. A: Which way did you like best?
36. T: I like the addition.
37. W: I like the addition it’s easier to me.
38. W: Thank you that was three.

[Arthur and Daniel’s group come up to present]

39. D: Now presenting...the scroll
40. A: So Charlie filled all of his balloons with 2 quarts of water. Warren filled each of his 6 balloons with 1 and ½ cups of water. Whose balloons contain the most water?

\[
\begin{align*}
1 \frac{1}{2} \times 6 &= 9 \\
\frac{1}{2} \times 6 &= 3 \\
1 \times 6 &= 6
\end{align*}
\]
Arthur and Daniel's group presented their problem. They added a theatrical drumroll accompanied by a rolled-up scroll as a prop to their presentation. The decision to include some originality into the presentation was acceptable (GR3E). Arthur read the problem from the scroll. He said, “Charlie filled all of his balloons with two quarts of water. Warren filled each of his six balloons with one and one-half cups of water. Whose balloons contain the most water?” They referred to the poster that they created when they spoke.

Daniel began by explaining that they multiplied one and one half by six. Arthur broke down the steps by explaining that they multiplied one half times six equaling 3 and one times 6 equaling six. He also said that they added them to get 9.

41. D: So, the first way we did it, we did one and \( \frac{1}{2} \) times 6.

42. A: First we did \( \frac{1}{2} \) times 6=3 and 1 times 6\( \times \) 6=6 and then we added them up and wrote 1 and \( \frac{1}{2} \) times 6\( \times \) 6=9.

43. D: \( \frac{1}{2} \) times 6 equals 9. \( \frac{1}{2} \) times 6 equals three and 1 times 6 equals 6.

44. A: We added it together and got 9.

45. D: The second way we did it was addition.

46. A: It was 9 divided by 1 and 1/2 equals 6.

47. D: So the answer we got was Warren has more water because Charlie only has two quarts to use and Warren has one cup more water.

48. D: These are all other questions we did. [Referring to all 4 problems they completed]

49. E: What about the other ones? GR3C [Arthur shook his head, no]

50. D: Lawrence.

51. L: I think you did a nice job.
52. D: Jadiah.

53. J: You explained it so well I did not have a question.

54. T: Any more questions or comments?

55. A: Edna.

56. E: You did a nice job organizing your poster.

[Caleb and Jadiah come up to the front of the room]

57. C: Camile has 6 water balloons.

58. J: Each is filled with four fluid ounces of water.

59. C: Bibi has 5 balloons.

60. J: Each is filled with one cup of water.

61. C: Whose balloons contain the most water?

62. C: So I did 6 times 4 is 24 fluid ounces because Camile has six water balloons and each is filled with 4 ounces of water. And then I did 5 times 8 is 40 fluid ounces because Bibi has five balloons with one cup of water and 1 cup has 8 fluid ounces. And 24 is less than 40.

63. J: So Bibi’s balloons contain the most water.

64. J: I did this. [Refers to diagram of the Magic G for liquid measurement]

65. J: Kind of like Ella did it.

66. J: Like the magic G but I did not do the quarts. P is for pints and c is for cups and o is the fluid ounces.

67. J: There is 16 fluid ounces per pint cuz one cup equals 8 fluid ounces So 2 times 8 is 16 Then down here Paris has 6 water balloons and all 6 have 4 ounces in them and if you add them together that equals 24 fluid ounces.

68. J: And Bibi has five water balloons and each one filled with one cup which equals 5 cups and 8 fluid ounces is equal to one cup and 3 times 8 is 24 so that’s just like 3 water balloons and there is five. And so, 24 is not as much as 40. So, Bibi has the most value and 5 times 8 is 40.

69. T: Questions or comments?
70. A: Arthur?

71. A: You organized really neat.

72. T: Okay thank you very much Jadiah and Caleb.

In Turn 46, Daniel explains that the second way that they figured this problem out was with addition. Arthur says, “It was 9 divided by 1 and 1/2 equals 6.” He does not explain how they added to get their answer.

Then in Turn 48, Daniel continues, “So the answer we got was Warren has more water because Charlie only has two quarts to use and Warren has one cup more. Ms. Littleton did not ask any questions. Edna asked, “What about the other ones? Arthur shook his head “No.” This group had time to finish all of the questions but did not present them, but knew that they could only present one question. They opened up the discussion to the other students. Daniel started by calling on Lawrence. Lawrence commented that the group had done a nice job. Then Jadiah commented, “You explained it so well, I did not have a question.”

Then Ms. Littleton stepped in to ask in Turn 55, “Any more questions or comments?” Edna provided a comment about their poster being well organized.

Then Jadiah’s group came up to present. Before presenting, Jadiah casually asked if they could squat down and enter as if they were appearing from below. (GR3A,C,D). Ms. Littleton demonstrating an equalized relationship (GR3A) said, “Go for it!” This demonstrated that they students did have decision making responsibility, even though they needed additional acknowledgement to do so. Caleb and Jadiah read the question by alternating the parts. They read, “Camile has six water balloons each
is filled with four fluid ounces of water. Bibi has five balloons; each is filled with one cup of water. Whose balloons contain the most water?

In Turn 63, Caleb begins by explaining, referring his representation on the poster, that he multiplied six times four to get 24 fluid ounces of water for Camile. He continues by explaining that he multiplied five times eight to determined that Bibi had 40 fluid ounces of water. He finalizes by saying, “And 24 is less than 40.” Then Jadiah adds, “So Bibi’s balloons contain the most water.”

Next, Jadiah directed the groups attention to the diagram of the Magic G that he used to figure out the second way of solving the problem, (See Figure 2). He continues with his explanation of the diagram. Jadiah says, “There is 16 fluid ounces per pint cuz one cup equals 8 fluid ounces So 2 times 8 is 16. Pointing to the poster he says, “Then down here Camile has 6 water balloons and all 6 have 4 ounces in them and if you add them together that equals 24 fluid ounces.” Jadiah continues,

And Bibi has five water balloons and each one filled with one cup which equals 5 cups and 8 fluid ounces is equal to one cup and 3 times 8 is 24 so that’s just like 3 water balloons and there is five. And so, 24 is not as much as 40. So, Bibi has the most value and 5 times 8 is 40. (Event 4, p.17)

When it was time to ask questions or comments, Arthur shared, “You organized really neat.” Then Ms. Littleton thanked the boys.

The presentations in this event provide evidence of the successful ways students work as partners in learning, taking turns presenting and asking questions of one another. Students and their teacher were very appreciative of one another for sharing their thinking and engaging with one another. Ms. Littleton thanked the students for their participation, did she praise. Students were observed to be enthusiastic about
presenting, respectful of one another, and willing to engage in giving and receiving comments and questions.

Results from the survey/questionnaire also indicate that students identified talking and explaining their mathematical ideas with others as beneficial to them, see Table 16. According to the Student Questionnaire Results, most students in this class like to learn from others. More than three fourths of the class indicated solving problems as easier when they worked with others. All three of the students interviewed confirmed this statement. Eighteen students including all three individuals involved in the interviews agreed that talking with others helped them to figure out that they did the problem correctly. Many students also agreed that comparing answers with other students helps them to see if their own answers are correct.

In his individual interview Arthur was asked if he liked talking about math during his individual interview, Arthur said, “It’s actually really fun to give all of the solutions.” (Individual Interview, p. 2). He also said that he would rather share and talk with other students about the problems rather than do them on his own. In his individual interview Arthur said, “Well it’s really helpful when someone does it really long so they can explain all, so they can basically say like I got this answer by doing it a certain way and they would explain it their way and it would be really long so I can understand it better.” (Individual Interview, p.2).

However, Arthur and seven others indicated that they did not feel that they understood math more when taking with other students, see Table 16. His high level of confidence about his mathematical ability was noted. It is believed that he was
trying to say that he felt that it was more helpful for other students listening to his ideas than it was for him to listen to their ideas.

Similarly, Chetan and nine others students did not feel that it was helpful for them when they explained their thinking. Furthermore, six students including Evelyn and Chetan prefer writing their answers to talking about problems. Yet in the interview Chetan said, “When I write it down [answer] I don’t really feel like I accomplished anything, so I kind of have to say it to someone, or to a friend next to me, so then I can understand it more and they can help me a little bit if I got it wrong” (Individual Interview, p. 1)

**Classroom Example 2-Event 5**

The second example of *Atmosphere Of Trust Is Present (GR3)* comes from the mathematical discourse that was part of Event 5 (Classroom Observation, June 10). During this lesson, Ms. Littleton and her students utilized discourse strategies that contributed to an strengthening an *Atmosphere Of Trust (GR3)* that existed in the classroom environment as they solved problems together in the classroom. This occurred while students received support from Ms. Littleton while sharing their knowledge in small groups and during full class discussions.

The dialogue below involves the conversations between Ms. Littleton and her students after she gave them the task, “Each group will receive a yard of ribbon. Use the ribbon to tie a bow around a pencil” (Event 5, p. 17). The lesson began with students sharing the length of each of their bows, by posting each on the whiteboard. This activity was followed by a task requiring students to wrap a box (Event 5, p. 17-20). Each group received a different size box to wrap. Groups were responsible for deciding how to
determine how many inches of ribbon they needed to wrap the box, including a bow. The following are the essential question and enduring understanding aligned with this lesson. Essential question: When do you need an exact measurement and when can you measure? Enduring understanding: Understanding measurement allows us to make better estimates during real life situations.

Ms. Littleton engages her students in joint reasoning as all groups share the lengths of their bows with the class. While students share, Ms. Littleton is again focused on listening to what students are saying, as she facilitates the discussion. Most of the time, students wait to be called on to offer their responses. This is very typical of all lessons observed in this study.

1. T: Each group will receive a yard of ribbon. Use the ribbon to tie a bow around a pencil.

2. T: The size of the bow and the length of the ribbon are up to you. After you make the bow and trim the ends, measure the ribbon you used. On the board record the length of yarn. Students began the task.

3. T: Alright at this time, if you can hear the sound of my voice, clap once. If you can hear the sound of my voice, clap twice. If you can hear the sound of my voice, clap three times.

4. T: After you have your measurement written, please have a seat.

5. T: Those of you that were outside, have a seat on the rug, waits 30 seconds for them to get ready.

6. T: At the count of 5 everyone should be seated, 5, 4, 3,2,1. GR3A

7. T: Group #1 tell me the measurement of your bow, nice and loud, Walter.

8. W: 22 inches.

9. T: Group #2.

10. J: We got, one foot nine inches.
11. T: Awesome, how many inches would that be?
13. T: Group #3.
14. E: We got 23 and 1/2 inches.
15. T: Four.
16. T: We got 2 feet and 2 and ½ inches.
17. T: How many inches would that be?
18. A: 26 and ½ inches.
19. T: Good, group #5.
20. S: 1 foot and 5 and ½ inches.
22. S: 17 and ½ inches.
23. T: Group #7
24. S: 10 and ½ inches.

26. T: Why did we end up, just a quick little lesson, these tape measures, I know that we have not used them that often, but they are not toys, you should not need them at this time, so I should not hear any clicking.

27. T: Why do you think we have all sorts of measurements for the bow?
28. C: Because you could have made a different sized bow.
29. T: Yeah, exactly, Caleb?
30. C: You could have had a different sized pencil.
31. T: A different sized pencil too, Elenor?
32. E: You cut your string longer or shorter than someone else’s.
33. T: Good.

34. T: Okay we are going to keep moving. Today each group will get a box to wrap.

35. T: So maybe you have not wrapped a box before but if you look at the screen (whiteboard projection of the task), you will wrap the box by having the string go around the width and also around the length, let me just cut this for you. (She demonstrated cutting the ribbon and how one would wrap it around a wrapped present).

36. T: You are going to first put it around the width, everyone see that I have it around the width.

37. T: Then you are first going, you are going to cross the string until it locks together like this, okay. Now you are ready for it to go around the length of the box.

38. T: It looks like I did not cut my string long enough because it looks like I did not cut enough to make a bow. I did a bad estimate here, you want to make sure you have enough to tie a bow after you wrap the ribbon around the width and the length.

39. T: Before I give you ribbon, you need to estimate, (clicked the presentation slide with steps of the tasks written out for students) you’re going to plan how to solve this problem with your group, you are going to estimate the length of the ribbon that you need. You are going to write about how you got your answer with your group. Once you have your plan and you have written out your plan, then you can measure and cut you ribbon to test your answer.

40. T: Mrs. Flanagan and I and I will help you cut the ribbon but we need to see you plan first and how you made your estimate. GR6G

Ms. Littleton initiates the activity by saying, “Group one tell me the measurement of your bow.” Walter responds, “22 inches.” The task is designed to encourage multiple solutions because each group creates their own bow. This is demonstrated as other groups share their measurements and multiple answers are accepted by the teacher. When a student provides a length that includes feet, she requires them to convert the answer. This is evident in Turn 10, when Ty provided, “We got 2 feet and 2 and a half inches.” Ms.
Littleton followed up by asking, “How many inches would that be?” Arthur, another group member jumped in and responded, “26 and one-half inches.”

In Turn 26, Ms. Littleton directs students’ thinking when she asks, “Why do you think we have all sorts of measurements for the bow?” Chevron answers, “Because you could have made different sized bows.” Caleb tried to generalize that the size of the curl on the bow might be larger if wrapped around a wider pencil. He said, “You could have had a different sized pencil.” Elenor add, “You could have had different sized pencils.” Student thinking is visible in their talk throughout this exchange. Ms. Littleton praises all groups for their input (GR3B). Then Ms. Littleton transitions students to the next part of the lesson by explaining the remaining steps involved in completing the task. She tells students in Turn 34, “Today each group will get a box to wrap.” She continues, “Your group’s job is to figure out how many inches of ribbon you need to wrap your box, including the bow.” She cuts the string to use for a demonstration. In the next few turns, Ms. Littleton provides additional guidance, stating her knowledge as she guides students with a demonstration about how a box is wrapped with a ribbon.

Ms. Littleton provides an opportunity for students to share in the decision-making responsibility by encouraging them to decide on the size of the ribbon needed, including the bow (GR3E). In Turn 39 below, she lets students know that they will be planning out home much ribbon then need for a bow that they design. She walks them through the process. As she is speaking, she realizes that she has cut the string too short and says,

It looks like I did not cut my string long enough because it looks like I did not cut enough to make a bow. I did a bad estimate here; you want to make sure you have enough to tie a bow after you wrap the ribbon around the width and the length. (Event 5, p.18)
In this instance, Ms. Littleton shares her mistake by casually acknowledges that she cut the ribbon too short to make a proper bow. She does not make a big deal of it, and neither do her students. In this instance, she allows herself to publicly admit to her mistake, acknowledging she does not know and do everything expertly. This allows a shift in her authority, allowing for more equalized roles in this classroom (GR3A). Without missing a beat, Ms. Littleton continues with the directions.

The expectation that has been established is that students work with one another by sharing and listening to their knowledge about the math involved in the task, as well as, their ideas about the planning of a solution. According to the Student Survey/Questionnaire results, 18 students agreed that listening to others about how they solved a problem was helpful to them (Student Questionnaire Results). All students interviewed, except for Evelyn and three other students agreed. However, when asked to reflect on why it is important to listen to what other students are saying when they share during the individual interview, Evelyn said, “it is important because that is how you learn your things in math, and it is good to know what their answers would be” (Individual Interview, p.3).

Additionally, Arthur provided information about how listening to the thinking of others impacts his own thinking, during his interview. The question posed inquired about whether or not he thought differently about a problem after hearing somebody else’s idea. Arthur responded, “I can think differently because I did it a whole different way and they did a whole other different way, and probably haven’t did that way before, and I can learn how they did it” (Individual Interview, p.2).
Also, when Ms. Littleton was asked to reflect on why listening was important in math instruction, she shared, “Because they're still learning from other students as they're hearing them discuss. So I’m hoping that they have their ears open.” (Individual Interview, p.6)

After she finishes with the explanation, she invites all students to contribute by posing questions to gain a better understanding of the task (GR3D). Beginning in Turn 41 below, students respond by asking specifics about the remainder of the task. She asks Jostos to repeat the directions to check for understanding (GR6D). Ms. Littleton answers their questions and again offers praise for their contributions.

41. T: Any questions?

42. T: Zandra?

43. Z: Do we use the same ribbon?

44. T: Good question.

45. T: You are not using the same ribbon you already used that was a test bow.

46. T: Any other questions?

47. T: Jostos can you repeat the directions for me?

48. J: You are going to talk about a plan about how long you ribbon is going to be. Then you are going to estimate it, write about how you are going to do it and then tell a teacher, then you measure the string and cut it.

49. T: Awesome.

50. T: Those steps are also on the paper I’m just handing out. So be sure when you get to the writing part you are recording your writing, each person is recording their own writing on their paper. You may use a ruler or a tape measure to help you with your estimate.
Establishing and cultivating an Atmosphere of Trust (GR3) supports students as they look critically at their own ideas and the ideas of others. The discourse in this lesson is another example of how students have learned that is appropriate to ask questions of one another and their teacher. While beginning the work of figuring out how to wrap their box, Ms. Littleton circulates the room to hand out boxes and ribbons. When she hands the ribbon to the first group, Paris *openly and casually* questions the size of the ribbon that Ms. Littleton has cut for her group (*GR3A, C*). Paris, not allowing any hierarchy of power that might exist between herself and the teacher states in a form of a question, “That does not look like 50 inches?” This interaction is a typical and expected exchange. Ms. Littleton realizes that she cut the string too short and turns to Ty, asking her to help measure a length closer to 50 inches this time. It is also acceptable in this classroom environment when Paris *casually praises* Ms. Littleton for cutting the bow closer to 50 inches. In Turn 52, Paris says, “That looks better.” Both of these instances demonstrate an equalized relationship between the student and her teacher. Paris is forthright in *openly* pointing out her teacher’s mistake and *offering praise* when it is corrected (*GR3B,C*). Ms. Littleton cultivates this sharing of authority by humbly acknowledging her mistakes.

51. T: You said 50 inches?

52. P: [Directed to the Teacher] That does not look like 50 inches?

53. T: Ty, can you measure 50 inches?

54. T: You have 36, how much more do you need?

55. P: That looks better. [string size]

56. P: [Paris takes the string from the T]
During the next part of the conversation, students begin discussing the task with one another in their small groups. Group members shared decision making and were able to suggest ideas about how to complete the task (GR3E). In Turn 57, Paris asks, “Should we start from the top?” She says, “Then we should start from the top, then do that crisscross it.” Paris wraps the box with the ribbon and then realizes they only have a small part of the ribbon left over for the bow. Then Paris says, “Uh oh, the bow is going to be so small. Madison grabs the box and manages to tie a small bow. The girls are pleased with the results. They praise their good work (GR3B). This demonstrates the highly cooperative culture and positive interactions among the groups, as they work together.

57. P: Should we start from the top?
58. T: There is nothing in the box.
59. P: Then we should start from the top, then do that crisscross it.
60. T: Oh, Ah.
61. P: So you wrap and then go under and then crisscross it.
62. P: Yeah, did I do the top or the bottom, I forgot.
63. M: The top.
64. M: Uh oh, the bow is going to be so small.
65. M: [Grabs box, ties the bow, experiences difficulty string is small] Success!
66. T: Yay!
67. M: [Singing] Boop-bit-e-boop-boop!
68. T: How did your 50 inches come out, is that your ribbon?
69. T: That is exactly how we wanted it.
70. T: So it’s just how you wanted it? [she questioned that the bow was very tiny]

71. T: Awesome, can you tell me exactly how you figured out your estimate?

72. M: We measured the box with the tape measure and then added a few inches.

73. T: So how did you measure the box?

74. P: With that. [pointing to the tape measure]

75. T: [gestures that she wrapped the tape measure around the box]

76. T: So you tied it around, you pretended that the tape measure was a ribbon?

77. T: Then we added seven inches.

78. T: Why did you add seven inches for the bow?

79. M: Because that was what we thought we needed it. [for the bow].

80. T: Because we had 43 and we thought 7 more would give us what we needed.

Ms. Littleton asking for greater understanding (GR6D) about their estimate asks, “Awesome can you tell me exactly how you figured out your estimate?” The students explain how they used the tape measure to practice wrapping the ribbon around the box. Then they explained how they added seven inches. Then in Turn 80, Ms. Littleton asks, “Why did you add seven inches for the bow?” When Madison said “Because that was what we thought it needed.” Ty added, “Because we had 43 and we thought 7 more would give us what we needed.” Students in this class respond well to talking with their teacher during mathematics.

According to Student Questionnaire Results, 19 of the students reported understanding math better when they talked with their teacher, including Anthony, Evelyn, and Chetan. Additionally, nine students agreed that they asked their teacher a lot of questions when they were learning math. This group also included Evelyn and Arthur.
On the other hand, twelve students and Chetan did not feel that they asked their teacher a lot of questions. Most of the questions students asked their teacher involved getting permission or clarifying directions. For example, in Event 1 Chetan asked, “Can I show it on the board?” (Event 1, p.1). In Event 3, Nicholas asked, “Will everyone in that entire group get one sheet of paper?” (Event 3, p.11)

The exchange with Ms. Littleton, Ty, Paris, and Madison again demonstrates the shared authority that exists among participants. When Ms. Littleton approached the group to check in on their progress, she noticed that the bow was tiny. In Turn 70, she asks, “Is that how you wanted it?” Ty responds by saying, “That is exactly how we wanted it.” Ms. Littleton accepted their thinking; she did not try to disagree or question their mathematical authority.

The dialogue continues below beginning with Turn 83. Ms. Littleton prompted students to *extend their thinking* by asking, “What if you did not have a tape measure, what would you do?” The girls all began to answer, at once. Ms. Littleton *casually* asked, “Can I have one person talk at a time?” (GR3A,C). Madison explained, “I would measure the length, and then I would do the width, and then I would do the length times two and the width times two.” Ms. Littleton probed to *extend her thinking* and asked, “Why would you do the length times two and the width times two?” Madison adds, “I would have to do that again.” “Why?” pushes Ms. Littleton. Madison says, “Because we only did the top length times two and the bottom times two and it would only be that [pointing to the top of the box and the bottom of the box] and not that [the four sides].” Madison adds, “And it would not go all around.” Ms. Littleton says, “So what it is telling you width
times two and length times two, what are you thinking about?” Madison says, “Perimeter.”

81. T: What if you did not have a tape measure, what would you do?

82. T: [All group members begins to explain…]

83. T: Can I have one person talk at a time? [smiling]

84. M: I would measure the length and then I would do the width and then I would do the length times two and the width times two.

85. T: Why would you do the length times two and the width times two?

86. M: And then I would have to do that again.

87. T: Why?

88. M: Because we only did the top length times two and the bottom times two and it would only be that [pointing to the top of the box and the bottom of the box] and not that [the four sides]

89. M: And it would not go all around.

90. T: So what it is telling you width times two and length times two, what are you thinking about?

91. M: The perimeter.

Additional questions used here to prompt further explanation would have been helpful to both check for understanding, as well as, make sure the entire group had participated in the decision making and had drawn similar conclusions (GR3E)

Summary

The mathematical conversations within Event 4 and 5, demonstrated evidence of an Atmosphere Of Trust among members of this classroom (GR3). Ms. Littleton facilitated this atmosphere by providing ample opportunities for students to talk about math with their teacher and peers. These casual conversations encouraging students to
openly engage often either in small groups or large groups (GR3A). Students ask questions, offered comments, explain solutions while interacting with peers and their teacher in very informal ways.

Also, during the class presentations in Event 4, students are given complete decision making to plan and conduct their presentations (GR3E). This included designing their solution, representing aspects of it on their poster and then choosing what to say. They also completely handled the answer and comments component. Audience members, including Ms. Littleton, were given the responsibility of asking appropriate questions and providing meaningful comments. All were treated as valued and capable members in this process (GR3D). Praise was offered by students and teachers (GR3B).

Ms. Littleton also encouraged students to develop An Atmosphere Of Trust (GR3) as evident in the design of both lessons. In Event 4, students first needed to work through the problems, share decision making responsibilities and present their solutions (GR3E). They also had to explain and justify their thinking to their peers and teacher. Students and Ms. Littleton engaged in casual interchanges to discuss their questions after students presented their poster to get clarification or push reasoning (GR3A). Some students had made mistakes and being questioning about their thinking was acceptable in this environment (GR3D).

Students also successfully provided positive feedback to their peers. They were very supportive of the work. However, the feedback was focused on the poster and the explanation and not specific to the group’s mathematical decision making. More modeling by Ms. Littleton to demonstrate more productive questions and comments would be beneficial.
In Event 5, students worked in groups of three to four students to decide how much ribbon they needed to wrap a box, including a bow. Students decided as a group how to wrap the box and then determined a length for the bow \((GR3E)\). They developed a plan together and received approval about their plan from the teacher. When Ms. Littleton circulated to investigate each group’s thinking, she engaged them in joint reasoning by asking questions to exchange ideas and check understanding. There were instances, however when students explained their answers and a follow up question seemed appropriate, yet Ms. Littleton did not ask one. The reason for this was unclear.

Students demonstrated an Atmosphere Of Trust during the discourse they exchanged in Event 5 \((GR3)\). A majority of the discussions took place during small groups interactions. These conversations were very conducive for casual interchanges demonstrating equalized relationships among the group of students \((GR3A)\). Also, it was apparent that students shared decision making responsibility as they interacted, deciding how to wrap the box and when measuring the amount of ribbon needed \((GR3E)\).

Throughout the seven events, students were willing contributors to the process listening to and sharing ideas, asking and answering questions and offering and receiving feedback. They were respectful of their teacher and peers. Overall, the positive atmosphere cultivated by Ms. Littleton welcomes students to take advantages of the many opportunities to engage in joint reasoning with others to find solutions and discuss mathematics. The overall positive attitude among students as evidenced in the interviews and questionnaires about working with others while problem-solving in mathematics supports the process used by Ms. Littleton in her fourth-grade classroom.
Everyone is Invited To Contribute (GR1)

The final most frequently used ground rule was Everyone Invited to Contribute (GR1). Ms. Littleton and her students utilized this ground rule to invite others to engage in discourse about the problems and ideas they discussed during events, see Table 14 and 15. Mrs. Littleton planned activities to include opportunities for students to talk with others about the mathematics they studied. During large and small group discussions, Mrs. Littleton used everyone is encouraged to contribute without being singled out (GR1A) on 35 occasions when asking questions directed at the class or during a small group discussion with students. Students used this element on four occasions. Mrs. Littleton used the second element students are chosen strategically by the teacher/student to contribute (GR1B) more frequently, using it 129 times. During discussions, she often called on students who had their hand raised to answer a question or directed a question at a student who was already explaining a solution or idea. Students used the element students are chosen strategically by the teacher/student to contribute (GR1B) on 21 occasions when prompted by their teacher to ask students for questions.

Classroom Examples

During the lessons, Ms. Littleton frequently invited students to participate in conversations Many examples of the first element from the ground rule Everyone Invited To Contribute (GR1) can be found throughout the study. Everyone is encouraged to contribute without being singled out (GR1A) occurred during the discussions throughout the study, even though they occurred less often. The examples of the types of questions that were used by Ms. Littleton as she utilized this element are described below. For example, during Events 1 and 2, students discussed the problems they had completed in
groups relating degrees in a circle to fractions, and measuring angles. Ms. Littleton also encouraged students to contribute without being singled out (GR1A) during the angle discussion in Event 3 while discussing liquid measurement, as well. All questions were also evident in the dialogue during Event 7 as facilitated by Ms. Littleton. At times, she repeated the same question to encourage more students to participate.

- Who would like to share their thinking? (Event 1, p. 1)
- Does anyone have any questions for Chetan? (Event 1, p. 1)
- T: Boys and girls do we have any questions or suggestions about how they are solving the problem? (Event 2, p. 5)
- Is there another unit we can find for four quarts? (Event 3, p. 7)
- What went really well [during the bow task]? (Event 7, p. 23)
- What would you do differently if you did the project again with a different size box? (Event 7, p. 24)
- What if we weren’t going to use a bow? But we needed to tie the ribbon around the width and around the length, and it needed to meet exactly where it crosses over. Would that be more difficult to figure out or less difficult to figure out? (Event 7, p. 25)

The open-ended questions directed toward inviting all students to contribute without being singled out (GR1A) allowed more opportunity for students to join the conversations. This was based on the wide variety of acceptable answers that were possible. Some of the responses are listed below:

- We started to use teamwork a lot more. (Event 7, p. 23)
- We started working together better. (Event 7, p. 23)
• Deciding who would do what and stuff. (Event 7, p. 24)

Some students commented on how they worked as a team and others commented on the actual problem, see additional comments below:

• We needed to get the ribbon down [to be able to wrap the box] so we had to like tape it down. (Event 7, p. 23)

• The measurement like the ruler, so we had to do it multiple times since we had 138 inches [referring to having to use a ruler when they needed to measure a length many inches beyond 12 inches] (Event 7, p. 23)

Ms. Littleton also invited everyone to contribute without being singled out (GRA) using other types of question. She asked, “What would you do differently if you did the project again with a different size box (Event 7, p. 25)? When Edna answered, “We would use a different size ribbon but the same thing. So, like do the same thing that was did but since the box is a little bit different size so, you have to use a different piece (Event 7 p. 25).

Ms. Littleton utilized the second element students are chosen strategically by the teacher/student to contribute (GR1B) more frequently. She used this element while following up with students about their statements or explanations. During Event 7, she asked Edna, “Can you tell us what you did?” Edna responded, “So I'd see if the box was big or small and then, I’d estimate to see if it would be more than a hundred inches or less than a hundred inches. Ms. Littleton strategically selected Elenor to ask another question to clarify her statement (GR1B). “She questioned, “So you would use a hundred inches as your benchmark?” Sometimes she started with a new question but chose a person to
start the conversation. Some examples of choosing students strategically to contribute are listed below: So first you tried three? (Event 1, p. 3)

- Why did you subtract 76? (Event 2, p. 4)
- Why did you decide to subtract it from 360? (Event 2, p. 5)
- Who can you find another unit that is equal to four quarts? Jaylissa? (Event 3, p. 9)

Although it can seem like a very traditional instructional technique to choose students to contribute (GR1B), the questions were reform-minded, and students were provided with many opportunities to contribute to the mathematical discussions which were included in all events. Ms. Littleton chose students to contribute (GR1B after a student raised their hand to answer a question she asked. This element was used by Ms. Littleton much more than everyone is encouraged to contribute without being singled out (GR1A), see Table 14. The lesson that had the most opportunities for students to contribute without being selected was in Event 7 because of the way the discussion was framed around both mathematical learning and collaborative learning experiences.

Ms. Littleton also invited students to contribute using the ‘turn and talk technique’ (p. 25). She initiated the use of this technique by inviting all to contribute without being singled out. The question was, “What if we weren’t going to use a bow? But we needed to tie the ribbon around the width and around the length, and it needed to meet exactly where it crosses over. Would that be more difficult to figure out or less difficult to figure out?” Although technique was used only once during the course of the study, students were well versed in it. As soon as she initiated the technique, students turned to a partner and shared their thinking. After students spoke for a few minutes Ms. Littleton she asked
students to raise a hand if they thought it would be less difficult or more difficult. After they showed their response, Ms. Littleton followed up with asking them to tell share why. A few students shared their thinking. Zandra said, “Because if it’s exactly, you have to get it like the exact measurements and you can’t like estimate or anything like that. So, it would be really hard to get it exactly” (p.26).

Evelyn also contributed to this discussion. She said, “Well when we had the bow, we had to add an extra, an extra number to it. And if we didn’t, we wouldn’t have to add that extra number” (p.26).

Students utilized this ground rule and elements much less frequently than their teacher, See Table 15. There were instances when they invited everyone to participate without strategically selecting students (GR3A). For example, during the bow lesson, students turned to their group members and asked questions to the entire group. One student said, “Can someone else do the wrapping because I did the bow (Event 5, p.14)?” The second student said, During the same lesson another student asked, “This is going to be seven inches all the way around this(box), okay?”

More often they chose fellow students strategically to contribute. This was most frequent during Event 4 when they used the protocol to structure the questions and comments. Presenters asked, “Does anyone have questions or comments” and then selected a student with their hand raised, to ask them a question or provide a comment (p.14). In comparison to the other lessons, the use of this second element (GR3B) occurred much more frequently, see Table 15. Moreover, when students were pointing out errors in reasoning, they selected their questions to specific students (GR3B). For example, when Paris asked her group member, “You measured the whole box?” to make sure he had
added correctly (Event 6, p. 20). This also occurred when Elenor questioned Chevron thinking when asking, “Did you say millimeters or millimeters” to point out an error (Event 4, p. 14).

**Summary**

Ms. Littleton used *Everyone Invited to Contribute (GRI)* was frequently used throughout the study to encourage a high level of participation among students. Mrs. Littleton planned activities to include opportunities for students to talk with others about the mathematics they studied.

Many examples of the first element from the ground rule *Everyone Invited To Contribute (GRI)*. *Everyone is encouraged to contribute without being singled out (GRIA)* occurred during many of the discussions throughout the study, even though they were not used very frequently. The more open-ended questions directed toward inviting all students *to contribute without being singled out (GRIA)* allowed more opportunity for students to join the conversations. This was based on the wide variety of acceptable answers that were possible. Ms. Littleton also *invited everyone to contribute without being singled out (GRIA)* using other types of question. Students were asked to share their thinking about how they solved problems. At times, she repeated the same question to encourage more students to participate in the conversations.

Ms. Littleton utilized the second *element students are chosen strategically by the teacher/student to contribute (GRIB)* more frequently. Generally she used this element while following up with students about their statements or explanations. Sometimes she started with a new question but chose a person to start the conversation
Although it can seem like a very traditional instructional technique to choose students to contribute (GR1B), the questions were reform-minded, and students were provided with many opportunities to contribute to the mathematical discussions which were included in all events.

However, as evident in Table 14, selecting students strategically to contribute (GR1B) was used by Ms. Littleton much more than everyone is encouraged to contribute without being singled out (GR1A). The lesson that had the most opportunities for students to contribute without being selected was in Event 7 because of the way the discussion was framed around both mathematical learning and collaborative learning experiences.

Ms. Littleton also invited students to contribute using the ‘turn and talk technique.’ She initiated the use of this technique by inviting all to contribute without being singled out. Although the technique was used only once during the course of the study, students were well versed in it.

Students utilized the ground rule Everyone Invited To Contribute (GR1) and the two elements within much less frequently than their teacher, See Table 15. There were instances when students invited everyone to participate without strategically selecting students (GR3A). For example, during the bow lesson, students turned to their group members and asked questions toward others in their group. One student said, “Can someone else do the wrapping because I did the bow” (Event 5, p.14)? During the same lesson another student asked, “This is going to be seven inches all the way around this(box), okay?” to let her group know what she would do next.

More often students chose fellow students strategically to contribute (GR1B). This was most frequent during Event 4 when they used the protocol to structure the
questions and comments. Presenters asked, “Does anyone have questions or comments” and then selected a student with their hand raised, to ask them a question or provide a comment (p. 14). In comparison to the other lessons, the use of this second element (GRB) occurred much more frequently, see Table 14. Moreover, when students were pointing out errors in reasoning, they selected their questions to specific students (GR3B). For example, when Paris asked her group member, “You measured the whole box?” to make sure he had added correctly (Event 6, p. 20). This also occurred when Elenor questioned Chevron thinking when asking, “Did you say millimeters or millimeters” to point out an error (Event 4, p. 14).

The following section includes an examination of the implementation of the remaining reform-based practices, as outlined in Chapter 1 of this document, i.e., Problem Posing, Active Learning with Authenticity, and Learning Through Interaction by Ms. Littleton. Ms. Littleton was observed using the reformed-based practices while engaging students in mathematical activity during seven events. Table 17 summarizes the implementation data about the practices listing the components supporting each one.

**Reform-Based Practices For Learning Mathematics With Understanding**

As the analysis below will show, Ms. Littleton, fulfilled her professional responsibility by implementing the Massachusetts State and Common Core Standards in mathematics, as dictated by her school district. Each of the reformed-based practices was embedded into the mathematics lessons and conversations she facilitated with students in a community of practice, see Table 17. Ms. Littleton also used the curriculum mapping tool Atlas to guide her unit planning and made daily decisions about how her instruction was carried out (Rubicon Atlas, 2018). The Atlas tool warehoused the scope and
sequence, content, skills, essential questions and enduring understanding developed by teachers in the district using the Common Core and Massachusetts state standards in math, English, social studies, and science (Massachusetts Department of Elementary and Secondary Education 2009; National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010). Common Assessments were also included in the Atlas tool. Teachers developed these assessments in grade level teams and entered them into the on-line planning tool curriculum map to use as a reference. The map has not yet contained lesson plans teachers used to implement the standards. Teachers in the district were aware of the Common Core Standards for School Mathematics document but have received little guidance from state and local administration about how to implement them into their mathematics instruction.

In addition to implementing the reform practices, Ms. Littleton instituted the structural changes that included providing opportunities for students to develop more autonomy and shared authority while learning mathematics. Along with the examination of the practices as implemented by Ms. Littleton are her perspectives, collected during interviews, about how she came to know and use reform-based practices. Perspectives about these methods from students will also be included.

During the individual interview, Ms. Littleton explained that her math classes and education classes had prepared her to teach mathematics. She spoke of one course in particular because it showed her how to use "more hands-on and not just book instruction all of the time" (Individual Teacher Interview, p. 1). Ms. Littleton described learning these techniques after acquiring experience teaching math, and while working with other teachers. She also felt that her math background prepared her for teaching. According to
Ms. Littleton, her pedagogical content knowledge was developed by engaging in “the questioning in those classes.” that she applied to her students in my class now, “even though that was extreme levels, it's still similar” (Individual Interview, p. 1). She added that she felt confident in her use of questioning and helping students explain what they were doing.

During the focus interview, students were asked to reflect on why Ms. Littleton asked them questions about their explanations. Arthur replied, “She might ask the question so you can understand how you are solving the problem and then you can think of other ways” (Student Focus Interview, p.1). Daniel added, “She asks questions to help people understand better.” Additionally, nearly all students reported understanding mathematics better when they talked about it with their teacher on the survey/questionnaire that they completed, see Table 16. Most students also found that listening to how other students solved problems was helpful.

Ms. Littleton also mentioned experiencing some difficulty with helping students who struggled. As, noted previously, she struggled with knowing how to "peel back” layers of content to expose the gaps students were dealing with because it effected “everything that we're trying to learn." (Individual Teacher Interview, p.1). However, she did mention that a professional development workshop offered during an in-district professional development day "opened up her eyes” to having to fill “the skills that her students might be missing” (Individual Teacher Interview, p. 3).

When asked how long she was using reformed-based mathematics practices, she asked, "What is reformed-based methodology?" After clarifying that reformed-based methodology included using methods to engage students more in learning, involving
problem-solving discussion, including real-life problems, Ms. Littleton was able to share her thinking about the ways she utilized the practices in her classroom. She explained how her instruction differed from traditional instruction. She described how she implemented the reformed-based methods, "It's more modeling, it's having kids discuss, having the kids show by drawing or using manipulatives" (Individual Interview, p.2). Ms. Littleton also reflected about her own readiness for using these methods in her math instruction. She said, "I mean I did not have a whole bag of tricks written like that. This definitely developed after I graduated, but at least I had the knowledge that I wanted to incorporate those things.” She continued, “I had a few that I started with and then build on it through experience.”

Ms. Littleton also spoke about engaging students in mathematical discourse. She acknowledged the complexities of mathematical conversations and the need to be able to follow students as they lead the conversation in new directions. Ms. Littleton also reflected about her use of questioning during these conversations. She described probing students’ thinking and reacting in-the-moment to help them think more clearly during mathematical conversations. She said, "I teach that way because I feel like it's the right way to do it, but, no one told me that that's what I needed to do" (Individual Interview, p. 9).

Table 17: Implementation of Reform Practices For Teaching Mathematics

<table>
<thead>
<tr>
<th>OBSERVATION PROTOCOL: REFORM-BASED METHODS</th>
</tr>
</thead>
<tbody>
<tr>
<td>LITTLETON</td>
</tr>
<tr>
<td>Problem Posing</td>
</tr>
<tr>
<td>Well Designed Problem*/Task**</td>
</tr>
</tbody>
</table>

419
<table>
<thead>
<tr>
<th>Enriches the Concept/Skill</th>
<th>*</th>
<th>*</th>
<th>*</th>
<th>*</th>
<th>*</th>
<th>*</th>
<th>*</th>
</tr>
</thead>
<tbody>
<tr>
<td>Provides Structure for Discussion</td>
<td>*</td>
<td>*</td>
<td>*</td>
<td>*</td>
<td>*</td>
<td>*</td>
<td>*</td>
</tr>
<tr>
<td><strong>Active Learning with Authenticity</strong></td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>6</td>
<td>7</td>
</tr>
<tr>
<td>Engages in Learning</td>
<td>*</td>
<td>*</td>
<td>*</td>
<td>*</td>
<td>*</td>
<td>*</td>
<td>*</td>
</tr>
<tr>
<td>Real Life Connections</td>
<td>*</td>
<td>*</td>
<td>*</td>
<td>*</td>
<td>*</td>
<td>*</td>
<td>*</td>
</tr>
<tr>
<td>Honors Mathematics As A Discipline</td>
<td>*</td>
<td>*</td>
<td>*</td>
<td>*</td>
<td>*</td>
<td>*</td>
<td>*</td>
</tr>
<tr>
<td><strong>Learning Through Interaction</strong></td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>6</td>
<td>7</td>
</tr>
<tr>
<td>Learning is Socially Constructed</td>
<td>*</td>
<td>*</td>
<td>*</td>
<td>*</td>
<td>*</td>
<td>*</td>
<td>*</td>
</tr>
<tr>
<td>Contributes to Learning Of Others</td>
<td>*</td>
<td>*</td>
<td>*</td>
<td>*</td>
<td>*</td>
<td>*</td>
<td>*</td>
</tr>
</tbody>
</table>

* Indicates occurrence of the component within Reform-Based Practice within each lesson

**Problem Posing**

Problem Posing is the second reformed based instructional practice described in Chapter 1 of this document. Ms. Littleton used *problem posing* to engage students in mathematics using conversations to explore the mathematics that she was teaching to her fourth graders. The components of Problem Posing included a *well-defined problem or task*, *enriches concepts and skills* and *provides a structure for discussion*. The components of Problem Posing were implemented during all of the seven events in this study, as noted in Table 17. As noted below, Ms. Littleton engaged using well-defined
problems during in events one and two. She utilized well-defined tasks in the events three through seven.

For this study, a well-defined task is a project-based problem that requires students to connect their learning to a real-life situation. A task can be complex and usually completed over the span of one or two class periods. Similarly, a well-defined problem challenged students to think beyond the skill. A well-defined problem also presented an opportunity for students to solve problems in more than one way. Both problems and tasks provided a level of complexity that allowed for rich discussions among participants. Each problem and task helped students convey their thinking to others and to exercise their knowledge of strategies used to find solutions. Additionally, the well-defined tasks Ms. Littleton used also required students to apply the skills learned earlier in the year. Samples of these tasks can be found later in this document (See Appendix K).

**Well Defined Problem**

For example, during Event 1, Mrs. Littleton assigned problems for students to complete in small groups. The problem completed required students to make the connections between angle measurements and fractions. For example, one of the questions included, “How many right angles would be equal to a full circle” (Event 1, p.1). Following the independent problem-solving work, Ms. Littleton reconvened the group to facilitate a conversation about their solutions. She asked, “Who would like to share out?” This question sparked several students to share how they determined their answer with little facilitation needed on the part of Ms. Littleton. The process of problem-
solving followed by a conversation was repeated again and again until all problems were completed.

The structure of the lesson was the same during Event 2; only students were given problems requiring them to solve for the missing angle (Event 2, p.4). Students needed to use the information they knew about the relationships among the angles to determine the value of the missing angle.

**Well Defined Task**

During Event 3, students completed a task that required them to answer one complex question that required them to identify the amount of liquid inside of a water balloon (pp. 11-13). All groups were assigned a different question, some more difficult than others. Students were expected to create a poster showing two methods used to solve the problem. They were advised to provide the details needed so that the description showed others how to solve the question. Tasks selected by Ms. Littleton were linked to the district curriculum (Rubicon Atlas, 2018). The standard corresponding to this task is referenced in the Atlas Curriculum Map (Grade 4). Below is a sample this information linking the assignment to the Massachusetts State Framework.

**Measurement/Data:** Solve problems involving measurement and conversion of measurements form a larger unit to a smaller unit.

Use the four operations to solve word problems involving distances, intervals of time, liquid volumes, masses of objects, and money, including problems involving simple fractions or decimals, and problems that require expressing measurements given in a larger unit in terms of a smaller unit. Represent measurement quantities using diagrams such as number line diagrams that feature a measurement scale. (Rubicon Atlas, 2018)

The task in this lesson required students to multiply and divide decimals and to understand the relationship of the size of the unit when manipulating these numbers.
During Event 5, students began the two-day task estimating the number of inches they needed to tie a bow for a present (See, pp. 17-19). After working in small groups, they joined their teacher for a discussion about the task. They discussed the different bows made and the size and lengths of each. During Event 6, students used this knowledge to determine how many inches they needed to wrap the entire box, including the bow (See pp. 20-22). This was followed by a conversation about the task which took place during Event 7 (See pp. 23-27). Solving problem-solving tasks required students to brainstorm strategies, negotiate answers and present solutions.

**Enriches Concepts and Skills**

The second component within Problem Posing is *enriched concepts and skills*. Problem Posing stimulated students to think beyond the grade level curriculum. As evident in the classroom observations, the content and skills needed to complete the problems and tasks were not introduced during the events. The problems and tasks were assigned to extend the students prior learning and required them to apply the content and skills in new and different ways. These tasks allowed students to shift their study away from practicing algorithms and calculations and toward reasoning about situations and applying methods they had learned earlier.

Problem Posing allowed Ms. Littleton to stimulate students learning by to shifting her instruction away from practicing how to find solutions toward reasoning about strategies and applying skills to new situations. All problems chosen by the teacher are designed to assist in meeting the standards, goals, and objectives within their grade level curriculum as indicated in curriculum maps (*Atlas Rubicon, 2010*). The Problem-Solving Unit reference materials in the Atlas Mapping Tool to extend the skills learned earlier in
the year. Table 17 reveals that all events focused on enriching the grade level concepts and skills. Ms. Littleton engaged students in discussion while they worked on tasks to process their ideas. Problems and tasks were used to extended learning and to strengthen problem-solving skills. She consistently conversed with students about the different ways they solved tasks and problems either during small group or as a class (See Table 14 and 15).

For example, during the capacity project in Event 3, students were extending their knowledge of adding, subtracting and multiplying decimals in situations where they had to select an operation and manipulate the information in the question to identify a logical number sentence and then determine the correct answer (p. 11-12). Additionally, students had to apply their knowledge of measurement to calculate the amount of ribbon needed to create a bow to and wrap a box (Event 6, p. 20-24). Students also had to be able to organize and communicate their ideas while representing their thinking on posters and during class discussions (Event 4, p. 13-15).

**Provides A Structure for Discussion**

The third component of Problem Posing is provides structure for discussion. The problem-posing approach helped Ms. Littleton establish a structure for engaging her students with peers while strengthening mathematical understanding. Ms. Littleton developed a practice by implementing practices to guide students through mathematical conversations around the problems they solved together (Table 14). For this study, the practices or structures she used have been defined as elements within the fourteen ground rules, as summarized earlier in this chapter. These ground rules defined the structure that became accepted as what it meant for Ms. Littleton and her students to learn and discuss
mathematics together in this classroom. This structure for *engaging in discussion* was implemented during all seven events in this study, as noted in Table 13. The problems and tasks chosen for her students provided Ms. Littleton with a structure for engaging students in conversations about strategies and solutions. Her work with students during mathematical discussion emphasized exchanging their knowledge and developing the greater understanding of how to solve the problems assigned to them. She also encouraged students to reflect on contributions and the ways they interacted while working together (Event 7, p. 23-26).

As described earlier, Ms. Littleton and her students implemented certain elements within the ground rules during their discussions, see Tables 14 and 15. Ms. Littleton engaged students in joint reasoning (GR6) by engaging them in a discussion about the problems and tasks they solved on 202 occasions. She also selected students to join the conversations (GR1B) on 168 occasions. Additionally, she encouraged students as they publically shared their knowledge (GR4) on 62 occasions, and offer multiple solutions (GR7) on 66 occasions. Ms. Littleton created an atmosphere of trust (GR2) as demonstrated on 62 occasions during events one through seven.

It was not clear whether or not Ms. Littleton was aware of how Problem Posing provided a structure for her discussions. When asked to reflect whether or not she used any structure to guide the implementation of her discourse. Ms. Littleton said,

I feel like there's a little bit of both. There are definitely things that have to be done and structured a lot of the questions that we discuss. And then, there's some that were just as we discover one thing, we're moving forward to something else, and it might fall in that order that I didn’t plan it to go. But that’s the way students are learning, so, I'm going to flow with that. (Individual Interview, p.6)
However, she was very aware of the need to follow where students were taking the discussion. She implemented in-the-moment decision making that allowed her to do this. This decision making was evident in the dialogue introduced earlier from Event 1, p.1) below:

1. D: Um, so, my first way is what I wrote, I know that 180 degrees is $\frac{1}{2}$, so that is $\frac{2}{4}$ and then we had $\frac{1}{4}$ left and that is 90 so I did 180 degrees plus 90 degrees equals 270 degrees, so $\frac{3}{4}$ of the circle is 270 degrees.

2. D: And then my other way, was um the full circle is 360 degrees so we are only leaving out $\frac{1}{4}$ so I subtracted the $\frac{1}{4}$ which is 90 degrees from the full circle which is 360 degrees and I did 360 degrees minus 90 degrees and got 270 degrees.

3. T: Devon just shared two different ways, one that was using subtraction why did Devon use subtraction to try to figure out the fraction of the circle?

4. T: Jostos?

5. J: Well, since the denominator is fourths then she just did fourths and then took away.

6. T: What do you mean?

7. J: Well, if the denominator is fourths then you could do all of the circle and take away only one fourth.

8. T: If the denominator is fourths then you could take away the one that is missing?

9. J: [Nods yes]

10. T: Awesome!

The student explains how he solved the problem using two methods. Ms. Littleton listens to his explanation and then in-the moment determines a key idea to point out to students. She that she wants to pull out the idea of using subtraction to answer the question. Then she asks, “Why did Devon use subtraction?” the discussion continues with
another Jostos interpreting what Devon was thinking. Ms. Littleton guides the discussion to support students to communicate their ideas effectively.

**Active Learning With Authenticity**

The third reformed based instructional practice is Active Learning With Authenticity. Ms. Littleton provided authentic experiences for students to learn real-world mathematics by solving real-life tasks that linked school mathematics with real-world mathematics. The components of Active Learning With Authenticity included: *engages in learning, making real-life connections, and honors mathematics as a discipline*. Students were engaged in Active Learning With Authenticity using various ground rules and elements during each of the observations, as noted in Table 14 and 15. The components of Active Learning With Authenticity were implemented during most of the seven events in this study, as noted in Table 17. Ms. Littleton used problems and tasks to *engage students in learning* and to *honor mathematics as a discipline* during all events. She connected the tasks to *real-life* problem solving during events three through seven.

**Engages In Learning**

The first component of Active Learning With Authenticity is *engages in learning*. Authenticity in learning occurred within the activity of students and discussions students participated while studying mathematics. She provided opportunities for students to *engage in learning* by including problems and tasks that required students to interact with the problems and one another in order to complete them. This required students to engage by exchanging ideas, actively planning and testing solutions. Ms. Littleton’s students engaged with peers while solving problems as much as they did with their teacher. Ms. Littleton asked questions after students contributed to the discussions, and
supported students to communicate their ideas. She also circulated the room while students worked in groups to gather information about their work together. Other interactions occurred when students engaged with their teacher to ask questions and offer feedback.

Again, Table 14 reveals that Ms. Littleton utilized elements within the ground rules to engage students by requiring them to share their ideas and solutions (GR6A) on 202 occasions. She used questioning to direct thinking (GR6B) on 52 occasions and to understand what they knew (GR6D) on 73 occasions. Questions were also used to encourage more students to engage by exchanging of ideas (GR6C) on 47. She prompted students to think creatively and share ways of solving problems (GR7A) on 63 occasions so that many strategies could be revealed and discussed. She asked questions to further their thinking (GR9B) on 13 occasions. Mrs. Littleton also questioned her students thinking by requiring them to explain their thought process and reasoning (GR11A) on 21 occasions. She also provided feedback to students when their thinking was viable and efficient (GR11E) on 14 occasions to demonstrate the importance of these skills.

Connects To Real-Life

The second component of Active Learning With Authenticity is making real-life connections. Authentic activities required students to make connections about how the math they used in school could be used outside of the classroom to solve real-world problems. Ms. Littleton engaged students in completing tasks that could also be applied in the world around them. Applying their learning to real-life experiences solidified the connections between learning math inside classrooms and in the real world.
Ms. Littleton included connections to situations where the mathematics discussed could be applied in *real life* during events three through seven. During Event 3 for example, students worked to determine how many water balloons they could fill with a given amount of water. One of the questions was, “*The package of Mel’s water balloons says that it holds 300 milliliters of water. How many balloons can he fill if he has two liters of water*” (Event 3, p.11)? This task required students to utilize real-world decision-making skills. They discussed the problem and determined the plan that they would use to solve the problem. Students contributed different ideas, and the group members negotiated with one another to select the strategy they used. Then they worked together to identify a solution that they all agreed upon (GR14B).

In Event 4, groups of students presented the two different strategies they used to solve their problem. While presenting, they referred to the representations drawn from the poster which to explain their strategies and solutions. After each group presented, the audience shared a comment and asked two questions. One student asked the question, “Did you get confused at any of the parts? (Event 4, p. 15). Another student asked, “Which way did you like best?” Ms. Littleton also interacted with the presenters during this Event. She checked for understanding when asking, “What was the question asking you” (Event 4, p. 14)? She also asked, “How did you come up with the four?” to encourage a more thorough explanation.

Ms. Littleton also had students complete a *real-life* measurement task in Event 5. Ms. Littleton directed students through a warm-up activity that required calculating the amount of ribbon needed to tie a bow around a pencil. Each group of students was given one yard or ribbon to design a bow. They worked together to create and measure the bow.
Classroom observations revealed that all groups reported out their solutions and discussed similarities and differences among them (GR11D). Ms. Littleton questioned to further students’ thinking. She asked, “Why do you think we had all sorts of measurements for the bows” (Event 5, p. 18).

For the second part of this task, each group was given a box. All boxes were a different size and shape. Ms. Littleton connected this activity to the real-life experience of wrapping a present. She provided a quick demonstration of how a ribbon was used to create a bow. She explained the expectations for completing the task.

You are going to plan how to solve this problem with your group; you are going to estimate the length of the ribbon that you need. You are going to write about how you got your answer with your group. Once you have your plan and you have written out your plan, then you can measure and cut your ribbon to test your answer. I will need to see your plan first to see how you made your estimate. (Event 5, p.18-19)

Students worked on this task during the end of Event 5 and during all of Event 6. While they worked, students negotiated amongst themselves who would wrap the ribbon, what size bow they would use, and how the plan would be written. Ms. Littleton supported students as they worked through during each phase of the task. She circulated to ask questions, *exchange ideas* and to understand their logic (GR6C). For example, when checking in about their plans, she asked, “So what is your plan” (Event 6, p. 21)? She asked multiple questions of each group as evident in the observation transcripts. During Event 6, she asked for further explanation, “How did you come up with that” (p. 22). Then she asked, “What do you think that you did that helped you find an accurate measure?”

Finally, during Event 7, Ms. Littleton engaged students in a reflective class discussion about their decision making while they completed the task. She also
encouraged students to make connections to this task with their own real-life experiences. This discussion included the open-ended question, “What went well” (Event 7, p. 23)? She also asked, “What were some of the challenges you faced?” and “So was your challenge measuring out that you needed?”

Then students exchanged ideas about how they might complete the task differently if they were asked to do the project again using a different sized box. Ms. Littleton sparked this discussion using several questions. This discussion was not about solutions but about learning from the work. Ms. Littleton said, “So I want you to think about this question for a second” (Event 6, p. 25). She continued by asking the question, “What if we weren’t going to use a bow?” Then she questioned, “Would that be more difficult to figure out or less difficult to figure out? Why?” She led students to talk about why the bow made calculating the total length more complex. Then students were encouraged to make the connection to real-life experiences that required measurement. She wanted them to think about when a person might need an exact measurement and not an estimate. She asked,

“And what’s the case in real life? Might you have to do something like this? Maybe not wrapping a box, but working with the material and going to make an estimate” (Event 7, p. 26)?

**Honors Mathematics As A Discipline**

The third component of Active Learning With Authenticity is *honors mathematics as a discipline*. Learning mathematics with authenticity also means that students come to understand mathematics in ways that mirror the structure and content of mathematics as a discipline. Ms. Littleton *honored mathematics as a discipline* by making connections
with the ideas, concepts, and notations found in the discipline of mathematics. She consistently used and emphasized accurate terminology and content with her students.

Table 17 reveals Ms. Littleton utilization of the mathematics mirroring the structure and content of mathematics as a discipline throughout the study. She honored mathematics as a discipline when she shared content and provided guidance to students about how to solve and explain problems using standard mathematical operations and notations. Ms. Littleton also guided her students to do the same. Students were required to explain their thinking by making connections among their thinking and mathematical content. They were also encouraged to provide explanations that included representations of their thinking using standard mathematical notation, often using numerical expressions, see Table 14 and 15.

Ms. Littleton honored mathematics as a discipline during Event 1, as described earlier, when she directed students thinking toward identifying the number of right angles contained in a circle. During the conversations, she guided students in making connections among concept within the discipline of mathematics. They explored the relationship of a right angle to one-fourth of a circle. While students shared out their solutions, a student contributed,

I know that 90 degrees is one-fourth of the 360-degree circle so I divided 90 by four, 360 degrees by 90 and I got four. I also made a circle and divided it into four pieces starting with 90 here, 90 here, 90 here, and 90 here. And there's four pieces.” (Event 1, p.1-2)

Ms. Littleton guided this connection when she asked, “Four right angles?” The student agreed. Students continued to share other ways of solving the problem and used conventional ways to add and subtract the 90 degrees.
Also in Event 2, introduced earlier, a similar discussion occurred while students were finding the measurement of supplementary angles (p. 4). Ms. Littleton stepped in when a student mistakenly calculated the problem. She led the student in clarifying and correcting his solution. He had added the two angle measurements instead of subtracting one from 180 degrees. During this part of the discussion, Ms. Littleton utilized the representation of the angles on the board and referred to it, using the mathematical vocabulary needed to explain how to find the accurate solution. As detailed earlier, Ms. Littleton engaged students in an activity that required them to identify standard units of measurements during the capacity task in Event 4. She assisted students in extending their understanding of measurement using the capacity problems to practice adding, subtracting and multiplying decimals. While students were working, she honored the mathematics by verifying viable solutions. When students needed re-direction, she stepped in to advise. She checked on their calculations, pointing out when errors were made and redirected their path. For example, she stepped in to ask a question during Event 3, she asked,

   Why did you choose to multiply? We were trying to figure out how many balloons we were trying to fill with 2000 liters of water, right. Since we have 2000 liters of waters and we are trying to get water to each balloon, should we be multiplying or dividing? (Event 3, p.14)

   **Learning Through Interaction**

   Learning Through Interaction is the fourth reformed-based practices for building understanding in mathematics. This practice was built on the idea that mathematics is a *socially constructed* endeavor. Ms. Littleton’s provided many opportunities for her students to interact as they solved problems alongside others in the community. As revealed in Table 14 and 15, Ms. Littleton facilitated the students’ interactions during
group work and provided some direction for explaining and presenting problems, evaluating findings, and challenging other students’ perspectives. Ms. Littleton used both components of the Learning Through Interaction included learning is socially constructed and contributes to the learning of others. The components of Learning Through Interaction were implemented during all of the seven events in this study, as noted in Table 17.

Ms. Littleton drew on many of the elements within the ground rules to encourage students to interact while learning, see Table 14 and 15. After solving the problem or task, students shared their knowledge (GR4A) on 21 occasions. Ms. Littleton restated contributions to help clarify ideas (GR4C) so that students could understand one another on 31 occasions, see Table 14. While students shared their thinking about tasks and problems, Ms. Littleton spent time questioning them about their contributions (GR11A) on 21 occasions. Students also questioned one another (GR11A) on 21 occasions. Moreover, Ms. Littleton encouraged students to learn from one another by encouraging them to offer many ways of solving problems (GR7A) on 63 occasions. Both Ms. Littleton and her students assisted in building on the ideas of other (GR8B) on seven occasions. Moreover, she always encouraged and chose students to contribute (GR1A and GR1B), and allowed students the freedom to share ideas about the work they were doing.

When asked to reflect on the process she used to share ideas and construct learning together, Ms. Littleton said,

I think it's important because it helps them to develop their thinking and the students really learn from each other. All the time I’ll hear some student start to talk about something, and you’ll hear the uh-huh moment that they're exploring things together and making their own conclusions rather than me telling them, okay. This is the formula. We're going to use the area. They find more ownership
in what they’re learning, and more understanding of it. (Individual Teacher Interview, p. 4-5)

**Learning Is Socially Constructed**

The first component of Learning Through Interaction is *learning is socially constructed*. Students were provided with many opportunities to *socially construct learning* with peers. The tasks and problems assigned had a level of complexity that required students to interact with others to complete them, see Table 17. Students interacted with others during all seven events. Ms. Littleton designed her lessons to include small group or partner problem solving followed by a class discussion. The social construction of learning occurred while students engaged in small and large discussions, shared and listened to multiple strategies and reflected on solutions.

Content needed to solve the problems or complete the tasks was not explicitly taught by Ms. Littleton during the lesson, as evident in the lesson transcripts. Students used what they knew about the topic and applied this knowledge to the task. The content was discussed when the need arose during conversations. Students wrestled with their shared ideas and strategized to solve a problem with peers. This small group approach provided students with the opportunity to discuss the problem with others before having to share their ideas during a full class discussion. When it was time for the class discussion, students presented their contributions in the context of what their small group had discovered or decided. Ms. Littleton and other students listened to explanations and interacted with one another with questions during the class discussions.

Ms. Littleton used open-ended questions, providing students with the freedom to share their thinking, often to begin discussions to allow students to explain their own thinking. For example, she asked, “Who would like to share out how you solved this
problem” (Event 2, p.4)? Similarly, she posed the questions, “Does anyone have any questions for him” (Event 1, p.1)? and, “So what do you think” (Event 6, p.22)?

At times, she did use questions to guide students’ participation more. For example, she began the group discussion during Event 1 with the statement, “We are sharing out because we're going to try to share different thinking, try not to share the same thinking” (p.1-2). She wanted students to share solutions while being mindful not to repeat strategies. When students did repeat, she reminded them of the expectation. She also steered them slightly when she asked a more pointed question, “Does anyone have questions for Evelyn about how she solved the problem” (Event 2, p.6)?

While students shared their thinking both in small and large group formats, Ms. Littleton encouraged them to build on the thinking of others, sharing multiple strategies to help increase the opportunity to socially construct their solutions together. For example, during Event 3 students discussed equivalent units of liquid measure before beginning the capacity task. Ms. Littleton started the interchange when she asked, “So four quarts is equal to what” (Event 3, p.9)? Students offered responses and explained their answers. Afterward, Ms. Littleton asked questions to prompt the student to explain. She inquired, “How did you figure that out” (Event 3, p.9)?

She also asked questions to draw more participation and to check additional students’ understanding. To do this, she asked questions to elicit understanding. This encouraged the sharing of a few more equivalent measures. In the exchange below, a student disagreed with another student over the content. Ms. Littleton used this opportunity to facilitate the discussion to allow students to construct meaning together.

Ms. Littleton asked, “Who can find another unit that is equal to four quarts?”
The first student offered “16 cups.”

A peer jumped in to disagree, “Shouldn’t it be 32, shouldn’t it be two pints in each quart, and she only did one pint in each quart?”

Ms. Littleton prompted more involvement from the class. She asked an open-ended question to invite more participation, “What do you think friends?

Another student spoke up, “I agree with Zandra because there is 4 cups so 16 cups because 4 times 4 is 16 cups.”

Ms. Littleton turned back to the student and said, “Does that (16 cups) make sense? Can you tell me why it makes sense?”

The student answered, “Because 4 times 4 is sixteen.”

Ms. Littleton checked for understanding, “Where did the four come from?”

She answered, “Four comes from the four pints and four quarts”

Discussions such as the one described above included students questioning other students. It was clear that students were listening as their classmates shared their thinking. This student did not hesitate to challenge the ideas shared by her peer. Another example of students questioning one another took place during Event 4. The first group to present labeled her solution using millimeters. One of her classmates disagreed with the use of that unit and asked, “Did you say millimeters or milliliters” (p.14)? Additionally, during a class discussion during Event 2, one student voiced her concern that a member of her group “had a different answer than the rest” of the table group (p. 6.). Moreover, while two students were working on their capacity problem task, a student disagreed with the strategy she was using and spoke up about it. She said, “No, that is not right, do you just want to go with my answer? I don’t think six and 200 is a correct answer. I think
mine would be more of a correct answer” (Event 3, p. 11-12). In all cases, Ms. Littleton stepped in to help the students respectfully negotiate the situation and construct knowledge by accurately determining a viable solution.

**Contributed To The Learning of Others**

The second component of Learning Through Interaction is *contributed to the learning of others*. Ms. Littleton facilitated a process that she used to engage students in learning from one another. When students shared their thinking or strategy, she ensured that they explained their reasoning, as evident in the classroom observations. This was done during all class discussions, as well as, during small group work. Ms. Littleton also encouraged students to interact with their classmates by asking questions of one another, and providing assistance, as needed. This was a priority because Ms. Littleton stated that learned more successfully from other students. She said, “It just helps them to understand it better where I might be trying to use the mathematical terms, that they're just not ready for” (Individual Interview, p. 5).

One example that included the use of a prompt to encourage students to offer an explanation that included reasoning or justification occurred during Event 5 (p. 19-20). Students were working in small groups to complete the Wrapping the Present Task. Ms. Littleton came over to check in with the group and proceeded to interact with students as they explained their solutions. She asked specific questions about their thinking to determine the process that they used for identifying a solution. All three group members joined in to explain the process used to *socially construct* the solution.

Teacher: Can you tell me exactly how you figured out your estimate?

Student 1: We measured the box with the tape measure and then added a few inches.
Teacher: So how did you measure the box?

Student 2: With that. (pointing to the tape measure)

Student 3: (gestured that she wrapped the tape measure around the box)

Teacher: So, you tied it around, you pretended that the tape measure was a ribbon?

Student 3: Then we added seven inches.

Teacher: Why did you add seven inches for the bow?

Student 1: Because that was what we thought it needed.

Student 3: Because we had 43 and we thought 7 more would give us what we needed.

Teacher: What if you did not have a tape measure, what would you do?

Teacher: Can I have one person talk at a time?

Student 1: I would measure the length and then I would do the width and then I would do the length times two and the width times two.

Teacher: Why would you do the length times two and the width times two?

Student 1: And then I would have to do that again.

Teacher: Why?

Student 1: Because we only did the top length times two and the bottom times two and it would only be that (pointing to the top of the box and the bottom of the box) and not that (the four sides). And it would not go all around.

Teacher: So, what it is telling you width times two and length times two, what are you thinking about?

Student 1: The perimeter.

Additionally, during the following class discussion about the measurement of an angle, a student stated the strategy he used to solve the following problem (Event 2, p.5).
Ms. Littleton followed up each of his statements to draw out the student’s thinking and help make his decisions more explicit to the class. The conversation ended with a question Ms. Littleton asked to engage other students in the conversation and prompt further interaction. This was a practice Ms. Littleton used (GR6C) as evident in Table 14.

Student: 76 plus 76 is 152 and then I subtracted 152 from 360 and I then I got 208 and then half of 208 is 104.

Teacher: Why did you decide to subtract it from 360?

Student: Because 360 is a full rotation.

Teacher: Hmm. Why did you subtract two 76 degree angles?

Student: Because if you only subtracted one it would get the wrong answer.

Teacher: How do you know?

Student: Because it would be different from 180 because 180 is only half of 360 but if you subtracted ½ of 360 it would give you the right answer.

Teacher: Interesting, so that is another way to think about it. Does anyone have questions for her about how she solved the problem?

Several times, Ms. Littleton asked students if they had any questions for the classmate who had just contributed to the conversations. This allowed students opportunities to interact and exchange ideas with one another (GR6C). During Event 1, Ms. Littleton solicited student input after each student shared their thinking. She asked, “Any questions” (Event1, p. 1-3)? She also asked, “Does anyone have questions for Evelyn about how she solved the problem” (Event 2, p. 5)? Moreover, she prompted, “Does anyone have questions or comments” (Event 4, p. 14)?

Sometimes, however, students did not respond to her requests. The greatest response occurred during Event 4 after students presented and the format for interacting included asking two questions and making one comment, see Table 15.
Students also contributed to the learning of others when asked to do so during group discussions. One example occurred during Event 2 after a student raised the concern about a discrepancy in the group’s solution. Student 1 said, “Our table group thought it was 104 because the straight angle on the bottom of the shape is 180 degrees and then if you do 76 minus 180 degrees then you get 104” (Event, 2, p. 5-6). Then she added, “She (Student 2) had a different answer than the rest of our table group.” The following is the dialogue the conversation that transpired.

1. Teacher: Did you talk to your table group about how you were solving the problem?”

2. Student 1: He (Student 3) has the same answer (as Student 2), but everyone else does not.

3. Teacher: Okay would you like to share your answer? Go ahead. GR1B GR6A GR7A

4. Student 2: At first, we did it the way Student 1 did it, and then we realized that we thought it wasn’t correct so F said we should probably do 180 plus 76 because 76 is part of the straight angle.

5. Teacher: Boys and girls do we have any questions or suggestions about how they are solving the problem?

6. Student 4: 76 is the acute angle (shows the angle with his hands), and 180 is the straight one (shows angle with his hands), so you should have subtracted instead of added.

7. Teacher: Does that make sense Student 2 and Student 3?

Above, Ms. Littleton asked, “Boys and girls do we have any questions or suggestions about how they are solving the problem?” to encouraged the class to help contribute to the learning of their classmates (GR6G). Instead of providing a strategy or answering the question for the students, Ms. Littleton turned the question toward the entire class. The Student 4 joined the conversation and offered a strategy for
identifying a correct solution. Ms. Littleton offered an additional statement afterward to clarify and check for understanding. She said,

So we have our 180 degrees here and the parallelogram drawn on the number line, this is our 76-degree angle and our 180-degree is the straight line, so it is part of the 180-degree angle. If you are adding 76 to 180 degrees what you are doing is saying we have an angle that starts here and goes all of the way around to here. (Event 2, p.6)

Then she asked, “Does that make sense?” and “How could you fix your answer?” The student responded, “Subtract 76 from 180.”

Again, for strategic discourse to be truly effective, it cannot be entirely teacher directed. As will be described below, Ms. Littleton set the course for student and teacher roles to evolve so that students could become more autonomous while they studied mathematics together.

**Shifting Authority Toward Shared Authority**

Ms. Littleton remarkably, and without any measurable training, used methods that are reforming how her students learn, not just what they learn, see Table 17. Ms. Littleton has used multiple practices that have established an environment that encourages equalized working relationships with and amongst her students.

Strategic discourse is built on the idea that both students and teachers are partners in learning and powerful sources of mathematical ideas and thinking. The roles were more equalized among Ms. Littleton and her students than in the traditional classroom. As examined earlier in this chapter, the teacher facilitated conversations allowing students to state their mathematical knowledge, share ideas and identify how they would solve problems. For example, students did this while relating fractions with degrees in a circle during Event 1 (p. 1-2). They also stated their mathematical knowledge, sharing
ideas and explained solutions during involving the capacity problems (Event, 4, p. 13-15). Students were not hesitant to take risks, make mistakes or raise questions. They carefully examined contributions and spoke up when they disagreed with solutions and strategies, even if it involved questioning the teacher. Ms. Littleton praised students again and again to acknowledge their hard work and valuable contributions (GR2A and GR2B). The level of participation and interaction among students and their teacher would not have been possible without the existence of the supportive environment established in Ms. Littleton’s classroom, see Table 14.

Cultivating the mathematical authority among students has taken time. Ms. Littleton modeled how to help others explain, rethink their strategies, and offer and receive help from their peers. For example, during Event 2 (p. 6), Edna raises a concern that the members of her group have different solutions. The discussion below occurs after Edna raises her concern.

1. E: Chevron had a different answer then the rest of our table group.
2. T: Did you talk to your table group about how you were solving the problem?
3. M: A has the same answer (as Chevron) but everyone else does not.
4. T: Okay would you like to share your answer? Go ahead.
5. C: At first, we did it the way Edna did it, and then we realized that we thought it wasn’t correct so, A said we should probably do 180 plus 76 because 76 is part of the straight angle.
6. T: Okay, GR6A
7. T: Boys and girls do we have any questions or suggestions about how they are solving the problem?
8. T: Arthur?
9. A: 76 is the acute angle (shows the angle with his hands), and 180 is the straight one (shows angle with his hands) so, you should have subtracted instead of added.
10. T: Does that make sense Chevron and A?

11. T: (Draws on the board) So, we have our 180 degrees here and the parallelogram drawn on the number line, this is our 76-degree angle and our 180 degrees is the straight line, so, it is part of the 180-degree angle.

12. T: If you are adding 76 to 180 degrees what you are doing is saying we have an angle that starts here and goes all of the way around to here.

13. T: Is that what we are looking at here (points to the extended angle beyond 180-degree straight angle) this larger angle?

14. T: Is that what we are looking at, this larger angle?

15. C: No.

16. T: No?

17. T: Does that make sense?

18. C: Yes.

19. T: So, A and Chevron how could you fix your answer?


21. T: Subtract what?

22. A: Subtract 76 from 180.

First, there is evidence that Edna has developed a concept of shared authority because she feels confident bringing a concern to the teacher in front of the class. Ms. Littleton takes the concern and provides time to discuss the problem. She begins this process by soliciting help from the remainder of the class. This shift in authority away from herself as an expert to sharing the expertise with the members of the community is powerful. She asks, “Boys and girls, do we have any questions or suggestions about how they are solving the problem?” Arthur steps up and tries to offer support to the students. In Turn 9 he explains a solution, “76 is the acute angle (shows the angle with his hands),
and 180 is the straight one (shows angle with his hands) so, you should have subtracted instead of added. Then Ms. Littleton clarifies the problem and solution for the students. Circling back to make sure that they understand why the problem could be answered in this way.

Ms. Littleton also helped students strengthen their mathematical authority while practicing how to provide explanations and justifications. She consistently provided opportunities for her students to engage in joint reasoning and allowed the freedom to select strategies and design solutions. She treated each as valuable and capable members of the learning community, as revealed in Table 14. The supportive environment thrived as Ms. Littleton provided students with multiple opportunities to engage in conversations about the problems and tasks that they worked on together with peers.

Although students were highly involved in classroom discussion, Ms. Littleton still maintained control of the conversations. At times, she did provide opportunities for students to direct parts of the conversations, but for the most part, she directed the course of the discussions. Students did have choices about which strategies to use while solving problems, but they did not have the freedom to solve problems or tasks that they had identified on their own. All activities were pre-set for them to complete.

**Summary**

Ms. Littleton implemented all four reformed-based practices including Problem Posing, Active Learning with Authenticity, Learning Through Interaction, and Strategic Discourse. Teachers implemented these practices along with the components of consistently throughout the study.
The components of the first reform-based practice Problem Posing included *well-designed problem or task enriches the concept or skill and provides structure for discussion*. All components were utilized during each of the seven events. Ms. Littleton utilized *well-designed problems or tasks* during all events to generate and *structure* mathematical *discussions* she facilitated with students as they solved problems together. All problems *enriched the concepts* of the topic addressed.

The components of the second reform-based practice Active Learning With Authenticity included *engages in learning, connects to real life, and honors mathematics as a discipline* were also implemented by both teachers. Ms. Littleton *engaged students in learning* while *connecting* learning tasks to *real-life* problems during all events.

Additionally, Ms. Littleton *honored mathematics as a discipline* by pointing out the mathematics that mirrored the structure and content of mathematics as a discipline throughout the study. Students were asked to explain their thinking, connected strategies to the discipline, and used logical numeric formats. Ms. Littleton *honored the mathematics as a discipline* during all events, except for events seven and seven.

Ms. Littleton also utilized the components of the third reformed based practice Learning Through Interaction were utilized by both teachers during each event to support the building of understanding among participants. The first component *learning is socially constructed* was encouraged as students interacted with other students to complete tasks in small groups and large groups, provided help to peers, asked questions, and interacted with the thinking of others.

*Contributes to the learning of others* is the second component of Learning Through. To help *contribute to the learning of others*, students asked questions,
compared ideas, shared multiple solutions and identified effective solutions. Ms. Littleton initiated opportunities for students to contribute to the learning of others when she asked students to offer ideas or strategies to assist their classmates to identify correct solutions. She also encouraged students to learn from one another by sharing multiple ways of solving problems or tasks.

Furthermore, Ms. Littleton has used methods that are reforming how her students learn not just what they learn, see Table 14. Ms. Littleton used multiple practices that have established an environment that encourages equalized working relationships with and amongst her students. Students stated their mathematical knowledge, share ideas and identified how they would solve problems. Students were not hesitant to take risks, make mistakes or raise questions. The level of participation was high because of the supportive environment established in Ms. Littleton’s classroom. Ms. Littleton helped students strengthen their mathematical authority while practicing how to provide explanations and justifications. She consistently provided opportunities for her students to engage in joint reasoning and allowed the freedom to select strategies and design solutions.

Although students were highly involved in classroom discussion, Ms. Littleton still maintained control of the conversations. At times, she did provide opportunities for students to direct parts of the conversations, but for the most part, she directed the course of the discussions. Students did have choices about which strategies to use while solving problems, but they did not have the freedom to solve problems or tasks that they had identified on their own. All activities were pre-set for them to complete.

The following chapter provides a cross-case analysis of the strategic discourse and reform-based practices used by the teachers and students, among both classrooms in
this case study, as they engage in mathematical conversations and solve problems together in their mathematics classrooms.
CHAPTER 6
CROSS-CASE ANALYSIS

This study was designed to examine the ways teachers implemented discourse practices as they provided reform-based instruction during their study of mathematics. Through investigating the strategic discourse practices used I hope to capture the essence of the exchanges among and between students and teachers as they engaged in problem-solving.

Applying the Vision for Mathematics Learning lens along with the socio-cultural theory, the research questions that guided this study helped to illuminate the specific elements of strategic discourse teachers and students used. Furthermore, these guiding questions revealed the ways teachers encouraged student interaction using problem posing accompanied by tasks connected to real-life situations. The research questions also provided information about teacher and student perspectives related to the strategic discourse used.

A cross analysis allows the researcher to deepen one's understanding and increase one's ability to generalize across cases (Miles & Hubermann, 1994). The purpose of this cross-case analysis was to draw out as much as possible to enable the reader to have a sense of what reformed-based discourse sounds like/looks like and what the results are for student understanding of mathematics. Comparing across cases can assist in providing more clarity and deeper understandings than possible through the examination of one case.

This cross-case study examined the ways two teachers encouraged students to engage in discourse while discussing their mathematical thinking and reasoning.
Additionally, investigating the ways students were engaged in discourse with their teacher and classmates was also examine. However, this investigation was not meant to scrutinize any of the teachers or students involved, but to provide a generalized picture of the ways strategic discourse practices were used by teachers in two classrooms. The comparison among the two classrooms provided much greater detail and a richer picture of this particular phenomenon that would have been possible with only one classroom. This research demonstrated the existence of ways teachers utilized strategic discourse in mathematics education. The discourse included in the examination includes both the student talk in response to teachers' questions and invitations, along with the talk that students initiate with their teacher and classmates.

Furthermore, cross-case study methodology assists in explaining the causal links in real-life situations that are too complex for a single survey or experiment (Yin, 1994). In line with this thinking, similarities across the two cases were examined. The comparison provided for the opportunity to discover patterns among the types of discourse two teachers and two groups of students were using while solving problems in math. This research established the existence of various types of strategic discourse already occurring in mathematics classes without formalized training or support from researchers.

The participants were chosen from the same school district to ensure commonalities among them. Both teachers were recommended by administrators based on their use of reformed based methodology including the use of mathematical conversations to study mathematics. Both had participated in the same district sponsored professional development which included a few curriculum mapping sessions and one
professional development opportunity in mathematics. On the contrary, the two teachers had differences among them in their mathematics education background and years of teaching.

The analysis reveals that both teachers and their students were implementing a variety of discourse practices in each of the classrooms, see Table 18. Additionally, this data reveals similarities which existed among the discourse elements implemented by teachers and students across the two cases, see Tables 19 and 20. Moreover, the incidence of strategic discourse practices involving the Developing Mathematical Knowledge and Encouraging Student Participation were more frequent among both teachers and their students. The analysis also revealed that the discourse practices used by the teachers and their students to Strengthening Critical Thinking were used significantly less in comparison.
Table 18: Ground Rules Frequency Comparison: Case 1 and Case 2

<table>
<thead>
<tr>
<th>EVENTS</th>
<th>Washington</th>
<th>Littleton</th>
<th>Washington-Students</th>
<th>Littleton-Students</th>
</tr>
</thead>
<tbody>
<tr>
<td>GR1</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>A</td>
<td>96</td>
<td>35</td>
<td>103</td>
<td>4</td>
</tr>
<tr>
<td>B</td>
<td>72</td>
<td>129</td>
<td>7</td>
<td>21</td>
</tr>
<tr>
<td>Total</td>
<td>168</td>
<td>164</td>
<td>110</td>
<td>25</td>
</tr>
<tr>
<td>GR2</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>A</td>
<td>18</td>
<td>42</td>
<td>3</td>
<td>7</td>
</tr>
<tr>
<td>B</td>
<td>10</td>
<td>20</td>
<td>0</td>
<td>7</td>
</tr>
<tr>
<td>Total</td>
<td>28</td>
<td>62</td>
<td>3</td>
<td>14</td>
</tr>
<tr>
<td>GR3</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>A</td>
<td>27</td>
<td>20</td>
<td>21</td>
<td>48</td>
</tr>
<tr>
<td>B</td>
<td>3</td>
<td>48</td>
<td>4</td>
<td>15</td>
</tr>
<tr>
<td>C</td>
<td>12</td>
<td>6</td>
<td>14</td>
<td>18</td>
</tr>
<tr>
<td>D</td>
<td>11</td>
<td>13</td>
<td>1</td>
<td>6</td>
</tr>
<tr>
<td>E</td>
<td>3</td>
<td>2</td>
<td>1</td>
<td>19</td>
</tr>
<tr>
<td>Total</td>
<td>56</td>
<td>89</td>
<td>41</td>
<td>106</td>
</tr>
<tr>
<td>GR4</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>A</td>
<td>21</td>
<td>21</td>
<td>43</td>
<td>113</td>
</tr>
<tr>
<td>B</td>
<td>1</td>
<td>5</td>
<td>16</td>
<td>46</td>
</tr>
<tr>
<td>C</td>
<td>31</td>
<td>23</td>
<td>6</td>
<td>11</td>
</tr>
<tr>
<td>D</td>
<td>2</td>
<td>13</td>
<td>1</td>
<td>10</td>
</tr>
<tr>
<td>Total</td>
<td>55</td>
<td>62</td>
<td>66</td>
<td>180</td>
</tr>
<tr>
<td>GR5</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>A</td>
<td>8</td>
<td>9</td>
<td>69</td>
<td>80</td>
</tr>
<tr>
<td>B</td>
<td>1</td>
<td>2</td>
<td>6</td>
<td>49</td>
</tr>
<tr>
<td>Total</td>
<td>9</td>
<td>11</td>
<td>75</td>
<td>129</td>
</tr>
<tr>
<td>GR6</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>A</td>
<td>76</td>
<td>202</td>
<td>147</td>
<td>230</td>
</tr>
<tr>
<td>B</td>
<td>49</td>
<td>52</td>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>C</td>
<td>40</td>
<td>47</td>
<td>4</td>
<td>18</td>
</tr>
<tr>
<td>D</td>
<td>35</td>
<td>73</td>
<td>7</td>
<td>21</td>
</tr>
<tr>
<td>E</td>
<td>8</td>
<td>3</td>
<td>7</td>
<td>1</td>
</tr>
<tr>
<td>F</td>
<td>40</td>
<td>8</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>G</td>
<td>23</td>
<td>16</td>
<td>3</td>
<td>1</td>
</tr>
<tr>
<td>Total</td>
<td>271</td>
<td>401</td>
<td>168</td>
<td>273</td>
</tr>
<tr>
<td>GR7</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>A</td>
<td>62</td>
<td>63</td>
<td>1</td>
<td>11</td>
</tr>
<tr>
<td>B</td>
<td>13</td>
<td>3</td>
<td>77</td>
<td>26</td>
</tr>
<tr>
<td>Total</td>
<td>75</td>
<td>66</td>
<td>78</td>
<td>37</td>
</tr>
<tr>
<td>GR8</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
The following section will address the first and second research question guiding this study and examine the strategic discourse practices used as they solve problems in mathematics.
Research Question #1

- Which types of strategic discourse do teachers use to guide mathematical thinking? Which types of strategic discourse are students using as they engage with their teacher and peers throughout the problem-solving process?

Strategic Discourse Practices Used To Guide Discourse

Strategic discourse practices formulated the structure for mathematical discussions in the two classrooms in this study. Teachers and their students implemented strategic discourse during classroom events. The frequency table reveals the number of times each strategic discourse practice was utilized. The strategic discourse practices are listed according to the elements supporting each Ground Rule one through fourteen. Moreover, the table represent the ground rules listed by the number of times each was implemented among teachers and students. The rationale for the inclusion of this data was to paint a picture of the types of discourse that comprised the talk in both classrooms. The following section examines the ways participants in this study implemented the discourse practices using the elements within these ground rules.

As introduced earlier in the study, the Exploratory Talk model contains fourteen ground rules developed to facilitate the discourse during mathematical discussions. In addition to the ground rules, the indicators were written by the researcher to offer a more complete understanding of the ways the ground rules were implemented during the conversations among teachers and students. Furthermore, to ensure a more focused discussion, the ground rules are organized into three broad themes. The three themes are Encouraging Student Participation, Developing Mathematical Knowledge, Strengthening Critical Thinking. Figure 1 reveals the frequency to which the ground rules in each theme
were implemented by teachers and students across the two cases. The frequency tables throughout the chapter are colored coded to assist in distinguishing between the three themes.

The first three ground rules, compose the theme Encouraging Students to Participate. *Everyone Invited To Contribute (GR1), Contributions and Opinions Treated Respectfully (GR2), and Atmosphere of Trust is Present (GR3)* were utilized by teachers and students to encourage participation and increase discourse in the classroom. The next five ground rules, coded in green, fall into the theme Developing Mathematical Knowledge. *Knowledge Is Made Public (GR4), Reasoning Is Visible In the Talk (GR5), Engage in Joint Reasoning (GR6), Multiple Solutions Are Encouraged (GR7), and Contributions Are Built on Prior Proposals (GR8)* all encourage students to interact to explore ideas and developing understanding together. The remaining ground rules, coded in orange, fall into the theme Strengthening Students' Critical Thinking Skills. *Ideas Extended Together (GR9), Listening Actively to Engage (GR10), Partners Engage Critically with Each Other (G11), Opinions are Considered Before Decisions Are Made (GR12), Ideas May Be Challenged with Counter Strategy (GR13) and Seek Agreement for Joint Decisions (GR14)*. All required teachers to create opportunities for students to push their understanding and interact critically with the mathematics content. Additionally, these ground rules promoted exploring contributions and determining the viability of ideas while analyzing effective solutions.
The examination of the data in this chapter, organized within the themes detailed above, begins with the implementation of the elements within the first theme Encouraging Student Participation. The second and third themes Developing Mathematical Knowledge and Strengthening Critical Thinking will follow.

**Theme 1: Encouraging Students To Participate**

The types of strategic discourse strategies that are aligned with the theme Encouraging Students To Participate promoted encouraging students to engage with one another in conversations while discussing concepts and solving problems. Lessons included several opportunities for students to engage in mathematical discourse as a whole class or within small groups.

The first three ground rules *Everyone Invited To Contribute (GR1), Contributions and Opinions Treated Respectfully (GR2), and Atmosphere of Trust (GR3)* comprised the theme Encouraging Students To Participate. Using the elements within each of these ground rules assisted teachers in promoting a high level of participation while engaging
in mathematical discourse, see Table 1. The following section describes how Mrs.
Washington and her students utilized the strategic discourse elements within the first
theme Encouraging Students To Participate.

**Mrs. Washington and Students**

Mrs. Washington utilized the ground rules in the first theme Encourages Students
To Participate 252 times during eight classroom events. Table 10 reveals Mrs.
Washington used the ground rules *Everyone Invited To Contribute (GR1)* on 168
occasions, *Contributions and Opinions Treated Respectfully (GR2)* on 28 occasions and
*Atmosphere of Trust (GR3)* on 56 occasions. As noted in Figure 6.1, Mrs. Washington’s
students also utilized the ground rules within this first theme Encouraged Students To
Participate 154 times during the eight-classroom events. They utilized the ground rules
*Everyone Invited To Contribute (GR1)* on 110 occasions, *Contributions and Opinions
Treated Respectfully (GR2)* on three occasions and *Atmosphere of Trust (GR3)* on 41
occasions.

Of the nine elements composing the ground rules in this theme, Mrs.
Washington's utilized *Everyone Encouraged To Contribute (GR1A)* 96 times, *Students
Are Chosen Strategically (GR1B)* 72 times, and *Interchanges Demonstrating Equalized
Relationships (GR3A)* 27 times most often while engaging students in mathematical
discourse, see Table 11. Students in Mrs. Washington's class frequently utilized *Everyone
Invited to Contribute Without Being Singled Out (GR1A)* 103 times and *Interchanges
Demonstrating Equalized Relationships (GR3A)* 21 times, see Table 11.
Table 19: Most Frequently Used Strategic Discourse Elements by Teacher

<table>
<thead>
<tr>
<th></th>
<th>WASHINGTON</th>
<th>LITTLETON</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Theme 1: Encouraging Student Participation</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>GR1A</td>
<td>Everyone Encouraged To Contribute (96)</td>
<td></td>
</tr>
<tr>
<td>GR1B</td>
<td>Students Are Chosen Strategically (72)</td>
<td>GR1B Students Are Chosen Strategically (129)</td>
</tr>
<tr>
<td>GR3A</td>
<td>Interchanges Demonstrating Equalized Relationships (27)</td>
<td>GR2A Praise is given for relevant ideas (40)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Theme 2: Developing Mathematical Knowledge</strong></td>
<td>GR4C Participants Offer Knowledge about Mathematics (21)</td>
<td></td>
</tr>
<tr>
<td>GR4C</td>
<td>Contributions Are Restated (31)</td>
<td>GR4C Contributions Are Restated (23)</td>
</tr>
<tr>
<td>GR6A</td>
<td>Ideas/Solutions Discussed w/ Others (76)</td>
<td>GR6A Ideas/Solutions Discussed w/ Others (202)</td>
</tr>
<tr>
<td>GR6B</td>
<td>Questions Posed to the Community To Direct Thinking (49)</td>
<td>GR6B Questions Posed to the Community Direct Thinking (52)</td>
</tr>
<tr>
<td>GR6C</td>
<td>Questions Posed To Encourage Exchange of Ideas (40)</td>
<td>GR6C Questions Posed To Encourage Exchange of Ideas (47)</td>
</tr>
<tr>
<td>GR6D</td>
<td>Community Asks Questions Understanding Thinking (35)</td>
<td>GR6D Community Asks Questions Understanding Thinking (73)</td>
</tr>
<tr>
<td>GR6F</td>
<td>Tasks Are Assigned To Initiate Working Together To Find Solutions (40)</td>
<td></td>
</tr>
<tr>
<td>GR6G</td>
<td>Assistance Is Offered To Help Work Through The Process Or Scaffold Learning (23)</td>
<td></td>
</tr>
<tr>
<td>GR7A</td>
<td>Many Ways of Solving Problems Encouraged (62)</td>
<td>GR7A Many Ways of Solving Problems Encouraged (63)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Theme 3: Strengthening Critical Thinking</strong></td>
<td>GR9B Questions Used to Further Thinking (37)</td>
<td></td>
</tr>
<tr>
<td>GR9B</td>
<td></td>
<td>GR11A All Have Opportunities to Question Others Ideas (21)</td>
</tr>
</tbody>
</table>

*Common elements to both teachers are highlighted*
Table 20: Most Frequently Used Strategic Discourse Elements by Students

<table>
<thead>
<tr>
<th>WASHINGTON</th>
<th>LITTLETON</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Theme 1: Encouraging Student Participation</strong></td>
<td><strong>GR1A Everyone Encouraged To Contribute (103)</strong></td>
</tr>
<tr>
<td><strong>GR3A Casual Interchanges Equalized Relationship (21)</strong></td>
<td><strong>GR3A Casual Interchanges Equalized Relationship (48)</strong></td>
</tr>
<tr>
<td><strong>Theme 2: Developing Mathematical Knowledge</strong></td>
<td><strong>GR4A Participants Offer Knowledge (79)</strong></td>
</tr>
<tr>
<td><strong>GR4B Strategies Are Explained in Words, And Or Pictures (46)</strong></td>
<td><strong>GR5A Justifications/Rationales are Provided (80)</strong></td>
</tr>
<tr>
<td><strong>GR5B Steps In Solutions Are Explained (49)</strong></td>
<td><strong>GR5A Justifications/Rationales are Provided (80)</strong></td>
</tr>
<tr>
<td><strong>GR6A Ideas And Solutions Discussed With Others (147)</strong></td>
<td><strong>GR6A Ideas and Solutions Discussed w/ Others (230)</strong></td>
</tr>
<tr>
<td><strong>GR6D Community Members Ask Questions To Try To Understand Thinking (21)</strong></td>
<td><strong>GR6D Community Members Ask Questions To Try To Understand Thinking (21)</strong></td>
</tr>
<tr>
<td><strong>GR7B Many Ways Solving Problems Shared (77)</strong></td>
<td><strong>GR7B Many Ways Solving Problems Shared (26)</strong></td>
</tr>
<tr>
<td><strong>Theme 3: Strengthening Critical Thinking</strong></td>
<td><strong>GR7B Many Ways Solving Problems Shared (26)</strong></td>
</tr>
</tbody>
</table>

*Common elements to both student groups are highlighted*
The following section examines the implementation of the most frequently used elements those used more than twenty times by Mrs. Washington and her students.

**Everyone Invited To Contribute (GR1)**

The elements within the ground rule *Everyone Invited to Contribute (GR1)* are used to encourage students to participate in the mathematical discourse. Along with the effective planning of lessons to include participation, the elements promoted a respectful environment where contributions were valued, and joint thinking along with problem-solving was expected. The following section describes how Mrs. Washington and her students utilized the strategic discourse elements within the first ground rule.

**Everyone is Encouraged To Contribute Without Being Singled Out (GR1A)**

Mrs. Washington consistently involved students using mathematical discussions. She *Encouraged Everyone to Contribute Without Being Singled Out (GR1A)* when she invited students to participate by engaging them in conversation about content and problems. She also allowed students to jump into conversations, not requiring them to raise their hands or wait to be called. Mrs. Washington students also *Encouraged Everyone to Contribute Without Being Singled Out (GR1A)*. Mrs. Washington students utilized the ground rule *Encouraged Everyone to Contribute Without Being Singled Out (GR1A)* by seizing on opportunities to participate. They also used the element *Casual Interchanges Demonstrating Equalized Relationship* often as they jumped into conversations, or made comments about the contributions made by students and teachers.

One strategic discourse strategy that Mrs. Washington used often to increase participation was the "Turn and Talk" strategy. Turn and Talk was used to encourage all students to engage with other classmates and ensured that all students had an opportunity
to express their thinking about a topic. The “Turn and Talk” questions were open-ended.

A few examples of the Turn and Talk prompts included "When is it important to know the area and perimeter of a space?" and, "Why are the product and sum not the same" (Event 5, p. 9-10)? After students talked amongst themselves, they shared answers to questions such as, "What did you talk to your friends about?" or "What were you chatting about" (Event 2, p.3)?

Mrs. Washington’s often implemented full class discussions and engaged all students in discourse about the topics and concepts that they were learning. She initiated discussions using a question or statement to draw students into the conversation. Examples of these questions included, "How did I collect my data," or "When is it important to know the area and perimeter of a space?" These questions were posed to invite all to participate in a conversation and encourage as many students as possible to join.

Mrs. Washington also revisited prior knowledge as a way to increase student involvement. For example, she supported students during a review of the data collection process before they collected and organized the data for themselves. While discussing the process, students shared what they knew about the topic. She used these strategies to ensure all students had the background for moving forward with the concept. During reviews, Mrs. Washington used open-ended question to draw students into the conversation. She asked, "How did I collect my data" (Event 1. p. 1)? Mrs. Washington also posed follow up questions to continue the engagement. She asked, "What is the purpose of a tally chart and what did it help me to figure out?" and, "Could I have collected my data another way?" Students answered the question by providing examples
of data collection methods that could be used. This warmup connected students with prior experiences needed for the next step in the lesson.

Mrs. Washington also framed questions in supportive ways to provide guidance and direction for students while participating in the discussion. She often offered context clues or repeated a part of the contribution. During the second Event for example, Mrs. Washington solicited possible data collection techniques from students.

One student suggested, "You could have used a pictograph" (Event 2, p.2). Mrs. Washington used “pictograph” in the follow up question.

She asked, "How could I have used a pictograph to collect my data?" Including part of the student's statement clarified the question. Embedding part of the student’s answer in her question directed attention to the idea and helped the whole class follow the conversation, keeping the participation level high.

Context clues were also used by Mrs. Washington to provide clarity to assist in understanding the question. For example, during a discussion about area and perimeter with students (Event 5, p.9). Mrs. Washington included a context clue to alert students to an earlier activity.

She asked, "How would you explain it, although you are in different spaces, how would you explain that you are all in the area?" This context clue "although you are in different spaces" was provided to remind students of the activity they participated in earlier in the day.

Mrs. Washington's students implemented the element Encouraged Everyone to Contribute Without Being Singled Out (GR1A) by joining the mathematical discussions when they had an idea or solution to share with the class. They were highly engaged in
the discourse and did not wait to be called on by their teacher or other students to participate.

**Students Are Chosen Strategically By The Teacher to Contribute (GR1B)**

Mrs. Washington used the element *Students Are Chosen Strategically By The Teacher To Contribute (GR1B)*. She encouraged participation just slightly less than she called on specific students. Students were chosen strategically after they answered a question or suggested an idea. After that, Mrs. Washington interacted with individual student about their ideas. This gave students opportunities to communicate their knowledge in depth. It also allowed Mrs. Washington to gather more information about their thinking.

During a discussion in Event 1 for example, Mrs. Washington asked students to tell her how tall she was and to explain how they figured this information out (p. 1-2). Students shared strategies.

The first student shared her answer, "You are 5 feet and one inch."

Mrs. Washington asked a follow-up question to prompt the student to justify her thinking. She asked, "How did you figure that out?"

The student said, "I know that 12 times five is 60." Mrs. Washington highlighted her thinking and asked, "So your strategy was multiplication?" The student provided additional information, "5 times 12 is 60 plus one is 61." Afterwards, another student shared his strategy. He said, "I was thinking that you could do 2 X12 = 24 and 24 plus one yard equals sixty, and then you just add one inch" (Event 1, p. 2).
Mrs. Washington strategically asked a follow-up question to the same student. She asked, "What does one yard equal?" After he answered 36, she concluded by saying, "So 24 plus 36 equals 60, plus one inch, wow!"

Additionally, during Event 4, Mrs. Washington encouraged students to consider the possibility of using an array to find the area of a shape. One student made the connection that an array could also be used to determine the perimeter of the shape. Mrs. Washington reacted to this comment with a probing question for her, "How can you do perimeter with an array" (Event 4, p.7)?

The student responded, "If you put four on the opposite sides and three on the bottom and top then add all of that up."

The following section describes how Mrs. Washington and her students utilized the strategic discourse elements within the third ground rule Atmosphere of Trust Is Present (GR3).

**Atmosphere of Trust Is Present (GR3)**

The elements within *Atmosphere Of Trust (GR3)* were used to create an environment that was supportive to students as they explored ideas and took mathematical risks. Establishing an *atmosphere of trust* allowed students to participate knowing that if and when they made mistakes, their contributions would still be valued. When teachers constructed a culture built on *Atmosphere of Trust* they provided opportunities for students to interact in positive ways with others. The teachers, in this study, cultivated an atmosphere of trust that resulted in students and teachers using this ground rule frequently, see Figure 6.1 and 6.2. The following section describes how Mrs.
Washington and her students implemented the strategic discourse elements within the first ground rule.

**Casual Interchanges Demonstrating Equalized Relationships (GR3A)**

Mrs. Washington and her students utilized the element *Casual Interchanges Demonstrating Equalized Relationships (GR3A)* as they interacted with one another and ideas during mathematical discourse with students. There was evidence of this *equalized relationship* in the ways Mrs. Washington promoted interactions with and among students during mathematical discussions. She supported students to participate, not insisting that they be called on to join the conversation. This allowed students to join conversations spontaneously when they had something to add. She also validated thinking by including their idea in her statements during discussion to highlight their ingenuity. At times, she put herself in the place of the learner by soliciting students’ assistance to help her to understand the concepts. In addition, Mrs. Washington teased students and made funny comments to raise the level of engagement among students.

Ample opportunities to talk through ideas and tasks were also provided. Additionally, she also promoted the ideology that each student required different things. This was evident in the differentiated lesson planning and questioning that she used. Likewise, the students accepted the possibility that the work would look different, and this was an acceptable way of life in the classroom.

Mrs. Washington celebrated when students contributed to the thinking. She built up their confidence and spotlighted their thinking using casual interactions and language. For example, Mrs. Washington would dramatically respond to their ideas with an animated, "Oh, what?", "Stand up lady!" and "Tell them!" when a student had something
interesting to say. One time when a student made a connection between a pictograph and tally chart, Mrs. Washington called out, "Oooooh! She followed this with the question, "How could it be similar and different at the same time?" prompting the student to provide her insights to the class (Event 2, p. 3). Mrs. Washington utilized these casual interactions to draw attention to creative thinking while motivating students to contribute.

Additionally, Mrs. Washington demonstrated her promotion of equalized relationships as she shifted the authority to her students. Students assumed the role of teacher, and vice versa. Mrs. Washington requested that students "write it out for me" or to explain to her when she asked questions such as, "what is he talking about?" On one occasion a student mentioned using division instead of multiplication. Mrs. Washington called out, "Get up here, I have no idea, what you are talking about!" to encourage the student to explain it to her and the class (Event 1, p. 2).

The equalized relationship established by Mrs. Washington also existed among students. Students demonstrated the equalized relationship in the ways that they interacted with one another and their teacher. Students joined conversations, explained their thinking and asked questions during mathematical discussions. The student also positively interacted with classmates. Although it was general knowledge that some students had different needs, students focused on themselves and demonstrated respect for one another.

Most of the discourse between and among students were facilitated by Mrs. Washington. During these exchanges, it was clear that everyone was part of the process. All students had the opportunity to participate and share their knowledge. They took turns participating in mathematical discussions. She invited students to interact with others by
asking them to comment about the contributions of their peers. During Event 4, she said,
For example, during the discussion in Event 4 a student jumped into the "What is an
array" conversation by commenting, "An array can help you find the area, Somebody
help me out with this" (p. 7). After a student responded that you can also use arrays to
find the perimeter, she asked, "Do you agree or disagree?" Most students joined by
responding "Yes!" to the first question. Then Mrs. Washington continued the
conversation by inviting students to share reasons to support why they agree or disagree.
Students took turns sharing.

Students demonstrated their acceptance of an equalized relationship in the ways
they interacted playfully with their teacher. Students responded well to Mrs.
Washington's humorous interactions, and a few times even made silly comments directed
toward her. For example, while figuring how tall their teacher was during Event 1, Ms.
Washington said, "I am tall aren't I" (p.1)? When students responded by shouting “no”,
Mrs. Washington went right along with it. These impromptu contributions by students
were always accepted as appropriate.

Additionally, students demonstrated the existence of an equalized relationship
through the ways they participated in discourse. Students were partners in learning. They
were given the authority to freely communicate. They freely communicated during
conversations where they openly asked questions. One example took place during the
cupcake lesson. Mrs. Washington invited students to question herself and their peers
about the task. She asked, "Any questions about what you are doing" (Event 3, p. 4-6)?
Students, without specific invitation, jump in to add to the conversation. During these
discussions students asked questions that pushed the task into a new direction. They
asked, "Can we do other graphs that we have not learned about but that we know about, like pie charts? And "Can it be a graph that we have not learned about yet?" Mrs. Washington invited these questions which demonstrated her willingness to allow student input to impact the outcome of the projects.

Additionally, students demonstrated an *equalized relationship* when they confidently let their teacher know when they disagreed with something that she said.

Once when Mrs. Washington purposely made the incorrect statement, "If you are using the same numbers you should have the same answer. The product and the sum should be the same." Several students reacted immediately to demonstrate their disagreement with her statement. They were adamant that she was wrong and that they were correct. This example was one of the instances that demonstrated the ease in which this teacher and her students interacted with one another.

The following section describes the implementation of the ground rules and elements within the first theme Encouraging Student Participation most frequently, more than twenty times, by Ms. Littleton and her students.

**Ms. Littleton and Students**

Overall, Mrs. Littleton used the ground rules within the first theme Encouraging Student Participation 315 times during seven classroom events. While she engaged students in mathematical conversations, Ms. Littleton utilized the ground rules *Everyone Invited To Contribute (GR1)* on 164 occasions, *Contributions and Opinions Treated Respectfully (GR2)* on 62 occasions and *Atmosphere of Trust (GR3)* on 89 occasions. Ms. Littleton's students also utilized the ground rules within the theme Encouraging Student Participation 145 times. They used the ground rules *Everyone Invited To Contribute*
(GR1) on 25 occasions, Contributions And Opinions Treated Respectfully (GR2) on 14 occasions and Atmosphere of Trust (GR3) 106 times during the study.

Of the nine elements composing the ground rules in this theme, Ms. Littleton's utilized four most often. She implemented the elements Everyone Invited To Contribute (GR1A) on 35 occasions, and Students Are Chosen Strategically By The Teacher To Contribute (GR1B) 129 times. She also implemented Praise Is Given For Relevant Ideas (GR2A) 42 times, as well as, Praise And Encouragement Offered (GR3B) on 48 occasions while engaging students in mathematical conversations. Ms. Littleton's students implemented two elements most often. They used Students Are Chosen Strategically By The Student To Contribute (GR1B) 21 times and Causal Interchanges Demonstrate Equalized Relationship (GR3A) 48 times.

The next following section describes the implementation of these elements, those used more than 20 times, within the first theme Encouraging Student Participation by Ms. Littleton and her students.

**Everyone Invited To Contribute (GR1)**

The elements within the ground rule Everyone Invited to Contribute (GR1) are used to inspire students to participate in mathematical discourse. Along with the effective planning of lessons that promote participation, the elements promoted a respectful environment where members contributions were valued, and joint thinking and problem solving was expected.

**Students Are Encouraged To Contribute (GR1A)**

Ms. Littleton utilized the element Encouraged Everyone to Contribute Without Being Singled Out (GR1A) when she invited all students to participate in conversation, as
seen in Table 14. She consistently promoted participation. Ms. Littleton initiated discussion using prompts directed toward the whole class. The questions posed to students were typically open-ended. During these discussions, solutions were explained and small group experiences were investigated.

Ms. Littleton easily conversed back and forth with her students, as well as, facilitated talk among students. She asked open ended questions to encourage students to interact with their contributions offered by their peers. These questions included, "Who would like to share out how you solved this problem?", "Would you like to share your thinking?" or, "What went well?" Students responded, sharing their solutions and answering additional questions. Additionally, before closing the lesson, Ms. Littleton invited additional contributions and said, "Alright, any other questions?" The open-ended questions allowed Ms. Littleton to encourage all to participate, with little restrictions placed on the type of ideas accepted.

Ms. Littleton also *encouraged students to contribute* during small group work. She and her students conversed about problems and task completed by students. During these discussions, Ms. Littleton encouraged students to participate in discussion using questions to probe their thinking. Although the questions Ms. Littleton asked in small group were not as open ended, but focused on assessing understanding and problem-solving abilities, students were free to join the conversations without being called on directly. During one small group session, students were calculating the amount of ribbon needed to wrap their box while working in small groups. Ms. Littleton encouraged them to participate in a conversation by asking, "What do you think that you did that helped you find an accurate measure?"
A student jumped in and responded, "We measured the length and width of the box and then added them" (Event 6, p. 22).

To continue the dialogue, Ms. Littleton invited other group members to contribute by asking, "Did you underestimate or over-estimate?"

One of the other group members responded, "Overestimate, by a little, not too much."

Ms. Littleton circulated from group to group to observe their interactions and encourage participation. She stepped into their conversations to check in on progress, set or reset direction and helped students communicate their ideas. A final question was posed to encourage students to participate by reflecting about the Wrapping The Present task. She asked, "What would you do differently if you had a different size box" (Event, 6, p.22)?

**Students Are Chosen Strategically By the Teacher/Student to Contribute (GR1B)**

Ms. Littleton also utilized the element *Students Are Chosen Strategically (GR1B)*. She selected particular students to share during mathematical discussions. She also directed her students to question other students. Table 14 reveals that Ms. Littleton strategically directed questions at specific students more often that she invited open contributions for the entire class.

Ms. Littleton planned her math classes to include a problem that students completed in small groups. She interacted with each of these small groups, strategically selecting students to check in with, as she monitored the progress. The small group work was followed up with a class discussion. Many questions during full group discussions were asked and then individual students were called on to contribute. Ms. Littleton
targeted specific questions to gather deeper explanations about students’ thinking or strategies for solving problems. She also questioned students to assist students to interpret their thinking for their class or to even rethink their solutions.

For example, a student shared her thinking about the number of right angles that would be equal to a full circle. The student said, "I know that there's 90 degrees is one part of a 360-degree circle" (Event 1, p. 1-2). Ms. Littleton questioned her to gather more insight about her explanation.

She asked, "You know that 90 degrees is what?"

The student responded by offering more information,

I know that 90 degrees is one-fourth of the 360-degree circle, so I divided 90 by four, 360 degrees by 90 and I got 4. I also made a circle and divided it into four pieces starting with 90 here, 90 here, 90 here, and 90 here. And there's four pieces" (Event 1, p.1).

Ms. Littleton also chose students strategically when she encouraged them to interact with the ideas of their peers. For example, she asked students to reflect on specific ideas presented. Ms. Littleton asked "Boys and girls do we have any questions or suggestions about how they are solving the problem?" or "Boys and girls do we have any questions or suggestions about how they are solving the problem?" Additionally, while students were finding missing angle measurements during Event 2, Ms. Littleton questioned, "She connected her thinking with what?" and "What else did we learn this year" (p. 2)?

When a student said, "Fractions."

She followed the response by saying, "Fractions, yes, you can use fractions. I saw some other people that also connected their thinking with fractions." This additional statement was offered by Ms. Littleton to facilitate additional exchanges among students.
Ms. Littleton's students also utilized the element *Students Are Chosen Strategically by encouraging Students To Contribute (GRIB)* frequently. During these instances, students asked their teacher or classmates questions about thinking or solutions shared during a mathematical conversation. Moreover, they openly questioned other students and their teacher when they needed clarification or disagreed with a solution.

In Event 3, for example, students shared units that were equivalent to four quarts. When one student offered "16 cups" as her answer, Ms. Littleton questioned her, "How did you figure that out" (Event 3, p.8)?

The student explained "Pint is two cups, and two quarts is 8 cups. 2, 4, 6, 8….

Another student spoke up and asked the teacher, "Shouldn't it be 32? Shouldn't it be two pints in each quart and she only did one pint in each quart."

Ms. Littleton skillfully directed the discussion back to the group by saying, "I don't know, what do you think?" When students did not respond immediately, she rephrased the question to encourage participation. She asked, "So why do you think she thinks it should be 16 and she thinks it should be 32? What do you think friends?"

A third student added, "I agree with her because there in a quart there is 4 cups, so 16 cups because four times 4 is 16 cups."

Ms. Littleton's students asked and answered questions from their peers proficiently on their own. Also in Event 3, a student offered a final equivalent measurement to four quarts saying, "128 fluid ounces."

This statement resulted in another student asking this student, "Wait! How did you know that?" She was impressed by his calculations.
Students also utilized the element students are chosen strategically by the student to contribute while they presented solutions to their capacity problems during Event 4. After sharing their solutions and strategies, students *strategically chose other students* to ask questions about their presentations. For example, after the second group presented their solution, they selected two students from the class to ask them a question about their presentation. The first question included, "Did you get confused at any of the parts?" One of the group members answered, "Yes. when Ms. Littleton came over and asked how did you come up with the four and I didn't even know how we came up with the four." The second student was selected, and he asked, "Which way did you like best?" Two group members responded by saying that they liked addition, due to that strategy being easier to use.

The following section describes the implementation of the elements within the second ground rule *Contributions And Opinions Treated Respectfully (GR2)* within the first theme Encouraging Student Participation among Ms. Littleton and her students.

**Contributions And Opinions Treated Respectfully (GR2)**

The elements within the ground rule *Contributions And Opinions Treated Respectfully (GR2)* included using praise to communicate to students about their important contributions and relevant ideas. Additionally, the elements provided positive feedback to students after they contribute to the mathematical discourse.

**Praise Is Given For Relevant Ideas (GR2A)**

Ms. Littleton provided feedback to students as they participated in mathematical discourse using the element *Praise For Relevant Ideas (GR2A)*. She often endorsed her students' relevant contributions by offering praise. Praise was implemented using
supportive and motivating words to encourage participation. Ms. Littleton used phrases such as, "Awesome" "Great" or "Nice." All contributions were validated in this way.

The following section describes the implementation of the third ground rule *Atmosphere Of Trust Is Present (GR3)* within the first theme Encouraging Student Participation among Ms. Littleton and her students.

**Atmosphere Of Trust Is Present (GR3)**

The elements within the ground rule *Atmosphere Of Trust (GR3)* were used to create an environment that is supportive to students as they explore ideas and take risks mathematically. An atmosphere of trust enables students to participate knowing that if and when they make mistakes, their contributions are valued. Both teachers in this study, cultivated an *Atmosphere of Trust*. They valued student contributions and often included opportunities for students to share what they know with others.

**Causal Interchanges Demonstrating Equalized Relationships (GR3A)**

One example of students using the element *Causal Interchanges Demonstrating Equalized Relationships (GR3A)* occurred when student spoke up during a mathematical discussion during Event 2. The exchange occurred when a student shared her concern. She told the teacher that half of the members in her group had a different and possibly incorrect solution than the other half. Raising this concern during the class discussion, the student stated, "She had a different answer than the rest of our table group" (Event 2, p.6)

Attempting to support the student, Ms. Littleton asked, "Did you talk to your table group about how you were solving the problem?"

The student responded, "He has the same answer as she does, but everyone else does not."
The teacher asked the student who had the other answer to explain the difference. She explained, "At first we did it the way they did it, and then we realized that we thought it wasn't correct so my partner said we should probably do 180 plus 76 because 76 is part of the straight angle."

The teacher treated this contribution respectfully and turned to the class to solicit their encouragement and advice. She asks, "Boys and girls do we have any questions or suggestions about how they are solving the problem?"

A student, not in the same group, offers his suggestion respectfully, "76 is the acute angle (shows the angle with his hands), and 180 is the straight one (shows angle with his hands), so you should have subtracted instead of added."

Next, Ms. Littleton turns to the group with the two answers and gently asks, "Does that make sense?" They answer “yes”, and the discussion moves on.

An additional example when students demonstrated evidence of the existence of an equalized relationship between students occurred during the capacity lesson in Event 3. A student raised a concern that someone has repeated an answer during the capacity lesson. The discussion took place while students were sharing multiple units used in measuring capacity.

One student presented, "Gallons" as a unit of measurement. Then another student immediately spoke up and said, "I said that" to acknowledge the repeat (Event 3, p. 8).

An additional example occurred when a student caught a peer’s error during the capacity problem presentations during. After a student used millimeters as a unit to quantify her answer, and a student in the audience respectfully pointed out the error asking, "Did you say millimeters or milliliters" Event 4, (p.14)? Ms. Littleton acknowledged the error and
the discussion moved on. Moreover, students also corrected their teacher Ms. Littleton. For example, this occurred while they worked on the capacity problems. Ms. Littleton projected the problem on the board with a spelling error. A student pointed out a spelling mistake on the whiteboard. Ms. Littleton simply said, "Thank you, I will fix that" (Event 3, p.10).

Ms. Littleton demonstrated her acceptance of an equalized relationship when she acknowledged making a mistake during the bow lesson. She did not estimate enough ribbon to tie a bow. While owning the mistake, she encouraged students to make a better decision and said, "I did a bad estimate here, you want to make sure you have enough to tie a bow after you wrap the ribbon around the width and the length."

**Praise and Encouragement Offered (GR3B)**

Ms. Littleton implemented the element *Praise and Encouragement Offered* (GR3B). She provided verbal feedback when they contributed. Table 14 reveals students responded well to the encouragement and willingly participated in mathematical discussions. Ms. Littleton utilized the element *Praise And Encouragement Offered* (GR3B). She encouraged participation by building trust with students. Praise and encouragement were offered by Ms. Littleton to her students when they contributed important ideas to mathematical discussions. This praise took the form of one-word responses, as noted above, and was used to acknowledge a student's contribution. However, the praise did not include feedback about the contribution but instead was offered as a thank you for being part of the conversation. The praise and encouragement in this ground rule usually included more than one-word comments. For example, when students shared an idea or solution, Ms. Littleton responded with either, "Ok, nice job, so
she connected her thinking with what?" or "Thank you for clarifying." She also offered comments such as, Good, try not to repeat." And "Awesome, so you are recognizing a relationship between different units of measure," Or "Neat handwriting."

Students also utilized the ground rule Atmosphere of Trust (GR3). They used the element Casual Interchanges Demonstrating Equalized Relationships (GR3A) most often in this ground rule. Students did not hesitate to engage in discourse during both small group or large group work. They asked questions, pointed out mistakes, offered comments and inquired about the thinking of others. They shared their thoughts and never hesitated to raise a concern.

As mentioned earlier, Ms. Littleton modeled this questioning culture. However, even though she questioned their thinking, Ms. Littleton treated students respectfully and encouraged their participation. At times errors were identified, and feedback was provided to students including advice with explanations and suggested strategies for arriving at the correct answer. She also encouraged her students to provide support to their peers. In these cases, students provided ideas about how to solve a problem. All of these strategies promoted a culture where equalized relationships existed.

**Summary of Theme 1: Encouraging Student Participation**

Both teachers utilized many of the elements to encourage every student to participate in mathematical discussions. All students frequently utilized all three ground rules in the first theme Encouraging Students to Participate in their discourse with students, see Figures 6.1. They provided support to one another during mathematical conversation to discuss concepts and solve problems. All teachers and students interacted at a high level throughout the study, according to the data.
The first ground rule *Everyone Invited To Contribute (GR1)* was utilized most frequently within this theme. Mrs. Washington consistently and *strategically chose students to participate* and *invited all students to openly contribute ideas* (GR1A and GR1B). This resulted in a high level of participation during classroom discussions. They consistently joined the conversations, almost always jumping in when they had something relevant to share.

Ms. Littleton and her students also frequently utilized the three ground rules in the first theme Encouraging Students to Participate in their discourse with students, see Figure 6.2. Of the three ground rules, she and her students utilized the first ground rule *Everyone Invited To Contribute (GR1)* most frequently. Ms. Littleton provided opportunities for students to participate in discussions both small and large group formats. Ms. Littleton *chose students to contribute (GR1B)* by selecting students to share ideas, as well as, invited students to contribute (GR1A) their thinking in both of these settings. Students also *Strategically Selected Students to Contribute (GR1B)* when engaging with peers during discussions and presentations.

Ms. Littleton utilized the ground rule *Contributions And Opinions Treated Respectfully (GR2)* using the element *Praise Is Given For Relevant Ideas (GR2A)*. She implemented the element *Praise For Relevant Ideas (GR2A)* frequently to provide feedback to students as they participated in the mathematical discourse. She validated students' relevant contributions by offering praise. This positive message was well received by students.

Mrs. Washington and her students utilized the elements within the third ground rule *Atmosphere of Trust (GR3)*. Both utilized the element *Casual Interchanges*
Demonstrating Equalized Relationships (GR3A) during mathematical discourse. There was evidence of this equalized relationship in the ways Mrs. Washington interacted with students during mathematical discussions. She encouraged full participation and wanted all to join the conversation, not waiting to be called on to join the conversation. She worked to make students feel comfortable and respected while studying mathematics together. She also explained, clarified, and reinforced concepts with her class. At times, she took on the role of the learner and not the expert. Mrs. Washington promoted a casual environment, often joking to encourage engagement. Students often contributed, even when they were unsure of answers. All contributions were accepted and respected by the community. Cultivating an atmosphere where all were valued members certainly resulted in a high level of participation among students in both classrooms.

Mrs. Littleton and her students also implemented the ground rule Atmosphere of Trust (GR3) very frequently while engaged in mathematical discourse. Ms. Littleton implemented the element Praise and Encouragement Offered (GR3B) most often, encouraging students as they contributed to the discussions. She treated students respectfully and always encouraged to participate. Ms. Littleton provided many opportunities for students to exchange ideas and to think while learning mathematics together. Students responded well to the encouragement as evident in the high level of participate in the data. Ms. Littleton's attention to precision meant that she questioned her students' thinking. However, these interactions were supportive and respectful. Often peers were given opportunities to help other peers in these situations.

Ms. Littleton's and Mrs. Washington's students also used the element Casual Interchanges Demonstrating Equalized Relationships (GR3A) often throughout the study.
They were comfortable in their role as partners in learning. They shared their thinking freely in large and small group settings. Mrs. Washington's students playfully joked with their teacher and their peers while engaging in conversations. Additionally, if someone contributed an unclear idea or error, Ms. Littleton's students did not hesitate to question it. Students were active participants who critically examined the contributions presented. It was clear that focusing on the correct answer was important, as was interacting with respect.

The following section explores the implementation of discourse strategies used by teachers with their students from the theme Developing Mathematical Knowledge.

**Theme 2: Developing Mathematical Knowledge**

The types of strategic discourse strategies that are aligned with the theme Developing Mathematical Knowledge promoted engaging students in mathematical conversations to develop greater knowledge of content, with others in the community. Primarily students broadened their knowledge of concepts and strategies when they worked with others during problem-solving. The elements within the five ground rules included in this theme encouraged students to share their knowledge with peers. Sharing knowledge meant that students provided rationales, justifications, and described steps in their solutions. Students were also encouraged to interact with others as they built this knowledge together. This required them to ask questions, interact with and build on ideas, generate multiple paths to solve problems and support one another in the process. All elements combined provided opportunities to discover, process and communicate ideas while students solved problems with others in mathematics. Both teachers in this study
implemented all ground rules five through eight to supported students to Develop Mathematical Knowledge.

Mrs. Washington supported her students in the development of mathematical knowledge using the ground rules in the theme Developing Mathematical Knowledge 432 times during the eight-classroom events, see Figure 1. Table 10 reveals that she utilized the ground rules; Knowledge is Made Public (GR4) on 55 occasions, Reasoning Is Visible in the Talk (GR5) on nine occasions, Engage In Joint Reasoning (GR6) on 271 occasions, Multiple Solutions Are Encouraged (GR7) on 75 occasions and Contributions Are Built on Prior Proposals (GR8) on 23 occasions.

The next section examines the implementation of the elements, those used more than twenty times, comprising the majority of the discourse within the second theme Developing Mathematical Knowledge among Mrs. Washington and her students.

Mrs. Washington and Students

Table 10 reveals Mrs. Washington’s implementation of the nineteen elements that composed the ground rules in this theme. Mrs. Washington's utilized eight of the elements frequently. She utilized Contributions Are Restated (GR4C) on 46 occasions, Ideas and Solutions Discussed with Others (GR6A) most frequently with 76 occurrences, Questions Are Posed to The Community to Direct Thinking (GR6B) 52 times, Questions Are Posed To Encourage Exchange of Ideas (GR6C), 40 times, Community Members Ask Questions to Understanding Thinking (GR6D) 73 times, Tasks Are Assigned to Initiate Working Together to Find Solutions (GR6F) 40 times, and Assistance is Offered to Help Work Through The Process or Scaffold Learning (GR6G) 23 times. Mrs. Washington also utilized Many Ways of Solving Problems Encouraged (GR7A) on 62 occasions.
Table 11 also reveals Mrs. Washington’s students utilized five elements frequently. They used *Participants Offer Knowledge About Mathematics (GR4A)* on 79 occasions and *Strategies Are Explained in Words And/Or Pictures (GR4B)* on 22 occasions. They also utilized *Justifications And Rationales Are Provided (GR5A)* on 69 occasions. *Ideas and Solutions Discussed with Others (GR6A)* were used most frequently by students marking 147 occurrences. Students also utilized *Many Ways Of Solving Problems Shared (GR7B)* 77 times during the eight events.

**Mrs. Washington and Students**

**Knowledge Is Made Public (GR4)**

Mrs. Washington consistently assisted students in Developing Mathematical Knowledge by having students explain their understanding to others using the ground rule *Knowledge Is Made Public (GR4)*. She posed questions and prompted thinking during discussions to reveal what students knew and did not know about a topic. She also encouraged students to propose solutions to the problems she presented. The explanations of these solutions included students using words, pictures and/or numbers to communicate their thinking and process they used to arrive at an answer. Additionally, Mrs. Washington frequently restated explanations students provided to verify their thought process and to direct the group's attention to key ideas contributed by each student. The following section explains the implementation of the most frequently used elements in the fourth ground rule.

**Participants Offer Knowledge About Mathematics (GR4A)**

Mrs. Washington's students frequently utilized the element *Participants Offer Knowledge About Mathematics (GR4A)* to demonstrate what they understood about the
mathematics discussed during discourse. Students responded to the teacher's questions, reacted to ideas and shared thoughts about the contributions of others.

Students offered their knowledge when they answered questions posed to them by their teacher. During Event 2 for example, Mrs. Washington asked, "What is the purpose of the tally chart" (p. 2-4)? One student shared his thinking by connecting the tally chart question to the data collection activity the group had completed earlier in the day. The student communicated his thinking by interpreting the data.

He said, "It said yes or no and whatever has yes tells how many people go the town library and not a lot of people go the to the town library."

Another example demonstrating students Offering Knowledge About Mathematics (GR4A) occurred during Event 9. Mrs. Washington asked, "Why is it important to know the area and perimeter of a space" (p. 9-10)?

One student said, "It helps to know the perimeter of your yard so that you know where to build the fence and where you are not going to put it up."

Mrs. Washington pressed for more and asked, "Would I need to know that or another reason? The same student responded again.

She said, "It is going to help you know how much fence to buy." An additional student reacting to this exchange came to the board and explained her thinking using pictures, numbers, and words. She drew the calculation that she used to determine the perimeter of the space. She labeled each side of the parallelogram with 5, 5, 7, and 7. She explained that the area was 35 and the perimeter was 24.
Encouraging more students to *Offer Knowledge (GR4A)* and to engage with the ideas being shared, Mrs. Washington asked, "Why did she get two different answers, her product did not equal her sum?"

A student jumped in to answer this question idea shared her knowledge and said, "Because you are multiplying the area and adding the perimeter."

**Strategies Are Explained in Words, Pictures And/Or Numbers (GR4B)**

Students explained their thinking using words and numbers when asked to share how they solved a problem. As noted in the examples above, students wanted to come up to the whiteboard to represent the number sentences they used to solve the problem, during their explanations. This was the case during Event 4; a student came to the board to share his explanation of an array. He referred to the representation he had drawn and said, "Basically it is a square. It's an eight by eight array" (p. 7-8). He continued, "Then you count by six, the rows and the columns, 6, 12, 18, 24, 30 and 36." Mrs. Washington praised his work, "Beautiful, it is great to give a visual."

Other times during the eight events, students explained their reasoning citing the numbers or numerals used in their strategies. For example, students would state, "I added five times on my fingers, and I got 60" and "I added 12 until I got to 48" or "I divided 61 by 12." Although some problems were solved using calculations and resulted in students simply stating the answer. Most solutions offered included numbers and words to explain their solutions.
Mrs. Washington used the element *Contributions Are Restated (GR4C)* frequently. She utilized this element to draw her students' attention toward a particular statement or question, or to emphasize an idea.

In Event 4 for example, Mrs. Washington used a student's contribution to frame her question. Using the student’s words helped to direct students’ attention to her question. After the student offered, "An array can help you find the area," Mrs. Washington restated the student’s phrase in a question (Event 4, p.6-10). She asked, "Oh! An array can help you figure out the area, is she right?"

After ideas were exchanged, Mrs. Washington followed up with another similar question. She asked, "So an array is something that can help you figure out the area, do you agree?" Students shared their thinking afterward.

Later in the same event, Mrs. Washington restated a contribution to assess the students thinking and to further develop the concept. The question she posed asked students to think about why the solution involving area and the perimeter was different when the shape was identical. A student responded, "For the perimeter, you are just adding and not doing multiplication." Another student added to this statement, "Because you are not doing multiplication you are finding the space of the outline, not all of the inside."

Next, Mrs. Washington asked, "Can I interpret what you are saying?" Then, she clarified this student's contribution so that all can understand the concept.
The following section explains the implementation of the most frequently used elements by Mrs. Washington and her students within the fifth ground rule Reasoning Is Visible In The Talk (GR5).

**Reasoning Is Visible In The Talk (GR5)**

Mrs. Washington students engaged in Developing Mathematical Knowledge as they shared their thinking using the ground rule *Reasoning Is Visible In The Talk (GR5)*. Students demonstrated their reasoning as they provided explanations about how they solved problems. Students described strategies, including the steps they used. At times, students provided greater insight into their thinking and solutions. These statements included justifications about why they included the steps they had taken.

**Justifications/Rationales Are Provided To Explain Thinking (GR5A)**

Students utilized the element *Justifications/Rationales Are Provided To Explain Thinking (GR5A)* as part of the discourse with their teacher and classmates. These illuminations occurred when prompted to explain their thinking.

In Event 2 for example, Mrs. Washington asked students to compare a tally chart with a pictograph. They were encouraged to think about the ideas that were being shared and share a justification. Several questions were posed to prompt students to contribute. Mrs. Washington asked, “We used pictographs after we collected the data right, why did we do that” (p. 3-4). After students discussed their thinking with a partner in a “turn and talk” they shared their ideas with the class. The first student shared his thinking. He said, “You can do the pictograph first, but it would be hard because it would take a lot more thinking.”
Another student stated a justification, “It would be hard to do it that way because the symbol means more than just one. “She then invited more ideas, “Girls what were you chatting about?”

One of the girls added, “A tally chart is probably easier.”

Mrs. Washington directed the discourse by posing the question, “What do we add to the tally chart to make them a little different from a tally chart?”

“A key,” said a student.

Mrs. Washington added a question, “So the purpose is to show the same data but with different symbols?”

A student justified her thinking. She said,

You don’t have to use a tally chart before you use a tally chart but it can be more efficient because if you could double check your work in the rough draft before you do your final draft and if you don’t do something before you would be messed up. (Event 2, p. 3-4)

Then another student reacted to this idea and said, “Are you saying that collecting the data in a tally chart is liked the first draft and putting your data into a pictograph is like making a final copy?”

The following section explains the implementation of the most frequently used elements by Mrs. Washington and her students within the sixth ground rule Engaging In Joint Reasoning (GR6).

**Engage in Joint Reasoning (GR6)**

Mrs. Washington consistently assisted students to Develop Mathematical Knowledge about the concepts they studied using the ground rule Engage in Joint Reasoning (GR6). Students were encouraged to build their knowledge by participating in lessons to encouraged sharing, discussing and questioning. Students also engaged in
discourse with others to review the mathematics they needed to solve problems. Also, prior knowledge was highlighted to reinforce old and new content while they solved real-world problems and tasks with others.

The following section explains the implementation of the most frequently used elements by Mrs. Washington and her students within the ground rule *Engage In Joint Reasoning* (*GR6*).

**Ideas And Solutions Are Discussed With Others (GR6A)**

Within the ground rule *Engage In Joint Reasoning* (*GR6*), Mrs. Washington used the element *Ideas And Solutions Are Discussed With Others (GR6A)* in a variety of ways throughout the study.

Mrs. Washington engaged students in discussion to solicit their ideas and questions about tasks and assignments. For example, during the introduction to the Cupcake Challenge in Event 3, Mrs. Washington communicated her expectations for the task. She explained to students that they would need to collect data using a survey. She also provided opportunities for students to ask questions and exchange ideas before handing out the directions to the task. Instead of telling the students, she asked, “What do you have to do first?”

Mrs. Washington did not respond when the student answered “tally marks” and instead put the question back to the student asking, “Tally marks about what?”

Another student then replied, “Like tally chart about like what kind of cupcakes,”

Mrs. Washington continued the discussion. She asked, “Why are you asking about cupcakes?” and, “What do you have to find out?”
Again, a student joined the conversation to share her idea. She asked, “Don’t we need a question?”

Instead of answering, Mrs. Washington turned and asked the class, “What does your question have to be?” Then students engaged in joint reasoning by taking turns to share their solutions. Mrs. Washington asked a few more questions to maintain the engagement. She asked, “Would that get you the same data?” or “What was the main idea of the question?”

Secondly, Mrs. Washington discussed ideas and solutions with students during mathematical discussions to review or study concepts they studied in math class. During the fourth event, for example, the class was discussing the meaning and purpose of arrays. A student had written an array representing 15 X 5 on the board, as part of his explanation. Mrs. Washington probed his thinking. She asked, “Can I solve this problem by just using the algorithm?” When students hesitated, she said, “I could do this whole array, or I could break it into two smaller ones, what two other problems can I use” (Event 4, p. 7-8)

A student added her idea, “By decomposing.”

Mrs. Washington followed up on the idea. She said, “What number am I decomposing?”

Then the student said, “15 and five.”

Again Mrs. Washington maintained the engagement by inviting all to share more of their thinking. She asked, “Into what two numbers?”

As shown above, students also utilized the element Ideas And Solutions are Discussed With Others (GR6A) while they Engaged In Joint Reasoning (GR6) during
conversations with their teacher and peers. In fact, they used the element *Ideas And Solutions Are Discussed With Others (GR6A)* slightly more frequently than their teachers demonstrating the high level of participation during discourse.

Students shared ideas and solutions to identify answers to problems. In Event 1, a student explained how he figured out that 60 inches was equal to five feet. He shared “I know that 12X5 is 60” (p. 1-2).

Another student built on this idea and stated how division could be used to solve the same problem. She said, “You start with 61 and divide it to get your answer.”

Students also shared their ideas during discussions and while involved in “Turn and Talk” activities to discuss their thinking with peers. In Event 2, for example, students were asked to discuss why they created pictographs after they collected data. Students talked with their partners and then exchanged their ideas with the class. One student shared his idea. He said, “Y can do the pictograph first, but is would be hard because it would take a lot more thinking” (Event 2, p. 3). Another student said, “It would be hard to do it that way because the symbol means more than just one.”

**Questions Are Posed to the Community to Direct Thinking (GR6B)**

Mrs. Washington also utilized the element *Questions Are Posed to the Community to Direct Thinking (GR6B)* frequently as part of her strategic discourse with students. She asked questions to direct thinking to initiate, facilitate or redirect a discussion. Her discourse included questions that encouraged students to engage about specific ideas, topics or concepts in math. Some of the questions used by Mrs. Washington included, “How did you figure it out?” “How could I have used a pictograph to collect my data? and “Why is it important to know the area and perimeter of a space?” Questions were
also used to direct attention toward an idea shared by a student. Additionally, they were used to encourage students to reflect on or evaluate their thinking or the thinking of others. She used these questions to encourage students to learn from others by comparing ideas. When comparing ideas, she asked, “Why did she get two answers?” and “Thinking about her product and her sum, why are they different?”

Along with using questions to direct thinking, questions were also used to redirect thinking. For example, when students struggled to understand the difference between a tally chart and a pictograph Mrs. Washington used questions to remind students of the content of a lesson completed weeks ago using pictographs. She asked, “When did we do our pictograph, before or after collecting data” (Event 2, p.4)

**Questions Are Posed To Encourage Exchange Of Ideas (GR6C)**

Mrs. Washington utilized *Questions Are Posed To Encourage Exchange Of Ideas (GR6C)* with students to Engage In Joint Reasoning (GR6) with one another during discourse. She provided opportunities for students to share ideas during each lesson, as well as invited multiple students to share their thinking on each question posed. Most questions were used to engage in conversation about the topics they studied. Students were encouraged to share their thinking out loud so that others could hear their thinking. Mrs. Washington also prompted students with questions to interact with the ideas and solutions shared by others.

Mrs. Washington encouraged the exchange of ideas by asking, “How did you figure that out?” and “Why do we use the same rules?” She also reminded students of prior activities when they struggled with concepts. For example, Mrs. Washington asked, “Remember when we were using things like paper clips and blocks?” or “This morning I
had you walk around the what” (Case A Transcript, Event 5, p. 9)? She tried diligently to encourage students to engage in discourse. Students responded well to these questions and often participates, as seen in Table 1.

Mrs. Washington also initiated an exchange of ideas by asking for input or by saying something that would spark a reaction. She would ask, “Is that fair to say?” or “Why did we do that?” She also asked contradictory questions to increase student participation. In Event 5 for example, she asked, “When you subtract you can’t have two answers in math, cmon” (p. 9-10)! Students responded to this strategy, reacted immediately to these statements and joined the conversations.

**Community Members Ask Questions to Try to Understand Thinking (GR6D)**

Mrs. Washington utilized the element *Community Asks Questions to Understand Thinking (GR6D)* with students. She used this element to decipher what students were thinking and to help students articulate their ideas more successfully. An example of this was drawn from an exchange during Event 1. Students were discussing Mrs. Washington’s height, and she asked questions to unveil their understanding of the strategies used by students. She asked, “How did you know that?” and “How did you decide to start with 48?” Additionally, when a student unexpectedly utilized division as a strategy, she asked excitedly, “What did you do?” The student wrote her explanation on the board and provided a detailed explanation of the strategy. Mrs. Washington continued to probe the student’s thinking as the class listened on. Trying to help other students understand the higher leveled thinking. Also, Mrs. Washington asked this question to also encourage her students to listen to important ideas and strategies that they might utilize in the future.
Tasks Are Assigned To Initiate Working Together To Find Solutions (GR6F)

Mrs. Washington used the element *Tasks Are Assigned To Initiate Working Together To Find Solutions (GR6F)* often. She incorporated problem-based learning activities into her lessons. Three of the eight lessons involved students working on tasks. The projects or tasks involved engaging students in an authentic activity to solve real-world problems with other students. For example, during Event 3, students completed a task that required them to collect and represent data about favorite cupcakes of their classmates. During this project, some of the students were also challenged to create an original cupcake that could turn out to be the best flavor in their parent’s bakery.

Additionally, during Event 6, students worked independently on the Let’s Plan It Out-Playground Design task. In this geometry based task, students designed a playground with given specifications around area and perimeter. Furthermore, during Event 8, students completed real-world tasks, of their choice, as part of their Independent Projects to culminate the year’s work. Although only three of the events included working on the tasks, other lessons involved setting the stage for an upcoming a project.

Assistance Is Offered To Work Through The Process Or Scaffold Learning (GR6G)

Mrs. Washington also used the element *Assistance Is Offered To Help Work Through The Process Or Scaffold Learning (GR6G)* to help her students successfully engage during discourse. She offered support when students needed help to articulate their explanation, identify a solution, or explain a strategy. In Event 3 for example, Mrs. Washington led a group of students through each step of the tasks, ensuring that students understood each component, along with the expectations for completing it.
Mrs. Washington provided written directions explaining the task, along with a rubric which was used to communicate the expectations for each project. She also provided opportunities for students to brainstorm ideas instead of having to solve problems independently. She used “turn and talks” so students could share their ideas before they completed tasks on their own. After they shared their thinking with a partner, students shared their ideas with the larger group.

Additionally, Mrs. Washington provided additional explanations when students needed further clarifications. She asked questions to help students to initiate the interactions. She broke the task down into manageable parts. “What is the question going to be?” When a group needed assistance setting up their bar graphs, including creating a title, figuring a scale, and naming each bar, she broke the task and coached them to move one step at a time. After a plan was discussed, students were instructed to complete these sections and check in with their teacher again to go over the remainder of the task later during the class.

Mrs. Washington also helped students when they shared their ideas during mathematical discussions. She was able to scaffold the conversation and her questions. Mrs. Washington layered questions according to difficulty. If someone was struggling, she decreased the complexity of the idea and asking a basic level question to help them to continue. She started with “What is the first thing?” or “Do you know what we did this morning?” to help them begin in a familiar place. They needed her to break down the problem into manageable parts. Mrs. Washington also repeated questions or ideas to reminded students of important pieces of information.
The following section explains the implementation of the most frequently used elements by Mrs. Washington and her students within the seventh ground rule *Multiple Solutions Are Encouraged* (GR7).

**Multiple Solutions Are Encouraged (GR7)**

Mrs. Washington utilized the ground rule *Multiple Solutions Are Encouraged* (GR7) while engaging in discourse with her class. Students were encouraged to share many solutions for solving problems. During these discussions, Mrs. Washington emphasized different ideas and often restated them to ensure that they were heard. She also prompted students to provide details about their ideas or solutions so that she and others could follow their thinking.

**Many Ways Of Solving Problems/Thinking Are Encouraged (GR7A)**

Mrs. Washington implemented the element *Many Ways Of Solving Problems/Thinking Are Encouraged* (GR7A). She encouraged students to share ideas, strategies, and solutions with one another. The sharing of ideas usually occurred during large group discussions. Mrs. Washington consistently encouraged her students to participate in these exchanges during every discussion.

Mrs. Washington prompted students to share many ways of solving problems and different ways of thinking after she posed a problem. Students were also invited students to share different strategies.

During Event 1 for example, as students were converting inches to feet to determine the height of their teacher. Mrs. Washington asked, “How tall am I, and how did you figure it out” (Event 1, p. 1-2)? After a student shared her thinking, Mrs. Washington encouraged additional students to participate when she said, “This is what is
in her brain, who did it a different way?” After another student shared his solution, Mrs. Washington again kept the conversation going by inviting additional students to share their ideas about how to solve the problem.

Similarly, during Event 3, students were discussing the purpose of a tally chart in the process of data collection. Mrs. Washington wanted students to share different ways that they data could be collected. She posed the question, “Could I collect my data in a different way” (p. Event 3, 4-6)? Students shared some thinking with the group. Then Mrs. Washington reworded the question to bring more students into the conversation. She said, “Could I have collected my data in a different way than a tally chart?”

Mrs. Washington also asked students to call on friends to help when a conversation was stalled and to encourage more shared thinking. This strategy was used to help students get out of a tough spot in a safe manner. Students did not have to admit to not having a different strategy or knowing an answer. Additionally, students’ ideas were always accepted. This culture of risk-taking fostered helped students to understand that it was okay to participate even if they were unsure of an answer.

A strategic discourse strategy Mrs. Washington used to facilitate the element Many Ways Of Thinking Are Encouraged (GR7A) was the "Turn and Talk." Mrs. Washington used this strategy to initiate an exchange of ideas. She asked students to turn to a partner to share their thinking about, “Why do we use 12 inches equals one foot?” or “Could I collect my data in another way?” After students talked amongst themselves, she asked questions such as, “What did you and your friends talk about?” or “Do you want to share your thinking?” to continue the sharing of ideas. Turn and Talk also insured that everyone had an opportunity to share their thinking. To encourage the most amount of
sharing, Mrs. Washington would insist that they shared with multiple peers using the statement, “Talk to someone different this time.”

**Many Ways Of Solving Problems/Thinking Is Shared (GR7B)**

Mrs. Washington’s students utilized the element *Many Ways Of Solving Problems/Thinking Is Shared (GR7B)* while engaging in discourse with their teacher and peers. Sharing multiple ways of solving problems was part of the practice of learning math in their classroom. Students responded to their teacher’s prompts by providing ideas and solutions.

Students offered multiple strategies for arriving at the same solution. Mrs. Washington prompted with questions such as, “Who did it a different way?” Students responded by joining discussions and shared their thoughts willingly.

During Event 1 for example, students were attempted to convert 61 inches to feet and inches. They offered multiple solutions, utilizing different computational strategies to solve the problem. The first student proposed, “12X6=60 and then 60 plus 1 is 61 inches” (p.1-2).

The second student added, “I added 12, five times on my fingers and I got 60, and as she did, I added one and knew it was five feet because I had my five fingers up.”

The third student suggested, “You are 61, I divided 61 by 12.” The final student to share added, “So, I was thinking that you could do 2X12 = 24 and 24 plus one yard = 60 inches and then you just add one inch.

Additionally, while discussing the meaning of an array, students shared different ways of thinking when they offered explanations and discussed how arrays could be used to determine the area and perimeter of a space. Similarly, students discussed the meaning
of area and perimeter as it related to building a fence. During this discussion, students offered thoughts regarding the importance of knowing the area and perimeter of a space, as a prerequisite for the completion of the Plan It Out activity.

The next section examines the implementation of the elements comprising the majority of the discourse, those used on more than twenty occasions, within the second theme Developing Mathematical Knowledge among Ms. Littleton and her students.

**Ms. Littleton and Students**

Ms. Littleton supported her students in the development of mathematical knowledge using the ground rules in the theme Developing Mathematical Knowledge 540 times during eight classroom events, see Figure 1. Additionally, Table 14 reveals Ms. Littleton utilization of the ground rules within this theme. She used *Knowledge is Made Public (GR4)* on 62 occasions, *Reasoning Is Visible in the Talk (GR5)* on 11 occasions, *Engage In Joint Reasoning (GR6)* on 401 occasions, *Multiple Solutions Are Encouraged (GR7)* on 66 occasions and *Contributions Are Built on Prior Proposals (GR8)* on 7 occasions.

Ms. Littleton’s students used the ground rules within the theme Developing Mathematical Knowledge 619 times during eight classroom events, see Table 14. The students utilized the ground rules *Knowledge is Made Public (GR4)* on 180 occasions, *Reasoning Is Visible in the Talk (GR5)* on 129 occasions, *Engage In Joint Reasoning (GR6)* on 273 occasions, *Multiple Solutions Are Encouraged (GR7)* on 37 occasions and *Contributions Are Built on Prior Proposals (GR8)* on 3 occasions, as noted in Table 15.

Of the nineteen elements composing the ground rules in this theme, Ms. Littleton utilized seven of them frequently. *Participants Offer Knowledge About Mathematics*
(GR4A) was used on 21 occasions, and Contributions Are Restated (GR4C) on 23 occasions. Ideas and Solutions Discussed with Others (GR6A) was used most frequently with 202 occurrences. Ms. Littleton used Questions Are Posed to The Community to Direct Thinking (GR6B) 52 times, Questions Are Posed To Encourage Exchange of Ideas (GR6C) 47 times, and Community Members Ask Questions to Understanding Thinking (GR6D) 73 times. Ms. Littleton also utilized Many Ways of Solving Problems Encouraged (GR7A) on 63 occasions.

Ms. Littleton’s students utilized Participants Offer Knowledge About Mathematics (GR4A) on 113 occasions and Strategies Are Explained in Words And Or Pictures (GR4B) on 46 occasions. They also utilized Justifications And Rationales Are Provided (GR5A) on 80 occasions and Steps In Solutions Are Explained (GR5B) on 49 occasions. Ideas and Solutions Discussed with Others (GR6A) were used most frequently by students marking 230 occurrences, and Community Members Ask Questions To Try To Understand Thinking (GR6D) was used on 21 occasions. Finally, students utilized Many Ways Of Solving Problems Shared (GR7B) 26 times during the course of the eight events.

The next section describes the implementation of the elements within the ground rules within the theme Developing Mathematical Knowledge among Ms. Littleton and her students.

Knowledge is Made Public (GR4)

Ms. Littleton consistently encouraged students in Developing Mathematical Knowledge. She used the element Knowledge Is Made Public (GR4) to emphasize the importance of sharing of their knowledge in their contributions. Ms. Littleton prompted
students to share their thinking during discourse to assess what they knew about the idea, skill or concept. Expecting students to explain the strategies they used to find solutions allowed her to assess their knowledge of the content embedded in the problems. Students explained their thinking and justified their strategies during discourse using this ground rule.

The following section describes the implementation of the most frequently used elements by Ms. Littleton and her students within the ground rule *Knowledge is Made Public* (GR4).

**Participants Offer Knowledge About Mathematics (GR4A)**

At times during the seven events in this study, Ms. Littleton stepped in to support students as they shared a mathematical idea or clarify a misunderstanding. In Event 1 for example, students shared their knowledge regarding how many right angles were in three-fourths of a circle a shared offered two proposals. The first student said, “I know that 180 degrees is ½, so that is 2/4 and then we had 1/4 left and that is 90 so I did 180 degrees plus 90 degrees equals 270 degrees, so ¾ of the circle is 270 degrees” (Event 1, p. 1-2). Next, the same student shared a second strategy, “My other way, was um the full circle is 360 degrees, so we are only leaving out ¼ so I subtracted the ¼ which is 90 degrees from the full circle which is 360 degrees, and I did 360 degrees minus 90 degrees and got 270 degrees.”

After she finished, Ms. Littleton offered her mathematical knowledge when she pointed out an important idea embedded in the student’s proposal. Ms. Littleton asked, “Why did she use subtraction to try to figure out the fraction of the circle?” After a
student shared the connection, Ms. Littleton restated the student’s effective identification of the viable strategy used to solve the problem.

Ms. Littleton also supported the building of student’s knowledge when she stepped in to clarify one of her student’s misconceptions. The class of students tried to identify the measurement of a supplementary angle, in a parallelogram, when given the measurement of the other. One student explained that she and her partner had added: “180 plus 76 because 76 is part of the straight angle” (Event 2, p. 6-7).

Another student stepped in to suggest to the student. He said, “You should have subtracted instead of added.”

Ms. Littleton followed his suggestion by publicly sharing her knowledge. She provided a clarification of her own. She said, “So we have our 180 degrees here, and the parallelogram is drawn on the number line, this is our 76-degree angle, and our 180 degree is the straight line, so it is part of the 180-degree angle” (Event 3, p. 8). She clarified, “If you are adding 76 to 180 degrees what you are doing is saying we have an angle that starts here and goes all of the way around to here.”

**Strategies Are Explained In Words, Pictures And/Or Numbers (GR4B)**

As Ms. Littleton’s students made their knowledge public as they utilized the element *Strategies Are Explained In Words, Pictures And/Or Numbers (GR4B)*. They provided explanations without prompting, often including justifications and rationales. These justifications and explanations, some noted in the examples above, often included words, pictures and/or numbers to communicate their knowledge. Students outlined the strategies they used to explain answers. For example, students explained as part of the
assignment in the capacity problem-solving task. The first group shared their work regarding the following problem:

Charlie filled all of his balloons with 2 quarts of water. Warren filled each of his six balloons with 1 and \( \frac{1}{2} \) cups of water. Whose balloons contain the most water?

Students recorded the following number sentences to support their thinking.

\[ 1 \frac{1}{2} \times 6 = 9 \]
\[ \frac{1}{2} \times 6 = 3 \]
\[ 1 \times 6 = 6 \]

Two group members participated in the explanation. Both referred to the number sentences on their posters. The first student began, “So, the first way we did it, we did one and half times six” (Event 4, p. 16).

The second said, “We did a half times six equals three and one times six equals six and then we added them up and wrote one and a half times six equals nine.”

Then the first student concluded, “The answer we got was Warren has more water because Charlie only has two quarts to use and Warren has one cup more water.”

Throughout the events in the study, students recorded their work using words, pictures and numbers to justify their solutions. All explanations shared by students during the problem presentations in Event 4 included explanations accompanied by words, pictures, and numbers to justify the solutions.

**Contributions Are Restated (GR4C)**

Ms. Littleton implemented the ground rule *Knowledge Is Made Public* by using the element *Contributions Are Restated (GR4C).* She restated contributions to summarize thinking and to emphasize meaningful ideas. During Event 1 for example, a student
explained knowing that there were four right angles in a circle. The student said, “I knew that a whole circle was 360 degrees, so I added 90 four times. Well, I added it 90, 90, 90, and I added them up and then got 270 and then I added 90 again and got 360” (Event 1, p. 1).

After the student explained his answer, Ms. Littleton restated a part of his strategy. Ms. Littleton asked, “So first you tried three?” to indicate a critical piece of information in the student’s explanation. She emphasized that although the student first added three, she did remember to add the additional 90 to arrive at 360 degrees.

Ms. Littleton also utilized Contributions Are Restated (GR4C) after she completed the ribbon lesson. While students reflected about their problem-solving work during a follow-up discussion, Ms. Littleton wanted students to apply their learning to real life. She asked them to consider the question, “Would it be better to do an over-estimate or under-estimate” (Event 7, p. 26)?

One student replied, “This is good for camping like you need to know like the estimate of how much wood your fire is going to need and--so you can wrap it up so the log does not roll away and you will have wood for your fire.”

Ms. Littleton asked, “So tying the logs together with some material?” Then she restated the question to help solicit additional ideas. She asked, “Would it be better to do an over-estimate or under-estimate there?”

The following section describes the implementation of the fifth ground rule Reasoning Is Visible In the Talk (GR5) utilized by Ms. Littleton’s students.
**Reasoning Is Visible In The Talk (GR5)**

Ms. Littleton students supported other students in Developing Mathematical Knowledge by demonstrating their thinking using the element *Reasoning Is Visible In The Talk (GR5)*. They offered explanations which provided insights for others about their thinking and decision making during problem solving. Again, these explanations included justifications and rationales that included steps they had taken to solve the problem.

The following section describes the implementation of the frequently used elements by Ms. Littleton’s students within the ground rule *Reasoning Is Visible In The Talk (GR5)*.

**Justifications/Rationales Are Provided To Explain Thinking (GR5A)**

Students utilized the element Justifications/Rationales Are Provided To Explain Thinking (GR5A) both in small group and large group situations. They provided justifications both verbally and in their written work, as well.

Students provided justifications to support their solutions for example, while they compared units for measuring liquids. Ms. Littleton asked, “If we had four quarts, what can we say four quarts is equal to” (Event 3, p. 9-10).

A student answered, “Eight pints.”

Ms. Littleton wrote four quarts equals eight pints on the whiteboard. Then she asks, “How did you see that?”

The student justifies her solution, “Because two pints are in each quart.”

Ms. Littleton encouraged the group to offer other units equal to four quarts. A second student said, “A gallon.”
When Ms. Littleton asked how she knew this answer. The student justified her answer, “Because inside of the G (gallon) there are four q’s and they represent four quarts, so there are four quarts in a gallon.”

**Steps In Solutions are Explained (GR5B)**

Students also utilized the element *Steps In Solutions are Explained (GR5B)* as part of their written explanations and while they justified their thinking during discussions. For example, students *Explained the Steps In Solutions (GR5B)* while they calculated the amount of ribbon needed to wrap their box and tie it with a bow. During problem solving groups, Ms. Littleton circulated the room to monitor their progress. She checked in with a group to ask, “Can you tell me exactly how you figured out your estimate” (Event 5, p. 20)?

A student explained the steps his group had taken. “We measured the box with the tape measure and then added a few inches.”

Then Ms. Littleton asked, “So how did you measure the box?”

The second student said, “With that (pointing to the tape measure).”

The third student demonstrated how she used the tape measure as a ribbon to wrap the box.

Ms. Littleton summarized their thinking. She said, “So you tied it around. You pretended that the tape measure was a ribbon?”

The third student added, “Then we added seven inches.”

Ms. Littleton asked, “Why did you add seven inches for the bow?”

The first student explained, “Because that was what we thought it needed.”
The third student continued, “Because we had 43 and we thought seven more would give us what we needed.”

During the same lesson, Ms. Littleton asked another group, “What is your plan? What did you come up with?” This group also explained the steps they had taken to arrive at a solution.

The first student who shared in this group said, “We came up with 40 inches.”

Ms. Littleton probed for more information. She asked, “How did you come up with that?” The second group member explained the steps the group went through to solve this problem. She said,

We measured the sides of the box (pointed to each of the four sides of the box) this way and this way…and this would be about nine, and ½ and this would be nine and ½, and then we added. All four sides equaled around 40 inches, and so that is the amount they requested” (Event 5, p. 20)

The following section explains the implementation of the most frequently used elements by Ms. Littleton and her students within the ground rule Engaging In Joint Reasoning (GR6).

**Engaging In Joint Reasoning (GR6)**

Ms. Littleton utilized the elements within the sixth ground rule *Engage in Joint Reasoning (GR6)* to assist students in Developing Mathematical Knowledge about the concepts studied. This was a critical part of the discourse among Ms. Littleton and her students. Ms. Littleton assisted students to build their knowledge by preparing lessons that encouraged participation through sharing, discussing and questioning during mathematical discourse. Ms. Littleton’s lessons included opportunities for students to learn content while solving real-world problems and tasks with others. Problem-solving sessions were followed by discussions about the knowledge developed and the thinking
that was constructed among students. The following section describes the implementation of the most frequently used elements by Ms. Littleton and her students within the ground rule *Engage In Joint Reasoning (GR6)*.

**Ideas And Solutions Are Discussed With Others (GR6A)**

Ms. Littleton used the element *Ideas And Solutions Are Discussed With Others (GR6A)* in different ways. She supported students as they discussed ideas about assignments and described strategies for problem solving.

Ms. Littleton introduced each task and provided a written description of the problem to students. For example, when Ms. Littleton introduced the Balloon Challenge, one of the several tasks assigned to students. Ms. Littleton said, “We are going to be working with a partner today on a couple of different problem-solving questions” (Event 3, p. 10-11). She continued, “These questions all have to do with water balloons and the amount of liquid inside of water balloons.”

For this lesson, Ms. Littleton provided an opportunity for students to review the expectations of the project. This brief discussion included discussing ideas about the project and clarifying directions for the assignment. Ms. Littleton asked, “What do you think that you should put on your poster?” Students responded with ideas including, “Explain how you got your answer” and “Show your work.” Ms. Littleton followed up by asking, “Would it be helpful to show two different ways to solve a problem?” They did not discuss strategies for solving the problems. She expected students to figure this part out on their own.

During the lesson above and throughout the study, Ms. Littleton engaged students in discussions to share their ideas and solutions while in small groups, or when sharing
their thinking during whole group discussions. As seen in Figure 6.1, Ms. Littleton utilized this ground rule very frequently, specifically this element, as did her students. This was a result of the opportunities students had to participate in small group problem-solving work and during full class follow up discussions.

While students worked on problems in small groups, they shared solutions and ideas with their teacher and other group members. Meeting with small groups during their problem-solving work, provided opportunities for Ms. Littleton to circulate and interact with small groups to discuss ideas and solutions.

In Event 3 for example, students were working on solving various capacity problems in small groups. While they were working, Ms. Littleton checked in with each group. She directed her questions to their specific problem and the challenges they faced. One discussion began when she asked, “So you did division here, and what did you do” (Event 3, p. 12-13)? The pair of students explained their thinking. Ms. Littleton probed their thinking further and asked, “We talked about why division was the best strategy, why did you do multiplication?” One student explained her reasoning. Ms. Littleton shared her knowledge and explained that division would work best in this situation. She explained, “Since we have 2000 liters of waters and we are trying to get water to each balloon, should we be multiplying or dividing?” When the students agreed that the operation should be division, she pushed them to explain this decision by asking “Why did you choose to multiply?” Even after the student answered the question, Ms. Littleton pressed her to check for understanding that multiplication was not the ideal strategy for this problem. The student was confused about the relationship between the remainder (200) and the number of balloons that could be filled (6). She tried to explain her
misunderstanding, and Ms. Littleton supported her thinking by encouraging her to
attempt to answer correctly, using division. The student had figured the answer to be 6
and 200, but the 200 was the portion that was left over since each balloon could hold 300
ounces. Ms. Littleton continued to discuss the idea and solution, but also reinforced the
mathematics embedded in the problem. She said,

Now we need to understand why our answer is right. So you have 2000 liters of
water, and you want to see how many groups of 300 you can make with that. You
have 200 left over at the end, can you make another group of 300 with it?

The students agreed that they could not fill another balloon and began to write
their solution on their poster. Before moving to another group, Ms. Littleton asked, “Can
you fill up a balloon with the left over two hundred?” When the student replied, “No,
can’t fill it up, so it stays as your left overs.” Mrs. Littleton persisted. She said, “Tell me
again why we are dividing and not multiplying?” Then the pair confirmed an accurate
solution and Ms. Littleton moved on.

Students also utilized the element Ideas And Solutions Are Discussed With Others
(Gr6A) when prompted by their teacher, as well as independently in small and large
group formats. Students provided solutions and ideas while explaining their thinking
about problems. For example, one student offered her idea about how to solve a problem
and said, “What we can do is do a multiplication problem to get the division problem”
(Event 3, p. 11).

Students also shared their ideas when they discussed reasons why each group had
a different sized bow during Event 5. Students shared multiple reasons and ideas. For
example, students shared, “Because you could have made a different sized bow,” and,
“You could have had a different sized pencil,” or, “You cut your string longer or shorter than someone else’s” (Event 5, p. 9).

Students also used the element *Ideas And Solutions Are Discussed With Others (GR6A)* when they shared their thinking about solutions. One example occurred while students were working in small groups during the wrapping the present task. They were discussing solutions with one another about the amount of ribbon needed to wrap their sized box. The first student offered, “6 plus 2 and $\frac{1}{2}$ that would be 8 and $\frac{1}{2}$, so we need to add those two together” (Event 2, p. 21).

The other student added, “That would be 18 inches, so 19 inches.”

The first student stepped in again and added, “No cuz those together would be one inch, those two halves, add that one inch to the nine inches and that is nine and nine plus nine is 18.” Students continued to share their ideas and solutions until they decided on their final solution.

**Questions Are Posed to the Community to Direct Thinking (GR6B)**

Ms. Littleton also utilized the element *Questions Are Posed to the Community to Direct Thinking (GR6B)* often as part of her strategic discourse with students. She asked questions to direct thinking whether to initiate, facilitate or redirect the lesson or discussion. The discourse Ms. Littleton facilitated consistently included questions that encouraged students to engage in conversations about the topics and concepts that she planned.

Ms. Littleton asked several questions to guide the discussions with students. She used questions such as “Why do you think that we have all sorts of measurements for the bow?” to direct students’ attention to a particular idea (Event 5, p.18). She asked open-
ended questions such as, “What were some of the challenges that you faced?” to invite students to share ideas on a given topic. Ms. Littleton also used questions to encourage students to share ideas instead of explaining the idea or process herself. She often allowed students to take the discussion in a direction that was meaningful to them. Questions such as, “Who would like to share out how you solved this problem?” or “What did you come up with?” helped to facilitate this conversation and consider the ideas that were important to them.

Ms. Littleton also used questions to focus the group’s attention toward a student’s contribution and to encouraged them to reflect on their ideas or the ideas of others. Ms. Littleton prompted this type of thinking when she asked questions such as, “She connected her thinking with what?”, and “Does that make sense?”, and “So how can you fix your answer?” or “Where did the four come from?”

Along with questions to direct the thinking, Ms. Littleton utilized questions that redirected thinking too. During a lesson, one student struggled to find the accurate measurement of a missing angle. Ms. Littleton questioned to redirect his thinking away from a misconception. The student said, “So we found 104, so we figured out that y, x is 104 so in all is 208” (Event 1, p. 4-5).

Ms. Littleton stepped in and asked, “Why did you find in all? What about the diagram is telling you to do x plus y?” The student looked puzzled and did not answer. Ms. Littleton continued to redirect his thinking, “Nothing, we are not adding x and y.” The student had not realized that he needed to subtract the two numbers to solve for the missing angle correctly.
Questions Are Posed To Encourage Exchange Of Ideas (GR6C)

Mrs. Littleton utilized Questions Are Posed To Encourage Exchange Of Ideas (GR6C) to encourage students to Engage In Joint Reasoning (GR6). She frequently encouraged the exchange of ideas and motivated multiple students to respond to her questions. Ms. Littleton encouraged students to exchange their ideas verbally so that the ideas could be shared with all. Students explicitly told how and why they took the steps that they did to solve a problem. After a student shared, Ms. Littleton prompted students to interact with one another about the ideas and solutions shared. Ms. Littleton usually prompted students by asking additional questions to help students reveal their thinking to others.

As mentioned earlier, students shared their solutions about problems they were working on that involved measuring angles. After students provided their answers, Ms. Littleton asked questions such as, “Why did you find all?” and “Are you taking 76 away from 180 or 180 away from 76” (Event 2, p. 4-5)? She also asked, “Why did you decide to subtract it from 360?” and “Why did you subtract two 76 degree angles?” or “Does that make sense?” All of these questions prompted students to make additional contributions and to elaborate on their thinking. As students exchanged their ideas, students listen to the various thoughts and strategies shared.

Ms. Littleton also encouraged students to interact with one another’s ideas during the mathematical discourse. After solutions were shared, Ms. Littleton often asked, “Does anyone have any questions for him?” Although students did not frequently act on these opportunities, Ms. Littleton did. She consistently asked questions such as, “You know that 90 degrees is what?” and “She connected her thinking with what?”
Community Members Ask Questions to Try to Understand Thinking (GR6D)

Ms. Littleton utilized the element *Community Asks Questions to Understand Thinking (GR6D)* to encourage her students to interact with one another. Most often, Ms. Littleton asked questions to understand students’ thinking as they shared their solutions to problems. During Event 3 for example, students used the “Big G” strategy to help determine equivalency in liquid measurement. Ms. Littleton asked questions to understand and illuminate the purpose of the strategy. She asked, “So why did you draw this” (Event 3, p. 8-9)?

One student justified his thinking and explained that each G or gallon has four q’s or quarts and each quart has two p’s or pints, and each p has 4c’s or cups in it. He added his own revelation. He said, “I kind of noticed something. It kind of times’.”

Seeking clarification about his thinking, Ms. Littleton asks, “What do you mean it times’?”

The student explained, “Because in the quarts it has two pints and in the two pints it has four cups, so it’s like two, and then it is ½ and then four.” Several other students were given the opportunity to explain the usefulness of the Big G strategy, each one bringing about new ways of interpreting how the representation was useful.

Additionally, Ms. Littleton used this element to uncover students’ thinking and to support students as they solved problems. In Event 4, while students were presenting their capacity problems, the class was also invited to ask questions to gain an understanding of the various ways their classmates solved the problems. During the presentations, Ms. Littleton prompted students to interact with one another. She asked, “Any more questions or comments” (Event 4, p. 14-17)? Additionally, Ms. Littleton also
asked questions to understand the students thinking. Her questions included, “What was the question asking you?” , “So what is the answer to that question?” and “How many balloons can he fill?” She skillfully used these questions to make their thinking more explicit to the community. These inquiries also served as models for the types of questions students could also ask others.

She promoted the element *Community Members Ask Questions to Try to Understand Thinking (GR6D)* among her students. Moreover, Ms. Littleton never hesitated to raise questions about their thinking, including when an incorrect answer or misconception was presented. She also encouraged students to ask questions of other students. If fact, students spoke up when they noticed an error or if they disagreed with a solution. During Event 1 for example, students were working on problems requiring calculations with angle measurements. Instead of helping the student to correct an error herself, Ms. Littleton encouraged students to assist their classmate instead. She asked, “Boys and girls, do we have any questions or suggestions about how they are solving the problem” (p.6)? Students joined the conversation and helped the student identify the correct solution.

Likewise, students in Ms. Littleton’s class used the element *Community Members Ask Questions To Try To Understand Thinking (GR6D)* either to learn more about a contribution or to question a solution. They questioned to learn more by asking, “Wait, how did you know that?” Students used more general questions when they were interested in what a peer had figured out. These questions included, “What did she get?” , “Which way did you like best?” and, “What was your second way?”
Students also asked questions when investigating the accuracy of a statement or solution. For example, a student questioned another student when she offered 16 cups as equivalent to 4 quarts. She asked, “Shouldn’t it be 32? Shouldn’t it be two pints in each quart and she only did one pint in each quart” (Event 3, p. 9-10).

Ms. Littleton directed this question back to the student who asked it. She asked, “I don’t know, what do you think?” When the student did not respond, she asked, “So why do you think Zandra thinks it should be 16 and Evelyn thinks it should be 32?”

Another student joined the conversation to state why she agreed with Zandra. She said, “I agree with Zandra because there in a so… in like in a quart there is 4 cups so 16 cups because four times 4 is 16 cups.”

Additionally, a student pointed out an error during the capacity presentations. She asked the presenter, “Did you say millimeters or milliliters?” when she realized that she had used the unit of linear measurement instead of the unit of liquid measurement (Event 4, p. 14). The presenter was unsure, so the teacher stepped in to let her know that it was in fact milliliters and not millimeters when measuring liquid.

The next section describes the most frequently used elements within the seventh Ground Rule Multiple Solutions Are Encouraged (GR7) among Ms. Littleton and her students are described in the following section.

**Multiple Solutions Encouraged (GR7)**

The elements within the ground rule *Multiple Solutions Encouraged (GR7)* were used to encourage students as they contributed solutions to problems in the learning community. Sharing different ways of thinking and solving problems was emphasized. Ms. Littleton utilized the element *Many Ways of Solving Problems Encouraged (GR7A)*
consistently to expand students understanding to include more than one way of solving a problem. The tasks assigned by Ms. Littleton were conducive to students generating multiple solutions. Students were provided the freedom to formulate strategies to solve the problem on their own or in problem solving groups.

**Many Ways of Solving Problems Encouraged (GR7A)**

Ms. Littleton utilized the element *Many Ways of Solving Problems Encouraged (GR7A)* to assist students to identify multiple strategies for the same problem. Students often solved problems initially with partners or in small problem-solving groups. After they completed a problem-solving task in small groups, the class would gather to share their thinking. Ms. Littleton expected students to discuss the problems and tasks that they worked on with their teacher or peers. She stressed contributing multiple solutions whenever possible. After a student shared their solutions, other students were invited to do so as a means of maintaining the conversation. Ms. Littleton praised their thinking and asked questions that helped make student’s thinking more elicit to others. Students were also encouraged to reflect on all contributions offered.

In the conversation during Event 1 for example, Ms. Littleton encouraged students to provide multiple ways for solving the same problem. Students worked to determine how many right angles would fit into a circle and willingly offered multiple solutions during the discussion.

The first student said, “I think it's four because there's four right angles in there. There's one, two, three and four” (Event 1, p. 1-2).

The second student said, “I know that 90 degrees is one-fourth of the 360-degree circle, so I divided 90 by four, 360 degrees by 90 and I got 4. I also made a circle and
divided it into four pieces starting with 90 here, 90 here, 90 here, and 90 here. And there's four pieces.”

Ms. Littleton continues the discussion. She said, “We have time for one more share.”

The third student said,

I know that 90 degrees is one-fourth of the 360-degree circle so I divided 360 degrees by 90 and I got 4. I also made a circle and divided it into four pieces starting with 90 here, 90 here, 90 here, and 90 here. And there's four pieces.

Many Ways Of Solving Problems/Thinking Is Shared (GR7B)

Ms. Littleton’s students utilized the ground rule Many Ways Of Solving Problems/Thinking Is Shared (GR7B) as a means to engage in discourse with their teacher and peers. Sharing multiple ways of solving problems was part of the students’ practice. All events included some type of problem that students solved paired with an opportunity to share these solutions with the entire class. Students often responded to their teacher’s requests to share multiple ideas by providing ideas, solutions, and strategies, see Table 15.

As part of the many ways thinking was shared, students took opportunities to reflect on the experiences they had while working on problems and tasks. During these experiences, students provided a variety of different insights that they developed while working with partners or small groups. In Event 7 for example, Ms. Littleton initiated a discussion that included a reflective component. She asked, “So after doing this once, now you have experience, and now, you have a different sizes box. What would you do
differently” (Event 7, p. 24-26)? Multiple students participated by sharing their thinking.

The following are the responses were offered by students:

- “Well, estimate and then add ten extra inches (for the bow).”
- “Add like 5 to 8 inches for like for the bow.”
- “I’d estimate to see if it would be more than a hundred inches or less than a hundred inches.”
- “We want to use like a regular ruler. We were only using a yard stick and tape measure.”

Each time students offered a reflection, Ms. Littleton followed up with a question to gather more information and clarify their insights.

**Summary of Theme 2: Developing Mathematical Knowledge**

Ms. Littleton and Mrs. Washington utilized many of the elements among the ground rules within the theme Developing Mathematical Knowledge. All elements in this theme provided opportunities for the community to interact while solving problems and to contribute to the learning of others while engaging in mathematical discussions. Both teachers in this study encouraged designing and sharing strategies and solutions with community members to build understanding of the mathematics they were studying.

Together both teachers and their students utilized four of the same ground rules in the second theme Developing Mathematical Knowledge frequently in their discourse with students, see Table 19. These ground rules made up the majority of the classroom discourse for both teachers and their students. Of the four ground rules that they used, the sixth ground rule *Engage In Joint Reasoning (GR6)* was used more frequently by
teachers and students than any of the others. Although the other three were frequently
used, the others were utilized by the teachers and students in varying frequencies.

Ms. Littleton and her students utilized the elements within the fourth ground rule
Knowledge Is Made Public (GR4) at the second highest level in this theme. Both Ms.
Littleton and her students utilized the element Participants Offer Their Knowledge About
Mathematics (GR4A) frequently while engaged in discourse with students. Ms. Littleton
offered her knowledge to emphasize a mathematical idea or to help clarify a
misunderstanding. Ms. Littleton’s students utilized the element Participants Offer Their
Knowledge About Mathematics (GR4A) almost four times more than their teacher.
Students also explained Strategies In Words, Pictures And/Or Numbers (GR4B)
frequently without prompting, including justifications and rationales to clarify their
thinking and solutions. Additionally, Ms. Littleton also utilized Contributions Are
Restated (GR4C) to summarize thinking or to emphasize meaningful ideas during
discourse.

Mrs. Washington students utilized the elements within the fourth ground rule
Knowledge Is Made Public (GR4) at the third highest level in this theme. Mrs.
Washington’s students utilized the ground rule Knowledge Is Made Public (GR4) a
similar amount of times, as seen in Figure 6.1. They utilized the element Participants
Offer Their Knowledge About Mathematics (GR4A) most often while engaged in
discourse. Students responded to the teacher’s questions, reacted to ideas and shared
thoughts about the contributions of others. Students also explained Strategies In Words,
Pictures And/Or Numbers (GR4B) when explaining their thinking and solutions. Mrs.
Washington utilized the element Contributions Are Restated (GR4C). She utilized this
element to draw her students’ attention toward a statement or question, or emphasize something that was shared either by herself or others in the class.

Mrs. Washington’s students utilized the ground rule *Reasoning Is Visible In The Talk (GR5)* frequently using the element *Justifications/Rationales Are Provided To Explain Thinking (GR5A)* while engaged in discourse with their teacher and peers. These explanations which sometimes included justifications were used by students to share how they solved the problem including the steps they had taken when prompted to do so by their teacher.

Likewise, Ms. Littleton’s students utilized the ground rule *Reasoning Is Visible In The Talk (GR5)* frequently using the element *Justifications/Rationales Are Provided To Explain Thinking (GR5A)*. They also consistently provided justifications, that included what they had done to solve the problem. At times, they also provided a rationale explaining why they had solved a problem a certain way and to support their thinking. The element *Steps In Solutions Are Explained (GR5B)* were also frequently used by students to communicate what they had done mathematically to figure out an answer.

Mrs. Washington and her students *Engaged In Joint Reasoning (GR6)* using the element *Ideas and Solutions Discussed With Others (GR6A)*. This element was used most frequently over the course of the eight events. Mrs. Washington facilitated discussions to encourage students to share their knowledge with one another. Students consistently engaged by sharing their *Ideas and Solutions With Others (GR6A)*. Students justified their solutions by sharing the steps they used to solve each problem. Mrs. Washington utilized questions to exchange ideas, understand students’ thinking, or to *direct the thinking of the student or class to engage students in joint reasoning (GR6B, GR6C, and*
Ms. Littleton and her students also utilized *Ideas and Solutions Discussed With Others (GR6A)* most frequently to *Engage In Joint Reasoning* over the course of the eight events. This first element *Everyone Invited To Contribute (GR6A)* was utilized most frequently of all of the elements within the ground rules by the teacher and her students. Ms. Littleton provided time for students to *discuss ideas and solutions* both in small and large group formats (GR6A). During each lesson, time was provided to work on solving problems in small groups. While working in small groups, students discussed their ideas and planned out how they would solve problems together. Then they shared their ideas and solutions during full class discussions. While sharing their knowledge Ms. Littleton’s students provided rationales, justifications, and steps in solutions to engage in joint reasoning with one another. Like Mrs. Washington, Ms. Littleton utilized questions to *exchange ideas, understand students’ thinking, or to direct the thinking* of the student or class to engage students in joint reasoning (*GR6B, GR6C, and GR6D*). Mrs. Littleton’s students asked questions of their peers and teacher as they worked to make sense of the mathematics and when attempting to *understand the thinking of others (GR6D)*.

Ms. Littleton and her students also frequently utilized the element *Many Ways Of Solving Problems /Thinking Are Encouraged (GR7A)* which was part of the ground rule *Multiple Solutions Are Encouraged (GR7)*. Ms. Littleton expected students to discuss problems and tasks they worked on with their teacher or peers, contributing multiple solutions whenever possible. After a student shared a solution, other students were invited to do so as a means to continuing the conversation. Students responded well to
this format by utilizing the element *Many Ways Of Solving Problems/Thinking Is Shared (GR7B)* by engaging in discourse and contributing multiple ways for solving the problem being discussed by the class.

Mrs. Washington also utilized the ground rule *Multiple Solutions Are Encouraged (GR7)* frequently while engaging in discourse with her class. Using the element, *Many Ways Of Solving Problems/Thinking Are Encouraged (GR7A)* she collected multiple strategies for arriving at the same solution. She emphasized the different ideas often by restating them to insure that ideas had been heard by all. Like Ms. Littleton’s students, Mrs. Washington’s students also utilized the ground rule *Many Ways Of Solving Problems/Thinking Is Shared (GR7B)* frequently while engaging in discourse with their teacher and peers. Students participated by responding to their teacher’s request to providing multiple ideas and solutions.

The following section explores the implementation of discourse strategies used by teachers and their students from the third theme Strengthening Critical Thinking.

**Theme 3: Strengthening Critical Thinking**

The types of strategic discourse strategies aligned with the third theme Strengthening Critical Thinking were used by teachers to extend students’ thinking beyond the expected understanding required to master grade-level content. Teachers used these elements to engage with mathematical content and with the ideas of others. The discourse strategies used encouraged students to question, disagree, compare and verify their shared thinking as a means of extending their mathematical content knowledge and to strengthening critical thinking skills. Students were engaged in real life problems which challenged them to consider input from peers and identify efficient and effective
strategies. Additionally, these critical thinking skills assisted students to develop stronger collaboration skills by learning to compromise and reach an agreement together.

A description of the ways Mrs. Washington and their students utilized the strategic discourse practices within the elements in the first theme Strengthening Critical Thinking are included in the following section.

Mrs. Washington and Students

Overall, Mrs. Washington used the ground rules in the third theme Strengthening Critical Thinking 135 times during the eight classroom events, see Figure 1. Of the eight ground rules in this theme, Mrs. Washington utilized *Ideas Are Extended Together* (GR9) on 49 occasions, *Partners Engage Critically With Each Other* (GR11) on 41 occasions, *Ideas May Be Challenged With A Counter Strategy* (GR13) on 18 occasions and *Seek Agreement for Joint Decisions* (GR14) on five occasions, as seen in Table 10.

Mrs. Washington’s students also utilized the ground rules within the theme in the third theme Strengthening Critical Thinking 88 times, see Table 10. Additionally, as revealed in Table 11, students implemented the ground rules *Ideas Are Extended Together* (GR9) on 16 occasions, *Partners Engage Critically With Each Other* (GR11) on 27 occasions, *Ideas May Be Challenged With A Counter Strategy* (GR13) on five occasions and *Seek Agreement for Joint Decisions* (GR14) on 18 occasions, while engaged in mathematical discourse with their teacher and peers.

Of the 19 elements composing the ground rules in this theme, Mrs. Washington’s utilized the ninth ground rule *Ideas Extended Together* (GR9) most often. Although Mrs. Washington utilized several other elements within the six ground rules, none were implemented above the designated frequency level of 20 for this study, see Table 10.
Additionally, Mrs. Washington’s students used several of the other elements, as well, but none were utilized above 20 instances, the designated frequency level for this study, see Table 11. The next section examines Mrs. Washington’s implementation of the ninth ground rule Ideas Extended Together (GR9).

**Ideas Extended Together (GR9)**

The elements within the ground rule *Ideas Extended Together (GR9)* assisted students to extend the ideas they contributed to conversations about mathematics. First, students were encouraged to reflect and examine thinking that they presented, or thinking presented by others. This ground rule was used when students had demonstrated an understanding of the math, and their teacher pushed them to think beyond by drawing conclusions or exploring additional skills. The prompt or questions used by teachers encouraged deeper thinking and more detailed explanations.

Mrs. Washington utilized *Questions Used To Further Thinking (GR9B)* on 37 occasions with her students. She encouraged students to expand their knowledge by responding to questions about their thinking and strategies.

She also utilized questions to further thinking to encourage students to educate their peers. When she found a solution interesting, she asked, “How did you figure that out?” to allow students to explore the thinking shared (Event 1, p.1).

Mrs. Washington asked questions to initiate the strengthening of critical thinking among her students. These questions included prompts to push students to make generalizations. For example, Mrs. Washington asked, “Why do you think everyone does that” (Event 1, p.1)? She also encouraged students to listen carefully to ideas and solutions presented by others, and then interact with one another about the ideas. The
interactions also included reflecting on the ideas to discuss the similarities and differences among them. For example, Mrs. Washington asked, “Could I have collected my data differently than a tally chart” (Event 2, p. 3)?

Also, Mrs. Washington questioned student to push them to provide detailed explanations for the actions they had taken. She directed questions directed at the steps students used in their strategies. For example, when students were converting inches to feet, she asked, “How did you decide to start with 48” (Event, p.1-2)?

Mrs. Washington also engaged students in conversations to discuss various representations of a solution. For example, Mrs. Washington asked, “What does it mean to visualize, what is a visual in math” (Event 4 (p.7-8)? A student responded, “Picture.” Then Mrs. Washington pushed the thinking from the visual representation of the problem to a numerical one. She asks, the group, “Can I also solve this problem (points to 15X15 on the whiteboard) using the algorithm?” Mrs. Washington continued, “I could do this whole array or break it down into two smaller ones, what would be the most logical?” A student joined in and extended the thinking.

She said, “By decomposing.”

This began a discussion, lasting a few minutes, among students and their teacher about the various combinations that could be used to decompose 15X15, complete with explanations.

The next section examines the element within the third theme Strengthening Critical Thinking that comprised the majority of the strategic discourse practices used by Ms. Littleton and her students.
Ms. Littleton and Students

Ms. Littleton utilized the ground rules within the third theme Strengthening Critical Thinking 86 times during eight classroom events, see Figure 1. Additionally, noted in Table 14, She used the ground rules Ideas Are Extended Together (GR9) on 17 occasions, Partners Engage Critically With Each Other (GR11) on 44 occasions, Opinions Are Considered Before Decisions Are Made (GR12) on two occasions, and Ideas May Be Challenged With A Counter Strategy (GR13) on five occasions.

Students also implemented the ground rules within the third theme 74 times, see Table 15. Students utilized Ideas Are Extended Together (GR9) on three occasions, Partners Engage Critically With Each Other (GR11) on 52 occasions, Opinions Are Considered Before Decisions Are Made (GR12) on 14 occasions, Ideas May Be Challenged With A Counter Strategy (GR13) on two occasions and Seek Agreement for Joint Decisions (GR14) on three occasions.

Of the 19 elements composing the ground rules in this theme, Ms. Littleton’s utilized All Have Opportunities To Question Each Others Ideas (GR11A) the most frequently, see Table 14. Although she used several other elements, none were used above 20 instances, the frequency level designated for this study. Ms. Littleton’s students also used several elements in this theme, but they did not utilize any of the ground rules at the designated frequency of above 20 instances, as noted in Table 15.

The next section examines the implementation of Partners Engage Critically With Each Other (GR11), the element comprising the majority of the discourse within the third theme Strengthening Student Thinking used by Ms. Littleton with her students.
**Partners Engage Critically With Each Other (GR11)**

Utilizing the element within the eleventh ground rule *Partners Engage Critically With Each Other (GR11)* provided the opportunity for Ms. Littleton to encourage students to interact critically with ideas and solutions while students solved mathematical problems. The elements encouraged students to evaluate and plan sophisticated solutions together. This also required students to speak out if they noticed errors in reasoning, or in support of their ideas when disagreements were involved. The following are the elements of this ground rule that were implemented by the Ms. Littleton and her students.

Ms. Littleton *Questioned Others Ideas (GR11A)* to clarify thinking and encouraged students to do the same. She often probed students’ contributions to encourage student to provide explanations or rationales. She used question such as, “So first you tried three?” to prompt students to reflect on their solution and explain their reasoning. Similarly, when a student was attempting to explain an answer, Ms. Littleton encouraged the student to interact critically with to explain his thinking.

He said, “So we found 104, so we figured out that y, x is 104 so in all is 208” (Event 1, p. 1-2).

Ms. Littleton asked, “Why did you find in all?

He replied, “Because x plus y is 208.”

Ms. Littleton pushed, “What about this diagram is telling you to do x plus y?”

The students responded “Nothing.”

Ms. Littleton followed up critically by pointing out, “Nothing, we are not adding x and y.” Students expected this type of interaction. The student continued to explain his thinking.
Likewise, Ms. Littleton supported students to question one another by initiating this process using questions such as, “Does anyone have any questions for her?” Students also questioned other students to draw attention to an error or to seek clarification.

**Summary of Theme 3: Strengthening Critical Thinking**

The most frequently used ground rule used by Mrs. Washington was Ideas Extended Together (GR9). Mrs. Washington implemented the element *Questions Used To Further Thinking* (*GR9B*) frequently to push their thinking while engaging in discourse about mathematical ideas and problems. Mrs. Washington engaged students in comparing ideas, explain their reasoning, make generalizations, and representing their thinking in different ways.

Ms. Littleton utilized the ground rule *Partners Engage Critically With Each Other* (*GR11*) most frequently with her students. She encouraged students to think critically and plan more sophisticated solutions together. Ms. Littleton used the element *All Have Opportunities To Question Each Others Ideas* (*GR11A*) most frequently with her students. This element required students to speak out if they noticed errors in reasoning. The element also encouraged students to support the validity of their ideas, even when disagreements ensued. Ms. Littleton modeled this type of questioning, and therefore it became part of their practice, as well.

Teachers and students were able to use some of the strategic discourse elements within this theme to strengthening their critical thinking. However, many elements were not consistently utilized by either teacher or their students. Therefore, the ground rules within the theme Strengthening Critical Thinking were used least often in comparison to the ground rules within the first two themes, (See Figure 6.3 and 6.4).
The following section will address the second research question guiding this study. The types of strategic discourse practices teachers identified as most successful during the individual teacher interview and teacher focus interview will be examined. Moreover, the similarities and differences among those recognized by teachers will be compared to those identified as important, by students.

**Research Question #2**

- According to the teacher, which types of strategic discourse are most successful? Are these similar or different from the types of strategic discourse identified by students?

**Successful Strategic Discourse Practices Identified By Teachers**

Strategic discourse practices guide the way students and teachers engage in mathematical discussions while solving problems in mathematics. Although Ms. Littleton and Mrs. Washington did not specifically name any of the strategic discourse strategies used during the interview sessions, they were able to share their perspectives about each of the strategies identified by the researcher. Once they were identified, benefits of each were discussed. This was also the case during the conversations during the individual and focus interviews with students.

During the individual and teacher focus interviews, both teachers discussed several important strategic discourse strategies used with students. The strategies were aligned to the elements within the ground rules they implemented throughout the study to facilitate mathematical conversations with students. These included *All Are Valued And Capable Members (GR3D), Ideas Are Discussed with Others (GR6A), Questions Are Posed To Direct Thinking (GR6B), Questioning To Try To Exchange Ideas (GR6C),*
Understand Thinking (GR6D), Assistance Is Offered To Try To Help Work Through The Process (GR6G), Many Ways Of Problem Solving Are Encouraged (GR7A), Turn And Talk To Someone About An Idea Or Concept (GR9A), and All Have Opportunities To Question Others Ideas (GR11A).

Students in Ms. Littleton's class also discussed the benefits of using the strategic discourse elements above, either during the interviews or on survey/questionnaires. These included, Ideas Are Discussed with Others (GR6A), Questions Are Posed To Direct Thinking (GR6B), Questioning To Try To Exchange Ideas (GR6C), Understand Thinking (GR6D), Assistance Is Offered To Try To Help Work Through The Process (GR6G), Many Ways Of Problem Solving Are Encouraged (GR7A), and All Have Opportunities To Question Others Ideas (GR11A).

In addition, Mrs. Washington's students discussed the benefits using the elements Ideas Are Discussed with Others (GR6A), Questioning To Try To Exchange Ideas (GR6C), Community Members Ask Questions to Understand Thinking (GR6D), Many Ways Of Problem Solving Are Encouraged (GR7A), Turn And Talk To Someone About An Idea Or Concept (GR9A) and Listening to understand other ways of thinking (GR10C).

The following section examines the important elements both teachers and students discussed during the individual and focus interviews. Moreover, results of the survey/questionnaire are also illuminated.

All Are Valued And Capable Members (GR3D)

During the individual teacher interview, Mrs. Washington reflected about the techniques she used to get students to engage in discussions. She wanted students to feel
comfortable while participating. She explained how she would throw herself into the discussions "as an equal" (Individual Teacher Interview, p. 7). She also explained how asking questions, and letting students know that she was genuinely interested in their thinking was beneficial. She also shared her thoughts about the ideas students contributed to discussions. She provided an example of what she would say. She honored their contributions when possible. She said, "You know what, I never thought about it that way." She also asked, “So why did you choose that way?"

Additionally, during the teacher focus interview, Mrs. Washington reflected about how she encouraged students to share their thinking and ideas. She reported wanting students to experience the math that they were learning. She also communicated how she expected all to participate in the mathematical conversations. This philosophy was evident in her statement, "You are not a passive participant in my classroom. I want you to share your ideas, and I want it to be okay that you make a mistake. That is something that has to be learned" (Focus Interview, p. 6).

Likewise, Ms. Littleton shared her thinking about wanting all students to participate in mathematical discussions. When asked to reflect on whether or not she used a structure for encouraging student participation she said, "There isn't really a structure, but if the student who I know doesn't normally make connections or doesn't normally share out, and their hand is up, I definitely try to call on them to give them that opportunity” (Individual Teacher Interview, p. 6).

**Ideas Are Discussed With Others (GR6A)**

Mrs. Washington discussed the benefits of students talking with teachers and peers. She found these experiences to be beneficial because they could "learn from each
other" and "not be afraid to say I don't understand what you are talking about" (Individual Interview, p. 2). She also noted the benefits of students helping others other students. She said, "If I can get them to work together maybe something that I said is told to them by a friend in a different way," and "now they get it." Also, Mrs. Washington reflected on the ways she strategically planned who contributed during these discussions to allow them to learn from one another. She wanted them to "hear from each other what's going on" (Individual Interview, p. 6).

Additionally, Mrs. Washington spoke about trying to encourage reluctant students to discuss their ideas with others. She reflected,

I have to get their response when it's a safe conversation for them. So in other words, they had a chance to hear other people, and they know that they're right. I would not call them first to start a conversation, they need to hear a few more people first, and then I can see it in their faces, and then I call on them, so we're building confidence that way (Teacher Focus Interview, p. 7).

Mrs. Littleton also discussed the importance of students discussing ideas with one another during the individual interview. When asked to reflect on why this was important, she said, "It helps them to develop their thinking and students really learn from each other" (Individual Interview, p. 4). Then she made the distinction between adult talk and students talk. She explained the positive impact that student to student conversations had on learning. Ms. Littleton explained,

I definitely think that there is a difference between adults talking to a student and a student talking to a student. It is extremely different. Sometimes they can just put it in kid-friendly language. It just helps them to understand it better where I might try using the mathematical terms they are not quite ready for. (Individual Interview, p. 4)

Students in Ms. Littleton's class also identified the importance of talking about math. During the individual student interview, Evelyn shared the positive effects of
discussing ideas with others. She said, "Some people need help, and it's good to share answers with others" (Evelyn Individual Student Interview, p. 1). Then she said, "And if it's wrong then Ms. Littleton can tell you and then it helps you to understand."

Likewise, students in Mrs. Washington's class acknowledged that talking about math and sharing ideas with others was important. For example, Gagan, said, "I can share my ideas with other people, and then they can change their ideas, and I can change my idea, so basically it is like putting ideas together and then coming up with one final idea" (Individual Student Interview, p. 2).

Similarly, Madison said, "I think it is good because I think that other people will hear what the ideas are and if I have another idea I explain it to them." (Individual Student Interview, p.1).

Benjamin also responded well to exchanging ideas while solving problems together. He said, "I like working with other people, that's what I am trying to explain" (Individual Interview, p. 3).

Moreover, most of Ms. Littleton's students and Mrs. Washington's students reported that they liked to share ideas and learning from others, see Survey/Questionnaire Table 12 and Table 16. Mrs. Washington's students also felt that it was helpful to others when they explained their thinking. Students reported that if their answer was not correct, they could find their mistake by talking with others.

**Questions Are Posed To Direct Thinking (GR6B)**

Both teachers utilized several different types of questions while engaged students in discussion. Ms. Littleton spoke about the ways she engaged students in questioning to
re-direct them when they had gone off track with their thinking. She reflected on this during the individual interview. She said,

"Sometimes I'll bring the whole class back and just like analyze what just happened, like why are we thinking this way and if it's the right way to think about this problem, for this skill. Then have students share out their thinking about it so they can analyze it together to find where the mistakes are because I still think that piece is important, like okay we made a mistake." (Individual Teacher Interview, p.5)

Ms. Washington reflected on the ways she helped her students to make real-life connections during math class. When asked to reflect on why this was important, she mentioned the area and perimeter lesson to explain that she wanted students to understand that there were "purposeful reasons for finding area" (Teacher Focus Interview, p. 5). She accomplished this task by having them walk around the perimeter of the classroom and stand in the area to help them conceptualize the concepts.

During the focus interview with Ms. Littleton's class, students were asked why they thought Ms. Littleton asked questions, to direct their thinking. They were provided with examples that included, "What would you have done differently?, and “What was challenging” (Focus Student Interview, p. 2)?

The first student responded by saying, "She asks the questions to help people understand better."

The second students said, "You might have something to say that other people agree with or that they hadn't thought of."

A third student added, "She might ask you what went well because then you could use that strategy next time."

During the individual student interviews Evelyn, a student in Ms. Littleton's class, was asked why she thought her teacher circulated to groups and asked questions while
they were working, she said that she thought that she did this, "To help you understand what you are doing."

**Questioning To Try To Exchange Ideas (GR6C) and Understand Thinking (GR6D)**

During the teacher focus interview, both teachers discussed ways they utilized questions during mathematical discussions. They often asked questions to exchange ideas and understand the thinking. During the focus interview, Ms. Littleton spoke about the ways she encouraged students to ask their peers questions after they shared their solutions.

Mrs. Washington also reflected about how she wanted her students to ask more questions. She said,

> My students will ask questions when they're reading somebody's writing. So, I'm fidgeting around with how do I get that into math; we have a common language in how to phrase questions in writing so I need to come up with how do you question when you're talking about math." (Teacher Focus Interview, p.2)

Ms. Littleton reflected about her use of questions. He interpreted the purpose of the questioning. He said,

> She wants to see, maybe you have thought one way and she wants to see another way to see which way pushes you, like if you had to pick another one that you thought would be challenging for you and she would probably be able to see that or that it was easy for you and she would probably be able to see that too. So she kind of wants to see what you are comfortable with and what is a challenge for you. (Student Focus Interview 2, p. 2)

Mrs. Washington students also shared their thinking about her use of questioning while discussing a math problem. The first student explained, "She wants to know if we actually know what she's talking about. She wants you to come up here and show that they know" (Student Focus Interview p. 6).
The second student reflected, "If we get that question wrong, we learn from our mistakes."

Moreover, many of Ms. Littleton's students reported asking their own questions to figure out what other people are thinking, see Table 16.

**Assistance Is Offered To Try To Help Work Through The Process (GR6G)**

Both teachers acknowledged assisting students as they articulated their mathematical thinking. Ms. Littleton explained how she listened closely to students to help decipher what they are trying to say. She wanted to help them communicate their ideas clearly. She described the process she used with students in math class. Ms. Littleton said, "I pull out what I understand when the thinking is illogical or ask others students what they think they have heard" (Teacher Focus Interview, p.1).

Mrs. Washington added that when students attempted to articulate their thinking they often need support. She explained her intervention. She said, "What I try to pull out what makes sense and it goes to what we're talking about for the day" (Teacher Focus Interview, p. 1). She continued, "I'll say, oh that fits, or what I hear you say is and I always repeat back, I said so that kind of fits in with what we're talking about today. Let's hold off on the other things but let's see how that piece fits into what we're talking about today"(Teacher Focus Interview, p.2).

Like their teacher, most of Ms. Littleton's students found it helpful to other students when they explained their thinking. They also reported asking questions helped their classmates to find a logical solution, see Table 16.
Many Ways Of Problem Solving Are Encouraged (GR7A)

Both teachers validated the importance of encouraging students to provide multiple ways of solving problems. When asked to discuss this strategy during the teacher focus interview, Mrs. Washington reflected about *encouraging multiple ways for solving problems*. She said,

> We did that too for multiplication. Each person uses a different strategy. So, it's a perfect example for me to say, so why did you decide to do it that way? What makes that strategy work for you? Does anybody have a question about that strategy? Or can they see themselves using that strategy? (Teacher Focus Interview, p. 2-3)

She continued by providing more examples of questions she used. She said, "I said so why would you think you would want to use that strategy? What did you like about it?"

Furthermore, Mrs. Washington added, "A lot of kids in math, it's about I'm going to get it done, I'm going to move on." She explained that her students have started to understand that identifying multiple solutions to problems is part of the process, but added that the process was not very systematic for them yet.

Also, Ms. Littleton reflected about the importance of students meeting her expectations to providing *multiple solutions to the problems* she assigned. She mentioned that students would sometimes avoid sharing solutions and instead solve the problems independently. They were rushing to complete the assignment more quickly. As she reflected about this, she recalled a time when she needed to remind students that "this was a group project" and that they "should both understand both strategies" (Focus Interview, p.4). According to Ms. Littleton, students needed to learn this critical expectation. She wanted them to put more value on finding solutions together.
During the focus interview sessions, students were asked to reflect on Ms. Littleton’s why they thought their teacher required them to provide more than one solution when discussing a problem. One student responded, "I think it was a good use of brain power instead of thinking of one way the challenge was to think of two ways" (Student Focus Interview, p. 1).

During the individual interviews, students were asked to provide similar feedback. Evelyn said that having to provide multiple solutions helped her "to understand it better" (Individual Student Interview, p. 3).

Jadiah reflected, "She wants to know if you know how to solve the problem, you know how to do the strategy, and she would want you to show your work. He continued, "She also wants me to share my thinking with the class" (Individual Student Interview, p.2). Jadiah also explained that sharing multiple strategies helped him to understand the problem a lot more. He said, "It gives me a lot of ways to do it."

Benjamin, a student from Mrs. Washington's class, reflected about why she wanted students to come up with more than one way to solve a problem. He said, "So the class can think oh I should have done it that way too" (Individual Student Interview, p. 3).

**All Have Opportunities To Question Others Ideas (GR11A)**

Mrs. Washington and Ms. Littleton consistently asked students questions while they engaged in mathematical discussions. During the individual interview, Mrs. Washington recalled a time when she was using questioning with students during the playground design task. She realized that students might have disregarded the dimensions of the playground and when reviewing their design said, "That is a beautiful garden, is it
going to fit?" (Individual Teacher Interview, p. 18) This simple question prompted the student to immediately identify her mistake and proceeded to fix the error independently.

Although Ms. Littleton consistently asked questions while discussing mathematics, she is also mindful of her students' reactions while questioning their thinking. When asked to reflect on the ways she uses and encourages questioning during the teacher focus interview, she said, "I have those students who really do share. They'll say I started this way but realized it was wrong so then we do it this way (Teacher Focus Interview, p. 7). However, Ms. Littleton also explained how students sometimes reacted in a different way. She reflected, "They have that idea that if they share something that is wrong, they think it's the end of the word. It's embarrassing." She spoke about how she tried to stress that this is okay. She wanted them to know that if they attempted a strategy that did not work, they should share their thinking so that the community can help them "get to the right spot."

Additionally, during the teacher focus interview, both teachers discussed how students could be encouraged to question one another more often during mathematical discussions. Ms. Littleton reflected on the need for more teacher modeling that included different types of questioning. She said,

Sometimes teachers just take the students' work for what it is. They say "yes you applied it correctly, but they don't necessarily say why did you put it this way. So, I think that the more teachers do it, then you can model for students to do it. (Teacher Focus Interview, p. 2)

During the individual interview, students in Ms. Littleton were asked why their teacher encouraged them to question other students about their ideas. Arthur explained that it was because they might not be sure how they "got" the answer to the problem
He also added, "They might do it in a different way or the same way, they might want to know my way."

Jadiah responded to the same question and said, "because some people might be like struggling" (Individual Student Interview, p. 3-4). He also mentioned that Ms. Littleton encouraged everyone to ask other students questions because if someone was confused about how they did the problem, they could ask how they did it, or to explain it again.

The next section details two additional discourse strategies discussed by Mrs. Washington during the interview process.

**Additional Discourse Strategies Identified By Teachers**

Mrs. Washington identified two other elements that she perceived as beneficial to her students. These elements included *Thinking Is Highlighted To Spotlight Different Ways Of Thinking (GR6E)* and *Listening To Understand Other Ways Of Thinking (GR10C)*.

**Thinking Is Highlighted To Spotlight Different Ways Of Thinking (GR6E)**

Mrs. Washington often stopped her discussions to draw attention to an interesting idea contributed by one of her students. During her interview, she shared how she highlighted "best practices" with her students. Mrs. Washington recalled a time when she pointed out an idea during the playground task. She said to the class, "Designers, I just had some really good conversation with her… she used a parallelogram as her slide" (Individual Teacher Interview, p. 6)! This was her way of bringing attention to a significant contribution.
**Listening To Understand Other Ways Of Thinking (GR10C)**

Mrs. Washington utilized the Turn and Talk strategy several times during the study. During her interview, Mrs. Washington described needing to practice "active listening," to show students how to talk to each other. Mrs. Washington emphasized that "one person has to actually listen and not talk while the other person talked" (Individual Teacher Interview, p.5).

Additionally, many of Mrs. Washington's students identified that listening to the ways their peers explained how they solved a math problem was helpful to them, see Table, 12.

The next section contains a description of the elements that were discussed by students that were not mentioned by their teacher in the interviews.

**Additional Strategies Identified By Students**

Both Mrs. Washington and Ms. Littleton's students provided insights about the element *Steps In Solutions Are Explained (GR5B)* communicated during the interview process and reported on the survey/questionnaire. Mrs. Littleton's students also provided insights about *Listening To Understand Other Ways Of Thinking (GR10C)* during conversations during the interview process and reporting on the survey/questionnaire.

**Steps In Solutions Are Explained (GR5B)**

During the individual student interview, Gagan explained why he thought Mrs. Washington asked students to explain their answers. He said, "She does it to see if we have been following what she is teaching us" (p. 2). Additionally, when asked why is it important that his teacher asks how he solved a problem in a certain way, Gagan said, "She wants to know that you know what you did."
Similarly, when asked why he thought Mrs. Washington asked him to explain his answer to the class Jadiah said, "I show them how I did my work, and they start to understand it a lot more, and if they don't understand it I still talk to them to see how they are doing and end up knowing what I am saying" (Student Individual Interview, p. 2).

Most students from Mrs. Washington's class also expressed that shared thinking helped them to figure out if they are on the right track, see Table 12 4.3. Similarly, most of the students in Mrs. Washington's class reported that it was helpful to them when they were asked to explain their thinking. Most also reported that sharing their thinking helped them to determine if they were on the right track.

Listening To Understand Other Ways Of Thinking (GR10C)

Listening to others share their thinking was part of the conversation in the student interviews. When asked why it was important to listen while someone was sharing their ideas, Evelyn reflected, "It is important because that is how you learn things in math, and it is good to know what their answer would be (Individual Student Interview, p. 3). Likewise, most students, from Ms. Littleton's class, claimed listening to how other students explain how they solved a math problem was helpful to them, according to the Survey/Questionnaire, see Table 16.

Summary

During the interviews, teachers discussed the positive impact strategic discourse strategies had on their instruction and student learning. Both teachers and students in this study discussed their perceptions regarding the strategies they used throughout the study. Ms. Littleton and Mrs. Washington both discussed utilizing nine of the same elements.

These included All Are Valued And Capable Members (GR3D), Ideas Are Discussed with
Others (GR6A), Questions Are Posed To Direct Thinking (GR6B), Questioning To Try To Exchange Ideas (GR6C) and Understand Thinking (GR6D), Assistance Is Offered To Try To Help Work Through The Process (GR6G), Many Ways Of Problem Solving Are Encouraged (GR7A), Turn And Talk To Someone About An Idea Or Concept (GR9A), and All Have Opportunities To Question Others Ideas (GR11A).

Mrs. Washington highlighted two additional strategies that she found beneficial. These included Thinking Is Highlighted To Spotlight Different Ways Of Thinking (GR6E) and Listening To Understand Other Ways Of Thinking (GR10C).

Students also highlighted six of the same elements as their teachers either during the interview or on the survey questionnaire. Mrs. Washington's students discussed Ideas Are Discussed with Others (GR6A), Questioning To Try To Exchange Ideas (GR6C) Understand Thinking (GR6D), Many Ways Of Problem Solving Are Encouraged (GR7A), Turn And Talk To Someone About An Idea Or Concept (GR9A), and Listening To Understand Other Ways Of Thinking (GR10C).

Ms. Littleton's students noted the benefits of eight of the same elements as their teacher. These included All Are Valued And Capable Members (GR3D), Ideas Are Discussed with Others (GR6A), Questions Are Posed To Direct Thinking (GR6B), Questioning To Try To Exchange Ideas (GR6C) and Understand Thinking (GR6D), Assistance Is Offered To Try To Help Work Through The Process (GR6G), Many Ways Of Problem Solving Are Encouraged (GR7A), and All Have Opportunities To Question Others Ideas (GR11A).

Students also spoke of additional elements they found beneficial during discourse not mentioned by their teachers during the interviews. Mrs. Washington students
discussed the benefits of the element *Steps In Solutions Are Explained (GR5B).* While Ms. Littleton students discussed the benefits of *Steps In Solutions Are Explained (GR5B)* and *Listening To Understand Other Ways Of Thinking (GR10C).*

The following section will address the third research question guiding this study and examine the ways teachers understand and use reform-based methodology to engage students in mathematical discourse while solving problems in math. Moreover, the four pillars of mathematical reform relate to Mrs. Washington and Ms. Littleton, as outlined in the first Chapter will be explored.

The following section will address the third research question guiding this study and examine the ways teachers understand and use reform-based methodology to engage students in mathematical discourse while solving problems in math. Moreover, the four pillars of mathematical reform relate to Mrs. Washington and Ms. Littleton, as outlined in the first Chapter will be explored.

**Research Question #3**

- In what ways do teachers come to understand and implement reform-based practices?

Ms. Littleton and Mrs. Washington fulfilled their professional responsibilities by implementing the Massachusetts State and Common Core Standards in mathematics, as dictated by their school district. While doing so, they utilized all four reform-based practices for building mathematical understanding, as outlined in the first Chapter of this document. The practices for building understanding; problem posing, authentic learning with, learning through interaction were embedded in the activities and conversations teachers facilitated with students in a community of practice. Teachers made the daily
decisions concerning instruction. The decided what students needed to learn, how they engaged in learning, and when and how they were assessed (Martin, Towers & Pirie, 2006; Warfield, Wood & Lehman, 2005). In addition to implementing these reform practices, Mrs. Washington and Ms. Littleton implemented structural changes that include providing opportunities for students to develop more autonomy and authority in the mathematics classroom with very little guidance from state and local administration (Boylan, 2010, Cuban, 2013).

The participants were purposely chosen for this study because they had been recommended by their school administration based on their knowledge of reformed based mathematics instruction, and their utilization of mathematical conversations with their students. Given this information, along with their experience teaching mathematics in the same school for four years, it was expected that similarities in their instruction would exist. However, given the differences in their educational background, teaching experiences and personal style, it was also expected that differences existed, as well.

The analysis below demonstrates the ways two teachers implemented reform-based teaching practices over the course of this study. Table 21 indicates whether or not the reform-based practices were part of each teacher’s instruction during each classroom event. Also included is a description of the methods used to implement each of reforms. Additionally, the analysis of the individual teacher interviews provided additional information about the ways teachers came to know the reform-based practices. The interviews also examined the ways the teacher’s experiences, thinking and perceptions about these methods impacted their instructional decision making.
This following analysis includes a description of the ways Mrs. Washington’s came to know and understand reform-based practices used to teach mathematics with understanding.

Mrs. Washington and Students

Mrs. Washington is a self-proclaimed non-traditional teacher in her late forties. She has been teaching in elementary schools in Massachusetts, for over 15 years. Mrs. Washington has a bachelor of science degree in Child and Adult Psychology and a master’s degree in Elementary Education. Mrs. Washington has taught third grade for thirteen years, along with two years teaching special education. During the times of this study, she was working as a third-grade general educator in a classroom alongside a full-time special educator during math and English Language Arts. During math classes, a special education teacher joined Mrs. Washington’s mathematics classes to support the special education students who have specialized education plans for mathematics. Mrs. Washington taught special education for a few years, then she taught grade five, before teacher third grade. During the individual interview, Mrs. Washington detailed her struggles with learning math, explaining that she struggled with concepts during her own sixth-grade experience. She said this experience makes her more determined to have students believe, "that they are able to do it [math]" (Teacher Individual Interview, p.7).

Mrs. Washington encouraged students to engage with others in conversations about mathematics often throughout the study. She described how talk was used with students to discuss their problem-solving strategies. Mrs. Washington said, "I think math is a social content. If we need to be able to solve world problems, then we should be able to do it in math class. When we do that I think you have to be able to talk and
communicate" (Teacher Individual Interview, p. 2). Mrs. Washington used problem-solving tasks as a way of developing social skills. She wanted students to learn how to interact because the tasks required students to "talk with other people" (Individual Interview, p. 8). The more advanced students, according to Mrs. Washington, may not need to practice all of the skills but they need" to learn how to agree on an explanation to explain their thinking."

When asked to reflect on her teaching, she said that having special education students made her realize that students needed the skills taught to them differently. She said:

I actually spend a lot of time thinking about the words that we're using, how are the words communicating a concept to them and how am I going to develop an opportunity to then take what we talked about and play with it. Let's take what they know and let's start applying it to everything that is going on. (Individual Teacher Interview, p.1)

When she moved from a special education teacher to a general education teacher, she noticed that students knew how to do the calculations in mathematics but did not understand why they were doing it. That was when she started teaching and thinking differently. She wanted students "to experience what they needed to learn and not just learn what they need to learn" (p.1-2).

Mrs. Washington also shared that she thought teaching was more about the process, not just answers in mathematics. During the individual interview, she mentioned spending a lot of time talking about the process with students. She also described being more comfortable with the experimental side of mathematics, acting it out, now that she has taught the content for five years. She also emphasized the use of hands-on experiences in her instruction. She felt strongly that students should be able to figure
things out. Mrs. Washington also explained using a spiraling curriculum and embedding skills within students problem-solving tasks and conversations. She purposefully exposed students to skills all year long and not just during units of practice.

This following analysis includes a description of the ways Mrs. Washington’s came to know and understand reform-based practices used to teach mathematics with understanding.

**Mrs. Littleton and Students**

Ms. Littleton has been teaching fourth grade for just over four years at Lakeview Elementary School. She is in her mid twenties. She earned a bachelor of arts degree in Mathematics and a master’s degree in Elementary Education. Ms. Littleton has also taken classes in special education but has not earned a degree.

During the individual interview, Ms. Littleton explained how her math classes and education classes had prepared her to teach mathematics. She spoke of one course in particular because it showed her how to use "more hand-on and not just book instruction all of the time" (Individual Teacher Interview, p. 1). Ms. Littleton described learning a lot from the experience of teaching math, and while working with other teachers. She also felt that her math background was helpful. According to Ms. Littleton, "It was the questioning that I would do in those classes that I can apply to my students in my class now, even though that was extreme levels, it's still similar." She added that she felt confident questioning students and helping them to try to explain what they were doing.

Ms. Littleton mentioned experiencing difficulty with helping students who struggled. She struggled with having to "peel back” their thinking to expose “the gap” because it's going to effect everything that we're trying to learn." (Individual Teacher
Interview, p.1). However, she mentioned that a professional development workshop offered by her district "opened up her eyes to the skills that her students might be missing" (Individual Teacher Interview, p. 3).

When asked how long she was using reformed-based mathematics practices, she asked, "What is reformed-based methodology?" After clarifying reformed-based methodology, Ms. Littleton shared how she utilized the practices in her classroom. She explained how her instruction differed from traditional instruction. She said, "It's more modeling, it's having kids discuss, having the kids show by drawing or using manipulatives" (Individual Interview, p.2). Ms. Littleton also reflected on her own readiness for using these methods in her math instruction. She said:

I mean I did not have a whole bag of tricks written like that. This definitely developed after I graduated, but at least I had the knowledge that I wanted to incorporate those things and had a few that I can get started with and then build on it through experience. (Individual Interview, p. 2)

Ms. Littleton also spoke about engaging students in mathematical conversations. She acknowledged the complexities of mathematical conversations and the need to follow students as they lead the conversation in new directions. When asked to reflect on the structure of the discussions she explained:

There are definitely things that have to be done and structured. And then there's some that we're just, as we discover one thing, we're moving forward to something else that I didn't plan it to go. But that the way students are learning, so I'm going to go with that. (Individual Interview, p. 6)

Ms. Littleton also reflected about her use of questioning during these conversations. She described probing students’ thinking and reacting in-the-moment to help them think more clearly during mathematical conversations. She said, "I teach that
way because I feel like it's the right way to do it, but I don't really like, no one told me that that's what I needed to do" (Individual Interview, p. 9).

The teachers have developed their own thinking around the methods they use to implement practices which are consistent with reform-based mathematics instruction. Mrs. Washington wants students to be successful, so she has developed methods that focus on strengthening their language skills as they engage in hands-on experiences to process the mathematics concepts and skills required of third graders. Ms. Littleton uses questioning to initiate thinking and talking about mathematics while using a variety of methods to fill in the gaps for her fourth graders.

The following section includes an examination of the implementation of the remaining reform-based practices, as outlined in Chapter 1 of this document, i.e., Problem Posing, Active Learning With Authenticity, and Learning Through Interaction by Ms. Littleton and Mrs. Washington. Both teachers were observed using the reformed-base practices while engaging students in mathematical activity during eight events. Table 21 summarized the implementation data about the practices listing the components supporting each one.

The following section includes an examination of the implementation of the remaining reform-based practices, as outlined in Chapter 2 of this document, i.e., Problem Posing, Active Learning with Authenticity, and Learning Through Interaction by Mrs. Washington. Mrs. Washington was observed using the reformed practices while engaging students in mathematical activity during eight events. Utilizing the Observation Protocol, each classroom event was analyzed to determine whether or not these practices were utilized as part of each teacher’s instructional practice. Table 21 summarizes the
implementation data about the practices listing the components supporting each one. The highlighted areas indicate the occurrence of each component of the reform-based practices. An * indicated when events included well-defined problems, and well-defined tasks occurred during the events with **.

Reform-Based Practices For Learning Mathematics With Understanding

Problem Posing

Problem posing was used to develop an understanding of concepts and to enhance reasoning and reflection skills (Cunningham, 2004, Lampert, 1990). The three components of problem-posing are well-defined problems or tasks, enriches concepts and skills and provides a structure for discussion. As described below, Mrs. Washington and Ms. Littleton utilized a combination of the problem-posing components throughout the study.

First, well-defined problems challenged students in ways that required them to think beyond the skill. The well-defined problems also presented opportunities for students to solve problems in more than one way. Additionally, well-defined tasks included project based problem that required students to connect their learning to a real-life situation. Tasks were complex and usually completed over the span of one or two class periods.

Mrs. Washington engaged students in discussions by posing a problem to generate discussion during events one and two, see Table 21. She used tasks during events three, six, seven, and eight. These well-defined tasks required students to solve authentic problems using skills learned throughout the year. Students completed these tasks while working in groups or pairs. While completing the final project during events seven and
eight, students were given the opportunity to work independently on tasks. During events four and five, the lesson helped to set the stage for the tasks completed during the next math class. The preliminary discussion included a problem that guided students to practice exploring a concept so that students were prepared to work on the task. The mathematical discourse facilitated by Mrs. Washington helped students to think about and apply the skills before having to complete the task on their own.

For example, during Events 1 and 2, Mrs. Littleton assigned problems for students to complete in small groups. The problems required students to make the connections between angle measurements and fractions (Case B Transcripts, Event 1, p.1). One question from Event 1 included, “How many right angles would be equal to a full circle.” The problem students completed during Event 2 required them to find a “missing angle” (Case B Transcripts, Event 2, p.4). Following the independent problem-solving work in these lessons, Ms. Littleton reconvened the group to discuss their solutions. She asked, “Who would like to share out?” This question sparked several students to share how they determined their answer with little facilitation needed on the part of Ms. Littleton.

Mrs. Littleton also engaged students in extending their discussions by posing problems or assigning tasks, see Table 21. During Events 1 and Event 2, she provided students with problems to complete in small groups. Following the independent practice, Ms. Littleton facilitated a conversation to discuss the strategies and solutions students used. The first part of the lesson in Event 3 helped to set the stage for the task students completed later that day. The follow-up task required students to complete a capacity problem involving students filling up water balloons. One problem read, “The package of Mel’s water balloons says that it holds 300 milliliters of water. How many balloons can
he fill if he has two liters of water?” Students solved the problem using two strategies which they presented to the class during math class, the next day (Case B Transcripts, Event 2, p. 7-13). On the following day, during Event 4, students presented their solutions to the task. As part of the task presentation, students answered two questions and received one comment from their peers (Case B Transcripts, p. 13-17). For example, one student asked another, “Did you get confused at any of the parts” (p. 15)? Additionally, students had to apply their knowledge of measurement to calculate the amount of ribbon needed to create a bow (Case B Transcript, Event 5, p. 17-20). To initiate their work, Ms. Littleton said, “The size of the bow and the length of the ribbon are up to you. After you make the bow and trim the ends, measure the ribbon you used” (p. 17). Then on the following day, students used the experiences to calculate the ribbon needed to wrap an entire box with a bow (Case B Transcript, Event 6, p. 20-22). Finally, during Event 7, students shared their learning experiences working on this task and then extended their learning to how what they learned could be applied in other real-life situations (Case B Transcript, p. 23-27).

Again, Mrs. Washington used either a well-defined problem or task during all events. Problems were posed during events one, two, four and five to engage students in thinking about the ideas and concepts embedded in the tasks, see Table 21. During Event 1 for example, Mrs. Washington posed the problem, “My doctor does not tell me how many inches tall I am. He tells me that I am 61 inches. He tells me that I am a certain amount of feet tall. How tall am I” (Case A Transcript, Event 1, p.1)? Mrs. Washington also poses a problem to engage students in a discussion about the different ways to represent data during the second event. Mrs. Washington posed the question, “Could I
have collected my data in a different way” (Case A Transcript, Event 2, p. 3)? Also, During Event 4, Mrs. Washington began the lesson by posing the question, “What is an array” (Case A Transcript, p. 7)? The discourse that occurred during this event helped to review the concept of an array and the ways arrays could be applied to measurement. Tasks required students to complete a project using the skills reviewed in the problems. Finally, students used their problem-solving skills when asked to “stand in the area,” and “walk the perimeter” in Event 5 to demonstrate their understanding of perimeter by walking around the perimeter of the classroom (Case A Transcript, p. 9).

Students engaged in tasks during four events including the third, sixth, seventh and eighth events. During Event 3, students completed a task that required them to collect and represent data about their favorite cupcakes (Event 3, p. 5). The more challenging task required groups to survey classmates about favorite flavors and even design a new flavor of cupcake. In Event 6, students were working on the playground/garden design (Case A Transcript, p. 6). The task required students to build a playground or garden with four rectangular shapes, three square shapes, two combined shapes and one polygon, see Appendix K. Students were asked to record their dimension in feet and yards. A brochure was also created once students had completed their design and labels. During Event 7, students completed a data measurement project that allowed them to identify a question, collect the necessary data and choose a graph to represent the results (Case A Transcript, p. 12). Finally, during Event 8, students chose a task from a variety of options. One task required students to design a mall using predetermined geometric shapes, area and perimeter (Case A Transcript, Event 8, p. 13). The most
difficult task required the student to compare three cell phone data plans to determine which was the most cost-effective.

Enriches concepts and skills, the second component of problem posing was also utilized by both teachers. Many of the lessons conducted by Ms. Littleton focused on enriching the concepts and skills students had learned earlier in the year (Atlas Rubicon, 2018). Mrs. Littleton also engaged students in extending their discussions by posing problems or assigning tasks, see Table 21. During Events 1 and Event 2, she provided students with problems to complete in small groups. Following the independent practice, Ms. Littleton facilitated a conversation to discuss the strategies and solutions students used. The first part of the lesson in Event 3 helped to set the stage for the task students completed later that day. The follow-up task required students to complete a capacity problem involving students filling up water balloons. One problem read, “The package of Mel’s water balloons says that it holds 300 milliliters of water. How many balloons can he fill if he has two liters of water?” Students solved the problem using two strategies which they presented to the class during math class, the next day (Case B Transcripts, Event 2, p. 7-13). On the following day, during Event 4, students presented their solutions to the task. As part of the task presentation, students answered two questions and received one comment from their peers (Case B Transcripts, p. 13-17). For example, one student asked another, “Did you get confused at any of the parts” (p. 15)? Additionally, students had to apply their knowledge of measurement to calculate the amount of ribbon needed to create a bow (Case B Transcript, Event 5, p. 17-20). To initiate their work, Ms. Littleton said, “The size of the bow and the length of the ribbon are up to you. After you make the bow and trim the ends, measure the ribbon you used”
Then on the following day, students used the experiences to calculate the ribbon needed to wrap an entire box with a bow (Case B Transcript, Event 6, p. 20-22). Finally, during Event 7, students shared their learning experiences working on this task and then extended their learning to how what they learned could be applied in other real-life situations (Case B Transcript, p. 23-27).

The first two lessons in Mrs. Washington’s classroom required students to extend their knowledge by converting linear measurement (Case A Transcript, Event 1, p. 1-2) and developing understanding about tools used to collect and display data (Case A Transcript, Event 2, pp. 2-3). During Event 4 and Event 5 students did not extend their learning but instead reviewed concepts and skills in preparation for upcoming tasks (Atlas Rubicon, 2018).

The well-defined tasks assigned by Mrs. Washington required students to apply and extend their skills in different ways to strengthen and enrich their prior learning and connect these skills to real life (Cupcake Challenge Lesson, Event 3, pp. 4-6). In Event 6 (Figure It Out, p. 11) students used arrays to design and calculate area and perimeters of object in gardens and playgrounds, see Appendix K. During Event 7, students completed a data measurement project that allowed them to identify a question, collect the necessary data and choose a graph to represent the results (Case A Transcript, p. 12). Finally, during Event 8, students chose a task from a variety of options. One task required students to use the knowledge they developed during the earlier design task to design a mall using predetermined geometric shapes, area and perimeter (Case A Transcript, Event 8, p. 13).
The third component within problem posing includes the structure used by teachers to support problem posing. Both teachers used the problems and tasks as a structure to frame their problem posing. The problems and tasks enabled the teachers to engage students in activity and conversation to assist in strengthening their understanding of the mathematics. The problems and tasks selected addressed skills embedded in their curriculum (Atlas Rubicon, 2018). According to the data collected in classroom observations, all lessons in every event included problem-solving and discussion, therefore using problem posing as a structure to engage students in conversations about mathematics was critical to the instructional practice among both of these teachers, see Table 21.

Teachers framed the structure for bringing students together to discuss the problems differently. Ms. Littleton began lessons by engaging students in small group problem-solving activities, then they came together to discuss their understanding (Case B Transcripts, Events, 1, 2, 3, 4, and 6). As students worked in small groups, she circulated the room to investigate their thinking and helped them to move forward, taking into consideration their understanding and ability to complete the problem or task. After students completed problems or tasks, Ms. Littleton dedicated time to engaging students in conversations about their solutions and ideas. During Events 5, students gathered to reviewed the units of liquid measurement before they worked on their task (Case B Transcripts, Event 5, p. 17-20). Moreover, students also gathered during Event 7 to discuss the results of their task in Event 6.

Mrs. Washington on the other hand, usually began each lesson talking with students about what they could expect to happen during their math class. Then Mrs.
Washington would pose a problem for students to discuss. As they talked, Mrs. Washington checked their contributions for understanding. During Events six through eight students did not engage in a group discussion. They were working on tasks that had already been assigned (Case B Transcripts, p. 11-17). During these times, Mrs. Washington circulated to interact with students and their ideas as they wrestled with tasks.

Neither teacher directly taught content while problem posing. The content and skills needed to complete the problems or tasks were skills that students had experienced earlier in the year. The problems and tasks required students to extend these skills authentically in new and different ways. During the discussions Mrs. Washington posed questions, described earlier, to investigate students thinking and then filled in the knowledge by guiding students to discuss areas where gaps emerged. She even repeated these questions and requested that many students answer them to be sure students did not miss the information shared (Walking the Perimeter Lesson, Event 5, p. 9-10). Mrs. Washington differentiated her assignments so that students could develop according to their academic needs. For example, during Event 3, different requirements were instituted for different groups of students based on their academic needs. The more challenging task required groups to take on additional responsibilities than the other two groups (Event 3, p. 5). Additionally, students in this classroom do not wait to be called on, they simply jump into the conversation when they have something to contribute (Case A Transcripts, Event 1, p. 1-3). This is an acceptable practice in the community.

On the other hand, Ms. Littleton started with a question that encouraged students to share the things they thought about during their problem-solving work. She asked,
“Who would like to share how they solved the problem?” (Case B Transcript, Event 1, p.1). Then students would take turns sharing their thinking. As they shared Ms. Littleton and the other students would listen and then ask questions about the contribution. Most of the questions originated from Ms. Littleton, but at times students posed questions seeking clarification (Case B, Transcript, 14-15).

Ms. Littleton also differentiated the capacity task during Event 3 by encouraging students to challenge themselves to identify two solutions and by assigning additional problems to some students (Case B Transcripts, p. 11-17). Students are given many opportunities to contribute in this classroom, however, they do wait to be called on by their teacher during class discussions (Case B Transcripts, Event 1, p. 1-3). This is not the case during small group instruction.

Table 21. Implementation of Reform Practices for Teaching Mathematics

<table>
<thead>
<tr>
<th>LITTLETON</th>
<th>EVENTS</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Problem Posing</strong></td>
<td>1</td>
</tr>
<tr>
<td>Well Designed Problem*/Task**</td>
<td>*</td>
</tr>
<tr>
<td>Enriches the Concept/Skill</td>
<td>*</td>
</tr>
<tr>
<td>Provides Structure for Discussion</td>
<td>*</td>
</tr>
<tr>
<td><strong>Active Learning with Authenticity</strong></td>
<td>1</td>
</tr>
<tr>
<td>Engages in Learning</td>
<td>*</td>
</tr>
<tr>
<td>Real Life Connections</td>
<td></td>
</tr>
</tbody>
</table>
Honors Mathematics As A Discipline

<table>
<thead>
<tr>
<th>Learning Through Interaction</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>Learning is Socially Constructed</td>
<td>*</td>
<td>*</td>
<td>*</td>
<td>*</td>
<td>*</td>
<td>*</td>
<td>*</td>
</tr>
<tr>
<td>Contributes to Learning Of Others</td>
<td>*</td>
<td>*</td>
<td>*</td>
<td>*</td>
<td>*</td>
<td>*</td>
<td>*</td>
</tr>
</tbody>
</table>

* Indicates occurrence of the component within Reform-Based Practice within each lesson

**Active Learning with Authenticity**

Active Learning With Authenticity is the second of four reformed-based practices for building understanding in mathematics. As described below, the components of Learning With Authenticity were implemented by each participant during all events, see Table 21. These components include included engages in learning, connects to real life, and honors mathematics as a discipline.

The first component engages in learning requires students to be active participants who are fully engaged in the process of building thinking. Authentic mathematical activity occurs within the activity of students as they discuss mathematics (Ball & Bass, 2000). Students engage with the content, as well as, with the community. Additionally, authentic learning occurred within the activity of students as they discussed mathematics with one another (Weiss et al., 2009). Authenticity was evident in the students’ contributions during this mathematical discourse, even if students were not always completely accurate. Both teachers provided opportunities for student to engage in learning in these two ways.
The first component of Active Learning With Authenticity is *engages in learning*. Authenticity in learning occurred within the activity of students and discussions students participated while studying mathematics. Ms. Littleton provided opportunities for students to *engage in learning* by using problems and tasks that required students to interact with the problems and one to complete them. This required students to engage by exchanging ideas, actively planning and testing solutions. The students engaged with peers while solving problems as much as they did with their teacher. Ms. Littleton asked questions after students contributed to the discussions, and encouraged students to communicate their ideas. She also circulated the room while students worked in groups to gather information about their work together. Other interactions occurred when students engaged with their teacher to ask questions and offer feedback. Students were consistently encouraged to join these conversations. Students responded well, always demonstrating a high level of participation in discussion, especially while using discourse elements to engage with one another, see Table 11 and 15.

Several of the elements of strategic discourse assisted Ms. Littleton in engaging students with the mathematics and one another, see Table 14. Below are some examples of how the elements that encouraged engagement with the mathematics and mathematical ideas of others were implemented.

Ms. Littleton utilized elements within the ground rules to *engage students* by requiring them to share their *ideas and solutions* (*GR6A*). Most of the elements were implemented a question or prompt. Mrs. Littleton questioned her students thinking by requiring them to explain their ideas (*GR6A*). During Event 1, one student shared her
solution. She said, “So I know that there's 90 degrees in one part of a 360 degree circle” Ms. Littleton stepped in to prompt her to explain her thinking more explicitly. She asked, “You know that 90 degrees is what” (Case B Transcripts, Event 1, p. 1). Ms. Littleton also used questioning to direct thinking (GR6B) and to understand what students knew about the mathematics (GR6D). For example, she asked, “Elenor connected her thinking with what? What else did we learn this year” (Case B Transcripts, Event 1, p. 2)? She also encouraged students to engage with the mathematics by pushing them to think creatively and share multiple ways of solving problems (GR7A). In Event 2 for example, Ms. Littleton asked, “Who would like to share out how you solved this problem” (Case B Transcripts, p. 1). Students were also challenged to come up with two ways of solving the capacity problems during Event 4 (Case B Transcripts, Event 1, p. 1). This also occurred during conversations when Ms. Littleton consistently asked for more students to share their thinking with the prompt, “Would you like to share your thinking” (Event 2 page 5)? Questions were also used by Ms. Littleton to encourage more students to engage by exchanging ideas (GR6C). For example, she often prompted students to discuss the thinking of others. She did this by prompting in general terms, “Any questions for Zandra” (Case B Transcripts, Event 1, p. 1)? She also asked more specific questions about the mathematics. For example, during Event 7, she asked, “So what would happen if you did an underestimate?” when students were reflecting about measuring the bow needed to wrap the box in the sixth event (Case B Transcripts, p. 27). Moreover, questions were used to engage students in the content to further their thinking (GR9B). For example, during Event 2, she asked, Boys and girls do we have any questions or suggestions about how they are solving the problem” (Case B Transcripts, p.
During Event 3 also asked students to consider one solution in light of another. She asked, “So why do you think Zandra thinks it should be 16 and Evelyn thinks it should be 32” (Case B Transcripts, p. 9)? Ms. Littleton also used questions to encourage more students to engage by *exchanging of ideas (GR6C)*. She often prompted students to discuss the thinking of others. She did this by prompting in general terms, “Any questions for Zandra” (Case B Transcripts, Event 1, p. 1)? She also asked more specific questions about the mathematics. For example, during Event 7, she asked, “So what would happen if you did an underestimate?” when students were reflecting about measuring the bow needed to wrap the box in the sixth event (Case B Transcripts, p. 27).

Mrs. Washington’s also implemented the second component of Learning Through Interaction. She used the component engages in learning to connect students with one another and the mathematics they study. The lessons planned during each event required students to interact with the mathematical content and their teacher to build knowledge and understanding. Also, students engaged with peers to discuss their thinking, most often during group discussions facilitated by their teacher. Students were consistently encouraged to join these conversations. Students responded well, always demonstrating a high level of participation in discussion, especially while using discourse elements to engage with one another, see Table 12 and 15.

Like Ms. Littleton, several of the elements of strategic discourse assisted Mrs. Washington in engaging students with the mathematics and one another. Below are some examples of how the elements that encouraged engagement with the mathematics and mathematical ideas of others were implemented.
As described earlier in this chapter, Mrs. Washington engaged students using elements to encourage them to *share ideas and solutions* (*GR6A*). Most of the elements were implemented using a question or prompt. Mrs. Washington questioned her students thinking by requiring them to explain their ideas. For example, she would pose a question and say, “How tall am I and how did you figure it out?” (*Case A Transcripts*, p. 1). She also used certain questions to get students to share their ideas. For example, during the cupcake challenge, students were looking at the graph, and she asked, “Now you will see some things that are very familiar to you, does that look familiar” (*Case A Transcripts*, Event 3, p. 4-6)? Mrs. Washington also used the ‘turn and talk’ strategy several times to initiate the sharing of ideas among students. During these instances, she used prompts such as, “Turn to someone and talk about why we use 12 inches equals 1 foot” (*Case A Transcripts*, Event 1, p. 1) and “So could we collect data using a pictograph or do we have to use the tally chart and then make the pictograph” (*Case A Transcripts*, Event 2, p. 3). The purpose was to get students to consider another student’s thinking. One of these questions included, “Are you are agreeing or disagreeing with her” in Event 2, (*Case A Transcript*, p. 2). “What is the main idea of their questions, Cameron and Aaron” was another questions that she asked while students were suggesting questions to go along with their surveys during Event (*Case A Transcripts*, p. 4-6)? *Mrs. Washington* also used *questioning* to engage students with the concepts while trying to *understand* what they knew about the mathematics (*GR6D*). For example, during the measurement lesson in Event 1 Mrs. Washington said, “I am 61 inches, but if I wanted to say how many feet I am tall and how do you figure it out” (*Case A Transcript*, Event 1, p 1)? A student immediately answers, “You are 5 feet and one inch.” Then Mrs. Washington asks, “How
did you figure that out?” to uncover the student’s thinking. Later in the same event Mrs. Washington prompted a student to initiate an explanation, “I have no idea, what you are talking about.” Mrs. Washington also engaged students in learning when spotlighting original ideas (GR6E) to draw attention to a novel way of thinking. For example, during Event 6, Mrs. Washington said, “Wait! Wait! So, your strategy was multiplication” to draw everyone’s attention to a student making the leap that instead of adding the feet that she could multiply them (Case A Transcripts, Event 1, p.1-)? Mrs. Washington also emphasized the element many ways of solving problems (GR7A) to engage students with both the mathematics and one another. She wanted students to challenge themselves to share a different solution. She asked, “Who did it in a different way” (Case A Transcripts, Event 1, p.1-2)? She also used the question, “Anyone have a different question” to continue the conversation (Case A Transcripts, Event 3, p. 4-6)? Finally, Mrs. Washington used questions to engage students in the content to further their thinking (GR9B). These questions were challenging for students. While students were discussing the Cupcake Challenge Task, Mrs. Washington asked, “Why would a line graph work (Case A Transcripts, Event 3, p. 4-6)? Students were suggesting various types of graphs and Mrs. Washington was trying to encourage them to choose a graph that best matched their data, which was an essential question noted in the third-grade curriculum (Atlas Rubicon, 2018). Similarly, she also asked So, my question was, could I collect my data another way? While they were discussing the Summer Reading Tally Chart during Event 1 (Case A Transcript, p. 2-4)

The second component of Active Learning With Authenticity is connects to real life. Activities that connect to real life allow students to make connections between the
way math is used in school and the world around us (Boaler, 1999; Bracha, Zemira, & Arami, 2002, and Weiss, Herbst, & Chen, 2009). Using activities that connect to real life helped students to practice using math the ways others would do to solve real-life problems.

All Mrs. Washington's lesson included connections to situations where the mathematics discussed could be used in real life, see Table 21. All of the lessons that Mrs. Washington conducted during the study connected math in the classroom with mathematics in the world outside of school. For instance, during the first events, students converted their teacher’s height from inches to feet and inches. Students answered the question, “Do you go to the library during the summer” (Case A Transcript, Event 1, p. 1). This data was collected in a Tally Chart and afterward reviewed by the class. During Event 2, students converted their teacher’s height from inches to feet and inches (Case B Transcripts, p. 2). Next, students collected data to identify and design favorite cupcakes, in Event 3, (Case B Transcripts, p. 4). They also determined the type of graph to illustrate their data. During Event 4 discussed how arrays could be used to find the area and perimeter (Case B Transcripts, p. 7). They also identified arrays in their environment, before completing the Array Project during the next class, see Appendix K. During Event 5 and 6 the task required students to design a playground or garden using their knowledge of arrays, area and perimeter (Case B Transcripts, pp. 9-11). During the remaining events, students completed individual project that also included connections to real life. These projects included comparing cell phone plans and designing a mall (Case B Transcripts, pp. 11-13).
Many of Ms. Littleton’s lessons included connections to situations where the mathematics discussed could be used in real life, see Table 21. Five out of seven lessons included a real-life connection. During Event 3, students completed capacity problems such as, how many water balloons could be filled with different amounts of water (Case B Transcript, Event 3, p. 11). As part of this task, students were asked to represent their two solutions on a chart paper and present their solutions to the class (Case B Transcript, Event 4, p. 14). Students practiced their measurement skills while estimating and then identifying the length of ribbon needed to make a bow and wrap a present (Case B Transcript, Event 5, p. 17). During Event 6, students used this knowledge to estimate the amount of ribbon needed to wrap a box, including the bow. Then they measured the exact length needed (Case B Transcript, p. 20). Finally, students reflected about their experiences working as a group and the understanding they developed, including how they might approach the same task in the future (Case B Transcript, Event 7, p. 23).

Additionally, Ms. Littleton encouraged students to think about other real-life situations where estimates are used or not used.

Honors mathematics is the third component of Active Learning With Authenticity. Honors mathematics as a discipline meant that students learned to understand mathematics in ways that mirrored the structure and content of mathematics as a discipline (Weiss et al., 2009). Mrs. Washington and Ms. Littleton both used mathematics that mirrored the structure and content of mathematics as a discipline throughout the study. As shown below, this occurred when they shared content and provided guidance to students about how to solve problems in a logical fashion. They also consistently honored mathematics as a discipline when the structure of mathematics
was highlighted during lessons both during large and small group work. Mrs. Washington and Ms. Littleton also guided their students to do the same. Students were required to explain their thinking by making connection among their thinking and mathematical content. They were also encouraged to provide explanations that included representations of their thinking often using a numeric format. The following are some of the situations when Ms. Littleton honored mathematics as a discipline during her work with students.

Ms. Littleton *honored mathematics as a discipline* during Event 1, as she directed students thinking toward identifying the number of right angles contained in a circle (Case B Transcripts, p. 1). During the conversations, she guided students in making connections among concept within the discipline of mathematics. Together they explored the relationship of a right angle to one-fourth of a circle. During the discussion, a student shared out his solution. He said,

I know that 90 degrees is one-fourth of the 360-degree circle so I divided 90 by four, 360 degrees by 90 and I got four. I also made a circle and divided it into four pieces starting with 90 here, 90 here, 90 here, and 90 here. And there's four pieces." (Event 1, p.1-2)

When the student did not identify the connection between 90 degrees and a right angle, Ms. Littleton stepped in with a reminder. She asked, “Four right angles?” The student agreed. The conversation continued as other students shared various ways of thinking about the problem and using conventional ways to add and subtract the 90 degrees from the 360-degree circle.

Also in Event 2, a similar discussion occurred while students found the measurement of supplementary angles (Case B Transcript, p. 4). Ms. Littleton stepped in when a student mistakenly calculated the problem. She led the student to clarify and correct his solution. He had added the two angle measurements instead of subtracting one
of the angles from 180 degrees. During this part of the discussion, Ms. Littleton referred to the representation written on the whiteboard. She used the representation to illustrate the missing angle. Along with this diagram, Ms. Littleton used the mathematical vocabulary needed to explain how to find the accurate solution.

Additionally, Ms. Littleton engaged students in an activity that required them to identify standard units of measurements during the capacity task in Event 3 (Case B Transcripts, p. 9). She assisted students in extending their understanding of measurement using the capacity problems to practice adding, subtracting and multiplying decimals. While students were working, she honored the mathematics by verifying viable solutions. When students needed re-direction during the follow up Balloon Task, she stepped in to advise. For example, she stepped in to make sure that the students were on the right track. She asked:

Why did you choose to multiply? We were trying to figure out how many balloons we were trying to fill with 2000 liters of water, right. Since we have 2000 liters of waters and we are trying to get water to each balloon, should we be multiplying or dividing? (Event 3, p. 14)

Students were struggling to identify the correct operation used to identify a logical answer. Ms. Littleton checked on their calculations, pointing out when errors were made and redirected their path. The following are some of the situations when Mrs. Washington honored mathematics as a discipline during her work with students.

During event one, Mrs. Washington honored mathematics as a discipline when she explained the purpose and use of standard measurement when determining lengths (Case A Transcript, Event 1, p. 1-2). During event two and three, she explained the differences between various elements in a pictograph, tally chart, and bar graph, emphasizing why one was more purposeful with the given set of data (Case A Transcript,
Events 2-3, p. 2-6). Mrs. Washington and her students honored mathematics as a
discussed while explaining the purpose of an array and how to use an array as a strategy
to solve one and two-digit multiplication problems (Case A Transcript, Event 4, p. 6-9).
Mrs. Washington also discussed using the decomposing strategy to break apart a large
problem into two more smaller problems and therefore decreasing the complexity of it.
Students represented their calculations both orally and in written form while sharing their
solutions. During event five, Mrs. Washington helped students to contemplate using “the
same numbers" in finding the area and perimeter but not getting the “same answer" (Case
A Transcript, Event 5, p. 10). Mrs. Washington honored mathematics as a discipline
when she explained that the numbers are added while identifying perimeter while
multiplying the numbers would be necessary for identifying the area. Finally, during
Event 6, Mrs. Washington discussed the attributes of polygons while students were
working on the design project (Case A Transcripts, p. 11). For example, she emphasized
that when using a square in the design, students would need to draw a shape with four
equal sides. Mrs. Washington was not observed using this component during the final
two events during independent student tasks.

Learning Through Interaction

Learning Through Interaction is the third of four reformed-based practices for
building understanding in mathematics. The two components embedded within Learning
Through Interaction include learning is socially constructed and contributes to the
learning of others. As described below, these components were implemented by each
participant during all events, see Table 21.
These components include learning is socially constructed and contributes to the learning of others. Learning Through Interaction is built on the idea that learning mathematics is a socially constructed endeavor (Ernest, 2004). Students learn alongside community members by observing and practicing the skills they needed to master their craft (Lave and Wenger, 1991, Barton & Hamilton, 2005; Gillies, 2014). As described below, Mrs. Washington and Ms. Littleton utilized a combination of the components of Learning Through Interaction during the study.

The first component of Learning Through Interaction is learning is socially constructed. Students interacted with other students to complete tasks in both small groups or large. They provide help to peers, ask questions, and promote each other's learning. Teachers helped guide the students' interaction during group work and provided direction for discussing and presenting problems, evaluating findings, and challenging other students' perspectives.

Mrs. Washington and Ms. Littleton conducted their discussions to enable students to impact the path of learning. As mentioned earlier, neither teachers explicitly taught the strategies or content needed to solve the problems or complete the tasks. The problems were presented, and students socially constructed ideas while sharing their strategies and offering suggestions regarding how others could solve the problem. Discussions were guided by the students who participated. Although both teachers maintained the direction of the lessons, ultimately knowing points in the dialogue where they wanted students to touch on, they facilitated the discussion to encourage student input. The following section describes the practices used by Ms. Littleton to promote the social construction of ideas.
When asked to reflect on the process she used to share ideas and construct learning together, Ms. Littleton said:

I think it's important because it helps them to develop their thinking and the students really learn from each other. All the time I’ll hear some student start to talk about something, and you’ll hear the uh-huh moment that they're exploring things together and making their own conclusions rather than me telling them, okay. This is the formula. We're going to use the area. They find more ownership in what they’re learning, and more understanding of it. (Individual Teacher Interview, p. 4-5)

Ms. Littleton provided students with many opportunities to socially construct learning with peers. The tasks and problems assigned had a level of complexity that required students to interact with others to complete them. As described earlier, Ms. Littleton designed her lessons to include small group or partner problem-solving work followed by a class discussion. The social construction of learning occurred while students engaged in these small and large discussions while they shared multiple strategies. Learning was socially constructed as students reflecting on the strategies and than determined solutions together in small groups. This was evident in the tasks involving the capacity problems, making a bow and wrapping the box, as described earlier.

Content needed to solve the problems or complete the tasks was not explicitly taught by Ms. Littleton during the lesson (Case B Transcripts, pp. 1-29). Students used what they had learned previously about the topic and applied this knowledge to the task. The content was discussed when the need arose during conversations. Students wrestled with their shared ideas about how to solve a problem with peers. This small group approach provided students with the opportunity to discuss the problem with others before having to share their ideas during a full class discussion. When it was time for the
class discussion, students presented their contributions in the context of what their small group had discovered or decided. Ms. Littleton and other students listened to explanations and interacted with one another with questions during the class discussions.

Ms. Littleton often used open-ended questions, providing students with the freedom to share their thinking, often to begin discussions to allow students to explain their own thinking. For example, she asked, “Who would like to share out how you solved this problem” (Event 2, p.4)? Similarly, she posed the questions, “Does anyone have any questions for him” (Event 1, p. 1)?” and, “So what do you think” (Event 6, p. 22)?

At times, she did use questions to guide students’ participation more. For example, she began the group discussion during Event 1 with the statement, “We are sharing out because we're going to try to share different thinking, try not to share the same thinking” (p.1-2). She wanted students to share solutions while being mindful not to repeat strategies. When students did repeat, she reminded them of the expectation. She also steered them slightly when she asked a more pointed question, “Does anyone have questions for Evelyn about how she solved the problem” (Event 2, p.6)?

While students shared their thinking both in small and large group formats, Ms. Littleton encouraged them to build on the thinking of others, sharing multiple strategies to help increase the opportunity to socially construct their solutions together. For example, during Event 3 students discussed equivalent units of liquid measure before beginning the capacity task. Ms. Littleton started the interchange when she asked, “So four quarts is equal to what” (Event 3, p.9)? Students offered responses and explained
their answers. Afterward, Ms. Littleton asked questions to prompt the student to explain. She inquired, “How did you figure that out” (Case B Transcript, Event 3, p. 9)?

She also asked questions to draw more participation and to check additional students’ understanding. To do this, she asked questions to elicit understanding. This encouraged the sharing of a few more equivalent measures during this lesson. In the exchange below, a student disagreed with another student over the content. Ms. Littleton used this opportunity to facilitate the discussion to allow students to construct meaning together.

Teacher: Who can find another unit that is equal to four quarts?

Student 1: “16 cups.

Student 2: “Shouldn’t it be 32, shouldn’t it be two pints in each quart, and she only did one pint in each quart?

Teacher: Ms. Littleton prompted more involvement from the class. She asked an open-ended question to invite more participation, “What do you think friends?

Student 3: I agree with Zandra because there is 4 cups so 16 cups because 4 times 4 is 16 cups.

Teacher: Does that (16 cups) make sense? Can you tell me why it makes sense?

Student 3: Because 4 times 4 is sixteen.

Ms. Littleton: Where did the four come from?

Student 3: Four comes from the four pints and four quarts.

Discussions such as the one described above included students questioning other students. It was clear that students were listening as their classmates shared their thinking. This student did not hesitate to challenge the ideas shared by her peer.
Another example of students questioning one another took place during Event 4. The first group to present labeled her solution using millimeters. One of her classmates disagreed with the use of that unit and asked, “Did you say millimeters or milliliters” (p.14)? Additionally, during a class discussion during Event 2, one student voiced her concern that a member of her group “had a different answer then the rest” of the table group (p. 6.). Moreover, while two students were working on their capacity problem task, a student disagreed with the strategy she was using and spoke up about it. She said, “No, that is not right, do you just want to go with my answer? I don’t think six and 200 is a correct answer. I think mine would be more of a correct answer” (Event 3, p. 11-12). In all cases, Ms. Littleton stepped in to help the students respectfully negotiate the situation and construct knowledge by accurately determining a viable solution. The next section describes the practices Mrs. Washington used to promote the social construction of learning.

Mrs. Washington designed her lessons to include discussions that enabled students to share their thinking about the topics they studied. This social construction of learning was cultivated using problems to incite interactions among Mrs. Washington and her students (Case A Transcripts, 1-18). The mathematical conversations that students participated in allowed students to develop their thinking and understanding by listening and talking with one another. Students also worked with other students to complete tasks during Events three, six, seven and eight in both small and large groups. These tasks provided another mechanism for the social construction of knowledge because they required students to plan and design solutions with the help of their teacher and peers.
Mrs. Washington emphasized the social construction of learning while facilitating conversations during mathematical discussions. As described above, Mrs. Washington typically posed a question that involved a specific skill or concept with students, as described earlier, and then spent several minutes discussing what students knew about it. She listened to their contributions and then posed additional questions to dissect what it was that they knew about the topic. Many students would have opportunities to make their knowledge public. Other students would listen. Mrs. Washington would step in with questions to uncover additional thinking. She would also step in to steer the conversation toward an idea that she wanted students to consider.

An example of this occurred during Event 2, Mrs. Washington facilitated a discussion among students while engaged in a discussion about a tally chart the students had completed earlier that morning (Case A Transcript, p. 2-4). Often in these discussions, Mrs. Washington encouraged students to think out loud about skills and concepts as a way of reviewing a topic. In this case, students would be applying skills discussing data collection before having to collect and represent their own data during an upcoming task. The following is the discussion that occurred during Event 1.

T: Could I have collected my data another way?

Turn and talk to someone near you.

Students turned and talked with a partner about the question.

T: So, my question was, could I collect my data another way.

S: You could have used a pictograph.

T: How could I have used a pictograph to collect my data?

S: So, you could say 1 smiley face equal one vote so than you would have wrote yes or not, and then you would have us draw a smiley face if we do.
T: Ok, if I had you draw the smiley face in the yes or no column is that still a tally chart and I or am I having you just put a different symbol in there?

S: Sort of, just the big difference is that you are using picture instead of tally’s.

T: Would that be different?

S: It would be different but it would still be a tally chart.

T: Ooh! How can it be different and similar at the same time? Somebody help us!

T: Student, please call on a friend to help!

T: Somebody help us out!

S2: It isn’t, she is just using a different symbol.

T: Thanks for the next question, what is she talking about?

T: Turn and talk to someone.

T: S2 just said it is pretty much the same just using a different symbol.

S3: It is different because it is a pictograph cuz at the bottom of the smiley face equals one vote, but you could do the same thing as the tally chart.

T: So are you agreeing or disagreeing with S2?

S3: I am agreeing with her.

T: When did we make pictographs, during the time we collected data or after the time we collected data on our last project?

The discussion was focused on a class graph completed by students earlier in the day.

Each student was asked to put a tally mark in the yes column if they went to the library in the summer, or in the no column if they did not go to the library in the summer. To begin the discussion, students were prompted to engage in a ‘turn and talk’ to discuss their ideas with one other person. Students talked with one another about their thinking (p. 2).
Students began sharing their ideas, suggesting a type of graph to use. A discussion among students, facilitated by their teacher about the similarities and differences among pictographs and tally charts ensued. The teacher’s questions prompted specific explanations about what students knew about a pictograph.

She explained what she heard the student contribute. Then she investigated their thinking when asking, “Would that be different” (p. 3)? Students join the conversation and try to explain. Mrs. Washington prompts other students to help one another out. She does not explain the difference but waits for students to attempt this. In Turn 17, she engages them in another turn and talk. Then after a minute or two of partners exchanging their thinking, Mrs. Washington asks, “So what do we think over here?” Mrs. Washington provides opportunities for other students to try to clarify the difference. Then she shifts the conversation a bit toward the critical aspect of tally charts being a tool that students use for collecting data posing another question. Then in Turn 22, she says, “So let me ask another question, when did we make pictographs, during the time we collected data or after the time we collected data on our last project?” One student answers, “Before.” Mrs. Washington does not respond to his incorrect answer. Instead, she looks to a classmate and asks, “Can you help him out?” Gagan steps in and corrects him saying, “After.” Another student confirms, “After, because we collected our data and then after you collect it you can actually do it.” Mrs. Washington continued to ask students to share during this conversation to confirm that they understood that the tally chart was used in the data collection process.

Mrs. Washington engaged students in the building of ideas with questions to spark their thinking and encourage an exchange of ideas. The turn and talk, used in the
example above, created a framework for students to begin the process of socially constructing their knowledge together. During the turn and talk, students shared what they knew about the topic. However, Mrs. Washington used the strategy more than once in a lesson, which usually meant that she was pushing students to build on their ideas together. This strategy was used during events one through five. Well-defined problems or tasks were also assigned during all events. During these times, students constructed knowledge by designing solutions together. Mrs. Washington also helped students to design and use contracts to guide their interactions while they worked collaboratively on assignments. The following section describes the practices used by Mrs. Washington as she encouraged student to contribute to the learning of others.

The second component of Learning Through Interaction is contributed to the learning of others. According to research, students whose experiences included interacting with others to complete tasks contributed to the learning of peers and achieved higher learning outcomes (Gillies, 2014). Many of the ground rules outlined in chapters four and five were utilized to help students interact with the ideas and thinking of teachers and peers in the learning community. Students expanded the thinking of their peers by making their mathematical knowledge public, making their reasoning visible in their talk, engaging in joint reasoning and sharing multiple solutions. Utilizing the ground rules helped teachers facilitate this process. Utilizing the ground rules also assisted students to learn how to promote each other's learning, accept responsibility and seek help from other members of the group.

Along with engaging students in all of the ground rules listed above, the teachers in this study also initiated the contributions to the learning of others in several ways; all
described earlier in this chapter. These included encouraging students encouraged students to interact with their peers to generate more ideas and solutions or to help a student get out of a situation where they were not sure of the answer. Students consistently volunteered their ideas and patiently waited while others shared theirs. Mrs. Washington invited students to ask other students to continue the conversation. Ms. Littleton invited students to question other students, especially when solutions were not viable. The following section describes the ways teachers in this study encourages students to contribute to the ideas of others.

Ms. Littleton facilitated a process that she used to engage students in learning from one another. When students shared their thinking or strategy, she ensured that they explained their reasoning, as evident in the classroom observations. This was done during all class discussions, as well as, during small group work. Ms. Littleton also encouraged students to interact with their classmates by asking questions of one another, and providing assistance, as needed. This was a priority because Ms. Littleton stated that learned more successfully from other students. She said, “It just helps them to understand it better where I might be trying to use the mathematical terms, that they're just not ready for” (Individual Interview, p. 5).

One example that included the use of a prompt to encourage students to offer an explanation that included reasoning or justification occurred during Event 5 (p. 19-20). Students were working in small groups to complete the Wrapping the Present Task. Ms. Littleton came over to check in with the group and proceeded to interact with students as they explained their solutions. She asked specific questions about their thinking to
determine the process that they used for identifying a solution. All three group members joined in to explain the process used to *socially construct* the solution.

T: Can you tell me exactly how you figured out your estimate?

S1: We measured the box with the tape measure and then added a few inches.

T: So how did you measure the box?

S2: With that. (pointing to the tape measure)

S3: (gestured that she wrapped the tape measure around the box)

T: So, you tied it around, you pretended that the tape measure was a ribbon?

S3: Then we added seven inches.

T: Why did you add seven inches for the bow?

S1: Because that was what we thought it needed.

S3: Because we had 43 and we thought 7 more would give us what we needed.

T: What if you did not have a tape measure, what would you do?

T: Can I have one person talk at a time?

S1: I would measure the length and then I would do the width and then I would do the length times two and the width times two.

T: Why would you do the length times two and the width times two?

S1: And then I would have to do that again.

T: Why?

S1: Because we only did the top length times two and the bottom times two and it would only be that (pointing to the top of the box and the bottom of the box) and not that (the four sides). And it would not go all around.

T: So, what it is telling you width times two and length times two, what are you thinking about?

S1: The perimeter.
Additionally, during the following class discussion about the measurement of an angle, a student stated the strategy he used to solve the following problem (Event 2, p. 5). Ms. Littleton followed up each of his statements to draw out the student’s thinking and help make his decisions more explicit to the class. The conversation ended with a question Ms. Littleton asked to engage other students in the conversation and prompt further interaction. This was a practice Ms. Littleton used (GR6C) as evident in Table 15.

S: 76 plus 76 is 152, and then I subtracted 152 from 360 and I then I got 208 and then half of 208, is 104.

T: Why did you decide to subtract it from 360?

S: Because 360 is a full rotation.

T: Hmm. Why did you subtract two 76 degree angles?

S: Because if you only subtracted one, it would get the wrong answer.

T: How do you know?

S: Because it would be different from 180 because 180 is only half of 360, but if you subtracted ½ of 360 it would give you the right answer.

T: Interesting, so that is another way to think about it. Does anyone have questions for her about how she solved the problem?

Several times, Ms. Littleton asked students if they had any questions for the classmate who had just contributed to the conversations. This allowed students opportunities to interact and exchange ideas with one another (GR6C). During Event 1, Ms. Littleton solicited student input after each student shared their thinking. She asked, “Any questions” (Event1, p. 1-3)? She also asked, “Does anyone have questions for Evelyn about how she solved the problem” (Event 2, p. 5)? Moreover, she prompted, “Does anyone have questions or comments” (Event 4, p. 14)?
Sometimes, however, students did not respond to her requests. The greatest response occurred during Event 4 after students presented and the format for interacting included asking two questions and making one comment, see Table 15.

Students also contributed to the learning of others when asked to do so during group discussions. One example occurred during Event 2 after a student raised the concern about a discrepancy in the group’s solution. Student 1 said, “Our table group thought it was 104 because the straight angle on the bottom of the shape is 180 degrees and then if you do 76 minus 180 degrees then you get 104” (Event, 2, p. 5-6). Then she added, “She (Student 2) had a different answer than the rest of our table group.” The following is the dialogue the conversation that transpired.

T: Did you talk to your table group about how you were solving the problem?”

S1: He (Student 3) has the same answer (as Student 2), but everyone else does not.

T: Okay would you like to share your answer? Go ahead.

S2: At first, we did it the way Student 1 did it, and then we realized that we thought it wasn’t correct so F said we should probably do 180 plus 76 because 76 is part of the straight angle.

T: Boys and girls do we have any questions or suggestions about how they are solving the problem?

S4: 76 is the acute angle (shows the angle with his hands), and 180 is the straight one (shows angle with his hands) so, you should have subtracted instead of added.

Teacher: Does that make sense Student 2 and Student 3?

Above, Ms. Littleton asked, “Boys and girls do we have any questions or suggestions about how they are solving the problem?” to encouraged the class to help contribute to the learning of their classmates (GR6G). Instead of providing a strategy or answering the
question for the students, Ms. Littleton turned the question toward the entire class. The Student 4 joined the conversation and offered a strategy for identifying a correct solution. Ms. Littleton offered an additional statement afterward to clarify and check for understanding. She said:

So, we have our 180 degrees here, and the parallelogram drawn on the number line, this is our 76-degree angle and our 180-degree is the straight line, so it is part of the 180-degree angle. If you are adding 76 to 180 degrees what you are doing is saying we have an angle that starts here and goes all of the way around to here. (Event 2, p.6)

Then she asked, “Does that make sense?” and “How could you fix your answer?” The student responded, “Subtract 76 from 180.”

The following section describes the practices Mrs. Washington used to encourage students to contribute to the learning of others.

As shown below, Mrs. Washington initiated opportunities for students to join her in supporting one another during the learning of mathematics. Table 11 reveal students offered their knowledge of mathematics and shared their solutions using words, pictures, and numbers (GR4B) on sixteen occasions. For example, during Event 4, Ben came up to explain the meaning of an array and to draw the image on the whiteboard (p. 7-8).

Additionally, in Event 6, a student came up to share her thinking to offer some advice to her classmates from making a similar mistake. She said:

If you are making your things (geometric shapes) super tiny, I added like my 4yards, 4 yards, and 3 yards and realized that I only used 11 of my 40 yards for my sides. So I am like…Wow! That is not very much; I can make those way bigger! (Event 7, p. 11)

As explained earlier in this chapter, Mrs. Washington provided opportunities for students to learn from others by inviting them to share their thinking so that others could
learn from them as they explained different ways of thinking. A couple of the elements that Mrs. Washington encouraged students offered their knowledge about mathematics. Students Offered Their Knowledge About Mathematics (GR4A) on 43 occasions. They also offered justifications and rationales (GR5A) on 69 occasions, and many solutions for problems (GR7B) on 77 occasions. All of these elements allowed students to demonstrate their knowledge to others in the community. The way Mrs. Washington guided discussions allowed student to contribute in these ways with her support and the support of classmates. All were helpful in providing ideas and support, as noted below when others struggled with the concepts.

Mrs. Washington invited students to help their classmates get out of a situation when they were not sure of the answer. She prompted these exchanges by suggesting that students call on a friend to help continue the conversation. Again, during Event 1, students were discussing why a tally chart would be a better choice for graphing the class data during Event 1. One student contributed his thinking and said, “You can do the pictograph, but it would be hard because it would take a lot more thinking” (Case A Transcript, Event 1, p.1). The second student clarified saying, “It would be too hard to do it that way because the symbol means more than just one.” Mrs. Washington confirmed the thinking and said, “He brings up a good point when we have been using the pictograph. The symbol has been used for more than one vote.”

Additionally, during the same lesson, Gagan steps in to help a classmate during the discussion about whether or not the tally chart was used before or after the pictogram in Event 2. When the student answers incorrectly, Mrs. Washington does not respond to his incorrect answer. Instead, she looks to a classmate and asks, “Can you help him out?”
Gagan steps in and corrects him saying, “After.” An additional student confirmed it, “Because we collected our data and then after you collect it you can actually do it” (Case A Transcript, Event 2, p. 3).

Moreover, during Event 5, Mrs. Washington asked a student if he was “standing in the area?” He replied, “No” (Case A Transcript, p.10). He was actually standing in the area, and Mrs. Washington asked another student to help him out. The students said it begins with “An A.” Afterwards, Mrs. Washington prompted, “So she just helped you, it begins with a [area].” A third student provided additional support to her classmate, “People might think it is the area because you are close to the outline (perimeter).”

Strategic discourse is built on the idea that both students and teachers are partners in learning and powerful sources of mathematical ideas and thinking. The roles were more equalized among Ms. Littleton and her students than in the traditional classroom. However, for strategic discourse to be truly effective, it cannot be entirely teacher directed. As will be described below, Ms. Littleton has set the course for student and teacher roles to evolve so that students could become more autonomous while they studied mathematics together.

**Shifting Authority Toward Shared Authority: Littleton**

Ms. Littleton has made use of methods that are reforming how her students learn, not just what they learn. As described above, Ms. Littleton has instituted the reform-based practices and several strategic discourse elements to successfully guide her instruction as she teaches mathematics with understanding. Additionally, and most remarkably, Mrs. Washington has established an equitable environment that encourages a partnership among her students and herself. This equalized environment has established a
heightened level of mathematical authority among students. Providing students more experience sharing their knowledge has provided greater expertise in communicating ideas and confidence in mathematics. Encouraging students to share thinking and support to their classmates has communicated to them that they are valuable contributors to developing understanding of mathematics. Both of these things have promoted greater mathematical authority.

As examined earlier in this chapter, the teacher facilitated conversations allowing students to state their mathematical knowledge, share ideas and identify how they would solve problems. For example, students did this while relating fractions with degrees in a circle during Event 1 (p. 1-2). They also stated their mathematical knowledge, sharing ideas and explained solutions during involving the capacity problems (Event, 4, pp. 13-15). Students were not hesitant to take risks, make mistakes or raise questions. They carefully examined contributions and spoke up when they disagreed with solutions and strategies, even if it involved questioning the teacher. Ms. Littleton praised students again and again to acknowledge their hard work and valuable contributions (GR2A and GR2B). The level of participation and interaction among students and their teacher would not have been possible without the existence of the supportive environment established in Ms. Littleton’s classroom, see Table 14.

Cultivating the mathematical authority among students has taken time. Ms. Littleton modeled how to help others explain, rethink their strategies, and offer and receive help from their peers. For example, during Event 2 (p. 6), Edna raises a concern that the members of her group have different solutions. The discussion below occurs after Edna raises her concern.
E: Chevron had a different answer then the rest of our table group.
T: Did you talk to your table group about how you were solving the problem?
M: A has the same answer (as Chevron) but everyone else does not.
T: Okay would you like to share your answer? Go ahead.
C: At first we did it the way Edna did it and then we realized that we thought it wasn’t correct so A said we should probably do 180 plus 76 because 76 is part of the straight angle.

T: Okay.

T: Boys and girls do we have any questions or suggestions about how they are solving the problem?
T: Arthur?

A: 76 is the acute angle (shows the angle with his hands) and 180 is the straight one (shows angle with his hands) so you should have subtracted instead of added.

T: Does that make sense Chevron and A?

T: (Draws on the board) So we have our 180 degrees here and the parallelogram drawn on the number line, this is our 76-degree angle and our 180 degree is the straight line, so it is part of the 180 degree angle.

T: If you are adding 76 to 180 degrees what you are doing is saying we have an angle that starts here and goes all of the way around to here.

T: Is that what we are looking at here (points to the extended angle beyond 180-degree straight angle) this larger angle?

T: Is that what we are looking at, this larger angle?

C: No.

T: No?

T: Does that make sense?

C: Yes.

T: So A and Chevron how could you fix your answer?

A: Subtract instead.

T: Subtract what?
A: Subtract 76 from 180.

First, there is evidence that Edna has developed a concept of shared authority because she feels confident bringing a concern to the teacher in front of the class. Ms. Littleton takes the concern and provides time to discuss the problem. She begins this process by soliciting help from the remainder of the class. This shift in authority away from herself as an expert to sharing the expertise with the members of the community is powerful. She asks, “Boys and girls, do we have any questions or suggestions about how they are solving the problem?” Arthur steps up and tries to offer support to the students. In Turn 9 he explains a solution, “76 is the acute angle (shows the angle with his hands) and 180 is the straight one (shows angle with his hands) so you should have subtracted instead of added. Then Ms. Littleton clarifies the problem and solution for the students. Circling back to make sure that they understand why the problem could be answered in this way.

Ms. Littleton also helped students strengthen their mathematical authority while practicing how to provide explanations and justifications. She consistently provided opportunities for her students to engage in joint reasoning and allowed the freedom to select strategies and design solutions. She treated each as valuable and capable members of the learning community, as revealed in Table 14. The supportive environment thrived as Ms. Littleton provided students with multiple opportunities to engage in conversations about the problems and tasks that they worked on together with peers.

Although students were highly involved in classroom discussion, Ms. Littleton still maintained control of the conversations. At times, she did provide opportunities for students to direct parts of the conversations, but for the most part, she directed the course
of the discussions. Students did have choices about which strategies to use while solving problems, but they did not have the freedom to solve problems or tasks that they had identified on their own. All activities were pre-set for them to complete.

**Shifting Authority Toward Shared Authority: Washington**

Mrs. Washington has made use of methods that are reforming how her students learn, not just what they learn. As described above, Mrs. Washington has instituted the reform-based practices and several strategic discourse elements to successfully guide her instruction as she teaches mathematics with understanding. Additionally, and most remarkably, Mrs. Washington has established an equitable environment that encourages a partnership among her students and herself. This equalized environment has established a heightened level of mathematical authority among students. Providing students more experience sharing their knowledge has provided greater expertise in communicating ideas and confidence in mathematics. (interviews liking math). Encouraging students to share thinking and support to their classmates has communicated to them that they are valuable contributors to developing an understanding of mathematics. Students have developed a greater responsibility evaluating their own thinking and the thinking of others (Huffered-Ackles, Fuson & Sherin, 2004). These experiences all have promoted greater mathematical authority.

Strategic discourse is built on the idea that both students and teachers are partners and powerful sources of mathematical ideas and thinking (Gee, 1991; Lampert, 1990). The relationship among Mrs. Washington and her students was untraditional and much more equalized than in traditional classrooms. As examined earlier in this chapter, there were several instances where the interactions were casual. Students jumped into
conversations and did not wait to be called on. They freely joined conversations and jokes went back and forth among teacher and students.

For example, when students were discussing the height of their teacher, she joked quite a lot about her short stature. When she asked, “When I take my shoes off, I am 61 inches tall. I am really tall aren’t I” (Case A Transcripts, Event 1, p.1). Students immediately chimed in and laughed while yelling, “No! You are not!” Mrs. Washington smiled and continued the conversation.

Mrs. Washington consistently encouraged her students to participate by sharing their mathematical expertise and ideas about many different topics and problems during the course of the study. Encouraging participation helped to demonstrate the value and worth of their ideas in the community. in the discourse and treated each as valuable and worthy members of the learning community. The supportive environment thrived as Mrs. Washington provided students with multiple opportunities to demonstrate their knowledge. She also provided them with the mathematical authority to share their thinking and engage in discussions about the topics they studied.

Students demonstrated an increase mathematical authority in the statements they made when sharing their knowledge. For example, a student stated, “I know that 5 X 12 equals 60” (Case A Transcripts, Event 1, p. 1). Also during the array project, Mrs. Washington shared a strategy she had for calculating a double-digit number using arrays. The problem was 15 times 5. She said, “If you have a mathematical equation that is too hard for you to solve, I could do this whole array or could do a break it into two smaller ones. What would be the most logical?” A student speaks up with a suggestion; she says, “You are decomposing.” Mrs. Washington asked and another student, “How many
multiplication problems am I doing?” Without hesitation, the student replies, “You are doing two multiplication problems, 15 times 5 and 5 times 5.”

In both situations, students step up and contribute. These are typical ways of participating for students. They do not hesitate, and never answer attempt to provide an answer.

Also, Mrs. Washington typically asked students to tell her what they thought they should do and does not just tell them. For example, when students were reviewing the directions for the survey task in Event 3, Mrs. Washington asked, “What do you think that you need to think about when you are going to make a bar graph” (Case A Transcripts, p. 6). Students respond by listing the several components of the bar graph. She finishes with, “Anything else we need?” The last students says, “You have to put the names of the things on the bottom.” She responds, “Good, that toes on the bottom.” She allows all of these contributions from students to help guide the work of their peers.

Additionally, as detailed in the previous section, students were also provided with the authority to provide help to their classmates, as needed. Reaching out to students in this way promoted the idea that they were valuable contributors to the mathematical knowledge among their peers.

Most students found that solving problems was easier when they worked with other students according to the Survey/Questionnaire Results, see Table 12). More than half reported that they asked questions of their classmates to help them find a logical solution. However, only four agreed that they helped their classmates when they are having trouble while solving math problems. Also, when asked about solving problems together with classmates, more than half of the students preferred to solve problems on
their own and not with others. Moreover, only six students reportedly liked to learn from others.

Cultivating their mathematical authority has taken time. Mrs. Washington has modeled helping students rethink their strategies, as well as, encouraged her students to offer and get help from their peers. Mrs. Washington also found ways to acknowledge students as competent thinkers when she brought attention to their ideas.

Although students were highly involved in classroom discussion, Mrs. Washington remained very much in control of the conversations. She guided the discussions, interjected her knowledge and interpreted what her students shared. Although students have not been actively engaged in planning, they have had several opportunities to collaborate with their teacher and peers. Moreover, students were given many opportunities to work on tasks independently, some of the assignments were highly teacher directed. The tasks in Event 7 and Event 8 were more open-ended and allowed students more room to complete the assignments independently.

**Summary**

The two teachers in this study implemented reformed-based practices including Problem Posing, Active Learning with Authenticity, Learning Through Interaction, and Strategic Discourse on several occasions during the study.

The components of the first reform-based practice Problem Posing included well-designed problem/task provides structure for discussion, enriches the concept or skill. Both teachers utilized problems and tasks as a means of structuring the mathematical conversations they facilitated with students. All problems enriched the concepts of the topic addressed.
The components of the second reform-based practice Active Learning With Authenticity included engages in learning, connects to real life, and honors mathematics as a discipline were utilized. Students in both classes successfully engaged with the content, as well as, with the community. Students were also included in activities connected to real life to practice using math the ways others would do to solve real-life problems. Furthermore, teachers honored mathematics as a discipline by using mathematics that mirrored the structure and content of mathematics as a discipline throughout the study. Students were also required students to explain their thinking and connect their strategies to the discipline of mathematics while representing their thinking using logical numeric formats.

The components of the third reformed based practice Learning Through Interaction were utilized for building understanding in mathematics learning is socially constructed and contributes to the learning from others were utilized. The first component learning is socially constructed was addressed as students interacted with other students to complete tasks in either small groups or large provide help to peers, ask questions, and promote each other's learning. Additionally, the ground rules implemented helped students contribute to the learning of others while sharing interacting with the ideas and thinking of teachers and peers in the learning community.

Additionally, Mrs. Washington and Ms. Littleton conducted well-planned lessons that included talk that was relevant and meaningful to their study of mathematics. Although the process of implementing strategic discourse lacked was missing the preplanning piece. Discourse occurred in in-the-
moment to moment interactions among students and teachers as they navigated problems and ideas (Boylan, 2010)

The following section will address the fourth research question guiding this study by examining the ways the two teachers perceive using strategic discourse improved the understanding of mathematics among their students. Moreover, the ways students perceive that their understanding was improved will also be explored.

**Research Question #4**

- In what ways do teachers believe that student understanding was improved was by the use of strategic discourse used in the classroom?
- In what ways do students believe that student understanding was improved by the use of strategic discourse in the classroom?

**Teachers Perspective Of Impact of Discourse on Students’ Understanding**

Mrs. Washington and Ms. Littleton discussed how they perceived understanding as improved as a result of using strategic discourse strategies during mathematical conversations. They also shared the ways specific strategies impacted the learning of their peers. The following is a summary of their thinking during the focus interview, and the individual interview process.

Ms. Littleton and Mrs. Washington both agreed engaging students in mathematical discourse with teachers and peers impacted students understanding of mathematics. During the teacher focus interview, Mrs. Washington said,

I believe so because when they make their own meaning, and even though we teach them, we can teach a concept all we want, but if it doesn't have personal meaning and they haven't been able to have the opportunity to put organization in their head, it's not going to stick. (Teacher Focus Interview, p. 7)
After Mrs. Washington shared her thinking, Ms. Littleton reflected about the ways engaging students in mathematical discourse with teachers and peers impacted their understanding of mathematics. Ms. Littleton said,

I think it's important because it helps them to develop their thinking and students really learn from each other. All of the time, I'll hear some student talk about something, and you hear like an "Ah-ha!" moment that they'll be exploring things together and making their own conclusions rather than me telling them. Okay, this is the formula we're going to use for the area. They then find more ownership while they're leaning and more understanding of it. (Individual Interview, p. 4)

Additionally, Ms. Littleton also shared how students easily connected problems with their own lives. She said, "I ask them to make connections to their own life. It's just crazy. The connection is a big thing that they can make when they are able to think on their own rather than just telling them" (Teacher Focus Interview, p. 8).

The following section is a summary of student’s beliefs about the impact strategic discourse strategies had on their learning and the learning of their peers.

**Impact of Strategic Discourse on Their Own Understanding**

Students from both classes discussed how they perceived understanding as improved using strategic discourse strategies during the focus interview and the individual interview process. During these discussions, students were asked how specific strategies, selected by the researcher based on observations of events, impacted students and learning.

During the individual interview with the fourth-grade students, students were asked to reflect on how discourse was helpful to their understanding. Arthur shared, "It's really helpful when someone does it really long, and they explain their way so I can understand better" (Individual Student Interview, p. 2). He shared that he preferred
talking with other students about problems instead of alone. He stated, "I can understand it better."

Likewise, Jadiah said, "I kind of have to say it to someone so, I can understand it more, and they can help me if I got it wrong" (Individual Student Interview, p.1). Additionally, as reported in the survey/questionnaire, most students indicated that comparing their answers with other students helped them see if my thinking was correct, see Table 16.

Furthermore, Table 16 indicated that most of Ms. Littleton's students reported helping classmates to find logical solutions when engaging with them during problem-solving.

Additionally, when Ms. Littleton's students were asked if finding multiple ways to solve problems helped others. One student responded, "Yes, it helps people understand and compare the second way" (Student Focus Interview 1, p. 1).

A second student reflected about offering multiple solutions for solving problems. He said, "It's like a good way to help you understand a problem in many ways. You think there is just one way, but there are millions of ways to solve problems (Student Focus Interview 1, p.1).

Moreover, students reflected about Ms. Littleton use of questioning to engage them in reflecting about their group assignments. One student said, "I think she asked the questions to help us understand."

Another student reflected, "She might ask you what went well so that you could use that strategy next time."
A third student said, "So you can understand how you're solving the problem and you can think of other ways" (Student Focus Interview 2, p. 2).

As mentioned earlier, Jadiah also explained that sharing multiple strategies helped him to understand the problem a lot more. He said, "it gives me a lot of ways to do it." All but one student indicated that talking about math helped them to understand more clearly, as noted in the student survey, see Table 16. Additionally, nineteen of the 21 students also reported understanding math more when they talked with their teacher. Finally, thirteen of 21 students reported that understanding math more when they talked with other students.

Table 12 revealed students in Mrs. Washington's class indicated that math helped them to understand more clearly, according to survey/questionnaire. Additionally, fifteen out of the twenty-one students indicated that they understood math more when they spoke about it with others. Other statements completed by students in Mrs. Washington's class revealed that close to half of the students thought that the statements were true and the remaining indicated that they were false. Therefore, students did not have an overall perception that informed this study.

**Summary**

Overall teachers perceived discourse as helping their students understand. Mrs. Washington explained the ways discourse helped students to make their own meaning by experiencing the problems and making sense of the concepts through discourse. Ms. Littleton explained discourse as helping students develop their thinking while they learned from one another. She also spoke about the ways discourse helped students to
explore ideas and draw their own conclusions. Additionally, she shared her perception that conversations allowed students to connect ideas to their own lives.

Mrs. Washington's class perceived math as helping them to understand more clearly. They also indicated that they understood more when they engaged in discourse with others. Ms. Littleton's students stated that the questions their teacher asked questions help them to understand. They also perceived that identifying multiple ways to solve problems as positive. Moreover, students reported understanding math more when they talked with their teacher and peers.

The final chapter will address the guiding research questions before summarizing key findings from the study. The chapter will also include an examination of the design methods to discuss the limitations surrounding this study. Additionally, implications for practice and recommendations future research and practical applications will be discussed.
CHAPTER 7

DISCUSSION

The purpose of this qualitative multiple case study was to increase understanding of the ways mathematical discourse was used to teach mathematics in the elementary classrooms. In order to do so, two teachers using reformed based methodology were selected to participate. In this final chapter, the research questions guiding this study will be revisited before summarizing key findings from the study. The chapter will also include an examination of the design methods to discuss the limitations surrounding this body of work. Additionally, implications for practice and recommendations future research and practical applications will be discussed.

The research questions guiding this study aimed to describe the types of reformed based methodology and discourse practices used by teachers with students in the elementary classroom. Additionally, the research questions included identifying important and impactful discourse practices according to the participants. Although this study attempted to investigate the importance of the practices and their impact on understanding as perceived by participants, teachers and students had difficulty clearly identifying and discerning the successful discourse practices they used or their impact on understanding. However, both provided perceptions regarding the strategic discourse practices described by the researcher during the interview process. As the study progressed, however, it became clear that focusing this examination on the specific strategies participants employed would be beneficial in understanding the specific types of discourse utilized by teachers and their students during mathematical conversations (Boaler, 2002; O’Connor, Michaels & Chapin, 2015; Lampert, 1990; Sherin, 2002;
Wood, 1998). As a result of an intense focus on the discourse practices, it became clear that the elements used by teachers could be categorized into three levels, or themes based on their complexity. Therefore, this study also attempted to discern the different themes based on the level of complexity of the discourse used, raising the notion that mathematical discussions become more focused on extending mathematical thinking (Cengiz, Kline & Grant, 2011; Franke, Turrou & Webb, 2015; Hiebert, & Grouws, 2007).

Moreover, the examination of the discourse amongst teachers and students also included the practices teachers initiated to increase student autonomy reducing their own authority (Boaler, 2002; King, 1992; Walshaw & Anthony; 2008; Wood, Cobb, & Yackel, 1991). The following sections will include a restatement of the hypothesis, followed by a summary of the key findings. Additionally, a discussion of the study’s limitations and implication for future research follow.

**Restatement of Hypothesis**

Over the past four decades, despite the recommendations to include mathematical discourse, elementary teachers attempting to engage students in solving problems together in classrooms has resulted in an increase in the talk, but educators have yet to develop a process for implementing purposeful mathematical conversations (Franke et al., 2015; Mercer, 2008). Furthermore, teachers have largely been left on their own to integrate discourse into their instruction, with little or no recommendations about what it is that they should be doing (Cochran-Smith & Fries, 2005; Putnam & Borko, 2000). Although researchers have provided examples of effective discourse, the process has not been made clear for use in classrooms without the support of researchers (Cengiz, et al., 2011; Franke et al., 2015). Furthermore, in general, teachers and students are unfamiliar
with discourse strategies, and teachers do not learn discourse practices that move students past the initial engagement (Franke et. al., 2015).

This study offered the opportunity to investigate the discourse practices two teachers and their students were using in the natural setting of their elementary mathematics classrooms. The elements linked to the ground rules of the Exploratory Talk Model provided a framework for examining the types of specific strategic discourse practices used among the participants (Mercer, 1999). The Exploratory Talk Model was most ideal to use in this study because it aligned to the practices used by the two teachers and their students and therefore provided a framework to guide the examination of the types of strategic discourse strategies being implemented.

It is important to note that the participants in this study were dedicated teachers who skillfully implemented lessons that provided several opportunities for students to build mathematical knowledge with others while engaging in discourse. The purpose of this discussion is not to critique their teaching, but to illuminate the successful ways discourse was used by teachers and students. This study also provided a voice for students and teachers as they described the impact these methods had on their experiences implementing mathematical discussions in their classroom. As a result of the analysis of the following themes emerged:

1. Reform-Based Instruction
2. Intentional Strategic Discourse
3. Sharing Mathematical Authority
4. Students and Teachers Inform Practice
The following sections provide a discussion of the key findings supporting these themes followed by a discussion of the study’s limitations and implications for future research. After examining a broad scope of literature, I presented a vision for teaching mathematics using four reformed based practices to teach understanding in a social framework in Chapter 1 of this document. It is my purpose to highlight the ways the two teachers in this study implemented aspects of all four of these practices while engaging in strategic discourse. Figure 9 provides a conception of learning in a community of practice focused on using reform-based methods to teach and learn mathematics with understanding. This conception illustrates the complexity of implementing strategic discourse to teach mathematics with understanding in a community of practice. This diagram denotes the four reform-based practices that guide the instruction. Within these four practices are the components that comprise each practice. Also included within each of the four practices are examples of words that represent the types of discourse occurring within the practice. Finally, at the core of the diagram is pyramid describing the level of complexity of the discourse activated by both teachers.
The next section begins with a summary of how teachers came to use the reform-based practices with their students.

**Reformed-Based Practices**

Findings reveal that the four reformed-based instructional practices of Problem Posing, Active Learning with Authenticity, Learning Through Interaction and Strategic Discourse were implemented by Mrs. Washington and Ms. Littleton throughout the classroom events. Additionally, the analysis of the individual teacher interviews provided additional information about the ways teachers came to know the reform-based practices.
These interviews examined the ways their various experiences, thinking and perceptions about these methods impacted their instructional decision making.

The in-depth analysis provided insight into how teachers came about knowing and using reform-based practices by introducing the participant’s perspective and understanding their world “as the participant views it and not as the researcher views it” (Rossman & Rallis, 2003, p. 181). Interviews also gave voice to the participants and help researchers investigate things that cannot be observed (Patton, 2002).

**Problem Posing**

Problem Posing was a practice used by both teachers to initiate discussions and develop an understanding of concepts while it enhanced reasoning and reflection skills (Cunningham, 2004, Lampert, 1990). As discussed previously, each teacher used a combination of well-defined problems and tasks to provide a structure for engaging students in discussion while developing mathematical concepts and skills, see Table 21. They used district curriculum maps to design problems and tasks around the standards, goals, and objectives within their grade level curriculum (Atlas Rubicon, 2018). Each problem and task required students to design solutions to problems that required them to build on the concept and skills while working together. This aligned with the research reporting benefits of using Problem Posing as a means for establishing a system of interaction to increase opportunities to engage students in shared thinking while collectively building an understanding of the learning community (Cobb et al., 2001; Hmelo-Silver, 2002; McClain & Cobb, 2001).

Mrs. Washington used problems during two events and tasks during the remaining six events. See Appendix K to review sample tasks. The well-defined problems and tasks
required students to solve authentic problems by building on real-world skills connected to the knowledge that they had learned previously. The problems and tasks used by Mrs. Washington were “rooted” in real-world contexts and created a high level of engagement amongst her students (Weiss et al., 2009, p. 276). Additionally, Mrs. Washington guided students to review and then practice applying the skills they needed before having to complete the complex task independently. Like Lampert (1990) she required students to justify their thinking by using their own words to describe how they solved the problem. Mrs. Washington also met with groups while they completed tasks to offer support, including providing information about the details of the assignment.

Mrs. Littleton used well-defined problems during the first two events and well-defined tasks during the remaining five events. Most math classes began with students first meeting in small groups to complete problems or tasks and then with the entire class to discuss thinking, strategies, and solutions. Problem-solving tasks were also assigned, and students worked in groups to develop solutions. These tasks required students to try to figure out the activity and apply skills learned earlier in the year. Similar to Dewey’s inquiry-based study, students formulated their own hypotheses and judged the validity of their solutions based on the strength of the hypotheses, as opposed to simply learning and following procedures that were provided to them by a teacher (1902). Ms. Littleton circulated the room to interact with groups while they worked. She used a type of discourse technique called “stepping in and stepping out” (Lampert, 1986). “Stepping in and stepping out” discourse was effective because it allowed Ms. Littleton to interact with students by discussing specific parts of the problem students were working on, as they needed assistance. She listened to what students discovered while they worked. She
only stepped in to address misconceptions or errors in reasoning. Each problem-solving session also included time to reflect, as a class, about the different ways they solved each task (Lampert, 1990).

**Active Learning with Authenticity**

Active Learning With Authenticity is the second of four reformed-based practices for building understanding in mathematics. The components of Active Learning with Authenticity include *engagement in learning, making connections to real life* and *honoring mathematics* as a discipline. As described below, both teachers designed tasks that were “rooted” in real-world contexts and authentic (Ball & Bass, 2000; Weiss et al., 2009, p. 276). Students in both classrooms had opportunities to share what was on their minds, which was in line with the definition of authenticity described as taking place within the activity of discussing mathematics among students by Ball & Bass (2000). Teachers challenged their students by *engaging* them in learning to discuss authentic real-world problems and tasks. Students were encouraged to engage in these conversations throughout each lesson. In discussions facilitated by their teacher, students talked through problems and shared different ways of thinking about the mathematical ideas.

Ms. Littleton engaged her students in authentic experiences by providing students with the opportunity to solve the problem or task and then to encourage students to share their authentic ideas and solutions with their classmates. Ms. Littleton consistently encourages students to share what they were thinking about; they did not just answer her questions or discuss what she wanted them to discuss. Ms. Littleton encouraged students to strategize and then discuss their ideas about how they to solved tasks with peers and
their teacher. Students were encouraged to ask questions about other students contributions and to share their reactions to them. Mrs. Littleton questioned her students thinking by requiring them to explain their ideas (GR6A). During Event 1, one student shared her solution. She said, “So I know that there's 90 degrees in one part of a 360 degree circle” Ms. Littleton stepped in to prompt her to explain her thinking more explicitly. She asked, “You know that 90 degrees is what” (Case B Transcripts, Event 1, p. 1). Ms. Littleton used questioning to direct thinking (GR6B) and promote these interactions. For example, she asked, “Elenor connected her thinking with what? What else did we learn this year” (Case B Transcripts, Event 1, p. 2)?

While engaged in discussions, Mrs. Washington questioned her students thinking by requiring them to explain their ideas. For example, she would pose a question and say, “How tall am I and how did you figure it out” (Case A Transcripts, p. 1). She also used the ‘turn and talk’ strategy several times to initiate the sharing of ideas among students. During these instances, she used prompts such as, “Turn to someone and talk about why we use 12 inches equals 1 foot” (Case A Transcripts, Event 1, p. 1). The purpose was to get students to consider another student’s thinking. Mrs. Washington also engaged students in learning when spotlighting original ideas (GR6E) to draw attention to a novel way of thinking. For example, during Event 6, Mrs. Washington said, “Wait! Wait! So, your strategy was multiplication” to draw everyone’s attention to a student making the leap that instead of adding the feet that she could multiply them (Case A Transcripts, Event 1, p.1-2)?

Additionally, all of the lessons that Mrs. Washington conducted during the study connected math in the classroom with mathematics in the world outside of school. For
instance, during the first event, students converted their teacher’s height from inches to feet and inches. Students answered the question, “Do you go to the library during the summer” (Case A Transcript, Event 1, p. 1). This data was collected in a Tally Chart and afterward reviewed by the class. During Event 2, students converted their teacher’s height from inches to feet and inches (Case B Transcripts, p. 2). Next, students collected data to identify and design favorite cupcakes, in Event 3, (Case B Transcripts, p. 4). They also determined the type of graph to illustrate their data. During Event 4 discussed how arrays could be used to find the area and perimeter (Case B Transcripts, p. 7). They also identified arrays in their environment, before completing the Array Project during the next class, see Appendix K. During Event 5 and 6 the task required student to design a playground or garden using their knowledge of arrays, area and perimeter (Case B Transcripts, pp. 9-11). During the remaining events, students completed individual project that also included connections to real life. These projects included comparing cell phone plans and designing a mall (Case B Transcripts, pp. 11-13).

Many of Ms. Littleton’s lessons included connections to situations where the mathematics discussed could be used in real life. During Event 3, students completed capacity problems such as, how many water balloons could be filled with different amounts of water (Case B Transcript, Event 3, p. 11). As part of this task, students were asked to represent their two solutions on a chart paper and present their solutions to the class (Case B Transcript, Event 4, p. 14). Students practiced their measurement skills while estimating and then identifying the length of ribbon needed to make a bow and wrap a present (Case B Transcript, Event 5, p. 17). During Event 6, students used this knowledge to estimate the amount of ribbon needed to wrap a box, including the bow.
Then they measured the exact length needed (Case B Transcript, p. 20). Finally, students reflected about their experiences working as a group and the understanding they developed, including how they might approach the same task in the future (Case B Transcript, Event 7, p. 23). Additionally, Ms. Littleton encouraged students to think about other real-life situations where estimates are used or not used.

Both teachers honored mathematics as a discipline when they represented and modeled mathematics in ways that mirrored the structure and content of mathematics as a discipline (Weiss et al., 2009). While discussing content, both teachers pointed out and used accurate ways to use standard mathematical notation. They also guided their students to do the same. Students were required to explain their thinking by making connections among their thinking and the mathematical content. Additionally, teachers encouraged students to provide explanations that included representations of their thinking along with noting numerical expressions.

Mrs. Washington honored mathematics as a discipline when she explained the purpose and elements of data and measurement and use of standard measurement (Case A Transcript, Event 1, p. 1). She also explained how various strategies could be used for different purposes. Furthermore, students provided solutions using representations of number sentences and mathematical notation both orally and in written form while explaining their solutions (Case A Transcripts, Event 2, p. 3).

Ms. Littleton honored mathematics as a discipline when sharing her knowledge of mathematical content with students during lessons. While students were working, she circulated the room, checking on their calculations and pointing out glitches in their thinking (Case B Transcripts, Event 3, p. 11). This occurred while small groups were
working on solving capacity problems. Ms. Littleton, also critically examined students’
answers, allowing students to examine the contributions of other students before she did
so, during full class discussions. She pointed out errors in reasoning during the discussion
about angles and measurement. She focused on the point of error and worked with
students to be sure that when their solution was corrected that they understood and could
articulate the accurate solution.

Students from both classrooms also engaged in mathematical experiences that
were similar to the process used by mathematicians (Lampert, 1990). When presented
with a problem or task students presented a hypothesis, justified their solution, and
reviewed the solution several times in a joint effort to find a solution or deeper
understanding (Case A Transcripts, Event 2, pp. 4-5). These activities took place during
the discussion segments which occurred after problem-solving in Ms. Littleton’s
classroom. Mrs. Washington’s students engaged her students in this way during class
discussions (Case A Transcripts, Event 1, pp. 1-2).

**Learning Through Interaction**

Vygotsky (1978), described the development of a child’s mind as occurring both
individually and socially through a process of developmental events that the student
gained by interacting with people, things, and the world. Successful interaction with
others allows students to develop a strong level of cognition while learning (Kilpatrick,
engaged students in Learning Through Interaction by providing experiences requiring
them to socially construct learning. Both teachers worked to build understanding while
engaging with ideas and one another during mathematical discourse through talk moves
to encourage and sustain interaction (Franke et al., 2015). They also persuaded students to contribute to the learning of others through a variety of the ground rules that encourage restating ideas, discussing solutions, asking questions to direct, exchange and understand ideas, as well as, sharing multiple ways of solving problems (Rojas-Drummond & Mercer, 2004). Both teachers also used clarifying questions to interact with students by validating solutions, helping to guide their thinking and effectively communicate their ideas (Gillies, 2004).

Mrs. Washington and Ms. Littleton treated learning as a social and communicative process, similar to the teachers examined by (Rojas-Drummond & Mercer, 2004). Like these teachers, Mrs. Washington and Ms. Littleton required students to take a more active and vocal role in the math class. They encouraged the exchange of ideas between students and expected them to support the learning of their peers. First, students were expected to engage with one another listening to the conversation. Ms. Littleton often asked students to interact with their peers by prompting them with, “Who would like share their thinking” (Case B Transcripts, Event 1, p.1)? Mrs. Washington asked students specific questions about their contribution or a contribution shared by a peer (Case A Transcripts, Event 1, p. 1). Secondly, teachers provided students with experiences to develop and share a variety of strategies for solving problems. As part of the capacity task during Event 3, Mrs. Littleton required students to report out two strategies and solutions (Case B Transcripts, Event 4, p. 14). While discussing data collected about visiting the library in the summer, Mrs. Washington encouraged a variety of students to share their thinking about the differences between a tally chart and a pictograph (Case A Transcript, Event 6, p. 11). She also had students reflect on the ideas
that were shared by their peers.

Both teachers used methods for Learning Through Interaction. The questions and tasks were designed to promote the *social construction of learning* through the joint problem-solving and classroom discussion about their solutions. Moreover, in the events observed, neither teacher directly taught students about a concept or topic or how to solve the problem or task. The content was provided only when it became clear that students had a gap in their thinking.

Ms. Littleton’s students socially constructed their learning first by trying out strategies and attempting to find solutions as they worked with peers to problems during small group work. Students designed and measured bows together during Event 5. They interacted during planning either to share responsibilities or to make decisions. In the discussion below students are working as a team to construct their learning while finding a solution (Case B Transcripts, pp. 19-22).

1. S1: So, 3 inches, 1/2 inch and 1 inch then 3 inches [measures box again].
2. S2: We don’t need to go around this [pointing to one side of the box].
3. S1: So, we [pretends to wrap the ribbon around the present]
4. S2: I want to make sure that you were right because it looks more than an inch.
5. S2: You measured the whole box?
6. S2: So, this is two 2 and ½ inches for two sides and 3 inches and 1/2 inches so that would be 7 inches so 9 and ½ inches. So just write 9 and ½ inches.
7. S3: You guys measured around the box right and then excess to make the ribbon?
8. S2: 6 plus 2 and ½ that would be 8 and ½.
9. S2: So, we need to add those two together.
10. S1: That would be 18 inches so 19 inches.

11. S2: Yeah.

12. S1: No cuz those together would be one inch, those two halves. Add that one inch to the nine-inch, and that is nine and nine plus nine is 18.

13. S2: Then we have to add it to what’s the measurement, 21 inches, right?


15. S2: So about 40 inches?

16. T: What is your plan? What did you come up with?

17. S2: We came up with 40 inches.

18. T: How did you come up with that?

19. S2: We measured the sides of the box [pointed to each side that they measured] this way and this way… and this would be about 9 and \( \frac{1}{2} \) and this would be 9 and \( \frac{1}{2} \) and then we added our bow.

20. T: And what was your bow?

21. T: Okay, what was your final answer. So, you need 40 inches or ribbon; I’ll be right back okay?

Ms. Littleton circulated the room to monitor their progress. She listened in as students constructed solutions guiding them using the “stepping in and out” method used by Lampert, verifying reasoning, providing content and redirecting students who veered off course (1990). During the students’ discussion about the task, she steps in (Turn 16) to ask the questions, “What is your plan?” and “What did you come up with?” Students explained their thinking. She went to get them the ribbon to test out their estimate.

After completing problems or tasks, students came together in a large group discussion to share their constructions with their classmates and teacher (Case B Transcripts, Event 7, p. 23).
In Mrs. Washington’s class, students *socially constructed* their learning by reviewing concepts, confirming understanding and then working on problems using the information. This was evident during the review of arrays before students made the connections about how to apply them to measurement (Case A Transcripts, Event 4, p. 6). Mrs. Washington also supported the construction of knowledge typically by asking a question to assess students knowledge of the concept. Then she provided students with an opportunity to solve, layering the conversation along the way. She was also providing additional content to consider while moving the entire group toward a reasonable solution. Mrs. Washington provided students with the kind of support moves she believed they needed while trying to stretch their thinking (Frank et al., 2015). Wood et al., also acknowledged this complex facilitation describing it as, 

walking a tightrope, on the one hand she needed to be sensitive when guiding children in their constructions not to make interventions that would inhibit their thinking, on the other hand, she wanted the students to develop the taken as shared meaning that form the basis for the mathematical communication. (1991, p. 608)

Mrs. Washington began with students explaining the *steps* they took to find their *solution* (*GR5B*). For example, this occurred when students provided a solution for converting her height from inches to feet during Event 2 (Case B Transcript, p. 2). Then when students needed *assistance* to help work through their thinking, Mrs. Washington used support moves to assist them in communicating their ideas either by *restating* the thinking for the student or having the student restate their own thinking (GR4C). In Event 4 for example, Mrs. Washington restated a student's contribution, “An array can help you find the area” to frame a question posed to the class (Event 4, p. 6-10). Using the student’s words helped to direct students’ attention to her question. After the student offered, "An array
can help you find the area,” Mrs. Washington restated the student’s phrase in a question. She asked, "Oh! An array can help you figure out the area, is she right?” The question encouraged students to think about why the solution involving area and the perimeter was different when the shape was identical. Students responded to her question. The first student said, "For the perimeter, you are just adding and not doing multiplication." A second student added, "Because you are not doing multiplication you are finding the space of the outline, not all of the inside."

If they struggled with the content then Mrs. Washington provided assistance by scaffolding through questions or directing their attention to a specific idea (GR6D and GR6G). For example, Mrs. Washington used opened ended questions such as, “Nathaniel, what were you thinking” (Case A Transcripts, Event 1, p.2). This question allowed him to share his own ideas with others (GR6A). He explains his solution. Mrs. Washington again asks a clarifying question, “How did you know that?” followed by “Can I ask you a question?” Nathaniel has some difficulty responded so Mrs. Washington helps him to work through the process by scaffolding his ideas by explains his thinking in her own words (GR6G). In Turn 65, Nathaniel explains his thinking. Then in Turn 68, she spotlights his thinking for all to notice.

Students were also encouraged to contribute to the learning of others during all events. They were consistently invited to share their ideas or solutions, often providing explanations and justifications to support their thinking. Also, both teachers initiated opportunities for students to support one another while identifying strategies to solve problems or during class discussions. First, students were expected to explicitly describe their thinking so that others could follow their line of reasoning (Rojas-Drummond &
Students shared how and why they solved their problems aloud. Students took turns sharing multiple ways of thinking or multiple strategies for solving the same problems. Ms. Littleton’s students shared the different strategies they used to show how they figured out how many right angles were in a circle (Case B Transcripts, Event 1, p. 1). Mrs. Washington shared the different ways they converted her height to inches (Case A Transcripts, Event 1, p. 1). During both of these examples, students contributed to the learning of other students by sharing different ways to solve the same problem. These experiences help to broaden student thinking and understanding because they have opportunities to explore strategies they had not thought of on their own.

Additionally, teachers also provided opportunities for other students to assist their peers when they struggled before they stepped in to correct the student themselves. Mrs. Washington initiated opportunities for students to contribute to the learning of others by inviting students to help classmates out of a situation where they were not sure of the answer. She prompted these exchanges by providing students with the opportunity to call on a friend to help continue the conversation. For example in Event 5 when she asked Gagan to “help him out” when a student struggled with knowing if he was standing in the area of the classroom (Case A Transcripts, p. 9). She also used spotlighting to enlighten students with the ideas of others (Lampert, 1990).

Ms. Littleton initiated the process of students contributing to the learning of others when using questions such as, “Does anyone have a question for them” Or “Class who can help her out? Sometimes students took opportunities to ask questions of their peers spontaneously without prompting. They did this to either to learn about someone else’s thinking or to question the accuracy of a solution or statement. However, there
were times when Ms. Littleton initiated an interaction and students did not respond. This concern is similar to the one raised by Franke et al., (2015). The authors reflected,

Inviting students to engage with others will not guarantee that students will, in fact, engage with each other, nor necessarily engage in ways that are supportive of mathematical learning. (2015, p.146)

This concern mirrored the finding concluded by Webb, Franke, Ing, Wong, Fernandez, Shin, & Turrou (2014) that even when teachers attempted to engage students with other students’ mathematical ideas, the results were inconsistent. Students did not always respond to these prompts. However, with encouragement students can participate in contributing to the knowledge of others. For example, students in Ms. Littleton’s class offered assistance to his peers by explaining a more viable solution. Students in one problem-solving group had two different answers. They could not decide if they should add the 76 degree angle to 180 degrees or subtract it. Arthur stepped in to help. He said, 76 is the acute angle [shows the angle with his hands] and 180 is the straight one [shows angle with his hands] so you should have subtracted instead of added. (Case B Transcripts, Event 2, p. 6). Ms. Littleton followed this exchange asking, “Does this make sense?” to the group? She followed this question with another question, “Caleb and Chevron how could you fix your answer?” The group anwered, “Subtract instead.” Ms. Littleton continued, “Subtract from what?” A group member answered, “Subtract 76 from 180,” and the discussion moved on.”

**Strategic Discourse**

According to Ms. Littleton, her math classes and education classes prepared her to teach mathematics. She also described her practice as evolving from her experiences teaching math, along with her math background (Teacher Individual Interview, p. 2). She
believed that encouraging students to talk about mathematics was important, especially talk among students (p. 4). Students are able to understand better when other students put their thinking in “kid friendly” language (p.5).

Ms. Littleton also expressed the value she found in the exploration of ideas among her students. She observed this leading to the development of their own conclusions, as opposed to her telling them. Other researchers indicated that students who developed their own ideas, developed their own ideas, and discussed solutions with other members of the learning community benefitted, as well (Boaler, 1999). Ms. Littleton also believed that as students developed conclusions and explored ideas would also retaining the process and being able to solve the problem in the future as opposed to when they were asked to match a formula to a problem (Teacher Individual Interview, p. 10). This is in line with Lave and Wenger’s (1991) description of learning being more sophisticated than simply acquiring skills or completing daily tasks as individual learners, occurring through a process in which students interact with one another and the content to strengthen thinking skills.

Ms. Littleton also mentioned that she was not told how to engage students in discussion. Instead, she explained how she had developed her instruction through practice and experience. She demonstrated an ability to interact with mathematical ideas successfully. Similar to the two teachers in the study by Franke et al., (2015), Ms. Littleton probed students’ thinking and reacted in-the-moment to help students to clarify ideas during mathematical conversations. She described this process during her individual interview when explaining how she had planned certain questions and discuss certain things and then the plan would shift because of the direction students would take.
the discussion. She explained this process, “we discover one thing, we're moving forward to something else, and it might fell in the order that I didn’t plan it to go. But that’s the way students are learning, so I'm going to go with that (p. 6).” Ms. Littleton demonstrated a high level of confidence in her knowledge of mathematical content. She also attributed the success that she has had with engaging students in mathematical discussions to her undergraduate courses where this type of questioning was modeled (Individual Teacher Interview, p. 1). Knowing the mathematics content assisted Ms. Littleton as they make more informed decisions when directing the discourse (Wood, 1998, Cenzin et. al, 2011). Secondly, she handled the spontaneity of discourse and effectively manage the course of the entire discussion while teaching the lesson is also advantageous (Cenzin et al., 2011; Franke et al., 2015; Mercer, 2000;).

Mrs. Washington valued bringing her students together to discuss mathematics. She identified a need for solving world problems and practicing that skill during math class. Mrs. Washington also valued talking and communicating. She used talk during problem-solving as a tool for joint reasoning, similar to Mercer, to develop her student’s communication and reasoning skills problem-solving during all events as a way of developing her student social skills through conversation see Table 14. Mrs. Washington wanted all students to learn how to articulate their thinking.

The students in both of these examples required some support from their teachers for productive dialogue to occur. Again, the process of negotiating with members of the community to find meaning in the thinking of one another is an important ingredient when learning in the situated perspective (Lave & Wenger, 1991).
She also believed that teaching more about the process, not just finding answers was important. Like the thinking of Hiebert and Grouws (2007). Mrs. Washington regarded her students’ ability to making sense of the mathematics through explaining their own thinking. Mrs. Washington also felt strongly that students should experience learning and apply this learning to everything that is going on (Individual Interview, p. 1). She also reported seeing math as a “social content area” because students need to solve problems but they also needed to talk and communicate with one another (p. 4). Additionally, she saw the benefits in students learning from her and then something that she said is told to them by a friend in a different way and then they are able to “get it.” She wanted learning to be authentic experiences through activities requiring a “reaching out of the mind” (Dewey, 1902). Additionally, she used a spiraling curriculum embedding skills within students problem-solving tasks and conversations. She purposefully exposed students repeatedly to skills all year long and not just during units of practice to build understanding.

After mining the data from this study, it became clear that the strategic discourse practices used by both teachers aligned to three themes. These themes served as a framework for describing the use and purpose of the strategic discourse practices used by teachers and students in this study. Each theme contained several key elements teachers used with students to engage in mathematical discussions. The themes framing the discourse were; Encouraging Students to Participate, Developing Mathematical Knowledge, and Strengthening Critical Thinking. Patterns among the most frequently used elements among teachers emerged within each theme. The elements within the first theme Encouraging Students to Participate simply promoted encouraging students to
engage with one another in conversations while discussing concepts and solving problems. Teachers utilized these elements to encourage students to talk with one another while building their understanding in math. The strategic discourse element that are aligned with the theme Developing Mathematical Knowledge promoted more rigor by engaging students in mathematical conversations to develop their knowledge of content. The elements within the third theme Strengthening Critical Thinking were used by teachers to strengthen critical thinking among their students.

Teachers utilized common elements within each of the three themes. These patterns became the shared mathematical norms that emerged among the two teachers practice as noted in the following sections (Cobb et al., 2001). These practices determined the norms that were developed and modified continuously by the teacher to support students as they communicated “different mathematical solutions, insightful mathematical solutions, efficient mathematical solutions and acceptable mathematical explanation” (Cobb et al., 2001, p. 124). As revealed in Figure 10 describes the elements most frequently used by teachers. As evident in the data, the theme with the most amount of frequently used elements by teachers is Developing Mathematical Knowledge, see Table 21. Both teachers emphasized learning and practicing mathematical content and that is reflected in the higher frequency of the ground rules in the theme Developing Mathematical Thinking among teachers.
Likewise, patterns were also evident among the discourse elements used by students with their teachers and peers. These patterns became the shared *mathematical norms* that emerged among the two groups of students as they engaged in discourse with their teacher and peers, see Figure 2 and 3. The norms governed how students interacted with the mathematics and each other while engaged in discourse. The norms that students developed as they practiced engaging in discourse also impacted how they interacted even when not prompted by teachers (Franke et al. 2015). Again, the most commonly used ground rules are part of the theme Developing Mathematical Thinking due to the teacher’s emphasis on learning and practicing mathematical content. A summary of each ground rule used from within the three themes will be described in the next section.
Student participation is essential to the success of mathematical discourse. Researchers are developing and studying ways to support teachers as they engage students in productive discourse with each other’s mathematical ideas is a complex process to sustain (Cengiz, 2011; Franke et al. 2015; Gillies, 2004; Hufferd-Ackles et al., 2004; Mercer 2000). Students do not always know or understand what is expected from them in order to participate (Sherin, 2001). Other times students face challenges around how to engage (Franke, 2015). Their participation can be inconsistent or nonexistent, as well (Webb et al., 2014). These challenges have included students being unable to: engage with a peer’s ideas; provide little or no detail about others’ thinking; or not address the mathematical ideas underlying a strategy shared by a classmate. However, utilizing the strategic discourse elements as norms for engagement, along with a culture that supported their participation, both Mrs. Washington and Ms. Littleton inspired their students to engage at a high level, as evident in Table 18.
The element *Students Are Chosen Strategically By The Teacher/Student To Contribute (GR1B)* was used most frequently by both teachers in the first theme *Encouraging Student Participation*. Ms. Littleton used this element to call on students to share his or her ideas or to follow up with a specific student (Case B Transcripts, Event 1, p. 3). This was usually the case when a student raised their hand to answer a question. The times that she followed up with a specific student was when she had a follow up question to seek clarification, justification or further explanation during small group work and class discussions. Mrs. Washington’s students usually did not wait to be called on which meant that she called on specific students to follow up with them after they contributed (Case A Transcripts, Event 5, p. 9). More of the events in Mrs. Washington’s classroom involved class discussions where she facilitated conversations that involved back and forth interactions among students and herself. Ms. Littleton’s students spent more of the time during the observed events working in small group setting and listened more to the conversations occurring among students.

The element *Casual Interchanges Demonstrating Equalized Relationship (GR3A)* was utilized by both groups of students, see Table 18. Mrs. Washington used banter back and forth with her students to help make learning interactive and fun. Mrs. Washington’s students joked back and forth with their teacher, and jumped in and out of conversations without invitation (Case A Transcript, Event 1, p. 1). Ms. Littleton encouraged the use of this element by welcoming students to comment, question or bring errors to the group’s attention Case B Transcript, Event 2, p. 5). It was evident that students had ownership of this element when they even raised questions to their teacher, as well as, corrected her when a mistake was made.
Although only one ground rule was utilized frequently by both teachers they dedicated a much of their class to discuss mathematics. The element *Everyone Invited to Contribute (GRIA)* was utilized often to openly welcome all to contribute the conversation. Likewise, Mrs. Washington’s students used this element very frequently as they simply jumped into conversations to contribute their thinking.

**Developing Mathematical Knowledge**

The purpose of Developing Mathematical Knowledge in this study is similar to the conception of researchers who believe that teachers and students who engage together in each other’s mathematical ideas, create opportunities to enrich their mathematical understanding (Franke et al., 2015). Additionally, engaging in discourse allows teachers learn about the development of student thinking and how to support their students. The students learn how to listen to another, how to ask a question, “that moves the mathematics forward, and how to position their ideas in relation to others’ ideas” (p. 146). Students need support to learn how to focus their mathematical reflection and reasoning on important mathematical concepts (Cengiz et al., 2011). Both teachers in this study used strategic discourse strategies in this theme to reveal students’ knowledge and reasoning so they and other students could listen to their thinking and try to understand their rationale. Additionally, students were encouraged to share multiple ways of solving problems so that they could experience new ways of thinking by listening to a variety of solutions to the same problem.

Teachers utilized six of the elements within the theme Developing Mathematical Knowledge. First, teachers utilized the element *Contributions Are Restated (GR4C)*. As students shared their knowledge and explained their strategies and solutions, Ms.
Littleton utilized the element *Contributions Are Restated (GR4C)* to highlight the information they provided for peers (Case B, Transcript, Event 3, pp. 8-9). Mrs. Washington also utilized the element *Contributions Are Restated (GR4C)* to bring her students attention to a contribution, verify the information or clarify the student’s thinking for the rest of the class (Case A Transcript, Event 2, p. 3). Chapin & O’Connor (2007) used a similar type of revoicing in their Academically Productive Talk model to restates a previous speaker’s words. According to these researchers, this form of revisiting ideas allowed more time for students to catch details, process them, and clarify their ideas than in more traditional formats.

Secondly, Ms. Littleton used the element *Ideas and Solutions Discussed with Others (GR6A)* very frequently while conducting lessons, to interact with students as they completed tasks and during class discussions to review the student’s problem-solving experiences (Case B Transcripts, Event 4. p. 14). Mrs. Washington used the element *Ideas and Solutions Discussed with Others (GR6A)* very frequently when talking with students about what they understood about ideas and how they could apply what they knew to a new activity or problem (Case A Transcripts, Event 4. p, 8). Teachers encouraged student to provide details about their thinking to helping make connections in their explanations and representations (Webb et al., 2014). Franke describes the correlation of seeking details in student’s ideas and the level of student engagement with the ideas of others,

> Attention to both the detail of the mathematics and the detail of each other’s mathematical ideas, thus requiring intricate, interactional work among the teacher, students, and mathematical content. (p. 129)
Thirdly, teachers used questioning to Develop Mathematical Knowledge. They used questions to engage students to direct discussions, exchange ideas and to understand thinking. The element *Questions Posed to the Community To Direct Thinking (GR6B)* was used when Ms. Littleton wanted students to consider something she had chosen to discuss (Case B Transcript, Event 7, p. 22). This included a concept or topic or an idea either to begin a lesson or help students when they had demonstrated an error in reasoning. She also posed questions when she was trying to prompt students toward the direction of a new path or idea. Mrs. Washington also used the element *Questions Posed to the Community To Direct Thinking (GR6B)* to guide students thinking toward a point that she wanted them to consider (Case A Transcript, Event, 5, p. 10). She also used this element to redirect their thinking when they struggled to understand a concept in one way and she was helping to explain it using a different path or example. Moreover, she used questions to redirect their thinking when they had gotten off course.

Both teachers encouraged students to interact this way quite often. Using practices to support exchanging or engaging with the ideas of others provides opportunities for students to participate, and encourage deeper engagement to engagement with the mathematical ideas presented by their classmates (Franke et al., 2015).

Mrs. Washington also used *Questions Posed to Encourage An Exchange of Ideas (GR6C)* to encourage students to share ideas and solutions with others. She encouraged the exchange of ideas during Event 5 by asking, “How did you figure that out?” and “Why do we use the same rules?” She also reminded students of prior activities when
they struggled with concepts. For example, Mrs. Washington asked, “Remember when we were using things like paper clips and blocks?” or “This morning I had you walk around the what” (Case A Transcript, p. 9)?

Mrs. Washington also used the element *Community Asks Questions To Understand Thinking (GR6D)* to ask students questions about their ideas and strategies. She used this element to decipher what student were thinking and to help students articulate their ideas more successfully. An example of this was drawn from an exchange during Event 1. Students were discussing Mrs. Washington’s height, and she asked questions to unveil their understanding of the strategies used by students. She asked, “How did you know that” and “How did you decide to start with 48” (Case A Transcript, pp. 1-2). Additionally, when a student unexpectedly utilized division as a strategy, she asked excitedly, “What did you do?” The student wrote her explanation on the board and provided a detailed explanation of the strategy. Mrs. Washington continued to probe the student’s thinking as the class listened on. Trying to help other students understand the higher leveled thinking. Also, Mrs. Washington asked this question to also encourage her students to listen to important ideas and strategies that they might utilize in the future. She modeled this element for her students. Students did not utilize this element frequently.

Ms. Littleton used the element *Questions Posed to Encourage An Exchange of Ideas (GR6C)* to encourage students to contribute ideas and to interact with a contribution offered by a peer. Ms. Littleton used questions to facilitate discussions to *engage students (GR6C)* in order to keep the conversation flowing among students. She asked, “So was
“your challenge measuring out the amount that you needed” or “So what would happen if you did an underestimate” to keep the conversation flowing among herself and students (Case B Transcript, Event 7, pp. 23-26).

Ms. Littleton also used *Community Asks Questions To Understand Thinking* (*GR6D*) to encourage students to ask questions of other students independently of the teacher. She modeled asking questions and prompted questions to initiate students asking questions of others using the element These questions were posed to students to lead student to interact with one another. Additionally, Ms. Littleton used these questions to prompt students to find out more information about how their classmates planned and solved problems. During Event 7 for example, Zandra shares the challenges that her group experienced and how they finally managed to wrap it around the box. Ms. Littleton follows up with a question about her explanation, “So was your challenge measuring out the amount that you needed” (Case B Transcripts, Event 7, p. 23). Ms. Littleton prompts Zandra to make the steps the group had taken more explicit to the rest of the community. Zandra explains, “It was like only 60, we have to do it multiple times because we have like two 60’s.” Ms. Littleton next question attempts to move this student toward the specific mathematical ideas involved in finding the solution to this problem. Ms. Littleton asked, “How’d you figure out that you needed to do 60 twice?” Zandra justifies her group’s thinking by explaining the steps taken to determine the length of the ribbon. They had estimated the sides of the box and then combined the estimates to arrive at 138 inches. Zandra does not communicate whether or not the group’s process connected the attributes of the three-dimensional figure with a strategy for finding their solution.
The final element utilized frequently in this theme by teachers was Many Ways of Solving Problems Encouraged (GR7A). As discussed below, each time a problem was discussed both Ms. Littleton and Mrs. Washington prompted students to share different strategies for solving the same problem. Allowing time for students to share different ways of thinking was valuable based on the time that was spend doing so in both classrooms. This element also showcased the variety of strategies students used. Mrs. Washington made a concerted effort to spotlight original and creative ideas, announcing each to her class.

Ms. Littleton encouraged students to engage with the mathematics by pushing them to think creatively and share multiple ways of solving problems (GR7A). In Event 2 for example, Ms. Littleton asked, “Who would like to share out how you solved this problem” (Case B Transcripts, p. 1). Students were also challenged to come up with two ways of solving the capacity problems during Event 4 (Case B Transcripts, Event 1, p. 1). This also occurred during conversations when Ms. Littleton consistently asked for more students to share their thinking with the prompt, “Would you like to share your thinking” (Event 2 page 5)? This element was used by Ms. Littleton to encourage more students to engage by exchanging ideas (GR6C). For example, she often prompted students to discuss the thinking of others. She did this by prompting in general terms, “Any questions for Zandra” (Case B Transcripts, Event 1, p. 1)?

Mrs. Washington also emphasized the element many ways of solving problems (GR7A) to engage students with both the mathematics and one another. She wanted students to challenge themselves to share a different solution. She asked, “Who did it in a different way” (Case A Transcripts, Event 1, p.1-2)? She also used the question, “Anyone
have a different question” to continue the conversation (Case A Transcripts, Event 3, p. 4-6)? Finally, Mrs. Washington used questions to engage students in the content to further their thinking (GR9B). These questions were challenging for students. While students were discussing the Cupcake Challenge Task, Mrs. Washington asked, “Why would a line graph work (Case A Transcripts, Event 3, p. 4-6)? Students were suggesting various types of graphs and Mrs. Washington was trying to encourage them to choose a graph that best matched their data, which was an essential question noted in the third-grade curriculum (Atlas Rubicon, 2018). Similarly, she also used the question, “Could I collect my data another way” while they were discussing the Summer Reading Tally Chart during Event 1 (Case A Transcript, p. 2-4)?

Teachers used more strategic discourse practices to develop mathematical knowledge when compared to the other two themes, see Table 19. However, this number was still limited. Even though the elements were underutilized, both teachers, discussions followed a clear mathematical path for students to follow. All conversations included grade level content that addressed several key points involving the topic. Moreover, students did not use a wide variety of elements either, but they were highly responsive to elements introduced by their teachers, always reacting with the high level of enthusiasm and participation. These positive results correlate directly to the positive culture that was cultivated by both teachers in these two classrooms.

Professional development that focuses on educating teachers about the research based discourse moves that are available to them is an essential first step (Cengiz et al., 2011; Franke, et al. 2015; Mercer, 2000; O’Connor, Michaels, & Chapin, 2015; Sherin, 2002, Wood, 1998). Explicitly teaching students the strategies and explicitly practicing
them with students is recommended. Furthermore, incorporating a wider variety of moves with students will help to strengthen the productivity in the discourse, as well.

**Strengthening Critical Thinking**

The elements within the third theme had a higher degree of complexity. The elements encouraged students to question, compare, disagree and verify their thinking as a means of extending their mathematical thinking. Additionally, students were encouraged to present arguments and the community interacted critically with their ideas. These practices are connected to the body of research directed at extend mathematical thinking using discussions (Hiebert & Grouws, 2007; O'Connor, Michaels, & Chapin, 2015; King, 1992). Hiebert and Grouwz (2007) describe these experiences as engaging in a “productive struggle” (p. 390). These experiences push students to think and figure out problems that are challenging for them. Extending mathematical thinking can be supported with additional talk moves used to encourage students to dig deeper when engaged while working with peers (O'Connor et al., 2015). Students also found success with the use of question stems to guide discussions (King, 1992). Moreover, researchers reported positive results when using a combination of “instructional actions” that included eliciting, supporting and extending all used to extend extending student thinking about mathematical ideas and strategies (Cengiz et al., 2011).

Although the teachers in this study utilized practices to engage in Strengthening Critical Thinking, these elements were used much less frequently in comparison to the elements in the theme Developing Mathematical Thinking. As a result, students did not have the opportunity to strengthen their critical thinking using the discourse practices initiated by their teachers to learn mathematics. Additionally, teachers did not use any
common elements frequently in this theme. In fact, each teacher utilized only one of the elements frequently during the study. Mrs. Washington utilized the element *Questions Used To Further Thinking (GR9B)* while Ms. Littleton utilized *All Have Opportunities to Question Others Ideas (GR11A)*. Additionally, neither class of students utilized any of the elements from this theme frequently while engaging in discourse with students.

However, both groups utilized a variety of elements a small number of times within the element *Partners Engage Critically With Each Other (GR11)*. Students in Mrs. Washington’s class *advocated for their way of thinking* through justifications (GR11C) and *compared solutions and ideas* under the guidance of their teacher (GR11D). Ms. Littleton’s students *questioned the ideas of others (GR11A)*, *disagreed with the thinking of others (GR11B)* and *advocated for their way of thinking* through justification (GR11C). They also utilized *input is collected before deciding on a strategy or solution* (GR12A).

As noted earlier, three themes that emerged in this study grow with increasing rigor beginning with Encouraging Student Participation, followed by Developing Mathematical Knowledge, and ending with Strengthening Critical Thinking. The acknowledgement of the three themes builds on the existing research supporting the need to include ways to instruct students using hierarchical modes to develop learning objectives at different levels complexity (Bloom, Engelhart, Furst, Hill & Krathwohl, 1956; Webb, 2002). This hierarchy describing the different levels of complexity as among the ground rules within each of the three themes are represented in Figure 7.3 below.
Although, the discourse elements within the Strengthening Critical Thinking theme were used infrequently, each time they were used teachers skillfully engaged student to extend their thinking. This suggests a potential for utilizing these practices to strengthen critical thinking more frequently in the future. Next, I look more closely at the idea of purpose in regard to the implementation of practices supporting the teachers use of strategic discourse.

**Intentional Strategic Discourse**

In agreement with research detailing the benefits to using discourse to support learning and understanding in mathematics, the findings in this study demonstrated the value teachers placed on using reform-based methodology and strategic discourse as evident in the amount of learning time dedicated to these methods. The analysis showed that teachers build a successful community for learning similar to that used by Lave and
Wenger (1991). They provided students with many opportunities to participate in activities to learn authentically with others while solving problems.

As evident in the observations conducted for each teacher, the mathematical discussions contained a clear mathematical path. Ms. Littleton reflected about this during her interview. She described how she definitely had things that she needed to be done, including the questions that needed to be discussed (Individual Interview, p. 6). Mrs. Washington also used questions to guide her mathematical discussions. Using questions are really good for me because they get me to “start thinking and driving” the conversations (Individual Teacher Interview, p. 2). Teachers drew on strategies using in-the-moment, real time decision making (Choppin, 2007; Franke et al, 2015). The teachers also used questioning and prompts to expose thinking, and offer further explanations. They skillfully engaged students by presenting mathematical concepts in interesting ways and connected to real life. They also consistently encouraged multiple solutions for problems. Students consistently responded well to their efforts, demonstrating a high level of participation. However, the discourse elements used during the implementation of the mathematical discourse were not intentional.

Analysis of the interview data collected exposed that the two teachers had a limited awareness of the strategies they were using. They were also unaware of possible additional moves that could have been utilized (Chapin & O’Connor, 2007; Rojas Drummond-Mercer, 2004; Sherin, 2002, Wood, 1998). Therefore, teachers did not select appropriate moves prior to discussions, nor did they reflecting about the effectiveness of the moves either. The discourse practices they were using were not intentional, so therefore the implementation of strategic discourse was extemporaneous.
Research suggests that the overall confidence in the teachers’ ability to successfully manage the complexities of productive discourse is low (Ding, Li, Piccolo, & Kulm, 2007). I do not concur with this point or some other conclusions drawn from this research. Specifically, research has uncovered how teachers are not skilled in guiding students or scaffolding while teaching mathematics (Ding et al., 2007). The two teachers in this study, were able to assist students with scaffolding, especially while talking about a problem that the students had already worked through on their own. In addition the research indicated difficulties teachers experienced with teachers leading students to identify multiple approaches for solving problems or elaborating on the other students’ ideas. Both teachers in this study were especially skilled in these areas. Furthermore, they also did not guiding students to identify just one solution. They stressed the importance of multiple ways of solving problems using a variety of strategies.

The teachers in this study need only to build on what they already know, to merge additional techniques with all that they are currently doing well. Researchers have reported success with focusing studies on expanding teachers pedagogical content knowledge with an ability to make momentary decisions to select an effective discourse strategy (Cengiz et al., 2011). Teachers who experience a higher level of success with this were those that had been using reformed based materials that had a professional development component included which provided them with experiences using these strategies during mathematical conversations. More recent research provides a collection of moves for teachers to draw from as they navigate their mathematical discourse (Cengiz et al. 2011; Franke et al., 2015). The researchers from these studies observed teachers developing a set of norms that shaped the ways they interacted with students during
mathematical conversations. Authors emphasized the importance of teachers attending to details in students’ explanations and learning how to use in-the-moment decisions about which of the moves would be successful. This study further emphasizes the need for strengthening expertise among students. Franke et al, suggested teachers build a referred to a “repertoire of pedagogical moves” used to support students when they struggled to engage effectively with the ideas of others (p. 143).

The findings from this study show that unplanned implementation of strategic discourse can result in a lack of complexity regarding the objectives for learning and the underutilization of productive moves among teachers. This connects to what researchers discovered when studying teachers as they used discourse to sustain engagement among students (Frank et al., 2015). Researchers reported that the teachers in their study never used the same series of moves more than once, even in response to the same challenges with students. This implied that the teacher moves were not part of a set of fully planned actions that could be applied repeatedly in the same way, but rather served as a collection of pedagogical moves that teachers drew from during discourse. Their results suggest that teachers should not be provided with a simple set of moves to follow in a lock step fashion. Instead they must learn and practice a variety of moves to build a repertoire. Then with this repertoire in their tool box, be able to draw from them as discourse situations arise, similar to the teachers studied by Franke, et. al., (2015). This is similar to Mercer’s conception which includes a teacher’s ability to draw from a variety of ground rules to implement for effective communication to occur (2000). According to Mercer, every new interaction creates a different context, and each context is re-created
through the interactions among participants and the different set of ground rules that are applied.

I concur that teachers should acquire of a collection or repertoire of moves, because the teachers in this study were unaware of the variety of moves available. Additionally, the few moves they did use were used repeatedly, and developed a small set of skills, as seen in Table 19. However, it is recommended that the development of these moves be conducted in a more focused and strategic manner based on the purpose each talk move serves. Most of the moves used by the two teachers in this study encouraged student participation and developed mathematical knowledge. A lot less of the moves used targeted the strengthening of critical thinking among students. Like the themes that emerged in this study, each talk move should be categorized and then implemented when they align with the intended purpose for each discussion. According to research in this topic, teachers who recognized the potential of situations through listening to student thinking and establishing clear goals about the mathematical ideas and concepts to pursue was critical to the successful implementation of productive discourse (Cengiz et al., 2011). These researchers also attributed the ability of the teachers to execute extending episodes to the ability to recognize the potential of a particular situation to do so. Careful listening to student thinking and establishing clear goals about the mathematical ideas was essential in the success of this process (Cengiz et al. 2011).

It is critical that teachers be able to move beyond developing knowledge of content toward strengthening critical thinking. The discourse practices used by teachers and students in this study were examined with this research using levels of complexities
among objectives and scope of learning activities. It was noted that teachers most frequently used elements in the second level of the hierarchy, only rarely using those within the highest level, see Figure 7.3. Furthermore, by not planning strategic discourse elements, teachers also did not consider the depth of knowledge required while using the elements that required critical thinking in the third theme. Therefore, teachers rarely provided opportunities for students to allow practice the skills needed to build critical thinking. Understanding and utilizing the learning hierarchies developed by experts has the potential for guiding educators in examining ways to increase the rigor embedded in learning activities (Bloom et. al, 1956; Webb, 2002).

The two teachers in this study used elements at all three levels. However, as noted earlier, some strategic discourse elements were used infrequently or never. As such, Franke et al., (2015) research asserts that engaging teachers in reflection can provide a means for growth. This reflection is necessary for identifying which of the discourse practices that are most successful for individual teachers and groups of students. teacher education and should be a part of every teacher development program. However, I argue that these teachers must have practiced a level of metacognition for the patterns in their discourse to surface. Therefore, an analysis of the natural discourse that each teacher utilizes as part of their instruction should be reviewed and supplemented with additional research based methods to enhance their instruction, not overhaul it.

**Sharing Mathematical Authority**

“Mathematical power” is defined by the NCTM (1989) as “an individual’s ability to explore, conjecture and reason logically, as well as the ability to use a variety of
mathematical methods effectively to solve non-routine problems” (p. 5). Discourse cultivates a process for helping students achieve mathematical power (Lampert, 1990). Researchers have described teachers who have success creating environments to help students to see themselves as having mathematical ideas to offer, that communication is valued, and that learning with others is part of learning mathematics (Walshaw & Anthony, 2008). The success of strategic discourse depends on the level of engagement and the willingness of the teacher to share authority with her students (Webel, 2010). However, some teachers can struggle with adapting their roles to allow students more instructional control (Hoek & Gravemeijer, 2011).

This was not the case in this study. Findings reveal that the two teachers studied willingly shared authority with students. Along with the reformed based methods, teachers demonstrated a proclivity for Sharing Mathematical Authority with and among their students. It is my purpose to highlight the importance of this shift in authority and how it fostered a partnership among students and teachers. The following section describes how authority was shared in the classrooms leading to a partnership that supported the discourse practices.

Clearly the high level of participation in discourse evident in this study was a direct result of the environment cultivated by the two teachers, along with their students in their attempt to socially construct knowledge in mathematics. A major factor attributing to this was the way teachers assumed less dominant teacher roles elevating the students’ roles. The elevated student role also allowed for a heightened level of shared mathematical authority among them. Teachers became partners with their students in learning. They did not conduct their classrooms in the traditional sense where teachers
handed down mathematical expertise to students. Neither did they expect students to simply receive this information: they required student’s active participation to the successful engagement, as summarized earlier in the Learning Through Interaction section of this document.

Again, the environment that teachers cultivated that included a prominent role among students involved opportunities to use knowledge, build greater understanding while helping each other find solutions, making sense together in the community of practice. Teachers also found ways to acknowledge students as competent thinkers who were strong sources of mathematical ideas (Choppin, 2007). The following describes the ways mathematical authority was shared among teachers and students in the two classroom communities.

The purpose of this summary is to reiterate the ways both teachers shared authority was shared in the two classrooms. First, Mrs. Washington encouraged students to maintain a very important role in classroom discussions. As she presented ideas, students were invited to jump into the conversation to add ideas throughout. Students were also encouraged to present their ideas and these contributions were always accepted and valued. Mrs. Washington used prompts such as, “Someone help me out” and “What is an array, turn and talk to someone” (Event 4, pp. 7-9. She also used questions such as, “Will that work” and “Why” to encourage students to contribute. She always acknowledged these contributions. For example, she said, “Excellent!” and “Beautiful.”

Mrs. Washington was not the only expert in the room, she did not correct students but encouraged them to review their thinking or created opportunities for other students to help. First, when a concept was difficult, she would act as if she was confused and ask
for help understanding. For example, while a students used division as a strategy to convert her height from inches to feet in Event 1, Mrs. Washington asked, “What did you do? I have not idea what you are talking about” (Case A Transcript, p. 2). Another example occurred during Event 4 when Mrs. Washington repeated a student’s contribution, “She just said, an array helps you find the area! Somebody help me out with this to get other students to explain why this made sense(Case A Transcripts, p. 7).

Mrs. Washington’s students demonstrated their heightened authority in different ways. They offered solutions to problems, stating their knowledge confidently. During Event 5 for example, Elena explained the reason why it was important to know the area and perimeter of a space. She said, “It will help you to know how much fence to buy” (Case A Transcripts, Event , p. 10). Additionally during this lesson Mrs. Washington asked why the answers were different when finding the area and perimeter, even if the lengths of the sides were the same. Olivia clarified, “Because you are multiplying the area and adding the perimeter. “Students also requested to come to the front of the room and represent their thinking for all to see by writing their solution on the whiteboard (Case A Transcripts, Event 4, p.7). Ben asked to come to the board to draw an array representing 6 by 6. Amy also came up to draw the diagram representing a shape with the area of 35 and a perimeter of 24 (Case A Transcripts, Event 5, p. 10).

The questionnaire and student survey data also suggested that students perceived having the mathematical authority to contributing to their own leaning and the learning of others. More than half of the students reported being helpful to others when explaining their thinking, see Table 12. Most also viewed their classmates as being able to support their success in solving problems. However nine students preferred learning on their own,
and only six liked learning from others. Much diversity among the perspectives of students existed within this data warranting further examination at a much broader scale of student thinking around this topic in the future.

Moreover, during the individual interviews a student spoke about the ways others looked to them for help. Ben thought Mrs. Washington asked him to share his ideas with his classmates because, “I am really good in math” (Individual Interview, p. 2). He also shared that Mrs. Washington asked him to explain his thinking because she wanted the other students to learn from him. Ben explained how other students supported his learning. He said, “I notice people who know more than me, then they can change my ideas and then they might have a better idea, and all of the people in my group will listen to them” (Individual Interview, p. 4).

A student also discussed the way students helped one another to build understanding. Jaylissa also perceived that Mrs. Washington wanted her to tell other students what she “meant” and students could see if they think the same way or if they had a “different way to explain the problem” (Individual Student Interview, pp.1-2).

Secondly, Ms. Littleton centered her conversations on the ideas and thinking of her students. The conversations were not controlled by Ms. Littleton, she simply facilitated students’ opportunities to share. Although she did interact with students as they shared their ideas, she often deferring answering by encouraging other students to support the construction of ideas among their students. Boys and girls do we have any questions or suggestions about how they are solving the problem? (Case B Transcript, Event 2, p. 6).
Students were encouraged to take a prominent role in this classroom. During Event 4, Ms. Littleton provided an experience where students were responsible for presented their solutions to the capacity problems to their peers. The students listening had the authority to ask the presenters two questions and offer one comment. A few of the questions were, “Did you get confused at any of the parts?” and “Which way did you like best” (pp. 15-17)? Comments included, “You did a nice job organizing your poster” and “You guys seem to know what you are doing.” They were serious about the process. Although one of the norms included raising a hand to talk, this did not limit participation.

During classroom discussions, Ms. Littleton initiated the process asking students to share their thinking. It was clear that they were listening to the exchange of ideas and did not hesitate to engage, to contribute an idea or strategy. They also were forthright when raising a concern with a solution, even if the concern involved their teacher’s line of thinking. For example, when some group members came up with a different answer, students raised the concern (Case B Transcripts, Event 2, p. 6). Also, while Ms. Washington was estimating the length needed for a ribbon, the group members questioned her, “That does not look like 50 inches” when they thought that the ribbon was too short Case B Transcripts, Event 5, p. 19).

Students demonstrated their heightened authority in different ways. They offered solutions to problems, stating their knowledge confidently. For example while explaining his solution to the capacity problem Arthur said,

So I did 6 times 4 is 24 fluid ounces because Camile has six water balloons and each is filled with 4 ounces of water. And then I did 5 times 8 is 40 fluid ounces because Bibi has five balloons with one cup of water and 1 cup has 8 fluid ounces. And 24 is less than 40. (Case B Transcript, Event 4, p. 15).

Additionally, during Event 3, Devon explained why she believed Zandra’s solution was
correct. She said, I agree with Zandra because there in a so…in like in a quart there is 4 cups so 16 cups because 4 times 4 is 16 cups (Case B Transcript, p.9)

Students often came to the board to represent their thinking while explaining their solutions. Chetan explained, Ok, so how many right angles would be equal to a full circle. If you put four right angles in a circle, can I show it on the board? (Case B Transcripts, Event 1, p.1). Ms. Littleton agreed and he began, “Okay. one, two, three, four. Yeah, I think it's four because there's four right angles in there. There's one, two, three and four.” Later in the same lesson Edna came to the board to explain her thinking, as well. She said, “I drew a circle and then I drew two 90 degree angles and I knew $\frac{1}{4}$ is 90 degrees and I needed $\frac{3}{4}$ so… I added 90 + 90 + 90 and that is $\frac{3}{4}$ so then I got 270 degrees (p. 3).

The student questionnaire survey data also suggested that students did see themselves as contributing to their own learning and the learning of others. The questionnaire and student survey data suggested that students perceived having the mathematical authority to contributing to their own leaning and the learning of others. Most students reported viewing themselves as helpful to their peers when experiencing trouble in solving problems, see Table 16. Many also viewed their classmates as being able to support their success in solving problems. Most also liked learning from others. Most students also liked to share their thinking aloud. However, some of the students preferred learning on their own which further demonstrated the diverse perceptions that existed in the data.

Moreover, during the individual interviews students spoke about the ways they looked to thers for help. Chetan said, “I kind of have to say it to someone, or to a friend
next to me, so then I can understand it more and they can help me a little bit if I got it wrong” (Individual Interview, p. 1).

Students also discussed ways that other students helped one another to build understanding together. Chetan said,

Sometimes I get it wrong and the other person gets it right, and through talking I get it a lot more, and then I do what they are trying to say on piece of paper, and then I come out with an answer, and I got it right, they help me understand the problem more. (Individual Student Interview, p.1)

Arhur also commented on this during the individual interview. He said that listening to his peers helped him to, “think differently” because when he had done it differently he could learn from how his peers solved their problem (Arthur, p. 2)

Furthermore, students Chetan and Evelyn also described helping their peers with problem-solving. Chetan said,

I show them how I did my work and they start to understand it a lot more and if they don’t understand it I still talk to them and see how they are doing and end up knowing what I am saying. (Individual Student Interview, p.1)

Evelyn explained that talking about strategies with a lot of people helped you to be aware of different ideas (Individual Student Interview, p. 2). Likewise, Aurther said,

Because they might do it in a different way or the same ways, and they did it in a different way, they might want to know my way (Individual Student Interview, p. 1).

Teachers were aware of this struggle for students and provided some insights about it during their joint interview. Mrs. Washington acknowledged that students needed to feel safe before sharing their thinking with the class. She explained helping these students to become more confident. She did this by by first letting other more confident students share and then when she saw “it in their face” that they were ready, she called on them to explain their thinking (Teacher Focus Interview, p. 7). Ms. Littleton explained that she supports students when they make a mistake so that they feel this is not
detrimental. She explained that she always let’s students know, “It's okay to share that you started out the wrong way or maybe you got stuck and you didn’t find the right answer. but to share their thinking so that we can help them get to the right spot.”

The partnerships that existed among teachers and their students were vital to the successful implementation of discourse. Insight about the perspective participants held about the practices they used while working together to solve mathematical problems using discourse is vital information to understand the benefits of these practices.

**Students and Teachers Inform Practice**

In the qualitative research method, there are multiple realities or multiple interpretations, not just one conception of reality or one interpretation (Guba and Lincoln, 1987). Gathering a variety of perspectives from teachers and students provided an opportunity to look inside the participants’ reality and communicate the subjects’ point of view (Bogdan & Biklen, 2007).

Rossman & Rallis describe the benefits of an in-depth interview as enabling the “participants perspective on the phenomena of interest” to “unfold as the participant views it and not as the researcher views it” (Rossman & Rallis, 2003, p. 181). Semi structured interviews allow for more casual conversation allowing participants to share their insights (Merriam, 1998). The purpose of the interviews in this study was to gather information about each teacher’s understanding and use of reform-based practices and their use of strategic discourse. The following section describes the insights that emerged among teachers and students.

As evident from the interviews, teachers agreed that talking with others about math was beneficial and led to understanding (Teacher Focus Interview, pp. 7-8).
Additionally, students also explained the value in having to explain their thinking and listening to others do the same. Unfortunately, teachers could not discern which elements within the strategic discourse were important or lead to understanding. This was a direct result of their lack of participation in professional development opportunities to educate them about effective, research based discourse practices they could draw from to encourage talk amongst their students. Without these experiences, the teachers were unable to answer the interview questions that required them to reflect on the practices they used and the impact on understanding. Instead for the purpose of the interviews, I made the decision to focus on the purpose and use of discourse as I interpreted the implementation of the practices and not based on the perceptions of teachers. As such, when asked more direct questions about specific practices during the interviews both teachers were able to offer some important information.

As described earlier in Chapter 5 of this document, Ms. Littleton had opportunities to participate in an individual and group focus interview. She discussed the importance of engaging in joint reasoning and the way these experiences helped students to develop their thinking skills. She also shared how students were able to take ownership of their learning when they figured out solutions to problems. Again, Ms. Littleton also described the importance of using mathematical talk. She described exploring things together and developing solutions gave students more ownership of the concepts and ideas. Moreover, she explained the benefits of students discussing problems together. She said that she believed that their understanding was deeper when they discussed the problems.
Likewise, three of Ms. Littleton’s students provided their perceptions during individual and group focus interviews. All students had the opportunity to participate in the Student Survey/Questionnaire. According to this data, many students said that they liked to share their thinking aloud. Most students also agreed that sharing their thinking helped them to figure out if they were solving the problem correctly. Six students, preferred learning on their own. None of the three students interviewed, preferred learning on their own. During an individual interview, one of the students described sharing thinking in math was helpful. All but one student agreed that explaining their thinking was helpful to others. Eleven students reported not liking math, whereas, ten responded that they did like math. Additional results are summarized in Chapter 5 in this document, see Table 16.

Mrs. Washington had opportunities to participate in an individual and group focus interview. She spoke about the importance of justification and focusing on the process and not just the answer. She also described her practice for carefully selecting students to share, noting that her struggling students benefitted from the layering of conversations, reviewing concepts and building on skills. She viewed the ‘turn and talk’ strategy as a very useful tool for this purpose. Also, Mrs. Washington reflected about using purposeful questioning and successfully enacting real time decision making to do so. Moreover, according to Mrs. Washington, questions provided her with maintaining the direction of the lesson.

Likewise, three of Ms. Littleton’s students provided their perceptions during individual and group focus interviews. All students had the opportunity to participate in the Student Survey/Questionnaire. When asked about why Mrs. Washington asked them
to explain their thinking two of the students interviewed both believed that it was important because it allowed Mrs. Washington to understand their thinking. Many of the remaining students believed it was helpful, according to the Student Questionnaire Survey. Students were also asked about talking with others in math. The students interviewed found this to be interesting and effective because they learned by listening to others. This was also indicated on the results of the Students Questionnaire. Most also agreed that it was helpful to them when asked to explain their thinking. A few students did not find it helpful, however. Most identified sharing their thinking as helpful to others. Only half of the students liked sharing their thinking aloud, including the three individuals interviewed. Six of the students reported being uncomfortable when having to share their thinking and ten saw it as challenging. Surprisingly, all seventeen students reported not liking math. Additional results are summarized in Chapte 4 in this document, see Table 13.

These insights demonstrate the importance of the impact that instructional practices have on the people who use them. How teachers and students perceive specific practices can also have an impact on their success. Some of the results were surprising in that it was not obvious through observation that students experiences dissatisfaction or struggles with some of the methods. This raises the question of whether or not students engage because they are asked to and not because they believe it to be effective or useful to them. If this survey had not been conducted, this insight may have been lost.

Investigating perceptions of students and teachers regarding the discourse practices, currently in use, are highly beneficial to improving current practice. However, this is only one piece of the puzzle. More in depth research focused on discovering
exactly what components cause discomfort and/or more challenging for some students
would be a worthwhile endeavor. Additionally, more research on identifying which
practices assist with developing understanding of mathematica is absolutely crucial once
teachers have developed their repertoire of strategic discourse practices so that they are
more informed about the practices in order to make a valid judgement.

Limitations relating to the research collected in this study are described in the
following section.

**Limitations**

Although case studies are useful in studying a process or innovation, as was the
case here, there are still limitations in this method (Merriam, 1998). One limitation is the
small sample size making it difficult to have representativeness or to offer results in
generalizable terms. Moreover, generalizations are difficult to justify through the use of
qualitative data, but the frequency tables presented in this study measured the
implementation of various types discourse elements used and also illuminated the
patterns existing across the cases (Guba & Lincoln, 1987). Even though the small number
of participants was a limitation, studying two groups enables a more intimate relationship
with participants allowing for a truer picture of the communities existing in each
classroom.

Another limitation of the study concerned the fact that teachers were not able to
identify the strategic discourse practices they used to guide their conversations with
students. This made it difficult to engage in conversations to identify those that were
important to leaning mathematics. Additionally, there were no interview questions that
listed the many strategic discourse practices they used. Only a few of the most used
practices were discussed with them during the focus interview. This limitation did not allow for an effective examination of the ways students and teachers interpreted the success of discourse practices. However, more data was available for students regarding the important discourse practices, but this was collected via the student survey/questionnaire.

A third limitation of this study is my familiarity with the participating teachers (Luft, 2007). I am an administrator in the district where the teachers worked and have collaborated with the teachers on district initiatives including curriculum mapping and resource selection committees. Though not a direct supervisor of the teachers in this study, my role may have impacted their performance levels. The results were interpreted with this in mind. This increased familiarity resulted in the use of more rigorous methodology to insure reliability. Furthermore, as a former math teacher and mathematics curriculum director, I was committed to being reflexive in my interpretations of observations (Rossman & Rallis, 2003). Throughout the study, I used the Exploratory Talk ground rules model adapted for the analysis to guide the data collection process. I was careful not to let my own biases as an administrator or curriculum leader cloud the authenticity of the data.

Additionally, the inclusion of rigorous methodology also strengthened the results. The methodology included various types of data including curriculum documents, individual and focus interviews, multiple videotaped observations, as well as, a student survey questionnaire. These sources of data provided multiple viewpoints from which to interpret the results. Attaining individual and group perceptions helped to reduce bias because the participants reported their own thinking, revealing their own truths. The
information gathered from these individual and focus interviews enhances the rich
description of the students, teachers, setting and practice making for a more
comprehensive description of the cases (Rossman & Rallis, 2003). By including different
sources of data, it was possible to achieve a greater understanding of the discourse used
among the teachers and students. For example, conducting multiple observations
provided a greater sense of the typical interactions among participants in both classroom.
Gathering input from both teachers and students also confirmed the use of discourse.
Furthermore, because I was the sole researcher on this project, the steps taken to
triangulate the data assisted in addressing the limitations.

**Implications**

This research is consistent with the work of other researchers who are developing
and studying how to support teachers in their efforts to engage students with each other’s
mathematical ideas (Hufferd-Ackles et al., 2004; Gillies, 2003; Mercer et al., 1999;
Sherin, 2002). However, gaps exist in the literature describing the implementation of
discourse used by teachers and students without the assistance of researchers (Cazden &
Beck, 2003; Cengiz et al., 2011; Franke, et al., 2015). This study assists in filling this
gap, as well as, creates the groundwork for further study. Using Mercer’s fourteen ground
rules suggested in the Exploratory Talk Model provide a structure for examining the
discourse practices that teachers were using in their classroom with students (1999). The
elements provided the categories used to code and measure the frequency to which
discourse practices were being used by teachers and students to engage in mathematical
discourse. These elements now serve as a reference for educators as they implement
mathematical discourse with intent. Moreover, the findings show that both teachers use particular elements in similar ways. However, there were elements that were used by one teacher or group of student and not the other. Investigating why this occurred is a potential area for future study.

As hypothesized, teachers were successfully using discourse to assist students to develop knowledge about key mathematics concepts by engaging in conversation with their teachers and peers. Interactions were productive and engaging. Teachers were highly successful at facilitated these interactions. However, teachers did so without planning their implementation prior to the discourse, nor did they reflect on the success of them afterward. Instead, teachers utilized the discourse elements spontaneously using real time decision making (Choppin, 2007). Consequently, teachers and their students utilized some elements often and others rarely or not at all. Although the teachers had a mathematical plan in mind they operated without a plan for guiding their discussions. Therefore, discussions were not as productive as they had the potential to be. These findings reveal that teachers who are unaware of the productive strategic discourse strategies and how they could be used to engage students thinking can limit students’ mathematical learning potential. Considering that the positive results regarding the discourse observed in the study, moving toward planning discourse with intension is recommended.

Another implication is that the two teachers in this study had little knowledge of the types of discourse available to them. It is important for educators to become familiar with the discourse elements and the purpose for using certain strategies. The elements
within each of the three themes, Encouraging Student Participation, Developing Mathematical Knowledge, and Strengthening Critical Thinking all have their part in mathematical discourse. However, knowing when and why each should be utilized is the work that needs to be completed. (Cengiz et al., 2011, Hiebert & Grouws, 2007; O’Connor, Michaels, & Chapin, 2015). This research builds on the literature in that it examines talk moves that go beyond simply engaging students in discussion, but demonstrate ways teachers can sustaining engagement and develop critical thinking (Cengiz et al., 2011; Franke et al., 2015). Teachers’ knowledge about mathematics and learning mathematics greatly influence the way they teach mathematics (Cengiz et al., 2011). This implies a critical need for education, especially around the types of discourse strategies that can be used with students.

I continue to be committed to learning how students and teachers can structure the discourse to consistently build student understanding of mathematics along with strengthening critical thinking. Future research should include an examination of the impact discourse elements have on understanding mathematics among students. Sample questions that would be included in this examination are: What are the discourse elements support developing mathematical thinking? Which discourse elements support critical thinking? Which discourse elements best support the learning of skills and concepts? Which discourse elements support problem solving? Which discourse elements lead to understanding?
Conclusions

The findings that emerged from this cross-case study stress the complexity involved in implementing mathematical discourse with students. Additionally, when teachers provided an environment that supported students to socially construct knowledge together, students engaged in high quality discussions that would not have been possible without the culture that existed in both classrooms. Assuming a less dominant teacher role fostered a partnership and elevated the role of students. This elevated role allowed students to build their mathematical power and assume a more equalized sharing of mathematical authority with their teachers. Students demonstrated their knowledge, developed greater understanding alongside others students and teachers in the community of practice.

Although teachers used many elements effectively, the majority of the discourse used by both teachers focused on Developing Mathematical Knowledge among their students. Teachers were spending too much time on the first two themes in lieu of the third. This was problematic because students experienced few opportunities to build critical thinking. It is essential that teachers understand the purpose of the ground rules and why the ways each could be used to enhance mathematical discussions. Researchers advise building a repertoire of moves that could be implemented in-the-moment to support students (Franke et al., 2015; Mercer, 2007). However, based on the findings of this study, I recommend teachers becoming more knowledgeable about moves, practicing them with students and then reflecting on the success or challenges as a means for building this repertoire. This work must occur before teachers can be expected to have a
repertoire to draw from. The ground rules developed as a result of this study stand as a protocol for teachers to refer to while implementing their own strategic discourse.

Finally, the findings from this study also reveal the value the personal perspectives of all participants regarding the discourse practices used to learn mathematics made in this study. Teachers and students shared the benefits of using talk to communicate ideas and to share thinking with others. Students shared preferences, along with challenges they encountered in the process. Future studies focused on investigating the perceptions of students and teachers regarding the effectiveness of individual talk moves would be highly beneficial. This type of investigation could likely include why teachers and students utilized certain elements less often or not at all.
Dear Student,

I want to tell you about a research study we are doing during the remainder of the school year. A research study is a way to learn more about something. I would like to find out more about how students use language to communicate their thinking in while solving math problems together in school. I also want to understand which of the strategies used are more useful to you and your teacher. You are being asked to join the study because you have demonstrated your ability to solve problems and your willingness to talk about your thinking with your teachers and peers in math class. Knowing more about how students use and think about discussion strategies to communicate ideas is important because it can help other teachers and students learn the strategies you recommend and help them learn math with greater understanding.

If you agree to join this study, you will be asked to complete problem-solving tasks and then discuss the strategies that you have used and why. You will also be asked to participate in four videotaped math lessons to capture the many ways you discuss your ideas with others. Additionally, you will be asked to complete a questionnaire for the purpose of sharing your feelings and opinions about how language used during problem solving helps you to understand mathematics with more understanding. You will also participate in a group interview with your classmates. This group interview, done in approximately two 30-minute sessions, will be conducted to gather feedback from all students about the strengths and weaknesses of discussion strategies. Some of you will be asked to participate in an individual interview about the same topics. This individual interview will last approximately 15-20 minutes.

Your parent or guardian knows about this study and we are asking if you would like to be part of it.

You do not have to join this study. It is up to you. You can say okay now and change your mind later. All you have to do is tell us you want to stop. No one will be mad at you if you don’t want to be in the study or if you join the study and change your mind later and ask to stop.

Before you say yes or no to being in this study, we will answer any questions you have. If you join the study, you can ask questions at any time. Just tell your parent or the researcher that you have a question.

If you want to be in this study, please write your name below.

Participant Name______________________________
Date__________________

Name of Person obtaining consent________________________
Date__________________
Dear Parent/Caregiver:

My name is Mary Coakley, and I am a student at the University of Massachusetts Amherst. I am currently employed in the Grafton Public Schools as an Assistant Principal at Millbury Street Elementary School.

I am working on a research study investigating the different ways that students and teachers discuss strategies and concepts in mathematics. Specifically, I am trying to determine which strategies are most effective for forwarding understanding. The information derived from this project will help strengthen teaching and learning for many teachers and students in the future.

Your child’s teacher has agreed to participate in my research study. As a participant in the study, _______________ has agreed to allow me to videotape her while she is teaching mathematics. Your child may be in the video footage that I collect. The video will be used to analyze teaching and learning practices and some of the footage may be used in my dissertation presentation. At no time will the students’ names, teacher’s name or school’s name be cited. Complete confidentiality will be protected at all times.

In addition, I will be conducting a survey and group interview with students. Students will be asked to provide input about the strategies they use and why. The survey will be completed individually and the interview will be conducted as a class.

Please indicate if you do or do not give permission to participate in the study. Also, sign and date the form, and return to ___________ as soon as possible. If you have any questions, please do not hesitate to contact me.

Sincerely,

Mary Coakley
(508) 839-0757
coakleym@grafton.k12.ma.us
APPENDIX C
RESEARCH PROJECT CONSENT FORM

Please check one in each section and return to your child’s teacher.

1. Videotaping
   _____ My child, _________________, has permission to be videotaped for this research project.
   ____ My child, _________________, does not have permission to be videotaped.

2. Survey/Questionnaire
   _____ My child, _________________, has permission to complete the survey/questionnaire as part of the research project.
   ____ My child, _________________, does not have permission to complete the survey/questionnaire.

3. Focus Interview
   _____ My child, ________________, has permission to participate in the group interview.
   ____ My child, ________________, does not have permission to be included in the group interview.

4. Individual Interview
   _____ My child, ________________, has permission to participate in the individual interview.
   ____ My child, ________________, does not have permission to be included in the individual interview.

Parent/Caregiver Signature: ________________________________________________

Date: __________________________
APPENDIX D
PARTICIPANT INFORMED CONSENT

I volunteer to participate in this qualitative study and understand that:

1. I will participate in one focus group interview and an semi-structured interview conducted by Mary Coakley.
2. I understand that the questions will address my view on the issues related to mathematics teaching and learning in my own classroom. I understand that the purpose of this research is to identify the benefits and challenges of students and teachers as they use discourse while engaging in problem solving.
3. The focus group interview will be videotaped to facilitate the analysis of the data.
4. The semi-structured interview will be audiotaped.
5. I will be observed and videotaped while teaching mathematics during the spring of 2015.
6. I understand that I will provide Mary Coakley with lesson plans for each lesson videotaped at least 24 hours before the videotaped observation.
7. I understand that I will reflect on my videotaped lessons and submit these reflections to Mary Coakley.
8. My name will not be used, nor will I be identified personally, in any way or at any time.
9. I understand that it will be necessary to identify participants in the study by general position and school district (e.g., a third-grade teacher from a suburban school district said…).
10. I may withdraw from part or all of this study at any time without consequence.
11. I have the right to review material prior to the oral exam or other publication.
12. I understand that the results from this study may be included in Mary Coakley’s doctoral dissertation and may also be included in manuscripts submitted to professional journals for publication.
13. I am free to participate or not to participate without prejudice.

Participant’s Signature ______________________________________

Date   _________________________________

Researcher’s Signature ______________________________________

Date   _________________________________

664
APPENDIX E
FOCUS GROUP INTERVIEW STUDENT PROTOCOL

Date:
Time:
Participants:

1. Looking at the video clip from a classroom session, how did ________ solve the problem? Explain.
2. Were you able to understand his thinking? How? Explain.
3. Why do you think he solved it this way? Explain.
4. Mrs. ____________ asked a question, what was it and why did she ask it?
5. Did any of the students ask questions during this video clip? Why or Why not?
6. Do you ever ask questions? When and why? Are these questions directed at your teacher or other students?
7. What questions could we have asked the student who shared the solution?
8. How might asking questions lead you to understanding more about someone else’s thinking?
9. Do you find listening to others explain their thinking helpful?
10. How do you feel when a teacher asks you to explain your mathematical thinking?
APPENDIX F
FOCUS GROUP INTERVIEW TEACHER PROTOCOL

Date: 
Time: 
Participants:

1. Looking at the video clip from a classroom session, how did _______ solve the problem? Explain.
2. Were you able to understand his thinking? Explain
3. Why do you think he solved it this way? Explain.
4. A question was asked by the teacher, what was it and why was it posed?
5. Would you have asked the same question again in the same situation? Why or why not? Explain.
6. Do you think that additional questions could have been asked? Why?
7. Could you give an example of other questions that may have been applicable in this situation? Explain.
8. How did the teacher questions lead to thinking and/or understanding? Did any questions prevent the thinking? Explain.
9. Did your questions invite discussion? How?
10. If planning for this lesson again, would you lead the discussion in the same way? What are you considering now that you did not consider before?
11. Were all students involved in the discussion? Why? Why not?
12. Did any of the students ask questions during this video clip? Why or Why not?
13. Were students invited to ask questions? When and why? Do you invite your own students to ask questions? When and why?
14. How might encouraging students to ask questions lead them to understanding more about someone else’s thinking? Mathematics?
15. How might encouraging students to ask questions lead them to understanding more about their own thinking?
16. Do you find listening to students explain their thinking helpful? Explain.
17. How do you think your students do when they attempt to explain their mathematical thinking?
18. Has their understanding been impacted?
19. Do the students find the mathematical talk useful? How do you know?
APPENDIX G
INDIVIDUAL TEACHER INTERVIEW PROTOCOL

Date:
Time:
Participant:

1. What has prepared you to teach mathematics?
2. How long have you used reform-based methodology to teach mathematics? Tell how this came about.
3. What are some of the challenges you face while teaching mathematics? Are they specific to mathematics? Why or why not?
4. Your students talk with you, other teachers and peers in mathematics, why do you feel that is important?
5. Are there any issues that arise with using math talk?
6. How is the talk structured in your classroom? Have students been taught how to engage in the talk? If yes, how so?
7. Is this something that you plan or does it emerge naturally?
8. Are there times when you think planning for talk could be beneficial? Why or why not? Please explain.
9. Have you notices whether or not certain strategies or techniques that you use elicit more thinking or understanding? Explain.
10. Do some students thrive in this environment more than others? Explain.
11. Are there some students who engage in talk more successfully than others? Why?
12. What would help you utilize math talk in the classroom more productive?
APPENDIX H
INDIVIDUAL STUDENT INTERVIEW PROTOCOL

Date:
Time:
Participant:

1. Looking at the video clip from a classroom session, how did _______ solve the problem? Explain.
2. Were you able to understand his thinking? How? Explain.
3. Why do you think he solved it this way? Explain.
4. Mrs. ___________ asked a question, what was it and why did she ask it?
5. Did any of the students ask questions during this video clip? Why or Why not?
6. Do you ever ask questions? When and why? Are these questions directed at your teacher or other students?
7. What questions could we have asked the student who shared the solution?
8. How might asking questions lead you to understanding more about someone else’s thinking?
9. Do you find listening to others explain their thinking helpful?

How do you feel when a teacher asks you to explain your mathematical thinking?
## APPENDIX I
STUDENT SURVEY/QUESTIONNAIRE PROTOCOL

Date:  
Time:  
Student Name:  

<table>
<thead>
<tr>
<th>Statement</th>
<th>Yes</th>
<th>No</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Talking helps me to understand math more clearly.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2. I am able to understand when another student explains their answer.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3. I understand math problems more when I talk about them with other students.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4. I ask my teacher a lot of questions.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5. I understand math better when I talk about it with my teacher.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6. It is helpful when I am asked to explain my thinking.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>7. Listening to how other students explain how they solved a math problem is helpful.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>8. I like to share my thinking aloud.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>9. I am uncomfortable when I have to share my thinking to the class.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>10. I prefer to solve problems on my own and not with others.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>11. Sharing my thinking helps me to figure out if I am on the right track.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>12. I ask questions so that I can figure out what other people are thinking.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>13. I ask questions to help others find a solution.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>14. Comparing my answers with other students helps to see if my thinking is correct.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>15. I prefer writing my answers down, not talking about them.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>16. I have trouble understanding how other students solve the problem.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>17. I appreciate learning from others.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>18. Solving problems are easier when I work with other students.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>19. I have trouble explaining my thinking in math.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>20. I do not like math.</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
### APPENDIX J

#### LESSON OBSERVATION PROTOCOL: PHASE 1

**CLASSROOM TALK** - Lampert (1990)

<table>
<thead>
<tr>
<th>Element</th>
<th>Conversing</th>
<th>Analyzing</th>
<th>Viability</th>
<th>Conjecturing</th>
<th>Generalizing</th>
</tr>
</thead>
<tbody>
<tr>
<td>Description</td>
<td>Problems are discussed and ideas exchanged</td>
<td>Offering differing ideas are encouraged</td>
<td>Solutions are justified and discussed to determine viability</td>
<td>Ideas extended and connected to other ideas</td>
<td>Knowledge is made explicit leading to generalizations</td>
</tr>
</tbody>
</table>

| Code | C | A | VJ | C+ | G |

**DIALOGIC TALK** - Gillies (2014)

<table>
<thead>
<tr>
<th>Element</th>
<th>Engaging</th>
<th>Questioning</th>
<th>Assisting</th>
<th>Feedback</th>
<th>Collaboration/Training</th>
</tr>
</thead>
<tbody>
<tr>
<td>Description</td>
<td>Share ideas Consider alternate perspectives</td>
<td>Teachers and students ask questions</td>
<td>Teacher helps students to building on ideas</td>
<td>Constructive and/or positive feedback</td>
<td>Students learn how to interact and discuss mathematics</td>
</tr>
</tbody>
</table>

| Code | E | Q | AS | F | CT |


<table>
<thead>
<tr>
<th>Element</th>
<th>Co-Reasoning</th>
<th>Questioning</th>
<th>Solutions/Justifications</th>
<th>Analyzing</th>
<th>Negotiating</th>
</tr>
</thead>
<tbody>
<tr>
<td>Description</td>
<td>Discussing and thinking about ideas together</td>
<td>Questions are asked to investigate thinking</td>
<td>Solutions to problems are discussed. Explanations are provided</td>
<td>Ideas are evaluated and extended together</td>
<td>Validity of ideas are negotiated</td>
</tr>
</tbody>
</table>

| Code | C | Q | SJ | A | N |
APPENDIX K
TEACHER DESIGNED TASKS

How much ribbon?
- We're going to investigate how much ribbon you would need to wrap a present.
- Let's start by thinking about the bow. Estimate how much ribbon you think you would need just for the bow.

Making a Bow
- Each group will receive a yard of ribbon.
- Use the ribbon to make a bow around a pencil.
- The size of the bow and the length of the ends are up to you.
- After you make the bow and trim the ends, measure the ribbon you used.
- On the board, record the length of yarn.

Bows
- What do you notice about the lengths of the bows?

Wrapping a box
- Today each group will get a box to wrap.
- Your group's job is to figure out how many inches of ribbon you need to wrap your box, including the bow.

1. Talk about a plan to solve the problem.
2. Estimate the length of ribbon you need.
3. Write about how you got your answer.
4. Measure and cut ribbon to test your answer.

Wrapping Wrap Up
- How accurate was your estimate?
- Did you underestimate or overestimate the actual amount of yarn that you needed?
- What would you do differently if you started this project again with a different sized box?
Array Museum

Project Based Learning

By: The Trend “E” Teacher

Project #1: Array Museum

Your class is making a math museum for parent night using arrays found in our world. You are to make a visual presentation showing as many arrays as possible. You are to show one array that has equivalent arrays that can be added together to equal your first array. For example, $6 \times 14 = (6 \times 10) + (6 \times 4) = 60 + 24 = 84$. You may use magazines, drawings, ads in newspapers and the Internet to represent your arrays. You are to include the numerical equation so that your parents understand your arrays. Your presentation may be on poster board, digital camera, photos, a slide show or PowerPoint presentation.
Let's Plan it out!!!

Directions: The town of has been donated a large piece of land. The land you are given is 40 yard by 80 yard rectangular shape. You have been given the opportunity to choose whether to build a garden or a playground. The town would love to see a variety of shapes in your plans. They also want to see what the measurements of shapes in your plans. They also want to see what the measurement of each shape of the land is by feet and yards.

You will create a brochure that will show all of your plans and measurements. There will need to be descriptions as well as a map of the space that you will create. Don’t forget to name your new piece of land as well!

Instructions (Step by Step):
1. Choose whether you would like to build a garden or a playground.
2. You will use be using a piece of graph paper to sketch your ideas or plans first.
   a. You must have in your sketch:
      i. 4 rectangular shapes
      ii. 3 square shapes
      iii. 2 combined shapes (i.e. forms an L or a T, etc)
      iv. 1 polygon (no area to be solved for this shape)
   b. Label what each of the shapes that you have chosen will be in your park or playground.
      i.e. If you are making a garden:
### APPENDIX L

**OBSERVATION PROTOCOL: PHASE 2**

<table>
<thead>
<tr>
<th>CODE</th>
<th>GROUND RULE</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>(GR1)</strong></td>
<td><strong>GROUND RULE 1 EVERYONE INVITED TO CONTRIBUTE</strong></td>
</tr>
<tr>
<td>(GR1A)</td>
<td>Everyone is encouraged to contribute without being singled out (T&amp;T)</td>
</tr>
<tr>
<td>(GR1B)</td>
<td>Students are chosen strategically by the teacher/student to contribute</td>
</tr>
<tr>
<td><strong>(GR2)</strong></td>
<td><strong>GROUND RULE 2 CONTRIBUTIONS OPINIONS TREATED RESPECTFULLY</strong></td>
</tr>
<tr>
<td>(GR2A)</td>
<td>Praise is given for relevant ideas</td>
</tr>
<tr>
<td>(GR2B)</td>
<td>Positive feedback regarding contributions</td>
</tr>
<tr>
<td><strong>(GR3)</strong></td>
<td><strong>GROUND RULE 3 ATMOSPHERE OF TRUST IS PRESENT</strong></td>
</tr>
<tr>
<td>(GR3A)</td>
<td>Casual interchanges demonstrating equalized relationship</td>
</tr>
<tr>
<td>(GR3B)</td>
<td>Praise and encouragement offered</td>
</tr>
<tr>
<td>(GR3C)</td>
<td>Questions asked openly and freely</td>
</tr>
<tr>
<td>(GR3D)</td>
<td>All are valued and capable members</td>
</tr>
<tr>
<td>(GR3E)</td>
<td>Decision making responsibility is shared</td>
</tr>
<tr>
<td><strong>(GR4)</strong></td>
<td><strong>GROUND RULE 4 KNOWLEDGE IS MADE PUBLIC</strong></td>
</tr>
<tr>
<td>(GR4A)</td>
<td>Participants offer their knowledge about mathematics</td>
</tr>
<tr>
<td>(GR4B)</td>
<td>Strategies are explained in words, pictures and/or numbers</td>
</tr>
<tr>
<td>(GR4C)</td>
<td>Contributions are restated</td>
</tr>
<tr>
<td>(GR4D)</td>
<td>Referrals are made to previous lessons, concepts or contributions</td>
</tr>
<tr>
<td><strong>(GR5)</strong></td>
<td><strong>GROUND RULE 5 REASONING IS VISIBLE IN THE TALK</strong></td>
</tr>
<tr>
<td>(GR5A)</td>
<td>Justifications/Rationales are provided to explain thinking</td>
</tr>
<tr>
<td>(GR5B)</td>
<td>Steps in solutions are explained</td>
</tr>
<tr>
<td><strong>(GR6)</strong></td>
<td><strong>GROUND RULE 6 ENGAGE IN JOINT REASONING</strong></td>
</tr>
<tr>
<td>(GR6A)</td>
<td>Ideas and solutions are discussed with others</td>
</tr>
<tr>
<td>(GR6B)</td>
<td>Questions are posed to the community to direct thinking</td>
</tr>
<tr>
<td>(GR6C)</td>
<td>Questions are posed to encourage exchange of ideas</td>
</tr>
<tr>
<td>(GR6D)</td>
<td>Community members ask questions to try to understanding thinking</td>
</tr>
<tr>
<td>(GR6E)</td>
<td>Thinking is highlighted to spotlight different ways of thinking</td>
</tr>
<tr>
<td>(GR6F)</td>
<td>Tasks are assigned to initiate working together to find solutions</td>
</tr>
<tr>
<td>(GR6G)</td>
<td>Assistance is offered to help work through the process or scaffold learning</td>
</tr>
<tr>
<td><strong>(GR7)</strong></td>
<td><strong>GROUND RULE 7 MULTIPLE SOLUTIONS ARE ENCOURAGED</strong></td>
</tr>
<tr>
<td>(GR7A)</td>
<td>Many ways of solving problems/thinking are encouraged</td>
</tr>
<tr>
<td>(GR7B)</td>
<td>Many ways of solving problems/thinking are shared</td>
</tr>
<tr>
<td><strong>(GR8)</strong></td>
<td><strong>GROUND RULE 8 CONTRIBUTIONS ARE BUILD ON PRIOR PROPOSALS</strong></td>
</tr>
<tr>
<td>(GR8A)</td>
<td>Refers to thinking of others in explanations</td>
</tr>
<tr>
<td>(GR8B)</td>
<td>Builds solutions on the ideas of others</td>
</tr>
<tr>
<td><strong>(GR9)</strong></td>
<td><strong>GROUND RULE 9 IDEAS EXTENDED TOGETHER</strong></td>
</tr>
<tr>
<td>(GR9A)</td>
<td>Turn and talk to someone about an idea or concept</td>
</tr>
<tr>
<td>(GR9B)</td>
<td>Questions used to further thinking</td>
</tr>
<tr>
<td>-------</td>
<td>-----------------------------------</td>
</tr>
<tr>
<td><strong>(GR10)</strong></td>
<td><strong>GROUND RULE 10 LISTENING ACTIVELY TO ENGAGE</strong></td>
</tr>
<tr>
<td>(GR10A)</td>
<td>Listening while others speak</td>
</tr>
<tr>
<td>(GR10B)</td>
<td>Listening to engage with ideas of others</td>
</tr>
<tr>
<td>(GR10C)</td>
<td>Listening to understand other ways of thinking</td>
</tr>
<tr>
<td>(GR10D)</td>
<td>Listening to critique solutions or thinking</td>
</tr>
<tr>
<td><strong>(GR11)</strong></td>
<td><strong>GROUND RULE 10 PARTNERS ENGAGE CRITICALLY WITH EACH OTHER</strong></td>
</tr>
<tr>
<td>(GR11A)</td>
<td>All have opportunities to question each others ideas</td>
</tr>
<tr>
<td>(GR11B)</td>
<td>Disagreeing with one another’s thinking is acceptable</td>
</tr>
<tr>
<td>(GR11C)</td>
<td>Participants advocate for their way of thinking</td>
</tr>
<tr>
<td>(GR11D)</td>
<td>Solutions and ideas are compared</td>
</tr>
<tr>
<td>(GR11E)</td>
<td>Thinking is verified as viable and efficient</td>
</tr>
<tr>
<td><strong>(GR12)</strong></td>
<td><strong>GROUND RULE 12 OPINIONS ARE CONSIDERED BEFORE DECISIONS ARE MADE</strong></td>
</tr>
<tr>
<td>(GR12A)</td>
<td>Input collected before deciding on a strategy or solution</td>
</tr>
<tr>
<td><strong>(GR13)</strong></td>
<td><strong>GROUND RULE 13 IDEAS MAY BE CHALLENGED WITH COUNTER STRATEGY</strong></td>
</tr>
<tr>
<td>(GR13A)</td>
<td>Ideas are challenges with another more viable solution</td>
</tr>
<tr>
<td>(GR13B)</td>
<td>Questions are used to facilitate debate (Do you agree? Do you disagree?)</td>
</tr>
<tr>
<td>(GR13C)</td>
<td>Disagreements used to find most effective and efficient solutions or strategies</td>
</tr>
<tr>
<td><strong>(GR14)</strong></td>
<td><strong>GROUND RULE 14 SEEK AGREEMENT FOR JOINT DECISIONS</strong></td>
</tr>
<tr>
<td>(GR14A)</td>
<td>Solutions are presented by the group</td>
</tr>
<tr>
<td>(GR14B)</td>
<td>Agreement is the goal</td>
</tr>
<tr>
<td>(GR14C)</td>
<td>Compromising is encouraged</td>
</tr>
<tr>
<td>(GR14D)</td>
<td>Thinking is confirmed by the group</td>
</tr>
</tbody>
</table>

Adapted from Exploratory Talk (Mercer, 1999)
## Teacher and Student Use of Ground Rule Elements

<table>
<thead>
<tr>
<th>EVENTS</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>TOTALS</th>
</tr>
</thead>
<tbody>
<tr>
<td>GR1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>A</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>B</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Totals</td>
</tr>
<tr>
<td>GR2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>A</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>B</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Totals</td>
</tr>
<tr>
<td>GR3</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>A</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>B</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>C</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>D</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>E</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Totals</td>
</tr>
<tr>
<td>GR4</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>A</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>B</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>C</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>D</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Totals</td>
</tr>
<tr>
<td>GR5</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>A</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>B</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Totals</td>
</tr>
<tr>
<td>GR6</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>A</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>B</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>C</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>D</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>E</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>F</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>G</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Totals</td>
</tr>
<tr>
<td>GR7</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>A</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>B</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Totals</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>--------</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>GR8</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>A</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>B</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>C</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>D</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Totals</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>GR9</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>A</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>B</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Totals</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>GR10</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>A</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>B</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>C</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>D</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Totals</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>GR11</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>A</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>B</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>C</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>D</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>E</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Totals</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>GR12</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>A</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Totals</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>GR13</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>A</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>B</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>C</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Totals</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>GR14</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>A</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>B</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>C</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>D</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Totals</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Memo: 11/18/16
Subject: Coding the strategic Discourse Teacher A

I began to code based on the discourse practices observed in the third grade classroom. I wanted to consider the types of discussions used by teachers and students. Most importantly, I wanted to be sure that the as I planned to code the discourse that it was focused on my research questions.

Several things need to be considered during this process. Reading through the dialogue, I first began to see patterns in the teachers words. I noted several categories to label the statements and questions that were made.
- Asking permission to share
- Calling on students
- Repeating their words
- Offering questions to insight thinking
- Prompting students

After reviewing my field notes, I decided to look more closely to identify the purposes or intention of the types of discourse used by the teacher. During the first two observations I observed the following:

The teacher is using the following:
- Drawing on prior knowledge
- Turn and Talk
- Restate
- Continuing the conversation
- Invitation
- Probing
- Clarification
- Assess Understanding

Questioning is used by this teacher to gain an understanding about what students know and can do mathematically. It is also used to push students to think critically about ideas and to probe their thinking by pushing them to think deeper.
## Problem Posing

<table>
<thead>
<tr>
<th>Teacher</th>
<th>Event 1</th>
<th>Event 2</th>
<th>Event 3</th>
<th>Event 4</th>
<th>Event 5</th>
<th>Event 6</th>
<th>Event 7</th>
<th>Event 8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Well Designed Problem/Task</td>
<td>W L</td>
<td>W L</td>
<td>W L</td>
<td>W L</td>
<td>W L</td>
<td>W L</td>
<td>W L</td>
<td>W X</td>
</tr>
<tr>
<td>Enriches the Concept/Skill</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>X</td>
</tr>
<tr>
<td>Provides Structure for Discussion</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>X</td>
</tr>
<tr>
<td>Active Learning with Authenticity</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Engages in Learning</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>X</td>
</tr>
<tr>
<td>Connects to Real Life</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>X</td>
</tr>
<tr>
<td>Honors Mathematics As A Discipline</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>X</td>
</tr>
<tr>
<td>Learning Through Interaction</td>
<td>Event 1</td>
<td>Event 2</td>
<td>Event 3</td>
<td>Event 4</td>
<td>Event 5</td>
<td>Event 6</td>
<td>Event 7</td>
<td>Event 8</td>
</tr>
<tr>
<td>Learning is Socially Constructed</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>X</td>
</tr>
<tr>
<td>Contributes to Learning Of Others</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>X</td>
</tr>
<tr>
<td>Strategic Discourse</td>
<td>Event 1</td>
<td>Event 2</td>
<td>Event 3</td>
<td>Event 4</td>
<td>Event 5</td>
<td>Event 6</td>
<td>Event 7</td>
<td>Event 8</td>
</tr>
<tr>
<td>Includes Purposeful Talk</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>X</td>
</tr>
<tr>
<td>Uses Standards For Practice</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>X</td>
</tr>
<tr>
<td>Effectively Communicates Ideas</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>X</td>
</tr>
</tbody>
</table>
### APPENDIX P
COMPARING CASE TO DISCOURSE AMONG TALK MODELS

#### CLASSROOM TALK

<table>
<thead>
<tr>
<th>Element</th>
<th>Lines</th>
<th>Conversing</th>
<th>Analyzing</th>
<th>Viability Justification</th>
<th>Conjecturing</th>
<th>Generalizing</th>
</tr>
</thead>
<tbody>
<tr>
<td>CODE</td>
<td>C</td>
<td>A</td>
<td>VJ</td>
<td>C+</td>
<td>G</td>
<td></td>
</tr>
</tbody>
</table>

Event 1.
Event 2.
Event 3.
Event 4.
Event 5.
Event 6.
Event 7.
Event 8.

**TOTAL**

#### EXPLORATORY TALK

<table>
<thead>
<tr>
<th>Element</th>
<th>Lines</th>
<th>Co-Reasoning</th>
<th>Questioning</th>
<th>Solutions Justifications</th>
<th>Analyzing</th>
<th>Negotiating</th>
</tr>
</thead>
<tbody>
<tr>
<td>CODE</td>
<td>C</td>
<td>Q</td>
<td>SJ</td>
<td>A</td>
<td>N</td>
<td></td>
</tr>
</tbody>
</table>

Event 1.
Event 2.
Event 3.
|--------|--------|--------|--------|--------|--------|--------|--------|-------|

<table>
<thead>
<tr>
<th>Element</th>
<th>Lines</th>
<th>Engaging</th>
<th>Questioning</th>
<th>Assisting</th>
<th>Feedback</th>
<th>Collaboration</th>
</tr>
</thead>
</table>

<table>
<thead>
<tr>
<th>CODE</th>
<th>E</th>
<th>Q</th>
<th>AS</th>
<th>F</th>
<th>CT</th>
</tr>
</thead>
</table>

|--------|--------|--------|--------|--------|--------|--------|--------|-------|

DIALOGIC TALK
APPENDIX Q
LEARNING MATH WITH UNDERSTANDING IN A COMMUNITY OF PRACTICE (COP)
REFERENCES


Cengiz, N., Kline, K., & Grant, T. J. (2011). Extending students’ mathematical thinking


hypotheses in video-supported research. *Video research in the learning sciences*, 239-254.


Gallego & S. Hollingsworth (Eds.), *What counts as literacy: Challenging the school standard* (pp. 118-138). New York: Teachers College Press.


Kirchner, P.S. (2002). How elementary school teachers learn to teach mathematics. ProQuest: Ann Arbor, MI.


Sykes, W. 1990. Validity and reliability in qualitative market research: A review.


