A SEMIOTIC ANALYSIS OF LINGUISTIC AND CONCEPTUAL DEVELOPMENT IN MATHEMATICS FOR ENGLISH LANGUAGE LEARNERS

Hyunsook Shin

University of Massachusetts Amherst

Follow this and additional works at: https://scholarworks.umass.edu/dissertations_2

Part of the Bilingual, Multilingual, and Multicultural Education Commons, Curriculum and Instruction Commons, Elementary Education Commons, Elementary Education and Teaching Commons, Language and Literacy Education Commons, and the Science and Mathematics Education Commons

Recommended Citation

This Open Access Dissertation is brought to you for free and open access by the Dissertations and Theses at ScholarWorks@UMass Amherst. It has been accepted for inclusion in Doctoral Dissertations by an authorized administrator of ScholarWorks@UMass Amherst. For more information, please contact scholarworks@library.umass.edu.
A SEMIOTIC ANALYSIS OF LINGUISTIC AND CONCEPTUAL DEVELOPMENT IN MATHEMATICS FOR ENGLISH LANGUAGE LEARNERS

A Dissertation Presented

by

HYUNSOOK SHIN

Submitted to the Graduate School of the University of Massachusetts Amherst in partial fulfillment of the requirements for the degree of

DOCTOR OF PHILOSOPHY

May 2019

College of Education
A SEMIOTIC ANALYSIS OF LINGUISTIC AND CONCEPTUAL DEVELOPMENT IN MATHEMATICS FOR ENGLISH LANGUAGE LEARNERS

A Dissertation Presented

by

HYUNSOOK SHIN

Approved as to style and content by:

_____________________________________________________
Margaret Gebhard, Chair

_____________________________________________________
Elizabeth McEneaney, Member

_____________________________________________________
Gonen Dori-Hacohen, Member

_____________________________________________________
Darrell Earnest, Member

_____________________________________________________
Jennifer Randall
Associate Dean of Academic Affairs
College of Education
DEDICATION

To my beloved family.
ACKNOWLEDGMENTS

"Life isn’t about waiting for the storm to pass; it’s about learning to dance in the rain" (Vivian Greene) is a quote that has been hanging on the wall of my house throughout my dissertation journey. It would not have been possible to dance in the rain without those who have supported me physically, intellectually, emotionally, and spiritually. I would like to give my special thanks to "Ms. Bright," who welcomed and helped me learn how her students learned language and mathematics simultaneously in her classroom. Ms. Bright, who also has a diverse cultural background, sympathized with my bicultural identity. Our conversations about Korean cuisine such as BBQ, kimchi and kim-bap always lifted my spirit whenever I felt homesick in this foreign country.

I would also like to express my deepest appreciation to each of my committee members. I thank Dr. Gebhard, my “academic mom,” for her guidance in pursuing SFL research. Although our academic views differed subtly at times, she was a wonderful resource and colleague with whom I can have a sophisticated discussion on SFL, teacher education, and school improvement with the most passionate heart. I am deeply grateful to Dr. Hacohen who supported me to broaden my understanding of discourse analysis. Dr. McEneaney, scholar in research methods, not only shared her expertise for my dissertation but also sincerely recognized my financial concerns and granted me numerous opportunities of assistantship. Dr. Earnest with whom I could pleasantly explore Vygotskian perspectives enabled me to be surrounded by warmhearted friends in “grad support group” on a regular basis.

In the grad-support group I could grow intellectually with kind encouragement while exchanging feedback and sharing food with Stephanie and Alicia who helped me stay connected to academia during my dissertation writing. As former math teachers, they
listened and sympathized with the importance of math discourse that I focused on. I am also indebted to SFL fellows, I-An, Kathryn and Shinji, without whom I might have been lonely in completing my dissertation. I-An earned her Ph.D. degree first and set a good example that I could emulate throughout this journey. Kathryn shared articles relevant to my research even when we could only seldomly meet. Kristen, Kathryn’s sister, also helped to save enormous time by transcribing audio/video files for me. Thanks to Shinji, I could have opportunities to extend SFL knowledge and apply it into our Korean and Japanese language classrooms at Smith College. Besides the fellowship, sharing delicious food with Marsha while releasing frustrations and anxiety about our seemingly-unceasing research was a console that could alleviate this worn journey.

I would love to express my deepest gratitude to my spiritual sisters at the Newman Center, especially Dottie and Megan. Dottie, my editor, was willing not only to proofread my paper but also to lift me in her prayers all the time. Megan and her mother Marge also covered me with God’s blessings and was kind enough to always invite my family to their house on big holiday seasons in the U.S. such as Easter and send Christmas cards every year.

Lastly, I give my everlasting thanks to my parents and beloved family. My mom’s healthy and wholesome Korean miso shipped from Korea not so much nourished all my family but shielded us from homesickness. Despite much adapting and adjusting, Miriam and Sarah—my two daughters—worked hard to make their own ways, which made me survive all emotional plight and concentrate on my work. With all my respect and love, I dedicate this work to Sangchil, my husband, who was the most supportive throughout the trajectory of my doctoral path. He untiringly supported me in deepening my understanding of Halliday’s and Vygotsky’s work and eventually passing the gateway to a world of scholarship and research.
In hindsight, my journey was not merely waiting for the storm but daringly walking right into the eye of the storm. This research is the seed of my academic endeavor and wisdom of how to bend flexibly but remain firmly entrenched in the ground. I conclude the paper by acknowledging the completion of my work as a fortune approved by God and a solid beginning for the rest of my scholarly journey throughout life.
ABSTRACT

A SEMIOTIC ANALYSIS OF LINGUISTIC AND CONCEPTUAL DEVELOPMENT IN MATHEMATICS FOR ENGLISH LANGUAGE LEARNERS

MAY 2019

HYUNSOOK SHIN, B.A., PUSAN NATIONAL UNIVERSITY
M.A., PUSAN NATIONAL UNIVERSITY
M.A., UNIVERSITY OF SYDNEY
Ph.D., UNIVERSITY OF MASSACHUSETTS AMHERST

Directed by: Professor Margaret Gebhard

This study explores how an elementary mathematics teacher supported English language leaners’ (ELLs’) academic language and concept development in the context of current high-stakes school reform. The conceptual frameworks informing this study include Halliday’s theory of systemic functional linguistics (e.g., Halliday & Matthiessen, 2014) and Vygotsky’s sociocultural theory of concept development (Vygotsky, 1986). Specifically, this study analyzes the interplay between academic and everyday language and how this interplay can facilitate the development of what Vygotsky referred to as “real” or complete concepts as students shift from “spontaneous” to more “scientific” understanding of phenomenon (Vygotsky, 1986, p.173). This year-long qualitative study combines case study methods with discourse analysis using SFL tools. Participants included an English-as-Second-Language teacher and her 14 ELL students. At the time of the study these students had varying degrees of English proficiency and were enrolled in a mix-aged classroom in an urban elementary school in Massachusetts. In SFL terms, the findings from this investigation indicate that the teacher used language in a structured way to interweave everyday language connected with familiar or “Given” information with academic language regarding “New” information. In addition, the data suggest that student talk, over time,
mirrored the way the teacher used language to “bind” everyday language representing spontaneous concepts with academic language representing mathematical concepts. Moreover, mathematics classroom discourse in this context often related multiple semiotic resources as “Token” to their meanings as “Value.” Drawing Halliday and Matthiessen’ (2014) concept of “decoding” and “encoding” activities associated with Token-Value relationships, students were guided in verbalizing mathematical reasoning that promoted both spontaneous and scientific concept development. In addition, the participant teacher made linguistic choices differently depending on the multisemiotic resources she used during instruction. The findings of this study suggest that teachers’ use of language plays a pivotal role in developing students’ language and mathematical conceptual knowledge simultaneously. Drawing teachers’ attention to the role discourse plays in classroom interactions and students’ disciplinary literacy development is especially consequential given the discourses of high-stakes testing, standardization, and accountability systems in K-12 schools in the United States.
## TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>ACKNOWLEDGMENTS</th>
<th>vi</th>
</tr>
</thead>
<tbody>
<tr>
<td>ABSTRACT</td>
<td>viii</td>
</tr>
<tr>
<td>LIST OF TABLES</td>
<td>xiv</td>
</tr>
<tr>
<td>LIST OF FIGURES</td>
<td>xvii</td>
</tr>
</tbody>
</table>

### CHAPTER

1. INTRODUCTION .................................................................................................................................................... 1

1.1 Statement of the Problem ......................................................................................................................... 1

1.2 Significance of the Study .......................................................................................................................... 4

1.3 Purpose of the Study and Research Questions ....................................................................................... 7

1.4 Organization of the Dissertation ............................................................................................................. 8

2. LITERATURE REVIEW .................................................................................................................................... 11

2.1 Introduction.................................................................................................................................................. 11

2.2 Systemic Functional Linguistics .............................................................................................................. 13

2.3 Genre-Based Pedagogy ............................................................................................................................... 16

2.4 Classroom Discourse ................................................................................................................................. 18

2.5 Methodology ............................................................................................................................................... 20

2.6 Research on Academic Language in Elementary School ........................................................................ 24

   2.6.1 Focus on Spoken Text .......................................................................................................................... 24

   2.6.2 Focus on Both Spoken and Written Text ............................................................................................ 27

   2.6.3 Focus on Written Text ........................................................................................................................ 30
4.3 Research Questions ....................................................................................................................... 86
4.4 The Unit of Analysis ....................................................................................................................... 87
4.5 The Researcher's Role .................................................................................................................. 88
4.6 Data Collection Procedures ........................................................................................................ 94
   4.6.1 Gaining Access and Socializing in the Field ................................................................. 94
   4.6.2 Selecting the Sample for the Study ............................................................................. 97
   4.6.3 Generating Data ........................................................................................................... 98
   4.6.4 Constructing a Data Record ...................................................................................... 103
   4.6.5 Summary of Data Collection ................................................................................ 105
   4.6.6 Data Analysis Procedure ....................................................................................... 106
4.7 Limitations of the Study Design ............................................................................................. 116

5. FINDINGS ........................................................................................................................................................... 119

5.1 Introduction ................................................................................................................................... 119
5.2 Teacher's Linguistic Scaffolding for Student Math Concept Development ........ 120
   5.2.1 Inviting Students to Engaging in Coding Activity ............................................... 123
   5.2.2 Facilitating Student Participation ........................................................................ 128
   5.2.3 Promoting Students' Reasoning .......................................................................... 135
5.3 Students’ Co-constructing Mathematical Concepts ....................................................... 140
   5.3.1 Establishing Math Concepts by Participating in Mathematics Discourse ................. 143
   5.3.2 Processing Math Concepts by Verbalizing Actions ............................................. 147
   5.3.3 Binding Everyday Concepts and Scientific Concepts .......................................... 153
5.4 Mathematics Classroom Discourse as Coding Activity .............................................. 158
5.5 Linguistic Choices Varying by Semiotic Resources .................................................... 169
   5.5.1 Language Use with Manipulatives .................................................................... 173
   5.5.2 Language Use with Visual Display ...................................................................... 176
   5.5.3 Language Use with Symbolism .......................................................................... 182
5.5.4 Language Use with Language as Main Semiotic Resource ............... 184

5.6 Summary ......................................................................................................................... 188

6. DISCUSSIONS ..................................................................................................................... 192

6.1 Introduction ....................................................................................................................... 192

6.2 Teacher's Linguistic Scaffolding .................................................................................... 192

6.3 Student Responses in Co-constructing Mathematical Concepts ......................... 198

6.4 Linguistic Choices Varying by Multiple Semiotic Resources ............................... 203

6.5 Summary and Limitations of the Study ................................................................. 207

7. CONCLUSIONS AND IMPLICATIONS .............................................................................. 212

APPENDICES

A. EMAIL 01 .......................................................................................................................... 217

B. CLASSROOM CONFIGURATION .................................................................................. 218

C. SAMPLE OF FIELD NOTES ......................................................................................... 219

D. PHOTOS .......................................................................................................................... 222

E. EMAIL 02 .......................................................................................................................... 226

F. INTERVIEW PROTOCOL ............................................................................................... 227

G. SAMPLE OF SELECTED SEGMENT .............................................................................. 229

H. ANALYSIS SAMPLE ....................................................................................................... 230

BIBLIOGRAPHY ................................................................................................................... 234
<table>
<thead>
<tr>
<th>Table</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Table 2.1 Mode continuum</td>
<td>14</td>
</tr>
<tr>
<td>Table 2.2 Relation between language and situation</td>
<td>15</td>
</tr>
<tr>
<td>Table 2.3 Approach to researching academic language development across content areas</td>
<td>23</td>
</tr>
<tr>
<td>Table 3.1 Language in everyday talk and math talk</td>
<td>55</td>
</tr>
<tr>
<td>Table 3.2 Examples of attributive-relational clauses</td>
<td>58</td>
</tr>
<tr>
<td>Table 3.3 Example of an identifying clause: decoding one</td>
<td>59</td>
</tr>
<tr>
<td>Table 3.4 Example of an identifying clause: encoding one</td>
<td>59</td>
</tr>
<tr>
<td>Table 3.5 Theme/Subject association</td>
<td>64</td>
</tr>
<tr>
<td>Table 3.6 Analysis of a clause complex as one extended clause</td>
<td>65</td>
</tr>
<tr>
<td>Table 3.7 The system of expansion</td>
<td>68</td>
</tr>
<tr>
<td>Table 3.8 MWP &amp; Answer</td>
<td>73</td>
</tr>
<tr>
<td>Table 4.1 Grade and English proficiency levels of student participants</td>
<td>98</td>
</tr>
<tr>
<td>Table 4.2 Transcription conventions</td>
<td>104</td>
</tr>
<tr>
<td>Table 4.3 Data Collection Matrix</td>
<td>106</td>
</tr>
<tr>
<td>Table 4.4 Episode Summary</td>
<td>109</td>
</tr>
<tr>
<td>Table 4.5 Classroom talk to mediate semiotic resources to math concepts</td>
<td>112</td>
</tr>
<tr>
<td>Table 4.6 The number of segments selected by semiotic resources</td>
<td>112</td>
</tr>
<tr>
<td>Table 4.7 The sample analysis of an incomplete clause</td>
<td>113</td>
</tr>
<tr>
<td>Table 4.8 Notations indicating relations between clauses</td>
<td>114</td>
</tr>
<tr>
<td>Table 5.1 Math word problem 01: Subtraction</td>
<td>129</td>
</tr>
</tbody>
</table>
Table 5.2 Math word problem 02: Division ................................................................. 136
Table 5.3 Math word problem 03: Division ................................................................. 163
Table 5.4 Math word problem 04: Subtraction ............................................................ 185
# LIST OF FIGURES

<table>
<thead>
<tr>
<th>Figure</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Figure 3.1 The dynamic nature of the ZPD</td>
<td>47</td>
</tr>
<tr>
<td>Figure 3.2 The interplay between everyday and scientific concepts</td>
<td>48</td>
</tr>
<tr>
<td>Figure 3.3 Situation types of language use</td>
<td>53</td>
</tr>
<tr>
<td>Figure 3.4 Semiotic extension of human consciousness in grammar</td>
<td>57</td>
</tr>
<tr>
<td>Figure 3.5 Language and concept development</td>
<td>60</td>
</tr>
<tr>
<td>Figure 3.6 Different ideational perspectives on symbolic processing</td>
<td>66</td>
</tr>
<tr>
<td>Figure 3.7 Direction of coding in clause complexes linked by a relation of cause</td>
<td>70</td>
</tr>
<tr>
<td>Figure 3.8 Upward and downward development of concepts</td>
<td>71</td>
</tr>
<tr>
<td>Figure 3.9 The interplay between math language and everyday language</td>
<td>76</td>
</tr>
<tr>
<td>Figure 4.1 Point of view on classroom discourse</td>
<td>84</td>
</tr>
<tr>
<td>Figure 4.2 Example of a thematic network</td>
<td>115</td>
</tr>
<tr>
<td>Figure 5.1 Thematic network of teacher talk</td>
<td>122</td>
</tr>
<tr>
<td>Figure 5.2 Thematic network of student response</td>
<td>140</td>
</tr>
<tr>
<td>Figure 5.3 Rosa’s writing: ‘534 – 428’</td>
<td>151</td>
</tr>
<tr>
<td>Figure 5.4 Rosa’s writing: ‘564–446’ &amp; transcript of her writing</td>
<td>157</td>
</tr>
<tr>
<td>Figure 5.5 Mathematics classroom discourse as coding activity</td>
<td>160</td>
</tr>
<tr>
<td>Figure 5.6 Wall chart in Ms. Bright’s classroom</td>
<td>164</td>
</tr>
<tr>
<td>Figure 5.7 Language and concept development through coding activity</td>
<td>168</td>
</tr>
<tr>
<td>Figure 5.8 Ideational perspectives on teacher talk construing multi-semiotic resources</td>
<td>170</td>
</tr>
</tbody>
</table>
Figure 5.9 Concept development through coding activity realized in lexicogrammar.......................................................... 173

Figure 5.10 Examples of linguistic choices varying by multiple semiotic resources.......................................................... 190
CHAPTER 1

INTRODUCTION

1.1 Statement of the Problem

Education reform movement in the U.S. has emphasized standardization and accountability for students’ academic achievement (Darling-Hammond, 2015; Frantz, Bailey, Starr, & Perea, 2014; Gebhard, 2019). Since the No Child Left Behind (NCLB) law was passed in the U. S. in 2001, educational policies have forced K-12 public schools to meet high-stakes standards and heighten the quality of schooling for all students including English language learners¹ (ELLs). In response, most states adopted new academic standards called Common Core State Standards (CCSS) for English language arts, literacy, and mathematics while 26 lead states endeavored to develop Next Generation Science Standards (NGSS). The CCSS and the NGSS, which focus on preparing students for college and career by the end of high school, intensify language use and language learning in content-area classrooms. The language-intensive standards require all students to communicate ideas based on evidence specific to content areas such as English language arts, social studies, science, mathematics (Lee, Quinn, & Valdes, 2013; Kibler, Walqui, & Bunch, 2015; Koelsch, Chu, & Banuelos, 2014). In particular, the requirement challenges ELLs who are developing language and literacy because they should meet the standards and be assessed by tests aligned with the

¹ This study defines ELLs as students “who have sufficient difficulty in speaking, reading, writing, or understanding the English language to be denied the opportunity to learn successfully in classrooms where the language of instruction is English” (National Center for Education Statistics [NCES], n.d.).
standards that highlight the importance of academic language across content subjects (Frantz et al., 2014).

The challenge ELLs face can be amplified under a circumstance where a high proportion of teachers accountable for their academic achievement are not prepared to work with those who predominantly speak a language other than English (Janzen, 2008). To be accountable for ELLs’ achievement gains, teaching practitioners and teacher educators should attend to the rapidly growing population of ELLs across the U.S. to support their academic language development in K-12 schools (e.g., DiCerbo, Anstrom, Baker, & Rivera, 2014; Gebhard, 2010; Janzen, 2008). However, content-area teachers who are required to provide linguistic scaffolding are frustrated with ways to incorporate academic language and curricular knowledge in their content-based classrooms (Lucero, 2012). In addition, studies address that content-area teachers tend to avoid using academic language and ELLs, in turn, lack opportunities to learn academic language (Brizuela & Earnest, 2008; Ernst-Slavit & Mason, 2011). Ernst-Slavit and Mason, for example, state that a teacher working with ELLs used ‘bottom/top number’ instead of ‘numerator/denominator’ in the mathematics classroom. The reduced exposure to academic language can result in ELLs’ failing to meet the language-intensive standards specific to subject matters and varying by grade. As grade levels increase, texts become more complex across content areas with intensified academic language demands (Frantz et al., 2014).

To facilitate content-integrated language learning, teachers should be equipped with not only knowledge of subject matters but also pedagogical content knowledge referring to knowledge for teaching subject matters (Shulman, 1986). In particular, teachers who work with ELLs need to have “disciplinary linguistic knowledge” corresponding to pedagogical content knowledge as well as content knowledge (Turkan, de Oliveira, Lee, & Phelps, 2014, p.1). Turkan and her colleagues
describe that disciplinary linguistic knowledge refers to knowledge required to unpack language specific to each discipline.

To support content-area teachers to have disciplinary linguistic knowledge, it is of great importance to inform how teachers can interact with students in moment-by-moment classroom practices. Little, however, has been known about ways teachers incorporate content and academic language to construct disciplinary concepts with ELLs. Gibbons (2003) demonstrated that two teachers interwove everyday language and academic language to guide English-as-a-second-language (ESL) students in using academic language to participate in science classroom discourse. Gibbons’ study, however, did not show explicitly the interconnectedness between language use and concept development despite connections between sociocultural theories of language and concept development: from sociocultural perspectives, language is a fundamental tool for concept development (Mercer & Littleton, 2007; Vygotsky, 1986) and language is a meaning-making system that works purposefully in a particular context (Halliday, 1978; Halliday & Matthiessen, 2014). In addition, little research has been conducted upon student talk in mathematics classroom discourse since most studies focused on teacher talk (Petkova, 2009). It is when students engage in classroom talk that they are ensured to develop “understanding of science, mathematics, any other subject as a whole” (Mercer & Littleton, 2007, p.115).

Given that ELLs’ participation in classroom discourse plays a central role for their academic success, studies on teacher-student interaction in the mathematics classroom can inform teaching practitioners and teacher educators of ways to narrow a widening achievement gap in mathematics between ELLs and non-ELLs. The discrepancy can result from standardized assessments aligned with the content standards emphasizing the importance of language use and language learning. According to Polat, Zarecky-Hodge, and Schreiber (2016), the National Assessment of Educational
Progress data show that fourth-grade and eighth-grade ELLs in the United States have been underperforming in mathematics assessment from 2003 to 2011 by two times and four times respectively compared with their non-ELL counterparts. In this regard, ELLs’ underperformance in mathematics achievement can relate to their academic language and concept development in the mathematics classrooms. The following section describes research strands in mathematics education and highlights the importance of this dissertation study.

1.2 Significance of the Study

Research in mathematics education for general-education students as well as linguistically, culturally diverse students has followed three different strands of inquiries. The first focus is on the use of multiple representations for better understanding of mathematical concepts. The second focus is on the importance of interactions and language in the mathematics classroom to support construction of disciplinary meanings. The third focus is on the knowledge base for teaching mathematics content to English language learners in particular.

First, recent mathematics studies in education have addressed the importance of connecting multiple representations of mathematical concepts in designing curriculum and instruction (e.g., Brizuela & Earnest, 2008; Chval, Pinnow, & Thomas, 2015; Dreher, Kuntze, & Lerman, 2016; Moreno & Duran, 2004; Tripathi, 2008). Tripathi (2008), for example, states that the use of multiple representations including verbal, pictorial, and algebraic representations enabled middle-school students to understand better and develop mathematical concepts by transforming and connecting one representation to another representation. Likewise, Dreher and her colleagues (2016) argue that teachers should provide students with linguistic guidance for connecting multiple representations because students are challenged in connecting and interpreting the
representations without support. These studies, however, have not explored how language mediates and connects multiple representations in mathematics instruction.

Second, some studies in mathematics education have placed attention on interactions and language as mediators in the construction of mathematical meaning. For example, research informed by a social semiotic perspective of communication attends to how configurations of language function to construct mathematical meaning and how teacher-student or student-student interactions unfolds in teaching and learning concepts of mathematics (e.g., Barwell, 2016; Chapman, 1995, 2003; de Oliveira & Cheng, 2011; Gonzalez, 2015; Herbel-Eisenmann & Otten, 2011; Morgan, 2006; Veel, 1999). Chapman (1995), for instance, shows that mathematics classroom discourse is characterized as unpacking abstract mathematical language into concrete everyday language. In this regard, teaching and learning mathematics can challenge English language learners (ELLs), who learn English as an additional language, in that the curricular subject is increasingly dependent on academic language as they progress in school (Schleppegrell, 2007). The growing dependence on academic language can result in marginalizing ELLs in English-mediated instruction, in which English is used as a medium of teaching for curricular subjects such as science, mathematics, social studies, and English language arts.

Third, research in mathematics education calls for the knowledge base for teaching the curricular content to ensure the rigor of schooling for ELLs (e.g., Barwell, 2005; de Oliveira & Cheng, 2011; Gebhard, Hafner, & Wright, 2004; Moschkovich, 2015; Takeuchi, 2015). Teacher educators maintain that mathematics teachers should be equipped with pedagogical knowledge—knowledge about teaching content knowledge—on top of their content knowledge when working with ELLs in that the students are required to learn and develop language and content knowledge of mathematics at the same time. Moschkovich (2015) and Takeuchi (2015) argue that mathematics
learning and language learning are inseparable and that teachers should expand ELLs’ engagement in mathematical activities using multiple representations (e.g., visual representations, symbolic representations) and multiple modalities (e.g., actions, drawing, speaking, writing). Gebhard and her colleagues maintain that ELLs should be taught mathematics through a language-based approach comprising multi-steps: reading a math word problem, presenting visual representations, narrativizing the problem, writing a formula corresponding to the problem, and solving the problem. These studies, however, did not examine how teacher talk and student responses contribute to language and concept development. Moschkovich’ and Takeuchi’ accounts did not provide linguistic analysis how linguistic choices teachers and students made construct mathematical meanings even though they view the essential role of language as mediator in teaching and learning mathematics.

This current study argues that linguistic analysis is required to understand how teachers use language to engage all students including ELLs in classroom talk and develop language and mathematical concepts because the unit of a concept is a word meaning (Vygotsky, 1986) and language is a meaning making system sensitive to a context in which language is used (Halliday, 1978). Teachers should provide opportunities for students to participate in mathematics classroom discourse through the active use of language to develop mathematical understanding (Moschkivich, 2012). Mercer and Littleton (2007), however, point out that “we cannot assume that all children naturally have access to the same opportunities for developing their use of language as a tool for learning, reasoning and solving problems” (p.2). To support ELLs to gain equal access to classroom talk in developing academic language and mathematical concepts, ways teachers use language should be a professional knowledge base, which is systematic, functional, and goal-oriented (Christie, 2002). In this regard, it is critical to examine how teachers can support ELLs’ conceptual
and linguistic development in content-area classrooms. It is also pivotal to explore how students interact with their teachers while participating in mathematical discourse as Chapman stated, “It [mathematics education] constitutes a series of social events in which teachers and learners interact, primarily through spoken language, to construct and share mathematical meanings” (Chapman, 1995, p.243). The subsequent section summarizes the problem statements and the significance of this current study as well as presents the purpose of this study along with research questions.

1.3 Purpose of the Study and Research Questions

Education reform policies have driven K-12 schooling to respond to high-stakes standards coupled with academic achievement assessments. The standards stress the importance of academic language, and all students including English language learners (ELLs) are required to communicate information specific to content areas at the grade level. The requirement, in particular, challenges ELLs who are learning language and content at the same time. This student population is exponentially growing across the states. Content-area teachers are expected to teach both language and content, but plenty of teachers are not prepared to work with ELLs. The standardized test outcomes show an enormous gap between ELLs and non-ELLs in mathematics. Studies on mathematics education attend to the use of multiple representations that support better understanding of mathematical concepts and language that construct mathematical meanings. Research in mathematics education also has focused efforts on ensuring the quality of education for ELLs, emphasizing the students’ participation in classroom talk to develop language and mathematical concepts.

However, there is little known about how teachers integrate language and concepts of
mathematics using multiple representations or multisemiotic resources (e.g., manipulatives, numerical symbolism, visual display, language) to support ELLs’ academic language and concept development in the elementary mathematics classroom.

Attending to three strands of research in mathematics education mentioned above, therefore, this study aims to examine how teachers use language to guide elementary ELLs in developing academic language and mathematical concepts simultaneously. Research questions are as follows:

(1) How can teachers use language to support English language learners’ academic language and concept development during mathematics instruction using language and other multisemiotic resources (e.g., manipulatives, diagrams, number sentences)?

(2) How do English language learners participate in language-focused mathematical instruction?

(3) How does classroom discourse vary in this context according to semiotic resources used to support English language learners’ language and mathematical concept development?

1.4 Organization of the Dissertation

This study presented seven chapters. The first three chapters outline the purpose of this study, provide a literature review on the topic of "academic language" development, and explain the theoretical frameworks informing this current study. Specifically, in Chapter 1, I address issues relevant to robust schooling for English language learners (ELLs) and the significance of this study. I also describe the purpose of this study, pose research questions, and outline the organization of this dissertation. In Chapter 2, I define what “academic language” is because there is no consensus on what this type of language is and how students should be taught "academic language." I also review the literature describing classroom-based practices focusing on academic language
development of ELLs in K-12 settings. In Chapter 3, I first outline Vygotsky’s (1986) theory of concept development. Vygotsky argued that children develop two types of concepts with and without schooling. These concepts include spontaneous and scientific ones respectively.

Spontaneous concepts develop “upwards” to make sense of scientific phenomena while scientific concepts develop “downwards” to regulate spontaneous concepts. The bottom-up or spontaneous concept development and the top-down or scientific concept development relate to the interplay between everyday language and academic language given that a word meaning is the unit of a concept (Vygotsky, 1986). Acknowledging word meaning as the unit of a concept, I also outline Halliday’s theory of language called systemic functional linguistics (SFL). This theory views language as a social semiotic, which works to make meanings sensitive to context in which language is used and develops (e.g., everyday versus academic one). In Hallidayan terms, scientific discourse typically involves a symbol or “Token” and the meaning or “Value,” both of which are connected by relational process (e.g., ‘is,’ ‘represent,’ ‘mean’). The symbolic relation between Token and Value is realized in “decoding” and “encoding” clauses, which I describe in detail in section 3.3.2. In addition, I provide an overview of a semiotic approach to mathematics instruction that attends to multisemiotic meaning-making systems such as visual display, symbolism, and language (e.g., Chapman, 1995; Lemke, 2003; O’Halloran, 2000).

Chapters from 4 to 7 describe research design, findings, discussions and implications. In particular, Chapter 4 describes the research methodology including data collection procedures and data analysis procedures. The data analysis draws on SFL to analyze classroom discourse, using the clause as the unit of analysis. Chapter 5 presents findings of this study to show how a teacher and her ELLs used language in constructing mathematical meanings through classroom talk and how mathematics classroom discourse varies depending on the nature of multiple semiotic resources at
play. This study first addresses findings in teacher talk and student response separately in section 5.2 and 5.3 to answer the research questions and later describes them collectively in section 5.4 because the classroom talk comprises interactions between teacher and student. In Chapter 6, I discuss the findings in relation to previous studies on ELLs’ academic language and concept development in mathematics and in K-12 settings in general. Chapter 6 also discusses limitations of this study. In Chapter 7, I conclude with a discussion of the implications of this study for literacy researchers, particularly those engaged in mathematics education and teacher preparation.
2.1 Introduction

A growing body of research has focused attention on educational policies and a distinctive student population who grows rapidly and requires pedagogical support in learning English as a second language in the US (e.g., Darling-Hammond, 2015; Hutchinson, 2013; Janzen, 2008). Darling-Hammond, for example, addressed that an achievement gap between English language learners (ELLs) and non-ELLs increased with No Child Left Behind (NCLB) law coupled with high-stakes standardized tests, highlighting that providing adequate education for students in the U.S., especially for ELLs is urgent. In addition, ELLs are challenged to meet Common Core State Standards (CCSS) that require an increasing demand on academic language development across content areas in K-12 schools (Dicerbo, Anstrom, Baker, & Rivera, 2014; Frantz, Bailey, Starr, & Perea, 2014; Lee, Quinn, & Valdes, 2013). For instance, Lee and her colleagues argue that language and science play a critical role in classroom science discourse involving science literacy, and visual representation and communication pertaining to science education, suggesting that ELLs should be supported in developing academic language across content subjects (e.g., math, science, social studies, English language arts).

Although there is consensus that developing ELLs’ academic language is critical, there has been controversy regarding the definition of academic language since Cummins’ (1979) distinction between ’cognitive academic language proficiency (CALP)’ and ’basic interpersonal communication skills (BICS)’ (see, Dicerbo, Anstrom, Baker, & Rivera, 2014). Cummins (1979) described CALP as a language associated with academic achievement and BICS as ”sociolinguistic aspects of
communicative competence or functional language skills” (p.198). According to Snow and Uccelli (2009), academic language is distinguished from language specific to each content subject, suggesting, “All-purpose academic language forms the core of content-area-specific language (p.114).” On the other hand, from the perspective of systemic functional linguistics (SFL), which views language as a meaning-making system to accomplish a particular purpose in a specific context (Halliday, 1994), academic language (AL) is defined as language used to support teaching and learning school-based subjects. Academic registers which involve discipline-specific words and grammar “vary by task, subject matter, and grade level” (Moore & Schleppegrell, 2014, p.92). In this current review AL is conceptualized from the SFL perspective in that SFL-based research has shown promising results in relation to teaching and learning AL by drawing on classroom-based data, which describe what is happening in classroom practices (e.g., Brisk, 2012; Chen & Jones, 2012; Gebhard, Harman, & Seger, 2007; Gebhard, Chen, & Britton, 2014; Gibbons, 2003).

Exploring moment-by-moment classroom practices can provide a critical insight into ways to improve AL. Therefore, the present review aims to examine emerging themes in literature on academic language development using systemic functional linguistics (SFL) according to curricular subjects (e.g., English language arts, math, science, and social studies). In reviewing the literature, this paper attends to scholarly findings that the achievement gap between ELLs and non-ELLs increased with grade levels (Butler, Stevens, & Castellon, 2007) and that there was a difference in instructional approaches between the elementary level and the secondary level (Evangelou, Taggart, Sylva, Melhuish, Sammons, & Siraj-Blatchford, 2008). In what follows are guiding questions posed in the current literature review:

1. What are the findings for SFL-based studies of academic language and literacy practice in K-12 contexts?
2. Are there differences based on grade levels? In other words, are there differences in the findings from SFL-based research conducted in elementary school vs. secondary school?

3. Are there differences in these findings based on disciplinary focus? In other words, are there differences in the findings from studies focusing on different discipline such as English language arts, math, science, and social studies?

In the subsequent sections, I start by outlining the conceptual framework informing this current review. I briefly overview systemic functional linguistics (SFL), genre-based pedagogy (GBP), and classroom discourse. I then describe the methodology used for inclusion and exclusion of literature. Next, I analyze how academic language develops in K-12 schools, focusing on empirical studies. Thereafter I examine emerging themes of studies on academic language that varies (or not) according to each subject matter. I close this review with conclusion, suggesting implications.

2.2 Systemic Functional Linguistics

Systemic functional linguistics (SFL), which is also called functional grammar, is a study to understand how language works and how people use language. SFL views language as social semiotic that works in a meaning-making system, which is socially constructed to accomplish a specific purpose in a particular context (Halliday, 1978, 1985, 1994). SFL refers grammar to a system of meanings, which are realized through a system of forms. Halliday argues that a text as a semantic unit is determined by a context in which the text is being created to simultaneously represent three kinds of meanings, that is, three metafunctions: ideational, interpersonal, and textual. Ideational metafunction construes experience. Interpersonal metafunction “enact social relationships”, and textual metafunction “creates relevance to context” (Halliday, 1994, p.36). The three metafunctions are actualized through three register variables sensitive to a context of situation: field, tenor, and mode. Field concerned with ideational metafunction is determined by
what is happening, involving *processes* (or verbs), *participants* (or nouns), and *circumstances* (or prepositional phrases of time, place, or manners). Tenor associated with interpersonal metafunction is determined by who is participating, concerning *mood* types (e.g., declarative, interrogative, imperative), *modality* (e.g., can, may, have to), *polarity*, and *appraisal*. Mode related to textual metafunction is determined by what role language is playing, which includes medium (e.g., spoken or written) and coherence of a text.

The language role that varies by interaction types (e.g., oral interaction, textual interaction) can be explained in terms of a continuum of mode in response to situation of language use (Martin, 2001). According to Martin, there can be a variety of dimensions between the two extreme ends of the mode continuum, as presented in Table 2.1.

<table>
<thead>
<tr>
<th>face to face</th>
<th>telephone</th>
<th>TV</th>
<th>Radio</th>
<th>letter</th>
<th>book</th>
<th>Stream of consciousness</th>
</tr>
</thead>
<tbody>
<tr>
<td>+aural</td>
<td>+aural</td>
<td>one way</td>
<td>one way</td>
<td>delayed</td>
<td>review</td>
<td>audience</td>
</tr>
<tr>
<td>+visual</td>
<td>-visual</td>
<td>aural &amp; visual</td>
<td>aural; written</td>
<td>feedback; feedback;</td>
<td>-visual</td>
<td>=self</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 2.1 Mode continuum (Martin, 2001, p.158)

Eggins (2004) describes the mode continuum by differences between spoken and written modes, showing the relation between language and situation of language use in Table 2.2. As shown in Table 2.2, spoken language has a dynamic structure that is characterized by spontaneity, open-endedness, context-dependence, and informality while written language has a synoptic structure that involves properties opposed to those of spoken language (Eggins, 2004, p.93). In addition, Eggins indicates that grammatical intricacy is characteristic of spoken language, which mostly
includes more clauses than written language. In SFL, a clause is a structural unit of lexicogrammar (or syntax), “to which functional configurations can be assigned” (Halliday, 1994, p.16). A clause is higher than a group or phrase and lower than clause complex in the hierarchy of constituency. Clause complex is “the grammatical and semantic unit formed when two or more clauses are linked together” (Eggins, 2004, p.255). The hierarchy of constituency ranges from morpheme through word, group, and clause, to clause complex. In contrast with spoken language, written language is lexically dense, involving nominalization that transforms processes (or verbs) into nouns functioning as a thing, or object.

<table>
<thead>
<tr>
<th>Spoken</th>
<th>Written</th>
</tr>
</thead>
<tbody>
<tr>
<td>Situation</td>
<td>Language</td>
</tr>
<tr>
<td>Interactive</td>
<td>Turn-taking organization</td>
</tr>
<tr>
<td>Face-to-face</td>
<td>Context-dependent</td>
</tr>
<tr>
<td>Language as an action</td>
<td>Open-ended</td>
</tr>
<tr>
<td>Without rehearsing what is going to be said</td>
<td>Spontaneity phenomenon (false starts, overlap)</td>
</tr>
<tr>
<td>Informal &amp; everyday</td>
<td>Everyday lexis</td>
</tr>
<tr>
<td></td>
<td>Grammatical complexity/intricacy</td>
</tr>
<tr>
<td></td>
<td>Lexically sparse</td>
</tr>
</tbody>
</table>

Table 2.2 Relation between language and situation (see Eggins, 2004, p.92-93)

The three register variables including mode determine a text contextualized in a particular context, allowing predictability of the text (Halliday & Hasan, 1985). A text is interpreted in both an
immediate and broader context. Text and context are intertextual in creating meanings with predictability. The increase of predictability regarding interpretation of a text is concerned with coherence of a text. Coherence is realized through cohesion that is “a set of linguistic resources...for linking one part of a text to another” (Halliday et al., 1985, p.48). The linguistic resources establish semantic relations within and beyond a clause, including reference (e.g., Tom...he...him), conjunction (e.g., in short, but, moreover, then), and lexical cohesion (Halliday, 1994, p.308). Lexical cohesion involves repetition, collocation, and synonymy. Repetition refers to repetition of an identical lexical item (e.g., negotiate, negotiation, negotiating, negotiator). Collocation refers to a tendency that lexical items occur together. Synonymy is concerned with lexical items that have specific-and-general relations (e.g., triangle, square/polygon) and part-and-whole relations (e.g., base, height/triangle). Lexical items associated with synonymy are called thematic items (Lemke, 1990).

2.3 Genre-Based Pedagogy

Genre-based pedagogy (GBP) developed from the concept of genre defined as “a staged, goal-oriented social process” (Martin, 1992, p.505), and started as writing strategies in Australia. This approach, grounded in SFL, aims to explore the relationships between language and language use in an educational context and to provide linguistic support for students who are required to talk, read, and write in a specialized way valued in school, highlighting that education comprises learning language, learning through language, and learning about language (Derewianka, 1990). In SFL, genre is also regarded as a text type that is characterized by a structural organization and distinctive lexicogrammatical features appropriate for the purpose and audience of the text (Derewianka, 1990). Genres typical of a schooling context include recount, narrative, instruction/procedure, information report, explanation, and argument. The narrative genre is organized in a specific order: Orientation ^ Sequence of Events ^ Complication ^ Resolution ('^')
refers that something next follows something previous). Lexico-grammatical features of this genre can entail human participants (e.g., I, we, friends), various process types (e.g., do, think, say), past tense, and temporal conjunctions (e.g., first, then, next, finally) (see for details, Derewianka, 1990).

GBP has focused attention to providing linguistic scaffolding by drawing on a curricular design called *teaching-learning cycle*, which is staged as follows: preparation ^ modeling & deconstruction ^ joint construction ^ independent construction of text (Derewianka, 1990; Hyland, 2004; Martin & Rose, 2012; Rothery, 1986). Linguistic scaffolding gradually reduces from the beginning to the last along with the cycle. At the beginning of the cycle, students are introduced to a target genre and encouraged to be aware of language use focused on the genre through classroom activities such as brainstorming, watching visual artifacts, making a word list and doing hands-on activities, considering purpose and audience of the text, the structural organization, and choices of language. In the deconstruction stage, students are provided direct and explicit instruction, by which the teacher can model and show how the text is organized by analyzing its structure and lexicogrammatical features. In the joint-construction stage, students are guided to research a topic they will write about and collect the topic-related information through individual, pair, or group work, and they are encouraged to contribute information while co-constructing the text with the teacher. At the last stage of the curriculum cycle, individual students are supported to write about a topic corresponding to the focused genre, referring to a model text and consulting with the teacher for feedback regarding their draft. Following this stage, reflection activities such as classroom presentation of students’ texts can be sequenced. GPB supporting students in the classroom is attending to classroom discourse as a mediator in schooling.
2.4 Classroom Discourse

Research in classroom discourse has been focused on how it is structured and why it matters in classroom practices (Cazden, 2001; Christie, 1995, 2004; Lemke, 1990; Wells, 1999). The *initiation, response, and feedback/follow-up* (IRF) exchange identified by Sinclair and Coulthard (1975) is regarded as typical of the structure of classroom discourse. The researchers address that meanings of the triadic exchange should be understood across the lesson or over long-time classroom activities. The lesson viewed as “the highest unit of classroom discourse” (Sinclair et al, 1975, p.59) comprises a series of *activity types or episodes*, whose boundaries among them are built by activity functions involving opening a lesson, review, closing, and etc. Each activity type is regarded as a genre (Lemke, 1990). Given an activity type as a unit of analysis, the work of classroom talk analysis should involve how meanings are created by the triadic exchanges in each activity (Lemke, 1990).

To interpret how meaning-making system works in science classroom discourse, Lemke, drawing on SFL as a theoretical framework, argues that it is imperative to understand how lexical items are semantically related to represent concepts of the subject matter. Semantically related lexis or thematic items are patterned in a specialized way. For instance, a *material* process realized through a verb follows an *actor* participant realized through a noun and is followed by a *goal* participant realized through a noun, as shown in the following clause: *The earth creates heat and light.* Thematic patterns are actualized through lexical cohesion, reference, and conjunction as well as the three register variables (i.e., field, tenor, and mode). In particular, the cohesive devices, involving repetition, synonymy, and collocation, enable teachers to support students’ learning by repeating, reformulating, and elaborating students’ responses, by which classroom discourse can be extended beyond the IRF exchange. In the mathematics classroom, for example, a student
articulates ‘it is the same,’ followed by ‘the difference is constant,’ which indicates that a teacher reformulates the student’s utterance ‘the same’ by drawing on a synonym ‘constant.’ According to Lemke (1990), students should be provided explicit instruction regarding cohesive relations, or thematic patterns that recur across texts within a specific subject. Lemke argues that it is prioritized to complete thematic patterns through extended discourse depending on pedagogic purposes of each activity type.

Classroom discourse that is purposeful in education plays a critical role as linguistic scaffolding, by which teachers support students’ understanding of curricular concepts. Lightbown and Spada (2006) state that from sociocultural perspectives, students’ learning results from social interaction in their zone of proximal development (ZPD). The ZPD is “the distance between the actual development level as determined by independent problem solving and the level of potential development as determined through problem solving under adult guidance, or in collaboration with more capable peers” (Vygotsky, 1978, p.86). Vygotsky (1986) states that teacher talk functions as a mediator to bridge students’ spontaneous/everyday concepts to scientific concepts. Fine-grained teacher talk can optimize students’ potential performance in their ZPD through dialogic interaction between teacher and students. Dialogic interaction, in this context, involves the interplay between spontaneous and scientific concepts. Developing scientific concepts is compared to learning a foreign language that requires classroom instruction. In other words, as students’ native language is a foundation in learning a foreign language, their spontaneous concepts play a role as foundation in developing scientific concepts. In sum, the conceptual framework informing this literature review is anchoring in sociocultural theory by drawing on systemic functional linguistics as theory of language, genre-based pedagogy, and classroom discourse that plays a role as a mediator of schooling.
2.5 Methodology

This review addresses literature from 1980 to 2015 because academic language has been a central issue since early 1980s (see, Dicerbo, Anstrom, Baker, & Rivera, 2014). Search engines used for the current review include Education Research Information Center (ERIC), Google Scholar, ProQuest, and Science Direct. Besides literature collected through these search engines, articles were included by thread search based on literature reviewed. Key terms in search of literature were systemic functional linguistics, academic/disciplinary language, and classroom talk, including descriptors equivalent to the key terms: functional grammar, academic literacy, dialogic approach, classroom discourse, teacher-student interaction, and so on.

In this review of research, I surveyed literature associated with school settings in English-dominant countries such as Australia, Canada, the U.K, and the US in that the countries have similar challenges in teaching and learning practice in conjunction with a demographic change caused by the increasing number of immigrant students (Janzen, 2008).

To ensure the rigor of studies included in this literature review, I drew only on published empirical peer-reviewed studies. Criteria for inclusion and exclusion were set by three phases. In the first phase of searching, I used key terms such as systemic functional linguistics, functional grammar, academic/ disciplinary language or literacy, dialogic, teacher-student interaction, and classroom talk/ discourse/ conversation pertaining to the elementary and the secondary settings. In the second phase, I selected literature drawing on systemic functional linguistics as a conceptual framework, but I omitted studies that used SFL only as an analytical tool. In the third phase, I explored research methods, examining data to confirm if they were focused on classroom practice of classroom talk and academic language development across content areas in K-12 settings.
In the initial passes through the three-phase survey, I selected articles that investigated language use in disciplinary instruction involving classroom-based practice. However, I excluded studies that addressed different aspects of educational challenges. For instance, I excluded research only focused on teacher education or professional development programs concerning characteristics of academic language. I also omitted research reports that presented how disciplinary documents including textbooks, assessment items, and student responses to tests are organized and what linguistic features are distinctive in the disciplinary texts. The present review drew on only classroom-based research toward students’ textual practices and oral class interactions, involving studies focused on “micro-scaffolding” practice, or moment-by-moment classroom practice within content areas (i.e., math, science, social studies, and English language arts) and “macro-scaffolding” practice relevant to curricular design and students' learning over time (Schleppegrell & O’Hallaron, 2011, p.10).

Literature discussed in this review involved analysis data of either written text or spoken text, which is represented by W and S respectively in Figure 3, while studies considering both data are represented by B in Figure 3. All of the studies drew on a range of qualitative research methods. Most of the studies were concentrated on either oral interaction in the classroom practice (n=8) or interplay between classroom talk and written texts (n=5). Nine studies analyzed written text. The present review examines fourteen studies at the elementary level to cover K-5 and eight studies at the secondary level ranging from grade 6 to 12, considering only four curricular subjects (i.e., science, mathematics, social studies, and English language arts) because the subjects are required in most U.S schools (Moje, 2009). In particular, this current review includes one study on social studies (i.e., history) because it is the only research that meets the criteria of inclusion. The majority of studies selected for discussion were published after 2000 (n=19) while a few of them were
before 2000 (n=3). The scholarly work was mostly conducted in the US (n=16), a few in Australia (n=4) and Canada (n=2). I, however, did not find any studies from the U.K.

Table 2.3 indicates that SFL-based research on academic language was conducted differently in elementary and secondary levels. At the elementary level, studies covered only science (n=7) and English language arts (n=7). Most research at this level incorporated either genre-based pedagogy (GBP) or sociocultural theory of learning coupled with SFL. Three out of fourteen studies analyzed the interplay between literacy practice and classroom talk. Research was more focused on academic literacy practice (n=7) than oral interaction in the classroom (n=4). The studies addressed ranging aspects within SFL theory. They also showed differences in designing research although all studies were grounded in qualitative research methods.

At the secondary level, research was conducted only within science (n=3), mathematics (n=4), and history (n=1), mostly focused on classroom talk. In addition, research in secondary settings aimed to investigate how talk played a role in teaching and learning subject matters, which is compared to studies in elementary school where supporting students’ literacy was a primary goal. Majority of the studies at this level highlighted lexical cohesion analysis linked to SFL theory. Most research drew on an inductive approach, compiling ethnographic data including classroom observations, field notes, audio/video-recording and transcripts, and interviews (Goetz & LeCompte, 1984). Only one study employed design-based research.

Table 2.3 also shows that the number of the studies varied according to content areas. Studies on academic language in science were the most prevalent (n=10) while research pertaining to history was the least (n=1). Studies in the mathematics classroom were mainly focused on teacher talk to support students’ understanding of mathematical concepts. Research on academic language in English language arts predominantly addressed a pedagogical approach to writing in
the discipline. Research on school science mostly involved analysis of classroom talk both at the elementary and at the secondary school levels. Table 2.3 summarizes the focus of articles included in the review. I discuss the findings of this review based on Table 2.3.

<table>
<thead>
<tr>
<th></th>
<th>Science</th>
<th>Math</th>
<th>History</th>
<th>Lang. Arts</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Elementary</strong></td>
<td>7</td>
<td></td>
<td></td>
<td>7</td>
<td>14</td>
</tr>
<tr>
<td></td>
<td>($S^2=3$; $W=2$; $B=2$)</td>
<td></td>
<td></td>
<td>($S=1$; $W=5$; $B=1$)</td>
<td></td>
</tr>
<tr>
<td><strong>Secondary</strong></td>
<td>3</td>
<td>4</td>
<td>1</td>
<td>7</td>
<td>8</td>
</tr>
<tr>
<td></td>
<td>($S=1$; $W=1$; $B=1$)</td>
<td>($S=3$; $B=1$)</td>
<td>($W=1$)</td>
<td>($S=1$; $W=5$; $B=1$)</td>
<td></td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td>10</td>
<td>4</td>
<td>1</td>
<td>7</td>
<td>22</td>
</tr>
<tr>
<td></td>
<td>($S=4$; $W=3$; $B=3$)</td>
<td>($S=3$; $B=1$)</td>
<td>($W=1$)</td>
<td>($S=1$; $W=5$; $B=1$)</td>
<td></td>
</tr>
</tbody>
</table>

Table 2.3 Approach to researching academic language development across content areas

In the subsequent section I discuss findings for SFL-based studies on academic language and literacy practice in K-12 contexts, categorizing the findings into three subsections: (1) research on academic language in elementary school; (2) research on academic language in secondary school; and (3) emerging themes in research on academic language in each subject matter. In first and second subsections, I overview research methods researchers draw on and examine how SFL-based studies at each school level addressed research foci required to underpin their work. Lastly, I discuss differences and/or similarities in disciplinary focus according to curricular subjects.

---

2 S and W indicate spoken and written text analysis respectively and B refers to spoken and written text analysis.
2.6 Research on Academic Language in Elementary School

At the elementary grade level, studies focus on English language learners (ELLs) whether they are in mainstream classrooms or in English-as-second-language (ESL) settings. Focusing on developing students' literacy abilities, the majority of the studies draw upon design-based research (see, Cohen, Manion, & Morrison, 2011) in that they involve SFL-based instruction/GBP as an intervention to enhance learners’ academic language (e.g., Brisk, 2012; de Oliveira & Lan, 2014; Gebhard, Harman, & Seger, 2007; Gebhard, Chen, & Britton, 2014; Harman, 2013). Besides, researchers describe research methods by various terms such as interpretive study (Gibbons, 2003), ethnographic and case-study approach (Honig, 2010), and action research projects (Haneda, 2000; Hodgson-Drysdale, 2014; Wells, 1993). The action research projects do not provide details regarding methods, stating that the studies are collaborative work between school and local university.

The overarching goals of studies are either to improve students' writing ability by analyzing students’ textual practices supported by genre-based pedagogy (GBP) or to examine the role of talk in developing students’ content knowledge by language. To achieve the goals, most researchers incorporate SFL coupled with sociocultural perspectives of learning. However, they address varying aspects within SFL according to their research foci: spoken text and written text.

2.6.1 Focus on Spoken Text

In regard to research focus on spoken text, studies demonstrate how linguistic resources function to make dialogic interaction meaningful in developing students’ content knowledge and academic language (AL). All studies in this strand employ an activity type as an analysis unit.
Wells (1993) analyzes teacher-student oral interaction in all activity types comprising one lesson, which include preparation, experiment, group-writing conference, clearing-up, and report writing. Each activity type involves varying participant formations (e.g., pair work, group work, individual work, whole-class discussion). He examines how use of language changes across all activities included in the lesson. Wells finds that field, tenor, and mode vary according to participant formation of classroom activities in the third-grade science classroom. The experiment activity by pair work is characterized by actions realized through material processes (e.g., do, measure, empty), which is compared to the group-writing conference characterized by reporting realized through mental and verbal processes (e.g., think, wonder, ask). Wells argues that the IRF sequence can be pedagogic depending on activity types. For example, the triadic exchange is appropriate for aligning a reviewing activity with its goal to check what students have learned.

Instead of examining one lesson, Gibbons (2003) has a close analysis of a specific activity type, *teacher-guided reporting*, which is observed to occur across all lessons of one curricular unit in the fourth-grade science classroom. During the teacher-guided reporting activity, students shared their learning verbally with the whole class while supported by the teacher. She investigates how teacher’s language use in the teacher-guided-report activity changes throughout the lessons. She highlights linguistic scaffolding that varies from context-embedded language to context-reduced language, addressing dialectic relationships between scientific concepts and everyday concepts. For example, ‘it sticks together’ is context-embedded language in that only co-participants who share a visual context of ‘it’ can recognize what ‘it’ refers to. In contrast, ‘when the north and south poles were facing one way, you felt the magnets attract’ is context-reduced language that enables interlocutors to interpret the meaning without relying on a visual context. Drawing on mode continuums to regard talk as spoken-like or written-like depending on degree of context-
detachedness of language, she explains that context-embedded language and context-reduced
language correspond to spoken-like talk and written-like talk respectively. She demonstrates that a
teacher takes up students’ spoken-like language (e.g., ‘stick together,’ ‘put away’) and switches it to
written-like language (e.g., ‘attract,’ ‘repel’) while elaborating and reformulating students’
utterances according to students’ development of academic concepts and language proficiency in
their ZPD.

Underpinning metalanguage use through teacher-student interaction, O’Hallaron, Palinscar
and Schleppegrell (2015) and Moore and his colleague (2014) employ design-based research
methods. Both studies examine whether use of metalanguage will raise students’ linguistic
awareness, by which students can reflect on texts required to read in school. O’Hallaron and her
colleagues explore how heightening critical awareness of appraisal lexis (e.g., interestingly,
fortunately) in science texts contributes to differentiating opinions from facts. Teachers raise
students’ awareness of how writers infuse their opinions with facts by engaging third-grade
students in reflecting on and in discussing appraisal lexis embedded in the texts.

Moore & Schleppegrell describe how metalanguage use engages students in meaningful
discussion of literary work in a range of classrooms (i.e., grade 3 to 5). They highlight Vygotsky’s
(1986) scientific/everyday concept formation and mediation by artifacts including written texts
and diagrams. They demonstrate that transforming literary metalanguage into learner-friendly
terms (e.g., doing, saying, showing, volume-up) enables students to verbally discuss how literary
characters are shaped by lexis associated with both processes and appraisal. The study indicates
that teachers’ reiteration of students’ utterances is vital in clarifying what they mean and in leading
their current oral interaction to more extended discourse. The findings suggest that use of learner-
friendly metalanguage coupled with scaffolding through reiteration enables learners to participate
in literary discussion and to provide their text-related experience orally as a knowledge co-constructor not only as a knowledge recipient. In sum, O’Hallaron and her colleagues (2015) as well as Moore and his colleague emphasize the importance of metalanguage in improving students’ awareness of language used in content-area texts.

2.6.2 Focus on Both Spoken and Written Text

Studies focused on both spoken text and written text call attention to how AL is developed either through multimodal meaning making of content knowledge or through implementation of SFL-informed pedagogy by the interplay between talk and textual practice (Chen & Jones, 2012; de Oliveira & Lan, 2014; Haneda, 2000; Hodgson-Drysdale, 2014). For instance, Haneda (2000), drawing on action research projects, addresses that science instruction comprises multimodal resources such as talking, writing, drawing, and hands-on activities, by which students develop science knowledge. Considering Vygotsky’s perspective on learning and Halliday’s SFL, Haneda (2000) examines two third-graders’ writing and teacher-student talk in a single lesson. She explores how teacher-student talk varying by classroom activities affects students’ subsequent writing. She finds that one of the two students orally changed processes and mood types according to activity types such as observing, describing, and explaining a phenomenon while the other did not. Their subsequent writings indicate the same difference between the two students. The findings show that a student who orally appropriated teacher-guided talk can represent knowledge through the subsequent drawing and writing. In other words, oral appropriation of content knowledge through teacher-student talk is prerequisite for such appropriation in writing. In the study, spoken interactions were within the ZPD of one student, which led to the student’s appropriation of knowledge while the interactions were out of the ZPD of the other student, which failed to result in the student’s appropriation of knowledge. Addressing the difference between the two students in
oral concept-making, the study concludes that teacher-student talk can enhance students’ concept making only when the interaction occurs within the ZPD.

Instead of analyzing one lesson, Hodgson-Drysdale (2014) analyzes teacher talk and student sample writings produced throughout a thirteen-week curricular unit comprising multiple lessons for fifth-graders. The analysis focuses on ideational meanings of processes, participants, and circumstances realized through teacher talk in whole-class discussion, hands-on activities and writing with the ones instantiated in students’ individual and group writing samples. Findings show that students have difficulty relating discipline-specific terms to their individual writing while they represent them better in the group writing. The researcher emphasizes that writing develops gradually and that students should be provided with long-term scaffolding coupled with explicit instruction. This study can contribute to explaining how an SFL-informed teacher can facilitate content instruction focused on language across a curricular unit. However, the study has a limitation in terms of appropriateness of an intervention. The writing instruction as intervention was not focused on teaching how a specific genre is organized and what linguistic features are distinctive in the genre. Instead, the teacher asked students only to write what they knew about the curricular concept ‘sedimentary’ at the end of the intervention. Furthermore, results of the study are limited because it included evidence of classroom talk drawn from field notes not from accurate transcripts based on audio-video-recordings.

In contrast, attending to transcripts drawn from audio-video recordings, de Oliveira & Lan (2014) conduct design-based research by using GBP as an intervention to examine how a teacher supports students’ science writing development. They argue that a case-study ELL student showed significant improvement in writing a procedural recount through four fifty-minute lessons in the mainstream fourth-grade science classroom. The argument is based on analysis of teacher-student
oral interaction and student-produced texts. The study describes how a teacher explicitly taught cohesive devices (e.g., first, then, finally) and names of experiment-related materials and how the student’s writing changed. The study showed how genre-based pedagogy is implemented in teaching the target genre knowledge, presenting that the student's final writing displayed not only the structural organization of the genre but also linguistic features distinctive to the text. However, a teacher-created model text provided in the study highlighted lexico-grammatical features but failed to specify the text organization. This failure makes unclear whether student writing with text organization was constructed by her understanding of the genre knowledge or mirrored the text organization presented on the screen.

Attending to Vygotsky’s development of scientific and spontaneous concepts and Halliday’s (1993) perspective on learning, Chen & Jones (2012) explore as a case study how students taught by an SFL-informed teacher respond to explicit grammar instruction, construct conceptual understanding, and apply it to their writing. Chen & Jones analyze classroom talk by lexical cohesion analysis to investigate conceptual understanding and student writing by lexical density to examine whether they can appropriate their grammar knowledge in writing. The analysis indicates that the lexis the students used was semantically related to what process, participant, and circumstance refer to and that texts they produced were lexically denser than their first drafts. Chen & Jones address the importance of metalanguage use in the English language arts classroom, arguing that use of metalanguage enables students to heighten language awareness and promote their writing ability. The findings imply that metalanguage use facilitates early-level elementary learners to develop both content knowledge and language, as Moore and Schleppegrell (2014) state.
2.6.3 Focus on Written Text

Focused on written text, studies address either what can be distinct genres in the science classroom or what genre knowledge supports or challenges students in their textual practice across different genres in the science and English language arts classroom. To identify distinct genres in learning science, one study employed an ethnographic and case-study approach without intervention. Six studies on students’ uptake or no uptake in relation to lexicogrammatical features (e.g., cohesive devices, nominalization) drew on genre-based pedagogy as an intervention for literacy development, using the teaching-learning cycle as a curricular design.

Embracing sociocultural views on learning as students’ participation in social practices that utilize multimodal resources available for them, Honig (2010) examines writing activities identified in the second-third-grade science classroom to classify students’ writing samples into genres rather than to see how a specific genre-related instruction works. The approach is based on the argument that science writing practices in general classrooms are characterized by a range of possibilities to diversify genres and generic features in students’ writing. The study identifies four distinct genres as findings: listing materials included in experiments, describing the experiments, creating a graphic organizer, and summarizing (the experiment).

Rather than identifying distinct genres in content classrooms, all the other six studies on academic writing development supported by GBP were conducted from a case-study approach and design-based research. They address students’ linguistic uptake and challenges by analyzing the literacy instruction that aims to teach particular text types such as recount, narrative, and explanation. Gebhard, Chen, & Britton (2014) demonstrate that teacher’s SFL metalanguage use supported third-grade students to engage in reading and writing and enhanced their literacy abilities across three different genres (i.e., biographical, historical, and scientific explanations) in
the ESL classroom setting. The study suggests that SFL metalanguage use is a promising tool especially for teachers who design a curriculum to heighten students’ awareness of functions of metalanguage across varying genres. The analysis indicates, however, that students’ control of nominalization can be challenging and require long-term scaffolding for over one year. Another linguistic challenge for ELLs rests in control of personal pronouns (e.g., I, we, you, she, he, they). Brisk (2012) finds that students ranging from grade three to five had difficulty appropriating personal pronouns according to various genres. The findings show predominant use of ‘I’ and ‘we’ in all genres, presence of ‘you’ as audience in imperatives of procedural genres, and less frequent involvement of third person pronouns in historical recounts and expositions.

Development of lexical cohesion is also regarded as challenging in producing literary writing (Harman, 2013). Harman examines how GBP supports fifth-grade ELLs in producing a literary genre for five months. Making a text cohesive and coherent in academic writing is realized through the skill of intertextuality, which is defined as borrowing “rhetorical and linguistic patterns” (p.126) from literature and interweaving them. Harman shows that genre-based pedagogy enabled fifth-grade students to incorporate sub-class lexis (e.g., father, mother, children) and whole-class lexis (e.g., family) and produce similar clause patterns, by which the student established cohesive meaning in the literary narrative. The findings imply that learners can appropriate lexicogrammatical knowledge into their academic writing rather than only reproduce expert-created texts and that this linguistic appropriation plays a pivotal role in the upper-level writing development.

Focusing on upper-level writing development, Gebhard, Harman, and Serger (2007) pinpoint that an argument genre is high-stakes for upper-level elementary schoolers because the genre involves modality, formal cohesive devices (e.g., therefore, however), and a specific text
organization. The study demonstrates how a teacher using GBP guided fifth-grade students to succeed in disagreeing on the policy to cancel recess time by writing well-developed texts with persuasive voices in a structural form. The findings suggest that the explicit scaffolding can lead students to control semiotic resources and accomplish a social purpose by writing in a way that community members value.

With respect to explicit scaffolding, Pavlak (2013) highlights using a variety of classroom activities aligned with each stage of the teaching-learning cycle to investigate how GBP can be implemented successfully in an underperforming urban school. Her eight-week study collaboratively designed by a teacher and a college-associated researcher describes what activities were constructed in developing third-grade students’ biography writing according to each phase of the cycle. The findings show that students took up vocabulary and text organization appropriate for the purpose of biography genre, addressing what words students borrowed from pre-read texts and whole-class discussions and how they organized their writing. Further discussion, however, is required to clarify how borrowing rhetoric expression from other texts contributes to establishing semantic relations among them.

2.7 Research on Academic Language in the Secondary School

At the secondary level, research on academic language involves only mathematics, science, and history, mostly focused on how classroom talk mediates content knowledge construction. Most studies drew on an inductive and interpretive approach, focused on language use realized through either talk or the interplay between talk and written texts (n=7). Only one study involved design-based research; this study examined literacy practice in the history classroom. One overarching goal is to analyze how a teacher construes curricular concepts or how language is used to bridge
students’ everyday experience to academic concepts of subject matters, using SFL. To achieve the goal, two studies highlight teacher talk that construes concepts in written text and compare the talk with the written text. Four studies address cohesion concerning how texts are coherently organized. One study drew on both Vygotsky’s views and Halliday’s perspectives on language use as theoretical framework (Zolkower & Shreyar, 2007). The studies describe varying perspectives within SFL according to research foci: (1) spoken text; (2) spoken and written texts; and (3) written text.

2.7.1 Focus on Spoken Text

Research focused on spoken text in the secondary settings addresses how a teacher construes content knowledge through classroom discourse. Studies underscore that teachers’ language use is patterned through configurations of academic registers that are context-specific, stating that the formations are relative to subject matters (Gonzalez, 2015; Herbel-Eisenmann, 2011; Lemke, 1988). To interpret concepts of curricular content of which students lack control and can misunderstand the concept, teachers can express one concept in a variety of ways that share identical semantic relations in different wording (Lemke, 1988). Lemke demonstrates the ways a teacher semantically relates words to concepts of science, which is the same as science texts, are asymmetrical with those students do. The finding suggests that semantic relations among words—that is to say, lexical cohesion—should be taught explicitly in the science classroom.

Implicit instruction pertaining to lexical cohesion can impede students’ understanding of disciplinary concepts that are construed differently by subtle meaning shifts (Herbel-Eisenmann & Otten, 2011). Herbel-Eisenmann and Otten attempts to explore mathematics classroom discourse. They find that specialized terms such as base and height of a triangle were used to represent either
measure of one side (i.e., extent) or one part of the polygon (i.e., entity) in two mathematics classrooms, in which the subtle meaning shifting occurring through lexical cohesion was not addressed explicitly. Teachers in the two classrooms showed similarities and differences in language choices. On the one hand, both teachers were ambiguous in using a being process in that a clause, ‘A is B’ can refer to dual meanings: (1) A is defined as B, in which A and B are reversible; and (2) A is attributed to B, in which A and B are irreversible. For example, in the following clause, ‘a square is a quadrilateral with four congruent sides and four right angles,’ ‘a square’ and ‘a quadrilateral with four congruent sides and four right angles’ can be replaced with each other without any difference of the meaning. In contrast, ‘rectangles’ cannot be replaced with ‘parallelograms’ in a clause, ‘rectangles are parallelograms.’ On the other hand, there were differences in use of thinking process, which can function to require students to reflect on mathematical concepts they learn. One teacher used thinking processes such as ‘notice’, and ‘think’ while the other did not use any thinking processes. The findings also show that subtle meaning shifts of mathematical terms with dual meanings were not explicitly explained, which can impede students’ understanding. For example, the meaning of base as entity in “cut…that full base right?...got cut in half” shifted to meaning of extent in “see that long base...what fraction is that of the base? Half…”

In addition, research finds that teacher talk begins with using everyday language reflecting everyday experience and shifts to academic language representing disciplinary concepts. Gonzalez (2015) examines how a teacher uses analogies to explain the ASA theorem (angle-side-angle theorem) by contradiction. The findings show that words used in the analogies concerning everyday experience were reiterated and collocated to develop cohesion and that clauses were linked by consequential conjunctions (e.g., because, therefore, if, then). The findings also imply that
analyses of teacher talk involving lexical cohesion (e.g., reiteration, collocation) and logical
conjunctions can provide guidance to ways teachers can bridge students’ everyday experience to
mathematical concepts.

Besides analysis of lexical cohesion and conjunctions, the research addresses that it is
significant to examine how a teacher uses language as a mediator to encourage students to verbally
represent their thoughts. The study implies that verbal thinking can play a critical role in construal
of content knowledge. Zolkower and Shreyar (2007) investigate what speech function a teacher
selects in a whole-class discussion and what role a teacher plays. They find that commands are
frequently used for demanding students’ participation in a whole-class discussion as a way they
give information based on their thoughts. The commands, however, are modulated in a way to tone
down the speeches: ‘you need to write it down’, compared to straightforward commanding ‘write it
down.’ The modulated commands engaged students in verbally expressing their thoughts in
response to open-ended tasks, in which the teacher plays a role as facilitator by repeating and
clarifying students’ statements rather than elaborating them with supplementary comments. The
findings suggest that the teacher can facilitate students’ verbal representation as a construal of
mathematical concepts with a tool of linguistic resources.

2.7.2 Focus on Both Spoken and Written Text

In studies focused on spoken and written texts, researchers address intertextual links
between the two modes. Chapman (1995) analyzes how teacher talk supports students to
understand the content knowledge in written texts. The analysis shows that teacher-student talk
shares similar thematic patterns with only differences in modes such as written-like talk vs.
spoken-like talk between the interactants, proposing that teacher talk contributes to
contextualizing written texts and unpacking mathematical concepts in them by less academic or informal words.

Another study focusing on the links between talk and written text examines how the two modes construct knowledge of physics and finds similarities and differences between classroom talk and a physics textbook (Young & Nguyen, 2002). Concentrated on process types, voices, and conjunctions, the analysis indicates that both talk and the textbook use material processes. The textbook, however, predominantly includes material processes realized through passive voices, nominalization concerned with transforming verbs to nouns, temporal conjunctions (e.g., first, next, when), and implicit consequential connections between clauses by using processes (e.g., cause, generate, establish). The linguistic discrepancies between spoken and written texts can challenge students in understanding concepts of subject matters.

2.7.3 Focus on Written Text

Studies focused on written text highlight demands on language that construct content knowledge. Seah, Clarke & Hart (2014) demonstrate that there are differences in addressing patterns of semantic relations among words between expert-created scientific texts and student-created texts. Students who learned matter states (i.e., solid, liquid, gas) associated with expansion and diffusion failed to explain how words such as particles, move, and expansion are semantically related. For instance, students’ explanation, “particles moved around making the shape expand” differs from a scientific account, “particles move further away from each other during expansion.” In a student’s response, the word ‘move’ was associated with a different circumstance (‘around’). The findings imply that it is critical to raise students’ awareness of language regarding how to tie participants, processes, and circumstances cohesively and semantically.
Critical language awareness affects how to interpret historical phenomena in that it is vital to recognize participants, historical events, times, and places (Achugar & Carpenter, 2012). Achugar et al. analyze student writing created in eight-period history classes and argue that explicit instruction based on linguistic support develops literacy abilities. The data analysis shows that linguistic scaffolding using metalanguage (e.g., verbs, references of participants) enables students to produce lexically dense and grammatically complex texts as well as improve their reading comprehensibility. Research focused on literacy practice indicates that understanding how language in text functions is imperative in reducing language demands to construe disciplinary knowledge in secondary school.

2.8 Main Themes in Research on Academic Language in Each Subject Matter

Research on academic language highlights a range of themes within each subject matter. For example, the themes in the school English class include raising critical language awareness by metalanguage use, improving writing ability by metalanguage use. Focusing on oral interactions and textual practices of content classrooms, studies address themes in two ways: (1) exploring features of teacher talk in developing academic language; and (2) examining how to support students’ literacy abilities. In what follows, I will discuss what themes are salient according to the focus of research—spoken text, written text, or both—within each subject matter: science, mathematics, history, and English language arts. Themes regarding research in history are considered restricted in one study, which is the only article included in the present review.

2.8.1 Science

Studies focused on classroom talk in the science classroom discuss the organizational structure of classroom discourse of the subject, specialized patterns in communicating
scientifically, mode shifting from spoken-like talk to written-like talk, and heightening critical awareness of language that creates scientific concepts. Wells (1993) analyzes that processes (e.g., doing, thinking, being) and tense shift according to students’ physical formations for classroom activities. Language plays a primary role in a whole-class discussion or a secondary role in a hands-on activity corresponding to goals of each activity type. In addition, understanding of the concepts of science requires identification and appropriation of lexicogrammatical features that have semantically relations to develop lexical cohesion.

Lemke (1988), focusing on ideational meanings, demonstrates how processes, participants, and circumstances are configured to have semantic relations: ‘ground’ as an agent, ‘heat’ and ‘light’ as subclass of energy, and ‘from the light energy’ as a circumstance of energy transformation in a clause, “the ground is now creating heat energy, from the light energy”. Lemke argues that it is imperative to teach and learn semantic relations among discipline-specific words in mastering academic subjects including science. Besides understanding such semantic relations, mode shifting between academic language and everyday language plays a vital role as linguistic scaffolding that enables students to connect their everyday experience to scientific concepts (Gibbons, 2003). Linguistic scaffolding in the science classroom should be provided to raise critical awareness of language that can shape how learners interact with scientific texts (O’Hallaron, Palinscar, & Schleppegrell, 2015). According to O’Hallaron and her colleagues, students should be taught explicitly to recognize that scientific texts, which are generally believed to only concern facts, are infused with authorial ideas through interpersonal adjuncts (e.g., interestingly, fortunately).

In studies focused on both talk and written text, researchers address mostly how teachers’ use of language is connected to students’ subsequent writing or construal of science textbooks (e.g., de Oliveira & Lan, 2014; Haneda, 2000; Hodgson-Drysdale, 2014; Young & Nguyen, 2002). They
highlight mode shifting from everyday language to academic language instantiated through teacher talk and students’ writing. Studies show that the mode shifting was salient in use of process, participant, and cohesive devices. For instance, de Oliveira et al. demonstrate that a student’s writing made a significant progress associated with inclusion of a variety of technical terms (e.g., dishwasher soap, float, sink) and logical connectives (e.g., first, next, finally), which are compared to the words included in the first writing (e.g., soap thingy, pour, when, then). One study at the secondary level examines differences in mode between textbooks and teacher talk concerning process types, participants, and conjunctions. Young et al. find that scientific register used in a physics textbook involves detached third-person participants, a range of relational processes (e.g., be, have, look), passive voice, and nominalization while teacher talk entails first-person participants, predominant use of process ‘be’, active voice and less layered expressions. Studies focused on the links between talk and written text suggest that students should be provided explicit instruction to recognize linguistic differences between spoken language and written language and to apply linguistic knowledge to understand concepts of science.

Research concerning literacy practice attends to text types typical of science classes and discrepancies between scientific texts and students’ writing. One study at the elementary level addresses that a ‘graphic organizer’, which is one of distinctive genres identified in a second-third-grade classroom, included general participants (e.g., we, they) and timeless present tense. The science literacy practices drew on multimodal resources such as drawing and writing in relation to scientific concepts (Honig, 2010). Research in middle school emphasizes semantic relations of domain-specific terms. Seah, Clarke, & Hart (2014) examine how scientific concepts are represented in scientific accounts and students’ writing. The findings show that students lacked understanding how participants, processes, and circumstances are semantically related to each
other to function in creating scientific concepts, proposing that analysis of semantic relations of content-specific terms—“content analysis” (Seah et al., 2014, p.959)—is critical in disciplinary instructions.

2.8.2 Mathematics

Studies on academic language in mathematics only considered the secondary settings, mainly focusing on how teachers use language to explain mathematical concepts associated with geometry and algebra. Only one of the studies considers the links between classroom talk and written text. In research focused on teacher talk is discussed how to construe mathematical concepts and engage students in learning the concepts. The construal of mathematical concepts involves identification of lexical cohesion among mathematical terms connected to everyday experience and mathematical concepts. Engagement in learning entails articulating the interpersonal metafunction and mode shifting from everyday language to disciplinary language.

Mathematical terms are lexically chained, which teachers construe at the paradigmatic level (Herberl-Eisenmann & Otten, 2011). For instance, teachers referred ‘base’ and ‘height’ not only to an entity of a polygon in teaching a geometric concept but also to an extent in referring to the formula of a polygonal area. Herberl-Eisenmann and Otten suggests that students should be provided explicit instruction to understand the lexical relations among the terms. In addition, teacher talk was structured coherently by reiteration and collocation, which allows a teacher to use analogies for guiding students to understand a mathematical concept of the ASA congruence theorem (Gonzalez, 2015). The teacher linked analogies pertaining students’ everyday experience to the theorem by drawing upon clauses with parallel structures.
With regard to students’ engagement in the mathematics classroom, researchers find that teachers can encourage students to recognize mathematical concepts by modulating mood types and linking everyday experience to mathematical concepts. The modulation of mood types associated with the interpersonal metafunction enables students to orally express their ideas and identify a target concept. For example, modulating imperatives into interrogatives and declaratives helped students to think mathematically and verbalize their thinking in the whole-class discussion (Zolkower & Shreyar, 2007). Besides such interpersonal metafunction, research focused on the linkage between talk and written text highlights mode shifting from everyday language to mathematical language, which contributes to students’ engaging in learning knowledge of mathematics. Chapman (1995) finds that the teacher shifted modes from everyday talk (e.g., *it was the same*) students provided into mathematical terms (e.g., *the difference pattern is constant*), maintaining the same thematic pattern with different wording. The analysis indicates that mode shifting played a role in transforming technical and abstract terms to context-dependent words, encouraging students’ engagement in learning.

### 2.8.3 History

Attending to high demand on language in history, research on academic language in the content area highlights that close reading and text analysis based on metalinguistic knowledge enable students to improve history literacy abilities (Achugar & Carpenter, 2012). The researchers demonstrate that metalanguage-based instruction associated with identification of historical participants and events helped students read historical documents carefully and retrieve discipline-specific terms (e.g., slavery, equality) to produce historical texts. The findings indicate that explicit explanation using metalanguage can lead students to have critical awareness of language, comprehend history texts, create lexically dense texts, and use syntactically complex clauses.
2.8.4 English

Studies in the English classroom, mainly focusing on literacy practices, address use of SFL-based metalanguage, linguistic challenges, and genre-based pedagogy. Research, considering talk or both talk and literacy practices, describes the impact of talk about language on students’ reading and writing (Chen & Jones, 2012; Moore & Schleppegrell, 2014). Talk about language associated with polarity and appraisal enables students to engage in learning how authors use language to shape characters in literary work by professing their everyday experience related to literary texts. The oral representation results in meaningful discussion between teacher and students (Moore & Schleppegrell, 2014). In addition, students’ understanding of metalanguage such as participants, processes, and circumstances leads to changes in terms of grammatical intricacy, lexical density, and transitivity in their subsequent writing (Chen & Jones, 2012). Furthermore, research focused on textual practices demonstrates that literacy instruction involving SFL metalanguage promotes knowledge of text organizations, which encourages students to recognize how lexis and grammar are patterned according to genres and improve their writing (Gebhard, Chen, & Britton, 2014).

With regard to linguistic challenges, however, studies show that there is some discrepancy in the appropriation of participants corresponding to different genres. Gebhard and her colleagues (2014) state that third-graders who participated in one-academic-year case study showed their understanding of ways person-participants (e.g., I, we, he, she, they) are used correspondingly in three different genres: historical, biological, and scientific explanations. In contrast, one study involving multi-graders from grade 3 to 5 indicates that appropriating person participants to genres is demanding, especially in relation to differentiated use of singular/plural pronouns and utilization of third-person pronoun(s) in genres including historical recount and exposition (Brisk, 2012).
In studies focused on analyzing teaching practices, researchers address how teachers implement genre-based pedagogy to develop reading and writing abilities. The studies highlight a curricular design using the teaching-learning cycle, emphasizing that explicit instruction including text deconstruction and joint-construction enables students to raise awareness of language and organize a text in a purposeful way (e.g., Gebhard et al., 2014; Gebhard, Harman, & Serger, 2007; Pavlak, 2013). The studies demonstrate that students accomplish knowledge of the ideational metafunction realized through participants, processes, and circumstances. In addition, one study describes that GBP-informed instruction supports students’ knowledge of lexical cohesion associated with paradigmatic and syntagmatic levels, suggesting that development of lexical cohesion is vital for students to create a context-specific text with lexicogrammatical features which can be contextualized in another context (Harman, 2013).

2.9 Summary

The findings from this review of research on academic language and literacy development using SFL in K-12 contexts show that the studies based on three strands of research foci address differences in contexts by school levels and by subject matters. At the elementary level, research mostly examines whether students’ writing changes with SFL-informed instruction, focusing either on students’ textual practices or on the link between oral interactions and writing practices. At the secondary level, the majority of studies investigate how teachers deconstruct curricular concepts by talk to accomplish goals of each activity type. The research in elementary school indicates that SFL-informed instruction results in promising effects in developing ELLs’ academic language and literacy abilities by raising linguistic awareness of how language use varies in constructing curricular concepts. The studies in secondary school find that lexical cohesion, which refers to semantic relationships among lexical items, is salient in words of teacher talk to construe the
concepts of subject matters. With respect to variations by subject matters, research foci feature discipline-specific aspects. For example, studies on science and mathematics mostly consider spoken texts while studies on English language arts and history mainly examine written texts although research in history/social studies classrooms is especially scant. The results indicate that teacher talk in science and mathematics classes is deconstructing and unpacking curricular concepts to support students’ concept development mainly in a way to shift modes from everyday language to disciplinary language. In the English classroom, textual practices involve students’ uptake in text organization and academic registers. In the history classroom, use of modality and lexical density are appropriated in students’ writing.
CHAPTER 3
THEORETICAL FRAMEWORK

3.1 Introduction

To explore how teachers can use language to support academic language and concept development for ELLs in the mathematics classroom, this study rests theoretically on sociocultural perspectives on concept and language development. In the subsequent sections, I first outline Vygotsky’s (1986) theory of concept development, which regards language as unit of a concept and addresses the interplay between spontaneous and scientific concepts in developing real, complete concepts. I then overview Halliday’s theory of language called systematic functional linguistics (SFL), which views language as a social semiotic meaning-making system that works to accomplish a particular purpose in a specific context. Next, I outline approaches to mathematics education from semiotic perspectives and summarize this chapter.

3.2 Vygotsky’s Theory of Concept Development

Attending to language as a critical tool in development, Vygotsky (1986) argues that a word meaning is the unit of verbal thought or a concept and that development of concepts is represented through the ability to use means such as words or signs in voluntary and purposeful ways. Word meanings are historically and socially constructed in that they entail generality to enable communication between participants in a specific community. In particular, linguistic interactions between adult and child are an essential foundation of the child’s language and concept development.
The concept development is not identical to learning. Rather, development and learning are inseparable in that mental development results from learning. Learning is viewed as “a necessary and universal aspect of the process of developing culturally organized, specifically human, psychological functions” (Vygotsky, 1978, p.90). Knowing word meanings as psychological functions is only the beginning moment of concept development, not the completion. For example, knowing meaning of a word 'subtraction’ does not indicate that the child can solve the subtraction problem by understanding the concept. The concept of subtraction, like any other concept, is learned through social interactions mediated by language and other semiotic means such as gestures and equations as the concept develops into an internalized concept. Thus, the mental development for solving problems requires the internalizing process of external knowledge through linguistic guidance from more knowledgeable others including peers. The guidance is effective only within the child’s zone of proximal development (ZPD), which is defined as “the distance between the actual development level as determined by independent problem solving and the level of potential development as determined through problem solving under adult guidance or in collaboration with more capable peers” (Vygotsky, 1978, p.86). The potential level of development indicating assisted performance can become the actual development level as individual performance once the child can independently perform some task faced at the potential development level, as shown in Figure 3.1. The ZPD viewed as a maturing or learning process, in which scaffolding through language and multi-semiotic resources plays a primary role, determines the child’s development of higher mental functions.
Figure 3.1 The dynamic nature of the ZPD (Bodrova & Leong, 2007, p.42)

Highlighting the child’s competence in assisted performance, Vygotsky (1986) emphasizes classroom instruction in development. According to him, children experience different levels of concept development with and without schooling. Without schooling, everyday/spontaneous concepts develop to be characterized as context-dependent and non-systematic, which prevents the child from using the concepts voluntarily. With schooling, scientific concepts develop to be characterized as abstract, detached, and mediated.

As such, spontaneous concepts differ from scientific concepts. The former evolves from everyday experience while the latter requires systematic instruction. In this sense, spontaneous concepts can include actions and everyday language evolving from everyday experience. Scientific concepts involve “verbal definition” (Vygotsky, 1986, p.148). Scientific concepts and spontaneous concepts are complementary in that spontaneous concepts should shape and be shaped by scientific concepts and vice versa. In other words, spontaneous concepts are used to make sense of scientific concepts while scientific concepts are used to regulate spontaneous concepts (Saxe, de
Kirby, Kang, Le, & Schneider, 2015), as shown in Figure 3.2. This figure indicates that actions are viewed as spontaneous concepts while formal definitions as scientific concepts.

![Diagram showing the interplay between everyday and scientific concepts](image)

**Figure 3.2** The interplay between everyday and scientific concepts (Saxe et al., 2015, p.17)

According to Vygotsky (1986), the directionalities of everyday and scientific concepts are opposite. At a certain point, these two concepts intersect and influence each other. He states, “the development of the child’s spontaneous concepts proceeds upward, and the development of his scientific concepts downwards, to a more elementary and concrete level” (p.193). Upward development of spontaneous concepts indicates that “everyday concepts, which have roots in children’s reactions to local situations, develop from the ‘bottom up,’ toward increasing levels of adequacy and generality” while downward development of scientific concepts suggests that “scientific concepts, introduced through explicit instruction and initially understood at a shallow level, develop from the ‘top down’ as children enrich and transform them through their conscious application to local situation” (Saxe et al., 2015, p.6). Within the interplay between the two concepts, which is required for real concept development, spontaneous concept formation is a
prerequisite for scientific concept formation. Vygotsky (1986) argues, “success in learning a foreign language [scientific concept] is contingent on a certain degree of maturity in the native language [spontaneous concept]” (p.195). Namely, the child develops scientific concepts within the ZPD in which his/her spontaneous concepts function as foundation for scientific concept formation.

The interplay between them results in development of a ‘real concept,’ which “must be taken only together with its system of relations that determine its measure of generality” (Vygotsky, 1986, p.173). The real concept of ‘flower,’ for instance, is formed when the child recognizes the system of the relation between ‘flower’ (i.e., scientific concept) and ‘rose’ (i.e., everyday concept) by viewing flower as more general concept and rose as subordinate concept of flower (p.173). A pseudoconcept, “i.e., a form of generalization that substantially differs from the real concept,” can be “an image that is by no means a simple sign of a concept,” which is given to children “in a ready-made form, from the speech of others” (Vygotsky, 1986, p.122). At the pseudoconcept development level, the child can utter ‘flower’ by imitating an adult’s speech without a real concept of flower as general name and he/she, as a result, has difficulty using the word to refer to various kinds of flowers such as rose, tulip, and dandelion. Thus, verbal communication with adults plays a powerful role in developing the child’s real concepts.

Real concepts are represented through the ability to use language functionally in describing concepts. Vygotsky (1986) writes: “the central moment in concept formation, and its generative cause, is a specific use of words as functional ‘tools’” (p.107). In the mathematics classroom, for example, the real concept of ‘regrouping’ in subtraction of ‘564–458’ can develop when the student first makes sense of regrouping through the action of ‘trading one tens for ten ones’ by stating ‘I traded the one tens for ten ones, so we were regrouping’ as inductive reasoning and later regulates the action as one instance of regrouping by the mathematical concept. To regulate the everyday
concept, the student can make deductive reasoning by saying ‘I crossed out the six [in the tens place] and put a five [to make fourteen ones] because she was regrouping.’ In other words, the ability to describe scientific concepts and everyday concepts with language is indicative of concept development.

Real concepts are formed through the coordination of spontaneous concepts and scientific concepts. For example, mathematical concepts comprise both “everyday mathematics” (Schliemann & Carraher, 2002, p251) based on everyday experience and school mathematics based on mathematical practices such as processing an algorithm. In this respect, the relation between spontaneous and scientific concepts is not hierarchical but complementary in value. Schliemann and Carraher (2002) show that students’ previous everyday experience associated with space was used for resources for making sense of scientific concepts mediated by visual representations (e.g., graphs).

Concept development follows learning, which is inseparable from development. In children’s learning and development, imitation, which is regarded as “pseudoconcepts” rather than real concepts, plays a significant role because “he [the child] does not create his own speech, but acquires the speech of adults” (Vygotsky, 1986, p.122). The child can produce spontaneous and creative speech through imitation.

Vygotsky (1986) describes as follows:

In the child’s development, imitation and instruction play a major role. They bring out the specifically human qualities of the mind and lead the child to new developmental levels. In learning to speak, as in learning school subjects, imitation is indispensable. What the child can do in cooperation today he can do alone tomorrow. (p.188)
In other words, imitation is a stepping-stone for the child’s development as opposed to simple reproduction. In particular, “deferred imitation” exists in the pathway to “creative and spontaneous speech” that adults produce (Speidel & Nelson, 1989, p.174). For example, a teacher utters, “You count back with me because we’re subtracting” and a student says, “I crossed out the six and put a five because she was regrouping” after several lessons. The student’s utterance is deferred imitation in that everyday language (e.g., ‘I crossed out the six and put a five’) comes in the primary clause just as in teacher’s speech ‘You count back with me’ and academic language ‘subtracting’ and ‘regrouping’ comes in the dependent clause. In the classroom, student talk that mirrors teacher talk is not a simple imitation but a critical process to the child’s development. Vygotsky (1986) states, “Verbal communication with adults thus becomes a powerful factor in the development of the child’s concepts” (p.123).

In sum, word meaning is the unit of a concept. The concept development interacts with learning within the child’s ZPD. The scope of the ZPD can vary according to ways of scaffolding. Concepts develop differently before and after schooling. Without schooling, spontaneous concepts evolve from everyday, concrete, personal, local, and context-dependent experience and develop upwards to make sense of scientific concepts. With schooling, scientific concepts, which are abstract, generalized, systematized, and context-detached, develop downwards to be connected to spontaneous concepts. The relations between spontaneous and scientific concepts are complementary, not hierarchical in value because spontaneous concepts are a critical foundation in developing scientific concepts and scientific concepts become concrete, practical, and applicable when they reach spontaneous concepts. To develop real concepts, the interplay between spontaneous and scientific concepts plays a critical role in a systematic instruction, in which language can play a primary role. Vygotsky, however, did not explicate how language makes
meaning to mediate concept development. In this regard, how language functions for meaning making can be explored in light of systemic functional linguistics as sociocultural theory of language.

3.3 Halliday’s Theory of Language

Systemic functional linguistics (SFL), which is also called functional grammar, aims to understand how multisemiotic resources including language work and how people use the resources to make meaning. SFL views language as social semiotic in that it is a meaning-making system that is socially constructed to accomplish a specific purpose in a particular context (Halliday, 1978, 1985, 1994; Halliday & Matthiessen, 2014). Language as social semiotic represents that language use shapes and is shaped by a social structure or a culture. SFL refers to grammar as a system of meanings, which are realized through lexicogrammar (or syntax). Lexicogrammar represents how meaning is structured. In lexicogrammar, a clause is a structural unit, “to which functional configurations can be assigned” (Halliday, 1994, p.16). A clause is higher than a group or phrase and lower than a clause complex in the hierarchy of constituency. A clause complex is “the grammatical and semantic unit formed when two or more clauses are linked together” (Eggins, 2004, p.255). The hierarchy of lexicogrammatical constituency ranges from word through group and clause to clause complex. In SFL, a clause is a central unit in lexicogrammar while a text is a semantic unit. In the following sections, I first define what a text is and describe how language in the text works differently in the context of situation. Second, I outline how a clause as a unit of message realizes meanings. Third, I move toward a clause complex above the clause to show how clauses are related in expanding meanings.
3.3.1 Text and Context

According to Halliday and Matthiessen (2014), text as a semantic unit refers to “an instance of the linguistic system,” which can be extended to “multimodal texts—instances of more than one semiotic system” (p.46). A text is contextualized in both an immediate and broader context in which the text is created and interpreted (Halliday & Hasan, 1985). Text and context are intertextual, which allows predictability. For example, a text as something that is spoken in the mathematics classroom can be predicted and understood by students from linguistic knowledge and from cultural, situational, and verbal context. A text is realized as an instance through register, which refers to a set of semantic variables according to context of situation where language is used (Halliday, 1978). Register comprises three variables: field, tenor, and mode.

The three register variables are instantiated in the continuum of language use according to situation types (Eggins, 2004). Situation types of three register variables can be described as shown in Figure 3.3.

<table>
<thead>
<tr>
<th>Field</th>
<th>Everyday</th>
<th>Technical</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tenor</td>
<td>Informal</td>
<td>Formal</td>
</tr>
<tr>
<td>Mode</td>
<td>Spoken</td>
<td>Written</td>
</tr>
</tbody>
</table>

Figure 3.3 Situation types of language use (see Eggings, 2004)

In Figure 3.3, the field includes two extreme situation types: everyday/concrete/ familiar and technical/abstract/unfamiliar situations. Field variables construct commonsense knowledge in an everyday situation but abstract and specialized knowledge in a technical situation. The two
extreme situations of the tenor continuum are informal, contextualized, and formal, decontextualized. Tenor variables represent power relations, degrees of contact, and affective involvement, three aspects of which range from equal to unequal power, from frequent to occasional contact, and from high to low affective involvement respectively. The mode continuum covers spoken through written discourse. Mode variables represent degrees of both visual-aural contact and immediacy of feedback from interactants. In other words, spoken discourse is characterized by dynamic face-to-face conversation with immediate feedback exchanges between interactants while written discourse is characterized as planning, editing, no visual-aural contact between interactants and limited feedback about writing. Despite differences in linguistic features between spoken and written discourse, there is no clear-cut distinction between them (Gibbons, 2006). For example, text message or email is more spoken-like in that they are characterized by contextualization and frequent exchanges with immediate feedback while presidential speeches feature more written-like talk in that the speeches are planned and edited discourse. In this regard, spoken discourse itself can be represented in a continuum between spoken-like and written-like discourse. For example, everyday discourse as spoken-like and mathematical discourse as written-like in the continuum can be described in relation to linguistic features as in Table 3.1.

<table>
<thead>
<tr>
<th>Linguistic Features</th>
<th>Everyday Discourse</th>
<th>Mathematical Discourse</th>
</tr>
</thead>
</table>
| **Examples**        | Altogether we have one hundred students.  
                     | I’m taking away twelve out of seventeen.  
                     | If you cross out eight you can’t do that. So you add, so you add ten to the ones. |  
|                     | There are a total of one hundred students at the school.  
                     | I’m subtracting twelve from seventeen.  
                     | I crossed out the six and put a five because she was regrouping. |
Table 3.1 Language in Everyday Talk and Math Talk

Different use of language in everyday discourse and mathematical discourse is concerned with registers, which are regarded as “ways of saying different things and tend to differ in semantics” (Halliday, 1978, p.35). As shown in Table 3.1, everyday registers include terms familiar to ordinary people (e.g., taking away, one hundred students altogether) while math registers are unfamiliar and domain-specific (e.g., subtracting a total of one hundred students, regrouping). Everyday discourse is displayed by contextualized language including exophoric references such as ‘that,’ which can be understood only by people who are participating in the conversation. Everyday discourse is also lexically sparse and grammatically intricate or wordy while mathematical discourse is lexically dense and grammatically simple. These linguistic choices are determined by context of situation in making meaning (Halliday, 1978, 1994). In other words, what language means is fully understood within the context of language use. The following section describes how meaning or message is realized in a clause.

3.3.2 Clause as a Unit of Message

Three register variables of field, tenor, and mode realize three meaning potentials or metafunctions—experiential, interpersonal, and textual—respectively. The three different metafunctions are represented simultaneously in a clause. Experiential and logical metafunctions constitute ideational metafunction. The logical metafunction concerning clause complexes above the clause is described in section 3.3.3. What follows outlines experiential, interpersonal, and textual metafunctions.

The experiential metafunction realized by field variables construes internal and external experience of reality, which is construed by the system of transitivity into “a manageable set of
PROCESS TYPES” (Halliday, 1994, p.106). The notion of transitivity is concerned with extension of process to other entities in a clause. A process “as set up in the grammar of the clause” comprises three components: “(i) the process itself; (ii) participants in the process; (iii) circumstances associated with the process” (ibid., p.107). In traditional grammar, processes refer to verbs and participants to nouns or noun phrases (e.g., ‘an equal number of balloons’) associated with processes. Circumstances correspond to adverbials or prepositional phrases (e.g., ‘between two and three’), which represent time, place, manner and so on.

As the nucleus of the transitivity system that realizes experiential meanings, types of process include material, mental, relating, verbal, behavioural, and existential processes. Material and relational processes, followed by mental and verbal ones, are most frequently used in both spoken and written texts, (Halliday and Matthiessen, 2014, p.215). This current study focuses on these four types of process, which are illustrated in Figure 3.4, because instructional registers, which this study examine, that target students’ language and content learning, mainly include the four processes (see Christie, 2002). What follows describes the four types of process: material, relational, mental, and verbal.
Material or doing processes mostly concern events, activities, and actions (e.g., taking away, subtracting, trading, regrouping). Clauses involving material processes—material clauses—construe "our experience of the material world" (Halliday & Matthiessen, 2014, p.245). Material clauses inherently involves one participant, which can be an animate or inanimate Actor. Material processes are subtyped into two: happening and doing process. In a material clause 'We went from ten to fifteen,' which a teacher can use in counting coins (i.e., dime and nickel), the outcome of the process 'went' happened to only one participant 'We.' A material process can impact two participants as in 'I crossed out the six,' in which 'I' and 'the six' are called Actor\(^3\) and Goal respectively. The result of the process 'crossed out' done by the Actor 'I' befell the Goal 'the six.'

---

\(^3\) This dissertation study uses capitalized terms such as 'Actor' in indicating functions of language.
Unlike material processes, relational processes (e.g., being verb, having verb) inherently involves two participants. Relational processes are most prevalently used in the mathematics classroom (O’Halloran, 2004; Veel, 1999). Relational processes are subcategorized into attributive and identifying ones. In an attributive-relational clause, the process relates a Carrier to an Attribute, as shown in Table 3.2. The attributive-relational process concerns three features: intensive, possessive, and circumstantial. The intensive relation is used to classify an entity or specify quality of an entity. The possessive relation links a Possessor to a Possessed while the circumstantial relation connects a Carrier to a circumstantial feature (Halliday & Matthiessen, 2014, p.268).

<table>
<thead>
<tr>
<th>Carrier</th>
<th>Relational</th>
<th>Attribute</th>
</tr>
</thead>
<tbody>
<tr>
<td>Eight</td>
<td>is</td>
<td>greater than five</td>
</tr>
<tr>
<td>I</td>
<td>have</td>
<td>five hundreds</td>
</tr>
<tr>
<td>It [the hour hand]</td>
<td>is</td>
<td>between the two and the three</td>
</tr>
<tr>
<td>Rectangles</td>
<td>are</td>
<td>parallelograms</td>
</tr>
<tr>
<td>Carrier</td>
<td>Relational</td>
<td>Attribute</td>
</tr>
</tbody>
</table>

Table 3.2 Examples of attributive-relational clauses

In the clause ‘eight is greater than five’ in Table 3.2, the process ‘is’ specifies the quality of the participant ‘eight’ as ‘greater.’ ‘Greater,’ though not a noun, also functions as a participant because the word ‘number’ is embedded in context after ‘greater.’ In the clause ‘I have five hundreds,’ the Carrier ‘I’ possesses the Attribute ‘five hundreds.’ In the clause ‘It is between the two and the three,’ the process ‘is’ relates the Carrier ‘It’ to the circumstantial feature ‘between the two and the three.’ The process ‘are’ in the clause ‘Rectangles are parallelograms’ relates ‘Rectangles’ as Carrier to ‘parallelograms’ as Attribute, where the two participants cannot be reversed into ‘Parallelograms are rectangles.’

In the identifying-relational clause, on the other hand, two participants can be reversible.
For example, an identifying clause ‘the hour is three o’clock’ can be reversed into ‘three o’clock is the hour.’ The two participants in the identifying clause refer to the same entity. One of the two corresponds to Token as expression while the other does to Value as meaning or content. One identifies the other. The relationship between Token and Value is symbolic in that the identifying clause construes a meaning or a Value ascribed to a symbol or a Token (Matthiessen, 1991, p.91).

For example, in an identifying clause ‘It [half of] is division,’ ‘It[half of]’ is a symbol that represents the meaning ‘division.’ Table 3.3 presents the grammatical analysis of an identifying clause in light of field including a participant and a process.

(What is half of?)

<table>
<thead>
<tr>
<th>Identified/Token: participant</th>
<th>Relational process</th>
<th>Identifier/Value: participant</th>
</tr>
</thead>
<tbody>
<tr>
<td>It [half of]</td>
<td>is</td>
<td>division</td>
</tr>
</tbody>
</table>

Table 3.3 Example of an identifying clause: decoding one

In Table 3.3, the Token is identified by reference to the Value. This identifying clause is a decoding one. As shown in Table 3.4, however, the identifying clause is an encoding one when a Value is identified by reference to a Token.

(What is division?)

<table>
<thead>
<tr>
<th>Identified/Value</th>
<th>Relational process</th>
<th>Identifier/Token</th>
</tr>
</thead>
<tbody>
<tr>
<td>It [division]</td>
<td>means</td>
<td>that a number is getting smaller</td>
</tr>
</tbody>
</table>

Table 3.4 Example of an identifying clause: encoding one

Table 3.4 shows that the Value ‘division’ is identified by reference to the Token ‘that a number is getting smaller.’ Division is the nominalized form of a process ‘divide.’ Halliday and
Matthiessen (2014) describe that the nominalization, by which an event realized in the process turns into an object, is always the Value (p.285). Figure 3.5 illustrates “the direction of coding” (Halliday & Matthiessen, 2014, p.280), which can also display language and concept development given that Token represents something concrete and context-dependent while Value does something abstract and context-detached.

This directionality determines whether the clause is operative/active or receptive/passive voice. The decoding clause is operative while the encoding one is receptive. In determining the voice, a substitute ‘represent’ instead of the verb ‘be’ can be used (see Halliday & Matthiessen, 2014, p.281). For instance, ‘half of represents division’ instead of ‘half of is division’ whereas ‘division is represented by that a number is getting smaller.’ The determination of the voice leads to identifying which one of the two participants in the identifying clause is Token or Value. In other words, the Subject of the decoding clause is a Token whereas that of the encoding clause is a Value, as shown in
Table 3.3 and 3.4. It is the identification of a Token-Value relationship that allows us to understand the relationship between lexicogrammar as expression and semantics as meaning (Matthiessen, 1991). In other words, the Token corresponds to an observable entity (e.g., action of counting back, the hour hand) while the Value does to something in mind or meaning (e.g., subtraction, the hour). Indexical representations (e.g., manipulatives) are Token-like while symbolic representations (e.g., number sentences) are Value-like. Language as a representation has a dual nature: everyday language as Token and academic language as Value. In the identifying clause ‘that a number is getting smaller represents division’ described above, ‘that a number is getting smaller’ is a Token realized in everyday language while ‘division’ is the Value realized in academic language. The Token indicates an instance of the Value as follows: ‘that a number is getting smaller’ as an instance of division.

In addition to determination of the voice of the clause, the direction of coding can play an important role in developing texts. Halliday and Matthiessen (2014) state that “decoding clauses can be used as a strategy for interpreting phenomena that have been observed” while “encoding clauses serve as a resource for presenting the steps in the organization of a text” as in a consequential explanation (p.283-284). For example, in the decoding clause ‘the action of counting back is subtraction,’ the Token ‘the action of counting back’ as an observable phenomenon is interpreted as the Value ‘subtraction.’ Relating one observable instance to the abstract concept can be used as a strategy to explain the concept. The Token-Value relationship can be regarded as part-whole in that the Token is one instance of the Value. In the encoding clause ‘subtraction is the action of counting back,’ the Value ‘subtraction’ is manifested by the Token ‘the action of counting back’ as one of various attribute potentials in explaining the abstraction concept ‘subtraction.’ The encoding clause can be concerned with deductive reasoning in that a generalized concept is applied to a
specific instance. In contrast, the decoding clause can be related to inductive reasoning because one representation is used to make sense of a generalized concept. Decoding the Token by reference to the Value can be seen as upward concept development, in which spontaneous concepts are used as resource to make sense of scientific concepts. Encoding the Value by reference to the Token can be viewed as downward concept development, in which scientific concepts are applied to regulate spontaneous concepts. The coordination of decoding the Token and encoding the Value can result in real or complete concept development.

In contrast to material clauses, mental clauses construe our experience of the inner world of our consciousness. Mental clauses always involve one conscious participant as Senser, which can be human or human-like (e.g., pets, toys in a story). The analysis model of a mental clause ‘how did she figure that out?’ is shown as Senser + Process + Phenomenon. The property of mental processes differentiated from material processes lies in “the potential for combining with projections” as in a clause ‘what hour do you think it would be?’ The projections take place in clause complexes, which will be described in section 3.3.

Verbal processes in the transitivity system realizing the experiential metafunction are often used to develop dialogic texts. Verbal clauses always include one participant as Sayer, which can accommodate addressee and “the content of what is said” (Halliday & Matthiessen, 2014, p.306). For example, in a clause ‘it is telling us how someone has more than someone else,’ the verbal process ‘telling’ associated with be verb (i.e., ‘is’) involves the Sayer ‘it’ accompanied by the addressee ‘us’ and the content of what is said ‘how someone has more than someone else.’ The content is projected by the verbal process. The verbal projection will be described more in section 3.3.3.

While the experiential metafunction expresses our experience of the external world and our own internal world, the interpersonal metafunction enacts social relations (Halliday, 1978). The
interpersonal meaning is realized through tenor variables, which is determined by who is participating. Tenor includes a system of choices in mood types (i.e., declarative, interrogative, imperative), and modality (e.g., can, may, have to) associated with certainty and usuality. The mood system, which is a focus of this current study, involves Subject and Finite. The Subject is the element that takes responsibility for information while the Finite “has the function of making the proposition finite” (Halliday & Matthiessen, 2014, p.144). The Subject corresponds to ‘subject’ in the traditional grammar and the Finite is concerned with the element of tense (e.g., past, present). These two elements determine mood types. For example, the Subject comes before the Finite in the declarative whereas the one comes after the other in the yes-no interrogative. The relationship between form and function in the system of mood types can be explained in the following example: (i) Count back with me; (ii) You count back with me. Clause (i) and (ii) can represent the same speech function (i.e., request) but the choice of mood types expresses different interpersonal meanings. The use of imperative mood in clause (i) can imply that the speaker is positioned as a leader in authority and the listener as one subject to authority. The use of declarative mood in clause (ii) can imply that the speaker is positioned as an expert who provides factual information to the listener (Eggins, 2004).

Enabling experiential and interpersonal metafunctions, the textual metafunction determined by the system of mode (e.g., monologic, dialogic, spoken, written) is concerned with “imposing order on the endless variation and flow of events” through Theme-Rheme structure (Halliday, 1994, p.106). The Theme is defined as “the element which serves as the starting point of the message; it is that with which the clause is concerned” while the Rheme is “the remainder of the message” (ibid., p.37). The Theme-Rheme structure develops flow of a text. The Theme “typically contains familiar, or ‘given’, information, i.e. information which has already been mentioned somewhere in the text or
is familiar from the context” whereas the Rheme “typically contains unfamiliar, or ‘new’, information (Eggins, 2004, p.299-300). For example, in a clause ‘We have five hundreds,’ the Theme ‘we’ as given information functions as starting point of the message while the Rheme ‘have five hundreds’ develops the Theme and includes new information. The Theme is associated with the Subject. In the interrogative ‘how many ones do you have?’ the Theme dissociated with the Subject contains new information ‘how many ones.’ The dissociated form is speaker-oriented because the speaker chooses to take new information as starting point of message. In a statement-like question ‘You have how many ones?’ the Theme is associated with the Subject and given information is placed in the Theme while new information in the Rheme. Table 3.5 shows the Theme/Subject association. The Subject-Theme association in the WH-interrogative is listener-oriented because the starting point of the message is already known to the listener (see Halliday & Matthiessen, 2014, p.120).

<table>
<thead>
<tr>
<th>Tenor</th>
<th>Mode</th>
<th>You</th>
<th>have</th>
<th>how many ones?</th>
</tr>
</thead>
<tbody>
<tr>
<td>Subject</td>
<td>Theme</td>
<td>Finite</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 3.5 Theme/Subject association

### 3.3.3 Clause Complex as a Sequence of Meaning

The logical function construes relations between clauses in a clause complex through two basic systems: taxis and logico-semantic relations (Halliday, 1978, 1994). As one basic system in the logical function, taxis is subdivided into parataxis and hypotaxis depending on degree of interdependency between clauses. Parataxis represents independent and equal relations between clauses as in the following example: (i) *I’m going to start at seventeen* (ii) *and get to twelve*, (iii) *so I’m subtracting*. The relationship between clause (i), (ii) and (iii) is logically symmetrical and each
clause is free or independent. The three clauses are loosely related by the linkers ‘and’ and ‘so.’ Hypotaxis, on the other hand, expresses the unequal relations between dominant clause and dependent clause as shown in the following example: (i) *You count back with me* (ii) *because we’re subtracting.* This clause complex is semantically tightened by the binder ‘because’ and the meaning is tighter than ‘we’re subtracting, so you count back with me.’

A clause complex linked hypotactically can be viewed in the Theme-Rheme structure as an extended clause because the dependent clause functions as a circumstance (Eggins, 2004, p.315). As shown in Table 3.6, clause complex (1) ‘*You count back with me because we’re subtracting*’ comprises a dominant clause ‘*You count back with me*’ and a dependent clause ‘*because we’re subtracting.*’ The dominant clause as the Theme is initiating the message of the clause while the dependent clause as the Rheme contains new information of subtraction. On the other hand, clause complex (2) entails unfamiliar information ‘subtracting’ in the Theme, which typically contains familiar information, while the Rheme involves familiar information ‘counting back.’ Clause complex (2) is speaker-oriented, which can make the listener find it hard to make sense of the new information ‘subtraction.’

<table>
<thead>
<tr>
<th>Theme</th>
<th>Rheme</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1) you count back with me</td>
<td>because we’re subtracting</td>
</tr>
<tr>
<td>(2) because we’re subtracting</td>
<td>you count back with me</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Familiar/given information</th>
<th>Unfamiliar/new information</th>
</tr>
</thead>
</table>

Table 3.6 Analysis of a clause complex as one extended clause
As the other basic system in the logical function, the logico-semantic system comprising projection and expansion “is specifically an inter-clausal relation—or rather, a relation between processes” (Halliday, 1994, p.216). In the projection, verbal or mental clauses project another clause as lexicogrammatical phenomenon or semantic phenomenon respectively. According to Matthiessen (1991), all types of process realized through the transitivity system can be combined with verbal or mental processes to be projected. Figure 3.6 cited from Matthiessen (1991, p.88) shows the examples of projection clauses and non-projection ones. The examples inside the circle indicate projection clauses involving verbal processes (e.g., ‘said,’ ‘showed’) and mental processes (e.g., ‘saw,’ ’knew’) and those outside the circle non-projection clauses.

Figure 3.6 Different ideational perspectives on symbolic processing (Matthiessen, 1991, p.88)
Projection involves two types: verbal and mental projections. Verbal projection represents what is said as “external semiosis” whereas mental projection does what is thought as “internal semiosis” (Matthiessen, 1991, p.86).

Halliday and Matthiessen (2014) describe the projection as follows:

The projection may be a representation of the content of a ‘mental’ clause—what is thought; we call such projections ideas. On the other hand, the projection may be a representation of the content of a ‘verbal’ clause—what is said; we call such projections locutions. Projection may thus involve either of the two levels of content plane of language—projection of meaning (ideas) or projection of wording (locutions). (p.509)

Given that what is thought and what is said are projected through mental and verbal processes respectively, verbal and mental processes in the projection clause can be seen to facilitate construction of wording and meaning respectively. In a verbal projection clause ‘you said we have to do what?’ the projecting clause ‘you said’ functions to show that the other clause ‘we have to do what’ is projected as what is said. The verbal clause ‘you said’ functions to facilitate the listener’s construction of wording to respond to the speaker. On the other hand, in the mental projection ‘What hour do you guys think it would be?’ the mental clause ‘do you guys think’ functions to project the content of the idea ‘what hour it would be.’

In addition, the relationship between the projecting and the projected in a projection clause nexus is symbolic because the content of what is said or what is thought is mediated by language and it, in turn, is a “semiotic phenomenon” (Halliday & Matthiessen, 2014, p.253). The projecting and the projected represent two different order of phenomena—“the lower order of abstraction” and “the higher order of abstraction”—just as Token and Value each represents the “lower expression” and the “higher content” in semantics (Matthiessen, 1991, p.89) Thus, the projecting
and the projected is similar to Token and Value. For example, the projecting (e.g., 'do you guys think') is the Token and the projected (e.g., 'what hour it would be') is the Value. The mental clause 'do you guys think' functions to facilitate deriving what is thought, that is, the relationship between the hour hand as Token and the hour as Value in this situation. In this regard, a question involving the mental projection can be viewed as demanding reasoning. The projection clause is compared with a single clause 'what hour would it be?' in which the listener is asked to identify the Value of the Token, not the relationship of the two entities.

Besides projection, expansion is included in the logico-semantic system. Expansion works to expand meaning through elaboration, extension, and enhancement in clause complexes. Table 3.7 presents the expansion system.

<table>
<thead>
<tr>
<th>Expansion</th>
<th>Subtypes</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>elaborating</td>
<td>expositing</td>
<td>It needs to pass the hour, it needs to pass that number.</td>
</tr>
<tr>
<td></td>
<td>exemplifying</td>
<td>What operation? I like when we add, subtract, multiply, divide.</td>
</tr>
<tr>
<td></td>
<td>clarifying</td>
<td>Someone has more than someone else. I we are comparing.</td>
</tr>
<tr>
<td>extending</td>
<td>adversative</td>
<td>The tens are cut up, but the ones are not.</td>
</tr>
<tr>
<td>enhancing</td>
<td>temporal</td>
<td>She had more when she broke them?</td>
</tr>
<tr>
<td></td>
<td>causal</td>
<td>You count back with me because we’re subtracting.</td>
</tr>
<tr>
<td></td>
<td>conditional</td>
<td>If it’s eighty-two, then that’s eight.</td>
</tr>
</tbody>
</table>

Table 3.7 The system of expansion

In elaboration, “one clause expands another by elaborating on it (or some portion of it): restating in other words, specifying in greater detail, commenting, or exemplifying” (Halliday &

\[4\]‘||’ indicates the border between two clauses.
Matthiessen, 2014, p.444). For example, meaning of a clause 'someone has more than someone else' is specified and clarified in the subsequent clause 'We’re comparing,' as shown in Table 3.7. In extension, the meaning of one clause ‘The tens are cut up’ is extended to another clause ‘the ones are not,’ which adds new information to the preceding clause. The addition is realized through an adversative relation marked by ‘but’ in this situation. In enhancement, “one clause expands another by embellishing around it: qualifying it with some circumstantial feature of time, place, cause or condition” (Halliday & Matthiessen, 2014, p.444).

As in Table 3.7, the meaning of a dominant clause ‘You count back with me’ is enhanced by reference to cause ‘because we’re subtracting.’ The relationship between the action of counting back in the dominant clause and the concept of subtraction in the dependent clause can be viewed as Token-Value relation because the action functions as a symbol of a concept ‘subtraction.’ In this Token-Value structure, the direction of coding represents encoding because the Value (i.e., subtraction) results in the Token (i.e., the action of counting back). In other words, the Value is identified by reference to the Token. On the other hand, in a clause complex ‘I’m going to start at seventeen and get to twelve, so I’m subtracting,’ the direction of coding is decoding because the action of counting back as Token results in the concept of subtraction as Value. Figure 3.7 illustrates the direction of coding in clause complexes developed through a relation of cause.
As shown in Figure 3.7, this current study regards clause complexes linked by a causal sequence (i.e., cause and effect or result and cause) as decoding/encoding process in that clauses combined in clause complexes represent Token in one clause and Value in the other. The decoding clause complex is similar to inductive reasoning while the encoding one to deductive reasoning. Inductive or deductive reasoning can be seen as upward or downward development of concepts as displayed in Figure 3.8.

Figure 3.7 Direction of coding in clause complexes linked by a relation of cause
(adapted from Halliday & Matthiessen, 2014, p.280)
The Token-Value structure, which “is the most difficult to come to terms with in the entire transitivity system,” (Halliday & Matthiessen, 2014, p.84) is dominant in scientific discourse such as mathematics classroom discourse that construes mathematical meanings constructed by multisemiotic resources or multiple representations. In scientific discourse including mathematics classroom discourse, it is imperative to identify what is Token or Value and understand how to connect Token to Value for students’ concept development.

In sum, language is a social semiotic system that simultaneously represents ideational metafunction comprising experiential and logical meanings, interpersonal metafunction, and textual metafunction. The experiential, interpersonal, and textual metafunctions encode content/experience, social participation, and relevance to context through field, tenor, and mode respectively. Register variables of field, tenor, and mode work in the continuum of language use, which varies sensitively according to situation types. Even within a spoken mode the continuum does work: spoken-like and written-like talk. The three register variables function to construct meaning in a clause.

A clause as a unit of message realizes the experiential, interpersonal, and textual meanings are realized in a clause as unit of message while a clause complex above the clause realizes the logical meaning. The experiential meaning is realized through the transitivity system including process, participant, and circumstance. The nucleus of the transitivity system rests in processes or verbs: material, mental, relational, verbal and so on. In particular, relational processes employing two participants (i.e., Token as expression and Value as meaning) are used most prevalently in mathematics classroom discourse. The Token involves multiple representations such as actions,
objects, images, numerical symbols, and language whereas the Value represents their meaning or content. The Token-Value structure is also seen in clause complexes concerning verbal and mental projections. Different types of clause complexes in a logical relation of time, cause or condition can be seen in Token-Value relation.

In SFL, what ‘text’ or discourse viewed as a semantic unit means is construed by lexicogrammar, which works specific to a context such as subject matters including mathematics, science, English language arts, and history. Each subject matter concerns different semiotic resources. Mathematics classroom discourse, which is the focus of this current study, can be viewed from a social semiotic perspective in that meaning is socially constructed through social interaction mediated by classroom talk and multiple semiotic resources contribute to making mathematical meanings (Chapman, 1995, 2003; Lemke, 2003; O’Halloran, 2000, 2004; Schleppegrell, 2007). In this respect, the subsequent section overviews mathematics classroom discourse from semiotic perspectives.

3.4 Semiotic Approach to Mathematics Classroom Discourse

From Halliday’s (1978) social semiotic perspective, mathematics is a multisemiotic system, which functions to construct meaning through language and other semiotic resources such as visual display and symbolism (O’Halloran, 2000). Visual display includes a picture of the analog clock and a number line. Mathematical symbolism includes numerical expressions, algorithms, and equation (e.g., $17 - 12 = 5$).

O’Halloran (2000) describes:

The mathematical symbolism contains a complete description of the pattern of the relationship between entities, the visual display connects our physiological perceptions to this reality [the mathematical symbolism], and the linguistic
discourse functions to provide contextual information for the situation described symbolically and visually. (p.363)

In other words, each multisemiotic resource functions differently to simultaneously construct or construe mathematical meaning potentials. For instance, answering a math word problem (MWP) requires constructing mathematical meaning by multisemiotic resources: making an answer-statement in language; representing an algebraic equation (e.g., $6 - 2 = 4$); and showing the solving process by using a number line, as presented in Table 3.8.

<table>
<thead>
<tr>
<th>MWP</th>
<th>Answer</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Language</strong></td>
<td>Dimitri is 2 years old. His brother Tony is 6 years old. Who is older? How many years older is he? Draw a number line to show your thinking.</td>
</tr>
<tr>
<td><strong>Symbolism</strong></td>
<td>$6 - 2 = 4$</td>
</tr>
<tr>
<td><strong>Visual Display</strong></td>
<td><img src="image" alt="Number line" /></td>
</tr>
</tbody>
</table>

Table 3.8 MWP & Answer
Answering the math word problem should be followed by construing the written text. Language in the written text of math word problem can be analyzed using SFL to identify what is being asked and what information is relevant to solve the problems (Huang & Normandia, 2008). Huang and Normandia state that a task is specified through mood types: declarative mood for making statements, interrogative mood for asking questions, and imperative mood for doing something related to problem solving.

Mathematics classroom discourse from the social semiotic perspective grounded in Halliday’s (1978) theory of language focuses attention on “social interaction: on how people construct systems of meaning, rather than on the systems themselves” (Chapman, 2003, p.129). An instructional approach to mathematics can include deconstruction of written texts, contextualization of mathematical concepts, and scaffolding students’ problem-solving. Mathematics instruction can be implemented by language-based approaches. For instance, Gebhard, Hafner, & Wright (2004) show that “language arts-based approaches” (p.42) can be appropriated for teaching math word problems. The approach includes contextualizing the math word problem by both talk and a list of math words, and collaborated writing such as teacher-student conference and peer editing.

In addition, mathematics classroom discourse attends to language as the mediator to bridge everyday concepts to mathematical/scientific concepts. Veel (1999) describes linguistic features specific to the mathematics class to show how mathematical meanings are construed in the mathematics classroom. First, teachers explain symbolic and visual representations mainly through spoken language to appropriate the mathematical knowledge to students’ levels. Second, language used in the mathematics classroom involves spoken construal of both technical terms (e.g., addition, subtraction, number line) and grammatical features (i.e., grammatical metaphor,
relational clauses, nominal groups). Grammatical metaphor can be realized by nominalization concerning re-configuration of meanings from an action to a concept (e.g., multiply/action vs. multiplication/concept). Relational clauses involving impersonal participants are predominant to explicate definitions of concepts and their attributives: ten tens blocks make a hundreds block. Extended noun groups are distinctive to mathematics language: a total of 100 students at the school.

Third, teacher’s language use is lexically dense while that of students include less content words. Last, the way for the teacher to interact with students is predominant to the Initiation-Response-Follow-up (IRF) sequence found by Sinclair & Coulthard (1975).

Attending to social interaction between teacher and students, Chapman (2003) highlights that “verbal language is the fundamental process for the shared construction of mathematical meaning” in schooling mathematics (p.131). She also addresses that language is a critical resource for all subject matters but each subject matter has “its own ways of speaking and behaving”, emphasizing the importance of examining “the way in which something is said, as well as what is being said” (Chapman, 2003, p.131). In other words, ways to speak in the mathematics classroom can vary by topics of lessons in conjunction with a range of activity types, such as “drawing graphs in algebra, working with calculators in arithmetic, and constructing angles in geometry” (Chapman, 2003, p.132). Various activity types can also include the use of manipulatives (e.g., base-10 blocks, coins, rulers, clocks) in elementary mathematics classroom. Mathematical concepts are not static but constructed through social interaction (Lemke, 2003; Chapman, 2003). Thus, ways that verbal language mediates social interaction varies according to activity types, which draw on different multisemiotic resources.

Chapman (1995) also demonstrates that mathematics classroom discourse is characterized as interweaving mathematical language with everyday language. In other words, teacher talk both
unpacks mathematical concepts constructed by abstract language in the mathematics textbook into concrete everyday language and bridges students’ everyday language to math language. In this interplay teacher talk connects math language including manipulative, symbolism, and visual display to everyday language to support students’ mathematical concept development. The interplay between math language and everyday language can construct mathematical concepts as shown in Figure 3.9.

![Figure 3.9 The interplay between math language and everyday language](adapted from Saxe et al., 2015, p.17)

As in Figure 3.9, everyday language characterized as concrete, context-dependent, and congruent is used as resources for making sense of math language. Math language characterized as abstract, detached, and incongruent is used as resources for regulating everyday language. This suggests that the coordination of everyday language and math language can construct mathematical concepts. Figure 3.9 is adapted from Saxe, de Kirby, Kang, Le, and Shneider (2015) in that linguistic
features of everyday language and math language described in Figure 3.9 share similarities with characteristics of ‘actions’ and ‘formal definition’ respectively.

Saxe and his colleagues (2015) argue that the interplay between actions and formal definitions develop mathematical concepts, grounded in Vygotsky’s (1986) theory of concept development. ‘Formal definition’ of number line is regarded as scientific concepts since the concepts are abstract and detached while actions are viewed as everyday concepts in that ‘counting, displacing, splitting’ actions are practical, empirical and experimental. Given word meaning as the unit of concepts (Vygotsky, 1986), this implies that mathematical concepts develop through the interplay between concrete, everyday language and abstract, math language.

In brief, mathematics classroom discourse is a multisemiotic meaning-making system that involves language and other semiotic resources (e.g., actions, objects, pictures, symbols). The multiple semiotic resources contribute to making mathematical meanings. In the mathematics classroom, classroom talk plays a fundamental role as mediator in relating multiple representations to mathematical concepts. In particular, teacher talk guides students in developing language and concepts. Mathematical concepts are developed through the interplay between everyday language and math language, in which there is no hierarchy in value.

3.5 Summary

Collectively, Vygotsky’s theory of development informs that concepts develop within the ZPD, in which scaffolding through language of more knowledgeable others plays a critical role. Given that the unit of concepts is word meaning, concepts are represented by language. Spontaneous concepts can be constructed by everyday language while scientific concepts can be constructed by domain-specific language including math language. Mathematical concepts can
develop through the coordination of both everyday language and math language. Halliday’s SFL provides a framework to explore how language is used functionally to construct mathematical concepts associated with multisemiotic resources in the mathematics classroom. SFL can show how mathematics classroom discourse interwoven by everyday language and math language unfolds to construct mathematical concepts mediated by multisemiotic resources.
CHAPTER 4

METHODOLOGY

4.1 Introduction

In chapter 4, I first describe the research design that combines a qualitative case-study approach with discourse analysis. I then present research questions of this study and describe the unit of analysis that indicates the scope of this study and my role as a qualitative researcher. Next, I expound data collection and data analysis procedures. Last, I address limitations of this study.

4.2 Research Design

The purpose of this dissertation study is to examine how teachers support ELLs’ academic language and mathematical concept development through mathematics classroom discourse in an urban elementary school. Viewing concepts as word meanings (Vygotsky, 1986), this study defines mathematical concepts as meanings of math words, “which cannot be assimilated by the child in a ready-made form but have to undergo a certain development” (p.146). Grounded in Vygotsky’s (1986) theory of concept development, this study views mathematical concept development as linguistically defining mathematical concepts and relating them to concrete phenomena. Linguistic definitions involve the interplay between everyday language and academic language. The interplay in linguistic definitions represents making sense of academic language with everyday language. From perspectives of systemic functional linguistics (SFL), this study defines academic language (AL) as language used to support teaching and learning school-based subjects (e.g., mathematics, science, social studies, English language acts). Academic registers which involve discipline-specific words and grammar “vary by task, subject matter, and grade level” (Moore & Schleppegrell, 2014, p.92). The relation between AL and mathematical concept development is inseparable but not
identical. Developing AL does not ensure mathematical concept formation to the fullest because real, complete concepts of mathematics develop through the coordination of everyday concepts involving everyday language and scientific concepts engaging AL (Vygotsky, 1986). In this sense, this study views real concepts of mathematics as developed only when students unpack AL into everyday language and repack everyday language into AL.

Based on the concepts of AL and mathematical concept development mentioned above, this study draws on a research design underpinned by qualitative research methodology based on the following assumptions: (1) reality is constructed by individuals; (2) each individual makes sense of a context from their own perspectives; (3) the researcher is a primary instrument in collecting and analyzing data (Creswell, 2014; Hancock & Algozzine, 2006; Merriam, 1998). In other words, I do not examine causal relationship between variables and a phenomenon to discover reality represented through meanings pre-existed in their world (see Bogdan & Biklen, 2016). Instead, I attempt to explore how individuals or participants create meanings in a context mirroring reality constructed by the participants involved in their social world. I intend to explore how the participants being studied perceive a particular situation, considering various data sources pertinent to the participants. In addition, I—a qualitative researcher—am intrinsically subjective in the procedures of data collection and analysis in that my own experience associated with the situation influences “all the judgments about coding, categorizing, decontextualizing and recontextualizing the data” (Starks & Trinidad, 2007, p.1376).

To investigate classroom discourse to mediate teaching and learning of ELLs’ academic language and mathematics, I use case study and discourse analysis based on SFL. This combination of case study and discourse analysis attempts an in-depth analysis of classroom activities, oral interactions between teacher and students, and the context in which the interactions unfold.
Specifically, case study is used to obtain in-depth understanding of a case of an elementary teacher’s support for her ELLs’ academic language and mathematical concept development bounded by activity and time (Bogdan & Biklen, 2016; Stake, 1995) in that different types of classroom discourse and activity emerge and develop for pedagogic purposes of a series of lessons over time. SFL-based discourse analysis examines lexico-grammatical patterns and meanings as well as structures of spoken and written texts used in mathematical activities (Christie, 2002; Gibbons, 2006; Halliday, 1994; Lemke, 1990; Martin & Rose, 2007; O’Halloran, 2004; Wells, 1999). The subsequent section describes more details of the case study and discourse analysis in relation to this current research.

4.2.1 Case Study

I draw on a qualitative case study approach for the following three reasons. First, a phenomenon under study is particularistic—focusing “on a particular situation, event, program, or phenomenon” (Merriam, 1998, p.29)—as aforementioned in the literature review of this dissertation. Little research has been conducted to examine how ELLs develop academic language and mathematical concepts in the elementary mathematics classroom. This class is provided in a pullout classroom setting. Examining this specific instance may contribute to illuminating a general issue associated with teaching both language and the content knowledge to ELLs in the mathematics classroom (Baxter & Jack, 2008; Merriam, 1998).

Second, this study holistically approaches the inquiry under study because the boundaries between the phenomenon of ELLs’ academic language and mathematical concept development and a context relevant to the phenomenon are not clearly obvious (Creswell, 2014; Dyson & Genishi, 2005; Merriam, 1998). This holistic approach focuses on meaning-making process of rather than
outcome of ELLs’ academic language and mathematical concept development. The process-focused examination contains various data sources such as interviews, participant observations, field notes, audiovisual records, photos, and student sample writing. Collecting such multiple data sources enables intense description of here-and-now situations. The description can show how the SFL-informed teacher works with ELLs to enhance their English proficiency and content knowledge of mathematics in a classroom setting, which may relate to a range of variables dependent on a here-and-now situation. Merriam (1998) states that case study is a suitable design “if the variables are so embedded in the situation as to be impossible to identify ahead of time” (p.32).

Third, the intent of the study is interpretive (Merriam, 1998) or instrumental (Stake, 1995). Interpretive case studies attempt to “illustrate, support, or challenge theoretical assumptions held prior to the data gathering” rather than just to describe “what was observed or what students reported in interviews” (Merriam, 1998, p.38). This study intends to obtain insight into how SFL and theory of concept development support to understand ELLs’ academic language and concept development in the mathematics classroom by analyzing this particular case. The intent is not “to develop an explanatory theory” grounded in the context (Starks & Trinidad, 2007, p.1374).

In sum, I perceive this case as particularistic in that it focuses on a particular group, place, and activity: one group of an SFL-informed teacher and her ELLs, an activity of teaching and learning academic language and mathematics, and a pullout classroom. Thus, I holistically approach the inquiry under study on ELLs’ academic language and mathematical concept development, intending to interpret the case from SFL and sociocultural perspectives.

4.2.2 Discourse Analysis

I utilize discourse analysis to examine how ELLs develop academic language and
mathematical concepts through their participation in mathematics classroom discourse. This approach is linguistically oriented discourse analysis, specifically based on systemic functional linguistics (SFL). The SFL-based discourse analysis explores what lexico-grammatical choices are made to make meaning in discourse and how the discourse is structured to achieve a specific purpose in a particular context (Eggin, 2004; Halliday, 1978, 1985, 1994; Martin, 1992). Thus, this study investigates what lexico-grammatical choices are made to construct meanings in mathematics classroom discourse and how classroom discourse is organized to accomplish a pedagogic purpose of mathematical activities for ELLs’ language and mathematical concept development throughout a curriculum unit across a sequence of lessons over the passage of time. One lesson comprises a series of episodes, which are defined as single classroom activity that is structured to accomplish a particular purpose (Lemke, 1990). Students can be organized differently by individual, pair, group or whole-class work. Episodes, for example, can include opening a lesson, copying notes, sharing ideas, deconstructing a mathematical concept or summarizing the lesson. An episode is a unit of classroom discourse.

In approaching discourse analysis, the concept of discourse varies by perspectives such as sociological, critical semiotic, and linguistic ones. Attending to the various approaches to discourse, I do not attempt to analyze discourse from sociological perspectives, in which researchers aim to understand how turn-taking sequence is maintained and how uses of a specific word function as cues to include or exclude members of a society (e.g., Hacohen, 2013; Sacks, Schegloff, & Jefferson, 1974). I also do not intend to examine discourse from critical semiotic perspectives including Foucauldian one, which explores discursive framework that scaffolds reality in certain ways (Cheek, 2004, p.1142). Cheek (2004) describes that discourses as certain ways of thinking exclude other ways of thinking in a specific context.
Using a linguistic approach, I take the concept of ‘text’ to mean discourse, based on SFL. A text refers to any instance of language or other semiotic resources that are “playing some part in a context of situation”, and it is also “an instance of social meaning in a particular context of situation” (Halliday & Hasan, 1985, p.10-11). A text can be spoken discourse, written discourse, gesture, visual images, or “combination of two or more communication modes” (Halliday & Mattheissen, 2014, p.46). I also use the term discourse in relation to a type of language-in-use varying according to social situation: classroom discourse (Christie, 2002).

With regard to ‘discourse analysis,’ this study views discourse analysis as interacting with “the analysis of grammar and the analysis of social activity” (Martin & Rose, 2007, p.4) in that discourse is “projection of meanings at a higher level” (Halliday, 1978, p.137), as shown in Figure 4.1.

![Figure 4.1 Point of view on classroom discourse (adapted from Martin & Rose, 2007, p.5)](image)

Halliday (1978) states:

A text, as well as being realized in the lower levels of the linguistic system, lexicogrammatical and phonological, is also itself the realization of higher-level
semiotic structures with their own modes of interpretation, literary, sociological, psychoanalytic and so on. (p.138)

Likewise, classroom discourse is realized in grammar, and it also realizes meanings of classroom activity. Classroom discourse analysis, which attempts to examine linguistic features and the meanings used in discourse, takes a central role in understanding meanings of classroom activity as “structured experience” (Christie, 2002, p.10). I focus on examining spoken and written texts used by the teacher and students because language plays a comprehensive role in providing contexts for most instances of semiotic resources (Barwell, 2016; Christie, 2002; Mercer and Littleton, 2007; Vygotsky, 1986). Thus, I attempt to analyze lexico-grammatical choices used in mathematics classroom discourse and the meanings or metafunctions: ideational, interpersonal, and textual metafunctions (see the detail in the ‘data analysis procedures’ section of this study). However, I do not intend to analyze grammatical patterns of other semiotic resources such as objects, gesture, visual images and numerical equations even though classroom discourse happening in the classroom of any subject matters is multimodal (Jewitt, 2009; Kress, Jewitt, Ogborn, & Tsatsarelis, 2001; Lemke, 2003; O’Halloran, 2000, 2005; Halliday & Matthiessen, 2014). I also attempt to examine how mathematics classroom discourse unfolds by semiotic resources to which classroom talk refers mainly in a particular episode, attending to the fact that linguistic patterns of classroom discourse “develop, change, and are modified over time” (Christie, 2002, p.5)

In sum, I draw on SFL-based discourse analysis to examine how ELLs develop academic language and mathematical concepts through their participation in mathematics classroom discourse. I regard an episode involving a single activity as a unit of classroom discourse and place attention on ways that classroom discourse unfolds across classroom activities or lessons comprising serial episodes over the passage of time. Based on SFL, I use the term discourse to refer
to both a text as any instance of semiotic resources making meaning and classroom discourse as a type of language used in the classroom, viewing classroom discourse interacting with grammar and classroom activity. However, I do not perceive discourse either as certain ways of thinking for excluding other ways of thinking or as a tool to interpret social action by analyzing turn-taking patterns concerned with adjacency pair or words indicating a membership of a society. Within the SFL framework, I focus on investigating patterns of lexico-grammatical choices used in mathematics classroom discourse and the meanings or metafunctions. Additionally, I focus on examining linguistic choices coupled with semiotic resources used in mathematics classroom discourse functioning to support ELLs’ linguistic and conceptual development in the mathematics classroom.

4.3 Research Questions

Drawing on qualitative case study and discourse analysis, this study aims to explore how a teacher supports ELLs’ academic language and mathematical concepts through their participation in mathematics classroom discourse in an urban elementary classroom. To examine classroom discourse that contextualizes on-going classroom activities happening in a particular classroom, it is imperative to investigate both teacher talk and student responses in that a meaning-making process in classroom practices essentially includes teacher-student interactions. Thus, this study poses the following research questions:

(1) How can teachers use language to support ELLs’ academic language and concept development during mathematics instruction using language and other multisemiotic resources (e.g., manipulatives, diagrams, number sentences)?

(2) How do English language learners participate in language-focused mathematical instruction?

(3) How does classroom discourse vary in this context according to multisemiotic resources used to support ELLs’ language and mathematical concept development?
4.4 The Unit of Analysis

To answer research questions, qualitative research determines the scope of the study, which is to pay close attention to a phenomenon being studied (Starks & Trinidad, 2007). Determining the scope is selecting the sample, that is, the unit of analysis (Merriam, 1998). The unit of analysis can guide researchers to answer, “what will be studied or what will not be studied in the scope of the research project” (Baxter & Jack, 2007, p.547). In educational research a classroom is frequently studied as the unit of analysis because it is “naturally bounded and geographically located” (Goetz & LeCompte, 1984, p.85).

The unit of analysis of this study is an elementary teacher’s support for her ELLs’ academic language and mathematical concept development in an elementary mathematics classroom. The unit of analysis is further bounded by activity and time. First, this study only examines classroom activities in which an elementary ESL teacher and her ELLs participated as a group or as a whole class. This study, however, excludes individual and pair activities to address the posed research questions. A whole class activity entails both Group 1 (i.e., the one mixed in grades and language proficiency) and Group 2 (i.e., the other only involving relatively-advanced third-graders). A group activity refers to the teacher’s instruction focused on either Group 1 or Group 2. Second, this study only investigates the teacher’s language used for instructing mathematics and the students’ participation in her mathematics classroom activities over the passage of time, specifically for sixty minutes once a week over 24 weeks from September 2014 to June 2015. This one-year study contributes to rigor of research in understanding how ELLs’ language and mathematical concepts develop through their engagement in classroom discourse across serial activity types across lessons over one academic year, over which learning and development can happen (Mercer & Littleton,
4.5 The Researcher’s Role

The research design includes the researcher’s role for entering the field studied in qualitative research (Dyson & Genishi, 2005). It is important to decide what role the researcher takes in the field because the researcher can influence how participants experience a phenomenon observed by the researcher and how to generate data (Creswell, 2014; Dyson & Genishi, 2005; Graue & Walsh, 1998; Merriam, 1998).

In qualitative research, the researcher is the primary instrument in selecting samples, conducting observations, generating and analyzing data. The qualitative research process is interpretive because it depends on the researcher’s perspective on the phenomenon being studied. The research, in turn, typically involves the researcher’s value, biases, past experiences and personal background, which can affect the interpretations of the phenomenon under study (Creswell, 2014; Merriam, 1998).

The nature of the interpretive research can shape the current research process in a way that I approach the inquiry being studied, conduct fieldwork, collect data, code themes in the data, and report the study. In particular, the inquiry this present study holds results from my lived experiences as a teacher who taught English as a foreign language in secondary schools in Korea for over twenty years before starting a doctoral program in September 2012.

As a novice teacher from 1991 to 1998, I participated in teacher professional development programs amounting to 240 hours provided by the government for secondary teachers. In addition, I actively visited several schools within the same school district to learn from classroom demonstrations how to teach effectively. I often questioned how teachers would support student
learning, attending to teacher-student oral interactions. In 2003 I participated in a master-level teacher education program in the University of Sydney, Australia. The one-year intensive program introduced me to sociocultural perspectives on second language teaching and learning, including Halliday’s functional grammar (i.e., systemic functional linguistics) and genre-based pedagogy (GBP). Back in a high school, I was challenged in implementing an SFL-based approach to reading and writing because I only had a glimpse of understanding about SFL and GBP. I had no experience to observe how an SFL-based approach supports students’ language and content knowledge development in K-12 settings.

In 2008 I was invited to work as a test item developer in a large project team led by Korea Institute for Curriculum and Evaluation. The project team involved college faculty and high-school teachers. The team worked to develop a new test system called the National English Aptitude Test (NEAT). The government planned to ultimately replace the previous Korean SAT with the NEAT after administering mock tests for over five years. The test system attempted to develop two-level English proficiency test items and evaluate students’ listening, speaking, reading and writing abilities. One level focused on the assessment of academic English proficiency while the other focused on that of non-academic, practical English proficiency. Developing the two-level tests resulted in a conflict among the team members because there was no shared understanding of academic versus non-academic English. Additionally, stakeholders such as students, parents, English teachers, and education policy makers were all frustrated with the potential replacement plan. In consequence the project for replacing the Korean SAT with the NEAT was suspended in 2012. The experience I had from the project planted a seed of the inquiry of this study: (1) what is academic English?; and (2) how can teachers support academic English development for students who speak English as a second language?
With the inquiry in mind, I started my academic journey as a doctoral student in September 2012. In the first semester I had an opportunity to explore how ELLs develop their literacy abilities by taking a course provided by Dr. Quinn. The course was theoretically guided by Halliday's functional grammar and GBP grounded in a sociocultural perspective on language and learning. The coursework was to write a paper about how ELLs learned to read and write after observing an ESL classroom and collecting student writing samples. Dr. Quinn introduced to me Ms. Bright, my focus teacher participant, who was taking her course as part of a graduate-level teacher professional development program. The program was funded by the government and provided for public school teachers in two urban districts in partnership with a state university in New England. Ms. Bright, who consented to participate in a research project led by Dr. Quinn and another faculty member, welcomed me into her classroom. When I introduced myself as a former high-school teacher in Korea to her, she professed that she came from Greece in her early childhood and that she was interested in various cultures. Besides the similar experience that both of us have, my identity as a female teacher might facilitate rapport with Ms. Bright. She introduced me to her students by stating that I, who came from Korea, would like to 'learn' how they learn to read and write English. The way to identify me with an adult learner might lead them to accept me into their classroom society.

In 2013 I was invited to voluntarily work for the research project. As a research assistant, I participated in collecting data from Ms. Bright’s classroom. The data collection allowed me to maintain a continuous relationship with her. I video-recorded her classroom practice and conducted a semi-structured interview. In addition, I was allowed to observe every Friday how she works to support ELLs’ language and literacy development. I recorded field notes with the intention of increasing the teacher’s and students’ levels of comfort, wishing them to take my presence in the
classroom for granted. The regular visits to Ms. Bright’s classroom not only helped her school staff and her students seem to regard me as a trustworthy visitor but also sensitized me to ways that the school calendar was mapped to prepare and administer standardized tests. The observations enabled me not only to establish positive rapport with Ms. Bright but also to have a better understanding of moment-by-moment classroom practices using SFL-based approach. The teacher was willing to use techniques informed by SFL and GBP to supporting students’ literacy activities related to science and social studies. I noticed that she explicitly taught genre stages and linguistic features of biographical and scientific genres.

At the end of the school year, however, Ms. Bright was asked to teach mathematics to one of her ESL groups during the upcoming 2014-2015 academic year, when facing completion of the graduate-level teacher professional program. Teaching mathematics and supporting English development can be a challenge to her. The challenge was evident when she invited me to work for the mathematics group, expressing her hesitation about deploying the SFL-base approach to mathematics. I encouraged her to use SFL-based approach to mathematics, exemplifying the organization and linguistic features of mathematics word problems (MWPs). She then seemed to decide to use the language-based approach to mathematics when she sent me an article associated with “linguistic challenges of mathematics teaching and learning” (Schleppegrell, 2007) through email. The challenge in SFL-based mathematics instruction promoted my inquiry of this study.

Throughout my fieldwork, the researcher role was in the continuum between ‘a collaborative partner’ and ‘an observer-as-participant.’ I took a role of the collaborative partner (Dyson & Genishi, 2005; Merriam, 1998) who works collaboratively with the participants in sharing and disseminating findings of research. Both Ms. Bright and I participated in the 2016 American Association of Applied Linguistics as co-presenters of our work done in her mathematics classroom.
Besides, throughout the on-going fieldwork my role of the collaborative partner was performed through emails in a twofold way. On the one hand, I searched and provided mathematics problem sets as well as standardized practice tests for Ms. Bright through emails to help her prepare for the mathematics instruction. On the other hand, I shared part of my field notes and photographs taken in the mathematics classroom when asked to help her write a coursework essay. In addition, I gave my reflections on her SFL-based approach to mathematics in order that she could add the reflections to her coursework paper as collaborative work. Moreover, I helped her with text analysis of students’ writing samples drawn from the mathematics classroom by using SFL.

Ms. Bright as collaborative partner was open to the classroom observations and supported the process of the IRB approval for a pilot study, which was further examined and became this dissertation research. She translated the original consent form written in Spanish into a simplified version for students’ parents in Spanish and distributed and collected the forms provided in both English and Spanish.

My role as collaborative partner, however, was different when I interacted with the teacher and students during class time in that the role is similar to ‘a complete participant’ who conceals his/her role and works “as a member of the group being studied (Merriam, 1998, 100). Rather, I took the role of ‘an observer as participant’ during the observations. An observer as participant known to the participants participates in their activities, which plays a secondary part in collecting data (Merriam, 1998). As an observer-as-participant, I recorded information regarding the activities as it occurred (Creswell, 2014). I attempted to keep a distance from the teacher and students, believing that my interactions with them interrupt the classroom activities and invalidate the research (Graue & Walsh, 1998). While being with students in the classroom, in particular, I tried to act like “a tolerated insider in children’s society” by avoiding both “evaluat[ing] children’s
behavior and feel[ing] responsible for them more than they [adults] do for their age peers” (Bogdan & Biklen, 2016, p.91). I felt accepted not only by Ms. Bright but also by her students, all of whom seemed to regard my presence in the classroom as routine until I left the research field.

I am aware of the possibility that my educational and cultural background, personal experience and interest can affect a research process of generating research questions, gaining access to conduct research, sampling the case, collecting data, analyzing and interpreting data. First, I am aware that I may benefit from Ms. Bright in obtaining permission from the students’ parents with the help of the teacher. The parents might be coerced to consent to their children’s participation in a study in order to show respect for the teacher although they would be reluctant to consent to the study conducted by an Asian adult unknown to them.

Second, the fact that Ms. Bright was observed while teaching her student would likely influence her classroom practices and do inadvertent harm to the teacher and students. I am aware that my fieldwork involving audio-video-recording might cause her to pay continuous attention to her work with the students and to produce an undesirable concern in the classroom despite my discretion not to develop any disruption. The young students might feel uncomfortable with the presence of an adult who is busy taking notes and audio-video-recording and in turn change their attitudes and behaviors. My presence itself may create a strain to the participants being studied, especially to the teacher. According to Bogdan and Biklen (2016), “feeling like they [teachers] are in a fish bowl can be difficult enough for teachers, but learning that they are the center of a discussion in a university class simply intensifies their uneasiness” (p.94). Ms. Bright might feel criticized when she had to spend class time disciplining students’ behaviors rather than focus on instructing mathematics to deal with the students mixed in grades and language proficiency levels.

Third, I am aware that my perception of SFL and GBP as well as sociocultural theory can
affect the process of data collection, analysis, and interpretation because I as researcher am the instrument for the process in qualitative research. Even before collecting data, my perspective on the theories and theoretical position may shape my views on the research field although the combination of the linguistic theory and the developmental theory helps me to verify data analysis and interpretation. Additionally, I might only generate and select particular data I perceive as important, unknowingly excluding the ones that are contradictory to my perspective. Merriam (1998) states that being biased in qualitative research can be inevitable because the researcher’s theoretical position and bias can work as a filter throughout the whole research process. To reduce the extent of my personal perspective on and theoretical position in the current research, I collected multiple data sources and triangulated them in analyzing and interpreting the data.

4.6 Data Collection Procedures

To organize data gathering, case study takes the following steps: selecting the sample for the study, entering the field, generating data, and constructing a data record (Creswell, 2014; Graue & Walsh, 1998; Stake, 1995). This study, however, took slightly different steps by first gaining access and then selecting the sample for the study. The gaining entry first provided me with an opportunity to get acquainted with the school, the school calendar, the classroom, and the participants being studied before selecting the participants, especially the student participants of the case.

4.6.1 Gaining Access and Socializing in the Field

Access to the field being studied was facilitated by Dr. Quinn who both the focus teacher participant and I know. In November 2012, I entered the field named Milltown Elementary School
located in a high-poverty urban district of New England, the US. The number of student enrollment across grades (i.e., K-5) is 467, out of whom the majority were Hispanics (57.8%). The total percent of students who speak a first language other than English and who are identified as English language learners is 40.7. The proportion of students in high needs\(^5\) is 74.9 percent out of all students. The school is designated as Title I School\(^6\) obtaining financial assistance from the local educational agency to ensure that the students meet high-stake state standards.

Knowing how a school calendar is mapped facilitated defining participants for study and planning data collection because students’ classroom activities were aligned with various events such as test preparation, administration, and assessment. For example, the classroom instruction on literacy practices was relatively intensive from September to December while it was more focused on preparation for standardized tests (e.g., PARCC, A-Net, ACCESS), administration, and assessment from January to June. The focus ESL class schedule worked so flexibly that it could accommodate ELLs’ classroom teachers’ requests for providing the students with supplementary class time associated with the standardized tests. In every school year, a classroom available in the school building seemed to be assigned to ESL teachers. It is evident that Ms. Bright worked at three different classrooms for three consecutive school years from 2012 to 2015. Besides, she shared the classrooms with different ESL teachers.

\(^5\) High needs refers to “a designation given by ESE [elementary & secondary education] to students who have been identified as receiving free or reduced-price lunch, students with disabilities, English language learners, or former English language learners” (Massachusetts Department of Elementary and Secondary Education, 2014, p.53).

\(^6\) Title I school is a designated school “with high numbers or high percentages of children from low-income families to help ensure that all children meet challenging state academic standards” (U.S. Department of Education, n.d.).
The classroom I observed was an atypical and small space of a large classroom partitioned by bookcases and cabinets, sharing the classroom with two other ESL teachers. In the space was set up a small whiteboard on an easel and a projector on the table, but there was no screen available. For a screen butcher paper was used to cover the windowpanes. The ELLs of the ESL classroom had no textbooks or workbooks specifically designed to support the students’ language and content knowledge development. The noise caused by the students of the two teachers frequently interrupted Ms. Bright’s instruction, embedded in my audio- and video-recorded data. The ESL classroom had ELLs pulled out of mainstreamed classrooms. The ELLs were also provided with supplementary classes to improve their English proficiency. The focus mathematics class, in particular, had three groups of students varying by grades and English proficiency in September and since October the class had had two groups of students: Group 1 with sixty-minute classes and Group 2 with thirty-minute classes. Ms. Bright’s mathematics classes were provided four times a week except for Wednesday, when the student participants worked with peers and teachers in their mainstreamed classrooms. Their peers in the mainstreamed classrooms had ninety-minute-long mathematics classes while the ELLs having supplementary mathematics classes. The ELLs of Group 1 and Group 2 only had four thirty-minute and sixty-minute mathematics classes respectively in their mainstreamed classrooms at their grade levels.

In the ESL classroom, Ms. Bright, who used to be an ELL, was working with ELLs pulled out of their mainstream classrooms. She had five-years of teaching experience at the time of my first visit to the school. She grew up in a district near Milltown School as an ELL. Her family came to the U.S. from Greece when she was a young pupil in elementary school. She is a fluent Greek speaker and has a beginning Greek literacy. Ms. Bright, whose husband is a Puerto Rican, can also speak Spanish fluently as well as read and write in Spanish. She helped her family business in restaurants
that her father ran before her marriage. She continued to work in a restaurant that her husband ran. After having two children, she decided to learn to become a teacher. Ms. Bright took a Bachelor of Arts degree in a college that provides a special scholarship program for those who are at nontraditional college age while working in the restaurant at the weekend. She continued to pursue her master’s degree by taking the ACCELA program in 2012. She completed the program and was granted an ESL license in February 2015. During her academic trajectory in the program, she was informed of SFL and GBP, which became a nexus between my theoretical interest and the field displaying an SFL-informed approach to language and content knowledge development.

4.6.2 Selecting the Sample for the Study

This study used a purposeful sampling method to select participants. The method is frequently drawn upon in qualitative research including case study and discourse analysis (Koerber & McMichael, 2008; Starks & Trinidad, 2007). Purposeful sampling aims to select an information-rich case for research (Merriam, 1998). To accomplish this purpose, the researcher should specify criteria defining the sample being investigated (Goetz & LeCompte, 1984).

The case for this current study is an elementary teacher who works to support her ELLs’ academic language and mathematical concept development in an elementary mathematics classroom. To select an information-rich case this study used three criteria. First, a teacher participant should teach ELLs both language and content knowledge in the mathematics classroom to ensure that language is used as mediator in teaching mathematics to them. Second, student participants should be ELLs and work with the SFL-informed teacher to learn academic language and mathematical concepts for one academic year starting from September 2014 and ending in June 2015. The one-year participation in the mathematics classroom can warrant that they
participate in on-going classroom discourse across the mathematics lessons occurring over the long passage of time. Third, an episode as unit of classroom discourse selected for the sample should be a whole-class discussion activity to ensure that the episode holds rich information regarding classroom talk between the teacher and students. In this study three types of a whole-class discussion activity were selected: (1) Group 1 (n=9-10) with a sixty-minute class; (2) Group 2 (n=5), who had only a thirty-minute class; (3) the combination of Group 1 and Group 2 (n=14-15). The two groups were divided into the students’ English proficiency levels: Group 1 with low proficiency and Group 2 with higher proficiency within Level 3 out of World-Class Instructional Design & Assessment (WIDA) English proficiency levels ranging from Level 1 to 6 (i.e., the highest level). Their grade levels were also mixed from 1 to 3, which is compared with those of Group 2 as shown in Table 4.1.

<table>
<thead>
<tr>
<th>No. of Each grade</th>
<th>Group 1 (n=9-10) 7</th>
<th>Group 2 (n=5)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gr. 1=1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Gr. 2=5-6</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Gr. 3=3</td>
<td></td>
<td>Gr. 3=5</td>
</tr>
</tbody>
</table>

Table 4.1 Grade and English proficiency levels of student participants

4.6.3 Generating Data

The inquiry under study as well as the sample selection prime a process of data generation and collection, in which data should be generated before being collected (Graue & Walsh, 1998).

7 The difference results from one student who first participated in the study and moved to another school in December 2014.
This study exploits basic strategies for generating data: collecting field notes, recording language, and interviewing (Dyson & Genishi, 2005). According to Dyson and Genishi (2005), field notes can be used as foundation of all data that are interconnected to one another to supplement information missed in some data. Written accounts describing who are involved in classroom discourse are critical when the researcher transcribes an audio-recorded file that can fail to identify the participants engaged in discourse. Recording language by audio/video recorders is central to capturing classroom talk and activities. Conducting an interview with the teacher participant enables the researcher to collect her perspective on SFL-informed pedagogy in her own words.

**Collecting field notes:**

Field notes, in a broad sense, can refer to all data including artifacts, interview transcripts, document, and photographs collected from and reflected on the fieldwork (Bogdan & Biklen, 2016). In this study the term ‘field notes’ is used to refer to written accounts recorded, photographs taken in the field, student writing samples collected, and emails exchanged with the teacher participant. The written accounts, which were typed after every observation, entail both descriptive, objective aspects of the fieldwork and reflective, subjective aspects of the work (Bogdan & Biklen, 2016).

During the classroom observations over 24-week period, I recorded descriptively and reflectively what happened in the field. In descriptive field notes I attempted to present in detail rather than I summarize and evaluate whatever I observed in the classroom. During on-site observations I jotted down times whenever I noticed changes in students’ entering and leaving as well as transitions in classroom activities. The descriptive field notes contain a description of the physical setting that displays the classroom seating arrangement and the researcher’s seating. The description also shows the configuration of furniture: desks, tables, a whiteboard on the easel, and bookcases. (see Appendix B). The descriptive field notes include students’ entry into and exit from
the classroom (see Appendix C) as well as the windowpanes covered with butcher paper to serve as a screen of the projector. In addition, the field notes entail board texts that display mathematical tasks or problems. Moreover, the field notes describe who were involved in classroom activities and what the participants said, which was also recorded with a voice recorder application for Android and a digital video recorder.

In the reflective field notes, I recorded not only accounts before and after the observations but also reasons for schedule changes or observation cancellation. Before the observations, I took notes of my activities before I visited the school and before the lesson started. After the observations, I wrote down ideas and speculation pertaining to classroom activities as shown in the field note aforementioned. I also recorded my reflection that I shared with the teacher so that I could explore ways to work as a collaborative partner in the field.

In addition to written field notes, photographs were taken mostly before the time the teacher collected the students and came into the classroom. The photographs contain a lesson objective, 'Math Message' and teaching materials including a wordlist hung on the paper rack (see Appendix D). I also took photographs of mathematics problems presented on the windowpane-screen. Twenty-four photographs are collected in total.

Samples of problem-solving work from the students of Group 1 were collected over five times. I collected and returned samples of student work two times before winter break after scanning them. I since then had collected and returned student work two times before the end of the academic year. At the last round of the collection Ms. Bright took photos of student writing that reflects learning from the mathematics classroom and shared them with me electronically. The samples collected amount to 577 pages in total.
Emails exchanged with Ms. Bright were stored in my personal email account and cellphone respectively. The emails show how both the teacher and the researcher worked collaboratively. For instance, I emailed her with teaching materials, especially for standardized test practices, which she modified to create similar problems for the students (see Appendix E). The total number of emails exchanged with Ms. Bright is forty-two.

In summary, the field notes collected in this study include descriptive accounts of on-going practices occurring during the lessons and reflective accounts of pre/post activities that the researcher performed before and after the visit to school. Besides the written accounts, the field notes contain photographs, student writing samples, and emails, which are supplementary in revealing the connections among them.

*Recording language by using audio-video-recorders:*

This study takes recording language by audio and video recorders as the core in data collection and analysis (Dyson & Genishi, 2005) because the inquiry of this study focuses on classroom discourse associated with ELLs’ academic language and mathematical concept development in the mathematics classroom. I utilized one voice recorder application of an Android cellphone and one digital video recorder, which I could afford.

From October to December of the school year, I recorded classroom talk by using the voice recorder application. The cellphone was placed on the bookcase next to me. Use of the unobtrusive audio-recorder is efficient (Graue & Walsh, 1998), and it can minimize students’ potential uneasiness caused by a video camera recording their talk and activities. The minimization is important because it was not the students themselves but their parents that consented to the participation in the research. In addition, the student participants, who began to work with the new
teacher to learn mathematics in the new school year, might feel more comfortable with the audio-recorder set unknown to them rather than a video camera set visibly to record them. In addition, I did not want two other ESL teachers to be uncomfortable. The length of audio files amounts to about 391 minutes.

From January to June of the year I utilized one digital video camera because the classroom was not spacious enough to set up multiple video cameras and because this study attempted to investigate classroom discourse happening between teacher and students, not the one occurring among individual students. The visual records, which allow the researcher to revisit what happened in the research field (Dyson & Genishi, 2005), contain not only classroom talk but also activities in conjunction with objects used for classroom activities. The camera was set up next to me sitting on a chair at the corner of the mathematics classroom. I sat close to the students of Group 1, who in turn became the focus of the video camera. I also used a camera phone to take photographs and took field notes to capture the details of classroom practices occurring out of the focus of the video recorder because the camera was placed in a broad context where both the teacher and the students were seen on the video records. At the time my presence in the classroom seemed to be taken for granted by the ELLs and the ESL teachers. The ESL teachers first noticed the video recording and expressed concern in regard that voices from their side of the classroom could be recorded. They later permitted using the video recorder when Ms. Bright explained to them the purpose of the recording: to capture Ms. Bright’s class for use in an educational conference, including only small amounts of visual records. The length of the video records amounts to about 530 minutes.

Interviewing:

I conducted one face-to-face interview with the teacher participant. After leaving the
research site, I set up the interview date in July 2015 when she was available after the end of the school year. The interview was audio-recorded with her consent. The interview, which lasted forty minutes, was formal in that it was scheduled beforehand (Rossman & Rallis, 2003). The interview as a supplement to field notes was conducted to capture her perspective on an SFL-informed approach to teaching both academic language and content knowledge of mathematics in her own words (Dyson & Genishi, 2005).

The type of the interview taken in this study is a semi-structured interview guided by a list of open-ended questions, including highly structured questions (Merriam, 1998). Even though the questions were scripted, I did not follow it verbatim. The questions were focused on her SFL-based approach implemented in the mathematics classroom (see Appendix F). In addition to the formal interview, I had informal interviews with the teacher when I was in the classroom. The informational interviews were described in the reflective field notes aforementioned in the section ‘collecting field notes.’

4.6.4 Constructing a Data Record

Data record construction and exit from the field are not temporally sequenced in qualitative research, in which the research process is recursive. Data records either can simultaneously take place while collecting data or can be constructed after leaving the research field (Dyson & Genishi, 2005; Goetz & LeCompte, 1984; Merriam, 1998). Written accounts, photos, student work collected during observations were transformed into a workable data record immediately after observing the research site. On the other hand, audio and video data from the site and the face-to-face interview with the teacher were organized to become retrievable after leaving the field.

The process of constructing a data record includes transforming raw data into a data record
accessible for analysis (Graue & Walsh, 1998, p.129). Before data transformation, I copied raw data as backup in case of the original being missed or damaged. The transformation in this study involves typing field notes, scanning student work in computer files, saving photos in a computer folder, and transcribing audio/video data.

To change all raw data collected into a manageable data record, I typed handwritten field notes into a word-processed notebook as immediately as possible after each observation. I scanned student work with dates recorded into computer files. I labeled the collected photos with the collection date and saved them as image files in a computer folder.

With respect to transcription, I took two phases: drawing on transcription conventions, and working on transcription. In the initial phase, I adopted and modified transcription conventions from Jefferson (1994), and Mercer and Littleton (2007) to increase readability, as shown below in Table 4.2. Information about non-verbal aspects and contextual information were described in italics in parentheses.

<table>
<thead>
<tr>
<th></th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>()</td>
<td>Inaudible sound</td>
</tr>
<tr>
<td>.</td>
<td>Pause</td>
</tr>
<tr>
<td>(...)</td>
<td>Long pause</td>
</tr>
<tr>
<td>(pl)</td>
<td>Incomplete utterance</td>
</tr>
<tr>
<td>[</td>
<td>Overlapped sound</td>
</tr>
<tr>
<td>(italics)</td>
<td>Contextual description</td>
</tr>
<tr>
<td>?</td>
<td>Interrogative or rising intonation</td>
</tr>
<tr>
<td>=</td>
<td>Latch</td>
</tr>
<tr>
<td>please</td>
<td>Emphasized words with underline</td>
</tr>
</tbody>
</table>

Table 4.2 Transcription conventions (see Mercer & Littleton, 2007)
The second phase working on transcription involves four steps. I first labeled the date of recording before transcribing the audio and video data. I then asked for transcription services from an English-native speaker, who worked with the modified conventions. The services enabled me to save time and gain transcripts workable for data analysis. Next, I revisited the audio and video data, comparing them with transcripts produced from the services to confirm if they would be transcribed appropriate enough to capture classroom talk and activities based on the given conventions. Last, I revised the transcripts to use pseudonyms for participants, school, and district. I also corrected errors while revisiting the audio and video data. For example, when student silence in response to teacher questions was treated as no turn taking, I revised it into a turn in the regard that teacher subsequent utterances can be contingent on student response. Silence can indicate that the student needs to be time to think, which is critical for ELLs who need linguistic scaffolding (Gibbons, 2009).

4.6.5 Summary of Data Collection

Qualitative research involves a serial transformation (Barrett, 2007). First, a phenomenon of interest is transformed into data collected by the researcher. Second, the raw data collected from the research field are transformed into data records workable for analysis. Last, the data records are transformed into findings to explain the focus phenomenon. The transformations are fundamentally oriented to answer research questions based on theories, focusing on the case under study. Table 4.3 presents a data collection matrix, which can provide a bird-eye view for the researcher by outlining research questions, theories, a unit of analysis, data and data collection periods (see LeCompte & Schensul, 2010).

<table>
<thead>
<tr>
<th>Research Questions</th>
<th>Theory</th>
<th>Unit of Analysis</th>
<th>Data (Collection Period)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1) How can teachers</td>
<td>SFL</td>
<td>Teacher’s</td>
<td>Field notes (once a</td>
</tr>
</tbody>
</table>
use language to support English language learners’ (ELLs) language and concept development during mathematics instruction using language and other multisemiotic resources?

- Vygotsky’s Concept Development
- language use to support her ELLs’ language and mathematical concept development in an elementary mathematics classroom
  - Bounded by activity and time:
  - whole-class discussion observed for sixty minutes per week over 24 weeks
  - Exclusion of students’ pair-work and individual work

How do ELLs participate in language focused mathematical instruction?

- SFL
- Vygotsky’s Concept Development

- Field notes (once a week over 24 weeks): written accounts, photos, classroom-based audio/video data
- Student work (five-round collection: November, December, April, June, & July)

How does classroom discourse vary in this context according to semiotic resources used to support English language learners’ language and mathematical concept development?

- SFL
- Vygotsky’s Concept Development

- Field notes (once a week over 24 weeks): written accounts, photos, classroom-based audio/video data
- Student work (five rounds: November, December, April, June, & July)

<table>
<thead>
<tr>
<th>294x680</th>
<th>202x665</th>
<th>294x652</th>
<th>294x639</th>
<th>294x625</th>
<th>294x598</th>
<th>294x584</th>
<th>294x570</th>
<th>294x557</th>
<th>294x543</th>
<th>294x529</th>
<th>294x515</th>
<th>294x501</th>
<th>294x488</th>
<th>294x474</th>
<th>294x460</th>
<th>294x446</th>
<th>294x432</th>
<th>294x419</th>
</tr>
</thead>
<tbody>
<tr>
<td>294x680</td>
<td>202x665</td>
<td>294x652</td>
<td>294x639</td>
<td>294x625</td>
<td>294x598</td>
<td>294x584</td>
<td>294x570</td>
<td>294x557</td>
<td>294x543</td>
<td>294x529</td>
<td>294x515</td>
<td>294x501</td>
<td>294x488</td>
<td>294x474</td>
<td>294x460</td>
<td>294x446</td>
<td>294x432</td>
<td>294x419</td>
</tr>
</tbody>
</table>

**Table 4.3 Data Collection Matrix**

**4.6.6 Data Analysis Procedure**

The approach to qualitative data analysis is interpretive (Bazeley, 2013; Creswell, 2014; Merriam, 1998). The interpretive analysis typically includes a process of consolidating, reducing and interpreting data (Merriam, 1998), given that qualitative research involves enormous
quantities of data. Qualitative researchers specify several stages for the analysis procedure. This current qualitative study hand-coded data and conducted data analysis according to six stages: organizing and reading data, coding data, selecting and categorizing data, conducting in-depth analysis and finding emerging themes, interrelating themes, and interpreting the themes (Bazeley, 2013; Creswell, 2014). The first three stages focus on capturing a holistic perspective on data and reducing the data while the other three stages focus on analyzing the reduced data in detail and interpreting findings in a local situation. In what follows, each stage is specified in detail.

Stage 1. Organizing and Reading Data

The core data source of this study is transcribed classroom talk that reveals how the teacher orally interacted with her students through a series of mathematics lessons during an academic year. I began data analysis by capturing a holistic perspective on the large quantities of data. I first read through all the transcripts from entire audio/video recordings. In the margins of the transcripts I wrote notes about emerging ideas while reviewing the field notes as a foundation that provides contextual information regarding the instructional practice of mathematics. I then reviewed all the photos and audio/video recordings as well as student work to make sense of the whole data and prepare for coding the data.

Stage 2. Coding Data

Coding starts with “taking text data or pictures gathered during data collection, segmenting sentences (or paragraphs) or images into categories, and labeling those categories with a term, often a term based in the actual language of the participant” (Creswell, 2014, p.198). For this initial coding, I organized the data into a form of ‘episode summary’ informed by Gibbons (2006), which serves to illustrate classroom activities cumulated for the one academic year. The summary
displays data collected from 19 lessons and facilitates data retrieval and in-depth analyses, by which this study can examine “meanings created within and across the episodes” (Gibbons, 2006, p.96). Meaning within each episode can show how the teacher and students construct mathematical meaning by using language with other semiotic resources. Meaning across episodes can indicate how the participants connect multiple semiotic resources in constructing mathematical concepts through a whole lesson or entire lessons occurred over the longitudinal period.

To segment the data into categories and name the categories, I created the episode summary table with four main columns: Episode number/Running time; HOW; WHAT; and Data. The episode summary involved three phases of data review. The first phase proceeded with reading the transcripts and segmenting each lesson into a series of episodes as shown in Table 4.4, using an episode as a unit of classroom discourse. The boundary of episodes is determined by “changes in activity type (structure or function)” or “changes in topic” (Lemke, 1990, p.50). Each lesson mostly comprised six to eight episodes (n=14/19). I typed episode numbers in the first column, and described teaching-learning process/classroom activity, interaction structure and salient channels of communication (e.g., spoken, written) under the headed HOW within the table. The HOW column presents how each lesson unfolds, and helps retrieve segments indicating whole-class discussion, on which this study focuses. As shown in Table 4.4, for example, ‘Interaction structure’ under HOW column shows that segments of episodes #01, #03, and #06 indicating whole-class discussion can be retrieved for analysis.
Table 4.4 Episode Summary

In the second phase, I re-read the transcripts and reviewed the audio/video recordings simultaneously with frequent pauses to write notes in the margins of the transcripts. The notes indicate foci of mathematical activities (e.g., deconstructing math language ‘a total of 100 students’, identifying place value of each digit in a two-digit number, regrouping tens into ones, telling time to five-minute interval). In addition, I highlighted the instructional register, that is, language focused on teaching and learning mathematics. In the episode summary table, I typed in the instructional register under the headed WHAT as shown in Table 4.4. The WHAT column also presents targeted mathematical concepts (e.g., ‘Understand place value of each digit in a four-digit number’). In particular, I marked running time indicating segments that load the targeted mathematical concepts.

In the third phase, I summed up all the data by typing data sources (e.g., field notes, photos,
video numbers, student work) relevant to each episode into the last column of the table. The Data
column works for data retrieval with ease, having highlighted data sources as an indicator in
retrieving data. For example, Episode #5 in Table 4.4 involves referring to Photo #24 as well as
some students’ work—erased in black for an ethical purpose—to better understand what happened
in the particular classroom practice.

Stage 3. Selecting and Categorizing Data

To reduce data and prepare in-depth data analysis, I first used two indicators: ‘whole-class’
work and segments indicated by ‘running time’ in the episode summary. I retrieved segments by
using the two indicators. For example, I retrieved segments in episode #03 that concerns both
‘whole-class’ work and ‘running time’ as in Table 4.4. Through the retrieval process, I finally
selected 58 segments. I then did a close reading on the 58 selected segments while attending to the
notes written and the instructional registers highlighted during the three-phase episode summary.
Majority of the segments were selected from lessons regarding both math word problems (n=22)
and place value (n=24). The rest of the segments dealt with counting money (n=3), telling time
(n=8), and two-dimensional shapes (n=1). The selected segments all involved learning academic
language, hands-on activity using objects or manipulatives, and representing mathematical
concepts through pictures and mathematical symbols.

Next, I attempted to categorize the segments into four main semiotic resources as Token:
language, manipulatives, visual display, and symbolism, all of which work to construct
mathematical concepts. According to Token⁸, I categorized the segments. In segments categorized

---

⁸ The capitalization of ‘Token’ is used to differentiate from ‘token’, indicating the function of
’signifier’ in SFL. The capitalized ‘Value’ indicates the function of ‘signified.’
as 'language,' classroom talk was used to mediate language as main semiotic resource to a mathematical concept. In other words, language is viewed as Token while the concept as Value. For example, the language ‘how many more’ presented in a math word problem is used as a Token to represent the Value of subtraction. In segments categorized as ‘manipulatives,’ ‘visual display,’ and ‘symbolism,’ classroom talk was used mostly to mediate objects (e.g., base-ten blocks, coins), pictures (e.g., number line, place-value chart), and symbolic representations (e.g., numbers, number sentence) respectively to mathematical concepts. Table 4.5 exemplifies classroom talk used to mediate semiotic resources (i.e., Token) to mathematical concepts (i.e., Value).

<table>
<thead>
<tr>
<th>Salient Semiotic Resource</th>
<th>Classroom Talk</th>
<th>Token</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Language</td>
<td>When we give away over here, what operation?</td>
<td>…She gave a few away… (MWP⁹)</td>
<td>Subtraction</td>
</tr>
<tr>
<td>Manipulatives</td>
<td>She’s gonna take ten tens for a hundred block.</td>
<td><img src="image" alt="Base-Ten Blocks" /></td>
<td>One hundred</td>
</tr>
<tr>
<td>Visual Display</td>
<td>If this is the hour hand, it’s still two o’clock.</td>
<td><img src="image" alt="Clock" /></td>
<td>Two o’clock</td>
</tr>
</tbody>
</table>

⁹ MWP in the parenthesis indicates a math word problem.
Table 4.5 Classroom talk to mediate semiotic resources to math concepts

<table>
<thead>
<tr>
<th>Symbolism</th>
<th>Is eight greater than five?</th>
<th>8&gt;5</th>
<th>Eight is greater than five</th>
</tr>
</thead>
<tbody>
<tr>
<td>Language</td>
<td>how many more, give a few away, half of</td>
<td>11</td>
<td></td>
</tr>
<tr>
<td>Manipulatives</td>
<td>tens &amp; hundreds blocks, coins</td>
<td>11</td>
<td></td>
</tr>
<tr>
<td>Visual display</td>
<td>number line, pictures of clocks, ‘72’ &amp; ‘272’</td>
<td>21</td>
<td></td>
</tr>
<tr>
<td>Symbolism</td>
<td>679, 12–7, 8&gt;5</td>
<td>15</td>
<td></td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td></td>
<td>58</td>
<td></td>
</tr>
</tbody>
</table>

Table 4.6 The number of segments selected by semiotic resources

This categorizing process revealed that some segments involved multiple semiotic resources. These segments were categorized as one of the four semiotic resources depending on which semiotic resource classroom talk mediates more or the most. As a result, the 58 segments are categorized as shown in Table 4.6, helping to prepare in-depth analyses.

Stage 4. Conducting In-Depth Analysis and Finding Emerging Themes

For in-depth analysis, I first took and examined three segments from each category in Table 4.6 (see Appendix G). I then selected and analyzed two more segment from each category to ensure that findings derived from a first-round analysis remain relatively constant with those from a second-round analysis. The rationale for selecting the twenty segments is that the segments mostly involve content-focused language.

Based on systemic functional linguistics (Halliday & Matthiessen, 2004, 2014), I conducted a clause-by-clause analysis of the sixteen segments in terms of ‘field,’ ‘tenor,’ and ‘mode’. Incomplete
clauses produced by the teacher were analyzed when the clauses were completed by students. For example, teacher talk such as “we have to figure out?” was analyzed as in Table 4.7. This utterance indicates that the teacher is inviting students to complete the clause by “how many more”.

| Field | Tenor | Mode | T
|-------|-------|------|---
| Senser | Subject | Theme | So, we have to figure out?
| Mental | Finite (modal) | Rheme | (Phenomenon)
|       |       |      |   |

Table 4.7 The sample analysis of an incomplete clause

Field, tenor and mode registers concern subcomponents. The field register, for example, was analyzed in terms of ‘process,’ ‘participant,’ and ‘circumstance’ as shown in Appendix H. In the tenor analysis, I only examined ‘Subject’ and ‘Finite’, whose order is significant in mood types (e.g., declaratives, interrogatives, imperatives). For instance, Subject comes before Finite in declaratives while Finite comes before Subject in interrogatives (Halliday & Matthiessen, 2004, p.143). The mode analysis concerned Theme and Rheme.

In addition to analyzing the three register variables, I investigated relations between

---

10 In Table 4.7, T refers to the teacher and S to a student.
clauses, which develop through the system of taxis (i.e., parataxis and hypotaxis) and logico-semantic relations (i.e., expansion and projection). In analyzing relations between clauses, I used notations following Halliday and Matthiessen (2014) as in Table 4.8. Tactic relations are coded as 1 and 2 or Greek alphabets α and β. The numerical codes indicate paratactic relations, in which statuses of the two clauses are symmetrical, while the alphabets indicate hypotactic relations. Logico-semantic relations between clauses concern extending meaning through expansion and projection. In expansion, ‘=,’ ‘+,’ and ‘×’ are used to indicate elaboration, extension, and enhancement respectively. For projection analysis, punctuation marks such as ‘‘ and “ are used. The single quote mark refers to a secondary clause that immediately follows a mental clause as a primary one while the square quote mark refers to a secondary clause introduced by a verbal clause (see Appendix H).

<table>
<thead>
<tr>
<th>Taxis</th>
<th>Logico-semantic relations</th>
</tr>
</thead>
<tbody>
<tr>
<td>Paratactic</td>
<td>Hypotactic</td>
</tr>
<tr>
<td>1 &amp; 2</td>
<td>α &amp; β</td>
</tr>
<tr>
<td>Elaboration</td>
<td>Expansion</td>
</tr>
<tr>
<td>=</td>
<td>+</td>
</tr>
<tr>
<td>Location</td>
<td>Idea</td>
</tr>
</tbody>
</table>

Table 4.8 Notations indicating relations between clauses

**Stage 5. Interrelating Themes**

Developing a qualitative narrative, researchers investigate relationships between specific themes and gain a holistic understanding of how each theme is interwoven to represent a phenomenon under study (Merriam, 1998). One of the strategies used to find relationships between specific themes is to visualize how themes are interrelated from particular/basic to general/global (Bazeley, 2013, p.192). In this stage, I attempted to visualize how particular themes are interconnected to be categorized and integrated into global themes pertinent to teacher talk,
student response and linguistic choices according to multi-semiotic resources. The network of themes contributes to developing a qualitative narrative that provides a holistic understanding of the phenomenon this study attends to (Bazeley, 2013). In visualizing themes, I first illustrated how themes in teacher talk are connected to represent one global theme. I then visualized themes found in student responses. Next, I interwove teacher talk and student responses to display the flow of classroom talk. Last, I illustrated themes that show linguistic choices made according to multi-semiotic resources. In reference to teacher’s language use, for instance, I combined ‘Declaratives to give contextual information’ and ‘Interrogatives to ask students to identify attribute, identity, or relationship of entities’ to represent a relatively general theme 'Inviting students to coding activity.' I then connected 'Inviting students to coding activity' to ‘Scaffolding linguistically for student math concept development.’ I finally created thematic networks that illustrate how themes are interrelated, as shown in Figure 4.2.

![Thematic Network Diagram](image)

Figure 4.2 Example of a thematic network

Stage 6. Interpreting the Themes

Stage 6 connects the interrelated themes or findings of this study to literature or theories to interpret the meaning of the findings, develop a thesis, and suggest action agendas for pedagogical

---

11 The term 'thematic network' is drawn from Bazeley (2013, p.193).
practices and future research (Bazeley, 2013; Creswell, 2014). In this stage, I first interpreted meaning derived from the findings of this study, drawing on Halliday’s theory of language and Vygotsky’s theory of concept development to explore how language works to develop concepts in the mathematics classroom. Next, I made conclusive remarks on how classroom talk supports ELLs’ linguistic and mathematical concept development in the instruction of mathematics involving multiple semiotic resources. Last, I explored implications that can be informative to teaching practitioners teacher educators, and researchers who work with all students including ELLs. The implications can also address pedagogy of teaching all subject matters (e.g., social studies, English language arts, science, mathematics).

4.7 Limitations of the Study Design

Limitations of this study concern data generation, data reduction for analysis, and generalizability. A first limitation relates to collecting field notes and recording language to generate data of this study, as mentioned in section 4.5.3. In the research site being studied, I was only able to observe a sixty-minute class once a week to collect field notes although the participants worked four 60-minute classes per week. Aware that this lack of accessibility can limit exploring how teachers can support ELLs’ language and mathematical concept development, I collected student work that had been done in my absence from the classroom to better understand the lessons I observed. In addition to the field notes, I used one digital audio/video recorder that was available in recording language and had difficulty capturing classroom talk accurately as marked in the transcripts. In particular, I sat back and recorded them in the back of the classroom because there was no room for a recorder in the front, so students often sounded weak to me. To better transcribe the audio/video data, I cross-checked if the field notes provided information relevant to retrieving the oral interaction.
Second, findings of this study can be limited to validating that the findings address how teacher and students interact orally in the mathematics classroom throughout the one academic year because I segmented transcripts, selected 58 segments, and analyzed 20 segments. To ensure the quality and trustworthiness of this study, I considered qualitative validity, using ‘validity strategies’: triangulation involving different data sources, thick description of the context under study, and one-year field work (Creswell, 2014). To facilitate triangulation, I created the episode summary, as described in section 4.5. The episode summary involved cross-checking a thick description and other different data sources. I used the summary to outline classroom activities that occurred during the prolonged time and confirmed if the segments I selected represented multi-semiotic activities happening in the mathematics classroom.

Third and last, findings of this study are limited because this research is a single case study involving one participant teacher who worked with ELLs in her mathematics classroom. She is not typical because she was informed of interdisciplinary linguistic knowledge from SFL-based teacher professional development programs. However, this case can be representative in the following two aspects. First, Ms. Bright can be typical in her math-specific content knowledge in that elementary teachers have comprehensive knowledge across content areas but limited content knowledge in mathematics (Good, 2009). Second, the participant ELLs, whose first language is Spanish, are representative in a sense that the dominant population of ELLs in the United States speak Spanish as their first language (Lopez, McEneaney, & Nieswandt, 2015). With the respect to this representativeness, the findings of this study can contribute to better understanding of how teachers can use English as a medium of instruction in teaching subject matter (e.g., mathematics, science, social studies, English language arts) and of how English learners’ participation in multisemiotic activities can support their linguistic and conceptual development in English-
mediated instruction of mathematics.
CHAPTER 5

FINDINGS

5.1 Introduction

This study aimed to examine how a teacher supported English language learners’ (ELLs’) academic language and mathematical concept development through mathematics classroom discourse in an urban elementary school by posing three research questions: (1) how can teachers use language to support English language learners’ academic language and mathematical concept development during mathematics instruction using language and other multisemiotic resources (e.g., manipulatives, diagrams, number sentences)?; (2) how do English language learners participate in language-focused mathematical instruction?; and (3) how does classroom discourse vary in this context according to semiotic resources used to support English language learners’ language and mathematical concept development?

This study, first, found that the teacher provided ELLs with linguistic scaffolding in inviting students to coding activity\(^\text{12}\), facilitating student participation, and promoting student reasoning. This study defines coding activity as relating multisemiotic resources to mathematical concepts through language based on SFL (Halliday & Matthiessen, 2014, p.280). Second, this study demonstrated that students played a role in co-constructing mathematical concepts by participating in multisemiotic activities, processing math concepts through verbalizing actions, and binding everyday concepts with academic concepts. Third, this study showed that student talk had

\(^{12}\) The notion of coding activity is derived from the transitivity system in SFL, as mentioned in chapter 3 (see Figure 3.5). The coding activity involves decoding a Token and encoding a Value.
no distinctive variations according to main semiotic resources (e.g., objects, pictures, symbols, language) that classroom talk refers to in mathematics classroom discourse. In contrast, teacher talk had variations in the ideational meaning that construes experience of inner and outer worlds. The subsequent sections discuss these findings in detail in the following order: (1) teacher’s linguistic scaffolding for students’ language and math concept development; (2) students’ co-constructing mathematical concepts; (3) mathematics classroom discourse as coding activity; (4) linguistic choices varying by semiotic resources; and (5) summary of findings.

5.2 Teacher’s Linguistic Scaffolding for Student Math Concept Development

“I think knowing what I know now about SFL and the genre-based pedagogy, it has helped me because it has helped me- it gave me an organization of how to teach, like in layers. So, understanding what to do first, what to do second. So some people might say, as probably you’ve heard, it’s a () or maybe genre-based pedagogy, I don’t know which one, is like a prescription, like you do it this way and then you do this and you do that. I think at the beginning people need structure so they can understand that the pieces- they can understand the whole thing and then it’s broken up.”

This excerpt is Ms. Bright’s response to my interview question, “do you think that your knowledge of that systemic functional linguistics and or genre-based pedagogy was helpful in teaching the students?” This response suggests that Ms. Bright benefited from SFL and genre-based pedagogy in her teaching practices. Her understanding of SFL and genre-based theory underlined in the script seems to lead her to design a curriculum that is ‘structured’ in stages or ‘pieces’ to accomplish a goal of classroom practice. This conceptualization may contribute to her being critically aware of language use in teaching.

13 The transcript was derived from a semi-structured interview with Ms. Bright that I had in July 16th, 2015 after the academic year ended.
mathematics, when verbal language plays a fundamental role in constructing mathematical meanings (Chapman, 2003, p.131). As mentioned in the quote, each classroom activity or episode, a series of which comprises one lesson as whole, was sequentially structured or ‘broken up’ into broadly three stages from beginning to end: inviting students to coding activity, facilitating student participation, and promoting students’ reasoning. Roles of the stages play in unfolding mathematical discourse can be weighed differently although all the stages comprise one episode. The episode of opening a lesson, for example, can focus on inviting students to coding activity whereas that of sharing ideas may highlight the stage of facilitating student participation or promoting student reasoning. Each stage involved distinctive linguistic choices that the teacher made.

Figure 5.1 illustrates how SFL-based analysis fleshed out findings pertaining to teacher talk in this study. The eight patterns of linguistic features on the left were salient in teacher talk. The linguistic features occurred across episodes. I, however, highlighted linguistic features that significantly worked to construct mathematical meaning in three stages. I categorized the three stages into three relatively general themes: ‘inviting students to engaging in coding activity,’ ‘facilitating student participation,’ and ‘promoting students’ reasoning.’ The category of ‘inviting students to engaging in coding activity’ entails ‘declaratives to give contextual information’ and ‘interrogatives to ask students to identify attribute, identity or the relationship of entities’, both of which were salient at the beginning of each episode. The category of ‘facilitating student participation’ includes four linguistic features: ‘incomplete clauses as sentence starter,’ ‘interrogatives associating Subject with Theme,’ ‘elaborating and enhancing clauses in interweaving Given with New information,’ and ‘imperatives to ask students to tell what they did.’ These four patterns occurred when the teacher engaged students in understanding a mathematical concept. I lastly categorized ‘interrogatives with why and how’ and ‘interrogatives involving projection’ into ‘promoting students’ reasoning’ in a sense that the questions can help students heighten their
reasoning competence. The three categories derive one general theme, ‘scaffolding linguistically for student concept development.’

![Thematic network of teacher talk](image)

Figure 5.1 Thematic network of teacher talk

As I present below, findings of this study address each episode as genre in that each episode is structured to accomplish a particular purpose (Christie, 2002; Lemke, 1990). Segments selected in the following sections are derived from three stages of episodes that comprise one lesson as whole. Subsequent sections describe teacher talk unfolded in three stages: inviting students to engaging in coding activity, facilitating student participation, and promoting students’ reasoning.
5.2.1 Inviting Students to Engaging in Coding Activity

This study defines coding activity as language use either in relating Token (e.g., 'base-ten block,' 'the hour hand,' 'number sentence,' 'division') to Value/meanings or relating Value to Token. To review from chapter 3, coding activities involved relational clauses that are used to identify Token or Value, as described section 2. When Token is identified, the clause is decoding the Token. In encoding clause, Value is identified (Halliday & Matthiessen, 2014, p.280). Use of the decoding clause is significant in the inviting stage. This inviting stage featured the coordination of experiential meaning and interpersonal meaning realized through field and tenor register variables respectively. Field variables represented contextual information comprising mathematical activities, mathematical entities/participants and circumstances. Tenor variables enabled the field variable to construct meaning when the teacher invited students to receive the contextual information and invited them to give information demanded.

At the beginning of an episode, Ms. Bright used declarative clauses and provided contextual information that was an imperative role in guiding students in identifying entities (e.g., number, place, value) of a mathematical concept. Contextual information can prelude coding activity that entails information collection. The collection comprises entities or participants, the relationships between them, and a certain circumstance in which they are situated. Coding activities required students to relate multiple semiotic resources (e.g., manipulatives, pictures, number sentences, language) to their meanings. After giving contextual information, the teacher asked students to find identities, attributes, or the relationship of entities by using WH-interrogatives. In inviting students to coding activity, circumstantial elements in the declarative clause and interrogative relational clauses highlighted coding activity that the teacher and students participated in, as shown in Excerpt 1.
Excerpt 1 Understanding place values in a two-digit number (E=Emma (Gr. 3); N=Nina (Gr. 3))

1. T: (i) When we write numbers, here in America, (ii) each place, Alba, has a value. (iii) Okay, for instance, this number here, these digits, in this digit are worth different amount or value okay? So, you, third grader already knows this. (iv) So, if I was going to say this place value, (v) Emma, what would that place value be called? (pointing to 2 in number '72')

2. E: Seven?

3. T: No I’m pointing to the… I’m pointing to the two.

4. E: Oh two? Then it’s one.

5. T: (i) So that’s ones. (ii) What would the seven’s place value be?


7. T: (i) The tens. So, listen to my question Emma. (ii) How many tens are in seventy-two?

8. E: Eight! Eight, eight.


10. T: Don’t shout. I’m going to repeat my question.

Ms. Bright prefaced the interaction by introducing entities (i.e., 'numbers,' 'place,' and 'value') as main participants relevant to this mathematical activity for understanding place value of each digit in a two-digit number '72' (1.15; ii). These clauses work for orientation of a math story that sets participants/entities, time and place. Specifically, a circumstance started by 'when' serves not only as a temporal circumstance but also as a locational circumstance indicated by 'here in America.' Turn 1-(ii), in which the teacher called a student’s name ‘Alba’ to attract her attention, works as a prelude to present a focus entity ‘place value’ (1-iv). Ms. Bright broke a math-specific language ‘place value’ into ‘place’ and ‘value’ to prepare students for understanding a mathematical concept of place value. Turn 1-(iii) indicates that the teacher first shifted everyday language

---

14 Excerpt 1 is retrieved from the transcript of a lesson collected in January 29th, 2015. In the excerpts of this study, T refers to ‘teacher.’

15 This study places natural numbers and Roman numerals in the parenthesis to indicate turns of dialog and a clause number. In (1-i & ii), ‘1’ indicates a turn number while ‘i’ and ‘ii’ refer to clause numbers.
'numbers' to academic to 'digits' and then ascribed a number to a quantity/amount or quality/value by using a relational process 'are.'

Ms. Bright proceeded to ask her students to decode the meaning of a number (i.e., 'Emma, what would that place value be called?' in turn 1-(v)). In turn 1-(iv). She prepared the students for the following question by using a verbal process 'say' and a main participant 'place value' in the if-led circumstantial clause. The teacher then selected another saying-verb 'be called.' The selection of the passive verb, which works as a transit to a relational process such as 'be' as in turn 5-(ii), can be intentionally chosen in that she could say, 'What would that place value be/represent?' She facilitated the question by using the same process type and placing a focus on a new academic language 'place value,' inviting students to a coding activity to identify what place the digit '2' represents. In this context, the students were asked to decode the identity of Token (i.e., the number symbol '2'). The teacher used another decoding clause in an interrogative mood (5-ii), where she selected 'be.' The choice of 'be' can be indicative of a "scientific discourse" that favors the identifying relational-clause structured as follows: Token/Identified ^16 Relational process ^ Value/Identifier (or Value/Identified ^ Relational process ^ Token/Identifier) (Halliday & Matthiessen, 2014, p.280). The subtle shift from 'say' through 'be called' to 'be' clause shows that the teacher provided ELLs with linguistic scaffolding that can allow them to participate in academic discourse.

Coding activity further proceeded with a question in reference to an attribute of a number (7). Ms. Bright asked Emma to ascribe a quality to the number symbol '72', saying "How many tens

16 The mark '^' indicates that something immediately follows the previous one.
are in seventy-two,” which can be instantiated as ‘How many tens does seventy-two have?’ The reference to the attribute can extend meaning-making of place value in a sense that students need to not only relate a number symbol to value through the decoding process but also identify attributes of the symbol.

In inviting students to identify attributes associated with a semiotic resource, Ms. Bright used WH-interrogatives including the having process combined with personal pronouns (e.g., ‘I,’ ‘you,’ ‘we’) which function to reduce information load of a message in a clause and invite students to identify contextual information demanded. In excerpt 2, for example, ‘how many digits do I have? (turn 1-ii) can be more familiar than ‘how many digits does this number have?’ because the former clause includes one mathematical entity while the latter does two (i.e., ‘how many digits’ & ‘this number’). Ms. Bright also favored the having-declarative clause when ascribing an attribute to a mathematical entity as in ‘we have three digits,’ in which ‘we’ can be replaced with an entity ‘this number.’ In other words, she invited students to identify the attribute of the entity, decreasing information load that the message delivers. The following segment occurred when students learned how to compare two 3-digit numbers using ‘>,’ ‘<’ and ‘=’ symbols.

Excerpt 2. Identifying the number of digits in a whole number ‘256’ (B=Bruno (Gr. 1))

1. T: (i) So here we have –(writes ‘256’ on the board) (ii) Bruno, how many digits do I have here? [One-(pointing ‘5’ in tens place of 256)
2. B: [Two? Five?
3. T: (i) Two, three. (ii) We have three digits. (…) In eng- when you're learning about- I think you're learning about this. (iii) We can make many numbers using zero through nine. (iv) We can make any number- any number using zero through nine. That’s it. That’s all we need. (v) So this is called a number. (vi) In it, we have three digits. (vii) Bruno, how many digits do we have in it?
4. B: Two?

17 The segment is derived from a class in March 12, 2015.
5. T: (writes ‘24’ on the board). (i) Two digits. (ii) That’s twenty-four. (iii) That would be a two-digit number. (points to the number ‘256’ written on the board) (iv) How many digits do I have here? (v) You said two. (vi) That’s one, two. (counting first two digits (i.e., 2 & 5 in 256) (vii) But I have another digit. (pointing ‘6’ in 256)


7. T: (i) How many total? (ii) You’re naming that number.

8. B: Three.

9. T: (i) You’re naming that number sweetie. (ii) That number is called six.


11. T: We have three digits.

In excerpt 2 above, Ms. Bright introduced a mathematical participant or entity using a having clause, writing the entity on the board instead of vocalizing the entity (turn 1-i). She then inquired an attribute of the entity (1-ii) while reserving the number entity, which indicates that the teacher attempted to decrease information load. The teacher counted (turn 3-i) and stated the number of the digits (turn 3-ii) when Bruno was confused about the meaning of ‘digit.’ In turn 3 (iii & iv), she articulated only ‘numbers’ and elaborated what digits mean, exemplifying ‘zero through nine’ rather than defining the meaning of digit. Pointing to the 3-digit number, she related the number symbol to language ‘number’ and classified the symbol as a family of the number (turn 3-v). She further provided contextual information required for the student to make sense of ‘digits’ by fronting the given information (i.e., ‘number’) and delaying the targeted academic language ‘digits’ (turn 3-vi). Ms. Bright asked Bruno to identify the attribute (i.e., the number of digits) ascribed to the number ‘256’ (turn 3-vii). Still the student named the number without recognizing the meaning of ‘digit’ (turn 4).

As indicated in turn 5-(i), Ms. Bright exemplified a 2-digit number ‘24’ and addressed that the number has two digits. To decode the token (i.e., ‘24’) presented on the board, she related the Value ‘two digits’ to its Token ‘24’ and ascribed the Value to the number (turn 5-ii & iii). She rather
reminded Bruno of her previous question asking for the Value of the given Token (turn 5-iv). She continued to explain the meaning of ‘digit’ (turn 5-v), counted only two digits of hundreds and tens places, and stopped to address the number of the digits (turn 5-vi). She pointed to the ones place ‘6,’ expecting the student to provide the Value ‘three digits’ as a right response (turn 5-vii). She attempted to further guide the student in understanding the meaning of ‘digit’ by replicating the question ‘how many digits?’ and stating what the student did was naming the number ‘6’ (turn 7-ii). Moreover, she decoded the digit six in ‘265’ by naming the number, that is language as Token (turn 9-ii). She rejected Bruno’s response (turn 9-i) to confirm whether or not he understood the concept of ‘digit.’ Ms. Bright finalized the segment by affirming the attribute of the entity (turn 11, ‘We have three digits’) when confirming that the student understood the meaning of ‘digit.’ As in excerpt 2, the segment indicates that the coding activity can be elaborated when the teacher guides students in verbalizing their ideas.

In sum, when inviting the students to engaging in coding activity, Ms. Bright gave her ELLs contextual information relevant to mathematical participants/entities in declarative clauses and asked them to link the entities as Token to the meanings as Value in interrogative clauses including relational processes (e.g., ‘represent’ as in ‘What does the six represent?’). In this initial stage, Ms. Bright introduced mathematical entities and asked the students to identify the meanings to prepare them for further developing the relationships between Token and Value by facilitating their participation in co-constructing mathematical concepts, as presented in the following section.

5.2.2 Facilitating Student Participation

This facilitating stage of teacher talk describes ways Ms. Bright engaged her students in mathematics discourse. She encouraged the students to participate in coding activity and asked them to verbalize their actions. The facilitating stage features the coordination of textual meaning and logical
meaning, specifically in terms of the Given-New information structure and the expansion nexus (e.g., elaboration, enhancement). In facilitating student participation, Ms. Bright favored the incomplete clause with rising intonation, Theme-Subject associating question, elaborating and enhancing clauses, and verbal-imperatives, all of which were aligned with the degrees that the students could understand mathematical concepts. In particular, the use of verbal-imperatives was significant in that it can occur not only in the stage of facilitating student participation in decoding Token for Value but also in the stage of promoting students to encode Value to Token by reasoning.

In facilitating student participation, Ms. Bright elaborated her utterances by adding some comment to a previous one and yielded the floor to students by using incomplete clauses. The elaborating and incomplete clauses first introduced familiar/everyday language and abstract/academic language later. Excerpt 3, for instance, Ms. Bright attempted to unpack language in a math word problem as in Table 5.1. This teacher-created problem, which can facilitate student understanding, included her name and another teacher’s name that the students already knew.

<table>
<thead>
<tr>
<th>Ms. Bright bought 10 gumballs. Miss Levin bought 25 gumballs. Who bought more? How many more did she buy?”</th>
</tr>
</thead>
</table>

Table 5.1 Math word problem 01: Subtraction

After presenting this problem, Ms. Bright asked students to find language that worked as clue in solving the problem as in excerpt 3.

Excerpt 3. Deconstructing language in a math word problem (C=Camila (Gr. 2))

__18 The segment was derived from a class in October 30, 2014.__

__19 The segment is derived from a class in October 30, 2014.__
To deconstruct the written text, Ms. Bright asked a question (1) and received a response (2). The teacher repeated Camila’s response and challenged a student to provide the rationale that ‘how many more’ would work as clue to solving the given math-word problem (3-i). Ms. Bright rephrased two clauses (i.e., ‘Ms. Bright bought 10 gumballs. Miss Levin bought 25 gumballs’) into one, ‘how someone has more than someone else’ (5-ii). She intentionally avoided using numbers with units such as ‘10 gumballs’ and simplified the written text, emphasizing ‘how,’ ‘someone,’ ‘more’ and ‘else.’ This simplified utterance with emphasis can hint that the problem concerns two participants compared. She later elaborated on and clarified the proceeding utterance by using written-like language ‘comparing’ (7-i) relevant to a concept of subtraction in this context. She then attempted to re-unpack the language with students by selecting incomplete clauses (7-iii; 9), which encourage student participation in mathematics discourse (8; 10). The unpacking and re-unpacking process indicates the interplay between Given information and New information.

The Given-New information interplay occurred in enhancing clauses concerning time, cause, or condition as in the elaborating clauses. Excerpt 4 features how the teacher developed discourse

---

20 In the excerpts of this study, ‘S’ and ‘Ss’ each indicate unidentified student and a group of students.
and enhanced meaning by using 'so' and 'because,' each of which works to link and bind a proposition/proposal with another one.

**Excerpt 4. Subtraction using a number line**

1. T: (i) I’m going to start here at seventeen (ii) and I need to get to twelve, (iii) so I’m going to subtract. Please listen! Here we go. (iv) You count back with me (v) because we’re subtracting. (vi) Count back.
2. S: I know. I didn’t count. I count back more, more.
3. T: I’m waiting. (.) Watch my, um, cursor. So the little- like an arrow. Watch it. Can everybody see it?
4. Ss: =Yes.
5. T: Okay. Watch me. Watch. (i) From seventeen to sixteen is (.) one. (ii) I’m going down by one. (iii) I’m subtracting one. Aren’t I?
6. Ss: Yes.
7. T: Then let’s go back. Okay, everybody. [One-
8. Ss: [One, two, three, four, five. Five years!

In excerpt 4, Ms. Bright attracted student attention using ‘here’ (1-i) and introduced two numbers (i.e., ‘17’ & ‘12’) as main participants of subtraction, using here-and-now/everyday language. She then attempted to connect everyday/Given information to math-specific language ‘subtract’ (1-iii). The Given information is paratactically linked to New information ‘we are going to subtract’ by the linker ‘so’ (1-iii). In the subsequent utterance, the teacher unpacked the academic language into everyday language ‘count back’ (1-iv) and repacked it to math language ‘subtracting’ by the binder ‘because’ (1-v). Binding two clauses can contribute the concept of subtraction to being closely related to the action of counting back. Linguistic scaffolding through the interplay between everyday language and academic language facilitated student participation (2). In turn 5, she repeated here-and-now/everyday language (5-i, ii) and connected it to math-specific language (5-)

---

21 The segment is derived from a lesson in November 13, 2014.
iii). She continued to call for student participation, inviting all students to count with her (7).

Ms. Bright also facilitated student participation by using statement-like questions and enhancing clauses. Excerpt 5 occurred when she utilized place-value blocks corresponding to ones and tens to teach a concept of regrouping defined as “exchanging 10 in one place-value position for 1 in the position to the left—or the reverse, exchanging 1 for 10 in the position to the right” (Van de Walle, Karp, & Bay-Williams, 2010, p.223). An understanding of this concept is required for addition and subtraction relevant to the traditional computation strategy ‘algorithm.’ The following segment showcases how the teacher used Theme-Subject association, verbal clause, enhancing clause of time, and elaborating clause to facilitate student participation.

Excerpt 5. Relationship between tens and ones (J=Juan (Gr. 3); E=Emma (Gr. 3))

1. T: So she said what, Juan?
2. J: ( )
3. T: Okay now a little bit louder because the fan- the heat just came out.
4. J: She said that (…) that you can trade (…) ten tens (sounds very weak)
5. T: (i) These are ten tens? (ii) These are not tens.
6. J: ( )
7. T: (nods her head) He got confused for a minute but guess what? He self-corrected! (i) Say it nice, (ii) turn around to your classmates (iii) and say it again. Loud sweetie, loud. <<<…>>> 23
8. J: She said that you can trade ten ones for one tens.
9. T: That’s right! We can trade ten of these-
   (Teacher picks up some tools)
10. T: (i) Take ten of these, Yoana. (ii) Take ten of them and (iii) put them on the table. So Juan is correct. Danna is also correct. They said the same thing in different ways. (iv) So when you’re thinking about today’s work, (v) you need to understand that you can trade. (vi) What’s another word that we’ve been learning about, other than trade? (vii) It starts with an R.
    (Danna raises her hand)

22 Excerpt 4 is derived from a class in February 12, 2015.

23 In all excerpts <<<…>>> indicates that some turns are omitted. Here four turns irrelevant to math content are omitted.
11. T: (i) You’re subtracting. Raise your hand if you think you might know. (ii) When you’re subtracting and you don’t have enough, (iii) you need to trade. 
   \textit{(Danna and Jay look excited and raise their hands)}
   \texttt{<<...>>}^{24}
13. Ss: Regrouping.
14. T: That’s right! (i) We’re regrouping! (ii) We’re taking some from one group and (iii) putting it into another! (iv) Okay, and that’s the same thing as trading!

Prior to this segment, Ms. Bright gained a student response when asking how many ones they needed to make a ten when using base-ten blocks. The teacher then asked Juan to restate what his peer said (1), placing known/Given information in the Theme and unknown/New information ‘what’ in the Rheme and associating the Theme with the Subject ‘she.’ The Theme-Subject association is indicative of a statement. This Given-New information structure facilitates student participation because the Subject ‘she’ is typically dissociated with the Theme in the interrogative such as ‘what did she say?’ The teacher guided Juan in perceiving the ones blocks, using another statement-like question ‘These are tens?’ (5-i) instead of a typical question ‘Are these tens?’ because the student failed to restate what his peer said. After the statement-like question (5-i), Ms. Bright demanded the student’s restatement in the verbal imperative (7-i, iii), which works as facilitator in helping students listen to others and report what they said. The facilitating remark supported Juan in being responsive to the teacher’s demand (8).

To further support students in understanding the relationship between ones and tens, Ms. Bright asked for two actions in a temporal order (10-ii, iii), commanding a second-grader named Yoana to take the ten ones first and put them on the table. This temporal enhancement between the

\texttt{24} In \texttt{<<...>>}, ten turns referring to regulatory registers are omitted.
two neighboring clauses is significant in the connection of manipulatives to mathematical concepts that involves procedural knowledge (Ma, 1999). The teacher then reiterated the procedural actions as in ‘today’s work’ (10-iv) and related them to ‘trade’ (10-v). The enhancing clause is used to interweave two actions with the meaning. In the following turn, she asked students to tell academic language equivalent to ‘trade,’ presenting a hint (10-vii). Before obtaining student response, Ms. Bright gave another clue (11-i, ii, iii), hinting that the academic language is related to subtraction. Her utterance of ‘subtracting’ and ‘trading’ can guide students in associating their actions with academic language by excluding the two given words. She eventually gained a right response, which is indicated by ‘Yes’ (12), followed by two verbal clauses (12-i, ii). Turn 12-(i) is a grammatical metaphor that represents a command saying ‘Repeat’ (12-ii). This modulated verbal imperative clause can facilitate student participation because the question allows students to either accept the offer by responding to the question or reject the offer by saying ‘no.’ By using verbal-imperatives, the teacher required students to collectively verbalize the targeted academic language ‘regrouping.’ She then articulated what they were doing (14-i) by repeating the language. In the immediate clauses (14-ii, iii), she unpacked the academic language, interweaving the language into everyday language. The teacher interwove again connecting academic language ‘regrouping’ back to everyday language ‘trading’ (14-iv). As indicated in excerpt 4, Ms. Bright facilitated student participation in mathematics discourse using Theme-Subject associating clause and verbal-imperatives as well as interweaving familiar/everyday language with academic language in Given-New information structure.

In this facilitating stage of teacher talk, four linguistic features were highlighted as follows: incomplete interrogative clauses (e.g., sentence starters), Theme-Subject associating interrogatives (e.g., ‘You have how many one?’), elaborating clauses (e.g., ‘What operation? Like add, subtract,’
multiply, divide?'), enhancing clauses (e.g., 'You count back with me because we're subtracting'), and verbal-imperatives (e.g., 'Tell me what she did'). This facilitating stage works as foundation for driving students to promoting their reasoning. Decoding through co-construction of making sense of Token precedes encoding Value through reasoning why present expression of action is taking place.

5.2.3 Promoting Students’ Reasoning

The stage of promoting students’ reasoning in teacher talk attends to the coordination of interpersonal meaning and logical meaning, specifically WH-interrogation (e.g., 'How do you know? 'Why did she put them in groups of ten?') and mental-projection interrogatives (e.g., 'Do you think that symbol goes there?'). Ms. Bright encouraged students to show their thinking or reasoning by asking why/how-questions and probing for information. How-interrogatives (e.g., 'How do you know?') are similar to why-interrogatives (e.g., 'Why did she put them in groups of ten?') in that both of them can be used to give a cue for students to find the relationship between Value and Token through justifying their answers as Value by reference to Token. The why/how-question sought for specification of an entity that the teacher had already mentioned. In response, students were positioned to validate a proposition of relating Token to Value by specifying the entity/Token through placing the Value in the clause of cause or manner. The mental-projecting interrogative served to probe information that students were expected to provide as a result of reasoning. Classroom talk is reiterative because promoting student reasoning presupposes a certain degree of student understanding that is established in the facilitating stage. Ms. Bright returns to the facilitating stage if she fails to obtain student response showing their reasoning. Excerpt 6, for
example, happened when the students reviewed a lesson that they had a week before to solve a math word problem presented in Table 5.2.

| Juan and Pam popped balloons. They each popped an equal number of balloons. Altogether (+) they popped _____ balloons. How many balloons did they each pop? |
|---|---|
| Gr.1=16 | Gr.2=64 | Gr.3=124 |

Table 5.2 Math word problem 02: Division

The students did not have this math class regularly because of standardized tests. In excerpt 6, Ms. Bright attempted to ask a student to explain how to solve a math word problem presented in Table 5.2, asking, ‘How do you know?’ (turn 7) and then facilitated student response by using a sentence starter (turn 9).

Excerpt 6. Showing problem-solving strategies (E=Emma (Gr. 3))

1. T: Okay, Emma, you’re gonna show us before it’s time to go.
   <<...>>,26
2. E: Okay, I, this, I underlined important words. ( ) And I drew this and then I wrote the answer statement and then I did this.
3. T: Algorithm?
4. E: Yeah. And the answer is sixty-two.
5. T: Check if it makes sense. Does it make sense?
7. T: How do you know?
8. E: Because ( )
9. T: And what was part of the given information, that they each popped an?
10. E: Equal number.
11. T: And sixty-two and sixty-two equal number?
12. E: Yeah.

25 The segment is derived from a lesson in December 18, 2014.

26 In <<...>>, six turns are omitted.
13. T: (i) When you put it together, (ii) altogether they popped?


This segment in excerpt 6 occurred when Ms. Bright prepared to wrap up the lesson as indicated in turn 1, where she asked Emma to share her work. Emma verbally listed her problem-solving procedures (turn 2), using an exophoric reference ‘this’ as in ‘And I drew this and then I wrote the answer statement and then I did this.’ The teacher clarified that the reference referred to math-specific language ‘algorithm’ (turn 3). Only listing the procedures, Emma gave her answer (turn 4). Ms. Bright attempted to ask Emma to make reasoning behind the answer by selecting imperative and yes/no interrogative (turn 5). When gaining ‘Yeah,’ the teacher changed the yes/no question into a how-question (turn 7). In response, Emma provided her rationale by using a circumstantial clause of cause proceeding with ‘Because’ (turn 8). She seems to fail to justify her answer, which is evident in the following utterance. This failure shows that the student cannot encode the Value to the Token. In turn 9 Ms. Bright directed Emma’s attention to the given math problem, using a sentence starter and prompting her response. In what follows (turn 11), the teacher asked the student to affirm that she had two equal numbers. Ms. Bright seems to emphasize the phrase ‘an equal number of balloons’ by selecting ‘it’ referring to ‘the two sixty-twos’ instead of ‘them’ as in ‘when you put it together’ (turn 13-i) and connects ‘put together’ and ‘altogether they popped?’ presented in the written text (turn 13-ii) while yielding the floor by using a sentence starter. As shown in excerpt 6, Ms. Bright facilitated student understanding when she failed to see students make reasoning in response to how-interrogatives.

Besides choosing how-questions to raise student reasoning, Ms. Bright favored why-interrogatives and mental-projecting interrogative. The use of why (e.g., ‘Why is it still nine o’clock?’) and mental-projecting interrogatives (e.g., ‘What hour do you guys think it would be?’) worked to
demand that students project an idea or a proposition "by a process of thinking" (Halliday & Matthiessen, 2014, p.516). Excerpt 7 showcases how the teacher used language to heighten student reasoning. The segment occurred when the teacher worked to teach how to read clocks to the hour using a dial-type instrument or an analog clock.

Excerpt 7. Tell the hour when the hour hand is between two numbers (Y=Yoana (Gr.2))

1. T: (i) I'm in between the nine and the ten. (ii) What hour do you guys [think it will be?
3. T: Nine. Why? (i) Yoana, why is it still nine o’clock (ii) when it’s still closer to the ten?
4. Y: Because it didn’t pass the ten.
5. T: It didn’t pass the?
7. T: (i) It didn’t pass the ten yet. (ii) What if I go here?
8. Ss: Eleven.

In turn 1-(i), Ms. Bright gave contextual information, pointing to a spot between two numbers of the analog clock projected on the screen. The contextual information is hypothetical, which is obvious because her finger represents the hour hand ‘I.’ She then inserted ‘do you guys think,’ which leads a question ‘what hour would it be?’ to become a proposition ‘what hour it would be’ (turn 1-ii). The proposition is a statement, which indicates that the Subject ‘it’ comes before Finite ‘would.’ The teacher required students to undergo a thinking process before asking why/how questions and project what they have in mind through mental-projection question. In comparison, the question ‘what hour would it be?’ does not demand explicitly that students think and reason. This single clause is used just to identify ‘it’ that represents the hour hand, not probing information that can result from a chain of reasoning. Following Yoana’s right response (turn 2), Ms. Bright

27 This segment is derived from a class in April 30, 2015.
recovered the ellipted part in her response and challenged the student to validate her proposition as in ‘why is it still nine o’clock?’ (turn 3-i). The teacher further challenged Yoana to make reasoning by binding a proposition ‘it is nine o’clock’ and another one ‘when it is still closer to the ten’ (turn 3-ii). The student might be confused about the hour that the hour hand represented because she was given two numbers ‘nine’ and ‘ten.’ Despite the challenge, the student verbalized the reason (turn 4) in response to the ‘why.’ Instead of confirming Yoana’s response, Ms. Bright used a sentence-starter without focus information ‘ten’ and asked her to affirm her idea (turn 5). The teacher reformulated the student response (turn 6-i) and attempted to confirm if the students understood a concept of the hour (turn 6-ii). The hypothetical question can imply ‘if I would go here to eleven, then what would happen?’ The use of if-then clause nexus can be intentional in that the nexus is typical of mathematical discourse (Huang & Normandi, 2008).

In sum, Ms. Bright provided verbal scaffolding for ELLs in the unfolding classroom talk including three stages: inviting students to engaging in coding activity, facilitating student participation, and promoting students’ reasoning. In inviting the students to engaging in coding activities, she gave them contextual information associated with mathematical participants/entities in declarative clauses and asked them to relate the entities to their Values in interrogative clauses. When facilitating student participation, Ms. Bright used incomplete clauses such as sentence starters, Theme-Subject associating questions (e.g., ‘You have how many hundreds?’ instead of ‘How many hundreds do you have?’), placing Given information first and New information later in elaborating and enhancing clauses, and verbal imperatives (e.g., ‘Tell me what you did’). In the stage of promoting students’ reasoning, the teacher significantly used how/why questions and mental-projection questions (e.g., ‘What hour do you guys think it would be?’). Verbal scaffolding that Ms. Bright provided for her ELLs seems to guide the ELLs in co-constructing mathematical concepts, as
presented in the subsequent section.

5.3 Students’ Co-constructing Mathematical Concepts

Students develop academic language and mathematical concepts while they participate in mathematical discourse and co-construct mathematical meaning with teachers and other students. Student participation can be supported by “what the teacher says or what the teacher attends to” (Kazemi & Hintz, 2014, p.23). Findings of this study in reference to student participation in mathematics classroom discourse show that English language learners were guided to use elliptical clauses omitting relational processes, material clauses, relational clauses, and enhancing clause nexuses. The selection of material clauses was prevalent in student talk and writing. These various linguistic features are categorized into three stages: establishing math concepts by participating in mathematics discourse, processing math concepts through verbalizing actions, and binding everyday concepts with scientific concepts, as illustrated in Figure 5.2.

![Thematic network of student response](image-url)

Figure 5.2 Thematic network of student response
With regard to student response, using elliptical clauses is typical of classroom talk because it is commonplace to ellipt what is previously mentioned in a question and provide an information focus (Halliday & Mattiessen, 2014, p.127). Despite typicality of elliptical clauses students use, I attend to their functions in teacher-student oral interactions. In particular, those occurred at the beginning of episodes can be of vital importance for students who are invited to participate in classroom discourse and develop content knowledge.

As shown in Figure 5.2, the first category ‘establishing math concepts by participating in mathematics discourse’ was significant in the initial phase of an episode, in which students were asked to respond to questions regarding attribute, identity, or relationship of entities. In response to teacher questions relevant to multiple semiotic resources, students mostly ellipted what were previously mentioned and provided information focus. For example, a student answered ‘Five hundreds’ in response to a teacher question ‘How many hundreds do you have?’, which can be said, ‘how many hundreds does this number have?’ As discussed in section 5.1, it is characteristic to replace ‘does this number have’ with ‘do you have’ in this mathematics classroom discourse. The replacement can function to invite students who are expected to take the responsibility for the proposition and to give information demanded. The demanded information was focused on the identification of hundreds place value of the number. The student response carried information focus that was hundreds place value. The having-verb or attributive-relational process served to identify attributes of an entity. In contrast, an identifying-relational process worked to relate Token to Value or vice versa, both of which refers to the same thing but represents ‘expression’ and ‘content’ respectively. In this respect, this study differentiates ‘elliptical clause that omits an identifying-relational process’ from ‘elliptical clause that omits an attributive-relational process,’
considering that the teacher favored relational clauses in initiating talk.

The second category ‘processing math concepts through verbalizing actions’ entails ‘material clauses to describe what they did’ and material clauses to report what they said.’ The two patterns concern what they did in solving problems or repeating what someone else said, by which students can reflect on their actions and process a concept through wording. In processing math concepts, they were encouraged to tell actions relevant to solving problems. When they reported someone’s utterance such as ‘you can trade the one hundreds for ten tens’, for instance, they could say ‘she said that you could trade the one hundreds for ten tens.’ The shift from ‘can’ to ‘could’ makes the report away from the original utterance, which indicates that students need to process the original remark to vocalize it.

The third category ‘binding everyday concepts with academic concepts’ engages ‘relating Token to Value or Value to Token’ and ‘enhancing clauses to show understanding’ in a sense that Token and Value each relate to everyday concepts and academic/scientific concepts. Both of them can be bound in enhancing clause nexus comprising Given information and New information as in a clause complex ‘We crossed out the six and put a five because she was regrouping.’ The enhancing clause nexus concerns a logico-semantic relation between a clause and the immediately following clause. For instance, the relationship between a clause ‘you count back with me’ and the subsequent clause ‘because we’re subtracting’ is semantically logical because one clause represents external experience while the other does something internal. The enhancing clause nexus, thus, can play a critical role in promoting student reasoning. All of the three categories presented in the middle of Figure 5.2 contribute to represent one general theme, ‘co-constructing mathematical concepts.’ The subsequent sections discuss these three stages in detail.
5.3.1 Establishing Math Concepts by Participating in Mathematics Discourse

At the beginning of an episode, students were invited to identify mathematical entities (e.g.,
manipulatives, pictures, number sentences, language) and establish a mathematical concept by
participating in mathematics classroom discourse that work to construct mathematical meanings.
Mathematics classroom discourse often requires students to relate semiotic resources to concepts
as well as to identify contextual information relevant to those semiotic resources. In SFL, semiotic
resources can be analyzed as Token and the concepts as Value as described in chapter 3.3. The
stage of establishing math concepts attends to student responses that feature ellipted clauses
concerned with relational processes (e.g., ‘be,’ ‘have,’ ‘represent’). These relational clauses decode
meanings of mathematical entities and ascribe quality or attributes to the entities. The relational
clause also represents the relationships of mathematical entities.

Excerpt 8, for example, displays that students were invited to relate Token to Value and
responded by elliptical clauses. The segment involves language (i.e., ‘altogether’) as a semiotic
resource.

Excerpt 8. Decoding language as Token (E=Emma (Gr.3))\textsuperscript{28}

1. T: Emma, point to other words that help you better understand the math word problem.
2. E: Altogether.
3. T: Altogether. What does altogether mean?
4. E: The- the group.
5. T: (i) The whole [.] group. (ii) So what does the whole group- what do they have altogether?
   (iii) How many balloons?

\textsuperscript{28} The segment is derived from December 11, 2014.
In turn 2, Emma responded by one word ‘Altogether,’ which indicates that ‘altogether is a clue,’ when asked to point to what words were clue in solving the math word problem (turn 1). Emma was commanded to act, but she vocalizes the clue word. Following the utterance, Emma was asked to decode meaning of ‘altogether’ (turn 3). The language ‘altogether’ becomes a mathematical entity as a Token, which students should decode meaning or Value behind the Token ‘altogether.’ The meaning refers to the sum of two equal parts, as in a problem ‘They (Juan and Pam) each popped an equal number of balloons. Altogether (+)29 they popped ___ balloons. How many balloons did they each pop?’ (see Table 5.2). The blank was used to differentiate the whole numbers by grade level. Emma in third grade had ‘124’ in the blank. Emma attempted to relate Token to Value indicated by ‘the group’ (turn 4), which is indicative of a shift from a circumstance (e.g., ‘altogether’) to an entity (e.g., ‘the group’). This response suggests that Emma needs to be guided in establishing a mathematical concept that involves identification of entities—the sum of two equal parts.

To support students in identifying ‘altogether’ to its mathematical meaning, Ms. Bright asked them to identify attributes of ‘the group.’ She elaborated on the student response by adding ‘whole’ to ‘the group’ (turn 5-i). She attempted to correct her question ‘what does the whole group’ but she rephrased the question as in ‘what do they have altogether?’ (turn 5-ii) while interweaving written-like/academic language and everyday language. She then linked the question to another question written in the problem (turn 5-iii). In response, students were asked to identify an attribute of the entity (i.e., ‘the whole group’), not its identity. The reformulation can be concerned with the level of student understanding or the ZPD (Vygotsky, 1978), probably because the teacher assumes that Emma needs to be guided in connecting ‘altogether’ to the sum of some parts. In turn

29 The symbol (+) was given in the original written text that Ms. Bright provided.
6, Emma ellipted 'the whole group has' and responded, *'one hundred twenty-four,'* addressing that she identifies one entity that possesses 124 balloons. The sequence of turn 5 and 6 indicates that students were asked not only to identify a value of a mathematical resource or representation but also to recognize attributes of the representation.

In addition to identifying attributes of a representation or an entity, students as participants were frequently connected to mathematical entities through the having-process (e.g., 'have' as in a clause, *'We have one hundred*) as if they held attributes of the entities in this mathematics classroom discourse. Student participants combined with the having-process were often ellipted in student response. Excerpt 9 occurred when Ms. Bright was working with grade-2 students while looking at their notebooks. The segment displays that students were first invited to decode the meaning of a mathematical entity (e.g., visual display of a number) as in turn 1-(ii). They were also asked to respond by reference to an attribute of the mathematical entity as in turn 5-(iii), *'how many hundreds do you have?* that means *'how many hundred does this picture have?* In response, students ascribed an attribute to the picture by elliptical clauses (e.g., turn 4, 6, 8, & 12).

Excerpt 9. Subtracting two 3-digit numbers (*'534–428'* by algorithm (A=Alba (Gr.2); C=Camila (Gr.2); Y=Yoana (Gr.2))

1. T:  Now, let’s see, (i) if your numbers here in your paper match the picture, the model I mean. Let’s start. (ii) What number do you have here now? (iii) Five hundred?
2. A:  Five hundred thirty-four.
3. T:  Hmm, (i) I don’t see five hundred thirty [...] (ii) the three, it has a slash.
4. C/Y: Five hundred twenty four!
5. T:  (i) You have five hundred twenty-four still [...] (ii) but it looks different, doesn’t it? (iii) How many hundreds do you have? (iv) Everybody, how many hundreds do you have?
7. T:  How many tens do you have?

30 The segment is derived from February 26, 2015.
8. C: Two.
9. T: You have three tens, really?
10. C: No two tens.

Prior to this segment, students were called to do pair work and draw pictures of ‘534−428’ in the notebook and show how to regroup the tens place and subtract the ones place. This segment only shows how students related a picture of 534 to the meaning and ascribed attributes to the picture. Ms. Bright first called for students’ attention, identifying a number with the pictorial representation (turn 1-i). She then invited the students to make sense of the value that the picture represents (turn 1-ii) through the having-clause, ‘what number do you have here now?’ in which ‘do you have here now’ indicates ‘the picture you drew in your notebook. The teacher was asking, ‘what number does this picture represent?’, inviting students to decode the meaning of the picture. Alba used an elliptical clause to attempt to decode the meaning (turn 2), which the teacher did not acknowledge while pointing out the slash the students marked on the picture (turn 3-i & ii). They were invited to receive information concerning an attribute ascribed to part of the picture (turn 3-ii). The picture contained a slash mark indicating that one of the three base-ten blocks was decomposed to regroup into ones. Camila and Yoana succeeded to decode the picture they had and articulated the number their picture represented after regrouping tens place (turn 4).

Acknowledging the students’ success in understanding the meaning represented in the picture (turn 5-i), Ms. Bright challenged students to assign an attribute to the picture, seeking for their agreement on the fact that there was a difference between the picture and the number they called (turn 5-ii). She demanded on information that pertained to the hundreds and tens place value as attribute of the 3-digit number ’534’ (turn 5-iii, iv, & 7). Students responded by elliptical clauses to offer the information (turn 6 & 8). In turn 9, the teacher attempted to confirm if the students
understood a concept of regrouping tens into ones marked in the picture, asking if the picture still represents three tens. Camila used a negative polarity (i.e., ‘No’) and offered her proposition in the ellipted form (turn 10). This student response ellipted ‘I have’ and provided focus information relevant to the attribute of the pictorial entity. As shown in excerpt 9, students were invited to participate in mathematics discourse and identify attributes of a mathematical entity and meaning associated with the entity when establishing mathematical concepts.

Collectively, in the stage of beginning to establish math concepts, students provided elliptical clauses (e.g., ‘Division’) in response to teacher questions (e.g., ‘Half of, what operation?’ indicating ‘what operation is half of?’), which demanded information focusing on identities or attributes of entities as Token. They identified a Token by reference to the Value and connected the Token to the Value by naming them. The identification of Token and Value prepared students to further develop the relationship between Token and Value in the stage of processing math concepts by verbalizing actions.

5.3.2 Processing Math Concepts by Verbalizing Actions

Given that students begin to establish math concepts by identifying the meaning of a mathematical entity and its attribute, they need to understand how entities function to construct meaning. This study defines ‘processing math concepts’ as further developing the Token-Value relationship by using entities and their attributes identified in the decoding activity. Understanding functions of entities can support students in processing mathematical concepts. In processing math concepts, students favored material clauses to describe how mathematical entities could make sense to them and report how they approached math problems. Students’ reports involved projection clause nexus that causes a change in wording to be “shifted away from reference to the
speech situation” (Halliday & Matthiessen, 2014, p.529). Besides projection of wording, the stage to
process math concepts highlights how students processed mathematical entities as Goal of action.
Students chose ‘you’ as Actor, which can work to visualize mathematical concepts. In this
mathematics classroom, ‘you’ excludes the teacher (turn 11 & 12) and instead may refer to students
who should take responsibility for the meaning-making process.

Excerpt 10. Connecting ten rods to unit cubes (L=Lucas; D=Danna; J=Juan in Gr. 3)\(^{31}\)

1. T: (i) Tell me, (ii) what’s something that is the same about these?
2. L: With the ones you can make hundreds, tens blocks.
3. T: (i) With the ones you can make a tens stick, that’s right, (ii) or (picks up and show a hundred
   block to Lucas) you can make a?
4. L: Hundreds block.
5. T: Right! Excellent job! (i) Now tell me (ii) what’s different about these two, Danna.
   \(<\ldots>\)\(^{32}\)
6. T: So how many ones do we need to make this ten?
7. D: ( )
8. T: Say you need
9. D: You need ten ones ( )=
10. T: =Stick. Right, so I know it gets kind of confusing. (i) So she said what, Juan?
    \(<\ldots>\>
11. J: She said that we can-
12. T: (i) No I didn’t say- you said the answer, not me. (ii) I'm telling you about-
13. J: I said ( )
15. J: She said that you can trade ten ones for one ten.

Excerpt 10 occurred when students learned about the relationship between the tens place
and the ones place using manipulatives ‘base ten blocks.’ Ms. Bright used the imperative clause
(turn 1-i) and relational clause (turn 1-ii), commanding Lucas to identify a relationship between

\(^{31}\) The segment is derived from a class in February 12, 2015.

\(^{32}\) In \(<\ldots>\>\>, five turn are omitted and after turn 10, six turns are omitted in \(<\ldots>\>\).
two entities (i.e., base ten blocks and base one cubes). He emphasized an entity (i.e., ‘the ones’) by fronting the entity in the clause and then connected it to ‘hundreds, tens blocks’ by using a material process ‘making’ (turn 2). The material clause can be instantiated in a relational clause, ‘ones can become tens and hundreds.’ The relational process ‘become’ connects to multiple mathematical entities which are inanimate. By using the material clause, Lucas attempted to invigorate and process the entities. Ms. Bright corrected Lucas’ response to list place values in increasing order of magnitude (turn 3-i & ii), but she facilitated their participation using an incomplete clause with the same material process ‘make’ Lucas used (turn 3-ii). Lucas completed the material clause by offering one entity (turn 4). In turn 5, Ms. Bright asked Danna to find differences between the entities. The teacher restated what Danna said, which is inaudible, and encouraged students to identify an attribute of an entity ‘ones’ (turn 6). Ms. Bright inserted ‘do we need’ between ‘how many ones’ and ‘to make this ten’ to split one clause ‘how many ones make ten?’ into two: ‘how many ones do we need?’ as a free clause and ‘to make this ten’ as a bound clause to support the free clause. The separation can serve to support students in making mathematical concepts. Turn 9 shows that Danna was guided to succeed in identifying attributes of the ones.

Students were asked to restate their peer’s statement, which happened when the teacher guided them in offering a response. She required Juan to re-voice Danna’s statement (turn 10-i). Juan used ‘we’ not ‘you,’ which can include the teacher (turn 11). Ms. Bright rejected being included to make students responsible for processing math concepts (turn 12-i). In turn 15 Juan reported Danna’s speech indirectly, in which her speech becomes remote from the original one that is situated in here-and-now context. This reported projection nexus can serve to process mathematical concepts through the shift from here-and-now/everyday concept to detached/scientific concept.
In addition to use of manipulatives as mathematical entity, students' selection of material clauses was prevalent in involving symbolic representations (e.g., math algorithm) as an entity, as shown in excerpt 11. The material clause functioned to visualize the entity that students could manipulate.

Excerpt 11. Explaining steps to solve ‘534−428’ by algorithm (R=Rosa in Gr. 2)33

1. R: First I [...] drew.
3. R: Drew. Drew a picture [...] and-
4. T: Just listen sweetie.
5. R: And crossed out four.
6. T: (i) You drew a picture [...] (ii) and you crossed out four. Oh. Okay. (iii) Keep going.
7. R: And then crossed out two from tens.
8. T: Two from the tens. Okay.
9. R: (i) But if you cross out eight (ii) you can’t do that (iii) so [...] so you add, so you add ten to the ones.
10. T: (i) You add ten to the ones? (ii) Well where’d you get that ten to add to the ones? [...] (iii) Where’d you get it?
11. R: From the-
12. T: From where?
13. R: From the tens.

Before this segment, students were asked to solve a subtraction problem (i.e., ‘534−428’) using a mathematical algorithm. They were asked to illustrate the problem-solving procedure as shown in Figure 5.3, which was produced by Rosa in grade 2.

---

33 The segment is derived from a class in February 26, 2015.
Rosa responded by an incomplete material clause (turn 1) when Ms. Bright asked Rosa to tell the procedures she took to solve the problem. The teacher supported Rosa with the word ‘picture’ she needed to complete the material clause (turn 2). The student involved an entity ‘picture’ as Goal that represented a number (i.e., ‘534’) and materialized the number, as shown in Figure 5.3. Rosa attempted to continue to explain the procedures but stopped (turn 3) because of someone’s interruption, which is evident in the teacher’s remark in turn 4. Rosa then engaged another entity ‘four’ in the hundreds place and visualized the entity associated with a material process ‘crossed out’ (turn 5), which indicates that she was subtracting from the left hand. The teacher acknowledged her response and only reiterated her statement (turn 6-i & ii), encouraging her to take the floor as indicated ‘keep going’ that means ‘keep telling’ (turn 6-iii). Rosa verbally described the subtraction of digits in tens place, which is realized in a material clause to visualize the subtraction as action (turn 7). Ms. Bright restated ‘two from the tens’ and let her continue to take the floor, which is indicated by ‘OK’ (turn 8). In Turn 9-(i), Rosa made reasoning to develop her argument through a relation of condition. She addressed that the audience ‘you’ could not subtract
8 out of 4 and have to regroup one tens in 534, failing to clearly verbalize the subtraction of 8 out of 4 (turn 9-ii). In turn 9-(ii), in particular, she used an unclear reference ‘that’ combined with ‘do,’ which indicates that she needs to develop language (e.g., ‘take away,’ ‘subtract’). Rosa, however, showed that one tens of 534 should be regrouped and added to ‘4’ in the ones (turn 9-iii). The single clause ‘you add ten to the ones’ is compared with a clause complex (turn 10-ii & iii). Repeating Rosa’s statement (turn 10-i), Ms. Bright demanded further information, attempting to visualize regrouping tens to ones by using a material process ‘get’ combined with a math entity ‘that ten’ (turn 10-ii). The teacher placed Rosa’s response (i.e., ‘to add the ten to the ones’) embedded in the primary action as ‘get that ten’ to support the student in processing the primary action. The teacher repeated and emphasized where-interrogative clause (turn 10-iii). In response, Rosa failed to complete her remark (turn 11). Ms. Bright broke the clause into a phrase ‘from where?’ while demanding on focus information (turn 12). Rosa provided the right response by an elliptical clause (turn 13) although she still needed to develop the concept of regrouping by binding a sequence of events or actions: ‘You can regroup one tens and add it to the ones because you cannot subtract 8 from 4.’ Excerpt 11 indicates that a series of actions realized through material processes (e.g., ‘drew,’ ‘crossed out,’ ‘add’) should be sequenced step-by-step when students solve a subtraction problem using a math algorithm.

In sum, when processing math concepts by verbalizing actions, the students favored material processes (e.g., ‘make’ as in ‘You can make hundreds, tens blocks’) in response to Ms. Bright’s requests (e.g., ‘Tell me what is something that is the same about these?’) to express how mathematical entities made sense to them. The students also used material processes (e.g., ‘trade’) to report what someone else did as in ‘She said that she can trade ten ones for one ten.’ Even when explaining how to solve a problem, the students selected a series of material processes (e.g., ‘drew,’
‘cross out,’ ‘add’) in complete clauses as opposed to elliptical clauses. The shift from elliptical to complete clauses can be indicative of students’ language development, which can promote students’ concept development in a clause as unit of message. Representing a message in a clause can enable students to make reasoning the Token-Value relationship using everyday language and academic language while binding everyday concepts and scientific concepts, as described in the following section.

5.3.3 Binding Everyday Concepts and Scientific Concepts

The stage of binding concepts attends to relational clauses that relate one entity to another and clause complexes integrating meaning through logico-semantic relations of cause and condition. Unlike the establishing stage, students were asked to find not only the meaning of multi-semiotic entities or resources but also apply the meaning to semiotic resources. Given that multisemiotic resources are “lower expression” or Token while the meanings “higher content” or Value (Halliday & Matthiessen, 2014, p.279), relating Token to Value is concerned with binding everyday concepts with scientific concepts. In this stage, students often used complete-relational clauses as opposed to ellipted-relational ones characterized in the establishing stage. They also used the encoding clause in binding concepts, which is significant because the use of the decoding clause was prevalent when the teacher invited students to coding activity at the beginning of an episode. With regard to clause complexing, students used clause complexes connecting an independent clause to a dependent one that represented cause or condition in developing a proposition realized in the independent clause. For example, a student developed her argument such as ‘it (the tens place of 82) is eight’ by connecting to a dependent clause ‘if that’s eighty-two,’ which serves to realize the logico-semantic relation of condition. The argument concerned a scientific concept or Value while the hypothetical statement was an everyday concept or Token,
both of which were combined to develop reasoning. Excerpt 12 shows how a student related Value to Token and combined one clause with another one in binding everyday concepts and scientific concepts. Prior to this segment, Emma was asked to identify how many tens 72 has and responded by first ‘eight’ and later ‘seven.’ She was challenged to validate her response.

Excerpt 12. Understanding place value of two digits in 72 (J=Juan & E=Emma in Gr. 3) 34

1. T: Can you repeat the question?
2. J: The question was that how many tens are in seventy-two.
3. T: Yes. (i) How many tens are in seventy-two. (ii) And to review, and some friends were chatting, (iii) Emma said, “Eight”. (iv) And then she goes “Nononono. Seven.” (v) So why did you change your answer? That’s what we wanted to know. Listen please.
4. E: (i) It’s seven (ii) because in seventy-two, in seventy-two there is only ten… Ten… Seven tens, not eight tens!
5. T: That’s right.
6. E: (i) And if it’s eighty-two, (ii) then that’s eight.
7. T: That’s right.

In excerpt 12, Ms. Bright called Juan to remind Emma of her question (turn 1). Juan told that ‘the question’ was represented by ‘how many tens are in seventy-two,’ that is, Value as content is identified by Token as expression (turn 2). The teacher repeated the question (turn 3-i) and reminded Emma what she stated previously through direct speech (turn 3-iii & iv). Ms. Bright encouraged Emma to show her reasoning by using the why-interrogative clause (turn 3-v). In response, Emma used an encoding clause that relates Value to Token, each of which is indicated by ‘it (the value of tens in 72)’ and ‘seven’ (turn 4-i). Subsequently she justified her argument by binding the causal enhancing clause to the preceding clause and intensified the argument using contrast ‘seven tens, not eight tens’ (turn 4-ii). Encouraged by the teacher’s positive evaluation (turn 5),

34 The segment is derived from a class in January 29, 2015.
Emma confidently showed her understanding by binding Token (turn 6-i) and Value (turn 6-ii) through a hypothetical logico-semantic relation. The use of a clause complex realizing causal and conditional meanings can contribute to promoting students’ reasoning by binding scientific concept or Value and everyday concept or Token.

When students developed an argument, student responses were structurally parallel to teacher talk in the way they placed familiar/everyday language first and combined it with abstract/academic language. Excerpt 13 happened when students were asked to solve the algorithm of ‘564−446’ and write to explain how to solve it after Donna in grade 3 showed her work on the board.

Excerpt 13. Regrouping tens for subtraction in the ones place (R=Rosa & A=Alba in Gr. 2; N=Nina (Gr. 3))

1. T:  Are you okay with talking about why we crossed out the six and made it a five? <<...>>
2. R:  (i) That [] we crossed out the six (ii) and put a five (iii) because she was regrouping.
3. T:  Agree or disagree?
4. Ss:  Agree.
5. T:  Interesting. Okay let’s see what third grade has to say. (i) Can you add to what she’s talking about? (ii) You start and Emma can add to what you said. So second grade? Listening ears.
6. A:  I got my listening ears on.
8. N:  ( )
9. T:  A little bit louder sweetie
10. N:  (looks at her notes) (i) She needed to cross out the six (ii) to put a five (iii) to make fourteen.

35 Excerpt 13 is derived from a class in February 12, 2015.
36 In <<...>>, eight turns are omitted.
In excerpt 13, the students were required to present their responses while referring to their notes. Turn 1 indicates that Rosa was called to present her writing to explain why the class collectively crossed out the digit ‘6’ in the tens place and put a ‘5.’ After the teacher interrupted to discipline a student, Rosa proceeded with explanation using her notes. She first restated Ms. Bright’s utterance as Given information (turn 2-i & ii) and then presented her rationale as New information (turn 2-iii), through which she combined an everyday concept with a scientific concept. The configuration of the Given-New information can support her in developing her argument (turn 2-i & ii) out of the logico-semantic relation of cause (turn 2-iii). The development of her argument is structurally identical to that of Ms. Bright’s as in a clause complex ‘you count back with me because we’re subtracting.’ Turn 2-(iii) also indicates that the students marked as ‘we’ learned from their peer marked as ‘she’ who regrouped tens in subtraction (turn 2-iii). After checking students’ understanding, Ms. Bright encouraged Nina to articulate her response by providing additional information (5-i). The process ‘add’ functioned as saying process such as ‘tell,’ which can support students in processing their peers’ responses and adding information based on the responses. In turn 10, Nina verbally provided additional information and justified her answer when she was asked to add her idea to Rosa’s. Nina made reasoning by developing her argument through temporal and causal clauses, integrating meaning tighter than Rosa because of ellipsis in turn 10-(ii) and (iii). Turn 10-(i) and (ii) represent Given information/Token while turn 10-(iii) does New information/Value. Nina linked three material clauses by using a binder ‘to’ as in ‘cross out the six to put a five and to make fourteen.’ First ‘to’ indicates temporal relation (e.g., ‘and then’) and second one the relation of purpose (e.g., ‘in order to’). The use of ‘to infinitive’ realizes meaning integrated tightly. Tight integration in meaning through the interplay between Given and New information can
serve to tightly bind everyday concepts and scientific concepts, which can result in heightening students’ reasoning.

In contrast, Rosa’s writing showcases that she first selected academic language ‘I regrouped it’ instead of everyday language ‘we crossed out the six and put a five,’ as shown in Figure 5.4.

![Figure 5.4 Rosa's writing: '564−446' & transcript of her writing](image)

(i) I regrouped it (ii) because if you don’t regroup it, (iii) it wouldn’t make sense.

This contrast can result from her use of multisemiotic resources in the written response including the lesson objective. Her writing displays the following components: solving a problem ^ problem-solving steps (e.g., presenting a visual representation) ^ writing to explain sequential steps. In solving the problem, Rosa computed the difference using a math algorithm (i.e., symbolic representation). She then visualized how to compute the problem using pictures of base-ten blocks, which indicate that she crossed out one tens out of six tens by a vertical wavy line and added ten ones that are marked dots. The written explanation proceeds with academic language ‘I regrouped

---

37 Her writing is presented with error corrections in spelling and punctuation to increase readability.
it' in the independent clause associated with the visual representation as well as the symbolic representation that can work to express 'cross out the six and put a five.' The association can indicate a decoding process that relates Token to Value. To make reasoning, she developed her argument through a causal clause complex 'because if I don't regroup it, it wouldn't make sense.' The causal nexus itself works to develop her argument through a conditional clause involving a negative polarity 'if I don't regroup it,' which is connected to another negative one 'it wouldn't make sense.' The negative clause complex can be replaced with a positive one such as 'if I regroup it, it would make sense.' As such, she introduced a concept of regrouping and attempted to justify her answer through a relation of cause although she did not provide an appropriate argument. Interweaving decoding and encoding clauses suggests that the student attempts to make reasoning. Given that Token and Value each relate to everyday concept and scientific concept, students attempts to bind the two concepts to make a complete concept.

5.4 Mathematics Classroom Discourse as Coding Activity

Classroom talk typically unfolds through teacher-student interactions. Linguistic features in teacher talk and student responses are interconnected as exchange in unfolding classroom talk. Figure 5.5 illustrates how teacher-student talk unfolded in mathematics classroom discourse to construct mathematical concepts. In Figure 5.5, student responses in rhombus were often realized in elliptical clauses and material clauses while the teacher talk contained various linguistic features. When student responses are appropriate, which is abbreviated as 'App.' in the figure, the responses help students establish math concepts, process the concepts, and bind everyday concepts and scientific concepts. The rectangle on the right mostly indicates linguistic choices that the teacher made to facilitate student participation. The rectangle on the left mainly shows how teacher talk invited students to coding activity and promote students' reasoning. The flow of classroom talk that
unfolded through decoding and encoding activities was not linear but iterative depending on students’ responses.

As shown in Figure 5.5, the initiation of classroom talk mostly began with asking what concepts multiple representations (e.g., manipulatives, pictures, symbols, language) were connected to. Ms. Bright asked students to relate Token or expression represented by multiple semiotic resources to Value or content (Halliday & Matthiessen 2014, p. 279). For example, a number symbol ’72’ is a Token while the Value is ‘seven tens plus two ones.’ The directionality in the relationship between Token and Value determines whether the clause is a decoding or an encoding one. When a relational process (e.g., ‘is,’ ‘represent’) relates Token to Value, the relational clause is a decoding one. This decoding activity supports students in making sense of Value with Token. When a relational process relates Value to Token, the relational clause is an encoding one. The encoding activity engages students in regulating Token with Value. In other words, encoding is assigning a specific representation/expression to Value. In the stage inviting students to coding activity, teacher talk was realized in the declarative clauses, serving to provide contextual information by introducing mathematical entities, their attributes, and the relationship between them. Following the declaratives, the teacher asked students to relate Token to Value, that is, decoding meaning of Token.
In response, students offered information by elliptical clauses in reference to Value, omitting what was mentioned in the teacher question. They were engaged in relating Token to Value, which is realized through a being process, that decoded meaning of Token. They were also asked to participate in identifying attributes of a Token. The identification frequently involved a having process combined with a personal pronoun. In finding attributes of a Token such as a 3-digit number ‘564,’ for example, students were asked in a way ‘how many hundreds do you have?’ or ‘how many hundreds are in five hundred sixty-four.’ By the use of ‘you’ as the Subject, the former question

---

38 In Figure 5.5, ‘App.’ and ‘Inapp.’ each refer to ‘appropriate response’ and ‘inappropriate response’ whereas S and Ss indicate ‘student’ and ‘students’ respectively.
can work better in engaging students as agent who is regarded accountable for offering information demanded to understand the relationship between attributes/parts and Token/whole.

In follow-up to student responses, Ms. Bright encouraged students’ participation by using facilitating talk. The facilitation involved a sentence starter with rising intonation that she used to ask students to complete a statement. She also introduced familiar/Given information as the departure of a message and placed a WH-element at the end by using statement-like questions such as ‘you have how many tens?’ instead of ‘how many tens do you have?’ In addition, she guided students in understanding the concepts through statements of elaboration, cause and condition. When developing academic language and concepts, for instance, she used everyday language and later combined it with math-specific language as in a statement ‘you count back with me because we’re subtracting.’ Moreover, Ms. Bright asked students to articulate what they had in mind by selecting verbal imperative clauses (e.g., ‘tell me’) or metaphorical imperative clauses (e.g., ‘can you add to what she’s talking about?’ ‘I would like Yoana to share’). The use of verbal-imperative clauses was significant in that students were encouraged to provide a complete clause as opposed to an elliptical one that students favored when responding to sentence starters, statement-like questions, and relational interrogatives. The selection of verbal-imperative clause was also distinctive in supporting students in reasoning. Student responses to verbal-imperative clauses were realized in material clauses that describe what students did to solve problems and report what their peers did. Verbalizing their experience in the outer world through material clauses can play a critical role in interpreting meaning of semiotic resources and processing mathematical concepts associated with semiotic resources. To be able to verbalize their actions to represent meaning of semiotic resources refers to making sense of Value or a mathematical concept of Token.

Once students verbalized to show their understanding of mathematical concepts, Ms. Bright
attempted to promote students’ reasoning by using mental-projection clauses, which can support students in constructing mathematical meanings through a process of thinking. For instance, she pointed to a spot between nine and ten on an analog clock projected on the screen and asked, ‘what hour do you think it would be?’ in which ‘do you think’ serves to shift a question ‘what hour would it be?’ into a proposition ‘what hour it would be.’ This mental projection works to require students to process this proposition explicitly through thinking and to project what they processed in mind. In response, students offered information focus such as ‘Nine o’clock,’ which means that ‘it would be nine o’clock.’ This hypothetical clause can be connected to ‘if the hour hand would be in between nine and ten,’ which results in clause-complexing ‘if-then.’ This clause complex includes a scientific concept ‘(it would be) nine o’clock’ combined with an everyday concept ‘if the hour hand would be in between nine and ten’ that is manifested as a visual representation. The combination can represent an encoding activity in a sense that a Value ‘the hour’ is identified by a visual representation as Token.

Collectively, this study found that mathematics classroom discourse contributed to students’ finding meanings of mathematical representations or entities/participants by decoding the representations, processing decoded meanings by verbalizing actions, and encoding mathematical concepts to exist as semiotic representations. The coordination of decoding and encoding activities occurred recursively when Ms. Bright and her students had whole-class work across episodes. What follows displays both decoding and encoding activities happening within an episode.

Excerpt 14 demonstrates how mathematics classroom discourse unfolds coding activities. This segment occurred when Ms. Bright differentiated two math word problems of subtraction and division by grade and presented each to lower graders (i.e., Gr. 1 & 2) and third graders. Table 5.3
displays the problem for third graders. On the board was a lesson goal presented for the third-graders: *Students can do divisions based on some math clue words: ‘half as many ....as’ and ‘altogether.’*

<table>
<thead>
<tr>
<th>Juan broke half as many balloons as Donna broke. Altogether they broke 124 balloons. How many balloons did Danna break?</th>
</tr>
</thead>
</table>

Table 5.3 Math word problem 0339: Division

Prior to excerpt 14, Ms. Bright asked students as a whole class to find clue words for solving the math word problem. A third-grader named Nina responded by ‘broke’ when asked what clue word helped them solve the problem. The student answered ‘That they like popped the balloons. They broke like half the balloons’ when asked again ‘how is the word helping you understand like what you have to do?’ The teacher rephrased a student response, stating ‘she said that Juan broke half of the balloons and Danna broke some, too.’ Ms. Bright then called Nina to come up and point to clue words on the wall chart presented in Figure 5.6. Nina pointed to ‘how many’ and ‘Juan broke half’ on the board, moved to the wall chart, and pointed to ‘half of.’

---

39 This problem is similar to but different from the one in Table 5.2 presented in December 18, 2014: ‘Juan and Pam popped balloons. They each popped an equal number of balloons. Altogether...’
Excerpt 14 focused on interpreting meaning of ‘half of’ in a whole-class discussion

Excerpt 14. Deconstructing meaning of ‘half of’ (N=Nina, E=Emma & J=Juan in Gr. 3)41

1. T: (i) So half of, so what operation? Remember that fancy word? (ii) Like when we add, subtract, multiply, divide. (iii) And Emma is saying?
2. E: Division
3. T: (i) Division! (ii) Now, division means that a number is getting bigger or smaller?
4. J: Bigger
5. T: Donna.
6. N: I know the answer.
7. T: (i) When we say division, (ii) is the whole piece of something getting bigger or smaller?
8. S: =Smaller
9. T: (i) Right, the group is getting bigger or smaller? (ii) Everybody, say it.
10. S: Mm, bigger!
11. Ss: Smaller!

________________________________________

40 This photograph was taken by the researcher of this study in September 30, 2014.

41 Excerpt 14 is derived from a class in December 11, 2014.
In inviting students to coding activity, Ms. Bright asked Nina to give information regarding the meaning of a linguistic expression ‘half of.’ Instead of saying, ‘what operation is ‘half of?’ the teacher placed ‘half of first and articulated ‘what operation?’ later (turn 1-i). This configuration is student-oriented in that the teacher guided the student in following information flow typically structured as Given information ^ New information. Ms. Bright then unpacked ‘operation’ in New information by exemplifying types of mathematical operations (turn 1-ii), and yielded the floor to Emma, who was called to the front with Nina (turn 1-iii). In response, Emma offered the meaning of ‘half of’ by an elliptical clause (turn 2). Emma related Token ‘half of’ to Value ‘division,’ which is realized in the decoding clause.

In follow-up, Ms. Bright facilitated student participation using various linguistic scaffoldings. In turn 3-(i), she revoiced the response and placed it as the departure of a message in the following statement (turn 3-ii). The teacher invited students to unpack the math concept ‘division’ while facilitating student participation through a statement-like question including options (i.e., ‘bigger’ and ‘smaller’) instead of giving information realized in a declarative mood (e.g., ‘Division means that a number is getting smaller’). In other words, she asked students to encode Value as in turn 1-(i). Noticing Juan fail to understand the concept (turn 4) and calling for students’ attention (turn 5), she declined Nina’s claim (turn 6). Instead she continued to facilitate student response by re-directing students towards the math concept ‘division’ (7-i) and shifting language from abstract to concrete (7-ii). Ms. Bright seems to materialize the concept using a relatively more concrete entity ‘the whole piece of something’ than ‘a number’ in turn 3-(ii) as to facilitate her question, by which she gained a right response from one student (turn 8). The entire class including that student was asked to confirm the answer (turn 9-ii) to a statement-like question (turn 9-i). The
shift of ‘the group’ from ‘the whole piece of something’ can be intentional because she made a lexical chain: ‘a number,’ ‘the whole piece of something,’ ‘the group,’ ‘my cookie,’ and ‘the number’ (turn 3, 7, 9, 17 & 22). This lexical chain can work to bridge the gap between everyday language and academic language. Students’ responses in turn 10 and 11 showed that some students were still confused about the concept.

Excerpt 15 represents the segment immediately following excerpt 14. Ms. Bright continued to work with her students to deconstruct the meaning of ‘division,’ assuming that there might be some students who were confused about the meaning.

Excerpt 15. Deconstructing meaning of ‘half of’ (N=Nina, E=Emma & J=Juan in Gr. 3)

12. J: I mean- smaller!
13. T: (i) Why do you say bigger first? (ii) Why did you say bigger? (iii) Why did you say bigger (iv) and then you changed your mind? (v) Is it because you heard him or (vi) did you just thought about what I was asking?
14. J: ( )
15. T: (i) Okay, what do you mean? (ii) Tell me what you mean. (iv) When we divide a group, whole group –
16. J: (i) I divide (ii) and I thought it was bigger and ( ) the um, the um, the ( ) (iii) then I realized that it was smaller.
17. T: (i) See what he said, then I realized. (ii) So that means that he really thought about it. (iii) So if I had a cookie, (iv) where’s my cookie? (…) 
18. (i) Here’s my cookie, (ii) let say this is my cookie (iii) and I need to divide it amongst all of us, (iv) would you get big pieces or little pieces?
19. S: Big.
21. T: (i) You would get little- (ii) Look at my cookie! (iii) You would get tiny pieces or big pieces?
22. Ss: Tiny pieces!
23. T: So the number is going to get?
25. T: Smaller. It will decrease.

To support students’ understanding, Ms. Bright placed attention to Juan’s utterance in turn
12 and challenged him to justify his answer by using a why-question repeatedly (turn 13-i, ii, and iii). She required Juan to make reasoning for changing his answer, limiting his argument to repetition of someone’s idea and his own thinking (turn 13-v & vi). Accepting that Juan answered out of his idea, which is indicated by ‘Okay,’ she used a WH-interrogative clause (turn 15-i) and then selected a verbal-imperative clause (turn 15-ii). The WH-question can result in an open response while the verbal-imperative clause can require the student to give information demanded.

Challenging the student, Ms. Bright attempted to elaborate on the division (turn 15-iv) but she was interrupted by Juan (turn 16). He sought to reason by enhancing meaning while linking clauses in temporal sequence (turn 16-i, ii & iii). The teacher appreciated his attempt of reasoning (turn 17-i & ii) and she aimed to explain the concept ‘division’ by a hypothetical object, which is marked by the tense ‘had’ (turn 17-iii). She then decided to utilize a concrete object (turn 17-iv).

Drawing on the concrete object representing a cookie (turn 18-i & ii), Ms. Bright guided students in physically perceiving the object, which is evident in a material clause including ‘divide’ (18-iii). In the subsequent question (18-iv), she selected ‘you’ referring to students as Actors associated with a material process ‘getting,’ which works to visualize the concept(18-iv). When noticing some students being confused as in turn 19, Ms. Bright acknowledged Nina’s offer (turn 21-i) and re-oriented students towards the concrete object (turn 21-ii). She then facilitated student participation using a statement-like question (turn 21-iii). Gaining a right answer, the teacher recalled ‘number’ asked beforehand in turn 3 and invited students to co-construct the concept by asking them to fill in focus information ‘smaller’ (turn 22). In follow-up to a right response such as ‘Smaller’ in turn 23, the teacher shifted language from everyday to academic ‘decrease’ to support students’ academic language development (turn 25). Excerpt 14 displays that the students developed the targeted concept to the degree that they were guided to connect Token (e.g., ‘half of’).
to Value (e.g., ‘division’) and then the Value to another Token (e.g., ‘It [the number] will decrease’ as in turn 25) at the clause level involving a relational process. In other words, the students could use an observable Token to make sense of an abstract Value and in turn apply the Value to another Token through decoding and encoding processes at the clause level, as illustrated in Figure 5.7.

Figure 5.7 Language and concept development through coding activity (adapted from Halliday & Matthiessen, 2014, p.280)

Although the third-grade students were guided to link Token and Value at the clause level, they failed to solve the problem. Following excerpt 14, Ms. Bright exchanged 27 turns with the third-graders to find another ‘clue word’ (i.e., ‘altogether’) and its meaning. She then elaborated on a student’s response ‘a group,’ stating, ‘You have two equal groups, two equal groups.’ This utterance indicates that Ms. Bright mistook the phrase ‘half as many balloons as’ for ‘as many balloons as’—‘Juan broke as many balloons as Danna’ rather than ‘Juan broke half as many balloons as Danna.’

42 Ms. Bright picked up this question from an online site, adjusted the numbers to wrong numbers and then tried to teach it
After exchanging a few more turns, she told them to continue the problem the next class and dismissed five third-graders who only had thirty-minute class. This episode shows that a subtle difference in language used in teaching math word problems can mislead all students including ELLs to struggle with the problem. In this respect, teachers need to have linguistic awareness in guiding students in linking representations including language to the meanings.

In sum, students were first guided to relate Token ‘half of’ to Value ‘division’ and later to relate Value ‘division’ to Token representing reduction of a number (i.e., ‘It will decrease’). In other words, they were supported to decode the meaning of a semiotic resource and encode or assign the meaning to another representation. The coding activities indicate that Ms. Bright provided verbal scaffolding for her students to bind everyday concepts and scientific concepts. The verbal scaffolding requires teachers’ linguistic awareness.

5.5 Linguistic Choices Varying by Semiotic Resources

Concerning linguistic choices according to semiotic resources, this study found that there was no significant change in student responses. In the decoding stage, students favored to respond by elliptical clauses and material clauses. In the encoding stage, they selected clause complexes linked by a relation of cause and condition regardless of types of semiotic resources that were centered on classroom talk. Likewise, teacher talk had no significant variations in linguistic choices in terms of interpersonal and textual meanings (e.g., association of Subject with Theme, why-interrogative clause). Even concerned with ideational meaning, teacher talk had no significant difference in that the teacher prevalently selected material and relational processes regardless of types of semiotic resource. Her talk, however, showed different patterns by semiotic resources in using the two types of process in a way to frequently select the one over the other or involve
different types of process (e.g., verbal, mental) between the material and the relational processes. The way to select mental or verbal processes also depends on whether the stage is a decoding or encoding process.

Figure 5.8 illustrates how teacher talk varied in engaging different types of multiple semiotic resources (i.e., manipulatives, visual display, symbols, and language), focusing on the ideational meaning represented by the configuration of processes, participants and circumstances. Including circumstances as participants combined with processes (Halliday & Matthiessen, 2014, p.329), this study considered the interplay between process and participant in construing multisemiotic resources to construct mathematical meanings. Figure 5.8 displays the configuration of processes and participants to make meanings of multi-semiotic resources. Drawing on Matthiessen (1991), the quadrants show how multiple semiotic resources are construed by lexicogrammar.

Figure 5.8 Ideational perspectives on teacher talk construing multi-semiotic resources (adapted from Matthiessen, 1991, p.87-88)
The horizontal line in the figure indicates the continuum of participants in field register variables from indexical to symbolic. Being indexical or symbolic is relative, not determinative. An indexical participant is one “that is actually connected to its object” while a symbolic participant is one “that is an arbitrary and conventional representation whatever it represents” (Matthiessen, 1991, p.70). Indexical participants are mostly concerned with everyday language whereas symbolic participants with academic language in that language works in the continuum of field register variables. The vertical line refers to the continuum of processes in field resources from happening/doing to being/having. According to Matthiessen (1991), semiosis can be expressed through a continuum ranging from ‘activity’ to ‘being’, each of which is actualized through material and relational processes. The process axis and the participant axis make meanings in the quadrants through the interplay between them. The arrows in the circle indicates that verbal and mental processes function to relate internal experience to external experience to project locutions and ideas. In schooling settings, language works not only as the main semiosis of a semiotic activity but also as the medium of teaching and learning occurring ubiquitously.

As shown in Figure 5.8, Ms. Bright significantly selected material and relational processes shifting registers from everyday to academic in reference to language as salient semiosis in talk. In construing meaning of manipulatives, lexico-grammatical configurations mostly involved material and relational clauses, which are characterized as indexical participants as well as acting and being processes. Regarding visual display, relational processes or material processes were prevalently used depending on direction of coding or degree of manipulativeness of visual representations (e.g., number line with high degree of manipulativeness versus pictures of analog clocks). Decoding activities involved relational processes predominantly while encoding activities included material
processes frequently. Visual display entailed characteristics of indexical and symbolic participants as well as those of being and doing processes. In making meaning of symbols (e.g., numbers, number sentences), teacher talk predominantly included relational and material clauses combined with symbolic participants such as numbers and symbols. Symbols were characterized as symbolic participants as well as acting and being processes. Besides material and relational processes, teacher talk involved verbal and mental processes, both of which worked mainly in the form of projection (e.g., *How do we know what we have to subtract?*).

Figure 5.8. only shows how Ms. Bright made linguistic choices to construe meanings of multiple semiotic resources with her students but does not illustrate how the choices could work to support concept development because the quadrants are two-dimensional. Given that these semiotic resources are represented in lexico-grammar in patterned ways, classroom talk can guide students in developing mathematical concepts as well as academic language. The quadrants imply that teacher talk in the mathematics classroom connected multisemiotic resources (e.g., manipulatives, visual display, symbols) to mathematical concepts in a patterned way. This patterned way of talk corresponding to semiotic resources can contribute to enabling the interplay between spontaneous and scientific concepts as to support ELLs’ language and mathematical concept development. As decoding activity, a relational process links Token to Value to make sense of scientific concepts including math concepts. As encoding activity, a relational process links Value to Token to regulate everyday concepts or assign Value as math concepts to Token as expressions. Figure 5.9 illustrates how lexicogrammar functions to develop math concepts.
Values or math concepts are characterized as abstract, detached, and symbolic. Tokens or expressions range from concrete, context-dependent, and indexical ones to abstract, detached, and symbolic ones, including manipulatives, visual representations, symbolic representations and language. The subsequent sections discuss linguistic choices the teacher made according to different semiotic resources.

### 5.5.1 Language Use with Manipulatives

With regard to manipulatives (e.g., base-ten blocks, coins) as salient resources in classroom talk, Ms. Bright alternately selected relational or having/being processes and material or doing processes, in which she favored material processes over relational processes. The alternate use of relational and material clauses worked to support students in identifying entities, finding attributes
of entities, and materializing mental computation. Ms. Bright, in particular, used questions to guide students in relating manipulatives as entity to meaning and identifying their attributes. When using material clauses, she supported the students in processing mathematical concepts as something going on in the outer world as opposed to in their inner/conscious world. Drawing on manipulatives as entities, she chose relatively indexical and here-and-now ones (e.g., 'coins,' 'a dime,' 'a nickel,' 'it,' 'I,' 'we'). Excerpt 15 displays how she supported her students in measuring values of coins drawing on realia (i.e., different kinds of coins). The measurement of coin values involves assigning each coin to a value, ordering the coins, and adding up the values (Van de Walle, Karp, & Bay-Williams, 2010, p.385). In Excerpt 16, Ms. Bright was working to organize coins by value and count the values with her students.

Excerpt 16. Measuring values of coins (B=Bruno (Gr. 1); A=Alba, Y=Yoana, R=Rosa & C=Camila in Gr. 2)43

1. T: I have pennies, I have a dime, and I have a nickel.
   
2. T: (…) Now, I sorted them already, (ii) as you can see, (iii) just like we shared. (iv) We sort first. (v) The second thing we do is what?

3. A: (looks at the screen and reads it) Count the coins that are worth the most.

4. T: Okay. (i) Which coins are worth the most right here? (ii) Bruno, point to it. (iii) Which coin is worth the most? (iv) Which one has the most value?

5. B: There’s a dime

6. T: (i) There’s a dime. (ii) So where do I start?

7. Y: With the dime.

8. T: (i) I start with the dime. (ii) So, let’s start with the dime. Everybody?

9. Ss: Ten

10. T: Hold on. I have a nickel.


---

43 Excerpt 16 is drawn from a class in April 2, 2015.

44 In <<...>>, ten turns are omitted.

174
13. T: (i) So what just happened? (ii) I changed the way?
14. C: We count!
15. T: We went from ten?
16. Ss: To fifteen.
17. T: (i) We went from ten, ten. (ii) Do I have another dime?
18. R: No.
19. T: (i) So, I can’t go ten, twenty, thirty. (ii) No, it’s different value. (iii) So what did I do?
   (iv) I changed the way?
21. T: We count. So, ten.
22. A: Twen-. No ten, [fifteen
23. Ss: [Fifteen, sixteen, seventeen, eighteen, nineteen.
24. T: So how much money do I have?
25. Ss: Nineteen.
26. T: Nineteen what? Dollars?
27. Ss: Cents!

Prior to this segment, Ms. Bright let students watch a video clip with captions that introduced how to count different types of coins and paused the video, which was presenting three steps for counting. In opening a whole-class discussion, she drew on realia as focus entities (i.e., pennies, a nickel, & a dime) as indicated in turn 1. The opening remark realized in a relational clause works to invite students to identify targeted mathematical entities (i.e., the coins connected to an introducer ‘I’. She then evoked a first step to measure different types of coins using a material process ‘sorted’ (turn 2-i) to bring inner experience ‘mental computation’ into outer world. Instead of explaining a second step, she engaged the students in participating in a materialized mental operation, which was realized in a material clause ‘count the coins’ (turn 3). Acknowledging a student’s response, Ms. Bright asked the students to recognize the value of each coin and compare the coin values by selecting relational processes (turn 4-i, iii, & iv). To facilitate this computing process, she invited Bruno to perform an action ‘point to it’ and show his understanding (turn 4-ii). When he articulated the name of the coin with the most value (turn 5), Ms. Bright affirmed his
response (turn 6-i) and required the students to prepare for a mental computation as indicated in turn 6 (ii) ‘where do I start?’ In response, Yoana offered the starting point ‘With the dime’ of the mental computation (turn 7).

Ms. Bright facilitated student participation in the mental math by selecting material processes (e.g., ‘start,’ ‘happen,’ ‘changed,’ ‘went/go,’ ‘do’ as in turn 8, 13, 15, 17, & 19), all of which work to realize “what we experience as going on 'out there' in the world around us” (Halliday & Matthiessen, 2014, p.214). The facilitation enabled the students to mentally add up the values (turn 9, 11, 12, 16, 22, 23). During the facilitating stage, Ms. Bright employed relational processes to ask students to identify a change in entities from dime to nickel (turn 10) and recognize a difference in their values (turn 19-ii). She guided the students in identifying difference in value by action ‘(we went from ten to) fifteen’ (turn 15 & 16). She also positioned them as agent of the mental computation while including not only herself but also all the students as marked ‘we count’ (turn 21).

In adding up the values of the coins, Ms. Bright selected the relational process ‘have’ to ask the students to offer the total value (turn 24). This relational clause is decoding one in that she was asking ‘what do these coins represent?’ Accepting their response, she challenged them to add the unit of money to the value (turn 26). When the students only offered the unit, she elaborated on the response by combining the unit ‘cents’ and the value (turn 28). The combination works to define the total value of the coins, that is ‘nineteen cents’ not ‘nineteen dollars.’

5.5.2 Language Use with Visual Display

When decoding meaning of a visual representation with low manipulativeness, Ms. Bright favored relational processes in connecting the representation to a number. For example, in drawing
on pictures of analog clocks that students are taught to read without manipulating the hour hand and the minute hand, she frequently used the being process to support students either in identifying where the hour/minute hands are located and or in relating the hands to the hour/the minute. When involving visual representations with high manipulativeness, on the other hand, Ms. Bright prevalently used verbal and material processes in guiding the students in verbally describing how they manipulated the pictures. The verbal process functions to project inner/outer experience through wording. In following up student responses, she often restated what they said, which was realized in material clauses. Salient entities in classroom talk involving visual representations were indexical and symbolic. Indexical entities entailed elements illustrated in the pictures (e.g., the hour hand, the minute hand, digits on the clock face, numbers used for counting) whereas symbolic entities involved numbers that the indexical entities represented. Reading clocks, for example, contained ‘30 minutes’ indicated by the six on the analog clock.

Excerpt 17 displays that Ms. Bright predominantly used the relational process to support students in reading clocks while using pictures of analog clocks with low manipulativeness. This segment happened when she was working to review how to read clocks that were illustrated in the worksheet. On the classroom wall, she presented an analog clock on the screen. Prior to this lesson, they learned how to read clocks to the hour and the half hour.

Excerpt 17. Reading clocks to the hour and the minute (R=Rosa, Y=Yoana, C=Camila in Gr. 2 & B=Bruno in Gr. 1)45

1. T: Rosa, what’d you get for the first one?
2. R: Five thirty.
3. T: Does everyone agree? Is it five thirty?

---

45 Excerpt 17 is derived from a class in April 30, 2015.
4. Ss: Yes.
5. T: Why did you decide on the five and not the six.
6. R: I decided on the five because, because the hour hand is not straight at the-
7. T: The hour did not?
9. T: The?
10. R: Six.
11. T: Exactly. Everybody, (i) the hour hand did not pass the six. You can look at it where I am
here or on your own paper. Exactly. (ii) Now you can really understand (iii) why the hour is five.
(iv) The hour is five hours, right? (v) Five hours have gone by. Okay? (vi) The minute hand is
what?
12. Y: ( ) minute.
13. T: Yeah, (i) what’s the minute hand? (..) (ii) what is it on?
15. T: (i) It’s on the six. (ii) What does the six represent? six o'clock?
17. T: What do you mean by thirty? What do you mean by that?
18. B: Right.
19. T: (i) What do you mean by thirty? (ii) Could you come up and show us? (iii) What do you
mean by thirty? <<...>>46
20. B: I don’t know.
21. T: Okay Bruno. (i) Yoana, tell us why you thought, (ii) why is that thirty? (iii) What do you
mean by thirty? (iv) I don’t see a three zero. (v) I see a six.
22. Y: ( )
   <<...>>
23. T: (i) Why did you say (ii) that means thirty (iii) when it’s on the six? Well, (iv) what is on
the six? (v) The hour hand or the minute hand?
24. Y: Minute
25. T: Okay, she’s right. (i) When the minute hand is on the six (ii) it’s thirty (iii) but how did
we get thirty? (iv) I don’t see three zero there. Bruno, you need to listen. This one was tricky for
you. Listen.
   ( )
26. T: The six
27. Y: The six
28. T: Represents
29. Y: Represents ( )
30. T: What? Thirty because?

46 In <<...>>, eight clauses in teacher talk are omitted and after turn 22 and 31, two and five turns
are omitted respectively.
31. T: (i) Start counting for us. (ii) Show us (iii) how you got thirty. Go.
32. Y: Five, ten, fifteen, twenty, twenty-five, thirty.
33. T: She got it!

In excerpt 17, Ms. Bright first selected relational processes such as ‘is’ in asking students to identify the hour or the minute represented by the hour hand or the minute hand in the pictures (turn 3; 11-vi; 13-i; 15-ii; 21-ii; 23-iii; 25-ii; 28). The identifying relational-clauses are decoding ones because it is Token that is identified. In turn 11-(vi), for instance, Ms. Bright asked students to relate the minute hand to the minute using a what-interrogative with a relational process (e.g., ‘The minute hand is what?’). The minute hand as Token was asked to be identified, in which she facilitated student response by selecting a statement-like question. The facilitation is concerned with the dissociation of the digit on the clock with the minute. As indicated in turn 15, she pointed out that the minute hand was on the six, which is dissociated with time that the hand represents. She, thus, required them to first recognize the indexical digit that the minute hand was on by using the relational process (turn 15-i; 23-iii; iv; 25-i).

Besides the relational process, Ms. Bright employed the verbal process (e.g., ‘mean,’ ‘say’) to facilitate student participation and support them in making reasoning. Turns from 23 to 33 display an encoding activity because the teacher asked students to relate Value to Token through a why-question. When Yoana provided a right response as in turn 16, for instance, the teacher used a verbal process ‘mean’ to ask her to explain why the minute hand on the six represents thirty on the clock (turn 17; 19-i). She then selected a material process ‘come’ combined with a verbal process ‘show’ to support the student in saying her thinking by action (turn 19). The use of a material process coupled with a verbal process occurred again when the teacher guided Yoana in pointing to digits on the clock on the screen, counting by 5, and telling the time (turn 31-i; ii). Turn 32 indicates
that the student was aware of the relationship between an indexical digit on the clock and a symbolic value, showing her mental process of counting on for multiples of 5 minutes.

Unlike pictures of analog clocks, Ms. Bright favored material processes while involving visual representations of base-ten models, especially guiding students in explaining how they acted on the representations. She also used verbal-imperative clauses mainly combined with material clauses to require the students to articulate their actions during a problem-solving process. Excerpt 18 occurred when Ms. Bright asked Rosa to tell how to solve a problem of ’534−428’ given in the traditional algorithm (see Figure 5.3).

Excerpt 18 Explaining a visual representation of subtraction ’534−428’ (R=Rosa, C=Camila in Gr. 2)47

1. T: So tell me what you did here. Explain it to me please. Go ahead.
2. R: I
   […]
3. T: Go ahead, Rosa. I’m listening.
4. R: I drew- I crossed out [.]
5. T: (i) Before you crossed out (ii) what did you do before (iii) cos you didn’t cross out first. (iv) Tell me (v) what you did first. First I-
6. R: First I regrouped.
7. T: (i) Before you regrouped, (ii) what did you do? (iii) So before you crossed out, (iv) before you regrouped, (v) what did you do?
8. C: Addition and then ( )
9. T: So first I-
12. R: Drew. Drew a picture [...] and-
13. T: Just listen sweetie.
14. R: And crossed out four.
15. T: (i) You drew a picture [...] (ii) and you crossed out four. Oh. Okay. (iii) Keep going.
16. R: And then crossed out two from tens.
17. T: Two from the tens. Okay.

47 Excerpt 18 is drawn from a class in February 26, 2015.
18. R: But if you cross out eight you can’t do that so [...] so you add, so you add ten to the ones.
19. T: (i) You add ten to the ones? Well (ii) where’d you get that ten to add to the ones? […] (iii) Where’d you get it?
20. R: From the t-
21. T: From where?
22. R: From the tens.
23. T: Yes! Like from the tens like area, right? Like from the place value. From that place value. Awesome.

As in turn 1-(i) and (ii), Ms. Bright selected verbal processes such as ‘tell’ and ‘explain’ to facilitate Rosa’s participation and for her to express how to solve the problem while using the base-ten representations. A material clause ‘what you did’ combined with the verbal process ‘tell’ was used for the student to verbally describe her actions on the visual representations. When Rosa did not complete the first clause and changed it into another (turn 4), the teacher restated Rosa’s second clause realized in a material one (turn 5-i; iii). She then required Rosa to recall a first action she took and tell the action (turn 5-ii; iv). Instead of providing additional information on top of Rosa’s response, Ms. Bright only revoiced her response realized through material processes such as ‘regrouped,’ ‘crossed’ and ‘drew’ (turn 7-i; iii; iv; 11). This series of material processes was used to require the student to complete the incomplete clause in turn 4 and show procedural actions (i.e., ‘drawing a picture,’ ‘crossing the four hundred models,’ ‘crossing the two tens models,’ ‘regrouping one tens into ten ones’). After accepting Rosa’s added information realized in a material clause ‘And crossed out the four’ (turn 14), the teacher restated her response and uttered, ‘Oh. Okay’ (turn 15), indicating that subtraction from the left to the right was not expected.

Ms. Bright, however, yielded the floor to require Rosa to verbally describe next action, choosing a verbal process ‘go’ (turn 15-iii). In response, Rosa used a material clause involving ‘two from tens’ (turn 16), in which ‘two’ and ‘tens’ refer to indexical pictures of two tens-models and the symbolic place value respectively. Restating Rosa’s response with a rising intonation (turn 19), Ms.
Bright asked her to clarify the addition of ten to the ones, still using a material process ‘get.’ Using a material process can work to help the student visualize and materialize her mental process of regrouping one tens to ten ones. Rosa’s picture indicates that she may mentally subtract the ones place without drawing a model of one tens. As presented in excerpt 18, Ms. Bright’s selection of material processes and verbal processes were significant when requiring students to explain how to solve a problem by using visual representations that they could act on.

5.5.3 Language Use with Symbolism

With respect to symbolic representations, Ms. Bright favored relational clauses over material clauses. The relational clauses were often combined with mental clauses, both of which worked to guide students in seeing, thinking and identifying elements in mathematical symbols. Mental processes were used to support the students in projecting their ideas. She also chose material clauses to facilitate students in identifying what is happening to symbols and act on them. Salient entities associated with material and relational processes mostly involved numbers, which can indicate both digits and their place value. In addition, operational symbols (e.g., +, −, ×, ÷) can be realized as material processes (e.g., ‘take away,’ ‘add’) or embedded as modifier to determine a preceding entity. For example, ‘plus four’ in an expression ‘ten plus four equals fourteen’ is post-modifying ‘ten’ while ‘ten plus four’ as a whole represents one entity that is connected to ‘fourteen.’ Excerpt 19 shows how Ms. Bright used language in employing the traditional algorithm for subtraction.

Excerpt 19. Subtracting ‘446’ from ‘564’ by the algorithm (Y=Yoana & A=Alba in Gr. 2)48

48 Excerpt 19 is drawn from a class in February 12, 2015.
1. T: Okay, (i) let’s see (ii) what you have here. (iii) How are you doing? ( ) (iv) Cause you can’t take away six from four.

2. S: ( )

3. T: That’s right! And so, (i) what does the new number become? (ii) Do you know that? (iii) Do you know (iv) what the new number becomes?


5. T: (i) It does become a five (ii) and it’s right there in the- in the what? In the [.] what column?


8. T: (i) It becomes a five in the tens (ii) but what happens to the number in the ones, that four? (iii) What happens to that digit?

9. Y: It becomes a three.

10. T: (i) It becomes a three? Um, (ii) but what are we really doing to those ones? (iii) What did you notice Hugo did here, second grade friends? (iv) What did you notice Hugo did here?

11. A: He took off one ten!

12. T: (i) He crossed off the ten, right? (ii) Here’s the ten. (iii) He crossed it off, (iv) made it a five like you said. I just read that from your paper, right? (v) But then how many ones does he have now?

13. A: Ten!

14. T: (i) He has ten for that ten (ii) but ten plus?

15. Y: Four

16. A: Four

17. T: Which equals?

18. S: Fourteen

19. A: Fourteen

20. Y: Fourteen

21. T: Yes! So that number really becomes- we add a ten to it.

Excerpt 19 occurred when Ms. Bright was working to subtract ones place of an expression '564−446.' The teacher used a mental clause combined with a relational clause (turn 1-i; ii) to require the students to perceive the expression as targeted entity. She then chose material clauses that were linked by a relation of cause (turn 1-iii; iv) to prompt the students to materialize a problem-solving process. She then predominantly selected relational processes such as 'become;' ‘is' and ‘have’ associated with numbers and place values (e.g., ones, tens) to connect the numbers to the values (turn 3-i; iv; 5-i; ii; 8-i; 9-i) or to link values to ‘he’ as possessor. Connecting the numbers to the values is a decoding activity.
Ms. Bright tended to use mental clauses combined with relational clauses when relating one symbolic entity to another (turn 3-iii; iv), which can work to facilitate the students in activating mental process. When using material clauses, she seemed to prompt them to perceive the subtraction as materialized action (turn 8-ii; iii), or she restated a student response to support them in relating the action to a mathematical process for regrouping one ten to ten ones (turn 12-i; iii; iv). The use of material processes seems to facilitate the students’ concept making of subtraction by visualizing the abstract concept. Based on the visualized concept through actions, the teacher and students co-constructed a number sentence (i.e., ‘10+4=14’) using relational process (turn 14-18). In turn 19, Ms. Bright shifted from academic language describing a number sentence ‘ten plus four equals fourteen’ to everyday language ‘we add a ten to it’ to unpack the symbolic expression. As indicated in excerpt 19, Ms. Bright favored relational processes over material processes in drawing on symbolic representations as salient semiotic resources in classroom talk. To facilitate students in mentally operating mathematical entities, she chose mental clauses combined with relational clauses or selected material clauses.

5.5.4 Language Use with Language as Main Semiotic Resource

When unpacking language in math word problems, Ms. Bright prevalently chose relational clauses to translate the language and guide students in understanding factual information in the problem. The relational process worked to connect language in the problem to the meaning, which often concerned mathematical operations. The relational process was also used to identify contextual and factual information relevant to the problem. In drawing on language as salient semiotic resource to solve math word problems, relational clauses were decoding ones, in which Token is identified. To facilitate student participation in classroom talk, the teacher selected material and verbal clauses between relational clauses. Material and verbal processes tended to
encourage the students to solve math word problems by doing and saying. Language, which included everyday language (e.g., students, fewer, absent) and math-specific language (e.g., less than, operation, subtraction, numbers), played a main role as semiotic resources.

Excerpt 20 happened when Ms. Bright taught second graders a math word problem in Table 5.4. At the beginning of the lesson she presented two problems: one for grade 1 and 2, and the other for grade 3.

In your class today, there were fewer students present than usual. Only 18 out of 26 students came to school. How many were absent?

Table 5.4 Math word problem 04: Subtraction

Ms. Bright began to work with second graders after releasing third graders in Group 2 who would have thirty-minute class instead of sixty-minute class for Group 1. A few third graders in Group 1 were still working on a different math word problem relevant to division.

Excerpt 20. Deconstructing ‘fewer’ (R=Rosa, C=Camila, A=Alba, Y=Yoana in Gr. 2)49

1. T: (i) So which words are going to help you?
2. R: Underlined ‘fewer’.
3. T: You're- And how did you know about that word?
4. C: It is on the chart under the ‘subtract.’
5. T: (i) It is on the chart, (ii) and it does mean what, Rosa? (iii) Rosa, what does it mean?
6. R: It means ( ).
7. T: Right, (i) and you have been learning about that word. (ii) So you know fewer means what?
8. S: ( )
9. T: Less or more?
10. A: Less!
11. T: (i) So fewer does mean less. (ii) So, I'm gonna underline it.

49 The excerpt 20 is drawn from a class in December 11, 2014.
12. T: (i) Camila, go up to the chart please (ii) cos some other math clue words are gonna help us. (iii) You need to talk nice and loud.
13. C: ( ) how many
14. T: Okay, (i) so, what does how many mean? (ii) You have to find?
15. C: The [..] ( ) and which is all together.
16. T: (i) So you have to find all together. (ii) What do you mean all together? (iii) Say it again. (iv) What do you mean?

(A beeping sounds from the speaker of the PA )

17. T: (i) So how many were ab-, so that’s giving us, (ii) we need a?
18. S: Number!
19. T: Good. (i) So circle that. (ii) Circle that. <<…>> (iii) Yoana, say it. (iv) Less than? Or more than? [...] (v) If some students are absent [...] (vi) we have fewer students, right? That’s a fancy word [...] (vii) that means into what? Into what? (viii) What’s that fancy word, fewer?
20. Y: ( )
21. T: (i) If you have it in a sentence, (ii) fewer means?
22. S: ( )
23. T: (i) Fewer means you subtract in this math word problem, right? Okay.

Ms. Bright first selected material and mental clauses in inviting students to find clue words in the problem, which are working as a Token (turn 1; 3). Once finding the Token, she used relational clauses to ask students to identify the meaning (turn 5-ii; iii; 7-ii; 9). In turn 9, the elliptical clause is a relational one such as ‘does fewer mean less or more?’ which can facilitate students in offering a response. In follow-up, Ms. Bright chose a relational clause to decode the meaning of ‘fewer’ (turn 11), finalizing a decoding activity by acting on the clue word through a material process ‘underline.’ The salient entities included relatively everyday language such as ‘words,’ ‘fewer,’ ‘more or less.’

50 In <<…>>, ten clauses in teacher talk are omitted and in turn 19 three clauses are omitted.
For another decoding, Ms. Bright employed material processes such as 'go,' 'are gonna help,' and 'find' in asking or facilitating student participation (turn 12-i; ii; 14-ii). She involved relational clauses in asking the students to define meaning of 'how many' (turn 14-i). Camila related 'how many' to 'altogether' (turn 13; 15), which indicate that she was confused probably because her first offer indicated in turn 4 (i.e., 'it is on the chart under subtract') was not accepted as right response. Ms. Bright used a material process 'find' to guide Camila in pointing 'altogether' and recognizing that the word was under addition of the wall chart (turn 16-i). The teacher required her to decode the meaning of 'how many' realized in relational clauses (turn 16-ii; iv). Ms. Bright inserted a verbal process 'say' between the relational processes, which can facilitate student participation (16-iii). Besides using verbal process, the teacher chose a material process and a relational process sequentially to seek for a student response (turn 17-i; ii). She finalized another decoding activity by acting on the target words 'how many' (turn 19-i; ii).

Turns in 19-(iii) to 23 display that Ms. Bright supported the students in participating in decoding activity for relating 'fewer' to 'subtraction.' She used a verbal process 'say' to invite Yoana's participation, providing her options 'less than or more than' she could choose (turn 19-iii; iv). Ms. Bright subsequently gave contextual information by connecting the target language 'fewer' to everyday experience realized in relational clauses (turn 19-v; vi). She guided the students in linking 'fewer' to a mathematical operation 'fancy word' (19-vii; viii). To engage the students in the decoding process, Ms. Bright related the students 'you' as possessor to 'it (the fewer)' and asked them to identify an operation relevant to 'fewer' (turn 21). She completed the decoding activity by relating 'fewer' to a mathematical symbol '−,' implying that 'fewer represents subtraction,' which indicates that she combined everyday language 'fewer' with academic language 'subtraction.' As shown in excerpt 20, when involving language as the main semiotic resource, mathematical
classroom discourse interwove everyday language and academic language. The following section summarizes the findings of this study.

5.6 Summary

This study found that the participant teacher made linguistic choices in various ways to support her ELLs’ academic language and mathematical concept development. When having whole-class work, the teacher first provided contextual information relevant to a targeted mathematical participant/entity as Token in the declarative and asked the students to connect the entity to the meaning as Value in the interrogative (e.g., ‘what did you get for the first one?’). At this initial stage, the students were guided to link Token (e.g., ‘a picture of the analog clock indicating five thirty’) to Value (e.g., ‘Five thirty’) using an elliptical clause (e.g., ‘Five thirty’), which is a decoding activity identifying a Token by reference to a Value. The response indicates ‘The picture represents five thirty.’

In facilitating student participation, Ms. Bright used incomplete clauses (e.g., sentence starters) and Theme-Subject associating questions (e.g., ‘the minute hand is what?’) rather than Theme-Subject dissociating questions (e.g., ‘what is the minute hand?’). In response, the students also responded by elliptical clauses (e.g., ‘Thirty’) rather than a complete clause ‘It’s thirty.’ Ms. Bright also facilitated the students’ participation in mathematical discourse by interweaving everyday language representing an observable phenomenon and academic language involving an abstract phenomenon as follows: ‘The hour hand did not pass the six. The hour is five hours.’ The preceding clause describes an observable phenomenon and the following one does an abstract one. The teacher further facilitated student participation in co-constructing mathematical meanings while asking them to verbalize their thoughts as in a question ‘what do you mean by thirty?’ In
response, students can use material processes (e.g., 'goes' as in 'The minute hand goes by five, like five, ten, fifteen, twenty, twenty-five, thirty') even though they failed to respond appropriately.

When the teacher attempted to promote students' reasoning, she selected questions involving mental processes (e.g., 'think' as in 'what hour do you guys think it would be?') or how/why questions (e.g., 'Why did you say that means thirty?'). In response, a student came up to the front and counted 'Five, ten, fifteen, twenty, twenty-five, thirty,' which can show that she still needs to develop her reasoning in telling the time. Once they were guided to express their argument, they could answer, '(it’s nine o’clock) because it[the hour hand] didn’t pass the ten' while binding a scientific concept and a spontaneous concept. The scientific concept corresponding to Value was linked to the spontaneous concept equivalent to Token. Namely, the Value is encoded to the Token, that is, an encoding activity.

In reference to linguistic choices according to multiple semiotic resources, student talk had no significant change. Teacher talk also had no salient variations in terms of the mood system (e.g., declarative, interrogative, imperative) and the Theme-Rheme structure or Given-New information structure. Regardless of semiotic resources, Ms. Bright often associated Subject with Theme in using WH-interrogatives as mentioned in patterns of teacher talk.

Concerning ideational meaning, however, teacher talk had similarities and differences according to four main semiotic resources (i.e., objects, pictures, mathematical symbols, and language). Teacher talk predominantly involved material and relational processes to cover the four semiotic resources, but patterns in which she used the processes varied by what semiotic resource classroom talk refers to. First, objects or manipulatives favored iterative use of material processes. Second, pictures or visual displays entailed relational process or material process, depending on the extent of manipulativeness of pictures (e.g., pictures of numbers with low manipulativeness,
pictures of analog clocks with high manipulativeness). Third, mathematical symbols concerned relational and material processes associated with numbers and symbols as participants. Last, language as metalanguage involved everyday field resources (e.g., 'count back,' 'half of') and academic field resources (e.g., 'subtract,' 'division'). The boundary between everyday language and academic language is determined by congruency in grammar. Being grammatically congruent indicates that what something means is directly connected to its object.

Figure 5.10 exemplifies linguistic choices that the teacher made according to multi-semiotic resources. The upper and bottom quadrants on the left construe the meanings of manipulatives.

![Diagram showing semiotics as activity and being](image)

Figure 5.10 Examples of linguistic choices varying by multiple semiotic resources (adapted from Matthiessen, 1991, p.88)

In referring to manipulatives, for example, teacher talk frequently involved material and relational processes (e.g., 'trade' and 'has') combined with indexical and concrete participants (e.g.,
‘she,’ ‘the one hundred block,’ ‘this one,’ ‘the minute hand’) as in clauses ‘She had to trade the one hundred block for how many tens?’ and ‘This one only has the minute hand and the hour hand while that one up there has the second hand.’ In reference to visual displays, teacher talk is exemplified in the axes of ‘semiosis as activity’ and semiosis as being’ because the teacher mostly engaged material clauses associated with both everyday language (e.g., ‘count back’) and academic language (e.g., ‘subtract’) when a referred picture is manipulative. When referring to pictures as static, the teacher favored relational processes (e.g., ‘is’) associated with both concrete and abstract participants (e.g., ‘the hour hand,’ ‘two o’clock’). In regard to symbols, teacher talk is presented in the upper and bottom on the right quadrants because the teacher prevalently selected material and relational processes involving abstract participants (e.g., ‘that zero,’ ‘five hundred fifty-eight,’ ‘number,’ ‘five’). In referring to language, teacher talk is exemplified around the axes of ‘everyday semiosis’ and ‘academic semiosis’ because the linguistic choices the teacher made are concerned with grammatical congruency. A clause ‘I need to divide my cookie’ is semantically congruent in that the clause explicitly indicates ‘who did what’ whereas another clause ‘division means that a number is getting bigger or smaller’ is not indicative of ‘who did what.’ The utterances inside the circle represent that both external experience (e.g., ‘how someone has more than someone else,’ ‘that symbol goes in there’) and internal experiences (e.g., ‘It’s telling us,’ ‘Do you think’) are connected by verbal process (e.g., ‘is telling’) or mental process (e.g., ‘think’). Verbal and mental processes can function as a facilitator in projecting what is said or what is thought.
CHAPTER 6

DISCUSSIONS

6.1 Introduction

To explore how teachers can support English language learners’ (ELLs’) academic language and concept development in the mathematics classroom, this study examined how an elementary teacher in an urban school used language in providing verbal scaffolding. This study also investigated how ELLs participated in mathematics classroom discourse. In addition, this study considered how mathematics classroom discourse varied by multiple semiotic resources. The subsequent sections discuss findings of this study in comparison with previous research in light of teacher’s linguistic scaffolding, student responses in co-constructing math concepts, and teachers’ linguistic choices varied by multi-semiotic resources (e.g., manipulatives, visual display, symbolic representations, language).

6.2 Teacher’s Linguistic Scaffolding

This study found that teacher talk unfolded in three stages: inviting students to coding activity, facilitating student participation, and promoting students’ reasoning. At the beginning of coding activity, teacher talk involved relational processes such as being or having verbs. Findings of this study corroborate other studies’ account that the use of relational processes is predominant in the mathematics classroom (Herbel-Eisenmann & Otten, 2011; O’Halloran, 2004; Shleppegrell, 2007; Veel, 1999). Herbel-Eisenmann and Otten demonstrated that a secondary math teacher selected an attributive relational process ‘are’ as in a clause ‘rectangles are parallelograms’ without an explicit explanation of the relationship between rectangles and parallelograms. They argued that students can be confused and regard ‘rectangles’ as equivalent to ‘parallelograms.’ The relational
process ‘are’ related a part to a whole by ascribing an attribute (e.g., ‘parallelograms’) to a participant (e.g., ‘rectangles’). O’Halloran quantified the frequency of relational processes selected and pointed out that attributive relational processes were selected most in mathematics classroom discourse. These studies, however, did not show how the relational process can work in developing mathematical concepts.

According to the findings of this study, the teacher used attributive relational clauses (e.g., ‘each place has value. These digits are worth different amount or value’) to provide information relevant to a target mathematical concept (e.g., place value) at the beginning of an episode. She also selected identifying relational clauses (e.g., ‘what is this place value [7 in 72] called?’) to invite her students to decode meaning of a mathematical entity/participant as Token (e.g., ‘this place value,’ that is, ‘7’), which is an immediately observable instance of reality. In addition, when supporting the students in delving into a concept associated with the entities/participants, the teacher employed attributive relational clauses (e.g., ‘how many digits do we have in this number?’), in which the relational process ‘have’ connected ‘how many digits’ to ‘we’ as possessor who takes accountability for understanding this mathematical concept. This study suggests that relational clauses can be chosen for teachers to provide linguistic scaffolding for ELLs who are required to receive contextual information and give information demanded in relation to decoding the Token at the initial stage of an episode. This initial stage involving relational clauses is of vital importance in a sense that teachers can engage students in identifying mathematical entities/participants as Token (e.g., tens and ones blocks, pictures of two tens, a 3-digit number, ‘half of’) and their meanings/Values, both of which can construct mathematical concepts. While guiding them in connecting the entities to the meanings, teachers can introduce academic language (e.g., regrouping, subtraction, place value, division) closely associated with mathematical concepts.
In the stage of facilitating student participation, the findings of this study support the scholarship based on systemic functional linguistics (SFL) in reference to the interplay between everyday language and academic language at the clause or clause complex level (Chapman, 1995; Gonzalez, 2015). These studies argued that teachers’ mode shifts from everyday language to academic language linguistically scaffolded students in constructing mathematical meanings, not discussing how teacher talk displayed the interplay between the two languages. Gonzalez also indicated that a math teacher interwove everyday language and math language by drawing on analogies concerning everyday experience, describing that conjunctions used frequently were associated with addition (e.g., ‘and’) and causation (e.g., ‘because,’ ‘so,’ ‘if...then’). These studies addressed that teacher talk elaborated on student responses, but only one of the studies described that teacher talk extended or enhanced meaning through the use of conjunctions (Gonzalez, 2015). Gonzalez, however, did not discuss how clauses or messages display through the Given-New information structure at the clause complex level.

This current study, thus, suggests that teacher talk realized in a clause complex needs to display everyday language as Given information in the first clause and academic language as New information in the following clause either in decoding a Token/mathematical entity for the Value or in encoding a Value/meaning to the Token. Given that everyday language describes observable entities or actions and academic language depicts abstract concepts, everyday language can be a Token and academic language a Value. In decoding the Token or encoding the Value, teachers can first introduce everyday language as Given information and later use academic language as New information. For instance, when decoding Token for Value in a clause complex ‘I’m gonna start at seventeen and get to twelve, so I’m going to subtract,’ teachers can first place everyday language as in ‘I’m gonna start at seventeen and get to twelve’ while delaying academic language as in ‘I’m going to
subtract.’ In encoding Value to Token in a clause complex ‘You count back with me because we’re subtracting,’ ‘You count back with me’ as Token expressed by everyday language comes first and ‘we’re subtracting’ as Value involving academic language comes later. The sequential choice of decoding-encoding clause complexes can contribute to developing a complete concept of subtraction. Information display in the order of Given and New information can enable teachers to support students to combine everyday language with academic language instead of defining and explaining the meaning of academic language (e.g., subtraction, place value). By placing everyday language familiar with students first and academic language later, teacher talk oriented toward listeners/students can facilitate their participation in mathematical discourse and guide them in expressing their thoughts verbally. In increasing student participation, teachers can also use sentence starters and statement-like questions (e.g., ‘One person has? ’ Juan, operation means what?’), which can facilitate students to fill out the missing element (e.g., ‘20 gumball’) or focus on the last part (e.g., ‘what’).

With respect to the teacher as a facilitator for students’ engagement, the findings of this study demonstrated that teacher talk prompted student participation through modulated commands (e.g., ‘Can you repeat? ’ ‘I’d like Yoana to share’). This result is consistent with Zolkower and Shreyar’s (2007) study in a sense that modulated commands a teacher selected make room for students to contribute to mathematical activity, engage them in expressing their thoughts verbally in the mathematics classroom, and allow them to think aloud mathematically. In addition to the use of modulated commands (e.g., declarative clause, interrogative clause), this study implies that the selection of verbal clauses (e.g., ‘Tell me what you mean,’ ‘Can you add to what she said?’ ‘I’d like Yoana to share’) can facilitate student participation and support ELLs’ academic language and concept development in a way that students can participate in a learning community by verbally
expressing observable actions, someone else’s saying and inner thinking. Findings of this current study, in particular, suggests that ELLs can linguistically participate in mathematical discourse when teachers encourage them to restate someone else’ utterance by asking ‘can you repeat?’ or ‘tell us what she said’ instead of using wait time (Gibbons, 2009; Kazemi & Hintz, 2014; Walsh, 2011) in order for them to have time before offering information demanded. Kazemi and Hintz state that “repeating gives students (including the sharer) an opportunity to hear the idea again” (p.50). Asking for replication of someone else’s statement and question can work as linguistic scaffolding for students who need to develop language and concepts simultaneously in that the students can still participate in mathematical practices for concept development by listening carefully to others and reporting what is said as well as reminding what information is demanded.

In the stage of promoting students’ reasoning, this study found that open-ended questions were selected. Studies argued that open-ended questions support students to do mathematical thinking and make reasoning (e.g., Kazemi & Hintz, 2014; Mercer & Littleton, 2007; Petcova, 2009). These studies, however, did not discuss how and when such questions prompt students’ reasoning. Findings of this study showed that the participant teacher used why/how questions in supporting students to develop their argument through a relation of cause and condition while encouraging them to make reasoning the relationship between Token and Value by using a clause complex often at the end of coding activity. For example, a student developed her argument using a causal statement ‘because it didn’t pass the ten’ as a response to a teacher question ‘why is it still nine o’clock when it is still closer to the ten?’ The teacher asked students to relate a Value/concept of the hour ‘why is it still nine o’clock’ to a Token/expression ‘when it is still closer to the ten.’ The student response is indicative of her concept of nine o’clock by reasoning the relationship between the hour hand and the hour. The reasoning was made when the student justified her own thinking by
binding an observable phenomenon (i.e., the location of the hour hand) and a concept of the hour. The location of the hour hand is a Token related to a spontaneous concept while the concept of the hour is a Value associated with a scientific concept. This linguistic interaction between teacher and student realized in why-and-because exchange can help students utilize scientific concepts to mediate everyday concepts.

In addition to why/how questions, teachers’ questions realized in mental clauses are significant in developing a reasoning ability (Herbel-Eisenmann & Otten, 2011). Findings of this current study support the claim that teachers’ selection of mental processes is important because the processes “require students to reflect on the activity in which they engaged” (p.446). They only pointed out that teachers’ use of mental processes such as ‘notice,’ ‘think’ would be concerned with students’ reflection on their activity. This current study, however, suggests that it is imperative to understand when to use mental-projection questions. This study found that teacher talk involved mental clauses in facilitating students’ reasoning of a concept as well as reflecting on a mathematical activity. For instance, the teacher participant asked, ‘what hour do you guys think it will be?’ before asking a why-question, and gained a student response ‘nine, nine.’ The use of the mental process supported the student in finding the Value (i.e., ‘nine o’clock’) to the Token (i.e., ‘it/the hour hand’). In the follow-up, the teacher asked the student to validate her answer using a why-question. Questions realized in mental processes that are followed by why-questions can work as a facilitator that can help students make reasoning concerning the relationship between Token and Value. The selection of mental projection also contributes students to linking one phenomenon relevant to a here-and-now situation (e.g., ‘do you guys think’) with “second-order phenomenon” realized in a statement such as ‘what hour it will be’ (Halliday & Matthiessen, 2014, p.514). This
linkage can facilitate students’ mathematical thinking and eventually develop mathematical concepts through teacher-student oral interaction.

6.3 Student Responses in Co-constructing Mathematical Concepts

According to findings of this study, ELLs engaged in constructing mathematical concepts through supported language the teacher provided while responding to teacher’s requests realized in questions or commands. At the beginning of an episode, students’ use of elliptical clauses (e.g., ‘division,’ ‘five hundreds’) in response to questions such as ‘what is half of?’ and ‘what’s here?’ indicates that the students were invited to identify mathematical entities/participants (e.g., ‘half of,’ ‘a picture,’ ‘division,’ ‘five hundreds’). In other words, the students were asked to decode Token for Value by relating words referring to concrete objects (e.g., half of, a picture) to meaning (e.g., division, five hundreds). The full responses (e.g., ‘half of is division,’ ‘it is five hundreds’) show that the students were guided to talk about meaning as opposed to something immediately observable in context. Talking about meaning can be realized in the relational clause that connects something observable in the context—‘half of’ and ‘the picture of five hundreds’—to a meaning or value. In this regard, findings of this study corroborate Painter’s (2007) argument that children are supported through context-bound language to talk about meaning rather than talk about immediately observable context.

In addition, the identification of mathematical entities/participants implies that the students began to establish a mathematical concept by offering a Value (e.g., ‘division’) related to a Token (e.g., ‘half of’). Using language as “functional equivalent,” which involves “the grouping of objects that are actually related to each other,” indicates that the students participated in categorizing concrete and factual entities into an abstract concept (Vygotsky, 1986, p.112). In this
sense, student participation in the categorization through talk using everyday language (e.g., half of) and academic language (e.g., division) supports Vygotsky’s (1986) theory of concept development, in which categorizing an abstract concept by reference to concrete entities can contribute to bringing into higher mental functioning such as verbal definition. In the initial stage of concept formation, use of functional equivalents enables oral interaction between teacher and student to co-construct a concept comprising everyday language and academic language because students before puberty share meaning of everyday language with teachers but they do not in using academic language teachers conceptualize (Vygotsky, 1986).

The identification of entities/participants also occurred when the students were asked to ascribe attributes to the entities after drawing pictures of 534 and 428 for subtraction of 3-digit numbers. For example, Ms. Bright asked how many hundreds were in a 3-digit number ‘534,’ stating ‘how many hundreds do you have?’ In response, a student provided an elliptical clause ‘five hundreds.’ The response indicates that the student identified an attribute ascribed to ‘534’: five hundreds is part of the number’s value (i.e., five hundreds, three tens, and four ones). The student connected the observable reality (i.e., the picture of five hundred) to the name realized in language. The identification of attributes can enable students to group different 3-digit numbers into one category, verbally express attributes of the numbers, and establish a mathematical concept of a 3-digit number that comprises a certain number of hundreds, tens and ones. While finding attributes, students participate in decoding concrete and observable entities/participants as Token to construct mathematical meanings or Values.

To further decode the meanings of mathematical entities, the students were guided to describe how the entities make sense to them by using material clauses (e.g., ‘you can use ten tens to make a hundred block’) as a response to a teacher’s demand ‘what do we connect these blocks with?’
Tell me.’ The response shows that the student used material processes (i.e., ‘use,’ and ‘make’) to explain the relationship between ten tens and one hundred: ten tens make a hundred. The selection of a material process ‘make’ seems to bond logical relation between two entities (i.e., ten tens and a hundred block) through direct experience realized in the material clause. Student responses realized at the complete clause level indicate that students linguistically construe the relationship of ten tens and a hundred block, both of which are connected to material reality. In using material clauses, students deal with mathematical concepts by concrete entities that they observe in real life. They also group the entities into one category (e.g., a value of one hundred) in a functional way despite different shapes of the two entities. Use of material processes can be distinctive in student talk as Wells (1999) demonstrated that students selected material processes significantly to talk about their actions while they were conducting a scientific experiment.

Findings of this study showed that students were supported to respond by material processes to report how they approached mathematical participants/entities as Token. For instance, Ms. Bright asked a student to verbally express what another student did in solving a problem, stating ‘you’re going to tell us what Hugo did.’ The teacher guided the student in using a material process as indicated in ‘what Hugo did.’ In response, the student chose material processes (e.g., ‘crossed out,’ ‘put,’) as in a clause ‘He crossed out a ten stick and put ten dots’ to verbalize her peer’s actions, by which the student can be supported to participate in constructing a mathematical concept of regrouping in a material way. This verbal description of actions can lead to higher mental functioning such as reasoning in a sense that verbalization of action can facilitate verbal thinking linked to reasoning the relation between Token and Value. Verbalizing actions realized in material clauses can bridge visualizing thoughts as in Vygotsky’s (1986) statement that “thinking in concepts does not exist beyond verbal thinking” (p.107). In this regard, it is of crucial importance
for students to report how they approach mathematical entities/participants in developing concepts as Haneda (2000) demonstrated that a student who verbally described the ideas taught could express them in writing as well.

In response to teacher questions realized in a verbal process (e.g., tell, repeat, share, add), students verbally expressed their actions at the clause level, which can lead to their thinking aloud for reasoning (Zolkiwer & Shreyar, 2007). The data analysis of this study showed that students were asked to relate Value to Token at the end of an episode although they sometimes related Token to Value when co-constructing number sentences by offering a value of a numerical expression. Relating Value (e.g., ‘the hour’) to Token (e.g., ‘nine o’clock’) as in a clause ‘it (the hour) is nine o’clock’ is an encoding process in which the Value is identified by reference to the Token and the concept of the hour is applied to a specific instance. The application implies that the abstract concept develops downward to regulate an immediately observable instance or everyday concept, as Vygotsky (1986) states that scientific concepts “gradually comes down to concrete phenomena” and regulate the phenomena (p.148). The encoding process realized in a relational clause can be expanded by a binder such as ‘because.’ In a clause complex ‘(it’s nine o’clock) because it didn’t pass the ten,’ the primary clause ‘it’s nine o’clock’ represents an abstract concept of the hour while the secondary clause ‘it didn’t pass the ten’ realizes an observable reality. The clause complex can work to identify the Value by reference to an observable Token. The identification indicates that the student can develop a scientific concept by encoding Value to Token when he/she develops his/her argument through a relation of cause and make reasoning a value/concept inductively by reference to attributes of the Token, that is, relating a part to a whole. This concept formation is embedded in the interplay between academic language and everyday language that students use in the mathematics classroom.
In interweaving everyday language with academic language, ways of students’ language use can mirror those of teachers. Findings of this study demonstrated that students’ concept formation was facilitated by their use of linguistic choices parallel to that of their teacher. For example, a student response ‘we crossed out the six and put a five because she was regrouping’ displays the same pattern in which everyday language comes in the Theme clause as the foundation of a message while academic language falls on in the Rheme clause as the development of the Theme. This pattern indicates that Given information realized in everyday language is used to make sense of New information represented by academic language. When facilitating students’ language and concept development simultaneously by modeling, Ms. Bright uttered ‘you count back with me because we’re subtracting.’ This utterance happened three-month earlier than the student’s use of the same pattern. Everyday language ‘you count back with me’ in the Theme clause realizes a spontaneous concept while academic language ‘we’re subtracting’ in the Rheme clause represents a scientific concept. The clause complexes which both the student and the teacher produced indicate that reasoning attributes of concrete and observable reality is made deductively by reference to an abstract concept, that is, through the part-and-whole relation. The parallel structure can be seen as an imitation that plays an essential role as a stepping stone in enhancing the student’s current level of development to higher level of development (Vygotsky, 1978, p.88). In particular, students’ “deferred imitation” between imitated language and spontaneous speech functions as a bridge to facilitate spontaneous speech (Speidel & Nelson, 1989, p.163). Deferred imitation suggests internalization of the linguistic structure for meaning-making of abstract/scientific concepts that are combined with spontaneous concepts through reasoning. This finding is in line with the argument that “internalization through imitation is not a matter of copying but entails an active, and frequently creative, reasoning process” (Lantolf & Thorne, 2007, p.213).
In the reasoning process, students' language use shows the increase in abstractness associated with academic language development at the grammatical level as well as the lexical level. In response to a teacher request 'are you okay with talking about why we crossed out the six and made it a five,' a student offered 'she needed to cross out the six to put a five to make fourteen.' The student’s response showed similarity to teacher talk such as 'you need to go past the number, past the hour in order to be that hour.' In the clauses above, the use of infinitive processes (i.e., ‘to put a five,’ ‘to make fourteen’ and ‘to be that hour’) increases abstractness caused by a non-finite process that is not limited by tense, Subject, or singularity/plurality. In reasoning the relationship between Token as spontaneous concept and Value as scientific concept, a student used this grammatical pattern after another student used the clause complex 'we crossed out the six and put a five because she was regrouping,' in which finite tense (i.e., past tense marked in ‘was’) and the Subject (i.e., ‘she’) exist. In other words, students developed academic language at the grammatical level by changing a clause (e.g., ‘because she needed to make fourteen’) into a group (e.g., ‘to make fourteen’).

6.4 Linguistic Choices Varying by Multiple Semiotic Resources

Multiple resources/representations (e.g., actions, manipulatives, visual images, symbolism, language) are signs as “means of internal activity” that are internally oriented to psychological operations (Vygotsky, 1978, p.55). According to Vygotsky (1978), signs such as multiple resources are psychological tools, of which language is defined as “the tool of tools” (p.53). Appropriate use of teacher talk can enable semiotic resources including language to work as a tool for concept formation. In this regard, using multiple semiotic resources is “mediated (indirect) activity” (Vygotsky, 1978, p.54).
This study found that linguistic choices Ms. Bright made varied according to semiotic resources, especially in terms of experiential meaning realized by the system of transitivity that concerns processes/verbs and participants/nouns. When using manipulatives as a salient resource in classroom talk, the teacher first selected the relational process 'have' combined with personal participants 'we' or 'you' as Subject: *'I have pennies, I have a dime, and I have a nickel.'* Through having-relational clauses, the teacher guided students in identifying mathematical entities/participants (e.g., base-ten blocks, coins) as Token indexically connected to material reality. She then used the relational process 'is/represents' to support the students to find attributes and values of each entity as well as to seek for the relationships, similarities, and differences between entities. For example, when asking students to find values of mathematical entities (e.g., pennies, nickel, dime) and compare them by value, Ms. Bright inquired *'which coin is worth the most (among pennies, nickel, and dime)?'* In response, a student offered *'there’s a dime'* connected to an immediate context where a wall chart of coins was displayed. Next, the teacher chose material processes (e.g., *'count,'* *'change,'* *'went'*) associated with personal participants such as *'we,'* *'you' and 'I'* as in *'I changed the way?'* or *'we went from ten?'* to solicit student participation by responses such as *'we count'* and *'to fifteen' respectively. The use of material processes can guide students in visualizing mental computation by action. Identifying mathematical participants and their attributes as well as processing mental computation realized in the material clause correspond to a developmental phase in which students group entities according to “immediate perception” (Vygotsky, 1986, p.111). According to Vygotsky, values or meanings of signs are given to students in this phase, who would unlikely find the meanings through direct experience.

Findings of this study show that the manipulativeness of visual representations determined teacher's selection of process types, which can be material or relational. Both of the processes
concern indexical and symbolic participants (e.g., the minute hand vs. minutes). When involving visual representations with higher manipulativeness (e.g., pictures of base-ten blocks), Ms. Bright chose material processes combined with verbal processes as in ‘Tell me what you did first,’ by which she facilitated students to produce a complete clause rather than an elliptical clause (e.g., ‘Regrouping’). She asked a student to explain what the student did on the picture. A student responded by ‘First I regrouped.’ Following up the student response, the teacher encouraged the student to unpack the academic language ‘regrouping’ by specifying her manipulation upon the visual representation by everyday language (e.g., ‘you drew a picture. And then you crossed out the four’). Ms. Bright used material processes to guide students in conceptualizing the procedure of mental computation for regrouping in a visible and materialized way mediated by talk. Meanings of specific actions verbally represented by everyday language (e.g., drawing a picture, crossing out a number, adding a number to another) are categorized into an abstract concept embedded in academic language (e.g., regrouping).

In contrast, she involved relational clauses as in ‘what’s the minute hand? What is it on?’ when Ms. Bright used visual representations with low manipulativeness (e.g., the picture of the analog clock) as the main resource in classroom talk. In response, a student verbalized ‘Thirty.’ In follow-up, she selected a verbal clause ‘what do you mean by thirty?’ by which she seems to facilitate students’ construction of wording in relating a Token (i.e., the location of the minute hand pointing to the six on the clock) to the Value ‘thirty minutes.’ Despite the discrepancy between students’ immediate perception of a digit 6 on the clock and the Value ‘thirty minutes,’ they associate the Token ‘6’ with the Value by identifying different functions of the minute hand and the hour hand. Grouping by function indicates students’ concept development shifted to relatively abstract phase from indexical phase (Vygotsky, 1986).
With respect to symbolism as the main semiotic resource in classroom talk, the use of relational or material clauses combined with mental clauses was significant. Participants associated with relational and material processes entailed symbolic entities (e.g., ten plus four, fourteen). In particular, the mental clause was a projecting one while the relational clause was a projected one. The projection nexus was realized in the interrogative clause as in ‘do you know what the new number becomes?’ The mental projection worked to encourage students to think and project their ideas about a change in two symbolic entities (e.g., change from six tens to five tens). In response to the mental projection clause, a student provided ‘five.’ In addition, the teacher included a symbolic participant ‘ten plus four’ when supporting students to make a number sentence (e.g., 10+4=14). The symbolism results from a change of a clause ‘add ten to four’ into a noun group ‘ten plus four’ on top of verbalizing the sign ‘=.’ When asking students to show a problem-solving procedure, the teacher chose mental clauses combined with material clauses as in ‘what did you notice Hugo did here?’ The response was realized in the material clause ‘he took off one ten.’ The use of mental processes functions to facilitate students in visualizing a mental computation procedure and identifying the relationship between symbolic mathematical entities. Teacher talk referring to symbolic representations can guide students in transitioning from “reliance upon external signs” to “entire operation of mediated activity” (Vygotsky, 1978, p.55). In other words, students verbally mediate external signs such as a number sentence, which can bridge to the internal reconstruction of a mathematical concept.

Concerning language as the main semiotic resource in classroom talk, Ms. Bright worked with students to solve math word problems. She significantly selected relational clauses to support students in finding linguistic expressions/Tokens associated with mathematical operations (e.g., addition, subtraction). She also favored relational clauses (e.g., ‘what is half of?’) to ask students to
identify contextual information relevant to the problems. Participants combined with relational clauses were indexical and symbolic ones (e.g., ‘half of’ vs. ‘division’), each of which represents an expression presented in the problem and meaning of the expression. In addition, the teacher used everyday language (e.g., students, pop) and academic language (e.g., operation, division, an equal number). When encouraging student participation, the teacher chose material clauses or verbal clauses as in ‘which words are going to help you?’ and ‘what do you mean altogether?’ The use of material (e.g., ‘help’) and verbal (e.g., ‘mean’) processes facilitated students to ‘do’ mathematics and ‘say’ their thinking for solving math word problems. Meaning of language as a salient semiotic resource in the mathematics classroom is situated in context (Moschkovich, 2015), in which language itself works as a clue for identifying the meaning of language. In identifying meaning, students can be guided to connect a linguistic expression (e.g., half of) to its meaning (e.g., division) rather than to be taught the meaning through teachers’ verbal definition. As a result, students can be supported to interweave everyday language with academic language and in turn develop mathematical concepts through a language-to-language association.

6.5 Summary and Limitations of the Study

Findings of this study suggest that teachers can support ELLs to perceive mathematical entities comprising both observable multi-semiotic resources as Token and their meanings as Value by choosing relational processes (e.g., is, has) in the initial stage of coding activity. In decoding the Token or encoding the Value, teachers can first introduce everyday language as known/Given information and later use academic language as unknown/New information. In the stage of facilitating student participation, teachers can provide information in the order of Given-New flow oriented to students. Teachers can also support students to learn academic language by reference to everyday language instead of explaining the meaning of academic language. The student-oriented
information display can facilitate their participation in mathematical discourse and guide them in constructing their thoughts verbally. In increasing student participation, teachers can also use sentence starters and statement-like questions. In addition, teachers’ use of verbal clauses can facilitate student in participating in a learning community by verbally expressing external actions/saying and inner thinking. ELLs can linguistically participate in mathematical discourse when teachers encourage students to restate someone else’s utterance instead of using wait time. In the stage of promoting students’ reasoning, it is imperative to understand when to use mental-projection questions. Questions realized in mental processes that are followed by how/why-questions can work as a facilitator that can help students make reasoning concerning the relationship between Token and Value.

In student response, the use of elliptical clauses was significant at the beginning of coding activity although student responses prevalently involved elliptical clauses across episodes in a sense that the elliptical clauses were equivalent to meanings or values of mathematical entities mentioned previously in teacher questions. The responses by elliptical clauses indicate that students can be guided through context-bound language to participate in talking about meaning and decoding concrete and observable entities/participants as Token to construct mathematical meanings or Values. Students can also process mathematical concepts using material or doing processes associated with concrete entities that they observe in real life. Verbal description of actions realized in the material clauses can lead to higher mental functioning such as reasoning in a sense that verbalization of action can facilitate verbal thinking linked to reasoning the relation between Token and Value realized in the clause complex. The clause complex can represent how students develop their own arguments through a relation of cause and condition and make inductive or deductive reasoning of the relationship between Token and Value. Inductive and
deductive reasoning processes can lead students to bind spontaneous concepts with scientific concepts through the interplay between everyday language and academic language. In particular, student talk can mirror teacher talk in a way to develop arguments by placing everyday language first and delaying academic language later. The combination of everyday and academic languages at the clause complex level can lead to concept formation through binding spontaneous concepts with scientific concepts.

Appropriate use of teacher talk can play an imperative role in mediating semiotic resources including language to mathematical concepts and working as a tool for concept formation. According to semiotic resources to which classroom talk refers, student talk may have no significant change in linguistic choices while teacher talk can vary by semiotic resources such as manipulatives, visual representations, symbolism, and language. First, in using manipulatives as a salient resource in classroom talk, teachers can use having/being verbs to guide students in finding indexical objects as participants and their attributes as well as connecting the objects to their concepts/values while choosing material processes to help students to visualize mental computation by action. Involving visual representations, teachers can make linguistic choices depending on manipulativeness of the representations. Second, visual representations with higher manipulativeness were operated by actions represented by material processes. A specific action as Token represented by everyday language is combined with a more abstract concept as Value expressed by academic language. On the other hand, visual representations with lower manipulativeness often comprise various entities/participants with different values. To guide students in identifying a specific entity and its value, teachers can involve relational clauses that represent visual participants/entities and their attributes as well as the values. Third, using symbolism as the main semiotic resource in classroom talk, teachers can combine relational or
material processes with mental processes, associating them with symbolic entities. In particular, the use of a mental process can facilitate students in visualizing a mental computation procedure and identifying the relationship between symbolic mathematical entities. Fourth and last, concerning language as a salient semiotic resource in classroom talk, teachers can work with students to solve math word problems using material processes combined with verbal clauses. They can also use relational processes when guiding students in finding a math concept equivalent to a linguistic expression ‘an equal number of.’ Participants distinctive in using language as a semiotic resource can include both indexical and symbolic ones. As mentioned above, teacher talk can mediate mathematical concepts to multiple representations in the continuum ranging from indexical representations to symbolic representations by using material or relational processes combined with mental or verbal processes.

Findings of this study can be limited in the following five aspects. First, this study only discussed classroom talk that mediates manipulatives, visual representations, symbolic representations, and language as semiotic resources in teaching and learning mathematics, excluding how other semiotic resources such as gestures and actions can contribute to making mathematical meanings. This exclusion is not because they are insignificant in constructing mathematical meanings but because they are beyond the scope of this study. Second, each semiotic resource was not analyzed at the grammar level although studies address how grammar of each semiotic resource realizes its meaning (e.g., Kress, Jewitt, Ogborn, & Tsatsarelis, 2001; O’Halloran, 2010). Third, this study did not examine how “regulative registers” for disciplining students impact on “instructional registers” pertinent to teaching and learning mathematical content (Christie, 2002, p.63). Fourth, the findings of this study could be limited to my understanding of mathematical concepts, which I view as word meanings grounded in Vygotsky’s theory of concept development.
Fifth and last, this study underexplored how teacher talk could impede students' language and concept development in an elementary mathematics classroom. To respond to research questions, this study only focused on how teacher talk can support students' linguistic and conceptual development.
CHAPTER 7

CONCLUSIONS AND IMPLICATIONS

This study aimed to explore how an elementary teacher used language to support academic language and concept development for her English language learners (ELLs) in the mathematics classroom. To accomplish this purpose, this study posed three research questions: (1) how can teachers use language to support ELLs’ academic language and concept development during mathematics instruction using language and other multisemiotic resources (e.g., manipulatives, diagrams, number sentences)?; (2) how do ELLs participate in language-focused mathematical instruction?; and (3) how does classroom discourse vary in this context according to multisemiotic resources used to support ELLs’ language and mathematical conceptual development? What follows briefly outlines the findings of this study, places concluding remarks, and suggests implications for literacy scholars, teacher educators, and teachers.

This study found that mathematics classroom discourse in this context involved three stages: (1) teacher’s inviting students to “coding” activity and students’ identifying mathematical entities/participants as Token and concepts/meanings as Value; (2) teacher’s facilitating student participation and students’ processing mathematical concepts by reporting their actions and someone else’s statements; and (3) teacher’s promoting reasoning and students’ binding spontaneous concepts with scientific concepts. Across these stages, the participant teacher talked in

51 ‘Coding activity’ in this study concerns identifying Token and Value in a clause or a clause complex, based on SFL.
a student-centered or listener-oriented way by placing everyday language as Given information first and academic language as New information later in a clause or a clause complex when engaging her ELLs in mathematics classroom discourse. This Given-New information structure enabled the students to construct academic language and mathematical concepts simultaneously. In response to teacher’s questions, students were guided to talk in elliptical clauses to find a Value or a meaning of a mathematical entity, produce complete clauses to report what was said or what they did. They then connected a clause to another to make a clause complex in response to why-/how-questions, expressing their reasoning about the relationships between Token and Value represented by everyday language and academic language respectively. According to multiple semiotic resources, linguistic choices Ms. Bright and her ELLs made had no difference except for linguistic choices used in making experiential meanings or talking about mathematical contents. The students predominantly chose material clauses irrespective of types of semiotic resources. Ms. Bright, however, selected different types of processes associated with various participants from indexical to symbolic. The teacher’s selection of participants from indexical to symbolic contributed to supporting the students in developing mathematical concepts. The selection of participants associated with processes was often recursive, and not linear from being indexical to symbolic. In other words, the teacher talked about a numerical expression (e.g., 564−468), which is symbolic, and then went back to pictures of the 3-digit numbers, which is less symbolic.

The findings of this study show that it is important to know that students, including ELLs, should be provided verbal scaffolding for their language and concept development, given that developing academic language and mathematical concepts is seen as inseparable (Moschkovich, 2015). First, teachers need to guide students in connecting everyday language to academic language to develop “real” or “complete” concepts comprising spontaneous and scientific concepts (Vygotsky,
To support language and concept development, teachers should interweave familiar/everyday language and abstract/academic language to engage students in relating multiple semiotic resources (i.e., Tokens) to the meanings (i.e., Values). Second, teachers should facilitate student participation in mathematical discourse by using sentence starters and statement-like questions, in which students can co-construct mathematical concepts by providing the required information in a word or a phrase. Teachers can also encourage student participation by asking them to report what they did or what someone said, which is realized in verbal clauses including saying verbs (e.g., tell, add, share) that can function to facilitate students in verbally projecting what is happening or what is said in a complete clause. For example, teachers can say ‘Tell me what Hugo did,’ ‘Can you add what she’s talking about?’ or ‘Yoana is going to share what she did’ while reducing potentials for students to respond by a word or a phrase (Halliday & Matthiessen, 2014, p.511). When responding to teacher’s requests involving verbal processes, students can be prepared to construct mathematical meanings. Third, teachers need to promote students’ reasoning of the relationships between Token and Value by asking students to respond to mental-projection questions or why/how questions. In response to mental-projection or why/how questions, students can construct mathematical meanings as the content of thoughts and develop their arguments through a relation of cause and condition, which is realized in a clause complex containing a Token as a spontaneous concept in one clause and a Value as a scientific concept in another. Fourth and last, it is critical to relate scientific concepts to spontaneous concept as observable equivalents in meaning rather than defining and explaining the scientific concepts in ways to use material processes associated with indexical participants (e.g., manipulatives, pictures with higher manipulativeness, everyday language) and relational processes with symbolic participants (e.g., pictures with lower manipulativeness, symbolism, academic language).
The conclusions of this study provide suggestions for literacy scholars, teacher educators, and teachers. First, literacy scholars would benefit from exploring the conceptual connections between sociocultural theories of language and concept development such as Vygotsky’s theory of concept development and Halliday’s SFL. The understanding of the conceptual connection can give insights into ways to use language interconnected with language and concept development. Literacy scholars can also examine how student participation in mathematics classroom talk can lead to developing academic language and in turn impacting mathematical concept development. In addition, studies can regard everyday language and academic language as complementary in real/complete concept development in that everyday language is used to make sense of scientific concepts while academic language is used to regulate spontaneous concepts. Moreover, literacy scholars can regard teaching and learning as science of language use given that a word meaning as the unit of concept. In the teaching-learning practice, a specific instance of verbal scaffolding involves a particular function for concept development and student responses indicate the degree of their concept development. Furthermore, scholars can provide opportunities for in-and-pre-service teachers to be reflective about ways of their oral interaction with students to convey content knowledge in the classroom practice.

Second, it is of great importance for teacher educators to encourage in/pre-service teachers to explore appropriate ways of verbal interaction such as presenting familiar/Given information represented through everyday language first and relating it to unfamiliar/New information expressed by academic language. In particular, pre-service teachers can reflect on their own teaching practices rather than just following their mentor teachers in that pre-service teachers are often positioned as learners as Zeichner and Liston (1987) argued. In addition, teacher educators can provide in/pre-service teachers with pedagogic knowledge in terms of not only conceptual
level but also linguistic level (i.e., how to make linguistic choices specific to pedagogical purposes, as described by Christie, 2002). Moreover, teacher educators can provide pre/in-service teachers with teacher education programs focusing on classroom discourse to support them in implementing state standards (e.g., CCSS, NGSS, and WIDA) into classroom practices in that those standards require students to communicate academic concepts specified by academic language (see Lee, Quinn, & Valdes, 2013; Lee, 2018).

Last, teachers can become aware that academic language and content of subject matters can be taught simultaneously because concepts are embedded in language. Academic language constructs scientific concepts including mathematical concepts. Studies report that teachers working with ELLs tend not to use academic language to facilitate the students’ participation in classroom discourse and understanding of targeted concepts (Brizuela & Earnest, 2008; Ernst-Slavit & Mason, 2011). To support ELLs to develop academic concepts, however, teachers should interweave everyday language and academic language rather than using only everyday language. Real/complete concept development occurs when students make sense of scientific concepts by verbally relating an observable phenomenon or a Token to its meaning or a Value and when they regulate everyday concept by linking a Value to a Token. When developing academic concepts and language, teachers can foreground everyday language as equivalents to academic language and interweave everyday language with academic language rather than defining and explaining academic concepts.
As far as I know, there is no literature about what genre MWPs would be. In consideration of their purpose, they can be organized recount-like organization, including numeric variables and units. The variables and units are required features. Recount usually consists of Orientation, Series of Events, and Evaluation. But MWPs are different from usual recount because circumstance concerning time and place is optional in orientation, which should involve participants and processes. Then it is followed by any event(s) with numeric variables, which are obligatory. Last, the text is finalized by information request question with unknown variables. So you can say that genre can be 'algebraic word problem'. The genre is organized by 'Orientation + Event(s) + Information Request Question'. Orientation + Event(s) are moves for contextualizing any problems. Typical features include participants, processes, known numeric variables, and WH-question with unknown variable(s).

The implication of your math lesson based on SFL or GBA can be:
- It contributes to raising awareness of both how MWP genre is organized and how lexicogrammatical features work to accomplish its purpose within a MWP text.
- The lesson helps support students’ language development as well as math content knowledge through frequent interactions with students in that the curriculum places high value on ‘joint-construction’. Positioning students as co-constructors of knowledge.
- It leads students to have ownership of math knowledge by being empowered with systemic scaffolding.
- It helps students to be well-prepared for higher-level academic life in that math is critical for high school graduation that involves MOAS Math text.

What else?
Hope it will be helpful for your writing. See you tomorrow.

Hyunsook
APENDIX B

CLASSROOM CONFIGURATION

(A: desks, B: easel, C: bookshelves, D: butcher paper, R: researcher, 1-15 referring to student chairs)
Oct. 07, 2014

[Journal]

I spent one and half hours searching for some MWPs in the internet, which I guess would be taught in next unit in second- and third-grade classroom. I downloaded and analyzed them. It took time to insert some pictures of visual aids for solving MWPs. After sending them to the teacher, I headed for Springfield. I found myself excited about observing the math class because I really wonder how the teacher will implement genre-based approach to teaching math, esp. math word problems at the same time that she supports her students’ language development.

I arrived in school at 1:10pm. Signed in the office and entered the classroom at 1:15. Took a photo of today’s MM. There was no new MWP presented. In the half space of the classroom, 2 teachers (maybe one ESL teacher and one assistant teacher) were doing math lesson with about 5 students, while letting the rest 4 students take a computer-based test.

I think that classroom management is still the most challenging to the teacher because of mixed-grade levels and differentiated entering and leaving the classroom. Today’s MWP might be hard for second graders, who are learning place value and ordering. The target MWP was addition involving coins such as nickels and dimes. Those units seemed difficult to lower-level students in relation to math in that those culture-embedded units represent 5 cents and 10 cents respectively.

After the lesson, I told the teacher that the units should be underlined (because she asked students to underline key words). She accepted that it would have worked better with the conversion of the units into 5 cents and 10 cents, saying that they are culture-related words. On my way back home, I thought that it might be better for $1.35 to be rephrased into 1 dollar and 35 cents, which can help students understand it better.

[Field Note]

<table>
<thead>
<tr>
<th>Time</th>
<th>Teacher/Student Activities</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>1:22</td>
<td>(T came in with 4 students: [Redacted], [Redacted], [Redacted], and [Redacted]) Ask L.O to read steps for solving MWP presented on the screen that was shown the previous week. (See photo_20140930_02&amp;03)</td>
<td>audio-recording from 1:22 to 1:53.42</td>
</tr>
<tr>
<td>Time</td>
<td>Event</td>
<td>Notes</td>
</tr>
<tr>
<td>-------</td>
<td>----------------------------------------------------------------------</td>
<td>----------------------------------------------------------------------</td>
</tr>
<tr>
<td>1:25</td>
<td>(came in.) L.A is asked to read Step 2.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( T: ) Why should we underline words and information?</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( L.A: ) You can underline numbers, you can underline ‘more than’, ‘how many’, altogether, ( ).</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( ) reads Step 4, adding some info to it, and ( ) reads Step 5.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( ) is asked to repeat what ( ) said.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( ) : Write a full sentence including answer.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( ) : After drawing a picture, you can write a number sentence.</td>
<td></td>
</tr>
<tr>
<td>1:40</td>
<td>Present a MWP on the screen. (see photo_20141007_02)</td>
<td></td>
</tr>
<tr>
<td>1:44</td>
<td>(Second graders (1 boy and 3 girls) and ( ) (Gr. 4) came in.)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Ask Jay to retell the problem so that the rest of the Ss listen to him.</td>
<td></td>
</tr>
<tr>
<td>1:49</td>
<td>(3 Ss in Gr. 2, ( ) (Gr. 3) and 2 more girls (( ) in Gr. 2 and a girl) came in.)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Ask ( ) to retell the given MWP, but he couldn’t.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Get him come to her and help him retell it.</td>
<td></td>
</tr>
<tr>
<td>1:51</td>
<td>Get St to copy the given MWP.</td>
<td></td>
</tr>
<tr>
<td>1:52</td>
<td>(( ) (Gr. 3) came in.)</td>
<td>17 students all came in except ( ) (Gr. 5)</td>
</tr>
<tr>
<td></td>
<td>While Ss are copying the MWP, T is helping them individually.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(Take a photo of the students who’re focused on writing.)</td>
<td></td>
</tr>
<tr>
<td>1:57</td>
<td>( T: ) Boys and girls, after copying, you underline key words and circle the question.</td>
<td>better to let them underline and circle by different colored pencils.</td>
</tr>
<tr>
<td></td>
<td>( T: ) OK, I’m coming around to check.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Check individuals’ deconstructing (i.e., underline and circle) (see, photo 20141007_02: Although ‘totals’ is underlined, it’s not be seen.)</td>
<td></td>
</tr>
<tr>
<td>2:07</td>
<td>Ask L.O to move to the chair in the back of the classroom.</td>
<td></td>
</tr>
</tbody>
</table>
Get Ss to see the underlined words and a circled question.
Ask them to show which operation is used by their fingers.
Have Ss look back at the MWP for making an answer statement.
Write the statement and ask them to write ‘Label answer statement’.

<table>
<thead>
<tr>
<th>Time</th>
<th>Event</th>
</tr>
</thead>
<tbody>
<tr>
<td>2:21</td>
<td>(Jay, Ashley, Emily, and L.A leave. L.O is asked to sit back and write the answer statement. He’s allowed to leave at 2:23.)</td>
</tr>
<tr>
<td>2:24</td>
<td>12 Ss left in the classroom are asked to look at the screen. T: Paula, come on up. Use your pencil and picture. Tell us your thinking so that we can hear you. Paula tells her thinking aloud and then pictures nickels and dimes (see, photo 20141007_03).</td>
</tr>
<tr>
<td>2:35</td>
<td>Ask Ss to label “Picture.” T: what else do we need? Angelika: We need a number sentence. She stops and is asked to come to the teacher for help. T: So Claud, come on up. Let him complete the equation. (see, photo 20141007_04) Complete the answer statement by filling in the numbers.</td>
</tr>
<tr>
<td>2:41</td>
<td>Check if Ss filled in the missing numbers in the answer statement.</td>
</tr>
<tr>
<td>2:44</td>
<td>T: Who can tell us what we learned today? Angelika, can you show us what we learned today?</td>
</tr>
<tr>
<td>2:47</td>
<td>Ss are released.</td>
</tr>
</tbody>
</table>
APENDIX D

PHOTOS

1. 20140930_01

2. 20140930_02
What should we think about when we solve MWP's? When we must solve MWP's we need to...

1. Read the story problem at least 2x
2. Think about the gist/big idea(s)
3. Reread and underline important words and circle any questions
4. Think about which operation to use (addition, subtraction, multiplication or division) based on math signal words

Answer statement comes from the question in the MWP to show your thinking.
WALT: Sort math words into different groups because each group has common features.

These words signal multiplication:
- multiplied by
- groups of
- twice as many
- times
- each

These words signal addition:
- in all
- sum
- all together
- total
- how many
- both
- plus

These words signal division:
- quotient
- every
- each
- how many
- each
- half of

These words signal subtraction:
- how many more
- more than
- fewer
- less than
- are left
- take away
- difference
Elise runs her own bakery company. This morning Elise's workers baked 801 key lime pies. They also baked some more after lunch. In total, they baked 900 key lime pies. How many key lime pies did Elise's workers bake after lunch?
APENDIX E

EMAIL 02

This is great! I will create similar MWP to do whole group and then students can do ones on their own that have the similar wordings. I like how there is also past tense used as well. This will support their listening comprehension and develop their expressive language, too.

Did you find any third grade MWPs from PARCC?

You can record any part you want. I suggest that maybe you can do a few different short videos and see which are more cleaner and have a better response from kids—just a thought.

Thanks!!

On Nov 16, 2014, at 12:29 AM, 신현숙 <sarah623@hotmail.com> wrote:

Hi [name]

I searched for PARCC practice test for second graders, but I got to know that there is no math test for them. Anyhow attached are MWPs with two-digit subtraction so that you can use them.

I’m going to your class on Tuesday, November 18 and 25. I wonder if I will be able to video-record your math group before Christmas holiday starts. I’d like you to let me know a specific unit you wants me to record so that you can use for your future professional development as a presenter.

Hope to talk more later.
APENDIX F

INTERVIEW PROTOCOL

* I’d like you to understand that this interview will be recorded for transcription.

1. How long have you been working as an ESL teacher?
2. When did you start and complete ACCELA program?
3. What is your highest degree? (e.g., MA, CAGS)
4. What kinds of teacher professional development programs have you taken? (e.g., reading, writing, leadership development other than degree programs like master)
5. How many hours did you get for teacher professional development programs?
6. What is your latest PD program? How many hours and when was it?

* I’d like to ask you about your working with the math group of students.

7. Did you have any challenge connected to something out of school in teaching math to your students? For example, things are Common Core State Standards, WIDA, policies, assessments, and so on.
8. Did you have any difficulty in relation to your school such as your principal, school curriculum, and classroom environment?
9. Do you think that your knowledge about SFL and genre-based pedagogy was helpful in teaching the students?
10. If yes, please tell me in what way, specifically.
11. You know the teaching-learning cycle that involves developing context, modeling and deconstructing context, joint-construction, independent construction, and reflection and evaluation. Do you think that the teaching-learning cycle was helpful in working with your math group of students? If yes, can you tell me in what way? (If no, why do you think it was not?)
12. You know that academic language development across all content areas is very important for all students, especially for English language learners. In what way can you develop students’ academic language in your math class?
13. Can you tell your students’ improvement in terms of thinking, or communicating mathematically, or responding by writing? Can you take an example?

14. Is there anything else you’d like to mention about your math class?
APENDIX G

SAMPLE OF SELECTED SEGMENT

1. 20141030 (audio #15) #198-212 (15:30-) Language (deconstruction of ‘how many more’)

1. T: Now [. ] which words are going to help us think about the operation in that example with Mrs. Bright and Ms. A?
2. S: I know it.
3. T: Camila! [. ] Which clue word will help us know what to do?
4. C: Ummm uh […] How many [. ] more?
5. T: Awesome. Awesome. How many more. =And how did she figure that out?
6. S: Cause, she ( ) asked her how many ( )
7. T: Right! So it’s telling us how ( ) someone has more than someone else. Is that right Camila?
8. C: Mmhmm.
9. T: And we’re comparing! We’re saying one person has [. ] everybody
11. T: Another person has-
13. T: So, we have to figure out-
14. S: How many are-
1. Language I: ‘how many more’ (Ex.01)

<table>
<thead>
<tr>
<th>T_1</th>
<th>Now which words are going to help us think about the operation in that example with Ms. Bright and Ms. Smile?</th>
</tr>
</thead>
<tbody>
<tr>
<td>Field</td>
<td>Actor Material: Initiator Senser Mental Circ.</td>
</tr>
<tr>
<td>Tenor</td>
<td>Subject Finite</td>
</tr>
<tr>
<td>Mode</td>
<td>Theme Rheme</td>
</tr>
</tbody>
</table>

| S_2   | I know it                                                                                                 |
| Field | Sensor Mental Phenomenon                                                                                   |
| Tenor | Subject Finite                                                                                            |
| Mode  | Theme Rheme                                                                                               |

| T_3   | Camila, which clue word will help us know what to do?                                                      |
| Field | Actor Material: initiator Senser Mental Phenomenon                                                        |
| Tenor | Subject Finite                                                                                            |
| Mode  | Theme Rheme                                                                                               |

| C_4   | Um, how many more                                                                                            |
| T_5-i | Awesome, awesome. How many more.                                                                           |
| T_5-ii| And how did you figure that out?                                                                           |
| Field | Cir. Sensor Mental Phenomenon Cir.                                                                          |
| Tenor | Finite Subject                                                                                              |
| Mode  | Theme Rheme                                                                                               |

<p>| S_6   | Cos she asked her how many                                                                                 |
| Field | Sayer Verbal Receiver Verbiage                                                                             |</p>
<table>
<thead>
<tr>
<th>Tenor</th>
<th>Subject</th>
<th>Finite</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Mode</td>
<td>Theme</td>
<td>Rheme</td>
<td></td>
<td></td>
</tr>
<tr>
<td>T_7-i</td>
<td></td>
<td></td>
<td>Right.</td>
<td></td>
</tr>
<tr>
<td>T_7-ii α</td>
<td>So it</td>
<td>'s telling</td>
<td>us</td>
<td></td>
</tr>
<tr>
<td>Field</td>
<td>Sayer</td>
<td>Verbal</td>
<td>Receiver</td>
<td></td>
</tr>
<tr>
<td>Tenor</td>
<td>Subject</td>
<td>Finite</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mode</td>
<td>Theme</td>
<td>Rheme</td>
<td></td>
<td></td>
</tr>
<tr>
<td>T_7-iii &quot;β 1</td>
<td>how someone</td>
<td>has</td>
<td>more</td>
<td>than someone else</td>
</tr>
<tr>
<td>Field</td>
<td>Carrier</td>
<td>Relational</td>
<td>Attributive</td>
<td>Qualifier</td>
</tr>
<tr>
<td>Tenor</td>
<td>Subject</td>
<td>Finite</td>
<td>Tenor</td>
<td></td>
</tr>
<tr>
<td>Mode</td>
<td>Theme</td>
<td>Rheme</td>
<td></td>
<td></td>
</tr>
<tr>
<td>T_7-iv</td>
<td>Is</td>
<td>that</td>
<td>right</td>
<td>Camila?</td>
</tr>
<tr>
<td>Field</td>
<td>Relational</td>
<td>Carrier</td>
<td>Attribute</td>
<td></td>
</tr>
<tr>
<td>Tenor</td>
<td>Finite</td>
<td>Subject</td>
<td>Residue</td>
<td></td>
</tr>
<tr>
<td>Mode</td>
<td>Theme</td>
<td>Rheme</td>
<td></td>
<td></td>
</tr>
<tr>
<td>C_8</td>
<td>Um</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>T_9-i =2</td>
<td>And we</td>
<td>'re comparing</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Field</td>
<td>Actor</td>
<td>Material</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Tenor</td>
<td>Subject</td>
<td>Finite</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mode</td>
<td>Theme</td>
<td>Rheme</td>
<td></td>
<td></td>
</tr>
<tr>
<td>T_9-ii α</td>
<td>We</td>
<td>'re saying</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Field</td>
<td>Sayer</td>
<td>Verbal</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Tenor</td>
<td>Subject</td>
<td>Finite</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mode</td>
<td>Theme</td>
<td>Rheme</td>
<td></td>
<td></td>
</tr>
<tr>
<td>T_9-iii &quot;β</td>
<td>one person</td>
<td>has?</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Field</td>
<td>Carrier</td>
<td>Relational</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Tenor</td>
<td>Subject</td>
<td>Finite</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mode</td>
<td>Theme</td>
<td>Rheme</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ss_10</td>
<td>Ten balls</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>-------</td>
<td>-----------</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>T_11</td>
<td>&quot;β&quot; Another person has?</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Field</td>
<td>Carrier Relational</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Tenor</td>
<td>Subject Finite</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mode</td>
<td>Theme Rheme</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ss_12</td>
<td>Twenty-five balls</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>T_13</td>
<td>So, we have to figure out?</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Field</td>
<td>Senser Mental Phenomenon</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Tenor</td>
<td>Subject Finite</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mode</td>
<td>Theme Rheme</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>S_14</td>
<td>How many more</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>T_15-1</td>
<td>Twenty-five is more</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Field</td>
<td>Carrier Relational Attribute</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Tenor</td>
<td>Subject Finite</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mode</td>
<td>Theme Rheme</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>T_15-ii</td>
<td>×2 So that number’s gonna go first</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Field</td>
<td>Actor Material Cir</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Tenor</td>
<td>Subject Finite</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mode</td>
<td>Theme Rheme</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>T_15-iii</td>
<td>Just watch</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Field</td>
<td>Behavioral</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Tenor</td>
<td>Finite</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mode</td>
<td>Theme</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>T_15-iv</td>
<td>Twenty-five balls first</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>T_15-v</td>
<td>Angelica, come up</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Field</td>
<td>Material</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Tenor</td>
<td>Finite</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mode</td>
<td>Theme</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>-----------</td>
<td>----------------</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>T_15-vi</td>
<td>and write the signal for subtraction</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Field</td>
<td>Material Goal Cir.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Tenor</td>
<td>Finite</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mode</td>
<td>Theme Rheme</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>T_15-vii</td>
<td>which we reach here</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Field</td>
<td>Range Actor Material Scope</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Tenor</td>
<td>Subject Finite</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mode</td>
<td>Theme Rheme</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>T_15-viii</td>
<td>Look up</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Field</td>
<td>Behavioral</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Tenor</td>
<td>Finite</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mode</td>
<td>Theme</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>T_15-ix</td>
<td>it’s gonna be subtraction</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Field</td>
<td>Token/Identifier Relational Value/Identified</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Tenor</td>
<td>Subject Finite</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mode</td>
<td>Theme Rheme</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>T_15-x</td>
<td>We’re gonna do subtraction</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Field</td>
<td>Actor Material Goal</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Tenor</td>
<td>Subject Finite</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mode</td>
<td>Theme Rheme</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
BIBLIOGRAPHY


Morgan, C. (2006). What does social semiotics have to offer mathematics education research? 

*Educational Studies in Mathematics, 61*, 219-245.


*Mathematical Thinking and Learning, 4*, 189-212.


