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MECHANICAL PERFORMANCE OF STRUCTURAL SYSTEMS WITH MISSING MEMBERS: FROM BUILDINGS TO ARCHITECTED MATERIALS

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MECHANICAL PERFORMANCE OF STRUCTURAL SYSTEMS WITH MISSING MEMBERS: FROM BUILDINGS TO ARCHITECTED MATERIALS

A Dissertation Presented

by

PANAGIOTIS PANTIDIS

Submitted to the Graduate School of the University of Massachusetts Amherst in partial fulfillment of the requirements for the degree of

DOCTOR OF PHILOSOPHY

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Structural Engineering & Mechanics
MECHANICAL PERFORMANCE OF STRUCTURAL SYSTEMS
WITH MISSING MEMBERS: FROM BUILDINGS TO
ARCHITECTED MATERIALS

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ACKNOWLEDGMENTS

Back in summer 2015, when I decided to leave my hometown and start my PhD studies at UMass, I could never imagine the challenges, experiences and emotions that this journey could offer. Now, moving towards the completion of this trip, I reflect with nothing but gratitude for this decision. Therefore, I want to thank first and foremost my supervisor, Dr. Simos Gerasimidis, for giving me this amazing opportunity. Simos has been an outstanding mentor, collaborator and friend for me over the past years. He introduced me to the world of research, he gave me the academic freedom to explore my interests and limits, and he exposed me to a series of challenges that have largely shaped who I am. For all the above, Simo, I want to express my deepest gratitude to you.

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support, patience, and love, as well as for our amazing interaction that makes me a better person. Finally, and most importantly, I want to thank my parents and my sister for their endless love and support. I will always love you.
Structural systems are potentially subjected to damage initiating scenarios throughout the course of their service time. Depending on the nature and extent of the damaging event, they may experience significant reduction or even complete loss of their mechanical performance. This dissertation delves into the mechanics of structural systems under the notion of missing members from their domain, investigating types of structural systems: a) multi-story steel framed buildings, and b) materials with a truss-lattice microstructure.

Part I of the dissertation investigates the performance of multi-story steel framed buildings under a column removal scenario, developing an analytical framework for their quasi-static robustness assessment. Two system-failure modes are taken into consideration: i) a yielding-type mechanism, where damage propagates as a series of beam plastic hinges or gravity connection failures, and ii) a stability-related mechanism, where a building column fails due to buckling. Validated against finite element results, this study demonstrates the capability of the method to assess the key features of the building performance such as the dominant collapse mode, the system capacity and the damage propagation path. Most importantly, it highlights that
governing mode is strongly dependent on the location of the initial damage scenario, emphasizing therefore the necessity for system-level approaches to identify correctly the building structural response.

Part II of the dissertation draws attention to the emerging class of architected materials with a truss-lattice microstructure, and performs a detailed study on the impact of missing struts to their elastic mechanical behavior. Considering a list of truss-lattice topologies with varying coordination numbers (connectivity) and accounting for a series of defect scenarios, this work identifies the effect of the various topological parameters that govern the degradation rates of the elastic constants, such as the lattice connectivity, anisotropy, and interrelation of the pristine elastic constants. This investigation is supported by numerical (finite element modeling), experimental (two-photon lithography) and analytical (elastic micromechanical models) approaches. The results revealed that the behavior of periodic imperfect truss-lattices with coordination number \( Z \geq 12 \) is almost indistinguishable from homogeneous materials, and demonstrated a clear trajectory between the topology coordination number and the least deleterious defect arrangement.
PREFACE

Published Content:

Part I has been partially adapted from [1, 2]:

   **Author’s contributions:** Derived analytical formulas, performed finite element simulations, analyzed data, wrote manuscript.

   **Author’s contributions:** Derived analytical formulas, performed finite element simulations, analyzed data, wrote manuscript.

Part II has been partially adapted from [3]:

   **Author’s contributions:** Performed finite element simulations and analytical calculations, analyzed data, partially wrote manuscript.
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PART I:
ROBUSTNESS OF STEEL-FRAMED STRUCTURES UNDER DISPROPORTIONATE COLLAPSE
CHAPTER 1

INTRODUCTION: PART I

1.1 Opening remarks

1.1.1 Disproportionate and progressive collapse

*Infrastructure resilience* is a multifaceted concept, revolving around the structural performance of an asset under a hazardous event, the interaction of the asset with the remaining properties of the broader infrastructure network, as well as the decision-making processes involved in the design of communities [4–6]. One of the main components of infrastructure resilience is *robustness*, and various definitions for this term have been proposed over the past decades [7–9]. In principle, robustness is a measure of structural insensitivity against propagation of damage within the structural system; a property which is affiliated with the ability of the structure to resist collapse following an *extreme* event. Such events, which are also termed as *abnormal*, are unforeseeable incidents which have a very low probability of occurrence and they have not been incorporated in the design of the infrastructure asset. Their origin may stem from nature (i.e. extreme earthquake or wind loading), malicious human activity (i.e. detonation of explosives, vehicle collisions), or accidental loads (i.e. unintended blasts, fires, construction errors). The interaction of the extreme event with the structural system can potentially initiate some form of damage in a localized part of the structure, a case which is commonly referred to as *initial damage scenario*.

Following the initial damage scenario, the structural components in the vicinity of the damaged region are typically subjected to extensive deformations, often in the inelastic regime, as load-redistribution mechanisms are activated and the structure...
strives towards a new equilibrium position. However, depending on the extent, magnitude and location of the initial damage scenario, as well as its interplay with the geometric and material characteristics of the structure, restriction of damage in this localized area may not always be feasible. In case damage propagates in a successive fashion, manifested as a chain reaction of sequential failures of structural components, then the structure is experiencing *progressive collapse*. This term refers to the type of damage progression throughout the system, which is characterized by this domino-type, mushrooming behavior. If the size of the resultant damaged part of the structure is disproportionately larger than the region which was directly affected by the initial damage scenario, then the structure is experiencing *disproportionate collapse* [10]. This term characterizes the relative size between the initially affected and ultimately damaged parts of the structure. Progressive collapse often results in disproportionate collapse [11], particularly when damage progresses throughout a significant part of the structural system. On the other hand, disproportionate collapse may be of immediate and not always progressive nature [12], marking a clear distinction between the two concepts.

### 1.1.2 Past incidents of disproportionate collapses

Historically, research interest has been drawn to the field of disproportionate collapse following collapse incidents of major impact to communities, which demonstrated a highly catastrophic nature. In view of that, three landmark events of crucial importance are subsequently described briefly, as they constitute pivotal points in the history of engineering research and construction practices.

- **Ronan Point Tower:**

Initiation of the community interest in the field of disproportionate collapse dates
back to the Ronan Point apartment incident (London 1968). On May 16 of that year, an accidental gas explosion at the 18th floor of the 22-story building triggered the detachment of a corner load-bearing precast concrete wall. Consequently, the floors above were left unsupported in their corner region and eventually collapsed to their lower counterparts, triggering a domino-type collision of the entire corner part of the building. The ensuing collapse was evidently both progressive and disproportionate. Due to its unprecedented and highly catastrophic nature the Ronan Point event marked a turning point in the history of construction design practices, resulting in the first wave of research and the subsequent development of provisions for the future prevention of such tragic events.

- **Alfred P. Murrah Building:**

  On April 19, 1995, the Federal Alfred P. Murrah Building in Oklahoma City was bombed by terrorists, who placed explosives of approximately 7000 pounds on a truck in the front face of the 9-story building. The massive blast destroyed three
perimeter columns of the structure, leading to the failure of the overlying transfer girder. Subsequently, the columns and floors supported by the transfer girder collapsed, resulting to the massive destruction of almost half of the building. This was another example of both progressive and disproportionate collapse, also accompanied by heavy human casualties. This bombing attack further heightened the need to incorporate in the prevailing design codes regulations for the enhancement of structural robustness against extreme events.

- **Events of 9/11:**

The event which steered the public interest and intensified regulative design approaches and scientific research towards the field of disproportionate collapse more than any other in the past, was indisputably the collapse of the World Trade Center Twin Towers in New York on September 9, 2001. After the aircraft collisions in the Towers, an extensive fire in the upper part of each building was generated and provoked additional excessive damage to the structural system. Consequently, col-
Figure 1.3: Collapse of the World Trade Center Twin Towers. View of the WTC Towers 1, 6 and 7 after the 9/11 events. (source: en.wikipedia.org)

Lapse of both 110-story buildings commenced and evolved in a pancake-fashion, with the upper part of the structure progressively colliding to the lower floors until the ground level. Following the events, several technical reports were conducted [13, 14] to investigate in detail the structural performance, to provide a well-established understanding of the damage progression path mechanics, and ultimately to assist in the prescription of regulative design guidelines.

1.1.3 Disproportionate collapse provisions

There have been many attempts to codify the lessons learned from each major collapse over the past decades and to develop regulative procedures to improve structural performance against disproportionate collapse. At the time being, the most two influential documents in the US are a) the GSA Alternate Path Analysis and Design Guidelines for Progressive Collapse Resistance [15], and b) the Unified Facilities Criteria - Design of Buildings to Resist Progressive Collapse [16]. These documents share a similar philosophy and they were developed to provide general guidelines for
design of federal buildings against disproportionate collapse. Among the methods included in these documents, the Alternate Path Method (APM) has notably dominated the field, and has been extensively adopted by the vast majority of the research community.

The core concept of APM is based on the notional removal of a vertical, load-bearing structural component, and the investigation of the capability of the remaining structure to sustain the loss of that component through the development of alternate load-carrying paths. APM therefore emphasizes on the redundancy of the structure, a property which describes its ability to bridge over loss of structural members through internally developed load-redistribution mechanisms, averting damage progression or separation of structural components at key locations. APM appertains in the direct design approach category, since it directly specifies the initial damage scenario (column or shear wall removal), as well as the structural analysis to be employed for the robustness assessment. Potential analysis types are a) linear static, b) nonlinear static, and c) nonlinear dynamic.

1.1.4 Literature review

In principle, three are the directions around which the research efforts on the field of disproportionate collapse have been primarily oriented: a) conducting experiments representative of a real column loss scenario, b) developing numerical models using the finite element approach and c) formulating analytical methodologies to describe the structural response based on closed-form expressions. In what follows, a brief overview of various noteworthy research projects which appertain in each category is presented.

Experimental investigation of full-scale structural disproportionate collapse is inherently hindered by practical limitations, such as the cost of assembling, loading
and demolishing a multi-storey composite framed structure. Nevertheless, significant attempts have been already made to gain insight in the actual structural behavior, focusing either on sub-assemblies of the structure or on critical structural components (moment or shear connections). Johnson et al. [17] conducted a half-scale experimental investigation of a three-bay by three-bay composite gravity floor system under four separate column removal scenarios, reporting maximum capacities which were less than the extreme event load combination. Dinu et al. [18] performed a downscaled experiment of a gravity floor subsystem with extended end-plate bolted beam-to-column connections. Failure was governed by the rupture of the beam end converging to the removed column, while the connections demonstrated excellent performance. However, they observed plastic deformations in the neighboring free edge columns, concluding that in a real scenario which accounts for the gravity load from the floors above, collapse could be initiated by the buckling of these columns. Gouverneur et al. [19] experimentally studied the response of a reinforced concrete slab strip, highlighting the beneficial effect of the catenary mechanism in the load-carrying capacity. Among others, Roddis and Blass [20], Weigand and Berman [21] and Oosterhof and Driver [22] experimentally evaluated the behavior of steel gravity connections of various types exposed to a typical column removal scenario.

The numerical approach involves the vast majority of the literature, mainly due to the practicality and efficiency of the finite element approach. Main and Sadek [23] numerically modeled and analyzed single-plate shear connections and gravity floor sub-systems, using both advanced and reduced modeling approaches. Kwasniewski [24] performed nonlinear dynamic analyses on a detailed 3D model of an 8-story steel framed building, identifying the modeling parameters with the greater impact on the final outcome and proposing a verification and validation approach to limit the analysis uncertainties. Li and El-Tawil [25] studied the response of a 10-story building.
using a higher- and a lower-sophistication finite element model of the structure against multiple column removal scenarios. Agarwal and Varma [26] numerically simulated two buildings with different gravity systems and performed corner compartment fire scenarios, focusing on the behavior of the gravity columns. They reported that the destabilization of these structural components governs the building behavior under a fire loading scheme and they highlighted the importance of adequate steel reinforcement in the slab to achieve a uniform redistribution of the load previously carried by the failed column to the adjacent columns, in order to maintain the overall structural integrity. Additional nonlinear dynamic finite element simulations of a 3D building structure under fire conditions were performed by Jiang and Li [27]. They investigated column buckling collapse modes associated with various heated column cases, focusing on the load redistribution mechanisms as well.

Compared to experimental and numerical investigations, analytical methods have not been developed to an equal extent. A landmark analytical methodology was developed in Imperial College of London [28, 29], which provided guidelines to evaluate structural robustness at successive levels of structural idealization, ranging from the simplest (beam level) to the more complicated (building level). This framework provides thorough insight in the structural response, since it entails the determination of the nonlinear static response both at the beam and the floor level, specified with sufficient accuracy. Additionally, Alashker and El-Tawil [30] proposed a design-oriented model to quantify the floor system robustness, purely based on analytical expressions. This methodology displayed sufficiently good consistency with numerical simulations at the final stages of collapse, yet there were significant discrepancies at the initial stages of loading. However, both of the aforementioned approaches do not explicitly address the response of the building columns, assuming that the latter are capable of maintaining their structural integrity under the load redistribution which follows the
column removal. This assumption is relaxed in the present investigation, where sufficient emphasis is placed on potential column instability phenomena. The correlation between disproportionate collapse and structural stability is thoroughly illustrated in the following section, where distinct stability-related collapse mechanisms following a column removal notion are exemplified.

1.1.5 Disproportionate collapse and structural stability

Stability can play a crucial role in the evolution of disproportionate collapse in steel framed structures. Structural instabilities are considered highly undesired from a design standpoint, since they manifest in a brittle way, provoking excessive damage propagation in an abrupt fashion. Therefore, it is of crucial importance for the structural integrity of the building that all potential instability related phenomena are captured and addressed in the building design.

Stability manifests itself through a wide spectrum of ways: at the member length-level (*short-wave instability*, referring to column buckling), at the member local-level (*local instability*, referring to flange buckling), and at the system level (*long-wave instability*, referring to system-global buckling). In the first case, the key structural components are the building columns which already have a high axial demand-to-capacity utilization ratio and they are typically located at the lower part of the building. As the structure strives towards obtaining a new equilibrium state following the column removal, the gravitational nature of the loading scheme imposes augmented demands on these columns and can potentially push them beyond their design limit state [31]. The second case may appear when the structure is subjected consequently to a column removal scenario and post-extreme event fire loading conditions in the vicinity of the removal. As shown in Gerasimidis et al. [32], collapse of a 15-story frame subjected to a ground floor column loss and subsequent fire loading was dom-
inated by flange buckling of the column facing the removal. Finally, regarding the third case, Gerasimidis et al. [33] demonstrated that very tall and slender frames are prone to a global loss of stability rather than having structural components experiencing individual buckling, with only limited appearance of fiber yielding in individual components of the building. These examples evidently showcase that structural instabilities constitute an actual threat for the building integrity following the notional removal of a load-bearing member, and they should always be accounted for during the building design.

1.2 Objectives and outline of Part I

Part I of this dissertation presents the development of an analytical framework to assess the quasi-static robustness of steel framed buildings under the notion of a removed column. This framework addresses the long-standing need in the field of disproportionate collapse for an analytical methodology with both a system-level and a member-level focus. The correlation between the column removal location and the ensuing collapse mechanism is thoroughly investigated, and it is demonstrated that methodologies with a system-level aspect are crucial to correctly estimate the building robustness against any column removal scenario. The novelty of the method lies in the utilization of a series of linear elastic analyses and closed-form expressions to describe the highly nonlinear performance of the damaged building, yielding as output the capacity of the structure as well as key features of the damage propagation path (sequence of beam plastic hinges or gravity connection failures, gradual degradation of the system stiffness, etc.). The proposed methodology is applicable to: a) steel moment-resisting frames under any column loss scenario, and b) steel and concrete composite gravity-framed systems under any interior gravity column removal. The framework takes into consideration stability phenomena, and in view of the context
of Section 1.1.5, the term stability refers to short-wave column instabilities.

The first part of the dissertation is based on previous work by the author in the field of robustness of buildings under disproportionate collapse [1, 2, 34–41]. Part I is organized as follows. Section 2 demonstrates the core principles and assumptions of the proposed methodology, and presents the derivation of the closed-form expressions. For the case of moment resisting frames, new disproportionate collapse curves termed as Euler-type curves are introduced, and the correlation between the traditional Euler curve for a compressed member (column) and the Euler-type curve for a compressed structural system (moment frame) is evidently illustrated. Section 3 includes the application of the analytical framework in 2D and 3D structural idealizations of prototype buildings. Section 4 demonstrates the numerical validation of the proposed method using a general purpose finite element software. The results from the analytical investigation are compared to the numerical findings, demonstrating the very good agreement between the two approaches. Finally, Section 5 summarizes the research accomplishments of Part I, presents the conclusions from the investigation, and provides suggestions for future research directions in the field of disproportionate collapse.
CHAPTER 2

ANALYTICAL FRAMEWORK ON THE ROBUSTNESS OF BUILDINGS WITH A COLUMN REMOVAL

2.1 2D steel moment-resisting framed structures (MRFs)

2.1.1 Yielding-type collapse mechanism for 2D MRFs

The first collapse mechanism which is incorporated in the proposed method is denoted as \textit{yielding-type collapse mechanism}. It is a ductile collapse mode and is manifested through the sequential formulation of plastic hinges as the gravity load is quasi-statically incremented in the structure. These hinges are formed at both ends of the beams which belong to the bays immediately adjacent to the removed column and at all floors above it. This region is termed as \textit{yielding-affected area} and it is depicted in Figure 2.1b. Essentially, as shown in Figure 2.1, the yielding-type collapse mode is activated when the number and location of the plastic hinges in the yielding-affected area are enough for the formation of a kinematic chain. The term \textit{plastic hinge} is used here to describe the flexural failure of a beam when the acting moment on this member reaches the member’s plastic capacity. In reality, due to the effect of hardening a plastic hinge never strictly occurs, but this assumption is considered well accepted in the field of disproportionate collapse and it is used for the purposes of the method.

To assess the sequence of the plastic hinge formulation and the capacity of the steel moment frame under a column loss scenario, the analytical method requires 2
Figure 2.1: Column removal and yielding-affected area in a typical MRF. 

a. 2D steel moment-resisting frame with an interior column removal scenario.  
b. The yielding-type mode is triggered when the number of plastic hinges suffices for the formation of a kinematic chain within the yielding-affected area (enclosed by the dashed line) 

independent linear elastic analyses. The first analysis is conducted on the entire damaged frame, where the column has been removed. The second analysis is conducted in the yielding-affected area. The variables of interest are the moments which are generated at the beams ends within the yielding-affected area in each analysis, which are then used as input in the proposed framework to calculate the yielding-type collapse load.

The analytical approach for the yielding-type mode is presented with the aid of the structure depicted in Figure 2.2. An interior column removal scenario is assumed to occur at the upper part of this multi-storey frame, with 3 beam series above the removal. It is noted that the method is developed in such a way that a generalization to any number of beams above the removal is feasible, accounting for interior, next-to-exterior and exterior column removal scenarios. Let us define the following terminology:
Figure 2.2: Structural idealizations of MRF incorporated to the 2D analytical framework. Example of a prototype structure with three beams above the removed column. 

**a.** Structure A: the structural system incorporated in the first linear elastic analysis (frame with column removal). 

**b.** Structure B: the structural system utilized in the second linear elastic analysis (yielding-affected area with hinges at the $k$ ends). 

**c.** Nomenclature of beam notation for this column removal scenario.

- Within the yielding-affected area, $k$ ends refer to the beams ends that abut the remaining structure and $m$ ends refer to the beams ends towards the midspan.

- $J$ is the set of all the beams that belong to the yielding-affected area, and $j$ is the indicator of each beam of the set.

- $SJ$ is the set of the strongest beams (maximum flexural capacity) in the yielding-affected area and $S_j$ a beam that belongs to the latter set.

The flexural capacity of a beam ($M_{j,\text{cap}}$) is defined as follows:

$$M_{j,\text{cap}} = w_j \times f_y$$

where $w_j$ is the plastic modulus of the section and $f_y$ is the material yield stress. Equation 1 provides the moment at which the entire section has reached the material yield stress and a plastic hinge has been formed.
Of immediate interest for the proposed method are the moments generated at both the $k$ and $m$ ends in the yielding affected area. A schematic evolution of the moments is depicted in Figure 2.3a-e, using the structural configuration of Figure 2.2. In these plots the horizontal axis represents the gravity load acting on the beams, and the vertical axis represents the moments generated on the beams ends. The area in the graphs below the horizontal axis shows the behavior of the beams at their $k$ ends (negative moments), and the area above the horizontal axis shows the $m$ ends behavior (positive moments). Therefore, each beam series in the yielding-affected area is represented by 4 lines, 2 $k$ lines and 2 $m$ lines. The steps for the analytical treatment of the yielding-type mechanism are presented below:

**Step 1:** The first step involves the application of an arbitrarily small load $q_{el,A}$ on Structure A (frame with removed column). This load is small enough that the structure remains in the linear elastic zone. This is Analysis A, from which the moments acting on each beam end are obtained $M^k_{j,A}$ for $k$ lines and $M^m_{j,A}$ for $m$ lines. The moment demand is always greater at the $k$ ends, therefore it is expected that the plastic hinges will first form there.

**Step 2:** Based on the superposition principle and the flexural capacity of each beam $M_{j,cap}$, the load that each beam requires to form a plastic hinge at the $k$ end ($q_{j,k}$) based on a linear estimation can be calculated. The maximum value of these loads is termed $q_k$ and defines the load at which all the beams have formed plastic hinges at their $k$ ends. The values of $q_{j,k}$ and $q_k$ are calculated as follows:

$$q_{j,k} = \frac{M_{j,cap}}{M^k_{j,A}} \times q_{el,A} \tag{2}$$
Figure 2.3: a-e. Schematic representation of the moment vs gravity load diagrams for the yielding-type collapse mode assessment in a prototype MRF with three beams above the removal.
\[ q_k = \max_{j \in J}(q_{j,k}) \]  

**Step 3:** For the load value \( q_k \) and based on the principle of superposition, the \( m \) ends moments can be calculated based on the following equation:

\[ M_{m,j,k} = \frac{q_k}{q_{el,A}} \times M_{m,j,A} \]  

**Step 4:** For load values beyond \( q_k \), the proposed method considers the moments at the \( k \) ends to remain almost constant, i.e. there is no additional moment at these locations for \( q > q_k \). This essentially implies that for load values \( q > q_k \) the structural system has been transformed into a system with plastic hinges at the beam \( k \) ends, and now a load redistribution takes place which increases the moments at the \( m \) ends as the load \( q \) is quasi-statically increased. The effect of this load redistribution captured by the second linear elastic analysis of the method (Analysis B). This analysis is conducted in the yielding-affected area, shown as Structure B in Figure 2.2, which is now isolated from the surrounding frame through hinges at the \( k \) ends. An arbitrarily small load \( q_{el,B} \) is applied on the system and the moments acting on each beam \( m \) end (\( M_{m,j,B} \)) are obtained.

**Step 5:** Based on superposition and the flexural capacity of each beam \( M_{j,cap} \), the additional load that each beam requires to form a plastic hinge at the \( m \) end (\( \Delta q_j \)) can be calculated. The maximum value of these loads is termed \( \Delta q_m \); this load value is added to \( q_k \) and provides the limit state load value \( q_m \), at which the beams have formed a kinematic chain and the yielding-type collapse mechanism is activated. These variables are calculated as follows:
\[ \Delta q_j = \frac{M_{j,\text{cap}} - M_{j,k}^m}{M_{j,B}^m} \times q_{el,B} \] 

\[ \Delta q_m = \max_{j \in J}(\Delta q_j) \] 

\[ q_m = q_k + \Delta q_m \] 

When the yielding affected area comprises a small number of beams, namely 3-4, this approach yields reasonably accurate results. However, as the number of the beams increases, the application of the superposition principle yields unconservative results and can significantly overestimate the actual frame collapse load. In fact, the capacity of Structure B is mainly determined by the capacity of the strongest beam, rather than all of the beams in the yielding-affected area; typically this is the first beam which lies directly above the column removal. One can reasonably think of this as follows: once the strongest beam - which according to the stiffness matrix formation has attracted the majority of the load - fails, the load that is already applied to the structure is redistributed to the weaker beams. These beams are expected to form plastic hinges at their \( m \) ends shortly after the failure of the strongest beam and definitely earlier than their initial linear projection. Thus, the method considers that the critical load at which the yielding-type mechanism is activated is the load at which the strongest beam forms the plastic hinges at its \( m \) ends. In case there are more than one beams sharing the strongest section, the first to form the \( m \) plastic hinges is assumed to be the critical. All the above lead to a modification of \( \Delta q_m \) as follows:
\[ \Delta q_m = \min_{S_j \in S_J} \left( \frac{M_{S_j,\text{cap}} - M_{S_j,k}^m}{M_{S_j,B}^m} \times q_{el,B} \right) \]  

(8)

The yielding-type collapse load \( q_m \) can be defined now as a yielding-type function \( C_b(\alpha) \), where the variable \( \alpha \) is the column removal location. The method is formulated in such a way as to include all possible column removal locations. Based on Equations 1 - 8, the yielding-type function obtains the following explicit form (see Appendix A for the step-by-step derivation of Equation 9):

\[
C_b(a) = f_y \times \left[ \max_{j \in J} \left( \frac{w_j}{M_{j,A}^k(a)} \times q_{el,A} \right) \right. \\
+ \left. \min_{S_j \in S_J} \left( \frac{w_{S_j} - M_{S_j,A}^m(a) \times \max_{j \in J} \left( \frac{w_j}{M_{j,A}^k(a)} \right)}{M_{S_j,B}^m(a)} \times q_{el,B} \right) \right] 
\]

(9)

It needs to be noted here that depending on the building design and in cases with many beams above the removal, it is possible that the strongest beam forms its \( m \) plastic hinges before some beams of the upper floors form their \( k \) plastic hinges. In other words, the load at which the strongest beam forms its \( m \) plastic hinge \( \left( \min_{S_j \in S_J} (q_{S_j}^m) \right) \) is less than the load at which the last \( k \) plastic hinge is formed \( (q_k) \). In this case, \( q_k \) is considered as the limit state load value at which the yielding-type mode is activated. For the case of \( \min_{S_j \in S_J} (q_{S_j}^m) < q_k \) the yielding-type function has the following form:

\[
C_b(a) = f_y \times \left[ \max_{j \in J} \left( \frac{w_j}{M_{j,A}^k(a)} \times q_{el,A} \right) \right] 
\]

(10)
2.1.2 Stability collapse mechanism for 2D MRFs

The stability collapse mode is governed by the buckling failure of one of the remaining columns of the steel frame, following the initial column removal. The critical failure load $P_{RC}$ for various types of columns is defined in Table 2.1, where $A_c$ is the column’s cross section, $I_c$ is the moment of inertia, $E$ is the modulus of elasticity, $f_y$ is the yield stress and $f_u$ is the ultimate stress of the material. The first column which exhausts its capacity-to-demand ratio is the critical component for the stability mechanism. Let us introduce here some useful notation definitions: let $C$ be the set of all the columns of the structure and $c$ be the indicator of each column of the set.

For the stability collapse mode, the axial force of each column ($P_{c.A}$) can be obtained through Analysis A. Based on the principle of superposition and the axial capacity of each column $P_{RC}$, the load of each column failure $q_c$ can be calculated. The minimum value of these loads is termed $q_{stability}$ and it is the limit state load value for the activation of this mechanism. Therefore, the expressions for the stability collapse mode are:

$$q_c = \frac{P_{RC}}{P_{c.A}} \times q_{el,A} \quad (11)$$

$$q_{stability} = \min_{c \in C} (q_c) \quad (12)$$

Similar to the yielding-type case, the stability collapse load can be defined as a stability function $C_c(\alpha)$, where the variable $\alpha$ is the column removal location. This function has the following form:
\[ C_c(a) = \min_{c \in C} \left( \frac{P_{Re}}{P_{c,A}(a)} \times q_{el,A} \right) \] (13)

It is important to note that the method assumes a linear increase of the column axial forces even for load values \( q > q_k \). In other words, the moment redistribution after the formation of plastic hinges at the \( k \) ends does not affect significantly the acting axial forces in the moment frame columns. The validity of this assumption is extensively discussed in the numerical validation of the method (Section 4).

2.1.3 Disproportionate collapse limit state function for 2D MRFs

The method intends to identify the collapse mechanism which is triggered first, therefore the comparison between the two collapse loads appears straightforward. Apparently, the minimum of the two is the critical load value. A new disproportionate collapse function is introduced here, given as the ratio of the stability over the yielding-type function:

\[ R(a) = \frac{C_c(a)}{C_b(a)} \] (14)

Values of \( R < 1 \) indicate that stability governs the frame collapse, while \( R > 1 \) implies the dominance of the yielding-type mode. In the present framework, where

<table>
<thead>
<tr>
<th>Column type</th>
<th>Condition</th>
<th>Critical load ( P_{Re} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Slender</td>
<td>( P_{Euler} &lt; A_c \times f_y )</td>
<td>( P_{Euler} = \frac{\pi^2 E \times I_c}{(k \times H_c)^2} )</td>
</tr>
<tr>
<td>Intermediate</td>
<td>( P_{Euler} &gt; A_c \times f_y )</td>
<td>( A_c \times f_y )</td>
</tr>
<tr>
<td>Stocky</td>
<td>( P_{Euler} &gt; A_c \times f_y )</td>
<td>( A_c \times f_u )</td>
</tr>
</tbody>
</table>

Table 2.1: List of critical load formulas for slender, intermediate-slenderness and stocky columns
damage is introduced in a discrete manner (complete column removal), $C_c$ and $C_b$ - and therefore $R$ - obtain a specific value. As a result, $R$ degenerates from a generic continuous function to a series of values, and it is therefore conceptually more straightforward to re-express it in a simpler fashion, namely as a factor. It is termed ductility limit state factor and it is denoted as $\mu$. It is a limit state factor since it distinguishes two different limit states, the yielding-type collapse mode and the stability collapse mode. Its critical value is termed as critical ductility limit state factor, it is denoted as $\mu_{\text{critical}}$ and it equals unity, representing from an analytical standpoint the case where the two collapse mechanisms are assumed to occur simultaneously.

Another comment can be made here regarding the rationale behind including the term ductility in the definition of $\mu$. It is widely accepted that ductile failure modes are far more desired than brittle ones, since they are not sudden and energy can be dissipated through excessive deformations. Research on the field of disproportionate collapse has been mainly focusing on the behavior of the building connections, attempting to provide enough ductility to the system to resist the large deflections imposed by the loading domain. A side intention of this work is to raise concern regarding the amount of ductility that should be provided to the system. This implies that the ductile beam-grillage design should not drive damage propagation into columns, or in other words, the stability of all the surrounding load-bearing components has always to be ensured. Therefore, an upper bound of ductility should be considered for the collapse mechanism not to switch to the undesired stability one, and for this reason the terminology critical ductility was adopted in the definition of $\mu_{\text{critical}}$.

For the case of $\min_{S_j \in S_J} \left( q_{S_j}^m \right) > q_k$, substituting Equations 9, 13 into Equation 14 provides the ductility limit state factor $\mu$: 

23
\[
\mu = \frac{1}{f_y} \times \min_{c \in C} \left( \frac{P_{re}}{P_{c,A}^{(a)}} \times q_{el,A} \right)
\]

\[
\times \max_{j \in J} \left( \frac{w_j}{M_{j,A}^{(a)}} \times q_{el,A} \right) + \min_{S_j \in SJ} \left( \frac{w_{S_j} - M_{S_j,A}^{(a)} \times q_{el,A}}{M_{S_j,B}^{(a)}} \times q_{el,B} \right)
\]  

(15)

For the case of \( \min_{S_j \in SJ} (q_{S_j}^{m}) < q_k \), substituting Equations 10, 13 into Equation 14 provides the alternative expression of the ductility limit state factor \( \mu \):

\[
\mu = \frac{1}{f_y} \times \min_{c \in C} \left( \frac{P_{re}}{P_{c,A}^{(a)}} \times q_{el,A} \right)
\]

\[
\times \max_{j \in J} \left( \frac{w_j}{M_{j,A}^{(a)}} \times q_{el,A} \right)
\]  

(16)

These expressions constitute the core of the proposed methodology for 2D steel moment-resisting frames. Each parameter on the right hand side of Equations 15, 16 is either known by the geometric and material properties of the structure \((f_y, w_j, w_{S_j}, P_{re})\) or can be obtained from the two linear elastic analyses \((M_{j,A}^{k}, M_{S_j,A}^{m}, M_{S_j,B}^{m}, P_{c,A})\).

These parameters are plugged into the closed-form expressions above and yield the value for \( \mu \), which explicitly estimates the governing collapse mechanism. The method provides information about the behavior of the structure both in the global level (overall collapse mechanism) and in the local level (behavior of columns and beams). Since the capacity-to-demand ratio of every column is a prerequisite for the calculation of the ductility limit state factor, the method detects the location of the buckled column when loss-of-stability is the failure mode. If a particular column is found to be the critical member for many removal scenarios, retrofitting this specific column can improve the response of the whole structure against many disproportionate collapse scenarios; therefore vulnerable areas of the structure can be identified. The method also calculates the capacity-to-demand ratio for all the beams, providing the sequence
of the plastic hinge formation inside the yielding-affected area. Therefore, the method provides ample insight on the load distribution within the structural system until the point of imminent collapse.

2.1.4 Euler-type curves for 2D MRFs

A more detailed observation of Equation 15 reveals another very interesting aspect of the proposed method. The distinction between the two specific collapse mechanisms resembles the well-known distinction of failure modes associated with a single member in compression. A column subjected to compressive stresses will either lose its stability under a critical load (buckling), or it will fail due to material yield stress exceedance (squashing). This well-known behavior is depicted in Figure 2.4a, where the vertical axis is the ratio of the axial stress acting on the column over the material yield stress and the horizontal axis is the slenderness of the column. Elastic buckling is the governing collapse mode of slender columns, material failure describes the failure of stocky columns and inelastic buckling applies in intermediate ones.

Moving from the component-level to the structure system-level, the method utilizes the results of Equation 15 and introduces new Euler-type curves for the moment-framed structure under the notion of a column removal. Previous work by Gerasimidis [42] indicated that column removals at the lower parts of the frames typically led to stability collapse modes, whereas column removals at the upper part of the frames typically resulted to yielding-type collapse modes. This observation signified a clear and strong correlation between the column removal location and the governing collapse mechanism. Since the analytical method has analytically treated each collapse mechanism, the introduction of the new Euler-type disproportionate collapse curve is straightforward.

The new Euler-type curve is presented in Figure 2.4b. The ductility limit state
Figure 2.4: Euler curve for a column vs Euler-type curve for an MRF.  

a. Euler curve for a column: slender members buckle vs stocky components which exceed their material yield stress.  

b. Euler-type curve for a 2D MRF: as the number of floors above the removed column increases, the collapse mode of the frame shifts from the ductile yielding-type to the brittle stability mode.
factor $\mu$ is plotted in the vertical axis of this graph. The variable plotted in the horizontal axis is the floors above the removal and it is represented by $F$, essentially defining the column removal location. Under a column removal scenario in the upper floors, the frame is expected to collapse due to the yielding-type mechanism ($\mu > 1$), while under a column removal in the lower floors stability is expected to be more critical ($\mu < 1$). Thus, if the $\mu$ values are plotted for column removals along a specific column row of the frame, the graph takes the form of Figure 2.4b and makes a clear graphical distinction between the two collapse modes. For example, if the interior column removals of a 18-story building are of interest, as shown in the inset images in Figure 2.4b, then a) the interior columns of each of the floors should be removed in turn separately, b) the $\mu$ value for each column loss scenario has to be calculated, and then c) the plot of these values formulates the Euler-type curve for this building and the specific column row. This graphical representation is instrumental to obtain at a negligible computational cost and time a sufficiently clear picture of the entire multi-story frame response, which would otherwise necessitate tedious numerical idealizations and substantially increased computational resources.

Apart from the similar collapse modes, another significant resemblance between the new Euler-type curve and the Euler curve has to be highlighted. The Euler curve depends only on the geometry and material properties of the single compressed column, which are the cross section area, the moment of inertia and the boundary conditions. Similarly, the new Euler-type curve depends only on the geometric and material characteristics of the structure. In a member-level, these variables define the response of the structural components individually (flexural capacity of each beam, axial capacity of each column). In the system-level, all the structural members are assembled and the load is distributed among the beams and the columns based on their stiffness. This is why $M^k_{j,A}$, $M^m_{Sj,A}$, $M^m_{Sj,B}$, $P_{c,A}$ are necessarily included in the
calculation of $\mu$, functioning as indicators of how the global stiffness matrix is initially formed (the column is removed) and eventually evolved (when plastic hinges have been formed at the $k$ ends). The application of the proposed method in Section 3 illustrate with further clarity and sufficient detail the analytical work presented in this section, and all the method assumptions are validated and extensively discussed.

2.2 3D steel and concrete composite gravity framed systems (GFSs)

2.2.1 Mechanical systems overview and details

This section demonstrates the extension of the proposed methodology to the 3D space, tailoring its implementation for steel and concrete composite gravity framed systems. Such vertical load framing systems comprise the gravity columns, the main girders, the secondary gravity beams (perpendicular to the girders, typically two secondary beams exist within each bay), rotationally pinned gravity connections and a concrete slab spanning rectangular bays. This structural system is widely employed in current design practices and a typical plan view of this layout is seen in Figure 2.5. The method has been developed to address the case of an interior gravity column being removed from the system, while the slab is continuous on the boundaries of the region which is immediately adjacent to the removed column. Henceforth, this region is termed as $2x2$ area, since it spans two bays in each direction around the removed column. It is indicated by the hatched area of Figure 2.5, assuming a column removal scenario at the C3 location. The system in Figure 2.5 has the moment frames at the building perimeter, however the method is also applicable on structural systems with a different placement of the moment frames as long as the removed column and the immediately surrounding columns belong to the gravity system. The
Figure 2.5: Plan view of the prototype structural system investigated in the proposed 3D analytical framework. The gravity system comprises gravity girders and beams, shear connections and gravity columns.

Structural components of prominent importance in the 3D space are the gravity (or shear) connections, the slab steel components (wire mesh, steel deck) and the building gravity columns. An overview of their performance characteristics is presented before the analytical interpretation of the collapse modes is developed.

The shear connections of the beam grillage are considered a highly vulnerable component of the gravity floor system. These connections are intended to transfer only shear forces, however in a disproportionate collapse scenario they are exposed to additional significant axial demands towards which they have not been designed. Shear
connection types which are commonly used in practice include shear-tab, welded-bolted or bolted-bolted single-angle and double-angle connections, etc. Each connection type has different stiffness, strength and ductility characteristics and therefore a generalization over all the different connection types can not be easily drawn. In this framework, the method assumes bolted-bolted double-angle connections; however, it is also applicable in cases where the gravity connections is of a different type but shares similar mechanical response features with this connection configuration. These features are the following: a) the gravity connections are assumed rotationally free, so that they do not transfer bending moments, b) they are assumed to be rigid with respect to the shear forces, and c) they have a bilinear axial force-displacement diagram, with the post-peak branch of the tensile force-displacement behavior following an immediate drop to zero forces, while a perfectly-plastic post-peak branch is followed for the compressive force-displacement behavior. The assumption regarding the tensile force-displacement behavior implies that a brittle failure mode governs the response of the double-angle once the tensile capacity is attained, and it is based on experimental findings by Oosterhof and Driver [22]. The experimental outcome of this project revealed that the failure mode of the bolted-bolted double-angle connections under loading conditions representative of a disproportionate collapse scenario was dictated by the propagation of a tear along the double angle, which in some cases was manifested in a sudden, brittle manner. Therefore, an immediate drop to zero force values was adopted for the tensile force-displacement diagram of the connections, as it was also suggested in their follow-up paper [43]. The bilinear perfectly-plastic behavior of the compressive force-displacement response of the gravity connection is also adopted in other studies, such as the one by Foley et al. [44] (bolted-bolted double-angle connections) and by Sadek et al. [45] (single-plate shear connections). Additionally, the axial capacity of the connections is significantly less than the axial capacity of the
gravity beams or girders. Therefore, the gravity beams are not expected to experience failure prior to their respectively attached tensile connections, rendering the tensile connections the weak link of the beam grillage (a similar assumption is adopted in the design-oriented model by Alashker and El-Tawil [30]).

Apart from the connections, an additional source of the gravity floor robustness stems from the slab steel components. These are the wire mesh reinforcement, which is located close to the top slab fiber acting in both directions and the steel deck, which is located at the bottom slab fiber and it is assumed to act only in one direction, parallel to the flutes (along the y-axis in Figure 2.5). The contribution of the steel deck perpendicular to the deck ribs is considered negligible. A bilinear elastic-perfectly plastic stress-strain relationship is adopted for the steel of the wire mesh and the deck. Finally, as in the 2D framework, the capability of the building gravity columns to withstand the load redistribution following the column removal is thoroughly examined, and as it is shown in Section 3, the number of floors above the removal is strongly related to the appearance of column instabilities.

### 2.2.2 Yielding-type collapse mechanism for 3D steel and concrete composite GFSs

The mechanics of the yielding-type mode are described herein using the prototype structure of Figure 2.5 and column C3 as the removed component. The capacity of the entire gravity floor is dictated by the capacity of the 2x2 area. For the sake of simplicity, the structure of Figure 2.5 is treated as a single-story building, and a generalization for more floors above the removal is given at the end of this section.

During the initial stages of loading, flexural (or composite) action governs the behavior in the 2x2 area. The connections at the perimeter supports are subjected to compression, while the remaining connections within the 2x2 area are subjected
to tensile forces. The connections located at the 2x2 area center (immediately above the removed column) are apparently subjected to the maximum tensile demand, compared to the remaining tensile connections. As the load increases, the wire reinforcement located at the upper slab fiber in the 2x2 area perimeter supports yields. This triggers a load redistribution within the system and additional flexural demands are generated towards the 2x2 area center. Eventually, the gravity connections at the 2x2 area center reach their maximum capacity in tension and fail, causing a load redistribution among the gravity beams. This load redistribution along with the continuous applied load increase, results to the failure of the immediately adjacent tensile gravity connections, triggering a new load redistribution among the remaining gravity beams. As the shear connections sequentially fail, another mechanism gradually emerges and becomes the primary load carrying mechanism. This mechanism is associated with large vertical deflections and it is known as membrane or catenary action. At this stage the slab steel components are fully activated in order to transfer the gravity load to the perimeter columns, through the development of significant membrane forces.

The method implementation requires a simple model of a single-story 2x2 area. Its structural configuration along with the adopted nomenclature are shown in Figure 2.6. This model is subjected to a series of elastic analyses, and the modeling assumptions of the 2x2 area model are:

- The gravity beams are modeled using 1st order beam elements, they are placed along their centerline, and they are assigned only elastic material properties (modulus of elasticity and Poisson’s ratio).

- The slab is assumed to have a constant depth and it is modeled with 4-node, reduced integration shell elements. The slab nodes are merged with the gravity beam nodes to ensure composite action (no slippage is allowed) and then the
Figure 2.6: Plan view of the 2x2 area structural idealization. The connections denoted in color are exposed to tensile demands and their response largely determines the structural behavior of the system.

The slab is offset upwards, at a distance so that the slab bottom fiber coincides with the upper fiber of the girder. The slab is assigned the concrete modulus of elasticity. The steel deck is simulated as a rebar at the bottom slab fiber, acting only in one direction.

- The connections are modeled with 1\textsuperscript{st} order beam elements and they are assigned the desired axial stiffness and a negligibly small moment of inertia, to achieve that moments are not transferred through these elements.

- The horizontal displacements in faces A and C along the x axis and in faces 1 and 7 along the y axis (Figure 2.6) are restrained. This assumption aims at representing the slab continuity and the restriction which comes from the
adjacent bays in the case of an interior gravity column loss.

- The columns at the perimeter of the 2x2 area are excluded from the model. Instead, the vertical (out-of-plane) displacement of the nodes referring to the columns location is restrained.

The yielding-type mode analytical interpretation is presented with the aid of two diagrams. The first is the $q$-$\delta$ curve of the gravity system, where $q$ is the gravity load and $\delta$ is the vertical displacement of the point immediately above the removed column (Figure 2.7). The second diagram includes the graph of each connection tensile force versus the gravity load and it is shown in Figure 2.8. Since a perfectly plastic bilinear behavior is adopted for the connections under compression, these components are not considered as crucial for the structural response as the connections experiencing tension and a steep drop in their axial force. The simple model has double symmetry and thus the two C4 connections are assumed to follow the same behavior, depicted by a single curve in Figure 2.8a. The same principle applies respectively for the four C1, the four C2 and the two C3 connections. In order to develop these graphs, the method performs 7 elastic analyses in the simple model of Figure 2.6. The model is accordingly modified in each analysis to address the failure of the connections and the progressive spread of the wire mesh and steel deck yielding. Figure 2.7b depicts the modifications applied in the prototype model for each analysis in terms of the excluded connections. The hatched areas in Figure 2.7c represent the regions of the yielded wire and deck. Deck yielding is simulated by excluding it from the slab section in the hatched regions, while wire yielding is simulated by assigning a negligible small tensile strength value in the respective shell elements. This way a load distribution path is created in the simplified numerical model as if the elements which appertain in the hatched regions are incapable of attracting any additional load in tension, approximating the
Figure 2.7: Analytical approach and steps for the yielding-type mode assessment in GFSs. a. A typical $q$-$\delta$ curve of the structural system up to the point of imminent collapse. $q$ is the gravity load, $\delta$ is the vertical displacement of the 2x2 area middle point. b. The table lists the goal of each linear elastic analysis, as well as the tensile connections which are excluded from the model in each analysis. c. The regions where the slab steel components are taken to have reached their yield point.
case of their slab steel components having already yielded. Apparently, the slab steel components are still capable of providing significant resistance after yielding until they reach their fracture criterion. However, in the regions where steel has yielded and follows a perfectly plastic post-peak branch, this resistance is provided mainly through their deformation and not through an increase in their internal forces. Therefore, given the fact that the tensile force of these elements cannot be increased, the specific simplified modeling technique seems appropriate to approximate the load redistribution path.

The yielding-type collapse mode is organized and presented in the following three steps:

**Step 1**: The first components which will reach a yielding limit state are the wire and steel deck at the 2x2 area perimeter supports. This is represented by a thin hatched strip in the perimeter of the respective model for Analysis I in Figure 2.7c. The width of the hatched region can be taken as 5% of the 2x2 area span, though a parametric analysis where the width was ranged between 5%-10% of the 2x2 area span showed a very small dependence on this percentage. Analysis I is conducted on this model by applying a small elastic load and obtaining the corresponding vertical displacement and the tensile forces generated on the connections C1-C4. Connections C3 and C4 are in the middle of a rectangular slab and have the same stiffness, therefore their tensile demand will be approximately the same. For the sake of the method description, connections C4 are assumed to attract slightly higher load than C3. Based on elasticity and the connection capacity, the load at which C4 fails can be calculated. For this load, the demand generated on the C1, C2 and C3 as well as the corresponding vertical displacement can be calculated (points A in Figures 2.7 and 2.8).

Connections C4 failure has an effect on the gravity system, which manifests in
Figure 2.8: Analytical idealization of the connection tensile forces in GFSs. Tensile force vs gravity load diagrams for a. C4, b. C3, c. C2 and d. C1 gravity connections. Due to double symmetry, a single curve suffices to describe the response of each connection family. The vertical jumps in the plots signify the load redistribution which occurs within the system, due to the failure of a connection family.

two ways: it triggers a load redistribution among the remaining beams to maintain the already applied load, as well as an increase in the vertical displacement. In other words, the system seeks a new equilibrium position. Since the connection failure is assumed abrupt, the forces of the C4s drop immediately to zero and this new equilibrium position is found for the same amount of applied load. To obtain the new equilibrium position, Analysis II is conducted on the respective model of Figure 2.7. This model excludes the failed connections C4 and accommodates a small rectangular yielded region in the center (the width of this region can be taken as 5% – 10% of
the 2x2 area span). The stiffness of this model is less than before and for this reason line II in Figure 2.7a has a smaller slope than line I. At the same time, the demand generated in the remaining connections (Figure 2.8) is greater than before. For the case of this rectangular slab, the new load distribution will result to connections C3 immediate failure (point B lies above the capacity threshold in Figure 2.8b). To obtain the equilibrium position after the C3s have failed, Analysis III is conducted on the respective model of Figure 2.7. From this analysis the slopes of lines III in Figures 2.7 and 2.8 are obtained, and points C lie along these lines at the level of connections C3 and C4 failure load. Overall, the horizontal plateau between points A and C in Figure 2.7 reflects the influence of connections C3 and C4 failures in the vertical displacement, and the vertical lines connecting points A and C in Figures 2.8c and 2.8d represent the increase in the tensile force of the remaining connections (C1 and C2).

**Step 2:** The method sets at this point the onset of the membrane action and at this stage the two load-carrying mechanisms (composite and membrane action) are assumed to be acting simultaneously. Composite action is still present since the connections C2 and C1 are intact, but due to the C4s and C3s failure its impact in the system capacity is deteriorated. The membrane action is assumed to be partially activated and the slab steel components progressively yield. This is accounted similar as before, by defining regions in the 2x2 area where the wire and the steel deck have yielded. Apparently, the transition from the composite to the membrane action is gradual and not distinct, however for the sake of the method development discrete points regarding this progression need to be established.

Analysis IV is performed on the respective model of Figure 2.7. Connections C3 and C4 are excluded and the yielded area has a rectangular pattern spanning between the intact connections C2. Similar to before, a small elastic load is applied and the
corresponding vertical displacement and the tensile force demand in connections C2 and C1 are obtained. Based on a linear projection to the connection capacity, the failure load of connection C2 can be calculated. For this load, the demand generated in the C1s and the corresponding vertical displacement can be also calculated (points D in Figures 2.7 and 2.8). Failure of connections C2 signifies a horizontal plateau in Figure 2.7 and an immediate increase in the force of the remaining connections C1 in Figure 2.8. To obtain this new equilibrium position, Analysis V is conducted on the respective model of Figure 2.7. This analysis provides the slopes of lines V and ultimately points E in Figures 2.7 and 2.8, which lie along these lines at the C2s failure load level.

**Step 3:** Based on the same principles Analysis VI is conducted on the respective model of Figure 2.7, where the wire mesh and the deck have yielded everywhere in the 2x2 area and the failed connections C4, C3 and C2 are excluded. Based on Analysis VI and the principle of superposition, a linear projection to the connection capacity provides the load of connections C1 failure, for which the corresponding vertical displacement can be calculated (points F in Figures 2.7 and 2.8). The ending point of the multi-linear curve in Figure 2.7 is obtained from the last analysis. Analysis VII is conducted on a model where all the connections (C1-C4) are excluded and the slab steel components are assumed to have yielded everywhere in the 2x2 area. This analysis provides the slope of line VII in Figure 2.7, thus point G is located along this line while its ordinate is the failure load of connections C1.

Overall, this mechanism is considered to be the dominant collapse mode once the following failure criteria have been met: a) the tensile connections within the 2x2 area have reached their capacity and failed, with the influence of all these failures being incorporated in the $q$-$\delta$ curve, therefore the ending point of the curve in Figure 2.7 is point G and not point F and b) the wire mesh and the steel deck have yielded.
everywhere within this region. The above criteria intend to describe a state where the two load-resisting mechanisms have reached their maximum capacity and beyond this point the totally applied load can be increased only at a negligible rate in a quasi-static framework. The analytical expressions for the yielding-type collapse load $C_{bl}$ and the corresponding total displacement $C_{bd}$ are respectively (see Table 2.2 for notation and Appendix B for a step-by-step derivation of Equation 17):

$$C_{bl} = C_{\text{max}} \times q_{el} \times \left[ \frac{1}{C_{4,OI}} + \frac{1}{C_{2,OIV}} \right]$$

$$C_{bd} = \delta_{VII}$$

So far the structure of Figure 2.5 has been treated as a single-story building and a generalization for more stories above the removal is required. Regarding the yielding-type mode, the method assumes that the response of a multi-story building subjected to column removals at different floors and along the same column row is expected to be approximately the same. This assumption is justified a) from the repetitive pattern of the gravity beam grillage layout, which encompasses the same gravity beams and connections at each floor, and b) from the pinned nature of the shear connections. Small differences are expected as the response of different floor column loss scenarios are investigated, nevertheless as it will be shown in Section 3 they are not considered to be of critical importance. Therefore, the two diagrams created by the single story 2x2 area model are representative of an interior gravity column removal at any floor in a steel framed and concrete composite multi-story building.

It is important to mention that a more realistic representation of the actual collapse of the building due to the yielding-type mode would require additional response
<table>
<thead>
<tr>
<th>Analysis</th>
<th>List of Variables</th>
</tr>
</thead>
</table>
| I        | Obtain $\delta_{OI}$: displacement from Analysis I  
           | $C_{4,OI}$, $C_{3,OI}$, $C_{2,OI}$, $C_{1,OI}$: connection forces from Analysis I  
           | $q_4$: connection C4 failure load  
           | Calculate $\delta_I$: displacement from Analysis I projection  
           | $C_{3,I}$, $C_{2,I}$, $C_{1,I}$: connection forces from Analysis I projection |
| II       | Obtain $\delta_{OII}$: displacement from Analysis II  
           | $C_{3,II}$, $C_{2,II}$, $C_{1,II}$: connection forces from Analysis II |
| III      | Obtain $\delta_{OIII}$: displacement from Analysis III  
           | $C_{2,III}$, $C_{1,III}$: connection forces from Analysis III  
           | Calculate $\delta_{III}$: displacement from Analysis III projection  
           | $C_{2,III}$, $C_{1,III}$: connection forces from Analysis III projection |
| IV       | Obtain $\delta_{OIV}$: displacement from Analysis IV  
           | $C_{2,IV}$, $C_{1,IV}$: connection forces from Analysis IV  
           | $q_2$: connection C2 failure load  
           | Calculate $\delta_{IV}$: displacement from Analysis IV projection  
           | $C_{1,IV}$: connection forces from Analysis IV projection |
| V        | Obtain $\delta_{OV}$: displacement from Analysis V  
           | $C_{1,OV}$: connection forces from Analysis V  
           | Calculate $\delta_V$: displacement from Analysis V projection  
           | $C_{1,V}$: connection forces from Analysis V projection |
| VI       | Obtain $\delta_{OVI}$: displacement from Analysis VI  
           | $C_{1,OVI}$: connection forces from Analysis VI  
           | $q_1$: connection C1 failure load  
           | Calculate $\delta_{VI}$: displacement from Analysis VI projection  
           | $\delta_{VI}$: displacement from Analysis VI projection |
| VII      | Obtain $\delta_{OVI}$: displacement from Analysis VII  
           | Calculate $\delta_{VII}$: displacement from Analysis VII projection |

Table 2.2: Obtained and calculated variables (displacements and connections tensile forces) for each analysis in the 3D yielding-type mechanism assessment.
features to be addressed. In particular, specific failure criteria regarding the fracture
of the wire mesh and the steel deck should be specified, to provide a more detailed
description of the gravity system response as it reaches the last stages of loading and
the slab steel components are closer to rupture. A relevant criterion was proposed by
Park [46] and it was adopted by Alashker and El-Tawil [30], according to which the
slab steel components reach their fracture elongation when the vertical displacement
at the slab center is equal to 10% of the shortest span length. The development
and incorporation of such empirical failure criteria is considered an essential future
enhancement of the method, but they are currently considered beyond the point of
the presented framework.

2.2.3 Stability collapse mechanism for 3D steel and concrete composite
GFSs

The stability mode is treated in a similar fashion as in the 2D case. A small gravity
load is applied to the building such as the structure remains in the elastic realm, and
the corresponding axial force of each column is obtained (depicted with the magenta
dots in Figure 2.9). This step can be accomplished in two ways: a) with the aid of
the gravity columns tributary areas, as shown in Figure 2.10a, or b) with an elastic
analysis on the entire building (Figure 2.10b), which should be modeled based on the
simplified modeling assumptions described in Section 2.2.2. The first option can be
executed based on hand calculations and it is very easy and quick in its application.
However, it has the disadvantage that the symmetrically arranged columns around
the removal have precisely the same demand, rendering difficult to predict which
one of these is actually the most critical. The second approach is only slightly more
accurate, however it is more time-consuming since it requires the development of a
building-level model.
Figure 2.9: Analytical method approach for the stability collapse mode in the 3D framework. A linear elastic analysis indicates the force demands in the building columns, and the column which exhausts first its utilization ratio, based on a linear projection to the member capacity, is the critical one.

Contrary to the yielding-type mode, the stability collapse load depends on the floor where the removed column is located. Based on the principle of superposition and the capacity of each column, the failure load of each column can be calculated (denoted as $q_c$ and shown in white circle dots in Figure 2.9). The capacity of each column is the minimum of the theoretical elastic and inelastic buckling capacity under purely axial load. The column which exhausts first its utilization ratio is the most critical one and the load at which this column buckles is termed as $q_{\text{stability}}$ (denoted with the filled red dot in Figure 2.9). Therefore, a new stability function can be defined:

$$C_c(a) = \min_{c \in C} \left( \frac{\beta \times P_{Re}}{P_c(a)} \times q_{el} \right)$$  \hspace{1cm} (19)
Figure 2.10: Approaches to obtain the column axial forces in GFSs. The axial force of each building column can be calculated based either on a. the *tributary areas* approach, or b. a building analysis utilizing simplified modeling assumptions.

The notation used here is the same as in the 2D framework: $C_c$ is the stability function, $\alpha$ is the location of the removed column, $C$ is the set of all the columns in the building, $c$ is an indicator of each column of the set, $P_{Re}$ is the column axial capacity (minimum of the elastic and inelastic buckling capacity), $q_{el}$ is the small elastic load which is applied to the structure, $P_c$ is the axial force generated on each column for this elastic load and $\beta$ is a reduction factor. This factor is introduced to address a reduction from the theoretical buckling capacity under purely axial load due to the following reasons. When the gravity floor system has reached large vertical deformations, significant horizontal forces are expected to be acting on the face of the columns where the connections are attached. These forces produce in turn a horizontal displacement at the top node of the column, a deformation which can cause an eccentricity regarding the column axial force and therefore reduce the capacity from the nominal one ($P_{Re}$). The influence of this phenomenon is expected to become even more profound when the slab is not continuous outside the 2x2 area and there is no restriction in the horizontal displacement from the adjacent bays. This phenomenon
is mitigated in the case of an interior column loss, nevertheless it is still present. The second source of reduction stems from the presence of a limited but not negligible moment in the gravity columns. The origin of this moment is a couple of forces of opposite sign which is inevitably generated at the top face of the columns at each floor, even with the assumption of rotationally pinned connections. At large deformations, where membrane action has become the dominant load-carrying mechanism, the slab steel components have yielded in tension throughout the entire 2x2 area. On the contrary, the gravity connections at the 2x2 area perimeter supports remain into compression throughout the complete evolution of the phenomenon, creating therefore this couple of forces and generating a bending moment at the top face of the surrounding column. The interaction of the column axial load, the moment, the horizontal force and the subsequently generated horizontal displacement which introduces a vertical load eccentricity, reduces the column capacity from the theoretical one under purely axial load, towards which the gravity columns have been designed. This justifies the introduction of the reduction factor $\beta$ in the 3D framework, and an extensive discussion of this factor along with the numerical validation of the method in the 3D space, is provided in Section 4.
CHAPTER 3
APPLICATION OF THE ANALYTICAL FRAMEWORK

3.1 Method application in 2D moment-resisting frames

3.1.1 Mechanical description of the 2D prototype structures

The analytical method is applied in a series of frames extracted from the SAC-FEMA Steel Project [47]. The frames correspond to the 20-story buildings in the cities of Boston and Seattle. Figure 3.1a shows the side view of the analyzed frames and Figure 3.1b depicts the top view of the building layout. The frame is oriented along the North-South direction. Both pre-Northridge and post-Northridge designs have been considered, with the side intention to investigate the impact of seismic related design guidelines to the disproportionate collapse response of the frames. The resulting 4 frames were analyzed for all possible column removal scenarios and they can be described by the following cases:

- Case A: 20 interior column removal scenarios at each floor of row D in Figure 3.1a.

- Case B: 20 next-to-exterior column removal scenarios at each floor of row E in Figure 3.1a.

- Case C: 20 exterior column removal scenarios at each floor of row F in Figure 3.1a.

Since the frames are symmetric around the notional line in the midspan of column rows C and D in Figure 3.1a, the aforementioned cases describe all possible column
Figure 3.1: Elevation view and geometric details of the MRFs. **a.** Front view of the moment-resisting frames. The frames are extracted from the SAC FEMA project and correspond to the Boston and Seattle designs, accounting both for pre- and post-Northridge guidelines. **b.** Top view of the building, the MRF is located at the perimeter and it is oriented along the North-South direction. **c.** Table with the values of each floor height.

removal scenarios. Therefore 240 analyses were conducted, 60 for each model, and the 3 new Euler-type curves for each frame were generated. The notation $Ci-P-Ca-N$ is henceforth utilized to refer to a specific model, where:

- **Ci:** Name of the city (Boston, Seattle)

- **P:** Northridge guidelines (pre-, post-)

- **Ca:** Case index (interior, next-to-exterior, interior)

- **N:** Number of floor where the column is removed (1 to 20)
All beam and column members have an I-profile section, and Table contains a detailed list of the members sections. The material assigned to all structural members is steel with elastic properties, since the analyses conducted remain purely in the elastic region. The modulus of elasticity is \( E = 200 \text{GPa} \) and Poisson’s ratio is \( \nu = 0.3 \). The base nodes are considered pinned, and lateral support is also provided at the 2 basement floors to account for the presence of the ground. A uniform, vertical, downwards, arbitrarily small load is applied to all the beams, in accordance with the requirements of the method regarding the loading. According to the prevailing regulations [15, 16], the nominal gravity load in the yielding-affected area has to be multiplied by the Dynamic Increase Factor \( \Omega_N \). The latter accounts for the dynamic nature of the disproportionate collapse phenomenon when a static analysis is conducted, and it is defined as follows:

\[
\Omega_N = 1.08 + 0.76/\left(\frac{\theta_{pra}}{\theta_y} + 0.83\right)
\]

(20)

where:

\[
\theta_{pra} = 0.0337 - 0.00086 \times (h_{beam}/2)
\]

(21)

\[
\theta_y = \frac{Z_{pl} \times f_y \times L}{6 \times E \times I}
\]

(22)

where \( \theta_{pra} \) is the plastic rotation angle given in the acceptance criteria tables of ASCE 41 (Life Safety level) and \( \theta_y \) is the yield rotation angle of the beam, given in
Equation 5.1 of ASCE 41. In the equations above, $h_{beam}$ is the beam section height, $Z_{pl}$ is the plastic section modulus, $f_y$ is the material yield stress, $L$ is the length of the beam, $E$ is the modulus of elasticity and $I$ is the beam moment of inertia. $\Omega_N$ is calculated for each beam of the frame and for a given removal scenario, the value of $\Omega_N$ used in the analysis is the maximum of the values above the removal.

3.1.2 Euler-type curves of the investigated structures

This section presents the results of the analytical method. The new Euler-type curves for each frame being are depicted in Figures 3.2 - 3.5. The three lines of data represent the interior, next-to-exterior and exterior column removal scenarios. The horizontal axis is the number of floors above the column removal and the vertical axis is the ductility limit state factor $\mu$, as calculated from Equations 15 or 16 for each column removal scenario. The points that are located above the threshold of $\mu_{\text{critical}} = 1$ indicate yielding-type collapse and the points that are located below the threshold represent loss-of-stability failure.

Observation of these figures results to the following notes:

- As the column is removed from top to bottom, $\mu$ is indeed decreasing as expected. There are no multiple jumps over the threshold; the yielding-type mode alters to the stability mode after a certain number of floors and then all the failures are attributed to column buckling. This characteristic appears universally, across all frames and column rows analyzed.

- Out of the 240 cases analyzed, the method predicted 137 yielding-type failures and 103 stability failures. Stability collapse has dominated more than 40% of the cases, a finding which clearly highlights the strong correlation between disproportionate collapse and this collapse mechanism. This finding is of ex-
 exceptional importance, because it signifies the true potential of a collapse mode which has not yet attracted extensive interest from the research community. The results of this extensive investigation reveal that columns which are primarily located at the bottom part of a structure are prone to lose their stability if a neighboring column is suddenly lost, and clearly highlight the necessity of accounting for this key response during the building design.

- The stability mechanism requires a specific number of floors to appear. This
Figure 3.3: Euler-type curves for the 20-story Seattle-Pre-Northridge moment-resisting frame. The two collapse mechanisms govern almost the same number of column removal scenarios.

The number is dependent on the frame design and the case examined, therefore it is not consistent among the models. For example, in the Boston-Pre-Exterior case, loss of stability occurs from the 1st to the 14th floor, while in the Boston-Post-Interior case this mechanism is activated only in the first (bottom) 4 floors. This finding creates a vague picture between the frames bottom part (stability vulnerable), upper part (yielding-type vulnerable) and their limits. Additionally, it was found that in some stability governed cases the location of the buckled column was not at the same floor as the removed column, but a few floors be-
Figure 3.4: Euler-type curves for the 20-story Boston-Post-Northridge moment-resisting frame. The Northridge design guidelines have substantially improved the performance of the frame, in the sense that more cases are shifted towards the desired, ductile collapse mechanism compared to the frame response prior to the Northridge design modifications.

- The curves for the exterior column removal scenarios are mainly located under the interior and next-to-exterior curves in all models. This implies that more exterior cases are below $\mu_{\text{critical}} = 1$, which signifies that the structure is more susceptible to lose stability under a corner column removal scenario compared to previous designs.
Figure 3.5: Euler-type curves for the 20-story Seattle-Post-Northridge moment-resisting frame. Again, the Northridge guidelines have enhanced the frame response, though at a pace notably lower than in the Boston frames.

to an interior column removal at the same floor. The latter is attributed to the limited capability of the structure to redistribute evenly the loads on the vertical load-bearing members when an exterior column is removed. In the exterior series of removals, the next-to-exterior row is the column row that will mainly attract the load that was previously carried by the exterior row. In the interior series of removals, there are two adjacent column rows that contribute to the load redistribution; therefore, these columns are burdened more evenly and in general less than in the exterior case. The curves for the exterior column
removal scenarios lay above the interior ones only in the upper 1-3 floors of every model. The reason is that in a cantilever yielding-affected area comprised by few beams, the load required to form all the plastic hinges is smaller than the load for the equivalent interior case. Therefore, the yielding-type collapse load (denominator of $\mu$) significantly decreases and the value of $\mu$ is increased.

- The post-Northridge design improved the performance of the buildings, in terms of taking cases away from the stability (brittle) to the yielding-type (ductile) collapse mechanism. This characteristic is apparent in both the Boston and Seattle building designs. However, this observation needs to be treated with caution, due to the different nature of the loads that resulted in the Post-Northridge guidelines (earthquake - horizontal loading) and the ones investigated in the disproportionate collapse scenario (gravity - vertical loading). The generalization of the notion that the seismic design improves the robustness of a building under a disproportionate collapse scenario requires further investigation and validation, though the findings reported here constitute a clear indication of the seismic design beneficial impact against the disproportionate collapse building response.

3.2 Method application in 3D gravity framed systems

3.2.1 Mechanical description of the 3D prototype structure

For the purpose of the method implementation in a 3D steel and concrete composite gravity framed structure, the Boston Pre-Northridge 9-story benchmark building from SAC-FEMA [47] was selected. The plan view of the structure is shown in Figure 2.5, and the interior gravity column in location C3 is the removed component along this column row. Since FEMA focused on the seismic structural response and
did not provide sufficient details for the design of the building gravity system, the
building geometry adopted herein is in alignment with the modifications provided
in the report by Foley et al [44]. These modifications refer to: a) the sections of
the gravity beams (W24x68 as main girders and W18x35 as secondary beams), b)
the orientation of the gravity columns (depicted in Figure 2.5) and c) the geometric
characteristics of the shear connections.

A detailed description of the approach by Foley et al. with regards to the gravity
connections can be found in their report [44], however the salient points of their
method are briefly presented here for the sake of completeness. In this report, a
parametric study on a variety of bolted-bolted double angle connections (web cleats)
was conducted. The varying geometric parameters were the thickness of the double
angle, the number of the bolt rows and the gravity beam at which the connection is
attached. Each bolt of the gravity connection was treated as a separate element, and
following approaches recommended by Shen and Astaneh-Asl [48], Liu and Astaneh-
Asl [49] and Thornton [50], Foley et al. [44] generated the nonlinear tension and
compression response of every bolt-element. The bolt-elements were then assembled
to derive a bilinear moment-rotation and axial force-displacement behavior for each
double angle connection. This way, each connection could be replaced by a single
structural component having the corresponding new bilinear response characteristics,
an approach which meets the requirements of the proposed analytical methodology.

In this work, a specific connection geometry from the work of Foley and his col-
leagues was adopted for all the shear connections of the building, the geometric details
of which are depicted in Figure 3.6. The selected connection has the minimum angle
thickness of the ones that Foley et al. investigated and the ratios of the connection ax-
ial capacity to the gravity beams axial capacities are 0.1 and 0.2 for the W24x68 and
W18x35 gravity beams respectively. The following modifications were also applied
Figure 3.6: Geometric details and response features of gravity connections. **a.** Side view and **b.** front view of the selected gravity connection (units in mm). The specific connection geometry is selected from Foley et al. [44] and is a double angle with 6.35 mm thickness. **c.** The axial force-displacement diagram of the equivalent spring element replacing the connection, as calculated by Foley et al. The tensile branch is modified to follow a sudden drop to zero forces once the connection tensile capacity has been reached.

to the response characteristics of the selected connection: a) the moment-rotation behavior was neglected, treating the connection as rotationally free, and b) the perfectly plastic post-peak branch in the tensile force-displacement curve was replaced by an abrupt drop to zero forces after the tensile capacity was reached, as shown in Figure 3.6c. The bilinear diagram of the compressive force-displacement response with a perfectly-plastic post-peak branch was retained.

According to the design of the building, the roof has a slightly reduced gravity
load compared to the typical floor. The region enclosed by the points C3, C4, E4 and E3 in Figure 2.5 has a penthouse at the roof level of the prototype building, which in turn has a gravity load slightly greater than the typical floor. However, the proposed analytical method assumes that the gravity load is the same at all floors above the removed column and therefore, to ensure the repetitive pattern of the gravity load, the entire roof was assigned the same vertically applied load as the floors below. Similar to the 2D case, the dynamic nature of the disproportionate collapse phenomenon is accounted for through the dynamic increase factor $\Omega_N$. This factor multiplies the gravity load which is applied in the 2x2 areas at all floors above the removal, and based on the specifications of the APM its value was calculated as 1.16.

3.2.2 Implementation of the proposed method in the 3D prototype structure

The analytical method is applied at the steel framed prototype structure of Figure 2.5 and column C3 is removed from the 1st to the 9th floor in turns, leading to 9 interior gravity column removal scenarios in the aggregate. As in the 2D framework, a column removal scenario was not performed in the building basement. The stability load is assessed based on the tributary areas approach, while the 7 elastic analyses were conducted with ABAQUS utilizing the simplistic modeling approach described in Section 2.2.2. The table on the right side of Figure 3.7 shows the values of the collapse load for each collapse mechanism and for all the column removal scenarios. The second column displays the collapse load values for the stability mode ($C_c$) and the third column the values for the yielding-type mode ($C_{bl}$). The observation of these values reveals a decreasing trend in the collapse load of the stability mode as the column is removed moving upwards along the height of the building. This is attributed to the fact that the utilization ratios of the gravity framed columns are higher in
the bottom part of the building, therefore the linear projection of the method identifies these columns as more prone to buckling compared to the columns at the upper building part. As a result, and since the yielding-type ultimate load is the same at each floor, the transition from the stability to the yielding-type collapse mechanism becomes apparent. The buckled column at the 1st floor could be any of the 1st floor C2, B3, C4 or D3, since these elements have the same tributary areas and they are considered equally vulnerable. The buckled column at the 1st floor could be any of the 1st floor C2, B3, C4 or D3, since these elements have the same tributary areas and they are considered equally vulnerable. The 2nd and the 3rd floor column removal scenarios constitute a transition zone, where the collapse loads are close to each other and the two collapse mechanisms are assumed to be activated almost simultaneously in both cases. Finally, collapse is clearly attributed to the yielding-type mode when a column loss scenario is performed from the 4th to the 9th floor.

Before proceeding with the validation of the method, the following comments can be made here regarding the behavior of the structure and the proposed analytical method:

- The gradual change in the dominant collapse mechanism illustrated in Figure 3.7 shows the same trend as found for the case of the 2D MRFs. It is therefore evident that different column removal scenarios will trigger different collapse mechanisms, and for the case of typical 3D steel and concrete composite buildings under an interior gravity column loss the most detrimental location of the column removal is at the bottom part of the structure. Here, the term detrimental has a twofold meaning: i) the collapse load for the 1st floor is less than the collapse load of any of the remaining column removal scenarios, and ii) this scenario is associated with structural instability phenomena which manifest an abrupt and catastrophic nature. Therefore, the extension of the method in the 3D space provides sufficient evidence that a) a comprehensive disproportionate collapse analysis should always account for stability-related failures and that
Figure 3.7: Results of the method application in 3D steel and concrete composite systems. The graph on the left shows the multi-linear $q$-$\delta$ curve which is representative of the yielding-type mode following the column removal scenario at each floor. The table on the right lists the building capacities against the two collapse modes. The 1$^{st}$ floor column removal scenario is governed by an instability failure of another column, the 2$^{nd}$ - 3$^{rd}$ floor cases constitute a transition zone, while collapse is attributed to the ductile failure mechanism for column removals at the 4$^{th}$ floor and above.

b) the analytical method developed herein offers an efficient and quick tool to discretize the regions where the two collapse mechanisms emanate.

- According to FEMA [47], a dead gravity load (D) of 96 psf and a live load (L) of 50 psf is applied at the typical floor of the prototype building. Based on Equation 3.5 of GSA [15], the structural system should be able to sustain a vertical downwards uniform load of $1.2D + 0.5L$, whereas this value should be multiplied by the dynamic increase factor $\Omega_N$ when the gravity load is applied at the 2x2 areas above the removed column. Therefore, the building is required to sustain a pressure load of 6.71 KPa at each floor, while the minimum collapse load according to the table of Figure 3.7 is 8.30 KPa. Therefore, it is concluded
that the building is capable of withstanding the prescribed demands with a safety factor of 1.24.

- The capacity of the building has been assessed utilizing the notion of a column being completely removed. The limitation of this approach has been identified in past research studies, such as in the work of Ellingwood [51], Sasani et al. [52], and McConell and Brown [53]. This notion can be characterized as an unrealistic approach towards the building robustness assessment, since in a real scenario (i.e. blast), damage is expected to affect multiple vertical structural components rather than being concentrated on a single one. In view of this, Gerasimidis and Sideri [54] developed a method accounting for partial distribution of damage in more than one columns, and demonstrated that the complete column removal approach can yield unconservative results on the actual building capacity, particularly in cases where column instability phenomena dominate structural collapse. One of the main advantages of the proposed methodology is the analytical approach of disproportionate collapse, which allows for the inclusion of scenarios with damage distributed on more than one columns. This can be done both with respect to the yielding-type mode by introducing semi-rigid vertical springs with an appropriately calculated stiffness at the locations of the central and perimeter columns in the 2x2 area model, as well as with respect to the stability mode by accounting for reduced column buckling capacities. A further development of the method accounting for partial distribution of damage is expected to lead to a more comprehensive understanding of the problem of disproportionate collapse and to a more accurate evaluation of the actual building capacity.

- The proposed methodology monitors the response of the weak links of the proto-
type structural system, the gravity columns and the gravity connections (the response of the latter is illustrated in Figures 4.12 and 4.13, along with their FEM validation). Thus, the method equips the designer with an easy-to-handle and efficient means of evaluation of the structural robustness against a hazardous disproportionate collapse event, without the need of performing exhaustive finite element simulations. One can then effectively capitalize on the method output and conduct parametric analyses at negligible computational time and cost, in order to tune the building design accordingly and drive the damage propagation path away from the columns and towards the ductile yielding-type mode. Therefore, incorporation of such methodologies in the industrial realm is not only anticipated to assist in the design of more resilient and robust structures, but also to expedite and facilitate the decision-making processes with regards to the design of such buildings.
CHAPTER 4
NUMERICAL VALIDATION OF THE
PROPOSED FRAMEWORK AND RESULTS
DISCUSSION

4.1 Numerical simulation approach for 2D MRFs

4.1.1 Finite element modeling assumptions and failure criteria

The numerical validation of the method was conducted using the finite element software ABAQUS/Standard [55]. Utilizing a quasi-static framework and accounting for both material and geometric nonlinearities, the robustness of the 240 moment frames is assessed and their capacities are compared to the analytical method findings. The material assigned to all structural members is steel with modulus of elasticity $E = 200 GPa$, Poisson’s ratio $\nu = 0.3$, yield stress $\sigma_y = 345$ MPa and ultimate stress $\sigma_u = 448$ MPa at uniaxial plastic strain 18%. Integration within the section was employed to capture the phenomenon of inelastic buckling. The effect of imperfections was simulated using lateral, horizontally applied concentrated forces at the level of each floor. The magnitude of the imperfection loads was three orders less than the vertical load. It has no real impact on the structural response, other than the initiation of a column buckling from a numerical standpoint once its capacity has been reached.

Beams and columns are simulated using B32OS beam elements. This is a 3D Timoshenko beam element, which also accounts for the warping degree of freedom (all sections have open I-profiles) and has 2 points in the Gaussian quadrature. A mesh of 10 elements/member in all the structural components was found adequate to monitor and safely capture the behavior of the frames. The same boundary conditions are
applied as in the elastic simulation of the system. A static, load-control push-down analysis is performed. Uniform vertical load is applied at all beams, incremented from zero until a critical load value, where a collapse mechanism is identified and failure occurs. The same values of the dynamic increase factor $\Omega_N$ are used as in the analytical method application.

In order to identify the dominant collapse mechanism, specific failure criteria are set for each mode. The yielding-type mechanism is the governing collapse mode when the following criteria are satisfied:

- The moment demand in every beam of the yielding-affected area is almost equal to its flexural capacity, both at its $k$ and $m$ ends.

- The vertical displacement above the column removal is excessively increased, before the stability mechanism has occurred in the model.

A representative example of a yielding-type dominated case is depicted in Figure 4.1 and corresponds to the Boston-Pre-Interior-18 scenario. ABAQUS output regarding the moment acting on each beam end and the evolution of the vertical displacement is shown in this figure, as functions of the applied load $q$. Once every beam reaches its flexural capacity at both ends, the vertical displacement (magenta curve) is increased excessively. This graph also serves as a justification of the notions behind the yielding-type approach of the method. It rationalizes the use of the superposition principle regarding the moment at the $k$ ends until plastic hinges are formed there. It justifies the assumption that the moments at the $k$ ends remain almost constant for load values $q > q_k$, which is a key assumption since the second linear elastic analysis is conducted on a substructure which has hinged ends as boundary conditions. It shows the variation in the slope of the moment curves at the $m$ ends for load values $q > q_k$, which is intended to be captured by the second linear elastic analysis. Finally,
Figure 4.1: FEM results for a typical yielding-type case, the Boston-Pre-Northridge-Interior-18 model.  

**a.** Evolution of the moments at the $k$ (solid) and $m$ (dashed) beam ends for the three beams above the removal. The frame capacity, $q_{y}$, is defined at the onset of the vertical deflection excessive increase at the point immediately above the removal (magenta line).  

**b.** Deformed shapes of the structure at (1) the elastic range and (2) the value of $q_{y}$. Deformation scale factor is 1.

It justifies the impact of the strongest beam on the overall response; once this beam fails, the weakest beam follows shortly after and the vertical displacement increases excessively.

The stability mechanism is considered to be met when all of the following are satisfied:

- The axial force in the column is almost equal to its axial capacity and at this point the horizontal displacement of the column middle point increases abruptly.
Figure 4.2: FEM results of a typical stability-governed case, the Boston-Pre-Northridge-Next-to-Exterior-1 model. 

**a.** Evolution of the buckled column axial force (blue curve) and column middle point horizontal displacement (magenta) vs the gravity load. Once the axial force reaches the column inelastic capacity, the horizontal displacement increases abruptly. Inset picture shows the deformed shape magnified by a factor of 5. 

**b.** Evolution of the column von Mises stresses. The entire cross section reaches the yield stress and the column buckles, followed by strain reversal and the stress relief at one column side (points 1-5).

- Normal stresses of the buckled column are consistent with inelastic column buckling theory.

- Negative eigenvalue messages appear in the ABAQUS message file, indicative of numerical singularities in the structure’s stiffness matrix.

An example of a numerically identified stability governed case is provided in Figure 4.2, corresponding to the Boston-Pre-Next-to-Exterior-1 scenario. Figure 4.2a depicts the axial force of the column along with the horizontal displacement of the column middle point, as functions of the applied load $q$. As soon as the axial force reaches the column critical load, the horizontal displacement is suddenly and excessively increased. Figure 4.2b depicts the buckled column von Mises stresses, a picture which is also a clear indication of inelastic buckling. When the axial load reaches the column’s inelastic buckling capacity, every in-section point reaches the material
yield stress $\sigma_y$. For an infinitesimally small load increment, bifurcation occurs and the horizontal displacement of the column middle point increases abruptly. Due to the geometric nonlinearities, a second-order moment is generated in the section and results in the increase of the compressive stresses in one flange and the appearance of reduced compressive stresses in the other (these stresses relieve the flange, which still remains in compression). This phenomenon is captured by ABAQUS by numerically integrating not only at the beam integration points, but at several points within the section as well. The I-profile used in ABAQUS has 13 in-section points, 5 in each flange and 3 along the web (notation in Figure 4.2b). A clear sign of inelastic buckling is depicted in this figure, where one flange (section points 9-13) and the web (section points 6-8) attract more compression after buckling, while the other flange (section points 1-5) is relieved.

4.1.2 Validation of the collapse mechanism

Figures 4.3 - 4.6 depicts the validation of the method regarding the collapse mechanism and the location of the buckled column. Each separate figure refers to a different frame and all 3 cases (exterior, next-to-exterior, interior) are included. The vertical axis is the location of the column removal and the horizontal axis is the location of the buckled column, in case of a stability collapse. The region below the thick black line is representative of the stability governed cases, while the region above is yielding-type dominated. The points on the graph with the blue diamonds are the analytical results, compared to the FEM output which is denoted in red circles.

- Boston Pre-Northridge

An almost absolute consistence between the FEM analysis and the analytical method can be observed in the Boston Pre-Northridge frame, where the buckled col-
Figure 4.3: Collapse mode validation for the Boston-Pre-Northridge models. The proposed method results are denoted with blue diamonds and the FEM results with red circles. The Boston-Pre-Next-to-Exterior-13 is the only case of the 240 analyzed in which discrepancy of the analytical and numerical results is observed. 

The column location is verified everywhere. The collapse mechanism is validated in almost every removal scenario; the only case which is not verified is the Boston-Pre-Next-to-Exterior-13. In this particular case the method slightly overestimates the flexural capacity of the beams in the yielding-affected area, predicting loss-of-stability, while ABAQUS indicates buckling immediately after the yielding-type collapse. The calculated value of the ductility limit state factor is $\mu = 0.976$, which is very close to the threshold $\mu_{\text{critical}} = 1$. Given these observations, it can be considered that in the zone where the collapse mechanism alters from the stability to the yielding-type mode, the two collapse modes are almost simultaneously activated. In this case, the collapse is attributed to both of them. Overall, the method predicts correctly the collapse mechanism in 239 out of 240 removal scenarios.

- **Seattle Pre-Northridge**

A complete validation between the FEM and the method results can be observed in the case of the Seattle Pre-Northridge frame. The Euler-type method predicts
accurately not only the collapse mechanism, but the location of the buckled column as well. In case of the column removals at the 5th floor, the weakest (buckled) column is found to be at the 6th floor. This is captured both by the method and the FEM approach.

- **Boston Post-Northridge**

  The method is completely validated in this frame, both in terms of the collapse mechanism and the buckled column location. In the Boston-post-exterior-2 scenario, the numerical estimation indicates simultaneous buckling in the 1st and the 2nd floor. For this scenario, the analytical method predicts that the difference in the $\mu$ values for the case of the 1st or the 2nd floor column being the critical one is 1.4%. The same picture is observed in the Boston-post-exterior-7 scenario (columns are located at the 6th and 7th floor, having a 0.6% difference in the respective $\mu$ values). This indicates that the method is capable of predicting the buckled column location even in cases where more than one columns buckle simultaneously. This advantage of the method
Figure 4.5: Collapse mode validation for the Boston-Post-Northridge models. Excellent agreement between the numerical and the analytical results is observed. In the Boston-Post-Exterior-2 and Boston-Post-Exterior-7 models both approaches predict almost simultaneous buckling of two columns, one being adjacent to the removal and one located in the floor below.

is attributed to the fact that the axial force of each column of the frame is considered for the calculation of $\mu$.

- **Seattle Post-Northridge**

The Seattle Post-Northridge frame was completely validated regarding the collapse mechanism, which is the main intention of the method. However, a more detailed description is required regarding the buckled column location for stability cases which demonstrated discrepancies between the FEM and the analytical estimation. The following description refers to the Seattle-Post-Exterior-5 model, and the same principles apply to the Seattle-Post-Exterior-3 & 4, Seattle-Post-Interior-3, 4 & 5 models. ABAQUS estimates buckling at the column of the 5$^{th}$ floor and nowhere else, in contrary to previous cases with two buckled columns. The method predicts buckling at the 1$^{st}$ floor; therefore, this discrepancy needs to be explained. Figure 4.7a shows the axial forces of the columns at the 1$^{st}$ and 5$^{th}$ floor, and it is worth noting that they both have the same cross section and capacity. It can be seen that the acting axial force of the 1$^{st}$ column is indeed higher than its 5$^{th}$ floor counterpart, therefore
Figure 4.6: Collapse mode validation for the Seattle-Post-Northridge models. The results coincide with respect to the nature of the collapse mechanism, and the observed discrepancies with regards to the location of the buckled column are investigated and explained in Figure 4.7.

This column is expected to be the critical (method prediction). However, the moment that is generated on the 5\textsuperscript{th} floor column by the vertical displacement of the cantilever yielding-affected area is much greater than the moment of the 1\textsuperscript{st} floor column. The presence of this moment is inherently incorporated in the von Mises stresses graph (Figure 4.7b). These enhanced nonlinearities lead to the buckling of the 5\textsuperscript{th} floor column, even though its axial force is slightly smaller than the capacity and the section has not reached the yield stress completely across its height. The simultaneous action of moment and axial force on the column is not yet captured by the analytical method. However, while accounting only for the axial load, the proposed methodology calculates a very small difference, namely 1.5\%, in the ratio of the $\mu$ values between the 1\textsuperscript{st} and the 5\textsuperscript{th} floor columns. This signifies that even within its current limitations, the method is able to identify the columns more prone to buckle with sufficient accuracy, while at the same time it is able to accurately predict the collapse mechanism of the structure.
4.1.3 Validation of the collapse load

Figures 4.8 - 4.9 depict the validation of the method regarding the collapse load for each removal scenario. In those graphs the vertical axis is the ratio of the collapse load calculated by the analytical method ($CL_{method}$) over the collapse load estimated by ABAQUS ($CL_{FEM}$). The horizontal axis is the floor at which each column is removed. Stability-governed cases are marked with the blue bars, while the yielding-type collapse load comparisons are denoted with the red bars.

- **Buckling cases**

A very good consistency of the collapse loads can be observed in the cases governed by the stability mode. The ratio of the loads is very close to 1 in all exterior and
interior cases, as well as for most of the next-to-exterior cases. The collapse load ratio is between 0.95 and 1.05 in 90/103 cases, while its value is greater than 1.10 only in 2/103 cases. This sufficiently justifies the notion of the linear projection of the axial demand on each column, which is not significantly affected by the moment redistribution. A further step, which would take into account the moment demand on the columns, is expected to further enhance the method accuracy regarding the buckling collapse load projection.

- **Yielding-type cases**

A favorable coincidence arises when comparing the collapse loads predicted by the analytical method to those calculated by the nonlinear analysis. The collapse load ratio is between 0.95 and 1.05 in 103/137 cases, while its value is greater than 1.10 in only 7/103 cases. Given the simplified procedures of the analytical method, where only 2 linear elastic analyses are employed to capture the complex and highly nonlinear phenomenon of moment redistribution, such an accuracy is considered acceptable. The method performance could be further improved if more than 2 elastic analysis were employed. A finer discretization of the phenomenon, utilizing more analyses to capture the progressive spread of plastic hinges, is expected to increase the accuracy of the proposed method. However, such modifications are expected to penalize the computational time, and therefore they were not incorporated in the current framework.

4.1.4 Discussion on the results and method principles

Following the extensive validation of the proposed methodology in the case of 2D MRFs, it is worth elaborating how was the expected influence of the material and geometric nonlinearities on the collapse load accommodated in the linear framework.
Figure 4.8: Validation of the frame capacities in the Pre-Northridge models. The ratios of the predicted capacities are prominently close to unity in the case of the exterior and interior column removals for both collapse mechanisms.
Figure 4.9: Validation of the frame capacities in the Post-Northridge models. Again, the analytical results match in general the numerical predictions more closely in case of the exterior and interior column removals.
of the method. When stability governs the frame collapse, the impact of the material nonlinearity on the collapse load is captured by accounting for the inelastic column buckling apart from the column Euler load (Table 2.1). When the yielding-type mechanism is governing, the impact of the material nonlinearity on the collapse load is captured by the second analysis that is performed. The latter is intended to capture the load redistribution which takes place when the material of the beams at the \( k \) ends reaches its yield stress.

Geometric nonlinearities are more difficult to capture, however their impact is not expected to be as crucial for the specific type of structures analyzed. For stability-dominated cases, very limited pre-buckling column deformations were observed from the numerical simulations, and the stability collapse loads were approached by the method linear projection with sufficient accuracy (Figures 4.8 - 4.9). However, the geometric nonlinearities constitute the reason of the discrepancy between the analytical method and FEM results regarding the buckled columns location. This trend, which is more apparent in the Next-to-Exterior models, is expected to be addressed with the incorporation of the column bending moments in the analytical methodology. Finally, the influence of geometric nonlinearities in the yielding-type mechanism is also anticipated to be of secondary importance, due to the following reason. Gravity loading, the main loading domain in a disproportionate collapse scenario, and the corresponding main deformation, which is vertical deflection of the beams above the removal, align along the same direction. Since no major load is assumed to act simultaneously and perpendicularly to the main deflection direction, such as wind or earthquake, insignificant second-order moment demand is expected to be generated on either the \( k \) or the \( m \) ends. As a result, any deviations of the analytical yielding-type capacity estimation against the corresponding numerical prediction in Figures 4.8 - 4.9 are mainly attributed to the method two-step discretization approach, rather than to the
impact of geometric nonlinearities.

4.2 Numerical simulation approach for 3D GFSs

4.2.1 Finite element modeling details

The validation of the method in the 3D framework requires the development of an advanced numerical simulation of the entire building, the details of which are presented in this section. ABAQUS/Standard was used for the numerical simulation of the structure, and a quasi-static push-down was conducted accounting for material and geometric nonlinearities. As in the 2D case, all beams and columns are modeled using the B32OS element and output from the 13 in-section points within an integration point is requested to monitor the phenomenon evolution with enhanced thoroughness. All beams are initially placed along their centerline and then the moment beams, which have a different section at almost every floor, are moved accordingly such as their upper fiber lies in the same plane as the upper fiber of the gravity girder. 10 elements are used for meshing each column of the building, while 20 and 18 elements are used for the discretization of each beam in the x- and y-direction respectively (Figure 2.5). As shown in the 2D framework, the combination of this element type and size for the column mesh is adequate for capturing column instabilities, while the refinement of the beam meshing compared to the 2D case is dictated by the presence of the overlying slab.

According to the design specifications provided by FEMA [47], the concrete slab has a thickness of 6.35 cm and lays on a steel deck of a 7.62 cm depth. The numerical idealization of the slab follows the notions developed and justified in Alashker et al. [56]. Instead of modeling the corrugated section, the slab is assigned an equivalent constant thickness of 10 cm, and as it was shown in Alashker et al. [56] this ap-
Figure 4.10: Representative images of the numerical model of the 3D prototype building developed in ABAQUS. **a.** 3D view of the complete model. **b.** 3D view of a single-story frame. The view angle has changed from a., to illustrate here the gravity beam grillage. **c.** 3D view of a 2x2 area single-story beam grillage.

The approach places more emphasis on the catenary stage of loading, which is of prominent importance here as well. The slab is modeled using the S4 element type, which is a fully-integrated four-node shell element. Five section integration points were used along the slab depth, to obtain a more detailed picture of the stresses development within the slab thickness. The slab is modeled at the beam centerline level, it follows the mesh discretization of the beams and it is merged with the beam nodes to assure composite action, averting slippage in the interface. It is then moved upwards by offset, at a distance such that its bottom fiber coincides with the upper fiber of the beams. The resulting model comprises 25000 B32OS elements for the beams and columns and 90000 S4 elements for the slab, and representative images of the building numerical idealization are shown in Figure 4.10.

The material assigned to the beams and columns has typical steel properties, with a modulus of elasticity $E = 210 \text{ GPa}$, Poisson’s ratio $\nu = 0.3$, yield stress $\sigma_y = 345 \text{ MPa}$, ultimate stress $\sigma_u = 448 \text{ MPa}$ and ultimate strain $\epsilon_u = 0.18$. 6x6-W1.4xW1.4 wire mesh and 2VLI22 steel deck are used, in accordance to Foley et al. [44]. The steel
deck has a bilinear perfectly plastic material model, with a modulus of elasticity $E = 210 \text{ GPa}$, Poisson’s ratio $\nu = 0.3$ and yield stress $\sigma_y = 276 \text{ MPa}$. It is embedded in the slab section acting as a rebar layer at the bottom fiber. It is assumed to be effective only in one direction, parallel to the flutes, having negligible load carrying capacity in the other direction. Based on Alashker et al., only 50% of the steel deck is considered to be fully activated and therefore contributing to the capacity of the slab. The concrete nominal tensile strength is neglected and the second source of the slab tensile capacity stems from the wire mesh. This contribution is incorporated in the slab through an equivalent stress-strain relationship which is assigned to the concrete, following Equation 1 from Alashker et al. [56]. Concrete is assigned an elastic modulus $E = 20 \text{ GPa}$, Poisson’s ratio $\nu = 0.2$ and compressive strength $\sigma_{\text{comp}} = 20 \text{ MPa}$.

The *CONCRETE DAMAGED PLASTICITY constitutive model is adopted for the concrete simulation, allowing the specification of a different response against tensile and compressive stresses. The dilation angle is set as 40°, the flow potential eccentricity $\epsilon$ is 0.1 (default ABAQUS value), the ratio of initial equibiaxial compressive yield stress to the initial uniaxial compressive yield stress is 1.16 (default ABAQUS value), the ratio of the second stress invariant on the tensile meridian to that on the compressive meridian is 0.667 (default ABAQUS value), and the viscosity parameter is set to $10^{-5}$.

The moment connections of the perimeter frames are assumed to be fully rigid, a behavior which was achieved by merging the nodes of the beams and columns at the points where they were converging. The building gravity connections (column-girder, column-beam and girder-beam) are modeled using the CONN3D2 connector element. This element imposes kinematic constraints between two nodes, by defining the dependency of the degrees of freedom (DOFs) of the second node to the DOFs of the first one. These relative displacements and rotations are referred to as components of
relative motion (CORMs) among the two connected nodes, whereas the first node is always taken to be located at the primary structural component (column, girder) and the second node to the secondary (girder, beam). The connector elements adopted herein employ the *SLOT and *ROTATION options from the connector-type library. The *SLOT option leaves independent the translational CORM along the connecting line referring to the connection response for axial loading and constraints the other two translational CORMs. The desired properties regarding the connection axial loading are modeled using the *Elasticity and *Failure behavior sub-options, specifying the intended axial stiffness, capacity and the feature of the abrupt drop once the maximum capacity is attained in tension. Essentially, this approach allows for the incorporation of the axial load-displacement diagram shown in Figure 3.6c into the detailed numerical model, directly defining the axial behavior of each connection. The *ROTATION option refers to the rotational CORMs and simulates the pinned-nature of the shear connection, since it leaves independent the CORM regarding this motion (beam bending around the major axis). The other two rotational CORMs are constrained.

4.2.2 Modeling limitations and comparison to other studies

The present study develops a numerical model at the building level, including all the primary components of the structure and permitting their material and geometric nonlinear response. Utilizing beam elements for the beams and columns as well as single connector elements for the gravity connections provides sufficiently accurate insight on the nonlinear inelastic response of the most critical components of the structure, rendering at the same time tractable and computationally efficient the running time of the numerical model. The sophistication of this model resembles the level of complexity of model M2 by Li and El-Tawil [25] and Alashker et al. [56].
The results of these studies indicated that parameters such as the inclusion of all the primary structural components as well as the careful sizing of the finite elements play the principal role in the numerical output of the building-level model, rather than the computationally exhaustive modeling of all the components (using for example shell elements). Therefore, emphasis here was placed in accounting for all the beams, columns and gravity connections of the building and correctly capturing their inelastic response, as well as ensuring that the load step size is adequately small to accurately capture the nonlinear response of all the structural components.

4.2.3 Validation of the yielding-type dominated cases

Based on the proposed method results, collapse for the cases of the 4th-9th floor had been attributed to the ductile collapse mechanism. Figure 4.11 depicts the analytical multi-linear $q$-$\delta$ curve (magenta line), along with the nonlinear load-displacement curves for the 4th to the 9th floor cases as extracted from ABAQUS. Figure 4.12 includes the 4 curves generated by the method regarding the connections (dashed lines), superimposed on the curves of all the connections in the 2x2 areas above the removed column for the 5th floor column removal scenario extracted from ABAQUS (solid lines). The response of the connections was found almost identical for all column removals, therefore only the results of this representative column removal scenario are illustrated here. Based on these two figures the following comments can be made:

- The FEM curves of the gravity connections response are prominently grouped together. This behavior is apparent both in the 2x2 area level (since they are symmetrically arranged around the removal) and at each floor level. This finding, in conjunction with the fact that all the numerical $q$-$\delta$ curves align very close to each other, justifies a core assumption of the method: the repetitive pattern of the gravity system as well as its structural configuration allow for
Figure 4.11: FEM validation of the $q$-\(\delta\) curve for the 3D yielding-type cases. The graph shows the \(q\)-\(\delta\) curves for the column removal scenarios between the 4\(^{th}\) and 9\(^{th}\) floor, as predicted by the analytical method (thick magenta line) and the simulation results. The analytical curve captures accurately the connections failure load, their impact to the system (indicated by the plateaus) and the degrading stiffness of the system (shown by the first two inset images). Finally, the ending point of the analytical curve, which indicates the point of imminent collapse, is also aligned on top of the numerical results path.

- The progression of failure in the connections as well as the load at which connections C4, C3 and C2 reach their maximum capacity are almost precisely assessed, while a rather small deviation exists regarding the C1s failure load. The correct estimation of the connection failure load is considered of crucial importance, since the connections failures trigger load redistribution mechanisms within the structural system and constitute limit states for the robustness of the structure. The analytical predictions align almost exactly with the FEM
Figure 4.12: FEM validation of the connections forces for the 3D yielding-type cases. The graph shows the tensile forces of all the gravity connections within the 2x2 area for the 5th floor column loss. The analytical prediction, shown with the dashed line, indicates the efficiency of the method to predict the failure load of the connections and the load re-distribution within the system (vertical jumps) which is triggered by the failure of a connections family. Due to the repetitive gravity system pattern, it is shown that a single curve is adequate to describe the response of every connections family.

C4, C3 and C2 curves throughout the entire phenomenon evolution, while the minor discrepancy regarding the C1 failure load is attributed to the more intense propagation of yielding in the slab steel components after the C2s have failed, leading to more pronounced non-linearities in the C1s curves. Additionally, the vertical displacement corresponding to each connection failure is also captured with sufficient accuracy, providing on the aggregate a comprehensive picture of the system response. Therefore, these observations further verify an-
other core analytical method principle, which is the technique of degenerating a highly nonlinear problem to a set of linear elastic analyses using only the elastic properties and appropriate estimations for the members limit states.

- The stiffness of the structure, represented by the slopes of the ascending branches in the FEM q-δ curves of Figure 4.11 is captured with very satisfying accuracy, especially until connections C2s fail. After this point minor discrepancies are expected, since the method employs discrete stages to account for the transition from the composite to the membrane action. Nevertheless, the stiffness of the multi-floor gravity system particularly at the initial stages of loading is accurately estimated.

- The method rationale regarding the impact of a sudden connection failure to the system is also prominently verified. This is evident from the presence and magnitude of the almost horizontal lines in the q-δ curves, as well as the presence and magnitude of the almost vertical lines in the connection force-load curves. The remarkably good consistency between the analytical and the numerical results reveals the capability of the method to capture with adequate accuracy the load redistribution mechanisms triggered by the failure of the connections, which include a rapid increase in the characteristic vertical displacement as well as a sudden increase in the demand of the remaining, still intact connections.

- The ending point of the analytical q-δ curve in Figure 4.11, which corresponds to the ultimate collapse load and the respective vertical displacement, aligns very close to the numerical q-δ curves. Therefore, the method is capable of assessing the system ultimate capacity and the vertical displacement corresponding to this limit state, as the gravity system approaches its complete failure.
4.2.4 Validation of the stability and transition zone cases

The analytical method had predicted that the stability collapse mechanism is triggered before the complete activation of the yielding-type mode the three bottom floor cases, calculating distinctly different collapse loads for the 1st floor and almost equal for the 2nd and 3rd floor column removal scenarios. Figure 4.13 shows the FEM $q$-$\delta$ curves for the bottom 3 column removal scenarios. It can be seen that these curves demonstrate a very good resemblance with the analytical multi-linear curve, however the corresponding analyses are terminated before the gravity system reaches the load achieved in the previous cases. To explain the source of this behavior, the axial force-load curves for all the connections in the 2x2 areas above the 1st floor removal are plotted in Figure 4.14. The general trend is the same as in the equivalent graph for the 5th floor removal, and the analytical curves capture accurately the behavior of the C4s, C3s and C2s connections. However, at the point of the numerical analysis termination ($q = 8.43\, KPa$), almost all of the C1s have still not reached their maximum capacity, which is clearly depicted in the embedded zoom-in picture inside Figure 4.14. This significant observation implies that there is a capacity reserve regarding the yielding-type mode and that the composite action has not completely vanished, since these intact connections can still contribute to the system robustness.

The 1st floor column removal scenario is governed by inelastic minor axis buckling of one of the columns adjacent to the removed, as it is demonstrated in Figure 4.15c. The 2nd and 3rd floor column removal scenarios are terminated due to numerical convergence problems, issued by the simultaneous unloading of the C1 connections and the buckling of a column adjacent to the removed one. This created a vague picture over which mode is dominant. However, the results were much more clear regarding the 1st floor case and they are presented below. Figure 4.15a shows the horizontal
Figure 4.13: FEM validation of the $q$-$\delta$ curve for the 3D stability cases. The graph shows the $q$-$\delta$ curves for the column removal scenarios between the 1st and 3rd floor, as predicted by the analytical method (thick magenta line) and the simulation results. The analytical curve captures with sufficient accuracy the trend of the numerical results, until the point of the analysis termination due to column instabilities.

Displacement of the middle node of the highlighted element in the column, against the applied gravity load. Point A in this graph signifies the onset of buckling, since after this point a very small increase in the applied load causes an abrupt increase in the horizontal displacement, a clear sign of the column instability. Figure 4.15b shows the evolution of the von Mises stresses from an integration point in the highlighted column element. This graph is remarkably important, since it reveals that the column section is subjected to non-uniform compression, as a result of an axial load and a bending moment combined action. Due to the presence of this moment, regions of the section reach the steel yield stress at different points along the vertical load increase process. The section yielding starts from the right side (Points 1), spreads towards the middle (Points 2) and finally the whole web reaches the yield stress (Points 3).
Figure 4.14: FEM validation of the connections forces for the 3D stability cases. The graph shows the tensile forces of all the gravity connections within the 2x2 area for the 1st floor column loss. Excellent agreement between the analytical and numerical results is demonstrated. It is clearly shown that at the point of analysis termination almost all of the C1 connections have not yet reached their failure threshold, illustrating the reserved capacity against the yielding-type mode.

As soon as this is occurring, the section is no longer capable of bearing any additional moment, loses its stability and buckles. As a result, the region denoted by Points 4 follows a steep increase in the von Mises stresses, while the outer left side indicated by Points 5 enters the tension zone (its compressive stresses are relieved).

Since the gravity connections are assumed not to transfer any bending moment, the origin of the moment inherently depicted at the von Mises stresses-load diagram of Figure 4.15b has to be investigated. Figure 4.16a shows a contour plot of the axial forces in the 2x2 area beam grillage (due to symmetry, only one quarter of the 2x2 area is shown). Three snapshots throughout the load incrementation process
Figure 4.15: FEM results for the 1st floor removal in 3D GFSs.  

**a.** Horizontal displacement of the buckled column middle point vs the gravity load. **b.** Evolution of the von Mises stresses throughout the member section until the column fails due to inelastic buckling. Output is extracted from an integration point of the highlighted element shown in the embedded picture. **c.** Deformed shape of the structure and location of the buckled column.

are included and correspond to different applied loads, as representative of the axial force evolution in the gravity beams. The first snapshot corresponds to a load of \( q = 4 KPa \) with all the connections being intact; the second corresponds to a load of \( q = 6 KPa \) where the connections C4 and C3 have failed, while the third snapshot corresponds to a load of \( q = 8 KPa \) and connections C2 have failed as well. This graph clearly shows that the beams at the 2x2 area perimeter supports and therefore the respective connections are always subjected to compression, even if the connections in the middle have failed or not. This observation leads to the following idealization,
which is explained herein and it is represented in Figure 4.16b. Provided that the wire mesh located at the top part of the slab in the 2x2 area perimeter has yielded and given the fact that the gravity connections are exposed to compression, a couple of forces of opposite direction is generated at the composite beam face ($F_{tension}$ and $F_{compression}$ respectively in Figure 4.16b). Due to force equilibrium, two forces of opposite sign are generated at the face of the column (F1 and F2 in Figure 4.16b). This couple of forces is applied at each floor level and it creates a moment demand along the column row, being thus the source of the moment which is present at the Figure 4.15b diagram.

4.2.5 Discussion of the results and method principles

Since the axial stiffness of the beam is much greater than the equivalent stiffness of the tensile part of the slab, the reaction of the beam compressive force at the column face (F1) should provoke an outward displacement at the top node of the column. This is verified by the graph of Figure 4.16c, where the undeformed (gray color) and deformed (dark green color) shapes of the buckled column are extracted from ABAQUS and they are simultaneously plotted: the column top node is evidently moving outwards. This finding is consistent with Sadek et al., who showed that using their idealizations including the framing, the deck and the slab an outward displacement was observed, contrary to idealizations in the same study in which the slab was not included and the displacement of the column top node was inwards (towards the removed column). The results presented here provide unambiguous evidence over the complicated loading conditions of the surrounding columns. These structural components are exposed to the concurrent action of a) a significant axial load from the floors above, b) an inevitable moment generated at the column top face (even with the assumption of rotationally free connections) and c) a significant
Figure 4.16: Outward displacement of buckled column due to compression in the beam ends at the 2x2 area perimeter. a. Beams at the 2x2 area support are always subjected to compression. b. Schematic idealization of the forces on the composite beam and column side, generating moment in the gravity columns. c. Deformed and undeformed FEM shape of the buckled column, demonstrating the outward horizontal displacement of the column top node.
horizontal force at the column top face, triggering an outward displacement at the column top node. The latter displacement can in turn cause an eccentricity regarding the axially applied load, further burdening the column. The combined action of these loading conditions evidently raises significant concerns over the actual column capacity and justifies for the adoption of the reduction factor $\beta$ in Equation 19. These conditions are not only interdependent with each other, but they are also strongly related to the design of each building. As a result, the derivation of a universal expression for the factor $\beta$ necessitates the investigation of a wide plethora of structural systems and it is a highly demanding task, which is though considered the necessary next step towards a more detailed and comprehensive description of the building behavior.

Overall, the column removal analyses at the bottom part of the building reveal both analytically and numerically a fundamentally different collapse mechanism, triggered by the inelastic minor axis buckling of a gravity column subjected to a combination of axial and moment demands. This collapse mode is activated before the complete exploitation of the gravity beam grillage capacity, rendering therefore the investigation of potential instability phenomena a necessary approach to accurately assess the building robustness under a column removal scenario at any floor. It should be also highlighted here that the current framework clearly adopts a conservative modeling approach with respect to the yielding-type mode. A small ratio of the connection-to-the-beams axial capacity is selected from Foley et al. [44], an abrupt drop in the post-peak axial force-displacement diagram of the tensile gravity connections is assumed, the gravity connections are assigned zero bending stiffness and capacity, and finally the ultimate collapse load is taken equal to the failure load of the connections $C_{1s}$, disregarding a potential increase in the system capacity based solely on the membrane action. This approach neglects a potential reserve in the structural
system robustness against the yielding-type mode and even with these conservative modeling assumptions, the stability mechanism evidently governs the behavior of at least 1 floor column removal scenario in a prototype 9-story building.
CHAPTER 5

SUMMARY, CONCLUSIONS AND FUTURE WORK: PART I

5.1 Summary and concluding remarks

The first part of the thesis has focused on addressing two long-standing gaps in the existent literature of the disproportionate collapse field: a) the lack of analytical frameworks for the robustness assessment of steel framed buildings under the notion of a column loss, and b) the analytical and numerical identification of the strong relation between disproportionate collapse and stability-related structural failure modes. Motivated by these objectives, a novel methodology was developed to assess key response features of the building performance up to the point of imminent collapse. Two broad combinations of structural systems - initial damage scenario were considered, a) 2D steel moment-resisting frames under any column loss and b) 3D steel-concrete composite gravity framed systems under any interior gravity column removal. For each structural idealization two collapse mechanisms were identified and addressed: a) a ductile, yielding-type mode, which encompasses damage concentration in the beams or connections above the removed component, and b) a brittle, stability-related failure mode, which is triggered by loss-of-stability of a building column.

The core of the proposed method lies in the segmental discretization of the structural response between critical limit states, and the implementation of an elastic analysis in each of these segments to obtain the variables of interest (moments, axial forces, etc.). In this regard, the proposed methodology constitutes a simplified robustness framework whose most salient features can be summarized as follows:
• The method provides closed-form expressions for the assessment of various aspects of the structural response, including: a) the building robustness, b) the progressiveness of failure along the damage propagation path, c) the impact of load-redistribution mechanisms during damage progression, and d) the location of the buckled column in the case of a stability-dominated collapse. This is a crucial advancement since it enables the quantitative appraisal of the structural performance based purely on explicit formulas and elastic analyses on simple structural idealizations. Consequently, the method can be compatible with broader resilience assessment frameworks, functioning as a means of the robustness-component estimation.

• The accuracy of the framework is extensively validated against high-fidelity numerical simulations of the entire structural system, demonstrating sufficiently satisfying consistency between the two approaches. As a result, the method constitutes a reliable alternative to the finite element method, reducing at the same time the computational labor time by orders of magnitude, particularly in the 3D case. The framework is applicable with any commercially available structural engineering package and more importantly it does not entail expertise in the field of disproportionate collapse from the user. Therefore, the method constitutes an easy-to-handle and efficient tool, appropriate for routine use by practitioners in the industry realm.

• The findings of this study provided conclusive evidence of the true potential of stability-related failures during a disproportionate collapse scheme. A clear correlation between the location of the removed column and the dominant collapse mode was revealed, with stability dominating column removal cases at the bottom part of the structures. System-level approaches are essential to capture
correctly the behavior of a multi-story building, as: i) the term *bottom part* is not rigorously defined and depends on the design of the building, and ii) the buckled column may not always be at the same floor as the removed one. In view of the abrupt, catastrophic nature of the stability mode and the results of this investigation, it becomes apparent that a comprehensive analysis against disproportionate collapse should always ensure that the building design does not steer damage towards the building columns.

5.2 Outlook and future work

5.2.1 Future suggestions for the improvement of the proposed methodology

Though the work reported in this document has laid new ground foundations for further advancements in the field of disproportionate collapse, there exist various aspects which can be further enriched in order to increase the practicality of the methodology and its appropriateness for industrial-oriented purposes.

The refinement of the analytical method with regards to the stability mode can be viewed as the next step, both in the 2D and the 3D framework. In both cases, the interaction of the bending moments with the axial forces, as well as the impact of any pre-buckling column deformations, are currently omitted from the framework. As it was discussed earlier in the manuscript, the detailed representation of the actual demands on the columns surrounding the removal would yield a more refined and prescriptive formula for the stability analytical function. This is considered a necessary prerequisite to increase the accuracy of the proposed methodology prior to its utilization for design purposes.

Furthermore, the applicability of the analytical method in the 3D space is limited
to interior gravity column removals in a specific structural layout. The universality of the method in the 3D space can be achieved by expanding the method to additional column loss scenarios (corner, exterior and next-to-exterior locations), as well as in other structural systems (i.e. where moment frames lie on the boundaries of the area immediately affected by the removed column). Additionally, the incorporation of a wider range of mechanical responses for the gravity connections is anticipated to increase the generality of the proposed framework.

5.2.2 Further suggestions for research in the field of disproportionate collapse

The findings of this study aspire to stimulate and intensify further research on the field of disproportionate collapse, setting new and broad objectives for future projects. Of immense importance is the experimental validation of scenarios expected to be dominated by column instabilities, such as ground-floor removals of high-rise buildings. Since full-scale experimental idealization of the entire structure is essentially prohibited by economic and practicality limitations, a promising alternative would be the fabrication of a first-story frame specimen and the imposition of appropriate loading and boundary conditions to approximate the continuity of the structure to the floors above. Such configuration should aim at a reliable representation of the true demands generated on the first story of a multi-story frame, and would provide vital and new information on the actual response of structures which are anticipated to collapse due to column instabilities, marking a new territory in the realm of disproportionate collapse experimental assessment. Finally, the experimental, numerical and analytical investigation on partial distribution of damage in structural systems, as well as the consideration of simultaneous removal of multiple columns, deem as the likely next steps towards a more realistic representation of the actual impact that
extreme events have on the structural robustness of steel framed buildings.
PART II:
ELASTIC MECHANICAL
PERFORMANCE OF DEFECT
CONTAINING TRUSS-LATTICE
MATERIALS
6.1 Opening remarks

6.1.1 Introduction to architected materials

The pursuit of materials with properties superior to the contemporary state-of-the-art has historically transformed the form of human activities, and materials which were revolutionary in their era have been used to landmark the various periods of human evolution (Ages of Stone, Copper, Bronze, Iron, Steel, Polymers). To facilitate their classification and to compare their performance in a systematic fashion, material property charts (also termed as Ashby charts) have been developed [57]. These charts map materials based on two of their properties, and any pair of properties can function as a viable candidate. The resulting charts share one common feature: they are not continuously populated. In other words, there exist holes (also termed as white spaces), which are either unattainable due to fundamental natural laws, or just empty, in which case their filling requires the acquisition of novel and more advantageous materials. In search of stronger, stiffer and more lightweight materials, one promising pathway is by tailoring the material architecture [58] - [59]. This approach enables to control precisely the spatial combination of materials or materials and space, and has opened up entirely new spaces in the material charts giving rise to the emergence of architected materials.

Architected materials are composites of two or more bulk materials, which follow a specific architectural pattern. Their rapid development has been enabled by recent breakthroughs in additive manufacturing technologies, such as [60] - [63], which have subsequently allowed for the exploitation of intrinsic properties only accessible
in the nanometer scale [64] - [66]. The synergistic integration between architecture, nano-size effects, bulk constituent properties and chemical composition has ultimately unlocked a vast space of unique and unprecedented properties [67] - [72]. The idea of tailoring architecture to achieve superior properties stems from biomimetic concepts and careful investigation of materials evolved in nature [73] - [78]. The hierarchical layout of many natural materials has also inspired the design of materials spanning multiple length scales in their microstructure [79] - [82], whereas principles of hierarchical design can also be observed in famous civil engineering structures such as the Eiffel Tower.

6.1.2 Lattice materials, deformation mechanisms and the role of connectivity

*Cellular* solids are a special class of architected materials, being the combination of a single material with space [83]. Incorporating the lattice design into cellular materials has enabled access to new territories in the material property charts, particularly in the low density space. A lattice is a connected network of struts [84], and this concept has diverse and far-reaching applications. In crystallography this term describes a grid of lines whose intersections provide the locations of the crystal atoms, whereas for structural engineers a truss-lattice is a structure comprised of struts which are either pin- or rigidly-jointed, typically used to create lightweight structures of exceptional stiffness and strength. Lattice materials employ the same topological concept, that of a connected array of struts, but differ from lattice structures with regards to the length scale. Their characteristic length scale (length of strut) is many orders of magnitude less than the overall macroscopic size, and as a result they can be treated as materials with their own effective properties.

Cellular materials have been traditionally categorized either as bending- or stretching-
dominated. This distinction depends on the primary deformation mechanism upon loading of the material. Bending-dominated topologies deform by bending of their ligaments, whereas in stretching-dominated topologies the primary deformation mechanism is axial loading of the struts (compressive forces in some members, tensile forces in others). For lattice materials with similarly situated nodes, that is materials in which the framework appears the same when viewed from any one of the nodes [85], the bending- or stretching-dominated classification generally depends on the rigidity of the pin-jointed version of the material [86]. The connectivity or coordination number $Z$, which is the average number of elements connected to a node, may serve as an indicator of rigidity. As it was shown in Deshpande et al. [86], the necessary and sufficient condition for rigidity of 2D and 3D networks with similarly situated nodes is $Z = 6$ and $Z = 12$ respectively. Topologies comprised of rigid unit cells are rigid and stretching-dominated, whereas topologies with lower coordination numbers can be either periodically rigid and in principle stretching-dominated, or non-rigid and bending-dominated [83].

The bending- or stretching-dominated classification has been shown to affiliate with the scaling laws of material mechanical properties, such as the Young’s modulus ($E_{\text{eff}}$) and the yield strength ($\sigma_{\text{y,eff}}$), with the relative density ($\bar{\rho} = \rho/\rho_s$) of the cellular solid [87]. For bending-dominated topologies the scaling laws are: $E_{\text{eff}} \propto \bar{\rho}^2$ and $\sigma_{\text{y,eff}} \propto \bar{\rho}^{1.5}$, whereas for stretching-dominated they are: $E_{\text{eff}} \propto \bar{\rho}$ and $\sigma_{\text{y,eff}} \propto \bar{\rho}$. Nevertheless, this classification has been recently re-visited by Meza et al. [88], who produced experimental data that showed remarkable departure from the traditional theories. Their work revealed a less significant impact of the topology in the scaling laws of the material properties, and concluded that additional parameters need to be quantified and accounted for in the analytical formulas, such as the contribution of the nodes.
6.1.3 Defects and imperfections in truss-lattice materials

Though truss-lattice materials have demonstrated outstanding mechanical properties, particularly when fabricated with nano-scale feature sizes, complete harnessing of their superior performance is hindered by defects and imperfections which inevitably appear during the fabrication processes. Whether fabrication follows a top-down (additive manufacturing) or bottom-up (self-assembly) route, the presence of defects penalizes the material response from the theoretically pristine state, in a fashion that questions the reliability of the material performance. As a result, the in-depth investigation of the complex interplay between topology, mechanical properties and types of imperfections that are introduced by the associated fabrication processes, is an essential prerequisite for understanding the flaw tolerance of the truss-lattice materials and the full exploitation of their mechanical properties in real world applications.

The majority of the research literature on the defect sensitivity of lattice materials is confined to the 2D space [89–96]. Studies on three-dimensional imperfect truss-lattices have been significantly less extensive [81, 88, 97–99], and only few investigations have been conducted tying their focus on geometric imperfections induced by specific fabrication processes, such as Selective Laser Melting [100] and Self Propagating Photopolymer Waveguide [101]. A wide range of morphological defects has been investigated in the aforementioned studies, such as the effect of missing members, non-uniform thickness in the ligaments, member waviness, cell-size variations, nodal perturbations and others. Despite this substantial body of research on the flaw tolerance of truss-lattice materials, the connection between defect-containing truss-lattice materials and well-established analytical models for continuum media is still a rather untapped region. With only very few exceptions, such as the work by Liu and Liang [102] on the elastic response of the 2D triangular lattice with defects, the
applicability of continuum based theories to 3D defected heterogeneous truss-lattice materials lies still as an open question.

6.2 Scope and outline of Part II

In view of the above, the scope of the second part of the dissertation is to investigate the elastic mechanical behavior of imperfect 3D truss-lattice materials with defects in the form of missing struts from the lattice domain, bridging the gap with continuum theories. The purpose of this investigation is to identify the contribution of geometry, topology, lattice connectivity and material anisotropy to the degradation rates of the elastic material properties. Two scenarios of defect spatial arrangement are considered: a) clusters of members being periodically removed from the lattice matrix, and b) random dispersion of missing struts. The study particularly emphasizes on the impact of connectivity $Z$ on the degradation of the elastic properties, and in view of this, the selected topologies span a wide range of $Z$ values. The findings from the finite element simulation approach are compared to analytical models for isotropic homogeneous materials with similar defect patterns, as well as experimental specimens of defect-containing truss-lattices fabricated with the two-photon lithography approach. Overall, the study reveals the range of connectivity at which truss-lattice materials respond to defects in a fashion which is almost indistinguishable from that of homogeneous ones, with the experiments confirming that this behavior can be realized in practice.

Part II of the dissertation has the following outline. After the Introduction, Section 7 presents the truss-lattice architectures to be investigated, discusses their elastic properties, and for each defect scenario it presents the details of the numerical, analytical and experimental characterization approach. Section 8 proceeds with the findings of the study, presenting first the case of periodic voids distribution and then the case
of random strut removal. Section 9 summarizes the second part of the dissertation, presents the main conclusions of the study, and discusses promising future research projects to further explore the mechanics of defect-containing architected materials.
CHAPTER 7

ANALYSIS METHODOLOGY

7.1 Truss-lattice topologies

Four architectures with varying coordination numbers $Z$ are considered in the present study. The values of the selected $Z$ as well as CAD models of the unit cell of each topology are shown in Figure 7.1. All of these architectures have similarly situated nodes and they exhibit at least cubic symmetry, if their struts have a circular cross section. Table 7.1 summarizes the general properties of the truss-lattices under consideration. The Tetrakaidecahedron architecture, which is also commonly referred to as Kelvin cell [103], is the least connected network with a coordination number $Z = 4$ and can be categorized as a non-rigid bending-dominated material. The Octahedron architecture has $Z = 8$ and it is a rigid bending-dominated topology. The Octet has $Z = 12$, and is rigid stretching-dominated material. Finally, the last topology is the recently proposed architecture by Gurtner and Durand [104] and it has $Z = 14$, being a rigid stretching-dominated material. This architecture is the only one that consists of two distinct families of struts, with uniform length and cross section area existing within each family. All of the architectures are represented within a cubic unit cell other than the Gurtner-Durand. This architecture was originally described within a tetrakaidecahedral unit cell, however a tetragonal unit cell is used here to emphasize the similarity between all topologies. The planes of cubic symmetry are parallel to the faces of the depicted unit cells for all topologies other than the Gurtner-Durand architecture. For this architecture the symmetry planes are normal to the vectors $\langle 1, 0, 1 \rangle$, $\langle 1, 0, 1 \rangle$, and $\langle 0, 1, 0 \rangle$, using the coordinate system defined in Figure 7.1.
Figure 7.1: Truss-lattice topologies under consideration. Depiction of the unit cells used in this study for the a. Gurtner-Durand, b. Octet, c. Octahedron, and d. Tetrakaidecahedron truss-lattice architectures.

The numerical results of this study are experimentally validated using the two-photon lithography approach, a fabrication technique which typically yields elliptical members in the truss-lattice domain. To maintain consistency between the numerical and experimental aspects of this investigation, the truss-lattice struts were assigned an elliptical cross section with a major axis that is twice the length of the minor axis, elongating the struts cross section in the y-axis (Figure 7.1). Apparently, one exception exists for the Gurtner-Durand struts that are parallel to the y-axis, in which case the strut major axis is assigned to be in the z-direction. The elliptical cross-section lowers the level of stiffness matrix symmetry from cubic (3 independent elastic constants) to orthotropic (9 independent elastic constants).

A choice has to be made over the geometric constant which is kept constant across all architectures. Among a variety of available choices, including but not limited to
Table 7.1: General geometric characteristics of the investigated truss-lattice architectures. Coordination number ($Z$), relative density ($\bar{\rho}$) and classification of the truss-lattice architectures under the Maxwell’s criterion and the primary deformation mechanism. In the calculation of the relative densities, the struts overlap in the nodal regions has been neglected. The strut aspect ratio is equal to $\lambda = 19.17$.

<table>
<thead>
<tr>
<th>Architecture</th>
<th>$Z$</th>
<th>$\bar{\rho}$</th>
<th>Maxwell’s criterion</th>
<th>Deformation mode</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tetrakaidecahedron</td>
<td>4</td>
<td>0.00833</td>
<td>Non-rigid</td>
<td>Bending</td>
</tr>
<tr>
<td>Octahedron</td>
<td>8</td>
<td>0.0333</td>
<td>Rigid</td>
<td>Bending</td>
</tr>
<tr>
<td>Octet</td>
<td>12</td>
<td>0.0666</td>
<td>Rigid</td>
<td>Stretching</td>
</tr>
<tr>
<td>Gurtner-Durand</td>
<td>14</td>
<td>0.0833</td>
<td>Rigid</td>
<td>Stretching</td>
</tr>
</tbody>
</table>

the relative density, unit cell volume and strut aspect ratio, the latter is selected. The strut aspect ratio $\lambda$ is defined as the ratio of the strut length to the radius of a circle that has the same area as the strut cross section. For all architectures, $\lambda = 19.17$. Maintaining the same strut aspect ratio for all architectures emphasizes on topological differences over geometric ones, whereas the wide range of the coordination numbers under investigation hinders the adoption of other parameters such as the relative density, which would result in extreme cross section choices for the architectures outliers (either very slender members for the Gurtner-Durand architecture or very stocky members for the Tetrakaidecahedron). The choice of a constant strut aspect ratio results in a different relative density $\bar{\rho}$ for each topology, and the $\bar{\rho}$ value of each topology are listed in Table 7.1. Since the Gurtner-Durand architecture has two families of equal length struts, the average strut aspect ratio was set to $\lambda = 19.17$. In addition to this constraint, each strut was set to occupy the same volume, irrespective of which family it belongs to. The latter constraint allows the change in relative density from strut removal to be determined by the number of struts removed alone, without having to consider which family they belong to. It should be noted that the Gurtner-Durand architecture was originally proposed with different cross-sectional dimensions for the two families of struts, chosen specifically so that the
Table 7.2: Elasticity tensor entries of the intact truss-lattices. The strut aspect ratio is $\lambda = 19.17$. The bulk material is assumed to have an elastic modulus equal to 2000 and a Poisson’s ratio of 0.3. For the excluded elastic constants: $C_{33} = C_{1111}$, $C_{2233} = C_{1122}$, and $C_{2323} = C_{1212}$, except for the Gurtner-Durand, where $C_{2323} = 12.3$ as a result of the orientation chosen for the elliptical cross-section on struts that are parallel to the y-direction. The universal anisotropy index, $A^U$, is provided as well. The bulk modulus can be calculated as $K = C_{iijj}/9$.

<table>
<thead>
<tr>
<th>Architecture</th>
<th>$C_{1111}$</th>
<th>$C_{2222}$</th>
<th>$C_{1122}$</th>
<th>$C_{1133}$</th>
<th>$C_{1212}$</th>
<th>$C_{3131}$</th>
<th>$A^U$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tetrakaidecahedron</td>
<td>2.06</td>
<td>2.10</td>
<td>1.97</td>
<td>2.01</td>
<td>0.0317</td>
<td>0.0172</td>
<td>0.988</td>
</tr>
<tr>
<td>Octahedron</td>
<td>12.3</td>
<td>12.4</td>
<td>5.91</td>
<td>6.00</td>
<td>6.09</td>
<td>6.07</td>
<td>0.524</td>
</tr>
<tr>
<td>Octet</td>
<td>24.5</td>
<td>24.7</td>
<td>11.8</td>
<td>12.0</td>
<td>12.3</td>
<td>12.2</td>
<td>0.531</td>
</tr>
<tr>
<td>Gurtner-Durand</td>
<td>37.9</td>
<td>39.7</td>
<td>11.7</td>
<td>13.4</td>
<td>12.2</td>
<td>13.7</td>
<td>0.0180</td>
</tr>
</tbody>
</table>

architecture yields maximal stiffness and isotropy. To achieve these properties, the ratio between the cross section areas of the shorter family of struts to the longer has to be 1.299. Although the latter geometric constraint is relaxed in the present study, the architecture adopted herein is still referred to as the Gurtner-Durand architecture, since these investigators were the first to propose this particular arrangement of struts.

Next, the elasticity tensors of the orthotropic intact truss-lattices were numerically calculated and the values of the tensors entries are shown in Table 7.2. The last column of Table 7.2 contains the universal anisotropy index which is calculated as:

$$A^U = 5\frac{\mu^V}{\mu^R} + \frac{K^V}{K^R} - 6 \quad (23)$$

where $\mu^V$ and $\mu^R$ are the Voigt and Reuss estimates for the shear modulus and $K^V$ and $K^R$ are the Voigt and Reuss estimates for the bulk modulus. This index takes a minimum value of 0 for isotropic materials, and increasing values for the index indicate departure from isotropy. Based on the values of Table 7.2, it can be seen that the isotropy outliers are the Gurtner-Durand architecture (least anisotropic) and the Tetrakaidecahedron (most anisotropic). The observed divergence from isotropy is
attributed to the ellipticity of the truss-lattice members, a feature which appears to have a rather negligible impact on the isotropy of the Gurtner-Durand architecture. Finally, it is important to note that each Octet elastic constant is almost exactly double the corresponding constant of the Octahedron. The particular correlation between their elastic constants results to the two architectures having nearly identical anisotropy, which is demonstrated by the almost same $A^U$ values for each topology. These two topologies have therefore same anisotropy but different coordination numbers, and this observation is very instructive as it provides the opportunity to isolate the contribution of the coordination number in the mechanical response of these truss-lattices once defects are introduced.

### 7.2 Finite element modeling approach

Numerical simulations were conducted using the general purpose finite element software ABAQUS/Standard. All struts were discretized as Timoshenko beam elements with one integration point per element (element type B31). Following a mesh convergence study, a single element per strut was found to be sufficient to characterize the small deformation elastic properties for the Gurtner-Durand, Octet, and Octahedron architectures. Five elements per strut were required for the Tetrakaidecahedron architecture, potentially due to the substantial bending deformations that are generated in this architecture. All simulations were conducted in the linear elastic regime, neglecting the effect of material and geometric nonlinearities. All finite element models consisted of a suitably sized supercell with periodic boundary conditions, rendering the numerical results free of edge effects (fixed or free material faces). The supercell sizes which were selected are associated with the various defect scenarios, and they are discussed in greater detail in the following sections.

For all architectures and defect patterns investigated (other than the non-uniform
void distribution), three effective elastic moduli were calculated: bulk ($K^*$), shear ($\mu^*$) and Young’s ($E^*$) modulus. The bulk modulus was calculated through the application of a hydrostatic pressure in the material supercell, based on Equation 24:

$$K^* = \frac{\sigma V}{\Delta V}$$  \hspace{1cm} (24)$$

where $K^*$ is the effective bulk modulus, $\sigma$ is the hydrostatic pressure, $V$ is the initial volume of the supercell and $\Delta V$ is the volumetric change of the supercell. $\Delta V$ was calculated as follows:

$$\Delta V = (1 - \epsilon_x)(1 - \epsilon_y)(1 - \epsilon_z)V$$  \hspace{1cm} (25)$$

where $\epsilon_i$ ($i = x, y, z$) is the average SVE strain in each of the three axes. The shear modulus was calculated by considering the strain energy stored within the supercell when subjected to the loading state of shear strain, as follows:

$$G^* = \frac{U_G}{2V \epsilon_G^2}$$  \hspace{1cm} (26)$$

where $G^*$ is the effective shear modulus, $U_G$ is the strain energy due to the shear loading (calculated by ABAQUS), $V$ is the initial volume of the supercell and $\epsilon_G$ is the imposed shear strain. Finally, the Young’s modulus was calculated by applying to the material a strain-controlled uniaxial stress state, based on Equation 27:

$$E^* = \frac{2U_E}{V \epsilon_E^2}$$  \hspace{1cm} (27)$$

where $E^*$ is the effective Young’s modulus, $V$ is the initial volume of the supercell and $\epsilon_E$ is the imposed axial strain. It has to be noted that due to the orthotropic nature of the materials, there exists a dependency on the orientation for the shear
and Young’s modulus. In this study, the shear modulus in the YZ plane and the Young’s modulus along the Y axis were probed, following the notation of Figure 7.1.

7.2.1 Truss-lattices with periodic voids

To investigate the effect of periodically distributed voids on the truss-lattice architectures, the properties of supercells comprised of $N \times N \times N$ unit-cells with $p$ voids of size $n \times n \times n$ unit cells were calculated, using the unit cell definitions given in Figure 7.1. With this description of a voided material, the void configuration of any architecture depends on three parameters: (i) the size of a void relative to the size of the unit-cell, defined by $n$; (ii) the void volume fraction, $f = p n^3 / N^3$; and (iii) the relative position of the voids. For the case of $p = 1$, subsequently referred to as the case of uniformly distributed voids, the voids fall on a simple-cubic lattice for all architectures that are defined on a cubic unit cell. For the Gurtner-Durand architecture the voids are located on a tetragonal lattice. For $p > 1$, the distribution of voids will subsequently be referred to as non-uniform, and in this case the relative position of the voids becomes an additional variable. A generic example of a voided tessellation with $p = 1$, $n = 3$ and $N = 6$, corresponding to a volume fraction $f = 0.125$, is shown in Figure 7.2a, whereas Figure 7.2b depicts the four truss-lattice topologies with the particular void arrangement. Section 8.1 reports the results of the investigation conducted to explore the effect of each of the three parameters that define the voided geometry.

7.2.2 Truss-lattices with randomly excluded members

Since the geometry of a truss-lattice material with randomly excluded struts is inherently stochastic, its behavior is estimated through the concept of the stochastic volume element (SVE) [105]. Two well-established criteria need to be satisfied in
Figure 7.2: Schematic view of a void configuration and its application in the truss-lattice architectures. **a.** Voided tessellation with $p = 1$, $n = 3$ and $N = 6$, corresponding to a volume fraction $f = 0.125$. The removed unit cells are highlighted in light blue color. **b.** Numerical models of the four truss-lattice topologies with the particular void geometry. As the coordination number decreases, the presence of the void in the central area of the supercell becomes visually more apparent.

In this study, preliminary analyses for the Young’s modulus of the Octet architecture with $f = 0.15$ were performed with supercell sizes $N = 6, 7$ and 8, where in the

order to obtain the property of a material that is independent from the property of the finite domain size and realization of the random field applied to that domain. The first criterion is termed *spatial averaging*, and requires a sufficiently large supercell to be investigated. If the supercell is too small then the periodicity of the randomly excluded struts will cause an error in the estimate, whereas the upper bound for the supercell size is established in such a way that the computational cost remains tractable. The second criterion, *ensemble averaging*, requires the investigation of a sufficiently large set of random defect realizations. The combination of spatial and ensemble averaging allows for an efficient and accurate estimate of the average properties for a stochastic material [105, 106].

In this study, preliminary analyses for the Young’s modulus of the Octet architecture with $f = 0.15$ were performed with supercell sizes $N = 6, 7$ and 8, where in the
context of random defects $f$ is the fraction of struts which are selected for exclusion. The results of this investigation are depicted in Figure 7.3. For each supercell size, ensemble averaging was performed on a set of 1000 random realizations of excluded struts. As it can be observed in Figure 7.3, all three supercell sizes returned the same mean value for the normalized Young’s modulus within 0.01%. Although larger than required for the Octet architecture, the supercell size $N = 8$ was selected for all numerical models with random defects. This allows for a consistent supercell size across all architectures, while also allowing for the possibility that other architectures may require larger supercell sizes than the Octet for convergence.

7.3 Experiments and analytical methods

7.3.1 Specimens fabrication and testing

The results of the numerical approach were validated against a series of experiments which were fabricated and tested by Andrew Gross (Harvard). Fabrication and
The post-process of the truss-lattice specimens was conducted in the Center For Nanoscale Systems (CNS) at Harvard University, while in situ mechanical testing was performed at Charles Stark Draper Laboratory. In what follows, a brief description of the fabrication route and testing methods is provided, whereas more details on the experimental aspect of this study can be found in Gross et al. [3].

The two-photon lithography (Nanoscribe GmbH) was adopted for the experimental validation of the FEM results. This is a top-down additive manufacturing approach, which was selected among the candidate techniques in order to have strut feature sizes as close as possible to the length scale most common to self-assembly techniques. Two-photon lithography was performed in the DiLL configuration with IP-Dip photoresist. After patterning, the specimens were developed in PGMEA (Baker BTS-220) for 20 minutes, followed by 5 minutes in IPA (J.T. Baker), and super-critical point $CO_2$ drying. The specimens were fabricated on a foundation from the same polymer material, in order to mitigate distortion triggered from post-fabrication shrinkage. A block was adhered to the specimens top surface, to provide well-defined boundary conditions and to ensure appropriate alignment of the nano-indenter. Following two-photon lithography, the polymer specimens were coated with $Al_2O_3$, to ensure consistent material properties throughout each specimen and between specimens. $Al_2O_3$ was deposited over the course of 500 atomic layer deposition cycles, using TMA and DI water as precursors. Measurement of the coating thickness on the substrate with a scanning ellipsometer (Gaertner Scientific, LSE-W) indicated that $58.5 \pm 0.4 \text{ nm}$ of $Al_2O_3$ was deposited. Postmortem scanning electron microscope (SEM) images suggested that the deposition rate of $Al_2O_3$ on the polymer specimens was nearly identical to that on the silicon substrate.

Mechanical testing of the specimens was performed with an in-situ SEM nano-indenter (Femto Tools). Displacement was applied by a linear piezo-flexure stage.
with capacitive position encoders, and the reaction force was measured by a MEMS capacitative force sensor with a resolution of 0.05 µN. Five load cycles were applied in the linear elastic range, and testing was terminated when the load reached 50% of the value at which the relationship between load and displacement diverged from linear for a pristine specimen of the same architecture. Pristine and defected specimens of two families of architectures were fabricated, the Octet and the Tetrakaidecahedron, and all defect defect scenarios were taken into consideration (uniform/non-uniform void distribution and randomly excluded members).

7.3.2 Elastic micromechanical models

The homogenized elastic behavior of materials with multiple phases of homogeneous constituents has been described by a number of analytical models with a micromechanical basis [107–109]. These models are applied in the present context to predict the degradation rates of the elastic constants of the two-phased defected truss-lattices, assuming that the first phase (matrix) is the pristine lattice and the second phase (inclusions) is the space which is introduced by the lattice struts removal. It is important to note here that there is a series of approximations along this process. In particular, these models: a) assume a continuous matrix and they are incapable of capturing the impact of topology (for example the coordination number $Z$ is excluded), b) assume an initially isotropic matrix, which contradicts with the orthotropic nature of the pristine truss-lattices, and c) they assume a spherical inclusion shape, introducing an additional approximation compared to the defected truss-lattices. They are nevertheless adopted in the present investigation with the aim to provide further insight on the elastic response of defect-containing truss-lattices, and interpretation of the results in Section 8 extensively reflects on these simplifications.
These models take as input a) the homogenized elastic constants in the pristine state of the lattice, which are selected from Table 7.2, and b) the defect volume fraction. Their output are the decreasing trends for the elastic constants under investigation. Due to the discrepancy between the isotropic nature of the analytical models and the orthotropic nature of the lattices, there exists a choice for which two elastic moduli should be selected as input to the elasticity solution. The bulk modulus is chosen due to its orientation independence, whereas for the case where orientation bias can not be avoided, the shear modulus of Equation 26 is probed (constant $C_{1212}$ in Table 7.2). Analytical models are adopted for lattices with uniformly distributed voids (i.e. $p = 1$) and randomly excluded struts. In contrast, the case of multiple voids with arbitrary relative position within a unit cell (i.e. $p = 2$) is examined only with numerical analyses and experiments, since micromechanical models are not readily available for this type of imperfection.

For the case of truss-lattices with uniformly spaced voids, the analytical approach developed by Nemat-Nasser and his colleagues [110–112] is adopted. Spherical voids of radius $R$ occupying the domain $\Omega$ on a simple cubic lattice with lattice constant $\Lambda = (4\pi R^3/3f)^{1/3}$ are assumed, and this arrangement introduces an additional approximation for the Gurtner-Durand architecture since for this lattice the voids in the numerical model are located on a tetragonal lattice rather than a simple cubic lattice. Calculation of the effective bulk and shear modulus requires the calculation of the average transformation strains for the cases of dilatational stress and shear loading respectively [111], and the detailed derivation of these equations can be found in Gross et al. [3]. For the case of truss-lattices with randomly excluded struts the selected analytical model is the Mori-Tanaka method [109]. This model assumes that a distribution of either random or aligned dilute ellipsoidal inclusions exist in a matrix with a uniform average stress. Since here we limit consideration to the case of an
isotropic matrix with spherical voids, the solution of the Mori-Tanaka model coincides with that of the Hashin-Shtrikman upper bound [113] for both the effective bulk and shear moduli, which therefore can be calculated as:

\[ K^* = K - f \left( \frac{1}{K} - \frac{3(1 - f)}{3K + 4\mu} \right)^{-1}, \]  
\[ (28) \]

\[ \mu^* = \mu - f \left( \frac{1}{\mu} - \frac{6(1 - f)(K + 2\mu)}{5\mu(3K + 4\mu)} \right)^{-1}, \]  
\[ (29) \]
CHAPTER 8

ELASTIC MECHANICAL PROPERTIES OF TRUSS-LATTICE MATERIALS WITH MISSING MEMBERS

8.1 Truss-lattices with voids

For the elastic response of truss-lattice materials with voids, the influence of three parameters is examined: the effects of void size, the impact of the void volume fraction, and the relative position between voids.

8.1.1 Effect of void size

For a homogeneous continuum material with uniformly distributed voids, the effect of a void is dependent only on the void volume fraction, with the absolute size of the void having no impact on the modulus reduction. Conversely, for heterogeneous materials such as the discrete truss-lattice materials considered here, the effect of a void that exists near the characteristic length scale of the material which is taken as the strut axial length, is expected to be dependent on the absolute size of the void. In other words, it is only when a separation of scales exists between the size of the void and the length scale of the material (i.e. when $n >> 1$) that the effect of the void will converge to the effect it would have in a homogenized representation of the heterogeneous material. To investigate such scale effects, the number of voids is fixed to $p = 1$ and the absolute void size $n$ ranges as $n = 2, 3, 4$. Then the Young’s, shear, and bulk moduli were computed while varying the supercell size $N$ in order to cover
Figure 8.1: Dependency of the Young’s modulus on the absolute void size. Results shown for a. Gurtner-Durand, b. Octet, c. Octahedron, and d. Tetrakaidecahedron. The higher connected Gurtner-Durand ($Z = 14$) and Octet ($Z = 12$) are essentially insensitive to the void size, whereas the least connected Octahedron ($Z = 8$) and Tetrakaidecahedron ($Z = 4$) are converging from below to the macroscopic (effective) modulus value.

a sufficiently informative range of volume fractions. The results of this investigation are plotted in Figures 8.1, 8.2 and 8.3 respectively.

Undoubtedly, the most striking trend from these plots is that for the more highly connected Octet and Gurtner-Durand architectures and across all elastic constants, there is essentially negligible sensitivity on the absolute void size $n$. This indicates that, under the specific assumptions on the void definition (unit-cell based), these topologies behave in a fashion which is identical to homogeneous materials, independent on the absolute void size. On the other hand, for the architectures with lower
Figure 8.2: Dependency of the shear modulus on the absolute void size. Results shown for the a. Gurtner-Durand, b. Octet, c. Octahedron, and d. Tetrakaidecahedron. The results indicate a void size independence for the highly connected Gurtner-Durand and Octet, in contrast to the least connected Octahedron and Tetrakaidecahedron.

coordination numbers, Tetrakaidecahedron and Octahedron, the trends for varying $n$ do not coincide with each other, indicating a dependency on the absolute void size when the void size scale is comparable to the truss-lattice characteristic length (strut) scale. This behavior is anticipated in heterogeneous media, and the investigation was extended for these two topologies as follows: for a fixed volume fraction $f = 0.125$, the absolute void and supercell size were simultaneously increased and the converging trends of the moduli were identified. The results are shown in Figure 8.4, where the elastic moduli seem to converge in an exponential-like form to that of the equivalent continuum material as $n$ increases. The rate of convergence though is different
Figure 8.3: Dependency of the bulk modulus on the absolute void size. Results shown for the a. Gurtner-Durand, b. Octet, c. Octahedron, and d. Tetrakaidecahedron (note that the y-axis range is different in plot d, due to the abrupt drop of the modulus at small f values). The observations from Figures 8.1 and 8.2 can be extrapolated here as well.

for each architecture and elastic constant, and a comparison of the curves for the Tetrakaidecahedron and Octahedron architectures shows that the shear moduli spans a larger range for the former, while the Young’s modulus spans a larger range for the latter. Thus, for architectures that are sensitive to void size effects, there are factors in addition to the coordination number that govern the response to voids.

Overall, these results are consistent with the idea that the effect of $Z$ decreases as the separation of scales between void size and material length scale increases. That is, as $n$ grows large, the response to defects depends solely on the elastic constants of the effective medium. It is for this reason that the results for the Octahedron
8.1.2 Effect of void volume fraction and comparison to analytical models

The effect of void volume fraction for a given modulus and across all topologies is shown in Figure 8.5 (bulk, shear) and Figure 8.6 (Young’s). The numerical results on these plots (solid curves) correspond to a void size $n = 3$, so that the results correspond to a void size in the scale dependent regime. The supercell size $N$ is varied between 5 and 10, obtaining a range of volume fractions between 0.027 - 0.216. The second curve (dashed lines) for each topology corresponds to the analytical estimations of the model by Nemat-Nasser et al. [111]. The most apparent trends from
the simulations are consistent with the literature on solids containing voids and two-dimensional truss-lattice materials [92, 93, 95]. Namely, the elastic properties degrade with increasing void volume fraction for all materials, and the sensitivity to defects tends to be less for architectures with higher coordination number. Additionally, across most of the range of void volume fraction reported, the ordering of both moduli follow the same sequence observed in Figures 8.1 - 8.4 at \( n = 3 \). A more in-depth observation of Figure 8.5 leads to the following observations and comments:

- As it was mentioned in Section 7.1, the Octet and the Octahedron architectures have the same level of anisotropy. Therefore, since the input variables in the analytical model are just the pristine elastic constants and the volume fraction, it is natural to expect almost identical analytical curves for the normalized effective moduli of these two topologies. This is verified in Figure 8.5a and 8.5b, where the analytical estimations for the two architectures coincide so closely that the Octet curves are visually obstructed.

- Since the coordination number is excluded from the analytical scheme, and the FEM results of the Octet architecture have been shown to be independent of the void size effects, the discrepancy between the analytical and numerical Octet curves can be fully attributed to the material anisotropy which is excluded from the analytical formula. A similar comment can be made for the Gurtner-Durand architecture, and this observation is further validated here since this topology is more isotropic than the Octet and the discrepancy between the analytical and numerical curves is generally smaller in this case (Figure 8.5a).

- Based on the above comments, it is safe to conclude that the gap between the Octahedron numerical and analytical curves is due to: a) the material anisotropy, from the Octahedron analytical curve until the level of the Octet
Figure 8.5: Investigation of the void volume fraction effect in a uniform void distribution. Solid lines correspond to FEM results and dashed lines to the analytical solution, for the a. bulk modulus and b. shear modulus. The Octahedron and Tetrakaidecahedron numerical curves correspond to a void size $n = 3$. The shear modulus numerical curves of the Octet and Gurtner-Durand architectures coincide so closely that the Gurtner-Durand curve is visually obstructed. The analytical curves for the Octahedron and Octet coincide so closely for both moduli that the Octet curves are visually obstructed. c-d. The percent difference ($\cdot \%$) between the numerical and analytical models for c. bulk modulus and d. shear modulus for the voided truss-lattices.

Numerical curve, and b) the coordination number, between the Octet and the Octahedron numerical curves. In other words, the discrepancy between the numerical curves of these two topologies is fully attributed to the low coordination number of the Octahedron and the associated void size effects, which are present here since $n = 3$.

These observations are of paramount importance since they provide a steady basis to isolate the effect of the material anisotropy and coordination number to the truss-
lattices mechanical response, identifying in a quantitative fashion the sources of discrepancy between the analytical predictions and numerical estimations. Nevertheless, although the coordination number and material anisotropy are crucial parameters in this investigation, there are additional factors which differentiate the considered architectures as well. In particular, the elastic constants of each architecture are related to each other in a fashion which is topology-dependent, and this is an aspect which must also be considered when interpreting these results. For example, the rapid reduction of the bulk modulus for the Tetrakaidecahedron at small void volume fractions may easily be perceived as a result of its low coordination number, however this behavior is present in the results of the analytical model as well (Figure 8.5a). The analytical model is insensitive to the coordination number and assumes an isotropic homogeneous material outside the voids, yet it is capable of capturing this sudden drop. This comparison reveals that the source for the precipitous drop in the Tetrakaidecahedron bulk modulus is more a consequence of the effective material being near the incompressible limit (i.e. $K >> \mu$, as it can be calculated from the elastic constants provided in Table 7.2), than the low coordination number of this architecture.

Finally, the percent error of the analytical model is depicted in Figure 8.5c and 8.5d, denoted as the modulus with a superscript $E$ and calculated as $(\cdot)^E = \left[\frac{(\cdot)_{\text{anal}} - (\cdot)_{\text{FEM}}}{(\cdot)_{\text{FEM}}}\right] \times 100\%$, where $(\cdot)$ is either $K$ or $\mu$ for the bulk and shear modulus respectively. These plots provide additional insight on the difference between the results of the numerical and analytical models for the effective bulk and shear moduli. It can be seen that the percent error between the analytical and numerical models is inversely proportional to the coordination number of the truss-lattice architecture. Furthermore, the analytical model error generally increases as the volume fraction $f$ increases. The rate of this increase is dependent on the topology, with the bulk modulus of the Tetrakaidecahedron being the only outlier from this trend. It is noted
Figure 8.6: Numerical dependence of the Young’s modulus on the void volume fraction for a uniform void distribution. For the Octahedron and Tetrakaidecahedron, where void size effects exist, the curves correspond to a void size \( n = 3 \).

that for the highly connected Gurtner-Durand architecture the difference between the analytical and numerical models is relatively small. Even at the largest defect volume fraction considered, \( f = 0.216 \), the analytical model predicts a value for the bulk and shear moduli only 5.5% and 12.3% higher than the numerical calculations, respectively, despite the number of differentiating factors between the two approaches (the analytical method excludes \( Z \) and assumes an isotropic matrix with spherical voids).

These observations provide further evidence that the elastic mechanical performance of defect containing truss-lattice materials with sufficiently high coordination numbers can be reasonably described by models developed for classical Cauchy continua.

Figure 8.6 presents the results on the Young’s modulus numerical investigation. The dependence of the Young’s modulus on the void volume fraction is observed to be similar to the other moduli reported in Figure 8.5. Again, the ordering of architectures by modulus is consistent with the sequence observed in Figures 8.1 - 8.4 at \( n = 3 \) across the range of void volume fraction reported here. Therefore, it can be inferred that void size effects and differences in the effective elastic constants are responsible
for the curve of the Octahedron lying beneath the Tetrakaidecahedron, while only the latter are responsible for the curve of the Octet lying above the Gurtner-Durand.

Experimental verification of the numerical models with the uniform void distribution was conducted by comparing experimental and FEM results for two families of voided lattices, the Octet and the Tetrakaidecahedron. For each of the two truss-lattice architectures, pristine and voided specimens based on a $9 \times 9 \times 9$ tessellation of unit cells were fabricated. The pristine Octet specimen is depicted in Figure 8.7a, with a normal view focusing on 9 unit cells shown in Figure 8.7b. For the specimens containing voids, 27 regularly spaced unit cells were removed to form a simple cubic arrangement of voids within the specimens. This can be visualized as a $3 \times 3 \times 3$ tessellation of the voided supercell depicted in Figure 8.7c, where the unit cell highlighted in light blue color is removed. The voided tessellations have a void volume fraction $f = (1/3)^3 = 0.037$. An SEM image of the voided Tetrakaidecahedron is shown in Figure 8.7d and 8.7e (close-up view). The presence of the voids can be seen in regions where there appears to be light filtering in from behind the lattice. The struts in both truss-lattice topologies are 4.75 $\mu$m long, and they have an elliptical cross section with major and minor diameters of 1140 and 555 $\pm 15$ nm respectively.

The experimental results were compared to an additional numerical model with void size $n = 1$ and supercell size $N = 3$, to preserve the impact of the void size effect between the two approaches. Consistent with the results from the void size study, this numerical model yields a lower prediction for the modulus of the Tetrakaidecahedron than the result reported in Figure 8.6, since this model uses a smaller void size. Comparison of the numerical and experimentally measured effective Young’s modulus values is shown in Figure 8.7f, and a very satisfactory agreement between the two approaches for both architectures can be observed. Potential sources of discrepancy are the compliance of the load frame, the finite size of the fabricated specimens vs the nu-
Figure 8.7: Experimental verification of the FEM approach for the case of uniform void distribution. 

**a.** 3D view of a pristine 9×9×9 Octet specimen. 20µm scale bar. **b.** Normal view focusing on 9 Octet unit cells. 5µm scale bar. **c.** Schematic of a voided 3×3×3 supercell, with the central unit cell removed (colored light blue). **d.** Normal view of a voided 9×9×9 Tetrakaidecahedron specimen. 50µm scale bar. **e.** High magnification image focusing on the voids located in the bottom corner of the specimen in (e). **f.** Comparison of the numerical and experimental results for the case of uniform void distribution in the Octet and the Tetrakaidecahedron architectures.

Numerically periodic idealization, the increased flexural stiffness of the composite beams in the experiment due to the alumina coating, variation in properties of the polymer, and geometric defects (e.g. nodal position) of the polymer template. Nonetheless, the agreement between the FEM and experimental results provides sufficient verification of the approach used for the numerical simulation. Furthermore, it is demonstrated that the effect of defects in experimental specimens with nanoscale feature sizes can be accurately captured with scale independent models of beam networks.
8.1.3 Effect of void relative positioning

Although the analysis of uniformly distributed voids is critical to understand the fundamental behavior of the different architectures to voids, in most self-assembled materials the void population will be disordered, motivating the investigation of non-uniform void distributions. Through this prism, the analysis on the effect of void relative position started by probing the Young’s modulus for a supercell with $N = 6$ that contains two voids ($p = 2$), each of size $n = 2$. This void configuration results in a defect volume fraction of $f = 2/27 \approx 0.0741$, and its schematic representation is shown in Figure 8.8. The first void is always placed at the center of the supercell, while taking advantage of symmetry in the relative void position and loading, the second void is placed in one of the 34 available positions in a sixteenth of the supercell. The
Figure 8.9: Young’s modulus dependence on voids relative position. Results shown for the a. Gurtner-Durand, b. Octet, c. Octahedron and d. Tetrakaidecahedron, following the voids arrangement of Figure 8.8. The color of each point corresponds to the normalized effective Young’s modulus and the dashed lines connect neighboring points which have equal azimuth angle \( \phi \), with the angle value given beside each line. The cases corresponding to the maximum and minimum effective modulus are also indicated.

Relative position of the two voids can be fully described by \( \rho \), \( \theta \) and \( \phi \), where \( \rho \) is the distance between the voids centers, \( \theta \) is the zenith angle and \( \phi \) is the azimuth angle (coordinate system defined in Figure 8.8b).

The numerical results of this investigation are plotted in Figure 8.9. It was revealed that the Young’s modulus in the loading direction is relatively insensitive to the azimuthal coordinate, \( \phi \), allowing for the presentation of the reduced modulus results with respect to only the radial and zenith spherical coordinates as shown in Figure 8.9.
Table 8.1: Statistical moments of the Young’s modulus variation with non-uniform voids distribution. The values in the column labeled Uniform Voids are interpolated from the trends of Figure 8.6. The results indicate that void volume fraction dominates the effective modulus reduction, nevertheless the impact of void relative positioning is non-negligible.

The data indicate that the most favorable position for the two voids is the same for every architecture, corresponding to the case of adjoining voids with the long direction of the adjoined void aligned to the loading direction. The most unfavorable position for the two voids is found to be architecture dependent. Simple statistical measures of these results are summarized in Table 8.1, which also includes the interpolated value of the modulus if a configuration of uniformly distributed voids existed at the same defect volume fraction. The results reported in Table 8.1 indicate that the uniform void distribution always has a slightly less deleterious effect on the modulus than the mean of all the non-uniform spacing cases, and tends to be near the center of the range of results for non-uniform void spacing. Given the mean and range for the non-uniform void spacing, it is clear that modulus reduction is dominated by void volume fraction and not void relative position, although the effect of the non-uniform spacing is certainly non-negligible. The importance of void volume fraction over void placement is further examined by calculating the reduction in bulk modulus for the three void arrangements associated with extremal values in Figure 8.9 for all architectures. The results for bulk modulus are consistent with those for Young’s in these extremal cases, further supporting the conclusion that the effect of void volume fraction dominates void arrangement.
Figure 8.10: Experimental validation of the non-uniform void distribution investigation. Results shown for two voided cases for the Octet and the Tetrakaidecahedron architectures. The void size is two unit cells ($n = 1$, $N = 3$, $p = 2$, resulting in $f = 0.074$), with the elongated side of the void placed either a. parallel ($\rho = L_C$ and $\theta = 0^\circ$) or b. normal ($\rho = L_C$ and $\theta = 90^\circ$) to the loading axis. c. Comparison of the normalized effective Young’s modulus values from experiments and simulation.

Experimental verification of the numerical models with non-uniform void distributions was conducted through examination of Octet and Tetrakaidecahedron specimens that are $3 \times 3 \times 3$ tessellations of the supercell depicted in Figure 8.7c, but with one additional unit cell removed adjacent to the central one in order to form an elongated void. Two orientations of the elongated void are considered for each architecture. In the first, the elongated dimension of the void is aligned with the loading axis, and in the second it is perpendicular to the loading axis. The former orientation corresponds...
to the most favorable void positioning for both architectures (\( \rho = L_C \) and \( \theta = 0^\circ \), where \( L_C \) is the length of the unit cell as defined in Figure 8.9 for each architecture), while the latter configuration is the most unfavorable for the octet topology (\( \rho = L_C \) and \( \theta = 90^\circ \)). Figure 8.10a shows an SEM image of a \( 9 \times 9 \times 9 \) Tetrakaidecahedron with the elongated void being parallel to the loading axis, and Figure 8.10b shows the fabricated \( 9 \times 9 \times 9 \) Octet specimen with the void being perpendicular to the loading direction. The volume fraction in both cases is \( f = 2 \times (1/3)^3 = 0.074 \).

The experimental values were compared to the numerical models of a \( 3 \times 3 \times 3 \) supercell with the same void arrangement and periodic boundary conditions, and comparison of the results is shown in Figure 8.10c. It can be seen that the experimentally measured values for all void arrangements lie within a few percent of the predicted values from the numerical models, with the numerical models consistently indicating a slightly lower estimation of the normalized Young’s modulus. The very good agreement between simulation and the specimens provides experimental support in the conclusion that the normalized modulus reduction is more sensitive to the void volume fraction than the relative void positioning, though the impact of the latter is confirmed to be non-negligible.

### 8.2 Truss-lattices with randomly excluded members

This section presents the results on the elastic mechanical response of truss-lattice materials which exhibit disordered defects in the form of randomly excluded struts. The numerical results on the effect of the volume fraction of randomly excluded members is shown in Figure 8.11 (bulk and shear) and Figure 8.12 (Young’s). In these plots, the data markers correspond to the mean values from the sets of 1000 simulations performed for each volume fraction of randomly excluded struts in an \( 8 \times 8 \times 8 \) periodic supercell. The dashed lines correspond to the results from the analytical
model of Mori-Tanaka (coinciding with the Hashin-Shtrikman upper bound), using Equations 28 and 29. The most apparent trend from the numerical results is that the rate of elastic property degradation with increasing void volume fraction is inversely proportional to the coordination number. This trajectory is particularly clear with regards to the Young’s modulus, though it is also present in the bulk and shear modulus plots. The Young’s modulus trends obtained here compare favorably with the literature.

Figure 8.11: Results on the bulk and shear modulus dependence on volume fraction for random defects. Solid lines correspond to the numerical results and dashed lines to the analytical solution, for the a. bulk modulus and b. shear modulus. The analytical curves for the Octahedron and Octet coincide so closely for both moduli that the curves for the octet are visually obstructed. c-d. The percent difference $(\cdot)^E$ between the numerical and analytical models for c. bulk modulus and d. shear modulus for the truss-lattice architectures with randomly excluded struts.
Figure 8.12: Numerical results on the Young’s modulus dependence on volume fraction for random defect distribution. A clear ordering between the coordination number and the slopes of the decreasing trends can be observed.

The trends shown in Figure 8.12 are in satisfying agreement with results reported in the literature. Wallach and Gibson [97] reported a linear decrease in the Young’s modulus of the Octet, with an approximate 17% reduction in the modulus for \( f = 0.10 \). A fairly similar behavior is observed here, following a linear decrease with a 19% reduction for the same volume fraction. The slight discrepancy is attributed to the different supercells analyzed in the two studies (fixed \( 6 \times 6 \times 1 \) supercells in [97] versus \( 8 \times 8 \times 8 \) periodic tessellations here), as well as the number of idealizations performed (100 in [97] versus 7000 in the present study). Additionally, Vajjhala et al. [114] reported a rough 50% modulus reduction for a 10% of defect in three-dimensional Tetrakaidecahedron materials, well-compared to the 54% reduction reported herein, given similar differences in the underpinning assumptions.

With the exception of the bulk modulus for the Tetrakaidecahedron, the relationships between the effective moduli and void volume fraction are nearly linear across the range of defect volume fraction investigated. As a result, these relationships can be characterized by the slope of the best fit line, a dimensionless number that physi-
Table 8.2: Slope values of the best fit lines for the FEM results regarding the reduction rates of the effective elastic moduli under random defect distribution. The values indicate the reduction rate in the elastic property per volume fraction of randomly excluded struts. It is observed that across all topologies, with the exception of the Tetrakaidecahedron, the reduction rates are similar across all moduli. 

<table>
<thead>
<tr>
<th>Architecture</th>
<th>$K$</th>
<th>$\mu$</th>
<th>$E$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tetrakaidecahedron</td>
<td>n/a</td>
<td>3.22</td>
<td>4.06</td>
</tr>
<tr>
<td>Octahedron</td>
<td>3.22</td>
<td>3.16</td>
<td>3.11</td>
</tr>
<tr>
<td>Octet</td>
<td>1.92</td>
<td>1.91</td>
<td>1.89</td>
</tr>
<tr>
<td>Gurtner-Durand</td>
<td>1.71</td>
<td>1.78</td>
<td>1.59</td>
</tr>
</tbody>
</table>

cally corresponds to the rate of reduction in the elastic property per volume fraction of randomly excluded struts. The absolute value of these slopes for each architecture and modulus are reported in Table 8.2. It is observed that for all of the architectures other than the Tetrakaidecahedron, the degradation rates are nearly identical across the three moduli. This consistency points toward the effect of coordination number. However, just as in the case of voids, the elastic properties of the effective-material must also be considered when analyzing the results. Indeed, it is again the nearly incompressible behavior of the Tetrakaidecahedron that greatly contributes to the increased sensitivity of its bulk modulus to random strut exclusions, which is also captured in the analytical model.

Further comparison between the analytical models and numerical results are instrumental in assessing the different contributions to the modulus reduction. Figure 8.11c-d show the relative error in the effective bulk and shear moduli respectively between the analytical models and numerical results. To illustrate the sources of the discrepancy between numerical and analytical curves, attention is drawn upon the response of the Octet and Gurtner-Durand architecture, which both have large coordination numbers but significantly different levels of anisotropy. For both architectures the error of the analytical model is quite low, and therefore it can be
concluded that the discrepancy is not extremely sensitive to anisotropy. Then the results for the Octahedron and Octet architectures are compared, the two truss-lattice materials which have nearly identical levels of anisotropy. The error of the analytical model is substantially larger for the Octahedron architecture, suggesting that the effect of coordination number dominates the error. These observations provide the strongest evidence yet that truss-lattice materials with lower coordination numbers are the most sensitive to randomly excluded struts. This remains true even when the effective elastic properties of the pristine material ensure a high sensitivity to defects, as in the case of the bulk modulus of the Tetrakaidecahedron. Moreover, the excellent agreement between the analytical models and the numerical results for the Octet and Gurtner-Durand architectures suggests that the critical observation from the preceding section on uniformly distributed voids can be extended to the case of randomly excluded struts. That is, for highly connected truss lattices (at least in the range of 12 – 14), the dependence of the elastic properties on random defects presents very strong similarities to that of continuous materials.

Truss-lattices with randomly dispersed defects were also experimentally tested for the Tetrakaidecahedron and Octet architectures. Three realizations of random defects were tested at defect volume fractions of \( f = 0.05, 0.1 \) and 0.15, on specimens with a tessellation size of \( 8 \times 8 \times 8 \). The defect realizations were chosen from the set of the 1000 realizations as the ones corresponding to the minimum, mean, and maximum reduction in stiffness, for each volume fraction. Figure 8.13 shows a gray-scale SEM image of the Octet experimental specimen with the defect realization that causes the mean reduction in stiffness at a defect volume fraction \( f = 0.15 \). The overlaid picture in fuchsia is an image from the numerical model, that shows the struts on the exterior surface for this particular realization of defects. It can be observed from the composite image that the intended defect realization is well captured in
Figure 8.13: SEM image of an Octet specimen with randomly excluded struts. The volume fraction is \( f = 0.15 \) and the specimen size is \( 8 \times 8 \times 8 \). An overlay of the struts on the exterior face for the prescribed defect realization is in fuchsia. 10 \( \mu m \) scale bar. The close-up image indicates the struts which are intentionally excluded, shown in light red. 5 \( \mu m \) scale bar.

The experimental specimen, which is further demonstrated in the close-up view in Figure 8.13.

The experimental results are plotted in Figure 8.14 with marker points, and the dashed lines correspond to the numerical estimation for the Young’s modulus of these two architectures. It is apparent that the measurements of the reduction in Young’s modulus with defect volume fraction from the experiments are in excellent agreement with the numerical results. Furthermore, no correlation exists between the defect realization and modulus reduction for the Octet specimens, with the effect of the selected defect realizations from the numerical study being outweighed by experimental scatter. This realization insensitivity provides strong evidence that sufficiently large experimental specimens have been tested to measure a material property, rather than
Figure 8.14: Experimental validation of the Young’s modulus dependence on volume fraction for random defect distribution. The experimental data are shown with markers and the numerical results with dashed lines, at various volume fractions of randomly excluded struts for the Octet and Tetrakaidecahedron architectures. The experimental specimens correspond to the defect realizations in the numerical study which experienced the minimum, mean, and maximum reduced modulus.

a property of the particular defect realizations. As a result of both the realization insensitivity and correspondence to simulation results, the behavior of the Octet architecture is experimentally confirmed to respond to the presence of random defects in a manner that is identical to a continuous material, at least for defect volume fractions up to $f = 0.15$. For the Tetrakaidecahedron specimens a stronger correlation between the particular defect realization and the elastic modulus exists, and it can be seen that the experimental results are sorted according to the numerical prediction at each defect volume fraction. Even with this larger dependency on the specific defect realizations, close matching was found between the average experimental response and the numerical results for the Tetrakaidecahedron architecture. The experimental values lie consistently slightly higher than the numerical estimation, and this behavior is attributed to the increased flexural stiffness of the specimens due to the alumina coating in the struts. This geometric characteristic is anticipated to have a greater im-
Random defects

Voids are preferable

correlation between truss-lattice coordination number and favorable defect pattern

So far, attention was confined to a single defect arrangement and subsequent investigation of the main parameters that dominate the degradation rates of the truss-
lattices effective moduli. In this section a slightly different perspective is adopted, aiming at further understanding of the truss-lattices response by comparing the effect that the two defect distributions have on their elastic properties. For this reason, the values of all moduli from the random strut removal are compared to their counterparts from the uniform void distribution, at the same defect volume fraction \( f = 0.125 \). The results of this comparison are shown in Figure 8.3, where the ratio of the effective modulus with random defects to the effective modulus with voids is plotted against the range of coordination numbers. The values of the effective moduli with random defects are linearly interpolated from the results at \( f = 0.1 \) and \( f = 0.15 \). The values of the effective moduli with a uniform void distribution have been calculated for a void size \( n = 15 \) (Figure 8.4), where the impact of void size effects are negligible for the least connected topologies, Octahedron and Tetrakaidecahedron. The data clearly reveal that as the coordination number increases, the random removal of struts becomes more preferential than the uniform void distribution. This trend exists across all architectures and moduli, with the Gurtner-Durand being the only architecture investigated for which random defect distribution is less detrimental than the uniform void arrangement for all elastic moduli. This finding provides preliminary yet strong evidence that highly connected lattice networks have a substantially improved performance to random defect distribution scenarios than defect arrangements which manifest in a periodic cluster fashion. Further investigation accounting for a wider range of defect volume fractions, elastic constants and coordination numbers is necessary to solidify this preliminary observation, as well as to shed light on the impact of additional factors (anisotropy, nodal overlap contribution, junction angle between struts, etc.) to the correlation between the truss-lattice topology and the least detrimental defect pattern.
CHAPTER 9

SUMMARY, CONCLUSIONS AND FUTURE WORK: PART II

9.1 Summary and concluding remarks

The second part of the dissertation delved into the correlation between the topological characteristics of defected truss-lattices and their elastic mechanical properties. The prime interest of this study was to shed light on the impact of the coordination number to the mechanical response of the truss-lattice under a defect distribution scenario, where defect was portrayed as struts being excluded from the material domain. In view of this, two defect patterns were considered, a) periodic arrangement of missing building blocks (uniform or non-uniform void distribution) and b) randomly missing members. Four truss-lattices of varying coordination numbers were considered and analyzed in their elastic regime for each defect arrangement.

The results of this investigation were supported by numerical (finite element method), analytical (elastic micromechanical models) and experimental (two-photon lithography, Nanoscribe GmbH) approaches, and revealed a characteristic of the truss-lattices which had not received adequate attention in past studies. In particular, it was found that the behavior of periodic imperfect truss-lattices with coordination number $Z \geq 12$ is almost indistinguishable from homogeneous materials. Under the presence of uniformly distributed voids, this observation was justified by the independence of the highly-connected lattices to the absolute void size (absence of void scale effects), in contrast to the response of less-connected architectures. Additionally, analytical micromechanical formulas which were developed for continuum media
with periodic spherical voids were applied to the voided lattices and their accuracy was shown to increase as the coordination number obtained higher values. Under the notion of randomly missing struts from the truss-lattice domain, the excellent correspondence of the numerically identified response of the highly connected truss-lattices to the Hashin-Shtrikman upper bound further solidified this finding. It was shown that the accuracy of those analytical models decreased substantially when applied to the imperfect lower-connected truss-lattice materials, demonstrating that additional topological factors should be embedded in the analytical estimations. Finally, it was also observed that the least detrimental arrangement of defects was correlated to the coordination number. In particular, elastic properties in higher-connected truss-lattices degraded significantly less in the case of random defects than in the case of voids, and vice versa for the less-connected truss-lattices.

9.2 Future research directions

This study demonstrated the prominent role of coordination number and material anisotropy in the elastic regime of defected truss-lattices, and provided evidence that additional topological parameters such as the incompressible nature of the pristine material largely dictates the response. To enhance our understanding on the mechanical response of imperfect truss-lattices, it is essential to further isolate each topology related parameter that impacts the mechanical properties. In view of this, the following suggestions can be made:

- Investigation on the mechanical response and comparison to analytical models for: i) initially isotropic truss-lattices, to monitor the impact of the gradually increasing anisotropy, and ii) architectures with the same coordination number but dissimilar topological characteristics, to further understand the contribu-
tion of topology. The goal of those studies is to shed additional light on the applicability of analytical schemes to truss-lattice materials with defects, and to pave the way for the development of new and versatile analytical frameworks.

- In conjunction with the defect scenario of missing struts, additional investigations should be performed on superimposed defect forms which represent typically observed imperfections in truss-lattice topologies. Indicatively, such defect forms could be the out-of-straightness of lattice struts, variations in the strut cross-section area and the misalignment of the nodes. The focus of these studies should be also expanded to the inelastic regime, in order to develop a robust understanding on the impact of each defect scenario on the strength and recoverability of defected truss-lattices.

Finally, a challenging yet incredibly useful next step is to develop design strategies to counteract the deteriorating effect of defects by *mechanically cloaking* the areas of large defect concentration. For example, in cases where the presence of cavities (voids) is desired across a lattice domain (to hide or store objects), the penalty in the mechanical properties can be canceled out by tuning appropriately the geometry or topology of the lattice members surrounding the voids. This can be achieved only once a deep understanding on the mechanics of defected truss-lattices has been established, accompanied also by advancements in topology optimization tools and computational algorithms.
APPENDIX A

DERIVATION OF THE YIELDING-TYPE MODE CAPACITY FOR 2D STEEL MOMENT FRAMES WITH A REMOVED COLUMN

In this section, the calculation of the capacity of 2D steel moment frames against the yielding-type mechanism is presented (Equation 9). The yielding-type function \( C_b(a) \) has been defined as the collapse load value \( q_m(a) \):

\[
C_b(a) = q_m(a)
\]  

(30)

Substituting \( q_m(a) \) from Equation 7 into Equation 30:

\[
C_b(a) = q_k + \Delta q_m
\]  

(31)

Substituting \( q_k \) from Equation 3 and \( \Delta q_m \) from equation 8 into Equation 31:

\[
C_b(a) = \max_{j \in J} (q_{j,k}) + \min_{s_j \in S_j} \left( \frac{M_{S_j,\text{cap}} - M_{S_j,k}^{m}(a)}{M_{S_j,B}(a)} \times q_{el,B} \right)
\]  

(32)

The first term of the right part of Equation 32 is expanded using Equations 1, 2:

\[
\max_{j \in J} (q_{j,k}) = f_y \times \max_{j \in J} \left( \frac{w_j}{M_{j,A}(a)} \times q_{el,A} \right)
\]  

(33)

Similarly to Equation 4, the moments at the \( m \) ends of the strongest beams for
The flexural capacity of the strongest beams is:

\[ M_{S_j,\text{cap}} = w_{S_j} \times f_y \]  

(35)

The second term of the right part of Equation 32 is expanded using Equations 34, 35:

\[
\min_{S_j \in SJ} \left( \frac{M_{S_{j,\text{cap}}} - M_{S_{j,k}}(a)}{M_{S_{j,B}}(a)} \times q_{el,B} \right) = \min_{S_j \in SJ} \left( \frac{w_{S_j} \times f_y - q_k \times \frac{M_{S_{j,A}}(a)}{q_{el,A}}}{M_{S_{j,B}}(a)} \times q_{el,B} \right)
\]  

(36)

Substituting Equations 2, 3 and 9 into Equation 36:

\[
f_y \times \min_{S_j \in SJ} \left( \frac{w_{S_j} - M_{S_{j,A}}(a)}{M_{S_{j,B}}(a)} \times \max_{j \in J} \left( \frac{w_j}{M_{j,A}(a)} \right) \times q_{el,B} \right)
\]  

(37)

Adding Equations 33 and 37, Equation 38 is obtained, which is identical to Equation 9:

\[
C_b(a) = f_y \times \left[ \max_{j \in J} \left( \frac{w_j}{M_{j,A}(a)} \times q_{el,A} \right) \right.
\]

\[
+ \left. \min_{S_j \in SJ} \left( \frac{w_{S_j} - M_{S_{j,A}}(a)}{M_{S_{j,B}}(a)} \times \max_{j \in J} \left( \frac{w_j}{M_{j,A}(a)} \right) \times q_{el,B} \right) \right]
\]  

(38)
APPENDIX B

DERIVATION OF THE YIELDING-TYPE MODE CAPACITY FOR 3D STEEL AND CONCRETE COMPOSITE GRAVITY FRAMES WITH A REMOVED COLUMN

This section presents the calculation of the yielding-type collapse load and corresponding ultimate displacement for the 2x2 area of a 3D steel and concrete composite gravity framed system, as described by Equations 17 and 18. Based on the principle of superposition, the subsequent analytical expressions can be derived for each analysis. In the following expressions \( C_{\text{max}} \) is the connection axial capacity and \( q_{\text{el}} \) is the elastic load applied in the respective analysis, which remains the same for all analyses.

**Analysis I:**

\[
q_4 = \frac{C_{\text{max}}}{C_{4,OI}} \times q_{\text{el}} \tag{39}
\]

\[
\delta_I = \frac{q_4}{q_{\text{el}}} \times \delta_{OI} \tag{40}
\]

\[
C_{3,I} = \frac{q_4}{q_{\text{el}}} \times C_{3,OI} \tag{41}
\]

\[
C_{2,I} = \frac{q_4}{q_{\text{el}}} \times C_{2,OI} \tag{42}
\]

\[
C_{1,I} = \frac{q_4}{q_{\text{el}}} \times C_{1,OI} \tag{43}
\]
Analysis III:

\[ \delta_{III} = \frac{q_4}{q_{el}} \times \delta_{OIII} \]  \hspace{1cm} (44)

\[ C_{2,III} = \frac{q_4}{q_{el}} \times C_{2,OIII} \] \hspace{1cm} (45)

\[ C_{1,III} = \frac{q_4}{q_{el}} \times C_{1,OIII} \] \hspace{1cm} (46)

Analysis IV:

\[ q_2 = \frac{C_{\text{max}} - C_{2,III}}{C_{2,OIV}} \times q_{el} + q_4 \] \hspace{1cm} (47)

\[ \delta_{IV} = \frac{q_2 - q_4}{q_{el}} \times \delta_{OIV} + \delta_{III} \] \hspace{1cm} (48)

\[ C_{1,IV} = \frac{q_2 - q_4}{q_{el}} \times C_{1,OIV} + C_{1,III} \] \hspace{1cm} (49)

Analysis V:

\[ \delta_V = \frac{q_2}{q_{el}} \times \delta_{Ov} \] \hspace{1cm} (50)

\[ C_{1,V} = \frac{q_2}{q_{el}} \times C_{1,Ov} \] \hspace{1cm} (51)

Analysis VI:

\[ q_1 = \frac{C_{\text{max}} - C_{1,V}}{C_{1,OVI}} \times q_{el} + q_2 \] \hspace{1cm} (52)

\[ \delta_{VI} = \frac{q_1 - q_2}{q_{el}} \times \delta_{OVI} + \delta_V \] \hspace{1cm} (53)
Analysis VII:

\[ \delta_{VII} = \frac{q_1}{q_{el}} \times \delta_{OVII} \]  \hspace{1cm} (54)

The yielding-type ultimate collapse load \( C_{bl} \) is:

\[ C_{bl} = q_4 + q_2 + q_1 \]  \hspace{1cm} (55)

The combination of Equations 39 to 55 provides the version of the yielding-type ultimate collapse load as shown in Equation 17:

\[ C_{bl} = C_{max} \times q_{el} \times \left[ \frac{1}{C_{4,0I}} + \frac{1 - \frac{C_{2,0III}}{C_{4,0I}}}{C_{2,0IV}} + \frac{1 - C_{1,0V} \times \left[ \frac{1 - \frac{C_{2,0III}}{C_{2,0IV}}}{C_{1,0VI}} + \frac{1}{C_{4,0I}} \right]}{C_{1,0VI}} \right] \]  \hspace{1cm} (56)

Finally, the yielding-type ultimate displacement \( C_{bd} \) is:

\[ C_{bd} = \delta_{VII} \]  \hspace{1cm} (57)
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