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INVESTIGATING THE BREAK IN THE CEPHEID
PERIOD-LUMINOSITY RELATION AND ITS IMPLICATIONS

A Dissertation Presented

by

CHOW-CHOONG NGEOW

Submitted to the Graduate School of the
University of Massachusetts Amherst in partial fulfillment
of the requirements for the degree of

DOCTOR OF PHILOSOPHY

May 2005

Department of Astronomy

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INVESTIGATING THE BREAK IN THE CEPHEID
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by

CHOW-CHOONG NGEOW

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*I dedicate this Thesis Dissertation to my parents, Yin-Ngee Ngeow
& Chiew-Hong Lim, and to my girl friend, Judy Chen.*

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ABSTRACT

INVESTIGATING THE BREAK IN THE CEPHEID PERIOD-LUMINOSITY RELATION AND ITS IMPLICATIONS

MAY 2005

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The Cepheid period-luminosity (PL) relation is a major component in the distance scale ladder. This relation has been considered to be linear (within period ranges of ~ 2 to ~ 100 days) and universal (i.e., metallicity independent) for a long time. However, recent work has strongly suggested that the PL relation for the Large Magellanic Cloud (LMC) Cepheids is non-linear with a discontinuity at 10 days. In addition, the LMC period-color (PC) relation is also shown to be non-linear. Besides this, the Cepheid PL relation could also be non-universal. The aim of this Thesis Dissertation is to investigate these two problems of the Cepheid PL relation, in particular to examine the non-linearity of the LMC PL relation and its implications in the distance scale studies, as well as for the stellar structure, pulsation and evolution.

The works for this Thesis Dissertation involve both the empirical and statistical analysis of the existing Cepheid data in different metallicity environments (Galactic, LMC and SMC) to characterize the break of the PL and PC relations, and the construction of stellar pulsation models using existing pulsation codes in attempt to understand the physics behind the break in the LMC PL and PC relations. The universality of the Cepheid PL relation is also studied with similar approaches. These works include: (a) developing the Fourier techniques to improve the Cepheid light curve fitting method; (b) testing the non-universality of the PL relation by comparing the Cepheid distances to *HST* observed galaxies with the Galactic and LMC PL relations and the statistical test of the

consistency of Galactic and LMC PL relation; (c) analyzing empirically and statistically for the non-linear PL, PC and AC (amplitude-color) relations for the Galactic, LMC and SMC Cepheid at maximum, mean and minimum light; (d) investigating the effect of non-linear PL relation in distance scale studies; and (e) constructing the Galactic and LMC models with the stellar pulsation codes in attempt to account for the non-linear and/or non-universal Cepheid PC and PL relations.

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CHAPTER 1

INTRODUCTION

The aim of this Thesis Dissertation is to investigate the observed non-linearity or the break in the Cepheid period-luminosity (PL) relations seen in the Large Magellanic Cloud Cepheids, and its implications for the distance scale applications and the stellar structure, pulsation and evolution. The work involved in this thesis project includes data collecting and analysis to further characterize the breaks, and pulsation modeling to understand the possible physical reasons behind such breaks in Cepheid PL relations. This chapter first presents a brief introduction to the Cepheid variables and their applications in astrophysics, followed by some discussion of the PL relation, including the evidence that suggests the existence of the break in PL relations, and the motivations to study this newly discovered phenomenon. The chapter concludes with the outlines of this Thesis Dissertation.

1.1 The Cepheid Variables

Cepheid variables are intrinsically pulsating stars named after their prototype, δ CEP, which was discovered by Goodricke in 1786 (see, e.g., Dewhirst & Hoskin 1997). The visual brightness of the Cepheid variables (hereafter Cepheids) varies periodically within days, which can go up to ~ 1 mag. This variation of the brightness is caused by the intrinsic radial pulsation of the stars. The period of the pulsation typically ranges from few days to about 3 months.

The changes of the Cepheid's brightness can be characterized by its light curve. The shape of the Cepheid light curves are skew-symmetric, with a sharp increase from minimum light to maximum light, then follow by a shallow decline back to the minimum. Sometimes, there is a secondary maximum (or "bump") located either on the ascendant or descendant branch of the light curve, which is known as the Hertzsprung progression (see Bono et al. 2000a and reference therein). In addition, the amplitudes of the light curve is larger in the visual bands (U, B, V, R, I) and decrease toward the infrared bands (J, H, K), as illustrated in Strohmeier (1972) and Madore & Freedman (1991). Figure 1.1 shows some examples of the V and I band light curves for the Galactic Cepheids.

Figure 1.1 –Some examples of the Cepheid light curves in V and I bands. The dots are observed data from Moffett & Barnes (1984), after folded with corresponding periods. The curves are fitted light curves using Fourier expansion techniques (see Chapter 2.2). The light curves for prototype Cepheid, δ CEP, are shown in upper-left panel.

Cepheids are yellow supergiant stars that populated in the upper-right region of the Hertzsprung-Russell (H-R) diagram. The yellowish color of Cepheids implies that the temperature of Cepheids is close to Solar, which is about 5500K. The term “supergiant” indicates that Cepheid is much more luminous and larger in size than the Sun. Table 1.1 summarized the ranges of the parameters for Cepheids, which are adopted from various sources (e.g., Strohmeyer 1972; Cox 1980).

Table 1.1 –The ranges of the fundamental parameters for classical Cepheids.

Parameter	Range
Period (days)	$\sim 1 - \sim 100$
Mass ($M_{\odot}$)	$\sim 3 - \sim 15$
Mean Luminosity ($L_{\odot}$)	$\sim 300 - \sim 45000$
Effective Temperature (K)	$\sim 4500 - \sim 6500$
Mean Radius ($R_{\odot}$)	$\sim 15 - \sim 200$
Absolute V Magnitude (mag.)	$\sim -0.5 - \sim -8$
V Band Amplitude (mag.)	$\sim 0.3 - \sim 1.2$
Radial Velocity Amplitude (km/s)	$\sim 30 - \sim 60$
Spectra Type	F6 – K2
Age (yrs)	$\sim 10^6 - \sim 10^8$

In terms of stellar evolution, Cepheids are young massive ($> \text{few } M_{\odot}$) stars that evolve away from the main-sequence and when they are in the instability strip – a region in the H-R diagram bounded by the blue (hot) and red (cool) edges where the stars pulsate – then the envelope of the stars

becomes unstable and the stars begin to pulsate. After leaving the main-sequence, these stars burn the hydrogen in a shell above the helium core and move across the H-R diagram nearly horizontally from hot to cool temperature, at the same time expand the radius to $> 15R_{\odot}$. In this phase, they cross the instability strip for the first time, known as the first crossing. After the triple-alpha process ignited at the end of the first crossing, the interior of the stars begins to burn helium in the core and the hydrogen in the shell, and the stars will exhibit a loop, known as the blue loop, in the H-R diagram. This blue loop crosses the instability strip for the second and third times. These crossings are summarized in Figure 1.2. The time scale for a $3.5M_{\odot}$ Cepheid to stay in the instability strip at the first, second and third crossings is $\sim 7 \times 10^4 \text{ yrs}$, $\sim 2 \times 10^6 \text{ yrs}$ and $\sim 4 \times 10^6 \text{ yrs}$, respectively. While for a $15M_{\odot}$ Cepheid, these time scales are $\sim 3 \times 10^4 \text{ yrs}$, $\sim 1 \times 10^6 \text{ yrs}$ and $\sim 6 \times 10^4 \text{ yrs}$, respectively (Bono et al., 2000c). More detailed descriptions of the evolution of Cepheids can be found, e.g., in Iben (1967) and Bono et al. (2000c).

Figure 1.2 –The schematic evolution of Cepheids (after main-sequence) in H-R diagram. The instability strip is indicated with the dashed lines. The first crossing for the massive stars through the instability strip is from point A (leaving main-sequence) to point B (where the triple-alpha process begins). Then these stars undergo a loop (the blue loop) that pass through the instability strip for the second and third times. These stars become Cepheids when they are in the instability strip. The curve that running from upper-left to lower-right corner is the main-sequence, and the location of the Sun in H-R diagram is also indicated.

There are other pulsating variables carrying the name of “Cepheid” or similar to it, which should not be confused with the classical (Type I) Cepheids (classified as DCEP in General Catalogue of

Variable Stars [GCVS], Kholopov et al. 1998) we considered in this Thesis Dissertation. In fact, the term “Cepheid” was once used in describing the regular or semi-regular pulsating stars, which include β Cephei, classical Cepheid, RR Lyrae and RV Tauri stars (Campbell & Jacchia, 1941). However, this general classification was broken down to different classes in the GCVS (Sterken, 1996). They are listed as follow (see, e.g., Strohmeier 1972; Feast 1996; Sterken 1996; Becker 1998):

- Type II Cepheid: Also known as W Virginis variables. The period-luminosity (PL) relation for these old Population II variables is parallel to the classical (Population I) Cepheids, but fainter by $\sim 1.5\text{mag.}$ in V-band (or four times fainter in luminosity). The mass for these variables are also lower than Type I Cepheids. They are classified as CW in GCVS.
- Dwarf Cepheid: Also known as δ Scuti or AL Velorum variables. Typically periods for these Population I variables are about 1 to 5 hours. Their V-band amplitudes are generally less than 0.3mag. They located near the main-sequence in H-R diagram but well within the instability strip. They are classified as DSCT in GCVS.
- β Cephei: Also known as β Canis Majoris variables. This is high mass variables with high surface temperature (spectra type of O8-B6). The pulsation period for this variables is less than a day, with typical value of few hours. Also, the V-band amplitudes for this type of variables are less than $\sim 0.3\text{mag.}$ They are classified as BCEP in GCVS.
- Anomalous Cepheid: These are RR Lyrae-like stars that mostly found in the dwarf spheroidal galaxies. They are low mass ($\sim 1.5M_{\odot}$) and metal poor stars with period in the range of ~ 0.3 to ~ 2 days. They are either originated from a single young stars or from a binary system with mass transfer (see, e.g., Bono et al., 1997).

The locations of these variable in the H-R diagram are labelled in Figure 1.3. The classical Cepheids also come along with certain sub-classes, classified as according to the mode of pulsation. Most Cepheids pulsate in the fundamental mode (FU), however there are some Cepheids pulsate in first-overtone mode (FO, classified as DCEPS in GCVS), or even higher mode¹. Some Cepheids even simultaneously pulsate in two modes, and referred as beat (or double mode) Cepheids. These higher mode Cepheids can be separated from FU Cepheids with several features: lower period, location in the plot of the PL relation, the shape of the light curves, the Fourier coefficients and others. However the Cepheids considered in this Thesis Dissertation are the classical fundamental mode Cepheids.

¹For FU Cepheids, the gas inside the star oscillate in fundamental mode (as in an open pipe). Similarly, the gas in FO Cepheids oscillate in first harmonics, and so on.

Figure 1.3 –The schematic H-R diagram and the locations for some pulsating variables as mentioned in the text. The curve that running from upper-left to lower-right corner is the main-sequence, and the location of the Sun in H-R diagram is also indicated.

1.1.1 Cepheids as Astrophysical Tools

Cepheid are useful tools and probes for studying astrophysics, which mainly come in two categories:

- (A) Distance scale study – Cepheids are primary distance indicators, and the Cepheid PL relations can be used to calibrate secondary distance indicators, such as the Tully-Fisher relation, Type Ia supernovae, fundamental plane and surface brightness fluctuations. This builds up the distance ladder and indicates the distance to more remote galaxies. Together with the measurements of the recession velocities of these more distant galaxies, the Hubble constant can be determined: this was the main goal of Hubble H_0 Key Project (Freedman et al., 2001).
- (B) Stellar pulsation studies – In terms of stellar astrophysics, the pulsational behavior of Cepheids (as well as other pulsating variables) helps in the understanding of stellar structure, pulsation and evolution. The pulsation models can be constructed with various input physics to mimic the observational quantities and behavior of the Cepheids, such as the period, light and velocity amplitudes, and etc. This kind of detailed comparisons between the outputs from the models with observation is crucial in understanding the internal structures and the physical

mechanisms (for example, the κ - or Γ -mechanism) that cause the stars to pulsate (e.g., see Ledoux & Walraven, 1958; Cox, 1980).

There are many different kind of studies that involve using the Cepheid variables, and a lot of them rely on the Cepheid PL relation to determine the distances. Some examples are given as below:

- Determining the Galactic abundance gradient – By studying the spectroscopic abundance for a sample of Cepheids in our Galaxy, at which the distances to these Cepheids can be obtained from the PL relations, Andrievsky et al. (2002a,b,c) found that the Galactic abundance gradient is flat around Solar neighborhood ($6.5\text{kpc} < R_G < 10\text{kpc}$), but exhibits a steep and shallow gradient at inner and outer part of the Galaxy, respectively.
- Determining the kinematics of Galactic rotation – Using the measured radial velocities, together with the distance measurements from adopted PL relation, the kinematic parameters for Galactic rotation such as the distance to the Galactic Center and the circular velocity of the Sun can be determined from fitting the data with models, as done in Metzger et al. (1998).
- Probing the star formation history – By Comparing the theoretical period distribution, calculated from stellar evolution and pulsation models, to the observations, Alcock et al. (1999) found that there is a recent burst of star formation that occurred in the Large Magellanic Cloud.
- Testing of the standard extinction law – Macri et al. (2001) studied the consistency of distance moduli and color excess for a number of extra-galactic Cepheids in 12 galaxies. Their results support the use of the standard extinction law in other galaxies to derive the distances.
- Probing the local velocity flow – The velocity dispersion in the Local Universe is derived using some nearby galaxies, with Cepheid distances and recession velocity (corrected for Virgo infall) by assuming a Hubble constant, in Ekholm et al. (2001). Their result supports other similar studies that the local Hubble flow is quiet and connected to the global Hubble flow at $R \sim 1.5\text{Mpc}$.
- Showing the discrepancy of using the EPM (Expanding Photosphere Method) in Type II supernova to determine distances – By comparing the EPM distance to SN1999em and the Cepheid distance to its host galaxy, NGC 1637, Leonard et al. (2003) showed that there is a large discrepancy between these two distance measurements (see also Appendix A).

1.2 The Cepheid Period-Luminosity Relation

The Cepheid PL relation was discovered by Leavitt in the early 20th century (see Fernie, 1969, for a review of the historical developments). After nearly a century of developments and studies, the basic physics for the Cepheid PL relations is well understood, and the techniques of using Cepheid PL relations to determine the distance to nearby ($< 30\text{Mpc}$) galaxies is well developed. This relation takes a very simple form:

$$M_{\lambda} = a_{\lambda} \log(P) + b_{\lambda}, \quad (1.1)$$

where M , a and b are the absolute magnitudes, slope and zero-point (ZP) in bandpass λ , respectively, and P is the pulsational period in days. Both of the slopes and ZP can be obtained either from the theoretical predictions or calibrated from the observations. The calibrating PL relation currently used is based mainly on the Large Magellanic Cloud (LMC) Cepheids, as applied by the H_0 Key Project team (Freedman et al., 2001) as well as in other studies (e.g., Saha et al., 2001b; Kanbur et al., 2003; Allen & Shanks, 2004).

In fact, the simplest way to derive equation (1.1) is by combining the period-mean density relation ($P \propto \rho^{-1/2}$) for the pulsators, the Stefan-Boltzmann law ($L = 4\pi R^2 T_e^4$) for assuming that Cepheids are close to black body radiation, and the mass-luminosity (ML) relation ($L \propto M^{\alpha}$) from stellar evolution. Together with the luminosity-magnitude and temperature-color conversions, these three equations yield the period-luminosity-color (PLC) relation, and the PL relation is obtained by projecting the PLC relation to the P-L plane (Madore & Freedman, 1991). Note that the Cepheid PL relation has always been considered to be a *linearly* defined function with period ranges of $\log(P) \sim 0.2$ to $\log(P) \sim 2.0$ (see the figures in, e.g., Tanvir, 1997).

In most cases, the Cepheid PL relation is assumed to be *universal*, i.e. the slope of the PL relation remains fixed, and the ZP is calibrated with independent means (such as independent distance measurements). The distance modulus to a target galaxy is found by obtaining the difference of the ZPs from the calibrated PL relation and the fitted PL relation to the Cepheids in the target galaxy. As mentioned, the most-widely used PL relations are calibrated with the large number and well observed Cepheids in the LMC, by adopting a distance modulus of $18.5 \pm 0.1\text{mag.}$ to the LMC (see, e.g., Freedman et al., 2001), because LMC is relatively nearby, contains a large number of Cepheids and the dispersion of the extinction is smaller than the Galactic Cepheids. However, there are numerous studies on the metallicity dependency of the Cepheid PL relation (see, for example,

Stothers, 1988; Freedman & Madore, 1990; Kochanek, 1997; Sasselov et al., 1997; Kennicutt et al., 1998; Caputo et al., 2000b; Freedman et al., 2001; Kanbur et al., 2003; Groenewegen et al., 2004; Romaniello et al., 2004; Sakai et al., 2004; Storm et al., 2004). Most of these studies suggest that the metallicity dependency enters the PL relation through the ZP, while the slope is still universal and fixed by the LMC Cepheids. Hence, a metallicity correction term is commonly applied to derive the Cepheid distances to nearby galaxies that have a different metallicity environment to that of the LMC (for examples, as in Freedman et al., 2001; Kanbur et al., 2003; Leonard et al., 2003). Furthermore, recent studies by Tammann et al. (2003) and Ngeow & Kanbur (2004) shows that the Galactic PL relation is *steeper* than the LMC PL relation, which suggests that the slope of the PL relation may also depend on the metallicity. Hence Cepheid PL relations may not be universal.

Besides the issue of metallicity dependency of the Cepheid PL relation, there are still remaining some issues that should be investigated further and settled (Feast, 2003) for the application of Cepheid PL relation. One of them is the non-linearity of the Cepheid PL relation, as discussed further in the next subsection.

1.2.1 The Non-Linearity of the Cepheid PL Relation

In the “Standard Candle” workshop held in Chile (Dec., 2002), Feast (2003) has raised the question regarding the non-linearity of the Cepheid PL relation, as we quote below:

“Are there non-linearities in the PL and PC [period-color] relations? Particularly is there a significant slope difference between short and long period ($>\sim 10$ days) Cepheids that would seriously affect the calibration and use of PL and PC relations?”

This is because the large datasets from the recent Optical Gravitational Lensing Experiment (OGLE, Udalski et al., 1999a) has suggested that the PL relation for the LMC Cepheids may not be linear (Tammann et al., 2002; Kanbur & Ngeow, 2004), in contrast to the linearity of the PL relations mentioned above. The non-linearity of the Cepheid PL relation is the main theme that will be investigated in this Thesis Dissertation.

The non-linearity of Cepheid PL relations has been recognized for some times, as the mean magnitudes for some of the long period Cepheids ($\log[P] \gtrsim 1.5$) do not follow the best-fit lines. In fact, some of the earlier works (include the works of Kukarkin, Shapley & Fernie) on the Cepheid PL relation have suggested or used a quadratic form, $M = \alpha + \beta \log(P) + \gamma [\log(P)]^2$, to describe the PL relation (Fernie, 1969). For example, semi-empirical derivation of the quadratic PL relations in the B- and V-band can be found in Fernie (1967). In addition, the composite PL relations at

mean and maximum light constructed with Cepheids in different galaxies, as presented in Sandage & Tammann (1968), also show some curvatures of the PL relations. In terms of theoretical modeling, Bono et al. (1999a) and Caputo et al. (2000a) have fit the quadratic PL relations to the pulsating periods and magnitudes obtained from the pulsational modelings.

The non-linearity of the PL relations, however, is more easily explained with the broken PL relations: there are two PL relations, for short and long period Cepheids respectively, with discontinuity (a break) at a specific period (e.g., at 10 days). Perhaps the earliest idea for the broken PL relation was proposed by Kukarkin in 1937, as quoted from Fernie (1969): “...he concluded that the relation was definitely non-linear...but the best fit was by two straight lines of different slope intersecting at a period of 10 days”. The modern version of the broken PL relations is reported in Tammann et al. (2002), that the OGLE LMC Cepheids follow different PL relations for periods shorter and longer than 10 days. This was also *independently discovered* by us in 2002. Compared to the previous datasets, the high quality and large number of Cepheids observed by the OGLE team, along with the estimated extinction to individual Cepheids, has made the detection of the broken PL relation at 10 days becomes possible. The empirical evidence for the break of the PL relations at $\log(P) = 1.0$ observed in OGLE LMC Cepheids is presented in Figure 1.4(a). The slopes of the long and short period PL relations in this figure are compared in Table 1.2. Except for the B-band with large error, as there are not many data points for the long period Cepheid, both of the V- and I-band slopes show roughly $\sim 2\sigma$ difference between the long and short period PL relations. Note that Caputo et al. (1999) also fit a broken PL relation to their results obtained from the theoretical models, but with a break at $\log(P) = 1.4$.

Table 1.2 –Comparison of the slopes (a) for the long and short period PL relations as seen in OGLE LMC Cepheids. The slopes are adopted from Tammann et al. (2002).

Band	$a(> 10\text{day})$	$a(< 10\text{ day})$	$a(> 10\text{ day}) - a(< 10\text{ day})$
B	-1.89 ± 0.62^a	-2.42 ± 0.08	0.53 ± 0.63
V	-2.48 ± 0.17	-2.86 ± 0.05	0.38 ± 0.18
I	-2.82 ± 0.13	-3.03 ± 0.03	0.21 ± 0.13

^a The large error for this slope is due to the small number of data points.

The reason for the break, or the non-linear nature, of the PL relation as seen in the LMC Cepheids is because the LMC period-color (PC) relation is also broken (Tammann et al., 2002; Kanbur & Ngeow, 2004), as shown in Figure 1.4(b). Recall that both of the PL and PC relations follow the PLC relation for Cepheid variables (see Madore & Freedman, 1991, for the basic physics

Figure 1.4 –The break of PL and PC relations in OGLE LMC Cepheids. *Left panel:* (a) The empirical evidence for the break of PL relations at $\log(P) = 1.0$ in BVI-bands. *Right panel:* (b) The reason that there is a break at $\log(P) = 1.0$ for PL relations is because the PC relations are also broken. These figures are adopted from Tammann et al. (2002) with permission.

of PLC relation). Consider that the PLC relation with the form of $M_V = a' + b' \log(P) + c'(V - I)$ is valid at any phase, and if the PC relation is broken at P_0 , i.e.,

$$(V - I) = x_S + y_S \log(P), \quad P < P_0,$$

$$(V - I) = x_L + y_L \log(P), \quad P > P_0.$$

Then, by substituting these PC relations to the PLC relation will lead to two PL relations:

$$M_V = a' + c'x_S + [b' + c'y_S] \log(P), \quad P < P_0,$$

$$M_V = a' + c'x_L + [b' + c'y_L] \log(P), \quad P > P_0.$$

Hence, understanding the physics behind the break in the LMC PC relation helps in the understanding of the break as seen in the LMC PL relation. In contrast to the LMC PC relation, the Galactic PC relation is fully consistent with a single regression line, i.e., there is no break in the Galactic PC, as well as the PL, relation (Tammann et al., 2003; Kanbur & Ngeow, 2004; Ngeow & Kanbur, 2004). Hence, the study of the broken, or the non-linear, PC and PL relation is not only important in distance scale studies (the effects of broken PL and PC relation should be taken into consideration when deriving the Cepheid distances), but also important in the study of stellar structure, evolution and pulsation that need to account for this newly discovered phenomenon.

Since the PC relation, as seen in Figure 1.4(b), are normally defined as $\langle X \rangle - \langle Y \rangle$, where $\langle \rangle$ denotes the intensity mean for the X and Y bandpass, then the PC relation at mean light is (close to) the average of the color for all phases. If we choose $(V - I)_i$ as the extinction corrected color at phase i , then a PC relation at this phase is: $(V - I)_i = d_i + c_i \log(P)$, and taking the average over a pulsation period, $i = 1 \dots J$, this becomes:

$$\langle V \rangle - \langle I \rangle = \frac{1}{J} \sum_{i=1}^{i=J} d_i + \frac{1}{J} \sum_{i=1}^{i=J} c_i \log(P). \quad (1.2)$$

It is clear that the average ZP and slope will be affected by their values at individual phase points. Any “anomaly” of the color at certain phases of the pulsation, e.g. at the maximum and/or minimum light, for some phase/period range would affect the mean light properties of the PC relation and hence produce the observed break in the PC and also the PL relations. For example, consider this relation: $V_{min} - V_{max} \propto \log(T_{max}) - \log(T_{min})$, as given in Simon et al. (1993), where $T_{max/min}$ is the temperature at maximum and minimum light, respectively. The left-hand-side of this relation is just the amplitude of the Cepheid, and the temperature terms on the right are directly related to the color. If, for some reasons, $\log(T_{max})$ vs period has a flat slope for a given period range, then there is a relation between the amplitude and $\log(T_{min})$, the amplitude-color (AC) relation, and vice versa. This flatness of the PC relation at maximum and/or minimum light in principle could influence the PC relation at mean light, and hence produce the broken PC relation. A mechanism that can produce a flat PC (or period-temperature) relation at maximum light is the interaction of the hydrogen ionization front (HIF) with the photosphere (defined at optical depth of 2/3) that halts the temperature of the photosphere at some characteristic values (Simon et al., 1993) which are independent of the global properties. Hence, the *preliminary* hypothesis to account for the non-linear LMC PC relation is:

- The long period (>10 days) LMC Cepheids behave like the Galactic Cepheids, where the HIF-photosphere interaction occurs at maximum light (Simon et al., 1993).
- The short period (<10 days) LMC Cepheids behave like the RR Lyrae variables, where the HIF-photosphere interaction occurs at minimum light (Kanbur, 1995; Kanbur & Phillips, 1996).

Another possibility is that the slopes for the long period Cepheids are steeper (i.e., the temperature of the Cepheids get much cooler as period increase) than the slopes for the short period Cepheids at certain phases (e.g., maximum and/or minimum), then the mean light PC relation could also be non-linear. These hypotheses will be examined in detail as presented in the later chapters in this Thesis Dissertation.

Besides the break of PL relations seen in LMC Cepheids, the SMC Cepheids also show a similar break at a period of 2 days from the EROS-2 data (Bauer et al., 1999). The authors carefully examine the possible errors or bias in the data that lead to this finding, and conclude that the break of PL relations seen in SMC Cepheids is real. They also list out 4 possible scenarios to explain the break seen in the EROS data, including the superposition of two Cepheid populations, mixing of two classes of Cepheids, and the shape of the instability strip from both evolutionary and pulsational models, but none of them can adequately explain the break in PL relations. However, a possible explanation is given in Baraffe et al. (1998), who explain that the change of the slope for short period SMC Cepheids (less than 2 days) is due to the evolutionary effect, that maybe different than the LMC Cepheids with, e.g., the HIF-photosphere interaction. Hence the non-linearity of the Cepheid PL relation may not unique to the LMC Cepheids, and different physics could be involved to account for the non-linearity. However, the non-linearity of the LMC PL relation will be the main focus in this Thesis Dissertation.

1.3 Structure of this Thesis Dissertation

Since Cepheid light curves are essential in studying the Cepheid variables and the distance scale application, accurately constructing the Cepheid light curves is important to extract useful information for the Cepheids, such as the mean magnitudes and the magnitudes at maximum/minimum light. In **Chapter 2**, we describe the Fourier techniques we developed that can be used to accurately construct the Cepheid light curves, including the Fourier expansion with simulated annealing techniques and the Fourier interrelations. These techniques are also used in other studies in this Thesis Dissertation.

The ideas of non-universality of the PL relations and the use of Galactic PL relation to calibrate the Cepheid distances have emerged as I began my dissertation work. To test this, we used *both* of the LMC and Galactic PL relations to calibrate the distances to $\sim$ two dozens *HST* observed galaxies (which span a wide range in metallicity) in **Chapter 3**, in contrast to the conventional approach that only used the LMC PL relation (Freedman et al., 2001). A major result from this study is: after applying a metallicity correction (Δ_Z) to the distance modulus obtained by using the LMC and Galactic PL relations, the *overall* difference in distances from the two PL relations used is negligible, as compared to the result obtained without using the Δ_Z , which shows a $\sim 10\sigma$ difference in distance when applying the two PL relations.

Encouraged by the results from Chapter 3, we then collect the Galactic Cepheids with independent distances to improve the calibration of the Galactic PL relations in **Chapter 4**. We used a rigorous statistical test (the *t*-test) to show that the Galactic Cepheids follow a different PL relation (i.e., the slopes) to their LMC counterparts at more than 95% confidence level. This is the first evidence from a statistical test that the empirical Cepheid PL relations is not universal, at least in the galaxies with metallicity comparable to Galactic and LMC values.

We then study the non-linearity of the LMC PL relation, and extend this study from optical to near infrared, as described in **Chapter 5** with rigorous statistical tests (*F*- and χ^2 -tests). The purpose of this chapter is two-fold: (a) to verify the non-linearity of the OGLE V-band PL relation with MACHO data; and (b) to extend the study of non-linear PL relation to the R- (from MACHO), J-, H- and K-band (from 2MASS). The results from rigorous statistical tests suggest that the VRJ- and H-band PL relations are better described with the broken PL relation, while the K-band PL relation is consistent with a single line.

We further investigated the PC and AC relations at maximum, mean and minimum light for the Galactic, LMC and SMC Cepheids in **Chapter 6**, since the PC and AC relations are connected as mentioned previously. We also apply a statistical test, the *F*-test, to examine the significance of the break at 10 days for these PC and AC relations. The results showed that the LMC PC relations are broken at maximum, mean and minimum light, however it only happens at maximum light for the Galactic PC relation and marginally at SMC PC(max) relation. The LMC PL relation is also shown to be broken at mean and minimum light, but not at the maximum light, with the *F*-test.

The impact or the implication of the non-linear LMC PL and PC relations in distance scale studies are discussed in **Chapter 7**, i.e., how does the non-linear PL and PC relations affect the distance scale measurements. We examine this by studying the linearity of the Wesenheit function, because it is equivalent to obtaining the Cepheid distance with the reddening-free distance modulus.

In this chapter we found that the Wesenheit function is indeed linear, hence the impacts of the non-linear PL and PC relations in distance scale applications is minimal (with a difference of $< 0.07\text{mag}$. in distance modulus when using the linear and broken PL relations).

In **Chapter 8**, we examine the Galactic PC and AC relations further by constructing Galactic pulsation models. One feature of the Galactic Cepheids is that the PC relation at maximum light for the long period Cepheids is flat. This is probably due to the HIF-photosphere interaction as described in the previous sub-section. By using the Florida pulsation codes, which include 1-D recipes for time-dependent convection calculations, we confirmed that the flatness of PC(max) relation for these long period Galactic Cepheids is caused by the proposed interaction.

The non-linear LMC PC and PL relations are further investigated in detail in **Chapter 9** with the methods and techniques developed in previous chapters. We have updated the empirical results from Chapter 6 due to the revision of the OGLE data, and we compared the Galactic and LMC PC relations in detail. We also extend the study from Chapter 8 to the LMC Cepheids by constructing the LMC pulsation models, with the same methodology and the stellar pulsational codes. The role of the HIF-photosphere interaction in explaining the non-linearity of the LMC PL and PC relation is also discussed.

The conclusions/summary of this Thesis Dissertation are presented in **Chapter 10**, along with the outlines of future works.

In **Appendix A**, I summarize the project that I was involved with Dr. Leonard, which compared the Cepheid distance and the EPM distance to a SN II in NGC 1637. **Appendix B** described the Fourier intrarelations, in addition to the Fourier interrelations as given in Chapter 2, that could potentially be used to reconstruct the Cepheid light curves.

CHAPTER 2

CEPHEID LIGHT CURVES RECONSTRUCTION WITH FOURIER EXPANSION AND INTERRELATIONS

Fourier decomposition is a well established technique used in stellar pulsation. However the quality of reconstructed light curves using this method is reduced when the observed data have uneven phase coverage. We use simulated annealing techniques together with Fourier decomposition to improve the quality of the Fourier decomposition for reconstructing the Cepheid light curves. This method restricts the ranges that Fourier amplitudes can take. The ranges are specified by well sampled Cepheids in the Galaxy and Magellanic Clouds. To reconstruct the Cepheid light curves observed by the *HST*, which typically consist of 12 V- and 4 I-band data points, we employ a direct Fourier fit to the 12 V band data using the simulated annealing method mentioned above. For the I-band data, we explicitly derive and use Fourier interrelations to reconstruct the I-band light curve. Error analysis of these light curves reconstruction procedures shows the errors are of the Fourier amplitudes are around $\sim 0.03\text{mag}$. We discuss advantages and drawbacks of these techniques when applied to *HST* Cepheid data and some other related issues.

2.1 The Cepheid Light Curve

The study of Cepheid light curves has two major applications: (a) distance determinations to nearby galaxies via the well calibrated period-luminosity (PL) relations (for example, see Madore & Freedman, 1985, 1991; Feast & Walker, 1987; Freedman et al., 2001; Saha et al., 2000, 2001b); (b) the understanding of the pulsational behavior of Cepheids by comparing the observed light curves to the theoretical counterparts (e.g., Davis et al., 1981; Simon & Davis, 1983; Buchler et al., 1990; Wood et al., 1997; Bono et al., 2002a). Therefore, reconstructing the Cepheid light curves from the observed photometric data is important regarding these two aspects.

Because Cepheid light curves are periodic, the data points from well observed Cepheids can be described by the n^{th} -order Fourier expansion, as first introduced by Schaltenbrand & Tammann in 1971:

$$m(t) = A_0 + \sum_{j=1}^n A_j \cos(j\omega t + \phi_j), \quad \omega = 2\pi/P, \quad (2.1)$$

where P is the period (in days) of Cepheid, and A_0 is the mean value of the light curve. The A_j and ϕ_j are the Fourier amplitudes and phases for j^{th} -order, respectively. Since the period of a Cepheid can be determined via other means and can be found in the literature, we can fold the time observation into phase as: $\Phi(t) = (t - t_0)/P - \text{int}[(t - t_0)/P]$, with t_0 being a common starting epoch. The value of Φ is from zero to one, corresponding to a full cycle of pulsation. Hence equation (2.1) can be written as (Schaltenbrand & Tammann, 1971):

$$m(t) = A_0 + \sum_{j=1}^n A_j \cos[2\pi j\Phi(t) + \phi_j]. \quad (2.2)$$

Equations (2.1) or (2.2) has remaining $2n + 1$ unknown parameters which require at least the same number of data points to solve for them. This technique has been widely used in the study of Cepheids and other pulsation variables (e.g., see Simon & Lee, 1981; Andreasen, 1988b; Morgan, 2003). As mention in Chapter 1.1, the Cepheid light cure is generally not in sinusoidal shape (which only required $n \leq 2$), hence the higher order ($n > 3$) terms are required to fully describe the light curve. Most of the observations on Galactic (e.g., Schaltenbrand & Tammann, 1971; Simon & Lee, 1981; Moffett & Barnes, 1985; Ferro et al., 1998) and Magellanic Clouds Cepheids (e.g., Andreasen & Petersen, 1987; Moffett et al., 1998; Udalski et al., 1999b,c) include large number of epochs that permit the higher order of Fourier expansion up to $n = 8$. However, some of these Cepheids have small number of observations that only permit lower order Fourier expansion. The Fourier expansion has been extensively used in studying the light curves of pulsating stars (see Antonello et al., 1987, and the reference therein). Some applications of equation (2.1) in the study of the Cepheids can be found, for example, in Simon & Davis (1983), Simon & Moffett (1985), Antonello & Poretti (1986), Andreasen & Petersen (1987), Buchler & Moskalik (1994), Poretti (1994) and Hintz & Joner (1997).

2.1.1 The Problems in Reconstructing the Light Curve

The traditional approach to solve the $2n+1$ parameters in equation (2.1) or (2.2) is using the least squares method, as in Simon & Lee (1981) and other studies. This method works well for Cepheids with large number of high-quality observations that uniformly sampled the light curves. However,

this method sometimes fails to produce acceptable light curves with small number of observations, due to the poor sampling of the data points, and produces some numerical bumps in the light curves. In Figure 2.1, we illustrate this problem with a *HST* observed Cepheid which has only 12 data points. The light curve for this Cepheid was reconstructed with the least squares method to solve the parameters in equation (2.1) by adopting $n = 4$, as indicated by solid curves. This light curve does not fit to the data well and exhibits numerical bumps around phase ~ 0.2 and ~ 0.5 . These numerical bumps are not physically associated with the true light curves, but result from the sparsely sampled data points. An alternate method to overcome the problem presented in Figure 2.1 is using a template of the light curves determined from well-sampled Cepheids (Stetson, 1996). The template-fitting method works well when determining the mean magnitudes for the Cepheids, however they cannot be used in the detailed structural study of the Cepheid light curves, such as determining the maximum & minimum light and the R_{ij} & ϕ_{ij} parameters¹.

In recent years, a great deal of effort has been made to discover extra-galactic Cepheids, and determine the distance, by using the *HST* as in Freedman et al. (2001, hereafter H0KP in this chapter) and Saha et al. (2001b). In most of the *HST* observations, in order to optimize the phase coverage of Cepheids within the *HST* observational windows, the number of V-band observations is chosen to be 12 (Freedman et al., 1994; Kennicutt et al., 1995). To correct for the extinction and reddening, I-band observations are included (see, for example, Kennicutt et al., 1995; Freedman et al., 2001). However, only 4 (or 5) I-band observations are needed, because the I-band amplitudes are smaller than in the V-band (Freedman, 1988; Freedman et al., 1994; Kennicutt et al., 1995). Existing techniques to estimate the mean from the sparsely sampled *HST* data include: (a) taking the phased weighted intensity mean (Saha & Hoessel, 1990) in V-band and using an empirical relation developed by Freedman (1988) or Labhardt et al. (1997) in I-band; and (b) adopting template fitting procedures (Stetson, 1996; Tanvir et al., 1999).

In this chapter we develop two techniques to reconstruct the Cepheid light curves. The first one, as given in Section 2.2, is applying the simulated annealing techniques to the Fourier expansion. This technique will be used to reconstruct most of the light curves studied in this Thesis Dissertation, including the V-band light curves from the *HST* observations. The second technique is referred as the Fourier interrelations, as discussed in Section 2.3, which is mainly used to reconstruct the *HST* I-band data. Since one of the main goals to develop these techniques is to reconstruct the *HST* data, we set $n = 4$ (compromise between the 12 V-band *HST* data and the goodness of fit) in this chapter

¹ $R_{ij} = A_i/A_j$ and $\phi_{ij} = \phi_i - i\phi_j$ (Simon & Lee, 1981).

for the purpose of illustration. The general applications of these techniques can be easily extended to higher order. We also present some potential advantages and drawbacks of the application of these techniques.

Figure 2.1 –An example of the reconstructed light curve to a Cepheid in NGC 1326A. The solid curve is the reconstructed light curve from the least squares solution. It is clear that the least-square solution for this Cepheid does not provide a satisfactory fit to the data. The dashed curve is the light curves reconstructed with the method described in Section 2.2. The original data are indicated with error bars.

2.2 Fourier Expansion with Simulated Annealing Techniques

Instead of *solving* for the unknown parameters in equation (2.2) with least squares method, we specify the range of each parameter in equation (2.2) and *fit* the data to obtain the best-fit values for each parameter. All the parameters are fit simultaneously with the simulated annealing method (for example, see Press et al., 1992) to minimize the corresponding χ^2 value: $\chi^2 = \sum [(m_{obs} - m_{fit})/\sigma_{obs}]^2$. The simulated annealing method will search the best fit values for a group of parameters within a reasonable amount of computational time. The dashed curve in

Figure 2.1 shows that by fitting the parameters in equation (2.2) to the observed data with an appropriate choice for the ranges, the reconstructed light curve has been greatly improved. In order to apply the simulated annealing method to fit the available data points, certain ranges of the fitted parameters have to be imposed. The range for the mean magnitude (A_0) term in equation (2.2) is trivial (say, $\pm 5\text{mag}$. from the arithmetic mean magnitude of the data points), and the ranges for the Fourier amplitudes (A_j) and phases (ϕ_j) are set to be: $A_j \in [a_j^{low}, a_j^{up}]$ and $\phi_j \in [0, 2\pi]$, respectively. Note that if the ranges for the Fourier amplitudes are unconstrained, e.g. $a_j^{low} = 0$ and $a_j^{up} = \text{large number}$, then this procedure is equivalent to applying the least squares method to obtain the parameters (preserved the detailed structures of the light curves but does not work well for poorly sampled data). On the other hand, if $a_j^{low}/a_1^{low} = a_j^{up}/a_1^{up} = A_j/A_1 = f(\log[P])$, $j \geq 2$, then this procedure is same as the template fitting method introduced by Stetson (1996, works well for poorly sampled data but does not preserved the detailed structures of the light curves). Hence, our method is constrained the ranges for the Fourier amplitudes such that the reconstructed light curves can be improved with poorly sampled data and preserved the detailed structures of the light curves at the same time. We do this, firstly, because we want to reconstruct the light curve to model it and, secondly, to capture any light curve shape changes due to metallicity.

Figure 2.2 –Two typical examples for the reconstructed light curves (in V-band) by using the simulated annealing method with unconstrained fit for the LMC Cepheids in similar periods. *Left:* (a) Well constructed light curve. The data points are distributed relatively uniform in phase. This LMC Cepheid has $\log(P) = 0.4576$. *Right:* (b) Poorly reconstructed light curve due to the bad phase coverage, and exhibits a numerical bump at phase ~ 0.5 . This LMC Cepheid has $\log(P) = 0.4580$.

In order to determine the proper ranges for the Fourier amplitudes, we use the following two datasets: (a) “calibrating set” Cepheids that consist mostly of the Galactic Cepheids with additional small amount of LMC/SMC Cepheids (Hendry et al., 1999); and (b) well-sampled OGLE (Optical Gravitational Lensing Experiment) LMC Cepheids (Udalski et al., 1999b). For the OGLE LMC Cepheids, we fit the photometric data with the unconstrained Fourier expansion, and eliminated those Cepheid with poorly reconstructed light curves (Ngeow et al., 2003). In Figure 2.2 we show the examples of well reconstructed and poorly reconstructed light curves for the OGLE LMC Cepheids. The plots of the V-band Fourier amplitudes as function of periods by combining these two datasets are presented in Figure 2.3. The plots of the I-band Fourier amplitudes are similar to Figure 2.3, but with lower values of the Fourier amplitudes (Ngeow et al., 2003) because the I-band amplitudes are generally smaller (Freedman, 1988). From this figure, it is clear that the Fourier amplitudes occupy certain ranges in the A_j vs. $\log(P)$ plots. Therefore we can determine the appropriate ranges of Fourier amplitudes for a given period from this figure. An example of the adopted ranges for the Fourier amplitudes in V-band are presented in Table 2.1 (see also Ngeow et al., 2003, for the adopted ranges of the I-band Fourier amplitudes), as these ranges are used in Chapter 3 to construct the V-band light curves for *HST* observed Cepheids. The ranges of the Fourier amplitudes are set to be started from zero (i.e., $a_j^{low} = 0$) to account for possible low amplitude Cepheids. Note that the upper limit of the ranges given in Table 2.1 is an approximation, since there is no exact upper limit for a given period. Experience shows that sometimes a slightly larger range of the Fourier amplitudes than the one given in Table 2.1 can reconstruct the light curve better. An iterative process can be made, if necessary, to find the most suitable upper limits to reconstruct satisfactory light curves. The reduction in ranges can improve the quality of the Fourier fit, as some of the numerical bumps are removed in the reconstructed light curve, provided that the problem of bad phase coverage is not too severe (see next sub-section).

The distributions of the Fourier amplitudes for OGLE LMC Cepheids and the “calibrating set” (or mostly Galactic) Cepheids appear to coincide, as seen in Figure 2.3. This may imply that the ranges of the Fourier amplitudes depend weakly on metallicity. However, this conclusion is only based on the analysis of two galaxies, and may not reflect the assumption that metallicity can affect the distribution of Fourier amplitudes. van Genderen (1978) showed that the upper limits of B band amplitudes are different for Cepheids in Galaxy/M31, LMC and SMC. In addition, Paczyński & Pindor (2000) showed that the OGLE LMC Cepheids, in the period range of $1.1 < \log(P) < 1.4$, have larger amplitudes than OGLE SMC Cepheids with same period ranges. Because the V-band amplitudes can be scaled from B and I-band amplitudes (Freedman, 1988), the different V-band

Figure 2.3 –The distributions of the V-band Fourier amplitudes (A_j) as function of $\log(P)$ for OGLE LMC Cepheids with well constructed light curves (crosses) and the Cepheids in the “calibrating set” (filled circles).

Table 2.1 –The adopted ranges for the V-band Fourier amplitudes.

Period Ranges	$A_1(V)$	$A_2(V)$	$A_3(V)$	$A_4(V)$
$0.0 < \log(P) \leq 1.0$	0.0-0.40	0.0-0.18	0.0-0.11	0.0-0.08
$1.0 < \log(P) \leq 1.1$	0.0-0.44	0.0-0.14	0.0-0.08	0.0-0.07
$1.1 < \log(P) \leq 1.2$	0.0-0.48	0.0-0.16	0.0-0.10	0.0-0.08
$1.2 < \log(P) \leq 1.3$	0.0-0.52	0.0-0.19	0.0-0.13	0.0-0.09
$1.3 < \log(P) \leq 1.4$	0.0-0.52	0.0-0.23	0.0-0.16	0.0-0.10
$1.4 < \log(P) \leq 1.5$	0.0-0.52	0.0-0.26	0.0-0.16	0.0-0.10
$1.5 < \log(P) \leq 1.6$	0.0-0.49	0.0-0.25	0.0-0.14	0.0-0.10
$1.6 < \log(P) \leq 1.7$	0.0-0.48	0.0-0.23	0.0-0.13	0.0-0.09
$1.7 < \log(P) \leq 1.8$	0.0-0.43	0.0-0.20	0.0-0.12	0.0-0.08
$1.8 < \log(P) \leq 1.9$	0.0-0.40	0.0-0.17	0.0-0.11	0.0-0.07
$1.9 < \log(P) \leq 2.0$	0.0-0.35	0.0-0.13	0.0-0.10	0.0-0.06

upper limits for Cepheids in different galaxies could also exist. Nevertheless, the ranges of Fourier amplitudes as given in Table 2.1 are assumed to cover the Fourier amplitudes for different metallicity

environments, because these ranges go from zero to the upper limit that is slightly larger than the ranges defined by OGLE LMC and “calibrating set” Cepheids. The detailed analysis of the relationship between the Fourier amplitudes and the metallicity environments will be presented in future work.

2.2.1 Results and Problems of the Fits

Using the Fourier expansion with the appropriate ranges for the Fourier amplitudes, some of the poorly reconstructed light curves can be improved and the results show good agreements to observational data. Figure 2.4 shows some examples of the improved reconstructed light curves when fitting equation (2.2) to the OGLE LMC V-band data by narrowing the Fourier amplitudes as given in Table 2.1. As can be seen from the figure, some of the obvious bumps (or dips) in the original reconstructed light curves can be removed with this simple technique, resulting in smoother light curves and better fits to the data. The estimation of the mean magnitudes would be closer to the *true* mean without the influence of the numerical bumps/dips. Note that for the cases with well-sampled data, both the least-squares solution and the simulated annealing method can fit the Fourier expansion to the data equally well.

Figure 2.4 –Examples of the reconstructed light curves fitted with the ranges of Fourier amplitudes given in Table 2.1. These light curves are indicated in solid curves. The dotted curves are the light curves resulted from unconstrained fit (equivalent to least squares fit). The periods of the OGLE LMC Cepheids are also given in upper-right corner. Original data points are also indicated.

In certain extreme cases that either the data points are heavily clustered in certain phases or there is a large gap between two adjacent data points, the Fourier expansion fails to construct a satisfactory light curve, regardless of the choice of the ranges for Fourier amplitudes. Figure 2.5 shows some examples of this case. By using either the initial ranges (dotted curves) or the narrower ranges (solid curves) to fit the Fourier amplitudes with observed data points, the reconstructed light curves are still not satisfactory due to the phase clustering of the original data. When using this technique to estimate means for the PL relations, these Cepheids should be treated with caution or rejected.

Figure 2.5 –Examples of the reconstructed light curves that exhibit numerical bumps/dips, regardless of fitting the data with smaller (solid curves, as given in Table 2.1) or larger (dotted curves) ranges of the Fourier amplitudes. This is mainly due to the bad phase coverage in the data, with gaps in between the data points. Original data points are also indicated.

2.3 The Fourier Interrelations

Because the I-band observation of HST typically consist of 4 epochs, equation (2.2) cannot be applied to reconstruct the light curves. However, statistical relationships between the j^{th} -order Fourier coefficients for V- and I-band have been introduced by Hendry et al. (1999) and by Kanbur & Nikolaev (2001), known as the Fourier interrelations. We explicitly derive and present them here because of their applicability in stellar pulsation studies. These allow the reconstruction of I-band light curves up to 4^{th} order. There also exist correlations of the first order Fourier amplitude (A_1) to subsequent higher order Fourier amplitudes in the same bands (for example, see Antonello et al.,

1987), known as the Fourier intrarelations (presented in Appendix B), however this approach is less preferable than Fourier interrelations.

The Fourier interrelations are the linear relation of j^{th} -order Fourier amplitudes and phases from V-band to I-band, and have the following form:

$$A_j(I) = \alpha_j + \beta_j A_j(V), \quad (2.3)$$

$$\phi_j(I) = \gamma_j + \eta_j \phi_j(V). \quad (2.4)$$

The coefficients of the interrelations are determined from the “calibrating set”, by minimizing the χ^2 value of the fit. Fourier interrelations for fundamental mode Cepheids are presented in Table 2.2. The table also shows the rms of the straight-line fit to the data. The fits were derived taking full account of errors in both variables with the standard model fitting procedures (for example, see Press et al. 1992, Chapter 15). The Fourier interrelations for equation (2.3) & (2.4) are clearly shown in Figure 2.6 & 2.7, respectively. The standard errors of the Fourier coefficients, derived from inverting the Hessian matrix, are also shown in the figures. Despite the appearance of strong correlations in the figures, the relations have large χ^2 per degree of freedom, i.e. very small statistical significance. Nevertheless, using interrelation to recover sparsely sampled light curves works quite well, as shown in Section 2.5. For the Galactic data in the “calibrating set”, we display bump and non-bump Cepheids with solid and open squares respectively (due to the $P_2/P_0 = 0.5$ resonance around 10 days on Hertzsprung progression, the periods of bump Cepheids normally lie in between 8 and 14 days.). For the LMC/SMC data in the “calibrating set”, we do not differentiate between bump and non-bump Cepheids.

The procedures for reconstructing the I-band light curves are similar to V-band light curves. Instead of fitting all the parameters (the mean magnitude and the Fourier parameters), we use the Fourier interrelations to obtain the I-band Fourier parameters from the V-band fit. This gives the shapes of the I-band light curves, and the mean I-band magnitudes are found by using the observed I-band data to minimize the corresponding χ^2 values.

Besides the Fourier interrelations among the “calibrating set” Cepheids, similar linear relations also exist for OGLE LMC and SMC Cepheids (Ngeow et al., 2003). There is little difference between the Fourier interrelations for the OGLE LMC and SMC Cepheids in the slopes of the best fit lines to that found for “calibrating set” Cepheids (Ngeow et al., 2003). The good agreement between them indicates that the *relative* changes in Fourier parameters are almost unaffected by metallicity.

Figure 2.6 –The Interrelations among the Fourier amplitudes in the “calibrating set” Cepheids. Crosses represent corresponding Fourier amplitudes and standard errors for each Cepheids. The dashed lines are the best-fit straight lines to the data, with the slopes and zero points given in Table 2.2. Solid and open squares are for bump and non-bump Cepheids.

This does not mean, however, that the light curve shape is independent of metallicity. Rather, it means that change in metallicity affects the Fourier parameters in different bands in the same way. Therefore, given a well-sampled light curve in V-band, one can estimate Fourier parameters in I-band reasonably well regardless of the metallicity of the parent galaxy. This result is useful in reconstructing the light curves of extra-galactic Cepheids in a broad range of metallicity environ-

Figure 2.7 –Same as Figure 2.6, except for the Interrelations among the Fourier phases.

ments. Hence, the Fourier interrelations given in Table 2.2 will be applied to reconstruct the I-band light curves for *HST* observed Cepheids.

The empirical relation that the earlier papers of H0KP used to obtain the I-band mean magnitude is based on the observational results of Freedman (1988), as the amplitude ratio of I- and V-band is about 0.5. However, Tanvir (1997) recommended use of an amplitude ratio of 0.6, because this ratio can improve the estimation of I-band mean magnitude. We note that the slopes in the Fourier interrelations (β term in Table 2.2) are approximately equal to 0.6, which is close to the Tanvir's

Table 2.2 –The coefficients of the Fourier Interrelations, as determined from “calibrating set”, for fundamental mode Cepheids^a.

Relation	α	β	χ^2
A_1	-0.006 ± 0.002	0.643 ± 0.007	2.69×10^2
A_2	0.001 ± 0.001	0.600 ± 0.011	9.74×10^2
A_3	0.000 ± 0.001	0.645 ± 0.023	6.05×10^1
A_4	0.000 ± 0.001	0.631 ± 0.039	4.85×10^1
Relation	γ	η	χ^2
ϕ_1	-0.178 ± 0.033	0.996 ± 0.001	2.21×10^2
ϕ_2	-0.048 ± 0.010	1.005 ± 0.003	6.89×10^1
ϕ_3	-0.015 ± 0.021	1.003 ± 0.004	1.02×10^2
ϕ_4	-0.001 ± 0.034	1.004 ± 0.006	7.10×10^1

^a Coefficients are corresponding to those defined in equation (2.3) & (2.4). Parameter χ^2 characterizes the goodness of fit: $\chi^2 = \sum_i (y_i - \alpha - \beta x_i)^2 / (\sigma_{y,i}^2 + \beta^2 \sigma_{x,i}^2)$.

value. This is the main reason why Fourier interrelation exists and work well in reconstructing the light curves (Section 2.5).

2.4 Error Analysis for Light Curve Reconstruction when Applied to *HST* Cepheids

In order to estimate the errors associated with the light curve reconstruction procedures presented in Section 2.2 & 2.3, we performed an error analysis based on Monte-Carlo simulations. We emphasize that we are simulating the photometric data published in the literature. It has been suggested that even this is not a real simulation of *HST* Cepheid data since we neglect the possibility of events such as “warm pixels” or cosmic rays. However we contend that such data points will either be rejected by the photometric reduction package used and, if not, the point sources responsible for these should not be used anyway.

Due to the large number of Cepheids, as well as the large number of observations in V- and I-band, the simulations are performed primarily with OGLE LMC Cepheids that have good light curves (as in Figure 2.2[a] and represented by crosses in Figure 2.3). These OGLE LMC Cepheid light curves, constructed by using all the available data points in both bands, are referred as the original light curves. We assume that when we fit a 4th-order Fourier expansion to these data, the resulting Fourier amplitudes are very close to their true values. In order to mimic the published *HST* photometric data, we performed three simulations for the error analysis. The procedures for each simulation are as follows:

- **Simulation 1:** First a Cepheid is picked randomly (with replacement) from the data set, then 12 points in the V-band and 4 points in the I-band were randomly selected without replacement. In addition to the OGLE LMC photometric errors, we add Gaussian noise of $\mu_{noise} = 0.05\text{mag.}$ and $\sigma_{noise} = 0.10\text{mag.}$ to these randomly selected data.
- **Simulation 2:** Same as in Simulation 1, but with larger Gaussian noise of $\mu_{noise} = 0.15\text{mag.}$ and keep $\sigma_{noise} = 0.10\text{mag.}$
- **Simulation 3:** We pick one Cepheid with large number of epochs ($N_V = 34$, $N_I = 188$), then 12 points in the V-band and 4 points in the I-band were randomly selected without replacement from this Cepheid. The additional Gaussian noise is same as in Simulation 1.

Then, the data from each simulation are used to reconstruct the light curves with the procedures described in Section 2.2 & 2.3, i.e. the 4th-order Fourier fit in the V-band and the Fourier interrelations in the I-band. The randomly selected 12 V points could of course be uniformly distributed or concentrated around one particular phase point. Mean magnitudes and the Fourier amplitudes obtained from our reconstruction procedures for these simulated data are then compared to the mean magnitudes and Fourier amplitudes from the original light curves. After running $N = 1000$ trials in each simulations, the error histograms for the mean magnitudes, as well as the Fourier amplitudes, in both V- and I-band are constructed. Gaussian distributions with parameters of μ and σ are then fitted to these error histograms, where μ represent the mean offsets between the simulated data and original data, and σ represent the errors in either the means or the Fourier amplitudes. The results of the Gaussian fits from each simulations for V- and I-band are presented in Table 2.3 & 2.4, respectively. The error histograms for the mean values, resulted from Simulation 1, are presented in Figure 2.8, with abscissa to be the difference between the simulated means and original means. Similarly, the errors histogram for Fourier amplitudes in V- and I-band are presented in Figure 2.9 & 2.10, respectively. The parameters from Gaussian fits are listed in the upper-left corners of these figures. We did not include the error histograms for Simulation 2 & 3 in this chapter, because they all look similar to the error histograms of Simulation 1 (Figure 2.8, 2.9 & 2.10).

From Table 2.3 & 2.4, it can be seen that the errors of the means and each Fourier amplitudes are consistent in all three simulations. As expected, Simulation 2 gave the largest errors because the Gaussian noise generated in this simulation is bigger than other two simulations. Simulation 3 has the smallest errors, because this simulation is performed for one star. The errors for Simulation 1 is in between these two cases, which is typical. In addition, Table 2.3 & 2.4 shows that the mean offsets of the simulated data and original data are very small. This can be seen in Figure 2.8-2.10,

Figure 2.8 –Histograms of offsets for mean magnitudes in OGLE LMC Cepheids from light curve reconstruction procedures. The x-axes are the offsets of the simulated means from the original values, and the y-axes are the normalized counts. The dashed curves are the fitted Gaussian distribution to the histograms, with the parameters given in upper-left corners. Left and right panels are for V- and I-band, respectively.

Table 2.3 –The results of error analysis from simulations in V-band. The Gaussian parameters μ and σ are mean off-sets and errors, respectively, for the means and the Fourier amplitudes.

Simulation	μ_{mean}	μ_{A1}	μ_{A2}	μ_{A3}	μ_{A4}
1...	0.0014	-0.0121	-0.0063	0.0002	0.0063
2...	0.0006	-0.0174	-0.0062	0.0013	0.0095
3...	-0.0018	-0.0226	-0.0154	-0.0071	0.0059
Simulation	σ_{mean}	σ_{A1}	σ_{A2}	σ_{A3}	σ_{A4}
1...	0.0312	0.0355	0.0330	0.0259	0.0210
2...	0.0337	0.0393	0.0332	0.0280	0.0228
3...	0.0256	0.0324	0.0272	0.0242	0.0172

Table 2.4 –Same as Table 2.3, but for the I-band.

Simulation	μ_{mean}	μ_{A1}	μ_{A2}	μ_{A3}	μ_{A4}
1...	0.0012	-0.0071	-0.0053	0.0018	0.0048
2...	0.0010	-0.0109	-0.0052	0.0020	0.0064
3...	-0.0042	-0.0132	-0.0039	0.0018	-0.0004
Simulation	σ_{mean}	σ_{A1}	σ_{A2}	σ_{A3}	σ_{A4}
1...	0.0283	0.0226	0.0195	0.0158	0.0130
2...	0.0348	0.0253	0.0206	0.0175	0.0145
3...	0.0251	0.0208	0.0163	0.0156	0.0108

Figure 2.9 –Histograms of offsets for the V-band Fourier amplitudes in OGLE LMC Cepheids from light curve reconstruction procedure. The dashed curves are the fitted Gaussian distribution to the histograms, with the parameters given in upper-left corners.

as all histograms are almost centered at zero. Therefore, no bias introduced by the reconstruction procedure, because all four Fourier amplitudes are tightly clustered around the “true” values given by the original well sampled Cepheid. The simulations suggests that with typical *HST* Cepheid data, it is meaningful to fit a 4th-order Fourier expansion and that our direct Fourier fitting procedures for estimating V- and I-band means are not biased.

2.5 Conclusion and Discussion for this Chapter

The light curve of a Cepheid can be reconstructed with the Fourier techniques described in this chapter. In particular, the quality of the reconstructed light curves can be improved by restricting the range of Fourier amplitudes can take in the Fourier fit. These ranges are obtained from well sampled Cepheid light curves in the Galaxy and Magellanic Clouds. This technique can be applied to the 12 V-band *HST* data with a 4th-order fit. While for the 4 I-band *HST* data, we apply the Fourier

Figure 2.10 –Same as Figure 2.9, but for the I-band.

interrelations to reconstruct the I-band light curves. Monte-Carlo simulations are also performed to estimate the errors in these light curves reconstruction procedures when applied to the *HST* data. Our simulations show that these reconstruction procedures are unbiased and the errors of the mean magnitudes and the Fourier amplitudes are around $\sim 0.03\text{mag}$.

In general, these relatively simple techniques can reconstruct the Cepheid light curves quite well, given that the data points are relatively uniform and well sampled. These techniques serve as an alternative method to obtain the mean magnitude of Cepheids besides phase weighted intensity mean and template fitting procedures. However, the detailed comparisons between these methods are beyond the scope of this chapter. Some examples of the reconstructed light curves for extra-galactic Cepheids with these Fourier techniques are presented in Figure 2.11, which show good agreements to the observed data points. We discuss some related issues as follow:

1. **The Correlations of Fourier Amplitudes:** It has been suggested that the higher order Fourier amplitudes shown in Figure 2.3 are correlated with the first order Fourier amplitude

Figure 2.11 – Examples of the reconstructed light curves in nearby galaxies. The V-band light curves (left panel) are reconstructed via 4th-order Fourier expansion, and the I-band light curves (right panel) are reconstruct via the Fourier interrelations.

(A_1) in the same bands (the Fourier intrarelations). These have been examined, for example, by Antonello et al. (1987) and in the Appendix B. However, the Fourier intrarelations are less preferable than Fourier interrelations for reconstructing the Cepheid light curves. The reasons are discussed in following examples and in Appendix B.

Here, we reexamine the results of Antonello et al. (1987) with our data sets and compare them to the Fourier interrelations. We used the data in Figure 2.3 and we re-plot the A_1 - A_2 Fourier intrarelations and the $A_1(V)$ - $A_1(I)$ Fourier interrelation in Figure 2.12. The “Calibrating set” Cepheids and OGLE LMC Cepheids are in left and right panels, respectively. In the figure, we distinguished Cepheids with different period ranges: (a) Cepheids with periods shorter than 8 days (short-period Cepheid, crosses); (b) Cepheids with period in between 8 to 14 days (bump Cepheid, triangles); and (c) Cepheids with periods longer than 14 days (long-period Cepheid, filled circles). The top two panels in Figure 2.12 are Fourier intrarelations in V- and I-band respectively, and the bottom panel is the Fourier interrelations of $A_1(V)$ and $A_1(I)$. It is clear from the figure that the scatter of A_1 - A_2 intrarelations are larger than the scatter of $A_1(V)$ - $A_1(I)$ interrelations. Furthermore, the short-period, bump and long-period Cepheids populate different regions in the plots of intrarelations, as is also seen in Antonello et al. (1987). In contrast, The tightness of correlations in the Fourier interrelations is clear, and less dependent

on period distribution. These two properties make Fourier interrelations more applicable in reconstructing the light curves than Fourier intrarelations.

Figure 2.12 –Comparisons of the correlations for A_1 - A_2 and $A_1(V)$ - $A_1(I)$. Crosses are for Cepheid with period shorter than 8 days; triangles are for Cepheids with period in between 8 to 14 days; and filled circles are for Cepheids with period longer than 14 days. Error bars are omitted for clarity. Left and right panels are for the “calibrating set” Cepheids and the OGLE LMC Cepheids, respectively.

2. The Template Light Curves and the Short Period Cepheids: From Figure 2.3, it can be seen that the distribution of the Fourier amplitudes at periods longer than 10 days (or $\log(P) > 1.0$) show certain trends, which can be used to construct templates light curves as a function of period, as given in Stetson (1996). We compared the second Fourier amplitude coefficient (A_2) calculated from Section 2.2 with the template light curves defined in Stetson (1996) for the “calibrating set” and the OGLE LMC Cepheids. The results is shown in Figure 2.13. Since the data in this figure are well sampled data, it is safe to assume that the progression of A_2 with period is a good representation of reality. The bottom panel shows the same Fourier amplitude calculated using the template method. It can clearly be seen that almost at all periods the range of A_2 at given period is larger than it would be predicted by the Stetson (1996) technique.

The relatively large ranges of the Fourier amplitudes for short period Cepheids ($\log(P) < 1.0$) is one reason for using the Fourier techniques presented in Section 2.2 to reconstruct the Cepheid light curves. In contrast to the long period Cepheids, the Fourier amplitudes for

short period Cepheids do not show any obvious trends but scatter around certain ranges (as given in Table 2.1). For example, at a given short period, $A_1(V)$ may occupy the range from ~ 0.1 to ~ 0.4 . Therefore, extra care has to be taken when applying template fitting to the Cepheids with periods less than 10 days. In fact, at periods shorter than $\log(P) < 0.85$ the template techniques do not apply (Stetson, 1996) but the Fourier techniques still holds good.

Figure 2.13 –Comparison of A_2 from Fourier techniques and template methods. The upper panels show the values of A_2 from direct Fourier fit with the method described in Section 2.2, and the lower panels show the values of A_2 by using the template light curves given by Stetson (1996). Since the template light curves are undefined for $\log(p) < 0.85$, Cepheids with period shorter and longer than $\log(p) = 0.85$ are represented as open circles and filled triangles, respectively. Left and right panels are for the “calibrating set” Cepheids and the OGLE LMC Cepheids, respectively.

Although the extra-galactic Cepheids discovered in the past *HST* observations generally have period longer than 10 days, some short period Cepheids have been discovered in Local Group galaxies IC 1613 (Dolphin et al., 2001) & Leo A (Dolphin et al., 2002) with *HST* and ground based observations, respectively. Hence for observing Cepheids at short period, the Fourier techniques are still useful to reconstruct the light curves. With the new installation of *ACS* (Advanced Camera for Survey) in *HST*, the discovery of more short period Cepheids in other galaxies is possible.

3. **The Effect of Metallicity:** One motivation for studying direct Fourier techniques to reconstruct Cepheid light curves is to deal with the possible metallicity dependence in Cepheids. Antonello et al. (2000) shown that the metallicity may affect the shapes of light curves close to 10 days, based on the comparison of Cepheid light curves in two galaxies. The appropriate

ranges of the Fourier parameters (as given in Table 2.1) are assumed to cover the possible ranges due to the different metallicity environments. In addition, the Fourier interrelations are shown to be only weakly dependent on the metallicity though this does not mean that the actual light curve shape is independent of metallicity.

The template light curves defined by Stetson (1996) are obtained from the Galaxy ($Z = 0.020$), LMC ($Z = 0.008$) and SMC ($Z = 0.004$), with sample of $\sim 30 - 45$ Cepheids in each galaxies. Stetson (1996) looked for and failed to find differences between the Galactic, LMC and SMC Cepheids in terms of their amplitudes and light curve shapes sufficient to change the estimation of mean magnitudes. However, much more data is available now. Paczyński & Pindor (2000) found statistically significant different amplitudes between the Galactic/LMC/SMC Cepheids in the period range $1.1 < \log(P) < 1.4$, in the sense that higher metallicity Cepheids have higher amplitudes. Moreover, in the period range $0.0 < \log(P) < 0.95$, the amplitudes ratio of R_{21} and R_{31} for Cepheids in LMC (Andreasen & Petersen, 1987) and SMC (Andreasen, 1988a) are found to be larger than the Galactic Cepheids. This may due to the different metallicity environments, at least for the case of the SMC (Buchler & Moskalik, 1994). Further some established workers in the field have found varying PC relations between the Galaxy, LMC and SMC and subsequently different PL relations (Tammann et al., 2002, 2003; Ngeow & Kanbur, 2004; Storm et al., 2004). Thus while it may be that template methods yield a sufficiently accurate mean in varying metallicity environments, we contend that the issue of light curve shape and metallicity needs to be revisited in the light of the data now currently available.

CHAPTER 3

THE EXTRA-GALACTIC CEPHEID DISTANCE SCALE FROM LMC AND GALACTIC PL RELATIONS

In this chapter, we re-calibrate the Cepheid distance to some nearby galaxies observed by the *HST* Key Project (KP) and the Sandage-Tammann-Saha (STS) group. We use much of the Key Project methodology in our analysis but apply new techniques, based on Fourier methods to estimate the mean of a sparsely sampled Cepheid light curve, to published extra-galactic Cepheid data. We also apply different calibrating PL relations to estimate Cepheid distances, and investigate the sensitivity of the distance moduli to the adopted calibrating PL relation. We re-determine the OGLE LMC PL relations using a more conservative approach and also study the effect of using Galactic PL relations on the distance scale.

For the KP galaxies after accounting for charge transfer effects, we find good agreement with an average discrepancy of -0.002 and 0.075mag. when using the LMC and Galaxy, respectively, as a calibrating PL relation. For NGC 4258 which has a geometric distance of 29.28mag. , we find a distance modulus of $29.44 \pm 0.06(\text{random})\text{mag.}$, after correcting for metallicity. In addition we have calculated the Cepheid distance to 8 galaxies observed by the STS group and find shorter distance moduli by -0.178mag. (mainly due to the use of different LMC PL relations) and -0.108mag. on average again when using the LMC and Galaxy, respectively, as a calibrating PL relation. However care must be taken to extrapolate these changed distances to changes in the resulting values of the Hubble constant because STS also use distances to NGC 3368 and NGC 4414 and because STS calibration of SN Ia is often decoupled from the distance to the host galaxy through their use of differential extinction arguments.

However, after correcting for metallicity effects, the difference between the distance moduli obtained using the two sets of calibrating PL relations becomes negligible. This suggests that Cepheids in the LMC and Galaxy do follow different PL relations and constrains the sign for the coefficient of the metallicity correction, γ , to be negative, at least at the median period $\log(P) \approx 1.4$, of the target galaxies.

3.1 Cepheids and the Extra-Galactic Distance Scale

The extra-galactic distance scale is one of the key problems in modern astronomy. One of the basic parts of the solution is the correlation between period and mean luminosity (PL) obeyed by classical Cepheids. The Hubble Space Telescope (*HST*) H_0 Key Project (Freedman et al., 2001, hereafter KP in this chapter) has used Cepheids (discovered either by ground-based telescopes or by *HST*) in 31 spiral galaxies, with 18 of them observed originally by KP, and the PL relation to estimate the distances to these galaxies. These distances were then used in turn to calibrate a host of secondary distance indicators¹ and hence estimate Hubble's constant. In parallel with this a team led by Sandage et al. (Saha et al., 2001b, hereafter STS in this chapter) has used the *HST* to discover Cepheids in spiral galaxies which host Type Ia supernovae. Cepheid distances to these galaxies were used to calibrate the Hubble diagram for Type Ia supernovae and hence estimate the Hubble's constant.

Though both groups used mostly *HST* observations to discover the Cepheids and similar methodologies, the KP team favor a short distance scale and a larger value of H_0 ($72 \pm 8 \text{ km s}^{-1} \text{ Mpc}^{-1}$, Freedman et al., 2001), whilst the STS group favor a long distance scale and smaller value of H_0 ($58.7 \pm 6.3 \text{ km s}^{-1} \text{ Mpc}^{-1}$, Saha et al., 2001b). The discrepancy in the value of the Hubble constant from these two groups is still unresolved (e.g., see Hendry & Kanbur, 1996; Beaulieu et al., 1997; Kochanek, 1997; Sasselov et al., 1997; Tanvir et al., 1999; Caputo et al., 2000b; Gibson et al., 2000; Gibson & Stetson, 2001). It is due only in part to the Cepheid distance scale. We note that the recent WMAP results (Spergel et al., 2003) find a Hubble constant very similar to the value obtained by KP. Nevertheless it is still very instructive to discover the reason behind the discrepancies in the Cepheid distance scale from the STS and KP groups. A proper understanding of the Cepheid PL relation in this regard is very important for an accurate local distance scale.

In this chapter we concentrate on re-calibrating the Cepheid distance to the target galaxies in both KP and STS groups with existing data because of the following factors:

1. *HST* Cepheid data is sparsely sampled with typically 12 and 4 points per Cepheid in the V- and I-band respectively (see Kanbur et al., 2003, for details). In order to apply the Cepheid PL relations, it is necessary to estimate the mean magnitudes from these data in both V- and I-band. We use the techniques of Fourier expansion and interrelations, as described in Chapter 2, to estimate the V and I mean magnitudes from sparsely populated light curves.

¹Including Tully-Fisher relation, surface brightness fluctuation method, Type Ia and II supernova luminosity and fundamental plane relation.

2. The average value of metallicities, defined as $12 + \log(O/H)$, in target galaxies is about $8.84 \pm 0.31dex$ (Freedman et al., 2001). This value is closer to the standard Solar value of $8.87 \pm 0.07dex$ (Grevesse et al., 1996) than the LMC value of $8.50 \pm 0.08dex$ (see reference in Ferrarese et al., 2000). Therefore, another approach is to use Galactic Cepheid PL relations as fundamental calibrating relations (Feast, 2001, 2003; Fouqué et al., 2003; Tammann et al., 2003; Thim et al., 2003). In addition, we used the LMC PL relation derived from the re-analyzed of the OGLE (Optical Gravitational Lensing Experiment) LMC Cepheids (Udalski et al., 1999b) on the basis of the quality of their V-band light curve, and develop more conservative LMC PL relations based on this analysis (Kanbur et al., 2003). This approach is different to the “sigma-clipping” methods currently used by Udalski et al. (1999a). We also investigate the sensitivity of extra-galactic Cepheid distances to the LMC PL relation. We compare the distances obtained when using both the LMC and Galactic PL relations.

3.2 General Methodology for Deriving the Cepheid Distance

The physical basis of Cepheid PL relations and their usage in determining the distance have been covered extensively in the literature (e.g., see Sandage & Tammann, 1968; Feast & Walker, 1987; Madore & Freedman, 1991; Freedman et al., 2001) and would not be repeated here, only a brief description will be presented. The application of the Cepheid PL relation to estimate distances involves the discovery and appropriate observation of Cepheids in a target galaxy that are sufficient to estimate their periods (in days) and mean magnitudes. Observations of extra-galactic Cepheids by *HST* are normally taken in the bandpasses of V and I, for the purpose of correcting the extinction/reddening. In order to avoid the incompleteness bias at the faint end of the observations (see Sandage, 1988; Lanoix et al., 1999a; Freedman et al., 2001), a period cut to the short period Cepheids in the target galaxy is commonly applied (see, for example, Freedman et al., 2001; Leonard et al., 2003). Then for the remaining N Cepheids, the following procedures are adopted to obtain the distance:

1. The PL relation that used to calibrate the Cepheid distance in bandpass λ can be expressed as: $M_\lambda = a_\lambda \log(P) + b_\lambda$, where a and b are the slope and zero-point, respectively. From the measurement of the apparent mean magnitudes for the i^{th} Cepheids in a target galaxy, m_λ^i , the *apparent* distance modulus in bandpass λ is $\mu_\lambda^i = m_\lambda^i - a_\lambda \log(P_i) - b_\lambda$. Then the distance modulus in bandpass λ for these Cepheids can be obtained by taking the unweighted mean to individual Cepheids, i.e. $\overline{\mu_\lambda} = \frac{1}{N} \sum_{i=1}^N \mu_\lambda^i$.

2. Since $\mu_0 = \mu_V - A_V = \mu_I - A_I$, the reddening-free distance modulus, or the Wessenheit function (Madore, 1982; Moffett & Barnes, 1986; Madore & Freedman, 1991; Tanvir, 1999; Leonard et al., 2003, , see Chapter 7 also for more details), can be derived as:

$$\mu_0 = \mu_V - R(\mu_V - \mu_I), \quad (3.1)$$

where $R \equiv A_V/E(V - I)$ is the ratio of total-to-selective absorption. Note that the validity of equation (3.1) is based on the assumption that there are no correlated measurement errors in the V and I bandpasses. Then any differences in the V- and I-band distance moduli are due to differential reddening (see Saha et al., 1996, 2001b, for details).

3. The reddening-free distance modulus for the i^{th} Cepheid can be calculated with equation (3.1), and it is straight forward to show that:

$$\overline{\mu_0} = \frac{1}{N} \sum_{i=1}^N \mu_0^i \quad (3.2)$$

$$= \overline{\mu_V} - R(\overline{\mu_V} - \overline{\mu_I}), \quad (3.3)$$

where $\mu_0^i = \mu_V^i - R(\mu_V^i - \mu_I^i)$.

4. Finally, due to the possible metallicity dependence of the Cepheid PL relation (see the discussions and reference in, e.g., Kennicutt et al., 1998; Freedman et al., 2001), metallicity corrections are added to μ_0 :

$$\mu_z = \overline{\mu_0} + \delta_z, \quad (3.4)$$

where $\delta_z = \gamma([O/H]_{ref} - [O/H]_{gal})$ with the usual definition of $[O/H] \equiv 12 + \log(O/H)$. The reference metallicity, $[O/H]_{ref}$, is $8.50dex$ and $8.87dex$ for using LMC and Galactic PL relations, respectively. Note that the value of γ is method-dependent, i.e. it depends on the bandpasses and calibrating PL relation adopted.

This procedure (see, for example, Freedman et al., 2001; Leonard et al., 2003; Kanbur et al., 2003; Tanvir, 1997), i.e. calculating the distance modulus for individual Cepheids and taking the unweighted average to be the final distance modulus to that target galaxy (equation [3.2]), is equivalent to fitting the V- and I-band PL relations to the data and obtaining the $\overline{\mu_V}$ and $\overline{\mu_I}$ that apply to equation (3.3). Other reasons for taking the unweighted mean are: (a) the photometric errors of the mean magnitudes are smaller than the expected width of the instability strip (which cause an intrinsic dispersion for the PL relation), which are, together with other systematic errors, the dominant part of the weight in a weighted mean (Leonard et al., 2003); and (b) it can be easily incorporated with the weighting scheme adopted by the STS group (Tanvir, 1997). Although both equations (3.2) and (3.3) give the same distance modulus, equation (3.2) assumes that $\mu_0 = \mu_V - R(\mu_V - \mu_I)$ is applicable to *individual* Cepheids in a target galaxy, while equation (3.3) assumes that $\mu_0 = \mu_V - R(\mu_V - \mu_I)$ is only applicable to an *ensemble* of Cepheids in the same target galaxy. Here we adopt the first approach (equation [3.2]) to calibrate the Cepheid distance, as this is also applied in KP.

For the parameters needed in this procedure, we discuss the adopted PL relations in the next section. For the value of R in equation (3.1) or (3.3), we adopt $R = 2.45$ from Cardelli et al. (1989) as in KP, although the STS team prefer a value of 2.43 (Saha et al., 2001b). For the value of γ in equation (3.4), some published values of γ range from -0.88 ± 0.16 & -0.56 ± 0.20 (Gould, 1994), $-0.44^{+0.10}_{-0.20}$ (Beaulieu et al., 1997; Sasselov et al., 1997), -0.4 ± 0.2 (Kochanek, 1997), -0.32 ± 0.21 (Freedman & Madore, 1990), -0.24 ± 0.16 (Kennicutt et al., 1998) to 0.27 (Caputo et al., 2000b). Strictly speaking, the metallicity dependence quoted in Kennicutt et al. (1998) is only applicable when the Madore & Freedman (1991) PL relation is used, and then only for the V and I bandpasses. Although there is some debate on the value for γ (see Freedman et al., 2001; Tammann et al., 2002, for details), we adopt $\gamma \equiv \gamma_{V,I} = -0.2 \pm 0.2 \text{ mag dex}^{-1}$ as in KP, who note that this value is in the middle of a number of different determinations of γ . Changes in metallicity affect the mean brightness of a Cepheid and the papers quoted above were aimed at quantifying this brightness shift due to metallicity. However, we now know that the Cepheid PL relation has a different slope in the Galaxy and LMC, and indeed may have two different slopes in the LMC itself. Thus it may be that metallicity affects not only the mean brightness of a Cepheid but also the way that mean brightness changes with period (the slope of the PL relation).

Since we are using the published photometry data to calculate the distance modulus, we share the same systematic errors as in the published results. These include the uncertainty in the zero point of the PL relation, calibration of photometric zero points, reddening and metallicity effects,

incompleteness bias and crowding. The order of a typical systematic error is around $\sim 0.1 - 0.2\text{mag}$. We discuss the calculation of random (or statistical) errors in the next subsection.

3.2.1 A Note on the Random Errors for μ_0

By definition, the random error for $\bar{\mu} = \frac{1}{N} \sum_{i=1}^N \mu^i$, due to purely statistical fluctuations, is:

$$\sigma^2 = \frac{1}{N(N-1)} \sum_{i=1}^N (\mu^i - \bar{\mu})^2. \quad (3.5)$$

This equation holds for the $\bar{\mu}_\lambda$ and $\bar{\mu}_0$ (from equation [3.2]). We can expand the expression for σ_0^2 by substituting the expression for μ_0^i and $\bar{\mu}_0$ to the above equation, then:

$$\begin{aligned} \sigma_0^2 &= \frac{1}{N(N-1)} \sum_{i=1}^N (\mu_0^i - \bar{\mu}_0)^2 \\ &= (1-R)^2 \sigma_V^2 + R^2 \sigma_I^2 - \text{CORR}_{VI}, \end{aligned} \quad (3.6)$$

where σ_V^2 and σ_I^2 are given by equation (3.5), and $\text{CORR}_{VI} = \frac{2(R-1)R}{N(N-1)} \sum_{i=1}^N (\mu_V^i - \bar{\mu}_V)(\mu_I^i - \bar{\mu}_I)$, a term for the correlated residuals from both bands (Tanvir, 1997; Freedman et al., 2001). This random error, as given in equation (3.5), is for the distance modulus μ_0 obtained from equation (3.2). Note that the use of equation (3.5) to estimate the random errors for the Cepheid distance modulus has been practiced in the literature (e.g., see Freedman et al., 2001; Kanbur et al., 2003).

On the other hand, if we apply the standard equation for the propagation of errors (POE) to equation (3.3), by ignoring the correlation term, then we have $\sigma_0^2(\text{POE}) = (1-R)^2 \sigma_V^2 + R^2 \sigma_I^2$. By comparing this expression to equation (3.6), we obtain:

$$\sigma_0^2(\text{POE}) = \sigma_0^2 + \text{CORR}_{VI}. \quad (3.7)$$

One immediately can see that $\sigma_0^2(\text{POE})$ is always greater than σ_0^2 since the term CORR_{VI} is most likely to be positive. This is because (a) $R > 1$ from the extinction curve; and (b) $\sum (\mu_V^i - \bar{\mu}_V)(\mu_I^i - \bar{\mu}_I)$ is mostly likely to be positive. The second condition is due to the existence of the period-luminosity-colour relation: if the V-band magnitude for a Cepheid is above (or below) the ridge-line of the V-band PL relation, then the corresponding I-band magnitude will also be above (or below)

the ridge-line of the I-band PL relation. The result is that if $(\mu_V^i - \overline{\mu_V})$ is positive (or negative), then $(\mu_I^i - \overline{\mu_I})$ is also positive (or negative), and hence the product of these two is positive. Further, if the extinction and/or the correlated errors (from measurement) make $(\mu_V^i - \overline{\mu_V})$ to be positive/negative, then $(\mu_I^i - \overline{\mu_I})$ is also going to be positive/negative, and again the product will be positive. Errors estimated from error propagation, i.e. using equation (3.3), will ignore the $CORR_{VI}$ term and hence resulted a larger random error than the random error estimated from equation (3.5). We will use the random error calculation from equation (3.6) in this chapter and in other subsequent chapters. Note that all the errors discussed here, the σ , are random errors only, which do not include the systematic errors such as the errors arise from the calibrations, the width of the instability strip, and others (see, e.g., the discussion by Saha et al., 2000).

3.3 *HST* Data and Analysis

The target galaxies that are selected in this study include: 16 KP galaxies, 8 STS galaxies and NGC 4258, which has an accurate geometrical distance measurement from water maser studies (Newman et al., 2001, hereafter WM galaxy in this chapter). The photometry data and the published periods for the Cepheids in each target galaxy were directly obtained from the corresponding published papers. There are two major photometry reduction packages used in reducing the data for these galaxies, the ALLFRAME (Stetson, 1994, 1996) and a variant of the DoPHOT (Schechter et al., 1993; Saha et al., 1996) package. The photometric data for the KP and STS+WM galaxies are obtained from the ALLFRAME (except for NGC 2541 and NGC 4321 which are only available in DoPHOT) and the DoPHOT, respectively. We emphasize that we did not repeat the photometric reduction from raw data. The list of the target galaxies can be obtained from Table 3.1.

Most of the *HST* observations of extra-galactic Cepheids contain only 12 V-band epochs and 4 (or 5) I-band epochs. These observations use a power-law time series sampling strategy to minimize the aliasing problem and maximize the phase coverage of the observed data points within the observing windows (Freedman et al., 1994; Kennicutt et al., 1995). In order to reduce the bias due to sampling procedures, STS and some early KP observations used phase-weighted intensity means to find mean magnitudes in the V-band. The I-band mean magnitudes were found via empirical relations developed by Freedman (1988) and Labhardt et al. (1997) for KP and STS observations, respectively. Other KP observations used a template-fitting procedure (Stetson, 1996) to obtain the mean magnitudes in both V- and I-band simultaneously. In addition to the methods mentioned, Ngeow et al. (2003) recently developed an alternative method to obtain the mean magnitudes, which

is based on Fourier techniques. Ngeow et al. (2003) give specific examples of situations when such a method could be preferable to template based techniques. Another motivation for developing these Fourier techniques is the possibility of reconstructing observed *HST* light curves to compare with models. We apply the Fourier method as described in Chapter 2, i.e. using the constrained Fourier expansion to the V-band data and the Fourier interrelation to the I-band data, to all target galaxies in order to be consistent in doing the analysis, and also to test this new method in the distance scale problem.

Although the filters installed on the *HST* are close to the standard Johnson and Kron-Cousins system, i.e. $F555W \sim V$ and $F814W \sim I$, conversions from the *HST* passbands to the standard photometry systems have been used by both KP and STS teams. For the case of the ALLFRAME photometry reduction in the KP galaxies, the conversions are made during the reductions with the formulae suggested by Holtzman et al. (1995), hence the published photometry of the Cepheids are in standard bands. A similar situation regarding the conversion holds for the two KP galaxies with DoPHOT photometry (NGC 2541 & NGC 4321) and the WM galaxy. For STS galaxies with DoPHOT photometric reductions, the conversions are applied after the reductions to the calculated mean magnitudes (see equations [5] & [6] in Saha et al. 1994 for WFPC; and equations [2] & [3] in Saha et al. 1996 for WFPC2). These conversions have been applied to STS galaxies in Table 3.4 & 3.5. The derived mean magnitudes in both V- and I-band for the Cepheids used in this study can be found in Kanbur et al. (2003). These mean magnitudes, together with the adopted PL relations (discussed in the next sub-section), can be used to obtain the Cepheids distances (μ_0) to the target galaxies.

There are two additional treatments for μ_0 before the final distance moduli can be adopted. They are the corrections for the charge-transfer efficiency (CTE) and the metallicity corrections, as we discuss below:

1. **CTE Correction:** The CTE for WFPC2 is a complicated issue. Simply speaking, it has been found that the performance of the WFPC2 depends on the exposure time, which lead to the “long-vs-short” exposure corrections, and other factors (see Stetson, 1998; Freedman et al., 2001, for more details). The published photometry data for KP galaxies were based on the Hill et al. (1998) calibration, but the final results presented in Freedman et al. (2001) are in the Stetson (1998) calibration. To convert the calibration from the Hill system to the Stetson system, a correction of $\delta_{CTE} = -0.07 \pm 0.02 \text{ mag.}$ is added to the distance modulus (Mould et al., 2000; Freedman et al., 2001). In contrast, the STS galaxies adopted the Holtzman et al.

Table 3.1 –The CTE and metallicity corrections for *HST* galaxies.(see text for details).

Galaxy (1)	N_{ceph} (2)	$[O/H]^a$ (3)	$\delta_z(LMC)^b$ (4)	$\delta_z(GAL)^b$ (5)	δ_{CTE} (6)
KP Galaxies					
NGC 925	72	8.55 ± 0.15	0.010	-0.064	-0.07
NGC 1326A	15	8.50 ± 0.15	0.0	-0.074	-0.07
NGC 1365	47	8.96 ± 0.20	0.092	0.018	-0.07
NGC 2090	30	8.80 ± 0.15	0.060	-0.014	-0.07
NGC 2541	29	8.50 ± 0.15	0.0	-0.074	-0.07
NGC 3031	17	8.75 ± 0.15	0.050	-0.024	0.0 ^c
NGC 3198	36	8.60 ± 0.15	0.020	-0.054	-0.07
NGC 3319	33	8.38 ± 0.15	-0.024	-0.098	-0.07
NGC 3351	48	9.24 ± 0.20	0.148	0.074	-0.07
NGC 3621	59	8.75 ± 0.15	0.050	-0.024	-0.07
NGC 4321	42	9.13 ± 0.20	0.126	0.052	-0.07
NGC 4414	8	9.20 ± 0.15	0.140	0.066	-0.07
NGC 4535	47	9.20 ± 0.15	0.140	0.066	-0.07
NGC 4548	24	9.34 ± 0.15	0.168	0.094	-0.07
NGC 4725	15	8.92 ± 0.15	0.084	0.010	-0.07
NGC 7331	13	8.67 ± 0.15	0.034	-0.040	-0.07
STS Galaxies					
IC 4182	27	8.40 ± 0.20	-0.020	-0.094	0.0 ^c
NGC 3627	25	9.25 ± 0.15	0.150	0.076	0.05
NGC 3982	14	(8.9 ± 0.4)	0.080	0.006	0.05
NGC 4496A	45	8.77 ± 0.15	0.054	-0.020	0.05
NGC 4527	13	(8.9 ± 0.4)	0.080	0.060	0.05
NGC 4536	31	8.85 ± 0.15	0.070	-0.004	0.05
NGC 4639	15	9.00 ± 0.15	0.100	0.026	0.05
NGC 5253	5	8.15 ± 0.15	-0.070	-0.144	0.0 ^c
WM Galaxy					
NGC 4258	15	8.85 ± 0.15	0.070	-0.004	0.0 ^c

^a $[O/H] \equiv 12 + \log(O/H)$, adopted from Ferrarese et al. (2000), except for NGC 3982 (Stetson & Gibson, 2001) and NGC 4527 (Gibson & Stetson, 2001).

^b $\delta_z = (-0.2 \pm 0.2) \text{ mag dex}^{-1}([O/H]_{ref} - [O/H])$. For LMC, $[O/H]_{ref} = 8.50 \text{ dex}$ and for Galactic, $[O/H]_{ref} = 8.87 \text{ dex}$.

^c Since the observations of NGC 3031, IC 4182 and NGC 5253 were done with WFPC (not WFPC2), CTE correction is not included. The reduction of NGC 4258 used Stetson's (1998) calibration (Newman et al., 2001), no CTE correction is needed.

(1995) calibration, which leads to a correction of $\delta_{CTE} = 0.05 \text{ mag}$. (see Saha et al. 2001a for example). The calibration used in the WM galaxy is in the Stetson system (Newman et al., 2001), hence no CTE correction is needed. Note that no CTE correction is added to the three galaxies (IC 4182, NGC 3031 & NGC 5253) observed with WFPC in the early days of *HST*. The CTE corrections for all target galaxies are summarized in the column 6 of Table 3.1. The CTE corrected distance modulus is referred as $\mu_{0,CTE}$.

2. Metallicity Correction: Due to metallicity differences between target galaxies and the LMC, metallicity corrections, as given in equation (3.4), are commonly applied to $\mu_{0,CTE}$ (as in Freedman et al., 2001). The metallicity of the target galaxies, in terms of $[O/H]$, is given in column 3 of Table 3.1, and the corresponding metallicity corrections with respect to the LMC, $\delta_z(LMC)$, are tabulated in column 4 for the same table. Although the (average) metallicity of the target galaxies is closer to the Galactic value than the LMC (hence the metallicity correction is small), there is no obvious reason for not applying the same metallicity correction when using a Galactic calibrating PL relation. Therefore, we apply the same correction, $\delta_z(GAL)$, to the distance moduli as in the case of using LMC PL relations. These metallicity corrections are listed in column 5 of Table 3.1 for each target galaxies.

3.3.1 The Adopted PL Relations

In order to apply the PL relations to *HST* data, we only look for the published LMC and Galactic PL relations that are available at the time of this study in both V- and I-band, and summarized in this subsection. All of the adopted PL relations are listed in Table 3.2 and 3.3, for the V- and I-band, respectively. Note that for the LMC PL relations in these tables, we adopt $\mu_{LMC} = 18.50\text{mag.}$, to be consistent with KP.

1. The LMC PL Relations: The LMC PL relations used by both KP team (before the publication of their final paper) and STS team are based on a homogeneous sample of 32 Cepheids (Madore & Freedman, 1991, hereafter MF91). Since these PL relations have been extensively used in determining Cepheid distances in the past, and the STS team still use these PL relations in their study (e.g., see Saha et al., 2001b), we adopt the MF91 relations as one of our calibrating PL relations in this study.

After the publication of PL relations derived from OGLE LMC Cepheids (Udalski et al., 1999a, hereafter U99), the KP team re-calibrated their Cepheid distances with these new LMC PL relations in their final paper, as well as the Cepheid distance to NGC 4258 by Newman et al. (2001). These reddening-corrected PL relations were derived using Cepheids with $\log(P) > 0.4$ to minimize possible contamination by first overtone pulsators and were "sigma clipped", resulting in ~ 650 Cepheids used in deriving the PL relations. In order to compare our results with published Cepheid distances, we also adopt the U99 PL relations. The new U99 PL relations have dramatically changed Cepheid distances compared to distances

derived from MF91 PL relations, in the sense that the derived distances are smaller with U99 PL relations (Freedman et al., 2001).

There are some criticisms about the U99 PL relations in the literature. The U99 PL relations are dominated by short period Cepheids (with $\log(P) < \sim 0.5$ and about 90% of them have period shorter than 10 days), and a lack of longer period Cepheids with $\log(P) > 1.5$ (Feast, 2001, 2003; Saha et al., 2001b). However, Freedman et al. (2001), by using Sebo et al. (2002) LMC data, claimed that the use of the OGLE LMC PL relations in estimating distances to target galaxies whose Cepheids all had periods longer than the longest period Cepheid observed by Udalski et al. (1999a) made little difference. Another potential problem of the U99 PL relations is the discovery of a break in the PL relation at $\log(P) = 1.0$, as shown in Tammann et al. (2002) and in Tammann & Reindl (2002), as the long and short period Cepheids follow different PL relations (see also Chapter 1.2.1). Hence we also adopt the PL relations for long period Cepheids from Tammann & Reindl (2002, hereafter TR02) since most of the extra-galactic Cepheids have periods longer than 10 days.

The OGLE LMC PL relations from U99 are obtained with using the sigma-clipping method, which is an iterative procedure whereby the data are fitted with a regression and then those points lying 2.5σ away from the regression line are removed. A new regression is fitted and the procedure continues for few cycles (Udalski et al., 1999a; Willick & Batra, 2000; Groenewegen, 2000). Therefore, it blindly removes all the outliers to reduce the scatter in the final fit. However, this approach may both remove some true Cepheids from the data and include some suspicious Cepheids in the final sample. In addition, Nikolaev et al. (2004) discussed some flaws associated with the sigma-clipping algorithm, including the implicit assumption of the normal distribution of the residuals and the sensitivity of the results to the chosen threshold of $k\sigma$.

Despite these potential problems, Kanbur et al. (2003) re-analyzed the same OGLE LMC data as in Udalski et al. (1999a) to derive the LMC PL relations, but without processing the sigma-clipping procedure. Instead, the fitted light curves (with constrained Fourier expansions mentioned in Chapter 2.2) for these OGLE LMC Cepheids were visually inspected, and eliminations of the Cepheids were carried out according to the following criteria: (a) Cepheid with $\log(P) < 0.4$; (b) Cepheid without V-band data; (c) Cepheids with unusual or poorly

constructed V-band light curves; (d) Cepheid with flat or low amplitude light curves²; and (e) Cepheids with mean magnitudes (after extinction correction) that are systematic fainter than the ridge-line at constant period (say, $\sim 1\text{mag.}$ in V-band). The detailed of these eliminations can be found in Kanbur et al. (2003). The extinction corrected PL relations obtained from the remaining samples are presented in Table 3.2 & 3.3 as K03, and the corresponding plots are given in Figure 3.1(a). In Figure 3.1(b), we plot out the positions of the eliminated Cepheids in PL plots. As can be seen from the figure, most of the outliers are rejected, as with sigma-clipping. However, some eliminated Cepheids fall along the ridge-lines and would not be rejected by the sigma-clipping method. Our philosophy is that it is better to select *bona fide* Cepheids, and remove dubious Cepheids that can be eliminated based on physical grounds. It is true that in the end there is little difference in our V- and I-band PL relation to that initially published by the OGLE team and used by the KP. Nevertheless, in the age of precision cosmology it is important to use appropriate data.

2. **The Galactic PL Relations:** Due to high extinction and less accurately known distances to Galactic Cepheids from other independent methods, Galactic PL relations have been considered less favorably in the extra-galactic distance scale application than LMC PL relations (e.g., Kennicutt et al., 1995). However, the idea of using Galactic PL relations has been revived recently (Feast, 2001, 2003; Tammann et al., 2003; Fouqué et al., 2003; Thim et al., 2003) because not only is the metallicity in many of the HST observed target galaxies closer to Galactic values, but also the current calibration of Galactic PL relations has recently been improved (e.g., see Feast, 2003; Fouqué et al., 2003). In addition, because of some possible problems associated with the OGLE LMC PL relations, including the break at 10 days and the dominance of short period Cepheids, and the possible small “depth effect” of LMC Cepheids (Groenewegen, 2000; Nikolaev et al., 2004), using the Galactic PL relations to calibrate Cepheid distances could be more desirable and can certainly provide an independent check of the LMC based scale which is independent of the LMC distance modulus: this is currently the largest source of systematic uncertainty in the extra-galactic distance scale ($\pm 0.1\text{mag.}$). This does not imply

²It has been pointed out that it may not be appropriate to eliminate those stars with a low pulsational amplitudes as this may bias the LMC PL relation. Such low amplitude Cepheids do exist and are seen in external galaxies. These low amplitude stars are not first overtone pulsators since their light curve Fourier parameters fall in the region occupied by fundamentals. They lie in the instability strip and do not have unusual colors for their period. Thus their luminosities, masses and temperatures are similar to other Cepheids of similar period. So why do they have such a low amplitude? It could be that they have a slightly different composition or are just entering or leaving the fundamental mode instability strip (Buchler & Kolláth, 2002). In this case it is our contention that they should be excluded from the sample since they are Cepheids undergoing a transition.

Figure 3.1 The LMC PL relations. Left: (a) The V- and I-band PL relations from the final OGLE LMC sample, after correction for reddening. The best fit PL relations are indicated in solid lines. The dashed lines show the 2σ dispersion, as expected from the width of instability strip. Right: (b) The locations of eliminated Cepheids in PL relations. The symbols are: filled circles = 3 Cepheids with unusual light curves; open circles = 34 Cepheids with poorly reconstructed light curves; filled squares = 43 Cepheids with low amplitudes and open triangles = 7 Cepheids that appear dimmer (by $\sim 1.0\text{mag.}$ in V-band) from the ridge-line at given periods.

that the systematic uncertainty associated with Galactic PL relations is smaller, but if this is true, the determination of Cepheid distances can be improved.

We pick three recent Galactic PL relations which have both V- and I-band PL relations. The first Galactic PL relation is derived from 28 Cepheids using the Barnes-Evans surface brightness technique (Gieren et al., 1998, hereafter GFG98). The updated version of GFG98 PL relations has become available just recently. This includes 32 Galactic Cepheids in their sample (Fouqué et al., 2003, hereafter FSG03). The last Galactic PL relation is adopted from Tammann et al. (2003), and includes the 28 Cepheids in GFG98 sample with an additional 25 Cepheids from Feast (1999), referred as T03 in Table 3.2 and 3.3. We did not include the Galactic PL relations resulting from the *Hipparcos* calibration because these PL relations assumed the V-band slopes are the same as in LMC PL relations, and hence calibrate the zero-point only (e.g., see Feast & Catchpole, 1997; Lanoix et al., 1999b; Groenewegen & Oudmaijer, 2000; Feast, 2003).

Table 3.2 –The adopted V-band Period-Luminosity relations.

Relation	Slope (a_V)	ZP (b_V)	σ_V	N
LMC (MF91)	-2.76 ± 0.11	-1.40 ± 0.05	0.27	32
OGLE LMC (U99)	-2.760 ± 0.031	-1.458 ± 0.021	0.159	649
OGLE LMC (K03)	-2.746 ± 0.043	-1.401 ± 0.030	0.223	634
LMC PL $_{>10d}$ (TR02)	-2.48 ± 0.17	-1.75 ± 0.20	0.16	~ 47
Galactic (GFG98)	-3.037 ± 0.138	-1.021 ± 0.040	0.209	28
Galactic (FSG03)	-3.06 ± 0.11	-0.989 ± 0.034	...	32
Galactic (T03)	-3.141 ± 0.100	-0.826 ± 0.119	0.24	53

Table 3.3 –The adopted I-band Period-Luminosity relations.

Relation	Slope (a_I)	ZP (b_I)	σ_I	N
LMC (MF91)	-3.06 ± 0.07	-1.81 ± 0.03	0.18	32
OGLE LMC (U99)	-2.962 ± 0.021	-1.942 ± 0.014	0.109	658
OGLE LMC (K03)	-2.965 ± 0.028	-1.889 ± 0.019	0.145	634
LMC PL $_{>10d}$ (TR02)	-2.82 ± 0.13	-2.09 ± 0.15	0.12	~ 47
Galactic (GFG98)	-3.329 ± 0.132	-1.435 ± 0.037	0.194	27
Galactic (FSG03)	-3.24 ± 0.11	-1.550 ± 0.034	...	32
Galactic (T03)	-3.408 ± 0.095	-1.325 ± 0.114	0.23	53

3.4 Results for the Calibrated Cepheid Distances

We use the *same* Cepheids in each target galaxy that were used by the original KP or STS study after various selection criteria (including the period-cut, color-cut and the quality of the Cepheids) had been applied. The periods of the Cepheids in final samples were taken from the corresponding papers. The Cepheid distances to these galaxies are obtained with the methods and procedures described in previous sections (see also Kanbur et al., 2003). The results of the distance moduli are presented in Table 3.4 and 3.5 when using both the LMC and Galactic PL relations respectively. Note that the distance moduli in these tables have been corrected for CTE (column 6 in Table 3.1) but not for metallicity effects, and all the errors in the tables are random (statistical) errors only.

3.4.1 Comparisons with Published Results

We compare our distance moduli, calculated using our Fourier techniques to estimate the V- and I-band means, with published distance moduli. The only two PL relations we can use in this comparison are the LMC PL relations from MF91 and U99, for STS and KP+WM galaxies, respectively. The results of these comparisons are: $\Delta = 0.005 \pm 0.015 \text{mag.}$ for the KP+WM galaxies; and $\Delta = -0.075 \pm 0.037 \text{mag.}$ for the STS galaxies. In overall, the difference is small among all of the 25 target galaxies, with a difference of $-0.020 \pm 0.017 \text{mag.}$ (corresponding to $\sim 1\%$ change in distance). This result implies that our method of calculating means is a viable alternative technique. Ngeow et al. (2003) show that our method has advantages in some situations. The largest and

Table 3.4 The Cepheid distance (with CTE correction) to nearby galaxies with different LMC PL relations.

Galaxy	MF91 ^a	U99 ^a	K03 ^a	TR02 ^a
NGC 925	29.801 ± 0.060	29.730 ± 0.060	29.718 ± 0.060	29.744 ± 0.060
NGC 1326A	31.102 ± 0.088	31.007 ± 0.088	30.998 ± 0.088	31.027 ± 0.088
NGC 1365	31.315 ± 0.052	31.191 ± 0.051	31.186 ± 0.051	31.218 ± 0.052
NGC 2090	30.424 ± 0.036	30.328 ± 0.036	30.319 ± 0.036	30.348 ± 0.036
NGC 2541	30.462 ± 0.072	30.355 ± 0.071	30.348 ± 0.071	30.378 ± 0.071
NGC 3031	27.887 ± 0.094	27.800 ± 0.093	27.791 ± 0.093	27.818 ± 0.093
NGC 3198	30.837 ± 0.060	30.728 ± 0.060	30.721 ± 0.060	30.751 ± 0.060
NGC 3319	30.593 ± 0.094	30.511 ± 0.092	30.501 ± 0.092	30.528 ± 0.093
NGC 3351	29.979 ± 0.090	29.914 ± 0.090	29.902 ± 0.090	29.927 ± 0.090
NGC 3621	29.259 ± 0.059	29.164 ± 0.059	29.155 ± 0.059	29.184 ± 0.059
NGC 4321	30.900 ± 0.067	30.769 ± 0.067	30.764 ± 0.067	30.798 ± 0.067
NGC 4414	31.236 ± 0.052	31.104 ± 0.043	31.099 ± 0.044	31.133 ± 0.045
NGC 4535	31.001 ± 0.054	30.879 ± 0.053	30.874 ± 0.053	30.906 ± 0.053
NGC 4548	30.867 ± 0.076	30.783 ± 0.078	30.773 ± 0.078	30.800 ± 0.078
NGC 4725	30.522 ± 0.066	30.399 ± 0.067	30.393 ± 0.067	30.426 ± 0.067
NGC 7331	30.891 ± 0.083	30.800 ± 0.078	30.791 ± 0.079	30.819 ± 0.079
IC 4182	28.365 ± 0.128	28.334 ± 0.130	28.318 ± 0.130	28.339 ± 0.129 ^b
NGC 3627	30.238 ± 0.083	30.134 ± 0.084	30.127 ± 0.084	30.157 ± 0.083
NGC 3982	31.714 ± 0.136	31.599 ± 0.134	31.593 ± 0.134	31.624 ± 0.135
NGC 4496A	30.959 ± 0.043	30.839 ± 0.043	30.833 ± 0.043	30.865 ± 0.043
NGC 4527	30.755 ± 0.129	30.635 ± 0.127	30.629 ± 0.128	30.661 ± 0.128
NGC 4536	30.961 ± 0.102	30.830 ± 0.102	30.826 ± 0.102	30.859 ± 0.102
NGC 4639	31.790 ± 0.108	31.664 ± 0.105	31.659 ± 0.106	31.692 ± 0.106
NGC 5253	27.881 ± 0.244	27.873 ± 0.246	27.854 ± 0.246	27.872 ± 0.245 ^c
NGC 4258	29.435 ± 0.058	29.384 ± 0.056	29.370 ± 0.056	29.393 ± 0.057

^a MF91 = Madore & Freedman (1991) PL relations; U99 = Udalski et al. (1999a) PL relations; K03 = Kanbur et al. (2003) PL relations; TR02 = Tammann & Reindl (2002) PL relations for $\log(P) > 1.0$. See Section 3.3.1 for details.

^b There are 12 Cepheids in this galaxy with period less than 10 days. If we use the PL_{<10d} relations, as given by Tammann & Reindl (2002), then $\mu = 28.340 \pm 0.130$, which is identical to the value with PL_{>10d} relations.

^c There are 2 Cepheids in this galaxy with period less than 10 days. If we use the PL_{<10d} relations, as given by Tammann & Reindl (2002), then $\mu = 27.876 \pm 0.246$, which is very close to the value with PL_{>10d} relations.

Table 3.5 –The Cepheid distance (with CTE correction) to nearby galaxies with different Galactic PL relations.

Galaxy	GFG98 ^a	FSG03 ^a	T03 ^a
NGC 925	29.765 ± 0.061	29.768 ± 0.060	29.833 ± 0.061
NGC 1326A	31.093 ± 0.090	31.070 ± 0.089	31.166 ± 0.091
NGC 1365	31.336 ± 0.053	31.283 ± 0.052	31.414 ± 0.053
NGC 2090	30.415 ± 0.038	30.392 ± 0.036	30.488 ± 0.039
NGC 2541	30.464 ± 0.075	30.430 ± 0.072	30.539 ± 0.075
NGC 3031	27.867 ± 0.098	27.854 ± 0.095	27.939 ± 0.098
NGC 3198	30.842 ± 0.061	30.805 ± 0.060	30.917 ± 0.061
NGC 3319	30.569 ± 0.096	30.560 ± 0.094	30.639 ± 0.096
NGC 3351	29.936 ± 0.090	29.946 ± 0.090	30.004 ± 0.090
NGC 3621	29.249 ± 0.060	29.226 ± 0.059	29.322 ± 0.061
NGC 4321	30.929 ± 0.067	30.869 ± 0.067	31.008 ± 0.067
NGC 4414	31.265 ± 0.063	31.204 ± 0.052	31.344 ± 0.064
NGC 4535	31.020 ± 0.055	30.969 ± 0.054	31.097 ± 0.055
NGC 4548	30.846 ± 0.078	30.834 ± 0.078	30.917 ± 0.078
NGC 4725	30.541 ± 0.067	30.490 ± 0.066	30.618 ± 0.067
NGC 7331	30.875 ± 0.091	30.858 ± 0.083	30.947 ± 0.092
IC 4182	28.285 ± 0.129	28.330 ± 0.128	28.346 ± 0.129
NGC 3627	30.237 ± 0.083	30.206 ± 0.083	30.311 ± 0.083
NGC 3982	31.726 ± 0.139	31.682 ± 0.136	31.802 ± 0.139
NGC 4496A	30.974 ± 0.043	30.926 ± 0.043	31.051 ± 0.044
NGC 4527	30.771 ± 0.132	30.723 ± 0.129	30.848 ± 0.132
NGC 4536	30.989 ± 0.102	30.929 ± 0.102	31.068 ± 0.102
NGC 4639	31.812 ± 0.112	31.758 ± 0.109	31.890 ± 0.113
NGC 5253	27.776 ± 0.243	27.846 ± 0.243	27.834 ± 0.242
NGC 4258	29.377 ± 0.062	29.401 ± 0.058	29.442 ± 0.062

^a GFG98 = Gieren et al. (1998) PL relations; FSG03 = Fouqué et al. (2003) PL relations; T03 = Tammann et al. (2003) PL relations. See Section 3.3.1 for details.

smallest difference comes from NGC 4639 and NGC 4414, with a difference of $-0.240 \pm 0.245\text{mag.}$ and $0.004 \pm 0.066\text{mag.}$, respectively.

We also compare our adopted results with published values by selecting the representative distance moduli obtained from both LMC or Galactic PL relations. We pick the distance moduli from the derived OGLE LMC (K03) PL relations in Section 3.3.1 (i.e. column 4 of Table 3.4) and FSG03 PL relations to represent the LMC and the Galactic results, respectively. The CTE-corrected distance moduli, $\mu_{0,CTE}$, are listed in Table 3.7. However, the metallicity corrections are not required in these comparisons because they are the same for published results and this work. The effect of metallicity corrections are discussed in Section 3.4.3. The results of the comparisons are plotted in Figure 3.2 and the average difference between our distance moduli and the published values are presented in Table 3.6 for both the LMC and Galactic PL relations. The overall comparisons indicate that our results are consistent with the published results. When using the LMC PL relation, our distance moduli agree well with the KP results, but show a large discrepancy for the STS results (-0.178mag.). A careful comparison of Table 3.4 with published results shows that this difference arises from two sources: $\sim 0.10\text{mag.}$ is due to the use of MF91 LMC PL relations by STS and $\sim 0.07\text{mag.}$ is due to differences in the V- and I-band means when calculating the distance moduli. However, when using the Galactic PL relations, our results are different by $\sim 0.1\text{mag.}$ compared to both KP and STS results. Clearly, the use of Galactic PL relations increases the distance moduli. Surprisingly, the distance modulus of NGC 4258 calculated from LMC PL relations is closer to the water maser distance although the metallicity of this galaxy is nearly identical to the Solar value.

Table 3.6 –Comparisons with the published results.

Case	$\Delta(\text{mag.})^a$
Using LMC PL relations	
16 KP galaxies	-0.002 ± 0.017
8 STS galaxies	-0.178 ± 0.039^b
All 25 galaxies	-0.059 ± 0.023
Using Galactic PL relations	
16 KP galaxies	0.075 ± 0.018
8 STS galaxies	-0.108 ± 0.038
All 25 galaxies	0.014 ± 0.024

^a Mean Difference, $\Delta = < \mu_{0,CTE}(\text{Here}) - \mu_{0,CTE}(\text{Pub.}) >$, the errors are the standard deviations of the means.

^b This difference is mainly due to the different LMC PL relations used. See text (Section 3.4.1) for details.

Figure 3.2 –Comparisons with published results. The symbols are: crosses = 16 KP galaxies; open circles = 8 STS galaxies; and filled circle = WM galaxy. Left:(a) Comparison of our results from the K03 LMC PL relations. The dashed line is the average difference of -0.06 . Right:(b) Comparison of our results from the Galactic PL relations. The dashed line is the average difference of 0.01 .

3.4.2 Distance Moduli from Various PL Relations

Since the distance moduli listed in Table 3.4 are calculated using different LMC PL relations, they can be compared. A quick comparison in Table 3.4 indicates that the MF91 PL relation always give the longer distance moduli, and the shortest distance moduli are from the K03 PL relations. The average difference between the distance moduli derived from MF91 and the OGLE LMC (K03), for all target galaxies in Table 3.4, is about $0.105 \pm 0.006\text{mag}$. Comparing this result with the previous case (i.e. Section 3.4.1) of using the same PL relations but different means magnitudes, the distance modulus is more sensitive to different calibrating PL relations. Also, the distance moduli from three OGLE LMC PL relations are consistent with each other within the statistical errors.

A similar situation exists when using Galactic PL relations, as shown in Table 3.5. The distance moduli from both GFG98 and FSG03 Galactic PL relations are consistent with each other (with a difference of $\sim 0.024 \pm 0.007\text{mag}$., as there are 26 common Cepheids in both samples), while FSG03 PL relations produce a shorter distance. However, the distance moduli from T03 PL relations are systematically further than the other two, although all three PL relations share some common Galactic Cepheids. Nevertheless, the average difference between the distance moduli from T03 and FSG03 is $\sim 0.097 \pm 0.008\text{mag}$., comparable to the difference seen in LMC PL relations.

Table 3.7 – Comparisons of the distance moduli to target galaxies after CTE and metallicity corrections.

Galaxy	$\mu_{0,CTE}(LMC)$	$\mu_z(LMC)$	$\mu_{0,CTE}(GAL)$	$\mu_z(GAL)$	$\mu_{0,CTE}(PUB)^a$	$\mu_z(PUB)^a$
NGC 925	29.718 ± 0.060	29.728	29.768 ± 0.060	29.704	29.80 ± 0.04	29.81
NGC 1326A	30.998 ± 0.088	30.998	31.070 ± 0.089	30.996	31.04 ± 0.10	31.04
NGC 1365	31.186 ± 0.051	31.287	31.283 ± 0.052	31.301	31.18 ± 0.05	31.27
NGC 2090	30.319 ± 0.036	30.379	30.392 ± 0.036	30.378	30.29 ± 0.04	30.35
NGC 2541	30.348 ± 0.071	30.348	30.430 ± 0.072	30.356	30.25 ± 0.05	30.25
NGC 3031	27.791 ± 0.093	27.841	27.854 ± 0.095	27.830	27.75 ± 0.08	27.82
NGC 3198	30.721 ± 0.060	30.741	30.805 ± 0.060	30.751	30.68 ± 0.08	30.70
NGC 3319	30.501 ± 0.092	30.477	30.560 ± 0.094	30.462	30.64 ± 0.09	30.62
NGC 3351	29.902 ± 0.090	30.050	29.946 ± 0.090	30.020	29.85 ± 0.09	30.00
NGC 3621	29.155 ± 0.059	29.205	29.226 ± 0.059	29.202	29.08 ± 0.06	29.11
NGC 4321	30.764 ± 0.067	30.890	30.869 ± 0.067	30.921	30.78 ± 0.07	30.91
NGC 4414	31.099 ± 0.044	31.239	31.204 ± 0.052	31.270	31.10 ± 0.05	31.24
NGC 4535	30.874 ± 0.053	31.014	30.969 ± 0.054	31.035	30.85 ± 0.05	30.99
NGC 4548	30.773 ± 0.078	30.941	30.834 ± 0.078	30.928	30.88 ± 0.05	31.05
NGC 4725	30.393 ± 0.067	30.477	30.490 ± 0.066	30.500	30.38 ± 0.06	30.46
NGC 7331	30.791 ± 0.079	30.825	30.858 ± 0.083	30.818	30.81 ± 0.09	30.84
IC 4182	28.318 ± 0.130	28.298	28.330 ± 0.128	28.236	28.36 ± 0.09^b	...
NGC 3627	30.127 ± 0.084	30.277	30.206 ± 0.083	30.282	30.22 ± 0.12	...
NGC 3982	31.593 ± 0.134	31.673	31.682 ± 0.136	31.688	31.72 ± 0.14	...
NGC 4496A	30.833 ± 0.043	30.887	30.926 ± 0.043	30.906	31.03 ± 0.14	...
NGC 4527	30.629 ± 0.128	30.709	30.723 ± 0.129	30.783	30.72 ± 0.12	...
NGC 4536	30.826 ± 0.102	30.896	30.929 ± 0.102	30.925	31.10 ± 0.13	...
NGC 4639	31.659 ± 0.106	31.759	31.758 ± 0.243	31.784	32.03 ± 0.22	...
NGC 5253	27.854 ± 0.246	27.784	27.846 ± 0.243	27.702	28.08 ± 0.20^c	...
NGC 4258	29.370 ± 0.056	29.440	29.401 ± 0.058	29.397	29.40 ± 0.09	29.47

^a Published distance moduli for KP galaxies are taken from Freedman et al. (2001), Table 4. The distance moduli for STS galaxies and WM galaxy are taken from series of STS papers and Newman et al. (2001), respectively.

^b Saha et al. (1994) assumed $A_V = A_I = 0$, hence $\mu = (\mu_V + \mu_I)/2$.

^c The paper (Saha et al., 1995) did not list out the final μ , hence this value is calculated via equation (3.1) with μ_V and μ_I given in the paper.

Table 3.8 –Comparisons of metallicity-corrected distance moduli.

Case	Δ (mag.) ^a
Without metallicity correction	
16 KP galaxies	-0.077 ± 0.005
8 STS galaxies	-0.070 ± 0.015
All 25 galaxies	-0.073 ± 0.006
With metallicity correction	
16 KP galaxies	-0.002 ± 0.005
8 STS galaxies	-0.003 ± 0.018
All 25 galaxies	-0.001 ± 0.007

^a Mean Difference, $\Delta = \langle \mu(LMC) - \mu(GAL) \rangle$, the errors are the standard deviations of the means.

3.4.3 The Effect of the Metallicity

As well as comparing our results with the published values, we discuss the distance moduli obtained when LMC (adopted from K03) and Galactic (adopted from FSG03) PL relations are used. First, comparing the distance moduli that are uncorrected for metallicity, $\mu_{0,CTE}(LMC)$ and $\mu_{0,CTE}(GAL)$ in Table 3.7, show that the distance moduli from LMC PL relations are always shorter than their Galactic counterparts (see Figure 3.3[a]). Summaries of this comparison are given in Table 3.8. This shows that the LMC PL relations will give a smaller distance modulus by $\sim 0.07 \pm 0.01$ mag. on average (or about 3.5% in distance) compared to the distance modulus obtained from Galactic PL relations. The negligible random errors suggest that the difference of 0.07 mag. might be significant, although it is small. However, when the metallicity corrected distance moduli from the two sets of PL relations ($\mu_z[LMC]$ and $\mu_z[GAL]$ in Table 3.7) are compared, the difference between them falls close to zero. The results are listed in Table 3.8 and shown in Figure 3.3(b).

Another approach is to use the Galaxy and LMC ($P > 10d$) as calibrating PL relations for metal rich and metal poor galaxies respectively. Thus if the seven SN calibrators of STS were forced on an LMC PL relation (U99), one would obtain an average distance decrease of 0.17 mag. However, if we force the five galaxies in their sample, which have on average the same metallicity as the Galaxy, onto the steep relation given in T03, the published STS distances are recovered for these galaxies to within 0.01 mag. Applying the same procedure to the 10 KP galaxies which are metal rich increases their distance moduli by about 0.16 mag. on average, whilst using TR02 for the remaining metal poor galaxies increases their distance moduli by 0.02 mag.

The use of Galactic PL relations to calibrate the Cepheid distance scale has been tried before by Paturel et al. (2002a) and Paturel et al. (2002b). The first paper, Paturel et al. (2002a), used the GFG98 sample and the method of “sosie” (Paturel, 1984) to determine Cepheid distances without

Figure 3.3 –Comparisons of distance moduli from LMC and Galactic PL relations. The symbols are: crosses = 16 KP galaxies; open circles = 8 STS galaxies; and filled circle = WM galaxy. Left:(a) Uncorrected for metallicity. The dashed line is the average difference of -0.07 . Right:(b) Corrected for metallicity. The dashed line is the average difference of -0.00 .

assuming the PL relations. The second paper, Paturel et al. (2002b), applied the *Hipparcos* calibrated PL relations (Feast & Catchpole, 1997) to find Cepheid distances. The mean offsets of their results and the published KP results are $0.161 \pm 0.029\text{mag.}$ and $0.027 \pm 0.016\text{mag.}$ for their first and second papers, respectively. The latter result is comparable to our work. However, the extra-galactic Cepheids used in both papers are slightly different than either the KP or STS groups, in contrast with our study in this chapter, because these authors applied a different method to deal with incompleteness bias. Also, these authors did not apply any metallicity corrections in their papers.

3.5 Conclusion and Discussion for this Chapter

In this chapter we re-calibrate the Cepheid distance to about two dozen nearby galaxies, including the KP and STS galaxies, and a water maser galaxy. We use much of the same methodology as the KP team: the same LMC distance, value of R and metallicity correction, and the same (number of) Cepheids along with the published photometries and periods. However our approach is different from the KP team in two aspects: (a) we estimate the mean magnitudes of sparsely sampled *HST* data from Fourier techniques (Ngeow et al., 2003) by reconstructing the Cepheid light curves; and (b) we use different sets of PL relations, including the LMC and Galactic PL relations, in distance determination. Overall, our results are consistent with each other and KP.

However, we find significantly shorter distances to the STS galaxies. The use of the new Fourier techniques to obtain V- and I-band means does not produce significant deviations from the existing methods. However the derived distance modulus is more sensitive to the calibrating PL relation that is adopted. The most striking result from this study is that the distance moduli derived from using the LMC and Galactic PL relations are indistinguishable after the metallicity corrections are applied. This provides strong support for the size and quantity of the metallicity dependence of the Cepheid PL relations and the non-universality of PL relations (Tammann et al., 2003).

Recent work has shown that the PL relation in the Galaxy and LMC have different slopes (Tammann et al., 2003; Ngeow & Kanbur, 2004; Sandage et al., 2004) and moreover that the PL relation in the LMC is "broken" at a period of 10 days (Kanbur & Ngeow, 2004; Sandage et al., 2004). Metallicity affects the mean brightness of a Cepheid and up to now, this has been the standard motivation for deriving and applying simple additive corrections to the PL relation to account for metallicity differences between calibrating and target galaxy. However it may be that metallicity also affects the slope of the PL relation. If this is so, then the use of a simple additive correction as given in equation (3.4) is not really appropriate because the PL relation may have a different slope in the target and calibrating galaxy. However, the right panels of Figure 3.3 show clearly that it works in the sense that the distance moduli differences obtained when using the Galactic and LMC as calibrating PL relations are driven close to zero. We try to investigate why this occurs in what follows.

If we want to compare the distance modulus obtained using LMC and Galactic calibrating PL relations (or any pair of PL relations) when using *exactly* the same (number of) Cepheids in the target galaxy, including the same periods and mean magnitudes, then the difference in distance modulus can be expressed as:

$$\Delta\mu_0 = [(R-1)\Delta a_V - R\Delta a_I] \frac{\sum \log(P)_i}{N} + (R-1)\Delta b_V - R\Delta b_I, \quad (3.8)$$

where $\Delta a_{(V,I)}$ and $\Delta b_{(V,I)}$ are the differences in slopes and zero-points for the two PL relations, respectively. Then, for these two PL relations, the change in the distance modulus is a simple linear function of the target galaxy period distribution ($\langle \log(P) \rangle \equiv \sum \log(P)_i / N$, hereafter mean period), under the assumption of constant R (i.e. the universality of Galactic extinction law, see observational verification for this assumption by Macri et al. 2001). The error in $\Delta\mu_0$ is the quadrature sum of the σ_μ from both PL relations for the target galaxy. If metallicity corrections of δ_z (from

Table 3.9 –The change of $\Delta\mu$ with different pairs of LMC and Galactic PL relations^a.

(LMC-GAL) pair	c	d	$d_z = d + 0.074$	$\langle \log(P) \rangle_0$	$\langle \log(P) \rangle_z$
(MF91-GFG98)	-0.257	0.369	0.443	1.4343	1.7218
(U99-GFG98)	-0.497	0.608	0.682	1.2231	1.3719
(K03-GFG98)	-0.470	0.561	0.635	1.1946	1.3521
(TR02-GFG98)	-0.439	0.548	0.622	1.2465	1.4149
(MF91-FSG03)	-0.006	0.041	0.115	6.8417 ^b	19.175 ^b
(U99-FSG03)	-0.246	0.280	0.354	1.1392	1.4399
(K03-FSG03)	-0.218	0.233	0.307	1.0673	1.4060
(TR02-FSG03)	-0.188	0.220	0.294	1.1678	1.5614
(MF91-T03)	-0.300	0.365	0.430	1.1859	1.4325
(U99-T03)	-0.540	0.595	0.669	1.1018	1.2388
(K03-T03)	-0.513	0.548	0.622	1.0692	1.2135
(TR02-T03)	-0.482	0.534	0.608	1.1085	1.2620

^a $\Delta\mu = c \langle \log(P) \rangle + d$ (i.e. equation [3.8]). $\langle \log(P) \rangle$ is the mean period for $\Delta\mu = 0$.

^b The large values of $\langle \log(P) \rangle$ are due to the small value of c in column 2. Hence the MF91 LMC PL relations will always produce a larger distance modulus, as compared to distance modulus from FSG03 Galactic PL relations.

equation [3.4]) are applied to equation (3.8), the metallicity of the target galaxies cancels out and leaves the difference between the LMC and Galactic metallicity. Adopting $[O/H]_{LMC} = 8.50dex$ and $[O/H]_{GAL} = 8.87dex$, along with $\gamma = -0.2 \pm 0.2mag. dex^{-1}$, the difference in metallicity-corrected distance modulus becomes:

$$\begin{aligned}\Delta\mu_z &= \Delta\mu_0 - 0.2(8.50 - 8.87), \\ &= \Delta\mu_0 + 0.074 (\pm 0.074).\end{aligned}\tag{3.9}$$

It is worth pointing out that equations (3.8) and (3.9), though straightforward to derive, have not been presented in the literature before to the best of our knowledge. Equation (3.9) is of course dependent on the metallicity correction law adopted and the slopes of the calibrating PL relations. We can re-write equation (3.8) as $\Delta\mu_0 = c \langle \log(P) \rangle + d$, where $\Delta\mu_0 = \mu_0(LMC) - \mu_0(GAL)$, $c = (R-1)\Delta a_V - R\Delta a_I$ and $d = (R-1)\Delta b_V - R\Delta b_I$. For the four LMC PL relations and the three Galactic PL relations considered in this chapter, there are a total of 12 combinations of (LMC, GAL) PL relations. The coefficients of c and d for each combination are listed in column 2 & 3 of Table 3.9. In the same table, we also list out the coefficients of d after applying the metallicity corrections (i.e. equation [3.9]), i.e. $\Delta\mu_z = c \langle \log(P) \rangle + d_z$ where $d_z = d + 0.074$.

To see what values of $\langle \log(P) \rangle$ would produce identical distance moduli from using either the LMC or the Galactic PL relations, we solve for $\langle \log(P) \rangle$ such that $\Delta\mu_0 = 0$ or $\Delta\mu_z = 0$. The

solutions are listed in columns 5 & 6 in Table 3.9 for the case of $\Delta\mu_0 = 0$ and $\Delta\mu_z = 0$, respectively. From Table 3.9, the mean period for two distance moduli to be identical is around $\langle \log(P) \rangle_0 \sim 1.1 - 1.2$, and $\langle \log(P) \rangle_z \sim 1.3 - 1.4$, except for the (MF91,GFG98) and (MF91,FSG03) pairs. Most of the extra-galactic Cepheids in our target galaxies have an observed mean period of ~ 1.4 (the mean period for 16 KP galaxies is 1.418; the mean period for 8 STS galaxies is 1.401; and the mean period for all 25 galaxies is 1.342; see also Kanbur et al. 2003). Therefore, without applying the metallicity correction, the Galactic PL relations will be expected to produce longer distances than the LMC PL relations in most of the (LMC,GAL) pair of PL relations. For example, the change of the distance modulus is $\sim 0.10\text{mag}$. when comparing the LMC PL relations (either U99 or K03) to the FSG03 Galactic PL relations in individual target galaxies. However, the LMC and Galactic PL relations will produce almost identical distance moduli after a metallicity correction to within $\sim 0.03\text{mag}$., for most of the target galaxies. The only exception is NGC 5253, because the mean period for this galaxy is 1.029, which is much smaller than the required $\langle \log(P) \rangle$ of ~ 1.4 . The T03 Galactic PL relation has a different slope to the GFG03 and FSG03 Galactic PL relations. Thus the median period in Table 3.9 required for a simple additive correction to be sufficient is approximately 1.2. This is slightly outside the range of mean period required to produce similar distance moduli after a simple metallicity correction when both Galactic and LMC calibrating PL relations are used.

If the result of this chapter, i.e. the near identical distance moduli from LMC and Galactic PL relations after metallicity corrections, is true, then this result can be used to constrain the sign for the coefficient of metallicity correction, the γ in equation (3.4). The value of $\gamma = 0.2 \pm 0.2\text{mag dex}^{-1}$ used in this study is adopted from Freedman et al. (2001), which is roughly the midrange value from several empirical studies (see Section 3.2). Since the difference of distance moduli between LMC and Galactic PL relations is about -0.07mag . without the metallicity correction (Table 3.8), a $+0.07\text{mag}$. metallicity correction is required to bring the two distance moduli to be identical. The correction of 0.074mag . in equation (3.9) is almost identical to this requirement, hence the value of γ should be around -0.2mag dex^{-1} and constrains the sign to be negative. In addition, if the Cepheid PL relations do indeed depend on metallicity, the result of this chapter suggests that a simple additive metallicity correction as in equation (3.4) is a good approximation to model the full complexity of the metallicity dependence of the Cepheid PL relation, provided the mean period of Cepheids in the target galaxy are in the appropriate range for the slopes and extinction laws adopted for the calibrating PL relations. Some researchers suggest using the LMC PL relations and Galactic PL relations for metal-poor and metal-rich galaxies, respectively, forgetting about

metallicity corrections until an solid understanding of this topic is obtained. This naturally begs the question of which calibrating PL relation to use if the metallicity of the target galaxy is in between the LMC and Galaxy. However our result does not, at the moment, provide evidence supporting one Galactic PL relation over another.

In summary, the above discussion suggests that: (a) metallicity corrections are necessary when using Cepheid PL relations to find distance moduli; (b) as a consequence, the PL relations do depend on metallicity; (c) hence, Cepheids in the LMC and Galaxy obey different PL relations; (d) the sign for the coefficient of metallicity correction (γ) has to be negative; and (e) both LMC and Galactic PL relations can be used to determine the distance modulus because either one of the PL relations would yield the same distance modulus after the appropriate metallicity correction. However, further study is needed to test these conclusions.

CHAPTER 4

PL RELATIONS FOR GALACTIC CEPHEID VARIABLES WITH INDEPENDENT DISTANCE MEASUREMENTS: COMPARISON TO THE LMC PL RELATIONS

In this chapter, we derive the period-luminosity (PL) relation for Galactic Cepheids with recent independent distance measurements from open cluster, Barnes-Evans surface brightness, interferometry and *HST* astrometry techniques. Our PL relation confirms the results of Tammann et al. (2003), which showed that the Galactic Cepheids follow a different PL relation to their LMC counterparts. Our results also show that the slope of the Galactic PL relation is inconsistent with the LMC slope with more than 95% confidence level.

4.1 The Need of the Galactic PL Relation

Tammann et al. (2003, hereafter T03 in this chapter) derived the Galactic PL relation by combining the Galactic Cepheids with independent distance measurements from open clusters/associations (Feast, 1999) and from the Barnes-Evans (BE) surface brightness techniques (Gieren et al., 1998). The resulting Galactic PL relations in T03 are steeper than the LMC PL relations commonly applied in distance scale applications (as in, e.g., Freedman et al. 2001). Similar conclusions are also reported in Fouqué et al. (2003).

The need to use the Galactic PL relation as a fundamental calibrating relation has become more desirable in recent years (Feast, 2003; Fouqué et al., 2003; Kanbur et al., 2003; Tammann et al., 2003; Thim et al., 2003; Storm et al., 2004), because of the following two main reasons: (a) The average value of metallicity (defined as $12 + \log[O/H]$) in target galaxies of the H_0 Key Project is $\sim 8.84 \pm 0.31 dex$ (Freedman et al., 2001), which is closer to the standard Solar value of $8.87 \pm 0.07 dex$ (Grevesse et al., 1996) than the LMC value of $8.50 \pm 0.08 dex$ (see reference in Ferrarese et al. 2000); and (b) There is evidence that the LMC PL relation is broken at 10 days (Tammann et al., 2002; Kanbur & Ngeow, 2004; Sandage et al., 2004), i.e., the short (< 10 days) and long period Cepheids in the LMC follow different PL relations. Due to these reasons, the calibrated Galactic PL relation will become important in future distance scale studies.

In Chapter 3, we compared the Cepheid distances to 25 *HST*-observed galaxies with the LMC and Galactic PL relation. The result shows that the overall difference between these two distances are negligible after applying the metallicity correction, which suggest that the PL relation could be depending on the metallicity. Encouraged by this result, we will compare the Galactic PL relation, derived from Cepheids with independent distance measurements, with the LMC PL relations in this chapter. Our analysis of the Galactic PL relation is similar to T03 but different in the following aspects:

1. In addition to the Cepheids from Feast (1999) and Gieren et al. (1998) that are used in T03, we include other recent distance measurements to Galactic Cepheids that are available in the literature (Section 4.2). These include 11 additional Cepheids that are not included in T03.
2. As most of the Cepheids we considered here have more than one independent distance measurement, we took the standard weighted-mean of the available distances as the final adopted distance to derive the PL relation.
3. We apply a statistical test, the *t*-test, to examine the consistency of the slopes for the Galactic and LMC PL relations.

4.2 Galactic Cepheids with Independent Distance Measurements

In order to calibrate the Galactic PL relation, the distances to the Galactic Cepheids need to be known via independent measurements (i.e., not depend on the *assumed* PL relation). There are a few ways to measure the independent distances to the Galactic Cepheids, and we collected these recent distance measurements from the literature. These are summarized as follows.

I. Distances from Open Cluster Techniques:

If the membership of a Cepheid in the open cluster can be verified (Turner, 1988), then the distance of this Cepheid is set to be equal to the distance of the open cluster. The distance of the open cluster can be found via the main-sequence fitting method (see, e.g., Turner et al., 1998; Feast, 1999; Turner & Burke, 2002; Hoyle et al., 2003).

The distances to the Cepheids in open cluster are adopted from table 3 of T03, where the authors adopted a distance modulus for the Pleiades of $\mu_{Pleiades} = 5.61 \pm 0.03\text{mag}$. (Stello & Nissen, 2001). Feast (1999) estimated that the uncertainty associated with cluster distance moduli is $\sim 0.15\text{-}0.20\text{mag}$. (see, e.g., Romeo et al. 1989), hence we assign an uncertainty of 0.20mag . to the open cluster distance moduli ($\mu_o[O]$) in Table 4.1. This error could incorporate

the uncertainty due to the location of the Cepheid in the cluster (far-side vs. near-side), the uncertainty due to metallicity corrections from cluster to cluster, etc. (see also Turner & Burke 2002). In addition, we also include other open cluster distances from Turner & Burke (2002) and Hoyle et al. (2003) in Table 4.1. Since these additional open cluster distances are based on $\mu_{Pleiades} = 5.56\text{mag.}$, we add a correction of $+0.05\text{mag.}$ to these distances¹ in Table 4.1 to be consistent with the T03 open cluster distances. However, it is unclear whether the distances given in Feast (1999, where the T03 adopted distances originate) and in Turner & Burke (2002) are totally independent to each other or not, as some of them were adopted from the same sources. For example, the open cluster distances to CV MON in Feast (1999) and Turner & Burke (2002) are all adopted from Turner et al. (1998). We have labelled the Turner & Burke (2002) distances that are dubious in this dependency regard in Table 4.1, and excluded them in obtaining the weighted-mean distances. For GT CAR, CG CAS and TV CMa in Turner & Burke (2002), which are not included in Feast (1999), there is no I-band data available in the literature. We exclude them in order to have a consistent number of Cepheids in B-, V- and I-bands.

II. Distance from Baade-Wesslink Method:

The Baade-Wesslink (BW) method involves the comparison of the angular size of the Cepheid and its radius to obtain the distance. The radius of the Cepheid is obtained via the integration of the velocity curve:

$$R(t) \propto \int V_r(t) dt.$$

There are two ways of obtaining the angular size of the Cepheids:

1. Barnes-Evans (BE) surface brightness techniques – The BE surface brightness techniques are first introduced by Barnes & Evans (1976) and frequently applied with the BW method to obtain the Cepheid distances. The idea behind the BE surface brightness techniques is simple: starting from the Stefan-Boltzmann law, $L = 4\pi R^2 \sigma T^4$, dividing by $4\pi D^2$ (where D is the distance) on both sides, we obtain the expression for the angular size (θ) after the $\log(L)$ - V_o and $\log(T)$ -($V - K$)_o conversions:

¹Thanks to G. A. Tammann for pointing out this correction to Hoyle et al. (2003) distances.

$$\log(\theta) = \alpha V_o + \beta(V - K)_o + \gamma.$$

The coefficients in this expression can be obtained with the independent calibration, such as using supergiant stars (Fouqué & Gieren, 1997). Detailed applications of this method can be found, e.g., in Gieren et al. (1993, 1997, 1998).

We adopt these distances from Fouqué et al. (2003), which is an updated version of Gieren et al. (1998), with additional Cepheids that are not included in Gieren et al. (1998). Fouqué (2003, private communication) has verified that the distances from Gieren et al. (1998) and Fouqué et al. (2003) are not independent of each other, as the distances from the later paper use the latest available data to update the distances in Gieren et al. (1998). The exception is CS VEL, where we include the distance from Gieren et al. (1998) since there is no distance given to this Cepheid in Fouqué et al. (2003). Note that T03 only included the distances from Gieren et al. (1998) but not from Fouqué et al. (2003). We also include the latest distance measurements using BE techniques from Barnes et al. (2003), who used a Bayesian statistical approach in their analysis.

2. Optical interferometry – By using optical interferometry, the angular sizes for some large and/or nearby Cepheids can be obtained from the measured visibility (Lane et al., 2002; Nordgren et al., 2002; Kervella et al., 2004a). These distances are adopted from Lane et al. (2002), Nordgren et al. (2002) and Kervella et al. (2004a). Note that for distances in Kervella et al. (2004a), we did not use the direct distance measurements for the four Cepheids (η AQL, W SGR, β DOR & l CAR) because the errors are asymmetric and large (except for l CAR). Instead we adopt the distances from their table 12. These are obtained from combining the interferometry data and the empirical period-radius relations from Gieren et al. (1998).

III. Distance from Astrometry/Parallax:

The distances to Galactic Cepheids can also be obtained from the trigonometric parallax with *Hipparcos* or the astrometry measurement from the *HST*. However, after converting the parallax to distance modulus, the error bars for the *Hipparcos* measurements are large, as can be seen in figure 2 of Madore & Freedman (1998). Therefore, the *Hipparcos* measurements are

not used in this study. For the *HST* astrometry measurements, currently there is only one Cepheid, δ CEP, with the distance measurement from *HST* astrometry (Benedict et al., 2002).

4.2.1 The Adopted Distances to the Galactic Cepheids

The selected Galactic Cepheids and the corresponding distances from these sources are summarized in Table 4.1. Since these distances are from independent measurements, we can take the weighted-mean among the available distances (i.e. column 2, 3, 5 & 6 in Table 4.1). Note that T03 only uses the distance moduli in column 2 & 4 of Table 4.1 in their paper.

In this work, we exclude all the possible non-fundamental mode Cepheids and those not classified as “DCEP” in the General Catalogue of Variable Stars (GCVS, Kholopov et al. 1998). Some of them are mentioned in T03, include: EV SCT, V1726 CYG, SZ TAU, QZ NOR, α UMi and V367 SCT. We further exclude EU TAU from Barnes et al. (2003) because it is a first-overtone Cepheid. Note that LS PUP is not classified as “DCEP” in the GCVS. We also exclude this Cepheid though it is included in the T03 sample. For Cepheids in Turner & Burke (2002) and Kervella et al. (2004a), we did not include the Cepheids that are classified as “DCEPS” in the GCVS, which includes SU CAS, GH CAR and Y OPH. The additional Cepheids included in our sample but not in T03 are labelled in Table 4.1.

In principle the number of Cepheids can be increased if we include the first overtone (FO) Galactic Cepheids in our sample, by using their fundamental mode periods in obtaining the PL relation. However, we prefer not to include the FO Cepheids due to the following reasons: (a) we want to avoid the contaminations from other types of Cepheids and select only the *bona fide* fundamental mode Cepheids; (b) we want to be consistent with T03, who excluded the non-fundamental mode Cepheids in their study; (c) the physics involved in fundamental mode and FO Cepheids is not the same (see, e.g., Antonello 1994; Antonello & Aikawa 1995; Bono et al. 1999b; Feuchtinger et al. 2000; Bono et al. 2002b) and (d) the results of microlensing surveys to the Magellanic Clouds suggest that the FO Cepheids follow their own PL relations (see, e.g., Udalski et al. 1999a). The theoretical studies from Bono et al. (1999a) and Baraffe & Alibert (2001) also suggest that the FO Cepheids follow different PL relations. Nevertheless, we list the FO Cepheids (classified as “DCEPS” in the GCVS) with recent distance measurements in Table 4.2 for completeness. Note that although BD CAS has been updated to “DCEPS” by Poretti (1994), there is no I-band data available for this Cepheid in the literature. We therefore exclude BD CAS in Table 4.2.

Table 4.1 – Distances to Galactic fundamental mode Cepheids.

Cepheid (1)	$\mu_o(O)^a$ (2)	$\mu_o(TB)^a$ (3)	$\mu_o(G98)^a$ (4)	$\mu_o(F03)^a$ (5)	$\mu_o(other)^a$ (6)	$\mu_o(w.m.)^h$ (7)
η AQL ⁱ	...	...	...	6.986 ± 0.052	7.526 ± 0.217 ^b ; 7.108 ± 0.148 ^c	7.025 ± 0.048
RX AUR ⁱ	...	...	...	...	11.101 ± 0.204 ^c	11.101 ± 0.204
U CAR	11.46	11.46 ± 0.04 ^g	11.069 ± 0.038	10.972 ± 0.032	...	10.984 ± 0.032
VY CAR	11.63	11.60 ± 0.09 ^g	11.419 ± 0.043	11.501 ± 0.022	...	11.503 ± 0.022
WZ CAR	...	...	12.980 ± 0.135	12.918 ± 0.066	...	12.918 ± 0.066
l CAR	...	...	8.941 ± 0.053	8.989 ± 0.032	8.670 ± 0.204 ^e	8.981 ± 0.032
CEa CAS	12.69	12.74 ± 0.15	...	...	12.63 ± 0.14 ^d	12.662 ± 0.091
CEb CAS	12.69	12.74 ± 0.15	...	...	12.63 ± 0.14 ^d	12.662 ± 0.091
CF CAS	12.69	12.74 ± 0.15	...	...	12.63 ± 0.14 ^d	12.662 ± 0.091
DL CAS	11.22	11.16 ± 0.04 ^g	...	...	10.99 ± 0.14 ^d	11.032 ± 0.115
V CEN	9.17	9.16 ± 0.04 ^g	9.302 ± 0.024	9.175 ± 0.063	...	9.175 ± 0.060
VW CEN	...	...	13.014 ± 0.042	12.803 ± 0.039	...	12.803 ± 0.039
XX CEN	...	...	10.847 ± 0.065	11.116 ± 0.023	...	11.116 ± 0.023
KN CEN	...	...	12.911 ± 0.060	13.124 ± 0.045	...	13.124 ± 0.045
δ CEP ⁱ	...	6.76 ± 0.10	...	7.084 ± 0.044	7.173 ± 0.048 ^b ; 7.181 ± 0.089 ^f	7.100 ± 0.029
X CYG ⁱ	...	10.30 ± 0.05	...	10.421 ± 0.016	10.209 ± 0.055 ^c	10.395 ± 0.015
SU CYG ⁱ	...	9.70 ± 0.06	...	...	...	9.700 ± 0.060
β DOR ⁱ	...	...	...	...	7.566 ± 0.153 ^c	7.566 ± 0.153
ζ GEM ⁱ	...	7.85 ± 0.10	...	...	7.794 ± 0.228 ^b ; 7.782 ± 0.211 ^e	7.732 ± 0.084
Z LAC ⁱ	...	...	...	11.637 ± 0.055	...	11.637 ± 0.055
T MON	11.14	11.05 ± 0.14 ^g	10.576 ± 0.067	10.777 ± 0.053	10.580 ± 0.068 ^c	10.721 ± 0.041
CV MON	11.22	11.12 ± 0.04 ^g	10.901 ± 0.046	10.988 ± 0.034	11.39 ± 0.21 ^d	11.003 ± 0.033
UU MUS	...	...	12.260 ± 0.092	12.589 ± 0.084	...	12.589 ± 0.084
S NOR	9.85	9.82 ± 0.04 ^g	9.918 ± 0.025	9.908 ± 0.032	...	9.907 ± 0.032
U NOR	...	...	10.769 ± 0.067	10.716 ± 0.060	...	10.716 ± 0.060
TW NOR	11.47	11.47 ± 0.08 ^g	...	...	11.38 ± 0.18 ^d	11.393 ± 0.134
V340 NOR	11.17	11.16 ± 0.11	11.498 ± 0.130	11.145 ± 0.185	11.23 ± 0.12 ^d	11.166 ± 0.070
BF OPH	...	...	9.496 ± 0.110	9.271 ± 0.034	9.265 ± 0.192 ^c	9.271 ± 0.033
UY PER	11.78	11.88 ± 0.50 ^g	...	...	...	11.780 ± 0.200
RS PUP	11.28	...	...	11.622 ± 0.076	11.160 ± 0.290 ^c	11.555 ± 0.069
VZ PUP	...	...	13.551 ± 0.036	13.083 ± 0.057	...	13.083 ± 0.057
AQ PUP	...	11.78 ± 0.10	12.750 ± 0.038	12.522 ± 0.045	...	12.397 ± 0.041
BN PUP	...	...	12.924 ± 0.051	12.950 ± 0.050	...	12.950 ± 0.050
GY SGE	12.65	12.74 ± 0.08 ^g	12.939 ± 0.071 ^h	...	...	12.650 ± 0.200
U SGR	9.07	8.94 ± 0.10 ^g	8.869 ± 0.015	8.871 ± 0.022	9.137 ± 0.158 ^c ; 9.13 ± 0.18 ^d	8.881 ± 0.022
W SGR ⁱ	...	...	...	...	7.933 ± 0.169 ^e	7.933 ± 0.169
X SGR ⁱ	...	...	...	...	7.553 ± 0.161 ^e	7.553 ± 0.161
WZ SGR	11.26	11.31 ± 0.04	11.262 ± 0.021	11.287 ± 0.047	12.001 ± 0.169 ^c ; 11.23 ± 0.16 ^d	11.316 ± 0.029
BB SGR	9.11	9.08 ± 0.08 ^g	9.238 ± 0.022	9.519 ± 0.028	9.805 ± 0.181 ^c	9.518 ± 0.027
RY SCO	...	...	10.469 ± 0.042	10.516 ± 0.034	9.911 ± 0.147 ^c	10.485 ± 0.033
KQ SCO	12.36	12.33 ± 0.25 ^g	...	...	...	12.360 ± 0.200
RU SCT	11.60	11.64 ± 0.14 ^g	...	...	11.41 ± 0.20 ^d	11.480 ± 0.141
T VEL	...	...	10.094 ± 0.023	9.802 ± 0.060	...	9.802 ± 0.060
RY VEL	...	...	12.100 ± 0.050	12.019 ± 0.032	...	12.019 ± 0.032
RZ VEL	11.19	11.27 ± 0.31 ^g	11.169 ± 0.025	11.020 ± 0.029	...	11.024 ± 0.029
SW VEL	12.08	12.04 ± 0.05 ^g	11.989 ± 0.056	11.998 ± 0.025	...	11.999 ± 0.025
CS VEL	12.59	12.59 ± 0.14 ^g	12.713 ± 0.144	...	...	12.671 ± 0.117
S VUL	13.24	13.24 ± 0.09 ^g	13.731 ± 0.095 ^h	...	...	13.240 ± 0.200
T VUL ⁱ	...	...	...	...	8.920 ± 0.146 ^c	8.920 ± 0.146
SV VUL	11.83	11.78 ± 0.05	12.325 ± 0.072 ^h	...	11.331 ± 0.081 ^c ; 10.98 ± 0.21 ^d	11.636 ± 0.041

^a $\mu_o(O)$ = open cluster distance from T03; $\mu_o(TB)$ = open cluster distance from Turner & Burke (2002), adjusted to $\mu_{Pleiades} = 5.61$ mag. by adding +0.05mag; $\mu_o(G98)$ = BE distance from Gieren et al. (1998); $\mu_o(F03)$ = BE distance from Fouqué et al. (2003); $\mu_o(other)$ = distance from other sources; $\mu_o(w.m.)$ = the weighted-mean distance for the entries in column 2, 3, 5 & 6, when available. Excepts for CS VEL, which we include the $\mu_o(G98)$ in obtaining the weighted-mean. ^b Distance measurements from interferometry: η AQL & ζ GEM are from Lane et al. (2002), and δ CEP is from Nordgren et al. (2002). ^c BE distance measurements from Barnes et al. (2003), who use a Bayesian approach in their analysis. ^d Open cluster distances from Hoyle et al. (2003), adjusted to $\mu_{Pleiades} = 5.61$ mag. by adding +0.05mag. ^e Interferometry distance measurements from Kervella et al. (2004a). ^f *HST* astrometric measurement from Benedict et al. (2002). ^g These distances are not included in calculating the weighted-mean distances, see text for details. ^h These distance moduli are not used in both Gieren et al. (1998) and T03 as they appear to be outliers in the PL plots. ⁱ These Cepheids are not included in T03.

Table 4.2 –Distances to Galactic first overtone Cepheids.

Cepheid	$\mu_o(O)^a$	$\mu_o(TB)^a$	$\mu_o(G98)^a$	$\mu_o(B03)^a$	$\mu_o(H03)^a$	$\mu_o(K04)^a$	$\mu_o(w.m.)^a$
GH CAR	...	10.99 ± 0.20	...	...	...	...	10.990 ± 0.200
SU CAS	7.12^c	7.11 ± 0.03^b	...	...	...	...	7.120 ± 0.200
V1726 CYG	11.02	11.02 ± 0.03^b	...	...	...	...	11.020 ± 0.200
QZ NOR	11.17	11.16 ± 0.11	11.095 ± 0.031	...	11.23 ± 0.12	...	11.169 ± 0.075
Y OPH	...	...	...	...	...	9.06 ± 0.21	11.060 ± 0.210
EV SCT	10.92	11.09 ± 0.07	11.066 ± 0.033	...	10.84 ± 0.15	...	11.026 ± 0.060
SZ TAU	8.72	8.71 ± 0.02^b	8.090 ± 0.042	8.74 ± 0.33	...	...	8.725 ± 0.171
EU TAU	...	...	...	10.27 ± 0.16	...	...	10.270 ± 0.160
α UMi	5.19	5.44 ± 0.05	...	...	...	...	5.425 ± 0.049

^a $\mu_o(O)$ = open cluster distance from T03; $\mu_o(TB)$ = open cluster from Turner & Burke (2002), adjusted to $\mu_{Pleiades} = 5.61$ mag. by adding +0.05mag.; $\mu_o(G98)$ = BE distance from Fouqué et al. (2003); $\mu_o(B03)$ = BE distance from Barnes et al. (2003); $\mu_o(H03)$ = open cluster distance from Hoyle et al. (2003), adjusted to $\mu_{Pleiades} = 5.61$ mag. by adding +0.05mag.; $\mu_o(K04)$ = interferometry distance from Kervella et al. (2004a); $\mu_o(w.m.)$ = weighted-mean distances. ^b These distances are not included in calculating the weighted-mean distances, since it is unclear whether these distances are independent to the distances given in Feast (1999) or T03. ^c This distance is adopted from Feast (1999), adjusting to $\mu_{Pleiades} = 5.61$ mag. by adding +0.04mag.

4.3 The Galactic PL Relation

The values of $\log(P)$, $(B - V)_o$ color, the color excess² $E(B - V)$, and the mean B, V and I-band³ magnitudes for the Cepheids in Table 4.1 are all taken from T03. The absorption-to-reddening coefficient, R , for individual Cepheids is derived using the prescription given in T03⁴: $R_V = 3.17(\pm 0.13) + 0.44[(B - V)_o - 0.78] + 0.05[E(B - V) - 0.42]$, $R_B = R_V + 1.00$ and $R_I = R_V - 1.28$. The B-, V- and I-band extinction-corrected absolute magnitudes for the 50 Cepheids in Table 4.1 can be calculated by adopting the weighted-mean distance moduli, as given in the last column of Table 4.1, and then fitted with least-square regressions to obtain the PL relation. The plots of the fitted Galactic PL relations are presented as solid lines in Figure 4.1, with the following expressions:

$$M_B = -2.594(\pm 0.106) \log(P) - 0.674(\pm 0.123), \sigma = 0.248 \quad (4.1)$$

$$M_V = -2.999(\pm 0.097) \log(P) - 0.995(\pm 0.112), \sigma = 0.226 \quad (4.2)$$

$$M_I = -3.303(\pm 0.094) \log(P) - 1.450(\pm 0.108), \sigma = 0.219 \quad (4.3)$$

The error bars in Figure 4.1 are obtained from the quadrature sum of the error estimates of distance modulus (given in the last column of Table 4.1), extinction (adopted from Fernie et al.

² $E(B - V)_{corr}$ in T03.

³ There are no mean I-band values for η AQL, δ CEP and ζ GEM in T03, hence we adopted the I-band mean magnitudes from Lanoix et al. (1999b).

⁴ There are other formulae for the R_V available in the literature. We choose the formula of R_V from T03 in order to be consistent with the work of T03. The detailed study of the sensitivity of PL relation to the selected R_V will be addressed in a future paper.

1995 database⁵) and mean magnitudes. We assign a conservative error of 0.05mag. to the mean magnitudes. This is reasonable because the mean magnitudes are derived from accurate and reliable light curves (Berdnikov et al., 2000).

Figure 4.1 –Galactic PL relation in B- (top panel), V- (middle panel) and I- (bottom panel) band. The lines are the fitted PL relations, as given in equation (4.1)-(4.3). The solid circles are for fundamental mode Cepheids as listed in Table 4.1. The open circles are the first overtone Cepheids from Table 4.2, plotted with their fundamental mode periods. The error bars include the errors in distance modulus, errors in extinction and an error estimation of 0.05mag. in mean magnitudes.

4.3.1 The Effect of FO Cepheids

In this section we investigate the effect of including the Galactic PL relations if we include the FO Cepheids that are listed in Table 4.2. The fundamental mode periods and the mean magnitudes in B-, V- and I-band for these Cepheids are taken from Berdnikov et al. (2000). The color excess for these Cepheids are calculated via the prescription given in T03: $E(B - V) = 0.951E(B - V)_F$, where $E(B - V)_F$ is taken from Fernie et al. (1995). Then the color, $(B - V)_o$, can be calculated

⁵<http://ddo.astro.utoronto.ca/cepheids.html>

via $(B - V) - E(B - V)$. The extinction corrections for these Cepheids are handled in the same way as in fundamental mode Cepheids. The fitted PL relations from the combination of 59 fundamental mode and first overtone Cepheids are:

$$M_B = -2.590(\pm 0.100) \log(P) - 0.665(\pm 0.111), \sigma = 0.265$$

$$M_V = -3.007(\pm 0.097) \log(P) - 0.962(\pm 0.108), \sigma = 0.257$$

$$M_I = -3.319(\pm 0.098) \log(P) - 1.406(\pm 0.108), \sigma = 0.260$$

By comparing these PL relations to equations (4.1)-(4.3), we see that including the FO Cepheids does not significantly alter or improve the PL relations. However, the dispersions of the PL relations (σ) have become larger than the PL relations without FO Cepheids. This is mainly due to the one outlier, GH CAR (with $\log[P_o] \sim 0.91$), as shown in Figure 4.1. After removing this Cepheid, the dispersions of the PL relations become comparable to those given in equations (4.1)-(4.3). Even though the PL relations with the FO Cepheids are almost identical to the PL relations without FO Cepheids, we prefer the solutions given in equations (4.1)-(4.3) to be the calibrated Galactic PL relations, as we have argued in Section 4.2.4.

4.3.2 The effect of open cluster distances from Turner & Burke 2002

Since it is unclear whether some of the open cluster distances given in Turner & Burke (2002) are totally independent of the distances given in Feast (1999) or not, we examine the changes of PL relations if we either exclude *all* open cluster distances from Turner & Burke (2002) or assume that these distances are totally independent of Feast (1999), and include them in obtaining the weighted-mean distances. Recall that equations (4.1)-(4.3) used some of the Turner & Burke (2002) distances that are either excluded in or independent of Feast (1999). For the former case, the PL relations with 49 fundamental mode Cepheids are:

$$M_B = -2.627(\pm 0.105) \log(P) - 0.623(\pm 0.123), \sigma = 0.240$$

$$M_V = -3.025(\pm 0.096) \log(P) - 0.954(\pm 0.112), \sigma = 0.220$$

$$M_I = -3.320(\pm 0.095) \log(P) - 1.420(\pm 0.110), \sigma = 0.216$$

For the latter case where we include *all* the open cluster distances from Turner & Burke (2002), the fitted PL relations for the 50 Cepheids in Table 2.1 become:

$$M_B = -2.619(\pm 0.107) \log(P) - 0.656(\pm 0.123), \sigma = 0.249$$

$$M_V = -3.024(\pm 0.097) \log(P) - 0.977(\pm 0.111), \sigma = 0.225$$

$$M_I = -3.328(\pm 0.098) \log(P) - 1.432(\pm 0.108), \sigma = 0.217$$

Therefore, the exclusion or inclusion of Turner & Burke (2002) distances produces almost identical PL relations. These PL relations also agree and are consistent with equations (4.1)-(4.3). However, there are some open cluster distances that are not included in Feast (1999) or T03 (e.g. for Cepheid AQ PUP), and there are certain cases where we have confidence to believe that the distances given in Turner & Burke (2002) and Feast (1999) are independent (e.g. for Cepheids CEa, CEB and CF CAS). Hence we continue to adopt equations (4.1)-(4.3) as the calibrated Galactic PL relations.

4.4 Comparison to Published Results

4.4.1 Comparison with other Galactic PL relations

We selected recent empirical PL relations from the literature that give *both* V- and I-band PL relations to be compared with our results, because the PL relations in these two bands are extensively used in extra-galactic distance scale studies (e.g., Freedman et al. 2001). For example, we exclude the PL relations from Barnes et al. (2003) or Hoyle et al. (2003) because they did not give the I-band PL relations in their papers. The selected PL relations, along with our results, are given in Table 4.3. These include the Galactic PL relations derived by Gieren et al. (1998, GAL-G98), Fouqué et al. (2003, GAL-F03) and Tammann et al. (2003, GAL-T03). We exclude the *Hipparcos*-based Galactic PL relations because these PL relations adopted the slopes from LMC PL relations, and calibrated the zero-points with *Hipparcos* data (as those used in, e.g., Lanoix et al. 1999b and Paturel et al. 2002b). From the table, it can be seen that our results are consistent with the GAL-G98 and GAL-F03 PL relation. However there is some discrepancy between our results and GAL-T03. In this situation, we can use the *t*-statistical test as mentioned in the next sub-section (see also, e.g., Zwilling & Kokoska 2000) to assess the significance of the difference in slopes under the null hypothesis that the slopes are the same. The results show that the *p*-value for the V and I-band slopes are 0.15 and 0.27, respectively. Hence the null hypothesis cannot be ruled out at the 95% confidence level, and our results are also consistent with the GAL-T03 PL relation.

Table 4.3 –Comparison of various PL relations^a.

PL relation	N	a_V	b_V	σ_V	a_I	b_I	σ_I	Ref. ^b
GAL-G98	28	-3.037 ± 0.138	-1.021 ± 0.040	0.209	-3.329 ± 0.132	-1.435 ± 0.037	0.194	1
GAL-F03	32	-3.06 ± 0.11	-0.989 ± 0.034	...	-3.24 ± 0.11	-1.550 ± 0.034	...	2
GAL-T03	53	-3.141 ± 0.100	-0.826 ± 0.119	0.24	-3.408 ± 0.095	-1.325 ± 0.114	0.23	3
GAL-Here	50	-2.999 ± 0.097	-0.995 ± 0.112	0.226	-3.303 ± 0.094	-1.450 ± 0.108	0.219	4
GAL-Theory1	...	-2.905	-1.183	...	-3.102	-1.805	...	5
GAL-Theory2	...	-2.45 ± 0.02	-1.50 ± 0.02	0.17	-2.78 ± 0.01	-2.02 ± 0.01	0.13	6
GAL-Theory3	...	-2.22 ± 0.01	-1.62 ± 0.01	0.14	-2.58 ± 0.01	-2.14 ± 0.01	0.10	7
LMC	~ 650	-2.760 ± 0.030	-1.458 ± 0.020	0.160	-2.962 ± 0.020	-1.942 ± 0.010	0.110	8

^a $M_{V,I} = a_{V,I} \log(P) + b_{V,I}$, and $\sigma_{V,I}$ is the rms dispersion. For LMC PL relation, assume $\mu_{LMC} = 18.50\text{mag}$.
^b Reference: [1] Gieren et al. (1998); [2] Fouqué et al. (2003); [3] Tammann et al. (2003); [4] Equations (4.2) & (4.3) from this work; [5] Baraffe & Alibert (2001), with $Z = 0.02$; [6] table 6 from Fiorentino et al. (2002), with $Y = 0.31$ and $Z = 0.02$; [7] table 6 from Fiorentino et al. (2002), with $Y = 0.28$ and $Z = 0.02$; [8] Freedman et al. (2001).

The small discrepancy of the PL slopes between our results and T03 is mainly due to the inclusion of additional distance measurements in this work (see Section 4.2 for details). If we take the arithmetic, unweighted-mean of the 39 Cepheids with $\mu_o(O)$ and $\mu_o(G98)$ in Table 4.1 (as these distance moduli are used in T03), the slopes of the fitted PL relations become steeper: $a_B = -2.750 \pm 0.123$, $a_V = -3.130 \pm 0.110$ and $a_I = -3.402 \pm 0.106$, which agree with the results of T03. However, by taking the weighted-mean distances for these 39 Cepheids, the PL slopes become shallower but still consistent with T03: $a_B = -2.721 \pm 0.125$, $a_V = -3.102 \pm 0.112$ and $a_I = -3.373 \pm 0.109$. Thus the different between our results and T03 is due to the inclusion of additional distance measurements in our study.

For completeness, we also include the recent theoretical PL relations in Table 4.3 from Baraffe & Alibert (2001, GAL-Theory1) and Fiorentino et al. (2002, GAL-Theory2 & GAL-Theory3) by adopting $Z = 0.02$. From the table, our empirical PL relations fairly agree with the theoretical PL relations from Baraffe & Alibert (2001). However, none of the empirical PL relations given in Table 4.3 agree with the GAL-Theory2 and GAL-Theory3.

4.4.2 Comparison with the LMC PL Relation

Since the PL relation is shown to be different in the LMC and Galaxy by T03 (also in Fouqué et al. 2003 and Kanbur et al. 2003), we verify this result by comparing our Galactic PL relation to the LMC counterpart, as those used by the H_0 Key Project Freedman et al. (2001). These LMC PL relations are obtained with ~ 650 Cepheids from the Optical Gravitational Lensing Experiment (Udalski et al., 1999a). For our Galactic PL relation, the difference in the V- and I-band slopes is: $\Delta a_V = 0.239 \pm 0.102$ and $\Delta a_I = 0.341 \pm 0.096$, which are 2.3σ and 3.6σ results, respectively. Are the slopes from Galactic Cepheids consistent with the slopes of the LMC PL relation? We also apply the t -statistical test to test for the equality in the slopes of the Galactic and LMC PL relation.

Assume that the Galactic data are (randomly) drawn from the same parent population, which is consist of the LMC data, then the distribution of the Galactic slopes should follow the t -distribution under the null hypothesis that $a_{GAL} = a_{LMC}$ (see Figure 4.2 for illustration). By calculating the corresponding t values, we can obtain the p -values (or the probability) under the null hypothesis. The results are:

Figure 4.2 –Illustration of the idea behind the t -test for the slopes of the Galactic PL relation.

For V-band slope, $p(V) = 0.017$.

For I-band slope, $p(I) = 0.001$.

The small p -values indicate that the null hypothesis can be rejected with more than 95% confidence level. Therefore the slopes of the V- and I-band Galactic PL relation are not consistent with their LMC counterparts. These statistical results strongly suggest that the PL relation is *not universal*, at least in the galaxies with metallicity that comparable to the Galactic or LMC values.

4.5 Conclusion and Discussion for this Chapter

By using the recent independent distance measurements to 50 Galactic Cepheids, we derive a Galactic PL relation in the B-, V- and I-bands. Our analysis differs from that in T03 due to the following aspects: (a) we include other recent independent distance measurements; and (b) we took the weighted-mean to the available distances. The results confirm that the Galactic PL relations are steeper than the LMC counterparts (Fouqué et al., 2003; Tammann et al., 2003). However, the steepness of the Galactic PL relation is still inconclusive (either as steep as in T03 or as shallow as in this study) because of the small number (~ 50) of Cepheids in the sample. Our Galaxy hosts ~ 500 Cepheids (Fernie et al., 1995), hence the Galactic PL relation can be improved if there are more independent distance measurements to Galactic Cepheids in the future, such as the Cycle 12 *HST* observations of the nearby Cepheids with astrometric measurements (10 Cepheids from Program 9879, with P.I.: G. F. Benedict, where 3 of them are not included in Table 4.1 [Benedict 2003, private communication]) or the future Space Interferometry Mission (SIM, launch in 2009)⁶ and the *Gaia* mission. The importance of having the accurately calibrated Galactic PL relation are two-fold:

1. Distance scale study: The distances to the high metallicity galaxies, either in the H_0 KP or in the future observations, should be obtained with the Galactic PL relations. The attempts of using the Galactic PL relations to calibrate the Cepheid distance to the H_0 KP target galaxies can be found in Kanbur et al. (2003).
2. Constrain on the pulsational/evolutionary models: The slopes from the theoretical Galactic PL relations are much *shallower* (Fiorentino et al., 2002) than the observed slopes as presented here, therefore the improvements for the theoretical Galactic PL relation is desired.

Finally, it is worth while to point out that the study by Heacox (2004), who applied the Bayesian analysis to the *Hipparcos* data, has found that the Galactic V-band slope is much shallower than the slopes found in this study or even in the LMC. Therefore, the improved Galactic PL relations from large sample is clearly desirable to solve this discrepancy in the future.

⁶<http://planetquest.jpl.nasa.gov/SIM/sim.index.html>

CHAPTER 5

THE NON-LINEAR PL RELATIONS FOR THE LMC CEPHEIDS FROM OPTICAL TO NEAR INFRARED

Recent studies, using the OGLE data, strongly suggest that the period-luminosity (PL) relation for the Large Magellanic Cloud (LMC) Cepheids shows a discontinuity at a period of 10 days. In this chapter, we apply statistical tests, the chi-square test and the F -test, to the Cepheid data from the MACHO project to test the non-linearity of the PL relation, and extend these tests to the near infrared (JHK) PL relations with 2MASS data. We correct the extinction for these data by applying an extinction map towards the LMC. Our results show that the VRJH-band PL relations are consistent with a two-line regression with a discontinuity at 10 days at the 99.5% confidence level, while the K-band PL relation is marginally consistent with a single-line regression. Besides the two-line regression, we also test the other forms of non-linearity, such as the piecewise regression and the polynomial fit, and the results from the statistical tests are essentially same as the two-line regression. The non-linearity of the V-band PL relation as seen in both of the OGLE and MACHO data, using different extinction maps, suggests that this non-linearity is real.

5.1 Recent Studies of the Non-Linear PL Relation

Cepheid period-luminosity (PL) relations play a crucial role in the distance ladder which can ultimately be used to determine Hubble constant. The current most commonly used calibrating PL relation is based on the Large Magellanic Cloud (LMC) Cepheids (see, e.g., Freedman et al., 2001; Saha et al., 2001b) and it is regarded as a linear function of $\log(P)$, where P is the pulsation period in days (see Chapter 1.2). However, recent studies (Tammann & Reindl, 2002; Kanbur & Ngeow, 2004; Sandage et al., 2004; Kanbur et al., 2005; Ngeow et al., 2005) have strongly suggested that the LMC PL relations are non-linear, i.e. there are two PL relations across the period range with discontinuity at 10 days (hence a break in the PL relation) for the short ($P < 10$ days) and the long period LMC Cepheids, respectively. Most of these studies are mainly based on the optical data (BVI-band) for the LMC Cepheids from the Optical Gravitational Lensing Experiment (OGLE,

Udalski et al., 1999a) database. This is illustrated in Figure 5.1 for the extinction corrected V-band PL relation with the OGLE LMC Cepheids. The existence of two LMC PL relations is further supported by the results from a rigorous statistical test (the F -test), as presented in Kanbur & Ngeow (2004), which shows that the V- and I-band LMC PL relations are better described by two PL relations. Since most of these studies, such as the the work of Tammann & Reindl (2002) and Kanbur & Ngeow (2004), are mainly based on the optical data (BVI-band) for the LMC Cepheids from the OGLE (Optical Gravitational Lensing Experiment, Udalski et al., 1999a,b) database, which are truncated at $\log(P) \sim 1.5$ due to CCD saturation, Sandage et al. (2004) and Kanbur et al. (2005) used additional data that are available from the literature, especially those with $\log(P) > 1.5$, to further support the existence of two PL relations in the LMC. The reason that the LMC PL relation is non-linear is because the period-color (PC) relation for LMC Cepheids is also non-linear across the 10 days period (Tammann & Reindl, 2002; Kanbur & Ngeow, 2004; Sandage et al., 2004; Kanbur et al., 2005). Recall that both of the PL and PC relations follow the PLC relation for Cepheid variables (see Madore & Freedman 1991 for the basic physics of PLC relation).

It is important to verify the non-linearity of the LMC PL relation from these earlier studies, which are mainly obtained from the OGLE data, with an independent dataset. Hence, we examine the non-linearity for the optical LMC PL relation with the Cepheids data obtained from the MACHO database (Alcock et al., 2000) in this chapter. In addition, we also investigate the LMC PL relations in the near infrared (NIR) bands (i.e., JHK-band) to see if these PL relations also show a non-linearity across the 10 days period.

5.2 LMC Data and the Extinction Corrections

The data for the fundamental mode LMC Cepheids are adopted from Nikolaev et al. (2004), which include the V- and R-band mean magnitudes, as well as the periods, from the MACHO database and the J-, H- and K-band mean magnitudes from the 2MASS database (Skrutskie, 1998). The mean magnitudes in V- and R-bands are computed from the complete light curves with the MACHO photometric data. The (random phase) magnitudes in J-, H- and K-bands are from the 2MASS single epoch observations, hence the mean magnitudes in these NIR bands are obtained with a random-phase correction function. The details for obtaining the mean magnitudes in all these bands can be found in Nikolaev et al. (2004). The locations of these Cepheids in the sky are plotted in Figure 5.2.

Figure 5.1 –The extinction corrected V-band PL relation for the LMC Cepheids from OGLE database, where the data are taken from Kanbur & Ngeow (2004). The solid lines are the fitted PL relation to long and short period Cepheids, while the dashed line is the fitted PL relation to all Cepheids.

The next step in the analysis is to obtain the extinction-corrected mean magnitudes for these Cepheids. This was done by applying an extinction map with the following values of R_λ (ratio of total-to-selective reddening, and $R_\lambda = A_\lambda/E[B - V]$), in order to be consistent with Nikolaev et al. (2004):

$$R_{V,R,J,H,K} = \{3.12, 2.56, 0.90, 0.53, 0.34\}.$$

The LMC extinction map we used is taken from Zaritsky et al. (2004). In brief, given the coordinates for each Cepheid and a search radius r , the extinction map returns the average value of extinction, $\overline{A_V} \pm \sigma_{A_V}$ (where $\sigma_{A_V} = \text{std. deviation}/\sqrt{N}$), from the stars bounded within the search radius. The extinction map allows to choose the $\overline{A_V}$ obtained from cool stars ($5500\text{K} < T_{\text{eff}} < 6500\text{K}$), hot stars ($12,000\text{K} < T_{\text{eff}} < 45,000\text{K}$) or both. Since Cepheid variables are cool stars, we only

Figure 5.2 –The distribution of the MACHO Cepheids in the sky. The boundary of the Zaritsky's extinction map is outlined as dashed lines. The three Cepheids without the extinction are marked as (red) squares with cross. The 8 outliers are marked as (red) filled circles. The dashed box roughly denotes the location of 30 Doradus. The box is centered at $(RA, DEC) = (84^{\circ}.675, -69^{\circ}.101)$, with the size of 0.5° .

use the extinction values from the cool stars. We use $r = 2'$ to ensure that there are sufficient number of stars within the search radius to obtain the $\overline{A_V}$ (usually more than 10 stars)¹. However, there are two Cepheids with locations beyond the extinction map and one Cepheid without any cool stars within the $r = 3'$ radius. For simplicity, we assume $\overline{A_V} = 0.3 \pm 0.1\text{mag}$. (roughly the mean extinction of LMC) for these three Cepheids. The locations of these three Cepheids are labelled in Figure 5.2 as squares with the cross. After the values of $\overline{A_V}$ for the Cepheids are obtained, the extinctions in other bands are calculated with the following relation: $A_{\lambda} = (\frac{R_{\lambda}}{R_V})\overline{A_V}$ with the errors $\sigma_{A_{\lambda}} = (\frac{R_{\lambda}}{R_V})\sigma_{A_V}$, where λ denotes the passband.

¹Only two Cepheids with enclosed stars that are less than 10

We then remove 8 outliers out of 1330 Cepheids (all of these outliers have $\log[P] < 0.75$) from the sample, as they are fainter by more than $\sim 1.2\text{mag.}$ and $\sim 1.0\text{mag.}$ in V- and R-band, respectively, to the fitted PL relations at the same period. It is possible that the extinctions for these outliers are underestimated because they are not obvious in the NIR PL relations. However the locations of these outliers are away from the 30 Doradus region in LMC, a well-known star-formation region with high extinction (see Figure 5.2). Another possibility is that they are Type II Cepheids that are mis-identified as the classical Type I Cepheids. However, it is clear that these outliers should be removed from our sample. In addition, we also remove the Cepheids with $\log(P) < 0.4$, in order to guard against the possible contamination from first overtone Cepheids (Udalski et al., 1999a) and the possible incompleteness at the faint end, as well as to be consistent with previous studies (Kanbur & Ngeow, 2004; Sandage et al., 2004). This leaves 1216 Cepheids for further consideration. Note that Udalski et al. (1999a) has applied the same period-cut to the OGLE LMC Cepheids to derive the PL relation that has been used in, e.g., Freedman et al. (2001).

5.3 Analysis and Results

The extinction-corrected VRJHK-band PL relations are plotted in the left panels of Figure 5.3-5.7, along with the weighted least squares regressions (Press et al., 1992), include the single-line regression for entire period range and the two-line regression for long and short period Cepheids respectively, fitted to the data. The long and short period Cepheids are separated at a period of 10 days. Sandage et al. (2004) has discussed the significance of the discontinuity at 10 days, which will not be repeated here. Other evidence that supports the discontinuity at 10 days are: (a) the works of Tammann et al. (2002), Kanbur & Ngeow (2004) and Sandage et al. (2004) strongly suggests that the LMC period-color (PC) relation is also non-linear with discontinuity at 10 days (see also upper panel of Figure 5.8); and (b) the Fourier amplitudes from the light curves of the short period ($P < 10\text{days}$) Cepheids are also radically different from their long period ($P > 10\text{days}$) counterparts (Udalski et al., 1999a; Ngeow et al., 2003; Sandage et al., 2004, and reference therein), which is a well established fact in stellar pulsation. It is worth to note that it is very difficult to statistically test the precise period that the discontinuity in the PL relation occurs because of the intrinsic width of the instability strip that causes the PL relation has an intrinsic dispersion. The confirmation of the discontinuity at 10 days has to be done with the stellar pulsation modelings, which is beyond the scope of this chapter. Hence, we follow the existing works (Tammann & Reindl, 2002; Kanbur & Ngeow, 2004; Sandage et al., 2004) and consider a discontinuity at 10 days in the chapter.

The weights that are used in the regressions are the quadrature sum of the photometric errors in mean magnitudes (σ_λ), the errors from the random phase corrections that are only applicable to JHK-bands (σ_{rp}), the errors from extinction (σ_{A_λ}), and the intrinsic dispersion due to the finite width of the instability strip (σ_{IS}). The dispersions of the PL relations in each band from Madore & Freedman (1991, hereafter MF91 in this chapter), $\sigma_{IS}(V, R, J, H, K) = \{0.29, 0.25, 0.16, 0.14, 0.13\}$, can be initially adopted for σ_{IS} . However, a comparison of the dispersions given in MF91 with Gieren et al. (1998), Persson et al. (2004) and Sandage et al. (2004) suggest that the VR-band and the JHK-band dispersions given in MF91 should be scaled by ~ 0.73 and ~ 0.88 , respectively. Hence we adopt the following values for the intrinsic dispersions:

$$\sigma_{IS}(V, R, J, H, K) = \{0.21, 0.18, 0.14, 0.12, 0.11\}.$$

The adopted σ_{IS} , such as those originally given in MF91, does not affect the results from the following statistical tests.

The results of the regressions are summarized in the bottom of the left panels in Figure 5.3-5.7. From these figures, the V- and R-band PL relations suggest that there is a discontinuity at 10 days, marginally in the J-band PL relation, but not obvious in the H- and K-band PL relations. The right panels of Figure 5.3-5.7 show the residuals of the fit as a function of period using single-line and two-line regressions. From the residual plots, both of the V- and R-band residuals show a trend at $\log(P) > 1.0$ when using the single-line regression. These trends are removed if two-line regressions are used to fit the data. These findings also support our assumption that the discontinuity of the PL relation occurs at a period of 10 days. Similarly, the J-band residuals show a marginal trend, which is not obvious in the H- and K-band.

5.3.1 Statistical Tests for the Data

To investigate the possible non-linearity of the PL relations in Figure 5.3-5.7, we applied the chi-square test and the F -test (as given in Weisberg, 1980) using the weighted least squares results. The null (H_O) and the alternate (H_A) hypothesis represent the reduced and the full models, respectively, where the reduced models are obtained by setting some parameters in the full models to certain specific values (e.g., zero, some values or equal to each others, see Weisberg 1980). Hence our reduced models is a one-line regression (two-parameters) fit to the entire period range, and the H_O is that one-line regression is adequate to describe the data. In contrast, the H_A is that the full model is necessary to fit the data. Here, we consider three different kinds of full models:

Figure 5.3 –PL relation and residual plots for the MACHO V-band. Left (a): The extinction corrected MACHO V-band PL relation. The dashed and solid lines are the fitted single-line and two-lines regressions, respectively. The regressions are given in the lower-right corner. The error bars are the quadrature sum of the photometric errors in mean magnitudes (σ_λ) and the errors from the extinction (σ_{A_λ}). Right (b): The residuals from using single-line (top panel) and two-lines (bottom panel) regression fit. The dashed line indicates the zero residual. The dotted lines are the 1σ dispersions with the values given in the lower-right corners.

Figure 5.4 –Same as Figure 5.3, but for the MACHO R-band PL relation.

1. A two-line regression (four-parameters) with discontinuity at a characteristic period, P_o , i.e.,

$$m = a^L \log(P) + b^L, P > P_o$$

Figure 5.5 –Same as Figure 5.3, but for the 2MASS J-band PL relation. The error bars are the quadrature sum of the photometric errors in mean magnitudes (σ_λ), the errors from the random phase corrections (that are only applicable to JHK-bands, σ_{rp}) and the errors from the extinction (σ_{A_λ}).

Figure 5.6 –Same as Figure 5.5, but for the 2MASS H-band PL relation.

$$m = a^S \log(P) + b^S, P < P_o$$

with $P_o = 10$ days.

2. A piecewise regression (three-parameters) that forces the two-line regression to be connected at P_o , i.e.,

Figure 5.7 –Same as Figure 5.5, but for the 2MASS K-band PL relation.

$$m = c + d \times \min[\log(P), \log(P_o)] + e \times \max[0, \log(P) - \log(P_o)],$$

such that the zero-point for long period PL relation is $c + d \log(P_o) - e \log(P_o)$.

3. A second-order polynomial (three-parameters) of the form of:

$$m = \alpha + \beta \log(P) + \gamma [\log(P)]^2,$$

which is used to fit all Cepheids.

The results of the weighted regressions for the two-line model have already been given in the left panels of Figure 5.3-5.7, while the results from the weighted regressions for both of the piecewise model and the polynomial model are presented in Table 5.1.

To apply the chi-square test and the F -test, we then calculated the weighted error sum of squares, $SSE = \sum [m(i) - \hat{m}(i)]^2 / [\sigma_\lambda^2(i) + \sigma_{rp}^2(i) + \sigma_{A_\lambda}^2(i) + \sigma_{IS}^2]$, for both H_O and H_A , where $m(i)$ is the observed magnitudes and $\hat{m}(i) = \hat{a} \log P(i) + \hat{b}$ is the fitted magnitudes from weighted least squares for i^{th} Cepheid. Under H_O , the difference of the SSE :

Table 5.1 –The results of the piecewise and polynomial regressions in each bands, where σ is the dispersion of the regression.

Bandpass	Piecewise regression				Polynomial fit			
	c	d	e	σ	α	β	γ	σ
$V \dots$	17.172	-2.846	-1.863	0.256	17.345	-3.435	0.477	0.258
$R \dots$	16.948	-2.985	-2.098	0.219	17.114	-3.545	0.449	0.221
$J \dots$	16.321	-3.157	-2.708	0.123	16.427	-3.502	0.267	0.123
$H \dots$	16.072	-3.223	-2.879	0.117	16.159	-3.505	0.215	0.109
$K \dots$	15.995	-3.194	-2.988	0.124	16.031	-3.317	0.100	0.124

$$\chi^2 = SSE(H_O) - SSE(H_A), \quad (5.1)$$

follows approximately a chi-square distribution with $\nu = n_{FL} - n_{RD}$ degrees of freedom (e.g, see Weisberg, 1980), where n_{FL} & n_{RD} are the number of parameters in full and reduced models, respectively. For large number of data points (N), this is essentially the same as applying a standard F -test (which many regression packages produce) under weighted least squares, where

$$F = \frac{\chi^2/\nu}{SSE(H_A)/(N - n_{FL})}, \quad (5.2)$$

which is approximately an $F_{(\nu, N-n_{FL})}$ distribution under the null hypothesis (see Chapter 6 also). The corresponding probability (p) for the χ^2 and the F -values, under H_O , can be obtained from consulting the standard statistical books or softwares. A large value of χ^2 and/or F (hence small p) indicates that the null hypothesis can be rejected. The results for the chi-square test and the F -test are presented in Table 5.2 and 5.3, respectively.

Table 5.2 –The results of the chi-square test, with H_O =one-line regression.

Bandpass	H_A =two-lines		H_A =piecewise		H_A =polynomial	
	χ^2	$p(\chi^2)$	χ^2	$p(\chi^2)$	χ^2	$p(\chi^2)$
$V \dots$	30.848	0.000	29.600	0.000	14.384	0.000
$R \dots$	33.397	0.000	31.943	0.000	16.861	0.000
$J \dots$	16.019	0.000	13.258	0.000	9.5534	0.002
$H \dots$	11.519	0.003	9.7781	0.002	7.5275	0.006
$K \dots$	4.9810	0.083	3.8766	0.049	1.6977	0.193

From these tables, the null hypothesis for the VRJ- & H-band PL relations are rejected with 99.5% confidence level, corresponding to the p -value to be smaller than 0.05, and hence either of the three different kinds of full models are needed to describe the data (except for the H-band PL

Table 5.3 –The results of the F -test, with H_O =one-line regression.

Bandpass	H_A =two-lines		H_A =piecewise		H_A =polynomial	
	F	$p(F)$	F	$p(F)$	F	$p(F)$
$V \dots$	12.320	0.000	23.643	0.000	11.375	0.001
$R \dots$	13.758	0.000	26.313	0.000	13.749	0.000
$J \dots$	13.375	0.000	22.074	0.000	15.825	0.000
$H \dots$	10.305	0.000	17.464	0.000	13.400	0.000
$K \dots$	3.7867	0.023	5.8909	0.015	2.5728	0.109

relation with polynomial fit in the chi-square test, however the value of $p = 0.06$ is marginal). Among the three full models, we favor the two-lines regression model instead of the polynomial fit because of the following reasons: (a) the residual plots in Figure 5.3 or Figure 5.4 suggest that the two-lines regression is more applicable; (b) the period-color (PC) relations are more consistent with two-lines regression (see Figure 5.8 below and Tammann & Reindl 2002; Kanbur & Ngeow 2004; Sandage et al. 2004, as well as Chapter 6 & 9); and (c) a simple period-mean density relation predicts a PL relation linear in $\log(P)$ but not in $[\log(P)]^2$. For the two-line regression model vs. the piecewise regression model, it is true that “true” non-linearity of the PL relation may be in the form of either models. However it is difficult to test this with the data, and the future theoretical calculations can be used to answer this problem. Here we adopt the two-line model as the form of non-linearity in order to be consistent with the studies in this Thesis Dissertation and in the literature (Tammann et al., 2002; Kanbur & Ngeow, 2004; Sandage et al., 2004). Nevertheless, the VRJ- & H-band LMC PL relations are shown to be non-linear. In contrast, the null hypothesis for the K-band PL relation cannot be rejected from the statistical tests. Hence the K-band PL relation is consistent with single-line regression. Therefore, the VRJ- and H-band PL relations are consistent with two PL relations with discontinuity at 10 days, while the K-band PL relation is marginally consistent with a single PL relation across the entire period range.

5.4 Conclusion and Discussion for this Chapter

By applying the statistical tests to the VRJHK-band PL relations obtained from the MACHO and 2MASS data, the VRJH-band PL relations show strong evidence that they are non-linear, with two PL relations discontinued (or broken) at 10 days, while the K-band PL relation is marginally consistent with the linear PL relation. The non-linearity of the extinction corrected V-band PL relations as seen from two independent datasets, one from the OGLE data (Tammann & Reindl, 2002; Kanbur & Ngeow, 2004; Sandage et al., 2004) and another from this study, strongly indicates that this non-linearity is real and not due to the artifacts of, for example, the extinction error.

Note that Persson et al. (2004) suggested the NIR PL relations (based on ~ 90 LMC Cepheids) are linear, which is probably due to the small number of short period Cepheids (~ 15) in their sample (compared to > 1000 short period Cepheids in our sample).

Figure 5.8 –The selected period-color relations, after extinction corrections, from the MACHO+2MASS LMC Cepheid data.

As mentioned in Chapter 1.2.1, the reason that the PL relation is non-linear is because the PC relation for the LMC Cepheids is also non-linear (Tammann & Reindl, 2002; Kanbur & Ngeow, 2004). This can be clearly seen from upper panel of Figure 5.8 for extinction corrected $(V - K)$ PC relations. Since both of the PL and PC relations obey the more general PLC relations, the morphology of the PC relation will affect the morphology of the PL relation (Madore & Freedman, 1991). Recall that the luminosity variation for the Cepheid variables in visual bands is dominated by the temperature variation (Cox, 1980). This can be seen from the perfect blackbody curves from Figure 5.9. However, as the wavebands move toward the infrared, the radius variation begin to dominate the variation in the infrared PL relations (Madore, 1985). Since there is no obvious reason that the period-radius relation for Cepheids should be non-linear (see, e.g., Bono et al., 1998; Gieren

et al., 1998, 1999), one would expect that the PL relation is linear in infrared. From this study, we see that the PL relation is non-linear at VRJ & H-band, and marginally linear at K-band suggests that the radius variation begin to dominate at K-band and longer wavelength. This evidence can also be seen from the lower panel of Figure 5.8, where the extinction corrected ($J - K$) PC relation becomes linear. These findings also constrain that the non-linearity of the PL relations is due the temperature (or color) variations as function of period. Kanbur & Ngeow (2004) and Kanbur & Ngeow (2005) have proposed a possible mechanism which is responsible for the non-linearity of the PC relation (and hence the PL relation). However the detailed discussion and investigation of this mechanism is addressed in Chapter 9.

Figure 5.9 –The blackbody curves at two selected temperature. The vertical lines denote the central position of the bandpasses in wavelength.

CHAPTER 6

PERIOD-COLOR AND AMPLITUDE-COLOR RELATIONS IN THE CLASSICAL CEPHEID VARIABLES I: THE EMPIRICAL RESULTS

In this chapter we analyze the behavior of Galactic, LMC and SMC Cepheids in terms of period-color (PC) and amplitude-color (AC) diagrams at the phase of maximum, mean and minimum light. We find very different behavior between Galactic and Magellanic Cloud Cepheids. Motivated by recent report of a break in LMC PC relations at 10 days, we use the F -statistical test to examine the PC relations at mean light in these three galaxies. The results of the F -test support the existence of the break occurred in LMC PC(mean) relation, but not in the Galactic or SMC PC(mean) relation. Furthermore, the LMC Cepheids also show a break at minimum light, which is not seen in the Galactic and SMC Cepheids. We further discuss the effect on the period-luminosity relations in the LMC due to the break in the PC(mean) relation.

6.1 Purposes for Studying the Period-Color and Amplitude-Color Relations at Maximum, Mean and Minimum Light

Simon et al. (1993) suggested that the period-color (PC) and amplitude-color (AC) relations for the Cepheid variables are intimately connected. This can be derived using the Stefan-Boltzmann law, along with the fact that the radius fluctuations are small in the optical (Cox, 1980), applied at maximum and minimum light to show that:

$$\log T_{max} - \log T_{min} = \frac{1}{10}(V_{min} - V_{max}), \quad (6.1)$$

where T_{max} and T_{min} refer to the effective temperature at maximum and minimum light, respectively. If, for some reason, T_{max} is independent of periods (i.e., the PC relation at maximum light is flat) and the color used is a good predictor of temperature, then equation (6.1) predicts a relationship between V-band amplitude and T_{min} , and thus a correlation between the V-band amplitude and

the observed color (after extinction correction) at minimum V-band light. Thus, higher optical amplitudes are associated with higher temperature amplitudes, which are in turn related to higher values of T_{max} and/or T_{min} . In this chapter we do not assert a causal relation between higher temperatures and higher amplitudes preferring to refer to these quantities as being “associated” (however, see Chapter 8). So, from equation (6.1), the form of the PC relation at maximum light is related to the form of the AC relation at minimum light. Conversely, equation (6.1) suggests that if T_{min} obeyed a flat relation with period, then there should be a relation between amplitude and color at maximum light. Hence, changes in the PC relations should be reflected in corresponding changes in the AC relations. The motivation for Simon et al. (1993) to derive equation (6.1) is to explain the observational facts that the Galactic Cepheids show a spectral type independent of period at maximum light (Code, 1947). This, and the possible physics behind the flat PC relation at maximum light, will be discussed more in Chapter 8.

Furthermore, Tammann et al. (2003) used Galactic and OGLE LMC/SMC Cepheid data to show that there is a difference in the PC relations in these three galaxies. Also, Tammann & Reindl (2002) and Tammann et al. (2002) show the existence of two PC relations, one for short ($P < 10$ days) and one for long ($P > 10$ days) period Cepheids, in the LMC (see Chapter 1 & 5). Motivated by this and the presence of high quality Magellanic Cloud Cepheid data from the OGLE project (Udalski et al., 1999a), we decided to investigate the properties of Magellanic Cloud Cepheids in terms of their PC and AC relations at maximum, mean and minimum light.

In addition to the two major reasons mentioned above, we list a number of other arguments motivating the present study:

1. Since the mean light is just that - the average over a range of values - interesting properties of Cepheids at mean light are due to the average of these properties at all pulsation phases. By investigating the phases of maximum and minimum light, we are studying those phases of stellar pulsation which contribute to the observed properties at mean light. Our interest lies in understanding the breaks in the LMC mean light Cepheid PC and period-luminosity (PL) relations at 10 days as shown in previous chapter. Similar to equation (1.2), let y_{ij} be the (absolute) magnitude of Cepheid variable stars, $i = 1, \dots, N_{star}$ at the j^{th} phase ($j = 1, \dots, J$) during a pulsation period P_i . Then we can formulate a PL relation at a particular phase as,

$$y_{ij} = a_j \log(P_i) + b_j, \quad (6.2)$$

where a_j, b_j are the unknown coefficients as a function of the phase j . If we define y_i as $\sum_{j=1}^{j=J} y_{ij}/J$, the average over the pulsation period, it is easy to show that $y_i = a_m \log(P_i) + b_m$, where a_m, b_m are the average over phase of the a_j, b_j in equation (6.2)¹. This will be true if y is measured in intensities and then the intensity mean converted to magnitudes or if y is measured in magnitudes. Of course the magnitude mean and intensity means are in general not equal to each other but the difference is small ($\sim 0.03\text{mag.}$) and constant over a wide range of periods (for example, see Gieren et al. 1998). Consequently, some insight into the mean light relation can be gained by studying PL relations at various phases, e.g. at maximum and minimum light. Since the PC relation affects the PL relation (see, e.g., Madore & Freedman 1991 for the basic physics of PC and PL relations), it is of interest to study the PC relation at various phases. Besides this, the maximum and minimum light are closely associated with the more interesting phases of stellar pulsation: the expansion/contraction velocity is close to its maximum value when the photosphere is passing through the mean radius of the star (see, e.g., Mihalas 2003 for the details).

2. The amplitude is a fundamental observational and theoretical quantity in stellar pulsation. Kanbur, Nikolaev & Ngeow (in-preparation) show that the amplitude is a very good descriptor of light curve shape, and the V-band amplitudes are correlated to the first Principal Component (*PC1*) of the light curve. In addition, Kanbur et al. (2002) demonstrated that *PC1* can explain over 90% of the variation in light curve shape. Thus the V-band amplitude can be taken to be a good descriptor of V-band light curve shape. A similar conclusion holds for the I-band. Because the optical brightness fluctuations of Cepheids are predominantly due to temperature fluctuations (Cox, 1980), it is thus instructive to examine AC diagrams at maximum and minimum V- and light. Furthermore, since AC relations are related to PC relations through equation (6.1), their study can serve as a useful complement to strengthen any conclusions reached using PC relations.

6.2 The Galactic, LMC and SMC Data

The standard Johnson-Cousins V- and I-band data for the fundamental mode Cepheids in the Galaxy, LMC and SMC were used in this study. For both of the LMC and SMC Cepheids, the periods, V- and I-band photometric data and the $E(B-V)$ values for every Cepheid were taken from

¹Note that a_m may not lie in between a_{max} and a_{min} , the slopes at maximum and minimum light, respectively. A similar conclusion also hold for the zero-point, b_m .

the OGLE (Udalski et al., 1999b,c) website. There are 771 and 488 fundamental mode Cepheids (as classified by OGLE team) in LMC and SMC, respectively. For Galactic Cepheid data, we considered only the Cepheids classified as “DCEP” in the General Catalog of Variable Stars (GCVS, fourth edition, Kholopov et al. 1998). The periods for the Galactic Cepheids are taken from the McMaster Cepheid database², and the $E(B - V)$ values are adopted from Tammann et al. (2003, listed as $E(B - V)_{corr}$ in their table 1). The photometric data for Galactic Cepheids were obtained from two sources: (a) Moffett & Barnes (1984), where actual data were downloaded from the McMaster Cepheid database; and (b) Berdnikov (1997)³ and Berdnikov & Turner (2001). For Cepheids that have entries in both Berdnikov (1997) and Berdnikov & Turner (2001), photometric data from these two sources were merged together to provide a better light curve. Since the bandpasses used in Moffett & Barnes (1984) are in Johnson V and I, we converted the Johnson I-band photometric data in this dataset to Cousins I-band with the color transformations given in Coulson et al. (1985).

The Cepheid photometric data in these three galaxies were then fitted with a Fourier expansion as given in equation (2.2). We used a simulated annealing technique to fit the data with this Fourier expansion, as described in Chapter 2.2. Thus our Fourier fits are carried out to published photometric data and our means are magnitude means. For Galactic data, we adopted a fifth order expansion ($n = 5$) to most of the Cepheids. However, in certain cases a fourth or sixth order Fourier expansion gave a better fit to the data. For OGLE LMC Cepheids, we fit the data with $n = 4$, as this dataset is also used in Ngeow et al. (2003) and in Kanbur et al. (2003). For OGLE SMC Cepheids, we mainly fit fourth or fifth order Fourier expansions to the data, while some of them were fitted with sixth order. All fitted light curves were then visually inspected (see Kanbur et al. 2003 for the selection criteria). Figure 7 of Ngeow et al. (2003) illustrates the improvement that can be obtained when using our fitting method to the observed light curves. We only selected those Cepheids with well fitted light curves in *both* V- and I-bands. In addition, we exclude Cepheids with $\log(P) < 0.4$ to avoid possible contamination from first overtone Cepheids (Udalski et al., 1999a). The numbers of Cepheids in the final samples are: 79 from Moffett & Barnes (1984) data; 75 from Berdnikov data; 634 from OGLE LMC data; and 391 from OGLE SMC data. From the Fourier fits we obtained the following quantities:

1. V- and I-band amplitude from the numerical maximum and minimum of the Fourier fits:

$$V_{amp} = V_{min} - V_{max}, I_{amp} = I_{min} - I_{max}.$$

²<http://dogwood.physics.mcmaster.ca/Cepheid/HomePage.html>

³Note that the column of $R - I_c$ should be swapped with column of $V - I_c$ in this dataset.

Figure 6.1 –The period-color and amplitude-color relations at phases of maximum, mean and minimum light for Galactic Cepheids. The open circles and filled circles correspond to short and long period Cepheids, respectively. The dashed and solid lines are the fitted period-color and amplitude-color relations to short and long period Cepheids, respectively. The number of short and long period Cepheids are $N_{short} = 113$ and $N_{long} = 41$. Left (a): The period-color relations. Right (b): The amplitude-color relations.

2. $(V - I)_{max}$: defined as $V_{max} - I_{phmax}$, where I_{phmax} is the I-band magnitude at the same phase as V_{max} .
3. $(V - I)_{mean}$: defined as $V_{mean} - I_{phmean}$, where I_{phmean} is the I-band magnitude at the same phase as V_{mean} and V_{mean} is the V band magnitude closest to m_0 , the mean value in equation (2.2).
4. $(V - I)_{min}$: defined as $V_{min} - I_{phmin}$, where I_{phmin} is the I-band magnitude at the same phase as V_{min} .

Finally, the colors at these three phases have been corrected for extinction using $A_{V,I} = R_{V,I}E(B - V)$. The values of R are: $R_V = 3.17$, $R_I = 1.89$ for Galactic data (Tammann et al., 2003), and $R_V = 3.24$, $R_I = 1.96$ for OGLE LMC and SMC data Udalski et al. (1999b,c). In fact, the results are unchanged for the values of R_V and R_I as long as $\Delta R \equiv R_V - R_I = 1.28$ as given in Tammann et al. (2003), because $(V - I)_o = (V - I) - \Delta R E(B - V)$. We apply the same extinction values of $A_{V,I}$ to the colors at maximum, mean and minimum light, since the quantities of $A_{V,I}$ should remain unchanged for any pulsation phases. In addition, our results are unchanged if we adopt $(\langle V \rangle - \langle I \rangle)_o$ for the mean color as in item (iii) above.

Figure 6.2 –The period-color and amplitude-color relations at phases of maximum, mean and minimum light for OGLE LMC Cepheids. The open circles and filled circles correspond to short and long period Cepheids, respectively. The dashed and solid lines are the fitted period-color and amplitude-color relations to short and long period Cepheids, respectively. The number of short and long period Cepheids are $N_{short} = 585$ and $N_{long} = 49$. Left (a): The period-color relations. Right (b): The amplitude-color relations.

Figure 6.3 –The period-color and amplitude-color relations at phases of maximum, mean and minimum light for OGLE SMC Cepheids. The open circles and filled circles correspond to short and long period Cepheids, respectively. The dashed and solid lines are the fitted period-color and amplitude-color relations to short and long period Cepheids, respectively. The number of short and long period Cepheids are $N_{short} = 334$ and $N_{long} = 57$. Left (a): The period-color relations. Right (b): The amplitude-color relations.

6.3 Analysis and the Results from the Statistical Test

Figures 6.1, 6.2 and 6.3 show the plots of $\log(P)$ vs. $(V - I)_o$ color and V_{amp} vs. $(V - I)_o$ color at maximum, mean and minimum light (see the definition in Section 6.2) for the Cepheids in Galaxy, LMC and SMC respectively. Open and closed circles in these figures are for short ($\log[P] < 1.0$) and long ($\log[P] > 1.0$) period Cepheids, respectively. In what follows short and long period Cepheids are always used in this sense. For all three data sets in the Galaxy, LMC and SMC, we can fit linear relations between $\log(P)$ and $(V - I)_o$ color and V_{amp} and $(V - I)_o$ color, both for the entire sample, and also for short and long period Cepheids separately. Tables 6.1 and 6.2 show these results for the fits to period-color and amplitude-color relations, respectively. In these tables, column 1 gives the phase at which the fit is made - either at maximum, mean or minimum light. Column 2 is labelled MRS and represents the mean residual sum of squares from the fit to the entire sample. The slope of the linear fit together with its associated error is given in column 3. Columns 4 and 5 give the residual mean sum of squares and slope plus error for the short period Cepheids. Columns 6 and 7 do this for the longer period Cepheids. Column 8-11 will be discussed shortly. In Figures 6.1-6.3 we have plotted the fitted lines for short and long period Cepheids as solid and dashed lines respectively.

What is of interest for the present study is whether these PC plots show statistical evidence of a change of slope at a period of 10 days. To investigate this, we can fit a regression line over the entire period range and then fit two regression lines, one separately for short and long period Cepheids. The former case is the reduced model while the latter case with two separate regression lines is the full model (see Chapter 5 also). We can write the model under consideration as:

$$Y = b_S W_S + b_L W_L + a_S Z_S + a_L Z_L + \epsilon. \quad (6.3)$$

Here Y is the dependent variable, in our case $(V - I)_o$ color at any of the three phases under consideration. W_S is an indicator variable which is 1 if the star's period is less than 10 days and 0 otherwise. W_L is an indicator variable which is 0 if the star's period is less than 10 days and 1 otherwise. The variable Z_S contains the independent variable, either $\log(P)$ or V_{amp} , but is zero if the period is greater than 10 days. Z_L is similar but is zero for periods less than 10 days. The parameters b_S , b_L , a_S and a_L are the zero-points and slopes for short and long period Cepheids, respectively. Thus what is of interest is if the data are consistent with $a_L = a_S$: the slope is same

for long and short period Cepheids. Weisberg (1980) shows that in this situation an appropriate F -test statistic can be formulated as described in the following equation (see Chapter 5 also),

$$F = \frac{(RSS_R - RSS_F)/[(N - 2) - (N - 4)]}{RSS_F/(N - 4)}, \quad (6.4)$$

where RSS_R , RSS_F are the residual sums of squares in the reduced (single line regression) and full model (two lines regression), respectively, and n is the number of stars in the entire sample. We refer the F statistic in equation (6.4) to an F distribution, $F_{2,N-4}$, under the null hypothesis that the two-parameter regression (i.e. a single line) is sufficient. The four-parameter regression (i.e. two lines) will have a smaller residual sum of squares. The probability of the observed value of the F statistic, $p(F)$, under the null hypothesis, gives the significance of this reduction in sum of squares. Thus, a “large” value of F , hence small $p(F)$, indicates that the null hypothesis can be rejected. Column 8 of Tables 6.1 and 6.2 gives the values of the F statistic and column 9 gives its probability value as described above. Note that because there are fewer observed long period Cepheids, the error on the slope for the long period PC relations is generally larger but this is automatically taken account of by the F -test.

It could also be that the short and long period slopes are similar but that the F statistic produces a significant result because the intercepts are different. In this case, we can also compare the statistical significance of differences in the slopes of these regressions by referring the quantity,

$$t = (a_S - a_L) \frac{1}{S_{a_S - a_L}}, \quad (6.5)$$

$$S_{a_S - a_L} = \sqrt{\frac{MS}{SS_S} + \frac{MS}{SS_L}}, \quad (6.6)$$

$$MS = \frac{SS_S + SS_L}{N_S + N_L - 4} \quad (6.7)$$

to a t distribution with $N_S + N_L - 4$ degrees of freedom. $p(t)$ is the probability of the observed value of the t statistic under the null hypothesis that the two slopes are equal. In the above formulae SS_S , SS_L , N_S , N_L , a_S , a_L are the residual sum of squares, the number of Cepheids and the slopes in the PC or AC relation for short and long period Cepheids, respectively. Columns 10 and 11 in Tables 6.1 and 6.2 give the values of t statistic and its associated $p(t)$ value from two-tail t distribution.

Our figures do not contain error bars on the estimated values of the $(V - I)_o$ color. These error bars are typically less than the size of the symbol. This includes errors due to photometry

Table 6.1 –Period-color relations at maximum, mean and minimum light.

Phase (1)	MRS_A^a (2)	$Slope_A^b$ (3)	MRS_S^a (4)	$Slope_S^b$ (5)	MRS_L^a (6)	$Slope_L^b$ (7)	F (8)	$p(F)$ (9)	t (10)	$p(t)$ (11)
Galactic										
Max	0.009	0.142 ± 0.031	0.006	0.389 ± 0.062	0.013	0.020 ± 0.105	7.95	0.00	3.40	0.00
Mean	0.008	0.264 ± 0.030	0.006	0.229 ± 0.062	0.013	0.373 ± 0.104	1.05	0.35	1.34	0.18
Min	0.007	0.310 ± 0.028	0.005	0.330 ± 0.058	0.013	0.309 ± 0.104	0.06	0.94	0.21	0.83
OGLE LMC										
Max	0.010	0.205 ± 0.020	0.010	0.234 ± 0.030	0.008	-0.031 ± 0.101	2.67	0.07	2.24	0.03
Mean	0.009	0.239 ± 0.018	0.008	0.152 ± 0.027	0.007	0.590 ± 0.097	13.19	0.00	4.10	0.00
Min	0.008	0.291 ± 0.018	0.008	0.204 ± 0.027	0.009	0.493 ± 0.109	10.38	0.00	2.73	0.01
OGLE LMC										
Max	0.013	0.324 ± 0.021	0.013	0.396 ± 0.039	0.011	0.207 ± 0.071	3.16	0.04	2.23	0.03
Mean	0.008	0.272 ± 0.017	0.007	0.229 ± 0.030	0.012	0.340 ± 0.074	1.77	0.17	1.65	0.10
Min	0.007	0.276 ± 0.015	0.006	0.282 ± 0.027	0.010	0.229 ± 0.066	0.42	0.66	0.87	0.38

^a MRS denotes mean of residual sum of squares from the fit. MRS_A , MRS_S and MRS_L are for the fits to all, short and long Cepheids in the sample, respectively.

^b Slopes for the period-color relations for all, short and long period Cepheids in the sample.

Table 6.2 –Amplitude-color relations at maximum, mean and minimum light.

Phase (1)	MRS_A^a (2)	$Slope_A^b$ (3)	MRS_S^a (4)	$Slope_S^b$ (5)	MRS_L^a (6)	$Slope_L^b$ (7)	F (8)	$p(F)$ (9)	t (10)	$p(t)$ (11)
Galactic										
Max	0.010	-0.098 ± 0.042	0.007	-0.313 ± 0.058	0.012	-0.169 ± 0.088	15.54	0.00	1.50	0.14
Mean	0.010	0.241 ± 0.042	0.007	0.011 ± 0.059	0.014	0.275 ± 0.095	10.98	0.00	2.65	0.01
Min	0.009	0.322 ± 0.041	0.007	0.145 ± 0.058	0.013	0.276 ± 0.089	9.76	0.00	1.36	0.18
OGLE LMC										
Max	0.010	-0.285 ± 0.023	0.007	-0.393 ± 0.023	0.007	-0.160 ± 0.058	93.12	0.00	3.62	0.00
Mean	0.011	0.016 ± 0.025	0.008	-0.118 ± 0.024	0.008	0.322 ± 0.064	95.99	0.00	6.42	0.00
Min	0.011	0.140 ± 0.025	0.009	0.017 ± 0.025	0.009	0.321 ± 0.065	92.66	0.00	4.33	0.00
OGLE SMC										
Max	0.010	-0.438 ± 0.021	0.007	-0.419 ± 0.018	0.010	-0.308 ± 0.070	80.45	0.00	1.79	0.07
Mean	0.012	-0.158 ± 0.023	0.007	-0.144 ± 0.019	0.017	0.034 ± 0.092	75.40	0.00	2.54	0.01
Min	0.012	-0.065 ± 0.023	0.008	-0.047 ± 0.020	0.011	0.104 ± 0.076	82.45	0.00	2.21	0.03

^a MRS denotes mean of residual sum of squares from the fit. MRS_A , MRS_S and MRS_L are for the fits to all, short and long Cepheids in the sample, respectively.

^b Slopes for the amplitude-color relations for all, short and long period Cepheids in the sample.

and reddening. A typical V-band photometric error quoted by the OGLE team is 0.015mag. and perhaps as much as 0.05mag. for reddening. If we add these in quadrature, a rough estimate of sigma for the photometry is 0.052mag. If we assume a similar figure for the I-band (with error of $\sim 0.001\text{mag.}$ to 0.005mag.) and again add up the errors in quadrature, then a typical error for the $(V - I)_o$ color is $\sim 0.07\text{mag.}$ Strictly speaking this error should be included in a weighted least squares fit when constructing our period-color or amplitude-color fits. The weights are the inverse of the error assigned to each data point. However if the weights are constant for each star then they cancel out in the calculation of the F statistic and in the derivation of the coefficients of the linear regression. Hence the only way that inclusion of errors in our least squares fits can affect the significance of our F statistic results is if the errors are systematically greater for large numbers of either short or long period Cepheids. If this is the case, it will affect not only our work but also published values of the LMC PL relations in the V- and I-band (e.g., in Udalski et al. 1999a).

We now discuss the assumptions required by the F -test. These are the error (ϵ) in equation (6.3) is constant for all the stars in our study, and further they are normally distributed with zero mean. To test these assumptions, we analyze the OGLE LMC data in greater detail. Figure 6.4 shows the plots of residuals from a single PC fit and two PC fits to the entire data against $\log(P)$. We see no discernible trend in the size of the residuals with period and are led to the conclusion that the residuals are homoskedastic (i.e. $\sigma \sim \text{constant}$). This supports our approach of adopting a constant σ to gauge photometric errors when doing a least squares fit. In addition, the residuals in Figure 6.4 from the fit with a single PC relation show a trend (though not in the size of the residuals) in long period Cepheids, which is reduced when using two PC relations for short and long period Cepheids to fit the data. In order to test that the residuals are normally distributed, we can plot the quantiles of the distribution resulting from the ordered residuals against the expected quantiles from a normal distribution: a qq -plot. If the residuals are indeed normally distributed then this plot should be close to the line $y = x$. Figure 6.5 shows such a plot. There is some small departure from normality at the extremes but we contend that this plot justifies our use of the F -test.

There is a great deal of information in Table 6.1 & 6.2, but for the context of the present study we summarize the results in the following section.

Figure 6.4 –Plots of the residuals of LMC period-color relation at mean light. Top: (a) The residuals from the long and short period PC relations as a function of $\log(P)$. Bottom: (b) The residuals from using a single PC relation as function of $\log(P)$. From the plots it is clear that the residuals are homoskedastic. In addition, the residuals from the fit with a single PC relation show a trend in long period Cepheids. This trend is reduced if two PC relations were used to fit the data.

6.4 The Period-Color and Amplitude-Color Relations

6.4.1 Results of the Period-Color Relations

For the Galactic Cepheids, the slope of the long period PC relation at maximum light is close to zero. Further, the overall slope at maximum light is the shallowest. This then supports the work of Code (1947). In addition, the F - and t -test indicates that the PC relation at maximum light is not consistent with a single slope. However, the mean and minimum light PC relations for the Galactic Cepheids are consistent with a single line.

For the LMC Cepheids, the slopes of the long period PC relation at maximum light is also close to zero, decreasing significantly from its value for short period Cepheids. The slope at mean and minimum light all increase when going from short to long period Cepheids. In the LMC, we see that the PC relation at maximum light is not consistent with a single line at the 93% confidence level.

Figure 6.5 –Plot of the ordered residual quantiles vs. normal quantiles, the *qq*-plot. The line is the case of $y = x$.

However, in terms of the F -test, the PC relation at mean and minimum light does show evidence to support a break at 10 days. Note that at both these phases, the slope of the PC relation for longer period Cepheids is much steeper than for shorter period Cepheids.

In the case of the SMC again the PC relation at maximum light is much flatter for longer period Cepheids than for shorter period. The relation at mean light for longer periods is steeper than at shorter period. Only the PC relation at maximum light shows (marginal) statistical evidence of a break at 10 days.

We note that the slope of PC relation at maximum light always decreases in going from short to long period Cepheids in all three galaxies. In contrast, the slope at mean and minimum light always increases except for the Galactic and SMC Cepheids at minimum light. We also make the interesting observation that the dispersion of the PC relation, whether for the entire sample or either short or long period group, is always the smallest at minimum light in these three galaxies. A glance at the F statistic for mean light for the LMC Cepheids, we reject the null hypothesis of

a single PC relation for long and short period LMC Cepheids at greater than the 99% confidence level, confirming the finding of the broken PC relation in LMC (Tammann et al., 2002; Tammann & Reindl, 2002). However, the Galactic and SMC mean light PC relation is consistent with one line.

6.4.2 Results of the Amplitude-Color Relations

The AC relations all show statistical evidence of two lines at greater than 99.9% confidence level with the F -test. At mean and minimum light, we see that the slopes of the AC relation increase significantly when going from short to long period Cepheids in all three galaxies. However at maximum light the slope becomes shallower for long period Cepheids. Also, the slopes of the AC relation at maximum light are always negative for both short and long periods Cepheids in Galaxy, LMC and SMC, which we do not see in the mean and minimum light. In addition, the slopes are flat for the short period Galactic Cepheids and long period SMC Cepheids at mean light, but not for the Cepheids in LMC.

We also remark on some important differences in the AC plane between Galactic and Magellanic Cloud Cepheids. At minimum light in the LMC and SMC, the slope of the AC relation is 0.017 ± 0.025 and -0.047 ± 0.020 , respectively, for short period Cepheids, whereas it is 0.145 ± 0.058 for Galactic Cepheids - more than 2σ difference. For longer period Cepheids in the LMC and SMC, this slope becomes positive (0.321 ± 0.065 and 0.104 ± 0.076), whilst the longer period AC slope in the Galaxy is always significantly above zero.

Even though Table 6.2 shows that AC relations are significantly different for short and long period Cepheids in the SMC, Figure 6.3(b) suggests that the effect is much reduced in the SMC as compared to, e.g. Figure 6.1(b). One possible reason for this is that SMC Cepheids gave lower amplitudes than LMC or Galactic Cepheids (Paczynski & Pindor, 2000). Equation (6.1) shows that even if the range of T_{max} or T_{min} is narrow, but if the amplitudes are not large, then T_{min} or T_{max} will not be driven to such low or high values.

6.4.3 Combining the PC and AC Relations

As discussed in Introduction, equation (6.1) predicts that if the PC relations is flat at maximum light, then there will be an AC relation at minimum light, and vice versa. We see some evidence to support this idea from Figure 6.1-6.3, and from Table 6.1 & 6.2. In summary:

- Galactic short period Cepheids: PC relation steep at maximum $\rightarrow$ AC $\sim$ flat/shallow at minimum.
- Galactic long period Cepheids: PC relation flat at maximum $\rightarrow$ AC relations $\sim$ steep at minimum.

PC relation steep at minimum $\rightarrow$ AC relation $\sim$ flat/shallow at maximum.

- LMC short period Cepheids: PC relation $\sim$ steep at maximum $\rightarrow$ AC relation flat at minimum.
- LMC long period Cepheids: PC relation flat at maximum $\rightarrow$ AC relations steep at minimum; PC relation steep at minimum $\rightarrow$ AC relation $\sim$ flat/shallow at maximum.
- SMC short period Cepheids: PC relation steep at maximum $\rightarrow$ AC relation $\sim$ flat/shallow at minimum.

6.5 The LMC PL Relations at Maximum, Mean and Minimum Light

Since the PL relation is affected by the PC relation (Madore & Freedman, 1991), it is of interest to examine the PL relation in the OGLE LMC Cepheids, which show a break in PC relation. Do the LMC data support a single PL relation or is the data consistent with a break at a period of 10 days as shown in Tammann & Reindl (2002)? We can use the F -test to investigate the LMC PL relations in the V- and I-band as in Section 6.3. The results of the F -test are shown in Table 6.3, where column 1 displays the phase and column 2 and 3 show the F statistic and its probability, under the null hypothesis of a single line, for the V-band PL relations. Similar results for I-band PL relations are listed in column 4 and 5 in Table 6.3. From the table, both V- and I-band PL relations at mean and minimum light are significant - that is the data are more consistent with a model where the slopes of the V- and I-band PL relation are different for short and long period Cepheids. However, the data are consistent with a single slope, in both V- and I-bands, for the PL relation at maximum light, even though the corresponding PC relation shows some evidence of a break at 10 days (see Table 6.1).

The actual slopes of the V- and I-band PL relation at maximum, mean and minimum light for short and long period Cepheids are given in Table 6.4. The second column in this table gives the overall slope. The third and fourth column give the short period and long period slopes, respectively. We clearly see that the slope at maximum light is virtually identical for short and long period Cepheids in both V- and I-bands. This is very different to the situation at mean and minimum light. How can the PC relation at maximum light show evidence of a break at $\log(P) = 1.0$ yet the PL relation in V- and I-band be consistent with one PL relation? This is certainly not the case for the PL relation at mean light in the LMC. This occurs because the LMC PC relation at mean light becomes steeper for long period Cepheids whereas at maximum light, the PC relation in the LMC becomes shallower for longer period Cepheids. Since the PC relation affects the PL relation, this shallow dependence at maximum light for longer period Cepheids suggests that the PL relation at that phase would not be affected too much.

Table 6.3 – F -test significance results for the LMC PL relation.

phase	$F_{V\ band}$	$P(F)_{V\ band}$	$F_{I\ band}$	$P(F)_{I\ band}$
Max	0.32	0.73	0.04	0.96
Mean	8.86	0.00	6.25	0.00
Min	17.5	0.00	19.0	0.00

Table 6.4 –Slopes of the V- and I-band PL relation at maximum, mean and minimum light for Cepheids in the LMC.

Phase	V slope (all)	V slope (short)	V slope (long)
Max	-2.74 ± 0.05	-2.73 ± 0.08	-2.97 ± 0.26
Mean	-2.75 ± 0.04	-2.95 ± 0.07	-2.35 ± 0.25
Min	-2.59 ± 0.04	-2.85 ± 0.06	-1.84 ± 0.26
Phase	I slope (all)	I slope (short)	I slope (long)
Max	-2.95 ± 0.03	-2.96 ± 0.05	-2.94 ± 0.19
Mean	-2.99 ± 0.03	-3.10 ± 0.04	-2.94 ± 0.18
Min	-2.88 ± 0.03	-3.06 ± 0.04	-2.33 ± 0.17

6.6 Conclusion and Discussion for this Chapter

We have presented in Figures 6.1-6.3 new observational characteristics of Cepheids. Following the work of Tammann & Reindl (2002), we apply rigorous statistical tests to show that at mean light, there is a change in the LMC PC relation between short ($\log[P] < 1.0$) and long period Cepheids, as shown in Figure 6.2(a). In addition, the LMC data also exhibit a change in the PC relation at minimum light. However, we find no such change at mean and minimum light in the Galactic and SMC Cepheids. We also find that the PC relations at minimum light generally show the smallest scatter among the three galaxies, as compare to the phases at maximum or mean light. We also study the amplitude-color diagrams at maximum, mean and minimum light and find very different behavior in the three galaxies for short and long period Cepheids. This different not only occurs within a given galaxy between the short and long period Cepheids, but also occurs from galaxies to galaxies. For example, the behavior of short and long period Cepheids in LMC is very different (i.e. Figure 6.2), and the behavior of Galactic and LMC Cepheids of short period is also very different. Thus we note that there is an effect with both period and metallicity. In addition, the PC relations clearly show greater structure than a simple two lines regression as used here. For example, the LMC data indicates another break at $\log(P) \sim 1.2$ (see, e.g., Figure 6.4). This will be the subject of a future paper.

We discuss some of the systematic effects that may affect our results, as follows:

1. Could reddening errors produce the results displayed in Figures 6.1-6.3? Consider the bottom panels of Figure 6.1(b) and 6.2(b) showing AC relations at minimum light for the Galaxy and

Table 6.5 – F -test significance results with color cuts for LMC Cepheids.

$(V - I)_o$ Range	F Max	$P(F)$ Max	F Mean	$P(F)$ Mean	F Min	$P(F)$ Min
0 - 1.9	2.67	0.07	13.2	0.00	10.4	0.00
0 - 1.1	2.95	0.05	14.0	0.00	10.7	0.00
0 - 1.0	4.45	0.01	10.4	0.00	7.86	0.00
0 - 0.9	3.34	0.04	7.69	0.00	6.38	0.00
0.3 - 1.9	2.67	0.07	13.2	0.00	10.4	0.00
0.4 - 1.9	2.48	0.08	14.8	0.00	12.2	0.00
0.5 - 1.9	2.41	0.09	16.3	0.00	13.0	0.00
0.5 - 0.9	3.16	0.04	10.4	0.00	8.76	0.00

LMC. In contrast to the Galactic counterpart, the AC relation for short period LMC Cepheids is flat. It is difficult to imagine a situation where the published reddening for these short period stars are in error to the extent of making this relation flat when it should have a slope like the Galactic relation. Furthermore, we use the $E(B - V)$ values from the literature, which would not only affect our results but also other published results that are based on these values.

2. Could outliers, perhaps stars that have been mis-classified as Cepheids, produce our significant results? We examine the OGLE LMC Cepheid data to investigate this question. The plot of mean $(V - I)_o$ color against $\log(P)$ in the middle panel of Figure 6.2(a) shows a number of stars which seem to have very red colors. We exclude these stars and repeat our analysis for PC relations in the LMC. We note that it is possible to reduce the significance of our result at mean light by excluding stars that are too red. However if we exclude stars that are too blue, the F test again becomes significant. This conclusion can be seen in Table 6.5, where we apply various color cuts to the LMC data and then calculate the F -test. Excluding the Cepheids with too red or blue color does not alter the results we have in Section 6.4.1.
3. Could we reduce the significance of our results by “judiciously” removing certain stars? Since we only consider stars with $\log(P) > 0.4$, if we extend this cutoff to 0.6 ($P = 3.98$ days) and repeat our analysis, we get very similar results. Also, excluding some of the longer period Cepheids, those with periods greater than $\log(P) > 1.4$ increases the significance of our results. Hence the period cut we applied in this study would not significantly alter the results or conclusions we have.

While excluding stars based on color or period cuts as mentioned above, in some cases, can reduce the significance of our F -test results at mean light (though never to a $p[F]$ value less than 0.1), such experiments have very little effect at minimum light. If we accept the assertion that

mean light properties of Cepheids are an average of Cepheid properties at all pulsation phases, then irrespective of mean light, Tables 6.1-6.2 and Figures 6.1-6.3 clearly demonstrate a significant change in Cepheid observational properties between short and long period Cepheids in the Galaxy, LMC and SMC. Further if we accept these exclusions, they would also have an effect on the PL relations in both V- and I-bands and hence on the currently accepted extra-galactic distance scale.

We now briefly summarize other implications from the present study. However, the detailed analysis of these topics are beyond the scope of this Thesis Dissertation, and will be addressed in future papers.

1. **Metallicity dependence on the PC relation:** From the period-mean density relation for a pulsator and the Stefan-Boltzmann law, it is easy to show that: $\log(P) = \alpha \log(L) + \beta \log(M) + \gamma \log(T_{eff}) + \text{const.}$ The parameters (α, β, γ) and the constant term can be determined from stellar pulsation calculations. For example:

$$\log(P) = 0.84 \log(L) - 0.68 \log(M) - 3.48 \log(T_{eff}) + \text{const.}, \text{ from van Albada \& Baker (1973).}$$

$$\log(P) = 0.93 \log(L) - 0.77 \log(M) - 3.54 \log(T_{eff}) + \text{const.}, \text{ from Simon \& Young (1997).}$$

$$\log(P) = 0.82 \log(L) - 0.60 \log(M) - 3.55 \log(T_{eff}) + \text{const.}, \text{ from Beaulieu et al. (2001).}$$

In addition, the luminosity and mass should also obey an M-L relation from stellar evolution calculations. This M-L relation predicts that lower metallicity Cepheids will have higher luminosity (see, e.g., Bono et al. 2000c) for given mass. Hence, at fixed period, lower metallicity Cepheids should be hotter than higher metallicity Cepheids. For example, Laney & Stobie (1986) showed that the long period SMC Cepheids are hotter than LMC Cepheids at given period by $\sim 200K$.

Since the Magellanic Clouds (MC) have lower metallicity than the Galaxy, Cepheids in the MC should be bluer than Galactic Cepheids. Tammann et al. (2003) have reported this finding in their paper. In this study, we also found that the short period MC Cepheids generally have bluer $(V - I)_o$ color than Galactic Cepheids, for a given period, at maximum, mean and minimum light. However, larger errors in the PC relations for long period Cepheids make such a statement more contentious for long period Cepheids. We intend to investigate this in detail in a future paper.

2. **Estimation of color excess:** Fernie (1994) used the theory of Simon et al. (1993) and the original suggestion of Code (1947) to establish a relation between $(B - V)$ color at maximum

light, the V-band amplitude, period and the color excess $E(B - V)$ for Galactic Cepheids. Figure 6.1 shows that another approach would be using the properties of Cepheids at minimum light to estimate the color excess, since the PC relations at minimum light in generally show a smaller dispersion (see Section 6.4.1). Furthermore, the Galactic PC relation at maximum light indicates a break at 10 days (Table 6.1), but not in the PC relation at minimum light. In future work, we plan to investigate the possibility of using this relation as a way to determine reddening.

CHAPTER 7

THE LINEARITY OF THE WESENHEIT FUNCTION: IMPLICATION OF THE NON-LINEAR LMC PL RELATION IN DISTANCE SCALE APPLICATION

There is strong evidence that the period-luminosity (PL) relation for the Large Magellanic Cloud (LMC) Cepheids shows a break at a period around 10 days. Since the LMC PL relation is extensively used in distance scale studies, the non-linearity of the LMC PL relation may affect the results based on this LMC calibrated relation. In this chapter we show that this problem can be remedied by using the Wesenheit function in obtaining Cepheid distances. This is because the Wesenheit function is linear although recent data suggests that the PL and the period-color (PC) relations that make up the Wesenheit function are not. We test the linearity of the Wesenheit function and find strong evidence that the LMC Wesenheit function is indeed linear. This is because the non-linearity of the PL and PC relations cancel out when the Wesenheit function is constructed. We discuss this result in the context of distance scale applications. We also compare the distance moduli obtained from $\mu_0 = \mu_V - R(\mu_V - \mu_I)$ (equivalent to Wesenheit functions) constructed with the linear and the broken LMC PL relations, and find that the typical difference in distance moduli is $\sim 0.03\text{mag}$. Hence, the broken LMC PL relation does not seriously affect current distance scale applications.

7.1 Non-Linearity of the PL Relation and the Distance Scale Application

Cepheid period-luminosity (PL) relation plays a major role in distance scale studies, which can ultimately be used to determine the Hubble constant. The calibrating PL relation currently used is mainly based on the Large Magellanic Cloud (LMC) Cepheids (see, e.g., Freedman et al., 2001; Saha et al., 2001b; Kanbur et al., 2003). The Cepheid PL relation has long been considered to be a linear function of $\log(P)$, where P is the pulsation period in days. However, the LMC PL relation is shown to be non-linear (Tammann & Reindl, 2002; Kanbur & Ngeow, 2004; Sandage et al., 2004; Ngeow et al., 2005) and consist of two PL relations, one for the long ($P > 10\text{days}$) and another for the short period Cepheids (see Chapter 5 for more details). This is illustrated in the lower panels of Figure

7.1 for the extinction corrected V-band LMC PL relation. In Tables 7.1 & 7.2, we collect the slopes and the zero-points (ZP) for the long ($\log[P] > 1.0$) and short period optical PL relations that are available in the literature¹, respectively. For completeness, we also include the linear, unbroken PL relation obtained using all Cepheids in these tables. The slopes and ZPs from different chapters are consistent and agree with each other. Note that these LMC PL relations have been corrected for reddening. The detailed investigation of the physics behind the broken LMC PL and PC relation is beyond the scope of this chapter (part of these are presented and discussed in Chapter 9), but it is of great interest for the studies of stellar pulsation and evolution.

Figure 7.1 –Comparison of the Wesenheit function with the extinction corrected V-band PL relation for the LMC Cepheids. On the top panels, we plot the Wesenheit function for the LMC data, which clearly show that the Wesenheit function is linear. We fit the data with a single line regression, using the least-squares method, as indicated on the figures. On the bottom panels, we show the extinction corrected V-band PL relations, and separate the data into the short (open circles) and long period (filled circles) Cepheids. The fitted PL relations for long, short and all period Cepheids are drawn as dashed, solid and dotted lines, respectively. We extend the short period PL relation to the longer period ranges in order to compare with the long period PL relations. The data in the left and right panels are taken from Kanbur & Ngeow (2004) and Kanbur et al. (2005), respectively.

The existence of two LMC PL relations suggests that in future distance scale studies, the appropriate LMC PL relation may need to be applied to the long and short period Cepheids, respectively (see, e.g., Kanbur et al., 2003). However, all of the previous applications of the LMC PL relation were based on the linear version (in a sense, it is an approximation of two PL relations). Hence an

¹The I-band PL relations from Kanbur & Ngeow (2004) are fitted with slightly different definition of the I-band mean magnitudes. Hence we refit the I-band PL relations with the conventional and reddening corrected I-band mean magnitudes. The results from the F -test remain unchanged.

Table 7.1 –Slopes for the LMC PL relations^a. Long and short periods are referred to $P > 10$ and $P < 10$ days, respectively.

Band	Period-Range	TR02	STR04	KN04	KNB05
V	All	-2.760 ± 0.031^b	-2.702 ± 0.028	-2.746 ± 0.043	-2.736 ± 0.036
V	Short	-2.86 ± 0.05	-2.963 ± 0.056	-2.948 ± 0.065	-2.937 ± 0.060
V	Long	-2.48 ± 0.17	-2.567 ± 0.102	-2.350 ± 0.252	-2.598 ± 0.161
I	All	-2.962 ± 0.021^b	-2.949 ± 0.020	-2.965 ± 0.028	-2.965 ± 0.024
I	Short	-3.03 ± 0.03	-3.099 ± 0.038	-3.096 ± 0.043	-3.090 ± 0.041
I	Long	-2.82 ± 0.13	-2.822 ± 0.084	-2.737 ± 0.179	-2.918 ± 0.112
B	All	...	-2.340 ± 0.037	...	...
B	Short	-2.42 ± 0.08	-2.683 ± 0.077	...	...
B	Long	-1.89 ± 0.62^c	-2.151 ± 0.134	...	...

^a The references are: TR02 = Tammann et al. (2002); STR04 = Sandage et al. (2004); KN04 = Kanbur & Ngeow (2004); KNB05 = Kanbur et al. (2005)

^b Since Tammann et al. (2002) does not give the results from the fit to all Cepheids, we adopted the slopes from Udalski et al. (1999a) because the same dataset is used in both papers.

^c The number of long period Cepheids in B -band is ~ 13 , hence the error is larger than the others.

Table 7.2 –Same as Table 7.1, but for the zero-points of the LMC PL relations, by assuming $\mu_{LMC} = 18.50\text{mag}$.

Band	Period-Range	TR02	STR04	KN04	KNB05
V	All	-1.458 ± 0.021^a	-1.451 ± 0.022	-1.401 ± 0.030	-1.412 ± 0.025
V	Short	-1.40 ± 0.03	-1.295 ± 0.036	-1.284 ± 0.041	-1.295 ± 0.038
V	Long	-1.75 ± 0.20	-1.594 ± 0.135	-1.795 ± 0.298	-1.523 ± 0.198
I	All	-1.942 ± 0.014^a	-1.896 ± 0.015	-1.889 ± 0.019	-1.890 ± 0.017
I	Short	-1.90 ± 0.02	-1.806 ± 0.024	-1.813 ± 0.027	-1.817 ± 0.026
I	Long	-2.09 ± 0.15	-2.044 ± 0.111	-2.109 ± 0.212	-1.909 ± 0.138
B	All	...	-1.160 ± 0.029	...	...
B	Short	-1.18 ± 0.05	-0.955 ± 0.049	...	...
B	Long	-1.65 ± 0.74^b	-1.364 ± 0.177	...	...

^a Since Tammann et al. (2002) does not give the results from the fit to all Cepheids, we adopted the zero-points from Udalski et al. (1999a) because the same dataset is used in both papers.

^b The number of long period Cepheids in B -band is ~ 13 , hence the error is larger than the others.

immediate question that arises is: how does the existence of two LMC PL relations affect previous studies? (A similar question was also asked by Feast 2003, see Chapter 1.2.1) In this chapter we show that if the Cepheid distance to a galaxy is derived using the Wesenheit function (or the equivalent μ_0 in equation [7.4]), for example in the H_0 Key Project (Freedman et al., 2001, and references therein) or in the Araucaria Project (Gieren et al., 2004; Pietrzyński et al., 2004), then the results might not be affected (see Section 7.3 for details). This is because the Wesenheit function is linear, even though the PL and PC relations that make up the Wesenheit function are not, as shown in Section

7.2.1. Given that the LMC PL relation is not linear, and the Wesenheit function has been applied in many published papers, we would like to examine the effect of broken LMC PL and PC relations on the application of the Wesenheit function to the distance scale. This will be examined mainly in the optical bands because the V- and I-band LMC PL relations have been frequently applied in the literature (as in, e.g., Freedman et al., 2001).

7.2 The Wesenheit Function

The Wesenheit function (e.g., see Freedman, 1988; Freedman et al., 1991; Groenewegen & Oudmaijer, 2000; Madore, 1976, 1982; Madore & Freedman, 1991; Moffett & Barnes, 1986; Opolski, 1983; Tanvir, 1997; Udalski et al., 1999a) is defined as $W = \text{magnitude} - R \times \text{color}$, where R is the ratio of total-to-selective absorption that has to be adopted. Note that the definition of W depends on the adopted R which may be different in different (interstellar) environments. A few variations of W used in the literature with different combinations of magnitudes and colors are:

$$W_V^{BV} = V - R(B - V), \quad R = A_V/E(B - V), \quad (7.1)$$

$$W_V^{VI} = V - R(V - I), \quad R = A_V/E(V - I), \quad (7.2)$$

$$W_I^{VI} = I - R(V - I), \quad R = R_I, \quad (7.3)$$

where B , V & I denote the (intensity) mean magnitudes. Similar definitions of W in near infrared bands can be found, for example, in Persson et al. (2004). The other definition of W , as given by van den Bergh (1975), is different than the one given in equation (7.1). The van den Bergh version of W replaces R by the slope of the constant-period line in the color-magnitude diagram (CMD). Madore & Freedman (1991, also in Moffett & Barnes 1986) have pointed out some problems with the van den Bergh version and the advantage of using R in the definition of W . The biggest advantage of using W is that it is reddening-free (see, for example, Madore & Freedman 1991), i.e. $W = V - R(B - V) = V_0 - R(B - V)_0 \equiv W_0$, where V_0 and $(B - V)_0$ denote the intrinsic visual magnitude and color, as the effect of interstellar extinction on the observed magnitude and color cancel out for a star (not only for Cepheids). Another advantage of using W is that the scatter in the W - $\log(P)$ plot is reduced (Madore, 1982; Madore & Freedman, 1991; Böhm-Vitense, 1997; Tanvir, 1997, 1999; Udalski et al., 1999a), as compared to the scatter in the V- or I-band PL relations (see Figure 7.1). The remaining scatter is due to the combination of photometric errors and the finite width of the instability strip (for example, see Brodie & Madore, 1980; Madore & Freedman,

1991; Böhm-Vitense, 1997; Gieren et al., 1998). Furthermore, an equivalent definition of W with the combination of absolute magnitude and color can be formulated, as $W_M = M_V - R(M_V - M_I)$, then the distance modulus can be obtained, i.e. $\mu_W = W - W_M$. It is straight forward to show that the following equation:

$$\mu_0 = \mu_V - R(\mu_V - \mu_I) \quad (7.4)$$

is equivalent to μ_W , where equation (7.4) is frequently applied in determining the extra-galactic Cepheid distances (see, for example, Allen & Shanks, 2004; Freedman et al., 2001; Kanbur et al., 2003; Saha et al., 2001b; Tanvir et al., 1999). Therefore, using equation (7.4) to obtain the distance is equivalent to obtaining the distance by fitting the W - $\log(P)$ plane with the W_M - $\log(P)$ relation.

7.2.1 Testing the Linearity of the Wesenheit Function

Both the Wesenheit function and equation (7.4) can be written as a combination of V- and I-band PL relations, and they are adopted from the LMC PL relations to derive distances. However, as we mentioned in Section 7.1, the LMC PL relations are not linear, hence the applicability of the Wesenheit function and equation (7.4) is immediately in question. In this sub-section we would like to test the linearity of the Wesenheit function as follows.

The PL relation in bandpass λ can be written as: $M_\lambda = b_\lambda^X + a_\lambda^X \log(P)$. The superscript X denotes the adopted period range, which is either for short (S , $\log[P] < 1.0$), long (L , $\log[P] > 1.0$) or all (A , short+long) periods. Then the V -($V-I$) Wesenheit function becomes (similar expressions can be derived for other magnitude-color combinations):

$$W^X = (1-R)b_V^X + Rb_I^X + [(1-R)a_V^X + Ra_I^X] \log(P), \quad (7.5)$$

The linearity of W demands that:

$$(1-R)a_V^S + Ra_I^S = (1-R)a_V^L + Ra_I^L \quad (\text{for slope}), \quad (7.6)$$

$$(1-R)b_V^S + Rb_I^S = (1-R)b_V^L + Rb_I^L \quad (\text{for ZP}). \quad (7.7)$$

Figure 7.2 –The residuals of W from Figure 7.1. The dashed line indicates the zero residuals. The residuals are scattered around the dashed lines, indicating that the Wesenheit function is linear.

By using the slopes in Table 7.1, we can calculate the values of the left-hand side and right-hand side in equation (7.6), as well as the slope for W^A by using the unbroken PL relation with equation (7.5). The results are summarized in Table 7.3 with different magnitude-color combinations. The adopted values for R in these combinations are: $R = A_V/E(B - V) = 3.24$ (Udalski et al., 1999b); $R = A_V/E(V - I) = 2.45$ (Freedman et al., 2001; Tanvir & Boyle, 1999); and $R = R_I = 1.55$ (Udalski et al., 1999a). The errors in Table 7.3 are estimated with the standard formula for propagation of errors, i.e. $\sigma^2 = (1 - R)^2 \sigma_{a_V}^2 + R^2 \sigma_{a_I}^2$. The same is done for the ZP in Table 7.4.

It can be seen immediately from Table 7.3 & 7.4 that the short period Wesenheit function is consistent with the long period Wesenheit function, as demanded by equation (7.6) & (7.7). Therefore, the Wesenheit function can be regarded as a linear function of $\log(P)$. Furthermore, the short and long period Wesenheit functions are also consistent with the Wesenheit function obtained from using all Cepheids in the LMC or the linear, unbroken PL relation. Note that the value of ~ -3.3 for the slope of the Wesenheit function (a_W) with I -($V - I$) combination also agrees with the

values given in Udalski et al. (1999a, $a_W = -3.277 \pm 0.014$) or in Udalski (2000, $a_W = -3.300 \pm 0.011$). The linearity of the Wesenheit function can be immediately seen from the top panels of Figure 7.1. The residuals of the W from the fitted regressions in Figure 7.1 are also plotted out as function of period in Figure 7.2. If the Wesenheit function is non-linear and can be broken into long and short period Wesenheit functions, as in the LMC PL or PC relations, then the residual plots are expected to show a trend for the long period Cepheids (as in the figure 4 from Kanbur & Ngeow 2004 for the PC relation). However, there is no obvious trend of the residuals seen from Figure 7.2. This further supports the linearity of the Wesenheit function.

Table 7.3 –Comparison of the slopes for W obtained using the Cepheids with short, long and all periods.

Dataset	Short	Long	All
$V-(V-I)$ Combination, $R = 2.45$.			
TR02	-3.276 ± 0.103	-3.313 ± 0.403	-3.255 ± 0.068
STR04	-3.296 ± 0.124	-3.192 ± 0.253	-3.307 ± 0.064
KN04	-3.311 ± 0.141	-3.298 ± 0.571	-3.283 ± 0.093
KNB05	-3.312 ± 0.133	-3.382 ± 0.360	-3.297 ± 0.079
$I-(V-I)$ Combination, $R = 1.55$.			
TR02	-3.293 ± 0.109	-3.347 ± 0.423	-3.275 ± 0.072
STR04	-3.310 ± 0.130	-3.217 ± 0.266	-3.332 ± 0.067
KN04	-3.325 ± 0.149	-3.337 ± 0.601	-3.304 ± 0.098
KNB05	-3.327 ± 0.140	-3.414 ± 0.379	-3.320 ± 0.083
$V-(B-V)$ Combination, $R = 3.24$.			
TR02	-4.286 ± 0.335	-4.392 ± 2.134	...
STR04	-3.870 ± 0.344	-3.915 ± 0.613	-3.875 ± 0.169

Table 7.4 –Same as Table 7.3, but for the zero-points.

Dataset	Short	Long	All
$V-(V-I)$ Combination, $R = 2.45$.			
TR02	-2.625 ± 0.066	-2.583 ± 0.468	-2.644 ± 0.046
STR04	-2.547 ± 0.079	-2.696 ± 0.335	-2.541 ± 0.049
KN04	-2.580 ± 0.089	-2.564 ± 0.676	-2.597 ± 0.064
KNB05	-2.574 ± 0.084	-2.469 ± 0.444	-2.583 ± 0.055
$I-(V-I)$ Combination, $R = 1.55$.			
TR02	-2.675 ± 0.069	-2.617 ± 0.492	-2.692 ± 0.048
STR04	-2.598 ± 0.083	-2.741 ± 0.352	-2.586 ± 0.051
KN04	-2.633 ± 0.094	-2.596 ± 0.711	-2.645 ± 0.067
KNB05	-2.626 ± 0.089	-2.507 ± 0.467	-2.631 ± 0.058
$V-(B-V)$ Combination, $R = 3.30$.			
TR02	-2.113 ± 0.206	-2.074 ± 2.543	...
STR04	-2.397 ± 0.220	-2.339 ± 0.810	-2.394 ± 0.132

A better and more sophisticated test of the linearity is using the F -test (Weisberg, 1980; Kanbur & Ngeow, 2004, see also Chapter 5 & 6). The null hypothesis in our F -test is that a single linear regression is sufficient, while the alternate hypothesis is that two linear regressions are needed to

describe the data. These regressions can be obtained with the standard least squares regression method. The setup and the formalism for the F -test is given in Chapter 6. By calculating the F -value with N data points (note that F is a function of N), we can obtain the probability, $p(F)$, under the null hypothesis, for the significance of the F -value. Recall that, in general, the null hypothesis can be rejected if F is large (e.g., $F > 4$) or $p(F)$ is small. For example, $p(F) < 0.05$ indicates that the null hypothesis can be rejected at 2σ level. By using the 634 Cepheids as given in Kanbur & Ngeow (2004), we found:

$$F = 1.473, p(F) = 0.230 \text{ for } W = V - 2.45(V - I),$$

$$F = 0.835, p(F) = 0.432 \text{ for } W = I - 1.55(V - I).$$

Similarly, for the 636 Cepheids in Kanbur et al. (2005) the results from the F -test are:

$$F = 1.840, p(F) = 0.160 \text{ for } W = V - 2.45(V - I),$$

$$F = 1.753, p(F) = 0.174 \text{ for } W = I - 1.55(V - I).$$

Plots of the $W = V - 2.45(V - I)$ as function of period for these two datasets are shown in the upper panels of Figure 7.1. The relatively small values of F and the large values of $p(F)$ indicate that the null hypothesis cannot be rejected. Hence the results from the F -test strongly suggest that the Wesenheit function is linear. The results *with* the extinction corrections (as given in Udalski et al. 1999b) or the removal of the obvious outliers in upper panels of Figure 7.1 and in Figure 7.2 are essentially the same.

By combining the results obtained from this sub-section, we found that, statistically, the Wesenheit function for the LMC Cepheids is indeed linear, although the LMC PL and PC relations are not. Figure 7.1 shows a comparison between the linear Wesenheit function and the broken V -band PL relation obtained from using two datasets. Therefore, for the LMC Cepheids, we conclude that:

Corollary A: The Wesenheit function is statistically linear, and the Wesenheit functions for short ($P < 10$ days), long and all (short+long) period Cepheids are approximately the same, i.e.,

$$W^S \sim W^L \sim W^A. \quad (7.8)$$

The fundamental reason that the Wesenheit function is linear is because the PC relation, e.g. $(V - I) = a + b \log(P)$, is also broken for the LMC Cepheids, in addition to the broken PL relations.

The non-linearities for both of the PL and PC relations cancel out when the Wesenheit function is formulated.

In the next sub-section, the result from the corollary A (equation [7.8]) are applied to the distance scale measurements, as well as investigate the accuracy of using the linear LMC PL relation vs. the broken LMC PL relation.

7.3 Wesenheit Function and the Distance Scale Application

For an ensemble of Cepheids in a target galaxy, the distance modulus to this target is: $\overline{\mu_0} = 1/N \sum \mu_0^i = \overline{\mu_V} - R(\overline{\mu_V} - \overline{\mu_I})$, where $\mu_0^i = \mu_V^i - R(\mu_V^i - \mu_I^i)$ with $\mu^i = m^i - a \log(P_i) - b$. This procedure is described in details in Chapter 3.2. If the PL relation is linear, then the single, unbroken PL relation (in both V- and I-band) can be adopted to fit to all Cepheids to obtain the distance modulus (μ_0^A). However, if the PL relation is non-linear, as in the case of LMC Cepheids, then the long period PL relation should be applied to the long period Cepheids to obtain the distance modulus (μ_0^L), and similarly for the short period Cepheids (μ_0^S). For example, attempts to use the broken LMC PL relations to calibrate the extra-galactic Cepheid distances can be found in Kanbur et al. (2003); Leonard et al. (2003); Thim et al. (2003, 2004).

As mentioned before, μ_0 from equation (7.4) is equivalent to $\mu_W = W - W_M$, i.e., $\mu_0 = \mu_W$. Since the Wesenheit function (both W and W_M) for LMC Cepheids is linear, then the unbroken, linear version of the PL relations is expected to be applicable to obtain the μ_0 to a target galaxy. Based on the corollary A (equation [7.8]) from the previous subsection, we also expect that the μ_0 obtained from using either the broken LMC PL relation (for long and short period Cepheids respectively) or the linear LMC PL relation (for all Cepheids) should agree well with each other. In other words, the difference in the distance modulus ($\Delta\mu_0$) when using either the linear or the broken PL relation should be small. This statement will be verified in the following discussion by quantitatively estimating the value $\Delta\mu_0$ should have and by examples. Even though $\Delta\mu_0$ is expected to be small, it has a *systematic* effect on the distance scale because of the different PL relations used.

Since the Cepheids discovered in most of the distant target galaxies from the *HST* (and some from the ground-based telescopes) are mainly the long period Cepheids, the difference in distance modulus ($\Delta\mu_0 = \mu_0^A - \mu_0^L$) when using either the linear or the broken long period LMC PL relation can be quantitatively estimated if the mean log-period ($\overline{\log[P]}_L$) of the long period Cepheids in the target galaxy is known. For the distance modulus obtained with equation (7.4), this difference is expressed as (Kanbur et al., 2003):

$$\begin{aligned}
\Delta\mu_0 &= [(R-1)\Delta a_V - R\Delta a_I] \overline{\log(P)_L} \\
&\quad + (R-1)\Delta b_V - R\Delta b_I, \\
&= -\Delta a_W \overline{\log(P)_L} - \Delta b_W,
\end{aligned} \tag{7.9}$$

where $\Delta a_{(V,I,W)}$ and $\Delta b_{(V,I,W)}$ are the differences in slopes and ZPs for the linear and the broken long period PL relations/Wesenheit function. The coefficients in equation (7.9), Δa_W and Δb_W , can be calculated using the slopes and ZPs for the $V-(V-I)$ Wesenheit function as given in Table 7.3 & 7.4, respectively. If $\overline{\log(P)_L} \sim 1.4$, then the value of $\Delta\mu_0$ estimated from equation (7.9) is around $\pm 0.03\text{mag.}$, corresponding to $\sim 1.5\%$ change in distance (in Mpc). Furthermore, for $1.0 < \overline{\log(P)_L} < 2.0$, equation (7.9) suggests that the maximum $\Delta\mu_0$ when using the linear or the broken long period PL relation from Table 7.1 & 7.2 is $\sim 0.07\text{mag.}$, as illustrated in Figure 7.3. A similar estimation can be done for the short period Cepheids (with $0.0 < \overline{\log[P]_S} < 1.0$) with maximum $\Delta\mu_0$ of $\sim 0.02\text{mag.}$ when using the linear or the broken short period PL relation. For example, Kanbur et al. (2003) calculated the distance moduli to 25 *HST* observed galaxies with the linear and the broken long period LMC PL relations (as $\sim 98\%$ of the Cepheids detected in these galaxies have period longer than 10 days). By comparing the distance moduli derived from the linear PL relation and the broken long period PL relation, the unweighted average of $\Delta\mu_0$ in these 25 galaxies is $-0.021 \pm 0.002\text{mag.}$, in the sense that the linear PL relation *systematically* produces slightly smaller distance moduli than the broken long period PL relation. Since the mean log-period for most of these galaxies is ~ 1.4 (Kanbur et al., 2003), then the difference of -0.02mag. is expected and agrees well with the above discussion.

There are some observed target galaxies that also contain certain number of short period Cepheids. In the near future, more short period Cepheids are expected to be discovered in the distant target galaxies due to the advancement of instrumentation (e.g., *HST/ACS*) or telescopes. To investigate the distance modulus obtained from using the broken PL relations to both long and short period Cepheids, we use the Cepheids in IC 4182 as an example, because this galaxy contains roughly equal number of short and long period Cepheids. We used the data from Gibson et al. (2000), which includes 13 and 15 short and long period Cepheids, respectively. We first use the linear, unbroken PL relation to fit the data for all 28 Cepheids to obtain μ_0^A . Then we applied the broken PL relation to the 13 short and 15 long period Cepheids separately, and obtain μ_0^S and μ_0^L from these two sub-samples. Finally, we apply the broken long/short period PL relation to the long/short period

Figure 7.3 –The difference in the distance modulus ($\Delta\mu_0$), when using the linear and broken LMC PL relations, as function of mean log-period ($\langle \log(P) \rangle$). The LMC PL relations with different datasets are given in the lower-left corner. The left and right panels show the plots of $\Delta\mu_0$ when using the broken short and long period PL relations, respectively, as compared to the linear PL relation.

Cepheids to obtain μ_0^i , and then use equation (3.2) to calculate the mean distance modulus ($\overline{\mu_0}$). The PL relations used, include the linear and the broken PL relations, are taken from Table 7.1 & 7.2, and the results are presented in Table 7.5. From this table, μ_0^A shows a good agreement to $\overline{\mu_0}$, with a difference of $\sim 0.013\text{mag.}$ or smaller². This result suggests that both of the linear and the broken LMC PL relations can be applied to obtain the Cepheid distances. Furthermore, Table 7.5 also shows that when $N_{short} \sim N_{long}$, the short period distance modulus agrees with the long period distance modulus (i.e., $\mu_0^S \sim \mu_0^L$), where the broken short and long period PL relations are applied to the 13 short and 15 long period Cepheids, respectively. However, this is generally not true when $N_{short} \ll N_{long}$ for most of the *HST* observed galaxies, as shown in Table 7.6 with the 4 shortest period Cepheids are arbitrary removed.

In short, when using the LMC PL relations to derive the Cepheid distances, we conclude that:

Corollary B: The distance moduli are approximately the same when using either the linear PL relation (to all Cepheids) or the means with broken PL relation (to long and short period Cepheids respectively), i.e.,

²Note that the median values of $\sim 28.25\text{mag.}$ for μ_0^A and $\overline{\mu_0}$ in Table 2.3 agree well to the distance modulus obtained from the TRGB (tip of red giant branch) method as given in Sakai et al. (2004, $\mu_{TRGB} = 28.25 \pm 0.06\text{mag.}$).

$$\mu_0^A \sim \overline{\mu_0}. \quad (7.10)$$

Both of the approaches will give similar and consistent distance moduli. Recall that the linear PL relation is the approximation of the broken PL relation, and the accuracy of using the linear PL relation is roughly $\pm 0.03\text{mag}$. Therefore, researchers have the freedom to use either the linear LMC PL relation as an approximation or using the correct broken LMC PL relation in deriving the Cepheid distances.

Table 7.5 –The distance modulus (μ_0) to IC 4182^a.

LMC PL relation	$\mu_0^S (N = 13)$	$\mu_0^L (N = 15)$	$\mu_0^A (N = 28)$	$\overline{\mu_0} (N = 28)$
TR02	28.262 ± 0.088	28.290 ± 0.067	28.267 ± 0.053	28.277 ± 0.053
SRT04	28.199 ± 0.087	28.232 ± 0.065	28.222 ± 0.053	28.217 ± 0.053
KN04	28.244 ± 0.087	28.250 ± 0.067	28.250 ± 0.053	28.247 ± 0.053
KNB05	28.238 ± 0.087	28.272 ± 0.069	28.253 ± 0.053	28.257 ± 0.054

^a The number in parentheses is the number of Cepheids used to derive the distance modulus. The superscripts ^S, ^L and ^A refer to the short, long and all period Cepheids, respectively. The errors are the random errors calculated from equation (3.6).

Table 7.6 –Same as Table 7.5, but the 4 shortest period Cepheids are arbitrary removed.

LMC PL relation	$\mu_0^S (N = 9)$	$\mu_0^L (N = 15)$	$\mu_0^A (N = 24)$	$\overline{\mu_0} (N = 24)$
TR02	28.160 ± 0.104	28.290 ± 0.067	28.228 ± 0.057	28.241 ± 0.058
SRT04	28.098 ± 0.105	28.232 ± 0.065	28.188 ± 0.058	28.182 ± 0.057
KN04	28.143 ± 0.105	28.250 ± 0.067	28.214 ± 0.057	28.210 ± 0.057
KNB05	28.138 ± 0.105	28.272 ± 0.069	28.218 ± 0.058	28.222 ± 0.059

7.4 Conclusion and Discussion for this Chapter

Due to the recent discovery of the non-linearity for the LMC PL and PC relations, the applicability of the LMC PL relation in distance scale studies is immediately in question. However, we showed that this problem can be remedied with the application of the Wesenheit function in distance scale applications. This is because the Wesenheit function for the LMC Cepheids is linear (corollary A), as shown in Section 7.2.1, although the LMC PL and PC relations are not. Therefore, the Cepheid distances obtained with the Wesenheit function or the equivalent μ_0 would not be affected with the recent finding of the broken PL and PC relations. We also found that the typical difference in distance modulus from using the linear or the broken PL relations is about 0.03mag. Hence, researchers can choose to apply either the linear or the broken LMC PL relations to obtain the Cepheid distances, without worrying that these two approaches will give inconsistent results, as both approaches are equally applicable in deriving the Cepheid distance (corollary B). Therefore,

the broken PL relation found in the LMC Cepheids will not seriously affect the previous applications of the linear LMC PL relation in distance scale studies, and the question raised by Feast (2003), as mentioned in Chapter 1.2.1, is essentially answered.

In addition to the non-linearity of the LMC PL relation, there are some recent studies also suggesting that the Cepheid PL relation is not universal, i.e., the Galactic PL relation is steeper than the LMC counterparts (Tammann et al., 2003; Fouqué et al., 2003; Kanbur et al., 2003; Ngeow & Kanbur, 2004; Storm et al., 2004). It is possible that the Wesenheit function may or may not depend on metallicity: Some preliminary studies for both viewpoints can be found in the literature (see, e.g., Moffett & Barnes, 1986; Caputo et al., 2000a; Baraffe & Alibert, 2001; Pietrzyński et al., 2004; Storm et al., 2004). However the detailed study of the metallicity dependency of the Wesenheit function is beyond the scope of this chapter. This chapter only studies the linearity of the Wesenheit Function for the LMC Cepheids.

CHAPTER 8

PERIOD-COLOR AND AMPLITUDE-COLOR RELATIONS IN THE CLASSICAL CEPHEID VARIABLES II: THE GALACTIC MODELS

In this chapter, we construct full amplitude non-linear hydrodynamical models of fundamental mode Galactic Cepheids and analyze the resulting theoretical period-color and amplitude-color relations at maximum, mean and minimum light. These theoretical relations match the general form of the observed relations well. This agreement is, to some extent, independent of the mass-luminosity relations used, pulsation codes, numerical techniques, details of the input physics and methods to convert theoretical quantities, such as bolometric luminosity and temperature, to observational quantities, such as V-band magnitudes or $(V - I)$ colors. We show that the period-color and amplitude-color properties of fundamental mode Galactic Cepheids with periods such that $\log(P) > 0.8$ can be explained by a simple application of the Stefan-Boltzmann law and the interaction of the photosphere with the hydrogen ionization front. We discuss the implications of our results for explaining the behavior of Galactic Cepheid period-color, and period-luminosity relations at mean light.

8.1 Cepheid PC & AC Relations, the HIF-Photosphere Interaction and the PL Relations

Simon et al. (1993, hereafter SKM in this chapter) used hydro-dynamical models to explain the observations of Code (1947): Galactic Cepheids show a spectral type independent of period at maximum light and a spectral type at minimum light that gets later as the period increases. SKM used data from Pel (1976) and Moffett & Barnes (1980, 1984) to show that Galactic Cepheids are such that higher amplitude stars are driven to cooler temperatures, and thus redder $(B - V)$ colors, at minimum light. This, according to equation (6.1) from Chapter 6 and the observational findings of Code (1947), is because of the range of temperatures at maximum light is independent of period for a large range of periods. This implies that the period-color (PC) relation at maximum light is

flat for the Galactic Cepheids, as found in SKM and Kanbur & Ngeow (2004, , hereafter KN in this chapter) for the $(B - V)$ and $(V - I)$ color, respectively.

The reason why Galactic Cepheids follow a spectral type that is independent of period at maximum light was explained in SKM: it is due to the interaction of the photosphere with the hydrogen ionization front (HIF). It is well-known that the partial hydrogen ionization zone (or the HIF) exists in the envelop of a pulsating star. As the star pulsates, this zone moves in and out in the mass distribution. Therefore, it is possible that the HIF will interact with the photosphere, defined at optical depth of $2/3$, at certain phases of pulsation. SKM suggested that this happened at maximum light for the Galactic Cepheids, as the HIF is so far out in the mass distribution that the photosphere occurs right at the base of the HIF. Together with the HIF is a sharp rise in opacity. At this point, the mean free path goes to zero. In the absence of any significant driving, even though the surrounding atmosphere has a non-zero inward velocity, this opacity wall prevents the photosphere from going deeper in the star and erases any “memory” of global stellar conditions. Thus for a large range of periods, the photosphere occurs at the base of the HIF at maximum light at a temperature which is independent of period. Therefore, at maximum light, this leads to a flat relation between period & temperature, period & color and period & spectral type, as seen in SKM and KN. At other phases, the HIF lies too far inside the mass distribution to interact with the photosphere. Since the HIF does not interact with the photosphere, the color (or the temperature) of the star follows the underlying PC relation. SKM further computed a sequence of radiative hydro-dynamical models for Galactic Cepheids to account for this explanation, and the results agreed with Code’s observations.

The flatness of the PC relation at maximum light, together with equation (6.1), implies a relation between amplitude and color (or temperature) at minimum light, i.e. the amplitude-color (AC) relation. Hence, KN performed a more detailed analysis of the PC and AC diagrams at maximum, mean and minimum light, not only to the Galactic Cepheids, but also to the Cepheids in Magellanic Clouds. Further, KN separated the data in to long ($P > 10$ days) and short period Cepheids in their analysis due to the recent finding of the break as seen in LMC period-luminosity (PL) relation at 10 days (Tammann et al., 2002; Tammann & Reindl, 2002; Kanbur & Ngeow, 2004). They found that the Galactic PC relation at maximum light displayed a statistically significant break at 10 days, but was consistent with a single line at mean and minimum light. In the LMC, the PC relation displayed this break at all three phases, though the statistical significance at maximum light was marginal. The SMC PC relation displayed similar properties to that in the Galaxy. In terms of flatness, both of the long period Galactic and LMC PC relations are flat at maximum light. Analysis of the Galactic Cepheid data in terms of AC diagrams confirmed the work of SKM, and extended it to the

case of $(V - I)$ color: there is a relation between amplitude and $(V - I)$ color at minimum V-band light in the long period Galactic Cepheids. For the LMC, short period Cepheids show no relation between V-band amplitude and $(V - I)$ color at minimum light, but long period Cepheids are such that higher amplitude stars are driven to redder $(V - I)$ colors and hence cooler temperatures at minimum light. At maximum light, short period Cepheids are such that higher amplitude stars are driven to bluer $(V - I)$ colors and hence hotter temperatures but long period Cepheids do not show such a relation.

An understanding of the PC and AC relations derived by KN are important, not only for stellar pulsation and evolution studies of Cepheids, but also because the Cepheid period-luminosity (PL) relation depends on the PC relation (see, e.g., Madore & Freedman, 1991). The PL relation will reflect significant changes in the PC relation. Hence in studying PC and AC relations in different galaxies, we are studying the universality of the Cepheid PL relation. Recent work by Tammann et al. (2003), Fouqué et al. (2003), Kanbur & Ngeow (2004), Ngeow & Kanbur (2004), Sandage et al. (2004) and Storm et al. (2004) have suggested that the PL relation in the Galaxy is significantly different from that in the LMC and, further, that the PL relation in the LMC is non-linear. In contrast, current observations indicate that the Galactic PL relation is linear (Tammann et al., 2003; Kanbur & Ngeow, 2004; Ngeow & Kanbur, 2004).

The purpose of this chapter (and the next chapter) is to confront these new observed characteristics of Cepheids with the latest stellar pulsation models and interpret the results in terms of the theory presented in SKM, which has been summarized above. This approach will ultimately yield a qualitatively deeper understanding some of the reasons behind the variation of the Cepheid PL relation from galaxy to galaxy. Our ultimate goal is to use our theoretical models to estimate the effect of this variation quantitatively. For this chapter we concentrate on Galactic Cepheid models which we compare with the same Galactic Cepheid data used in KN.

8.2 Methods, Models and Code Description

Our models and methods improve upon those in SKM in the following respects: First of all, they contain a formulation to model 1-D time dependent turbulent convection (Yecko et al., 1998; Kolláth et al., 2002), in contrast to the purely radiative models used in SKM. In addition, we construct more models so that we can examine in greater detail the PC and AC characteristics of Cepheids. Secondly, we investigate the pulsation properties as a function of other phase points. When investigating PC and AC relations at mean light, KN defined mean $(V - I)$ color as $V_{mean} - I_{phmean}$ where V_{mean}

is the V-band magnitude closest to the value of A_0 , the mean magnitude obtained from a Fourier decomposition, and I_{phmean} is the I-band magnitude at this same phase. The reason for doing this is because the color of the star at a certain phase is precisely the V-band magnitude minus the I-band magnitude at that same phase. In contrast, a definition of mean color such as $\langle V \rangle - \langle I \rangle$ folds in the phase difference (albeit small) that exists between the V- and I-band light curves. We adopt the definitions given in KN for color at maximum, mean and minimum light. However, there are two phases when the V-band magnitude is closest to the value of A_0 : once on the ascending branch and once on the descending branch of the light curve. The color (or the temperature), in both models and theory, need not necessary be the same at these two phases. In KN's analysis of the observed data, the descending branch mean was adopted as the "mean". We pay specific attention to this detail in our results and discussion section. Finally, we look at $(V - I)$ colors whereas SKM studied predominantly $(B - V)$ colors. The $(V - I)$ color is a good indicator of temperature (Gonzalez & Wallerstein, 1996; Tammann et al., 2003) and is a crucial color for the existing calibration of the extra-galactic distance scale (e.g., Freedman et al. 2001).

The numerical techniques and physics included in these models are detailed in Yecko et al. (1998) and Kolláth et al. (2002). The **Florida** pulsation code has been used successfully to model double mode Cepheid pulsations (Kolláth et al., 1998), Galactic first overtone Cepheids (Feuchtinger et al., 2000), Cepheid mass-luminosity (ML) relations (Beaulieu et al., 2001) and the study of Magellanic Clouds Cepheids (Buchler et al., 2004). The code takes the mass, luminosity, effective temperature, hydrogen and metallicity abundance (by mass), M, L, T_e, X, Z , as input parameters. In this study, we used $(X, Z) = (0.70, 0.02)$ as a representation of Galactic hydrogen and metallicity abundance (hence we also used the corresponding opacity table). For the remaining parameters, mass and luminosity are connected by an adopted ML relation, hence it is only necessary to choose the mass and the effective temperature for computing a Cepheid model. In this study, we used two ML relations, which are calculated from evolutionary models appropriate for intermediate-mass stars:

1. The ML relation given by Chiosi (1989), which was also used by Simon & Kanbur (1995) in their theoretical study of long period Cepheids:

$$\log L = 3.61 \log M + 0.924. \quad (8.1)$$

2. The ML relations given by Bono et al. (2000a):

$$\log L = 0.90 + 3.35 \log M + 1.36 \log Y - 0.34 \log Z, \quad (8.2)$$

Table 8.1 –Input parameters for Galactic Cepheid models with periods obtained from a linear analysis. The periods, P_0 and P_1 , are referred to the fundamental and first overtone periods, respectively. Similarly for the growth rate, η . Both of the mass and luminosity are in Solar unit, the temperature is in K and the period is in days.

M	$\log(L)$	T_e	P_0	η_0	P_1	η_1
ML Relation from Bono et al. (2000a)						
11.5	4.279	4830	43.3924	0.042	27.58	-0.164
11.1	4.228	4900	37.8090	0.039	24.40	-0.157
10.8	4.188	4950	34.4156	0.037	22.25	-0.133
10.5	4.147	5000	30.8189	0.035	20.26	-0.119
9.55	4.009	5075	23.4835	0.025	15.69	-0.094
9.45	3.994	5265	20.0414	0.028	13.55	-0.051
8.60	3.856	5300	15.8313	0.022	10.78	-0.040
7.70	3.696	5332	12.1076	0.015	8.291	-0.036
7.30	3.618	5440	10.0124	0.014	6.886	-0.017
7.30	3.618	5490	9.69213	0.014	6.670	-0.007
7.00	3.557	5490	8.83394	0.013	6.087	-0.010
6.80	3.515	5485	8.31328	0.011	5.733	-0.014
6.45	3.438	5545	7.12090	0.011	4.921	-0.007
6.10	3.357	5580	6.16131	0.009	4.267	-0.006
6.00	3.333	5590	5.59661	0.006	3.884	-0.007
ML Relation from Chiosi (1989)						
10.0	4.534	5150	66.9094	0.144	38.47	-0.413
8.50	4.279	5100	44.8456	0.129	27.38	-0.238
8.00	4.184	5090	38.5673	0.116	23.98	-0.196
7.40	4.061	5050	32.4902	0.096	20.55	-0.171
7.20	4.019	5250	26.1109	0.081	16.99	-0.115
7.00	3.975	5400	21.8715	0.060	14.45	-0.077
6.30	3.810	5390	16.9148	0.055	11.37	-0.048
5.50	3.597	5350	12.4229	0.038	8.445	-0.051
5.10	3.478	5396	10.0049	0.030	6.851	-0.041
5.00	3.447	5420	9.37928	0.028	6.436	-0.036
4.90	3.416	5440	8.80955	0.026	6.056	-0.033
4.80	3.383	5470	8.21323	0.025	5.657	-0.027
4.50	3.282	5460	7.06755	0.017	4.878	-0.037
4.40	3.247	5490	6.56235	0.016	4.536	-0.032
4.20	3.174	5560	5.60322	0.015	3.887	-0.022
Input parameters from SKM						
4.57	3.306	5707	10.5472	0.035	7.234	-0.000
5.44	3.578	5550	6.25949	0.022	4.340	0.020

$$= 3.35 \log M + 0.726.$$

The units for both M and L are in Solar unit. Note that equation (8.1) gives a much larger L/M ratio than equation (8.2). The range of L/M ratio covered by equations (8.1) and (8.2) is reasonably broad and encompasses a fair selection of ML relations in the literature. Our aim for this chapter is not to test one ML relation against another, but rather, to show that the physical effect we are interested in is, to a large extent, independent of the ML relation chosen.

The effective temperature, T_e , was chosen to ensure a good fundamental growth rate and a stable first overtone mode. After a linear non-adiabatic analysis (Yecko et al., 1998) which yields the normal mode spectrum, an initial perturbation, which is scaled to match the fundamental mode linear velocity eigenvector, is applied and the model followed until a stable limit cycle is reached. In some cases, this means following the pulsations for about 1000 cycles. This results in a full amplitude Cepheid oscillating in the fundamental mode. Following SKM, we can compute the temperature at an optical depth $\tau = 2/3$ and also the temperature and opacity profiles as a function of depth at various phases of the pulsation.

In addition to the two ML relations used, we have also computed two models using the parameters (M, L, T_e) from table 1 of SKM, in order to compare the results based on purely radiative and the turbulent convective models, and further broaden the range of ML relation considered. The input parameters for these models and for the two ML relations used are presented in Table 8.1.

We compare theoretical quantities to observable quantities using a number of prescriptions:

1. We convert observed, extinction corrected $(V - I)$ to $(B - V)$ colors using the formula given in Tammann et al. (2003), and then convert the $(B - V)$ index to temperatures using the formula given in SKM.
2. We use the BaSeL atmosphere database¹ (Lejeune, 2002; Westera et al., 2002) to construct a fit giving temperature as a function of $(V - I)$ color, with and without a $\log(g)$ term. Where a $\log(g)$ term is used, we obtain this, at the appropriate phase, from the models. This database is also used in, for example, Beaulieu et al. (2001) and Cordier et al. (2003) for the study of Magellanic Cloud Cepheids.

¹<http://www.astro.mat.uc.pt/BaSeL/>

3. These techniques were also used to obtain the bolometric corrections required to convert theoretical luminosity curves into theoretical V-band light curves. Simon & Kanbur (1995) have shown that these corrections are small.

The qualitative thrust of our results does not change according to the details of the method used to convert colors to temperatures and luminosity to V-band light. In what follows, we report on results using method (ii) above with a $\log(g)$ term. One caveat here is that we use hydrostatic equilibrium atmospheres. The effective temperature and effective gravity for these atmospheres are obtained from the relevant quantities at the photosphere in the models. For the effective gravity we use

$$g = \frac{GM}{R^2} - \frac{du}{dt},$$

where du/dt is again given by the instantaneous structure of the models. Such an approach has been widely used in the literature in recent years for both Cepheids and RR Lyraes (see, for examples, SKM, Simon & Kanbur 1995, Bono et al. 1999a, Sandage et al. 1999, Beaulieu et al. 2001, Caputo et al. 2002, Kovács 2003, Ruoppo et al. 2004). Some justification based on detailed modeling may be found in Kolláth et al. (2000) and Keller & Mutschlecner (1970). We note that at most phases the du/dt term is small but can become large at certain phases during the ascending and descending branches. For the models considered here, this occurs for three models with $\log(P)$ between 1.5 and 1.65 at phases close to maximum light.

8.3 Results for the Galactic Models

The results from the model calculations are summarized in Table 8.1, 8.2 and 8.3. Table 8.1 gives the input parameters, as well as the computed periods and growth rates from the linear non-adiabatic analysis for the fundamental and first overtone modes for the models constructed in this chapter. We note that the models used generally had reasonably high fundamental mode growth rates and negative first overtone growth rates. This puts our models well inside the fundamental mode instability strip for all periods considered. Table 8.2 gives the temperatures at the maximum/minimum luminosity at full amplitude for the corresponding models in Table 8.1. Table 8.3 gives similar quantities but for the ascending and descending branch means. In Table 8.3, $\langle L \rangle$ is the average luminosity obtained from the light curve in one pulsating cycle. L_{mean} is the luminosity

that is obtained from the closest temporal grid points (from the light curve) to the $\langle L \rangle$, on both of the ascending and descending branches. T_{mean} are the corresponding temperatures at these grid points. Since $\langle L \rangle$ and L_{mean} may not be equal to each other, we obtain the temperatures that correspond to $\langle L \rangle$ by interpolation between L_{mean} and the adjacent luminosity that bracket the $\langle L \rangle$. These temperatures are referred as T_{mean}^{inter} in Table 8.3. Note that the temperatures at mean light on the descending branch is normally cooler than the mean light temperatures on the ascending branch.

Figure 8.1 –The period-color (PC) relations for the Galactic Cepheids. The crosses are the observed data points, which are taken from Kanbur & Ngeow (2004). The open and solid squares are the Galactic models with Bono et al. (2000a) and Chiosi (1989) ML relations, respectively, whilst the solid circles are results for models computed with the ML relation used in SKM. The temperatures from the models are converted to the $(V - I)$ color using the BaSeL database with the $\log(g)$ term. Left (a): PC relations at maximum and minimum light. Right (b): PC relation at mean light, for both of the acceding and descending branches.

8.3.1 The PC Relations

In this sub-section we compare the PC relations from the model calculations and the observations. Figure 8.1 shows a four panel plot of log-period against extinction corrected $(V - I)$ color for Galactic Cepheids as given in KN. Superimposed on this are values obtained from our models, where the theoretical temperatures are converted to $(V - I)$ colors using method (ii) described above. The solid and open squares represent models based on the Chiosi (1989) and Bono et al. (2000a) ML relations respectively, whereas the solid circles refer to models computed with the ML relation used in SKM. The left panels are for maximum (top) and minimum (bottom) light respectively, whilst the

Table 8.2 –Temperatures at maximum and minimum light from full-amplitude non-linear model calculations. The periods, luminosity and temperature are in days, $L_{\odot}$ and K, respectively.

P	L_{max}	T_{max}	L_{min}	T_{min}
ML Relation from Bono et al. (2000a)				
43.3924	23323.69	5301.44	15265.45	4653.25
37.8090	20547.96	5355.92	13844.78	4759.77
34.4156	18675.64	5399.02	12811.00	4681.60
30.8189	17046.03	5446.49	11592.02	4719.65
23.4835	12259.96	5456.64	8535.450	4820.82
20.0414	12503.63	5720.76	7897.603	4948.82
15.8313	8828.696	5664.44	6018.911	5025.59
12.1076	5616.756	5525.95	4402.925	5135.11
10.0124	4470.551	5531.94	3703.182	5274.68
9.69213	4484.612	5585.30	3647.089	5303.57
8.83394	3872.188	5619.60	3172.488	5312.02
8.31328	3501.343	5612.30	2906.126	5322.39
7.12090	2954.314	5673.40	2436.595	5396.37
6.16131	2434.291	5700.83	2059.419	5459.73
5.59661	2260.885	5691.47	2023.329	5513.33
ML Relation from Chiosi (1989)				
66.9094	39933.45	5468.07	25451.85	4807.32
44.8456	24179.71	5738.10	12478.80	4707.13
38.5673	19927.26	5762.38	9964.144	4696.57
32.4902	15175.07	5724.16	7818.562	4720.46
26.1109	13968.71	5886.81	7294.068	4949.57
21.8715	13020.35	6056.17	7076.027	5176.93
16.9148	8608.982	5999.40	4782.056	4961.41
12.4229	5070.216	5846.81	3081.266	4998.02
10.0049	3724.848	5785.06	2465.684	5074.73
9.37928	3427.493	5770.41	2320.390	5115.94
8.80955	3130.614	5748.80	2187.659	5156.00
8.21323	2843.303	5729.82	2062.474	5217.61
7.06755	2080.732	5572.38	1722.053	5295.72
6.56235	1885.999	5565.48	1587.056	5332.63
5.60322	1580.301	5697.21	1315.624	5383.56
Input parameters from SKM				
10.5472	2276.417	5894.64	1660.255	5430.83
6.25949	5084.652	6068.32	2974.031	5170.89

Table 8.3 –Temperatures at mean light from full-amplitude non-linear model calculations. See text for the meanings of $\langle L \rangle$, L_{mean} , T_{mean} and T_{mean}^{inter} . The periods, luminosity and temperature are in days, $L_{\odot}$ and K, respectively.

P	$\langle L \rangle$	$L_{mean}(asc)$	$T_{mean}(asc)$	$L_{mean}(des)$	$T_{mean}(des)$	$T_{mean}^{inter}(asc)$	$T_{mean}^{inter}(des)$
ML Relation from Bono et al. (2000a)							
43.3924	19774.9	19653.607	5076.72	19774.061	4719.16	5085.42	4719.24
37.8090	17280.2	17078.646	5114.30	17267.319	4759.22	5130.82	4760.52
34.4156	15390.5	15555.296	5159.32	15346.989	4772.37	5143.81	4776.56
30.8189	14018.4	14150.570	5205.60	14024.699	4833.74	5191.91	4833.08
23.4835	10192.1	10183.759	5239.51	10191.410	4931.29	5240.69	4931.39
20.0414	9841.01	9929.1677	5464.55	9828.9718	5093.87	5451.69	5095.78
15.8313	7180.69	7186.0066	5454.37	7176.7521	5161.14	5453.34	5162.00
12.1076	4954.75	4958.1150	5429.69	4955.1704	5254.78	5428.68	5254.65
10.0124	4147.76	4153.0371	5518.32	4150.1024	5390.79	5515.96	5389.95
9.69213	4150.28	4159.2783	5580.60	4141.0319	5433.01	5576.70	5436.31
8.83394	3600.06	3608.3159	5575.65	3611.7288	5439.77	5571.56	5435.18
8.31328	3270.50	3261.5882	5558.95	3271.9206	5435.65	5563.73	5435.04
7.12090	2738.62	2734.9282	5623.67	2739.4820	5492.58	5625.95	5492.13
6.16131	2271.74	2265.2783	5651.82	2275.3634	5533.36	5660.78	5531.01
5.59661	2149.59	2149.7132	5646.80	2151.3451	5559.70	5646.70	5558.46
ML Relation from Chiosi (1989)							
66.9094	34954.3	35127.180	5412.97	34950.980	4992.63	5404.79	4992.77
44.8456	19688.7	19843.004	5446.15	19660.664	4902.60	5434.34	4904.75
38.5673	15650.0	15888.680	5439.08	15652.258	4867.57	5417.09	4867.35
32.4902	11522.9	11331.742	5312.26	11511.286	4796.29	5335.61	4797.57
26.1109	10436.0	10650.701	5571.95	10432.457	4983.82	5541.43	4984.29
21.8715	9979.12	10039.792	5765.91	9909.8995	5244.96	5756.71	5258.68
16.9148	6648.78	6722.9877	5704.33	6649.7902	5213.82	5688.05	5213.56
12.4229	3945.54	3988.1625	5571.12	3927.3390	5146.88	5555.92	5153.68
10.0049	3007.79	3028.3688	5566.36	3004.3297	5237.81	5557.55	5239.67
9.37928	2802.28	2810.9339	5570.03	2810.0216	5283.35	5566.27	5278.95
8.80955	2605.42	2606.5476	5570.03	2609.8551	5319.08	5569.55	5316.25
8.21323	2417.35	2413.1797	5579.92	2422.3123	5369.50	5581.37	5366.15
7.06755	1914.78	1919.5566	5532.01	1908.3662	5401.51	5527.21	5406.84
5.56235	1764.77	1759.8178	5554.42	1763.3763	5440.10	5559.56	5441.30
5.60322	1490.55	1495.4280	5647.41	1495.1755	5507.79	5641.55	5503.47
Input parameters from SKM							
10.5472	2022.70	2028.8310	5837.90	2031.3158	5634.15	5832.38	5627.78
6.25949	3777.86	3731.9089	5710.18	3785.7844	5373.31	5726.55	5369.91

Figure 8.2 –The plots of $\log(T)$ vs. $\log(P)$ for the Galactic data and models. The symbols are same as in Figure 8.1. The conversion from observed $(V - I)$ color to temperature is done with the BaSeL database. Left (a): The $\log(P)$ - $\log(T)$ plots at maximum and minimum light. Right (b): The $\log(P)$ - $\log(T)$ plots at mean light, for both of the ascending and descending branches.

Figure 8.3 –The V-band amplitude-color (AC) relations for the Galactic Cepheids. The symbols are same as in Figure 8.1. The bolometric model light curves are converted to V-band light curves with the bolometric corrections obtained from BaSeL database. Left (a): AC relations at maximum and minimum light. Right (b): AC relation at mean light, for both of the acceding and descending branches.

right panels are for the means of the ascending and descending branches, respectively. We see that there is very good agreement between models and observations in this diagram. Figure 8.2 shows the same quantities but on the $\log(P)$ - $\log(T)$ plane. The observed $(V - I)$ colors for the Galactic Cepheid

Table 8.4 $-\log(T)$ vs $\log(P)$ regression at maximum, mean and minimum light for the Galactic Cepheid data. σ is the dispersion from the regression of the form: $\log(T) = a + b \log(P)$.

phase	σ	Slope, b	Zero-point, a
All Periods			
Max	0.016	-0.033 ± 0.005	3.807 ± 0.005
Mean	0.017	-0.062 ± 0.006	3.801 ± 0.005
Min	0.022	-0.077 ± 0.007	3.786 ± 0.007
$\log(P) > 0.8$			
Max	0.017	-0.014 ± 0.009	3.787 ± 0.009
Mean	0.019	-0.068 ± 0.010	3.808 ± 0.010
Min	0.023	-0.076 ± 0.012	3.786 ± 0.012
$\log(P) < 0.8$			
Max	0.014	-0.109 ± 0.023	3.862 ± 0.016
Mean	0.014	-0.131 ± 0.023	3.846 ± 0.016
Min	0.019	-0.193 ± 0.031	3.865 ± 0.022

data are converted to the temperature by using the BaSeL database mentioned in previous section, where the $\log(g)$ terms for the observed colors are approximated by using $\log(g) = 2.62 - 1.21 \log(P)$ (Beaulieu et al., 2001). We note that the qualitative nature of our results is the same in Figure 8.1 and 8.2: the flat nature of the relation at maximum light, the non-zero slope at minimum light and an amalgam of these at mean light.

Recall that KN analyzed a large sample of Galactic Cepheid data and showed that the PC relation was consistent with a single line at mean and minimum light, but at maximum light there was significant statistical evidence for a break at 10 days. Figures 8.1 & 8.2 suggests that this break actually occurs at $\log(P) \approx 0.8$. As in KN, we can perform a two line regression, for Cepheids with periods smaller and greater than $\log(P) = 0.8$, and compare with a one line regression across the entire period range. This is what is meant by short and long period in the following paragraph. Table 8.4 shows the phase, overall dispersion, slope and error at that phase and then these same quantities for the short and long period Cepheids respectively. From Figure 8.2 and Table 8.4, we can conclude the following results:

1. The overall slope in the $\log(P)$ - $\log(T)$ plane at maximum light is flat and is significantly different from the slope at mean and minimum light.
2. The short period slopes in this same plane are significantly different from zero at all three phases. In contrast, the long period slope at maximum light in this plane is close to zero and is significantly different from the slope at mean and minimum light.
3. If we reject all Cepheids with $\log(P) < 0.8$ and test, using the methodology of KN, whether the remaining data are consistent with a single line or with two lines broken at a period of 10 days,

we find that in the $\log(P)$ - $\log(T)$ plane, even at maximum light, the data are consistent with a single line. Thus the broken PC relation at maximum light for Galactic Cepheids described by KN is due primarily to the different PC relation obeyed by short ($\log[P] < 0.8$) period, fundamental mode Cepheids.

We note that this good agreement exists for maximum, mean and minimum light. The exception is perhaps the longer period models constructed with the Bono et al. (2000a) ML relation, with $\log(P) > 1.3$, at maximum light. We discuss these models in Section 8.4. Since the observed data exhibit very similar characteristics in both period-color and period-temperature planes, that is, a distinctly non-linear relation with a sudden break at a period of $0.75 < \log(P) < 1.0$ and since the models agree well with the data, it follows that existing models contain the physics to explain these observations. We study this in Section 8.3.3.

8.3.2 The AC Relations

Figure 8.3 depicts plots of the AC relations for both models and data, at maximum, minimum, ascending and descending branch means. The bolometric corrections obtained from the BaSeL database are used to construct the V-band amplitudes from the model light curves. We see good agreement between models and data in these diagrams, though models constructed with our code using the Bono et al. (2000a) ML relation have smaller amplitudes even though these models match observations in the AC diagrams. Models constructed using the Chiosi (1989) ML relation with our code generally have larger amplitudes. We caution here that the computed amplitudes, unlike the periods, depend strongly on the strength of the artificial viscosity and of the assumed turbulent eddy viscosity.

Table 8.5 gives the results from a linear regression of V-band amplitude against $\log T$ at the three phases of interest for the Galactic Cepheid data, taken from Figure 8.2 & 8.3. Equation (6.1) shows that, if the variation of $\log T_{max}$ with period has a shallow slope close to zero, then an equation of the form $\log T_{min} = a + bV_{amp}$ is such that b is about -0.10 . From Table 8.5, we see that the overall slope is -0.079 , which is very close to the theoretically expected value. Accounting for the error on the slope shows that the actual slope is statistically indistinguishable from the theoretical value of -0.10 . In addition, Table 8.5 implies that the overall slope for V_{amp} - $\log T_{max}$ relation is close to zero across the entire period range. This strongly supports the validity of equation (6.1) and our interpretation of the predictions arising from this equation if either $\log T_{max}$ or $\log T_{min}$ obey a flat relation with period. Figure 8.4 argues this graphically with a two panel plot displaying V-band amplitude against $\log T_{min}$ and $\log T_{max}$ on the left and right panels, respectively. An AC relation

Table 8.5 –V-band amplitude (V_{amp}) vs. $\log T_{min}$ regression for the Galactic Cepheid data. σ is the dispersion from the regression of the form: $\log T_{min} = a + bV_{amp}$.

phase	σ	Slope, b	Zero-point, a
All Periods			
Max	0.018	0.011 ± 0.007	3.770 ± 0.006
Mean	0.021	-0.050 ± 0.009	3.784 ± 0.007
Min	0.024	-0.079 ± 0.010	3.778 ± 0.008
$\log(P) > 0.8$			
Max	0.017	0.018 ± 0.008	3.757 ± 0.007
Mean	0.021	-0.048 ± 0.009	3.776 ± 0.009
Min	0.023	-0.073 ± 0.011	3.766 ± 0.009
$\log(P) < 0.8$			
Max	0.015	0.038 ± 0.015	3.760 ± 0.011
Mean	0.017	-0.007 ± 0.017	3.761 ± 0.012
Min	0.023	-0.040 ± 0.023	3.760 ± 0.016

may also occur if either $\log T_{max}$ or $\log T_{min}$ has a shallower relation with period than the other quantity.

8.3.3 The Interaction of the HIF and Photosphere

In Figures 8.5-8.7, we present the plots of the temperature and opacity profiles, i.e. temperature and log opacity against location in the mass distribution respectively, for a representative selection of the models in Table 8.1. We display plots for a long period ($\log[P] > 1.0$), short period ($\log[P] < 1.0$) and a 10 day period model. The mass distribution is measured by the quantity $Q = \log(1 - M_r/M)$, where M_r is the mass within radius r and M is the total mass. Each panel shows the temperature and opacity profiles at maximum, mean and minimum light with dotted, solid and dashed curves, respectively. For each model, we also include the plots for the case when the mean is on the ascending and descending branch respectively. The photosphere is marked with a filled circle in these figures. Figure 8.8 illustrates the luminosity and temperature profiles against mass distribution in one pulsating cycle.

In general, the HIF sweeps back and forth in the mass distribution as the star pulsates. A naive way of thinking about this would be that the HIF is furthest out and furthest in, in the mass distribution, at maximum and minimum light respectively (SKM). Our models indicate that the situation is more complicated than this, and we summarize the main results from Figure 8.5-8.7 as follow:

1. At minimum light, the photosphere is closest to the HIF at short period but moves progressively away from the base of the HIF as the model period increases. This is in concordance with the lowest panel of Figures 8.1 and 8.2 which display the period temperature relation for

Figure 8.4 –The plots of $\log(T)$ vs. V amplitudes (V_{amp}) for the Galactic data and models. The symbols are same as in Figure 8.1. The conversion from observed $(V - I)$ color to temperature is done with the BaSeL database (see text for details). Left (a): The plot of V_{amp} - $\log(T_{min})$, The dashed line is *not* the best-fit regression to the data, but illustrates how closely the overall trend of the data follow a slope of -0.1 , as predicted from equation (6.1). This line is constructed with $\log(T_{min}) = 3.78 - 0.1V_{amp}$. Right (b): The plot of V_{amp} - $\log(T_{max})$. The dashed line show the average value of $\log(T_{max})$, where $\langle \log(T_{max}) \rangle = 3.78$.

Figure 8.5 –The temperature and opacity profiles for a long period model. The dotted, solid and dashed curves are for the profiles at maximum, mean and minimum light, respectively. The filled circles are the location of the photosphere at $\tau = 2/3$ for each phases. The mean light profiles at the ascending and descending branch are the solid curves that lie close to the profiles at maximum light (dotted curves) and minimum light (dashed curves), respectively. Left (a): Temperature profile. Right (b): Opacity profile.

Figure 8.6 –Same as Figure 8.5, but for model with period at 10 days.

Figure 8.7 –Same as Figure 8.5, but for model with short period.

Figure 8.8 – Illustrations of the temperature profiles at different pulsation phases, where $q = M_r/M$. The upper and lower panels are the light curves and the temperature profiles at various phases, respectively. The lower-left panel shows the changes of temperature profiles from minimum light to maximum light (ascending branch), and the lower-right panel shows the similar temperature profiles in descending branch. In lower panels, the dashed curves represent the temperature profiles at maximum and minimum light, while the solid curves represent the temperature profiles at other phases as indicated in upper panels.

models and data as one single line such that longer period models have cooler photospheric temperatures at minimum light. However, at maximum light, the photosphere lies at the base of the of the HIF for all the models presented.

2. We can quantify this by defining the quantity Δ to be the “distance”, in units of Q , between the photosphere and the point in the HIF where dT/dQ is a maximum. Figure 8.9 illustrates this and Figure 8.10 portrays the results when we plot Δ against $\log(P)$ at maximum, minimum and the ascending and descending branch means. In Figure 8.10, the error bars are estimated from the coarseness of the grid points around the location of HIF. We see clearly that at maximum light, Δ is constant over a large period range, whereas at minimum light, Δ gets larger as the period increases. The value of Δ at the phases of ascending and descending branch means is a mixture of the behavior at maximum and minimum light. There is an indication that for long period models ($\log(P) > 1.3$) at maximum light, the photosphere starts to become disengaged from the base of the HIF. As the period increases, so does the L/M ratio and, in general, the effective temperatures become lower. Kanbur (1995) showed that such changes push the HIF further inside the mass distribution, making it harder to interact with the photosphere even

at maximum light when it is closer to the surface. This can cause a non-zero slope in the $\log(P)$ - $\log(T_{max})$ plane at long periods. This was also noted by SKM.

3. The HIF at mean light is not always between its locations at maximum or mean light. Figure 8.8 illustrates our point and shows various locations on the ascending and descending branch of a typical light curve from our models. The lower two panels show the temperature profile at each of these phases. We see that the HIF at phase b is actually further out in the mass distribution than at maximum light and conversely the HIF is further in in the mass distribution at the phase corresponding to mean light on the descending branch than at minimum light. Moreover, the nature and shape of the HIF seems to change significantly during the pulsation period.
4. At short/long period, the HIF at ascending/descending branch mean light is located in front / behind its location at maximum/minimum light.
5. We also note that the nature of the HIF changes when it is located outside of its locations at either maximum or minimum light. These changes in the HIF occur at period ranges corresponding to the location of bumps on the ascending or descending branches of the light curve: the Hertzsprung progression.
6. If we define "amplitude of HIF oscillation" as the range of $\log(1 - M_r/M)$ values occupied by the HIF, then there is a correlation between this and the resulting amplitude of the photospheric light curve.

8.3.4 Cepheids with $\log(P) < 0.8$

Figure 8.1 and the results of Tables 8.4 & 8.5 suggest that Cepheids with $\log(P) < 0.8$ do not obey a flat relation with period at maximum light. Our current grid of models does not include any models with such periods, but Figures 8.10 does indicate that the photosphere will be located at the base of the HIF at all phases of pulsation as is the case with RRab stars (Kanbur, 1995; Kanbur & Phillips, 1996). Figure 8.10 also indicates that as the period gets shorter the "distance" in the mass distribution between the HIF and the photosphere is the same at maximum and minimum light. This does not necessarily mean that there will be a flat PC relation at both these phases as explained by Kanbur (1995) and Kanbur & Phillips (1996) for the case of RRab stars. These short period, fundamental mode Cepheids will be investigated in a future paper.

Figure 8.9 –Illustration of the location of HIF and photosphere, and the way to calculate Δ . The crosses represent the zones in the model calculations. The location of the photosphere, at $\tau = 2/3$, and the HIF are marked with filled circles. The location of the HIF is defined as the zone with the steepest gradient in the temperature profile. Therefore, Δ is the “distance” (in terms of $\log[1 - M_r/M]$) between the photosphere and HIF. Upper and lower panels illustrate the cases that the photosphere is far away and close to the HIF, respectively.

8.4 Light Curve Structure of the Models

It is desirable to compare the theoretical light curves from models *quantitatively* to the observed light curves, for example by Fourier decomposition (Payne-Gaposhkin, 1947; Simon & Kanbur, 1995) or Principal Component Analysis (Kanbur et al., 2002). The three panels of Figure 8.11 present the plots of the Fourier amplitudes A_1 & A_2 (top and bottom left-side panels respectively) and $\phi_{21} = \phi_2 - 2\phi_1$ (right-side panel) against $\log(P)$ for the observed data and the model light curves. The Galactic Cepheid data, which are the same as those used in KN, and the V-band model light curves are subjected to a Fourier decomposition of the form (i.e, equation [2.1] or [2.2]): $V(t) = A_0 + \sum[A_k \cos(2k\pi t/P + \phi_k)]$, where P is the period. We used a simulated annealing technique, as

described in Chapter 2, to perform this Fourier decomposition. This method significantly reduces numerical noise in the Fourier decomposition of sparsely sampled periodic data (Ngeow et al., 2003).

In terms of these diagrams, there is much better agreement between models and observations when we use the Chiosi ML relation (equation [8.1]). These diagrams also illustrate that the agreement between models and data in the $\log(P)$ -temperature or $\log(P)$ -color plane is a necessary, but not sufficient, condition for agreement between observed and theoretical light curves. This also emphasizes that the physical effect of interest in this chapter - the interaction of the photosphere and HIF - is a fundamental physical effect which is, to some extent, independent of the ML relation used.

The **Florida** code used in this study contains a recipe to calculate the time dependent turbulent convection whilst SKM found a similar result using a purely radiative, dynamically zoned code. Our results imply that this interaction is also independent of numerical techniques and physics included in the code. What is needed is the presence of hydrogen and an accurate description of the opacity when hydrogen starts to ionize.

At first sight, the left panels of Figure 8.11 might lead one to question the ML relation in equation (8.2). However, the numerical recipe to model convection contains seven dimensionless parameters (the α parameters, Yecko et al., 1998), many of which can influence the amplitude. A more detailed study and comparison of the effect of ML relations may remove this discrepancy. For example, the two models constructed with the ML relation used in SKM, shown as black dots in Figure 8.11, do lie closer to the observations in the Fourier plane. This may also explain the discrepancy between observations and theory in the $\log(P)$ -($V - I$)_{max} plane for $\log(P) > 1.3$ when the Bono et al. (2000a) ML relation is used.

8.5 Conclusion and Discussion for this Chapter

By looking at the way the Cepheid photosphere, the region where the Cepheid continuum is generated, interacts with the HIF, we have provided a simple qualitative physical explanation for the observed PC properties of fundamental mode Galactic Cepheids with $\log(P) > 0.8$. This explanation relies on the fact that the opacity in a Cepheid becomes very high when hydrogen starts to ionize. This acts as a wall and prevents the photosphere going any deeper, and leads to a flat relation between period and temperature at the phase when the HIF interacts with the photosphere. For Galactic Cepheids this is observed at maximum light. At other phases, the photosphere is located away from the opacity wall and its temperature is related to the global properties of the star and

Figure 8.10 –The plots of Δ as function of $\log(P)$ for the Galactic models. The symbols are same as in Figure 8.1. The dashed lines represent (roughly) the outer boundary of the HIF.

Figure 8.11 –The plots of first two Fourier amplitudes, and ϕ_{21} as function of period for the observational data and models. The symbols are same as in Figure 8.1. Upper Left (a): Plot of A_1 vs $\log(P)$. Lower Left (b): Plot of A_2 vs $\log(P)$. Right (c): Plot of ϕ_{21} vs $\log(P)$.

hence its period. This explains, convincingly, why the Galactic Cepheid period-temperature relation is flat at maximum light and has a non-zero, single slope at mean and minimum light.

The qualitative nature of this idea remains true whether the pulsation code is purely radiative (as in SKM), or has a numerical recipe to model time dependent turbulent convection as in this work. The interaction between the photosphere and HIF may also provide some explanation for the suggestion made by Kervella et al. (2004a), that the region where spectral lines are formed do not necessarily move homologously with the region where the K-band continuum is formed. In addition, because we have used two ML relations with a wide range of L/M ratio in our study, and because the same physical effect is present in models constructed with either relation, the qualitative nature of our result is independent of the ML relation used. However, the ML relation used and its variation with metallicity, will dictate how the interaction of the HIF with the photosphere changes with Cepheids in a different metallicity environment. In the next chapter, we will investigate how these ideas can be used to explain the PC, AC and PL properties of fundamental mode LMC Cepheids.

We have also found evidence that the non-linear nature of the Galactic PC relation at maximum light reported by KN is due to short period fundamental mode Cepheids with $\log(P) < 0.8$. These short period stars follow distinctly different PC relations and deserve further detailed study.

What bearing do the results of this chapter have on the Cepheid PC and PL relations at mean light? It is clear that PC relations at different phases contribute to the PC relation at mean light, as shown in equation (1.2). Ngeow & Kanbur (2005 – in preparation) compute PC relations for Galactic and Magellanic Cloud Cepheids at all phases between 0 and 1 and show that, for example for Galactic Cepheids, as the phase approaches maximum light, the slope of the PC relation becomes flatter (see also Chapter 9.4). Since the mean light PC relation affects the mean light PL relation (Madore & Freedman, 1991), our aim of understanding the reasons behind changes in the PC and AC relations at different phases has a direct bearing on understanding at least one cause of the possible variation of the mean light Cepheid PL relation from galaxy to galaxy. Below we discuss how our results are pertinent to studies of the variability of the Cepheid PL relation from galaxy to galaxy.

The evidence that shows the slope of the LMC PL relation at mean light is significantly different to the slope of the mean light PL relation in the Galaxy has been provided in Tammann et al. (2003), Ngeow & Kanbur (2004) and Storm et al. (2004). Furthermore, the mean light LMC PL relation suffers a change at a period of 10 days whilst the mean light Galactic PL relation data is consistent with a single slope with current data (Tammann et al., 2003; Fouqué et al., 2003; Kanbur & Ngeow, 2004; Ngeow & Kanbur, 2004; Sandage et al., 2004; Storm et al., 2004). However, the results from

this chapter and also from KN and Ngeow & Kanbur (2005 – in preparation) imply that the mean light PC relation is affected by changes in the PC relation at different phases during a pulsation cycle, as given in equation (1.2). Thus the study of PC relations at various phases impacts on the variability of the mean light PL relation from galaxy to galaxy.

In this chapter, we have updated and extended the work of SKM to provide an account of a simple physical mechanism, the interaction of the photosphere and HIF, which can change the properties of PC relations for Galactic Cepheids and so affect the PL relation. Specifically, a new result of our work is that for Galactic Cepheids with $\log(P) > 0.8$, the maximum light PC relation is flat, i.e., the HIF and photosphere are engaged at maximum light. We also study the changes in AC relations because they are linked to PC relations through equation (6.1). A flat PC relation at maximum light leads to an AC relation at minimum light of slope ≈ -0.1 from equation (6.1), as seen in Figure 8.4 and Table 8.5. However, a Cepheid in the LMC will have a different ML relation, usually in the sense that LMC ML relations have higher L/M ratios. Then in order to compare Galactic and LMC Cepheids of the same period, the LMC Cepheid needs to be hotter (KN). Kanbur (1995) and Kanbur & Phillips (1996) found that such changes in the (M, L, T_e) triplet describing the model changes the relative location of the HIF and hence the phase and the range of periods at which they interact. Our preliminary hypothesis is that for LMC models, it is only after $\log(P) > 1.0$ that the HIF and photosphere are engaged at phases around maximum light, leading to a flat PC relation. This flat PC(max) relation for long period ($\log[P] > 1.0$) LMC Cepheids could be one of the causes for the non-linear nature of the mean light LMC PL relation (KN). Empirically, KN provide evidence that the LMC PC relation is flat for $\log(P) > 1.0$ whereas this crossover period is $\log(P) \approx 0.8$ in the case of the Galaxy, as found in this chapter. In case of SMC Cepheids, KN also provide evidence that the maximum light PC relation is not flat even for long period Cepheids. Our contention is, using some of the results of this chapter, that amplitudes in SMC Cepheids are not high enough to force an interaction between the photosphere and HIF, as the SMC Cepheids have lower amplitudes than the LMC (Paczynski & Pindor, 2000). This must wait confirmation from SMC models.

Groenewegen et al. (2004) recently reported a metallicity dependence in the zero point of Cepheid PL relation at mean light. They used 34 Galactic Cepheids with individual metallicity measurements and then supplemented this sample with primarily long period Magellanic Cloud Cepheids to show the existence of a quadratic term in $\log(P)$ in the PL relation. When they used primarily Galactic Cepheids in their sample, they found no evidence of a quadratic term. They interpreted these results as being due to a metallicity dependent zero point in the PL relation. These results are also consistent with the Cepheid LMC PL relation having different slopes for long and short period Cepheids as

suggested by KN and Sandage et al. (2004). Sandage et al. (2004) also plotted amplitude-mean color relations in luminosity bins. However, KN's AC relations were along the instability strip. It will be interesting to apply the precepts behind equation (6.1) and multi-phase AC relations in luminosity and/or period bins, that is, across the instability strip.

CHAPTER 9

PERIOD-COLOR AND AMPLITUDE-COLOR RELATIONS IN THE CLASSICAL CEPHEID VARIABLES III: THE LMC MODELS AND THE NEW EMPIRICAL RELATIONS

In this chapter the new empirical LMC period-color (PC), amplitude-color (AC) and period-luminosity (PL) relations at maximum, mean and minimum light are presented with the updated LMC Cepheids data. The F -tests are performed to these relations and it was found that these empirical relations are non-linear with a discontinuity (or a break) at a period of 10 days, expect for the PL relation at maximum light. The multi-phase PC relations for the Galactic and LMC Cepheids are compared. The results show that the slopes of the LMC long and short period PC relations as a function of pulsational phase are qualitatively similar to the Galactic counterparts but quantitatively different. In particular the LMC slopes are steeper and shallower in the long and short period ranges than the Galactic slopes. These differences lead to a non-linear LMC PC relation at most of the pulsational phases but only a small fraction (in pulsational phases) of the Galactic PC relations are non-linear. In addition, it is also found out that the Galactic and LMC PC relations are flat at maximum light for the long period Cepheids, while this happen around phase of ~ 0.8 (with phase zero being the maximum light) only for the short period LMC Cepheids. Further, both of the Galactic and LMC PC relations are non-linear at maximum light and also around phase of ~ 0.8 , which are not seen before.

Following the Galactic models constructed in Chapter 8, we also constructed the LMC models with the two adopted ML relations. The location of the photosphere relative to the hydrogen ionization front (HIF) is investigated. It is found out that at maximum light, the behavior of the photosphere-HIF interactions for the LMC models is similar to the Galactic counterparts, where the photosphere is located next to the base of the HIF. While at minimum light, both of the LMC and the Galactic long period models show a similar behavior that the photosphere is disengaged from the HIF. But there are some indications for the photosphere-HIF interactions to occur at the short period LMC models, which is not seen in the Galactic short period models. Comparisons of the theoretical quantities with observations for the PC, AC, period-temperature and period-Fourier

parameters relations show that the models are generally agree with the data, however these models have smaller amplitudes and cooler temperatures at maximum light. It is desired to improve the stellar pulsation codes before the final conclusion can be drawn for the physical reason behind the non-linear LMC PL and PC relations.

9.1 The Motivations

The evidence of the non-linear period-luminosity (PL) and period-color (PC) relations for the LMC Cepheids have been presented in Chapter 5 & 6. A *preliminary* hypothesis to account for the non-linear PC relation is the interaction between the photosphere and the hydrogen ionization front (HIF) at the outer surface of the Cepheids. In this hypothesis, the long period LMC Cepheids will behave like the Galactic Cepheids where the photosphere-HIF interactions occur at the maximum light, and the short period LMC Cepheids will behave like the RR Lyrae variables where the photosphere-HIF interactions occur at the minimum light.

The methods and the techniques that are needed to investigate this hypothesis and the non-linearity of the PC and PL relations are well developed in previous chapters. These include: (a) the light curve construction techniques to reconstruct the full Cepheid light curves; (b) the statistical test (the F -test and/or the chi square-test) to examine the non-linearity of the empirical relations; (c) the studies of the empirical PC and the amplitude-color (AC) relations that provide some hints for the photosphere-HIF interactions; and (d) the use of stellar pulsation code to construct the pulsation models to investigate the photosphere-HIF interactions. In this chapter these methods and techniques are combined coherently to investigate in detail the non-linear LMC PL and PC relations and the role of the photosphere-HIF interactions that may cause these non-linearity. This chapter mainly divided into two parts: empirical studies with the updated LMC Cepheids data, and the LMC models construction with the same stellar pulsation codes used in the previous chapter. Note that throughout this chapter, by long and short period Cepheids we mean Cepheids with periods greater and less than 10 days, respectively.

9.2 Updated LMC Data

In order to construct the PL, PC and AC relations at the phases of maximum, mean and minimum light, it is necessary to construct the full Cepheid light curve from the available V- and I-band photometric data points. This was done with the n^{th} -order Fourier expansion (equation [2.2]) as described in Chapter 2. In Chapter 6, we have constructed the light curves of the fundamental

mode Cepheids in the Large Magellanic Cloud (LMC) by using the extensive photometric dataset in the OGLE (Optical Gravitational Lensing Experiment) database. However, the dataset used in Chapter 6 was downloaded in 2002, prior to the updated version of the dataset that was available after April 24, 2003 (OGLE website, Udalski 2004 [private communication]). The updated version includes additional V- and I-band data for most of the Cepheids. In addition, the periods have been refined using the complete set of photometric data by the OGLE team. Due to these reasons, we decided to repeat the light curve construction with the updated data and periods. Other differences between this chapter and Chapter 6 are: (a) since the Cepheids in OGLE database are truncated at $\log(P) \sim 1.5$, due to the saturation of the CCD detector for the longer period (hence brighter) Cepheids (Udalski et al., 1999a), therefore we include some additional LMC Cepheid data from Moffett et al. (1998) and Barnes et al. (1999) which extend the period coverage to $\log(P) > 1.5$; and (b) in Chapter 6, the Fourier fit was done only to $n = 4$, whereas here we fit the data with higher order Fourier expansion.

The photometric data of all Cepheids, comprising 771 from OGLE and 14 from Moffett et al. (1998)+Barnes et al. (1999), were mainly fit with $n = 4$ to $n = 8$ Fourier expansions in the V- and I-band. However, for some of the OGLE long period Cepheids ($P > 11.5$ days), it is found out that the quality of the fitted light curves can be improved upon by using a higher order Fourier expansion, hence we extend the fit to $n = 12$ for these long period Cepheids. All the fitted light curves were visually inspected and the best-fit light curves from the different orders of the Fourier expansions were selected. The extinction is corrected with the standard procedure, i.e. $m_0^{(V,I)} = m_{(V,I)} - R_{(V,I)}E(B - V)$ with $R_V = 3.24$ and $R_I = 1.96$ (Udalski et al., 1999b). The values of $E(B - V)$ for each OGLE Cepheids are taken from the OGLE database (Udalski et al., 1999b), while for the Cepheids in Moffett et al. (1998)+Barnes et al. (1999), the values of $E(B - V)$ are adopted from Sandage et al. (2004). Finally, the extinction correction for the color is just $(V - I)_0 = (V - I) - (R_V - R_I) \times E(B - V)$.

9.2.1 Selection of the Cepheids

There are total of 785 LMC Cepheids in our sample. In order to guard against possible contamination from the first overtone Cepheids (Udalski et al., 1999a) and to be consistent with the previous studies (Chapter 6; Kanbur & Ngeow, 2004; Sandage et al., 2004; Udalski et al., 1999a), we remove the Cepheids with $\log(P) < 0.4$. This leaves 743 Cepheids in our sample. Before we use this sample to derive the PC, AC and PL relations, we need to clean-up the sample by removing the “bad” Cepheids. We first remove 9 Cepheids in the OGLE database without the V-band photometry, or

the number of V-band data is too low to fit a $n = 4$ Fourier expansion. We then remove 21 Cepheids with poorly fitted or unacceptable light curves (mainly in the V-band) in the sample, such as those with a large scatter of data points or with bad-phase coverage (large gaps between the phased data points), because the maximum and minimum values (as well as the means) from these light curves cannot be accurately determined.

Color-cuts are then applied to the remaining Cepheids to remove some outliers. We first plot out (as in Figure 9.1[a]) the extinction corrected PC relation at mean light, defined as $(V - I)_0^{\text{mean}} = [A_0(V) - A_0(I)] - (R_V - R_I)E(B - V)$ with A_0 obtained from equation (2.2)¹. The plot shows that there are number of outliers in the period-color plane, mostly with $\log(P) < 1.0$. The presence of these outliers is probably due to: (i) their extinction are either over or under estimated in the OGLE database; (ii) they have blue or red companions that cannot be resolved due to the problems of blending; or (iii) other unknown physical reasons. Detailed investigation of these outliers are beyond the scope of this chapter, but it is clear that they should be removed from the sample. However, they are classified as fundamental mode Cepheids because their amplitude ratio ($R_{21} = A_2/A_1$, Simon & Lee, 1981) follows the distribution of other normal fundamental mode Cepheids as evident from Figure 9.1(d). These outliers are removed with the adopted color-cut of $0.35 < (V - I)_0^{\text{mean}} < 0.95$, a compromise between maximizing the number of Cepheids in the sample and excluding the Cepheids with unusual color. Note that this color-cut is only applied to the short period Cepheids because there are not many long period Cepheids in the sample. Furthermore, the PC (mean) relation for the long period Cepheids is steeper than the short period Cepheids, hence as the period increases the color becomes redder, which can be excluded from the color-cut.

Some Cepheids with unusually low V- and I-band amplitudes were found in the sample. Their amplitudes are typically 2 ~ 3 times smaller as compared to the amplitudes of other Cepheids at given period. Some examples of the light curves for these low amplitude Cepheids are given in Kanbur et al. (2003). In addition, most of the light curves for these low amplitude Cepheids can be fitted with $n = 4$ Fourier expansion, while other Cepheids with “normal” amplitude may require higher order fits. These low amplitude Cepheids do exist in some galaxies (see, for example Pietrzyński et al., 2004). Kanbur et al. (2003) has briefly discussed some possible physical reasons for these Cepheids to have such low amplitudes, such as they are just entering or leaving the fundamental mode instability

¹We use this definition because it is the average of the color at all phases (from zero to one).

Figure 9.1 –The various relations and the color-magnitude diagram (CMD) for the LMC Cepheids, after removing Cepheids with $\log(P) < 0.4$ and those lacking of V-band photometry. The symbols are (see text for details): filled circles = Cepheid with poor fit of the light curves; open squares = the rejected Cepheids after color-cut of $0.35 < (V-I)_0^{\text{mean}} < 0.95$; open triangles = the excluded Cepheids after the amplitude cut; filled triangles = the removed Cepheids for possible duplicity; and open circles = three longest period Cepheids, which are the outliers in PC(max) relation. The dots and the crosses are the remaining good Cepheids from OGLE and Moffett et al. (1998)+Barnes et al. (1999), respectively.

strip (Buchler & Kolláth, 2002)² or they have different chemical composition³. It is also possible that they are first overtone Cepheids but mis-identified as fundamental mode Cepheids, because the

²Few of them do lie near the boundaries of the instability strip as shown in Figure 9.1(c).

³Cepheids in lower metallicity galaxies will have lower amplitudes (see, e.g., van Genderen, 1978; Paczyński & Pindor, 2000)

first overtone Cepheids normally have lower amplitudes and the light curves can be well fitted with lower-order Fourier expansions. Again, the detailed investigation of these low amplitudes Cepheids are beyond the scope of this chapter. Here, we apply a conservative amplitude cut of 0.3mag. in the V-band to remove the low amplitude Cepheids (see Figure 9.2). Note that the practice of removing the low amplitude Cepheids has been done in the literature, for example Pietrzyński et al. (2004) applied a cut of 0.4mag. to the Cepheids in NGC 6822. Besides that, we also remove OGLE-286532 and HV-2883 as they are clear outliers in the V-band amplitude vs. $\log(P)$ plot, as shown in Figure 9.2.

Figure 9.2 –The V-band amplitudes as function of $\log(P)$ for the Cepheids after removing the Cepheids without V-band data, the Cepheids with poorly fit light curves and the Cepheids with color-cut. The dashed line is the adopted amplitude cut. The two outliers in the long period range are also identified.

Some of the possible duplicated Cepheids were then removed in the OGLE database by consulting table 4 of Udalski et al. (1999b). A preliminary analysis of the PC relation for the remaining Cepheids reveals that the three longest Cepheids should be removed from the sample, because they are clear outliers in the PC plot at maximum light (see upper panel of Figure 9.3). Without these three longest

Figure 9.3 –The long period LMC PC relations at maximum and minimum light. Upper panel (a): The PC relation at maximum for the long period LMC Cepheids. The dashed line is the regression to all long period Cepheids, with the form of $(V - I)_0^{max} = 0.261 \log(P) + 0.207$, while the solid line is the regression to the Cepheids without the three longest period Cepheids, with the form of $(V - I)_0^{max} = 0.039 \log(P) + 0.470$. Lower panel (b): The PC relation at mean light for the long period Cepheids. The dashed and the solid lines are the regressions with and without the three longest period Cepheids, respectively.

period Cepheids, the PC(max) relation for the long period Cepheids is flat, which is consistent with the results found in Chapter 6. The hypothesis of the HIF-photosphere interaction also suggests the flatness of the PC relation at maximum light for long period Cepheids. However, as the period gets longer (with $\log[P] > 1.8$), the photosphere disengages from the HIF (Simon et al., 1993). It is clear from upper panel of Figure 9.3 that these three Cepheids bias the slope of the PC(max) relation by making the slope becomes steeper. Even though these three Cepheids fall on the ridge line of the PC relation at mean light, as shown in lower panel of Figure 9.3, they should be excluded from the sample because they are the outliers of the PC(max) relation. Hence, the final sample consists of 636 LMC Cepheids that will be considered further. The locations of the outliers from various selection

criteria are shown in Figure 9.1 for the PC(mean) relation, V-band PL relation, R_{21} -log(P) relation and the color-magnitude diagram (CMD).

9.3 The New Empirical Relations

To construct the empirical PC, AC and PL relations, we used the following quantities from the Fourier fits for the Cepheids as obtained from previous section:

- V-band amplitude: the difference of the numerical maximum and minimum from the Fourier expansion, $V_{amp} = V_{min} - V_{max}$.
- $(V - I)^{max}$: defined as $V_{max} - I_{phmax}$, where I_{phmax} is the I-band magnitude at the same phase as V_{max} .
- $(V - I)^{mean}$: defined as $A_0(V) - A_0(I)$, where A_0 are obtained from equation (2.2). This is very similar to the conventional definition of the mean color, $\langle V \rangle - \langle I \rangle$, where $\langle \rangle$ denotes the intensity mean.
- $(V - I)^{phmean}$: defined as $V_{mean} - I_{phmean}$, where I_{phmean} is the I-band magnitude at the same phase as V_{mean} . V_{mean} is the V-band magnitude closest to $A_0(V)$, the mean value in equation (2.2).
- $(V - I)^{min}$: defined as $V_{min} - I_{phmin}$, where I_{phmin} is the I-band magnitude at the same phase as V_{min} .
- $V^{max/min}$ and $I^{max/min}$: defined as the numerical maximum/minimum from the Fourier expansion.
- V^{mean} and I^{mean} : defined as the mean value in equation (2.2), the A_0 .

Note that these quantities (except the V_{amp}) have been corrected for extinction as mentioned in previous section.

To test the non-linearity of the PC, AC and PL relations, or the “break” at a period of 10 days, we apply the F -test as given in Chapter 5 & 6 (see Chapter 6 for the detailed formalism and the setup for the F -test). Simply speaking, the null hypothesis in the F -test is single line regression is sufficient, while the alternate hypothesis is that two lines regressions with a discontinuity (a break) at 10 days is necessary to fit the data. The probability $p(F)$, under null hypothesis, can be obtained with the corresponding F values and the degrees of freedom. In general, the large value of

F (equivalent to the small value of $p[F]$) indicates that the null hypothesis can be rejected. For our sample, $F \sim 3.0$ when $p(F) = 0.05$ (the 95% confident level), therefore the null hypothesis can be rejected if F is greater than 3 with more than 95% confident level and the data is more consistent with the two-line regression.

The empirical LMC PC and AC relations at maximum, means and minimum light for all, long and short period Cepheids are summarized in Table 9.1 & 9.2, and the corresponding plots are presented in Figure 9.4 & 9.5, respectively. A glance of Table 9.1 and Figure 9.4 suggests that the LMC PC relations are broken at maximum, means and minimum light. These are confirmed with the F -test results with $F_{PC}(\text{max, mean, phmean, min}) = \{7.88, 9.69, 18.0, 13.1\}$. Similarly, the F -test results for the AC relation are: $F_{AC}(\text{max, mean, phmean, min}) = \{132, 153, 150, 162\}$. Hence, the LMC PC and AC relations are non-linear (hence broken) at maximum, means and minimum light. Note that the flatness of the long period PC(max) relation as given in Table 9.1 (0.039 ± 0.071) is in good agreement with the slope found in Chapter 6 (-0.031 ± 0.101). Recall that equation (6.1) predicts that if the PC relation is flat at maximum light, then there is a correlation between the amplitude and the color at minimum light. This is seen in Table 9.1 (and in Figure 9.5) for the long period AC(min) relation, with a slope of 0.241 ± 0.049 .

Table 9.1 –The LMC period-color relation in the form of $(V - I) = a \log(P) + b$, and σ is the dispersion of the relation.

Phase	a	b	σ
All, $N = 636$			
Maximum	0.177 ± 0.017	0.323 ± 0.012	0.098
Mean	0.229 ± 0.013	0.478 ± 0.009	0.074
Phmean	0.248 ± 0.014	0.489 ± 0.010	0.081
Minimum	0.300 ± 0.013	0.530 ± 0.009	0.075
Long, $N = 58$			
Maximum	0.039 ± 0.071	0.470 ± 0.087	0.095
Mean	0.320 ± 0.053	0.386 ± 0.065	0.071
Phmean	0.409 ± 0.067	0.319 ± 0.083	0.091
Minimum	0.357 ± 0.057	0.486 ± 0.070	0.076
Short, $N = 578$			
Maximum	0.263 ± 0.029	0.272 ± 0.018	0.097
Mean	0.153 ± 0.022	0.523 ± 0.014	0.074
Phmean	0.141 ± 0.023	0.552 ± 0.014	0.078
Minimum	0.210 ± 0.022	0.582 ± 0.014	0.073

The fitted LMC PL relations at maximum, mean and minimum light are presented in Table 9.3. The plots of the PL relations at maximum/minimum light and at mean light are shown in Figure 9.6 & 9.7, respectively. The F -test results for these PL relations are: $F_V(\text{max, mean, min}) = \{1.53, 8.50, 17.9\}$, and $F_I(\text{max, mean, min}) = \{0.12, 7.16, 19.7\}$. The large F -values for both V-

Table 9.2 –The LMC amplitude-color relation in the form of $(V - I) = aV_{amp} + b$, and σ is the dispersion of the relation.

Phase	a	b	σ
All, $N = 636$			
Maximum	-0.283 ± 0.019	0.646 ± 0.015	0.092
Mean	0.019 ± 0.019	0.617 ± 0.014	0.091
Phmean	0.064 ± 0.021	0.608 ± 0.016	0.098
Minimum	0.148 ± 0.021	0.622 ± 0.015	0.097
Long, $N = 58$			
Maximum	-0.207 ± 0.049	0.714 ± 0.048	0.083
Mean	0.154 ± 0.050	0.631 ± 0.049	0.085
Phmean	0.242 ± 0.061	0.589 ± 0.059	0.103
Minimum	0.241 ± 0.049	0.693 ± 0.048	0.083
Short, $N = 578$			
Maximum	-0.419 ± 0.019	0.729 ± 0.014	0.076
Mean	-0.130 ± 0.018	0.708 ± 0.013	0.074
Phmean	-0.099 ± 0.109	0.708 ± 0.014	0.078
Minimum	-0.008 ± 0.020	0.716 ± 0.014	0.079

Figure 9.4 –The period-color (PC) relations for the LMC Cepheids at maximum, means and minimum light. The open and filled circles are for short and long period Cepheids, respectively. The solid and dashed lines are the fitted PC relations for the short and long period Cepheids, respectively.

and I-band PL relations at mean and minimum light strongly indicate that the PL relations at these two phases are not linear, and the data is better described with the broken PL relation. However, the small F -values at maximum light, with corresponding p -values of 0.217 and 0.883 for the V- and I-band PL(max) relations respectively, show that the null hypothesis of the F -test cannot be rejected. Hence there is no observed break seen in the PL(max) relation and the data is consistent with single line regression.

Figure 9.5 –Same as Figure 9.4, but for the amplitude-color (AC) relations.

Figure 9.6 –The LMC PL relation at maximum (left panel) and minimum (right panel) light. The upper and lower panels are for the V-band and the I-band PL relations, respectively. The open and filled circles are for short and long period Cepheids, respectively. The dashed lines are the fitted PL relations for the long period Cepheids. While the solid lines are the fitted PL relations for the short period Cepheids, and extended to the longer period range to compare with long period PL relations.

9.3.1 Testing of the Outliers

Could the outliers that were eliminated in the previous section influence the results of the non-linear PL and PC relations? We investigate this by considering the PC(mean) relation with the color-cuts, amplitudes cuts and the elimination of the three longest period Cepheids. In Table 9.4, we present the slopes for the short period PC relation from the least squares regression, along with

Figure 9.7 –Same as Figure 9.6, but for the LMC PL relations at mean light.

Table 9.3 –The period-luminosity relation in the form of $m_{(V,I)} = a_{(V,I)} \log(P) + b_{(V,I)}$, and $\sigma_{(V,I)}$ is the dispersion of the relation.

Phase	a_V	b_V	σ_V	a_I	b_I	σ_I
All, $N = 636$						
Maximum	-2.783 ± 0.045	16.700 ± 0.032	0.259	-2.965 ± 0.030	16.379 ± 0.021	0.169
Mean	-2.736 ± 0.036	17.088 ± 0.025	0.206	-2.965 ± 0.024	16.610 ± 0.017	0.140
Minimum	-2.547 ± 0.035	17.270 ± 0.025	0.202	-2.841 ± 0.024	16.741 ± 0.017	0.138
Long, $N = 58$						
Maximum	-2.941 ± 0.189	16.868 ± 0.233	0.254	-2.995 ± 0.129	16.411 ± 0.159	0.173
Mean	-2.598 ± 0.161	16.977 ± 0.198	0.216	-2.918 ± 0.112	16.591 ± 0.138	0.150
Minimum	-2.182 ± 0.166	16.893 ± 0.205	0.223	-2.528 ± 0.119	16.403 ± 0.147	0.160
Short, $N = 578$						
Maximum	-2.680 ± 0.077	16.640 ± 0.048	0.260	-2.946 ± 0.050	16.368 ± 0.031	0.169
Mean	-2.937 ± 0.060	17.205 ± 0.038	0.203	-3.090 ± 0.041	16.683 ± 0.026	0.138
Minimum	-2.818 ± 0.057	17.429 ± 0.036	0.194	-3.030 ± 0.039	16.852 ± 0.024	0.131

the results from the F -test with different color-cuts. For comparison, the slope for the long period PC(mean) relation is 0.343 ± 0.034 . From the table, we can see that the F -test result for the color-cut of $0.00 < (V - I)_0^{\text{mean}} < 5.00$ (i.e., all Cepheids are included) already suggested the two PC relations

are better than one PC relation. The F -test results are robust for further color cuts, i.e., all other color-cuts show significant non-linearity of the PC(mean) relations at a period of 10 days. Table 9.5 shows the similar results for different amplitude-cuts, and again the F -test results are robust with different amplitude-cuts. The F -value for the PC(mean) relation which include the 3 longest period Cepheids is 12.15, which is comparable to 9.69 when these longest period Cepheids are excluded. Therefore, the removal of the outliers with various selection criteria does not influence the empirical results presented in this chapter.

Table 9.4 –The slopes of the short period PC relation with different color-cut and the results of the F -test. Only the short period Cepheids within the color ranges are used in the F -test.

Color-cut	Slopes _S	N_{short}	F	$p(F)$
0.00-5.00	0.179 ± 0.031	648	4.273	0.014
0.00-1.25	0.169 ± 0.029	646	6.048	0.002
0.00-1.15	0.165 ± 0.027	644	7.313	0.001
0.00-1.05	0.182 ± 0.023	637	7.865	0.000
0.00-0.95	0.159 ± 0.022	632	13.25	0.000
0.00-0.85	0.161 ± 0.021	628	14.30	0.000
0.00-0.75	0.137 ± 0.019	595	26.85	0.000
0.35-1.25	0.164 ± 0.028	643	6.701	0.001
0.35-1.15	0.160 ± 0.026	641	8.128	0.000
0.35-1.05	0.177 ± 0.023	634	8.936	0.000
0.35-0.95	0.155 ± 0.021	629	15.10	0.000
0.35-0.85	0.156 ± 0.020	625	16.42	0.000
0.35-0.75	0.132 ± 0.018	592	31.50	0.000

Table 9.5 –The slopes of the PC relation with different V-band amplitude (AV)-cut and the results of the F -test. Only the Cepheids with V-band amplitude greater than the amplitude-cut are used in the F -test. The subscripts L and S refer to the long and short period Cepheids, respectively.

AV-cut	Slopes _S	N_S	Slope _L	N_L	F
0.20	0.154 ± 0.021	628	0.343 ± 0.034	65	15.23
0.25	0.152 ± 0.021	626	0.340 ± 0.034	64	15.45
0.30	0.149 ± 0.021	617	0.340 ± 0.034	64	15.93
0.35	0.152 ± 0.022	603	0.340 ± 0.034	64	14.78
0.40	0.143 ± 0.022	587	0.341 ± 0.035	63	16.83
Exclude OGLE 286532 & HV 2883					
0.20	0.154 ± 0.021	628	0.330 ± 0.037	63	13.09
0.25	0.152 ± 0.021	626	0.326 ± 0.037	62	13.35
0.30	0.149 ± 0.021	617	0.326 ± 0.037	62	13.81
0.35	0.152 ± 0.022	603	0.326 ± 0.037	62	12.78
0.40	0.143 ± 0.022	587	0.327 ± 0.038	61	14.71

9.4 Comparisons of the LMC and Galactic PC relations

Since the properties of the PC relation (the slope and the zero-point) at mean light are the average of the entire pulsation cycle, as shown in equation (1.2), then the non-linear LMC PC

relation (at mean light) seen in Tammann et al. (2002), Kanbur & Ngeow (2004), Sandage et al. (2004) and Ngeow et al. (2005) could be due to the behavior of the PC relation at certain phases of the pulsation (see the discussion in Chapter 1.2.1). In contrast to the LMC Cepheids, the Galactic PC relation (at mean light) is linear (see, e.g., Tammann et al., 2003; Kanbur & Ngeow, 2004, and Chapter 6 & 8). Therefore it is important to investigate and compare the LMC and Galactic PC relation at various phases, especially at the maximum and minimum light, to find out the differences that lead to the non-linear LMC and linear Galactic PC relation at mean light.

It is shown that the LMC PC relation at the maximum is flat for the long period Cepheids. However, the flatness of the PC(max) relation is also seen in the Galactic long period Cepheids (Simon et al., 1993; Kanbur & Ngeow, 2004; Kanbur et al., 2004, , see Chapter 6 & 8 also). The flatness of these PC relations has resulted in the non-linear behavior of the PC relation at maximum light for both of the Galactic and LMC Cepheids, as supported from the F -test (see Chapter 6 for the Galactic results)⁴. The F -test results for the PC relation at minimum light, however, are different between the LMC and the Galactic Cepheids, where the LMC PC(min) relation is non-linear but the Galactic PC(min) relation is not. It is possible that the minimum light behavior leads to the non-linear LMC PC relation at mean light.

The empirical Galactic and LMC PC relations are compared in Figure 9.8. It is clear from this figure that the PC(max) relations are broken, as the PC(max) relations are flat for the long period Cepheids but not for the short period Cepheids. At minimum light, both of the Galactic long and short period PC relations are consistent and agree with each other, leading to a linear PC(min) relation across the entire period range. However the slope of the LMC PC(min) relation is shallower for the short period Cepheids than the long period Cepheids, which leads to the non-linearity of the PC(min) relation. A similar situation is also occurs for the mean light PC relation.

To further investigate the non-linear PC relation at mean light for the LMC Cepheids and the difference between the Galactic and LMC PC relation, we calculated the multi-phase PC relations from phase zero to one. This is done by shifting the fitted light curves from Fourier expansion (equation [2.2], with the same t_0 in V- and I-band) to a common starting epoch. A convenient choice of this epoch is the (V-band) maximum light. The phase that corresponds to the V-band

⁴Note that although the flatness of the PC(max) relation occurs at $\log(P) \sim 0.8$ for the Galactic Cepheids, as suggested in Chapter 8 and in Kanbur et al. (2004), the period cut of $\log(P) = 1.0$ will be used to separate the long and short Galactic Cepheids in this chapter, in order to be consistent with the LMC data.

Figure 9.8 –Comparison of the empirical Galactic (dashed lines) and LMC (solid lines) PC relations at maximum (upper panel), mean (middle panel) and minimum (lower panel).

maximum light⁵, denoted as Φ_{max} , can be obtained from the fitted V-band light curves. Then, the light curves in *both* V- and I-band can be phase-shifted according to the following relations:

$$\phi_k(\text{new}) = 2\pi k\Phi_{max} + \phi_k(\text{old}), \quad (9.1)$$

where $\phi_k(\text{old})$ are from equation (2.2). Hence, the zeroth phase of the new light curves corresponds to the (V-band) maximum light. Since the Cepheid light curves are skew-symmetric with slow declination from maximum to minimum, the phase of minimum light is located roughly in between phases ~ 0.6 and ~ 0.8 . Table 9.6 summarized the averages of Φ_{min} , the phase at minimum light, from the samples.

⁵We assume the V-band light curve traces the luminosity curve, hence the maximum light occurs at the phase of minimum V-band light curve fitted from the Fourier expansion.

Table 9.6 – The average values of the ϕ_{min} .

Period-Range	GAL	LMC
All	0.675	0.733
Long	0.641	0.665
Short	0.687	0.740

Figure 9.9 – Variations of the slopes for the empirical Galactic and LMC PC relations as a function of pulsational phase. The solid and open triangles are for the Galactic and LMC PC relations, respectively. The dashed lines represent the zero slope, i.e., the slopes near these lines will have a flat PC relation. The maximum light occurs at the zeroth phase.

The results of the multi-phase PC relations are presented in Figure 9.9, where the slopes for the Galactic (filled triangles) and LMC (open triangles) PC relations as a function of pulsational phase (Φ) are compared. The overall behavior of the slopes for the PC relation (as a function of pulsational phase) is similar in both of the LMC and the Galactic Cepheids for all (top panel), long ($\log[P] > 1.0$, middle panel) and short ($\log[P] < 1.0$, bottom panel) period Cepheids. As mentioned before, the slopes for the PC(max) relation for both of the long period Galactic and LMC Cepheids are flat, which are seen in the middle panel as the slopes approach zero around phase zero ($\Phi = 0.0$),

corresponding to the phase of maximum light). This flatness of the slopes has extended to phase at $\Phi \sim 0.1$, then the slopes gradually increase until $\Phi \sim 0.8$ and quickly drop back to zero. In addition, the slopes of the long period LMC PC relation are generally steeper (larger values) at most phases than the Galactic counterparts. For the short period Cepheids, the slopes of both Galactic and LMC PC relations stay nearly constant at most phases except around $\Phi \sim 0.0$ and $\Phi \sim 0.8$. Note that the slopes for both Galactic and LMC short period Cepheids also approach zero at $\Phi \sim 0.8$ (Galactic: slope = 0.091 ± 0.065 at $\Phi \sim 0.80$; LMC: slope = 0.031 ± 0.022 at $\Phi \sim 0.84$). However, the plots of the PC relation around $\Phi \sim 0.8$ (see lower-left panel of Figure 9.12) show that the short period LMC PC relation is indeed nearly flat around this phase, but the short period Galactic PC relation can be broken further into two PC relations with a break at $\log(P) \sim 0.8$. The short period Galactic PC relation has a positive slope for $\log(P) < 0.8$ and a negative slope for $0.8 < \log(P) < 1.0$, hence the overall slope is close to zero as seen in the lower panel of Figure 9.9. This behavior of $\Phi \sim 0.8$ seen in the short period Cepheids is a new observational feature that has not been reported in the literature before. In contrast to the long period PC relations, the short period Galactic PC relations have steeper slopes than the LMC counterparts at all phases.

To further compare the difference between the Galactic and the LMC PC relations, the difference of the slopes for long and short period PC relations ($\Delta_{\text{slope}} = \text{slope}_L - \text{slope}_S$) are plotted as a function of pulsational phase in Figure 9.10. Even though Δ_{slope} appeared to be similar for the LMC and the Galactic PC relations⁶, a careful examination of Figure 9.10 reveals that, except the differences near phase zero, only a small fraction (about ~ 0.15 at $\Phi \sim 0.8$) of the pulsational phases that Δ_{slope} of the Galactic PC relations is different from the zero difference. In contrast, the fraction of the pulsational phases that Δ_{slope} is different from $\Delta_{\text{slope}} = 0$ (at $\Phi \sim 0.8$) for the LMC PC relations is ~ 0.35 , much larger than the Galactic value. This results in a mean difference of -0.034 and 0.167 for the Galactic and the LMC PC relations, respectively, which in part leads to a non-linear LMC PC(mean) relation and a linear Galactic PC(mean) relation. Note that the deviations near $\Phi \sim 0.0$ are due to the flatness of the PC relations for the long period Cepheids at this phase, which do not really affect the mean light PC relation.

Finally, the F -tests are applied to the Galactic and LMC multi-phase PC relations. The plots of the F -values as a function of phase are presented in Figure 9.11. In this figure, the dashed lines represent the 95% confidence level (or $p[F] = 0.05$), corresponding to $F \sim 3.0$. Therefore, the F -

⁶Besides the similar shapes of Δ_{slope} , the amplitudes of the differences for the Galactic and LMC PC relations are also similar, with amplitude, $|\Delta_{\text{slope}}(\text{MAX}) - \Delta_{\text{slope}}(\text{MIN})|$, of ~ 0.9 .

Figure 9.10 –Difference of the slopes for the long and short period PC relation as a function of phase, for the LMC (top panel) and the Galactic (bottom panel) Cepheids. The dashed lines represent the zero difference.

values above the dashed line indicate that the PC relation is non-linear, while the F -values below this line mean the PC relation is linear. From this figure, it can be seen that the LMC PC relations are not linear in most of the pulsational phases, except around $\Phi \sim 0.15$ and $\Phi \sim 0.95$. In contrast, the Galactic PC relations are linear in a large fraction of the pulsational phases, from $\Phi \sim 0.15$ to $\Phi \sim 0.70$ and again at $\Phi \sim 0.88$. Therefore, these make the LMC PC(mean) relation to be non-linear and the Galactic PC(mean) relation to be linear. Although the non-linearity of the Galactic and LMC PC relations at maximum light (with large F -values) has been mentioned previously, Figure 9.11 shows that this non-linearity extends in pulsational phases around the maximum light in both galaxies. Further, the non-linear PC relations around $\Phi \sim 0.8$ are not only seen in the LMC Cepheids but also occur for the Galactic Cepheids. Again, this is another new feature that has not been mentioned previously in the literature.

In short, the analysis performed in this section shows that:

Figure 9.11 –Comparison of the F -values for the PC relation as a function of phase for the LMC (upper panel) and the Galactic (lower panel) Cepheids. The dashed lines represent the 95% confidence level (or $p[F] = 0.05$), corresponding to $F \sim 3.0$. Hence, the F -values above the dashed line indicate that the PC relation is non-linear, while the F -values below this line suggest the PC relation is linear.

1. The shapes of the slopes as a function pulsational phase for the long and short period Galactic PC relations are comparable over a large fraction of the pulsational phases.
2. The long and short period LMC PC relations are steeper and shallower than the Galactic counterparts, respectively.
3. Long period Galactic and LMC Cepheids show a flat PC relation near and at maximum light, while this happens near $\Phi \sim 0.8$ for the short period LMC PC relation.
4. The average difference between the slopes for long and short period PC relation is larger in the LMC Cepheid than the Galactic Cepheids.
5. The LMC PC relations are non-linear at most of the pulsational phases, but the Galactic PC relations are not.

6. Besides the phases near maximum light, both of the Galactic and LMC PC relations are non-linear around $\Phi \sim 0.8$.

Although most of the discussion in this section is focused on the slopes of the PC relations, the zero-points of the PC relations also show similar properties. The snapshots of the LMC and Galactic PC relations at various pulsational phases are compared in Figure 9.12. The non-linearity with a discontinuity or a break at 10 days for the PC relation is clearly seen at the phase of 0.82 for both galaxies, as well as in most of the other phases for the LMC PC relations. The Galactic PC relation at $\Phi = 0.82$ (and $\Phi = 0.0$) hints there is another discontinuity at $\log(P) \sim 0.8$, however the detailed investigation is beyond the scope of this chapter.

9.5 Methods and LMC Models

The stellar pulsation codes, which include a 1-D turbulent convection recipe (Yecko et al., 1998), used to construct the LMC models are the same as in Chapter 8. Briefly speaking, the codes take the mass (M), luminosity (L), effective temperature (T_{eff}) and chemical composition (X, Z) as input parameters. The chemical composition is set to be $(X, Z) = (0.70, 0.008)$ to represent the LMC hydrogen and metallicity abundance (by mass). The mass and luminosity are obtained from the ML relations calculated from evolutionary models. The T_{eff} are chosen to ensure the models oscillate in the fundamental mode and located inside the Cepheid instability strip. The pulsation periods for the models are obtained from the linear non-adiabatic analysis (Yecko et al., 1998). All other parameters used in the pulsation codes, including the α parameters for the turbulent convection recipes, that were used in the Galactic models are kept the same when constructing the LMC models, with the only “variable” parameter being the metallicity. The only difference between this study and Chapter 8, besides the metallicity, is the value set for the artificial viscosity parameter, C_q . In this study, we set $C_q = 16.0$ for the LMC models to improve the shape of the theoretical light curves, in contrast to the value of 4.0 used for the Galactic models.

In Chapter 8, the ML relations are adopted from Chiosi (1989) and Bono et al. (2000c). In order to be consistent with previous chapter, the ML relations used in this chapter will also be adopted from these two sources. However, Chiosi (1989) only provided two ML relations, one for $Z = 0.20$ which are used in Chapter 8, and another one for $Z = 0.001$. Hence we have to adopt the second ML relation for the LMC models. Even though the LMC metallicity is higher than $Z = 0.001$, the LMC is still considered as a low metallicity system in the literature. Hence the Chiosi (1989) ML relation can be approximately applied for the LMC models. In summary, the ML relations used are:

Table 9.7 –Input parameters for LMC Cepheid models with periods obtained from a linear analysis. The periods, P_0 and P_1 , are referred to the fundamental and first overtone periods, respectively. Similarly for the growth rate, η . Both of the mass and luminosity are in Solar units, the temperature is in K and the period is in days.

M	$\log(L)$	T_{eff}	P_0	η_0	P_1	η_1
ML Relation from Bono et al. (2000c)						
11.0	4.375	5050	46.4155	0.124	28.98	-0.118
10.0	4.236	5100	35.6727	0.091	22.92	-0.093
9.50	4.161	5250	28.2406	0.094	18.68	-0.046
9.10	4.099	5260	25.3960	0.082	16.92	-0.042
8.75	4.042	5310	22.3804	0.076	15.07	-0.027
8.40	3.982	5380	19.3886	0.071	13.20	-0.008
7.95	3.902	5330	17.7750	0.055	12.09	-0.027
7.00	3.717	5410	12.6085	0.035	8.722	-0.018
6.55	3.620	5490	10.2940	0.031	7.183	-0.006
6.40	3.587	5485	9.81474	0.027	6.853	-0.010
6.00	3.493	5510	8.37226	0.020	5.866	-0.014
5.90	3.468	5500	8.12498	0.017	5.691	-0.018
5.80	3.443	5525	7.69466	0.017	5.400	-0.015
5.70	3.418	5560	7.23505	0.017	5.090	-0.009
5.30	3.312	5600	6.01283	0.012	4.244	-0.009
ML Relation from Chiosi (1989)						
7.20	4.272	5380	40.2561	0.275	24.51	-0.162
6.80	4.192	5380	35.4374	0.264	21.91	-0.122
6.20	4.063	5410	28.2629	0.225	17.94	-0.076
5.95	4.005	5420	25.6378	0.211	16.43	-0.060
5.40	3.869	5510	19.4314	0.170	12.82	-0.015
5.15	3.803	5510	17.5523	0.160	11.66	-0.007
4.65	3.660	5490	14.3143	0.131	9.611	-0.010
4.20	3.518	5510	11.3659	0.101	7.729	-0.011
4.00	3.450	5545	10.0011	0.089	6.854	-0.005
3.95	3.432	5540	9.77393	0.085	6.701	-0.008
3.80	3.378	5550	8.94637	0.075	6.157	-0.009
3.70	3.341	5575	8.31297	0.070	5.745	-0.005
3.65	3.322	5570	8.10751	0.066	5.605	-0.008
3.60	3.302	5530	8.09994	0.058	5.583	-0.022
3.60	3.302	5600	7.71463	0.065	5.352	-0.001

1. ML relation given in Chiosi (1989):

$$\log(L) = 3.22 \log(M) + 1.511. \quad (9.2)$$

2. ML relation given in Bono et al. (2000c):

$$\begin{aligned} \log(L) &= 0.90 + 3.35 \log(M) + 1.36 \log(Y) - 0.34 \log(Z), \\ &= 3.35 \log(M) + 0.886. \end{aligned} \quad (9.3)$$

The units for both M and L are in Solar units. Note that these two ML relations cover reasonable broad L/M ratios given in the literature. The input parameters for the LMC models with these ML relations and the periods calculated from linear non-adiabatic analysis are given in Table 9.7.

After the full amplitude models are constructed from the pulsation codes, the temperature and the opacity profile can be plotted in terms of the internal mass distribution ($\log[1 - M_r/M]$, where M_r is mass within radius r and M is the total mass) at a given phase of pulsation. As in Chapter 8, the locations of the HIF (sharp rise in the temperature profile) and photosphere (at optical depth $\tau = 2/3$) can be identified in the temperature profile. To quantify the HIF-photosphere interaction (if the photosphere is next to the base of the HIF or not, see also Chapter 8.3.3), we calculate the “distance”, Δ , in $\log(1 - M_r/M)$ between the HIF and the photosphere from the temperature profile. The definition of Δ can be found in Figure 8.9. A small Δ means there is HIF-photosphere interaction, and vice versa.

The theoretical quantities from the models can be compared to the observed quantities using the following prescriptions:

1. As in Chapter 8, we use the BaSeL atmosphere database (Lejeune, 2002; Westera et al., 2002) to construct a fit giving temperature and effective gravity as a function of $(V - I)$ color. The effective gravity is obtained at the appropriate phase from the models. These prescriptions are used to convert the temperatures to the $(V - I)$ colors. The bolometric corrections (BC) are obtained in a similar manner.
2. We use the prescriptions given in Beaulieu et al. (2001) to convert the observed colors to the temperatures appropriate for the LMC data as follows:

$$\begin{aligned}
 \log(g) &= 2.62 - 1.21 \log(P), \\
 \log(T_{eff}) &= 3.91545 + 0.0056 \log(g) - 0.2487(V - I)_0, \\
 \Delta T &= \log(T_{eff}) - 3.772, \\
 BC &= -0.0153 + 2.122\Delta T - 0.02 \log(g) - 11.65(\Delta T)^2.
 \end{aligned}$$

Note that these functions are also obtained from the BaSeL atmosphere database.

9.6 Results from the Models

The effective temperatures for the full amplitude models in Table 9.7 at the corresponding maximum and minimum light (or luminosity) are given in Table 9.8. For the effective temperatures at

mean light, Chapter 8 showed that the temperatures for the mean light at ascending and descending branch of the light (or luminosity) curve are not the same, hence Table 9.9 gives the effective temperature at these phases for our LMC models. The layout of Table 9.9 is the same as Table 8.3 from Chapter 8. Following Chapter 8, the locations of the photosphere can be identified in the temperature and opacity profiles. These are done in Figure 9.13-9.15 with a $\log(P) > 1.0$, a $\log(P) = 1.0$ and a $\log(P) < 1.0$ model, respectively. The left panels of Figure 9.13-9.15 are the temperature profiles, while the right panels are the opacity profiles. The photospheres are marked as filled circles in these figures. Finally, the plots of the Δ , the “distance” between the photosphere and the HIF from the temperature profiles, as function of pulsating periods for the LMC models are presented in Figure 9.16 with the two ML relations used. In Chapter 8, it is found that the distribution of Δ as a function of period is independent of the adopted ML relation. This is also seen in the LMC models as shown in Figure 9.16.

Figure 9.13-9.15 and Figure 9.16 show that at maximum light, the photosphere lies at the base of the HIF for all of the models. Although there is a slight deviation at some longer period models, the location of the photosphere is close to the HIF within the error bars (which are defined as the coarseness of the grid points around the location of the HIF). As in Chapter 8 for the Galactic models, the closeness of the photosphere to the base of the HIF, which is almost independent of the pulsation periods, results the flat PC relation for the long period LMC Cepheids seen in previous sub-sections. In case of the minimum light, even though Figure 9.16 shows that $\Delta(MIN)$ are nearly constant across the period range and the photosphere is near the base of the HIF, as in the case of maximum light, $\Delta(MIN)$ do show a shallow correlation with period after 10 days. Judging from the error bars of $\Delta(MIN)$ and from Figure 9.13-9.15, there is tentative evidence that the photosphere is disengaged from the HIF for $\log(P) > 1.0$. Hence the temperatures or the colors at minimum light follow the global properties with a steeper long period PC(min) relations⁷. For the short period models, there are some indications that the photosphere is next to the base of the HIF at both maximum and minimum light, which could produce a shallower slope of the PC relation as compared to the long period counterparts. Furthermore, the observational data, as shown in lower panel of Figure 9.9 and in Figure 9.12, suggest that the photosphere is engaged with the HIF at $\Phi \sim 0.8$ with a flat PC relation. This phase is close to the minimum light as shown in Table 9.6 (the average of the phases at minimum light for short period LMC Cepheids is 0.74). The right panels of Figure 9.16 show that the behaviors of Δ - $\log(P)$ plots at mean lights seems to be the mixture

⁷The slopes of the Δ - $\log(P)$ relation may not be correlated with the slopes of the PC relation

Table 9.8 –Temperatures at maximum and minimum light from full-amplitude non-linear model calculations. The periods, luminosity and temperature are in days, $L_{\odot}$ and K, respectively.

P	L_{max}	T_{max}	L_{min}	T_{min}
ML Relation from Bono et al. (2000c)				
46.4155	27481.28	5445.00	18329.74	4826.00
35.6727	19742.83	5434.01	14321.20	4962.65
28.2406	16894.15	5502.35	12093.99	4978.72
25.3960	14460.62	5562.28	10570.70	5010.85
22.3804	12772.75	5615.34	9283.257	5069.43
19.3886	11263.84	5687.98	8143.460	5145.20
17.7750	9034.392	5561.39	7044.743	5147.04
12.6085	5637.467	5528.15	4800.840	5278.11
10.2940	4371.170	5537.42	3839.001	5377.80
9.81474	4004.569	5518.39	3580.396	5383.61
8.37226	3217.688	5596.34	2929.354	5437.17
8.12498	3032.639	5581.53	2793.441	5438.96
7.69466	2864.745	5602.37	2640.439	5467.54
7.23505	2705.430	5639.03	2490.271	5504.98
6.01283	2105.589	5669.57	1986.586	5567.91
ML Relation from Chiosi (1989)				
40.2561	23500.98	5821.99	11853.38	4947.11
35.4374	19604.60	5830.18	9898.124	4962.26
28.2629	14661.29	5875.98	7659.896	5057.96
25.6378	12836.04	5884.01	6916.299	5108.02
19.4314	9538.226	5985.21	5555.179	5303.60
17.5523	8035.700	5921.07	4947.967	5142.58
14.3143	5490.345	5783.01	3639.964	5164.11
11.3659	3850.971	5814.49	2722.473	5242.47
10.0011	3286.030	5817.00	2377.950	5300.52
9.77393	3136.139	5801.77	2301.751	5296.73
8.94637	2738.003	5798.30	2071.106	5331.19
8.31297	2499.416	5794.49	1917.696	5365.34
8.10751	2374.713	5780.35	1850.048	5369.40
8.09994	2235.292	5723.18	1796.586	5355.41
7.71463	2271.393	5805.49	1772.270	5408.34

of the behavior at maximum and minimum light, although the Δ for the ascending and descending means follow the maximum and minimum light, respectively.

The temperature profiles from the Galactic and the LMC models are compared in Figure 9.17 at maximum and minimum light. The upper panels of Figure 9.17 show that at maximum light, the photosphere is not far from the base of the HIF in both of the Galactic and the LMC models. In contrast, the photosphere is further away from the HIF in the Galactic models than the LMC models at the minimum light. The plots of the Δ - $\log(P)$ relation from the Galactic and LMC models at maximum and minimum light are also compared in Figure 9.18. It can be seen from the figure that at maximum light, the behavior of both Galactic and LMC models are similar, where the photosphere are near the base of the HIF. At minimum light, the long period models show that the

Table 9.9 –Temperatures at mean light from full-amplitude non-linear model calculations. See text for the meanings of $\langle L \rangle$, L_{mean} , T_{mean} and T_{mean}^{inter} . The periods, luminosity and temperature are in days, $L_{\odot}$ and K, respectively.

P	$\langle L \rangle$	$L_{mean}(asc)$	$T_{mean}(asc)$	$L_{mean}(des)$	$T_{mean}(des)$	$T_{mean}^{inter}(asc)$	$T_{mean}^{inter}(des)$
ML Relation from Bono et al. (2000c)							
46.4155	24249.2	24423.695	5330.06	24282.129	4882.55	5319.42	4880.66
35.6727	17207.7	17078.471	5293.90	17188.051	4922.99	5305.14	4924.47
28.2406	14484.7	14466.869	5457.79	14529.348	5073.65	5459.55	5069.64
25.3960	12540.8	12461.087	5443.66	12544.735	5096.58	5452.77	5096.18
22.3804	10997.1	10996.587	5493.44	11039.123	5157.95	5493.51	5152.87
19.3886	9587.02	9560.1922	5545.87	9618.0583	5232.85	5549.58	5228.51
17.7750	7983.35	7988.5457	5473.94	7994.9159	5213.83	5473.06	5211.77
12.6085	5208.60	5208.1564	5493.34	5202.6440	5349.00	5493.36	5350.84
10.2940	4170.31	4162.1135	5560.81	4174.0993	5450.34	5564.43	5449.06
9.81474	3860.04	3861.4313	5556.01	3852.8715	5446.51	5555.34	5449.11
8.37226	3109.66	3106.7337	5570.85	3110.1806	5481.59	5572.55	5481.37
8.12498	2939.41	2935.0599	5553.67	2939.6504	5476.34	5556.38	5476.23
7.69466	2775.73	2775.1245	5581.66	2777.4057	5501.67	5582.05	5500.77
7.23505	2618.53	2620.1965	5620.29	2618.8674	5534.39	5619.15	5534.20
6.01283	2051.97	2050.8394	5646.29	2051.3277	5583.97	5647.27	5584.50
ML Relation from Chiosi (1989)							
40.2561	18754.3	18580.019	5704.60	18824.542	5144.86	5718.84	5141.42
35.4374	15896.6	15885.248	5749.07	15861.179	5147.12	5750.16	5158.63
28.2627	11157.8	11188.594	5692.54	11143.155	5111.26	5688.21	5112.18
25.6378	10388.8	10366.128	5782.33	10358.696	5184.81	5785.59	5188.79
19.4314	7743.89	7741.3087	5871.14	7750.7288	5332.20	5871.60	5330.76
17.5523	6538.19	6537.3819	5833.02	6532.0823	5310.84	5833.20	5312.28
14.3143	4564.83	4621.4163	5750.64	4565.7876	5280.59	5732.80	5280.34
11.3659	3289.17	3288.4584	5711.02	3281.3799	5328.36	5711.31	5331.32
10.0011	2811.35	2823.6619	5726.94	2800.7972	5376.66	5721.20	5381.50
9.77393	2702.54	2712.0209	5715.81	2692.4263	5378.63	5711.29	5383.51
8.94637	2385.34	2395.4576	5704.28	2388.3903	5412.32	5699.14	5410.62
8.31297	2189.29	2193.3351	5708.03	2188.7842	5448.47	5705.90	5448.80
8.10751	2094.38	2103.6034	5698.27	2093.0120	5449.90	5693.36	5450.83
8.09994	2004.90	1998.5077	5648.26	1999.8838	5419.02	5651.76	5422.70
7.71463	2004.07	2009.2157	5713.42	2004.9007	5490.38	5710.72	5489.80

photosphere is disengaged from the HIF, while the behavior of the short period models are different between the Galactic and LMC models. The photosphere of the short period LMC models seems to be located closer to the HIF at minimum light, but it is not the case for the short period Galactic models. This could lead to shallower slopes of the PC(min) relation seen in the LMC Cepheids as compared to the Galactic counterparts. In short, there is some tentative evidence from the models that the LMC long period Cepheids behave like the Galactic Cepheids, while the short period LMC Cepheids behave like the RR Lyrae stars at the minimum light. However, these findings are still not conclusive (see below).

Several theoretical quantities from the models can be compared with the observational data. This includes the pulsation periods, the V-band amplitudes and the Fourier parameters, the temperatures and colors at the maximum, mean and minimum light. These are the PC plots, the AC plots, the period-temperature plots and the Fourier parameters plots. The temperatures in Table 9.8 & 9.9, after conversion to the $(V - I)$ colors as mentioned in previous section, are superimposed along with the observed LMC PC relations as plotted in Figure 9.19. Similarly, Figure 9.20 show the same

quantities but on the $\log(T)$ - $\log(P)$ plane with the observed $(V-I)$ colors converted to temperatures using the prescriptions given in Section 9.5. The theoretical bolometric light curves are converted to the V-band light curves with the bolometric corrections obtained from the BaSeL database mentioned previously. From the theoretical V-band light curves, the amplitudes can be estimated and these are plotted in Figure 9.21 along with the colors from models to compare with the empirical AC relations. The Fourier parameters, obtained from equation (2.2), of the theoretical V-band light curves can also be obtained with $n = 6$ Fourier expansion. These Fourier parameters are compared with the observational data in Figure 9.22. Several features are noticed from Figure 9.19-9.22:

1. The general trends of the models qualitatively match the observational data.
2. The models with the ML relation from Chiosi (1989), with lower L/M ratio, seems to match the observations better. These models also tend to lie near the envelopes of the PC, AC, $\log(T)$ - $\log(P)$ and $A_i(V)$ - $\log(P)$ relations defined by the observational data.
3. The slopes of the period-color (or period-temperature) relations at maximum and minimum light from the models roughly match the observational data, i.e., the theoretical PC(max) relation is approximately flat but there's a relation at the minimum light.
4. The temperatures from the models with Bono et al. (2000c) ML relation is cooler (hence redder) than the models with Chiosi (1989) ML relation and the observed data at maximum light. In contrast, the temperatures (or the colors) from the models with these two ML relations coincide and are consistent with each other, and they are located near the blue edge of the observed data at minimum light.
5. The means at the descending branches are in better agreement with the observed data than the means at the ascending branches. This is because the observed means, $(V-I)_0^{\text{phmean}}$, are obtained mostly from the descending branches.
6. The amplitudes of the theoretical light curves (in both of the bolometric and V-band light curves) are smaller than the observations at given period, especially for the models with ML relation from Bono et al. (2000c). These can be seen from the AC relations as given in Figure 9.21 and the left panels of Figure 9.22.

Overall, some agreements and disagreements are found between the theoretical quantities and the observational data. It is also found out that there are some minor problems associated with the pulsation codes when the LMC models are constructed, which include the smaller amplitude of

the model light curves and the cooler temperatures at the maximum light (especially with Bono et al. 2000c ML relation). From equation (6.1), a lower amplitude implies that the temperature at maximum light should be cooler, hence these two problems are not independent. Varying other parameters in the pulsation codes, including the α parameters, does not improve the situations. Given that there are some successes of using the codes in Chapter 8 for the Galactic models and the codes have been used in several papers (Kolláth et al., 1998; Feuchtinger et al., 2000; Beaulieu et al., 2001; Buchler et al., 2004), we believe that the qualitative nature of the photosphere-HIF interactions as shown in Figure 9.16 and 9.18 still hold. This is in part because Figure 9.16 & 9.18 suggest that the behaviors of Δ as function of periods are nearly independent of amplitudes, as the models with Chiosi (1989) ML relation have higher amplitudes (although still smaller than the observations) than the models with Bono et al. (2000c) ML relation. However, better codes that fix these problems or the 3-D convection codes are needed to fully investigate the physics behind the non-linear LMC PL and PC relations in the future (see Chapter 10.2).

9.7 Conclusion and Discussion for this Chapter

Using the updated Cepheid data from the OGLE database, supplemented with some longer period Cepheids from Moffett et al. (1998) and Barnes et al. (1999), we have derived the new empirical relations for the LMC Cepheids after various selection criteria. F -statistical test are applied to these empirical relations to test their non-linearity at a period of 10 days. The results from the F -test strongly suggest that the LMC PC, AC and PL relations are non-linear at maximum, mean and minimum light, except for the LMC PL relation at maximum light. The high-quality data for the Galactic and LMC Cepheids allows the detailed study and comparisons of the PC relations as a function of pulsational phase. It is found out that the slopes of the LMC PC relations are steeper and shallower than the Galactic counterparts for the long and short period Cepheids, respectively. Further, the Galactic PC relations are linear over a large range of pulsational phases, except at/near phase of maximum light (with $\Phi = 0.0$) and at $\Phi \sim 0.8$. In contrast, the LMC PC relations are non-linear at most of the phases. These different behaviors have lead to the non-linear LMC PC relation and the linear Galactic at mean light. In addition, the non-linear PC relations at $\Phi \sim 0.8$ seen in both of the Galactic and LMC Cepheids are new features that has not been reported in the literature before. In fact, the non-linear PC relation at $\Phi \sim 0.8$ might be the largest contribution for the non-linear LMC PC relation seen at mean light.

The interaction between the HIF and photosphere at various phases, especially at maximum and minimum light, is attempted to account for the non-linear LMC PC relation. This is done by constructing the pulsation models for the LMC Cepheids with the same stellar pulsation codes as used in Chapter 8. The ML relations are also adopted from the same sources as in Chapter 8, which include the Chiosi (1989) and Bono et al. (2000c) ML relations. Following Chapter 8, the “distance” (Δ) between the photosphere and the HIF in temperature profiles is estimated. The plots of the Δ - $\log(P)$ relation from the Galactic and the LMC models are compared. The behavior of Δ at maximum light are similar in both of the Galactic and LMC models, where the photosphere is near the base of the HIF in large range of the periods. This leads to a flat PC(max) relation seen in the long period Galactic and LMC Cepheids. While at minimum light, the LMC models suggest that the photosphere is disengaged from the HIF for the long period Cepheids, which is also seen in the Galactic models. This makes the effective temperature at the photosphere follows the global PC relation appropriate for the long period Galactic and LMC Cepheids. However there are some indications that the photosphere is near the HIF for the short period LMC models, in opposite to the Galactic counterparts. This could force the short period LMC Cepheids to have shallower slope of the PC(min) relation.

Due to small number of LMC models, it is impossible to derive the theoretical PC, AC and PL relations and compare directly to the empirical relations. However, these LMC models can be qualitatively compared to the observations by converting some physical quantities to the observable quantities and vice versa, such as the temperature-color conversion. These comparisons were done with the PC, AC, period-temperature relations at maximum, mean and minimum light, as well as the comparisons of the Fourier parameters from theoretical light curves with observations. The theoretical quantities from the models generally agree with the observations, but it is found out that these models tend to have smaller amplitudes and (hence) the temperature is cooler at maximum light than the real Cepheids. Nevertheless we argued that the qualitative nature of the photosphere-HIF interaction is not seriously affected with these minor problems.

Finally, as mentioned in previous chapters, the behavior of the PC relations will affect the PL relations. This can be seen from Figure 9.23, where the F -test results as a function of phase are compared between the LMC PC and PL relations. From this figure it can be seen that the non-linearity of the PC relation traces the non-linearity of the PL relations at most of the phases except near $\Phi \sim 0.0$ (corresponding to the maximum light). The discrepancy near $\Phi \sim 0.0$ is due to the flat

PC relation at maximum light for the long period Cepheids⁸. The plots of the LMC PL relations at various phases (same as in Figure 9.12) are presented in Figure 9.24. The non-linearity of the LMC PL relations at certain phases is clearly visible from this figure, especially at $\Phi = 0.82$.

In conclusion, the steeper and the shallower slopes for the long and short period LMC Cepheids, as compared to the Galactic counterparts, have made the LMC PC, and hence the PL, relation to be non-linear at mean light. The photosphere-HIF interaction could be accounted for the behavior of the PC relations at certain phases, such as the long period LMC Cepheids at maximum light (which behave like Galactic Cepheids) and the short period LMC Cepheids at/near minimum light (which behave like RR Lyrae stars). At these phases, the photosphere is engaged with the HIF (where the photosphere locates next to the base of the HIF) that produce a flat PC relation, as seen from the observational data. In other phases, the photosphere is disengaged from the HIF and follows the global properties of the Cepheids. The detailed study of the multi-phase PC relations, as presented in Section 9.4, suggested that the non-linear nature of the LMC PC relation is more complicated than the HIF-photosphere interactions at maximum and minimum light. In other words, the HIF-photosphere interactions at maximum and minimum light probably only partially account for the non-linearity of the LMC PC relation. The global properties of the Cepheids at various phases of the pulsation, such as the metallicity dependency of the PC relations, and other factors could also play an important role in the non-linear LMC PC relation. It is desired to use better codes with more models to fully investigate the physics behind the non-linear LMC PC relation in the future.

⁸If the PC(max) relation is not flat for the long period Cepheids, then we expect the F -test results near $\Phi \sim 0.0$ would not show any non-linearity.

Figure 9.12 – Comparisons of the PC relations for the LMC and Galactic Cepheids at various phases.

Figure 9.13 –The temperature (left panel) and the opacity (right panel) profiles for a long period LMC model. The dotted, solid and dashed curves are for the profiles at maximum, mean and minimum light, respectively. The filled circles are the location of the photosphere at $\tau = 2/3$ for each phases. The mean light profiles at the ascending and descending branch are the solid curves that lie close to the profiles at maximum light (dotted curves) and minimum light (dashed curves), respectively.

Figure 9.14 –Same as Figure 9.13, but for a 10-days period LMC model.

Figure 9.15 –Same as Figure 9.13, but for a short period LMC model.

Figure 9.16 –The plots of Δ as function of $\log(P)$. The open and solid squares are the models calculated with Bono et al. (2000c) and Chiosi (1989) ML relation, respectively. The dashed lines represent (roughly) the outer boundary of the HIF.

Figure 9.17 –Comparisons of the temperature profiles between the Galactic and LMC models at maximum (upper panel) and minimum (lower panel) light. The solid and dashed curves represent the temperature profiles from the Galactic and the LMC models, respectively. The left and right panels are for the short and long period models, respectively, with the periods labelled at the upper left corners. The filled (for the Galactic models) and open (for the LMC models) circles are the locations of the photosphere in these models.

Figure 9.18 –Comparisons of the $\Delta\text{-log}(P)$ plots between the Galactic (left panel) and the LMC (right panel) models at maximum (top panel) and minimum (bottom panel) light.

Figure 9.19 –The LMC PC relations with the results from the models. The crosses are the observed data points as given in Figure 9.4. The open and solid squares are the models calculated with Bono et al. (2000c) and Chiosi (1989) ML relation, respectively. The temperatures of the models are converted to the $(V - I)$ color using the BaSeL database. Left panel: PC relations at maximum and minimum light. Right panel: PC relations at mean light for both of the ascending and descending means. The mean colors for the observed data are the $(V - I)_0^{\text{phmean}}$ as given in Figure 9.4.

Figure 9.20 –The plots of $\log(T)$ - $\log(P)$ relations for the LMC data and models. The symbols are the same as in Figure 9.19. The conversion of the $(V - I)$ colors to the temperature are done using the equations given in Beaulieu et al. (2001). Left panel: $\log(T)$ - $\log(P)$ relations at maximum and minimum light. Right panel: $\log(T)$ - $\log(P)$ relations at mean light for both of the ascending and descending means.

Figure 9.21 –The LMC AC relations with the results from the models. The crosses are the observed data points as given in Figure 9.5. The open and solid squares are the models calculated with Bono et al. (2000c) and Chiosi (1989) ML relation, respectively. The bolometric light curves from models are converted to V-band light curves with the BC obtained from the BaSeL database. Left panel: AC relations at maximum and minimum light. Right panel: AC relations at mean light for both of the ascending and descending means. The mean colors for the observed data are the $(V - I)_0^{\text{phmean}}$ as given in Figure 9.5.

Figure 9.22 –The Fourier parameters as function of periods for the LMC observational data (crosses) and the models, where open and solid squares represent the LMC models calculated with Bono et al. (2000c) and Chiosi (1989) ML relation, respectively. Upper Left: Plot of A_1 vs $\log(P)$. Lower Left: Plot of A_2 vs $\log(P)$. Right: Plot of ϕ_{21} vs $\log(P)$.

Figure 9.23 –Comparison of the F -values for the PL and PC relation as a function of phase. The dashed lines represent the 95% confidence level (or $p[F] = 0.05$), corresponding to $F \sim 3.0$. Hence, the F -values above the dashed line indicate that the PL/PC relation is non-linear, while the F -values below this line suggest the PL/PC relation is linear.

Figure 9.24 -Comparisons of the LMC V- and I-band PL relations at various phases.

CHAPTER 10

CONCLUSION AND FUTURE WORKS

In the final chapter of this Thesis Dissertation, we outline the main results from this Thesis Dissertation and some of the related future works.

10.1 Conclusion of this Thesis Dissertation

The aim of this Thesis Dissertation is to investigate the non-linearity and/or the non-universality of the Cepheid PL relations. The non-linear of the Cepheid PL relation, i.e. there are two PL relations with discontinuity at 10 days, is seen in the LMC Cepheids and it is because the LMC PC relation is also non-linear. These investigations have been conducted with two approaches: (a) empirical and statistical analysis of the existing observational data; and (b) constructing stellar pulsation models with existing pulsation codes.

To empirically study the non-linearity of the Cepheid PL and PC relations, it is not only desired to analyze the data in terms of the mean magnitudes, as conventionally used, but also at other phases such as the maximum and minimum light. To improve the light curve fitting method, we developed the Fourier expansion procedures with simulated annealing techniques and the Fourier interrelations, which can also be applied to the sparsely populated *HST* data. The *HST* data have been used to calibrate the Cepheid distances to about two dozen galaxies with both the LMC and Galactic PL relations. This is because the idea of the non-universality of the Cepheid PL relation has emerged when I began my thesis work. To improve the Galactic PL relation and further to test the non-universality of the PL relation, independent distances to 50 Galactic Cepheids were collected to calibrate the Galactic PL relation and a statistical test was performed to show that the Galactic Cepheids follow a different PL relation to their LMC counterparts. This is the *first evidence* from a statistical test that the empirical Cepheid PL relation is not universal.

The non-linearity of the LMC PL relation is due to the non-linear period-color (PC) relation observed in the LMC Cepheids. The photometric data of the Galactic, OGLE LMC and SMC Cepheid were used to study the PC relation not only at mean light, but also at maximum and

minimum light. We apply the F -statistical test to examine the significance of the break at 10 days for these PC relations. The results showed that the LMC PC relations are broken at maximum, mean and minimum light. However it is only broken at maximum light for the Galactic PC relation and marginally for the SMC PC(min) relation. The tests to the LMC PL relations also show the PL relations are broken at mean and minimum light, and marginally at maximum light. Again, this is the *first evidence* from rigorous statistical tests that the empirical LMC PC and PL relations are not linear. Since the previous study of the non-linear LMC PL relation is based on the OGLE data, the non-linear V-band LMC PL relation is further verified with the MACHO data. The study of the non-linear LMC PL relation is also extended further to the near infrared bands with 2MASS data. The effect of the non-linear PL and PC relation on the distance scale applications is found that this effect is minimal (with a difference of $< 0.07\text{mag.}$ in distance modulus), but it is important for the studies of stellar pulsation and evolution. A working hypothesis for the non-linear PC relation is the interaction between the hydrogen ionization front (HIF) and the photosphere, which can alter the slope of the PC relation. Hence we have run a sequence of Galactic models using the existing pulsation codes to investigate the connection between the HIF-photosphere interaction and the flatness of the PC relation. The Galactic models have confirmed the flatness of the PC relation at maximum light is due to this HIF-photosphere interaction.

Comparisons of the multi-phase PC relations between the Galactic and LMC Cepheids show that there are some difference between the Galactic and LMC PC relations, which lead to the observed linear Galactic and non-linear LMC PC(mean) relations, respectively. The LMC models provide some tentative evidence that the non-linear LMC PC relation is due to the fact that the photosphere is engaged with the HIF for short period LMC Cepheids but disengaged for the long period LMC Cepheids. However, other factors could also affect the non-linearity of the LMC PC relations. Even though it is needed to wait and use the improved stellar pulsation codes to finally draw the conclusion for the physics behind the non-linear LMC PC and PL relation, the works done in this Thesis Dissertation have established the firm foundation (both empirical evidence and theoretical modeling) to further understand and tackle the problems of the non-linear PC and PL relations.

10.2 Future Works

To further understand and characterize the non-universality and non-linearity of the Cepheid PL relation, we plan to conduct the following studies:

A. Empirical/Observational Studies: Current work on the break for the PL and PC relations is based on the Galactic, LMC and SMC Cepheids (Kanbur & Ngeow, 2004). We will expand the sample to include more galaxies that span a much wider range of metallicity, and an observational program using the ground-based telescopes will be constructed for this purpose. There exist some high quality photometric datasets that are available in the literature. For example, IC1613 from the Optical Gravitational Lensing Experiment; NGC300 & NGC6822 from the Araucaria Project; and M31 & M33 from the DIRECT Project. These datasets need to be analyzed in the context of PL, PC and amplitude-color relations, as in Kanbur & Ngeow (2004), in order to investigate the non-universality and non-linearity of the Cepheid PL relation. In addition, it is vital to have another galaxy with metallicity close to the LMC value, to see if there is a corresponding break for this galaxy. Our target galaxy is IC4182, and we have proposed a *HST-ACS* Cycle 14 observation, because this galaxy is nearby, face-on and has a metallicity close to LMC.

B. Theoretical Studies: The pulsation codes that are currently used have to include a 1-D recipe to model the time dependent 3-D turbulent convection. This recipe contains some dimensionless parameters related to important physical quantities (Yecko et al., 1998). These parameters are carefully chosen based on observational constraints and physical reasons. An extension of the project is to use the extensive observational data to better constrain or calibrate these parameters. Furthermore, a 3-D convection model will be constructed with existing codes (ZEUS-MP, Norman, 2000), in order to access the accuracy of using the 1-D recipe in stellar pulsation models.

As mentioned in Chapter 4, the Galactic PL relation from pulsational studies does not agree with the empirical results. Therefore, we plan to construct Galactic pulsation models with the full 3-D convection models and/or use the better calibrated parameters in the 1-D recipe, and compare them to the observations. We can also construct the models that are relevant to the LMC, as there are some minor problems associated with the current 1-D pulsation codes (see Chapter 9.6 for details), or even to the lower metallicity galaxies. This will allowed the detailed investigation on the non-linear and non-universal nature of the PL relations from pulsation modeling. Beside these, an additional outcome from this study is that we can compare the theoretical M_V - and M_I -band light curves (for given period) to the *HST* observed extra-galactic Cepheids. This allow us to *directly* obtain the distances to the target galaxies *independent of*

the adopted LMC distance: this is one of the largest systematic errors in the distance scale and circumvents a number of steps in the distance ladder.

The theoretical works mentioned above are not only beneficial to the study of Cepheid variables, but also can be extended to study other types of pulsational variables such as RR Lyrae and Type II Cepheids.

Finally, after these problems for the Cepheid PL relations are solved, one application of the improved PL relations is the re-calibration of the distances to Type Ia and II supernovae that can be used to estimate the Hubble constant.

APPENDIX A

THE DISTANCE TO NGC 1637: COMPARISON OF THE CEPHEID DISTANCE WITH THE EXPANDING-PHOTOSPHERE METHOD

¹Opportunities of comparing two primary distance indicators are rare. The discovery of SN 1999em in NGC 1637 provides such opportunity to compare the Expanding-Photosphere Method distance and the Cepheid distance for the first time. The EPM distance to SN 1999em has been measured by three independent groups, with the average value of $D_{EPM} = 7.86 \pm 1.57(\text{random})\text{Mpc}$. Using Hubble Space Telescope Cycle 10 observations of NGC 1637, we have discovered several dozen Cepheid variable stars, and derived $D_{Cepheid} = 11.33 \pm 0.21 \pm 1.10\text{Mpc}$. This represents the first direct comparison between the EPM applied to a well-observed, spectrally and photometrically normal, SN II-P and the distance determined through Cepheid variables. The result shows that the Cepheid distance is 44% greater than the EPM distance.

A.1 Introduction

Opportunities to directly compare distances estimated by two *primary* extra-galactic distance indicators are rare. The discovery of SN 1999em allows the comparison of the EPM (Expanding-Photosphere Method) distance to the supernova (SN) and the Cepheid distance to its host galaxy, NGC 1637 (Figure A.1), as SN 1999em is a Type II-Plateau event (Hamuy et al., 2001; Leonard et al., 2002; Elmhamdi et al., 2003).

The previous comparison of the EPM distance and Cepheid distance has been presented by Leonard et al. (2002), and showed that $D_{Cepheid} \sim D_{EPM}$. However, the five SN in the sample are either poorly observed or they are spectrally or photometrically peculiar Type-II SN. In contrast, SN 1999em is a well-observed and normal Type-II Plateau SN. This motivated us to have the first direct comparison of EPM and Cepheid distance. More details of this work are presented in Leonard et al. (2003).

¹Based on the talk presented in “Stellar Candles for Extra-Galactic Distance Scale” Workshop, Chile 2002.

Figure A.1 –The *HST* image of NGC 1637, taken ~ 2 years after the explosion of SN 1999em.

A.2 Comparisons of the Distances

1. **The EPM Distance:** The basic ideal of EPM distance has been covered in Eastman et al. (1996), Hamuy et al. (2001) and Leonard et al. (2002) and won't be repeated here. However, a brief description of the method is presented as followed.

The EPM distance is a variant of Baade's method that used in the study of Galactic Cepheids. In brief, the distance to the Type-II SN can be found by equating the two fundamental equations. The first equation comes from the expansion of the photosphere, where the angular size of the SN can be expressed as:

$$\theta = \frac{R}{D} = \frac{v_{phot}(t - t_0)}{D}, \quad (\text{A.1})$$

where v_{phot} is the expansion velocity of the photosphere and t_0 is the epoch of SN explosion.

The second equation arrives from the conservation of flux:

$$\theta = \sqrt{\frac{f_\lambda}{\zeta_\lambda(T_c)^2 \pi B_\lambda(T_c) 10^{-0.4A(\lambda)}}}, \quad (\text{A.2})$$

where f_λ is the observed flux at wavelength λ , $B_\lambda(T_c)$ is the black-body at temperature T_c (which can be obtained from the color of the SN), corrected for the extinction, $10^{-0.4A(\lambda)}$. Because in reality the photosphere of SN is not a perfect black-body, a factor of ζ has to be

included in equation (2) to account for this deviation. ζ is often referred as dilution factor, but sometimes it is also called as distance correction factor, because $D_{true} = \zeta D_{measure}$ (Eastman et al., 1996).

The EPM distance to SN 1999em has been measured individually by three independent groups: $D = 7.5 \pm 0.5 \text{Mpc}$ (Hamuy et al., 2001); $D = 8.2 \pm 0.6 \text{Mpc}$ (Leonard et al., 2002) and $D = 7.8 \pm 0.3 \text{Mpc}$ (Elmhamdi et al., 2003). The errors of these measurements are random errors only. If combine the three measurements together, then the EPM distance to SN 1999em is:

$$D_{EPM} = 7.86 \pm 1.57 \text{ Mpc.}$$

The error is taken to be 20% of the measurement because from the study of the EPM Hubble diagram, (Hamuy, 2001) found that the error in EPM distance is about 20%. However, the EPM distance can have uncertainty up to $\sim 60\%$, due to theoretical uncertainty of parameter ζ can have (Leonard et al., 2003). Different modelling groups can have different values of ζ , but only one group, Eastman et al. (1996), published the values of ζ for the application of deriving EPM distance. These values are applied by the three groups mentioned above to derive the EPM distance to SN 1999em.

2. **The Cepheid Distance:** The Cepheid distance is obtained from the *HST* observations to NGC 1637 (*HST* Cycle 10 observation). The photometry reductions are performed by using the *HSTphot* reduction package (Dolphin, 2000). In order to check the reduction process and analysis, we performed the same reduction procedures to archival data of NGC 3351 (Graham et al., 1997), and found the Cepheid distance to this galaxy is $30.08 \pm 0.06 \text{mag}$. Compared this distance to the Cepheid distance of $30.00 \pm 0.09 \text{mag}$., given in the final paper of HST Distance Scale Key Project (KP, Freedman et al., 2001), the two results are in good agreement. This comparison assured our photometric reduction and analysis are robust and reliable.

After performing the reduction and analysis, the final sample consist of 35 Cepheids in NGC 1637. These Cepheids are used to derive the Cepheid distance to the galaxy. The Cepheid PL relations used in this study are taken from Udalski et al. (1999a), and assume the LMC distance modulus of 18.50mag ., in order to be consistent to the KP. The PL relations are:

$$M_V = -2.760 \log(P) - 1.458, \quad (\text{A.3})$$

$$M_I = -2.962 \log(P) - 1.942, \quad (\text{A.4})$$

The final distance modulus has to be corrected for extinction and reddening, via the Wesenheit function (Madore, 1982):

$$\mu_0 = \mu_V - R(\mu_V - \mu_I), \quad (\text{A.5})$$

where $R = A_V/(A_V - A_I) = 2.45$. In addition, the metallicity of NGC 1637 is higher than LMC, as the value of $12 + \log[O/H]$ for this galaxy is 9.08 (Leonard et al., 2003), hence a correction of $0.12 \pm 0.12 \text{mag.}$ is applied to its distance modulus. The final Cepheid distance to NGC 1637 is:

$$D_{\text{Cepheid}} = 11.33 \pm 0.21 \pm 1.10 \text{ Mpc.}$$

The systematic error includes: uncertainties in slope and zero point of PL relations, metallicity correction, aperture correction and WFPC2 zero point correction. It is necessary to emphasize that the total errors of Cepheid distance can have is about 10% (or less), either from this work or from KP final results (Freedman et al., 2001), and significantly smaller than the total errors of EPM distance can have.

3. **Comparison of the Two Distances:** The EPM distance and Cepheid distance to NGC 1637 are given in previous two sub-sections, and can be readily compared. The result of the comparison shows that the Cepheid distance is greater than the EPM distance:

$$D_{\text{Cepheid}} \sim 1.44 D_{\text{EPM}}$$

Hence, which distance is likely to be more accurate? Given that the Cepheid distance has only 10% of error, and compared to the error of EPM distance, which can be as large as 60%, the Cepheid distance is seems to be more accurate and to be the true distance to NGC 1637.

Further evidence to support the Cepheid distance is likely to be correct is from the literature search of other independent distance measurements to this galaxy. There are four distance indicators favor the longer distance regime ($D > 10\text{Mpc}$), include (see Leonard et al. 2003 for details): Tully-Fisher, Inner Ring Structure, Spectral-Fitting Expanding Atmosphere Method & Standard Candle Method for SN II-P. In contrast, only one distance indicator, the Brightest Red-Supergiant, favors the short distance regime ($D < 10\text{Mpc}$).

Given that the Cepheid distance has much smaller error than EPM distance, and there are more evidences favor the longer distance regime, the Cepheid distance is likely to be the accurate distance to NGC 1637.

A.3 Conclusion & Discussion

Using EPM to derive distance to Type-II SN is still an emerging method, because it has total uncertainty of $\sim 60\%$ due to the theoretical uncertainty of the dilution factor. In contrast, using Cepheids to derive the distance to their host galaxy is a well-developed method, and only has uncertainty at the 10% (or less) level. The discovery of SN 1999em in NGC 1637 allowed the first direct comparison between the EPM distance and the Cepheid distance. From this study, it is found that: *The Cepheid distance is 44% greater than the EPM distance*, and the Cepheid is likely to be more accurate.

If the Cepheid distance to NGC 1637 is correct, then what does it implies to Hubble constant? In Hamuy (2001) study, the EPM Hubble diagram of 9 Type-II SN yielded $H_0 = 71 \pm 9 \text{ km s}^{-1} \text{ Mpc}^{-1}$. However, if this EPM Hubble diagram is re-calibrated using the Cepheid distance of NGC 1637, the resulted Hubble constant becomes: $H_0 = 49 \pm 6 \pm 11 \text{ km s}^{-1} \text{ Mpc}^{-1}$. The smaller value of Hubble constant is expected because the Cepheid distance is larger than EPM distance, resulting a smaller Hubble constant. The large systematic error is also resulted from the calibration with only one calibrator.

Certainly, the results of larger Cepheid distance or smaller Hubble constance are based on one comparison of the two distance indicators to NGC 1637, and clearly more similar comparisons are needed in the future to have final conclusion of the offsets between Cepheid and EPM distance and the resulted Hubble constant.

APPENDIX B

THE FOURIER INTRARELATIONS

The technique of Fourier interrelations, as described in Chapter 2.3, are the linear relations between the Fourier parameters in equation (2.2) in different wavebands. Similarly, there exist another group of linear relations between the Fourier parameters in the same waveband, known as the Fourier intrarelations. The reason for developing the Fourier intrarelations is same as in Fourier interrelation, mainly to reconstruct the I-band light curves that only contain 4 epochs as observed by *HST*. Therefore, the Fourier intrarelations are the linear relation between the 1st order Fourier parameter and the higher order Fourier parameters in one particular waveband, since 4 data points only permit the 1st order Fourier expansion. The Fourier intrarelations have the following expression for either V or I bands:

$$A_j = a_j + b_j A_1, \quad j = 2, \dots, 4 \quad (\text{B.1})$$

$$\phi_j = c_j + d_j \phi_1, \quad j = 2, \dots, 4 \quad (\text{B.2})$$

As in the case of Fourier interrelations, the coefficients are determined from the “calibration set” Cepheids (see Chapter 2.2). The results of the fits to the data with equation (B.1) & (B.2) are presented in Table B.1, which only listed out the 3rd and 4th order Fourier intrarelations. The 2nd order Fourier intrarelations were omitted because for sparse, 12 epoch V-band data, we can fit the data with 2nd order Fourier expansion and expand to 4th order with Fourier intrarelations to reconstruct V-band light curves. Then we can use the Fourier interrelations to reconstruct the I-band light curves from the V-band light curves. The plots for the Fourier intrarelations in “calibrating set” Cepheids are presented in Figure B.1(a) & B.1(b) for Fourier amplitudes and phases, respectively. From these figures, though a relation clearly exists, it may not be linear though we show the best fit linear relation. In the plots of A_3 against A_1 for both V- and I-band, there are some stars which lie well below the best fit linear relation. The A_4 verses A_1 plots also show some evidence of a non-linearity. Since the error bars have been plotted on these diagrams, the trends described here are real. This non-linearity may due to differences in long and short period Cepheids.

Table B.1 –The coefficients of the Fourier Intrarelations, as determined from “calibrating set”, for fundamental mode Cepheids. Coefficients are corresponding to those defined in equation (B.1) & (B.2). χ^2 parameter is same as in Table 2.2.

Relation	a	b	χ^2
$A_3(V)$	-0.043 ± 0.002	0.281 ± 0.006	3.97×10^2
$A_4(V)$	-0.021 ± 0.002	0.139 ± 0.006	2.97×10^2
$A_3(I)$	-0.025 ± 0.002	0.275 ± 0.010	2.77×10^2
$A_4(I)$	-0.014 ± 0.002	0.145 ± 0.010	1.98×10^2
Relation	c	d	χ^2
$\phi_3(V)$	2.544 ± 0.020	3.017 ± 0.008	2.42×10^3
$\phi_4(V)$	0.202 ± 0.034	4.061 ± 0.013	1.40×10^3
$\phi_3(I)$	3.122 ± 0.027	3.043 ± 0.011	1.72×10^3
$\phi_4(I)$	0.887 ± 0.044	4.122 ± 0.020	9.02×10^3

The Fourier intrarelations in OGLE LMC Cepheids have also been found, and presented in Ngeow et al. (2003). The possible non-linearity of the Fourier intrarelations that seem on the “calibrating set” Cepheids also show up in the plots of OGLE LMC intrarelations, as some stars which lie below the best fit linear regression and the indications of non-linearity in the A_4 vs. A_1 plots. Furthermore, the slope of the best fit line increases from the “calibrating set” Cepheids to the LMC Cepheids. The difference in slope between “calibrating set” and LMC Cepheids found in the intrarelations plots is real and is probably attributable to the metallicity differences. Due to the possible of non-linearity and metallicity dependence on the parent galaxy, the Fourier intrarelations are less preferable than the Fourier interrelations (Chapter 2.3) for reconstructing the I-band light curves with only few data points available.

Figure B.1 –The intrarelations among the Fourier amplitudes and Fourier phases in the “calibrating set” Cepheids. The dashed lines are the best-fit straight lines to the data.

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