April 2021

Phenomenology of Fermion Production During Axion Inflation

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A Dissertation Presented

by

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Submitted to the Graduate School of the

University of Massachusetts Amherst in partial fulfillment

of the requirements for the degree of

DOCTORATE OF PHILOSOPHY

February 2021

College of Natural Sciences

Department of Physics

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ACKNOWLEDGEMENTS

I would like to thank my advisor, Lorenzo Sorbo, for his tremendous patience during this process. His guidance and advice has been invaluable. I would like to express my appreciation to every member of my committee for their time. I would also like to thank Emily Barrett for providing support and encouragement. Thank you to all my friends and family for helping me through this process for all these years.

ABSTRACT

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FEBRUARY 2021

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We study the production of fermions through a derivative coupling to an axion inflaton and the effects of the produced fermions on the scalar and tensor metric perturbations. We show how such a coupling can arise naturally from supergravity with an axion-like field driving large-field inflation and small instanton-like corrections. We present analytic results for the scalar and tensor power spectra, and estimate the amplitude of the non-Gaussianties in the equilateral regime. The scalar spectrum is found to have a red-tilted spectral index, small non-Gaussianities, and can be dominant over the vacuum contribution. In contrast, the tensor power spectrum from the fermions is always subdominant compared to the vacuum contribution. The pseudoscalar coupling will favor production of one chirality of the fermions over the other, and therefore, it is expected that the resulting gravitational waves will be chiral. However, the parity-odd component of the power spectrum is shown to be subdominant to the parity-even component. The combined results of the scalar and tensor spectra allow to lower the energy scale of inflation, effectively lowering the tensor-to-scalar ratio. Chaotic inflation, in which the inflationary potential is quadratic, is currently ruled out as it predicts too large a tensor-to-scalar ratio. We show how this fermion-axion model can allow for quadratic inflation which agrees with all current measurements.

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Chapter 1

Inflation and cosmological perturbations

1.1 Introduction

The standard model of cosmology assumes the cosmological principle: that spacetime is isotropic and homogeneous. Though the stars, galaxies and small-scale structures of the universe seem to violate this principle, on large scales, $\gg 10$ Mpc, it holds quite well. The best measurement of the isotropy and homogeneity of the universe comes from the cosmic microwave background radiation (CMB). The temperature variations in the CMB are isotropic to about one part in 10^4 [1] after subtracting the dipole due to our galaxy's motion relative to the cosmological rest frame. Measurements of the matter content of the universe - namely the baryonic, radiation, dark matter, and dark energy - from Planck [1], have determined that the universe is spatially flat. The spatial curvature of the universe is characterized the density parameter, $\Omega \equiv \frac{\rho}{\rho_{\rm crit}}$, the ratio of the energy density of the universe to the critical density – the density which separates an open universe from a closed one. Observations show that $\Omega = 1.00^{+0.07}_{-0.03}$, suggesting a flat universe.

The homogeneity and flatness of the universe are examples of fine-tuning issues which require very specific initial conditions in the standard big bang cosmology. These observations are explained by postulating that the early universe underwent a period of rapid expan-

sion called inflation [2, 3]. During inflation, any initial spatial curvature or inhomogeneities would be quickly tamed, leading to the seemingly fine-tuned initial conditions. Further, quantum fluctuations of the matter fields during inflation will be stretched to superhorizon scales where they freeze, effectively becoming classical degrees of freedom. These will then seed density perturbations in the metric, giving rise to the observed inhomogeneities in the CMB, and in our present universe.

There is strong observational evidence supporting the hypothesis that the early universe went through a rapid period of accelerated expansion. Inflation provides a simple explanation for the observed red-tilted, approximately Gaussian and adiabatic density fluctuations [4, 5]. The inflationary scenario also generically predicts primordial gravitational waves, which can be measured or constrained through the B-mode polarization of the cosmic microwave background. The relative magnitudes of the power spectra of the tensor perturbations to the scalar density perturbations is called the tensor-to-scalar ratio, r. Current measurements restrict this to $r \lesssim 0.09$ [6, 7], which tightens to $r \lesssim 0.06$ when the consistency relation $n_t = -r/8$, as appropriate for vacuum fluctuations from slow-roll inflation, is imposed. Future experiments are expected to reach $\sigma_r \sim 0.001$ [8], which motivates the study of mechanisms for the inflationary expansion and for the production of the primordial perturbations that go beyond the most minimal slow-roll inflationary models.

The production of secondary particles during inflation can alter the signatures in the CMB dramatically. These particles will generate additional perturbations to the metric which will contribute to the scalar and tensor power spectra. This allows for these theories to make definite predictions to be compared with measurements. The sourced fields can also back react on the inflaton, and, in some cases, allow for inflation on steep potentials [9]. There is a rich range of phenomenology to be studied in this field.

We show in this work how the production of fermions has many novel properties. Due to Pauli blocking, long wavelength fermions can not be generated in large quantity. As such, fermion production has been studied comparatively little. However, we give a model in which the produced fermions fill the Fermi sphere up to a large momentum, therefore allowing for a large total occupation number. While large production of bosons can lead to large non-Gaussianities, putting those models in tension with measurements, we find that

our model of fermion production avoids this. This model is shown to allow for the lowering of the energy scale of inflation, thereby lowering the tensor-to-scalar ratio. In particular, this can be used to bring the currently ruled out chaotic inflation back into agreement with all current measurements. This is significant, as chaotic inflation is among the simplest models of inflation and can be embedded quite easily in supergravity [10, 11, 12, 13]. This opens the door for further study not only on chaotic inflation, but to other models of inflation as well.

1.2 Observables of inflation

The background spacetime metric for a homogeneous-isotropic universe is the Friedmann-Robertson-Walker (FRW) metric, $\mathrm{d}s^2 = -\mathrm{d}t^2 + a(t)^2\delta_{ij}\mathrm{d}x^i\mathrm{d}x^j$, characterized by Hubble parameter $H \equiv \dot{a}/a$, where $a(t)$ is the scale factor giving the physical distance between points. The evolution of the scale factor is determined by the Einstein field equations,

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = \frac{1}{M_P^2}T_{\mu\nu}\,, \tag{1.2.1}$$

where $M_P = (8\pi G)^{-1/2} = 2.435 \times 10^{18}$ GeV is the Planck mass. Assuming the matter content of the universe is an isotropic fluid with energy density ρ and pressure p, this reduces to the Friedmann equations,

$$H^2 = \frac{\rho}{3M_P^2}\,, \tag{1.2.2}$$

$$\dot{H} = -\frac{1}{2M_P^2}(\rho + p)\,. \tag{1.2.3}$$

From eq. (1.2.3), we see that if $p = -\rho$, then H is constant, and the scale factor is exponential, $a(t) = \mathrm{e}^{Ht}$. This can lead to inflation.

The simplest model of inflation is single-field slow-roll inflation, in which inflation is driven by a scalar field known as the inflaton, ϕ. A scalar field has energy density $\rho =$

$\frac{1}{2}\dot{\phi}^2 + V$, and pressure $p = \frac{1}{2}\dot{\phi}^2 - V$. In order for the inflaton to drive inflation, it must have a potential energy which dominates over its kinetic energy, $\frac{1}{2}\dot{\phi}^2 \ll V$, which will lead to $p \approx -\rho$. The suitability for the inflaton potential to support inflation is characterized by the slow-roll parameters,

$$\epsilon \equiv \frac{M_P^2}{2}\left(\frac{V'}{V}\right)^2 , \qquad \eta \equiv M_P^2 \frac{V''}{V} , \tag{1.2.4}$$

where $'$ denotes derivative with respect to the inflaton ϕ. Inflation requires $\epsilon, |\eta| \ll 1$. Current measurements give the constraints on the slow-roll parameters, $\epsilon < 0.0042$, and $\eta = -0.0124^{+0.0033}_{-0.0052}$ [1]. The simplest picture of an inflationary potential is one with a high energy and gradual slope, such that the inflaton will slowly roll down it during inflation – hence the name slow-roll inflation. Though this feature is present in most models of inflation, we will show that suitable particle production can act as friction against the inflaton rolling down its potential, leading to inflation even in steep potentials.

The field ϕ will have a homogeneous background part, as well as inhomogeneous perturbations on top of this, $\phi(\mathbf{x}, t) = \phi_0(t) + \delta\phi(\mathbf{x}, t)$. These perturbations in the inflaton, $\delta\phi(\mathbf{x}, t)$ will generate perturbations in the metric.

The perturbations are characterized by the correlators of the fields corresponding to the scalar perturbations, ζ, and tensor perturbations, h_{ij}. These are typically given in terms of the power spectra,

$$\mathcal{P}_\zeta(k) = \frac{k^3}{2\pi^2}\langle\zeta(\mathbf{k}_1)\zeta(\mathbf{k}_2)\rangle' , \qquad \mathcal{P}_t(k) = \frac{k^3}{2\pi^2}\langle h_{ij}(\mathbf{k}_1)h^{ij}(\mathbf{k}_2)\rangle' , \tag{1.2.5}$$

where the $'$ denotes correlators without the factor of $(2\pi)^3\delta(\mathbf{k}_1 + \mathbf{k}_2)$, and $k = |\mathbf{k}_1| = |\mathbf{k}_2|$. These can be modeled as an approximate power law, $\mathcal{P}(k) \sim k^{n(k)}$, with a generally slowly varying exponent called the spectral tilt. Conventionally, the scalar and tensor spectral tilts are defined as

$$n_s - 1 = \frac{\partial \ln \mathcal{P}_\zeta(k)}{\partial \ln k}\,. \qquad n_t = \frac{\partial \ln \mathcal{P}_t(k)}{\partial \ln k}\,. \tag{1.2.6}$$

The scalar power spectrum is found to be nearly scale-invariant. Measurements from Planck find $n_s = 0.9649 \pm 0.0042$, where $n_s = 1$ would define a scale-invariant spectrum. Measurements of the tensor perturbations are generally reported in terms of the tensor-to-scalar ratio,

$$r \equiv \frac{\mathcal{P}_t(k)}{\mathcal{P}_\zeta(k)}\,, \tag{1.2.7}$$

where Planck constrains $r \lesssim 0.06$.

Non-Gaussianities of the curvature perturbations are found in the higher-order correlators. In this thesis, we consider the bispectrum for the scalar modes, given in terms of the three-point function,

$$B_\zeta(\mathbf{k}_i) = \langle \zeta_{\mathbf{k}_1} \zeta_{\mathbf{k}_2} \zeta_{\mathbf{k}_3} \rangle'\,. \tag{1.2.8}$$

The density and inflaton perturbations are related through the gauge relation [14]

$$\zeta = -\frac{H}{\dot{\phi}_0}\delta\phi + O(\delta\phi^2)\,. \tag{1.2.9}$$

Therefore, one can compute correlators of the scalar perturbation in terms of $\delta\phi$.

1.3 Quantization of the inflaton field and predictions of single-field slow-roll inflation

We now examine single-field slow-roll inflation, where inflation is driven by a scalar, ϕ. We derive the equations of motion for perturbations of ϕ in a de Sitter background (an FRW spacetime with scale factor $a(t) = \mathrm{e}^{Ht}$), and show how to quantize the field in such a background. Finally, we give the predictions for the CMB observables defined in the previous section for this inflation scenario. The action for the inflaton is

$$S_\phi = \int \mathrm{d}^4x\sqrt{-g}\big(-\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi)\big)\,. \tag{1.3.1}$$

First, we will treat ϕ as a spatially homogeneous field $\phi_0(t)$ in a background de Sitter space. This is effectively the infinite wavelength mode of the inflaton. The equations of motion are

$$\ddot{\phi}_0 + 3H\dot{\phi}_0 + V'(\phi) = 0\,. \tag{1.3.2}$$

We see that the expansion of space acts as a friction term, $3H\dot{\phi}$. Since the slow-roll approximation requires $\dot{\phi}^2 \ll V$, we can expect that $\ddot{\phi}_0$ is negligible, leaving $3H\dot{\phi}_0 \approx -V'$. Combining this with the Friedmann equation (1.2.2), $3M_P^2H^2 \approx V$, the slow-roll condition becomes,

$$\epsilon = \frac{M_P^2}{2}\frac{(V')^2}{V^2} \ll 1\,, \tag{1.3.3}$$

reproducing the condition given in (1.2.4). The zero mode of the inflaton drives inflation so long as $\epsilon < 1$. The rolling of the field down its potential can provide the energy to source secondary particles and is much of the focus of the remainder of this work. The perturbations of the inflaton generate the inhomogeneities in the early universe and can have interesting

interactions with other fields.

The vacuum of quantum field theory is full of fluctuations of particles of all momenta appearing and annihilating. During inflation, space is expanding exponentially, and therefore, the wavelength of the quantum fluctuations are also stretched exponentially. The horizon radius, H^{-1}, remains nearly constant during inflation, and so eventually, the wavelengths will leave the horizon. When this happens, the modes become "frozen". The modes become classical in the sense that the commutator vanishes. Inflation expands the universe by a factor of $\sim \mathrm{e}^N$, where N is referred to as the number of e-folds. In order to explain the observed homogeneity of the universe, the number of e-folds between when the largest observable scales left the horizon and the end of inflation needs to be around 50-60. Once inflation ends, the horizon begins to expand faster than the scale factor and eventually, the modes reenter the horizon. These density perturbations then seed matter density perturbations.

We now turn to the production of the quantum fluctuations of the inflaton. It will be convenient to work in conformal time, τ, defined by $\mathrm{d}t = a\mathrm{d}\tau$. Conformal time ranges from $-\infty$ to 0. Derivatives with respect to conformal time are denoted $'$. In these coordinates, the background metric is $\mathrm{d}s^2 = a^2(\tau)(-\mathrm{d}\tau^2 + \delta_{ij}\mathrm{d}x^i x^j)$, and the scale factor is $a(\tau) = -\frac{1}{H\tau}$.

We perturb ϕ about a homogeneous background, $\phi(\mathbf{x},\tau) = \phi_0(\tau) + \delta\phi(\mathbf{x},\tau)$, and expand the action to second order in $\delta\phi$. We will examine the case for the nearly massless inflaton, in which we can ignore the $V''(\phi)$ term. This gives the equations of motion for $\delta\phi$,

$$\delta\phi'' + 2aH\delta\phi' - \nabla^2\delta\phi = 0\,. \tag{1.3.4}$$

To solve this, we Fourier transform $\delta\phi$,

$$\delta\phi(\mathbf{x},\tau) = \int \frac{\mathrm{d}^3k}{(2\pi)^{3/2}}\delta\phi_k(\tau)\mathrm{e}^{i\mathbf{k}\cdot\mathbf{x}}\,. \tag{1.3.5}$$

This puts the equation in the form

$$\delta\phi_k'' + 2aH\delta\phi_k' + k^2\delta\phi_k = 0\,. \tag{1.3.6}$$

By making the substitution $\delta\phi = \sigma/a$ we can cast this equation into the form

$$\sigma_k'' + \Big(k^2 - \frac{a''}{a}\Big)\sigma_k = 0\,. \tag{1.3.7}$$

Modes whose wavelength is much smaller than the horizon, that is $\frac{k}{aH} = -k\tau \gg 1$, are insensitive to the expansion of space. Therefore we expect that these modes will be oscillating modes as in Minkowski space. Indeed, in this short wavelength limit, (1.3.7) is simply $\sigma_k'' + k^2\sigma_k = 0$, the Klein-Gordon equation for a massless scalar in Minkowski space. From this, we require that in the UV limit, the solution to (1.3.7) matches onto

$$\sigma_k(\tau) \sim \frac{\mathrm{e}^{-ik\tau}}{\sqrt{2k}}\,, \qquad -k\tau \gg 1\,. \tag{1.3.8}$$

This choice of normalization is known as the Bunch-Davies vacuum. With this boundary condition, the full solution for the inflaton perturbations is

$$\delta\phi_k(\tau) = \frac{\mathrm{e}^{-ik\tau}}{a\sqrt{2k}}\Big(1 - \frac{i}{k\tau}\Big)\,. \tag{1.3.9}$$

For superhorizon scales, ones which satisfy $-k\tau \ll 1$, the perturbations become constant, with magnitude

$$|\delta\phi_k(\tau)| = \frac{H}{\sqrt{2k^3}}\,, \qquad -k\tau \ll 1\,. \tag{1.3.10}$$

The inflaton field is quantized in the standard way,

$$\delta\hat{\phi}_{\mathbf{k}}(\tau) = \delta\phi_k(\tau) a_{\mathbf{k}} + \delta\phi_k^*(\tau) a^\dagger_{-\mathbf{k}}\,, \tag{1.3.11}$$

with creation and annihilation operators satisfying the commutation relations, $[a_{\mathbf{k}}, a^\dagger_{-\mathbf{k}'}] = \delta^3(\mathbf{k}-\mathbf{k}')$. $a^\dagger_{\mathbf{k}}$ and $a_{\mathbf{k}}$ diagonalize the early-time Hamiltonian. As mentioned in Section 1.2, the perturbations in the inflaton field generate scalar perturbations in the gravitational field. These two field perturbations are related by the gauge relation (1.2.9). This then allows to compute the scalar power spectrum defined in (1.2.5),

$$\mathcal{P}_\zeta(k) = \frac{H^2}{8\pi^2\epsilon M_P^2}\,. \tag{1.3.12}$$

Since the modes freeze out when crossing the horizon, we evaluate the power spectrum at $k = aH$. Since H is nearly constant during inflation, $\mathrm{d}\ln k = \mathrm{d}k/k \approx \mathrm{d}a/a = (\dot{a}/a)\mathrm{d}t = H\mathrm{d}t$. Therefore, the spectral index for single-field slow-roll inflation is approximately

$$n_s - 1 = \frac{\partial \ln \mathcal{P}_\zeta(k)}{\mathrm{d}\ln k} \approx 2\eta - 6\epsilon\,. \tag{1.3.13}$$

The process is much the same with the tensor modes. The quanta of gravitational waves are massless spin-2 particles, and as such, have two polarization states, the $+$ and $\times$ polarization states. The field is decomposed into a polarization basis (the sum is over $\lambda = +, \times$)

$$h_{ij}(\mathbf{k},\tau) = \sum_\lambda h_\lambda(\mathbf{k},\tau) e^\lambda_{ij}(\mathbf{k})\,. \tag{1.3.14}$$

Proceeding as was done for the scalar modes, the tensor power spectrum is found to be

$$\mathcal{P}_t(k) = 2\frac{H^2}{\pi^2 M_P^2}\,, \tag{1.3.15}$$

where the factor of 2 reflects the sum over the two polarization states. It is worth noting that the tensor spectrum only depends on the energy scale of inflation, $H^2 \sim V/3M_P^2$. As the inflaton rolls down its potential, the energy density decreases, generating a tilt to the spectrum given by its spectral index,

$$n_t = -2\epsilon\,. \tag{1.3.16}$$

With (1.3.12) and (1.3.15), the tensor-to-scalar ratio defined in (1.2.7), is

$$r = 16\epsilon\,. \tag{1.3.17}$$

We finish this section by discussion the predictions of chaotic inflation [15]. Chaotic inflation uses a quadratic inflationary potential, $V = \frac{1}{2}m_\phi^2\phi^2$, and so is a natural candidate to investigate. In order for the quadratic potential to be "flat" enough to support inflation, we must have $\epsilon \ll 1$ where $\epsilon = \frac{2M_P^2}{\phi^2}$. Therefore, chaotic inflation is a "large-field" model, requiring $\phi \gtrsim M_P$. In particular, to leading order in slow-roll, $\phi = 2M_P\sqrt{N}$, which for $N = 60$, gives $\phi \sim 15M_P$. This then gives $\epsilon = \frac{1}{2N}$, and similarly $\eta = \frac{1}{2N}$, which comfortably satisfies the slow-roll condition.

Chaotic inflation predicts a spectral tilt $n_s - 1 = -\frac{2}{N}$, giving $n_s \approx 0.967$, which is in agreement with the measurements from Planck. Unfortunately, the good news ends there. The predicted tensor-to-scalar ratio is $r = \frac{8}{N} \approx 0.13$, which is too large for the measured upper limit, $r \lesssim 0.06$ (see fig. 1.1). As such, the simplest model of chaotic inflation is ruled out by observations. However, we've taken the time to discuss it here as in Section 4, we show how when the inflaton is coupled to fermions, this can lead to the possibility of returning chaotic inflation to within observational bounds. An overview of this process is

given in the next section.

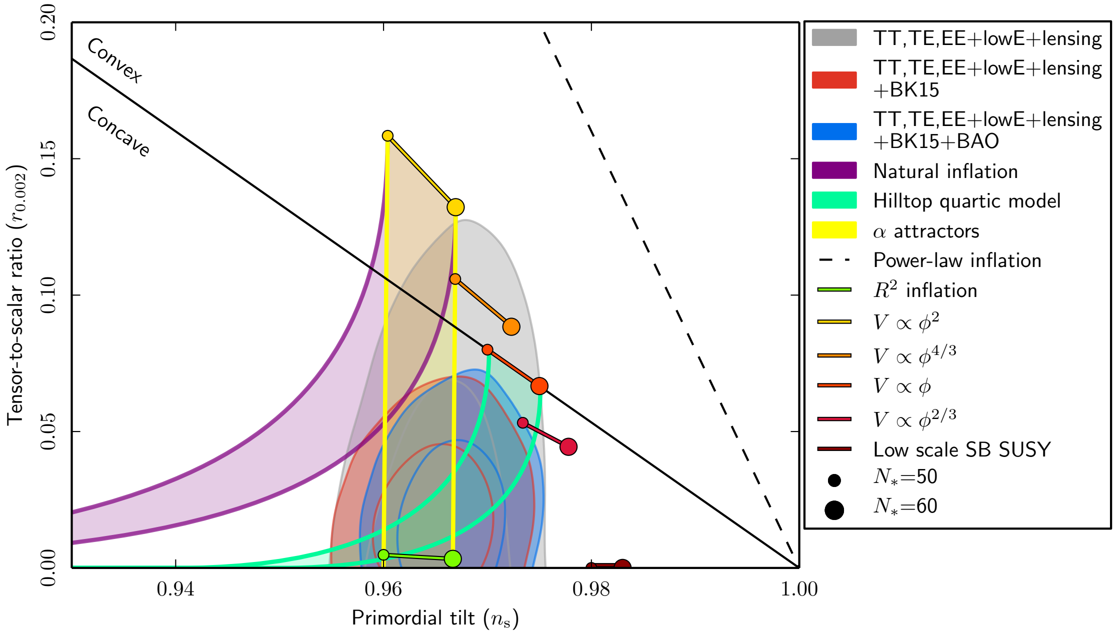

Figure 1.1: The current constraints from the Planck Collaboration on the tensor-to-scalar ratio, r, and the scalar spectral tilt, n_s [1]. Note that ϕ^2 inflation is currently ruled out as it predicts too large a tensor-to-scalar ratio.

1.4 Models of particle production during inflation and overview

The results of the previous section apply to when the inflaton is the only significant source for gravitational perturbations. When we account for other fields, sourced by the expansion of space, or by direct coupling to the inflaton, these results will be modified, sometimes significantly. If the vacuum and sourced contributions to the density perturbations are statistically independent, the two-point function is $\langle \zeta_k \zeta_{k'} \rangle = \langle \zeta_k^{\text{vac}} \zeta_{k'}^{\text{vac}} \rangle + \langle \zeta_k^{\text{sourced}} \zeta_{k'}^{\text{sourced}} \rangle$. Therefore, the power spectrum will be the sum of the two contributions, $\mathcal{P}_\zeta(k) = \mathcal{P}_\zeta^{\text{vac}}(k) + \mathcal{P}_\zeta^{\text{sourced}}(k)$ [16].

Scenarios where particle production occurs during inflation can allow to decouple inflationary observables from the shape of the potential. In these scenarios, the rolling inflaton provides the energy necessary for the generation of quanta of a secondary field whose presence can affect the spectra of scalar and tensor perturbations. A model which has been widely studied is that involving an additional scalar with an interaction $\Delta\mathcal{L} = g^2(\phi - \phi_0)^2\chi^2$ [17,

18]. As ϕ rolls down its potential and approaches ϕ_0, the scalar χ momentarily becomes massless and is produced copiously. This is because during this time, the effective mass evolves nonadiabatically, causing particle production. While a single event of particle production [19] can lead to features in the power spectra, a continuous process can generate an additional quasi-scale invariant component for the spectrum of scalar perturbations, or even provide a channel for the dissipation of the inflaton's energy that can lead to inflation even if the potential does not satisfy the slow-roll conditions [20, 21, 9, 22].

Production of gauge fields and fermions can occur in axion (or natural) inflation. Slow-roll inflation requires a flat potential; in axion inflation this flatness is a natural consequence of an approximate shift symmetry [23]. Originally motivated as a possible solution to the strong CP problem [24], axions, or pseudoscalar, fields are ubiquitous in string theory, and monodromy [25, 26, 10, 27, 28] or alignment [29, 30, 31, 32] effects can make them inflaton candidates, giving rise to vacuum primordial gravitational waves within the reach of future experiments.

Due to the approximate shift symmetry, the axion inflaton φ must couple derivatively to matter fields. At mass-dimension five, the possible couplings are

$$\Delta\mathcal{L} = \frac{\varphi}{f} F\tilde{F} + \frac{\partial_\mu \varphi}{f}\, \bar{X}\gamma^\mu\gamma_5 X \;, \tag{1.4.1}$$

to gauge fields and fermions X, respectively. Here F is the usual gauge-field field-strength tensor, $\tilde{F}$ is its dual, and f is a scale known as the axion decay constant. This coupling of the axion to gauge fields leads to exponentially large gauge field amplification, with several possible phenomenological consequences (see [33] for a review). These include steep inflation [9], thermal inflation [34, 35], magnetic field production [36, 37, 38, 39, 40, 41], large non-Gaussianity [42, 16, 43], chiral gravitational wave production [44, 45, 46, 47, 48, 49], instant preheating [50, 51], and the generation of primordial black holes [52, 53, 54].

The fermionic coupling has not attracted as much attention. Due to Pauli blocking, fermions cannot undergo the same exponential amplification as the gauge fields. Furthermore, on the one hand, massless fermions are conformal and therefore cannot be created gravitationally through the expansion of the Universe [55]. On the other hand, very heavy

fermions decouple, so that, in the absence of the coupling (1.4.1), only fermions with mass $m \approx H$ are produced in sizable quantities. With only one scale in the problem, the Hubble scale, the energy density is $\sim H^4$, which is too small to produce observable effects (with the possible exception of super-heavy dark matter [56, 57]). During axion inflation, however, this conclusion does not follow due to the presence of the additional scale $\dot{\varphi}/f$.

The first studies of this system have been performed in reference [58], and, more recently, in references [22, 59, 60, 61]. In that study, solutions of the Dirac equation including the effects of the homogeneous rolling inflaton mode were obtained along with the occupation number of the fermions during inflation and the oscillations in the phase immediately following inflation [58]. The effect of the axion-fermion coupling is to helically-bias the production of fermions leading to a net helicity asymmetry. This helicity asymmetry then leads to the possibility that the chiral fermions could lead to successful leptogenesis [62]. Subsequently, ref. [63] studied the possibility that these fermions source gravitational waves during inflation, and found that in this system a chiral component of the spectrum of primordial tensors is generated. Finally, ref. [64] found that these chiral fermions can generate circularly polarized photons, or V-modes after reheating.

In this thesis, we study the regime $\dot{\varphi}/f \gg H$. Because fermion modes can be populated up to $k \sim \dot{\varphi}/f$, the energy density can be parametrically larger[1] than H^4, as first noticed in reference [58]. We also identify a regime in which the sourced contribution to the power spectrum dominates the vacuum contribution, yet the non-Gaussianity is beneath current observational bounds. This behavior is in striking contrast to the analogous effect in systems with strong bosonic particle production. This difference can be understood by noting that phenomenologically interesting results require one to populate a large number of fermion modes since the occupation of these modes is restricted by Pauli exclusion, and their sum is uncorrelated and becomes increasingly Gaussian by the central limit theorem. Bosonic systems, conversely, allow for large occupation numbers *per mode*, which add coherently leading to sourced n-point functions that are generically related to the two-point function by $\langle \delta\varphi^n \rangle \sim \left(\langle \delta\varphi^2 \rangle \right)^{n/2}$ [65]. As the fermion-axion coupling is increased, eventually one enters

[1]On the contrary, in the opposite $\dot{\varphi}/f \ll H$ regime the effect of this coupling leads to a fermion energy density that is much smaller than H^4, and thus is negligible.

a regime of strong backreaction where the evolution of the inflaton zero-mode is controlled by particle production; this is the fermionic analogue of steep inflation studied in reference [9]. We expect that the above argument that non-Gaussianity remains small still holds in the regime of strong backreaction.

This thesis is organized as follows: In Chapter 2 we study the production of fermions during axion inflation, the backreaction on the inflaton evolution, and the fermion contribution to the scalar spectrum and non-Gaussianities. We show that the spectrum is quasi-scale-invariant and highly Gaussian. In Chapter 3 we calculate the tensor power spectrum of the sourced fermions and show it to be small compared with the vacuum spectrum. Finally, in Chapter 4, we show how these results can allow to lower the energy scale of inflation, thus lowering the tensor-to-scalar ratio. In particular, this allows for chaotic inflation to have a lower value of r and be in agreement with observation. We give a supergravity construction of this fermion model with axion inflaton which generates a quadratic potential for the inflaton.

Chapter 2

Phenomenology of fermion production during axion inflation

In this chapter we study the production of fermions through a derivative coupling with a pseudoscalar inflaton and the effects of the produced fermions on the scalar primordial perturbations. We present analytic results for the modification of the scalar power spectrum due to the produced fermions, and we estimate the amplitude of the non-Gaussianities in the equilateral regime. Remarkably, we find a regime where the effect of the fermions gives the dominant contribution to the scalar spectrum while the amplitude of the bispectrum is small and in agreement with observation. We also note the existence of a regime in which the backreaction of the fermions on the evolution of the zero-mode of the inflaton can lead to inflation even if the potential of the inflaton is steep and does not satisfy the slow-roll conditions.

Axion, or natural, inflation is a class of models of inflation in which the flatness of the inflation potential is protected by an (approximate) shift symmetry [23]. This class of models naturally gives rise to the relatively large-amplitude primordial gravitational waves targeted by next-generation experiments (see, for example, [25, 26, 31, 28]). The shift symmetry that protects the form of the potential is respected by the coupling of the axion to matter, and requires that the axion couples derivatively to other matter fields in order to facilitate reheating.

This model was first studied in [58]. In the present work we study the axion-fermion system in a different field basis from that paper. Using an axial transformation of the fermions $\psi \to e^{-i\phi\gamma_5}\psi$, we show (see section 2.1 below) that the Lagrangian with a pseudoscalar derivatively coupled to a fermion is equivalent to a Lagrangian in which the fermion has a time-dependent mass term $m\,\bar{\psi}\,e^{-2i\gamma_5\phi}\psi$ — see eq. (2.1.5). The main motivation for this reformulation is that it makes the behavior in the limit $m \to 0$ clear. In this limit, the coupling of ψ to the inflaton manifestly vanishes. In the basis used in ref. [58], the coupling also vanishes in this limit. However, this vanishing is only apparent *after* integrating the interaction by parts, and then using the Dirac equation. While the physics in either basis must of course be the same, we demonstrate that in the limit of interest ($\dot{\phi}/f \gg H$) the Hamiltonian in the basis of ref. [58] does not have a convenient perturbative expansion. Consequently, the conclusions of this work regarding the occupation numbers are different to the conclusions reached in the work ref. [58].

The main focus of this chapter is on the contribution that the non-vacuum fermion modes give to the spectrum and bispectrum of the scalar metric perturbations. We find that, in the regime $\mu \equiv m/H \lesssim 1$, $\xi \equiv \dot{\phi}/(2fH) \gg 1$, the occupation number of the fermions scales as μ^2/ξ for momenta up to a cutoff $\approx H\,\xi$, so that the total number density scales as $\mu^2\,\xi^2$. The modification to the power spectrum scales as $\frac{H^4}{f^4}\mu^2\xi^2\log\xi$, and the bispectrum as $\frac{H^6}{f^6}\frac{\mu^2}{\xi}$. This implies that in the regime of large $\mu\,\xi$ the system can be in a regime where the two-point function of the scalar fluctuations dominates over the vacuum contribution while the parameter f_{NL}, which measures the departures from Gaussianity, is small and in agreement with constraints from observations. This result is surprising if one considers that the origin of the sourced spectrum is quadratic in the (Gaussian) fermion field, which might lead one to expect strong non-Gaussianiaty. However, this result can be explained in terms of the central-limit theorem: the numerous fermion modes that contribute to the sourced spectrum sum incoherently, leaving a (quasi) Gaussian signal.

While most of our analysis is performed in the regime where the backreaction of the produced fermions on the background dynamics is negligible, we consider also the situation where this is not the case. and we find that strong backreaction effects can allow slow-roll inflation even if the potential for ϕ does not obey the usual slow-roll conditions $|V'| \ll V/M_P$,

$|V''| \ll V/M_P^2$. The argument we have just presented suggests that the perturbations should also be highly Gaussian in this regime.

This chapter is organized as follows. In section 2.1 we discuss the quantization of ψ on the time-dependent background provided by the (quasi) de Sitter geometry along with the rolling inflaton, and we evaluate the resulting occupation number. As expected, the parity-violating nature of the system implies different occupation numbers for the two helicities of ψ. We demonstrate that in the limit $m \to 0$ the occupation number of fermions of both helicities vanishes, as a consequence of the conformal and chiral symmetry of the system. In section 2.2, we study the backreaction of the fermions on the zero-mode, or homogeneous, inflaton background. In section 2.3 we analyze the modifications to the inflationary power spectrum induced by the presence of a nonvanishing occupation number for the fermions, while in section 2.4 we study the bispectrum. In section 2.5, we explore the possibility of slow-roll inflation on steep potentials in the limit of very strong backreaction.

2.1 Fermion production during inflation

In this section we study the production of fermions during axion inflation and obtain solutions to the Dirac equation for a fermion coupled to the slowly-rolling ($\dot{\phi}$ =constant) pseudoscalar. In particular, we compute the resulting occupation number for the right- and the left-handed components of the fermion.

We consider the theory of a pseudoscalar inflaton ϕ interacting with a Dirac fermion X through a derivative interaction with coupling constant $1/f$

$$\mathcal{L} = a^4 \left\{ \bar{X} \left[i \left(\tilde{\gamma}^\mu \, \partial_\mu + \frac{3}{2} \frac{a'}{a} \tilde{\gamma}^0 \right) - m - \frac{1}{f} \tilde{\gamma}^\mu \, \gamma^5 \, \partial_\mu \phi \right] X + \frac{1}{2} \left(\partial \phi \right)^2 - V\left(\phi\right) \right\} . \qquad (2.1.1)$$

Here the $\tilde{\gamma}$-matrices in flat Friedmann-Lemaître-Robertson-Walker spacetime with scale factor a are related to those in Minkowski spacetime by $\tilde{\gamma}^\mu = \gamma^\mu / a$, while $\gamma^5 = i \, a^4 \, \tilde{\gamma}^0 \tilde{\gamma}^1 \tilde{\gamma}^2 \tilde{\gamma}^3 = i \, \gamma^0 \gamma^1 \gamma^2 \gamma^3$. We neglect metric fluctuations[1] and treat the background as fixed de Sitter spacetime.

[1] More precisely, we study scalar metric perturbations in the spatially flat gauge, neglecting the presence of the shift and lapse scalar factors which provide slow-roll suppressed contributions to the spectra.

Throughout this chapter and the following one, we use conformal time and "mostly minus" signature for our metric, and we use the Dirac representation for the γ matrices. Specifically,

$$\gamma^0 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} , \; \gamma^i = \begin{pmatrix} 0 & \sigma_i \\ -\sigma_i & 0 \end{pmatrix} , \; \gamma^5 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} . \tag{2.1.2}$$

The fermions are canonically normalized by redefining $Y = X\, a^{3/2}$, so that

$$\mathcal{L} = \bar{Y} \left[i\, \gamma^\mu\, \partial_\mu - m\, a - \frac{1}{f} \gamma^\mu\, \gamma^5\, \partial_\mu \phi \right] Y + \frac{1}{2} a^2 \eta^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - a^4 V\left(\phi\right) . \tag{2.1.3}$$

Next, we perform one more redefinition of the fermion field,

$$Y = \mathrm{e}^{-i\gamma^5 \phi/f}\, \psi\, , \tag{2.1.4}$$

which yields the Lagrangian

$$\mathcal{L} = \bar{\psi} \left\{ i\, \gamma^\mu\, \partial_\mu - m\, a \left[\cos\left(\frac{2\phi}{f}\right) - i\gamma^5 \sin\left(\frac{2\phi}{f}\right) \right] \right\} \psi + \frac{1}{2} a^2 \eta^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - a^4 V\left(\phi\right) . \tag{2.1.5}$$

The latter field redefinition is motivated by two considerations. First, as discussed in the introduction, by writing the Lagrangian in terms of ψ it is apparent that the inflaton decouples from the fermion in the limit $m \to 0$. This decoupling is not as evident when the Lagrangian is in the form of eq. (2.1.3). Second, in order to determine the occupation number for the fermions we resort to the usual technique of the Bogolyubov coefficients, which relies on the diagonalization of the portion of Hamiltonian that is quadratic in the fields. In the formulation of eq. (2.1.3) the momentum conjugate to ϕ, which is needed to compute the Hamiltonian, is given by $\Pi_\phi = a^2\, \dot{\phi} - \frac{1}{f} \bar{Y} \gamma^0 \gamma^5 Y$, which contains a term that is quadratic in the fermion field (this should be compared with the simpler expression $\Pi_\phi = a^2\, \dot{\phi}$ obtained in the formulation in eq. (2.1.5)). This leads to a different definition of the quadratic part of the Hamiltonian which, in turn, leads to the unphysical result that certain modes of the fermion are excited by the rolling of the inflaton even in the limit $m \to 0$, where we would

expect these degrees of freedom to decouple. These two issues are related, in the sense that the perturbation theory based on the quadratic fermion Hamiltonian obtained from eq. (2.1.3) blows up at a finite time in the massless limit. In what follows, we work with the Lagrangian in eq. (2.1.5).

In order to determine the Bogolyubov coefficients for the fermions ψ, and therefore their particle number, we must second quantize ψ in the presence of the time dependent background induced by the rolling of ϕ. To do so we first focus only on the ψ-dependent part of the Lagrangian, approximating ϕ as a homogeneous, time-dependent background: $\phi(\mathbf{x},\,\tau) \simeq \phi_0(\tau)$. From the slow-roll condition $\dot{\phi}_0 \simeq$constant,[2] we have

$$\phi_0(\tau) = -\frac{\dot{\phi}_0}{H}\,\log\left(\tau/\tau_{\rm in}\right)\,, \tag{2.1.6}$$

where $\tau_{\rm in}$ is related to the initial value of ϕ. We have verified that, consistent with the fact that the fermion is derivatively coupled to the inflaton, our results do not depend on the value of $\tau_{\rm in}$.

Relegating the details of our derivation to Appendix A, we find that, by decomposing

$$\psi = \int \frac{\mathrm{d}^3k}{(2\pi)^{\frac{3}{2}}} e^{i\mathbf{k}\cdot\mathbf{x}} \sum_{r=\pm}\left[U_r(\mathbf{k},\,\tau)\,a_r(\mathbf{k}) + V_r(-\mathbf{k},\,\tau)\,b_r^\dagger(-\mathbf{k})\right]\,, \tag{2.1.7}$$

with

$$U_r(\mathbf{k},\,\tau) = \frac{1}{\sqrt{2}}\begin{pmatrix} \chi_r(\mathbf{k})\,u_r(x) \\ r\chi_r(\mathbf{k})\,v_r(x)\end{pmatrix}\,,\quad V_r\,(\mathbf{k}) = C\,\bar{U}_r\,(\mathbf{k})^T\,,\qquad C = i\gamma^0\gamma^2 = \begin{pmatrix} 0 & i\sigma_2 \\ i\sigma_2 & 0\end{pmatrix}\,,$$

$$\chi_r(\mathbf{k}) \equiv \frac{(k + r\,\sigma\cdot\mathbf{k})}{\sqrt{2\,k\,(k+k_3)}}\,\bar{\chi}_r,\qquad \bar{\chi}_+ = \begin{pmatrix}1\\0\end{pmatrix},\quad \bar{\chi}_- = \begin{pmatrix}0\\1\end{pmatrix}\,, \tag{2.1.8}$$

[2]We denote $\dot{} \equiv d/dt$, $' \equiv d/d\tau$, with t cosmic time and τ conformal time. We note that $\dot{\phi}_0$ varies at second order in slow roll, but we disregard this small effect here.

(where k_3 is the z-component of the vector $\mathbf{k}$), the mode functions are given by

$$
\begin{aligned}
u_r(x) =& \frac{1}{\sqrt{2x}} \left[\mathrm{e}^{ir\hat{\phi}(x)}\, s_r\,(x) + \mathrm{e}^{-ir\hat{\phi}(x)}\, d_r\,(x) \right] , \\
v_r(x) =& \frac{1}{\sqrt{2x}} \left[\mathrm{e}^{ir\hat{\phi}(x)}\, s_r\,(x) - \mathrm{e}^{-ir\hat{\phi}(x)}\, d_r\,(x) \right] ,
\end{aligned} \tag{2.1.9}
$$

which satisfy the normalization condition $|u_r|^2 + |v_r|^2 = 2$, with

$$
s_r\,(x) = \mathrm{e}^{-\pi r \xi}\, W_{\frac{1}{2}+2ir\xi,\, i\sqrt{\mu^2+4\xi^2}}(-2ix)\,, \qquad d_r\,(x) = -i\,\mu\, \mathrm{e}^{-\pi r\xi}\, W_{-\frac{1}{2}+2ir\xi,\, i\sqrt{\mu^2+4\xi^2}}(-2ix)\,, \tag{2.1.10}
$$

where $W_{\mu,\,\lambda}(z)$ denotes the Whittaker W-function. We have also defined

$$
\hat{\phi}\,(x) \equiv \frac{\phi_0}{f} = -2\xi \log\left(x/x_{\mathrm{in}}\right)\,, \tag{2.1.11}
$$

with

$$
x \equiv -k\tau\,, \quad x_{\mathrm{in}} \equiv -k\tau_{\mathrm{in}}\,, \quad \mu \equiv \frac{m}{H}\,, \quad \xi \equiv \frac{\dot{\phi}_0}{2fH}\,. \tag{2.1.12}
$$

We work in de Sitter spacetime, $a(\tau) = -1/H\tau$, and disregard subleading corrections in slow roll.

With the knowledge of the mode functions, we diagonalize the quadratic Hamiltonian for the fermions, which reads

$$
H_{\psi}^{(2)} = \int \mathrm{d}^3x\, \bar{\psi} \left[-i\,\gamma^i\, \partial_i + m_R - i\gamma^5\, m_I \right] \psi\,, \tag{2.1.13}
$$

where the subscript $_{\psi}$ indicates that we are considering only the fermionic part of the Hamiltonian and the superscript $^{(2)}$ indicates that we are considering only the part of the Hamiltonian that is quadratic in the field fluctuations (in section 2.3 below we consider the cubic and quartic part of the Hamiltonian). The quantities m_R and m_I are defined as

$$
m_R \equiv m\, a\, \cos\left(\frac{2\phi_0}{f}\right) \quad , \quad m_I \equiv m\, a\, \sin\left(\frac{2\phi_0}{f}\right)\,. \tag{2.1.14}
$$

Long but straightforward computations show that $H_\psi^{(2)}$ takes the form

$$
\begin{aligned}
H_\psi^{(2)} &= \sum_{r=\pm} \int \mathrm{d}^3 k \left(a_r^\dagger(\mathbf{k}),\, b_r(-\mathbf{k})\right) \begin{pmatrix} A_r & B_r^* \\ B_r & -A_r \end{pmatrix} \begin{pmatrix} a_r(\mathbf{k}) \\ b_r^\dagger(-\mathbf{k}) \end{pmatrix}, \\
A_r &\equiv \frac{1}{2}\left[m_R\left(|u_r|^2 - |v_r|^2\right) + k\left(u_r^* v_r + v_r^* u_r\right) - i\, r\, m_I\left(u_r^* v_r - v_r^* u_r\right)\right], \\
B_r &\equiv \frac{r\, e^{ir\varphi_{\mathbf{k}}}}{2}\left[2\, m_R u_r\, v_r - k\left(u_r^2 - v_r^2\right) - i\, r\, m_I\left(u_r^2 + v_r^2\right)\right],
\end{aligned}
\tag{2.1.15}
$$

with $e^{i\varphi_{\mathbf{k}}} \equiv (k_1 + i\, k_2)/\sqrt{k_1^2 + k_2^2}$. Note that eq. (2.1.15) implies that whenever B_r is nonvanishing, the operators $a_r^\dagger$ and $b_r^\dagger$ do not create energy eigenstates. Therefore, they should not be interpreted as ladder operators associated with a single-particle state. The matrix appearing in the first line of eq. (2.1.15) can be diagonalized as

$$
\begin{gathered}
\begin{pmatrix} A_r & B_r^* \\ B_R & -A_r \end{pmatrix} = \begin{pmatrix} \alpha_r^* & \beta_r^* \\ -\beta_r & \alpha_r \end{pmatrix} \begin{pmatrix} \omega & 0 \\ 0 & -\omega \end{pmatrix} \begin{pmatrix} \alpha_r & -\beta_r^* \\ \beta_r & \alpha_r^* \end{pmatrix}, \\
\omega \equiv \sqrt{k^2 + m_R^2 + m_I^2},
\end{gathered}
\tag{2.1.16}
$$

where the Bogolyubov coefficients α_r and β_r read

$$
\begin{aligned}
\alpha_r &= \mathrm{e}^{ir\varphi_{\mathbf{k}}/2}\left[\frac{1}{2}\sqrt{1 + \frac{m_R}{\omega}}\; u_r + \frac{1}{2}\sqrt{1 - \frac{m_R}{\omega}}\; \mathrm{e}^{-ir\theta}\; v_r\right], \\
\beta_r &= r\, \mathrm{e}^{ir\varphi_{\mathbf{k}}/2}\left[\frac{1}{2}\sqrt{1 - \frac{m_R}{\omega}}\; \mathrm{e}^{ir\theta}\; u_r - \frac{1}{2}\sqrt{1 + \frac{m_R}{\omega}}\; v_r\right],
\end{aligned}
\tag{2.1.17}
$$

where $\mathrm{e}^{i\theta} \equiv (k + i m_I)/\sqrt{k^2 + m_I^2}$. It is then straightforward to see that the operators

$$
\begin{pmatrix} \hat{a}_r(\mathbf{k}) \\ \hat{b}_r^\dagger(-\mathbf{k}) \end{pmatrix} = \begin{pmatrix} \alpha_r & -\beta_r^* \\ \beta_r & \alpha_r^* \end{pmatrix} \cdot \begin{pmatrix} a_r(\mathbf{k}) \\ b_r^\dagger(-\mathbf{k}) \end{pmatrix}
\tag{2.1.18}
$$

diagonalize the Hamiltonian, with $\hat{a}_r^\dagger(\mathbf{k})\, \hat{a}_r(\mathbf{k})$ and $\hat{b}_r^\dagger(\mathbf{k})\, \hat{b}_r(\mathbf{k})$ describing number operators of, respectively, particles and antiparticles with energy $\sqrt{k^2 + m_R^2 + m_I^2} = \sqrt{k^2 + m^2 a^2}$.

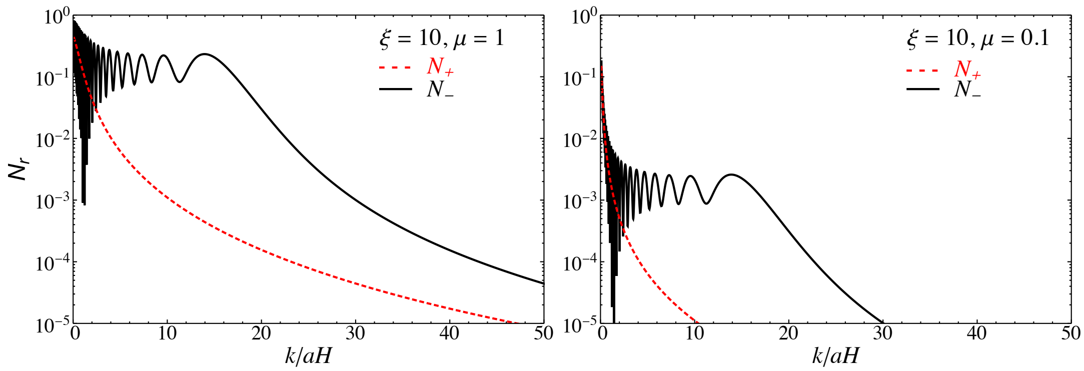

Figure 2.1: The occupation number of $r = -1$ (top, solid curves) and $r = +1$ (bottom, dashed curves) fermions, N_r as a function of the momentum k, for $\xi = 10$ and $\mu = 1$ (left panel) and $\mu = 0.1$ (right panel).

The occupation number of helicity-r particles (and antiparticles) is then

$$\begin{aligned} N_r \equiv |\beta_r|^2 &= \langle 0|\hat{a}_r^\dagger(\mathbf{k})\,\hat{a}_r(\mathbf{k})|0\rangle = \langle 0|\hat{b}_r^\dagger(\mathbf{k})\,\hat{b}_r(\mathbf{k})|0\rangle \\ &= \frac{1}{2} - \frac{m_R}{4\omega}\left(|u_r|^2 - |v_r|^2\right) - \frac{k}{2\omega}\,\mathrm{Re}\,(u_r^* v_r) - \frac{r\,m_I}{2\omega}\,\mathrm{Im}\,(u_r^* v_r)\ . \end{aligned} \tag{2.1.19}$$

We now discuss the main properties of the functions $N_r(k)$. In figure 2.1, we show the occupation number of the $r = +1$ and $r = -1$ fermions at the end of inflation ($\tau = -1/H$) for $\xi = 10$ with $\mu = 1$ (left) and $\mu = 0.1$ (right). First, let us focus on fermions with $r = +1$. For those particles the occupation number drops rapidly to zero as k gets larger than m. The reason for this behavior is that the presence of a nonvanishing mass leads to the breaking of conformality and the generation of fermions on the de Sitter background. For momenta larger than m the fermions are approximately conformal and the occupation number becomes smaller. This phenomenon is purely gravitational and affects both the left- and the right-handed modes, but for the modes with $r = -1$ it is overwhelmed by the effects of nonvanishing ξ. In fact, modes with $r = -1$ have nonvanishing occupation number for k as large as $2\xi H$. We interpret this as the consequence of the fact that the excitation of those fermion modes is induced by the coupling to the pseudoscalar inflaton.[3] We also note that

[3]These results rely on the assumption that $\xi > 0$. Changing the sign of ξ has the effect of exchanging the occupation numbers of the $r = +$ and $r = -$ modes.

the occupation number of the interesting $r = -1$ mode displays high frequency oscillations as a function of the momentum k. The two panels of figure 2.1 show that those oscillations happen around a value of the occupation number that is approximately given by μ^2/ξ.

By evaluating analytically eq. (2.1.19) in various limits we observe that both N_+ and N_- vanish as μ^2 in the limit $\mu \to 0$. This is consistent with the decoupling of ψ from the inflaton for $m = 0$. More specifically, one obtains

$$N_r \simeq \frac{\mu^2}{4\,x^2}\,, \qquad \mu \ll x \ll 1\,. \tag{2.1.20}$$

In the regime of moderate $\mu \lesssim 1$ and large $\xi \gg 1$ we find that the occupation number of the $r = -1$ modes is oscillating about a constant that is well approximated by μ^2/ξ for modes with $x \lesssim \xi$ before dropping as $\xi^2\,\mu^2/x^4$ for $x \gtrsim \xi$. As a consequence, for this range of parameters the total number density of the modes with $r = -1$ scales, for $\mu \lesssim 1$, as $\frac{\mu^2}{\xi} \times \xi^3 \sim \mu^2\,\xi^2$ that can be parametrically larger than unity per Hubble volume.

Moving to the regime of large μ, in figure 2.2 we show the occupation number for the $r = +1$ and $r = -1$ modes for $\mu = 10$ and $\xi = 10$ (left) and for $\mu = 10$, $\xi = 1$ (right). Remarkably, even if the occupation number for the $r = +1$ modes is smaller than that of the $r = -1$ ones, both occupation numbers are of order unity despite the fact that the mass of the fermions is much larger than the Hubble scale. This means that the coupling to the inflaton prevents the decoupling of fermions with $m \gg H$ (for comparison, the occupation number of fermions with $\mu = 10$ and $\xi = 0$, not plotted, is at most of the order of 10^{-5}). Of course, the occupation number of the fermions decreases (as $\sim \xi^2/\mu^2$) when μ becomes much larger than ξ.

A numerical evaluation of the total number density of $r = -1$ fermions yields

$$\int \mathrm{d}^3k\, N_-(k) \simeq 52\,H^3\mu^2\,\xi^2\,, \qquad \xi \gg \mu\,, \qquad \xi \gg 1. \tag{2.1.21}$$

The main conclusion of this section is that a nonvanishing value of ξ leads to nontrivial behavior of the fermions. Chiral fermions are copiously produced even if $m \ll H$ (as long as $\mu^2\,\xi^2$ is large enough), and even very heavy fermions with $m \gg H$ can be produced with

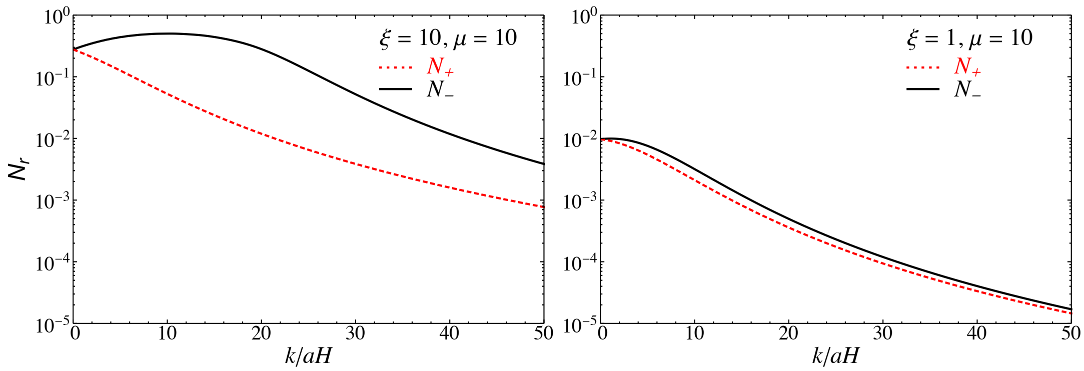

Figure 2.2: Same as figure 2.1, but for $\xi = 10$, $\mu = 10$ (left) and for $\xi = 1$, $\mu = 10$ (right).

large occupation numbers as long as $\mu \lesssim \xi$. We now move on to compute the effect of these fermions on the inflaton.

2.2 Backreaction

In this section we examine the backreaction of the produced fermions on the homogeneous, or background, inflaton. The equation of motion for the inflaton, derived from the Lagrangian (2.1.5), reads

$$\phi'' + 2\frac{a'}{a}\phi' - \Delta\phi + a^2\,V'(\phi) = \frac{2\,m}{f\,a}\bar{\psi}\left[\sin\left(\frac{2\phi}{f}\right) + i\gamma^5\cos\left(\frac{2\phi}{f}\right)\right]\psi\,. \qquad (2.2.1)$$

We seek to establish the conditions under which the backreaction of the produced fermions on the background dynamics is negligible. We do this by imposing that the right hand side of the equation above, evaluated in the Hartree approximation, is much smaller than $a^2\,V'(\phi)$. Using manipulations analogous to those of section 2.1 above, the quantity

$$\begin{aligned}\mathcal{B} &\equiv \frac{2\,m}{f\,a}\langle\bar{\psi}\left[\sin\left(\frac{2\phi}{f}\right) + i\,\gamma_5\,\cos\left(\frac{2\phi}{f}\right)\right]\psi\rangle \\ &= \frac{2}{f\,a^2}\int\frac{\mathrm{d}^3q\,\mathrm{d}^3p}{(2\pi)^3}e^{i(\mathbf{q}-\mathbf{p})\mathbf{x}}\langle\bar{\psi}(\mathbf{p})\left[m_I + i\,\gamma_5\,m_R\right]\psi(\mathbf{q})\rangle\,,\end{aligned} \qquad (2.2.2)$$

can be written as

$$\mathcal{B} = 4\frac{m\,H^3}{f}\,a^2\,\sum_r r\int \frac{y\,\mathrm{d}y}{2\pi^2}\,\mathrm{Im}\left\{s_r\,d_r^*\right\}\,. \tag{2.2.3}$$

Quite remarkably, the integral can be computed analytically after regulating it with a hard cut-off at a finite and large Λ (see [22] for details). The integral turns out to have a logarithmic divergence for large Λ. As we discuss in greater detail in subsection 2.3.1 below, we can deal with the divergence either by simply subtracting the divergent part, or by adiabatic regularization. The result does not change in the limit of large ξ, and in the regime $\mu \lesssim 1 \ll \xi$ we obtain

$$\mathcal{B} \simeq -\frac{8}{\pi}\,\frac{H^4}{f}\,a^2\,\mu^2\,\xi^2\,. \tag{2.2.4}$$

By imposing that the backreaction of the fermions on the zero mode of the inflaton is negligible, $\mathcal{B} \ll a^2\,V'(\phi) \simeq 3\,H\,\dot{\phi}\,a^2$, we derive a first condition on the parameter space of the model:

$$\mu^2\,\xi \ll \frac{f^2}{H^2}\,, \tag{2.2.5}$$

where we emphasize that f should be much larger than H in order for the effective field theory to be valid at energy scales of the order of H.

As a second condition for negligible backreaction we impose that the energy density of the produced fermions gives a negligible contribution to the expansion rate of the Universe. The energy density in fermions is computed in [22] and reads

$$\rho_\psi = 16\,\pi^2 H^4 \mu^2 \xi^3\,, \tag{2.2.6}$$

so that by requiring it to be subdominant with respect to the energy in the inflaton one obtains the parametric constraint

$$\mu^2\,\xi^3 \ll \frac{M_P^2}{H^2}\,. \tag{2.2.7}$$

It is easy to check that during slow roll, $\dot{\phi} \ll H\, M_P$, this condition is satisfied as long as eq. (2.2.5) holds.

2.3 Power spectrum

Fermions with a nonvanishing occupation number backreact on the fluctuations of the inflaton, therefore modifying the primordial scalar perturbations. In this section we compute this effect to leading order. As we will show, at the level of approximation that we are using this modification is scale invariant, and therefore it is unobservable in the spectrum because it is degenerate with the vacuum contribution generated by the inflationary expansion of the Universe. However, an observable effect is potentially generated in the bispectrum, which in single-field inflation is slow-roll suppressed to a currently unobservable level. A precise calculation of the bispectrum is very challenging, but in section 2.4 below we use the results of this section 2.3 to estimate its magnitude.

In order to focus on the physics, we only present the main steps of our calculation of the leading order correction to power spectrum in this section. The details of the calculations can be found in [22].

We compute the leading order modifications to the power spectrum of the fluctuations of the inflaton using the in-in formalism (see, e.g. [66]). To do so, we define the perturbation $\delta\phi(\mathbf{x},\,\tau) = \phi(\mathbf{x},\,\tau) - \phi_0(\tau)$ and we expand the interaction Hamiltonian to second order in $\delta\phi$

$$\begin{aligned} H_{\rm int} \supset & -\frac{2am}{f}\int \mathrm{d}^3x\,\bar{\psi}\left[\sin\left(2\,\frac{\phi_0}{f}\right) + i\,\gamma^5\cos\left(2\,\frac{\phi_0}{f}\right)\right]\psi\,\delta\phi \\ & -\frac{2am}{f^2}\int \mathrm{d}^3x\,\bar{\psi}\left[\cos\left(2\,\frac{\phi_0}{f}\right) - i\,\gamma^5\sin\left(2\,\frac{\phi_0}{f}\right)\right]\psi\,\delta\phi^2 \equiv H_\psi^{(3)} + H_\psi^{(4)}\,, \end{aligned} \qquad (2.3.1)$$

where we have neglected the contribution from the inflaton self-interactions, whose effects are

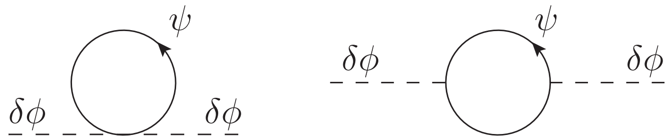

Figure 2.3: The two diagrams that contribute at leading order to the two-point function of $\delta\phi$.

slow-roll suppressed. We then use $H_{\rm int}$ to compute the modification to the power spectrum

$$\delta P_\zeta\left(\tau,\,k\right)\Big|_{-k\tau\ll 1} = \frac{k^3}{2\pi^2}\,\frac{H^2}{\dot\phi_0^2}\,\sum_{N=1}^{\infty}\left(-i\right)^N\int^{\tau}{\rm d}\tau_1\ldots\int^{\tau_{N-1}}{\rm d}\tau_N \\ \times\left\langle\left[\left[\cdots\left[\delta\phi^{(0)}\left(\tau,\,\mathbf{k}\right)\delta\phi^{(0)}\left(\tau,\,\mathbf{k}'\right),\,H_{\rm int}\left(\tau_1\right)\right],\cdots\right],\,H_{\rm int}\left(\tau_N\right)\right]\right\rangle',\tag{2.3.2}$$

where we have used the relation $\zeta = -H\,\delta\phi/\dot\phi_0$ between the fluctuations of the inflaton and the scalar perturbation of the metric, and the prime denotes the correlator stripped of the $\delta^{(3)}(\mathbf{k}+\mathbf{k}')$ associated with momentum conservation.

In evaluating the expression eq. (2.3.2) we use the mode functions for ψ found in section 2.1 above, eqs. (2.1.7) through (2.1.10). Regarding the mode functions of $\delta\phi$, we use those of a massless field in de Sitter space:

$$\delta\phi^{(0)}\left(\mathbf{x},\,\tau\right) = \int\frac{{\rm d}^3k}{\left(2\pi\right)^{3/2}}\,{\rm e}^{i\mathbf{k}\cdot\mathbf{x}}\left[\delta\phi_k^{(0)}\left(\tau\right)a_{\mathbf{k}} + \delta\phi_k^{(0)*}\left(\tau\right)a_{-\mathbf{k}}^{\dagger}\right]\,,\tag{2.3.3}$$

with

$$\delta\phi_k^{(0)}\left(\tau\right) = \frac{H}{\sqrt{2k}}\left(i\,\tau+\frac{1}{k}\right){\rm e}^{-ik\tau}\,.\tag{2.3.4}$$

The two parts of the interaction Hamiltonian $H_\psi^{(3)}$ and $H_\psi^{(4)}$ describe a cubic $\bar\psi\,\psi\,\delta\phi$ vertex and a quartic $\bar\psi\,\psi\,\delta\phi^2$ vertex. Those two vertices can be used to draw the two diagrams shown in figure 2.3, which contribute to eq. (2.3.2) at leading order in the $1/f$ expansion. We discuss these diagrams in the next two subsections.

2.3.1 Quartic loop

The first diagram in figure 2.3 gives

$$\delta P_\zeta^{(4)}\left(\tau,\, k\right) = i\,\frac{k^3}{2\pi^2}\,\frac{H^2}{\dot\phi_0^2}\,\frac{2m}{f^2}\int^\tau \mathrm{d}\tau_1\, a(\tau_1)\int\frac{\mathrm{d}^3p\,\mathrm{d}^3q\,\mathrm{d}^3w}{(2\pi)^3}\Bigg\langle\Bigg[\delta\phi^{(0)}\left(\tau,\,\mathbf{k}\right)\delta\phi^{(0)}\left(\tau,\,\mathbf{k}'\right),$$
$$\bar\psi\left(\tau_1,\,\mathbf{p}\right)\left[\cos\left(2\,\frac{\phi_0(\tau_1)}{f}\right) - i\,\gamma^5\sin\left(2\,\frac{\phi_0(\tau_1)}{f}\right)\right]\psi\left(\tau_1,\,\mathbf{q}\right)$$
$$\delta\phi^{(0)}\left(\tau_1,\,\mathbf{w}\right)\,\delta\phi^{(0)}\left(\tau_1,\,\mathbf{p}-\mathbf{q}-\mathbf{w}\right)\Bigg]\Bigg\rangle', \tag{2.3.5}$$

which, with some algebra and in the large scale limit $-k\tau\to 0$, can be simplified to

$$\delta P_\zeta^{(4)}\left(\tau,\, k\right) = \frac{2H^5 m}{f^2k^3\pi^2\dot\phi_0^2}\int^\tau\frac{\mathrm{d}\tau_1}{\tau_1}\,\left[\cos\left(k\tau_1\right)+k\tau_1\sin\left(k\tau_1\right)\right]\left[\sin\left(k\tau_1\right)-k\tau_1\cos\left(k\tau_1\right)\right]$$
$$\times\int\frac{\mathrm{d}^3p}{(2\pi)^3}\left\langle\bar\psi\left(\mathbf{p}\right)\left[\cos\left(2\,\hat\phi\right)-i\,\gamma^5\sin\left(2\,\hat\phi\right)\right]\psi\left(\mathbf{p}\right)\right\rangle'_{\tau_1}. \tag{2.3.6}$$

By inserting the expressions for the mode functions of the fermions into this equation we finally obtain

$$\frac{\delta P_\zeta^{(4)}\left(\tau,\,k\right)}{P_\zeta^{(0)}} = \frac{4H^2\mu}{f^2\pi^2}\int_x\frac{\mathrm{d}x_1}{x_1^4}\,\left[\cos\left(x_1\right)+x_1\sin\left(x_1\right)\right]\left[x_1\cos\left(x_1\right)-\sin\left(x_1\right)\right]$$
$$\times\int\mathrm{d}x_p\,x_p\sum_r\Re\left[d_r^*\left(x_p\right)s_r\left(x_p\right)\right], \tag{2.3.7}$$

where we have normalized this contribution to P_ζ by the vacuum term $P_\zeta^{(0)} = H^4/(4\,\pi^2\,\dot\phi_0^2)$, we have introduced the dimensionless integration variables $x_1\equiv -k\tau_1$ and $x_p\equiv -p\tau_1$, and where the functions $d_r(x)$ and $s_r(x)$ are given in eq. (2.1.10).

We proceed to evaluate the two integrals that appear in eq. (2.3.7). The integral in $\mathrm{d}x_1$ diverges when the lower limit of integration x is sent to 0 (remember that $x=-k\tau=k/H$ as we want to evaluate the power spectrum at the end of inflation, $\tau=-1/H$). In fact one

finds

$$\int_x^{\infty} \frac{\mathrm{d}x_1}{x_1^4}\,\left[\cos\left(x_1\right)+x_1\sin\left(x_1\right)\right]\left[x_1\cos\left(x_1\right)-\sin\left(x_1\right)\right]\Big|_{x\to 0}$$
$$\simeq \frac{1}{3}\log(x)+\frac{3\,\log 2+3\,\gamma_E-7}{9}+\mathcal{O}(x^2)\,, \tag{2.3.8}$$

where $\gamma_E \simeq .577$ is the Euler-Mascheroni constant. This infrared divergence is a consequence of the fact that the fermions have a nonvanishing average density that keeps sourcing the fluctuations of the inflaton even when they are outside of the horizon. This divergence is regulated by the finite amount of e-foldings between the time when the inflaton mode leaves the horizon and the end of inflation.

The integral in $\mathrm{d}x_p$ is much more challenging, and is quadratically divergent in the ultraviolet. As it was the case for the integral in section 2.2, it is possible to compute it analytically after introducing a UV regulator that sets the upper limit of integration to some finite and large Λ (see [22]). The divergent part of the integral reads

$$\int_0^{\Lambda}\mathrm{d}x_p\,x_p\sum_r\Re\left[d_r^*\left(x_p\right)s_r\left(x_p\right)\right]=\mu\left(\Lambda^2-\left(\mu^2-8\xi^2+1\right)\log\Lambda\right)+\mathcal{O}(\Lambda^0)\,. \tag{2.3.9}$$

Now, we have at least two different ways of dealing with this divergence. We can subtract from the exact integral its adiabatic part, or we can simply subtract by hand the part that diverges when $\Lambda\to\infty$. The adiabatic subtraction might be problematic, as the adiabatic contribution turns out to dominate the physical one at momenta of order H [67]. Since these momenta contribute to the finite part of the integral, adiabatic subtraction can induce spurious components into our integral.

As discussed in [22], adiabatic subtraction does indeed introduce spurious contribution which scales as $\mu\,\xi^2$ and is therefore large at large ξ. However, this contribution is subdominant at sufficiently large ξ, as there is a physical contribution which scales as $\mu\,\xi^2\log(\xi)$. In this limit, and setting $\tau=-1/H$ to have quantities computed at the end of inflation, we

get the simple result

$$\frac{\delta P_\zeta^{(4)}(k)}{P_\zeta^{(0)}}\bigg|_{\text{end of inflation}} \simeq \frac{32\, m^2\xi^2\, \log\xi}{3\pi^2 f^2}\, \log(H/k)\,. \tag{2.3.10}$$

2.3.2 Cubic loop

The second diagram in figure 2.3 gives

$$\begin{aligned}
\frac{\delta P_\zeta^{(3)}(\tau,k)}{P_\zeta^{(0)}} &= -\frac{2k^3}{H^2}\frac{m^2}{f^2}\int^\tau \mathrm{d}\tau_1 a(\tau_1)\int^{\tau_1}\mathrm{d}\tau_2 a(\tau_2)\\
&\times\int\frac{\mathrm{d}^3p\mathrm{d}^3q}{(2\pi)^3}\left(\delta(\mathbf{k}+\mathbf{p}-\mathbf{q})+\delta(-\mathbf{k}+\mathbf{p}-\mathbf{q})\right)\\
&\times\left[\sin\left(\frac{2\phi_0(\tau_1)}{f}\right)+i\gamma^5\cos\left(\frac{2\phi_0(\tau_1)}{f}\right)\right]_{ij}\left[\sin\left(\frac{2\phi_0(\tau_2)}{f}\right)+i\gamma^5\cos\left(\frac{2\phi_0(\tau_2)}{f}\right)\right]_{ab}\\
&\times\left(\delta\phi^{(0)}(\mathbf{k},\,\tau)\,\delta\phi^{(0)}(\mathbf{k},\,\tau_1)^*-\delta\phi^{(0)}(\mathbf{k},\,\tau_1)^*\,\delta\phi^{(0)}(\mathbf{k},\,\tau_1)\right)\\
&\times\Big\{\delta\phi^{(0)}(\mathbf{k},\,\tau)\,\delta\phi^{(0)}(\mathbf{k},\,\tau_2)^*\,\langle\bar\psi(\mathbf{p},\,\tau_1)_i\,\psi(\mathbf{p},\,\tau_2)_b\rangle'\,\langle\psi(\mathbf{q},\,\tau_1)_j\,\bar\psi(\mathbf{q},\,\tau_2)_a\rangle'\\
&-\delta\phi^{(0)}(\mathbf{k},\,\tau_2)\,\delta\phi^{(0)}(\mathbf{k},\,\tau)^*\,\langle\bar\psi(\mathbf{q},\,\tau_2)_a\,\psi(\mathbf{q},\,\tau_1)_j\rangle'\,\langle\psi(\mathbf{p},\,\tau_2)_b\,\bar\psi(\mathbf{p},\,\tau_1)_i\rangle'\Big\}\,,
\end{aligned} \tag{2.3.11}$$

where we have already normalized to the vacuum power spectrum. With some work it is possible to evaluate the fermionic part (details can be found in [22]) and write

$$\begin{aligned}
\frac{\delta P_\zeta^{(3)}(\tau,\,k)}{P_\zeta^{(0)}} &= \frac{m^2}{2\,f^2\,k^3}\int^\tau\frac{\mathrm{d}\tau_1}{\tau_1^2}\int^{\tau_1}\frac{\mathrm{d}\tau_2}{\tau_2^2}\int\frac{\mathrm{d}^3p\,\mathrm{d}^3q}{(2\pi)^3p\,q}\left(\delta(\mathbf{k}+\mathbf{p}-\mathbf{q})+\delta(-\mathbf{k}+\mathbf{p}-\mathbf{q})\right)\\
\times\sum_{rs}&\left(1+r\,s\,\frac{\mathbf{p}\cdot\mathbf{q}}{p\,q}\right)(\sin k\tau_1-k\tau_1\,\cos k\tau_1)\,\Big\{(-i-k\tau_2)\;e^{ik\tau_2}\;(r\,s\,v_r(-p\tau_1)\,v_s(-q\tau_1)\\
&+u_r(-p\tau_1)\,u_s(-q\tau_1))\,(r\,s\,v_r^*(-p\tau_2)\,v_s^*(-q\tau_2)+u_r^*(-p\tau_2)\,u_s^*(-q\tau_2))+h.c.\Big\}\,,
\end{aligned} \tag{2.3.12}$$

where we recall that u_r and v_r are given in eqs. (2.1.9) and (2.1.10).

The computation of the above integral is extremely challenging (even an estimate is challenging, as each term appearing in it is rapidly oscillating), so we must resort to a number of approximations. First, we approximate the integrand assuming $p \gg k$, which implies $p \simeq q$. We expect this to generate at most an $\mathcal{O}(1)$ error in our final result. At this point the integral still contains products of four Whittaker functions. To simplify the

integral we Wick-rotate the time variables and we use simple approximations (that can be obtained in the limit $\xi \gg 1$ by dealing carefully with the branch cuts in the definition of the Whittaker functions) that bring the mode functions to the form

$$\begin{aligned} s_r(-iy) &\simeq A_{1,r}\, y^{-i\sqrt{\mu^2+4\xi^2}}\, e^{y} + B_{1,r}\, y^{i\sqrt{\mu^2+4\xi^2}}\, e^{-y}\,, \\ d_r(-iy) &\simeq A_{2,r}\, y^{-i\sqrt{\mu^2+4\xi^2}}\, e^{y} + B_{2,r}\, y^{i\sqrt{\mu^2+4\xi^2}}\, e^{-y}\,, \end{aligned} \tag{2.3.13}$$

with A_i and B_i constants that depend on μ and ξ. We show in [22] that in the $\mu \lesssim 1$, $\xi \gg 1$ regime of interest, the $r = s = +$ contribution is exponentially suppressed with respect to the $r = s = -$ contribution.

We finally recognize that the terms proportional to A_1 and A_2 correspond to the "vacuum" part of the modes we are considering, i.e. the part of modes that do not vanish and behave as positive frequency only as $p \to \infty$, and we subtract this part by hand from the mode functions, effectively keeping only the part proportional to B_1 and B_2 only.

After these manipulations we obtain the scaling

$$\frac{\delta P_\zeta^{(3)}(k)}{P_\zeta^{(0)}}\Big|_{\text{end of inflation}} \propto \frac{m^2}{f^2}\,\mu^2\,\sqrt{\xi}\,|\log(k/H)| \tag{2.3.14}$$

which in the regime $\mu \lesssim 1$, $\xi \gg 1$ is subdominant with respect to contribution $\delta P_\zeta^{(4)}$ found in the previous subsection.

2.3.3 Summary for the power spectrum

We conclude this section by summarizing our main result: the first diagram in figure 2.3 dominates the modification to the power spectrum of scalar perturbations in this model, with

$$\delta P_\zeta\,(k)\,\big|_{\text{end of inflation}} \simeq P_\zeta^{(0)}\,\frac{32\,m^2\,\xi^2\,\log\xi}{3\,\pi^2\,f^2}\,\log(H/k)\,. \tag{2.3.15}$$

The scaling we find is consistent with the fact that the leading contribution to δP_ζ is approximately proportional to $1/f^2$ and to the total number of fermions $\sim \mu^2\,\xi^2$.

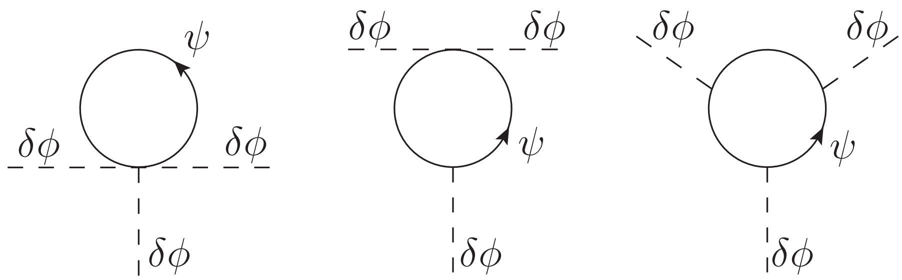

Figure 2.4: The three diagrams that contribute at leading order to the three-point function of $\delta\phi$.

2.4 Non-Gaussianity

As we saw above, the calculation of the fermionic contribution to the two-point function of the inflaton is challenging, and for the cubic diagram we could only obtain what we consider to be a reasonable estimate. As one can expect, the calculation of the three-point function is even more challenging. There is a new operator, besides the cubic and the quartic interaction Hamiltonians $H_\psi^{(3)}$ and $H_\psi^{(4)}$ given in eq. (2.3.1) above, that contributes to the three-point function. It is a quintic interaction Hamiltonian

$$H_\psi^{(5)} = -\frac{4ma}{3f^3}\bar{\psi}\left[\sin\left(\frac{2\phi_0}{f}\right) + i\gamma^5\cos\left(\frac{2\phi_0}{f}\right)\right]\psi\,\delta\phi^3\,, \tag{2.4.1}$$

which leads to a new $\bar{\psi}\,\psi\,\delta\phi^3$ vertex. Using the vertices generated from $H_\psi^{(3)}$, $H_\psi^{(4)}$, and $H_\psi^{(5)}$ we obtain, at leading order in $1/f$, the three diagrams of figure 2.4.

2.4.1 The quintic diagram

As we argue below, the first of these diagrams gives the leading contribution to the bispectrum. Fortunately, this contribution can be calculated analytically; after some long calculations that can be found in [22], we find the following expression

$$\begin{aligned}\langle\delta\phi(\mathbf{k}_1,\,\tau)\delta\phi(\mathbf{k}_2,\,\tau)\delta\phi(\mathbf{k}_3,\,\tau)\rangle' =& \frac{3H^6m}{2f^3}\int^\tau \mathrm{d}\tau_1\, a(\tau_1)f(k_1,k_2,k_3,\tau_1)\\ &\times\int\frac{\mathrm{d}^3p}{(2\pi)^{9/2}}\sum_r\frac{r}{-p\tau_1}\Im\left\{(d_r^*(-p\tau_1)s_r(-p\tau_1)\right\}\,,\end{aligned} \tag{2.4.2}$$

where

$$f(k_1, k_2, k_3, \tau_1) = \frac{1}{k_1^3 k_2^3 k_3^3} \cdot \left[\tau_1 \left(k_1 k_2 k_3 \tau_1^2 - k_1 - k_2 - k_3\right) \cos(\tau_1(k_1 + k_2 + k_3))\right.$$
$$\left. - \left(\tau_1^2(k_1 k_2 + k_1 k_3 + k_2 k_3) - 1\right) \sin(\tau_1(k_1 + k_2 + k_3))\right] . \qquad (2.4.3)$$

Since most of the dynamics occurs at momenta $-k\tau \sim \xi \gg 1$, which is well within the horizon, we expect the non-Gaussianities to be of equilateral shape. Therefore we estimate the magnitude of the bispectrum by setting $k_1 = k_2 = k_3 \equiv k$. As in the two-point functions, the integral in $\mathrm{d}\tau_1$ is logarithmically divergent as $-k\tau \to 0$, giving

$$\langle \delta\phi(\mathbf{k}_1,\, \tau)\delta\phi(\mathbf{k}_2,\, \tau)\delta\phi(\mathbf{k}_3,\, \tau)\rangle'\Big|_{\text{equilateral}} \simeq -\frac{3H^5 m}{f^3 k^6} \log(-k\tau) \cdot \frac{4\pi}{(2\pi)^{9/2}}$$
$$\times \sum_r r \int \mathrm{d}y\, y\, \Im\left\{s_r^*(y) d_r(y)\right\} , \qquad (2.4.4)$$

where the integral in $\mathrm{d}y$ is the same one appearing in eq. (2.2.3). This is as expected because the operator appearing here is the third derivative with respect to ϕ of the fermionic Lagrangian, which is identical in form to the one that contributes to eq. (2.2.3); that is to say, it is identical to the first derivative of that same part of the Lagrangian (times minus one). As we have discussed above, the integral in $\mathrm{d}y$ in the equation above is divergent as $y \to \infty$; we obtain in the limit $\xi \gg 1$, $\mu \lesssim 1$

$$\langle \delta\phi(\mathbf{k}_1,\, \tau)\delta\phi(\mathbf{k}_2,\, \tau)\delta\phi(\mathbf{k}_3,\, \tau)\rangle'\Big|_{eq} = -\frac{3H^6 \mu^2}{(2\pi)^{7/2} f^3 k^6} \log(-k_*\tau) \, \left[8\pi\xi^2 + 12\xi \log\left(\xi/\Lambda\right)\right.$$
$$\left. + \mathcal{O}(\xi) + \mathcal{O}(\Lambda^0)\right] , \qquad (2.4.5)$$

where eventually we drop the logarithmically divergent term.

2.4.2 The remaining two diagrams

Next, we would need to evaluate the remaining two diagrams in figure 2.4. The calculation of the second of them turns out to be of a complexity comparable to that of the cubic contribution to the power spectrum. The third one should be even more complicated, so that a complete evaluation of the bispectrum would be prohibitively difficult. We can,

however, infer some scaling properties which allow us to claim that, compared to the first diagram, these two diagrams should give negligible contributions to f_{NL}^{eq}.

Our first observation is that each cubic vertex gives a contribution $\sim \frac{m}{f}\bar{\psi}\,\psi$, each quartic vertex gives a contribution $\sim \frac{m}{f^2}\bar{\psi}\,\psi$, and each quintic vertex gives a contribution $\sim \frac{m}{f^3}\bar{\psi}\,\psi$. Our next, and most crucial, observation is that $\bar{\psi}\,\psi$ oscillates with amplitude $\propto \frac{m}{\xi}$ for momenta up to $-k\tau \simeq \xi$. This implies that each fermionic loop integral, which goes as d^3k, gives a contribution $\sim \xi^3$.

Once we apply these scalings to the backreaction term of section 2.2 (which would be represented diagrammatically by a tadpole) we obtain a scaling $\frac{m}{f} \times \frac{m}{\xi} \times \xi^3 \simeq \frac{m^2\,\xi^2}{f}$ which indeed is the result found in eq. (2.2.4). Next, we can check whether our scalings work in the calculation of the two diagrams of figure 2.3. For the first (quartic) diagram we obtain the scaling $\frac{m}{f^2} \times \frac{m}{\xi} \times \xi^3 \sim \frac{m^2\,\xi^2}{f^2}$ which agrees with result presented in eq. (2.3.10). For the cubic diagram, on the other hand, we would expect the scaling $(\frac{m}{f})^2 \times (\frac{m}{\xi})^2 \times \xi^3 \sim \frac{m^4}{f^2}\xi$ which is in disagreement; the result (2.3.14) scales $\sim \frac{m^4}{f^2}\sqrt{\xi}$. A possible explanation of this disagreement is that this diagram contains a term $\sim (\bar{\psi}\,\psi)^2$ where each factor $\bar{\psi}\,\psi$ is *oscillating* with amplitude $\sim 1/\xi$, so that interference effects might reduce the overall amplitude of the integral. Finally, as to the bispectra, we observe that the amplitude $\sim \frac{m^2}{f^3}\,\xi^2$ of the quintic diagram (2.4.4) agrees with the scalings outlined above, as it emerges as the product $\frac{m}{f^3} \times \frac{m}{\xi} \times \xi^3$.

Based on the this discussion, the second diagram of figure 2.4 should scale as $\frac{m}{f} \times \frac{m}{f^2} \times (\frac{m}{\xi})^2 \times \xi^3 \sim \frac{m^4}{f^3}\,\xi$, and the third diagram in that figure should scale as $(\frac{m}{f})^3 \times (\frac{m}{\xi})^3 \times \xi^3 \sim \frac{m^6}{f^3}$. This may be further suppressed if the same phenomena that are reducing the amplitude of the cubic contribution to the spectrum are at work here. Nonetheless, even without this suppression, both quantities are subdominant with respect to the contribution from the quintic diagram to the bispectrum. To sum up, the condition $\mu^2 \ll \xi$ allows to neglect the second diagram of figure 2.3 in the computation of the power spectrum, and the second and third diagrams of figure 2.4 in the computation of the bispectrum.

2.4.3 Summary for the bispectum

To summarize, we argue that the parameter f_{NL}^{eq} for this model, in the limit $\xi \gg 1$, $\mu \lesssim 1$ we are interested in, is obtained from eq. (2.4.5) where the divergent part is dropped or renormalized away by adiabatic subtraction; we expect these to agree when $\xi \gg 1$, as happens for the contribution (2.3.10) (see [22]).

Using the relation $\zeta = -H\,\delta\phi/\dot{\phi}_0$ and the relationship

$$-\frac{H^3}{|\dot{\phi}_0|^3}\langle\delta\phi\,\delta\phi\,\delta\phi\rangle' = \frac{9}{10}(2\pi)^{5/2} f_{NL}\left[\frac{H^2}{\dot{\phi}_0^2}\left(\frac{H}{2\pi}\right)^2\left(1+\frac{32\,m^2\,\xi^2\,\log\xi}{3\,\pi^2\,f^2}\,\log(H/k)\right)\right]^2\frac{1}{k^6} \tag{2.4.6}$$

between the bispectrum and the parameter f_{NL}, where we have accounted for the fact that eq. (2.3.15) is also contributing to the power spectrum, we therefore obtain

$$f_{NL}^{eq} \simeq \frac{\frac{40\,H^2\mu^2\xi^3}{3\pi f^2}\log(H/k)}{\left(1+\frac{32\,H^2\,\mu^2\,\xi^2\,\log\xi}{3\,\pi^2\,f^2}\,\log(H/k)\right)^2}\,. \tag{2.4.7}$$

We show the value taken by the non-linear parameter as a function of parameter space in figure 2.5, where H is determined as a function of m/f and ξ by imposing the measured normalization of the spectrum of scalar perturbations ($P_\zeta = 2.2{\cdot}10^{-9}$ [68]) and we have taken $\log(H/k) = 60$. In plotting figure 2.5 we have used the exact expressions of the quartic and quintic diagrams found in [22]. We see that there is significant parameter space consistent with $f_{NL}^{\rm eq} = -4 \pm 43$ [69]. In the figure we also show the region where $|\dot{\phi}_0| < f^2$, and the effective quantum field theory description of the rolling axion with a fixed decay constant f is under control (we also need to impose $H < f$; this condition is satisfied wherever $|\dot{\phi}_0| < f^2$). We note that, for a fixed value of m, the non-Gaussianity first grows with growing ξ, and then it decreases. To understand this, we recall that fermion modes of chirality $r = -1$ are produced with momentum up to $\sim 2\,\xi H$, as we discussed after eq. (2.1.19). Therefore, as ξ increases we increase the number of fermion modes that are produced, and these then source the inflaton perturbations. The sourced perturbations are non-Gaussian, which explains the initial growth of the non-Gaussianity parameter with ξ. However, as ξ keeps growing, the

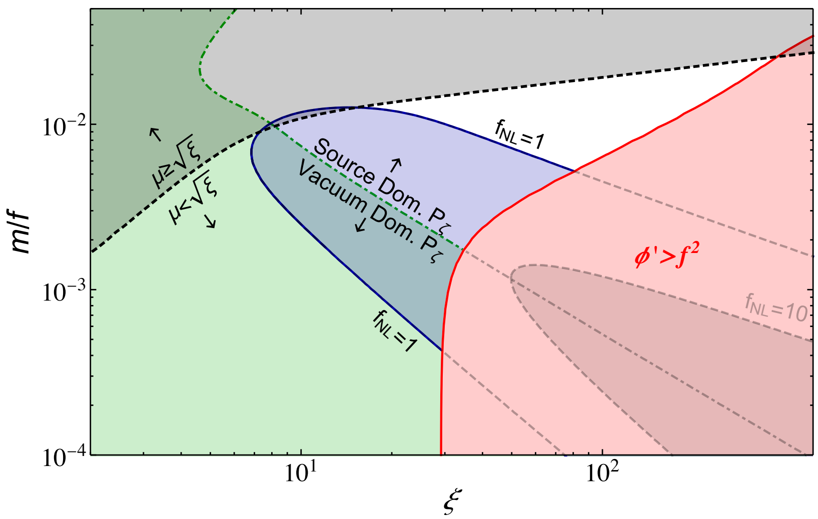

Figure 2.5: Contour lines of non-linear parameter f_{NL} evaluated on exactly equilateral configurations. We indicate the regions of parameter space in which the vacuum and the sourced perturbations dominate the scalar power spectrum. The figure also shows the region where $\phi' > f^2$ (where the effective quantum field theory description of the rolling axion with a fixed decay constant f is inappropriate) and the region $\mu > \sqrt{\xi}$, where the diagrams that we have neglected in the calculation of the power spectrum and bispectrum are not negligible (see discussion in subsection 2.4.2). The region where the motion of the inflaton is controlled by the backreaction of the produced fermions (where our analysis of the perturbations is invalid) lies at large values of $m/f \gtrsim .1$, outside of the region of parameter space covered by this plot.

contributions from the various fermion modes add up in an uncorrelated way to each other, and their contribution becomes more and more Gaussian (due to the central limit theorem) as their number grows.[4] This argument, and the trend in figure 2.5, leads us to argue that the perturbations should be Gaussian also in the regime of strong backreaction, where our computation of the perturbations is invalid.

[4]We note that this differs from the mechanism of non-Gaussian inflation perturbations sourced by a vector field [42]. In that case the monotonic growth of non-Gaussianity with ξ is due to the fact that the amplitude of the gauge modes grows exponentially with ξ.

2.5 Fermion production and inflation on a steep axionic potential

One might wonder whether the dissipation associated to the production of fermions in the regime of strong backreaction could allow for inflation on a steep potential, along the lines of [9]. In particular, we would now like to address the question of whether fermion production would allow slow-roll inflation when the inflaton has an axionic potential $V_{\rm ax}(\phi) = \Lambda^4 \left(\cos(\phi/f) + 1\right)$ where the axion constant f is much smaller than M_P. It is indeed well known that, for $f < M_P$, the axion potential $V_{\rm ax}$ is too steep to support successful slow-roll inflation. On the other hand, it is conjectured that potentials like $V_{\rm ax}$ with $f > M_P$ cannot be realized in UV-complete theories involving gravity, so that a mechanism able to produce enough inflation when $f < M_P$ would be of great interest.

We will therefore consider in this section a model with a cosine-like potential $V(\phi) = V_{\rm ax}(\phi)$, so that $V'(\phi) \simeq V(\phi)/f$. Also, in this section only, we will denote the coupling between fermions and the inflaton as α/f with α a dimensionless coefficient. This implies that $\xi = \alpha\dot\phi/(2Hf)$. Given the level of approximation of the discussion in this section, we will set to one all the $O(1)$ and $O(2\pi)$ factors.

The equation for the zero mode of the inflaton, eq. (2.2.1), including backreaction reads

$$\ddot\phi_0 + 3H\dot\phi_0 + V_{\rm ax}'(\phi_0) \simeq \frac{\alpha}{f} H^4 \mu^2 \xi^2 = \frac{\alpha^3}{f^3}\mu^2 H^2 \dot\phi_0^2\,, \tag{2.5.1}$$

where we recall that $\mu \equiv m/H$, with m being the fermion mass, and that ϕ_0 denotes the inflaton zero mode.

We now assume that, in the regime of strong backreaction, the last term on the left hand side of the equation above is balanced by the term on the right hand side (rather than by the Hubble friction), so that, using $V_{\rm ax}' \simeq V_{\rm ax}/f \simeq H^2 M_P^2/f$ we get the slow-roll equation

$$\dot\phi_0 = \frac{f\,M_P}{\alpha^{3/2}\mu}\,. \tag{2.5.2}$$

We now must impose a number of conditions for our theory to be valid.

1. First, in order for the effective field theory to make sense, we must work below the cutoff f of the theory. This implies, in particular $\dot{\phi}_0 \ll f^2$, which gives the first constraint

$$\alpha^{3/2} \gg \frac{M_P}{f\,\mu} \gg 1\,, \tag{2.5.3}$$

where the second inequality emerges from requiring $f \ll M_P$ and the assumption $\mu \lesssim 1$.

2. Consistency requires $3H\dot{\phi}_0 \ll V'(\phi_0)$, which gives

$$\alpha^{3/2} \gg \frac{f^2}{H\,M_P\,\mu}\,. \tag{2.5.4}$$

3. Moreover, the energy density in fermions must be much smaller than that in the inflaton. The energy density of fermions can be computed directly (see [22]), and we obtain the scaling

$$\rho_\psi \propto H^4\mu^2\xi^3 \sim \frac{HM_P^3}{\alpha^{3/2}\mu}\,, \tag{2.5.5}$$

which is subdominant with respect to the energy in the inflaton $\sim H^2M_P^2$, as long as

$$\alpha^{3/2} \gg \frac{M_P}{H\mu}\,. \tag{2.5.6}$$

Note that this condition is stronger than that of eq. (2.5.3), since $f \gg H$.

4. Of course, we must have inflation, which requires $\dot{\phi}_0^2 \ll H^2M_P^2$; this gives the additional constraint

$$\alpha^{3/2} \gg \frac{f}{H\mu}\,, \tag{2.5.7}$$

which is again automatically satisfied if eq. (2.5.6) is valid, since $f \ll M_P$.

5. Finally, we must impose that we have a sufficient number of efoldings. To have 60

efoldings the inflaton must span $\Delta\phi = 60\,\dot{\phi}_0/H \ll f$ since the inflaton potential has period $\sim f$. This gives the condition

$$\alpha^{3/2} \gg 60\,\frac{M_P}{\mu H}\,, \tag{2.5.8}$$

which is stronger by a factor 60 than eq. (2.5.6).

To sum up, we have inflation with the inflaton motion controlled by backreaction in our model if the two conditions (2.5.4) and (2.5.8) are satisfied. The arguments presented at the end of section 2.4 lead to the conclusion that the perturbations in this regime are likely to be Gaussian to a high degree. However, the computations performed in this work are invalid in this regime.

2.6 Discussion

In [37] it was shown that the (derivative) coupling of a pseudoscalar inflaton to a gauge field would lead to the exponential amplification of the modes of one helicity of the gauge field. In the present paper we have discussed how an analogous coupling to fermions can lead to a fermion number density that is parametrically (although not exponentially) larger than unity in units of the Hubble radius. This can lead to a rich phenomenology, which is all the more interesting because fermions, because of Pauli blocking and because of conformality in the limit $m \to 0$, do not usually give any relevant effect during inflation. Such a phenomenology is significantly different from that induced by the amplification of the modes of a gauge field because the large number of produced particles lives, in the fermionic case, at large momenta, whereas in the case of the gauge field these modes live close to the horizon scale.

This chapter has focused, in particular, on the effect of the backreaction of the produced fermions on the zero mode and on the fluctuations of the inflaton in the regime $\mu \equiv m/H \lesssim 1$, $\xi \equiv \dot{\phi}/(2fH) \gg 1$. The backreaction on the zero mode turns out to be negligible for $\mu^2\,\xi \ll f^2/H^2$. In the regime in which the fermionic contribution to the power spectrum of scalar metric perturbations is subdominant with respect to the vacuum contribution, the corrections to P_ζ and to the non-Gaussianity parameter f_{NL} scale respectively as

$(H^2/f^2)\mu^2\,\xi^2$ and $(H^3/f^3)\,\mu^2\,\xi^3$. Remarkably, however, in the regime $\xi \gg 1$ the fluctuations sourced by the fermion become approximately Gaussian, as a consequence of the fact that many fermion modes with large momenta contribute incoherently to the scalar perturbations. This leads to the interesting situation where the spectrum of perturbations is dominated by its sourced component, rather than by its vacuum one, and yet the non-Gaussianities are small and in agreement with observations. In this regime, the measured power spectrum does not yield the combination $H^4/\dot{\phi}^2$, but rather the combination $H^2\,m^2/f^4$.

We have also shown that the backreaction of the fermions on the zero mode of the inflaton can be strong enough to allow for inflation even on a steep scalar potential. This is especially relevant in models of natural inflation with a cosine potential. Potentials with this shape are ubiquitous in string theory and enjoy properties of radiative stability that make them especially attractive. However, these potentials are conjectured to be always too steep to support successful slow-roll inflation. In the system considered in this paper, however, it would be possible to obtain 60 e-foldings of inflation on these steep axionic potentials thanks to the slow-down of the inflaton induced by the production of fermions. The price to pay for this is a large value of the dimensionless parameter α that appears in section 2.5. A similar mechanism was discussed in [9], with the fermions replaced by a gauge field. That work contained also an estimate of the amplitude of the primordial perturbations, which was found to be too large to agree with observations in the simple case of a single species of gauge field. We expect that an estimate of the spectrum of perturbations generated in the regime where fermions strongly backreact on the inflaton zero mode will be even more difficult. However, given the peculiarities that we have encountered in this study, we can expect a different parametric dependence of the power spectrum that might lead to better agreement with observations.

Chapter 3

Gravitational waves from fermion production during axion inflation

In this chapter, we present analytic results for the gravitational wave power spectrum induced in models where the inflaton is coupled to a fermionic pseudocurrent. We show that although such a coupling creates helically polarized fermions, the polarized component of the resulting gravitational waves is parametrically suppressed with respect to the non-polarized one. We also show that the amplitude of the gravitational wave signal associated to this production cannot exceed that generated by the standard mechanism of amplification of vacuum fluctuations.

Given the rich phenomenology of the scalar perturbations found in Chapter 2, it is important to characterize how fermions source gravitational waves in the regime where $\dot{\varphi}/f \gg H$. As was noticed in reference [58] and confirmed in reference [22], the produced fermions have a helicity asymmetry, which was used for leptogenesis in reference [62]. This helicity asymmetry raises the possibility that the spectrum of sourced gravitational waves has a chiral component. Sourced production of gravitational waves in this context was previously studied[1] in reference [63]. However, the fermion basis used in that work was leading to pathologies as $m \to 0$, whereas in this work we use the basis introduced in Chapter 2, in which perturbation theory remains valid as the fermion mass m becomes

[1] Gravitational wave production by non-chiral fermions has also been studied in reference [**Figueroa:2013vif**].

small. We review this basis in section 3.1.

This chapter is organized as follows. In section 3.1, we introduce our theory, and working in the Arnowitt-Deser-Misner (ADM) [70] formalism, we solve the gravitational constraint equations to second order. We then use these solutions to obtain the interaction Lagrangian to fourth order in fluctuations. From this interaction Lagrangian, we obtain one $\mathcal{O}(\gamma\Psi^2)$ vertex and seven $\mathcal{O}(\gamma^2\Psi^2)$ vertices, where γ and Ψ respectively denote, schematically, the tensor and the fermion degrees of freedom. In section 3.2, we use these interactions to compute eight loop diagrams in the in-in formalism [14, 66]. The $\mathcal{O}(\gamma^2\Psi^2)$ interactions lead to seven one-loop one-vertex diagrams, which we evaluate in section 3.2.2, while the $\mathcal{O}(\gamma\Psi^2)$ generates a two-vertex loop, which we evaluate in section 3.2.2. We discuss our results in section 3.2.3; we show that the chirally asymmetric contribution is subdominant, and that the total sourced contribution to the tensor-to-scalar ratio is beneath the vacuum component. Details of our calculations can be found in [59]. We work in natural units where $\hbar = c = 1$, and $M_{\mathrm{Pl}} = 1/\sqrt{8\pi G}$ is the reduced Planck mass.

3.1 Fermion-graviton interactions during axion inflation

The aim of this chapter is to compute the amplitude of the tensor modes sourced by fermions in a model where a pseudoscalar inflaton φ is derivatively coupled to a fermion X of mass m, as described in the action (3.1.1) below. At leading order in perturbation theory, fermions source gravitational wave power at one-loop. At one-loop, diagrams of two topologies are possible. The first topology—the cubic loop—is a two-vertex diagram that is generated by two cubic order vertices consisting of one gravitational wave and a fermion bilinear. The second topology—the quartic loop—is a one-vertex diagram generated by a quartic-order vertex consisting of two gravitational waves and a fermion bilinear (see figure 3.1 below). To find the required interactions, we therefore need to expand the full action to quartic order in fluctuations.

3.1.1 Starting action

We consider a theory containing a pseudoscalar inflaton φ with a shift-symmetric coupling to a fermion X and minimally coupled to gravity, so that our action, in mostly minus convention, reads

$$S = \int d^4x\sqrt{-g}\left[\frac{M_{\rm Pl}^2}{2}R + \frac{g^{\mu\nu}}{2}\partial_\mu\varphi\partial_\nu\varphi - V(\varphi) + \bar{X}\left(i\gamma^\mu D_\mu - m - \frac{1}{f}\partial_\mu\varphi\gamma^\mu\gamma_5\right)X\right]. \tag{3.1.1}$$

As discussed in Chapter 2, (see appendix A for more details), the use of these fields makes apparent the shift-symmetric nature of the inflaton-fermion interaction, but it obscures the fact that such interaction vanishes as $m \to 0$. It is therefore convenient to redefine the fermion according to

$$X \to \Psi = e^{i\gamma_5\frac{\varphi}{f}}X\ , \tag{3.1.2}$$

which puts the fermion action in the form

$$S_\Psi = \int d^4x\sqrt{-g}\,\bar{\Psi}\left[i\gamma^C e_C{}^\mu\left(\partial_\mu + \frac{1}{2}\omega_{\mu AB}\Sigma^{AB}\right)\Psi - m\,\cos\left(\frac{2\varphi}{f}\right) + im\,\sin\left(\frac{2\varphi}{f}\right)\gamma_5\right]\Psi, \tag{3.1.3}$$

where greek letters are spacetime indices $\mu,\nu \in \{0,1,2,3\}$, capital roman letters are 4D Lorentz indices $A,B,C \in \{0,1,2,3\}$. In this paper we will also use lower case roman letters from the start of the alphabet as spatial Lorentz indices $a,b,c \in \{1,2,3\}$, and finally roman letters from the middle of the alphabet as spatial spacetime indices, $i,j,k \in \{1,2,3\}$. The generator of local Lorentz transformations is $\Sigma^{AB} = \frac{1}{4}\left[\gamma^A,\gamma^B\right]$, and the spin connection is $\omega_\mu{}^{AB} = e^A{}_\nu\nabla_\mu e^{B\nu}$, where $e^A{}_\nu$ is the vierbein.

3.1.2 The action in ADM form

Because certain components of the metric are constrained degrees of freedom whose values depend on the fermion bilinears (as well as the other dynamical degrees of freedom), gravitationally-mediated fermion-graviton couplings are generated when these constraints

are eliminated from the action. In order to perform this analysis, it is convenient to decompose the metric using the ADM formalism. The key advantage of this formulation is that the constrained degrees of freedom enter the theory algebraically; their equations of motion are algebraic constraints. The metric in ADM form reads

$$ds^2 = N^2 d\tau^2 - h_{ij}(dx^i + N^i d\tau)(dx^j + N^j d\tau)\ , \tag{3.1.4}$$

where N is the lapse and N^i is the shift. For the background metric we choose $N = a$, so that τ denotes conformal time. Derivatives with respect to τ are represented with primes. The spatial indices, i, j, k, ... are raised and lowered with h_{ij}, so that $N^i \equiv h^{ij}\, N_j$, $h^{ij}\, h_{jk} = \delta^i_k$. Finally, $\det[g] = -N^2 \det[h]$.

In these coordinates, the action for the purely bosonic sector of the theory (involving gravity and the inflaton) becomes

$$S_{\rm B} = \int d\tau\, d^3x\, N\sqrt{h}\left[\frac{M_{\rm Pl}^2}{2}\left({}^{(3)}R + K^{ij}\,K_{ij} - K^2\right) + \frac{\pi_\varphi^2}{2\,N^2} - \frac{1}{2}h^{ij}\partial_i\varphi\partial_j\varphi - V(\varphi)\right]\ , \tag{3.1.5}$$

where $\pi_\varphi \equiv \varphi' - N^j\partial_j\varphi$ and

$$K_{ij} \equiv -\frac{1}{2N}\left(h'_{ij} - {}^{(3)}\nabla_i N_j - {}^{(3)}\nabla_j N_i\right), \quad K = K^i_{\ i}\,. \tag{3.1.6}$$

The fermionic action, S_F, in these coordinates reads

$$S_F = \int d^4x\, \mathcal{L}_F, \tag{3.1.7}$$

where (see appendix B)

$$
\begin{aligned}
\mathcal{L}_F = a^3 \Bigg\{ & i\bar{\Psi}\gamma^0 \Big[\partial_0 + \left(\partial_i N - N^j\, K_{ij}\right) e_b{}^i \Sigma^{0b} \\
& + \frac{1}{2} e^c{}_k \left(\partial_0 e_b{}^k - \left(N K^k{}_m - {}^{(3)}\nabla_m N^k\right) e_b{}^m\right) \eta_{ac}\Sigma^{ab}\Big]\Psi \\
& + i\bar{\Psi}\left(\gamma^a N e_a{}^k - \gamma^0 N^k\right)\Big[\partial_k - K_{ik} e_b{}^i \Sigma^{0b} + \frac{1}{2}\left(e^c{}_i \partial_k e_b{}^i + {}^{(3)}\Gamma^m{}_{ik} e^c{}_m e_b{}^i\right)\eta_{ac}\Sigma^{ab}\Big]\Psi \\
& - N m \bar{\Psi}\left[\cos\left(\frac{2\varphi}{f}\right) + i\sin\left(\frac{2\varphi}{f}\right)\gamma_5\right]\Psi\Bigg\}. \qquad (3.1.8)
\end{aligned}
$$

In these expressions, ${}^{(3)}\nabla_i$ denotes the three dimensional covariant derivative, η_{ab} is the spatial part of the Minkowski metric, $\eta_{AB} = \mathrm{diag}[1, -1, -1, -1]$, and the spatial vielbeins satisfy $\delta_{ab} e^a{}_i e^b{}_j = h_{ij}$. The total action is the sum $S = S_B + S_F$.

3.1.3 Constraints

When written in terms of the ADM decomposition, one can see that the lapse N and the shift N_i enter in the action, eqs. (3.1.5) and (3.1.8), without time derivatives (in the case of the lapse, spatial derivatives are also missing from the action). This implies that the corresponding Euler-Lagrange equations are constraints. The equation of motion for the lapse is the Hamiltonian constraint

$$
\begin{aligned}
0 = \frac{\delta S}{\delta N} = & \frac{M_{\mathrm{Pl}}^2}{2}\, {}^{(3)}R - \frac{1}{2} h^{ij}\partial_i\varphi\partial_j\varphi - V - \frac{M_{\mathrm{Pl}}^2}{2}(K^{ij}K_{ij} - K^2) - \frac{1}{2N^2}\pi_\varphi^2 \\
& + \frac{i}{2} e_a{}^i (\bar{\Psi}\,\gamma^a\,\partial_i\Psi - \partial_i\bar{\Psi}\,\gamma^a\,\Psi) - \frac{1}{4} e_a{}^i\, e_{bj}\, {}^{(3)}\nabla_i e_c{}^j\, \epsilon^{abc}\,\bar{\Psi}\gamma^0\gamma^5\Psi \\
& - m\,\bar{\Psi}\Big[\cos\left(\frac{2\varphi}{f}\right) + i\sin\left(\frac{2\varphi}{f}\right)\gamma^5\Big]\Psi\,, \qquad (3.1.9)
\end{aligned}
$$

while the equation of motion for the shift is the momentum constraint

$$
\begin{aligned}
0 = & \frac{\delta S}{\delta N_i} - {}^{(3)}\nabla_j \frac{\delta S}{\delta({}^{(3)}\nabla_j N_i)} = \frac{1}{N}\pi_\varphi\partial_i\varphi + \frac{i}{2}(\bar{\Psi}\,\gamma^0\,\partial_i\Psi - \partial_i\bar{\Psi}\,\gamma^0\,\Psi) \\
& + \frac{1}{4} e^a{}_j\, {}^{(3)}\nabla_i e^{bj}\,\epsilon_{abc}\,\bar{\Psi}\gamma^c\gamma^5\Psi + M_{\mathrm{Pl}}^2\, {}^{(3)}\nabla^j\,(K_{ij} - h_{ij}\,K) + \frac{1}{4}{}^{(3)}\nabla^j\left(e^a{}_i\, e^b{}_j\,\epsilon_{abc}\,\bar{\Psi}\gamma^c\gamma^5\Psi\right). \qquad (3.1.10)
\end{aligned}
$$

In these expressions, ϵ^{abc} is the "flat" three-dimensional Levi-Civita tensor, with the convention $\epsilon^{123} = +1$.

We work in the spatially flat gauge where $\det[h_{ij}] = a^6$ and the dynamical scalar fluctuation degrees of freedom are in the fluctuations of the inflaton. We parametrize the tensor perturbations of the metric as[2]

$$h_{ij} = a^2 \left(\mathrm{e}^{\gamma}\right)_{ij} = a^2 \left[\delta_{ij} + \gamma_{ij} + \frac{1}{2}\gamma_{im}\gamma_{mj} + \dots\right] , \tag{3.1.11}$$

where $a(\tau)$ is the scale factor. The transverse-traceless nature of the tensor modes, $\delta^{ij}\gamma_{ij} = \gamma_{ij,j} = 0$, implies that $\det\left[e^{\gamma}\right] = 1$. Similarly, the spatial vielbeins are expanded in terms of the tensor perturbations as

$$e^a{}_i = a\,\delta^{ak} e^{\frac{1}{2}\gamma_{ki}} = a\,\delta^{ak}\left[1 + \frac{1}{2}\gamma_{ki} + \frac{1}{8}\gamma_{kj}\gamma_{ji}\dots\right]. \tag{3.1.12}$$

Our goal is to determine the effective cubic $\gamma\bar{\Psi}\Psi$ and quartic $\gamma\gamma\bar{\Psi}\Psi$ component of the Lagrangian, where γ schematically denotes the graviton. As is well known, in order to determine the action to n-th order in the fluctuations, the solutions to the constraint equations are required at order $n-2$; terms of order n and $n-1$ simply multiply lower-order constraint equations [71]. Thus, to obtain the action up to fourth order in the fluctuations, we require solutions for the constraints (the lapse and shift) up to quadratic order. Note that Lorentz invariance means that the fermion fields only begin to contribute to the constraint equations at quadratic order. We therefore solve the above constraints in eqs. (3.1.9) and (3.1.10) perturbatively, to second order in the fluctuations, before plugging them back into the original action.

To facilitate a perturbative solution, we expand the lapse and shift functions as

$$\begin{aligned} N &= a\left(1 + \alpha^{(1)} + \alpha^{(2)} + \dots\right) , \\ N_i &= \partial_i\theta^{(1)} + \partial_i\theta^{(2)} + \dots + \beta_i^{(1)} + \beta_i^{(2)} + \dots , \end{aligned} \tag{3.1.13}$$

where the superscript denotes the order of the expansion, and where $\beta_i^{(1,2)}$ are transverse,

[2] Repeated lower roman indices are summed with the Kronecker delta: $x_i x_i \equiv \sum_{i,j=1}^{3} \delta^{ij} x_i x_j$

$\partial_i^{(1,\,2)}\beta_i = 0$. We also expand the inflaton as $\varphi(\tau,\,\vec{x}) = \varphi_0(\tau) + \delta\varphi(\tau,\,\vec{x})$, and we treat the fermions as first order quanties, so that fermion bilinears are of second order.

To zeroth order, the Hamiltonian constraint reduces to the Friedmann equation

$$\mathcal{H}^2 = \frac{1}{3\,M_{\rm Pl}^2}\left(\frac{{\varphi_0'}^2}{2} + a^2\,V(\varphi)\right)\,, \qquad \mathcal{H} \equiv \frac{a'}{a}\,, \tag{3.1.14}$$

while the momentum constraint is automatically satisfied. At first order, we obtain [14]

$$\beta^{(1)} = 0\,, \qquad \alpha^{(1)} = \frac{\varphi_0'}{2\,\mathcal{H}\,M_P^2}\,\delta\varphi\,, \qquad \Delta\theta^{(1)} = -\frac{\varphi_0'^2}{2\,M_P^2\mathcal{H}^2}\left(\frac{\mathcal{H}\,\delta\varphi}{\varphi_0'}\right)'\,. \tag{3.1.15}$$

Since we are not interested in the perturbations sourced by fluctuations of the inflaton, we drop these from now on. Ignoring inflaton fluctuations, the second order constraints read

$$\begin{aligned}
\alpha^{(2)} &= \Delta^{-1}\left\{\frac{1}{8\mathcal{H}}\partial_j\left[(\partial_j\gamma_{\ell i})\gamma_{i\ell}'\right] + \frac{ia}{4M_{\rm Pl}^2\mathcal{H}}\left[\bar\Psi\gamma^0\Delta\Psi - (\Delta\bar\Psi)\gamma^0\Psi\right]\right\}\,,\\
\beta_j^{(2)} &= \Delta^{-1}\left\{\frac{1}{2}\Delta^{-1}\partial_j\partial_k\left[(\partial_k\gamma_{\ell i})\gamma_{i\ell}'\right] - \frac{1}{2}\left[(\partial_i\gamma_{jk}')\gamma_{ki} + (\partial_j\gamma_{\ell i})\gamma_{i\ell}' - (\partial_i\gamma_{jk})\gamma_{ki}'\right]\right.\\
&\quad + \frac{ia}{M_{\rm Pl}^2}\partial_j\Delta^{-1}\left[\bar\Psi\gamma^0\Delta\Psi - (\Delta\bar\Psi)\gamma^0\Psi\right]\\
&\quad \left. - \frac{a}{M_{\rm Pl}^2}\left[i\left(\bar\Psi\gamma^0\partial_j\Psi - (\partial_j\bar\Psi)\gamma^0\Psi\right) - \frac{1}{2}\epsilon_{ijk}\partial_i(\bar\Psi\gamma^k\gamma^5\Psi)\right]\right\}\,,\\
\theta^{(2)} &= \Delta^{-1}\left\{-\frac{1}{16\mathcal{H}}\left[\gamma_{ij}'\gamma_{ij}' + (\partial_j\gamma_{kq})\,\partial_j\gamma_{qk}\right] - \frac{ia}{4M_{\rm Pl}^2\mathcal{H}}\left(\bar\Psi\gamma^0\partial_0\Psi - (\partial_0\bar\Psi)\gamma^0\Psi\right)\right.\\
&\quad \left. - \frac{a^2}{M_{\rm Pl}^2\mathcal{H}}V(\varphi_0)\Delta^{-1}\left\{\frac{1}{8\mathcal{H}}\partial_j\left[(\partial_j\gamma_{\ell i})\gamma_{i\ell}'\right] + \frac{ia}{4M_{\rm Pl}^2\mathcal{H}}\left[\bar\Psi\gamma^0\Delta\Psi - (\Delta\bar\Psi)\gamma^0\Psi\right]\right\}\right\}\,,
\end{aligned} \tag{3.1.16}$$

where $\Delta = \partial_i\partial_i$ is the spatial Laplacian, Δ^{-1} is its inverse, and we note that $\epsilon_{123} = -1$. We have used the linear equation of motion for the fermion to simplify the solution for $\theta^{(2)}$; the details of this calculation are given in appendix B.

3.1.4 Explicit form of the fermion action, and fermion-GW interactions

We insert the solutions to the constraint equations for N and N^i to second order (eqs. (3.1.14), (3.1.15), and (3.1.16)) into the action, eq. (3.1.5) + (3.1.7), and then expand order by order in the fluctuations. This gives the quadratic action $S^{(2)} = S_\gamma^{(2)} + S_F^{(2)}$ for the free

gravitons and fermions, the cubic action $S_F^{(3)}$ describing the $\mathcal{O}\left(\Psi^2\gamma\right)$ interactions, and the quartic action $S_F^{(4)}$ describing the $\mathcal{O}\left(\Psi^2\gamma^2\right)$ interactions.

The quadratic action for the gravitons reads

$$S_\gamma^{(2)} = \frac{M_{\rm Pl}^2}{8}\int d^4x\, a^2\left[\gamma'_{ij}\gamma'_{ij} - \partial_k\gamma_{ij}\partial_k\gamma_{ij}\right]\,, \tag{3.1.17}$$

while the quadratic action for the fermions is

$$S_F^{(2)} = \int d^4x\left[i\bar{\psi}\left(\gamma^0\partial_0 + \gamma^a\partial_a\right)\psi - ma\,\cos\left(\frac{2\varphi}{f}\right)\bar{\psi}\psi + ima\,\sin\left(\frac{2\varphi}{f}\right)\bar{\psi}\gamma_5\psi\right]\,, \tag{3.1.18}$$

where we have rescaled the fermion field according to $\psi \equiv a^{3/2}\Psi$.

At cubic order we find

$$S_F^{(3)} = -\frac{i}{2}\int d^4x\,\gamma_{ij}\,\bar{\psi}\,\gamma^i\,\partial_j\,\psi \equiv \int d^4x\,\mathcal{L}^{(3)}\,, \tag{3.1.19}$$

and some straightforward, but lengthy, algebra leads to the quartic order action

$$\begin{aligned} S_F^{(4)} = \int d^4x \Bigg\{ & \frac{i}{16}\gamma_{ab}\,\gamma_{bk}\left(\bar{\psi}\gamma^a\partial_k\psi - \partial_k\bar{\psi}\gamma^a\psi\right) \\ & + \frac{1}{16}\gamma'_{aj}\gamma_{jb}\epsilon_{abc}\bar{\psi}\gamma^c\gamma^5\psi + \frac{1}{16}\epsilon_{abc}\gamma_{kc}\left(\partial_a\gamma_{kb}\right)\bar{\psi}\gamma^0\gamma^5\psi \\ & + \frac{i}{4}\left(1 - \frac{V}{4\,H^2\,M_{\rm Pl}^2}\right)\Delta^{-2}\partial_m\left(\partial_m\gamma_{kn}\,\gamma'_{kn}\right)\left(\bar{\psi}\gamma^0\Delta\psi - (\Delta\bar{\psi})\,\gamma^0\psi\right) \\ & - \frac{\Delta^{-1}}{8}(\gamma'_{jk}\,\partial_j\gamma_{ik} - \gamma_{jk}\,\partial_j\gamma'_{ik} - \gamma'_{kj}\,\partial_i\gamma_{kj})\left[\epsilon_{aic}\partial_a(\bar{\psi}\gamma^c\gamma^5\psi) + 2\,i\left(\bar{\psi}\gamma^0\partial_i\psi - \partial_i\bar{\psi}\gamma^0\psi\right)\right] \\ & - \frac{i}{32\,aH}\left(\partial_i\gamma_{jk}\,\partial_i\gamma_{jk} + \gamma'_{ij}\gamma'_{ij}\right)\Delta^{-1}\left(\bar{\psi}\gamma^0\,\Delta\psi - \Delta\bar{\psi}\gamma^0\psi\right) \\ & - \frac{i}{16\,aH}\left(\bar{\psi}\gamma^0\psi' - \bar{\psi}'\,\gamma^0\psi\right)\,\Delta^{-1}\partial_i\left(\partial_i\gamma_{kj}\,\gamma'_{kj}\right)\Bigg\} \equiv \int d^4x\,\mathcal{L}^{(4)} \equiv \sum_{i=1}^{7}\int d^4x\,\mathcal{L}_i^{(4)}\,, \end{aligned} \tag{3.1.20}$$

where $H = \dot{a}/a$ is the Hubble parameter and $\epsilon_{123} = -1$. $\mathcal{L}_1^{(4)}$, $\mathcal{L}_2^{(4)}$, $\mathcal{L}_3^{(4)}$ refer to the three terms in the first two lines of eq. (3.1.20), while the remaining $\mathcal{L}_i^{(4)}$ refer to the other four lines (one term per line). Note that the interactions $\mathcal{L}_4^{(4)}$, $\mathcal{L}_5^{(4)}$, $\mathcal{L}_6^{(4)}$, $\mathcal{L}_7^{(4)}$ arise from integrating out the non-dynamical constraints (the second order parts of the lapse and shift). From the

cubic and quartic Lagrangian densities we find the interaction Hamiltonian densities

$$\begin{aligned} H^{(3)}_{\rm int}\,(\tau) &= -\int d^3x\, \mathcal{L}^{(3)} \;\;, \\ H^{(4)}_{{\rm int},i}\,(\tau) &= -\int d^3x\, \mathcal{L}^{(4)}_i \;\;,\;\; (i=1,\ldots 7) \;\;. \end{aligned} \tag{3.1.21}$$

To proceed, we expand the tensors in Fourier space as

$$\gamma_{ij}(\mathbf{x},\,\tau) = \sum_\lambda \int \frac{d^3k}{(2\pi)^{3/2}}\,\gamma^\lambda_{\mathbf{k}}(\tau)\,\Pi^\lambda_{ij}\,(\mathbf{k})\;e^{i\mathbf{k}\cdot\mathbf{x}},\;\gamma^\lambda_{\mathbf{k}}(\tau) = \frac{\sqrt{2}}{a(\tau)\,M_{\rm Pl}}\,{\rm t}^\lambda_{\mathbf{k}}(\tau), \tag{3.1.22}$$

where the field ${\rm t}^\lambda_{\mathbf{k}}$ is canonically normalized. The sum is over the right-handed ($\lambda = +1$) and left-handed ($\lambda = -1$) tensor polarizations, with the polarization tensors satisfying

$$\Pi^\lambda_{ij}\,(\mathbf{k})^* = \Pi^{-\lambda}_{ij}\,(\mathbf{k}) = \Pi^\lambda_{ij}\,(-\mathbf{k})\,,\quad \Pi^\lambda_{ij}\,(\mathbf{k})\,\Pi^{\lambda'}_{ij}\,(\mathbf{k}) = 2\delta_{\lambda,-\lambda'},\quad \epsilon_{abc}\mathbf{k}_b\Pi^\lambda_{cd}\,(\mathbf{k}) = i\lambda k\Pi^\lambda_{ad}\,(\mathbf{k})\,. \tag{3.1.23}$$

We also Fourier transform the fermions according to

$$\psi(\mathbf{x},\tau) = \int \frac{d^3k}{(2\pi)^{3/2}}\;\psi_{\mathbf{k}}(\tau)\;e^{i\mathbf{k}\cdot\mathbf{x}}\,. \tag{3.1.24}$$

In terms of the fields ${\rm t}^\lambda_{\mathbf{k}}$ and $\psi_{\mathbf{k}}$, the quadratic action is

$$\begin{aligned} S^{(2)} = \int dt \Bigg[&\int d^3k \Big(i\bar\psi_{\mathbf{k}}\left(\gamma^0\partial_0 + i\gamma^a k_a\right)\psi_{\mathbf{k}} - ma\cos\left(\frac{2\varphi}{f}\right)\bar\psi_{\mathbf{k}}\psi_{\mathbf{k}} + ima\sin\left(\frac{2\varphi}{f}\right)\bar\psi_{\mathbf{k}}\gamma_5\psi_{\mathbf{k}}\Big) \\ &+ \sum_\lambda \frac{1}{2}\int d^3k\left(\partial_0{\rm t}^\lambda_{-\mathbf{k}}\partial_0{\rm t}^\lambda_{\mathbf{k}} - \left(k^2 - \frac{a''}{a}\right){\rm t}^\lambda_{-\mathbf{k}}{\rm t}^\lambda_{\mathbf{k}}\right)\Bigg]\,, \end{aligned} \tag{3.1.25}$$

and we note that the kinetic terms are canonically normalized.

Inserting eqs. (3.1.22) and (3.1.24) in the interaction Hamiltonians eq. (3.1.21), we obtain the Fourier space Hamiltonian densities we report in appendix C.

3.2 Fermion contributions to the tensor power spectrum

In this section, making use of the interaction Hamiltonians derived in the previous section, we compute the fermion contribution to the gravitational wave two-point correlation function. After quantizing the free theory, we introduce the in-in formalism and compute the cubic and quartic loops generated by the interactions derived in section 3.1. Finally, we end this section by showing how simple scaling arguments concur with our results.

3.2.1 Quantization

We canonically quantize the theory by expanding the fields into modes

$$\hat{\mathrm{t}}_{\mathbf{k}}^{\lambda}(\tau) = \mathrm{t}_{k}^{\lambda}(\tau)\, a_{\mathbf{k}}^{\lambda} + \mathrm{t}_{k}^{\lambda,*}(\tau)\, a_{-\mathbf{k}}^{\lambda,\dagger} \quad , \quad \psi_{\mathbf{k}}(\tau) = \sum_{r=\pm}\left(U_{\mathbf{k}}^{r}(\tau)\, b_{\mathbf{k}}^{r} + V_{-\mathbf{k}}^{r}(\tau)\, c_{-\mathbf{k}}^{r,\dagger}\right) \quad , \tag{3.2.1}$$

where the creation-annihilation operators for the tensor modes satisfy the commutation relations

$$\left[a_{\mathbf{k}}^{\lambda},\, a_{\mathbf{k}'}^{\lambda'\,\dagger}\right] = \delta_{\lambda\lambda'}\,\delta\left(\mathbf{k}-\mathbf{k}'\right), \tag{3.2.2}$$

and the fermionic operators satisfy anti-commutation relations

$$\{b_{\mathbf{k}}^{r},\, b_{\mathbf{k}'}^{r'\,\dagger}\} = \{c_{\mathbf{k}}^{r},\, c_{\mathbf{k}'}^{r'\,\dagger}\} = \delta_{rr'}\,\delta\left(\mathbf{k}-\mathbf{k}'\right). \tag{3.2.3}$$

The mode functions $\mathrm{t}_{k}^{\lambda}(\tau)$, and the spinors $U_{\mathbf{k}}^{r}(\tau)$ and $V_{-\mathbf{k}}^{r}(\tau)$ are solutions of the Euler-Lagrange equations of motion that follow from the action in eq. (3.1.25). We further decompose the 4-component fermionic spinors into helicity states

$$U_{\mathbf{k}}^{r}(\tau) = \frac{1}{\sqrt{2}}\begin{pmatrix} u_{k}^{r}(\tau)\ \chi_{r}(\mathbf{k}) \\ r v_{k}^{r}(\tau)\ \chi_{r}(\mathbf{k}) \end{pmatrix} \quad , \quad V_{\mathbf{k}}^{r}(\tau) = C\, \bar{U}_{\mathbf{k}}^{r}(\tau)^{T} \quad , \tag{3.2.4}$$

where $C = i\gamma^0\gamma^2$ is the charge-conjugation operator, and the spinors $\chi_r(\mathbf{k})$ are explicitly given by

$$\chi_r(\mathbf{k}) \equiv \frac{k + r\boldsymbol{\sigma}\cdot\mathbf{k}}{\sqrt{2k(k+k_z)}}\bar{\chi}_r \quad , \quad \bar{\chi}_+ = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad , \quad \bar{\chi}_- = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \, , \tag{3.2.5}$$

where k_z is the $z-$component of $\mathbf{k}$, and σ_i are the Pauli matrices. Note that $\chi_r(\mathbf{k})$ are helicity eigenspinors which satisfy $\mathbf{k}\cdot\boldsymbol{\sigma}\chi_r(\mathbf{k}) = rk\chi_r(\mathbf{k})$. We use the Dirac representation for the γ matrices,[3]

$$\gamma^0 = \begin{pmatrix} \mathbb{1} & 0 \\ 0 & -\mathbb{1} \end{pmatrix} \quad , \quad \gamma^i = \begin{pmatrix} 0 & \sigma^i \\ -\sigma^i & 0 \end{pmatrix} \quad , \quad \gamma^5 = \begin{pmatrix} 0 & \mathbb{1} \\ \mathbb{1} & 0 \end{pmatrix} \, . \tag{3.2.6}$$

To obtain solutions to the classical mode equations, we approximate the background inflationary spacetime as de Sitter space and take the evolution of the inflaton to be rolling at a constant speed in cosmic time. This implies $\varphi_0(\tau)/f = \varphi_0^{\rm in}/f - 2\xi\log(x/x_{\rm in})$, with $x \equiv -k\tau$ and $x_{\rm in} \equiv -k\tau_{\rm in}$, where $\tau_{\rm in}$ is some reference time. With these approximations, the mode functions for the fermion field are given by

$$\begin{aligned} u^r(x) =& \frac{1}{\sqrt{2x}}\left[\mathrm{e}^{ir\varphi_0/f}\, s^r(x) + \mathrm{e}^{-ir\varphi_0/f}\, d^r(x)\right] , \\ v^r(x) =& \frac{1}{\sqrt{2x}}\left[\mathrm{e}^{ir\varphi_0/f}\, s^r(x) - \mathrm{e}^{-ir\varphi_0/f}\, d^r(x)\right] , \end{aligned} \tag{3.2.7}$$

which satisfy the normalization condition $|u^r|^2 + |v^r|^2 = 2$, with [58, 22]

$$s^r(x) = \mathrm{e}^{-\pi r\xi}\, W_{\frac{1}{2}+2ir\xi,\, i\sqrt{\mu^2+4\xi^2}}(-2ix) \, , \quad d^r(x) = -i\,\mu\,\mathrm{e}^{-\pi r\xi}\, W_{-\frac{1}{2}+2ir\xi,\, i\sqrt{\mu^2+4\xi^2}}(-2ix) \, , \tag{3.2.8}$$

where $W_{\mu,\lambda}(z)$ denotes the Whittaker W-function and

$$\mu \equiv \frac{m}{H} \, , \quad \xi \equiv \frac{\dot{\varphi}_0}{2fH} \, . \tag{3.2.9}$$

[3] In these expressions, $\mathbb{1}$ denotes the 2×2 identity matrix.

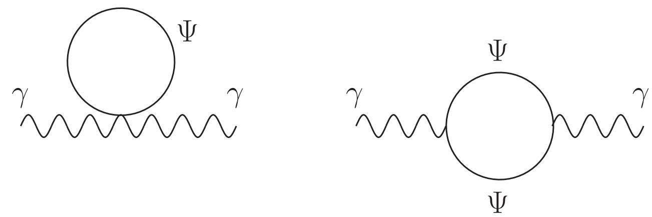

Figure 3.1: The two diagrams that contribute at leading order to the two-point function of the graviton γ.

In the same approximation, the tensor mode functions read

$$\mathrm{t}_{\mathbf{k}}^{\lambda}(\tau) = \frac{1}{\sqrt{2k}} \left(1 - \frac{i}{k\tau}\right) e^{-ik\tau} \,. \tag{3.2.10}$$

In both cases, the integration constants have been chosen so that the solutions match onto the appropriate Bunch-Davies vacuum solution at early times, $x \to \infty$.

3.2.2 Fermion loop-corrections to the gravitational wave power spectrum

The interaction Hamiltonians derived above allow us to compute the leading order contributions from the produced fermions to the two-point function of the graviton. These are computed using the in-in formalism, where the correlation function of an operator $\hat{O}_1 \ldots \hat{O}_n(\tau)$ at time τ is given by

$$\begin{aligned} \left\langle \hat{O}_1 \ldots \hat{O}_n(\tau) \right\rangle = \sum_{N=0}^{\infty} (-i)^N \int^{\tau} d\tau_1 \ldots \int^{\tau_{N-1}} d\tau_N \\ \times \left\langle \left[\left[\ldots \left[\hat{O}_1^{(0)} \ldots \hat{O}_n^{(0)}(\tau)\,,\, H_{\text{int}}(\tau_1) \right], \ldots \right], H_{\text{int}}(\tau_N) \right] \right\rangle . \end{aligned} \tag{3.2.11}$$

The interactions in section 3.1 result in two classes of diagrams: there are seven *quartic loop* diagrams, illustrated in the left panel of figure 3.1, one for each of the seven vertices generated by the quartic action (3.1.20), and one *cubic loop* diagram with two vertices generated by the cubic action (3.1.19), illustrated in the right panel of figure 3.1. We discuss these diagrams in the next two subsections.

Quartic loops

We begin with the left diagram of figure 3.1. The seven terms in the quartic action lead to seven quartic contributions to the graviton spectrum of the form

$$\left\langle \gamma_{\mathbf{p}_1}^{\lambda_1}(\tau)\,\gamma_{\mathbf{p}_2}^{\lambda_2}(\tau)\right\rangle_i^{(4)} = -\frac{2i}{M_{\rm Pl}^2\,a(\tau)^2}\int^\tau d\tau_1\left\langle\left[\mathrm{t}_{\mathbf{p}_1}^{\lambda_1}(\tau)\,\mathrm{t}_{\mathbf{p}_2}^{\lambda_2}(\tau),\,H_{\mathrm{int},\,i}^{(4)}(\tau_1)\right]\right\rangle\,,\qquad i=1,\,\ldots,7\,, \tag{3.2.12}$$

where the interaction Hamiltonians $H_{\mathrm{int},i}^{(4)}$ are given in eq. (C.0.2). Remarkably, all these diagrams can be computed exactly. The details of the calculation, as well as the exact results, are presented in [59]. Here we summarize the main issues one encounters when performing this calculation. At the end of this section we present the expression of the sum of the quartic loops in the limit $\mu \ll 1 \ll \xi$.

First, several of the terms in the interaction Hamiltonian contain the nonlocal operator Δ^{-1}, the inverse of the Laplacian. When evaluating eq. (3.2.12) one often encounters the expectation value of quantities evaluated at vanishing momentum that, when acted upon by Δ^{-1}, lead to a undetermined "0/0" that needs to be regularized. To deal with this limit we follow the prescription given in [72]: these undetermined quantities are schematically given by

$$\frac{1}{|\mathbf{q}_1-\mathbf{q}_2|^2}f(\mathbf{q}_1-\mathbf{q}_2,\,\mathbf{p}_1)\delta(\mathbf{q}_1-\mathbf{q}_2)\,,\qquad f(0,\,\mathbf{p}_1)=0\,, \tag{3.2.13}$$

where $\mathbf{p}_1$ is an external momentum. We regularize eq. (3.2.13) by setting $\mathbf{q}_1 = \mathbf{q}_2 + \boldsymbol{\epsilon}$, where we eventually send $\boldsymbol{\epsilon} \to 0$. Since f is a scalar, it depends only on $\boldsymbol{\epsilon}\cdot\mathbf{p}_1$, p_1^2, and ϵ^2. We then impose that $\boldsymbol{\epsilon}$ approaches zero in a direction that is orthogonal to $\mathbf{p}_1$, so that $\boldsymbol{\epsilon}\cdot\mathbf{p}_1 = \mathcal{O}(\epsilon^2)$. With this convention, all the operators containing Δ^{-1} give finite and unambiguous results.

Secondly, many integrals contributing to the graviton two-point function are divergent in the ultraviolet. We deal with these divergences as we did in Chapter 2, by introducing an ultraviolet cutoff Λ and by subtracting all the terms that are divergent as $\Lambda \to \infty$. As we have discussed in Chapter 2, we expect the result of this procedure to be equivalent to that obtained by adiabatic subtraction in the limit $\xi \gg 1$.

After long calculations, which can be found in [59], we obtain the leading contribution from the quartic diagrams

$$\sum_{\text{vertices}} \left\langle \hat{\gamma}^{\lambda_1}_{\mathbf{p}_1}(\tau)\hat{\gamma}^{\lambda_2}_{\mathbf{p}_2}(\tau)\right\rangle_{\text{quartic}} \simeq -\frac{8\,H^4 \log(-p_1\tau)}{9\pi\, M_{\text{Pl}}^4\, p_1^3}\, \mu^2\, \xi^3\, \delta(\mathbf{p}_1+\mathbf{p}_2)\, \delta^{\lambda_1,\lambda_2}\,, \tag{3.2.14}$$

in the limit $\mu \ll 1 \ll \xi$ and for superhorizon modes $-k\tau \ll 1$. We note that this contribution is parity-even. Parity-odd terms are associated to the operators $H^{(4)}_{\text{int},2}$, $H^{(4)}_{\text{int},3}$ and $H^{(4)}_{\text{int},5}$, which contain the Levi-Civita symbol. However, the contributions from $H^{(4)}_{\text{int},2}$ and from the parity-odd part of $H^{(4)}_{\text{int},5}$ vanish identically after angular integrations, so that the only parity-odd contribution to the tensor power spectrum is given by $H^{(4)}_{\text{int},3}$ and yields

$$\left\langle \hat{\gamma}^{\lambda_1}_{\mathbf{p}_1}(\tau)\hat{\gamma}^{\lambda_2}_{\mathbf{p}_2}(\tau)\right\rangle_{\text{parity-odd}} \simeq \lambda_1\, \frac{H^4}{3\, M_{\text{Pl}}^4\, p_1^3}\, \mu^2\, \xi^2\, \delta(\mathbf{p}_1+\mathbf{p}_2)\, \delta^{\lambda_1,\lambda_2}\,, \tag{3.2.15}$$

which is sub-leading, by a factor $1/\xi$, with respect to the parity-even component.

Cubic loop

Next we consider the cubic loop, shown on the right side of figure 3.1. There is a single contribution to this diagram, given by

$$\left\langle \gamma^{\lambda_1}_{\mathbf{p}_1}(\tau)\, \gamma^{\lambda_2}_{\mathbf{p}_2}(\tau)\right\rangle^{(3)} = -\frac{2}{M_{\text{Pl}}^2\, a(\tau)^2} \int^{\tau} d\tau_1 \int^{\tau_1} d\tau_2 \left\langle \left[\left[\mathrm{t}^{\lambda_1}_{\mathbf{p}_1}(\tau)\, \mathrm{t}^{\lambda_2}_{\mathbf{p}_2}(\tau),\, H^{(3)}_{\text{int}}(\tau_1)\right],\, H^{(3)}_{\text{int}}(\tau_2)\right]\right\rangle\,, \tag{3.2.16}$$

where $H^{(3)}_{\text{int}}(\tau)$ is given by eq. (C.0.1). Unlike those appearing in the quartic loops, the integrals in the cubic loop are prohibitively difficult to evaluate exactly. Therefore, we apply the same sequence of approximations developed in Chapter 2 to the present calculation. Here we outline these approximations; the details of the calculation are presented in [59].

We start by setting the external momenta to zero; as discussed in Chapter 2 we expect this approximation to generate at most an $\mathcal{O}(1)$ error. Next, since the functions appearing in the integrals are rapidly oscillating, we perform a Wick rotation on the time integration variables, so that the Whittaker functions appearing in the fermion mode functions now have real argument and are exponentially increasing or decreasing. Next, we approximate

those Whittaker functions as linear combinations of monomials times exponentials, with special attention given to the branch cuts. The explicit form of those approximations are given in [59], and we have verified their validity in the regime $\xi \gg 1$ we are interested in. These approximate expressions contain a part that behaves like positive frequency (we schematically denote the coefficient of this part by $\mathcal{A}$) and a part that behaves like negative frequency (whose coefficient is denoted schematically by $\mathcal{B}$). Explicit expressions for $\mathcal{A}$ and $\mathcal{B}$ can be found in [59].

Once the above approximations are in place, the integrals can be computed analytically. We find a divergence in the limit $\tau_2 \to \tau_1$, although it is only present in the $\mathcal{A}^2$ term. Since this term corresponds to positive frequency, "vacuum only" modes, we subtract them. Once this component is subtracted, we are left with the final result

$$\left\langle \hat{\gamma}^{\lambda_1}_{\mathbf{p}_1}(\tau)\hat{\gamma}^{\lambda_2}_{\mathbf{p}_2}(\tau)\right\rangle^{(3)} \sim \mathcal{O}(0.1) \times \frac{H^4 \delta(\mathbf{p}_1+\mathbf{p}_2)\delta^{\lambda_1,\lambda_2}}{M_{\rm Pl}^4\, p_1^3}\mu^2\, \xi^3\, \log(-p_1\tau), \tag{3.2.17}$$

which has the same parametric dependence as the contribution from the quartic loop.

3.2.3 Scaling of our result

The gravitational wave power spectrum is related to the two point function as

$$\langle \gamma^{\lambda_1}_{\mathbf{p}_1}(\tau)\, \gamma^{\lambda_2}_{\mathbf{p}_2}(\tau)\rangle = \frac{2\pi^2}{p_1^3}\, \mathcal{P}_t^{\lambda}\, \delta(\mathbf{p}_1+\mathbf{p}_2)\, \delta^{\lambda_1,\lambda_2}\ . \tag{3.2.18}$$

The contribution from the produced fermions, using the results eqs. (3.2.14) and (3.2.17) we derived in the previous subsections, is

$$\delta P_t^{\lambda} \simeq \mathcal{O}(0.01)\, \frac{H^4}{M_{\rm Pl}^4}\, \mu^2\, \xi^3\, \log(-p_1\tau). \tag{3.2.19}$$

We now compare this sourced gravitational wave signal to the vacuum contribution, $\mathcal{P}_t^{\rm vacuum} \sim 0.1\, H^2/M_{\rm Pl}^2$. Their ratio can be written as

$$\frac{\delta\mathcal{P}_t}{\mathcal{P}_t^{\rm vacuum}} \simeq 0.1\, \frac{\mu^2\, \xi^3\, H^2}{M_{\rm Pl}^2}\, \log(-p_1\tau)\, . \tag{3.2.20}$$

As discussed in Chapter 2, the quantity $\mu^2\,\xi^3\,H^2/M_{\rm Pl}^2$ corresponds to the ratio between the energy density in fermions and the total energy density in the Universe, which must be much smaller than unity. We thus conclude that an axion-like inflaton coupled to fermions cannot produce tensor modes that dominate over the spectrum of vacuum fluctuations, in contrast to the scenario in which an axionic inflaton is coupled to gauge fields.

More specifically, if we require that the energy in fermions does not exceed $\mathcal{O}(10\%)$ of the background energy, then the correction induced by the sourced component to the amplitude of the tensor spectrum is at most of $\mathcal{O}(1\%)$, irrespective of the amplitude of the tensor-to-scalar ratio. This constraint, along with the current observational constraint $r \lesssim 0.06$, already puts the sourced component below the $r \simeq 10^{-3}$ sensitivity of Stage-4 [8] CMB experiments. More importantly, both the sourced and the vacuum contributions are, to a first approximation, parity-even and scale invariant, so that it would be difficult for observations to tell from one component from the other. While theoretically the sourced tensors contain a distinctive parity-odd component, such a component is suppressed by a factor $1/\xi$ with respect to the parity even one. Therefore, since $\xi \gtrsim 10$, the parity odd component would make up less than 0.1% of the full tensor spectrum, which sends the level of parity violation in this model well below the threshold of detectability.

Finally, in Chapter 2 we determined the overall scaling, as a function of the parameter ξ and μ, of the diagrams that are relevant for the scalar (bi)spectrum. Here we apply analogous arguments to the diagrams that led to the result in eq. (3.2.19) for the tensor spectrum.

Our first observation is that the cubic interaction (3.1.19) gives a contribution $\sim \gamma_{ij}\,\bar\psi\,\Gamma\,p\,\psi$, whereas each quartic interaction in eq. (3.1.20) gives a contribution $\sim \gamma_{ij}^2\,\bar\psi\,\Gamma\,p\,\psi$, where p schematically denotes a quantity that scales as fermion momentum and Γ denotes some combination of the Dirac γ-matrices (exceptions are $\mathcal{L}_2^{(4)}$ and $\mathcal{L}_3^{(4)}$ which contain no dependence on the fermion momentum, but only on the graviton momentum). The contribution from $\langle\bar\psi\,\gamma^i\,\hat p_i\,\gamma^5\,\psi\rangle$ is ultimately zero, due to the asymmetry of the Levi-Civita symbol. A numerical evaluation shows that the combinations $\langle\bar\psi\,\gamma^i\,\hat p_i\,\psi\rangle$ and $\langle\bar\psi\,\gamma^5\,\gamma^0\,\psi\rangle$ (which appear in both the cubic and quartic gravitational vertices) oscillate with amplitude μ^2/ξ, for momenta up to $-k\tau \simeq \xi$. Moreover, the fermion part of the operators in $\mathcal{L}^{(4)}_{4,\,5,\,6,\,7}$ can always

be brought to a form $\langle \Im \left(\partial_k \bar{\psi}\, \gamma^0 \psi\right)\rangle$ (see [59]) – here $\Im$ denotes the imaginary part), which can also be seen to scale as μ^2/ξ for momenta up to $-k\tau \simeq \xi$. (We note for comparison that the bilinears which appear in the diagrams involving fluctuations of the inflaton, $\langle \bar{\psi}\, \psi\rangle$ and $\langle \bar{\psi}\, \gamma^5 \psi\rangle$, oscillate instead with amplitude μ/ξ. These were considered in Chapter 2.) Since all fermion bilinears in the $\gamma\bar{\psi}\psi$ and the $\gamma\gamma\bar{\psi}\psi$ sector scale as μ^2/ξ, each fermionic line in the diagrams of figure 3.1 contributes μ^2/ξ.

We recall that each interaction Hamiltonian (with the exception of $\mathcal{L}_2^{(4)}$ and $\mathcal{L}_3^{(4)}$, as noted above) also carries a power of p, and therefore each vertex gives an additional power of ξ. Furthermore, every fermionic loop integral, which goes as d^3k, gives a contribution $\sim \xi^3$.

Once we apply these scalings to the quartic diagrams, we have a scaling (μ^2/ξ) (one fermion line) times ξ (one vertex) times ξ^3, giving an overall scaling $(\mu^2/\xi) \times (\xi) \times (\xi^3) \sim \mu^2\, \xi^3$. The quartic diagrams involving $\mathcal{L}_2^{(4)}$ and $\mathcal{L}_3^{(4)}$ are respectively vanishing (as a consequence of the symmetries of the operator) and scaling as $(\mu^2/\xi) \times (1) \times (\xi^3) \sim \mu^2\, \xi^2$—the factor of (1) instead of (ξ) originates from the fact that this vertex does not contain a power of the fermion momentum. These results are in agreement with the direct calculations presented in [59].

For the cubic gravitational diagram, a naive implementation of these scalings would read $(\mu^2/\xi)^2$ (two fermion lines) times ξ^2 (two vertices) times ξ^3, giving an overall scaling $\sim \mu^4\, \xi^3$. This would disagree by a factor of μ^2 with the result obtained by the direct (albeit approximate) calculation in [59]. The different scaling with μ can, however, be understood as follows. Each fermion line in fact scales as $\mathcal{O}(1)+\mathcal{O}(\mu^2)$, although in the quartic diagrams the $\mathcal{O}(1)$ contribution is always divergent and therefore is removed by regularization. The cubic diagram, with two fermion lines, has terms of order $\mathcal{O}(1)$, $\mathcal{O}(\mu^2)$, and $\mathcal{O}(\mu^4)$. The first is again removed by renormalization, leaving the $\mathcal{O}(\mu^2)$ term which arises from interference.

Given the parity violating nature of the system, one can expect a parity violating tensor spectrum $\delta\mathcal{P}_t^{+1} \neq \delta\mathcal{P}_t^{-1}$, which we did indeed find. As mentioned above, however, this is subdominant by a factor $1/\xi$ with respect to the parity-even part.

3.3 Discussion

Axion, or natural inflation is a class of models for slow-roll inflation where the required flatness of the potential is protected from radiative corrections by an approximate shift symmetry. This shift symmetry means that any axion-matter couplings must be via derivatives, and the lowest dimension couplings of an axion inflaton to gauge fields and fermions are given by eq. (1.4.1). These couplings are typically employed for reheating in these models. The recent literature (see [33] for a review) has shown that the coupling of the axion to gauge fields can lead to a rich phenomenology during inflation. Analogous studies for fermions are more scarce [58, 63, 22, 60].

We worked in the ADM basis, in which the nondynamical metric perturbations N and N^i are integrated out via the corresponding energy and momentum constraints. We have solved these constraints perturbatively in the fermionic field Ψ and in the gravitational waves γ. We thus obtained the $\mathcal{O}\left(\gamma\bar{\Psi}\Psi\right)$ and $\mathcal{O}\left(\gamma\gamma\bar{\Psi}\Psi\right)$ interactions, which we used to compute the contribution to the gravitational wave spectrum from the diagrams shown in figure 3.1. The left diagram is technically simpler; after regularizing it we were able to evaluate it exactly, as described in [59]. The right diagram is much more involved, necessitating the approximations discussed in [59], analogous to those made for the cubic diagram in reference [22]. Although the computations were very involved, in section 3.2.3 we presented some simple scaling arguments that correctly capture the scaling of the result with the parameters, μ and ξ, of the model.

Our main conclusion is that, in contrast to the scenario in which the axion inflaton is coupled to vector fields, the gravitational waves sourced by the fermions cannot be greater than the vacuum gravitational waves. This conclusion holds also in the regime, studied in Chapter 2, where the strong backreaction of the fermion degrees of freedom controls the dynamic of the zero mode of the inflaton.

Chapter 4

Reviving chaotic inflation and supergravity construction

Having computed the scalar and tensor power spectra for the fermion model in the previous two chapters, we now present an application of this process to reviving chaotic inflation. Processes of particle production during inflation can increase the amplitude of the scalar metric perturbations. We show that such a mechanism can naturally arise in supergravity models where an axion-like field drives large field inflation. In this class of models one generally expects instanton-like corrections to the superpotential. We show, by deriving the equations of motion in models of supergravity with a stabilizer, that such corrections generate an interaction between the inflaton and its superpartner that can lead to copious production of fermions during inflation. In their turn, those fermions source inflaton fluctuations, increasing their amplitude, and effectively lowering the tensor-to-scalar ratio for the model, as discussed in Chapters 2, and 3. This allows, in particular, to bring the model where the inflaton potential is quadratic to agree with all existing observations.

Cosmological observations restrict the space of viable inflationary models in various directions. The measurement of the spectral index gives $n_s - 1 \simeq -1/30$ with a $\sim 10\%$ uncertainty, with no appreciable running. This, together with constraints on nongaussianities (the parameter f_{NL} is about four orders of magnitude smaller than its value in a fully nongaussian distribution), and with the fact that isocurvature modes are below the 5%

level [1], severely constrains non-vanilla models of inflation. However, the class of models of inflation that arguably taste the most like vanilla, those described by a monomial potential $V \propto \varphi^n$, are either ruled out or under significant pressure from the constraints on the tensor-to-scalar ratio $r \lesssim .06$ [73]. In particular, the simplest choice for a monomial, the chaotic inflation with quadratic potential, is ruled out at the $\sim 4\sigma$ level.

In this chapter we will argue that particle production can bring the model of chaotic inflation $V(\varphi) = \frac{m^2}{2}\varphi^2$ (plus, as we will see, corrections that we will require to be negligible) to agree with all constraints from observations. This possibility was already considered in [74, 75], that discussed a system where an auxiliary scalar χ gets an oscillating mass-squared through a coupling to the inflaton, leading to periodic production of quanta of χ. Remarkably, in our work we will see that one can resurrect chaotic inflation by simply embedding it in a supersymmetric setting, and including a small, instanton like correction to the superpotential. These ingredients – monomial inflation with small instanton corrections in supersymmetry – are expected in models where the inflaton is an axion-like degree of freedom whose potential is generated by monodromy [25, 26], see [33] for a review. In particular, the quadratic form of the inflaton potential is generated in the axion–four-form system of [76, 10, 27, 77]. Note also that [78, 28] have shown that this axion–four-form system can be brought to agree with observations by the inclusion of higher dimensional operators that flatten the potential at large field values, similarly to the effect [25, 26, 79, 80] observed in string theory constructions. In this work we will assume, however, that such flattening does not occur for the observationally relevant range of field values.

Going into the specifics of our scenario, we will show that the addition of an instanton-like $\sim e^{-\phi}$ component to the superpotential, where ϕ is a superfield whose imaginary component gives the axion-like inflaton, leads to a coupling of the inflaton to its fermionic partner, the inflatino, that can be written in the form $\sim \bar{\psi}\left(\gamma^\mu\gamma^5\partial_\mu\varphi\right)\psi$. The rolling inflaton thus provides a time-dependent contribution to the fermionic Lagrangian that leads to the generation of quanta of ψ [62, 58]. The quanta of ψ, in their turn, source fluctuations of the inflaton. The phenomenology associated to this fermion-inflaton system has been studied in [22, 59]. In the first of those papers it was shown that there is a regime where the inflaton fluctuations sourced by the produced fermions dominate over the standard ones originating

from the amplification of the inflaton's vacuum fluctuations. Remarkably, and in contrast with the case in which gauge fields are amplified by the rolling axion-like inflaton [42], the statistics of the inflaton perturbations is quasi-gaussian, and in agreement with observations. Also, as we will see, the spectral index in this model turns out to be the same as in standard chaotic inflation, and therefore agrees with observations.

The existence of an additional component of scalar perturbations increases, for fixed values of the parameters, the amplitude of the power spectrum P_ζ. If such a component is sizable, therefore, we must lower the energy scale of inflation in order to fit the observed value of P_ζ. This has the consequence of lowering the amplitude of the tensor perturbations (in Chapter 3 it was checked that the fermions do not source significantly the tensor modes), and of bringing the model to agree with the current constraints on the tensor-to-scalar ratio r. This is one of the main results of this chapter.

As a warm-up, in Section 4.2 we will consider a globally supersymmetric model, with a superpotential $W \propto \phi^2$, that can lead to chaotic inflation, with a small contribution $\sim e^{-\phi}$. While we work directly in the regime of supersymmetry with a single chiral superfield, it is worth noticing that this same construction can be realized [81] by supersymmetrizing the axion–four-form system of [76, 10, 27]. Mapping the resulting fermionic Lagrangian to that studied in [22], and imposing theoretical as well as observational constraints, we find that this scenario can agree with observations, leading in particular to a tensor-to-scalar ratio that can be as small as $r \simeq .007$, about a factor 8 below the current bound.

Given that the inflaton has Planckian excursions, however, the assumption of global supersymmetry is not appropriate, and one has to go to the full supergravity description. We perform such an analysis in Section 4.3. We consider models that are free from the η-problem [82] by choosing a Kähler potential that depends only on the combination [83, 11] $\Phi + \bar{\Phi}$, where the inflaton ϕ is in the imaginary part of the chiral superfield Φ. In order to design a potential that is dominated by the quadratic term and whose flatness at large values of the inflaton is not spoiled by the supergravity correction, we consider models of inflation with a stabilizer [11, 12, 13] superfield S. Since this system features two superfields Φ and S, we must diagonalize the dynamics of two fermions, the inflatino and the stabilizerino, that is given in general terms in [84]. We do so by generalizing the analysis of [85, 86] to the case

where the system contains a pseudoscalar component, but with the simplifying assumption that, thanks to the presence of the stabilizer, the superpotential vanishes on shell. To our knowledge, such a calculation is new in the literature, and is our other main result.

After diagonalizing the fermions, and in the regime where the inflaton potential has small oscillations superimposed to a large monomial component, we find that the dynamics of this system is identical – up to simple redefinitions of parameters – to the globally supersymmetric one. Thus the parameter space contains a viable region where the scalar potential is essentially quadratic, but r can be as small as a factor ~ 8 below the current constraints, also in the case in which the model is embedded in supergravity.

4.1 Fermion production during inflation, and the amplitude of tensor-to-scalar ratio

Let us start by reviewing the results of Chapters 2, and 3. Those chapters contain the study of a system consisting of a pseudoscalar inflaton φ with arbitrary potential $V(\varphi)$ generated by the breaking of the shift symmetry $\varphi \to \varphi$+constant, along with a fermion Y of mass m_ψ. Including the shift symmetric coupling of lowest dimensionality of the inflaton to Y, the fermionic component of the Lagrangian takes the form

$$\mathcal{L}_Y = \bar{Y}\left[i\gamma^\mu\partial_\mu - m_\psi - \frac{1}{f}\gamma^\mu\gamma^5\partial_\mu\varphi\right]Y\,, \tag{4.1.1}$$

where f is a constant with the dimensions of a mass. It is convenient to define a new fermion field ψ, related to Y by

$$\psi = e^{i\gamma^5\varphi/f}Y\,, \tag{4.1.2}$$

in terms of which the fermionic Lagrangian reads

$$\mathcal{L}_\psi = \bar{\psi}\left\{i\gamma^\mu\partial_\mu - m_\psi\left[\cos\left(\frac{2\varphi}{f}\right) - i\gamma^5\sin\left(\frac{2\varphi}{f}\right)\right]\right\}\psi\,. \tag{4.1.3}$$

The expression (4.1.3) shows that, in the limit $m_\psi \to 0$, the fermionic degree of freedom

decouples from the inflaton. On the other hand, the form (4.1.1) of the fermionic Lagrangian emphasizes the shift-symmetric nature of the fermion-inflaton coupling.

As shown in Chapter 2, the interaction described above, in a quasi-de Sitter background with Hubble parameter H, leads to the generation of chiral quanta of ψ with an occupation number that is constant, and given approximately by $.1\,(m_\psi/H)^2$, for momenta up to $\sim |\dot{\varphi}|/f$. The fermions can thus have a very large number density $\sim 10^{-2}\left(\frac{m_\psi}{H}\right)^2\left(\frac{|\dot{\varphi}|}{f}\right)^3 \gg H^3$, and can affect the dynamics of the inflaton background and of its perturbations. In this chapter we will be interested in the regime in which the fermions do not affect significantly the background dynamics, but provide the main source of inflaton perturbations.

An especially interesting result of Chapter 2 is that, even in the regime in which the component sourced by the fermions dominates the inflaton perturbations, the statistics of those perturbations is very close to gaussian, and in agreement with the constraints from Planck [87]. This is due to the fact that, even if the process $\bar{\psi}\,\psi \to \delta\varphi$ is a $2 \to 1$ process that would naturally lead to non gaussian statistics, fermions from a broad set of momenta participate to the process, and gaussianity is re-obtained as an effect of the central limit theorem. The bottom line is that the model of Chapters 2, and 3 can lead to a regime where the perturbations are sourced by the fermions, and still their properties are in agreement with observations.

Since the amplitude of the sourced perturbations has a functional dependence on the parameters of the system that is different from the standard case, this set up has the potential of reviving models of inflation whose potential would be otherwise ruled out by CMB constraints.

We will focus here on the model of inflation where the potential has the simplest functional form: a quadratic potential. In the standard case in which the perturbations are from the vacuum, this model's prediction for the spectral index is in agreement with data, but is ruled out by the amplitude of the tensor modes, since it predicts $r = 8/N$, where N is the number of efoldings, that for $N \lesssim 60$ requires $r > .13$, whereas Planck/Keck constrains $r < .06$.

The presence of fermions in the dynamics in the system naturally calls for a supersymmetric construction. In the next section we will construct a globally supersymmetric model

where we obtain the desired features, before moving on to a construction in supergravity that is more complicated, but more appropriate, since we are discussing large field inflation.

4.2 A model in global supersymmetry

As a warm up, let us consider a globally supersymmetric theory with a single chiral superfield Φ and superpotential

$$W = \frac{\mu}{2}\,\Phi^2 + \Lambda^3\, e^{-\sqrt{2}\Phi/F}\,, \tag{4.2.1}$$

where μ, Λ and F are parameters with dimensions of mass. The corresponding Lagrangian reads

$$\begin{aligned}\mathcal{L} = &-\partial_\mu\phi\,\partial^\mu\phi^* - \left|\mu\phi - \sqrt{2}\frac{\Lambda^3}{F}e^{-\sqrt{2}\phi/F}\right|^2 \\ &+ \bar{\psi}\left[i\gamma^\mu\partial_\mu - \mu - \Re\left\{2\frac{\Lambda^3}{F^2}e^{-\sqrt{2}\phi/F}\right\} + i\Im\left\{2\frac{\Lambda^3}{F^2}e^{-\sqrt{2}\phi/F}\right\}\gamma^5\right]\psi\,,\end{aligned} \tag{4.2.2}$$

where ϕ is a complex scalar and ψ is a four-component Majorana fermion.

To proceed, we assume[1] that the real part of the field ϕ is stabilized to $\Re\{\phi\} = 0$ and we thus redefine $\phi = i\varphi/\sqrt{2}$, obtaining our final Lagrangian

$$\begin{aligned}\mathcal{L} &= -\frac{1}{2}\partial_\mu\varphi\,\partial^\mu\varphi - V(\varphi) + \bar{\psi}\left(i\gamma^\mu\partial_\mu - \mu - 2\frac{\Lambda^3}{F^2}\cos(\varphi/F) - 2\frac{\Lambda^3}{F^2}i\gamma^5\sin(\varphi/F)\right)\psi\,,\\ V(\varphi) &= \frac{\mu^2}{2}\varphi^2 - 2\,\mu\frac{\Lambda^3}{F}\varphi\,\sin(\varphi/F) + 2\frac{\Lambda^6}{F^2}\,.\end{aligned} \tag{4.2.3}$$

The fermionic part of this Lagrangian is analogous, with the identifications $F = f/2$ and $m_\psi = 2\,\Lambda^3/F^2$, to the Lagrangian (4.1.3), with the addition of a mass term μ for the fermions (that, as we will see below, can be neglected), while, neglecting the cosmological constant $\sim \Lambda^6/F^2$, the scalar potential is that of chaotic inflation with oscillating corrections.

We thus see that the simple superpotential eq. (4.2.1) can already lead to the kind of system outlined in Section 4.1 above: a model of quadratic inflation (with small corrections)

[1] This is by no means a consistent assumption, and we will make it in this section that has only illustrative purposes. In Section 4.3 below, on the other hand, we will consistently minimize the full potential of the model.

with a sizable coupling of fermions to the inflaton, where the power spectrum of scalar perturbations may be dominated, in some region of parameter space, by the fermion production and might thus be in agreement with Planck constraints. To make sure that this is the case, however, we must explore the constraints on the parameter space available to the system.

We will assume that the potential is dominated by its quadratic part, and that fermions give a negligible contribution to the background dynamics, so that all the results from chaotic inflation will carry over. In particular, we will have the approximate slow-roll relations

$$\dot{\varphi} \simeq -\sqrt{\frac{2}{3}}\mu M_P\,, \qquad \varphi \simeq 2\,M_P\sqrt{N}\,, \qquad H \simeq \mu\sqrt{\frac{2}{3}N}\,, \tag{4.2.4}$$

where $N \simeq 60$ is the number of efoldings until the end of inflation.

The strength of fermion production is measured by the dimensionless parameter ξ, that takes the value

$$\xi \equiv \frac{|\dot{\varphi}|}{4FH} = \frac{1}{4\sqrt{N}}\frac{M_P}{F}\,. \tag{4.2.5}$$

We will assume, as is expected to be the case in UV-complete theories of gravity [88, 89], that the parameter F is sub-Planckian, and small enough that $\xi \gtrsim 1$.

Due to the presence of the term proportional to μ in the fermionic sector of the Lagrangian, the present system is different, as discussed above, from that of Chapters 2, and 3. As shown in Chapter 2, however, fermion production happens for momenta up to $k_{\rm cutoff} \simeq 2H\xi$. As a consequence, since slow roll requires $\mu \ll H$ whereas, since $k_{\rm cutoff} \gtrsim H$, the effect of the parameter μ does not affect the dynamics of fermions to any significant level, and we can safely neglect it.

Let us now list the constraints on our parameter space.

- *Monotonicity of potential.* In order for the oscillating term in the potential not to spoil the monotonicity of the quadratic part during inflation we require

$$2\frac{\Lambda^3}{F^2\mu} < 1\,. \tag{4.2.6}$$

- *Backreaction.* One can neglect the backreaction of the fermions on the inflating background provided the condition $m_\psi^2\,\xi \ll 3\pi\,F^2$ is satisfied [22]. In our model, this corresponds to the condition

$$\frac{1}{3\pi\sqrt{N}}\frac{\Lambda^6\,M_P}{F^7} \lesssim 1\,. \tag{4.2.7}$$

- *Validity of effective field theory.* The term proportional to $\sin(\varphi/F)$ in the scalar potential generates oscillations of frequency $\omega = \dot{\varphi}/F$ in the Hubble parameter. By requiring that physics occurs at scales below the cutoff $4\pi F$ of the axionic effective field theory, we obtain the constraint $\omega \lesssim 4\pi F$, that translates into

$$\mu M_P \lesssim 2\sqrt{6}\,\pi F^2\,. \tag{4.2.8}$$

- *Small m_ψ approximation.* The results of [22, 59] have been obtained assuming $m_\psi \ll H$, that in terms of our parameters reads

$$\frac{\Lambda^3}{F^2} \ll \mu\sqrt{\frac{N}{6}}\,, \tag{4.2.9}$$

  that is identically satisfied when the condition of monotonicity of the potential, eq. (4.2.6) is satisfied. Also, eq. (4.2.9), along with the requirement $\xi \gg 1$, implies that a second condition of perturbativity required in [22, 59], namely that $m_\psi \ll H\sqrt{\xi}$, is identically satisfied.

- *Tensor modes.* The Planck-Keck constraint [73] $r < .06$ rules out standard quadratic inflation. As discussed in [59], the amplitude of tensor modes in our model has essentially the same expression as in the standard case, which gives a constraint

$$\frac{4}{3\pi^2}\frac{\mu^2}{M_P^2}N < .06\,P_\zeta\,, \tag{4.2.10}$$

  where P_ζ is the scalar power spectrum.

- *No oscillations in scalar perturbations.* Oscillations in the potential will induce os-

cillations in the power spectrum of scalar perturbations. This phenomenon has been studied in detail in the case in which the scalar spectrum is generated by the standard mechanism of amplification of vacuum fluctuations of the inflaton. The constraint on the amplitude of those oscillations, δn_s, can be approximately written as $\delta n_s \lesssim 10^{-3}\sqrt{M_P/F}$ [90, 1], where in our model $\delta n_s \simeq 6\sqrt{2\pi}\,N^{1/4}\frac{\Lambda^3}{\mu F^2}\sqrt{F/M_P}$ [91], which would provide a strong additional constraint on our model. However, these constraints do not hold in the regime we will be interested in, where the scalar perturbations are sourced by the fermion field. As a consequence, we will not consider them in this analysis.

- *Nongaussianities.* There are two potential sources of nongaussianities. First, those induced by the presence of the fermion bath in interaction with the inflaton, that have been shown in Chapter 2 to be negligible. Second, there is a possibility of resonant nongaussianities [91] induced by the small oscillations in the inflaton potential. As in the point above, however, the existing estimates of the amplitude of this effect do not hold in the regime of sourced perturbations we are interested in, and we will ignore them here.

Once we fix the number of efoldings to $N = 60$, our theory has three parameters, namely Λ, μ and F. We can eliminate one of them by imposing that the power spectrum takes its observed value $P_\zeta = 2.2 \times 10^{-9}$, using the expression, valid for our system,

$$P_\zeta = \frac{H^4}{4\pi^2\dot{\varphi}^2}\left(1 + \frac{8}{3\pi^2}\frac{m_\psi^2}{F^2}\xi^2 N\log\xi\right) = \frac{\mu^2\,N^2}{6\pi^2\,M_P^2}\left[1 + \frac{2}{3\pi^2}\frac{\Lambda^6\,M_P^2}{F^8}\left|\log\left(4\sqrt{N}\frac{F}{M_P}\right)\right|\right]. \tag{4.2.11}$$

In particular, it is convenient to use the normalization of the power spectrum to eliminate Λ. Once we do this, we can plot the constraints enumerated above on a two-dimensional plot, see Figure 4.1. As one can see, there is a portion of parameter space that satisfies all the constraints above, and that extends from $F \simeq 3 \times 10^{-4}\,M_P$ to $F \simeq 10^{-3}\,M_P$ and where μ can be as small as $1.3 \times 10^{-6}\,M_P$. This implies, using eq. (4.2.10), that the tensor-to-scalar ratio in this model of quadratic inflation with corrections can be as small as $\sim .007$, i.e.

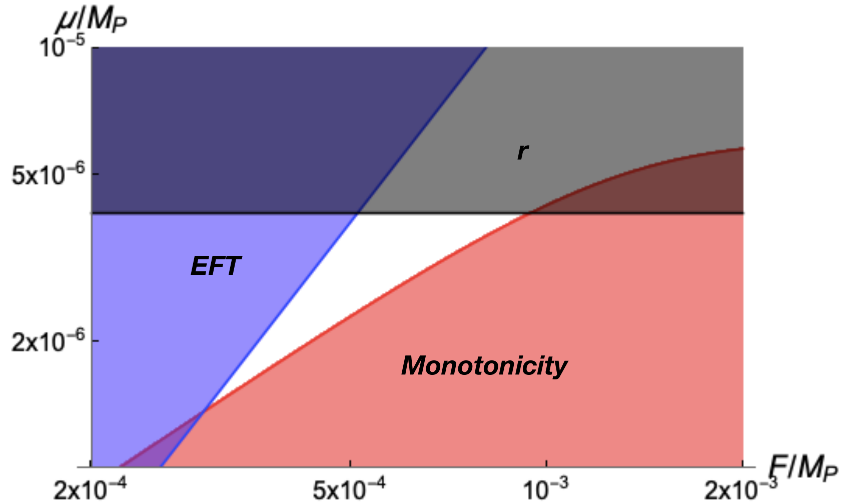

Figure 4.1: The parameter space for the model (4.2.3). The shaded region marked by an "r" is excluded by the observational bound $r < .06$ on the amplitude of the tensor modes. The region labelled "EFT" is excluded by the constraint (4.2.8), and the region labeled "Monotonicity" is excluded by the constraint (4.2.6).

about an order of magnitude below the present bounds.

Another well-constrained quantity we have not talked about is the spectral index, $.961 \lesssim n_s \lesssim .969$ [92]. Its expression for this specific model is essentially the same as the standard chaotic inflation scenario, $n_s - 1 = d\log P_\zeta/dN \simeq 2/N$ so that by assuming the standard value $N = 60$ the spectral index is automatically in agreement with observations.

To summarize this section, the globally supersymmetric model with superpotential (4.2.1), with the assumption that the real component of the inflaton is stabilized, leads to a model of quadratic inflation that, thanks to inflaton-inflatino interactions, is compatible with all the existing phenomenological constraints.

Of course, this model with global supersymmetry is not quite suitable for chaotic inflation, where the fields can get Planckian values. In the next section we will thus turn our attention to the more appropriate construction of a model of supergravity where fermions can source the spectrum of scalar perturbations.

4.3 The full construction in supergravity

Even before worrying about the role of fermions, the construction of models of inflation in supergravity is famously [82] a nontrivial task. In this paper we will consider models with a stabilizer [11, 12, 13], that allow to design essentially any potential. The down side of these models is that they need at least two superfields – the inflaton and the stabilizer, which makes the analysis of the fermionic sector quite cumbersome.

In Section 4.3.1 below, we will study in general terms the equations of motion for the fermionic degrees of freedom in models with an inflaton and a stabilizer superfields. Then, in Section 4.3.2, we will specialize our equations to the case of a superpotential leading to quadratic inflation with small oscillations, and we will show that the analysis of parameter space performed in the globally supersymmetric model of Section 4.2 above can be directly applied to the full supergravity construction.

4.3.1 Equations for fermions in models of supergravity with a stabilizer

We start from a theory of two chiral multiplets coupling to the supergravity multiplet. Of the two spin-1/2 matter fields, one is the goldstino and can be gauged away. We are thus left with two helicity-1/2 fermions, the transverse component of the gravitino, $\theta = \gamma^i \psi_i$ [93, 94, 95], and the fermion Υ [84], a linear combination of the fermions in the matter multiplets. The longitudinal, helicity-3/2 component of the gravitino will play no significant role (it gets a mass proportional to the superpotential [93, 94, 95], which vanishes in the models with stabilizer we are interested in), and we will ignore it here. The derivation of the equations of motions for fermions in general supergravity models can be found in [84]. In particular, eq. (9.20) in that paper provides the equations of motion for the fermions:

$$
\begin{aligned}
&(\hat{\partial}_0 + \hat{B} + i\gamma^i k_i \gamma^0 \hat{A})\,\theta - \frac{4}{\alpha a} k^2 \Upsilon = 0\,, \\
&(\hat{\partial}_0 - i\gamma^i k_i \gamma^0 \hat{A} + \hat{B}^\dagger + a\hat{F} + 2\dot{a} + \frac{a}{M_P^2}\mathbf{m}\gamma^0)\Upsilon + \frac{1}{4} a\alpha \Delta^2 \theta = 0\,,
\end{aligned}
\tag{4.3.1}
$$

where, for a Kähler potential K and a superpotential W, and considering the two superfields Φ_i, $i = 1, 2$, and their scalar components ϕ_i and $\phi^i = (\phi_i)^*$, one has the quantities

$$
\begin{aligned}
& m \equiv \mathrm{e}^{\frac{K}{2M_P^2}} W \,, \qquad \mathbf{m} \equiv \Re\{m\} - i\,\Im\{m\}\,\gamma^5 \,, \\
& m^i = \Big(\partial^i + \frac{1}{2M_P^2}\partial^i K\Big)\, m \,, \qquad m^{ij} = \Big(\partial^i + \frac{1}{2M_P^2}\partial^i K\Big)\, m^j - \Gamma^{ij}_k\, m^k \,, \\
& \hat{\partial}_0 = \partial_0 - \frac{i}{2} A_0^B \gamma^5 \,, \qquad A_0^B = \frac{i}{2M_P^2}(\phi'^i \partial_i K - \phi'_i \partial^i K) \,, \\
& H^2 = \frac{1}{3M_P^2}\left(|\dot{\phi}|^2 + V\right) \,, \qquad V = m_i (g^{-1})^i_j m^j - 3\frac{|m|^2}{M_P^2} \,, \qquad |\dot{\phi}|^2 \equiv g^i_j\, \dot{\phi}_i\, \dot{\phi}^j \,, \\
& \alpha = 3M_P^2\Big(H^2 + \frac{|m|^2}{M_P^4}\Big) \,, \qquad \alpha_1 = -3M_P^2\Big(H^2 + \frac{2}{3}\dot{H} + \frac{|m|^2}{M_P^4}\Big) \,, \qquad \alpha_2 = 2\dot{\mathbf{m}}^\dagger \,, \\
& \hat{A} = \frac{1}{\alpha}(\alpha_1 - \gamma^0 \alpha_2) \,, \qquad \hat{B} = -\frac{3}{2}\dot{a}\hat{A} + \frac{a}{2M_P^2}\mathbf{m}\gamma^0(1 + 3\hat{A}) \,, \\
& \xi_i \equiv m_i - \gamma^0 g_i^j \dot{\phi}_j \,, \qquad \Delta \equiv 2\frac{\sqrt{V}|\dot{\phi}|}{\alpha} \,, \\
& P_R = \frac{1}{2}(1 - \gamma^5) \,, \qquad P_L = \frac{1}{2}(1 + \gamma^5) \,, \qquad \Pi_{ij} = \frac{1}{\alpha}(m_i g^k_j \dot{\phi}_k - m_j g^k_i \dot{\phi}_k) \,, \\
& \hat{F} = -\frac{4}{\alpha \Delta^2 \det g}\left(\xi^k P_R\, (g^{-1})^l_k m_{li} \Pi^{ij} \xi^\dagger_j + \xi_k P_L (g^{-1})^k_l m^{li} \Pi_{ij} \xi^{\dagger j}\right) \,,
\end{aligned}
\tag{4.3.2}
$$

where a prime denotes a derivative with respect to the conformal time while an overdot is a derivative with respect to cosmic time, and where $\partial^i = \partial/\partial\phi_i$, and $\partial_i = \partial/\partial\phi^i$. Also, $g^i_j \equiv \partial^i \partial_j K$ is the Kähler metric and $\Gamma^{ij}{}_k = (g^l{}_k)^{-1}\partial^i g^j{}_l$ is the Kähler connection. The scalars satisfy the equations of motion $g_i^j(\ddot{\phi}_j + 3H\dot{\phi}_j + \Gamma^{kl}_j \dot{\phi}_k \dot{\phi}_l) + \partial_i V = 0$. Further, we have the relation $\alpha_1^2 + \alpha_2^\dagger \alpha_2 + \alpha^2 \Delta^2 = 1$.

We will denote the two chiral superfields by $\Phi = \Phi_1$, and $S = \Phi_2$ (with scalar components ϕ and s, respectively), and we choose a minimal Kähler potential for S, but keep a general potential for Φ,

$$
K(\Phi,\, \bar{\Phi}; S,\, \bar{S}) = \mathcal{K}(\Phi,\, \bar{\Phi}) + S\bar{S} \,. \tag{4.3.3}
$$

For the superpotential, we use a stabilizer model,

$$
W = S\, f(\Phi) \,, \tag{4.3.4}
$$

where $f(\Phi)$ is an arbitrary function and S is stabilized at 0. A consequence of this is that $m|_{s=0} = 0$, and therefore the mass of the longitudinal helicity-3/2 component of the gravitino, $m_{3/2} = |m|/M_P^2$ vanishes.

The scalar potential is

$$\begin{aligned} V &= \mathrm{e}^{\frac{K}{M_P^2}} \left[(g^{-1})^i_j \left(\partial_i W^* + \frac{\partial_i K}{M_P^2} W^* \right) \left(\partial^j W + \frac{\partial^j K}{M_P^2} W \right) - 3\frac{|W|^2}{M_P^2} \right] \\ &= \mathrm{e}^{\frac{K}{M_P^2}} \left[\left| f(\phi) + \frac{|s|^2}{M_P^2} f(\phi) \right|^2 + g^\phi_\phi |s|^2 \left| f'(\phi) + \frac{\partial^\phi K}{M_P^2} f(\phi) \right|^2 - 3\frac{|s|^2}{M_P^2} |f(\phi)|^2 \right]. \end{aligned} \tag{4.3.5}$$

Differentiating gives $\partial_s V|_{s=0} = 0$, $\partial_s^2 V|_{s=0} \geq 0$, and $\partial_s \partial_{\bar{s}} V|_{s=0} \geq 0$ showing that $s = 0$ is a stable critical point of the potential. Therefore, from here on we set $s = 0$, and the scalar potential is simply $V = \mathrm{e}^{\frac{\mathcal{K}}{M_P^2}} |f(\phi)|^2$.

With these choices, we have

$$\begin{aligned} &m^s = \mathrm{e}^{\frac{\mathcal{K}}{2M_P^2}} f(\phi)\,, && m^{s\phi} = \mathrm{e}^{\frac{\mathcal{K}}{2M_P^2}} \left[f'(\phi) + \tfrac{\partial^\phi \mathcal{K}}{M_P^2} f(\phi) \right], && m^\phi = m^{ss} = m^{\phi\phi} = 0 \\ &\xi^s = \mathrm{e}^{\frac{\mathcal{K}}{2M_P^2}} f(\phi)\,, && \xi^\phi = -\gamma^0 g^\phi_\phi \dot{\phi}^*\,, && \Pi^{s\phi} = \tfrac{1}{\alpha} \mathrm{e}^{\frac{\mathcal{K}}{2M_P^2}} f(\phi) g^\phi_\phi \dot{\phi}^*\,, \end{aligned} \tag{4.3.6}$$

A bit of calculation shows that

$$\hat{F} = \frac{V - |\dot{\phi}|^2}{2V|\dot{\phi}|^2} \left(\partial^\phi V \dot{\phi} + \partial_\phi V \dot{\phi}^* \right) + \frac{V + |\dot{\phi}|^2}{2V|\dot{\phi}|^2} \left(\partial^\phi V \dot{\phi} - \partial_\phi V \dot{\phi}^* \right) \gamma^5 . \tag{4.3.7}$$

Let us now proceed to diagonalize the equations of motion for the fermions. The system (4.3.1) can be derived from the Lagrangian [86]

$$\begin{aligned} \mathcal{L} = &-\frac{\alpha a^3}{4k^2} \bar{\theta} \Big[\Big(\gamma^0 \hat{\partial}_0 + i\gamma^i k_i \hat{A} + \gamma^0 \hat{B} \Big) \theta - \frac{4k^2}{a\alpha} \gamma^0 \Upsilon \Big] + \\ &- \frac{4a}{\alpha \Delta^2} \bar{\Upsilon} \Big[\Big(\gamma^0 \hat{\partial}_0 - i\gamma^i k_i \hat{A} + \gamma^0 \hat{B}^\dagger + a\gamma^0 \hat{F} + 2\dot{a}\gamma^0 + \frac{a}{M_P^2} \gamma^0 \mathbf{m} \gamma^0 \Big) \Upsilon + \frac{1}{4} a\alpha \Delta^2 \gamma^0 \theta \Big], \end{aligned} \tag{4.3.8}$$

where, following [84], we use the convention $\bar{\theta} = i\theta^\dagger \gamma^0$ for barred spinors. We canonically

normalize the fermions defining

$$\theta = 2\frac{i\gamma^i k_i}{\sqrt{\alpha a^3}}\tilde{\theta}\,, \qquad \Upsilon = \frac{\Delta}{2}\Big(\frac{\alpha}{a}\Big)^{1/2}\tilde{\Upsilon}\,. \tag{4.3.9}$$

The Lagrangian with normalized fields (and taking $s = 0$) is

$$\begin{aligned}\mathcal{L} = \bar{\tilde{\theta}}\Big[\Big(-\gamma^0\partial_0 + i\gamma^i k_i\frac{\alpha_1}{\alpha} - \frac{i}{2}A_0^B\gamma^0\gamma^5\Big)\tilde{\theta} + i\Delta\gamma\cdot k\gamma^0\tilde{\Upsilon}\Big] + \\ + \bar{\tilde{\Upsilon}}\Big[\Big(-\gamma^0\partial_0 + i\gamma^i k_i\frac{\alpha_1}{\alpha} + \Big(\frac{i}{2}A_0^B - a\hat{F}_5\Big)\gamma^0\gamma^5\Big)\tilde{\Upsilon} + i\Delta\gamma\cdot k\gamma^0\tilde{\theta}\Big]\,.\end{aligned} \tag{4.3.10}$$

where

$$\hat{F}_5 = \frac{V + |\dot{\phi}|^2}{2V|\dot{\phi}|^2}\Big(\partial^{\phi}V\dot{\phi} - \partial_{\phi}V\dot{\phi}^*\Big)\,. \tag{4.3.11}$$

Note in particular that $\hat{F}_5$ is pure imaginary, and that it vanishes for real ϕ.

Let us write the Lagrangian (4.3.10) in the compact form

$$\mathcal{L} = \bar{X}\Big[-\gamma^0\partial_0 + i\gamma\cdot k\,N + M\Big]X\,, \tag{4.3.12}$$

with $X = (\tilde{\theta}, \tilde{\Upsilon})^T$ and $N = N_1 + N_2\gamma^0$, where

$$N_1 = \begin{pmatrix} \alpha_1/\alpha & 0 \\ 0 & \alpha_1/\alpha \end{pmatrix}, \qquad N_2 = \begin{pmatrix} 0 & \Delta \\ \Delta & 0 \end{pmatrix}, \qquad M = \begin{pmatrix} -\frac{i}{2}A_0^B & 0 \\ 0 & \frac{i}{2}A_0^B - a\hat{F}_5 \end{pmatrix}\gamma^0\gamma^5\,. \tag{4.3.13}$$

We now redefine the fields in such a way as to remove the factor of N in front of $i\gamma^i k_i$. Using the relation $\alpha^2 - \alpha_1^2 = \alpha^2\Delta^2$, we can see that $N^\dagger N = N_1^2 + N_2^2 = 1$, so N is unitary. Therefore, we can define $N = \mathrm{e}^{2\Psi\gamma^0} = \cos 2\Psi + \gamma^0 \sin 2\Psi$ where Ψ is a 2×2 hermitian matrix [96]. We choose

$$2\Psi = \begin{pmatrix} 0 & \pi - \sin^{-1}\Delta \\ \pi - \sin^{-1}\Delta & 0 \end{pmatrix}. \tag{4.3.14}$$

It is straightforward to check that $\cos 2\Psi = N_1$, and $\sin 2\Psi = N_2$. After redefining $X = \mathrm{e}^{-\Psi\gamma^0} Z$, the Lagrangian takes the form

$$\mathcal{L} = \bar{Z}\Big[-\gamma^0\partial_0 + i\gamma\cdot k + \tilde{M}\Big]Z\,, \tag{4.3.15}$$

where the new matrix $\tilde{M}$ reads

$$\begin{aligned}\tilde{M} &\equiv \mathrm{e}^{\Psi\gamma^0}(M - \partial_0\Psi)\,\mathrm{e}^{-\Psi\gamma^0}\\ &= \frac{1}{2}\begin{pmatrix} \big[-iA_0^B + a\,(1-\alpha_1/\alpha)\,\hat{F}_5\big]\gamma^0\gamma^5 & -\frac{\alpha}{\alpha_1}\Delta' + a\,\hat{F}_5\Delta\gamma^5 \\ -\frac{\alpha}{\alpha_1}\Delta' + a\,\hat{F}_5\Delta\gamma^5 & \big[iA_0^B - a\,(1+\alpha_1/\alpha)\,\hat{F}_5\big]\gamma^0\gamma^5 \end{pmatrix}.\end{aligned} \tag{4.3.16}$$

Furthermore, we can remove the $\gamma^0\gamma^5$ term by redefining the fields as

$$Z = \begin{pmatrix} \mathrm{e}^{i\sigma_1\gamma^5} & 0 \\ 0 & \mathrm{e}^{i\sigma_2\gamma^5} \end{pmatrix}\begin{pmatrix}\psi_1 \\ \psi_2\end{pmatrix}, \tag{4.3.17}$$

where, in order for the $\gamma^0\gamma^5$ terms to vanish, σ_1 and σ_2 must satisfy

$$\begin{aligned}\partial_0\sigma_1 &= -\frac{1}{2}A_0^B - i\frac{a}{2}\,(1-\alpha_1/\alpha)\,\hat{F}_5\,,\\ \partial_0\sigma_2 &= \frac{1}{2}A_0^B + i\frac{a}{2}\,(1+\alpha_1/\alpha)\,\hat{F}_5\,.\end{aligned} \tag{4.3.18}$$

Once we choose σ_1 and σ_2 that satisfy these equations, we are at last left with a coupled set of fermions with a mass matrix of the form $\begin{pmatrix} 0 & M_1 + iM_2\gamma^5 \\ M_1 + iM_2\gamma^5 & 0\end{pmatrix}$, where M_1 and M_2 are defined below. Such a system can be completely diagonalized in terms of the rotated fields

$$\begin{aligned}\chi_1 &= \frac{1}{\sqrt{2}}(\psi_1 + \psi_2)\,,\\ \chi_2 &= \frac{1}{\sqrt{2}}(\psi_1 - \psi_2)\,,\end{aligned} \tag{4.3.19}$$

giving the final Lagrangian

$$\mathcal{L} = (\bar{\chi}_1, \bar{\chi}_2)\left[-\gamma^0\partial_0 + i\gamma\cdot k + a\begin{pmatrix} M_1 + iM_2\gamma^5 & 0 \\ 0 & -M_1 - iM_2\gamma^5\end{pmatrix}\right]\begin{pmatrix}\chi_1 \\ \chi_2\end{pmatrix}, \tag{4.3.20}$$

where

$$\begin{aligned} M_1 &= -\frac{\alpha}{2\alpha_1}\dot{\Delta}\cos(\sigma_1+\sigma_2) + \frac{i}{2}\hat{F}_5\Delta\sin(\sigma_1+\sigma_2)\,, \\ M_2 &= -\frac{\alpha}{2\alpha_1}\dot{\Delta}\sin(\sigma_1+\sigma_2) - \frac{i}{2}\hat{F}_5\Delta\cos(\sigma_1+\sigma_2)\,, \end{aligned} \tag{4.3.21}$$

that depend only on the combination

$$\sigma \equiv \sigma_1 + \sigma_2\,, \qquad \dot{\sigma} = i\frac{\alpha_1}{\alpha}\hat{F}_5\,, \tag{4.3.22}$$

and where α, α_1, and Δ are given in (4.3.2), and $\hat{F}_5$ is given in eq. (4.3.11). Thus, we see that we have a system of two decoupled fermions with the same mass. This is a general result, assuming only a stabilizer model superpotential where the Kähler potential is minimal in S. The scalar potential and fermion dynamics are determined by the choice of function $f(\Phi)$ and Kähler potential, $\mathcal{K}(\Phi,\,\bar{\Phi})$. This allows a great deal of freedom in constructing a model with fermions coupled to an inflaton with choice of inflationary potential. For example, taking ϕ to be real will make $M_2 = 0$ and $M_1 = -\frac{\alpha}{2\,\alpha_1}\Delta$.

In the next section, we show how this can be used to recover, in a full supergravity setting, the Lagrangian of Section 4.2.

4.3.2 Quadratic inflaton potential, plus small oscillations – analysis of the parameter space

We now show how we can recover the Lagrangian (4.2.3) from the full supergravity theory in (4.3.20) with the choice

$$f(\Phi) = \mu\Phi + \hat{\Lambda}^2 \mathrm{e}^{-\frac{\sqrt{2}\Phi}{F}}\,, \qquad \mathcal{K}(\Phi,\,\bar{\Phi}) = \frac{1}{2}(\Phi+\bar{\Phi})^2\,. \tag{4.3.23}$$

We have three parameters, μ, F, and $\hat{\Lambda}$ with the dimensions of mass. Here, we write $\hat{\Lambda}$ to distinguish the parameter of this section from the Λ of Section 4.2. We take $\phi = \frac{1}{\sqrt{2}}(\rho + i\varphi)$ so that the scalars are canonically normalized. During inflation, φ will act as the inflaton while ρ will oscillate near its minimum and will not play a significant role in the scalar potential. The choice (4.3.23) gives the scalar potential

$$V = \mathrm{e}^{\frac{\rho^2}{M_P^2}} \left[\frac{\mu^2}{2}(\rho^2 + \varphi^2) + \sqrt{2}\mu\hat{\Lambda}^2 \mathrm{e}^{-\frac{\rho}{F}} \left(\rho \cos\frac{\varphi}{F} - \varphi \sin\frac{\varphi}{F} \right) + \hat{\Lambda}^4 \mathrm{e}^{-\frac{2\rho}{F}} \right] . \tag{4.3.24}$$

We will take there to be a hierarchy of scales, $\rho \ll F \ll M_P \lesssim \varphi$. As we will see below, therefore, ρ will be nonzero, but can be made sufficiently small within a certain parameter range. As mentioned in Section 4.2, $F \ll M_P$ is motivated by embedding this model in a UV-complete theory of gravity. The scalar potential is then well approximated by

$$V \simeq \frac{\mu^2}{2}\varphi^2 - \sqrt{2}\mu\hat{\Lambda}^2 \varphi \sin\frac{\varphi}{F} + \hat{\Lambda}^4 . \tag{4.3.25}$$

This potential is structurally similar to the one given in (4.2.3), namely chaotic inflation plus small oscillations. Matching gives the relation

$$\hat{\Lambda}^2 = \sqrt{2}\Lambda^3/F , \tag{4.3.26}$$

so that monotonicity of the potential requires

$$\sqrt{2}\frac{\hat{\Lambda}^2}{\mu F} < 1 . \tag{4.3.27}$$

Once this condition is satisfied, we can use the slow-roll approximation (4.2.4) to describe the evolution of φ at zeroth order.

We can now solve for $\rho(t)$ from the equations of motion obtained after linearizing in ρ the potential (4.3.24)

$$\ddot{\rho} + 3H\dot{\rho} + \frac{\sqrt{2}\mu\hat{\Lambda}^2}{F}\varphi \sin\frac{\varphi}{F} = 0 , \tag{4.3.28}$$

where we will treat φ as approximately constant except in the rapidly oscillating $\sin(\varphi/F)$ term where we use the leading order in slow-roll $\varphi(t) \simeq \varphi(0) - \sqrt{\frac{2}{3}}\mu M_P t$. Neglecting the decaying term from the homogeneous solution of eq. (4.3.28), and requiring

$$\frac{F}{M_P} \ll \frac{M_P}{\varphi} \propto \frac{1}{\sqrt{N}}\,, \tag{4.3.29}$$

that is equivalent to the large-ξ approximation, we obtain

$$\rho(t) \simeq \rho_0 + \frac{3}{\sqrt{2}}\frac{\hat{\Lambda}^2}{\mu M_P}\frac{F}{M_P}\varphi \sin\frac{\varphi}{F}\,, \tag{4.3.30}$$

where ρ_0 is an integration constant. We have verified, by solving numerically the exact system of coupled equations for ρ and ϕ, the accuracy of the approximation (4.3.30) and that the constant ρ_0 is much smaller than F.

The requirement $\rho \ll F$ gives, therefore, the additional constraint

$$\frac{\hat{\Lambda}^2}{\mu M_P} \ll \frac{M_P}{\varphi} \simeq \frac{1}{\sqrt{N}}\,. \tag{4.3.31}$$

Now we move on to $\varphi(t)$, for which we want to go beyond the slow-roll approximation. The function $\varphi(t)$ satisfies the approximate equation

$$\ddot{\varphi} + 3H\dot{\varphi} + \mu^2\varphi\left(1 - \sqrt{2}\frac{\hat{\Lambda}^2}{\mu F}\cos\frac{\varphi}{F}\right) = 0\,, \tag{4.3.32}$$

that we can solve perturbatively in $\hat{\Lambda}^2$, defining $\varphi = \varphi_0 + \hat{\Lambda}^2\varphi_1 + O(\hat{\Lambda}^4)$, [97, 91]. By linearizing the equation for φ in $\hat{\Lambda}^2$, and keeping the leading terms in the approximation $\varphi \gtrsim M_P \gg F$ and in the slow roll approximation, the equation for φ_1 reads

$$\ddot{\varphi}_1 + \sqrt{\frac{3}{2}}\frac{\mu\varphi_0}{M_P}\dot{\varphi}_1 - \sqrt{2}\frac{\mu\varphi_0}{F}\cos\frac{\varphi_0}{F} = 0\,, \tag{4.3.33}$$

where, again, we treat φ_0 as constant except inside the rapidly oscillating $\cos(\varphi_0/F)$ term.

The solution, ignoring the decaying mode, is

$$\varphi(t) \simeq \varphi_0(t) - \frac{3}{\sqrt{2}} \frac{\hat{\Lambda}^2}{\mu M_P} \frac{F}{M_P} \varphi_0(t) \cos \frac{\varphi_0(t)}{F} \,. \tag{4.3.34}$$

We see from (4.3.31) and from $F \ll M_P$ that $\hat{\Lambda}^2 \varphi_1 \ll \varphi_0$, therefore, we are comfortably within the perturbative region.

Now we turn our attention to the remaining quantities in the fermion mass in eq. (4.3.20), starting with $\hat{F}_5$. Using (4.3.30),(4.3.34), along with the approximations (4.3.31), this gives

$$\hat{F}_5 \simeq \frac{-\sqrt{3} i \hat{\Lambda}^2}{M_P} \frac{\varphi}{F} \sin(\varphi/F) \,. \tag{4.3.35}$$

Note that we are using φ and not φ_0 in the above expression. At the order we are considering, they are equivalent. Continuing with σ, to leading order in slow-roll, $\alpha_1/\alpha \simeq -1$, so that $\dot{\sigma} \simeq -i\hat{F}_5$. When integrating $\dot{\sigma}$, we will treat φ as constant outside of the $\sin(\varphi/F)$. We will not be interested in the constant of integration as it is simply a constant phase in the fermion fields, so that we obtain

$$\sigma \simeq -\frac{3\hat{\Lambda}^2}{\sqrt{2}\mu M_P^2} \varphi \cos(\varphi/F) \,. \tag{4.3.36}$$

Performing the same approximations for Δ, we obtain

$$\Delta \simeq \sqrt{\frac{2}{3}} \frac{2M_P}{\varphi} \Big(1 - \frac{2M_P^2}{3\varphi^2} + \frac{3}{\sqrt{2}} \frac{\hat{\Lambda}^2}{\mu M_P} \frac{\varphi}{M_P} \sin(\varphi/F) \Big) \,, \tag{4.3.37}$$

and

$$\dot{\Delta} \simeq \frac{4}{3} \frac{\mu M_P^2}{\varphi^2} - 2\sqrt{2} \frac{\hat{\Lambda}^2}{F} \cos(\varphi/F) \,. \tag{4.3.38}$$

Finally, inserting (4.3.35), (4.3.36), (4.3.37), and (4.3.38) into (4.3.21), we get the fermion

mass,

$$M_1 + iM_2\gamma^5 \simeq \mu\Big(\frac{2M_P^2}{3\varphi^2} - \sqrt{2}\frac{\hat{\Lambda}^2}{\mu F}\big(\cos(\varphi/F) + i\sin(\varphi/F)\gamma^5\big)\Big)\,. \tag{4.3.39}$$

By translating to the parameters of Section 4.2 using the identification (4.3.26), we recover the fermionic part of the Lagrangian of equation (4.2.3). In the supergravity case, the constant part of the fermionic mass (i.e., corresponding to the term proportional to μ in the first line of eq. (4.2.3)) is slow-roll suppressed, and we can neglect it here as we did in Section 4.2.

To conclude, with the redefinition (4.3.26) the plot in Figure 4.1 applies also to the supergravity model. In particular, this shows that the supergravity model defined by eqs. (4.3.3), (4.3.4) and (4.3.23) there is a regime of parameter space where the data can be in agreement with all CMB constraints while the inflaton potential is, up to corrections that we want to be negligible, simply quadratic.

4.4 Discussion and conclusions

Standard chaotic inflation is ruled out by experiment. It predicts too large a value for the tensor-to-scalar ratio. The tensor spectrum is determined by the energy scale of inflation, which in the simple model of quadratic inflation is fixed by the normalization of the scalar spectrum. We have shown in this chapter that a source-dominated scalar spectrum can allow to lower the energy scale of inflation, thereby bringing chaotic inflation back into the observationally allowed regime.

In Chapters 2, and 3 it was shown that fermions coupled to an axion inflaton can lead to a source-dominated scalar spectrum and a vacuum-dominated tensor spectrum. More specifically, since the vacuum perturbations and sourced perturbations of the scalar modes are statistically independent, the power spectrum is the sum, $\mathcal{P}_\zeta = \mathcal{P}_\zeta^{\rm vacuum} + \mathcal{P}_\zeta^{\rm sourced}$, and similarly for the tensor spectrum. Therefore, the fermion-sourced model with $2.2 \times 10^{-9} \simeq \mathcal{P}_\zeta^{\rm sourced} \gg \mathcal{P}_\zeta^{\rm vacuum} \propto V \propto \mathcal{P}_t$, allows one to lower the energy scale of inflation. With $\mathcal{P}_t$ dominated by the vacuum perturbations one can then lower the value of the tensor-to-scalar

ratio.

This chapter contains two main results. First, we have shown that the model of [22, 59] can be effectively constructed from a globally supersymmetric model with superpotential (4.2.1). This superpotential generates a quadratic scalar potential (and can be easily generalized to any potential able to support large field inflation), plus small oscillations. The fermion sector produces the inflaton-fermion coupling studied in [22, 59] with a negligible additional fermion mass term. In particular, this applies naturally to the model of [76, 10, 27], that naturally leads to a quadratic inflaton potential using monodromy. Thus, the analysis from [22, 59] applies, allowing for the lowering of r while maintaining n_s unaffected and without generating large non-Gaussianities. While the model is subject to a number of constraints, there is a region, in white in Figure 4.1, where those constraints are all satisfied.

Second, we have examined supergravity with two chiral multiplets with one of the scalars acting as a stabilizer. In Section 4.3.1 we have written down the general equations of motions for the fermions in this class of models. Remarkably, the two helicity-1/2 states in the theory behave identically, as fermions with mass $M_1 + iM_2\gamma^5$, where the generally time dependent terms M_1 and M_2 are given in eq. (4.3.21). Specializing to the case where the superpotential consists of a slowly varying component and quickly oscillating term, we have shown in Section 4.3.2 that the equations for the fermions in the full supergravity theory reduce to those obtained in the case of the globally supersymmetric model, in agreement with the intuition from the equivalence theorem [98, 99, 100]. It would be interesting to see whether these results extend to more general classes, beyond those with a stabilizer, of models of axion inflation in supergravity. The final result is that, in a class of relatively simple models of inflation in supergravity, the potential can be essentially quadratic while the theory is compatible with all existing observations.

Appendix A

The fermion mode functions and their occupation numbers

In order to solve the equation of motion of the fermion

$$\left\{i\,\gamma^\mu\,\partial_\mu - m\,a\left[\cos\left(\frac{2\phi}{f}\right) - i\gamma^5\sin\left(\frac{2\phi}{f}\right)\right]\right\}\psi = 0\,, \tag{A.0.1}$$

and to evaluate its occupation number in the ψ basis, it is convenient to first solve the equation of motion of the fermion in the Y basis

$$\left[i\gamma^\mu\partial_\mu - am - \frac{1}{f}\gamma^0\gamma^5\partial_0\phi_0\right]Y = 0. \tag{A.0.2}$$

Eq. (A.0.2) follows from the Lagrangian in eq. (2.1.3). We decompose Y as

$$Y(\mathbf{x},\,t) = \int\frac{\mathrm{d}^3k}{(2\pi)^{\frac{3}{2}}}e^{i\mathbf{k}\mathbf{x}}\sum_{r=\pm}\left[\tilde{U}_r(\mathbf{k},t)a_r(\mathbf{k}) + \tilde{V}_r(-\mathbf{k},t)b_r^\dagger(-\mathbf{k})\right],$$

with

$$\tilde{U}_r(\mathbf{k},t) = \frac{1}{\sqrt{2}}\begin{pmatrix}\chi_r(\mathbf{k})\,\tilde{u}_r(k,t)\\ r\chi_r(\mathbf{k})\,\tilde{v}_r(k,t)\end{pmatrix},\quad \tilde{V}_r(\mathbf{k},t) = \frac{1}{\sqrt{2}}\begin{pmatrix}\chi_r(\mathbf{k})\,\tilde{w}_r(k,t)\\ r\chi_r(\mathbf{k})\,\tilde{y}_r(k,t)\end{pmatrix}, \tag{A.0.3}$$

where $\chi_r(\mathbf{k})$ is a helicity-r two-spinor, $\sigma \cdot \mathbf{k}\, \chi_r(\mathbf{k}) = r\, k\, \chi_r(\mathbf{k})$, which we normalize as $\chi_r^\dagger(\mathbf{k})\, \chi_s(\mathbf{k}) = \delta_{rs}$, and which can be written explicitly as

$$\chi_r(\mathbf{k}) = \frac{(k + r\,\sigma\cdot\mathbf{k})}{\sqrt{2\,k\,(k+k_3)}}\,\bar{\chi}_r, \qquad \bar{\chi}_+ = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad \bar{\chi}_- = \begin{pmatrix} 0 \\ 1 \end{pmatrix}. \tag{A.0.4}$$

Courtesy of the invariance under charge conjugation, we can impose $\tilde{V}_r(\mathbf{k}) = C\,\bar{\tilde{U}}_r(\mathbf{k})^T$, where $C = i\gamma^0\gamma^2$, which implies

$$\tilde{w}_r = \tilde{v}^*_{-r} \;\;,\;\; \tilde{y}_r = \tilde{u}^*_{-r}\,, \tag{A.0.5}$$

where we have used $i\sigma_2\chi_r^*(\mathbf{k}) = -r\chi_{-r}(\mathbf{k})$.

The Dirac equation, after defining

$$\mu \equiv \frac{m}{H}\,, \quad \xi \equiv \frac{\dot{\phi}_0}{2\,f\,H}\,, \quad x \equiv -k\tau\,, \tag{A.0.6}$$

gives the following system

$$\begin{aligned} \partial_x\tilde{u}_r &= i\,\frac{\mu}{x}\,\tilde{u}_r + i\left(1 + \frac{2\xi}{x}\,r\right)\tilde{v}_r\,, \\ \partial_x\tilde{v}_r &= -i\,\frac{\mu}{x}\,\tilde{v}_r + i\left(1 + \frac{2\xi}{x}\,r\right)\tilde{u}_r\,. \end{aligned} \tag{A.0.7}$$

The system is solved by [58]

$$\tilde{u}_r = \frac{1}{\sqrt{2x}}\,(s_r + d_r) \;\;,\;\; \tilde{v}_r = \frac{1}{\sqrt{2x}}\,(s_r - d_r)\;, \tag{A.0.8}$$

with

$$s_r = \mathrm{e}^{-\pi r\xi}\,W_{\frac{1}{2}+2ir\xi,\,i\sqrt{\mu^2+4\xi^2}}(-2ix)\,, \quad d_r = -i\,\mu\,\mathrm{e}^{-\pi r\xi}\,W_{-\frac{1}{2}+2ir\xi,\,i\sqrt{\mu^2+4\xi^2}}(-2ix)\,, \tag{A.0.9}$$

where $W_{\alpha,\beta}(z)$ denotes the Whittaker function, and where the integration constants have been determined by imposing the normalization $|\tilde{u}_r|^2 + |\tilde{v}_r|^2 = 2$ and the positive frequency

condition

$$\lim_{x\to\infty} u_r\left(x\right) = \lim_{x\to\infty} v_r\left(x\right) = \mathrm{e}^{i\left(x+2r\xi\ln(2x)-\frac{\pi}{4}\right)}. \tag{A.0.10}$$

We can now use these results to compute the mode functions of the field ψ. Recalling that

$$Y = \mathrm{e}^{-i\gamma^5\phi/f}\,\psi\,, \tag{A.0.11}$$

and decomposing

$$\psi = \int \frac{\mathrm{d}^3k}{(2\pi)^{\frac{3}{2}}} e^{i\mathbf{kx}} \sum_{r=\pm}\left[U_r(\mathbf{k},t)a_r(\mathbf{k}) + V_r(-\mathbf{k},t)b_r^\dagger(-\mathbf{k})\right]\,, \tag{A.0.12}$$

we have

$$U_r\left(\mathbf{k},\tau\right) = \mathrm{e}^{i\gamma^5\phi/f}\,\tilde{U}_r\left(\mathbf{k},\tau\right)\,,\quad V_r\left(\mathbf{k},\tau\right) = \mathrm{e}^{i\gamma^5\phi/f}\,\tilde{V}_r\left(\mathbf{k},\tau\right)\,. \tag{A.0.13}$$

Next, decomposing $U_r\left(\mathbf{k},\tau\right)$ and $V_r\left(\mathbf{k},\tau\right)$ as we did for $\tilde{U}_r\left(\mathbf{k},\tau\right)$ and $\tilde{V}_r\left(\mathbf{k},\tau\right)$ above, we obtain the relationship

$$\begin{aligned} u_r(k,\tau) &= \cos\left(\frac{\phi}{f}\right)\tilde{u}_r\left(k,\tau\right) + i\,r\,\sin\left(\frac{\phi}{f}\right)\tilde{v}_r\left(k,\tau\right)\,,\\ v_r(k,\tau) &= \cos\left(\frac{\phi}{f}\right)\tilde{v}_r\left(k,\tau\right) + i\,r\,\sin\left(\frac{\phi}{f}\right)\tilde{u}_r\left(k,\tau\right)\,, \end{aligned} \tag{A.0.14}$$

which gives the expression of u_r and v_r in terms of s_r and d_r presented in eq. (2.1.9) in the main text. It is straightforward to see that the normalization condition $|\tilde{u}_r|^2 + |\tilde{v}_r|^2 = 2$ implies that also $|u_r|^2 + |v_r|^2 = 2$. Moreover, one can see that the positive frequency condition (A.0.10) implies

$$\lim_{x\to\infty}\tilde{u}_r\left(k,\tau\right) = \lim_{x\to\infty}\tilde{v}_r\left(k,\tau\right) = \mathrm{e}^{i\left(x+2r\xi\ln(2x)-\frac{\pi}{4}\right)}\,\mathrm{e}^{-2\xi ir\ln(x/x_{\rm in})} = \mathrm{e}^{i\left(x+2r\xi\ln(2x_{\rm in})-\frac{\pi}{4}\right)}\,, \tag{A.0.15}$$

where we have used $\phi = (\dot{\phi}_0/f)\log(x_{\rm in}/x)$, and this shows that the subdominant $\log(x)$ term has disappeared from the exponent.

We can now use these results to diagonalize the Hamiltonian for the fermions. We define

$$m_R \equiv m\,a\,\cos\left(\frac{2\phi_0}{f}\right) \quad , \quad m_I \equiv m\,a\,\sin\left(\frac{2\phi_0}{f}\right) \,, \tag{A.0.16}$$

so that the fermionic part the Hamiltonian reads

$$H_{\text{free}} = \int \mathrm{d}^3x\,\bar{\psi}\left[-i\,\gamma^i\,\partial_i + m_R \,-i\gamma^5\,m_I\right]\psi\,. \tag{A.0.17}$$

Using the decomposition (2.1.7), performing long algebraic manipulations, and using properties such as

$$\chi_r(-\mathbf{k}) = -r\,e^{ir\varphi_{\mathbf{k}}}\,\chi_{-r}(\mathbf{k})\,, \quad e^{i\varphi_{\mathbf{k}}} \equiv \frac{k_1 + i\,k_2}{\sqrt{k_1^2+k_2^2}}\,; \qquad i\sigma_2\chi_r^*\left(\mathbf{k}\right) = -r\chi_{-r}\left(\mathbf{k}\right)\,, \tag{A.0.18}$$

we eventually obtain

$$\begin{aligned}
H_{\text{free}} &= \int \mathrm{d}^3k\,\left(a_r^\dagger\left(\mathbf{k}\right),\,b_r\left(-\mathbf{k}\right)\right)\begin{pmatrix} A_r & B_r^* \\ B_r & -A_r \end{pmatrix}\begin{pmatrix} a_r\left(\mathbf{k}\right) \\ b_r^\dagger\left(-\mathbf{k}\right)\end{pmatrix}\,,\\
A_r &= \frac{1}{2}\left[m_R\left(|u_r|^2 - |v_r|^2\right) + k\left(u_r^* v_r + v_r^* u_r\right) - i\,r\,m_I\left(u_r^* v_r - v_r^* u_r\right)\right]\,,\\
B_r &= \frac{r\,e^{ir\varphi_{\mathbf{k}}}}{2}\left[2\,m_R u_r\,v_r - k\left(u_r^2 - v_r^2\right) - i\,r\,m_I\left(u_r^2 + v_r^2\right)\right]\,.
\end{aligned} \tag{A.0.19}$$

We next diagonalize the Hamiltonian. We find that the matrix in eq. (A.0.19) above has eigenvalues $\pm\omega$, with

$$\omega \equiv \sqrt{k^2 + m_R^2 + m_I^2}\,, \tag{A.0.20}$$

so that the diagonalization will be realized by finding two numbers α_r and β_r for which

$$\begin{pmatrix} A_r & B_r^* \\ B_R & -A_r \end{pmatrix} = \begin{pmatrix} \alpha_r^* & \beta_r^* \\ -\beta_r & \alpha_r \end{pmatrix}\begin{pmatrix} \omega & 0 \\ 0 & -\omega \end{pmatrix}\begin{pmatrix} \alpha_r & -\beta_r^* \\ \beta_r & \alpha_r^* \end{pmatrix}\,. \tag{A.0.21}$$

This transformation can be interpreted as a definition of the operators that create and annihilate the quanta that diagonalize the Hamiltonian at the time t, see eq. (2.1.18). (The

coefficients α and β are conventionally denoted as Bogolyubov coefficients.) The above equation is solved by

$$\alpha_r = \mathrm{e}^{ir\varphi_{\mathbf{k}}/2+i\lambda_r}\left[\frac{1}{2}\sqrt{1+\frac{m_R}{\omega}}\,u_r+\frac{1}{2}\sqrt{1-\frac{m_R}{\omega}}\,\mathrm{e}^{-ir\theta}\,v_r\right],$$
$$\beta_r = r\,\mathrm{e}^{ir\varphi_{\mathbf{k}}/2-i\lambda_r}\left[\frac{1}{2}\sqrt{1-\frac{m_R}{\omega}}\,\mathrm{e}^{ir\theta}\,u_r-\frac{1}{2}\sqrt{1+\frac{m_R}{\omega}}\,v_r\right], \qquad \mathrm{e}^{i\theta}\equiv\frac{k+im_I}{\sqrt{k^2+m_I^2}}\,, \tag{A.0.22}$$

where λ_r is arbitrary and real. We get the occupation number

$$\begin{aligned} N_r = |\beta_r|^2 =& \frac{1}{2}-\frac{m_R}{4\omega}\left(|u_r|^2-|v_r|^2\right)-\frac{k}{2\omega}\,\mathrm{Re}\,(u_r^* v_r)-\frac{r\,m_I}{2\omega}\,\mathrm{Im}\,(u_r^* v_r) \\ =& \frac{1}{2}-\frac{\mu}{2x\sqrt{x^2+\mu^2}}\,\mathrm{Re}\,[s_r^*\,d_r]-\frac{1}{4\sqrt{x^2+\mu^2}}\,\left[|s_r|^2-|d_r|^2\right], \end{aligned} \tag{A.0.23}$$

where the first line corresponds to the expression (2.1.19) in the main text.

Appendix B

Computation of the fermion-gravitational wave interactions

In this appendix we discuss the steps that lead from eq. (3.1.3) to eq. (3.1.8). We begin by writing down the vielbeins for the line element in eq. (3.1.4), $g_{\mu\nu} = \eta_{AB} e^A{}_\mu e^B{}_\nu$. These are given by

$$e^A{}_\mu = \begin{pmatrix} N & 0 \\ N^i e^a{}_i & e^a{}_i \end{pmatrix} , \tag{B.0.1}$$

where the spatial components $e^a{}_i$ satisfy $\delta_{ab} e^a{}_i e^b{}_j = h_{ij}$. Starting from (B.0.1), we can also write

$$e_{A\mu} = \eta_{AB} e^B{}_\mu = \begin{pmatrix} N & 0 \\ N^i \eta_{ab} e^b{}_i & \eta_{ab} e^b{}_j \end{pmatrix} , \quad e_A{}^\mu = \begin{pmatrix} \frac{1}{N} & -\frac{N^j}{N} \\ 0 & e_a{}^j \end{pmatrix} , \quad e^{A\mu} = \begin{pmatrix} \frac{1}{N} & -\frac{N^j}{N} \\ 0 & \eta^{ab} e_b{}^j \end{pmatrix} , \tag{B.0.2}$$

and one can indeed verify that the product $g^{\mu\nu} = e^\mu_A e^\nu_B \eta^{AB}$ is the inverse of the metric in eq. (3.1.4).

In terms of the vielbein, the spin connections are obtained from

$$\omega_\mu{}^{AB} = e^A{}_\nu \nabla_\mu e^{B\nu} = e^A{}_\nu \left(\partial_\mu e^{B\nu} + \Gamma^\nu{}_{\sigma\mu} e^{B\sigma}\right) \ , \tag{B.0.3}$$

where $\Gamma^\nu{}_{\sigma\mu}$ are the usual Christoffel symbols associated with the metric $g_{\mu\nu}$. Using eq. (B.0.1) and (B.0.2), we find the components of the spin connection in ADM coordinates

$$\begin{aligned}
\omega_0{}^{0b} &= \left(\partial_i N - N^j K_{ij}\right)\eta^{ab} e_a{}^i \ , \\
\omega_0{}^{ab} &= e^a{}_k \partial_0 e_c{}^k \eta^{bc} + \left(-N h^{ik} K_{km} + {}^{(3)}\nabla_m N^i\right) e^a{}_i e_c{}^m \eta^{bc} \ , \\
\omega_i{}^{0b} &= -K_{ki} e_c{}^k \eta^{bc} \ , \\
\omega_i{}^{ab} &= e^a{}_k \partial_i e_c{}^k \eta^{bc} - {}^{(3)}\Gamma^m{}_{ki} e^a{}_m e_c{}^k \eta^{bc} \ .
\end{aligned} \tag{B.0.4}$$

Inserting these in the action (3.1.3) we obtain an expanded form of the fermion action in ADM coordinates

$$S_F = \int d^4x \, \mathcal{L}_F \ , \tag{B.0.5}$$

where

$$\begin{aligned}
\mathcal{L}_F = a^3 \Bigg\{ & i\bar{\Psi}\gamma^0 \Big[\partial_0 + \left(\partial_i N - N^j K_{ij}\right) e_b{}^i \Sigma^{0b} \\
& + \frac{1}{2} e^c{}_k \left(\partial_0 e_b{}^k - \left(N K^k{}_m - {}^{(3)}\nabla_m N^k\right) e_b{}^m\right) \eta_{ac} \Sigma^{ab}\Big]\Psi \\
& + i\bar{\Psi}\left(\gamma^a N e_a{}^k - \gamma^0 N^k\right)\left[\partial_k - K_{ik} e_b{}^i \Sigma^{0b} + \frac{1}{2}\left(e^c{}_i \partial_k e_b{}^i + {}^{(3)}\Gamma^m{}_{ik} e^c{}_m e_b{}^i\right)\eta_{ac}\Sigma^{ab}\right]\Psi \\
& - N m \bar{\Psi}\left[\cos\left(\frac{2\varphi}{f}\right) + i \sin\left(\frac{2\varphi}{f}\right)\gamma_5\right]\Psi \Bigg\} ,
\end{aligned} \tag{B.0.6}$$

and $\Sigma^{AB} = [\gamma^A, \gamma^B]/4$. The full action is the sum of the bosonic part in eq. (3.1.5) and the fermionic part in eq. (B.0.5).

We are interested in the interactions between the fermions and the tensor modes of the

metric, and so we expand the spatial part of the vielbein as

$$e^a{}_i = a\,\delta^{ak} e^{\frac{1}{2}\gamma_{ki}} = a\,\delta^{ak}\left[\delta_{ki} + \frac{1}{2}\gamma_{ki} + \frac{1}{8}\gamma_{kj}\gamma_{ji} + \dots\right] \,, \tag{B.0.7}$$

$$e_a{}^i = a^{-1}\,\delta^k_a \delta^{ij} e^{-\frac{1}{2}\gamma_{kj}} = a^{-1}\,\delta^k_a\delta^{ij}, \left[\delta_{ij} - \frac{1}{2}\gamma_{ij} + \frac{1}{8}\gamma_{i\ell}\gamma_{\ell j} + \dots\right] \,, \tag{B.0.8}$$

which leads to the following components of the spin connection (expanded here to quadratic order in tensors)

$$\omega_0{}^{0b} = a^{-1}\left(\partial_a N + \frac{\mathcal{H}}{N} N_a\right)\eta^{ab} \,, \tag{B.0.9}$$

$$\omega_0{}^{ab} = -\frac{1}{8}\eta^{ac}\eta^{bd}\left(\gamma'_{cj}\gamma_{jd} - \gamma_{cj}\gamma'_{jd}\right) - \frac{a^{-2}}{2}\left(\partial_d N_c - \partial_c N_d\right)\eta^{ac}\eta^{bd} \,, \tag{B.0.10}$$

$$\omega_i{}^{0b} = \frac{a\mathcal{H}}{N}\eta^{bc}\left(\delta_{ci} + \frac{1}{2}\gamma_{ci} + \frac{1}{8}\gamma^2_{ci}\right) - \frac{1}{2aN}\eta^{bc}\left(\partial_i N_c + \partial_c N_i\right) + \frac{a}{2N}\eta^{bc}\left(\gamma'_{ic} + \frac{1}{2}\gamma_{in}\gamma'_{nc}\right) \,, \tag{B.0.11}$$

$$\begin{aligned}\omega_i{}^{ab} = & -\eta^{ac}\eta^{bd}\Big(\frac{1}{2}(\partial_d\gamma_{ic} - \partial_c\gamma_{id}) + \frac{1}{8}\left(\gamma_{dk}\partial_i\gamma_{kc} - \gamma_{ck}\partial_i\gamma_{kd}\right) \\ & + \frac{1}{4}(\partial_d\gamma^2_{ic} - \partial_c\gamma^2_{id}) + \frac{1}{4}\left(\gamma_{dk}\partial_c\gamma_{ki} - \gamma_{cm}\partial_d\gamma_{mi}\right) + \frac{1}{4}\left(\gamma_{ck}\partial_k\gamma_{id} - \gamma_{dk}\partial_k\gamma_{ic}\right)\Big) \,.\end{aligned} \tag{B.0.12}$$

These relations, along with the expansion of the lapse and shift in eqs. (3.1.13), are inserted into the constraint equations (3.1.9) and (3.1.10). The part of the Hamiltonian constraint equation quadratic in field fluctuations is

$$\begin{aligned}4\mathcal{H}M^2_{\rm Pl}\Delta\theta^{(2)} + \frac{M^2_{\rm Pl}}{4}\left[(\gamma')_{ij}(\gamma')_{ij} + (\partial_j\gamma_{kq})\,\partial_j\gamma_{qk}\right] = -4a^2\alpha^{(2)}V(\varphi_0) \\ + 2a\left[\frac{i}{2}\left(\bar\Psi\gamma^a\partial_a\Psi - (\partial_a\bar\Psi)\gamma^a\Psi\right) - m\bar\Psi\left[\cos\left(\frac{2\varphi_0}{f}\right) - i\gamma^5\sin\left(\frac{2\varphi_0}{f}\right)\right]\Psi\right],\end{aligned} \tag{B.0.13}$$

while the quadratic part of the momentum constraint reads

$$\begin{aligned}0 = 2M^2_{\rm Pl}\mathcal{H}\,\partial_j\alpha^{(2)} + M^2_{\rm Pl}\left[-\frac{1}{2}\Delta\beta^{(2)}_j - \frac{1}{4}(\partial_i\gamma')_{jk}\gamma_{ki} - \frac{1}{4}(\partial_j\gamma_{\ell i})(\gamma')_{i\ell} + \frac{1}{4}(\partial_i\gamma_{jk})(\gamma')_{ki}\right] \\ - a\left[\frac{i}{2}\left(\bar\Psi\gamma^0\partial_j\Psi - (\partial_j\bar\Psi)\gamma^0\Psi\right) - \frac{1}{4}\epsilon_{jab}\partial_a\left(\bar\Psi\gamma^b\gamma^5\Psi\right)\right] \,.\end{aligned} \tag{B.0.14}$$

Eqs. (B.0.13) and (B.0.14) can be solved to find the quadratic order lapse and shift

$$
\begin{aligned}
\alpha^{(2)} &= \Delta^{-1}\left\{\frac{1}{8\mathcal{H}}\partial_j\left[(\partial_j\gamma_{\ell i})(\gamma')_{i\ell}\right]+\frac{ia}{4M_{\rm Pl}^2\mathcal{H}}\left[\bar{\Psi}\gamma^0\Delta\Psi-(\Delta\bar{\Psi})\gamma^0\Psi\right]\right\},\\
\beta_j^{(2)} &= \Delta^{-1}\left\{\frac{1}{2}\Delta^{-1}\partial_j\partial_k\left[(\partial_k\gamma_{\ell i})(\gamma')_{i\ell}\right]-\frac{1}{2}\left[(\partial_i\gamma')_{jk}\gamma_{ki}+(\partial_j\gamma_{\ell i})(\gamma')_{i\ell}-(\partial_i\gamma_{jk})(\gamma')_{ki}\right]\right.\\
&\qquad+\frac{ia}{M_{\rm Pl}^2}\partial_j\Delta^{-1}\left[\bar{\Psi}\gamma^0\Delta\Psi-(\Delta\bar{\Psi})\gamma^0\Psi\right]\\
&\qquad\left.-\frac{a}{M_{\rm Pl}^2}\left[i\left(\bar{\Psi}\gamma^0\partial_j\Psi-(\partial_j\bar{\Psi})\gamma^0\Psi\right)-\frac{1}{2}\epsilon_{ijk}\partial_i(\bar{\Psi}\gamma^k\gamma^5\Psi)\right]\right\},\\
\theta^{(2)} &= \Delta^{-1}\left\{-\frac{1}{16\mathcal{H}}\left[(\gamma')_{ij}(\gamma')_{ij}+(\partial_j\gamma_{kq})\,\partial_j\gamma_{qk}\right]-\frac{ia}{4M_{\rm Pl}^2\mathcal{H}}\left(\bar{\Psi}\gamma^0\partial_0\Psi-(\partial_0\bar{\Psi})\gamma^0\Psi\right)\right.\\
&\qquad\left.-\frac{a^2}{M_{\rm Pl}^2\mathcal{H}}V(\varphi_0)\Delta^{-1}\left\{\frac{1}{8\mathcal{H}}\partial_j\left[(\partial_j\gamma_{\ell i})(\gamma')_{i\ell}\right]+\frac{ia}{4M_{\rm Pl}^2\mathcal{H}}\left[\bar{\Psi}\gamma^0\Delta\Psi-(\Delta\bar{\Psi})\gamma^0\Psi\right]\right\}\right\},
\end{aligned}
\tag{B.0.15}
$$

where $\Delta = \partial_i\partial_i$ is the spatial Laplacian, and Δ^{-1} is its inverse. In deriving these solutions, we have disregarded fluctuations of the inflaton field because we are only interested in the interactions between gravitational waves and fermions. The inclusion of inflaton fluctuations introduces terms that are quadratic in the inflaton fluctuations, as well as terms quadratic in the first order perturbation to the lapse and shift. We have also made use of the linear order equation of motion for the fermion. This induces corrections to the action that begin at fifth order in fluctuations and are thus irrelevant here.

We are now ready to evaluate the action, eq. (B.0.6), on the constraint surface and eliminate the non-dynamical lapse and shift. Inserting the solutions to the constraints (B.0.15) into the full action, eq. (3.1.5) + (3.1.8), we expand the result to quartic order. This results in an action for the dynamical fields, ψ and γ, consisting of a quadratic (free) part, $S^{(2)}$ and cubic and quartic parts, $S_F^{(3)}$ and $S_F^{(4)}$, which describe the interactions of a fermion bilinear with one and two gravitational waves, respectively. Because the constraint equations are derived from the variation of the action with respect to the lapse and shift, one can use their equations of motion before substituting in their solutions. This results in the cancellation of a large number of terms, and leaves the result we report above in eqs. (3.1.19) and (3.1.20).

Appendix C

Interaction Hamiltonian

In this appendix we write the explicit forms of the interaction Hamiltonian terms eq. (3.1.21), obtained by inserting the decompositions (3.1.22) and (3.1.24) into the actions in eqs. (3.1.19) and (3.1.20). For the cubic term, we find

$$H_{\rm int}^{(3)} = -\frac{1}{2\sqrt{2}M_{\rm Pl}}\frac{1}{a(\tau)}\sum_{\lambda}\int\frac{\prod_{i=1}^{3}d^3k_i}{(2\pi)^{3/2}}\mathrm{t}_{\mathbf{k}_1}^{\lambda}\bar{\psi}_{\mathbf{k}_2}\gamma^c\psi_{\mathbf{k}_3}\Pi_{cj}^{\lambda}(\mathbf{k}_1)(\mathbf{k}_2+\mathbf{k}_3)_j\,\delta^{(3)}(\mathbf{k_1}-\mathbf{k}_2+\mathbf{k}_3)\;. \tag{C.0.1}$$

The various terms contributing at quartic order are

$$
\begin{aligned}
H^{(4)}_{\text{int},1} =& \frac{1}{8M_{\text{Pl}}^2}\frac{1}{a^2}\sum_{\lambda\lambda'}\int\frac{\prod_{i=1}^4 d^3k_i}{(2\pi)^3}\mathrm{t}^{\lambda}_{\mathbf{k}_1}\mathrm{t}^{\lambda'}_{\mathbf{k}_2}\bar{\psi}_{\mathbf{k}_3}\gamma^c\psi_{\mathbf{k}_4}\Pi^{\lambda}_{cm}(\mathbf{k}_1)\Pi^{\lambda'}_{mj}(\mathbf{k}_2)(\mathbf{k}_3+\mathbf{k}_4)_j \\
&\quad\times\delta^{(3)}(\mathbf{k_1}+\mathbf{k}_2-\mathbf{k}_3+\mathbf{k}_4)\,, \\
H^{(4)}_{\text{int},2} =& \frac{1}{8M_{\text{Pl}}^2}\frac{1}{a}\sum_{\lambda\lambda'}\int\frac{\prod_{i=1}^4 d^3k_i}{(2\pi)^3}\left[\left(\frac{\mathrm{t}^{\lambda}_{\mathbf{k}_1}}{a}\right)'\mathrm{t}^{\lambda'}_{\mathbf{k}_2}\bar{\psi}_{\mathbf{k}_3}\gamma^c\gamma^5\psi_{\mathbf{k}_4}\epsilon^{abc}\Pi^{\lambda}_{aj}(\mathbf{k}_1)\Pi^{\lambda'}_{jb}(\mathbf{k}_2)\right] \\
&\quad\times\delta^{(3)}(\mathbf{k_1}+\mathbf{k}_2-\mathbf{k}_3+\mathbf{k}_4)\,, \\
H^{(4)}_{\text{int},3} =& \frac{1}{8M_{\text{Pl}}^2}\frac{1}{a^2}\sum_{\lambda\lambda'}\int\frac{\prod_{i=1}^4 d^3k_i}{(2\pi)^3}\left[\mathrm{t}^{\lambda}_{\mathbf{k}_1}\mathrm{t}^{\lambda'}_{\mathbf{k}_2}\bar{\psi}_{\mathbf{k}_3}\gamma^0\gamma^5\psi_{\mathbf{k}_4}\lambda' k_2\Pi^{\lambda}_{bk}(\mathbf{k}_1)\Pi^{\lambda'}_{kb}(\mathbf{k}_2)\right] \\
&\quad\times\delta^{(3)}(\mathbf{k_1}+\mathbf{k}_2-\mathbf{k}_3+\mathbf{k}_4)\,, \\
H^{(4)}_{\text{int},4} =& -\frac{i}{2\,aM_{\text{Pl}}^2}\left(1-\frac{V}{4M_{\text{Pl}}^2H^2}\right)\sum_{\lambda\lambda'}\int\frac{\prod_{i=1}^4 d^3k_i}{(2\pi)^3}\left[\mathrm{t}^{\lambda}_{\mathbf{k}_1}\left(\frac{\mathrm{t}^{\lambda'}_{\mathbf{k}_2}}{a}\right)'\bar{\psi}_{\mathbf{k}_3}\gamma^0\psi_{\mathbf{k}_4}(k_3^2-k_4^2)\right. \\
&\left.\times\frac{(\mathbf{k}_1+\mathbf{k}_2)\cdot\mathbf{k}_1}{|\mathbf{k}_1+\mathbf{k}_2|^4}\Pi^{\lambda}_{jk}(\mathbf{k}_1)\Pi^{\lambda'}_{jk}(\mathbf{k}_2)\right]\delta^{(3)}(\mathbf{k_1}+\mathbf{k}_2-\mathbf{k}_3+\mathbf{k}_4)\,,
\end{aligned}
$$

$$
\begin{aligned}
H^{(4)}_{\text{int},5} =& -\frac{1}{4M_{\text{Pl}}^2}\frac{1}{a}\sum_{\lambda\lambda'}\int\frac{\prod_{i=1}^4 d^3k_i}{(2\pi)^3}\left\{\mathrm{t}^{\lambda}_{\mathbf{k}_1}\left(\frac{\mathrm{t}^{\lambda'}_{\mathbf{k}_2}}{a}\right)'\right. \\
&\times\left[(\bar{\psi}_{\mathbf{k}_3}\gamma^c\gamma^5\psi_{\mathbf{k}_4})\frac{\epsilon^{aic}(\mathbf{k}_3-\mathbf{k}_4)_a}{|\mathbf{k}_1+\mathbf{k}_2|^2}-2i\frac{(\mathbf{k}_3+\mathbf{k}_4)_i}{|\mathbf{k}_1+\mathbf{k}_2|^2}(\bar{\psi}_{\mathbf{k}_3}\gamma^0\psi_{\mathbf{k}_4})\right] \\
&\left.\times\left[(\mathbf{k}_1)_j\Pi^{\lambda}_{ik}(\mathbf{k}_1)\Pi^{\lambda'}_{jk}(\mathbf{k}_2)-(\mathbf{k}_2)_j\Pi^{\lambda}_{jk}(\mathbf{k}_1)\Pi^{\lambda'}_{ik}(\mathbf{k}_2)-(\mathbf{k}_1)_i\Pi^{\lambda}_{jk}(\mathbf{k}_1)\Pi^{\lambda'}_{jk}(\mathbf{k}_2)\right]\right\} \\
&\times\delta^{(3)}(\mathbf{k_1}+\mathbf{k}_2-\mathbf{k}_3+\mathbf{k}_4)\,, \\
H^{(4)}_{\text{int},6} =& -\frac{i}{16M_{\text{Pl}}^2\,aH}\sum_{\lambda\lambda'}\int\frac{\prod_{i=1}^4 d^3k_i}{(2\pi)^3}\left[\left(\left(\frac{\mathrm{t}^{\lambda}_{\mathbf{k}_1}}{a}\right)'\left(\frac{\mathrm{t}^{\lambda'}_{\mathbf{k}_2}}{a}\right)'-\frac{\mathbf{k}_1\cdot\mathbf{k}_2}{a^2}\mathrm{t}^{\lambda}_{\mathbf{k}_1}\mathrm{t}^{\lambda'}_{\mathbf{k}_2}\right)\right. \\
&\quad\left.\times\bar{\psi}_{\mathbf{k}_3}\gamma^0\psi_{\mathbf{k}_4}\frac{k_3^2-k_4^2}{|\mathbf{k}_3-\mathbf{k}_4|^2}\Pi^{\lambda}_{ij}(\mathbf{k}_1)\Pi^{\lambda'}_{ij}(\mathbf{k}_2)\right]\delta^{(3)}(\mathbf{k_1}+\mathbf{k}_2-\mathbf{k}_3+\mathbf{k}_4)\,, \\
H^{(4)}_{\text{int},7} =& \frac{i}{8M_{\text{Pl}}^2a^2H}\sum_{\lambda\lambda'}\int\frac{\prod_{i=1}^4 d^3k_i}{(2\pi)^3}\left[\left(\bar{\psi}_{\mathbf{k}_3}\gamma^0\partial_0\psi_{\mathbf{k}_4}-\partial_0\bar{\psi}_{\mathbf{k}_3}\gamma^0\psi_{\mathbf{k}_4}\right)\mathrm{t}^{\lambda}_{\mathbf{k}_1}\left(\frac{\mathrm{t}^{\lambda'}_{\mathbf{k}_2}}{a}\right)'\right. \\
&\quad\left.\times\frac{\mathbf{k}_1\cdot(\mathbf{k}_1+\mathbf{k}_2)}{|\mathbf{k}_1+\mathbf{k}_2|^2}\Pi^{\lambda}_{ij}(\mathbf{k}_1)\Pi^{\lambda'}_{ij}(\mathbf{k}_2)\right]\delta^{(3)}(\mathbf{k_1}+\mathbf{k}_2-\mathbf{k}_3+\mathbf{k}_4)\,. \qquad \text{(C.0.2)}
\end{aligned}
$$

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