Material Property Heterogeneity in Dimensional Lumber and its Relationship to Mass Timber Performance

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MATERIAL PROPERTY HETEROGENEITY IN DIMENSIONAL LUMBER AND ITS RELATIONSHIP TO MASS TIMBER PERFORMANCE

A Dissertation Presented

by

FIONA A. O’DONNELL

Submitted to the Graduate School of the University of Massachusetts Amherst in partial fulfillment of the requirements for the degree of

DOCTOR OF PHILOSOPHY

May 2021

Civil Engineering
DEDICATION

To my parents - my most fervent supporters and greatest inspirations
ACKNOWLEDGMENTS

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Thank you to my family and friends for providing encouragement, perspective, and love throughout my studies. I’m lucky to have you on my team.
ABSTRACT
MATERIAL PROPERTY HETEROGENEITY IN DIMENSIONAL LUMBER
AND ITS RELATIONSHIP TO MASS TIMBER PERFORMANCE

MAY 2021

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According to the Environmental Protection Agency, buildings account for 38% of the United States’ carbon dioxide emissions, providing architects and structural engineers a unique opportunity to mitigate a significant factor driving climate change by implementing innovative and sustainable technology in infrastructure design. Wood and mass timber products are becoming an increasingly popular alternative building material due to their economic and environmental benefits. The natural growth of wood leads to highly heterogeneous material properties. Defects such as checks, knots, and localized slope of grain contribute to some of this variation; however, wood properties vary significantly even in clear wood. Using mass timber products, like Cross Laminated Timber (CLT), creates an averaging effect of constituent board material properties and reduces the effects of defects. Thus, this research aims to understand and characterize the influence of the material property heterogeneity of constituent boards on CLT panel behavior and performance.

Specifically, a two-dimensional and three-dimensional probabilistic model for the distribution of knots in dimensional lumber is developed, which allows for the simulation of synthetic samples calibrated to any softwood species. Additionally, parallel and perpendicular compressive properties of Eastern hemlock are experimentally determined.
to serve as input for a constitutive model to predict Eastern hemlock constituent board and CLT behavior. Nonlinear three-dimensional finite element models are developed to investigate the impact of knots on effective stiffness and strength, stress path, and location of yielding initiation. The relationship between knot defects and reduced strength and stiffness of dimensional lumber is fundamental to visual grading methods. The correlation between knot defect geometry and MOE/MOR was investigated in Eastern hemlock and Sitka spruce. Finally, the Variability Response Function (VRF) is applied to CLT to investigate the impact of lengthwise variability in MOE of constituent boards on the variance in displacement response of CLT.

Basic conclusions include:

- Development of a probabilistic model for the distribution and geometry of knots in dimensional lumber
- Orthotropic compressive properties of Eastern hemlock
- Knots have a greater impact on effective stiffness and strength in tensile loading than compressive loading
- A method to understand the influence of constituent board properties on CLT performance
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CHAPTER 1

INTRODUCTION

1.1 Background and Motivation

Cross Laminated Timber (CLT) is a massive engineered wood composite panel employed in large-scale construction of floor, wall, and roof assemblies. CLT panels are prefabricated multi-layer panel wood products composed of dimensional lumber. Each panel is fabricated from at least three layers of parallel boards glued together at their surfaces with an adhesive under pressure. Alternate layers of dimensional lumber boards are placed cross-wise, providing the product a high level of in-plane stability.

Figure 1. Conventional lay-up of a CLT panel with mid-layers oriented orthogonally to the parallel layers (Buck et al., 2016)

CLT products have been part of the European market for the past three decades, and have recently entered the United States market. They have received a significant level of interest due to their technical capabilities, cost-competitiveness, and sustainable properties. Since its introduction in the 1990s, CLT has become a viable alternative to traditional building materials like steel and reinforced concrete (Risen, 2014). CLT construction has been predominately used in low- to mid-rise structures for public occupancy buildings. However,
since the world’s first tall timber building was constructed in 2009, several tall timber structures have been built worldwide.

CLT construction is becoming recognized for its environmental attributes and as a way to mitigate climate change. The impacts of global climate change are already being felt in the United States and are projected to intensify in the future. However, the severity of future impacts will depend largely on the actions taken to reduce greenhouse gas emissions (USGCRP, 2018). According to the United States Environmental Protection Agency, buildings account for thirty-eight percent of the carbon dioxide emissions in the United States (United Stated Environmental Protection Agency, 2009). This provides a unique opportunity to architects and structural engineers to mitigate a significant factor driving climate change by implementing innovative and sustainable technology in infrastructure design.

In structural applications, the most important properties of wood are stiffness and strength. Lumber sawn from a log, regardless of species, has significant variation in mechanical properties. The manufacturing process can also be subject to considerable variation. Grading standards have been developed to sort lumber quality for a particular end-use application by requiring conformance to a specified set of parameters. There are two approaches to the grading of structural lumber: visual grading and mechanical grading. Visual grading sorts lumber into grades by its visible characteristics such as knots, wane, slope of grain, and other natural and machining features. Mechanical grading sorts lumber by a machine that evaluates its mechanical properties using a nondestructive test. In the United States, most structural lumber is produced from softwood species that have been visually graded (Green & Hernandez, 1998).

Currently, mass timber products are fabricated from high quality and high-grade wood species like Douglas-fir and Southern Pine. However, mass timber’s composite nature provides an opportunity to utilize traditionally low-value species typically considered inadequate for structural purposes. Finding applications for underutilized species creates the potential for a
promising market for low-value wood species that are abundant in the United States. Additionally, finding commercial markets for low-value woods supports national forest management strategies to improve forest health while giving rise to more sustainable building practices and increased job opportunities in rural areas of the United States (Brashaw et al., 2012.; Lyon & Bond, 2014).

1.2 Research Objectives

The overarching goal of this research is to examine the influence of the variability of constituent board material properties on Cross Laminated Timber performance. This is studied by modeling and characterizing defects and heterogeneous material properties in lumber and studying their impact on wood stiffness and strength at the dimensional lumber and CLT panel scale. To this end, a series of numerical models have been developed and experimental testing has been performed. Specific objectives include:

1. Characterize knot geometry in dimensional lumber and develop a probabilistic model for the distribution of knots to allow for simulation of synthetic boards calibrated to a particular species
2. Experimentally evaluate orthotropic compressive properties of Eastern hemlock for use in a constitutive finite element model
3. Develop three-dimensional finite element models at the dimensional lumber scale
4. Experimentally and analytically investigate the correlation between knot defects and effective stiffness and strength of dimensional lumber
5. Quantify the variability in displacement response of a CLT panel as a result of spatial variation of MOE in the constituent boards
This research predominantly investigates the structural viability of Eastern hemlock as the primary constituent for CLT panel fabrication. In the United States, Eastern hemlock grows along the East Coast from New England to northern Alabama and Georgia, as well as in the Great Lake states. Typical Eastern hemlock is coarse and uneven in texture with considerable shake. Currently, Eastern hemlock is considered to be inadequate for structural applications and is used primarily for lumber and pulpwood (Ross & USDA Forest Service., 2010). Although Eastern hemlock was chosen as the focus of this study, many of the methods and analysis techniques developed through this research are transferable to other softwood species.

1.3 Literature Review and Associated Work

Numerous researchers have studied the mechanical properties of wood over many decades. Knots are acknowledged to be responsible for stiffness and strength reduction in structural lumber. Knots are the result of branches embedded in the trunk of a tree and significantly influence the surrounding wood causing variable density, grain distortion, and material discontinuity in the vicinity of knots (Bodig and Jayne, 1982). The influence of knots depends on their size, location, shape, resultant slope of grain, and type of stress to which the member is subjected. Knots typically have a greater impact on tensile properties than compressive properties, and a greater impact on strength properties than stiffness properties (Ross & USDA Forest Service., 2010). There have been numerous research efforts to numerically model the structural performance of knots in wood (see, for example, Foley, 2003, Baño et al., 2011, and Xu, 2002).

The characterization of the longitudinal fiber orientation is also of interest. Philips, Bodig, and Goodman developed a flow grain analogy, in which the longitudinal fiber
orientations around knots are modeled by a fluid mechanical formulation (1981). Foley later expanded this to consider the dive angle by employing a polynomial model (Foley, 2003). Based on these formulations, others have investigated methods to measure and reconstruct knot geometry. Specifically, Kandler, Lukacevic, and Füssl developed a deterministic reconstruction of knot geometry based on surface fiber angle measurements (Kandler et al., 2016a).

Because tree growth can be manipulated by forest management techniques, there is potential to influence the heterogeneity of lumber by implementing new forest management practices. Thus, research has been performed to understand how management may impact knot size, distribution, and other stem characteristics (Zhang et al., 2006). Specifically, Lemieux, Beaudoin, and Zhang (2001) established the relationship between external branch parameters and internal knot morphology in Black Spruce. Further, a stochastic model to simulate branch and knot formation in loblolly pine was developed by Trincado and Burkhart (Trincado & Burkhart, 2009).

Monte Carlo simulations have also modeled the heterogeneous material properties of wood. Kline (1986) developed a second-order Markov model for generating spatial variability in MOE. Monte Carlo simulations have also been applied to engineered products, predominately glue-laminated (glulam) timber beams, for reliability analysis (e.g., Foschi and Barrett 1980; Schaffer et al. 1986). More recently, a prediction model for the bending properties of glulam was developed using knot and MOE distributions as the main input variables (Lee et al., 2005). An alternative method to study the variation of material properties was developed by Shinozuka (1987), which uses a deterministic
function to describe the structure’s boundary conditions and the random material property's spectral density function to evaluate the variance of the response of the system.

1.4 Organization of Dissertation

The dissertation consists of a series of stand-alone chapters. Therefore, there is inevitably some repetition of material, predominately in the introduction section of each chapter. A summary of each chapter is provided below:

1. The current chapter (Chapter 1) discusses the background information and the guiding motivation for the work provided in this dissertation. Research objectives are defined and a literature review is provided.

2. Chapter 2 is divided into two sections. Section 2.3 – 2.5 provides a characterization of knot defect geometry and a two-dimensional probabilistic model to describe the distribution of knots in dimensional lumber. Section 2.6 – 22.8 expands this model into three-dimensions.

3. Parallel and perpendicular compressive properties of Eastern hemlock were experimentally determined to serve as input for a constitutive model to predict Eastern hemlock constituent board and CLT panel behavior. Chapter 3 provides the methodology and results of the experimental tests.

4. Chapter 4 details nonlinear three-dimensional finite element models developed to investigate the influence of knot defects on effective stiffness and strength, stress path, and yielding initiation in dimensional lumber.

5. Chapter 5 investigates the correlation between knot defect geometry and Modulus of Elasticity (MOE)/Modulus of Rupture (MOR) in Eastern hemlock and Sitka spruce dimensional lumber.
6. A method to investigate the variability in displacement response of a CLT panel is presented in Chapter 6. Typically, problems involving stochastic material properties are solved by Monte Carlo, which is computationally expensive for complex systems. Instead, Chapter 6 provides a new analysis method by applying a Variability Response Function (VRF) to CLT geometry.

7. Chapter 7 provides a summary of the findings of this research and details for possible future work.
CHAPTER 2

PROBABILISTIC MODEL FOR KNOT DEFECT GEOMETRY AND DISTRIBUTION

2.1 Introduction

Compared to traditional structural materials like concrete and steel, wood is highly heterogeneous. This is caused, in part, by naturally occurring defects such as knots, resulting in localized regions of low stiffness and strength. While the knot material is stiffer than the surrounding clear wood, the knot defect causes the clear wood fibers to deviate around the knot to create low stiffness regions. The use of composite wood products and mass timber leads to the potential of an averaging effect of these material properties and enabling the use of traditionally low-value materials to enter the market. Finding a high volume of commercial use for traditionally low-value woods supports national forest management strategies to improve forest health while also producing green jobs in rural communities (Brashaw et al.). However, a significant limitation on researching traditionally low-value woods as constituents in composite mass timber products is the cost of materials, time, and fabrication. Thus, a stochastic model is developed and presented in this chapter to help mitigate these issues and allow for two and three-dimensional synthetic dimensional lumber models. A natural extension of this model is to combine many individual boards to create synthetic composite mass timber products, such as Cross Laminated Timber panels. The model is calibrated to Eastern hemlock, a traditionally low value yet abundant species in the Northeast of the United States.

Because the significant heterogeneity of wood material properties provides unique structural challenges, the influence of defects on wood mechanical properties has long been of interest. At the global scale, knots form from the growth of branches and act as the
branch’s internal attachment to the tree stem. Since the growth of trees can be manipulated by forest management techniques, research has been performed to understand how these interventions impact knot size, distribution, and other stem characteristics (Zhang et al., 2006). Specifically, Lemieux, Beaudoin, and Zhang (2001) established the relationship between external branch parameters and internal knot morphology in Black Spruce. Further, a stochastic model to simulate branch and knot formation in loblolly pine was developed by Trincado and Burkhart (Trincado & Burkhart, 2009). At the local scale, the existence of knots results in displaced and distorted longitudinal fibers. Because wood is a highly orthotropic material, these fiber deviations create localized regions of low strength. Significant work has been done to understand and model these longitudinal fiber deviations around knots. Philips, Bodig, and Goodman developed a flow grain analogy in which the longitudinal fiber orientations around knots are modeled by a fluid mechanical formulation (Phillips et al. 1981). Foley later expanded this to consider the dive angle by employing a polynomial model (Foley 2003). Based on these formulations, others have investigated methods to measure and reconstruct knot geometry. Specifically, Kandler, Lukacevic, and Füssl developed a deterministic reconstruction of knot geometry based on surface fiber angle measurements (Kandler et al., 2016a). Much of this work is limited to the characterization of knot geometries and their influence on the local material properties. In this paper, the distribution of knots is studied at an intermediate scale, focusing on knots in dimensional lumber. By fitting distributions and geometric models to the knot geometry characterization, researchers can overcome some limitations of the costs associated with data measurement and collection and limited sample sizes by creating synthetic knot geometry. This allows for the creation of large scale models of knot defects in both
dimensional lumber and composite wood products to better understand the global influence of knots on effective material properties.

Section 2.2 of this paper describes the measurement methods and the geometric characterization used as inputs to the stochastic models. The model parameters, calibration, and validation are described in Section 2.3 - 2.5 for the two-dimensional model and Section 2.6 - 2.8 for the three-dimensional model.

2.2 Knot Idealization: Measurement Methods and Geometric Characterization

Due to wood's natural growth, the distribution of knot defects can vary significantly for dimensional lumber of the same species. To study this, a total of eighty-three Eastern hemlock 12 ft (365.8 cm) long nominal 2 in (5.1 cm) x 4 in (10.2 cm) boards were purchased in three separate batches from two local mills in Western Massachusetts. When purchasing from the source mill, boards were selected visually to avoid extreme warp. Thirty-one of these boards were randomly selected for further processing and investigation for this study. At the University of Massachusetts Wood Mechanics lab, the boards were jointed and planed to have final dimensions of 3.3 cm in thickness, 8.1 cm in width, and 365.8 cm in length (Kaboli, 2019). Each board was also graded according to the Northeastern Lumber Manufacturers Association (NELMA) grading rules for structural lumber (NELMA, 2013). Figure 2 shows selected examples of the distribution of knots for the thirty-one Eastern hemlock boards. Several geometric parameters influence the distribution of the defects, including diameter, depth, the direction of growth, knot center coordinates, and orientation.
Figure 2. Photographs of selected Eastern hemlock samples and associated grades according to the NELMA guidelines. Photos for only side A are provided for the Select Structural, Grade 1, and Grade 2 samples, while photos for both sides A and B are provided for the Grade 3 sample. The categorization of the depth of knots are called out for the Grade 3 sample, showing an interior non-through knot (a), an edge through knot (b), and an interior through knot (c).

To begin characterizing the distribution of knots in dimensional lumber, each knot is categorized as an interior knot, edge knot with a visible center, or edge knot without a visible center. Interior knots are knots whose total surface area is located on the board. Edge knots with a visible center are knots where the knot's center is located on the board, but a portion of the surface area is outside the board. Edge knots without visible centers are knots where the center and a portion of the knot's surface area is outside the board, while a portion of the knot is located on the board. Edge knots are due to the cutting of larger lumber into smaller boards, causing some knots to be divided between boards. An example of each classification of knot is shown in Figure 2.
Figure 3. Geometric parameter definitions and coordinate system. Sides A and B are the board's flat sides, and sides C and D are the board's edge sides. The longitudinal, tangential, and radial axes capture the boards' length, width, and depth, respectively. The first knot is an interior knot, the second knot is an edge knot with a visible center, and the third knot is an edge knot without a visible center.

For each board, the total number of knots, $n$, as well as the number of interior knots, $n_I$, edge knots with a visible center, $n_{EC}$, and edge knots without a visible center, $n_{EWC}$, are recorded such that

$$n = n_I + n_{EC} + n_{EWC}.$$  \hspace{1cm} (Eq. 1)

For interior knots and edge knots with a visible center, the knot center coordinates, $c_x$ and $c_y$, are recorded with respect to the longitudinal, $x$, and tangential, $y$, axes. For interior knots, the minimum, $d_{min}$, and maximum, $d_{max}$, diameters and the angle of the maximum diameter with respect to the longitudinal axis, $\theta_{max}$, were recorded. For edge knots with visible centers, the minimum and maximum diameters were calculated by

$$d_{min_i} = \min(d_{1i}, d_{2i})$$

$$d_{max_i} = \max(d_{1i}, d_{2i})$$

for $i = 1, \ldots, n_{EWC}$  \hspace{1cm} (Eq. 2)

where
\[ d_{2i} = 2 \left( \frac{w_i}{2} + \frac{l_i^2}{8w_i} \right) \]  \quad \text{for } i = 1, \ldots, n_{EW_C} \quad \text{(Eq. 3)}

The visible axis length is represented by \( d_1 \), the length of the knot on the edge of the longitudinal axis is \( l \), and the knot's width along the tangential axis is \( w \).

Edge knots without visible centers were assumed to be circular and the minimum and maximum diameters were

\[ d_{\text{min}i} = d_{\text{max}i} = 2 \left( \frac{w_i}{2} + \frac{l_i^2}{8w_i} \right) \]  \quad \text{for } i = 1, \ldots, n_{EW_C} \quad \text{(Eq. 4)}

The knot center coordinate along the longitudinal axis is estimated for knots without visible centers. The knot center coordinate along the tangential axis is calculated by

\[ c_{yi} = w_i - \frac{d_{\text{max}i}}{2} \]  \quad \text{for } i = 1, \ldots, n_{EW_C} \quad \text{(Eq. 5)}

These parameters were measured for knots appearing on all four sides of each sample. However, knots appearing on sides C and D were neglected in this analysis (Figure 3).
Each knot is also classified with respect to its depth. Through knots appear on more than one face of the board, while non-through knots appear only on one face of the board. Only through knots located on both sides A and B are included in the analysis presented in this paper. Depending on growth rate and direction, through knots have different knot center coordinates, minimum and maximum diameters, and angle of the maximum diameter with respect to the longitudinal axis on each side of the board. An example board with through knots is shown in Figure 2, where each through knot pair is emphasized.

For each through knot, a three-dimensional vector, $\mathbf{G}$, describing the direction of growth is defined. The three-dimensional vector originates at the knot center coordinate on side A and points towards the corresponding knot center coordinate on side B. A schematic diagram of the vector is shown in Figure 3. The $x$-component of the vector is parallel to the longitudinal axis and is calculated by subtracting the knot center coordinates along the $x$-axis on side A and B,

$$
G_x = c_{xB} - c_{xA} \quad \text{(Eq. 6)}
$$

where $c_{xA}$ and $c_{xB}$ are the knot center coordinates along the longitudinal axis for side A and side B, respectively. Similarly, the $y$-component of the vector is parallel to the tangential axis and is calculated by subtracting the knot center coordinates along the $y$-axis on side A and B,

$$
G_y = c_{yB} - c_{yA} \quad \text{(Eq. 7)}
$$

The $z$-component is parallel to the radial axis and equal to the thickness of the board,

$$
G_z = t_b \quad \text{(Eq. 8)}
$$

where $t_b$ is the thickness of the board.
The vector is then normalized to the thickness of the board, such that the resulting vector, $G$ is

$$G = \left[ \frac{g_x}{t_b}, \frac{g_y}{t_b}, 1 \right]. \quad \text{(Eq. 9)}$$

For each board, the mean of the normalized direction vector is calculated by,

$$\mu_G = \left[ \frac{\sum_{i=0}^{N_t} G_{x_i}}{N_t}, \frac{\sum_{i=0}^{N_t} G_{y_i}}{N_t}, 1 \right] \quad \text{(Eq. 10)}$$

where $\mu_G$ is the mean of the direction vector for a given board, $G_{x_i}$ and $G_{y_i}$ are the longitudinal and tangential components of the direction vector of a through knot, and $N_t$ is the number of through knots on a given board. To orient each board in the same direction, side A is defined such that the longitudinal component of the mean direction vector, $\mu_{G_x}$, is positive.

These observed parameters are used to analyze the distribution and geometry of knot defects. For convenience and consistency, through knots are associated with side A. Two categories of knots are designated to describe the distribution of knot defects. Category A includes both through knots, appearing on sides A and B, and non-through knots, which appear only on side A. Category B includes only non-through knots that appear on side B.

### 2.3 Two-Dimensional Probabilistic Model

A two-dimensional model for synthetic knot geometry was developed. The model assumes that all synthetic knots are through knots and are circular or elliptical cylinders with depth along the radial axis, equal to the board's thickness. The two-dimensional model is calibrated to knots in Category A.
For this paper, the following notation,
\[ p \sim (\mu, \sigma), \]
will be used to indicate that the stochastic parameter, \( p \), is related to the observed properties within the parenthesis, \( \mu \) and \( \sigma \). The appropriate distributions will be chosen based on the calibration data, discussed in Section 2.4 of this paper.

### 2.3.1 Number of knots

The number of knots on each board is a discrete non-negative random variable. It is assumed that all knots have a depth equal to the thickness of the board. Thus the number of synthetic knots on side A of the board is equal to the number of synthetic knots on side B. The number of knots, \( n \), is a nonnegative discrete random variable and is determined by the distribution of knots per unit area in Category A, \( v \), multiplied by the area, \( A \), in which knot centers may be located:

\[ n = vA. \]  
\[ \text{(Eq. 11)} \]

Knot defects are not only distributed along the width, \( w_b \), and length, \( l_b \), of the board, but also extend slightly past these boundaries to account for edge knots without a visible center. Thus, an additional length in both the longitudinal, \( x_l \), and tangential, \( y_l \), directions are defined. The area in which knot centers are located is then:

\[ A = (l_b + 2x_l)(w_b + 2y_l). \]  
\[ \text{(Eq. 12)} \]
2.3.2 Diameters

The number of synthetic knots is divided into two categories: circular knots and elliptical knots. The number of circular and elliptical knots are discrete random variables, $n_c$ and $n_e$, respectively. The number of synthetic circular knots is related to the percentage of knots that are circular in the observed data rounded to the closest integer,

$$n_c = \text{round}(np_c)$$  \hspace{1cm} (Eq. 13)

where $p_c$ is the ratio of circular knots to total knots in Category A,

$$p_c = \frac{n_c}{n},$$  \hspace{1cm} (Eq. 14)

and $n_c$ is the number of circular knots observed in Category A, and $n$ is the total number of knots observed in Category A.

The remaining knots are defined as elliptical,

$$n_e = n - n_c.$$  \hspace{1cm} (Eq. 15)

The diameters of circular knots, $d_c$, are a continuous positive random variable,

$$d_c \sim (\mu_{d_c}),$$  \hspace{1cm} (Eq. 16)

where $\mu_{d_c}$ is the mean and standard deviation of the circular knot diameter in Category A.

The synthetic minimum and maximum axis lengths of elliptical knots, $d_{e_{min}}$ and $d_{e_{max}}$, are correlated continuous positive random variables,
\[ d_{e \text{min}} \sim (\mu_{d \text{min}}, \sigma_{d \text{min}}) \]  
\[ d_{e \text{max}} \sim (\mu_{d \text{max}}, \sigma_{d \text{max}}), \]  
(Eq. 17)

where the mean and standard deviations of the minimum and maximum elliptical knot axis lengths for Category A are \( \mu_{d \text{min}}, \sigma_{d \text{min}}, \mu_{d \text{max}}, \) and \( \sigma_{d \text{max}}, \) respectively. These random variables are then concatenated into vectors of minimum and maximum diameters,

\[ D_{\text{min}} = [d_c, d_{e \text{min}}] \quad \text{and} \quad D_{\text{max}} = [d_c, d_{e \text{max}}] \]  
(Eq. 18)

where \( D_{\text{min}} \) is the minimum diameter and \( D_{\text{max}} \) is the maximum diameter of all the synthetic knot defects.

### 2.3.3 Depth

To determine the depth of each knot, a probability of being a through knot, \( p_T \), is associated with the diameter of each knot on side A. As stated previously, each knot is assumed to have a depth, \( D \), equal to the board thickness. Thus, the probability of being a through knot is deterministic and equal to one,

\[ P_{T_i} = 1 \quad \text{for} \quad i = 1, \ldots, n \]  
(Eq. 19)

For the two-dimensional model, the number of through knots, \( n_T \), is equal to the number of knots,

\[ n_T = n, \]  
(Eq. 20)

and the number of non-through knots, \( n_{N_T} \), is zero.

### 2.3.4 Direction of Growth

The two-dimensional model for knot defect distribution models knots as circular or elliptical cylinders with a depth equal to the board's thickness. The knot center
coordinates are the same at both sides A and B, and the normalized direction of growth is deterministic,

\[ G_i = [0, 0, 1]. \quad \text{for } i = 1, \ldots, n \quad \text{(Eq. 21)} \]

### 2.3.5 Knot Center Coordinates

Knot center coordinates along the longitudinal and tangential axis, \( c_x \) and \( c_y \), are then determined for each knot. The knot center coordinates are independent continuous random variables. The knot center coordinate along the longitudinal axis for each generated knot on side A is generated first and is bound by the length of the board, \( l_b \), plus the additional length, \( x_l \), in both the positive and negative directions along the longitudinal axis:

\[ 0 - x_l \leq c_x \leq l_b + x_l. \quad \text{(Eq. 22)} \]

Similarly, the knot center coordinate in the tangential axis for each generated knot on side A is bound by the width of the board, \( w_b \), and the additional length, \( y_l \), in the positive and negative tangential direction:

\[ 0 - y_l \leq c_y \leq w_b + y_l. \quad \text{(Eq. 23)} \]

The direction of the growth vector for the two-dimensional model is \([0, 0, 1]\). Therefore, the knot center coordinates on side A equal the knot center coordinates on side B.

### 3.3.6 Orientation of knots

The orientation of knots with respect to the longitudinal axis is discretized for practicality. Since the observed knot defects are approximated to be elliptical or circular, the angle measurement requires a lack of precision. The orientation of generated elliptical knots is defined by the angle of the maximum knot diameter with respect to the longitudinal
axis, $\theta_e$. For the two-dimensional model, elliptical knots' orientation is a discrete random variable with the values $-45^\circ$, $0^\circ$, $45^\circ$, and $90^\circ$. Because circular knots are not angled towards any direction, $\theta_c$ is equal to zero. The circular and elliptical knot angles with respect to the longitudinal axis are then combined to $\theta$, which represents all knots in the two-dimensional model:

$$\theta = [\theta_c, \theta_e].$$

(Eq. 24)

2.3.7 Comments

If overlapping knots are detected during sample generation, then the knot center coordinates are regenerated using the same method described above, and the overlapping check is repeated.

Finally, because the knots are generated over an extended area, $A_g$, some knots may be exterior knots. Thus, each knot is categorized as an interior or exterior knot. Generated knots with any portion of the knot area on the board area are defined as interior knots. Likewise, any generated knot in which no portion of the knot is located on the board's area is defined as an exterior knot. If a knot is determined to be an exterior knot, its parameters are removed from the synthetic knot data set.

Thus, knot defects are defined by four stochastic integer parameters, and twelve stochastic vectors describing the distribution of knots over an area, $A$, have been defined. The integers are the total number of knots generated on side A, $n_A$, the number of knots generated that appear only on side B, $n_B$, the number of generated through knots, $n_T$, and the number of non-through knots, $n_{NT}$. The five vectors each have a length $n$ and define the generated knot geometry. These include the probability of being a through knot, $P_T$, the depth, $D$, the minimum and maximum knot diameters for the two-dimensional
model, $d_{\text{min}}$ and $d_{\text{max}}$, the normalized direction of growth vector, $G$, the knot center coordinates, $c_x, c_y$, and the angle of the maximum knot diameter with respect to the longitudinal axis, $\theta$.

### 2.4 Two-Dimensional Model Calibration

The distribution of knot defects described above was calibrated to 31 Eastern hemlock samples randomly selected from a larger stock of boards acquired for testing and evaluating. Each board was 12 ft (365.8 cm) long, 3.2 in (8.1 cm) wide, and 1.5 in (3.8 cm) thick. The knot idealization methods described in Section 2.2 were used to characterize the sample's distribution of knot defects.

The selected boards were graded by the Northeastern Lumber Manufacturers Association (NELMA) grading rules (NELMA, 2013). The board classifications ranged from Select Structural to Grade 3, as defined. The grade depends on a relationship between the knot center coordinates and the maximum knot width, $w_m$, on the board perpendicular to the longitudinal axis. Boards with the smallest knots are categorized as Select Structural, while boards with the largest knots are described as Grade 3 (Figure 2).
The distribution of knot defects is described by a series of parameters: number of knots per unit area \( \nu \), knot center coordinates \( c_x \) and \( c_y \), the direction of growth vector normalized to the thickness of the board \( G \), minimum and maximum diameters \( d_{min} \) and \( d_{max} \), depth \( D \), and orientation of the maximum diameter with respect to the longitudinal axis \( \theta \). The distributions chosen to calibrate this stochastic model to Eastern hemlock are detailed in the following sections. The two-dimensional model use knots in Category A for calibration.

### 2.4.1 Number of knots

The number of interior knots and edge knots with visible centers is normalized to a unit area, \( \nu \):

\[
\nu = \frac{n_I + n_{ewc}}{w_b l_b}.
\]  
(Eq. 25)

The resulting distribution from applying the above equation to the 31 Eastern hemlock samples is shown in Figure 7. Among the 31 boards, there was a total of 249 knot defects
in Category A. The mean and standard deviation of the number of knots per unit area are $2.7E-3 \pm 1.8E-3$.

Figure 7. Distribution of the number of knots per unit area for the 31 Eastern hemlock samples.

An empirical distribution is used to model the number of knots per unit area:

$$v \sim EMP(v).$$  \hfill (Eq. 26)

Multiplying the distribution by the synthetic board area, $A$, yields the number of knots per board. The number of knots on side A is

$$n = vA.$$  \hfill (Eq. 27)

For the two-dimensional model, the number of knots on side A is equal to the number of knots on side B. The length and width of the samples are 365.76 cm and 8.128 cm, respectively. The chosen extra length in each direction is 5 cm. Thus, the area in which knots are located, $A$, is equal to 6811.8 cm$^2$.

2.4.2 Diameters

Both circular knot diameters and elliptical knot major and minor axis lengths are modeled as positive continuous random variables. However, circular knots have equal minimum and maximum diameters, while elliptical knots have highly correlated minimum
and maximum knot diameters. Of the 249 Category A knots, 100 were considered circular, meaning they have equal minimum and maximum diameters, while 149, with unequal maximum and minimum diameters, were classified as elliptical. The minimum and maximum elliptical knot diameters have a correlation coefficient, \( \rho_d \), of 0.94. The mean and standard deviations for circular knot diameters and minimum and maximum elliptical knot diameters are shown in Table 1. The mean aspect ratio of elliptical knots is 0.84 ± 0.15.

Table 1. Mean and standard deviation of both circular and elliptical knot diameters for Categories A and B for the 31 Eastern hemlock boards used to calibrate this model.

<table>
<thead>
<tr>
<th></th>
<th>mean ± standard deviation (cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Circular Knot Diameter</td>
<td>1.23 ± 1.19</td>
</tr>
<tr>
<td>Minimum Elliptical Knot Diameter</td>
<td>1.64 ± 0.99</td>
</tr>
<tr>
<td>Maximum Elliptical Knot Diameter</td>
<td>1.95 ± 1.16</td>
</tr>
</tbody>
</table>

The distributions for the knot diameters for circular and elliptical knots are shown in Figure 8 and Figure 9. Figure 8 provides the distributions for the circular knot diameters in Category A, \( d_c \).

For the two-dimensional model, knots are modeled as either circular or elliptical. Of the 249 knot defects on side A of the 31 Eastern hemlock samples, 100 were circular. The percentage of circular knots, \( p_c \), is then 40%. The number of elliptical knots, \( n_e \), is found by subtracting the number of circular knots, \( n_c \), from the total number of knots.

An exponential function truncated at the maximum circular knot diameter observed in the data models the diameter for circular knots such that,

\[
d_c \sim \exp(\mu_{d_c}) < \text{max}(d_c),
\]

(Eq. 28)

where \( \mu_{d_c} \) is the mean circular knot diameter, 1.23 cm, and \( \text{max}(d_c) \) is the maximum observed circular knot diameter, 6.04 cm. A histogram of the 100 circular knots from the
input calibration data superimposed with the exponential probability density function is presented below.

Figure 8. Truncated exponential distribution for the diameter of circular knots used in the two-dimensional model simulates the diameter of circular knots on the 31 sample Eastern hemlock boards. The distribution is truncated at the largest observed circular knot diameter.

Similarly, the minimum and maximum diameters for elliptical knots are modeled by correlated exponential distributions truncated at the maximum observed minimum and maximum knot diameters.

\[
\begin{bmatrix}
d_{\text{e} \text{min}} \\
d_{\text{e} \text{max}}
\end{bmatrix}
\sim N\left(\begin{bmatrix}
\mu_{d_{\text{e} \text{min}}} \\
\mu_{d_{\text{e} \text{max}}}
\end{bmatrix}, \begin{bmatrix}
\sigma_{d_{\text{e} \text{min}}}^2 & C_{\text{min, max}} \\
C_{\text{min, max}} & \sigma_{d_{\text{e} \text{max}}}^2
\end{bmatrix}\right) < \begin{bmatrix}
\max(d_{\text{e} \text{min}}) \\
\max(d_{\text{e} \text{max}})
\end{bmatrix}
\]  
(Eq. 29)

where \(\mu_{d_{\text{e} \text{min}}}, \mu_{d_{\text{e} \text{max}}}, \sigma_{d_{\text{e} \text{min}}}^2,\) and \(\sigma_{d_{\text{e} \text{max}}}^2\) are the mean and variance of the minimum and maximum elliptical knot axis lengths, respectively, while \(C_{\text{min, max}}\) is the covariance of the minimum and maximum knot axis lengths, or 1.12 cm².

Histograms and probability density functions are presented in Figure 9.
Figure 9. Truncated exponential distribution for minimum and maximum knot diameters used in the two-dimensional model to simulate the minimum (a) and maximum (b) elliptical knot diameter on the 31 sample Eastern hemlock boards. Both distributions are truncated at the largest observed minimum and maximum knot diameter, respectively.

2.4.3 Depth

The depth of knot defects, $D$, is correlated to the knot's diameter with a correlation coefficient of 0.40. Knots with larger diameters have a higher probability of being a through knot than knots with smaller diameters. However, for the two-dimensional model, the depth of each knot is deterministic. All knots are considered to be through knots.

2.4.4 Direction of Growth

The normalized direction of the growth vector, $G$, is a continuous random variable for the longitudinal and tangential components and a deterministic variable for the radial component. However, the direction of growth is deterministic for the two-dimensional model. Each knot is considered a through knot with the same knot center coordinates on sides A and B. Thus, the normalized direction of growth is

$$G = [0, 0, 1].$$  \hspace{1cm} (Eq. 30)
2.4.5 Knot Center Coordinates

The knot center coordinates are continuous random variables. Knot center coordinates are positive and located within the board's area for interior knots and edge knots with visible centers. However, in the case of edge knots without centers, knot centers occur outside the board's domain. The distributions of the knot center coordinate locations along the longitudinal and tangential axes are presented in the following two histograms. The red vertical lines represent the boundary of the board. Knot centers outside this boundary are edge knots without visible centers.

For the two-dimensional model, the knot center coordinates along the tangential and longitudinal axis are independent continuous variables modeled by two uniform distributions. The knot centers are bound by the length of the board and the additional length along the longitudinal axis and the board's width and the additional length along the tangential axis. Thus,

\[ c_x \sim U(−x_t, l_b + x_t) \]  \hspace{1cm} \text{(Eq. 31)}

and

\[ c_y \sim U(−y_t, w_b + x_y). \]  \hspace{1cm} \text{(Eq. 32)}

Here, the length and width of the board are 365.76 cm and 8.128 cm. The extra length in both the tangential and longitudinal axes is 5 cm. The uniform probability density and the uniform probability density normalized to the sample board's width are superimposed on the observed data histograms in Figure 10.
Figure 10. Uniform distribution for knot center coordinate locations for both through and non-through knots on side A. Given the board’s comparatively small width compared to the added length to allow for edge knots without visible centers, the uniform PDF is normalized for the tangential Axis Coordinate (a). There are fewer knots generated outside the longitudinal axis boundaries (b). The distributions for the knot center locations on side B are similar.

While the observed data does not show knot center coordinate locations past the boundaries of the board's area, knots were likely located in areas extending past the board's dimensions because each board is cut from a larger piece of wood. The knot's visibility is a function of the knot center coordinate and the diameters of the knot.

### 2.4.6 Orientation

The orientation of knots is defined by the angle of the maximum diameter with respect to the longitudinal axis. This angle was difficult to measure precisely; therefore, approximate angles of -45°, 0°, 45°, or 90° were used. These angles are significant because they influence the board’s grade classification according to NELMA rules. Of the 149 elliptical knots in category A, nine were at a -45°, 101 were at a 0°, 17 were at a 45°, and 22 were at a 90° angle with respect to the longitudinal axis.
An empirical distribution models the distribution of elliptical knot angles with respect to the longitudinal axis. The angle of elliptical knots, $\theta_e$, is a vector of $n_{eg}$ values sampled from the empirical distribution. The angle for circular knots, $\theta_c$, is a vector of $n_{cg}$ zeros. The final vector for the angle of knots with respect to the longitudinal axis is then,

$$\theta = [\theta_c, \theta_g].$$  \hspace{1cm} (Eq. 33)

### 2.4.7 Summary of Defined Parameters

A summary of the defined parameters for the probabilistic models is provided below. The equation or distribution used to calibrate each parameter to Eastern hemlock is also included.

<table>
<thead>
<tr>
<th>Variable Name</th>
<th>Definition</th>
<th>Distribution or Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Board Geometry</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$w_b, l_b, d_b$</td>
<td>Width, length, and depth of the synthetic board, along the tangential, longitudinal, and radial axes, respectively</td>
<td></td>
</tr>
<tr>
<td>$x_l, y_l$</td>
<td>Additional length added to the longitudinal and tangential board dimensions to account for edge knots without centers.</td>
<td></td>
</tr>
<tr>
<td>$A$</td>
<td>Area of synthetic board</td>
<td>$A = (l_b + 2x_l)(w_b + 2y_l)$</td>
</tr>
<tr>
<td><strong>Number of Knots</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$v$</td>
<td>Number of knots per unit area</td>
<td>$v \sim EMP(v)$</td>
</tr>
<tr>
<td>$n_I$</td>
<td>Number of interior knots</td>
<td></td>
</tr>
<tr>
<td>$n_{EC}$</td>
<td>Number of edge knots with visible knot centers on the board</td>
<td></td>
</tr>
<tr>
<td>$n_{EW}$</td>
<td>Number of edge knots with centers located outside the board dimensions</td>
<td></td>
</tr>
<tr>
<td>$n$</td>
<td>Number of synthetic knots</td>
<td>$n = vA$</td>
</tr>
<tr>
<td><strong>Knot Geometry</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$w_i, l_i, d_1, d_2$</td>
<td>Dimensions used to define minimum and maximum axis lengths of edge knots without visible centers</td>
<td></td>
</tr>
<tr>
<td>$p_c$</td>
<td>Ratio of circular knots to total knots in category A</td>
<td>$p_c = \frac{n_c}{n}$</td>
</tr>
<tr>
<td>Symbol</td>
<td>Description</td>
<td>Formula</td>
</tr>
<tr>
<td>----------</td>
<td>-----------------------------------------------------------------------------</td>
<td>------------------------------------------------------------------------</td>
</tr>
<tr>
<td>$n_c$</td>
<td>Number of synthetic circular knots</td>
<td>$n_c = \text{round}(np_c)$</td>
</tr>
<tr>
<td>$n_e$</td>
<td>Number of synthetic elliptical knots</td>
<td>$n_e = n - n_c$</td>
</tr>
<tr>
<td>$\mu_{d_c}$</td>
<td>Mean of the observed approximate circular knot diameters</td>
<td></td>
</tr>
<tr>
<td>$d_c$</td>
<td>Diameters of synthetic circular knots</td>
<td>$d_c \sim \exp(\mu_{d_c}) &lt; \text{max}(d_c)$</td>
</tr>
<tr>
<td>$\mu_{d_{\text{min}}}, \sigma_{d_{\text{min}}}, \mu_{d_{\text{max}}}, \sigma_{d_{\text{max}}}$</td>
<td>Mean and standard deviation of the observed minimum and maximum axis lengths of approximate elliptical knots</td>
<td></td>
</tr>
<tr>
<td>$\mathcal{C}_{\text{min, max}}$</td>
<td>Covariance of the minimum and maximum axes lengths for knots approximated to be elliptical in Category A</td>
<td></td>
</tr>
<tr>
<td>$d_{e_{\text{min}}}, d_{e_{\text{max}}}$</td>
<td>Minimum and maximum axis lengths of synthetic elliptical knots</td>
<td>$[d_{e_{\text{min}}}, d_{e_{\text{max}}}] \sim N \left( \begin{bmatrix} \mu_{d_{e_{\text{min}}}} \ \mu_{d_{e_{\text{max}}}} \end{bmatrix}, \begin{bmatrix} \sigma_{d_{e_{\text{min}}}}^2 &amp; \mathcal{C}<em>{\text{min, max}} \ \sigma</em>{d_{e_{\text{max}}}}^2 &amp; \mathcal{C}<em>{\text{min, max}} \end{bmatrix} \right) \right) &lt; \begin{bmatrix} \text{max}(d</em>{e_{\text{min}}}) \ \text{max}(d_{e_{\text{max}}}) \end{bmatrix}$</td>
</tr>
<tr>
<td>$D_{\text{min}}, D_{\text{max}}$</td>
<td>Minimum and maximum axis lengths of combined circular and elliptical knots</td>
<td>$D_{\text{min}} = [d_c, d_{e_{\text{min}}}]$</td>
</tr>
<tr>
<td>$D_{\text{max}} = [d_c, d_{e_{\text{max}}}]$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Knot Depth**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Formula</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_T$</td>
<td>The probability of a knot being a through knot</td>
<td>$P_T = 1$</td>
</tr>
<tr>
<td>$n_T$</td>
<td>Number of through knots</td>
<td></td>
</tr>
<tr>
<td>$n_{NT}$</td>
<td>Number of non-through knots</td>
<td></td>
</tr>
<tr>
<td>$D$</td>
<td>Depth of knots</td>
<td></td>
</tr>
</tbody>
</table>

**Direction of Growth**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Formula</th>
</tr>
</thead>
<tbody>
<tr>
<td>$g_x, g_y, g_z$</td>
<td>Components of the direction of growth vector along longitudinal, tangential, and radial axes</td>
<td></td>
</tr>
<tr>
<td>$G$</td>
<td>Normalized direction of growth vector</td>
<td>$G = [0, 0, 1]$</td>
</tr>
</tbody>
</table>

**Knot Center Coordinates**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Formula</th>
</tr>
</thead>
<tbody>
<tr>
<td>$c_x, c_y$</td>
<td>Knot center coordinates along the longitudinal and tangential axes</td>
<td>$-x_l \leq c_x \leq l_b + x_l$ $-x_l \leq c_y \leq l_b + x_l$</td>
</tr>
</tbody>
</table>

**Orientation of Knots**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Formula</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta_c$</td>
<td>Angle of synthetic circular knots</td>
<td>$\theta_c = [0]$</td>
</tr>
</tbody>
</table>
\[ \theta_e \text{ Angle of synthetic elliptical maximum knot axis length with respect to the longitudinal axis } \quad \theta_e \sim EMP(\theta_e) \]

\[ \theta \text{ Angle of maximum knot axis length with respect to the longitudinal axis } \quad \theta = [\theta_c, \theta_e] \]

2.5 Two-Dimensional Model Validation

The model is validated by comparing the distributions and statistics for the generated knot geometry to the observed knot geometry. Additionally, the observed distribution of grades is compared to the generated sample distribution of grades.

With the described distributions, two-dimensional Eastern hemlock boards can be simulated for any board’s length and width. Table 3 shows synthetic samples created using the two-dimensional model calibrated to the 31 Eastern hemlock samples. Each synthetic sample was graded according to NELMA grading guidelines. These samples can be qualitatively compared to the samples in Figure 2.

Table 3. Synthetic Eastern hemlock boards simulated using the two-dimensional model calibrated to the 31 Eastern hemlock samples. Each synthetic board is graded according to NELMA grading guidelines.
The mean number of knots per board for 1000 generated samples is 9.39. The observed data shows a mean of 9.70 for the same size board. For the 1000 generated sample boards, 9496 knots were generated, of which 3212 (34%) were circular. The mean knot diameter for the generated samples was 1.01 cm, compared to 0.96 cm for the input calibration data. Elliptical knots are 66% of the generated knots. The mean for the minimum and maximum knot diameters for the 6284 generated elliptical knots were 1.48 cm and 1.97 cm, respectively. The input calibration data shows a mean of 1.47 cm for the minimum elliptical knot diameters and 1.84 cm for the maximum elliptical knot diameters.

The following table presents statistics for the synthetic knot distributions based on 1000 generated sample boards compared to the 303 knot defects on the 31 Eastern hemlock samples.

Table 4. Mean and standard deviations for two-dimensional model synthetic Eastern hemlock boards and the 31 Eastern hemlock samples used to calibrate the model. The synthetic data serves as the model's output data; the observed data from the 31 Eastern hemlock boards is the input data to the model.

<table>
<thead>
<tr>
<th></th>
<th>Synthetic Data mean ± standard deviation</th>
<th>Observed Data mean ± standard deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Knots</td>
<td>9.39 ± 6.77</td>
<td>9.70 ± 10.30</td>
</tr>
<tr>
<td>Circular Knot Diameter (cm)</td>
<td>1.01 ± 0.99</td>
<td>0.96 ± 1.10</td>
</tr>
<tr>
<td>Minimum Elliptical Knot Diameter (cm)</td>
<td>1.48 ± 0.94</td>
<td>1.47 ± 0.99</td>
</tr>
<tr>
<td>Maximum Elliptical Knot Diameter (cm)</td>
<td>1.97 ± 1.59</td>
<td>1.84 ± 1.25</td>
</tr>
</tbody>
</table>

Each of the generated samples was also graded. For simplification, it was assumed that all the synthetic knots were tight sound knots. The resulting grade distribution for 1000 samples is shown in Figure 11. For the synthetic samples, 223 were Select Structural, 181 were Grade 1, 216 were Grade 2, 263 were Grade 3, and 119 were lower quality than Grade 3.
Figure 11. NELMA Grade distribution for 1000 synthetic sample boards.

The distribution of grades for the 1000 generated boards is slightly different from the observed samples' distribution of grades (Figure 6). This may indicate that the sample size of 31 Eastern hemlock boards is insufficient to describe the knot defect distribution. If only 31 boards are generated, the distribution of grades is highly variable. Samples showing the range of grade distribution is shown in Figure 12. The model can be better calibrated and thus more accurate by increasing the sample size for the input calibration data. Additionally, the board's defined grade is very sensitive to small changes in the parameters of a single knot. Thus, it is challenging to match a grade distribution directly from a probabilistic knot model.
Figure 12. Grade distribution for synthetic data sets with 31 samples. The distributions for each figure change, suggesting that 31 samples is not a large enough sample size to capture the knot defect distribution.

Approximately 12% of the generated boards are classified as grades lower than Grade 3, although not observed in the input data. This is because each board's grade is determined by limitations of the maximum width of the knot on the board, perpendicular to the longitudinal axis. This means the grade classification is dependent on the knot center coordinate along the tangential axis, the maximum and minimum knot diameters, and the angle of the maximum knot diameter with respect to the longitudinal axis. For Grade 3 boards, the maximum allowable knot widths for edge knots and interior knots are 4.45 cm and 6.35 cm, respectively. Because the maximum knot diameter is 7.5 cm, knots can be
generated that do not fit within the allowable widths and are thus lower quality than grade 3.

A limitation of the model described above is that it neglects the correlation between the number of knots, \( n_K \), and the average knot size per board. The average size of the knot defects are described by the mean of the maximum knot diameters per board, \( d_{\text{max mean}} \), and the mean of the minimum knot diameters, \( d_{\text{min mean}} \), per board. The two variables \( d_{\text{max mean}} \) and \( d_{\text{min mean}} \) are then

\[
d_{\text{max mean}} = \text{mean}(d_{\text{max}}) \quad \text{(Eq. 34)}
\]

\[
d_{\text{min mean}} = \text{mean}(d_{\text{min}}) \quad \text{(Eq. 35)}
\]

with a length equal to the number of boards with at least one knot. The correlation coefficient between the number of knots per board and mean maximum knot diameter per board, \( \rho_{n_K,d_{\text{max mean}}} \), is -0.50. Similarly, the correlation coefficient between the number of knots per board and the mean minimum knot diameter per board, \( \rho_{n_K,d_{\text{min mean}}} \), is -0.57.

![Figure 13](image.png)

Figure 13. There is a moderate correlation, -0.50 and -0.57, between the number of knots per board and the mean minimum and maximum knot diameter on a given board. This relationship is neglected in the two-dimensional model.
2.6 Three-Dimensional Probabilistic Model

A three-dimensional synthetic knot model is also developed. For simplicity, the model assumes that all synthetic knots are circular. A distribution is used to determine each knot's depth, resulting in only a portion of the synthetic knots to be through knots. The knot center locations from surface A to B can differ, meaning the knot's depth may no longer be parallel to the radial axis. These distributions are calibrated to knots in Category A and B.

For this paper, the following notation,

\[ p \sim (\mu, \sigma), \]

will be used to indicate that the stochastic parameter, \( p \), is related to the observed properties within the parenthesis, \( \mu \) and \( \sigma \). The appropriate distributions will be chosen based on the calibration data, discussed in Section 2.7 of this paper.

2.6.1 Number of knots

The number of knots on each board is expressed as the number of knots on side A, \( n_A \), which includes both through knots and non-through knots, and the number of non-through knots on side B, \( n_B \). The number of knots, \( n_A \), is a nonnegative discrete random variable and is determined by the distribution of knots per unit area in Category A, \( v_A \), multiplied by the area, \( A \), in which knot centers may be located:

\[ n_A = v_A A. \]  \hspace{1cm} (Eq. 36)

Knot defects are not only distributed along the width, \( w_b \), and length, \( l_b \), of the board, but also extend slightly past these boundaries to account for edge knots without a visible center. Thus, an additional length in both the longitudinal, \( x_l \), and tangential, \( y_t \), directions are defined. The area in which knot centers are located is then:
\[ A = (l_b + 2x_l)(w_b + 2y_l). \]  \hspace{1cm} \text{(Eq. 37)}

Figure 14. Board dimensions with added length, \( y_l \) and \( x_l \), in the longitudinal and tangential directions to accommodate knot centers located outside the board's area, allowing for edge knots without visible centers.

The number of knots on side B is generated independently of the number of knots on side A of the board, by

\[ n_B = v_B A, \]  \hspace{1cm} \text{(Eq. 38)}

where \( v_B \) is the number of knots per unit area for Category B.

2.6.2 Diameters

The knot diameters are continuous positive random variables. For simplicity, all knots are considered as cylindrical, such that the surface area of each knot is circular. The synthetic diameters for knots on side A, \( D_A \), and side B, \( D_B \), are expressed by:

\[ D_A \sim (\mu_{D_A}, \sigma_{D_A}) \]  \hspace{1cm} \text{(Eq. 39)}
\[ D_B \sim (\mu_{D_B}, \sigma_{D_B}). \]

\( \mu_{D_A} \) and \( \sigma_{D_A} \) are the mean and standard deviation of the average knot diameter, \( D_A \), for each knot in Category A, and \( \mu_{D_B} \) and \( \sigma_{D_B} \) are the mean and standard deviation of the average knot diameter, approximating each knot as circular, in Category B.

The following section discusses the method for determining whether a given knot on side A is a through knot. If a knot on side A is a through knot, a cylinder models the
defect such that the knot diameter on side A is equal to the corresponding knot diameter on side B.

### 2.6.3 Depth

The depth of each knot is related to the knot diameter such that larger knots have a higher probability of being a through knot than smaller knots. The probability of a given knot being a through knot, $p_T$, is related to the diameter of each knot on side A:

$$P_{T_i} \sim (D_{A_i}) \quad for \ i = 1, \ldots, n_A \quad (Eq. \ 40)$$

The number of through knots, $n_T$, and non-through knots, $n_{NT}$, is stochastic. The sum of the number of through knots and non-through knots is equal to the number of knots on side A,

$$n_A = n_T + n_{NT}. \quad (Eq. \ 41)$$

After determining the probability of a knot being a through knot, two vectors describing each knot’s depth are defined. Through knots have a depth equal to the thickness of the board, $t_b$. For simplicity, non-through knots are assumed to have a depth equal to one half the thickness of the board, $\frac{t_b}{2}$. Thus the depth of the knots on side A is defined as

$$d_{A_i} = \left[ \begin{array}{c} t_b, \\ \frac{t_b}{2}, \end{array} \right] \quad through \ knots \quad non - through \ knots \quad for \ i = 1, \ldots, n_A \quad (Eq. \ 42)$$

where $t_b$ is the thickness of the board. Similarly, the depth of the knots on side B is

$$d_{B_i} = \left[ \begin{array}{c} t_b, \\ \frac{t_b}{2}, \end{array} \right] \quad through \ knots \quad non - through \ knots \quad for \ i = 1, \ldots, n_B + n_{NT} \quad (Eq. \ 43)$$
2.6.4 Direction of Growth

Knots are caused by branches growing from the trunk of the tree, causing knots to be directed at an angle through the board's depth when processed into dimensional lumber. The model considers knot growth direction by modeling the longitudinal and tangential components of the normalized direction of growth vector as continuous random variables and the radial component as deterministic. There is a tendency for the growth of all the knot defects on a board to be oriented in similar directions, such that the longitudinal growth vectors are pointing in the same direction. Thus each board has a mean and standard deviation direction of growth along the longitudinal and tangential axes. The mean direction of growth for the synthetic board is modeled by

\[
\mu_{G_x} \sim (G_{x_{\text{min}}}, G_{x_{\text{max}}})
\]

(Eq. 44)

\[
\mu_{G_y} \sim (G_{y_{\text{min}}}, G_{y_{\text{max}}})
\]

for the longitudinal and tangential components of the normalized direction of growth vector, respectively. The standard deviation of the direction of growth for the longitudinal and tangential components of the synthetic board are then modeled by

\[
\sigma_{G_x} \sim (\sigma_{G_{x_{\text{min}}}}, \sigma_{G_{x_{\text{max}}}})
\]

(Eq. 45)

\[
\sigma_{G_y} \sim (\sigma_{G_{y_{\text{min}}}}, \sigma_{G_{y_{\text{max}}}}).
\]

For each through knot, the direction of growth along the longitudinal and tangential axes is then modeled by the mean and standard deviation direction of growth per board parameters while the direction of growth along the radial axis is one,

\[
G_i \sim \left( \left( \mu_{G_x}, \sigma_{G_x} \right), \left( \mu_{G_y}, \sigma_{G_y} \right), 1 \right)
\]

for \( i = 1, \ldots, n_T \)

(Eq. 46)
For simplicity, the direction of growth for non-through knots is equal to zero along the longitudinal and tangential axes, and one-half along the radial axis,

$$G_i = \begin{bmatrix} 0, 0, \frac{1}{2} \end{bmatrix} \quad for \ i = 1, \ldots, n_T \quad (Eq. 47)$$

### 2.6.5 Knot Center Coordinates

Knot center coordinates along the longitudinal and tangential axis, $c_x$ and $c_y$, are then determined for each knot. The knot center coordinates are independent continuous random variables. The knot center coordinate along the longitudinal axis for each generated knot on side A is generated first and is bound by the length of the board, $l_b$, plus the additional length, $x_l$, in both the positive and negative directions along the longitudinal axis:

$$0 - x_l \leq c_x \leq l_b + x_l. \quad (Eq. 48)$$

Similarly, the knot center coordinate in the tangential axis for each generated knot on side A is bound by the width of the board, $w_b$, and the additional length, $y_l$, in the positive and negative tangential direction:

$$0 - y_l \leq c_y \leq w_b + x_y. \quad (Eq. 49)$$

The corresponding knot center coordinates for through knots with growth from side A to side B is calculated by adding the knot center coordinates on side A to the longitudinal and tangential components of the direction of growth vector multiplied by the thickness of the board. The longitudinal and tangential knot center coordinates for through knots on side B are then,

$$c_{x_{B_i}} = t_b G_{x_i} + c_{x_{A_i}} \quad for \ i = 1, \ldots, n_T \quad (Eq. 50)$$

$$c_{y_{B_i}} = t_b G_{y_i} + c_{y_{A_i}} \quad for \ i = 1, \ldots, n_T \quad (Eq. 51)$$
Finally, the knot center coordinates for the knots only on side B are determined independently, the same way as the knots on side A. The knot center coordinates along the longitudinal and tangential axis are generated by

\[ 0 - x_l \leq c_{x_{B_i}} \leq l_b + x_l \quad \text{for } i = 1, \ldots, n_{NT} \]  
(Eq. 52)

\[ 0 - y_l \leq c_{y_{B_i}} \leq w_b + x_y \quad \text{for } i = 1, \ldots, n_{NT} \]  
(Eq. 53)

### 2.6.6 Comments

If overlapping knots are detected during sample generation, then the knot center coordinates are regenerated using the same method described above and the overlapping check is repeated.

Finally, because the knots are generated over an extended area, \(A_g\), some knots may be located fully outside the board domain. Any synthetic knot in which no portion of the knot is located on the board area is removed from the synthetic knot data set.

In summary, knot defects are defined by four stochastic integer parameters and eleven stochastic vectors describing the distribution of knots over an area, \(A\), have been defined. The integers are the total number of knots generated on side A, \(n_A\), the number of knots generated that appear only on side B, \(n_B\), the number of generated through knots, \(n_T\), and the number of non-through knots, \(n_{NT}\). The five vectors each contain either \(n_A\), for knots on side A, or \(n_B + n_T\) values, for the total knots appearing on side B, and define the generated knot geometry. These include the probability of being a through knot, \(P_T\), the depth, \(d_A\) and \(d_B\), circular knot diameters on sides A and B for the three-dimensional model, \(D_A\) and \(D_B\), the normalized direction of growth vector, \(G\), the knot center coordinates, \(c_{x_A}, c_{y_A}, c_{x_B}\) and \(c_{y_B}\).
2.7 Three-Dimensional Model Calibration

The model for the distribution of knot defects described above was calibrated to 31 Eastern hemlock samples randomly selected from a larger stock of boards acquired for tests and evaluation. Each board was 12 ft (365.8 cm) long, 3.2 in (8.1 cm) wide, and 1.5 in (3.8 cm) thick. The knot idealization methods described in Section 1 were used to characterize the sample's distribution of knot defects.

The selected boards were graded by the Northeastern Lumber Manufacturers Association (NELMA) grading rules (NELMA, 2013). The board classifications ranged from Select Structural to Grade 3 (Figure 15). The grade depends on a relationship between the knot center coordinates and the maximum knot width, \( w_m \), on the board perpendicular to the longitudinal axis. Boards with the smallest knots are categorized as Select Structural, while boards with the largest knots are described as Grade 3 (Figure 2).

![Figure 15. Distribution of grades for the 31 Eastern hemlock boards used to calibrate this model, as defined in the NELMA grading guidelines.](image)

The distribution of knot defects is described by a series of parameters: number of knots per unit area \( (\nu) \), knot center coordinates \( (c_x \text{ and } c_y) \), the direction of growth vector normalized to the thickness of the board \( (G) \), knot diameter \( (D) \), and depth \( (d) \). The
distributions chosen to calibrate this stochastic model to Eastern hemlock are detailed in
the following sections.

2.7.1 Number of knots

The number of interior knots and edge knots with visible centers is normalized to a unit
area, $\nu$:

$$\nu = \frac{n_I + n_{ewc}}{w_b l_b}.$$  \hfill (Eq. 54)

The resulting distribution from applying the above equation to the 31 Eastern hemlock
samples is shown in Figure 16. Among the 31 boards, there were 249 knot defects in
Category A and 130 knot defects in Category B. The mean and standard deviation of the
number of knots per unit area are $2.7E-3 \pm 1.8E-3$ and $1.4E-3 \pm 2.6E-3$ for Categories A
and B, respectively. The mean number of knots per unit area is larger for Category A, as
through knots are included in Side A.

![Figure 16. Distribution of the number of knots per unit area for the 31 Eastern hemlock samples. There are typically more knots in Category A because side A considers both through and non-through knots visible on side A (a) while Category B considers only non-through knots visible on side B (b).]
An empirical distribution is used to model the number of knots per unit area:

\[ \nu_A \sim EMP(\nu_A) \]  \hspace{1cm} (Eq. 55)

\[ \nu_B \sim EMP(\nu_B). \]  \hspace{1cm} (Eq. 56)

Multiplying the distribution by the synthetic board area, \( A \), yields the number of knots per board. The number of knots on side A and side B is

\[ n_A = \nu_A A \]  \hspace{1cm} (Eq. 57)

\[ n_B = \nu_B A. \]  \hspace{1cm} (Eq. 58)

The length and width of the samples are 365.76 cm and 8.128 cm, respectively. The chosen extra length in each direction, \( x_l \) and \( y_l \), is 5.0 cm. Thus, the area in which knots are located, \( A \), is equal to 6811.8 cm\(^2\).

### 2.7.2 Diameters

Knot diameters are modeled as positive continuous random variables. Assuming circular knots is an appropriate simplification, at least in the selected Eastern hemlock samples, since the mean aspect ratio of elliptical knots is 0.84 ± 0.15 for knots on side A and 0.75 ± 0.17 for knots on side B. The observed circular knot diameter for each knot is calculated by an average of the measured minimum and maximum knot diameters. The mean and standard deviations of the average knot diameters of the calibration data are presented in Table 5. The mean knot diameter in Category A is much larger than the mean diameter in Category B because Category A considers both through and non-through knots. Through knots tend to be larger than non-through knots.
Table 5. Mean and standard deviation of observed circular knot diameters for Categories A and B for the 31 Eastern hemlock boards used to calibrate this model.

<table>
<thead>
<tr>
<th></th>
<th>mean ± standard deviation</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(cm)</td>
<td>Category A</td>
<td>Category B</td>
</tr>
<tr>
<td>Average Knot Diameter</td>
<td>1.57 ± 1.4</td>
<td>0.78 ± 0.82</td>
<td></td>
</tr>
</tbody>
</table>

The circular knot diameters are modeled by an exponential function truncated at the maximum knot diameter observed in the data. The knot diameter for knots in Category A are modeled by

\[ D_A \sim \exp(\mu_A) < \max(D_A) \]  \hspace{1cm} (Eq. 59)

and the knot diameter for knots in Category B are modeled by

\[ D_B \sim \exp(\mu_B) < \max(D_B). \]  \hspace{1cm} (Eq. 60)

The exponential probability density function superimposed on the input calibration data on sides A and B is shown in the following figure.

(a) Diameter of knot defects on side A  \hspace{1cm} (b) Diameter of knot defects on side B

Figure 17. Truncated exponential distribution for minimum and maximum knot diameters is used to simulate the knot diameters on sides A (a) and B (b). Both distributions are truncated at the largest observed knot diameter on sides A and B, respectively.
2.7.3 Depth

The depth of knot defects, $d$, is correlated to the knot's diameter with a correlation coefficient of 0.40. Knots with larger diameters have a higher probability of being a through knot than knots with smaller diameters. For simplicity, the depth of the knot defects is considered to be a binary variable. All through knots have the same depth, and all non-through knots are considered to have the same depth. The relationship between the depth and mean diameter of each knot in the observed data set is shown in Figure 18. Of the 379 observed knots, 191 are through knots.

Knots in Category B are always non-through knots since they, by definition, only appear on side B. However, side A knots can be through or non-through knots. The probability of being a through knot for knots on side A is related to each knot's diameter. A linear regression is fit to the observed data for the diameter of knots on side A and their depth, as shown in Figure 18, with a maximum value of one. The parameters associated with this linear regression are the slope, $m$, and the y-intercept, $b$,

$$p_T = m D_A + b \leq 1.$$  \hspace{1cm} \text{(Eq. 61)}

Fit to the observed data for through and non-through knots on side A, the slope and y-intercept are 0.11 $cm^{-1}$ and 0.61, respectively.
Figure 18. Probability of a given knot being a through knot dependent on the knot diameter on side A. This captures the phenomenon that larger knots have a higher probability of going through the board's full depth.

Once a probability of being a through knot is defined, the depth is stochastically determined. As defined in Section 2.6.3, through knots have a depth equal to the depth of the board, and non-through knots have a depth equal to half the board's depth.

2.7.4 Direction of Growth

The normalized direction of growth vector, \( G \), is a continuous random variable for the longitudinal and tangential components and a deterministic variable for the radial component. Histograms for the normalized longitudinal and tangential components of the direction vector for the 191 through knots in the data set are provided below in Figure 19. The mean normalized direction of growth vector is 0.44 for the longitudinal component and 0.06 for the tangential component. The mean for the longitudinal component is much larger than the mean for the tangential component since boards were defined such that the mean normalized longitudinal component of the direction vector per board is positive. The normalized direction of growth vector for the radial component is one.
As discussed previously, knot defects on a given board tend to be inclined in similar directions. Thus, the mean direction of growth and standard deviation of growth for knots on a given board are determined. The mean normalized direction of growth per board for the longitudinal and tangential direction vectors is modeled by a uniform distribution bound by the minimum and maximum mean direction vector along each axis. Thus, along the longitudinal axis, the mean normalized direction of growth vector is

$$\mu_{G_x} \sim U(G_{x_{\text{min}}}, G_{x_{\text{max}}})$$

(Eq. 62)

and along the tangential axis, the mean normalized direction of growth vector is

$$\mu_{G_y} \sim U(G_{y_{\text{min}}}, G_{y_{\text{max}}})$$

(Eq. 63)

Of the thirty-one Eastern hemlock samples, only twenty-four boards have more than two through knots. These twenty-four boards were used to calibrate the distribution of the mean and standard deviation of the direction of growth vector per board. For the twenty-four observed boards, the minimum and maximum mean normalized direction of the growth vector is 0.24 and 2.22 along the longitudinal axis and -1.17 and 1.21 along the tangential axis. Histograms of the observed data overlain with the uniform probability density function are provided in the following figures.
Figure 19. Uniform probability density function fit to the mean per board of the normalized direction vector. This parameter captures the tendency for knots to grow at a similar angle for a given board. Considering the branching patterns of trees, the longitudinal component, \( G_x \), is always positive (a), however the tangential component, \( G_y \), can be either negative or positive (b).

Similarly, the standard deviation of the mean direction of growth per board is modeled by a uniform distribution bound by the minimum and maximum observed standard deviation per board. Along the longitudinal axis, the standard deviation of the direction of growth vector per board is

\[
\sigma_{G_x} \sim U \left( \sigma_{G_{x_{\text{min}}}}, \sigma_{G_{x_{\text{max}}}} \right) \quad \text{(Eq. 64)}
\]

and along the tangential axis, the standard deviation of the direction of growth vector per board is

\[
\sigma_{G_y} \sim U \left( \sigma_{G_{y_{\text{min}}}}, \sigma_{G_{y_{\text{max}}}} \right). \quad \text{(Eq. 65)}
\]

The minimum and maximum standard deviation for the twenty-four boards with more than two through knots were 0.25 and 2.74 along the longitudinal axis and 0.40 and 1.78 along the tangential axis.
A normal distribution is used to generate the normalized direction of growth along the longitudinal and tangential axis for each through knot from the previously defined mean and standard deviation of the longitudinal and tangential components of the direction of growth vector. The radial component of the normalized direction of growth vector is one. Thus, the normalized direction of growth vector for through knots is

\[ G_i \sim \left[ N(\mu_{Gx}, \sigma_{Gx}), N(\mu_{Gy}, \sigma_{Gy}), 1 \right] \quad \text{for } i = 1, \ldots, n_T \quad \text{(Eq. 66)} \]

For non-through knots, knots appearing only on side A or B, the normalized direction of growth along the tangential and longitudinal axes are assumed to be equal to the mean direction of growth along each axis, while the radial component is equal to the one half,

\[ G_i \sim \left[ \mu_{Gx}, \mu_{Gy}, \frac{1}{2} \right] \quad \text{for } i = 1, \ldots, n_T \quad \text{(Eq. 67)} \]

### 2.7.5 Knot Center Coordinates

The knot center coordinates are continuous random variables. Knot center coordinates are positive and located within the board's area for interior knots and edge knots with visible centers. However, in the case of edge knots without centers, knot centers occur outside the board's domain. The distributions of the knot center coordinate locations along the longitudinal and tangential axes are presented in the following two histograms. The red vertical lines represent the boundary of the board. Knot centers outside this boundary are edge knots without visible centers. The mean knot center coordinate for Category A is 184.8 cm along the longitudinal axis and 4.23 cm along the tangential axis. Similarly, for Category B, the mean is 196.1 cm along the longitudinal axis and 4.21 cm along the tangential axis.
The knot center coordinates along the tangential and longitudinal axis on side A are independent continuous variables modeled by two uniform distributions. The knot centers are bound by the length of the board and the additional length along the longitudinal axis and the board's width and the additional length along the tangential axis. Thus,

\[ c_{x_A} \sim U(-x_l, l_b + x_l) \]  \hspace{1cm} (Eq. 68)

\[ c_{y_A} \sim U(-y_l, w_b + x_y). \]  \hspace{1cm} (Eq. 69)

Here, the length and width of the board are 365.76 cm and 8.128 cm. The extra length in both the tangential and longitudinal axes is 5.0 cm. The uniform probability density and the uniform probability density normalized to the sample board's width is superimposed on histograms of the observed data.

Figure 20. Uniform distribution for knot center coordinate locations for both through and non-through knots on side A. Given the board's comparatively small width compared to the added length to allow for edge knots without visible centers, the uniform PDF is normalized for the tangential Axis Coordinate (a). There are fewer knots generated outside the longitudinal axis boundaries (b). The distributions for the knot center locations on side B are similar.
While the observed data does not show knot center coordinate locations past the boundaries of the board's area, knots were likely located in areas extending past the boards' dimensions because each board is cut from a larger piece of wood. The knot's visibility is a function of the knot center coordinate and the diameters of the knot.

For through knots, the knot center coordinates on side B are related to the knot center coordinates on side A and the direction of growth vector,

\[
C_{x_{B_i}} = C_{x_{A_i}} + G_{x_i} t_b \\
C_{y_{B_i}} = C_{y_{A_i}} + G_{y_i} t_b
\]

for \( i = 1, \ldots, n_T \) (Eq. 70)

The knot center coordinates for knots only on side B are independent continuous random variables modeled by two uniform distributions,

\[
C_{x_B} \sim U(-x_l, l_b + x_l) \quad \text{(Eq. 71)}
\]

\[
C_{y_B} \sim U(-y_l, w_b + x_y). \quad \text{(Eq. 72)}
\]

### 2.7.6 Summary of Defined Parameters

A summary of the defined parameters for the probabilistic models is provided below. The equation or distribution used to calibrate each parameter to Eastern hemlock is also included.

<table>
<thead>
<tr>
<th>Variable Name</th>
<th>Definition</th>
<th>Distribution or Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Board Geometry</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( w_b, l_b, t_b )</td>
<td>Width, length, and thickness of the synthetic board, along the tangential, longitudinal, and radial axes, respectively</td>
<td></td>
</tr>
<tr>
<td>( x_l, y_l )</td>
<td>Additional length added to the longitudinal and tangential board dimensions to account for edge knots without centers.</td>
<td></td>
</tr>
<tr>
<td>( A )</td>
<td>Area of synthetic board</td>
<td>( A = (l_b + 2x_l)(w_b + 2y_l) )</td>
</tr>
<tr>
<td><strong>Number of Knots</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \nu_A, \nu_B )</td>
<td>Number of knots per unit area in Categories A and B</td>
<td>( \nu_A \sim EMP(\nu_A) ) ( \nu_B \sim EMP(\nu_B) )</td>
</tr>
<tr>
<td>Symbol</td>
<td>Description</td>
<td></td>
</tr>
<tr>
<td>--------</td>
<td>-------------</td>
<td></td>
</tr>
<tr>
<td>( n_I )</td>
<td>Number of interior knots</td>
<td></td>
</tr>
<tr>
<td>( n_{EC} )</td>
<td>Number of edge knots with visible knot centers on the board</td>
<td></td>
</tr>
<tr>
<td>( n_{EWC} )</td>
<td>Number of edge knots with centers located outside the board dimensions</td>
<td></td>
</tr>
<tr>
<td>( n_{A}, n_{B} )</td>
<td>Number of synthetic knots in Categories A and B</td>
<td></td>
</tr>
</tbody>
</table>

Knot Geometry

- \( w_l, l_l, d_1, d_2 \): Dimensions used to define minimum and maximum axis lengths of edge knots without visible centers.
- \( \mu_{d_{cA}}, \mu_{d_{cB}} \): Mean of the observed approximate circular knot diameters in Category A and Category B.
- \( D \): Diameters of synthetic circular knots.
  - \( D_A \sim \exp(\mu_{DA}) < \max(D_A) \)
  - \( D_B \sim \exp(\mu_{DB}) < \max(D_B) \)
- \( \mu_{DA}, \sigma_{DA}, \mu_{DB}, \sigma_{DB} \): Mean and standard deviation of the observed diameters of knots in Categories A and B, assuming circular knots.

Knot Depth

- \( P_T \): The probability of a knot being a through knot.
  - \( p_T = 0.11D_A + 0.61 \leq 1 \)
- \( n_T \): Number of through knots.
- \( n_{NT} \): Number of non-through knots.
- \( d_{A}, d_{B} \): Depth of knots on side A and side B.

Direction of Growth

- \( g_x, g_y, g_z \): Components of direction of growth vector along longitudinal, tangential, and radial axes.
- \( G_{x_{min}}, G_{x_{max}}, G_{y_{min}}, G_{y_{max}} \): Minimum and maximum longitudinal component of the observed direction of growth vector.
- \( \sigma_{G_{x_{min}}}, \sigma_{G_{x_{max}}}, \sigma_{G_{y_{min}}}, \sigma_{G_{y_{max}}} \): Minimum and maximum longitudinal component of the observed direction of growth vector.
- \( \mu_{G_x}, \mu_{G_y} \): Mean synthetic direction of growth for a board along longitudinal and tangential axes.
  - \( \mu_{G_x} \sim U(G_{x_{min}}, G_{x_{max}}) \)
  - \( \mu_{G_y} \sim U(G_{y_{min}}, G_{y_{max}}) \)
- \( \sigma_{G_x}, \sigma_{G_y} \): Standard deviation of the synthetic direction of growth for a board along longitudinal and tangential axes.
  - \( \sigma_{G_x} \sim U(\sigma_{G_{x_{min}}}, \sigma_{G_{x_{max}}}) \)
  - \( \sigma_{G_y} \sim U(\sigma_{G_{y_{min}}}, \sigma_{G_{y_{max}}}) \)
- \( G \): Normalized direction of growth vector.
  - \( G_l \sim N(\mu_{G_x}, \sigma_{G_x})N(\mu_{G_y}, \sigma_{G_y}, 1) \)
  - \( G_l \sim \left[ \mu_{G_x}, \mu_{G_y}, \frac{1}{2} \right] \)

Knot Center Coordinates

- \( c_{xA}, c_{yA} \): Knot center coordinates along the longitudinal and tangential axes for knots on side A.
  - \( c_{xA} \sim U(-x_l, l_b + x_l) \)
  - \( c_{yA} \sim U(-y_l, w_b + x_y) \)
\[c_{x_B}, c_{y_B}\] Knot center coordinates along the longitudinal and tangential axes for knots on side B

\[
\begin{align*}
C_{x_Bt} &= C_{xA} + G_{x} t_b \\
C_{y_Bt} &= C_{yA} + G_{y} t_b \\
C_{x_B} &\sim U(-x_l, l_b + x_l) \\
C_{y_B} &\sim U(-y_l, w_b + x_y)
\end{align*}
\]

2.8 Three-Dimensional Model Validation

The model is validated by comparing the distributions and statistics for the synthetic knot geometry to the observed knot geometry. Additionally, the observed distribution of grades is compared to the synthetic samples’ distribution of grades.

With the described distributions, three-dimensional Eastern hemlock boards can be simulated for any board's length and width. Table 8 presents an example of the synthetic samples calibrated to the 31 Eastern hemlock samples. Each synthetic sample was graded according to NELMA grading guidelines. These samples can be qualitatively compared to the photos of selected samples in the table below.

Table 7. Photographs of selected Eastern hemlock samples and associated grades according to the NELMA guidelines.

<table>
<thead>
<tr>
<th>SS</th>
<th>Photographs</th>
<th>Grades</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td><img src="image1" alt="Sample 1" /></td>
<td><img src="image2" alt="Grade 1" /></td>
</tr>
<tr>
<td>2</td>
<td><img src="image3" alt="Sample 2" /></td>
<td><img src="image4" alt="Grade 2" /></td>
</tr>
<tr>
<td>3</td>
<td><img src="image5" alt="Sample 3" /></td>
<td><img src="image6" alt="Grade 3" /></td>
</tr>
</tbody>
</table>
Table 8. Synthetic Eastern hemlock Sample generated using the three-dimensional model calibrated to the 31 Eastern hemlock boards. Close up of selected knots are presented to demonstrate the three-dimensional geometry of the modeled knots.

<table>
<thead>
<tr>
<th>Synthetic Sample</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a)</td>
</tr>
<tr>
<td>(b)</td>
</tr>
<tr>
<td>(c)</td>
</tr>
</tbody>
</table>

![Diagram](image)

The stochastic model for the distribution and geometry of knot defects is validated by comparing the statistics of the synthetic data to the observed data. The parameters used
for validation are the number of knots and knot diameters only on side A, only on side B, and through knots. For 1000 generated samples, the synthetic data describes the input calibration data well. The mean number of knots on side A only and side B only is 3.97 and 5.24 for the synthetic boards and 3.61 and 2.45 for the 31 observed samples, respectively. Similarly, the mean number of through knots is 7.23 for the synthetic boards and 6.16 for the observed samples. The following table presents statistics for the synthetic knot distribution and geometries based on 1000 generated boards compared to the 303 knot defects on the 31 Eastern hemlock samples.

Table 9. Mean and standard deviations for three-dimensional model synthetic Eastern hemlock boards and the 31 Eastern hemlock samples used to calibrate the model. The synthetic data serves as the model's output data; the observed data from the 31 Eastern hemlock boards is the input data to the model.

<table>
<thead>
<tr>
<th></th>
<th>Synthetic Data mean ± standard deviation</th>
<th>Observed Data mean ± standard deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Knots (side A only)</td>
<td>3.97 ± 3.13</td>
<td>3.61 ± 7.50</td>
</tr>
<tr>
<td>Number of Knots (side B only)</td>
<td>5.24 ± 8.76</td>
<td>2.45 ± 3.67</td>
</tr>
<tr>
<td>Number of Through Knots</td>
<td>7.23 ± 5.11</td>
<td>6.16 ± 4.33</td>
</tr>
<tr>
<td>Knot Diameter (side A only) (cm)</td>
<td>0.92 ± 0.79</td>
<td>0.87 ± 0.85</td>
</tr>
<tr>
<td>Knot Diameter (side B only) (cm)</td>
<td>0.72 ± 0.67</td>
<td>0.76 ± 0.77</td>
</tr>
<tr>
<td>Knot Diameter (Through Knots) (cm)</td>
<td>1.73 ± 1.42</td>
<td>1.74 ± 1.16</td>
</tr>
</tbody>
</table>

Each of the synthetic samples was also graded. For simplification, it was assumed that all the synthetic knots were tight sound knots. The resulting grade distribution for 1000 samples is shown in Figure 21. A comparison can be made between the output grades from the synthetic data and the input grades of the 31 Eastern hemlock boards (Figure 6).
The distribution of grades for the 1000 generated boards is slightly different from the observed samples' distribution of grades (Figure 6). This may indicate that the sample size of 31 Eastern hemlock boards is insufficient to describe the knot defect distribution. If only 31 boards are generated, the distribution of grades is highly variable. Samples showing the range of grade distribution is shown in Figure 22. The model can be better calibrated and thus more accurate by increasing the sample size for the input calibration data. Additionally, the board's defined grade is very sensitive to small changes in the parameters of a single knot. Thus, it is challenging to match a grade distribution directly from a probabilistic knot model.
Figure 22. Grade distribution for synthetic data sets following the three-dimensional model with 31 samples. The distributions for each figure change, suggesting that 31 samples is not a large enough sample size to capture the knot defect distribution.

Approximately 12% of the synthetic boards shown in Figure 21 are classified as grades lower than Grade 3, although not observed in the input data. This is because each board's grade is determined by limitations of the maximum width of the knot on the board, perpendicular to the longitudinal axis. This means the grade classification is dependent on the knot center coordinate along the tangential axis, the maximum and minimum knot diameters, and the angle of the maximum knot diameter with respect to the longitudinal axis. For Grade 3 boards, the maximum allowable knot widths for edge knots and interior knots are 4.45 cm and 6.35 cm, respectively. Because the maximum knot diameter is 7.5
cm, knots can be generated that do not fit within the allowable widths and are thus lower quality than grade 3.

A limitation of the model described above is that it neglects the correlation between the number of knots, \( n \), and the average knot size per board. The average size of the knot defects are described by the mean of the maximum knot diameters per board, \( \mu_{D_{\text{max}}} \), and the mean of the minimum knot diameters, \( \mu_{D_{\text{min}}} \), per board. The correlation coefficient between the number of knots per board and the mean maximum knot diameter per board is -0.50. Similarly, the correlation coefficient between the number of knots per board and the mean minimum knot diameter per board is -0.57.

![Graph showing correlation between number of knots and mean diameters](image)

Figure 23. There is a moderate correlation, -0.50 and -0.57, between the number of knots per board and the mean minimum and maximum knot diameter on a given board. This relationship is neglected in the two-dimensional model.

2.9 Conclusion

Two-dimensional and three-dimensional probabilistic models for the distribution of knot defects are proposed and calibrated to the input knot geometry of 31, 12 ft (365.76 cm) long, 3.2 in (8.128 cm) wide, and 1.5 in (3.81 cm) thick Eastern hemlock sample boards.
The two-dimensional model generates a distribution knot defects, modeled as circular or elliptical cylinders with depth parallel to the radial axis, equal to the board's thickness. The three-dimensional model generates circular defects with variable depth and growth angle. Both models are calibrated to stochastic knot geometry data thirty-one Eastern hemlock samples. An empirical distribution is used to model the number of knots per board and the maximum knot diameter angle with respect to the longitudinal axis. Truncated exponential distributions are used to model the circular and elliptical minimum and maximum knot diameters. The model is validated by comparing the grading distribution according to NELMA guidelines of the generated samples and the 31 Eastern hemlock samples. The distribution of grades for the Eastern hemlock boards and generated boards are similar. When only 31 samples are generated, the distributions are highly variable, indicating that the sample size of 31 boards is too small to represent knot data distribution. The results also show that the statistics describing the mean and standard deviation for the number of knots and knot diameters for the probabilistic model are consistent with those observed in the 31 Eastern hemlock samples.

This work allows for the generation of synthetic dimensional lumber following geometric knot characteristics. While the model was calibrated to Eastern hemlock in this paper, the methods discussed here should be transferable to other species. A natural extension of this work is to combine multiple individual boards into mass timber products, like Cross Laminated Timber. The influence of knots on these products can then be investigated computationally at a large scale.
CHAPTER 3

ORTHOTROPIC COMPRRESSIVE PROPERTIES OF EASTERN HEMLOCK

3.1 Introduction

Mass timber construction is a category of framing styles typically characterized by the use of large composite wood products for wall, floor, and roof construction. Mass timber products are alternative building materials to the traditional steel and reinforced concrete, which release substantial amounts of carbon emissions during production. These products have been part of the European market for the past three decades as an abundant, renewable, and recyclable alternative to traditional building materials. They have recently entered the market in the United States.

Mass timber is becoming recognized for its environmental attributes and as a way to mitigate climate change. The impacts of global climate change are already being felt in the United States and are projected to intensify in the future. However, the severity of future impacts will depend largely on the actions taken to reduce greenhouse gas emissions (USGCRP, 2018). According to the United States Environmental Protection Agency, buildings account for thirty-eight percent of the carbon dioxide emissions in the United States (United Stated Environmental Protection Agency, 2009). This provides a unique opportunity to architects and structural engineers to mitigate a significant factor driving climate change by implementing innovative and sustainable technology in infrastructure design.

Currently, mass timber products are fabricated from high quality and high-grade wood species like Douglas-fir and Southern pine. However, the composite nature of mass timber provides an opportunity to utilize low-value species typically considered inadequate for structural purposes. Finding applications for underutilized species creates the potential for a promising market for low-value wood species that are abundant in the United States.
Additionally, finding commercial markets for low-value woods supports national forest management strategies to improve forest health while giving rise to more sustainable building practices and increased job opportunities in rural areas of the United States.

The research described in this chapter is part of a larger project that aims to understand the structural mechanics of wood at the mass timber scale. Another stage of this research involves the development of three-dimensional finite element models of knots in clear wood. The clear wood is modeled by orthotropic linear elastic behavior with orthotropic yielding and isotropic hardening. Thus, orthotropic wood properties are necessary as input to the constitutive material model. However, very few species have all orthotropic properties defined, as it is typically only studied for research purposes.

The overarching research project primarily focuses on Eastern hemlock because it is a local and under-utilized species. Eastern hemlock grows along the East Coast from New England to northern Alabama and Georgia, as well as in the Great Lake states. Typical Eastern hemlock is coarse and uneven in texture with considerable shake. Currently, Eastern hemlock is considered to be inadequate for structural applications and is used primarily for lumber and pulpwood (Ross & USDA Forest Service., 2010).

The goal of this work is to define a full set of compressive orthotropic properties of Eastern hemlock to serve as input for a constitutive model in the later stages of this research project. This paper presents the methods and results of experimental testing to determine both the stiffness and strength material properties of clear specimens of Eastern hemlock. Specifically, the longitudinal, radial, and tangential moduli of elasticity and compression yield strengths are determined through clear specimen material testing per ASTM D143-09.
### 3.2 Experimental Program

For the purposes of this paper, wood is defined as a homogeneous (defect-free), orthotropic material with independent mechanical properties along three mutually perpendicular axes: Longitudinal, Tangential, and Radial. Additionally, the strength of wood is dependent on the direction of loading. Thus, the elastic behavior of wood can be described through twelve elastic properties: three moduli of elasticity, three moduli of rigidity, and six Poisson’s ratios. Six axial yield strengths can describe the strength behavior, three for tension loading and three for compression loading, and three shear strengths. These properties, and the associated coordinate system, are described in Table 10 and Figure 24 below.

![Coordinate System](image)

**Figure 24. Coordinate System**
The Wood Handbook provides the longitudinal modulus of elasticity, $E_L$, of Eastern hemlock to be equal to 8,300 MPa for 12% moisture content. The remaining material properties are not published in the literature. The goal of this research is to evaluate the orthotropic stiffness and compressive strength properties of Eastern hemlock. These properties are highlighted in green in Table 10. The testing method used to perform the experimental testing, ASTM D143-09, is also noted in the table.

### Table 10. Orthotropic Material Properties

<table>
<thead>
<tr>
<th>Stiffness</th>
<th>Longitudinal Axis</th>
<th>Radial Axis</th>
<th>Tangential Axis</th>
<th>LR Plane</th>
<th>RT Plane</th>
<th>LT Plane</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E_L$ = 8,900 psi</td>
<td>$E_R$ (ASTM D143-09)</td>
<td>$E_T$ (ASTM D143-09)</td>
<td>$G_{LR}$</td>
<td>$G_{RT}$</td>
<td>$G_{LT}$</td>
<td></td>
</tr>
<tr>
<td>Strength</td>
<td>$F_{CL}$ (ASTM D143-09)</td>
<td>$F_{CR}$ (ASTM D143-09)</td>
<td>$F_{CT}$ (ASTM D143-09)</td>
<td>$F_{TT}$</td>
<td>$F_{VLR}$</td>
<td>$F_{VRT}$</td>
</tr>
</tbody>
</table>

### 3.3 Materials and Methods

The material property testing was performed in accordance with ASTM D143-09. Each specimen was cut from larger 2 in x 4 in Eastern hemlock lamstock that had previously been constituents of Cross Laminated Timber Panels that had been fabricated and tested at the University of Massachusetts Amherst as part of a previous study. The specimens were taken from clear, straight-grained sections of the panels that were not impacted by the previous testing. Each small scale sample was conditioned in a humidity chamber to twelve percent moisture content. Samples that were not being tested immediately remained in the humidity chamber to reduce the influence of varying moisture content since temperature and moisture content significantly impact wood material
properties. Each test was performed on a 30K Material Test System (MTS®) screw-driven universal testing machine.

### 3.3.1 Longitudinal Properties

The stiffness and strength along the longitudinal axis were investigated in accordance with Section 9 of ASTM D143-09. Thirty samples were used for longitudinal testing. The specimens had dimensions with length 1 in x 1in x 4 in. The specimens were processed such that the end grain surfaces were parallel to each other and at right angles to the longitudinal axis.

<table>
<thead>
<tr>
<th>E L</th>
<th>Length, ( L_L ) (in)</th>
<th>Width, ( L_T ) (in)</th>
<th>Depth, ( L_R ) (in)</th>
<th>Loaded Plane</th>
<th>Number of Specimens</th>
</tr>
</thead>
<tbody>
<tr>
<td>E L</td>
<td>4</td>
<td>1</td>
<td>1</td>
<td>RT</td>
<td>30</td>
</tr>
</tbody>
</table>

As shown in Figure 25, a spherical bearing plate was used to obtain a uniform distribution of load over the specimen's ends. The load was applied continuously at a crosshead rate of 0.012 in/min until a maximum crosshead displacement of approximately 0.1 in or until the proportional limit was well passed, as indicated by the load-displacement curve. An extensometer was attached to the center of the testing specimen to record the strain throughout the elastic region of the test and was removed when a load of approximately 3,000 lb was reached. Each failure mode was classified by the fracture surface's appearance as described in Figure 10 of ASTM D143-09. Photos of each specimen were taken before and after testing.
3.3.2 Perpendicular Properties

The radial and tangential modulus of elasticity was experimentally investigated in accordance with ASTM D143-09 Section 12: Compression Perpendicular to Grain. Sixty specimens were processed, thirty to be used for radial stiffness and strength tests and thirty to be used for tangential stiffness and strength tests. Each specimen had dimensions 2 in x 2 in x 6 in, as specified by ASTM D143-09.

<table>
<thead>
<tr>
<th></th>
<th>Length, $L_L$ (in)</th>
<th>Width, $L_T$ (in)</th>
<th>Depth, $L_R$ (in)</th>
<th>Loaded Plane</th>
<th>Number of Specimens</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E_T$</td>
<td>6</td>
<td>2</td>
<td>2</td>
<td>LR</td>
<td>30</td>
</tr>
<tr>
<td>$E_R$</td>
<td>6</td>
<td>2</td>
<td>2</td>
<td>LT</td>
<td>30</td>
</tr>
</tbody>
</table>

The compression loading was applied through a 2 in wide metal bearing plate located at the specimen's center. The load was continuously applied at a crosshead movement rate of 0.012 in/min until a maximum displacement of 0.1 in was achieved, at
which time the test was stopped. Photos of each specimen were taken before and after testing.

![Test Set-Up Schematic](image1)

![Photo of test set-up](image2)

**Figure 26. Perpendicular Material Property Test Set-Up**

3.4 Analysis and Results

Longitudinal results are provided in Section 3.4.1 and perpendicular results are presented in Section 3.4.2

3.4.1 Longitudinal Results

Thirty specimens were tested along the longitudinal axis. However, only ten failed with an acceptable failure mechanism and were used in the following analysis. The specimens that resulted in an unacceptable failure mechanism presented brooming failure at one or both of the ends. Figure 27 (a) displays the stress-strain plot resulting from the crosshead displacement. The red Xs denote the maximum stress, which defined the longitudinal strength of each specimen. The mean longitudinal strength for the ten samples was 5.46E03 psi ± 620.40 psi. The wood strength reported in the Wood Handbook is
5.41E03 psi. In Figure 27 (b), the stress-strain plot resulting from the extensometer data is displayed. The portions of the curves highlighted in red were considered linear and were used to calculate the stiffness. The mean longitudinal modulus of elasticity was determined to be 1.15E06 psi ± 2.944E05 psi, while the longitudinal modulus of elasticity in the Wood Handbook is reported to be 1.20E6 psi.

3.4.2 Perpendicular Results

Forty specimens were tested to determine the perpendicular stiffness and strength: twenty along the tangential axis and twenty along the radial axis.

Figure 28 presents the stress-strain response for the compression test along the tangential axis. The portion of the stress-strain response highlighted in red was considered linear elastic and used to calculate the modulus of elasticity, $E_T$. In accordance ASTMD143, the strength was defined as the stress at 0.025 strain. The red Xs denote this in the following figure. The mean tangential compression stiffness was determined to be
9.67E3 ± 4.76E4, while the mean tangential compression strength was determined to be 1.06E3 ±2.74E2.

Figure 28. Eastern hemlock compression test results along the tangential axis. The red sections highlighted in red indicate the stress-strain response used to calculate the stiffness. The red Xs indicate the strength value.

Figure 29 presents the stress-strain response for the specimens tested in compression along the radial axis. As above, the portions of the response highlighted in red represent assumed linear elastic behavior used to calculate the Modulus of Elasticity along the Radial axis, $E_R$. The red Xs denote the strength defined by the stress and 0.025 strain. The mean radial compression stiffness was determined to be 5.44E3 psi ± 2.01E4 psi, while the mean radial compression strength was determined to be 6.57E2 psi ± 1.40E2 psi.
3.4.3 Summary

Table 13 provides a summary of the Eastern hemlock material properties found in this study. The longitudinal stiffness and strength agree with the wood handbook, providing confidence towards the methods used to determine the tangential and radial stiffness and strength.

Table 13. Summary of Eastern hemlock compression stiffness and strength along the longitudinal, tangential, and radial axes.

<table>
<thead>
<tr>
<th>Axis</th>
<th>Number of Samples</th>
<th>Modulus of Elasticity (psi)</th>
<th>Strength (psi)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Longitudinal</td>
<td>10</td>
<td>1.15E6 ± 2.94E5</td>
<td>5.46E3 ± 6.20E2</td>
</tr>
<tr>
<td>Tangential</td>
<td>20</td>
<td>9.67E3 ± 4.76E4</td>
<td>1.06E3 ± 2.74E2</td>
</tr>
<tr>
<td>Radial</td>
<td>20</td>
<td>5.44E3 ± 2.01E4</td>
<td>6.57E2 ± 1.40E2</td>
</tr>
</tbody>
</table>

Due to the heterogeneity in wood, a significant limitation of these results is the small specimen size. In an attempt to better understand the accuracy and precision of the results, the current confidence interval was calculated as well as the number of specimens.
required to reach a 95% confidence interval. The data collected in this study is only to be used as input to a constitutive model to predict CLT panel behavior. The sample size required to reach a 95% confidence interval is not within the scope of this research.

Table 14. Confidence Interval of the experimental results as well as the number of samples required to reach 95% confidence.

<table>
<thead>
<tr>
<th>Material Property</th>
<th>Confidence Interval</th>
<th>Number of samples required to reach 95% Confidence</th>
</tr>
</thead>
<tbody>
<tr>
<td>Longitudinal</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Stiffness</td>
<td>44.3%</td>
<td>141</td>
</tr>
<tr>
<td>Strength</td>
<td>80.2%</td>
<td>28</td>
</tr>
<tr>
<td>Tangential</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Stiffness</td>
<td>34.3%</td>
<td>425</td>
</tr>
<tr>
<td>Strength</td>
<td>61.2%</td>
<td>111</td>
</tr>
<tr>
<td>Radial</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Stiffness</td>
<td>43.4%</td>
<td>239</td>
</tr>
<tr>
<td>Strength</td>
<td>75.5%</td>
<td>70</td>
</tr>
</tbody>
</table>

3.5 Conclusions

The orthotropic axial compression stiffness and strength properties for Eastern hemlock were investigated and evaluated experimentally in accordance with ASTM D143-09. Ten specimens for the parallel to grain tests and twenty specimens for the perpendicular to grain tests were evaluated. Small-scale, clear, straight-grained specimens were used. The longitudinal Modulus of Elasticity, $E_L$, was found to be 1.15E6 psi with 44.3% confidence, while the strength, $f_L$, was found to be 5.46E3 psi with 80.2% confidence. The perpendicular compression stiffness and strength properties along the tangential and radial axes were also determined. The tangential Modulus of Elasticity and Strength was found to be 9.67E3 psi and 1.06E3 psi, respectively, while the radial Modulus of Elasticity and Strength were found to be 5.44E3 psi and 6.57E2 psi. The stiffness values were found at a 34.3% and 43.4% confidence interval for tangential and radial axes, respectively. The stiffness and strength values determined for Eastern hemlock in this paper act as an initial study and should be applied with caution in appropriate circumstances.
CHAPTER 4
A FINITE ELEMENT APPROACH TO INVESTIGATING THE INFLUENCE OF KNOTS ON CROSS LAMINATED TIMBER

4.1 Introduction

Knots are associated with localized regions of low stiffness and strength. Traditionally, visual grading rules have been used as a tool to categorize the mechanical quality of dimensional lumber by grouping them into quality classes based primarily on knot geometry and ascribing corresponding strength properties to these specimens. Using knot geometry to predict mechanical properties is appropriate when the goal is to eliminate the lowest quality lumber in a large sample of boards; however, it is typically not accurate when considering a single board and knot geometry. To understand the influence of knots' geometrical parameters on mechanical performance deterministically, a finite element approach is required.

Various methods for modeling knots in dimensional lumber have been proposed. A knot is caused by the growth of a branch that has become incorporated in the bole, or trunk, of a tree. Knots impact a wood member's mechanical properties by interrupting the continuity and direction of wood fiber (Ross and USDA Forest Service, 2010). Therefore, many models have been developed to model the fiber orientation surrounding knots. Phillips et al. developed a model for the deviation of fibers around knots based on laminar flow theory around a solid elliptical obstacle (Phillips et al., 1981). Foley expanded on this model by the addition of a dive angle through Shigo’s knot growth model (Foley, 2003). Several authors have applied these theories in two-dimensional and three-dimensional FE models in which knots are modeled by circular, elliptical, cylindrical, or conical shapes (Guindos & Guaita, 2013; Kandler et al., 2016b). Because knots interrupt the continuity of
longitudinal fibers, it has been proposed that knots are unable to carry tension loads. Baño used FEM to model scots pine beams with knots in bending and found that an adherent knot model was found to best model the experimental behavior when the knot was located on the compression side, while a hole model was most representative of behavior when the knot was on the tension side (Baño et al., 2011). According to Boatright and Garrett, however, fiber deviations around knots prevent shear failure at an early state and causes the effect of knots to be less severe than the effects of a hole of equal size in clear wood (Boatright & Garrett, 1979a, 1979b).

While many models for the influence of knots in dimensional lumber exist, many focus on the small scale and localized impact of the defects. Thus, scaling these models to consider large-scale mass timber products is computationally intensive and inefficient. The benefit of mass timber products, like Cross Laminated Timber (CLT), is the averaging effect of their heterogeneous material properties and their efficiency at load sharing. A large-scale finite element model of knots in the constituent boards and CLT panel is required to characterize this behavior.

The purpose of this study is to understand how knot defects may influence the mechanical properties and structural performance of CLT panels as a function of the defect size, location, and shape. Knot defects can be considered at three scales: a single knot defect, multiple knots within a single dimensional lumber board, and knots within a full CLT panel. The geometry and distribution of a singular knot defect were investigated in Chapter 2. This scale provides input parameters for evaluating knot influence on mechanical properties in clear wood on the larger scales. Using finite element methods,
the impact of knots on stress distributions, effective stiffness, and effective strength can be analyzed over the larger scales.

This chapter demonstrates a model to investigate knots at the constituent board level. Both stiffness and strength are considered. The model presented in this work is calibrated to Eastern hemlock material properties and knot geometry. However, the models may be applied to other softwood species as well.

![Knot Defects on Clear Wood](image)

**Figure 30.** The influence of knot defects on the mechanical behavior of clear wood was investigated at three scales.

### 4.2 Model Geometry

In Chapter 2, the knot geometries for approximately 300 knots on 31 Eastern hemlock boards were measured. These geometries consider the knot center coordinates, minimum and maximum diameters, depth, and the grade of these boards. The 31 Eastern hemlock boards were then used as constituent boards of 4 CLT panels tested in 4-point bending. These boards were used to create the geometry of the finite element model. Each knot’s geometry and location were tracked with respect to each of the scales, allowing for
a detailed analysis of the influence of knots on mechanical properties at both the board and CLT panel scale.

The most significant aspect of knots is the discontinuity and redirection of fibers around stiff inclusions. The changes in continuity and direction of fibers depend on a variety of stochastic factors: size, orientation, location, and proximity to other knots. The relationship between fiber orientation and knot geometry is shown below in Table 15 for various Eastern hemlock samples. As shown, even for knots with similar geometric properties, the orientation of fibers surrounding the knot defect can be very different. Foley developed a method of modeling the local displacement of fibers around knots based on flow theory and Shigo’s knot formation theory (Foley, 2003). This algorithm allows for consideration of the fiber deviation around a knot both in the longitudinal-tangential plane as well as the dive angle in the radial direction. However, this method may be computationally intensive when used on a large scale. Thus, a simplified method is developed and utilized in this project.
Knots create discontinuity and redirection of fibers around the defect, creating localized regions of low stiffness and strength. Knots occur in trees where the continued growth of the bole has surrounded a branch. When both branch and bole are alive and growing, an intergrown knot occurs. Alternatively, when the branch has died and the bole continues to grow, enclosing the dead limb, an encased knot occurs.

In order to better characterize the impact of knots on mechanical properties, it is important to distinguish between intergrown and encased knot defects. Knots disrupt the clear wood fiber continuity and direction, impacting the material properties along a given axis. Encased knots typically tend to have less cross-grain than intergrown knots, and therefore generally have less impact on the clear wood's mechanical properties. However, in encased knots, the bole fibers are not continuous with the fibers of the encased knot, resulting in a poor connection between the knot defect and the clear wood. Encased knots
resist very little stress. In the case of intergrown knots, the fibers between the knot and the clear wood are completely intergrown, resulting in a much tighter connection and the ability to transmit stresses.

The simplest method of modeling the impact of knots in wood boards is to use two materials such that the board is modeled as clear wood with stiffness $E_{CW}$ and a circular stiff inclusion with stiffness $E_K$, as shown in Figure 31. Knots have a greater impact on tensile properties than compressive properties (Ross & USDA Forest Service., 2010). For the purposes of this model, it is assumed that knots are capable of load transfer in compression. Thus, knots are idealized as stiff inclusions in compression loading.

![Figure 31. Geometric model of knot defect idealized as a stiff inclusion in clear wood for compression loading](image)

However, knots are assumed to not be capable of load transfer in tension and are therefore idealized by holes in axial tension.
Ten of the thirty-one Eastern hemlock boards previously mentioned were randomly selected to create the geometries used in this analysis. Table 16 presents each of the geometries with the knots modeled as holes. The same boards were also modeled with the knots as stiff inclusions to compare the behavior. Each board is 144 in (36.6 m) long, 3.2 in (8.1 cm) wide, and 1.5 in (3.8 cm) deep. The knots are modeled as either cones or cylinders. Through knots have a depth equal to the depth of the board, while non-through knots are assumed to have a depth equal to half the board's depth.
Table 16. Finite element model geometry for Eastern hemlock samples with knots modeled as holes.

<p>| | | | | | | | | | |</p>
<table>
<thead>
<tr>
<th></th>
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<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>6</td>
<td>7</td>
<td>8</td>
<td>9</td>
<td>10</td>
</tr>
</tbody>
</table>
4.3 Loading and Boundary Constraints

This model studies the influence at the dimensional board scale. However, the behavior of the dimensional lumber as a constituent of a CLT panel is also considered. Therefore, each board is modeled in axial compression (knots idealized by stiff inclusions) or tension (knots idealized by holes) to mimic the loading experienced by the top and bottom layers of a three-layer CLT panel in bending. In a parallel study conducted by Kaboli at the University of Massachusetts Wood Mechanics Lab, four Eastern hemlock CLT panels were tested in four-point to failure (Kaboli, 2019). Load-displacement results are presented in Figure 33.

![Figure 33. Four-point bending test results for Eastern hemlock CLT panels tested at UMass Amherst Wood Mechanics Laboratory (Kaboli, 2019)](image)

The mean maximum applied load at failure was calculated to be $9.5 \frac{k}{ft} \left(1.39 \frac{KN}{cm}\right)$ (Kaboli, 2019). Thus, via beam theory calculations, the internal tension and compression load experienced by the top and bottom lamellas at failure was calculated to be $2.5 ksi \ (17.2 \ Mpa)$ (Figure 34). Figure 35 presents the axial compression loading condition of the boards.
Figure 34. The internal compression and tension loads in the top and bottom lamella calculated from full-scale experimental testing.

The board is also restrained against rigid body motion. A frictionless support restraining displacement along the longitudinal axis is located on the right end surface at \( x = L \), where \( L \) is the sample's length. The node located at \( x = L, y = w, z = 0 \), where \( w \) is the sample's width, is restrained against displacement along all axes. The node at \( x = L, y = 0, z = 0 \) is restrained against displacement in the radial direction.

Figure 35. Boundary constraints for three-dimensional finite element model of wood board with applied axial tension or compression

4.4 Mesh

The mesh is automatically generated using quadratic elements. Locations of high interest, such as boundaries where material properties change, are refined using curvature and proximity size functions in ANSYS. Example meshes in the longitudinal-tangential plane with refinement around the knots are shown below.
4.5 Material Properties

The clear wood is modeled as an orthotropic material, with independent material properties in the directions of three mutually perpendicular axes: longitudinal, tangential, and radial. Twelve constants, nine of which are independent, are needed to characterize the elastic behavior of wood: three moduli of elasticity, $E_L$, $E_T$, $E_R$, three shear moduli, $G_{LT}, G_{LR}, G_{RT}$, and six Poisson’s ratios, $\mu_{TL}, \mu_{LR}, \mu_{RT}, \mu_{TL}, \mu_{RL}, \mu_{TR}$.

The MOE for the three mutually perpendicular axes were experimentally evaluated in Chapter 3. The Wood Handbook provides material property values for the longitudinal
modulus of elasticity, elastic ratios, and Poisson’s ratios for various species (Ross and USDA Forest Service, 2010). However, since Eastern hemlock is not currently used for structural purposes, Poisson’s ratios are not available in the Wood Handbook and must be approximated by using species with similar properties to Eastern hemlock to inform the shear moduli and Poisson ratios. Specifically, the Poisson ratios of Eastern hemlock are assumed to be equal to the Poisson ratios Western hemlock, which are published in the Wood Handbook (Ross and USDA Forest Service, 2010). The ratio of tangential-radial Shear Modulus to longitudinal MOE is 0.003 for Western hemlock. This ratio was used in conjunction with the longitudinal MOE reported in the Wood Handbook, $9.24E6 \text{ Mpa}$, to determine the tangential-radial shear modulus for Eastern hemlock such that $G_{RT} = 2.80E4 \text{ Mpa}$. The longitudinal-radial shear modulus for Eastern hemlock was determined experimentally by Bahmanzad at the University of Massachusetts Wood Mechanics Lab (Bahmanzad, 2019). The remaining shear modulus, $G_{LT}$, was determined through personal communications with Kaboli who performed experimental testing at the UMass Wood Mechanics Lab. The value presented in Table 17 is a preliminary result that was available at the time of the FEM development. Kaboli provided an updated value for $G_{LT}$ (8.55E4 \text{ Mpa}) in a recent publication (Kaboli, 2019). The qualitative results presented in Section 4.6 are not expected to change significantly, given the modest difference in the preliminary and published results of $G_{LT}$. The clear wood is therefore modeled by the orthotropic linear elastic properties provided in Table 17.

<table>
<thead>
<tr>
<th>Modulus of Elasticity, $E$ (Mpa)</th>
<th>Poison Ratio, $\mu$</th>
<th>Shear Modulus, $G$ (Mpa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>L $7.93E6$</td>
<td>LT $0.423$</td>
<td>LT $1.11E5$</td>
</tr>
<tr>
<td>T $6.67E5$</td>
<td>TR $0.422$</td>
<td>TR $2.80E4$</td>
</tr>
<tr>
<td>R $3.75E5$</td>
<td>LR $0.485$</td>
<td>LR $3.98E5$</td>
</tr>
</tbody>
</table>
The clear wood strength properties are modeled by the Hill Yield Criterion, which is governed by the failure surface:

\[
\sigma_o = \sqrt{H(a_L - a_T)^2 + F(a_T - a_R)^2 + G(a_L - a_R)^2 + 2N\tau_{LT}^2 + 2L\tau_{RT}^2 + 2ML\tau_{LR}^2}
\]  

(Eq. 73)

The Hill Yield Criterion assumes orthotropic yielding and isotropic hardening. This model does not account for the differences in yield strength in tension and compression (ANSYS, 2013). The clear wood strength properties are provided in the table below. The orthotropic axial strength properties for Eastern hemlock were determined by experimental testing in Chapter 3. The parallel to grain (LT) shear strength is reported in the Wood Handbook as 7.31 Mpa for Eastern hemlock at 12% moisture content. The shear strength values are experimentally determined from Eastern white pine rolling shear tests that were available via personal communications with Kaboli. Since the development of the FE model, rolling shear strength values for Eastern hemlock have been published (1.41E3 Mpa) (Kaboli, 2019). The hardening modulus presented in Table 18 is a numerical place holder for elastic perfectly plastic behavior.

Table 18. Clear wood strength properties assuming orthotropic yielding and isotropic hardening governed by the Hill Potential theory

<table>
<thead>
<tr>
<th>Axial Strength (Mpa)</th>
<th>Shear Strength (Mpa)</th>
<th>Hardening Modulus (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>L 3.83E4</td>
<td>LT 7.31E3</td>
<td>LT</td>
</tr>
<tr>
<td>T 4.97E3</td>
<td>TR 1.73E3</td>
<td>TR</td>
</tr>
<tr>
<td>R 4.13E3</td>
<td>LR 1.73E3</td>
<td>LR</td>
</tr>
</tbody>
</table>

Material properties for knot defects are difficult to determine experimentally and could not be found in the literature. Therefore, the material properties for the knots modeled by stiff inclusion are assumed. The stiff inclusions are modeled with isotropic bilinear properties. Preliminary studies showed that the stress concentrations surrounding stiff inclusions are not sensitive to the MOE of the stiff inclusion, as long as the MOE of the
stiff inclusion is much larger than the longitudinal MOE of the clear wood. Therefore, the MOE of knot defects, $E_K$, is approximated by

$$E_K = \text{mean} \left( \frac{E_L}{E_R} \right) \frac{E_L}{E_T}.$$  \hspace{1cm} (Eq. 74)

Using the values for $\frac{E_L}{E_T}$ and $\frac{E_L}{E_R}$ provided in the Wood Handbook for Western hemlock and the longitudinal MOE of Eastern hemlock, $E_K$ is calculated to be $2.30E8$ $Mpa$. Poisson’s ratio is assumed to be 0.4. For simplicity, it is assumed that the knots have a much higher yield stress than the clear wood. Therefore, the yield strength was selected to be much greater than the clear wood yield strength and is a numerical placeholder. Again, the hardening modulus was chosen to model elastic perfectly plastic behavior. Stiffness and strength properties for the knots modeled as stiff inclusions are presented in Table 19.

<table>
<thead>
<tr>
<th>Table 19. Knot properties modeled by isotropic bilinear behavior</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Modulus of Elasticity (Mpa)</strong></td>
</tr>
<tr>
<td><strong>Poisson Ratio</strong></td>
</tr>
<tr>
<td><strong>Yield Strength (Mpa)</strong></td>
</tr>
<tr>
<td><strong>Hardening Modulus (Mpa)</strong></td>
</tr>
</tbody>
</table>

4.6 Results

The ten board geometries were modeled in accordance with the material properties and boundary conditions detailed above. Each geometry was modeled in axial tension (knots modeled as holes) and axial compression (knots modeled as stiff inclusions).

4.6.1 Elastic Results

Results show that stress concentrations occur at the boundary between knot defects and clear wood. In tension, when knots are modeled as holes, it can be seen that the stresses flow around the defect. However, in compression knots are modeled as stiff inclusions, and the stresses flow through the stiffer knot defect material. Two example normal stress
contours in the longitudinal direction are provided in the figures below. Tensile loading results in much higher stress concentrations than axial compression due to the differences in knot model.

Figure 38. Selected example of edge knot normal stress in longitudinal axis due to axial tension (top) and axial compression (bottom). Stress contour values are provided in psi.
Additionally, edge knots typically result in higher stress concentrations than interior knots.

Figure 39. Selected example of interior knot normal stress in longitudinal axis due to axial tension (top) and axial compression (bottom). Stress contour values are provided in psi.

The effective stiffness longitudinal MOE, $E_L$, for each sample is calculated such that:

$$
\overline{E_L} = \frac{\overline{K}L}{A},
$$

(Eq. 75)

where

$$
\overline{K} = \sum \frac{P_{nodal}}{\delta_x}.
$$

(Eq. 76)

$\overline{K}$ is the effective stiffness, $P$ is the applied nodal loads, $\delta_x$ is the longitudinal displacement at the nodes, $A$ is the cross-sectional area, and $L$ is the length of the board. In axial compression the knots are modeled by stiff inclusions, and the effective longitudinal MOE
increases approximately 0.6% from the clear wood longitudinal MOE. However, in axial tension knots are modeled as holes, and the effective longitudinal MOE decreases approximately 8.7% from the longitudinal MOE of the clear wood (Figure 40). Note sample 6 has a significant decrease from the clear wood MOE in axial tension due to two large edge knots in the board geometry.

![Figure 40. Effective MOE for ten boards in axial tension and axial compression. Longitudinal MOE of clear wood is indicated by the black dashed line.](image)

The effective stiffness is also investigated in conjunction with the number of knots and knot volume. Figure 41 does not show a meaningful relationship between the number of knots and the effective longitudinal MOE, indicated by the very low slope of the linear best fit lines.
Figure 41. Relationship between effective longitudinal MOE and number of knots. Linear best fit lines and correlation coefficients are provided.

The correlation coefficients, $\rho$, and slope of the linear best fit line are provided in Table 20 for both tension and compression loading conditions.

Table 20. The slope of the linear best fit line and correlation coefficients for the relationship between knot geometry and effective longitudinal MOE.

<table>
<thead>
<tr>
<th>Slope of linear best fit line, $m$</th>
<th>Number of Knots</th>
<th>Compression</th>
<th>Tension</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>4.70E3 $Mpa$</td>
<td>$1.51E4 Mpa$</td>
<td></td>
</tr>
<tr>
<td>Knot Volume</td>
<td>47.5 $Mpa/cm^3$</td>
<td>$9.57E2 Mpa/cm^3$</td>
<td></td>
</tr>
<tr>
<td>Correlation coefficient, $\rho$</td>
<td>Number of Knots</td>
<td>Compression</td>
<td>Tension</td>
</tr>
<tr>
<td></td>
<td>0.45</td>
<td>$-0.11$</td>
<td></td>
</tr>
<tr>
<td>Knot Volume</td>
<td>0.49</td>
<td>$-0.76$</td>
<td></td>
</tr>
</tbody>
</table>

The relationship between effective MOE and knot volume is also investigated in Figure 42. Knot volume is defined as the sum of each knot volume along the board’s length, assuming knots are modeled by cylinders or cones. The relationship between knot volume and effective MOE is much stronger in axial tension than axial compression.
4.6.2 Strength Results

The Wood Handbook indicates that the presence of a knot has a much greater effect on most strength properties than on stiffness (Ross & USDA Forest Service, 2010). Thus, strength behavior was studied by investigating the nodal failure at 17.2 Mpa (2.5 ksi) axial tension and compression load. Figure 43 and Figure 44 present Sample 1 nodal results. Nodes highlighted in red represent nodal failure. Nodal failure occurs at the interface between knot defects and clear wood. Edge knots or knots close to the board edge show more nodal failure than interior knots. This can be seen by comparing the knot failure surrounding knots A and B, which are interior knots close to the edge of the board, to knot C, which is located at the center of the board. There is a 5% nodal failure along the board length when the axial tension load is equal to 17.2 Mpa (Figure 43). When the beam is in axial compression, the nodal failure is only 0.5% (Figure 44).
Figure 43. Sample 1 in axial tension (17.2 MPa). Red nodes represent nodal failure.
Figure 44. Sample 1 in axial compression (17.2 MPa). Red nodes represent failure.
Nodal analysis was performed for all ten samples (Figure 45). The mean nodal failure in axial tension is $14.68\% \pm 29.68\%$, while the mean nodal failure in axial compression is $0.11\% \pm 0.19\%$. The strength capacity of Sample 4 is greatly influenced by a large edge knot. The difference in the performance between tensile and compressive loading is explained by the differences in how the knot properties are modeled. The results are well aligned with the Wood Handbook which states that, in general, knots have a greater influence on strength in tension than in compression (Ross & USDA Forest Service., 2010).

![Figure 45. Percent nodal failure at 17.2 Mpa (2.5 ksi) axial loading for knots modeled in tension (knots modeled by holes) and compression (knots modeled by stiff inclusions)](image)

This concept can also be analyzed by considering the load required to induce 1% nodal failure. Recall, in the four-point bending tests on the Eastern hemlock CLT panels, the mean maximum applied load was $1.39 \frac{KN}{cm} \left(9.5 \frac{k}{ft}\right)$. The internal tension and compression load experienced by the top and bottom lamellas at failure was calculated to be $17.2 \ Mpa (2.5 \ ksi)$. In axial compression, knots are modeled as stiff inclusions and the strength capacity is increased, such that the mean load to induce 1% nodal failure is $26.75 \pm 3.10 MPa$, resulting in a bending load of $1.69 \frac{KN}{cm} \left(11.6 \frac{k}{ft}\right)$. In axial tension, the
knots are modeled as holes, and the strength capacity is decreased. The mean load to induce 1\% nodal failure is $8.62 \pm 4.55 \, \text{Mpa}$, resulting in a bending load of $0.55 \frac{KN}{cm} \left(3.75 \frac{k}{ft}\right)$.

Because MOE can be determined without destructive testing, grading rules make assumptions on strength capacity based on MOE. Figure 47 investigates the relationship between strength and MOE by considering the correlation coefficients between the effective MOE and the load at 1\% nodal failure. Tensile loading shows a moderate relationship between stiffness and strength, while compressive loading does not.
Finally, the relationship between grade and effective material property is considered. Each sample was visually graded according to NELMA guidelines. Select structural boards, indicated by SS in Figure 48, have the smallest knot defects, while Grade 3 boards have the largest defects. Tensile results show a meaningful relationship between effective MOE and grade. This is indicated by a large slope of the linear best fit line and a strong correlation. However, no meaningful relationship could be detected between effective MOE and grade in compression loading.
Figure 48. Relationship between grade and effective MOE. Linear best fit lines and correlation coefficients are also provided.

This relationship is also studied for the strength case. Again, there is a more meaningful relationship considering tensile load, than compressive loading.

Figure 49. Relationship between grade and load at 1% nodal failure. Linear best fit lines and correlation coefficients are also provided.
Table 21 presents the slope of the linear best fit line and correlation coefficients for the relationship between visual grade according to NELMA guidelines and effective material property for both elastic and strength cases.

Table 21. The slopes of the linear best fit lines and correlation coefficients for the relationship between grade and effective property.

<table>
<thead>
<tr>
<th>Slope of linear best fit line, $m$</th>
<th>Compression</th>
<th>Tension</th>
</tr>
</thead>
<tbody>
<tr>
<td>Effective MOE</td>
<td>$1.28E4 \text{ Mpa}$</td>
<td>$-4.82E5 \text{ Mpa}$</td>
</tr>
<tr>
<td>Load at 1% nodal failure</td>
<td>$8.69E3 \text{ Mpa}$</td>
<td>$-2.29E3 \text{ Mpa}$</td>
</tr>
<tr>
<td>Correlation coefficient, $\rho$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Effective MOE</td>
<td>0.30</td>
<td>-0.86</td>
</tr>
<tr>
<td>Load at 1% nodal failure</td>
<td>0.30</td>
<td>-0.54</td>
</tr>
</tbody>
</table>

4.7 Conclusions

A three-dimensional finite element model was developed to investigate the influence of knots at the dimensional lumber scale. Each board was composed of two materials: clear wood and knots. The clear wood was modeled as an orthotropic linear elastic material with orthotropic yielding and isotropic hardening governed by Hill Potential theory. Knots were considered as either holes, for axial tensile loading, or stiff inclusions, for axial compressive loading. The stiff inclusions were considered to be isotropic, governed by isotropic bilinear behavior. An axial load of $17.2 \text{ Mpa}$ was applied to model the top and bottom lamella's internal load at failure in a three-layer CLT panel composed of Eastern hemlock dimensional lumber.

Ten sample Eastern hemlock geometries were analyzed. The results show that the influence of knots differs in tensile and compressive loading. In compressive loading, stress is able to flow through the knot. However, the stress must flow around the knot in tensile

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loading (knots modeled by holes). The knot location also has a significant impact on stress concentrations; edge knots cause much higher stress concentrations than interior knots with the same dimensions, particularly in tension. The largest stress concentrations form at the interface between knot defects and clear wood. The occurrence of knots in axial compression leads to and increase in effective MOE and strength. However, knots in tensile loading lead to a decrease in effective MOE and strength.

The assumptions made about the ability of knots to transfer loads and the influence of knots on effective material properties significantly impact grading guidelines, board end-use, and value. Modeling knots as either stiff inclusions or holes represents the extreme behavior; the connection between knots and clear wood can perfectly transfer axial load or the connection between knots and clear wood allows no load transfer. Understanding the knots’ ability to transfer and carry load is important when considering the impact of knots on structural lumber. If knots are capable of carrying load in tension, however, and can be modeled more accurately by stiff inclusions, the results indicate that no meaningful relationship between visual grade and wood MOE/strength. This could mean that grading guidelines do not allow for the efficient use of wood, particularly in mass timber products.

4.8 Future Work

The efficient use of lumber is integral to the goals of sustainable construction as applied to mass timber. Cross laminated timber is naturally forgiving of defects due to the composite behavior of the technology. However, it is important to characterize and understand the influence of knots on effective stiffness and strength at the CLT panel scale. Therefore, the finite element model of a single board must be expanded into a three-dimensional full-scale CLT panel geometry. As previously mentioned, the knot defects
were tracked from thirty-one Eastern hemlock boards to their location within four CLT panels. These CLT panels were then destructively tested in 4-point bending. These CLT panels will inform the geometry of the finite element model at the CLT panel scale. A significant limitation of this work is the computational cost of a strength analysis of a large geometry with a very small mesh, making it potentially infeasible to model every knot at the CLT panel scale. Therefore, a second model will be considered in which each board will be divided into a number of discrete sections and assigned an effective modulus of elasticity based on smaller-scale FEM. This model will be used only for elastic analysis but will inform which knots are critical, causing the largest stress concentrations and whether merging the geometry in this way provides an adequate understanding of the panel's structural behavior under four-point bending. The experimental tests will be used to validate the models.

Further, a three-dimensional probabilistic model was developed in Chapter 2 for the distribution of knot defects in dimensional lumber. Because the current sample size of 31 Eastern hemlock boards and four CLT panels is quite small, the probabilistic model can be applied to create synthetic geometries. By automating the formulation of the FEM, a much larger sample size can be considered. This will allow for a reliability analysis of both dimensional lumber and CLT panels.
5.1 Introduction

The natural growth of wood leads to highly heterogeneous material properties in sawn lumber. Defects such as knots and localized slope of grain contribute to some of this variation; however, wood properties can vary even in clear wood. This is due to the constantly changing environmental factors that influence tree growth, such as soil conditions, moisture, and growing space (Ross and USDA Forest Service, 2010). Knots are the result of branches embedded in the trunk of a tree and significantly influence the surrounding wood causing variable density, grain distortion, and material discontinuity in the vicinity of knots (Bodig and Jayne, 1982). Knots have long been acknowledged to be responsible for stiffness and strength reduction in structural lumber. However, models to capture the influence of knots in structural lumber are complex. Nonetheless, there have been numerous research efforts to model the structural performance of knots in wood (see, for example, Foley, 2003, Baño et al., 2011, and Xu, 2002).

Grading is used to categorize sawn lumber as it is processed to determine the mechanical quality and potential use for each board. In the United States, the American Lumber Standard Committee (ALSC) maintains a voluntary product standard, called the American Softwood Lumber Standard (PS 20-10), which prescribes the ways in which stress-grading principles can be used to formulate grading rules designated as conforming to the American Lumber Standard (American Softwood Lumber Standard, Voluntary Product Standard PS 20-10, 2010). This standard is implemented through an internationally recognized consensus accreditation and certification program, with the purpose of
providing uniform, industry-wide grade marking and inspection of softwood lumber. The National Grading Rule establishes the lumber classifications and grade names for visually graded dimension lumber. Visual requirements are developed by rules-writing agencies, which write and publish grading rule books and are certified by the ALSC Board of Review (Ross & USDA Forest Service., 2010). In North America, design values for the mechanical properties of visually graded major commercial softwood dimensional lumber species are established by testing of a representative sample of full-size members as specified by ASTM D1990 in-grade testing procedure (American Society for Testing and Materials, 2014).

Visual grading techniques are developed on the premise that mechanical properties of lumber differ from properties of clear wood due to growth characteristics that can be seen and evaluated by eye. Thus, the growth characteristics can be used to sort lumber into stress grades. Typical features that are used as limiting characteristics distinguishing each grade level include knots, slope of grain, checks and splits, shake, density, decay, heartwood and sapwood, pitch pockets, and wane (Ross & USDA Forest Service., 2010). This chapter specifically focuses on the geometry of knots as a limiting factor for stiffness and strength. Knots cause localized cross grain, interrupt the continuity of grain, and are associated with regions of low stiffness and strength. However, low-stiffness regions generally represent a small volume of the board length and overall board stiffness is not greatly impacted by the presence of knots. Knots have a greater effect on most strength properties than stiffness. Specifically, tensile strength is more greatly impacted by the presence of knots than compression loading. The influence of knots in bending depends on
whether the knot is located in the tensile or compressive side of the beam; knots located along the neutral axis have little or no effect (Ross & USDA Forest Service., 2010).

The goal of this study is to quantify the relationship between knot geometry and bending Modulus of Elasticity (MOE) and Modulus of Rupture (MOR) in Eastern hemlock and Sitka spruce. The geometry of knots is defined through various methods in an attempt to find the best indicator of dimensional lumber stiffness and strength in bending. Regardless of the knot characterization method investigated in this chapter, results for the three cases studied do not provide a meaningful relationship between knot defects and Modulus of Elasticity or Modulus of Rupture in either of these species. As previously mentioned, no relationship between knot geometry and MOE is expected. The lack of relationship between knot geometry and MOR can be explained by the very limited data sets.

5.2 Materials and Data

Three data sets comprising mechanical test data and the corresponding geometric characterization of knots were examined in this study. The first data set contained twenty-seven Eastern hemlock (EH, *Tsuga canadensis*) boards purchased from local mills in Massachusetts (USA), which were processed and tested at the Building and Construction Technology (BCT) Wood Mechanics Lab at the University of Massachusetts Amherst (UMass Amherst). The remaining data sets were provided by the Centre for Timber Engineering at Edinburgh Napier University. The fifty-six specimens in the second data set came from an eighty-three-year-old stand of Sitka spruce (*Picea sitchensis*), here referred to as Birkley wood (SS-BW) located in Kielder forest (England), while the fifty-six specimens in the third data set came from a fifty-seven-year-old stand located on the
Baronscourt estate (SS-BC) in County Tyrone, Northern Ireland. These two datasets from the UK were both from long-running forestry experiments (rotation length and spacing, respectively) and contain a wider range of strength, stiffness, and knot size than is typical of UK grown spruce.

5.2.1 Data Set 1: Eastern hemlock - EH

A total of eighty-three Eastern hemlock 12 ft long nominal 2 x 4 boards were purchased in three separate batches from two local mills in Western Massachusetts. When purchasing from the source mill, boards were selected visually to avoid extreme warp. At the UMass Amherst Wood Mechanics lab, the boards were jointed and planed to have final dimensions of 33 mm (1.3 in) in thickness, 81 mm (3.2 in) in width, and 3658 mm (144 in) in length (Kaboli, 2019). Each board was then visually graded according to the Northeastern Lumber Manufacturers Association (NELMA) grading rules for structural lumber (NELMA, 2013). Twenty-seven of these boards were randomly selected for further processing and investigation for this study. Ninety percent of the boards were flat-sawn.

Knot defects were characterized using visual methods described in Chapter 2 and in accordance with Figure 50. Each knot is defined by its knot center coordinates along the longitudinal and tangential axes, maximum and minimum diameters, angle with respect to the longitudinal axis, and depth. Knots visible on only one face are considered non-through knots, while knots visible on two or more faces are considered through knots. Non-through knots are assumed to have a depth equal to half the board’s depth, and through knots are assumed to have a depth equal to the depth of the board, allowing for a three-dimensional
knot geometry to be characterized. Knot characterization was performed only on the flat sides of the boards.

Nondestructive, flatwise three-point bending tests were conducted on all boards to evaluate longitudinal Modulus of Elasticity (MOE) along the full span of the board. Tests were performed in accordance with ASTM D198 with a clear span of 3531 mm (ASTM, 2014). The test speed 25.4 mm/min and load was applied to 20% of the expected proportional limit to avoid permanent deformation of the samples as they were to be used in the fabrication of cross laminated timber panels in a future study.

Figure 50. Idealized Eastern hemlock board geometry and knot geometry characterizations. Ninety percent of boards were flat-sawn. Each board was tested in non-destructive flatwise three-point bending to determine the bending modulus of elasticity.

5.2.2 Data Set 2: Sitka spruce (Birkley wood) – SS (BW)

The physical and mechanical test data for the second data set was provided by the Centre for Timber Engineering at Edinburgh Napier University. The samples were taken from an eighty-three-year-old stand of Sitka spruce, known as Birkley wood, located in Kielder forest. A report was published on the effect of rotation length on the grade recovery
and wood properties, in which three hundred boards from thirty trees were analyzed (Moore & Lyon, 2008).

The parent tree, log, and radial position of each of the board was recorded to allow for linking of the materials in each step of production. For a subset consisting of fifty-six samples, parameters describing the knot defect geometry were also collected. A detailed description of the data collection method is provided by Moore and Lyon (2008).

The 56 boards had nominal dimensions of 100 mm x 47 mm x 3000 mm. As part of the previously mentioned study, each board was photographed on all four sides and the size and position of the knots were determined using image processing and analysis (Moore & Lyon, 2008). Each flat side was denoted as either A or B, while the edges were labeled as C or D.

Assuming an elliptical geometry, the maximum and minimum diameters, \( D_{\text{max}} \) and \( D_{\text{min}} \), and the angle of the maximum diameter with respect to the longitudinal axis, \( \theta_{\text{max}} \), were recorded for each knot (Figure 51). In contrast to the Eastern hemlock data set, the knot depth was not measured, providing only a two-dimensional knot characterization. Because knot depth is not recorded, the knot characterizations on the parallel flat sides are considered independent of one another for grading purposes.

Mechanical testing of the timber was performed at Edinburgh Napier University. Each board was tested destructively in four-point-bending in accordance with EN408 to determine the local MOE (LocMOE), global MOE (GloMOE), and Modulus of Rupture (MOR) (Moore & Lyon, 2008). LVDTs were placed on either side of the specimen on a cradle that spans the 500 mm central portion of the beam, and the curvature of this length is what defines the Local MOE. Global MOE captures the Modulus of Elasticity along the
board’s full span and is defined by the displacement of the middle of the board with respect to the undisplaced location of the supports. These tests were performed as edgewise bending tests with loads applied at 600 mm from the supports. The global span is the distance between the supports, 1800 mm, and the local span is the central 500 mm between the applied loads (Figure 51).

Figure 51. Idealized knot geometry and test configuration for Sitka spruce boards. A majority of the boards were flat sawn. Each knot is characterized by the maximum and minimum diameters, $D_{\text{max}}$ and $D_{\text{min}}$, and the angle of the knot with respect to the longitudinal axis, $\theta_{\text{max}}$. Close-up shows board configuration during edgewise four-point bending tests and knot geometry characterization for Sitka spruce Birkley wood data set. Global and local Modulus of Elasticity and Modulus of Rupture were calculated.
5.2.2 Data Set 3: Sitka spruce (Baronscourt) – SS (BC)

The physical and mechanical test data for the third data set was also provided by the Centre for Timber Engineering at Edinburgh Napier University. Samples were taken from a fifty-seven-year-old stand of Sitka spruce, located on Baronscourt estate in County Tyrone, Northern Ireland. A report was published on the effect of early re-spacing on the physical and mechanical properties, in which four different re-spacing techniques were compared to a control (Moore et al., 2009).

The methods used for the geometric characterization and material property testing of this data set mirror the methods used in the previous data set. From the original Baronscourt estate stand, seventy-two trees were felled and processed into two hundred and five 3.7 m long sawn logs. The logs were processed into 1706 structural timber boards with cross-sectional dimensions of 100 mm x 47 mm and kiln-dried. A subset of fifty-six boards was selected for further mechanical testing and assessment of knots. Each piece of timber was photographed on all four faces and the size and location of each knot were determined using image analysis software.

As in the previous data set, destructive edgewise four-point bending tests were conducted on each board using a Zwick Z050 testing machine in accordance with the procedures described in EN 408. Global MOE, local MOE and MOR were calculated from the experimental results. The test set up is provided above in Figure 51.

A more detailed summary of the methods is provided in Moore et al. (2009).

5.2.3 Summary of Data

A summary of the three data sets is provided below in Table 22 and Figure 52.
<table>
<thead>
<tr>
<th>Data Set</th>
<th>Source</th>
<th>Location of Testing</th>
<th>Number of Samples</th>
<th>Type of Test</th>
<th>Span ($m$)</th>
<th>Material Properties Measured</th>
</tr>
</thead>
<tbody>
<tr>
<td>Eastern hemlock (EH)</td>
<td>Massachusetts, USA</td>
<td>UMass Amherst</td>
<td>27</td>
<td>3-pt flatwise bending</td>
<td>3.5</td>
<td>$MOE_{glo}$</td>
</tr>
<tr>
<td>Sitka spruce (BW)</td>
<td>Northumberland, England</td>
<td>Edinburgh Napier</td>
<td>56</td>
<td>4-pt edgewise bending</td>
<td>1.8</td>
<td>$MOE_{glo}$, $MOE_{loc}$, $MOR$</td>
</tr>
<tr>
<td>Sitka spruce (BC)</td>
<td>Country Tyrone, Northern Ireland</td>
<td>Edinburgh Napier</td>
<td>56</td>
<td>4-pt edgewise bending</td>
<td>1.8</td>
<td>$MOE_{glo}$, $MOE_{loc}$, $MOR$</td>
</tr>
</tbody>
</table>

Figure 52. Scaled dimensions of the Eastern hemlock (EH) and Sitka spruce (SS) samples. The Eastern hemlock boards have dimensions 81 mm x 33 mm x 3658 mm and the Sitka spruce boards have dimensions 100 mm x 47 mm x 3000 mm. The Eastern hemlock boards were tested elastically in three-point bending to calculate the longitudinal Modulus of Elasticity. The Sitka spruce Boards were tested to failure in four-point edgewise bending to evaluate global and local Modulus of Elasticity and Modulus of Rupture.

A comparison of the Eastern hemlock, Sitka spruce (BW), and Sitka spruce (BC) show that the knot statistics in terms of number and size of knots are comparable, suggesting that Eastern hemlock and Sitka spruce are qualitatively similar to one another.
in that regard. Figure 53 shows photographs of example knot geometries for Eastern hemlock and Sitka spruce.

Figure 53. Selected examples of Eastern hemlock and Sitka spruce (BC) flat side knot geometry.

The number of knots is normalized to the area of the board, such that

\[ N = \frac{n}{w_b l_b} \]  \hspace{1cm} (Eq. 77)

where \( N \) is the normalized number of knots, \( n \) is the number of knots on the surface area of the board, and \( w_b \) and \( l_b \) are the length and width of the board, respectively. The results of this calculation are provided in Figure 54 for the flat sides of the three data sets.

(a) Flat Side A \hspace{1cm} (b) Flat Side B
Figure 54. The number of knots on a board normalized to the board's surface area, for the flat sides of twenty-seven Eastern hemlock, fifty-six Sitka spruce (Birkley wood), and fifty-six Sitka spruce (Baronscourt) samples.

The average knot diameter for each knot is calculated by,

$$
\overline{D} = \frac{D_{max} + D_{min}}{2}.
$$

(Eq. 78)

Figure 55 provides the distributions of the average knot diameters for the flat side faces of the three data sets.

This data is also provided in Table 23 for both flat and edge sides of the data sets. From a qualitative perspective, the distributions of these parameters are similar among the three data sets.
Table 23. Eastern hemlock, Sitka spruce (BW), and Sitka spruce (BC) knot geometry statistics, including the normalized number of knots and mean diameter.

<table>
<thead>
<tr>
<th></th>
<th>Number of Samples</th>
<th>Normalized Number of Knots, $N_{mean \pm std, X \times 10^{-3}}$</th>
<th>Average Diameter $mean \pm std (cm)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Eastern hemlock</td>
<td>Flat A</td>
<td>27</td>
<td>3.8 ± 3.5</td>
</tr>
<tr>
<td></td>
<td>Flat B</td>
<td>27</td>
<td>3.2 ± 2.0</td>
</tr>
<tr>
<td>Sitka spruce (BW)</td>
<td>Flat A</td>
<td>56</td>
<td>5.2 ± 1.8</td>
</tr>
<tr>
<td></td>
<td>Flat B</td>
<td>56</td>
<td>5.4 ± 1.7</td>
</tr>
<tr>
<td></td>
<td>Edge C</td>
<td>56</td>
<td>5.0 ± 2.6</td>
</tr>
<tr>
<td></td>
<td>Edge D</td>
<td>56</td>
<td>5.7 ± 3.2</td>
</tr>
<tr>
<td>Sitka spruce (BC)</td>
<td>Flat A</td>
<td>56</td>
<td>4.7 ± 1.8</td>
</tr>
<tr>
<td></td>
<td>Flat B</td>
<td>56</td>
<td>4.8 ± 2.4</td>
</tr>
<tr>
<td></td>
<td>Edge C</td>
<td>56</td>
<td>6.1 ± 3.8</td>
</tr>
<tr>
<td></td>
<td>Edge D</td>
<td>56</td>
<td>5.4 ± 3.7</td>
</tr>
</tbody>
</table>

The mechanical properties from the three data sets are also compared. The results for the global MOE (GloMOE) and local MOE (LocMOE) are provided in Figure 56.
Figure 56. Modulus of Elasticity resulting from experimental bending tests. Eastern hemlock (EH) was tested in three-point flatwise bending (span length, 3.5 m), while both Sitka spruce (SS-BW and SS-BC) were tested in four-point edgewise bending (span length, 1.8 m).

Both Sitka spruce data sets were tested to failure in edgewise four-point bending to evaluate the Modulus of Rupture (MOR). The resulting histograms, mean, and standard deviations are shown in Figure 57.
Figure 57. Distribution of Modulus of Rupture for Sitka spruce (BW) and Sitka spruce (BC) tested in edgewise four-point bending (span length, 1.8 m).

Although these data sets contain different species grown in different environments, in the context of mechanical properties and knot geometry, the three data sets are similar when considering the statistical properties. Thus, the same statistical methods and analytical techniques developed in this paper are applicable for all three data sets when investigating knots' influence on mechanical properties.

5.3 Knot Geometry

Knots can be characterized in a number of ways when considering their influence on the effective mechanical properties. A common method uses a Knot Area Ratio (KAR), which describes the ratio between the knot area and the total board area. Using two-dimensional knot geometry, the Surface Knot Area Ratio ($KAR_s$) can be defined, while the Cross-Sectional Knot Area Ratio ($KAR_x$) can be defined using three-dimensional knot geometry. Knots located in highly stressed regions can have a larger influence on board capacity than knots located in low-stress regions. To capture this effect, weighting factors
can be calculated and applied based on the moment diagram along the span during loading or by considering the stress profile through the depth of the beam.

5.3.1 Surface Knot Area Ratio

The Surface Knot Area Ratio \((KAR_s)\) represents the amount of knot area on the surface of the board compared to the surface area of the board along the longitudinal-tangential plane of the board. This parameter only considers the two-dimensional knot geometry by neglecting the depth of the knot through the board. For simplicity, a two-dimensional surface meshing method was used to calculate the Surface Knot Area Ratio. As shown in Figure 58, a two-dimensional mesh is generated along the surface of the board. The red markers highlight the knot boundary on the face of the tangential-longitudinal plane, while nodes highlighted green are within the knot boundary. For each tangential cross-section along the longitudinal axis, the surface Knot Area Ratio is calculated by

\[
KAR_s(x) = \frac{n_b(x)}{n_y}
\]  

(Eq. 79)

where \(n_b(x)\) is the number of nodes within the knot boundary along the tangential axis (highlighted in blue in Figure 58), \(n_y\) is the number of nodes along the tangential axis, and \(KAR_s(x)\) is the surface Knot Area Ratio along the longitudinal axis. Of course, the \(KAR_s\) can also be calculated using the maximum and minimum knot diameters and the equation of an ellipse. The results for both methods are indistinguishable, particularly at fine mesh sizes.

The advantage of the two-dimensional surface knot area method is that it only requires the knot geometry visible on the surface of the board, which is easily accessible through a number of manual or automated techniques.
Figure 58. A mesh is generated over the surface of the board and the number of nodes within the knot boundary divided by the number of nodes in the tangential cross-section determines the Surface Knot Area Ratio ($KAR_s$) along the longitudinal axis (Eq. 79).

### 5.3.2 Cross-Sectional Knot Area Ratio

The Cross-Sectional Knot Area Ratio ($KAR_x$) captures the three-dimensional knot geometry in a given radial-tangential cross-section along the longitudinal axis. A three-dimensional mesh is generated within the board boundaries Figure 59. The red markers indicate the knot geometry boundary through the depth of the board on the radial-tangential plane. Nodes highlighted in green indicate the region inside the knot boundary. For each cross-section, the number of mesh nodes within the knot boundary is divided by the number of nodes in the cross-section mesh such that, the Cross-Sectional Knot Area Ratio is

$$KAR_x(x) = \frac{n_b(x)}{n_y n_z}$$

(Eq. 80)

where $n_b(x)$ is the number of nodes within the knot boundary, $n_y$ and $n_z$ are the number of nodes in the tangential and radial axes, and $KAR_x(x)$ is the Cross-Sectional Knot Area Ratio along the longitudinal axis.
Figure 59. A three-dimensional mesh is generated and the Cross-Sectional Knot Area Ratio ($KAR_x$) is calculated by dividing the number of nodes within the knot boundary by the number of nodes in the cross-section (Eq. 80).

The two methods do not yield different results for cylindrical through knots. However, the Surface Knot Area Ratio and Cross-Sectional Knot Area Ratio will vary in the case of through knots growing in a direction, $G$, through the depth of the board, conical through knots, and non-through knots.

### 5.3.3 Span Weighted Knot Area Ratio

Knots are considered to cause localized regions of low stiffness and strength in wood, through disturbance in the grain around the knots, and possible discontinuities in the wood in the form of dead knots, included bark and drying cracks. The influence of knots on the effective mechanical properties of the overall board is related to the location of the knot. For example, a knot is expected to cause a more significant decrease in effective stiffness and strength if it is located in a region of high stress. A span weighted surface Knot Area Ratio ($SKAR_s$) and a span weight cross-sectional Knot Area Ratio ($SKAR_x$) is
defined to account for this relationship between knot location and regions of high stress due to loading:

\[ SKAR_s(x) = S_lKAR_s(x) \]  \hspace{1cm} (Eq. 81)

\[ SKAR_x(x) = S_lKAR_x(x) \]  \hspace{1cm} (Eq. 82)

where \( S_l \) is a discrete weighting factor applied along the length of the beam such that regions of high stress for a defined loading condition will have a larger weight than regions of low stress. These span weights are determined by calculating the ratio between the theoretical displacement of a beam with a localized region of low stiffness to the theoretical displacement of a beam with homogeneous stiffness. A localized region of low stiffness is shifted along the beam span length and the beam deflection is calculated. The ratio of the maximum displacement of the beam with a localized region of low MOE \( (y_i) \) and the maximum displacement of a beam with homogenous MOE \( (y_0) \) is calculated such that

\[ R_{span_i} = \frac{y_i}{y_0}, \]  \hspace{1cm} (Eq. 83)

where \( i \) relates to the location of the low MOE region. In this case, it is assumed that the ratio between the low MOE region \( (E_L) \) and the MOE of the remainder of the beam and the homogenous beam \( (E_0) \) is 0.5:

\[ E_L = 0.5E_0. \]  \hspace{1cm} (Eq. 84)

The choice of the ratio \( E_L/E_0 \) will impact the resulting span weight factors; as \( E_L/E_0 \) decreases, the span weight factors increase.

The global span weights, \( S_{glo} \), are the maximum ratio for each beam:

\[ S_{glo_i} = \max(R_{span_i}). \]  \hspace{1cm} (Eq. 85)

These span weights are shown in Figure 60 for three-point bending and Figure 61 for four-point bending.
The local MOE is the modulus of elasticity in the center 500 mm of the beam. The local span weights are calculated with respect to the LVDT displacement, \( d_{LVDT} \), such that

\[
S_{loc_i} = \frac{d_{LVDT} - d_{max}}{D_{LVDT} - D_{max}} \tag{Eq. 86}
\]

where \( D_{LVDT} \) and \( D_{max} \) are the displacements of the homogenous beam and \( d_{LVDT} \) and \( d_{max} \) are the displacements of the beam with a localized region of low stiffness. Results are presented in Figure 61 for four-point bending.
5.3.4 Depth Weighted Knot Area Ratio

In bending, extreme fibers experience larger stresses than those close to the neutral axis. Thus, defects located on the tension or compression strand are expected to have a larger influence on the board's effective mechanical properties than knots close to the neutral axis. To account for this, a weight related to the moment of inertia is calculated. This depth weight, $D$, is determined by

$$D = \left(\frac{2\bar{y}}{d}\right)^2$$

(Eq. 87)

where $\bar{y}$ is the distance between the knot center coordinate along the depth of the board and the neutral axis and $d$ is the depth of the beam. The depth weight factors for the Sitka spruce board geometry are shown in Figure 62. This analysis is only applied to the Sitka spruce boards, as knot geometries along the edge side of the Eastern hemlock boards were
not recorded. Additionally, the Eastern hemlock samples are less suited to this type of analysis because they were flatsawn and tested in flatwise bending.

Figure 62. Depth weights are modeled by the assumed stress profile through the depth of the board. Depth weights are applied to Knot Area Ratios at each cross-section along the longitudinal axis.

Knots have a greater impact on tensile properties than compressive properties (Ross & USDA Forest Service., 2010). The depth weighting factors applied in this chapter neglect this effect, since the orientation of the boards during testing was not recorded.

The depth weights are applied by,

\[
DKAR_s(x) = D_l KAR_s(x) \quad \text{(Eq. 88)}
\]

\[
DKAR_x(x) = D_l KAR_x(x) \quad \text{(Eq. 89)}
\]

5.4 Relationship between Knot Geometry and Effective Material Properties

The relationship between knots and the effective mechanical properties of dimensional lumber in bending is investigated visually and numerically in the following sections. Knot geometry is characterized by four ways: surface knot area ratio \((KAR_s)\), cross-sectional knot area ratio \((KAR_x)\), span weighted knot area ratio \((SKAR_s\) or \(SKAR_x)\),
and depth weighted knot area ratio ($DKAR_s$ or $DKAR_x$). Regardless of how the knot geometry is characterized, the results suggest that the relationships between the knot characteristics and elastic material properties are modest at best. In general, and as expected, the results do not show any meaningful relationship between the knot characteristics and the Modulus of Elasticity. When considering the strength properties, there is a slight increase in correlation coefficients when considering knot characteristics and Modulus of Rupture, but the relationships are inconsistent, and no meaningful relationship was detected. This is likely because no distinction was made between the tension and compression sides of the board in edgewise bending.

5.4.1 Comparison between Surface Knot Area Ratio and Cross-Sectional Knot Area Ratio

As described in Sections 5.3.1 and 5.3.2, the knot geometry can be characterized in two-dimensions by the Surface Knot Area Ratio or in three-dimensions by the Cross-Sectional Knot Area Ratio. The Surface Knot Area Ratio considers the knot geometry on the faces of the board, while the Cross-Sectional Knot Area Ratio also considers the depth of the knots through the boards.

The maximum $KAR_s$ and $KAR_x$ is defined as the maximum Knot Area Ratio along the length of the board. When considering the Eastern hemlock data set, the correlation coefficient, $\rho$, between the Modulus of Elasticity and the maximum $KAR_s$ is $-0.43$ and $-0.38$ for Flat Sides A and B of the boards, respectively. When the three-dimensional $KAR_x$ is considered, the correlation coefficient is $-0.45$. 
Figure 63. Relationship between longitudinal Modulus of Elasticity and Maximum Cross-Sectional Knot Area Ratio (a) and Maximum Surface Knot Area Ratio (b). Because there is not a meaningful difference between the correlation coefficients for the Surface Knot Area Ratio and the Cross-Sectional Knot Area Ratio, the Surface Knot Area Ratio is sufficient in characterizing the knot geometry and the depth of knots through the thickness of the board can be neglected.

The correlation coefficients show a moderate correlation between the Modulus of Elasticity and the Surface and Cross-Sectional Knot Area Ratios. More significantly, however, there is not a notable difference between the $KAR_s$ and $KAR_x$, suggesting that surface characterizations of knot area are equally predictive for elastic properties as characterizations that include knot depth. Similar relationships are also observed between the Modulus of Elasticity and the summation of the Knot Area Ratio along the length of the board. Thus, for simplicity, the remainder of this paper only considers $KAR_s$.

5.4.2 Relationship Between Knot Characteristics and Stiffness Properties

The relationship between the elastic material properties and the observed knot geometries is investigated through the correlation coefficient and the slope of the linear best fit line. While only selected relationships have been presented in the following figures,
the correlation coefficients are provided in the tables for all relationships and the scatter plots of the remaining parameters are similar.

Figure 64 presents the relationship between the global Modulus of Elasticity and the global maximum Surface Knot Area Ratio for the flat sides of the Eastern hemlock and the edge sides of the Sitka spruce (BW) and Sitka spruce (BC). Recall, these sides represent the tension and compression sides of the boards in bending. The maximum Surface Knot Area Ratio is defined as the largest Surface Knot Area Ratio within the global span, the span between the two supports. The maximum Surface Knot Area Ratio reflects the critical knot, used to define the grade according to visual grading methods.

For all data sets, the slope of the best fit line and the correlation coefficients are negative, indicating a negative relationship between knot geometry and modulus of elasticity. However, it is clear that each data set has a different relationship between the Knot Area Ratio and the Modulus of Elasticity. The Eastern hemlock results show a very similar slope between the linear regression lines on Flat Sides A and B, as well as similar correlation coefficients. In contrast, both Sitka spruce data sets have a notable difference between Edge Sides C and D with respect to the relationships between the Modulus of Elasticity and the Knot Area Ratio. This is most dramatically seen in the Sitka spruce (BW) data set, which has a moderately high correlation coefficient for Edge Side C, $\rho = -0.67$, and approximately zero correlation for Edge Side D. Sitka spruce (BC), in contrast, has zero correlation for Edge Side C and $-0.29$ for Side Edge Side D. As previously mentioned, knots have a greater impact on local tension than local compression, which could be an explanation for the results showing a much higher correlation coefficient on Side D than Side C. However, the naming convention was not consistent in each data set a
(i.e., Edge Side C was not always the tension strand and Edge Side D was not always the compression strand) and no efforts were made to orient the boards such that the limiting defect was consistently tested in tension. Therefore, a much higher correlation on one side, as opposed to the other, is not expected.

Figure 64. Relationships between Global Modulus of Elasticity and Maximum Global Surface Knot Area Ratio for Eastern hemlock, Sitka spruce (Birkley wood), and Sitka spruce (Baronscourt) data sets. The linear best fit line plus/minus one standard deviation is plotted. The slope, $m$, of the linear best fit lines and the correlation coefficient, $\rho$, are provided. Results do not show a consistent relationship between Knot Area Ratio and Modulus of Elasticity among the three data sets.

This relationship can also be investigated along the local span, the center 500 mm of the beam. Further, while the maximum Surface Knot Area Ratio describes the critical knot, the summation of the Surface Knot Area Ratio provides an understanding of the knots along the board. Correlation coefficients for these parameters are provided below in Table 24. The results show a very small difference when considering the total Surface Knot Area Ratio and the maximum Surface Knot Area Ratio. Similarly, there is not a significant difference in results when comparing the global span knot geometries to the global MOE and the local span knot geometries to the local MOE. Additionally, total $KAR_s$ has a very
low slope for the linear best fit, indicating that there is no meaningful relationship between the summation of the Surface KAR and the MOE.

Table 24. Slope of linear best fit and Correlation Coefficient, $\rho$, between Modulus of Elasticity and Surface Knot Area Ratio on Compression and Tension sides of Boards in Bending. Maximum $KAR_s$ is defined as the largest $KAR_s$ value in either the global or local span. Total $KAR_s$ is defined as the summation of the $KAR_s$ along either the global or local span of the board.

<table>
<thead>
<tr>
<th></th>
<th>Total $KAR_s$</th>
<th></th>
<th>Maximum $KAR_s$</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Local $\rho$</td>
<td>$\rho$ slope</td>
<td>Local $\rho$</td>
<td>$\rho$ slope</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>EH</td>
<td>Flat Side A</td>
<td>$N/A$</td>
<td>$N/A$ -0.49</td>
<td>$N/A$</td>
</tr>
<tr>
<td></td>
<td>Flat Side B</td>
<td>$N/A$</td>
<td>$N/A$ -0.41</td>
<td>$N/A$</td>
</tr>
<tr>
<td>SS (BW)</td>
<td>Edge Side C</td>
<td>$-0.49$</td>
<td>$-0.64$ -0.01</td>
<td>$-0.59$ -6.7</td>
</tr>
<tr>
<td></td>
<td>Edge Side D</td>
<td>$-0.17$</td>
<td>$-0.01$ 0.00</td>
<td>$-0.19$ -2.5</td>
</tr>
<tr>
<td>SS (BC)</td>
<td>Edge Side C</td>
<td>$-0.05$</td>
<td>$-0.18$ 0.00</td>
<td>$0.03$ -0.48</td>
</tr>
<tr>
<td></td>
<td>Edge Side D</td>
<td>$-0.05$</td>
<td>$-0.26$ 0.00</td>
<td>$-0.05$ -0.43</td>
</tr>
</tbody>
</table>

As described in Section 5.3.3, the span weight Knot Area Ratio, $SKAR_s$, is considered to account for the assumed larger influence of knots on effective mechanical properties in high-stress regions. Figure 65 presents the relationships between the longitudinal MOE and the maximum span weight Surface Knot Area Ratio for the global tension and compression strands of each data set. The correlation coefficients and slope of the linear best fit line are not consistently or meaningfully increased by considering the span factor.
Figure 65. Relationships between Global Modulus of Elasticity and Maximum Span Weighted Global Surface Knot Area Ratio for Eastern hemlock, Sitka spruce (Birkley wood), and Sitka spruce (Baronscourt) data sets.

The correlation coefficients for the local and global spans for the total and maximum Span Weighted Surface Knot Area Ratio are provided below.

Table 25. Slope of linear best fit, \( m \), and Correlation Coefficients, \( \rho \), between Modulus of Elasticity and Surface Knot Area Ratio considering Span Factor on Compression and Tension sides of Boards in Bending.

<table>
<thead>
<tr>
<th></th>
<th>Total ( SKAR_S )</th>
<th>Maximum ( SKAR_S )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Local</td>
<td>Global</td>
</tr>
<tr>
<td></td>
<td>( \rho )</td>
<td>slope</td>
</tr>
<tr>
<td><strong>EH</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Flat</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Side A</td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.47</td>
<td>-0.01</td>
</tr>
<tr>
<td></td>
<td>0.08</td>
<td>0.00</td>
</tr>
<tr>
<td></td>
<td>Edge</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Side C</td>
<td></td>
</tr>
<tr>
<td></td>
<td>-0.14</td>
<td>0.00</td>
</tr>
<tr>
<td></td>
<td>Edge</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Side D</td>
<td></td>
</tr>
<tr>
<td></td>
<td>-0.05</td>
<td>0.00</td>
</tr>
</tbody>
</table>
The relationship between the knot geometries on the board's surfaces parallel to the loading and the longitudinal modulus of elasticity is also investigated. In this case, only Sitka spruce Flat Sides A and B are presented, as knot measurements for the edges of the Eastern hemlock knot geometry were not recorded.

Figure 66. The relationship between MOE and the knot geometry on the board’s surfaces parallel to loading (Flat Side A and Flat Side B) are studied for the two Sitka spruce data sets.

The relationship between the global MOE and the global maximum Surface Knot Area Ratio is provided in Figure 67.
Figure 67. Relationships between Global Modulus of Elasticity and Maximum Global Surface Knot Area Ratio for board sides parallel to loading of Sitka spruce (Birkley wood) and Sitka spruce (Baronscourt) data sets.

The correlation coefficients for the remaining parameters are provided in Table 26.

The results indicate no detectable meaningful relationship between the Surface Knot Area Ratio and the Modulus of Elasticity.

Table 26. Slope of linear best fit and Correlation Coefficients, \( \rho \), between Modulus of Elasticity and Surface Knot Area Ratio for board sides parallel to loading.

<table>
<thead>
<tr>
<th></th>
<th>Total ( KAR_s )</th>
<th>Maximum ( KAR_s )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Local ( \rho )</td>
<td>Global ( \rho )</td>
</tr>
<tr>
<td></td>
<td>slope</td>
<td></td>
</tr>
<tr>
<td>SS</td>
<td>Flat Side A</td>
<td>( -0.22 ) -0.01</td>
</tr>
<tr>
<td></td>
<td>Flat Side B</td>
<td>( -0.23 ) -0.01</td>
</tr>
<tr>
<td>SS (BC)</td>
<td>Flat Side A</td>
<td>( -0.01 ) -0.00</td>
</tr>
<tr>
<td></td>
<td>Flat Side B</td>
<td>( -0.04 ) -0.00</td>
</tr>
</tbody>
</table>

Finally, the depth weight factor described in Section 5.3.4 is considered. The results for the global maximum depth weighted Surface Knot Area Ratio is provided in Figure 68.
Figure 68. Relationships between Global Modulus of Elasticity and Maximum Global Surface Knot Area Ratio considering Depth Factor for board sides parallel to loading of Sitka spruce (Birkley wood) and Sitka spruce (Baronscourt) data sets.

The correlation coefficients between the local, global, total, and maximum Surface Knot Area Ratio are presented in Table 27. There is not a significant increase in the relationships when considering the Depth Weight.
Table 27. Slope of linear best fit and Correlation Coefficients, $\rho$, between Modulus of Elasticity and Surface Knot Area Ratio considering Depth weight for board sides parallel to loading.

<table>
<thead>
<tr>
<th></th>
<th>Total $DKAR_s$</th>
<th>Maximum $DKAR_s$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Local</td>
<td>Global</td>
</tr>
<tr>
<td>$\rho$</td>
<td>$\rho$ slope</td>
<td>$\rho$ slope</td>
</tr>
<tr>
<td>SS (BW)</td>
<td>Flat Side A</td>
<td>$-0.18$ $-0.02$</td>
</tr>
<tr>
<td></td>
<td>Flat Side B</td>
<td>$-0.12$ $-0.01$</td>
</tr>
<tr>
<td>SS (BC)</td>
<td>Flat Side A</td>
<td>$-0.02$ $-0.00$</td>
</tr>
<tr>
<td></td>
<td>Flat Side B</td>
<td>$-0.07$ $-0.01$</td>
</tr>
</tbody>
</table>

While the Wood Handbook acknowledges a low relationship between knots and Modulus of Elasticity, which aligns with the findings in this study, caution should be applied when drawing meaningful conclusions from the results presented in this paper. There were meaningful limitations in this study, specifically related to the small sample size and neglecting the distinction between tension and compression for the edgewise bending tests.

5.4.3 Relationship between Knot Characteristics and Strength Properties

The relationship between the measured knot geometries and the strength properties is also considered. The Modulus of Rupture was measured for only the Sitka spruce boards; thus, the Eastern hemlock samples are neglected from this aspect of the study.

Figure 69 presents a scatter plot for the maximum Surface Knot Area ratio and the Modulus of Rupture. The correlation coefficients for the maximum $KAR_s$ and the MOR are not consistent between the Birklsey wood and Baronscourt Sitka spruce data sets, although there is a moderate correlation for Side C of the Birklsey Wood. This suggests the
moderate correlation for one side of the Sitka spruce (BW) boards does not reflect a real, meaningful relationship between maximum Surface Knot Area Ratio and Modulus of Rupture.

Figure 69. Relationships between Modulus of Rupture and Surface Knot Area Ratio for Sitka spruce (Birkley wood) and Sitka spruce (Baronscourt) data sets.

Mirroring the elastic behavior response, the correlation coefficients are not increased when considering the span weights to account for the knot defect's location with respect to the applied load on the beam span.
Correlation coefficients for the total Surface Knot Area Ratio and the Modulus of Rupture are provided in Table 28. The results indicate that considering regions of high stress through the depth of the board does not increase the relationship between knots and MOR in a consistent or meaningful way.

Table 28. Slope of linear best fit and Correlation Coefficient, $\rho$, between Modulus of Rupture and Total and maximum Surface Knot Area Ratios considering and neglecting the span weight parameter.

<table>
<thead>
<tr>
<th></th>
<th>Total $KAR_s$</th>
<th>Total $SKAR_s$</th>
<th>Maximum $KAR_s$</th>
<th>Maximum $SKAR_s$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\rho$ slope</td>
<td>$\rho$ slope</td>
<td>$\rho$ slope</td>
<td>$\rho$ slope</td>
</tr>
<tr>
<td>SS (BW)</td>
<td>Edge Side C</td>
<td>-0.51 -0.04</td>
<td>-0.44 -0.08</td>
<td>-0.60 -40</td>
</tr>
<tr>
<td></td>
<td>Edge Side D</td>
<td>-0.14 -0.00</td>
<td>-0.17 -0.02</td>
<td>-0.24 -13</td>
</tr>
<tr>
<td>SS (BC)</td>
<td>Edge Side C</td>
<td>-0.12 -0.01</td>
<td>-0.22 -0.01</td>
<td>-0.12 -5.2</td>
</tr>
<tr>
<td></td>
<td>Edge Side D</td>
<td>-0.37 -0.01</td>
<td>-0.08 -0.00</td>
<td>-0.37 -18</td>
</tr>
</tbody>
</table>
The relationship between the Modulus of Rupture and the knot characteristics on the board's faces parallel to loading is investigated in Figure 71 and Figure 72. Further, there is a notable decrease in correlation when considering the depth weights.

(a) Sitka spruce (BW)
(b) Sitka spruce (BC)

Figure 71. Relationships between Modulus of Rupture and Surface Knot Area Ratio for sides parallel to loading for Sitka spruce (Birkley wood) and Sitka spruce (Baronscourt) data sets.

(a) Sitka spruce (BW)
(b) Sitka spruce (BC)

Figure 72. Relationships between Modulus of Rupture and Surface Knot Area Ratio considering depth factor for sides parallel to loading for Sitka spruce (Birkley wood) and Sitka spruce (Baronscourt) data sets.
Table 29 provides the correlation coefficients for the total and maximum Surface Knot Area Ratios for the Sitka spruce data sets. Considering the maximum Surface Knot Area Ratio results in a slightly higher correlation coefficient than considering the total Surface Knot Area Ratio. However, these relationships are modest and inconsistent at best.

Table 29. Slope and Correlation Coefficient, $\rho$, between Modulus of Rupture and Total and maximum Surface Knot Area Ratios neglecting and considering the depth weight.

<table>
<thead>
<tr>
<th></th>
<th>Total $KAR_s$</th>
<th>Total $DKAR_s$</th>
<th>Maximum $KAR_s$</th>
<th>Maximum $DKAR_s$</th>
</tr>
</thead>
<tbody>
<tr>
<td>SS (BW)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Flat Side A</td>
<td>$\rho$ = -0.47, slope = -0.03</td>
<td>$\rho$ = -0.38, slope = -0.07</td>
<td>$\rho$ = -0.51, slope = -49</td>
<td>$\rho$ = -0.31, slope = -64</td>
</tr>
<tr>
<td>Flat Side B</td>
<td>$\rho$ = -0.28, slope = -0.01</td>
<td>$\rho$ = -0.24, slope = -0.03</td>
<td>$\rho$ = -0.32, slope = -34</td>
<td>$\rho$ = -0.18, slope = -51</td>
</tr>
<tr>
<td>SS (BC)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Flat Side A</td>
<td>$\rho$ = -0.25, slope = -0.01</td>
<td>$\rho$ = -0.29, slope = -0.03</td>
<td>$\rho$ = -0.27, slope = -23</td>
<td>$\rho$ = -0.18, slope = -39</td>
</tr>
<tr>
<td>Flat Side B</td>
<td>$\rho$ = -0.19, slope = -0.01</td>
<td>$\rho$ = -0.12, slope = -0.01</td>
<td>$\rho$ = -0.22, slope = -21</td>
<td>$\rho$ = -0.20, slope = -46</td>
</tr>
</tbody>
</table>

The relationship between knots and strength properties is integral to North American grading rules and standards (American Society for Testing and Materials, 2014; NELMA, 2013; Ross & USDA Forest Service., 2010). While the results presented in this paper could not identify a relationship between knot geometry and strength capacity, extreme caution should be applied when drawing any meaningful results from these results due to significant limitations in the data sets. The sample size was limited both in number of samples, species, and size of lumber. The size effect was not considered and the distinction between the influence of knots in compression and tension was neglected.

5.4.4 Grading Simulation

The relationship between knots and effective material properties is particularly significant when considering its relevance to grading. Grading guidelines use knot defects’
size and location as a method to separate the lowest quality material. Thus, an additional analysis is performed to simulate the grading of the boards and to determine the output MOE properties. An indicating property is defined by one of the four KAR characteristics (total $KAR_s$, maximum $KAR_s$, total $SKAR_s$, or maximum $SKAR_s$). A grading threshold, which represents percent yield, is then selected. The boards are ranked by indicating property. The boards are then split into two categories by the grading threshold, such that a percentage of the boards fall below the grading threshold and a percentage of the boards pass the grading threshold. The mean output MOE and confidence intervals are calculated. This analysis was performed on both elastic and strength properties.

Table 30 provides an example of this analysis for the Eastern hemlock and Sitka spruce (BW and BC) data sets. The indicating property is the maximum Knot Area Ratio. The grading threshold is defined as 50%, such that the half of the boards with the largest KARs fall below the 50% grading threshold and the half of the boards with the smallest KARs pass the 50% grading threshold. The boards are sorted by the indicating property and the means of the output MOE, and 67% and 95% confidence intervals are calculated.

Results show that the mean MOE for the lower half of the boards is less than the mean MOE of the whole data set, and the mean MOE for the boards above the grading threshold is greater than the mean MOE of the whole data set for all three species. However, in many cases, the confidence intervals for the boards that fall below the grading threshold overlap with the confidence intervals for the whole data set. Aligned with the expected results, these results suggest that KAR is a weak indicating property for MOE.

Identical analyses were performed to consider all possible grading thresholds. However, the results did not show KAR as a strong indicating property. While only one
side of each data set and two indicating properties are presented in Table 30, the data set provided is representative of the results for the remaining samples and knot characterizations.

Table 30. Modulus of Elasticity of the Sitka spruce (Baronscourt) data set considering the Maximum Knot Area Ratio neglecting the weight factor and considering the weight factor. The grading threshold is defined as 50%. Mean MOE and corresponding 67% and 95% confidence intervals are provided.

<table>
<thead>
<tr>
<th></th>
<th>Neglecting Span Weight Factor</th>
<th>Considering Span Weight Factor</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\mu_{MOE}$ (GPa)</td>
<td>$\mu_{MOE}$ (67% CI) (GPa)</td>
</tr>
<tr>
<td>EH A</td>
<td>ALL</td>
<td>9.7</td>
</tr>
<tr>
<td></td>
<td>50% FAIL</td>
<td>8.6</td>
</tr>
<tr>
<td></td>
<td>50% PASS</td>
<td>10.7</td>
</tr>
<tr>
<td>SS (BW) C</td>
<td>ALL</td>
<td>9.0</td>
</tr>
<tr>
<td></td>
<td>50% FAIL</td>
<td>8.1</td>
</tr>
<tr>
<td></td>
<td>50% PASS</td>
<td>9.9</td>
</tr>
<tr>
<td>SS (BC) C</td>
<td>ALL</td>
<td>7.2</td>
</tr>
<tr>
<td></td>
<td>50% FAIL</td>
<td>7.0</td>
</tr>
<tr>
<td></td>
<td>50% PASS</td>
<td>7.4</td>
</tr>
</tbody>
</table>

An identical analysis is performed considering the Modulus of Rupture. Table 31 presents the results for the maximum KAR as the indicating property and 50% as the grading threshold. Consider the Maximum KAR neglecting the weighting factor for the Sitka spruce (BC) data set. The mean MOR considering all 56 boards is 33.5 GPa, the mean MOR for the boards that do not pass the grading threshold is 30.8 GPa, and the mean MOR for boards that pass the grading threshold is 36.2 GPa. However, both the 67% and 95% confidence intervals for the boards that pass the grading threshold overlap with the whole data set's confidence intervals.
While only a subset of the results is presented in this paper, the analysis was performed to consider all grading thresholds and indicating properties. The results presented are representative of the remaining samples and analyses.

Table 31. Strength properties of the Sitka spruce (Baronscourt) data set considering the maximum Knot Area Ratio neglecting the span weighting factor and considering the span weighting factor. The grading threshold is 50%. Mean Modulus of Rupture and corresponding confidence intervals are provided.

<table>
<thead>
<tr>
<th></th>
<th>Neglecting Span Weight Factor</th>
<th>Considering Span Weight Factor</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\mu_{MOE}$ (GPa)</td>
<td>$\mu_{MOE}$ (67% CI) (GPa)</td>
</tr>
<tr>
<td>SS (BW) C</td>
<td>ALL</td>
<td>45.3</td>
</tr>
<tr>
<td></td>
<td>50% FAIL</td>
<td>38.9</td>
</tr>
<tr>
<td></td>
<td>50% PASS</td>
<td>51.7</td>
</tr>
<tr>
<td>SS (BC) C</td>
<td>ALL</td>
<td>33.5</td>
</tr>
<tr>
<td></td>
<td>50% FAIL</td>
<td>30.8</td>
</tr>
<tr>
<td></td>
<td>50% PASS</td>
<td>36.2</td>
</tr>
</tbody>
</table>

5.5 Conclusions

The goal of grading lumber is to identify and segregate the lowest quality lumber. In visual grading methods, boards are rejected by making assumptions for the stiffness and strength capacity by evaluating visual features, such as knots. The relationship between knots and mechanical properties of dimensional lumber is integral to visual grading methods. In this study, the relationship between knots and effective bending Modulus of Elasticity and Modulus of Rupture is investigated for Eastern hemlock and Sitka spruce.

Knots are characterized in four ways: (1) summation of the KAR (knot area ratio), (2) summation of KAR considering a weighting factor, (3) the maximum/critical KAR, (4) the maximum/critical KAR considering a weighting factor. As expected, no relationship could be found between bending MOE and knot geometry. Regardless of knot characterization
method studied, correlation coefficients and slopes of the linear best fit lines provide no meaningful or consistent indicator of a relationship between knots and MOR.

While the Wood Handbook acknowledges a limited relationship between knots and overall board stiffness, the relationship between knots and strength properties is integral to North American grading rules and standards (American Society for Testing and Materials, 2014; NELMA, 2013; Ross & USDA Forest Service., 2010). Although a relationship between the knot geometry characterization investigated in this chapter and Modulus of Rupture could not be identified for the two data sets considered, extreme caution should be implemented when drawing meaningful conclusions from the results presented in this chapter. The results were greatly impacted by limitations and inconsistencies in the data sets, and likely caused the lack of relationship between knot geometry and MOR.

Specifically, published grading rules are based on very large, representative, data sets. The data sets discussed in the paper include only eighty-three Eastern hemlock boards and two sets of fifty-six Sitka spruce board. There were inconsistencies with testing methods; the Eastern hemlock boards were tested in flatwise bending, while the Sitka spruce samples were tested in edgewise bending. Additionally, as mentioned previously, knots have a greater impact on tensile strength than compression strength. This effect was neglected in this study and boards were not oriented such that the limiting defect was exposed to tension during the bending tests. It should also be noted that additional, compounding factors, were neglected in this study including, but not limited to: size effect, density, moisture content, and cross grain.
CHAPTER 6
APPLICATION OF VARIABILITY RESPONSE FUNCTION TO CROSS LAMINATED TIMBER

6.1 Introduction

Understanding the influence of randomly heterogeneous material properties on structural systems' performance is essential for reliable and efficient designs. This is a primary consideration in mass timber products, which are composed of boards with heterogeneous material properties. Specifically, Cross Laminated Timber (CLT) panel design is often driven by serviceability, displacement, and vibration. Therefore, the variability in displacement response of a CLT panel is of interest for both reliability and sensitivity analysis. The variability in displacement response is a result, in part, of the randomly heterogeneous modulus of elasticity in the constituent boards.

CLT panels are gaining popularity in the United States as a cost-effective and renewable alternative to traditional building materials. Specifically, Cross Laminated Timber (CLT) is a large-scale mass timber wood composite product composed of bi-directionally oriented layers of dimensional lumber boards. CLT panels are typically employed in floor, wall, and roof assemblies. Due to the natural growth of wood, material properties can vary significantly within a single board and within an ensemble of boards. Mass timber products, however, also allow an averaging of the randomly heterogeneous material properties. In composite products, like CLT, converting the random constituent material properties into effective material properties is an essential procedure in facilitating the analysis of variance of the response of the structure.

Typically, problems related to probabilistic and heterogeneous properties are solved by Monte Carlo simulations which are computationally intensive, particularly for
complex systems. Kline (1986) developed a second-order Markov model for generating the spatial variability in MOE. Further, Monte Carlo simulations have been applied to engineered products, predominately glue-laminated (glulam) timber beams, for reliability analysis (e.g., Foschi and Barrett 1980; Schaffer et al. 1986). More recently, a prediction model for the bending properties of glulam was developed using knot and MOE distributions as the main input variables (Lee et al., 2005).

This paper proposes a new analysis method by applying the Variability Response Function (VRF) to CLT geometry. The VRF method, first developed by Shinozuka (1987), convolves the structure’s deterministic boundary conditions and the spectral density function, such that changes to the spectral density function can be made without requiring multiple Monte Carlo simulations. This paper summarizes the VRF method and equations for applying the method to CLT products. Three case studies are provided as examples of the application of the method. However, this method is applicable to any three-layer CLT panel including lay-ups with combined species or lumber grades. This method is particularly valuable when considering the reliability and suitability of uncertified species for use in CLT.

6.2 Variability Response Function

The variability in the response of structural systems is driven by spatial variability in the system’s material properties, among other sources of variability such as geometry and loading. This variability in response is an important factor when considering the reliability and performance of structural systems and materials. Traditionally, these variabilities are studied by Monte Carlo simulation, which involves the simulation of random fields describing the random system parameters. The accuracy of the results is
dependent on the accuracy of the simulation algorithm of the random fields. Although this is a universal approach, it is computationally expensive, particularly for complex systems. Shinozuka (1987) proposed an alternative method to study the response variability for determinate linear structures using the spectral density function of the material property spatial variability. This method, referred to as the variability response function (VRF) method, uses a deterministic function to describe the structure’s boundary conditions and the spectral density function of the random material property to evaluate the variance of the response of the system. For the structural system studied in this paper, the variability in response refers to the variability in displacement due to spatial variability of modulus of elasticity. The VRF method assumes that random system parameters, often material properties, can be described by a statistically homogenous random field $f(x)$. In the case of variable MOE, this is expressed by

$$ E(x) = E_0 [1 + f(x)] $$

(Eq. 90)

where $E_0$ is the expected value of $E(x)$ and $f(x)$ is a stationary one-dimensional stochastic field with mean zero.

The fundamental expression for the variability of the response of a stochastic system using the VRF method is:

$$ \text{Var}[u(x)] = \int_{-\infty}^{\infty} S_f(\kappa) VRF(x, \kappa) d\kappa $$

(Eq. 91)

where $u(x)$ is the displacement of the system at a location $x$, $S_f(\kappa)$ is the spectral density function of the random field $f(x)$ which models the system stochasticity, and $k$ indicates the wavenumber (Teferra, 2012). The Variability Response Function ($VRF$) is a function that depends on the deterministic properties of the structural system, like loading and
boundary conditions, and identifies the sensitivity of a response quantity to the correlation structure of a property of the structure that is modeled as a homogenous random field.

This method was expanded to statically indeterminate structures by approximating the VRF through a Green’s function formulation (C.G. & Shinozuka, 1989; Kardara et al., 1990). These papers provide a small number of analytic solutions to stochastic structural systems, mainly for simple linearly elastic structures under static loading. Teferra expanded the VRF concept to include nonlinear statically determinate and indeterminate beam applications (Teferra, 2012).

The existence of a VRF for the effective material properties of a heterogeneous material for a statically determinate structure is proven by Arwade and Deodatis (2011). This method works by homogenizing a heterogeneous material through the equivalence of elastic strain energy in the heterogeneous and homogenous bodies. This work was later expanded to include statically indeterminate beams (Teferra et al., 2014).

The analytical evaluation of the VRF is cumbersome, even for very simple stochastic systems. Therefore, a Fast Monte Carlo simulation technique to approximate the VRF is also introduced as an extremely efficient numerical approach requiring a small sample size (Deodatis & Shinozuka, 1989). This approach was used in previously mentioned papers as a method to approximate the VRF for statically indeterminate structures. The Fast Monte Carlo method is clearly articulated by Miranda (2009) and is summarized below.

A random field of the form

\[ s(x) = \sqrt{2} \cos(\kappa x + \phi) \]  

(Eq. 92)
with random phase angles $\phi$ uniformly distributed in $[0, 2\pi]$, and spectral density function $S_f(k)$, generates a response variance, $Var[u(x)]$, such that the VRF is defined as

$$VRF(x, \kappa) = Var[u(x)].$$  \hspace{1cm} \text{(Eq. 93)}$$

If the response variance is computed independently through Monte Carlo simulations using realizations of $s(x)$, then the VRF can be estimated for every wavenumber of interest. The implementation of the fast Monte Carlo simulation is as follows:

1. Fix $x$
2. Fix $\kappa$
3. Generate $N_{sim}$ sample realizations of the random field described in Eq. 92 using:

$$s^{(j)}(x) = \sqrt{2} \cos(\kappa x + \phi^{(j)})$$

$$\phi^{(j)} = \left(j - \frac{1}{2}\right) \frac{2\pi}{N_{sim}}$$

for $j = 1, 2, ..., N_{sim}$.
4. For each sample $s^{(j)}(x)$ compute a sample deflection $u^{(j)}(x)$ from a deterministic structural analysis of the beam with bending stiffness $EI^{(j)}(x) = EIo/(1 + s^{(j)}(x))$.
5. Estimate the variance of the $N_{sim}$ sample responses $u^{(j)}(x)$ using the ensemble averages.
6. Calculate the value of the VRF using Eq. 93.
7. Repeat steps 2-6 for different values of $\kappa$ over a sufficiently wide range of wavenumbers.
6.3 Application of the Variability Response Function to Cross Laminated Timber

The natural growth of wood leads to highly heterogeneous material properties in sawn lumber. The elastic modulus can vary significantly between boards of the same species and even along the length of individual boards. Grading schemes have been developed to characterize wood quality and group boards of similar stiffness and strength. However, there is still significant variability in these categories. Mass timber products are composite beams, columns, and panels made of dimensional lumber. These composite products have an averaging effect on the material properties of the dimensional lumber, decreasing both the natural variation of material properties and, consequently, the variation in the structural performance of the wood. While it is clear that these composite products influence the variability of wood material properties and structural performance, no method other than the computationally intensive Monte Carlo method, to the authors' knowledge, has been used to show the variation in displacement response of a wood composite product as a result of random material properties, particularly the spatial variation of the modulus of elasticity.

This paper shows how the VRF method can be applied to mass timber products to understand how variability in the MOE of the constituent boards influences the variance of the displacement response of the composite product. While this method could be applied to other mass timber products, equations in this paper are derived to calculate the variability response function of a three-layer Cross Laminated Timber (CLT) panel in three-point bending. CLT panels consist of several layers of dimensional lumber boards stacked in orthogonal alternating directions, bonded with structural adhesives. They consist of an odd number of layers, typically three to seven. Constituent boards can vary in dimensions from
5/8 in (15.9 mm) to 2 in (50.8) in thickness and 2.4 in (61.0 mm) and 9.5 in (241.3 mm) in width. Panel sizes can vary by manufacturer, but typical widths range from 2 ft (609.6 mm) to 10 ft (3048 mm), with lengths up to 60 ft (18.3 m) (Karacabeyli & Brad, 2013).

![Figure 73. 406 mm wide three-layer Eastern hemlock Cross Laminated Timber panel fabricated at the University of Massachusetts Amherst Wood Mechanics Lab.](image)

The CLT panel considered for the purposes of this paper is modeled as a simply-supported beam in three-point bending, with varying stiffness, $EI_p(x)$. This loading condition was selected for simplicity and consistency with the experimental work on CLT panels at the University of Massachusetts Amherst Wood Mechanics Lab.

![Figure 74. Schematic of CLT panel in three-point bending](image)

The beam displacement is determined by equivalence of work and strain energy such that displacement is expressed by:
The panel stiffness is calculated by considering the effective properties of the top and bottom layers, which depend on the constituent boards' properties. Given the mean and variance of the MOE of the constituent boards, the effective MOE statistics for each layer of the panel and for the effective stiffness of the whole CLT panel can be calculated. The mid-layer, with orthogonally oriented boards, is excluded from the calculations as it is assumed to provide negligible bending stiffness. The properties of the constituent board can be defined by experimental material testing or assumed based on board species and grade.

The spectral density of the stiffness of the panel, $S_{EI_p}(k)$ is required to calculate the variability of the response of the system (Eq. 91). The covariance function and the spectral density function contain the same information about spatial variability and are related through the Fourier Transform,

$$S_l(x) = \mathcal{F}[C_l(\xi)]$$

(Eq. 95)

The covariance function of the MOE of the constituent boards can be estimated from experimental testing of the spatially varying MOE in the constituent boards and is modeled by an exponential decay function such that

$$C_l(\xi) = \sigma_l^2 e^{-\beta \xi}.$$  

(Eq. 96)

where $\xi$ is the lag, $\sigma_l^2$ is the variance of the constituent board, and $\beta$ is the rate of decay. Given the covariance function of the MOE of the constituent boards, the covariance function of the MOE of the effective layers and the covariance function of the stiffness of the CLT panel can be calculated. Consequently, the spectral density function of the MOE
of the layer and the stiffness of the CLT panel can be determined using the Fourier Transform.

Table 32 provides the variable naming conventions and an illustration of the composite product as it is fabricated from dimensional lumber, to layers, and finally a complete CLT panel.

Table 32. Constituent board, layer, and panel material properties.

<table>
<thead>
<tr>
<th>Material Property</th>
<th>Statistics</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Constituent Boards</strong></td>
<td>Mean of top boards: ( \mu_{1t}, \mu_{2t}, ... \mu_{Nt} )</td>
</tr>
<tr>
<td></td>
<td>Variance of top boards: ( \sigma_{1t}^2, \sigma_{2t}^2, ... \sigma_{Nt}^2 )</td>
</tr>
<tr>
<td></td>
<td>Covariance top boards: ( C_{1t}(\xi), ... C_{Nt}(\xi) )</td>
</tr>
<tr>
<td>MOE</td>
<td>Mean of bottom boards: ( \mu_{1b}, \mu_{2b}, ... \mu_{Nb} )</td>
</tr>
<tr>
<td></td>
<td>Variance of bottom boards: ( \sigma_{1b}^2, \sigma_{2b}^2, ... \sigma_{Nb}^2 )</td>
</tr>
<tr>
<td></td>
<td>Covariance bottom boards: ( C_{1b}(\xi), ..., C_{Nb}(\xi) )</td>
</tr>
<tr>
<td><strong>Layers</strong></td>
<td>Effective MOE of top/bottom layer: ( E_t(x), E_b(x) )</td>
</tr>
<tr>
<td></td>
<td>Effective mean of top/bottom layer: ( \mu_{E_t}, \mu_{E_b} )</td>
</tr>
<tr>
<td></td>
<td>Effective variance of top/bottom layer: ( \sigma_{E_t}^2, \sigma_{E_b}^2 )</td>
</tr>
<tr>
<td></td>
<td>Covariance top/bottom layer: ( C_{E_t}(\xi), C_{E_b}(\xi) )</td>
</tr>
<tr>
<td><strong>Panel</strong></td>
<td>Effective Panel Stiffness: ( EI_p(x) )</td>
</tr>
<tr>
<td>EI</td>
<td>Effective mean Panel Stiffness: ( \mu_{EI_p} )</td>
</tr>
<tr>
<td></td>
<td>Effective variance of Panel Stiffness: ( \sigma_{EI_p}^2 )</td>
</tr>
<tr>
<td></td>
<td>Covariance function of Panel Stiffness: ( C_{EI_p}(\xi) )</td>
</tr>
</tbody>
</table>

Consider a CLT panel with \( N \) constituent boards in each layer. Assuming the cross-sectional area of each of the constituent boards is equal, the constituent board material properties are used to calculate the effective layer MOE by

\[
\overline{E_L} = \frac{\sum_{i=1}^{N} E_i(x)}{N} \quad \text{(Eq. 97)}
\]
Assuming stationarity, the mean of the effective MOE for each layer is

\[ \mu_{EL} = \frac{\sum_{i=1}^{N} \mu_i}{N} \]  

(Eq. 98)

and the variance of the effective MOE for each layer is

\[ \sigma_{EL}^2 = \frac{\sum_{i=1}^{N} \sigma_i^2}{N^2}. \]  

(Eq. 99)

The covariance function for the effective layer is then

\[ C_{EL}(\xi) = \frac{\sum_{i=1}^{N} C_i(\xi)}{N^2} \]  

(Eq. 100)

and the spectral density function of the effective layer is

\[ S_{EL}(k) = \frac{2}{\pi N^2} \left( \sum_{i=1}^{N} \sigma_i^2 \beta_i \left( \frac{1}{k^2 + \beta_i^2} \right) \right). \]  

(Eq. 101)

For a CLT panel in bending, the panel bending stiffness, \( EI_p \), governs bending displacements. The CLT panel's middle layer is neglected in analysis, as it is assumed to provide negligible bending stiffness.

![Cross-section of CLT panel with defined geometric properties](image)

Figure 75. Cross-section of CLT panel with defined geometric properties

The effective panel stiffness is defined by

\[ EI_p(x) = \frac{4NAh_b^2}{(n(x) + 1)^2} \left[ n(x)^2 \overline{E}_i(x) + \overline{E}_b(x) \right]. \]  

(Eq. 102)
where \( n(x) \) is

\[
n(x) = \frac{E_b(x)}{E_t(x)}
\]  
(Eq. 103)

and \( A \) is the cross-sectional area of the constituent boards,

\[
A = h_b b_b
\]  
(Eq. 104)

However, using this definition of \( EI_p(x) \), the mean panel stiffness cannot be expressed analytically using the expectation operator. Therefore, for simplicity, it is assumed that the neutral axis is constant at the mid-plane of the panel. Consequently,

\[
n(x) = \frac{\mu_{E_b}}{\mu_{E_t}}
\]  
(Eq. 105)

Applying this assumption, the effective panel stiffness is:

\[
EI_p(x) = NAd_b^2 [n^2 \overline{E_t}(x) + \overline{E_b}(x)].
\]  
(Eq. 106)

where

\[
d_b = \frac{2h_b}{n + 1}
\]  
(Eq. 107)

The mean panel stiffness is then

\[
\mu_{EI_p(x)} = NAd_b^2 (n^2 \mu_{E_t} + \mu_{E_b})
\]  
(Eq. 108)

and the variance is

\[
\sigma_{EI_p(x)}^2 = N^2 A^2 d_b^4 (n^4 \sigma_{E_t}^2 + \sigma_{E_b}^2).
\]  
(Eq. 109)

The covariance function of the panel is defined by

\[
C_{EI_p}(\xi) = N^2 A^2 d_b^4 [n^4 C_{E_t}(\xi) + C_{E_b}(\xi)]
\]  
(Eq. 110)

and the spectral density function of the panel is

\[
S_{EI_p}(k) = \frac{2A^2 d_b^4 n^4}{\pi} \left( \sum_{i=1}^{N} \sigma_i^2 \beta_{it} \left( \frac{1}{k^2 + \beta_i^2} \right) \right) + \frac{2A^2 d_b^4}{\pi} \left( \sum_{i=1}^{N} \sigma_i^2 \beta_{ib} \left( \frac{1}{k^2 + \beta_i^2} \right) \right).
\]  
(Eq. 111)
Consider the general case of a homogenous three-layer CLT panel, where each of the constituent boards has the same statistical properties for the modulus of elasticity, such that \( \mu_{1t}, \mu_{2t} \ldots \mu_{Nt} = \mu_1, \mu_2 \ldots \mu_N = \mu_b \), \( \sigma_{1t}^2, \sigma_{2t}^2 \ldots \sigma_{Nt}^2 = \sigma_1^2, \sigma_2^2 \ldots \sigma_N^2 = \sigma_b^2 \), and \( C_{1t}(\xi), C_{2t}(\xi) \ldots C_{Nt}(\xi) = C_1(\xi), C_2(\xi) \ldots C_N(\xi) = C_b(\xi) \). In this case, the ratio, \( n \), is equal to 1. The mean MOE for each layer is

\[
\mu_{E_t} = \mu_b, \tag{Eq. 112}
\]

the variance is

\[
\sigma_{E_t}^2 = \frac{\sigma_b^2}{N}, \tag{Eq. 113}
\]

the covariance function is

\[
C_{E_t} (\xi) = \frac{C_b(\xi)}{N}, \tag{Eq. 114}
\]

and the spectral density function is

\[
S_{E_t} (\xi) = \frac{2\sigma_b^2 \beta_b}{\pi N^2} \left( \frac{1}{k^2 + \beta_b^2} \right) \tag{Eq. 115}
\]

The mean of the effective panel stiffness can then be expressed by

\[
\mu_{Elp(x)} = 2NAh_b^2 \mu_b \tag{Eq. 116}
\]

The variance is expressed by

\[
\sigma_{Elp(x)}^2 = 2NA^2 h_b^4 \sigma_b^2 \tag{Eq. 117}
\]

and the covariance is

\[
C_{Elp} (\xi) = 2NA^2 h_b^4 C_b(\xi) \tag{Eq. 118}
\]

and the spectral density function is

\[
S_{Elp} (k) = \frac{4NA^2 h_b^4 \sigma_b^2 \beta_b}{\pi} \left( \frac{1}{k^2 + \beta_b^2} \right) \tag{Eq. 119}
\]
Using these properties, the variance of the displacement response of a three-layer CLT panel can be calculated using Eq. 91.

### 6.4 Case Study: Eastern hemlock CLT Panel

This method can be applied to a diverse range of species and CLT panel layups. Consider a CLT panel composed of one species with three boards in the longitudinal layers. The CLT panel is loaded in three-point bending, with a force of $100\,N$ applied at mid-span. The random effective panel stiffness, $\overline{EI_p(x)}$, is a function of the random MOE of the composite boards. If the constituent board properties are known, the mean and variance of the layer MOE and the panel stiffness can be calculated by applying Eq. 112 – Eq. 119. These values are then used to calculate the spectrum and the VRF of the system. Eq. 91 can then be applied to evaluate the variability in the system's response, or more simply, the variability in displacement due to the applied load.

The spectrum, $S_{\overline{EI_p}}(k)$, is first evaluated using Eq. 119. The VRF for the panel is then evaluated across a full range of phase angles and wavenumbers using the Fast MC method described in Section 6.2.

The application of these calculations is straightforward if the statistical properties of the MOE for the constituent boards are known. The mean and variance of the MOE may be found in the literature for some species. However, a benefit of this method is the ease at which species currently considered uncertified for structural purposes can be evaluated with respect to reliability and potential for use in mass timber products. For the purposes of this study, the statistical properties of the elastic modulus in Eastern hemlock were evaluated experimentally. Eighty-one 12 ft long nominal 2 in x 4 in boards were purchased in three separate batches from two local mills in Western Massachusetts. When purchasing
from the source mill, boards were selected visually to avoid extreme warp. At the University of Massachusetts Amherst Wood Mechanics lab, the boards were jointed and planed to have final dimensions of 33 mm (1.3 in) in thickness, 81 mm (3.2 in) in width, and 3658 mm (144 in) in length. Nondestructive, flatwise three-point bending tests were conducted on all boards to evaluate bending MOE along the full span of the board. Tests were performed in accordance with ASTM D198 (ASTM, 2014). The resulting mean MOE of the Eastern hemlock samples was 9299.8 $MPa$ and the standard deviation was 1937.1 $MPa$.

A subset of four boards was selected to be used for a more detailed study of the variation in MOE along the length of the board. A total of nineteen bending tests with a moving span of 762.0 $mm$ were performed along the length of each board (Figure 76 a). The covariance function for each board was also calculated (Figure 76 b).

![Figure 76](image-url)

Figure 76. Results of experimental testing on selected Eastern hemlock samples. (a) Variation in modulus of elasticity and (b) covariance function
By taking the ensemble average of the covariance functions and fitting an exponential decay fit, a decay rate, $\beta$, of 0.00371 $mm^{-1}$ is determined.

The experimentally determined mean, variance, and decay rate of the constituent boards can be used to calculate the top and bottom layer MOE statistical properties and the panel stiffness statistical properties for a CLT panel with three boards oriented in the longitudinal direction (Table 33). By applying Eq. 112 – Eq. 117, the mean and variance of the panel stiffness, $\mu_{E_{I_p}}$ and $\sigma^2_{E_{I_p}}$, are determined to be $2.51E11 N mm^2$ and $4.55E20 N^2 mm^4$, respectively.

Table 33. Statistics of layer modulus of elasticity and panel stiffness for a three-layer Eastern hemlock CLT panel with three boards in width

<table>
<thead>
<tr>
<th>Material Property Statistics</th>
<th>Geometry</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Constituent Boards</strong></td>
<td></td>
</tr>
<tr>
<td><strong>Mean MOE (MPa)</strong></td>
<td>$9.30E3$</td>
</tr>
<tr>
<td>$\mu_{1t}, \mu_{2t}, \mu_{3t} = \mu_{1b}, \mu_{2b}, \mu_{3b}$</td>
<td></td>
</tr>
<tr>
<td><strong>Variance MOE (MPa$^2$)</strong></td>
<td>$3.75E6$</td>
</tr>
<tr>
<td>$\sigma_{1t}^2, \sigma_{2t}^2, \sigma_{3t}^2 = \sigma_{1b}^2, \sigma_{2b}^2, \sigma_{3b}^2$</td>
<td></td>
</tr>
<tr>
<td><strong>Decay Rate (mm$^{-1}$)</strong></td>
<td>$0.00371$</td>
</tr>
<tr>
<td>$\beta_{1t}, \beta_{2t}, \beta_{3t} = \beta_{1b}, \beta_{2b}, \beta_{3b}$</td>
<td></td>
</tr>
</tbody>
</table>

| **Layers** | |
| **Mean MOE (MPa)** | $9.30E3$ |
| $\mu_{E_{I_t}}, \mu_{E_{I_b}}$ | |
| **Variance MOE (MPa$^2$)** | $1.25E6$ |
| $\sigma_{E_{I_t}}^2, \sigma_{E_{I_b}}^2$ | |
| **Decay Rate (mm$^{-1}$)** | $0.00371$ |
| $\beta_{E_{I_t}} = \beta_{E_{I_b}}$ | |

| **Panel** | |
| **Mean Stiffness (N mm$^2$)** | $2.51E11$ |
| $\mu_{E_{I_p}}$ | |
| **Variance Stiffness (N$^2$ mm$^4$)** | $4.55E20$ |
| $\sigma^2_{EI_{p}}$ | |
| **Decay Rate (mm$^{-1}$)** | $0.00371$ |
| $\beta_{EI_{p}}$ | |

The spectrum is then evaluated across an appropriate range of wave numbers such that $k = [0, 0.05]$. The VRF is evaluated across the same wavenumbers and a full range of phase angles, $\phi = [0, 2\pi]$. Results for the spectrum and VRF are presented in Figure 77.
The variance of the mid-span displacement is then calculated, resulting in a variability response of $3.02 \times 10^{-6}$ mm.

![Figure 77. Spectrum and VRF for Eastern hemlock board with three boards in width.](image)

The same analysis is repeated for panels with varying number of boards, $N$. As intuitively expected, the variability of the displacement response decreases as the number of boards increases due to an averaging affect.

<table>
<thead>
<tr>
<th>Number of boards in width, $N$</th>
<th>Panel Width, $w_p$ (mm)</th>
<th>Mean Panel Stiffness, $\mu_{Ep}$ ($N \text{ mm}^2$)</th>
<th>Variance of Panel Stiffness, $\sigma^2_{Ep}$ ($N \text{ mm}^2$)$^2$</th>
<th>Coefficient of Variation, $COV_{Ep}$</th>
<th>Standard Deviation of Response, $\sigma_{Ep}$ (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>243.8</td>
<td>2.51E11</td>
<td>4.55E20</td>
<td>0.085</td>
<td>0.0174</td>
</tr>
<tr>
<td>5</td>
<td>406.4</td>
<td>4.18E11</td>
<td>7.58E20</td>
<td>0.066</td>
<td>0.0081</td>
</tr>
<tr>
<td>7</td>
<td>569.0</td>
<td>5.85E11</td>
<td>1.06E21</td>
<td>0.056</td>
<td>0.0049</td>
</tr>
<tr>
<td>9</td>
<td>731.5</td>
<td>7.53E11</td>
<td>1.36E21</td>
<td>0.049</td>
<td>0.0033</td>
</tr>
<tr>
<td>25</td>
<td>2032.0</td>
<td>2.09E12</td>
<td>3.79E21</td>
<td>0.030</td>
<td>7.23E-4</td>
</tr>
<tr>
<td>35</td>
<td>2844.8</td>
<td>2.93E12</td>
<td>5.31E21</td>
<td>0.025</td>
<td>4.36E-4</td>
</tr>
</tbody>
</table>

The VRF Method can be validated by comparing the variability of the response of the system via the VRF Method to the Monte Carlo Method. The Monte Carlo Method
works by generating constituent board samples with random MOE profiles following a zero-mean stationary Gaussian process with a one-sided spectral density function specified by

\[
S_{Eb}(k) = \frac{2\sigma_{Eb}^2 \beta_{Eb}}{\pi(k^2 + \beta_{Eb}^2)}.
\]  
(Eq. 120)

The samples are then generated by

\[
E(x) = a\sigma_{SEb} \cos(k'x) + b\sigma_{SEb} \cos(k'x)
\]  
(Eq. 121)

where \(a\) and \(b\) are vectors of Gaussian distributed random numbers with a length equal to the number of harmonics considered. \(\sigma_{SEb}\) is expressed by

\[
\sigma_{SEb} = \sqrt{S_{Eb} \Delta k}.
\]

The constituent board samples can then be used to calculate the displacement response of the system using traditional homogenization techniques and beam theory. A comparison of the VRF Method and Monte Carlo Method results are shown in Table 35.

<table>
<thead>
<tr>
<th>Number of boards in width, (N)</th>
<th>VRF Method: Variability of Response, (VAR[\Delta_p]) (mm)</th>
<th>Monte Carlo: Variability of Response, (VAR[\Delta_p]) (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>3.02E-4</td>
<td>3.14E-4</td>
</tr>
<tr>
<td>5</td>
<td>6.53E-4</td>
<td>7.35E-4</td>
</tr>
<tr>
<td>7</td>
<td>2.38E-5</td>
<td>2.61E-5</td>
</tr>
</tbody>
</table>

6.5 Case Study: Hybrid CLT Panel

A benefit of mass timber is that the constituent boards can be oriented to optimize structural performance. The advantage of the VRF method is that novel layouts can be efficiently studied in order to investigate the reliability and performance. A hybrid CLT panel can be fabricated by placing different species in the top and bottom layers, such that traditionally higher value species can be placed on the tension layer and traditionally lower
value species can be on the compression layer of the panel. The variance of the displacement response in three-point bending can be investigated via the VRF method.

Consider a hybrid three-layer CLT panel in three-point bending with \( 100 N \) of applied load at midspan. The top layer consists of Eastern hemlock and the bottom layer consists of spruce. The statistics for the modulus of elasticity of Eastern hemlock were determined experimentally as discussed above. Similarly, nondestructive, flatwise three-point bending tests were conducted on two spruce boards to evaluate bending MOE. Tests were performed in accordance with ASTM D198 (ASTM, 2014). While this is a very limited sample size, it is sufficient for the purpose of this paper, which works to show the applicability of this method to CLT panels. No covariance data was measured, so in order to illustrate the influence of the modulus of elasticity spectrum on the displacement response, a much smaller decay rate is assumed for the spruce boards than the Eastern hemlock boards. Determining the layer modulus of elasticity mean and variance and the panel stiffness mean and variance is straightforward by applying Eq. 98 – Eq. 109. The covariance functions and spectra for the layer and panel are determined by Eq. 100 – Eq. 101 and Eq. 110 – Eq. 111, respectively. Table 36 presents the layer modulus of elasticity statistics and the panel stiffness statistics for a hybrid Eastern hemlock-spruce panel with three boards in width. Note, the decay rate is not provided for the panel because the effective covariance function is not an exponential decay.
Table 36. Experimentally determined Eastern hemlock and spruce modulus of elasticity statistics. Statistics of layer modulus of elasticity and panel stiffness for a three-layer hybrid CLT panel with three boards in width.

<table>
<thead>
<tr>
<th>Constituent Boards</th>
<th>Material Property Statistics</th>
<th>Geometry</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Mean MOE (MPa)</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(\mu_{t1}, \mu_{t2}, \mu_{t3})</td>
<td>9.30E3</td>
<td></td>
</tr>
<tr>
<td>(\mu_{b1}, \mu_{b2}, \mu_{b3})</td>
<td>1.34E4</td>
<td></td>
</tr>
<tr>
<td><strong>Variance MOE (MPa^2)</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(\sigma_{t1}^2, \sigma_{t2}^2, \sigma_{t3}^2)</td>
<td>3.75E6</td>
<td></td>
</tr>
<tr>
<td>(\sigma_{b1}^2, \sigma_{b2}^2, \sigma_{b3}^2)</td>
<td>3.78E6</td>
<td></td>
</tr>
<tr>
<td><strong>Decay Rate (mm^-1)</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(\beta_{t1}, \beta_{t2}, \beta_{t3})</td>
<td>0.00371</td>
<td></td>
</tr>
<tr>
<td>(\beta_{b1}, \beta_{b2}, \beta_{b3})</td>
<td>0.00100</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Layers</th>
<th>Material Property Statistics</th>
<th>Geometry</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Mean MOE (MPa)</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(\mu_{E_t})</td>
<td>9.30E3</td>
<td></td>
</tr>
<tr>
<td>(\mu_{E_b})</td>
<td>1.34E4</td>
<td></td>
</tr>
<tr>
<td><strong>Variance MOE (MPa^2)</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(\sigma_{E_t}^2)</td>
<td>1.25E6</td>
<td></td>
</tr>
<tr>
<td>(\sigma_{E_b}^2)</td>
<td>1.26E6</td>
<td></td>
</tr>
<tr>
<td><strong>Decay Rate (mm^-1)</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(\beta_{E_t})</td>
<td>0.00371</td>
<td></td>
</tr>
<tr>
<td>(\beta_{E_b})</td>
<td>0.001</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Panel</th>
<th>Material Property Statistics</th>
<th>Geometry</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Mean Stiffness (N mm^2)</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(\mu_{EI_p})</td>
<td>2.96E11</td>
<td></td>
</tr>
<tr>
<td><strong>Variance Stiffness (N^2 mm^4)</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(\sigma_{EI_p}^2)</td>
<td>5.44E20</td>
<td></td>
</tr>
</tbody>
</table>

The spectrum and VRF are then evaluated across \(k = [0, 0.05]\) and a full range of phase angles, \(\phi = [0, 2\pi]\). Results for the spectrum and VRF are presented in Figure 78 for a range of CLT panel widths. The variance of the mid-span displacement is then calculated in accordance with Eq. 91.
Table 37 presents the displacement response variance and the coefficient of variation for a three-layer hybrid CLT panel with increasing \( N \). The variance of the displacement decreases as the width of the panel increases. This is a result of the averaging effect of composite products. Note that although the hybrid panel has a larger variance in stiffness than the Eastern hemlock panel (e.g. \( 5.44E20 \ (N \text{ mm}^2)^2 \) compared to \( 4.55 \ (N \text{ mm}^2)^2 \) for a 3 board width), the panel displacement response variability is smaller. This is due to the longer correlation length of the spruce constituent boards.
Table 37. Effective stiffness property statistics and variance of displacement response of hybrid CLT panel of varying width.

<table>
<thead>
<tr>
<th>Number of boards in width, ( N )</th>
<th>Panel Width ( w_p ) (( mm ))</th>
<th>Mean Panel Stiffness, ( \mu_{E_I} ) (( N \ mm^2 ))</th>
<th>Variance of Panel Stiffness, ( \sigma_{E_I}^2 ) ( (N \ mm^2)^2 )</th>
<th>Coefficient of Variation, ( COV_{E_I} )</th>
<th>Variability of Response, ( \sigma_{\Delta p} ) (( mm ))</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>243.8</td>
<td>2.96E11</td>
<td>5.44E20</td>
<td>0.078</td>
<td>0.0154</td>
</tr>
<tr>
<td>5</td>
<td>406.4</td>
<td>4.93E11</td>
<td>9.07E20</td>
<td>0.061</td>
<td>0.0071</td>
</tr>
<tr>
<td>7</td>
<td>569.0</td>
<td>6.90E11</td>
<td>1.27E21</td>
<td>0.052</td>
<td>0.0043</td>
</tr>
<tr>
<td>9</td>
<td>731.5</td>
<td>8.88E11</td>
<td>1.63E21</td>
<td>0.046</td>
<td>0.0030</td>
</tr>
<tr>
<td>25</td>
<td>2032.0</td>
<td>2.47E12</td>
<td>4.53E21</td>
<td>0.027</td>
<td>6.39E-4</td>
</tr>
<tr>
<td>35</td>
<td>2844.8</td>
<td>3.45E12</td>
<td>6.35E21</td>
<td>0.023</td>
<td>3.86E-4</td>
</tr>
</tbody>
</table>

6.6 Case Study: Influence of Covariance Function

One of the key advantages of the VRF method is that the influence of the constituent properties on the panel response can be efficiently studied. This allows the effectiveness of novel layups and concepts to be studied before or without costly experimental testing. For example, consider a panel made out of Eastern hemlock boards on both the top and bottom layers. However, assume that the covariance function of the bottom layers can be manipulated by forest management techniques such that there is a longer correlation length, or lower rate of decay, \( \beta_{Nb} \). By applying Eq. 97 – Eq. 111, the layer and panel effective properties can be determined. Note that in this scenario, the mean and variance of the MOE for the constituent boards and layers, and the mean and variance of the panel stiffness are equal to the values presented in Table 33. However, \( \beta_{Ec} \) is experimentally determined to be 0.00371 \( mm^{-1} \) and \( \beta_{Eb} \) is selected to equal to 0.00100 \( mm^{-1} \). The spectrum and VRF are evaluated across \( k = [0, 0.05] \) and a full range of phase angles, \( \phi = [0, 2\pi] \). Results are presented in Table 38.
Table 38. Effective stiffness property statistics and variance of displacement response of Eastern hemlock panel with manipulated covariance function of bottom layer constituent boards

<table>
<thead>
<tr>
<th>Number of boards in width, N</th>
<th>Panel Width, $w_p$ (mm)</th>
<th>Mean Panel Stiffness, $\mu_{EI_p}$ ($N \text{ mm}^2$)</th>
<th>Variance of Panel Stiffness, $\sigma^2_{EI_p}$ ($N \text{ mm}^2$)</th>
<th>Coefficient of Variation, $COV_{EI_p}$</th>
<th>Standard Deviation of Response, $\sigma_{\Delta p}$ (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>243.8</td>
<td>2.51E11</td>
<td>4.55E20</td>
<td>0.085</td>
<td>0.0227</td>
</tr>
<tr>
<td>5</td>
<td>406.4</td>
<td>4.18E11</td>
<td>7.58E20</td>
<td>0.066</td>
<td>0.0105</td>
</tr>
<tr>
<td>7</td>
<td>569.0</td>
<td>5.85E11</td>
<td>1.06E21</td>
<td>0.056</td>
<td>0.0064</td>
</tr>
<tr>
<td>9</td>
<td>731.5</td>
<td>7.53E11</td>
<td>1.36E21</td>
<td>0.049</td>
<td>0.0044</td>
</tr>
<tr>
<td>25</td>
<td>2032.0</td>
<td>2.09E12</td>
<td>3.79E21</td>
<td>0.030</td>
<td>9.42E-4</td>
</tr>
<tr>
<td>35</td>
<td>2844.8</td>
<td>2.93E12</td>
<td>5.31E21</td>
<td>0.025</td>
<td>5.69E-4</td>
</tr>
</tbody>
</table>

As expected, the standard deviation of the displacement response of the system decreases as the number of boards along the width of the panel increases. The most notable result is made by comparing the results of the CLT layup to the results of the case study presented in Section 6.4. The only difference between the CLT panels investigated in Section 6.4 and 6.6 is that in Section 6.4 $\beta_{Eb}$ is equal to $\beta_{Et}$, while in Section 6.6 $\beta_{Eb}$ is less than $\beta_{Et}$. Thus, a comparison of these two models provides an indication of covariance sensitivity. Results show that the standard deviation of displacement response of the panel modeled in Section 6.6 is approximately 1.3 times the standard deviation of the displacement response in Section 6.4.

While three examples were provided in this paper, the equations and methods presented can be applied to any three-layer CLT panel with combined species or lumber grades.

6.7 Conclusions

The primary contribution of this paper is that the variability response function (VRF) method can be successfully applied to a Cross Laminated Timber (CLT) panel to
investigate the variability in displacement response due to spatial variation in Modulus of Elasticity of constituent boards. The variance of the displacement response is calculated by considering the spectrum of the Modulus of Elasticity of the constituent boards and the VRF, which considers the deterministic boundary conditions of the system. This method is advantageous for conducting studies on the sensitivity of the CLT panel displacement to the constituent board material properties since the deterministic boundary conditions are decoupled from the spectrum of the modulus of elasticity resulting in a simple numerical integral that negates the need for additional computationally intensive Monte Carlo simulations. These findings advance the ability to understand the relationship between displacement response and spatial variation of material properties in wood composite products since previously the only method shown to investigate this was Monte Carlo Simulations.

Three examples are provided to show the efficacy of this VRF approach. The examples consist of a CLT panel in three-point bending. However, the methods and equations derived in this paper could be applied to any bending loading condition by modifying the deterministic displacement equation. Similarly, the derived equations are applicable to a three-layer CLT panel with any cross-sectional dimensions, number of boards, and species. The third case study specifically addresses the sensitivity of covariance to the displacement response of the system.

The analytical method presented in this study has significant implications on the ability to understand the displacement response of CLT panels to the constituent board material properties. Novel CLT panel layups can efficiently be studied, increasing opportunities for traditionally low-value species to be considered for suitability in CLT
panels. The VRF method is particularly well suited to reliability analysis of CLT panels as well as sensitivity analysis of constituent board material properties on effective CLT panel stiffness. Further, this method can be used to inform a diverse range of problems regarding mass timber, including grading of mass timber, forest management practices, and CLT panel design. An additional implication of these findings is that the VRF method has the potential to also be applied to other mass timber products of interest.

Future work should include expanding the application of the VRF method to five, seven, and nine layer CLT panels, which reflects the panel lay-ups most typically used in mass timber construction applications.
CHAPTER 7

CONCLUSIONS AND FUTURE WORK

7.1 Conclusions

Basic conclusions found within the framework of this thesis are summarized below:

- The primary contribution of Chapter 2 is the development of a two-dimensional and three-dimensional probabilistic model for the distribution and geometry of knots in dimensional lumber. This model can be calibrated to any softwood species and allows for the creation of synthetic wood boards.

- Orthotropic compressive properties of Eastern hemlock are experimentally determined. The longitudinal Modulus of Elasticity, $E_L$, was found to be $1.15 \times 10^6$ psi, while the strength, $f_L$, was found to be $5.46 \times 10^3$ psi. The tangential Modulus of Elasticity and Strength were found to be $9.67 \times 10^3$ psi and $1.06 \times 10^3$ psi, respectively, while the radial Modulus of Elasticity and Strength were found to be $5.44 \times 10^3$ psi and $6.57 \times 10^2$ psi.

- Three-dimensional non-linear finite element models of dimensional lumber are developed. Clear wood is modeled by orthotropic linear elasticity and orthotropic yielding with isotropic hardening. Knots are considered to be either holes or stiff inclusions modeled by isotropic bilinear behavior. This distinction between tension and compression loading, and associated knot model, plays a significant role in the results. Compressive results (knots are modeled as stiff inclusions) indicate that knots have a much smaller impact of effective stiffness and yielding point than tensile loading (knots modeled by holes). As expected, the stress paths show that when knots are modeled by stiff inclusions the stress flows through the knot while when knots are modeled by holes the stress must flow through the
remaining cross-section of the wood. This results in an increase in compressive stiffness and strength and a decrease in tensile stiffness and strength. Finally, stress paths show that stress concentrations are largest at edge knots.

- The relationship of knot defects to Modulus of Elasticity and Modulus of Rupture is studied in Eastern hemlock and Sitka spruce. The geometry of knots is characterized in four ways: (1) summation of the KAR (knot area ratio), (2) summation of KAR considering a weighting factor, (3) the maximum/critical KAR, (4) the maximum/critical KAR considering a weighting factor. The relationship between knot geometry and MOE/MOR are studied through slope of the linear best fit line, correlation coefficient, and confidence interval. Regardless of the knot characterization method, results do not show a meaningful or consistent relationship between knot defects and Modulus of Elasticity or Modulus of Rupture in either of these species. Results showing no relationship between knots and MOE is expected. A lack of relationship between knot geometry and MOR is likely due to large limitations in the data sets.

- A method is presented to apply the variability response function (VRF) method to a Cross Laminated Timber (CLT) panel to investigate the variability in displacement response due to spatial variation in modulus of elasticity of constituent boards. The variance of the displacement response is calculated by considering the spectrum of the modulus of elasticity of the constituent boards and the VRF which considers the deterministic boundary conditions of the system. Results show that as the number of boards increases, the coefficient of variation for the bending stiffness of the panel decreases. The primary
impact of this paper is the development of a method to understand the influence of constituent board material properties on CLT performance, allowing for the modeling of novel CLT lay-ups and increasing the opportunity for traditional low-value species to be considered as constituents in CLT panels.

In general, this thesis presents a series of analytical methods to investigate how knots influence the heterogeneity of dimensional lumber, and in turn, how the heterogeneity of dimensional lumber affects the performance of mass timber products. The models created are calibrated to Eastern hemlock; however, they are applicable to any softwood species.

### 1.2 Future Work

The development of mass timber products from traditionally low-value woods is dependent on understanding how the heterogeneous material properties of dimensional lumber influence the mechanical performance of mass timber. Further, a precise and validated understanding of the influence of knots on mass timber is vital for the development of effective and efficient grading schemes. Recommendations for future work to further the goals of efficient use of wood material in mass timber applications are outlined below:

- The development of three-dimensional finite element models of CLT panels is required to understand and characterize the averaging of mechanical properties in mass timber products. Future work should include expanding the FEM presented in Chapter 4 to the CLT panel scale and to include strength behavior.
- Grading standards are created through large-scale testing initiatives and probabilistic analysis. Thus, it is perhaps more important to understand the
mechanical behavior of knots in wood within a probabilistic perspective than within a single board or CLT panel. To this end, finite element models can be created using the probabilistic model for knot geometry and distribution developed in Chapter 2 in order to perform a reliability analysis. Stochastic FEMs calibrated to Eastern hemlock should be developed in order to investigate the feasibility of CLT panels with Eastern hemlock constituent boards. Further, a robust testing program should be performed on a large sample size to evaluate wood grade and performance, to analyze the influence of knots on mechanical behavior.

- The variability response function method presented in Chapter 6 should be expanded to include 5, 7, and 9 layer CLT panels, which reflects the commonly used lay-ups in construction applications.
- The existence of a variability response function has been formally proven for statically determinate beam structures following nonlinear constitutive laws. Future work should involve applying the VRF method to understand how heterogeneous material properties at the dimensional lumber scale influences the variation in strength performance at the CLT panel scale.
- Future work should also be dedicated to developing grading standards for wood material with the specific end-use of mass timber production. Current methods rely on sawn lumber applications and the grading methods are likely not appropriate for efficient use of material in mass timber construction.
- While advancing the technical understanding of the mechanics and behavior of wood and mass timber products is necessary, a significant limitation to the use of local species in wood construction in the Northeast of the United States is a lack of production of mass timber products in the region. Future efforts should include the
dissemination of this work to the public, government, and construction industry in the region in order to help facilitate local production.
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