X-BAND PHASED-ARRAY WEATHER-RADAR POLARIMETRY TESTBED

William Heberling IV
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X-BAND PHASED-ARRAY WEATHER-RADAR POLARIMETRY TESTBED

A Dissertation Presented

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The heavens declare the glory of God;
the skies proclaim the work of His hands.
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ABSTRACT

X-BAND PHASED-ARRAY WEATHER-RADAR POLARIMETRY TESTBED

MAY 2022

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Phased-array weather radar have potential to replace reflector dish radar in major weather radar networks such as NEXRAD, providing faster update times and greater scan flexibility. However, the use of electronic scanning introduces polarization errors on weather radar measurables, requiring polarimetric bias calibration. The sources of polarimetric bias have been described theoretically, but experimental verification is still limited. Additionally, no standard method of calibration for polarimetric bias exists for phased-arrays.

Therefore, the University of Massachusetts Amherst (UMass) presents a fully operational X-Band phased-array weather radar polarimetric testbed. The testbed evaluates the calibration of a planar dual-polarization X-band phased-array radar through simultaneous operation with a co-located mechanically-scanned polarimetric reference radar. A detailed description of both radar systems is provided, as well as radar installation and data collection procedures.
In addition, this research proposes a novel method of phased-array polarimetric calibration which improves on prior work by decomposing bias sources into independent linear terms. This modular approach is practical, easy to implement, and adaptable to any radar system.

Finally, this research delivers a detailed demonstration of both the calibration method and the weather radar polarimetric testbed. The calibration method is evaluated by direct comparison of weather observations using the testbed.

Notable results include: the polarimetric calibration improves the phased-array’s correlation with the dish radar in all cases. However, the linear correction appears insufficient to calibrate all variables over the entire scan range. Increasing the time offset between observations decreases the correlation between radars in every case. Lastly, this evaluation provides insight into the effects of ground clutter, rain attenuation, and noise on a direct comparison between radars.
List of Contributions

Original contributions of this work include:

- A novel method of phased-array polarimetric calibration which improves on prior work by decomposing bias sources into independent linear terms. This modular approach:
  - Breaks down a complex problem into simple independent parts.
  - Incorporates both theoretical corrections and existing practical calibration techniques.
  - Supports arbitrary tilt and roll of the radar antenna.
  - Implements corrections as linear solutions applied in real time.
  - Is adaptable to many types of phased-array radars.

- An evaluation of the calibration method using a fully operational X-Band phased-array polarimetric testbed, comprised of two dual-polarized weather radars. This experiment:
  - Successfully implements the modular approach model on a phased-array radar.
  - Evaluates the model by direct comparison of weather observations.
  - Contributes valuable phased-array weather radar data.
  - Provides insight into the model’s merit through statistical data analysis.
Publications

Publications resulting from this work [1, 2, 3]:


This work was also presented at the American Meteorological Society (AMS) Annual Meeting in 2019, 2020, and 2021.
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CHAPTER 1
INTRODUCTION

1.1 Motivation: Phased-arrays in Meteorology

1.1.1 Limitations of Parabolic Antenna Radars

All operational weather radar systems currently use parabolic reflector antennas, including the NEXRAD network in the US and the OPERA in Europe. Parabolic antennas have been used for decades due to their simplicity and reliability, however their use in meteorology does impose some limitations. Most critical of these is scan update interval. Typical parabolic antenna radars performing volumetric scans take around five minutes per update. The reflector must be physically rotated and tilted to each scan beam, mechanically limiting scan rate. In the case of severe, rapidly evolving storms, mechanical scanning often does not provide the temporal resolution necessary for accurate modeling and prediction of weather phenomenon. A faster scan update rate could allow for earlier, more accurate prediction in potentially dangerous situations such as flash flooding and tornadoes.

Parabolic antennas are also limited in flexibility when it comes to scan pattern. They can only form a single beam, whose beamwidth, gain, and sidelobes are determined by the physical characteristics of the reflector and therefore cannot change. They typically can only track a single target at a time. In addition, because the reflector must move mechanically from each beam to the next, they must scan beams sequentially.
1.1.2 Phased-Array Antennas

A phased-array antenna consists of many radiating elements arranged in a grid, whose amplitude and phase can be controlled individually. By manipulating these amplitudes and phases, an interference pattern resembling an antenna beam can be formed. In addition, the properties of the beam such as scan direction, beamwidth, gain, and sidelobes can be controlled. This is known as electronic beam scanning.

While phased-array technology has been available for many years, its application in meteorology is in its infancy. The promise of this technology is being tested and evaluated through fixed radar testbeds, such as the National Weather Radar Testbed (NWRT) [5, 6] and the current Advanced Technology Demonstrator [7, 8], mobile Doppler radars for rapid scanning [9, 10, 11], and low-cost, low-power polarimetric phased-arrays [12, 13, 14, 15]. Phased-array weather radars allow for rapid scanning and faster update times (high temporal resolution of severe weather). This is possible due to electronic scanning in which the beam can move nearly instantaneously to any scan direction. In addition, multiple-target or simultaneous-beam scanning becomes possible. Phased-arrays also eliminate moving parts from the system, potentially reducing routine maintenance.

To be viable for meteorological applications, phased-array antenna radar must meet certain requirements. The most critical of these is the ability to make high-fidelity dual-polarization measurements, which are necessary for accurate rainfall estimates, for hydrometeor classification, and for morphological analysis of storm structure. As a result, operational weather radars require rigorous initial and periodic calibration. Performing such calibration with a phased-array is significantly more complex than for a reflector antenna due to the varying beam gain, beam shape, and polarization properties with scan angle. New techniques must be developed and tested to ensure data quality is not degraded by the introduction of phased-array radars.
1.1.3 Polarimetric Error

One of the greatest challenges facing phased-array radar application in meteorology is the existence of dual polarization measurement biases inherent to electronic scanning. They appear when scanning off the principal planes of a phased-array and are significant enough to exceed the tolerances required by most meteorological studies. These polarization biases are dependent on the scan angle and thus any correction or calibration must also depend on scan angle.

The measurement biases in a phased-array weather radar can be described as a combination of three main phenomena:

1. The amplitude and phase pattern of the radiating elements is non-uniform over radar scan angle.

2. The radiated field polarizations of horizontally and vertically excited elements become non-orthogonal when electronically scanning off the principal planes.

3. If the array is tilted or rolled, its field polarizations no longer represent global horizontal and vertical.

A reliable method to model and calibrate each of these phenomena is a main deliverable of this research and critical to the advancement of phased-arrays in meteorology.

1.1.4 Research Goals

The major goal of this project is to study the behaviors and limitations of phased-array weather radar polarimetry through the implementation and operation of a radar testbed. Observations of a polarimetric planar phased-array utilizing bias correction will be validated through systematic comparison with a co-located and synchronized mechanically-scanned radar. Research objectives include:
• To design and construct a weather radar polarimetric testbed comprised of a dual-polarization phased-array radar and a co-located mechanically-scanned reference radar.

• To develop a practical calibration model specifically for polarimetric phased-array weather radar using antenna theory and various prior research works.

• To evaluate the calibration model’s performance through direct comparison of weather observations using the weather radar polarimetric testbed.

### 1.1.5 Project Timeline

The full project timeline is shown in Figure 1.1. Assembly of the testbed was performed in 2017-2018, with data being collected using the testbed in the fall of 2018 and 2019. The polarimetric correction model was developed and refined throughout 2019 and 2020, and data comparison of weather observations was performed in 2021. Each of these stages of research is discussed in more detail in the following chapters.

**Figure 1.1.** Project timeline for research on the X-Band Phased-Array Weather Radar Polarimetry Testbed.
1.2 Summary of Prior Literature

Provided here is a summary of prior literature on the topic. Early research in polarimetric calibration was intended for parabolic dish reflector antenna radars. In 2002, Moisseev et. al. described the problem as a matrix transformation between the actual scatter and the measurements of the antenna. They also studied point target, light rain, and vertical scan calibration for dish radar [16].

In 2003, the National Severe Storm Laboratory (NSSL) at NOAA deployed the first phased-array weather radar, known as the National Weather Radar Testbed (NWRT). This single polarization S-Band radar demonstrated that rapid updates of weather data were possible with electronic scanning [5, 6]. The NWRT was decommissioned in 2016 and has been replaced by the Advanced Technology Demonstrator (ATD).

The first mobile phased-array weather radar was built by the Naval Postgrad School in 2005 [10]. It operated at X-Band and like the NWRT only supported a single polarization.

In 2009, Moisseev’s transform matrix approach to polarimetry was adapted to phased-array radar by Zhang, Doviak, and Zrnić et. al. [17]. By this point, interest was growing in phased-array antennas as an option for weather radars. In their work, they explain the unique challenges faced when performing polarimetric calibration of a Planar Phased-Array Radar (PPAR). Using ideal dipole radiators, they present a theoretical correction to weather parameters. In 2011, they expand that study to consider cylindrical arrays as a possible alternative to PPAR [18, 19].

From 2011 to 2013, the NEXRAD WSR-88 system was upgraded to dual polarized radars (WSR-88D), allowing for more accurate qualitative precipitation estimates (QPE) and hydrometeor classification [20]. Leading up to this, several methods for calibration of polarimetric dish radar were described and experimentally verified, in-
cluding vertical scans, light rain scans, drop size distribution fitting, and the scattering
matrix form\[20, 21, 22, 23, 24\].

In 2013, Lei, Zhang, and Doviak describe a projection matrix approach for PPAR
calibration using ideal patch and aperture elements \[25\]. That work in particular is
foundational to the research presented here. A detailed discussion of their approach
is provided in Chapter 4.

In 2014, the Microwave Remote Sensing Laboratory (MIRSL) at the University
of Massachusetts Amherst (UMass) built the Phase Tilt Weather Radar (PTWR).
This X-Band phased-array radar supported dual polarization measurements. It could
electronically scan along a single axis and was mechanically scanned in the other axis.
The PTWR was deployed in Arlington, Texas in the spring of 2014 \[13, 14\].

Design of the ATD began in 2014 and the radar was installed in Norman, OK in
2018. The ATD is a dual-polarized phased-array weather radar at S-band. It was
designed with the NEXRAD system in mind and will be used in the future to evaluate
phased-array radar as the next upgrade to major weather radar networks \[7, 8\].

In 2016 and 2018, Aboserwal, Salazar and Fulton et. al challenge the use of con-
tentional polarization coordinates famously set forth by Ludwig in 1973 and propose
instead a coordinate system based on the weather scatters’ frame of reference (grav-
ity, the ground) \[26, 27, 28\]. They also consider the effects of tilting the PPAR, but
do not consider roll.

Most prior works assume ideal radiating elements, and do not provide an ap-
proach to incorporating physical measurements of radar components. Similarly, they
lack a method which allows for integration of real-world calibration techniques. While
these prior works have described polarimetric bias sources theoretically, experimental
verification of these theories is still needed. Weather observations collected by a po-
larimetric phased-array weather radar are still limited. Additionally, no standardized
method of calibration for polarimetric bias exists for phased-arrays, though some have been proposed.

1.3 Description of Chapters

Chapter 1: Introduction Describes the motivation for this research. It presents the limitations of parabolic antenna radars currently in use, the advantages of phased-array technology, and the challenges of correcting for polarimetric errors in phased-arrays. It then summarizes prior literature seeking to address these challenges.

Chapter 2: Preliminaries Provides an overview of weather radar theory, including the radar range equation, volume reflectivity, doppler velocity and polarimetric variables. It also covers phased-array antenna theory, including linear and planar phased-arrays.

Chapter 3: Instrument Specification and Operation Details the instruments used in this research. It describes the hardware components of the Skyler phased-array radar, and the parabolic reflector UMass Experimental X-Band (UMaXX) radar. Radar installation and data collection procedures are described as well.

Chapter 4: Bias Correction Model Begins by discussing prior work on phased-array weather radar in detail: how it contributes to this research and its limitations. Next, it presents a novel method of characterizing polarimetric bias in phased-arrays based on a decomposition of bias sources. Finally, it describes a partial correction calibration model which implements the novel approach.

Chapter 5: Calibration Results Explains the execution the calibration method presented in the previous chapter. It first describes the data processing techniques used for Skyler and UMaXX radars. Next, it displays various plots
comparing data from the two radars to evaluate the correction model. Finally, it discusses several challenges discovered while performing this analysis.

Chapter 6: Conclusions Presents a summary of this research, conclusions, and potential future work it enables. Appendices include a full list of radar data collected, an optional correction of cross-polarized radiation, and a method for dealing with the effects of a wet radome.
CHAPTER 2
PRELIMINARIES

2.1 Weather Radar Range Equation

In meteorology, the radar range equation is used to describe the power received and is given by:

\[ P_r = P_t \frac{G_t G_r \lambda^2 \sigma}{(4\pi)^3 R^4} \]  \hspace{1cm} (2.1)

where \( P_t \) is the peak transmitted power, \( G_t \) is the transmit antenna gain, \( G_r \) is the receive antenna gain, \( \lambda \) is the wavelength, \( R \) is the range, and \( \sigma \) is the radar cross-section of the target [29].

2.2 Reflectivity Factor

Volume reflectivity is defined as the total expected backscattering cross section per unit volume. For a spherical hydrometeor with diameter \( D \), \( \sigma_b(D) \) is the backscattering cross section. \( N(D, r) \), known as the drop size distribution, represents expected density of drops with diameter \( D \) within the unit volume at vector range \( r \). Volume reflectivity can be found by integrating the product of \( \sigma_b(D) \) and \( N(D, r) \) for all drop sizes, resulting in units of \( m^{-1} \). 

\[ \eta(r) = \int_0^\infty \sigma_b(D) N(D, r) dD \]  \hspace{1cm} (2.2)

When the targets (water drops) are small compared to the wavelength, the radar is said to be in the Rayleigh regime, and backscattering cross section \( \sigma_b(D) \) can be approximated as:

\[ \sigma_b(D) = \frac{\pi^5}{\lambda^4} |K_w|^2 D^6 \]  \hspace{1cm} (2.3)
where $K_w$ is the complex refractive index of water at wavelength $\lambda$. Substituting for $\sigma_b(D)$ in (2.2) gives:

$$\eta(r) = \frac{\pi^5}{\lambda^4} |K_w|^2 \int_0^\infty N(D, r) D^6 dD$$  \hspace{1cm} (2.4)$$

The integral portion of this equation is known as the reflectivity factor $Z$, which can be rewritten in discrete form:

$$Z = \int_0^\infty N(D, r) D^6 dD = \frac{1}{\Delta V} \sum_{i=1}^N D_i^6$$ \hspace{1cm} (2.5)$$

Reflectivity factor has units of $m^3$, however it is typically converted to use $mm^6/m^3$ instead, representing drop sizes in millimeters over volume in meters. Because the reflectivity factor depends on the sixth moment of drop size, it is heavily influenced by the presence of large hydrometeors. It also spans a very wide range, and thus is usually reported logarithmically as dBZ, where $0 \text{ dBZ} = 1 \text{ mm}^6/m^3$.

Therefore, to find the radar range equation for volumetric targets, first take the total backscattering constant as the product of reflectivity and the volume illuminated by the transmit pulse. Assuming a circularly symmetric Gaussian beam shape and a rectangular pulse, the volume’s backscattering constant is:

$$\sigma_v = \frac{\beta^2 \pi R^2}{8(\ln 2)} \cdot \frac{c \tau}{2} \cdot \eta$$ \hspace{1cm} (2.6)$$

where $\beta$ is antenna half-power beamwidth, $\tau$ is the transmit pulse length, and $c$ is the speed of light. Substituting into (2.1) gives:

$$P_r = P_t \frac{G_t G_r \beta^2 c \tau \lambda^2 \eta}{1024(\ln 2)^2 \pi^2 R^2}$$ \hspace{1cm} (2.7)$$

Finally, substituting (2.4) into (2.7) gives the volumetric radar range equation typically used in weather radar.

$$P_r = P_t \frac{G_t G_r \beta^2 c \tau \pi^3 |K_w|^2}{1024(\ln 2)^2 \lambda^2 R^2} Z$$ \hspace{1cm} (2.8)$$
This equation relates power received at a given range to reflectivity factor $Z$. All other terms are assumed constant during radar operation.

### 2.3 Polarimetric Variables

From measured radar signals, certain useful weather variables can be determined. The first of which is reflectivity factor $Z$ (hereafter referred to as reflectivity), which as shown above, is proportional to received power. In each equation, $E$ represents the received electric field signals.

\[
Z \propto P = < E(t) E^*(t) > 
\]  
(2.9)

In addition to reflectivity, pulse radars also measure doppler velocity ($v_d$) as a phase shift in received signal. However, measuring the phase shift directly is difficult, so instead the signal from one pulse is compared to that of the following pulse which has a time delay of $T_P$. This method is called pulse-to-pulse doppler.

\[
v_d \propto \angle < E(t) E^*(t - T_P) > 
\]  
(2.10)

A radar which transmits only a single polarization is limited to these two weather variables. However, nearly all weather radars make use of dual-polarization, where both a horizontally and vertically polarized signal is transmitted and received. By comparing the two polarizations, several more useful weather variables can be measured. Differential reflectivity ($Z_{dr}$) is simply the ratio of vertical and horizontal reflectivity factors ($Z_v$ and $Z_h$) and provides a more accurate estimate of drop size and rain rate.

\[
Z_{dr} = 10 \log(Z_h/Z_v) 
\]  
(2.11)
Polarization correlation coefficient \( (\rho_{hv}) \) is useful for hydrometeor classification and is determined by:

\[
\rho_{hv} = \frac{\langle E_h E_v^* \rangle}{\sqrt{\langle |E_h|^2 \rangle \langle |E_v|^2 \rangle}}
\]  \hspace{1cm} (2.12)

where \( E_h \) and \( E_v \) are the electric fields measured by the radar in the horizontal and vertical polarizations respectively. The complex angle of \( \rho_{hv} \) is known as the differential propagation phase \( (\phi_{dp}) \), and is equal to the phase difference between horizontal and vertical receive signals.

\[
\phi_{dp} = \angle \langle E_h E_v^* \rangle = \phi_h - \phi_v
\]  \hspace{1cm} (2.13)

This measurement is useful for determining attenuation and some rainfall estimates.

### 2.4 Phased-Array Theory

Phased-array theory is discussed in detail in Balanis’ textbook on antenna theory, some of which is summarized here [30]. Phased-array antennas are constructed from multiple radiating elements arranged in some geometrical configuration. The total field pattern of the array is then the vector sum of the field patterns of all the elements (neglecting coupling). By changing the excitation amplitude and phase of each element, one can create regions of constructive interference in the desired scan direction, and destructive interference elsewhere. This is known as electric beam steering. The individual pattern of the elements and their geometrical arrangement also determine the properties of the array, but these usually cannot be changed once the array is built. Individual elements do not have to be identical, but in most practical cases they are identical to simplify design and manufacturing.

The total far field pattern of a phased-array is typically described as the multiplication of two terms: The field of a single element (relative to the center of the array), and the array factor.
\[ E(\text{total}) = [E(\text{single element})] \times \text{[array factor]} \] (2.14)

This is known as the pattern multiplication principle. For example, the array factor of a simple two-element array along the z-axis (shown in Figure 2.1) is given by:

\[ AF = 2 \cos \left[ \frac{1}{2} (kd \cos \theta + \beta) \right] \] (2.15)

where \(k\) is the wave number, \(d\) is the distance between elements, and \(\beta\) is the difference in excitation phase between elements.

\[ \psi = kd \cos \theta + \beta \] (2.17)

\textbf{Figure 2.1.} Array of two dipole elements positioned along the z-axis (Image from Balanis [30]).

In a linear uniform array with \(N\) identical elements, each excited with equal magnitude and progressive phase difference \(\beta\), the array factor can be constructed from the summation of each element’s contribution:

\[ AF = \sum_{n=1}^{N} e^{j(n-1)\psi} \] (2.16)

\[ \psi = kd \cos \theta + \beta \] (2.17)
This can be rewritten and normalized as:

\[ AF_n = \frac{1}{N} \left[ \frac{\sin \left( \frac{N}{2} \psi \right)}{\sin \left( \frac{\psi}{2} \right)} \right] \approx \frac{\sin \left( \frac{N}{2} \psi \right)}{\frac{N}{2} \psi} \]  

(2.18)

For a uniform linear broadside array, the half power beamwidth is:

\[ \Theta_h = \cos^{-1} \left[ \cos \theta_0 - 0.443 \frac{\lambda}{(L + d)} \right] - \cos^{-1} \left[ \cos \theta_0 + 0.443 \frac{\lambda}{(L + d)} \right] \]  

(2.19)

where \( \theta_0 \) is the scan angle, and \( L \) is the length of the array. Directivity for a large array is given as:

\[ D_0 = 2N \frac{d}{\lambda} \approx 2L \frac{\lambda}{\lambda} \]  

(2.20)

where \( L \) is the length of the array.

Finally, this can be extended to a two-dimensional rectangular grid of elements, known as a planar array. In this dissertation, antenna broadside is defined as the direction orthogonal to the planar array aperture. Here, the array factor is the product of the array factors for two linear arrays, one along each axis. In a two-dimensional planar array in the x-y plane, the array factor is described by:

\[ AF_n(\theta, \phi) = \left[ \frac{1}{M} \frac{\sin \left( \frac{M}{2} \psi_x \right)}{\sin \left( \frac{\psi_x}{2} \right)} \right] \left[ \frac{1}{N} \frac{\sin \left( \frac{N}{2} \psi_y \right)}{\sin \left( \frac{\psi_y}{2} \right)} \right] \]  

(2.21)

where

\[ \psi_x = kd_x \sin \theta \cos \phi + \beta_x \]  

(2.22)

\[ \psi_y = kd_y \sin \theta \sin \phi + \beta_y \]  

(2.23)

Each axis has its own progressive phase, \( \beta_x \) and \( \beta_y \), meaning the main antenna beam can be steered in two dimensions. In a scanning planar phased-array, calculating
the exact gain and beamwidth can quickly become very complex. However, for a large array scanning near broadside, the beamwidth can be approximated using the following formulas:

$$\Theta_h = \sqrt{\frac{1}{\cos^2 \theta_0 \left[ \Theta_x^2 \cos^2 \phi_0 + \Theta_y^2 \sin^2 \phi_0 \right]}}$$

$$\Psi_h = \sqrt{\frac{1}{\Theta_x^2 \sin^2 \phi_0 + \Theta_y^2 \cos^2 \phi_0}}$$

where $\Theta_h$ is the half power beamwidth in the elevation ($\phi$) plane, and $\Psi_h$ is the half power beamwidth in the azimuth ($\theta$) plane. $\Theta_x$ and $\Theta_y$ are the half power beamwidth of the linear array for the x and y-axes respectively and are calculated using (2.19). The beam solid angle is:

$$\Omega_A = \Theta_h \Psi_h$$

For large planar arrays near broadside, directivity can be approximated as:

$$D_0 = \pi \cos \theta_0 D_x D_y \simeq \frac{\pi^2}{\Omega_A (rad^2)}$$

where $D_x$ and $D_y$ are the directivities of linear arrays of length $L_x$ and $L_y$ respectively.
CHAPTER 3
INSTRUMENT SPECIFICATIONS AND OPERATION

3.1 Testbed Design

This research seeks to study the viability of phased-array antennas for weather radar polarimetry. This is achieved through development of a calibration model for polarimetric phased-array weather radar, and by validation of the calibration model through the implementation and operation of a practical radar testbed. The testbed consists of two dual-polarized, X-band weather radars: a planar phased-array antenna radar (PPAR) and a mechanically-scanned parabolic reflector antenna radar.

By co-locating the two radars and operating them synchronically, radar weather observations can be directly compared to evaluate the calibration model’s performance. Specifically, the electronically scanning PPAR will use the mechanically-scanned dish radar as a control reference. Both radars are dual-polarized and operate at X-band, with a frequency separation of approximately 200 MHz. The hardware specifications of each of the two radars are described in detail below.

3.2 Skyler Specifications

3.2.1 Background Information

The phased-array used in this study is the first prototype of the “Skyler” radar system [15] developed by Raytheon (Hereafter simply referred to as Skyler). Skyler is a low-power, X-band, planar phased-array radar (PPAR) developed for weather and aviation applications. Earlier publications have referred to this system as the Low Power Radar, or LPR [31, 32, 33]. As a PPAR, Skyler is capable of electronically scanning in
two dimensions (2D), covering 90 degrees in azimuth and 30 degrees in elevation. It also supports dual-polarization measurements in Alternate-Transmit/Alternate Receive (ATAR) mode.

Figure 3.1. Photo of the Skyler Radar with the planar radome removed.

3.2.2 Array Architecture

The Skyler prototype consists of 2560 dual-polarized antenna elements divided into 20 tiles of 128 elements each. The tiles are based on an integrated, air-cooled, active panel concept [34]. Although the prototype employs liquid cooling, subsequent prototypes are air-cooled. The tiles and individual elements can be seen in Figure 3.1. The antenna enclosure box is 59” wide, 68.5” tall, and 13.5” deep. Inside this
enclosure are the major components of the radar: The driver amplifier, beamformer assembly, up/down converter, digital receiver, network switch, and power distribution board. The datalogger computer and liquid coolant chiller pump are external to the enclosure. A diagram of the rear view of the enclosure is shown in Figure 3.2. Figures 3.3 and 3.4 show simplified block diagrams of the radar signal path on transmit and reception respectively. Each component will be described in detail below.

![Figure 3.2. Rear view of the Skyler radar enclosure backend.](image)

### 3.2.3 Antenna Elements

The radiating elements are square microstrip patches which are slot-fed (Figure 3.5). The dimensions of each element are 332 x 320 mil (8.43 x 8.13 mm). The elements are arranged in an array of 64 by 40 elements (4 by 5 tiles) with a design scan...
Figure 3.3. Block diagram of signal generation and control on radar transmission. Data lines are shown in blue, while power lines are shown in red.

Figure 3.4. Block diagram of signal acquisition on radar reception. Data lines are shown in blue, while power lines are shown in red.

range of 90 degrees in azimuth and 30 degrees in elevation. Patch frequencies range from 9.56 to 9.64 GHz. The maximum transmit power per element is approximately 17 dBm. The array is illuminated with a uniform taper upon transmission, and with a 20-dB Taylor distribution upon reception.
3.2.4 SiGe T/R Modules

Each element is controlled by a custom Silicon-Germanium (SiGe) Monolithic Microwave Integrated Circuit (MMIC) chip. These SiGe chips include two low noise amplifiers, a power amplifier, and a programable attenuator and phase shifter. They also contain switches for transmit/receive modes and horizontal/vertical polarizations. As a result, each element has individual amplitude, phase, and polarization control on both transmission and reception. The circuit diagram of a single SiGe chip is shown in Figure 3.6.

The SiGe chips only support one polarization at a time, thus the polarization scheme employed by the prototype radar is limited to Alternate-Transmit/Alternate-Receive (ATAR). Later generations of the MMIC support two simultaneous channels. Because the prototype radar employs the ATAR mode, the co-polar correlation coefficient cannot be measured at zero time lag. The method described in [35] is used to estimate its magnitude and phase. There are 128 of these chips arranged in a 16 by
8 grid on a circuit board. Built into the SiGe board is a 128-way RF power divider, and each of the 20 boards is fed by a 20-way divider/combiner.

![Diagram of the internals of a single SiGe MMIC chip.](image)

**Figure 3.6.** Diagram of the internals of a single SiGe MMIC chip.

### 3.2.5 PLCC Boards

Each of the 20 subpanels is controlled by a Power and Logic Circuit Card (PLCC). These cards are all networked over 100 Mbit ethernet to a 24-port network switch inside the radar box. The PLCCs receive amplitude, phase, and polarization settings from the host computer based on the desired dwell. Next, the PLCCs program the SiGe boards with the updated settings. They also provide +48V power to the SiGe boards. This +48V power for all 20 PLCCs is provided by a central Power and Data Circuit Card (PDCC). The PDCC provides power and timing information to all internal components of the radar. It is locked to the same 10 MHz reference clock as the up/down converter and the digital receiver.
3.2.6 Up/Down Converter

The up/down converter assembly converts between RF center frequency of 9.6 GHz and the 60 MHz intermediate frequency of the digital transceiver. It does this in two stages, with the second IF at 2100 MHz. The local oscillators (LO) are at 11.7 GHz and 2160 MHz. Both LOs are locked to a 10 MHz external reference clock. Because the radar operates in ATAR, the up/down converter has only a single channel. A diagram of the up/down converter is shown in Figure 3.7. For simplicity, attenuation and amplification steps are omitted from this figure.

Figure 3.7. Diagram of the two stage up/down converter in the Skyler radar. For simplicity, amplification and attenuation components are not shown.
3.2.7 Ettus USRP

The digital transceiver originally installed by Raytheon has been replaced with an Ettus N210 Universal Software Radio Peripheral (Ettus USRP). The Ettus USRP is a software-defined radio (SDR) which uses software to emulate hardware components. It contains a Xilinx Spartan Field Programmable Gate Array (FPGA), a 100 MS/s dual channel ADC, a 400 MS/s DAC, ethernet connectivity, and has a 100 MHz clock rate, locked to a 10 MHz external reference.

The Ettus has been customized to transmit an arbitrary waveform. It can construct single or dual PRF sequences, as well as linear and non-linear frequency modulated chirp waveforms with a duty cycle up to 20%. On the receive side, the Ettus converts the 60 MHz signal from the up/down converter to baseband, performs analog to digital conversion, and sends the data via ethernet to the data logger PC.

3.2.8 On-board Software / Networking

The various components of the Skyler radar communicate through an internal digital network. At the center of this network is a 24 port network switch, which is connected to each of the 20 PLCCs and the PDCC via 100 Mbit ethernet. It also connects to the Ettus USRP and the host computer via Gbit ethernet. Figure 3.8 shows a block diagram of this network.

In operation, the host computer initiates a radar scan by first computing the beamforming parameters for each dwell in the scan. A dwell is a series of pulses with a particular beam direction, polarization mode, and pulsing mode. The host computer sends the amplitude, phase, and polarization data for the first dwell to the PLCCs and sends timing commands to the PDCC and Ettus USRP. The radar executes the dwell and the Ettus USRP sends the sampled receive signal back to the host computer. This is repeated for all dwell in the scan. A datalogging program
on the host computer processes the raw sampled data into second moment weather products. It then stores these moments along with the raw data in real time.

![Block diagram of the internal network of the Skyler radar.](image)

**Figure 3.8.** Block diagram of the internal network of the Skyler radar.

The software Graphical User Interface (GUI) allows the radar operator to adjust settings for polarization type, PRF dwell, scan parameters, pulse type, and data collection prior to operation. It also allows for starting and stopping radar transmission, as well as providing a live readout of the radar status such as temperature, power usage, array orientation, and GPS location (Figure 3.9).

### 3.2.9 Liquid Cooling and Chiller

This Skyler prototype model uses liquid cooling for heat dissipation. Heat pipes run along metal panels which fit inside the array between the SiGe chips and the
PLCC boards. They are connected to an external chiller pump, which must always be running when the radar is used.

### 3.3 UMaXX Specifications

#### 3.3.1 Background Information

The mechanically-scanned parabolic reflector radar used is a reconstructed and updated version of the MA-1 radar. This radar was originally built by the Engineering Research Center for Collaborative Adapting Sensing of the Atmosphere (CASA) as part of the Integrated Project 1 (IP1), a network of four short range radars deployed...
in south-west Oklahoma in 2006 [36]. After being redesigned and rebuilt in 2018, the radar has been rebranded as the UMass eXperimental X-band (UMaXX) Radar. The upgrading of MA-1 into the new UMaXX radar was mostly completed by fellow MIRSL student Jezabel Vilardell Sanchez, and a full description of her work can be found in her master's thesis [37]. A summarized description of the hardware is presented here.

3.3.2 Dish Antenna

UMaXX uses a dual linear polarized, X-Band parabolic reflector dish with a 122 cm (4 ft) diameter. It has an aperture efficiency of 0.6, 38.5 dBi gain and a 1.85 degree beamwidth. This dish is mechanically scanned in elevation using a motion servo driver controller and worm gear drive made by the IDC corporation, and scanned in azimuth using an RPM PSI direct-drive rotator (RT-0507) positioner. This system allows the antenna to scan in elevation from -5 to 90 degrees relative to the horizon and rotate continuously the full 0 to 360 degrees in azimuth.

3.3.3 Magnetron Transmitter

The transmitter is a magnetron cavity resonator fed by a pulse modulator. The modulator board is a Raytheon MK II line type modulator using pulse forming networks (PFNs). The magnetron used is an M1458 X-Band with a frequency range of 9380 to 9440 MHz and a peak transmit power of 12 kW. It outputs to the antenna via waveguide. Since magnetrons transmit pulses with random phase, the radar operates in cohere on receive mode by sampling the transmit pulse leakage. As a cavity resonator, the magnetron’s center frequency drifts with temperature, usually stabilizing once the radar has been running for a few minutes. The downconverter is tuned to this steady state frequency.
3.3.4 Downconverter

Originally, the MA-1 radar used a triple downconversion receiver implemented on a custom board. For UMaXX, this was removed and replaced with a much simpler single downconversion receiver. A free-running DRO at 9.35 GHz and an image-reject mixer are used to convert from the 9.41 GHz transmit frequency to the first IF at 60 MHz, which is then presented to an Ettus Research N210 digital receiver. The Ettus then performs the final downconversion to baseband internally, with the help of an external 10 MHz reference LO.

3.3.5 Ettus USRP

Many of the improvements to UMaXX are made possible due to the incorporation of the Ettus N210 Universal Software Radio Peripheral (Ettus USRP). The Ettus USRP is a software-defined radio (SDR) which uses software to emulate hardware components. It contains a Xilinx Spartan Field Programmable Gate Array (FPGA), a 100 MS/s dual channel ADC, a 400 MS/s DAC, ethernet connectivity, and has a 100 MHz clock rate, locked to a 10 MHz external reference. Using its internal FPGA, the Ettus USRP acts as both the baseband receiver and the transmit trigger pulse generator. It also provides all the calibration and Linear Depolarization Ratio (LDR) mode control signals.

3.3.6 LDR Mode

Under normal operation, the UMaXX radar transmits and receives in both vertical and horizontal polarizations simultaneously, known as Simultaneous Transmit Simultaneous Receive (STSR) mode. However, it has been modified to also support cross-polarization measurements using an LDR mode. In this mode, only horizontal polarization is transmitted while both polarizations are received. This is achieved within the waveguide structure using an electro-mechanical switch to redirect the vertical transmit signal into a load. While such a configuration is not ideal, as half
of the transmit power is wasted, it was deemed the best solution given the physically small radar enclosure and placement of the antenna waveguide feeds.

Figure 3.10. UMaXX waveguide duplexer structure, updated to support LDR mode (Image from Sanchez, et al [37]).

3.3.7 Networking

The main acquisition program runs on the radar control computer and controls both the pedestal positioner as well as the Ettus interface. The radar operator inputs system parameters such as scan type, pulse scheme, pulse duration, LDR mode, etc. The system then executes the scan and raw data is streamed to the radar PC from the Ettus. Weather moments are also processed in real time and Plan Position Indicator (PPI) display images are generated as .png files. These PPI images are then uploaded to a CASA server where data can be view live via an online portal. While installed on
the Orchard Hill Tower, the UMaXX control PC can be accessed remotely through a fiber optic link.

**Figure 3.11.** Photo of the UMass Experimental X-band Radar (UMaXX) installed inside the Orchard Hill tower radome. Also pictured: Casey Wolsieffer (left) and William Heberling (right).

### 3.4 Testbed Joint-Operation

#### 3.4.1 Tower Installation

The two radars were successfully installed on top of a tower located on Orchard Hill at the University of Massachusetts Amherst (UMass) campus (Fig. 3.11). The tower is approximately 21 meters (70 feet) tall. UMaXX was installed inside the tower radome, while the phased-array radar was mounted to the tower deck with a tilt angle of 15 degrees so that its lowest scan angle corresponds to zero degrees elevation, as shown in Figure 3.13. The liquid chiller pump was also installed in the tower. The completed testbed allows for the co-located radars to be operated simultaneously.
and remotely. Radar coverage for both radars is shown in Figure 3.14. UMaXX is typically operated with a maximum range of 60 km (37.3 mi) and Skyler with a maximum range of 40 km (24.9 mi). System parameters for both the Skyler and the UMaXX radars are enumerated in Table 3.1 [34, 31].

While UMaXX continuously operates throughout the year, Skyler was only run periodically due to limited data storage and temperamental software. Also, because Skyler sits outside the tower radome, it has little protection against harsh winter weather. Therefore, it was removed from the tower for storage each November and reinstalled in the spring.

![Skyler installed on the Orchard Hill Tower](image)

**Figure 3.12.** Photo of Skyler installed on the Orchard Hill Tower.

### 3.4.2 Data Collection Schemes

The core of this research is based on joint-operation measurements collected by both the Skyler and UMaXX radar systems. Such data was recorded over two years from the top of the Orchard Hill tower on UMass campus in a variety of weather
Table 3.1. Radar System Characteristics

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Skyler</th>
<th>UMaXX</th>
</tr>
</thead>
<tbody>
<tr>
<td>Center Frequency</td>
<td>9.6 GHz</td>
<td>9.41 GHz</td>
</tr>
<tr>
<td>Peak/average Power</td>
<td>125 W / 23 W</td>
<td>12.5 kW / 12.5 W</td>
</tr>
<tr>
<td>Bandwidth</td>
<td>&lt;6 MHz</td>
<td>5 MHz</td>
</tr>
<tr>
<td>Polarization</td>
<td>Dual H/V (ATAR)</td>
<td>Dual H/V (STSR)</td>
</tr>
<tr>
<td>Beamwidth</td>
<td>1.9° Az, 2.1° El</td>
<td>1.85°</td>
</tr>
<tr>
<td>Scan Range</td>
<td>±45° Az, 0-30° El</td>
<td>Hemisphere</td>
</tr>
<tr>
<td>Range Resolution</td>
<td>60 m</td>
<td>30-150 m</td>
</tr>
<tr>
<td>Waveforms</td>
<td>Pulse, NLFM, LFM</td>
<td>Pulse</td>
</tr>
<tr>
<td>Sequences</td>
<td>Single/Dual PRF</td>
<td>Single/Dual PRT</td>
</tr>
</tbody>
</table>

conditions. Various scan modes were also used and are available for analysis. Broad system calibrations can be made by averaging this data, while situational corrections
can be verified for specific data subset cases. A complete timeline of all data collection events for 2018 and 2019 are shown in Tables 3.2 and 3.3 below.

3.4.3 Initial Data Processing

During operation, both radars collect and store raw sampled horizontal (H) and vertical (V) receive signals and processed second moment data in real time. Afterwards, this data is reprocessed to include range correction, velocity unfolding, basic noise removal, and a rough reflectivity calibration. It is then reformatted into Network Common Data Form (NetCDF) files. This format is commonly used in meteorology and is compatible with a wide range of software packages and libraries.
### Table 3.2. Data Collection 2018 Timeline

<table>
<thead>
<tr>
<th>Date</th>
<th>Task</th>
</tr>
</thead>
<tbody>
<tr>
<td>August 2017</td>
<td>Skyler assembled in lab</td>
</tr>
<tr>
<td>March 2018</td>
<td>UMaXX pedestal + antenna assembled in lab</td>
</tr>
<tr>
<td>May 2018</td>
<td>Skyler installed on truck</td>
</tr>
<tr>
<td>June 2018</td>
<td>SPOTTR 2018 Mobile Deployment</td>
</tr>
<tr>
<td>July 17, 2018</td>
<td>Vertical data collected from truck bed</td>
</tr>
<tr>
<td>August 1, 2018</td>
<td>Vertical data collected from truck bed</td>
</tr>
<tr>
<td>August 20, 2018</td>
<td>Phased Tilt Radar removed from tower</td>
</tr>
<tr>
<td>August 24, 2018</td>
<td>UMaXX installed in tower</td>
</tr>
<tr>
<td>October 4, 2018</td>
<td>Skyler installed in tower</td>
</tr>
<tr>
<td>October 11, 2018</td>
<td>Data collection with both radars</td>
</tr>
<tr>
<td>October 27, 2018</td>
<td>Data collection with both radars</td>
</tr>
<tr>
<td>November 3, 2018</td>
<td>Data collection with both radars</td>
</tr>
<tr>
<td>November 7, 2018</td>
<td>Drone flight experiment</td>
</tr>
<tr>
<td>November 9, 2018</td>
<td>Data collection with both radars</td>
</tr>
<tr>
<td>November 13, 2018</td>
<td>Data collection with both radars</td>
</tr>
<tr>
<td>November 30, 2018</td>
<td>Skyler removed from tower</td>
</tr>
</tbody>
</table>

### Table 3.3. Data Collection 2019 Timeline

<table>
<thead>
<tr>
<th>Date</th>
<th>Task</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jan-April 2019</td>
<td>Repairs in lab (PLCCs)</td>
</tr>
<tr>
<td>May 2019</td>
<td>Skyler reinstalled on truck</td>
</tr>
<tr>
<td>June 2019</td>
<td>SPOTTR 2019 Mobile Deployment</td>
</tr>
<tr>
<td>July 22, 2019</td>
<td>Vertical data collected from truck bed</td>
</tr>
<tr>
<td>July 25-30, 2019</td>
<td>Skyler installed in tower</td>
</tr>
<tr>
<td>August 13, 2019</td>
<td>Data collection with both radars</td>
</tr>
<tr>
<td>August 19, 2019</td>
<td>Data collection with both radars</td>
</tr>
<tr>
<td>August 21, 2019</td>
<td>Data collection with both radars</td>
</tr>
<tr>
<td>August 28, 2019</td>
<td>Data collection with both radars</td>
</tr>
<tr>
<td>September 2, 2019</td>
<td>Data collection with both radars</td>
</tr>
<tr>
<td>September 4, 2019</td>
<td>Data collection with both radars</td>
</tr>
<tr>
<td>September 12, 2019</td>
<td>Data collection with both radars</td>
</tr>
<tr>
<td>September 24, 2019</td>
<td>Data collection with both radars</td>
</tr>
<tr>
<td>September 26, 2019</td>
<td>Data collection with both radars</td>
</tr>
<tr>
<td>October 7, 2019</td>
<td>Data collection with both radars</td>
</tr>
<tr>
<td>October 27, 2019</td>
<td>Data collection with both radars</td>
</tr>
<tr>
<td>October 31, 2019</td>
<td>Data collection with both radars</td>
</tr>
<tr>
<td>November 4, 2019</td>
<td>Skyler removed from tower</td>
</tr>
</tbody>
</table>

Each NetCDF file stores scan in azimuth at a single elevation. Weather variables are each stored as an array of arranged azimuth verse range bin. Variables stored
are horizontal reflectivity ($Z_h$), differential reflectivity ($Z_{dr}$), doppler velocity ($v_d$),
doppler spectrum width ($w$), cross-polar correlation coefficient ($\rho_{hv}$), differential phase
($\phi_{dp}$), and linear depolarization ratio (LDR).

The file also stores header information such as date, time, pulse mode, pulse length
and repetition frequency (PRF), polarization mode, scan type, azimuth, elevation, as
well as radar tilt, roll, latitude, and longitude.

While this initial processing is straightforward, it is extremely helpful to have
both Skyler and UMaXX data stored in the same format. This allows all the data to
be easily accessed, filtered, modified, and displayed when performing more complex
calibration and analysis as described later on.

3.4.4 Quasi-Vertical Data

Quasi-vertical data was collected by operating the Skyler radar while installed on
the truck bed. The array rested around six degrees from lying horizontally and was
scanned across its full range ($\pm 45$ Az, $\pm 15$ El). This was done during periods of
persistent, uniform light rain so that hydrometeors can be assumed spherical. Data
was collected for several hours on July 17th and August 1st of 2018, as well as July
22nd, 2019. Such data is usually useful for calibration purposes due to relatively
uniform targets at all scan angles.

However, the effects of water droplets collecting the Skyler’s planar radome were
found to be severe, since the radome was nearly horizontal with the ground. Thus,
the quasi-vertical data was not used in the final calibration or data analysis. Instead,
measurements made from the tower in light rain were used for system characterization,
as discussed in section 5.1.1. An expanded description of wet radome effects on a
planar phased-array is included in the Appendix.
3.4.5 Mobile Operation

During the spring of 2018 and 2019, the Skyler radar was temporarily transferred to a truck-bed mounted fully mobile configuration. The phased-array was installed on an azimuthal pedestal and could also be tilted from standing upright to pointing completely vertical. The liquid cooler, power supplies and PC controller were all installed in the truck as well and powered by an on-board generator. The entire system is capable of deploying or evacuating in under 5 minutes.

Mobile operations were performed using the Skyler truck in collaboration with Purdue University as part of the Students of Purdue Observing Tornadic Thunderstorms for Research (SPOTTR) program, with the goal of making close range observations of potentially tornadic storms in the midwestern United States [38]. Data was collected during June of 2018 and 2019 during many severe weather events. This data provides additional phased-array measurements in weather situations unavailable on campus.

![Photo of the Skyler radar installed on the truck for mobile operation.](image)

**Figure 3.15.** Photo of the Skyler radar installed on the truck for mobile operation.
UMass provided assistance in driving, maintaining, and operating Skyler for the duration of the program, and also trained Purdue students in deployment of the radar. In addition, the calibration method described by this research was used when processing Skyler data for the SPOTTR program. However, as the UMaXX radar was not present as an external reference, this mobile data less useful for the direct comparison approach used to analyze the calibration performance. Therefore, mobile data will not be included in later data discussions, but may be useful in future works.
CHAPTER 4

BIAS CORRECTION MODEL

4.1 Research Background

4.1.1 Summary of Prior Works

In order to fully explain this research, it is necessary to review the existing literature of prior works. From 2011 to 2013, an upgrade was performed on the NEXRAD WSR-88 system to dual polarized radar (WSR-88D), which allowed for more accurate qualitative precipitation estimates (QPE) and hydrometeor classification [20]. Prior to this, several methods for calibration of polarimetric dish radar were described and experimentally verified, including vertical scans, light rain scans, drop size distribution fitting, and scattering matrix form [21, 22, 23, 24, 20].

In 2002, Moisseev et. al. described the problem of polarimetric calibration by relating the measured scattering matrix $M$ to the actual scattering matrix $S$ through a $2 \times 2$ transform matrix:

$$M = T^T ST$$

In that paper, they conclude that for a dish radar requiring $Z_{dr}$ accuracy of 0.1 dB, point target calibration can be used. However, for accurate LDR measurements, light rain and vertical calibration is needed [16].

By 2009, research into phased array antennas as an option for weather radars was growing. In particular, Planar Phased-Array Radars (PPAR) have great potential as the next upgrade to major weather radar networks such as NEXRAD, providing faster update times and scan flexibility.
The research in this dissertation is mainly built on the foundation laid out by Zhang, Doviak, and Zrnić et. al. in their 2009 paper on phased-array radar polarimetry [17]. In that paper, they introduce the existence of scan angle dependent polarimetric biases unique to PPAR which present a challenge for calibration. They propose an approach similar to Moisseev’s, using a projection transform matrix \( P \) to relate measured \( S \) and intrinsic \( S' \) scattering matrices.

\[
S = P^T S' P
\]  

They demonstrate a theoretical correction for ideal dipole radiators in both ATSR and STSR modes. They also relate these biases to weather parameters, such as reflectivity, differential reflectivity, differential phase, and correlation coefficient. In 2011 they expand the 2009 study to include doppler bias removal and consider cylindrical array architecture as a possible alternative to PPAR for weather applications [18, 19]. However, comparison of cylindrical and planar phased-array weather radars is beyond the scope of this dissertation and will not be discussed further here.

Polarization biases introduced by PPAR are related to the polarization of the radiated electric fields, and thus are dependent on the radiating elements used. Lei, Zhang, and Doviak consider this in 2013, laying out projection matrix approaches for PPAR using ideal patch and aperture elements [25]. That work is highly relevant here because the PPAR used in this dissertation (Skyler) has patch radiator elements.

Significant to the research in this dissertation is the precise definitions of coordinates and cross-polarization. Aboserwal, Salazar and Fulton et. al approached this issue in 2016 and 2018, describing in detail the polarization definitions famously set forth by Ludwig in 1973 [26, 27]. They concluded that for weather radar applications, none of Ludwig’s definitions are particularly useful and instead, a coordinate system based on the weather scatters’ frame of reference (gravity and the horizon) should be used. In those papers they also considered the effects of tilting the PPAR, which is
discussed in this dissertation as well. We now provide a more detailed description of the correction approach presented in the works of Moisseev, Zhang, Doviak, Zrnić, and Lei.

4.1.2 Projection of Fields from a Planar Phased-Array

The projection of fields radiated by elements of a planar phased-array follows the descriptions provided by Zhang, et al. [17] and subsequent works [18, 25, 39]. Those works define a projection matrix to relate the broadside transmitted wave to the wave incident at a given scan direction:

\[ \vec{E}_i = P \vec{E}_t \]  
(4.3)

where \( \vec{E}_t \) is the magnitude of the far-zone electric fields transmitted along the array broadside direction by the horizontally and vertically polarized radiators, and \( \vec{E}_i \) is the far-zone electric fields incident on the target at a given scan angle after being projected onto the local horizontal and vertical directions. This projection occurs upon transmission and again upon reception of the radar echo. At the target, the propagation and backscattering characteristics of the radar signal are described by the same intrinsic scattering parameters employed by conventional reflector antenna radars. The resulting projected scattering parameters are related to their intrinsic counterparts through:

\[ S(p) = P^T S' P \]  
(4.4)

where \( S(p) \) is the scattering matrix (including propagation effects) that would be measured by a PPAR scanning off of broadside, \( S' \) is the intrinsic scattering matrix (also including propagation effects) that would be measured by the PPAR scanning at broadside, and \( P \) is the matrix that projects the radiated fields at broadside (no
polarization biases) $\vec{E}_t$, into the incident fields in the scatter’s scan direction and local coordinates (including polarization biases), $\vec{E}_i$:

$$\vec{E}_i = \begin{bmatrix} E_{i\phi} \\ E_{i\theta} \end{bmatrix} = \mathbf{P} \begin{bmatrix} E_{t\theta} \\ E_{t\phi} \end{bmatrix} = \mathbf{P} \vec{E}_t$$  \hspace{1cm} (4.5)

and where

$$\mathbf{P} = \begin{bmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{bmatrix} = \begin{bmatrix} E^{(h)}_\phi & E^{(v)}_\phi \\ -E^{(h)}_\theta & -E^{(v)}_\theta \end{bmatrix}.$$  \hspace{1cm} (4.6)

The superscripts (h) and (v) denote the polarization of the wave excited at the element. In (4.4), $\mathbf{P}^T$ is the transpose matrix that projects the far field back to the PPAR upon reception. The intrinsic scattering matrix, $\mathbf{S}'$, represents the co-polar and cross-polar backscattering (including propagation effects) and is given by

$$\mathbf{S}' = \begin{bmatrix} s'_{hh} & s'_{hv} \\ s'_{vh} & s'_{vv} \end{bmatrix}.$$  \hspace{1cm} (4.7)

It is worth noting that the subscripts here denote the common polarization assignments in spherical coordinates, namely $h$ is parallel to unit vector $\hat{\phi}$ and $v$ is parallel to $-\hat{\theta}$. Lei et al. [25] showed that for microstrip patch radiators, the projection matrix is given by

$$\mathbf{P} = \begin{bmatrix} \sin \theta \cdot g^{(h)}(\theta, \phi) & \cos \theta \sin \phi \cdot g^{(v)}(\theta, \phi) \\ 0 & \cos \phi \cdot g^{(v)}(\theta, \phi) \end{bmatrix}.$$  \hspace{1cm} (4.8)

where it was assumed that the array face lay in the $y$-$z$ plane and where the broadside direction was along the $x$-axis (see Figure 4.1). Here, $g^{(h,v)}$ represents the element’s radiated electric field pattern (less a dipole term).

The authors then demonstrate a theoretical correction for radiators in both alternate transmit/simultaneous receive (ATSR) and simultaneous transmit/simultaneous
receive (STSR) modes. They showed that $S'$ could, in principle, be retrieved from $S(p)$ by means of a correction matrix, $C$,

$$S' = C^T S(p) C$$  \hspace{1cm} (4.9)$$

where $C = P^{-1}$.

The authors also present a set of equations relating intrinsic second moment weather products to the corresponding measured weather products using the projection matrix. From this projection matrix it can be shown that for a given intrinsic reflectivity, $Z_h'$ and $Z_v'$, the reflectivity measured by the phased-array is given by:

$$Z(p) = p_{11}^4 Z_h' + p_{21}^4 Z_v' + 2 \sqrt{Z_h' Z_v'} Re[\rho_{hv}] p_{11}^2 p_{21}^2$$  \hspace{1cm} (4.10)$$

Similarly, it can be shown that for a given intrinsic differential reflectivity, $Z_{dr}'$, the $Z_{dr}$ measured by a phased-array for ATSR systems is given by:

$$Z_{dr}(p) = 10 \log \left( \frac{p_{11}^4 Z_{dr}' + p_{21}^4 + 2 \sqrt{Z_{dr}' Z_{dr}'} Re[\rho_{hv}] p_{21}^2 p_{22}^2}{p_{12}^4 Z_{dr}' + p_{22}^4 + 2 \sqrt{Z_{dr}' Z_{dr}'} Re[\rho_{hv}] p_{12}^4 p_{22}^2} \right)$$  \hspace{1cm} (4.11)$$

Similar relations can be found for $\rho_{hv}$ and $LDR$:

$$\rho_{hv}(ATSR) = \frac{p_{11}^2 p_{12}^2 Z_{dr}'^{1/2} + p_{11}^2 p_{22}^2 \rho_{hv}}{p_{11}^2 \cdot \sqrt{p_{12}^4 Z_{dr}' + p_{22}^4 + 2 p_{12}^2 p_{22}^2 Re[\rho_{hv}] Z_{dr}'^{1/2}}}$$  \hspace{1cm} (4.12)$$

$$LDR(p) = 10 \log \left( \frac{p_{11}^4 p_{12}^4 Z_{dr}' + p_{21}^4 p_{22}^4 + 2 p_{11}^2 p_{12}^2 p_{21}^2 p_{22}^2 \sqrt{Z_{dr}' Z_{dr}' Re[\rho_{hv}]}}{p_{11}^4 Z_{dr}' + p_{21}^4 + 2 p_{21}^2 p_{22}^2 \sqrt{Z_{dr}' Re[\rho_{hv}]} Z_{dr}'^{1/2}} \right)$$  \hspace{1cm} (4.13)$$

In general, the projected polarimetric products depend upon the intrinsic products, elements of $P$, and upon the differential reflectivity, $Z_{dr}$, and the co-polar correlation coefficient’s ($\rho_{hv}$) magnitude and phase.
4.1.3 Limitation of Prior Works

Prior works mainly rely on ideal radiating elements. While a theoretical description of polarimetric biases is essential, in practice it is useful to account for non-ideal hardware components using experimental calibration measurements. Thus, it is desirable to allow measurements of non-ideal radar components to be incorporated while still making use of known electromagnetic field theory of dipole radiators.

Similarly, prior works lack a method which allows for integration of real-world calibration techniques. It is advantageous for a calibration process to accommodate whatever physical measurements are available. This could include near-field chamber, far-field chamber, or in-situ field characterization of individual elements or of the full array beam pattern.

Next, while some works consider the possibility of a tilted array, there is no simple approach to correcting arbitrary tilt and roll or describing its interaction with other sources of bias for any phased-array.

The issues described above can be traced back to a single cause: prior works present polarimetric bias as a single projection matrix, containing all bias sources. In this form, individual bias sources are not separable. This is undesirable, as the individual role and effect of each bias source is not apparent. Calibration in practice may be overly complex or convoluted as a result. Adding, removing, or adapting the correction for a specific source of bias is difficult. For example, if the antenna gain pattern is measured experimentally, but the array is operated at a tilted angle, it is not obvious how to incorporate both practical measurements and geometrical theory to the correction. Based on these limitations, we devised a new approach for the novel correction method used in this dissertation. This method fulfills the following requirements:

- Can incorporate both known theoretical corrections and physical measurements including existing calibration techniques.
• Allows correction of arbitrary tilt and roll for any phased-array.

• Applies corrections in the form of simple, linear equations that can be implemented in real time.

• Can be easily adapted to many types of phased-array radars and calibration processes.

This is accomplished by containing each source of bias within its own linearly independent module. The method is described in detail in the next section.

4.2 Decomposition of Bias Projection Components

This research proposes a new approach to describing and correcting polarization distortions in phased-arrays, which incorporates and expands on several methods described in previous works. This model separates bias contributors into independent terms which can be customized for various phased-array configurations and allows for inclusion of cross-polarized radiation and the effects of a wet radome covering the array face. Below is a summary of this bias correction model. This work has also been published in IEEE Transactions on Geoscience and Remote Sensing [1].

We now reproduce the above projection methodology expressed in terms that are more easily separated. To do so, a precise definition of coordinate systems and of cross-polarization is necessary. Aboserwal et al. [27] have approached this issue, describing in detail the polarization definitions famously set forth by Ludwig [28] and the implications of their use for weather radar. We agree with their conclusion that for weather radar applications by a PPAR, none of Ludwig’s definitions are exactly appropriate and instead, a coordinate system based on the weather scatterers’ frame of reference (gravity and the horizon) should be used.

For this dissertation, it is important to clarify the use of the term “broadside.” When describing single element antennas such as reflector dish, horn, or patch anten-
Figure 4.1. Coordinate system. World-relative coordinates and unit vectors are unprimed, array-relative coordinates and unit vectors are primed. The array face lies in the $y'$-$z'$ plane centered at the origin.

nas, broadside is often used interchangeably with boresight to describe the center of the main antenna beam. However, in the case of a planar phased-array antenna, this work defines broadside as the direction orthogonal to the planar array aperture. The ability to electronically scan the phased-array means the main beam will not always point in the broadside direction. The direction of the main antenna beam is described as boresight, scan direction, or beam direction in this work.

Following prior works, we begin with the $z$-axis parallel to gravity (vertical) and the $x$-axis pointed towards the horizon. We then place the planar array in the $y$-$z$ plane with the array center at the origin, such that antenna broadside is parallel to the $x$-axis. In the far field, spherical coordinates are used, where the world-relative horizontal polarization is aligned with $\hat{\phi}$ and the 'vertical' polarization is aligned with $-\hat{\theta}$. Additionally, it is useful to define a tilted array coordinate system because phased-array antennas are often operated with the planar array tilted upward in
elevation. Thus for some tilt angle $\delta$, a new coordinate basis is formed using primed coordinates $x'$, $y'$, $z'$, $\phi'$, and $\theta'$, as indicated in Figure 4.1.

We express the projection matrix as the product of three sources contributing to polarization bias. Each of these sources is itself represented by a matrix. The sources are:

**Main Beam Gain and Relative Phase ($G$):** This is simply the amplitude and phase pattern of the radiating elements with scan angle. It is expressed in terms of the native polarizations radiated by the elements of the PPAR.

**Radiated Dipole Polarizations ($D$):** This considers the effect of the radiated polarization of the dual-polarized elements with scan angle, and it relates the native polarization of the elements to the array-relative azimuth and elevation directions ($\phi'$, $\theta'$). Element polarizations are expressed in terms of equivalent electric dipoles or magnetic dipoles depending upon the nature of the element.

**Array Tilt and Roll Projection ($T$):** Finally, the PPAR antenna is often tilted. The effects above are all associated with the array-relative scan angle, but the intrinsic scattering must be related to world-relative coordinates.

With this framework, the final projection matrix is given by:

$$P = TDG$$

(4.14)

where the each of the left-multiplied matrices may be viewed as occurring in sequence during transmission, beginning with $G$ and ending with $T$. This matrix multiplication is non-commutative. The $G$ matrix occurs first because it is a diagonal matrix representing the scaler amplitude and phase pattern of the antenna. Once multiplied by the $D$ matrix, it has been converted to vector ($\phi'$, $\theta'$) space. Finally, the $T$ matrix must occur last (left-most), since it converts to the world-relative coordinate basis, but
and $D$ are in the array-relative basis. In the following, we describe the individual terms.

### 4.2.1 Main Beam Gain and Relative Phase

First, the calibration of the array hardware must be included. The radiating elements and associated electronics introduce a scan angle dependent variation in antenna amplitude and phase. These are analogous to the $g^{(h,v)}$ terms described in [25], though not identical as will be seen shortly. Additionally, variation of gain or phase length in the up/down converter and changing beam shape and size introduce more scan pattern variations between horizontal and vertical channels. In theory, each of these could be characterized and corrected separately, but doing so would be complex and aggregation of errors could make the result useless. Therefore, in practice it is best to calibrate via laboratory or field measurements. By characterizing all of these effects at once through experimental measurements using the physical antenna, the process is simplified and error in the correction is reduced. In addition, periodic recalibration can mitigate the effects of any drift in scan pattern over of time. This recalibration can be performed in the field without much difficulty as often as once a month.

The results can then be represented as the amplitude and phase correction for both $H$ and $V$ channels at each scan angle. For an array consisting of ideal dual-polarized radiators:

$$
\mathbf{G} = \begin{bmatrix}
g_{hh}(\theta', \phi') & 0 \\
0 & g_{vv}(\theta', \phi')e^{j\beta(\theta', \phi')}
\end{bmatrix}
$$

(4.15)

Where $g_{hh}$ and $g_{vv}$ are the one-way voltage gains for the horizontal and vertical channels respectively, and $\beta$ is the system differential phase between the $H$ and $V$ channels, all of which are a function of array relative scan angle $(\theta', \phi')$. Thus, all measurements used to calibrate $\mathbf{G}$ must be converted to scan angles relative to the array face. In practice, calibration was performed by averaging light rain data collected during nor-
mal operation of the phased-array. Details of this process are discussed in Chapter 5.

For simplicity, we assume no cross-polarized radiation from the elements \((g_{hv} = g_{vh} = 0)\). The case of cross-polarized radiation can be handled and is discussed later in the appendices.

While using weather observations to determine the } \mathbf{G} \text{ matrix (as is done in Chapter 5), the radome covering the array face is likely wet. If the radome is a planar sheet attached to the array face, then the presence of water can yield a direction-dependent attenuation and excess phase. In principle, this can be incorporated into } \mathbf{G}. \text{ We discuss the treatment of a planar wet radome in the Appendix. If the array lies inside a spherical radome, then the treatment of water on the radome is identical to that for reflector antennas, and no special consideration is needed for the phased-array.}

### 4.2.2 Radiated Dipole Polarizations

While the } \mathbf{G} \text{ matrix accounts for spatially varying amplitude and phases introduced by array elements, it does not consider the orientation of their radiated fields. This is the purpose of the dipole matrix } (\mathbf{D}). \text{ It projects the H and V channel voltages into their respective } \hat{\phi}' \text{ and } \hat{\theta}' \text{ vector components, where } \hat{\phi}' \text{ and } \hat{\theta}' \text{ are the array-relative spherical coordinates as indicated in Figure 4.1. For practical radiators, the “horizontal” and “vertical” polarized fields become non-orthogonal when electronically scanning off the principal planes, introducing coupling between the polarizations. For a microstrip patch or waveguide aperture element, the equivalent sources are magnetic currents whereas for crossed dipole elements, the equivalent sources are electric currents.}

Figure 4.2 illustrates the orientation of electric fields as a function of radiation direction for both horizontally polarized and vertically polarized electric current sources and magnetic current sources. In general, the } \mathbf{D} \text{ matrix is expressed as
\[
D = \begin{bmatrix}
\hat{u}^{(h)} \cdot \hat{\phi}' & \hat{u}^{(v)} \cdot \hat{\phi}' \\
\hat{u}^{(h)} \cdot \hat{\theta}' & \hat{u}^{(v)} \cdot \hat{\theta}' \\
\end{bmatrix},
\]

where the vectors \( \hat{u}^{(h,v)} \) are the far-field unit vectors of the fields produced by the equivalent current sources for each polarization. The co-polarized and cross-polarized unit vectors for both types of current sources are given in [27] (Table III). The far-field unit vectors for \( \hat{z}' \)-oriented and \( \hat{y}' \)-oriented magnetic current sources (corresponding to H-polarized and V-polarized microstrip patch antennas, respectively) are

\[
\begin{align*}
\hat{u}^{(h)} &= \hat{\phi}' \\
\hat{u}^{(v)} &= \frac{\cos \theta' \sin \phi' \hat{\phi}' - \cos \phi' \hat{\theta}'}{\sqrt{\sin^2 \phi' \cos^2 \theta' + \cos^2 \phi'}} 
\end{align*}
\]

As a result, for magnetic dipole radiators \( D \) is given by

\[
D_m = \begin{bmatrix}
1 & -\frac{\sin \phi' \cos \theta'}{\sqrt{\sin^2 \phi' \cos^2 \theta' + \cos^2 \phi'}} \\
0 & \frac{\cos \phi'}{\sqrt{\sin^2 \phi' \cos^2 \theta' + \cos^2 \phi'}} \\
\end{bmatrix}
\]

while for electric dipole radiators, \( D \) is given by

\[
D_e = \begin{bmatrix}
\frac{-\cos \phi'}{\sqrt{\sin^2 \phi' \cos^2 \theta' + \cos^2 \phi'}} & 0 \\
\frac{-\sin \phi' \cos \theta'}{\sqrt{\sin^2 \phi' \cos^2 \theta' + \cos^2 \phi'}} & 1 \\
\end{bmatrix}
\]

The denominators in the above matrix entries are normalizations for the unit vectors and may also be written as \( \sqrt{1 - \sin^2 \theta' \sin^2 \phi'} \). We see that for magnetic dipole radiators, the horizontal polarization aligns with the unit vector \( \hat{\phi}' \) while the vertical polarization does not align with \( \hat{\theta}' \). For electric dipole radiators, the vertical polar-
ization aligns with \( \hat{\theta}' \) while the horizontal polarization does not align with \( \hat{\phi}' \). Similar to the discussion in [25], the matrices \( D_m \) and \( D_e \) are related:

\[
(D_m^T)^{-1} \propto D_e \tag{4.21}
\]

\[
(D_e^T)^{-1} \propto D_m.
\]

For a PPAR consisting of microstrip patch radiating elements, the product of \( DG \) is given by

\[
DG = \begin{bmatrix}
    g_{hh}(\theta', \phi') & \frac{\sin \phi' \cos \theta'}{\sqrt{\sin^2 \phi' \cos^2 \theta' + \cos^2 \phi'}} g_{vv}(\theta', \phi') \\
    0 & \frac{-\cos \phi'}{\sqrt{\sin^2 \phi' \cos^2 \theta' + \cos^2 \phi'}} g_{vv}(\theta', \phi')
\end{bmatrix},
\]

where the system differential phase term associated with \( g_{vv} \) has been suppressed for clarity. This is equivalent to the projection matrix described in (4.8) and as described in [17, 25], however there is a notable difference in representation. In (4.8), the definitions of the horizontally polarized element pattern term \( g^{(h)} \) did not include a \( \sin \theta' \) term representative of a Hertzian dipole radiation pattern, so the far field pattern was defined as \( E_\phi \approx \sin \theta' g^{(h)}(\theta', \phi') \). In (4.22), the \( \sin \theta' \) term is incorporated into \( g_{hh} \). Similarly, for the vertically polarized element pattern, the same ‘\( \sin \theta' \)-like’ term is incorporated into \( g_{vv} \) and is accounted for in the normalizations of the unit vectors in \( D \). The relationship between the two representations can be written

\[
g_{hh} = \sqrt{\sin^2 \theta' : g^{(h)}} \tag{4.23}
\]

\[
g_{vv} = \sqrt{\cos^2 \theta' \sin^2 \phi' + \cos^2 \phi' : g^{(v)}}.
\]

While this seems like a trivial difference in notation, it is significant. The \( G \) matrix represents the amplitudes and relative phase that are measured, for example, by PPAR observing a collection of spherical scatterers uniformly distributed over the field of view without any projection. That is, it represents the direct response of the PPAR.
in its native polarization basis as defined by the radiating elements and thus represents
the “raw” amplitude and phase measurement of the phased-array. This matrix can
be obtained through routine measurements of natural targets averaged over space
(range) and time, for example in light precipitation. The remaining components of
the projection matrix are determined entirely by geometry.

4.2.3 Array Tilt and Roll

The above results are specific to a PPAR lying in the y-z plane with the broadside
direction along the x-axis. In this case, the array relative scan angles, \( \theta', \phi' \), and
unit vectors, \( \hat{\theta}' \) and \( \hat{\phi}' \), are equal to the world-relative scan angles, \( \theta, \phi \), and unit
vectors \( \hat{\theta} \) and \( \hat{\phi} \). It is common, however, to tilt the array such that the broadside
direction points toward the middle of the range of desired elevation coverage so as to
optimize the scan range in elevation. This is a rotation about the y-axis by an angle
\( \delta \) as shown in Figure 4.1. In a mobile operation, it is conceivable that there might
be a small roll angle as well (a rotation about the x-axis by an angle, \( \gamma \)) so this is
also considered here. The coordinate rotation is achieved by converting to cartesian coordinates (via matrix $T_1$), performing the rotations ($T_2$), and then converting to world-relative spherical coordinates ($T_3$).

\[
\begin{bmatrix}
\hat{\phi} \\
\hat{\theta}
\end{bmatrix} = T \begin{bmatrix}
\hat{\phi}' \\
\hat{\theta}'
\end{bmatrix}, \quad T = T_3 T_2 T_1 \tag{4.24a}
\]

\[
T_1 = \begin{bmatrix}
-sin \phi' & cos \phi' cos \theta' \\
cos \phi' & sin \phi' cos \theta' \\
0 & -sin \theta'
\end{bmatrix} \tag{4.24b}
\]

\[
T_2 = \begin{bmatrix}
\cos \delta & 0 & -\sin \delta \\
\sin \gamma \sin \delta & \cos \gamma & \sin \gamma \cos \delta \\
\cos \gamma \sin \delta & -\sin \gamma & \cos \gamma \cos \delta
\end{bmatrix} \tag{4.24c}
\]

\[
T_3 = \begin{bmatrix}
-sin \phi & cos \phi & 0 \\
\cos \theta cos \phi & \cos \theta sin \phi & -\sin \theta
\end{bmatrix} \tag{4.24d}
\]

It is also possible to relate the array-relative scan angle $(\phi', \theta')$ to world-relative scan angle $(\phi, \theta)$ using

\[
\cos \theta = \sin \theta' \cos \phi' \cos \gamma \sin \delta - \sin \theta' \sin \phi' \sin \gamma + \cos \theta' \cos \gamma \cos \delta \tag{4.25a}
\]

\[
\tan \phi = \frac{\sin \theta' \cos \phi' \sin \gamma \sin \delta + \sin \theta' \sin \phi' \cos \gamma + \cos \theta' \sin \gamma \cos \delta}{\sin \theta' \cos \phi' \cos \delta - \cos \theta' \sin \delta} \tag{4.25b}
\]

The final resulting matrix, $P = TDG$, projects the radiated fields of the tilted PPAR into world-relative coordinates. It is worth noting that while the radiators exhibit particular alignments with the array-relative unit vectors, $\hat{\theta}'$ and $\hat{\phi}'$, once the array is tilted, these unit vectors are no longer aligned with the world-relative $\hat{\theta}$ and $\hat{\phi}$. Thus all elements of the final projection matrix are non-zero. The action of this
projection on the intrinsic scattering parameters, $S_{\prime}$, and on the polarimetric second-moment products is the same as that derived in prior works [17, 25] and is discussed shortly.

4.3 Correction to Polarimetric Variables

The expressions describing the biased observations of the PPAR in terms of intrinsic values and the projection matrix were derived for ATSR mode (also valid for ATAR mode) in [17, 25]. With some algebraic manipulation, their expressions for reflectivity factor, differential reflectivity, and co-polar correlation coefficient can be expressed as follows:

$$Z_{h}^{(p)} = Z_{h} \left| p_{11} \right|^4 (1 + \Delta Z_{h})$$  \hspace{1cm} (4.26)

$$Z_{dr}^{(p)} = Z_{dr}^{\prime} \left| \frac{p_{11}}{p_{22}} \right|^4 \left( \frac{1 + \Delta Z_{h}}{1 + \Delta Z_{v}} \right)$$  \hspace{1cm} (4.27)

$$\rho_{hv}^{(p)} = \rho_{hv}^{\prime} \left( \frac{p_{11}p_{22}^{\prime}}{|p_{11}||p_{22}|} \right)^2 \frac{(1 + \Delta \rho_{hv})}{\sqrt{(1 + \Delta Z_{h})(1 + \Delta Z_{v})}}.$$  \hspace{1cm} (4.28)

Where, as before, ($p$)-superscripts represent projected variables observed by the PPAR, primes denote intrinsic values (that would be observed by a mechanically scanning radar), and $p_{ij}$ are elements of the projection matrix, which may be complex-valued. In each of these expressions, the projected variable is expressed as the product of the intrinsic variable, a projection-dependent term involving diagonal elements of a projection matrix only, and a target-dependent term involving the off-diagonal elements of the projection matrix along with intrinsic variables $Z_{dr}^{\prime}$ and $\rho_{hv}^{\prime}$. These target-dependent terms are given by
\[ \Delta Z_h = \frac{|p_{11}|^4 Z_{dr}^{-1} + 2Re[\rho^\prime_{hv} p_{11}^2 p_{21}^2 Z_{dr}^{-1/2}]}{|p_{11}|^4}, \quad (4.29) \]
\[ \Delta Z_v = \frac{|p_{12}|^4 Z_{dr}^{-1} + 2Re[\rho^\prime_{hv} p_{12}^2 p_{22}^2 Z_{dr}^{-1/2}]}{|p_{22}|^4}, \quad (4.30) \]
\[ \Delta \rho_{hv} = \frac{p_{11}^2 p_{12}^2 Z_{dr}^{1/2} + p_{12}^2 p_{21}^2 \rho^\prime_{hv} + p_{21}^2 p_{22}^2 Z_{dr}^{-1/2}}{p_{11}^2 p_{22}^2 \rho^\prime_{hv}}. \quad (4.31) \]

The Linear Depolarization Ratio (LDR) is defined in terms of world-relative horizontal and vertical polarizations. \( LDR_h \) is defined as the ratio of vertically polarized power received to horizontally polarized power received when horizontal polarization is transmitted. \( LDR_v \) is the ratio of horizontally polarized power received to vertically polarized power received when vertical polarization is transmitted. Either measurement requires that the radar transmit and receive in alternating polarizations so as to measure both co-polarized and cross-polarized echoes. With this same definition one can define the LDR of a PPAR.

Expressions for \( LDR_h \) and \( LDR_v \) were presented in [25] in the absence of any cross-polarized scattering. Their expressions indicated only the lower bound on LDR that could be observed given the projection and non-orthogonality of polarizations and were therefore independent of the intrinsic LDR. By including the possibility of cross-polarized scattering, the \( LDR_h \) observed by a PPAR can be expressed in a manner consistent with the other variables:

\[ LDR_h^{(p)} = LDR_h \frac{|p_{11} p_{22} + p_{12} p_{21}|^2}{|p_{11}|^4} \left( \frac{1 + \Delta LDR_h}{1 + \Delta Z_h} \right) + LDR_{h,\text{min}} \quad (4.32) \]

where

\[ \Delta LDR_h = \frac{2Re[(\rho^\prime_{hx} p_{11} p_{12} + \rho^\prime_{vx} p_{21} p_{22} Z_{dr}^{-1/2})(p_{11} p_{22} + p_{12} p_{21})^*]}{LDR_h^{1/2} |p_{11} p_{22} + p_{12} p_{21}|^2} \quad (4.33) \]
\[ LDR_{h,\text{min}} = \frac{|p_{11}|^2 |p_{12}|^2 + |p_{21}|^2 |p_{22}|^2 Z_{dr}^{-1} + 2Re[\rho^\prime_{hv} p_{11} p_{12} p_{21} p_{22}^2 Z_{dr}^{-1/2}]}{|p_{11}|^4 (1 + \Delta Z_h)} \quad (4.34) \]
where $\rho'_{hx}$ and $\rho'_{vx}$ are the intrinsic cross-polar correlation coefficients. In obtaining these expressions, it has been assumed that $s'_{hv} = s'_{vh}$ as is the case for reciprocal media. Because $LDR_h$ necessarily involves cross-polar measurements, the projection-dependent correction term includes off-diagonal elements of the projection matrix. The expression for $LDR^{(p)}_h$ differs from the other variables in that there is also an additive term. The term $LDR_{h,min}$ is the same term as was derived by [25]. Namely, it is the minimum LDR that can be observed due to the non-orthogonality of polarizations off of the principal planes. It can also be readily shown that

$$LDR_v^{(p)} = LDR^{(p)}_h Z^{(p)}_{dr}$$

(4.35)

$$LDR_{v,min} = LDR_{h,min} Z^{(p)}_{dr}.$$  

(4.36)

### 4.4 Partial Correction Model

An “exact” correction of the biased variables can be achieved, in principle, through the use of the inverse of the projection matrix, $P^{-1} = C$, and by solving for the intrinsic variables as functions of the projected variables. This is implemented in equations (4.26 - 4.28) and (4.32) which include the target dependent terms given in (4.29 - 4.31) and (4.33). Note that the projection matrix itself inverts cleanly, as it is not far removed from the identity matrix: diagonal elements near unity magnitude and off-diagonal elements small. Still, while it is possible to use the full expressions for correction, it is also desirable to correct observations with minimal calculation so as to minimize further uncertainties introduced by the combinations of measured variables.

In practice, the $p_{21}$ term is non-zero, but it is small. As it appears in equations either squared or raised to the fourth power, it seems reasonable to neglect it. In this case, $\Delta Z_h \approx 0$, and the correction for $Z_h^{(p)}$ is simply to divide it by $|p_{11}|^4$. This makes
sense since the horizontally polarized fields from the patch elements are well aligned with the $\hat{\phi}'$ direction which is also only slightly misaligned with $\hat{\phi}$.

Neglecting $p_{21}$ also simplifies the equation for $Z_{dr}$, however given the small dynamic range of $Z_{dr}$, and the precision to which measurements are typically desired, caution is in order. Nonetheless, it is of interest to consider if it may be further simplified. In the next chapter, it will be shown experimentally that $p_{12}$ is also small for much of the scan range, except in the upper corners of the scan area. Again, however, it appears in the equation for $Z_{dr}^{(p)}$ either raised to the fourth power or squared. If it could be neglected, the correction for $Z_{dr}^{(p)}$ would be to multiply it by $|p_{22}/p_{11}|^4$.

Examining (4.28) for $\rho_{hv}^{(p)}$, the first term multiplying the intrinsic value has an amplitude of exactly unity, but captures the projected system differential phase. That is, $\rho_{hv}^{(p)} = |\rho_{hv}^{(p)}| \exp(j\phi_{dp}^{(p)})$, where $\phi_{dp}^{(p)}$ is the projected differential phase. The second term adjusts both amplitude and phase. If both $p_{21}$ and $p_{12}$ could be neglected, the expression would include only the first term. Thus the correction would compensate the system differential phase only.

Finally, if $p_{21}$ can be neglected, the expression for $LDR_{h}^{(p)}$ reduces to

$$LDR_{h}^{(p)} \approx LDR_{h}' \frac{|p_{22}|^2}{|p_{11}|} (1 + \Delta LDR) + LDR_{h,\text{min}}$$  \hspace{1cm} (4.37)

$$\Delta LDR \approx \frac{2Re[\rho_{hv}^{(p)} p_{12} p_{22}^*]}{LDR_{h,v}'^2 |p_{22}|^2}$$  \hspace{1cm} (4.38)

$$LDR_{h,\text{min}} \approx \frac{|p_{12}|^2}{|p_{11}|}.$$  \hspace{1cm} (4.39)

These expressions reveal that the $LDR_{h}$ measurement is primarily limited by $p_{12}$, and that estimates will be biased high even for intrinsic LDR values above the minimum.

Based on these approximations, a partial correction model using only projection dependent terms ($\mathbf{P}$ matrix terms) can be formed. This is highly desirable, as target dependent terms introduce potentially cascading measurement errors and add non-linearity, making inverting the correction equations much more complicated, espe-
cially in implementation. The partial corrections are a simplified version of equations (4.26 - 4.28) and (4.32):

\[
Z_h^{(p)} = Z_h' |p_{11}|^4
\]

\[
Z_{dr}^{(p)} = Z_{dr}' \frac{|p_{11}|}{|p_{22}|}
\]

\[
\rho_{hv}^{(p)} = \rho_{hv}' \left( \frac{p_{11} p_{22}^*}{|p_{11}||p_{22}|} \right)^2
\]

\[
LDR_h^{(p)} = LDR_h' \left( \frac{|p_{22}|}{|p_{11}|} \right)^2 + \frac{|p_{12}|^2}{|p_{11}|^2}
\]

The merit and limitations of this partial correction will be examined experimentally in the next chapter.

This chapter discussed prior work on phased-array weather radar polarimetry before presenting the novel decomposition approach to characterizing polarimetric bias. This method provides a simple and adaptable calibration technique by separating each bias contributor into independent linear terms, which can then be implemented in real time. It also provides a way to perform a linear partial correction to weather products by neglecting target dependent terms. Next, experimental data using this approach will be presented and discussed.
5.1 Skyler Data Corrections

The quality of Skyler measurements and the bias correction model are assessed here. This is done through a direct comparison of data from the UMaXX and Skyler radars in a variety of modes and weather conditions in order to verify the calibration of phased-array polarimetric measurements. Since large amounts of data was collected using both radars, large scale statistical analysis will be used to look for persistent, systematic patterns in collected data. Additionally, several specific observation cases will be examined. In all the following discussions of Skyler datasets, data referred to as ‘calibrated’ has been corrected using the partial correction method described earlier, where the $P$ matrix is constructed from $G$, $D$, and $T$. Data referred to as ‘uncalibrated' has received simple radar factor and range corrections but has not received any polarimetric bias corrections.

Here UMaXX is used as a ’truth’ reference, and the error in corrected and uncorrected Skyler data will be compared with weather radar requirements. While informative, this approach may not yield definitive results due to the small scale of biases compared to other potential measurement errors. Thus, challenges faced while performing this evaluation will also be discussed in order to provide additional insight. While some mobile deployment and vertical weather observations were collected using Skyler, they will not be incorporated into this analysis, but may be of interest for future works.
5.1.1 Measurement of Skyler Projection Matrix

In order to determine the projection matrix for the bias correction method described in Chapter 3, the system behavior must be accurately characterized. This is done using experimental measurements to obtain the $G$ matrix. Many approaches exist for calibrating the amplitude and phase of the beam-peak. It could be measured in a near-field chamber or far-field range, formulated from individual element patterns, or determined in-place from routine measurements.

Here we consider the latter case in which the radar observes weather targets of opportunity. For this, the radar was installed atop a 21 m tower on the UMass Campus observing a 90° sector to the North. The radar was tilted 15 degrees so that the elevation scan range is from 0° to 30° in elevation. To collect data for antenna calibration, the radar was operated in a short-range mode so that observations could be obtained at the highest elevation angles while remaining sufficiently below the freezing level or overtopping the precipitation. In addition, observations at close range minimize the impact of attenuation and differential propagation phase. Operation at short range necessitates use of short pulses rather than longer “chirp” waveforms necessary for sensitivity at long range. A 2 microsecond pulse length was employed. Radar echoes were range corrected by a factor of $R^2$ and averaged over ranges where Signal to Noise Ratio (SNR) exceeded 10 dB for each scan direction. Generally this corresponds to ranges less than about 3 km. The averaging was performed over 57 complete scans of the volume, and the average power versus scan angle is shown in Figure 5.1. Due to complex terrain and buildings and trees in the vicinity, the lower few elevation angles were contaminated by ground clutter (much of this through sidelobes). Nonetheless, the average patterns elsewhere are fairly uniform, varying by a few dB over the entire scan range.

Due to the symmetry of the antenna construction, we expect that the amplitude and phase patterns are even functions of array-relative azimuth and elevation scan
Figure 5.1. Measured 2-way relative power and relative phase response of Skyler prototype antenna to light rain on 26 September 2019. Power measurements are relative to the horizontal channel at broadside, and relative phase is taken to be zero at broadside. The lower elevations are corrupted by nearby ground clutter.

angle. Therefore, in Figure 5.2 we replace the contaminated lower elevation angles, with data from the complementary high elevation angles. Furthermore, we consider the even part of the observations by averaging mirrored versions in azimuth and elevation. We take the differential phase measured at broadside to represent the system (electronics) differential phase, which is removed so that remaining differential phase is associated with the antenna scan characteristics. Any backscatter differential phase, if present, would be similarly removed. We also apply modest smoothing (averaging over a $5^\circ \times 5^\circ$ window). This represents our estimate of the $\mathbf{G}$ matrix.

Using the measured $\mathbf{G}$, the final projection matrix is constructed given that the antenna elements are microstrip patches (magnetic current dipoles implying the $\mathbf{D}_\mathbf{M}$ matrix is used.) and the array is tilted to $15^\circ$. The result is shown in Figure 5.3. A few observations can be made. First, the element $p_{21}$ is non zero, and this is due exclusively to the tilt of the array. For an ideal patch radiator in the vertical plane,
Figure 5.2. Even part of 2-way relative power and relative phase response of Skyler prototype antenna to light rain with corrupted elevations replaced with complementary high elevation observations.

$p_{21}$ is identically zero. It is still small compared to the other elements. Second, the elements $p_{12}$ and $p_{22}$ both involve $g_{vv}$, however, it is not obvious in $p_{12}$, as the dipole and tilt effects overwhelm the impact of the amplitude pattern.

### 5.1.2 Evaluation of Partial Correction Model

With knowledge of $\mathbf{P}$, the expected weather product biases can be calculated from the second moments, assuming known intrinsic targets. These are shown in Figure 5.4 for the case of a uniform intrinsic reflectivity factor of $Z_{h}^{'} = 30$ dBZ, differential reflectivity of $Z_{dr}^{'} = 1$ dB, co-polar correlation of $\rho_{hv}^{'} = 0.9$, and differential phase of $\phi_{dp}^{'} = 0^\circ$. Also shown is the lower limit of observable linear depolarization ratio, $LDR_{h,min}$, which is due principally to the non-orthogonality of the polarizations radiated by the H and V elements. Also shown in Figure 5.4 are contour lines indicating the bounds of acceptable bias for the various weather products. The contour levels are $Z_{h}^{bias} = -1$ dB, $Z_{dr}^{bias} = \pm 0.2$ dB, $\rho_{hv}^{bias} = 0.01$, and $\phi_{dp}^{bias} = \pm 3.6^\circ$. 
The reflectivity and differential reflectivity bias limits are based on requirements for adequate estimation of rain rate [21]. The correlation coefficient bias requirement may be overly stringent but the range of this parameter is rather small. The differential phase bias requirement here is based on 1 dB of path-integrated attenuation error assuming $A_{h,dB} = 0.28 \phi_{dp}$ (at S-band, where attenuation is negligible, differential phase accuracy is set at 2.5° to obtain acceptable specific differential phase, $K_{dp}$ [40]. The same requirement at X-band, where the wavenumber is about three times greater, would be about 7.5°.)

The results of Figure 5.4 indicate that without correction, reflectivity bias is acceptable over the central portion of the scan range and that correlation coefficient magnitude is only seriously affected in the upper corners of the scan range. Differential reflectivity and differential phase, however, require correction over most of the scan range. It can also be observed that a phase-only correction of $\rho^{(p)}_{hv}$ would be
Figure 5.4. Weather product biases assuming the projection of $P$ and uniform intrinsic weather variables: $Z_h' = 30$ dBz, $Z_{dr}' = 1$ dB, $\rho_{hv}' = 0.9$, $\phi_{dp}' = 0^\circ$, and $\text{Tilt} = 15^\circ$. Dashed contour lines indicate the boundaries between regions of acceptable and unacceptable bias (see text).

In order to evaluate the merit of the partial correction model described in section 4.4, corrections can be applied to results of Figure 5.4 using equations (4.40 - 4.43). The expected remaining bias is shown in Figure 5.5, and is equivalent to the target dependent terms in equations (4.29 - 4.31) and (4.33). The partial correction of $Z_h$ is sufficient to achieve 1 dB accuracy over the entire field of view, except for extreme upper corners. Partial correction of $Z_{dr}$ is sufficient to achieve 0.2 dB accuracy over most of the lower elevations, but more precise correction is necessary, particularly in the upper corners of the scan range. A similar result was found by [18]. With respect to the differential phase, the phase-only correction perfectly compensates it.
Figure 5.5. As in Figure 5.4 for the case of $\phi'_{dp} = 0^\circ$ with projection-dependent corrections (only) applied.

in the case of zero intrinsic differential phase. Figure 5.6 shows the same partial correction of polarimetric variables in the case of $\phi'_{dp} = 90^\circ$. This intrinsic differential phase is consistent with approximately 25 dB of path-integrated attenuation and is representative of the likely maximum value where measurements are possible. We find that the $Z_{dr}$ bias is decreased but the $\phi_{dp}$ bias is increased in the upper corners of the scan range.

Since we have decomposed the projection into components due to the radar’s amplitude and phase properties, antenna properties, and geometry it is of interest to evaluate the relative impact of the individual components. Figure 5.7 illustrates the bias remaining if we compensate only for amplitude and phase ($G$), and ignore projection ($TD$) entirely. Surprisingly, this less-than-partial correction performs better than the partial correction involving the full transformation matrix. It appears that
Figure 5.6. As in Figure 5.4 for the case of $\phi_{dp} = 90^\circ$ with projection-dependent corrections (only) applied.

A fortuitous cancellation of error sources is responsible for this result. That is, the error in $P$ incurred by ignoring $DT$ has opposite sign to the target-dependent error terms in this case. These compensating errors are associated with the off-diagonal elements of $P$, as the off-diagonal elements of $G$ are zero for ideal elements. While it appears to perform better in this case, we cannot claim that this is a general result.

In all the following discussions of Skyler datasets, data that is referred to as ‘calibrated’ has been corrected using the partial correction equations to weather products, where the $P$ matrix is constructed from $G$, $D$, and $T$. Data referred to as ‘uncalibrated’ has received simple radar factor and range corrections but has not received any polarimetric bias corrections.
Figure 5.7. As in Figure 5.4 but with $P = G$ corrections (only) applied.

5.2 UMaXX Data Corrections

The UMaXX radar has continued operating from Orchard Hill Tower with only brief downtimes for minor maintenance. It collects a full volumetric dual-pol scan at three-minute intervals. The UMaXX radar data is to be used as a ‘truth value’ reference in calibrating the Skyler radar. However, ground clutter, partial beam blockage, and radome effects were found to be significant, so a method of correcting for such effects was necessary as part of processing UMaXX data to remove experimental variables introduced by UMaXX itself. This is especially important to do before making meaningful comparisons between UMaXX and Skyler datasets. These corrections are of the same type traditionally performed on dish weather radars; however, the implementation may vary. These corrections are described in detail in Casey Wolsieffer’s
Master’s thesis [41], but a summary is provided here. Corrections applied to the UMaXX data are:

1. Ground clutter filter at lower elevations.
   This uses a decision tree classifier trained on pre-classified data. The resulting filter is largely dependent on differential phase, velocity, and co-polar correlation. The result is the flagging and removal of data bins containing ground clutter.

2. Partial beam blockage at lower elevations.
   This uses an approximation of the radar’s main beam and compares it to a digital elevation model (DEM) of the terrain surrounding the UMaXX tower. The percentage of beam energy blocked at each scan angle is converted to reflectivity loss in dB and stored in a lookup table. The result is accurate reflectivity measurements at scan angles where the radar beam is partially blocked by terrain.

3. Differential reflectivity and phase system bias.
   This averages light rain observations versus scan angle to remove system and/or radome biases to differential reflectivity and phase. These biases were significant for UMaXX. The result is the removal of system differential reflectivity and phase biases.

   In particular, the last correction may account for much of the uncertainty in the following comparison of $Z_{dr}$. As shown in Figure 5.8, variations in $Z_{dr}$ appear periodic in azimuth with the five peaks aligning with the five panels making up the UMaXX radome. A similar pattern also exists in differential phase. These errors are significant (greater than 1 dB in $Z_{dr}$ and 5 deg. in $\phi_{dp}$), and while a correction was applied, residual error may still be a large source of error in the following radar comparisons.

   Attenuation correction to both reflectivity and differential reflectivity is discussed in Casey’s thesis. These corrections are available but were disabled for the purposes
Figure 5.8. Data for light rain (20-22 dBZ) taken over the course of several rain events throughout July, 2019 (figure from C. Wolsieffer [41]). Z_{dr} data taken near the radar was binned by azimuth and averaged for four elevations. At all four elevations, five peaks can be made out which likely correlate with the five panels that make up the radome.

The reason is because both radars are X-band and co-located, they will experience the same attenuation through heavy rain. Ignoring attenuation effects on both radars removes the possibility of introducing more error through the attenuation correction process. Thus, no attenuation correction was applied to UMaXX.

5.3 Radar Data Comparison

While many datasets were collected using both radars, for the purpose of this analysis we will limit our scope to a select few datasets. This data was chosen as most useful for a direct comparison of the two radars. Therefore, criteria for selection are datasets of longer length, similar radar scan modes, with weather targets of significance. Datasets were not selected based on bias correction or comparison results.
The datasets used consist of two days of rain data collected in August of 2019 and two days in September of 2019. Analysis will be done on the all-inclusive dataset of all four days. In addition, specific weather events will be examined in more detail.

### Table 5.1. Skyler Tower Data List

<table>
<thead>
<tr>
<th>Date</th>
<th>Time Start</th>
<th>Time End</th>
<th>Pulse</th>
<th>Pol Mode</th>
<th>PRF</th>
</tr>
</thead>
<tbody>
<tr>
<td>08/21/19</td>
<td>23:08:29 UTC</td>
<td>00:18:19 UTC</td>
<td>50 us chirp</td>
<td>HH VV ATAR</td>
<td>Dual 2403 / 1602 Hz</td>
</tr>
<tr>
<td>08/28/19</td>
<td>19:14:33 UTC</td>
<td>23:36:56 UTC</td>
<td>50 us chirp</td>
<td>HH VV ATAR</td>
<td>Dual 2403 / 1602 Hz</td>
</tr>
<tr>
<td>09/02/19</td>
<td>17:52:20 UTC</td>
<td>23:01:08 UTC</td>
<td>50 us chirp</td>
<td>HH VV ATAR</td>
<td>Dual 2403 / 1602 Hz</td>
</tr>
<tr>
<td>09/26/19</td>
<td>18:17:55 UTC</td>
<td>18:39:45 UTC</td>
<td>50 us chirp</td>
<td>HH VV ATAR</td>
<td>Dual 2403 / 1602 Hz</td>
</tr>
</tbody>
</table>

### Table 5.2. UMaXX Tower Data List

<table>
<thead>
<tr>
<th>Date</th>
<th>Time Start</th>
<th>Time End</th>
<th>Pulse</th>
<th>Pol Mode</th>
<th>PRF</th>
</tr>
</thead>
<tbody>
<tr>
<td>08/21/19</td>
<td>23:00:25 UTC</td>
<td>00:20:47 UTC</td>
<td>1 us</td>
<td>HH+VV STSR</td>
<td>Dual 2400 / 1600 Hz</td>
</tr>
<tr>
<td>08/28/19</td>
<td>20:12:05 UTC</td>
<td>02:57:43 UTC</td>
<td>1 us</td>
<td>HH+VV STSR</td>
<td>Dual 2400 / 1600 Hz</td>
</tr>
<tr>
<td>09/02/19</td>
<td>22:28:02 UTC</td>
<td>23:17:14 UTC</td>
<td>1 us</td>
<td>HH+VV STSR</td>
<td>Dual 2400 / 1600 Hz</td>
</tr>
<tr>
<td>09/26/19</td>
<td>17:39:56 UTC</td>
<td>20:04:09 UTC</td>
<td>1 us</td>
<td>HH+VV STSR</td>
<td>Dual 2400 / 1600 Hz</td>
</tr>
</tbody>
</table>

In order to directly compare data between Skyler and UMaXX, an algorithm was implemented which searches through each Skyler netcdf datafile and pairs it with a UMaXX file with matching elevation and a timestamp difference within a specified interval. Next, the Dual-Radar Cross-checking Exclusion (DRaCX) filter is applied. This filter ensures only range bins for which both radars collected weather data is included. The DRaCX filter first excludes any Skyler data that does not fall within: $Z_h$ from 20 to 60 dBZ, $Z_{dr}$ from -5 to +5 dB, $\rho_{hv}$ from 0.8 to 1.0, and SNR > 10 dB. For UMaXX the limits are: $Z_h$ from 20 to 60 dBZ, $Z_{dr}$ from -5 to +5 dB and $\rho_{hv}$ from 0.5 to 1.0. These parameters are tabulated in Table 5.3. Then, DRaCX cross-checks the two filters. Thus, for a given range and time, data bins which do not pass the filters for both radars are excluded. The filter for UMaXX is more relaxed because this filter is applied after the UMaXX noise filtering process described in the previous section. After applying DRaCX, all remaining Skyler and UMaXX datapoints can be directly compared for each weather product.
Table 5.3. DRaCX Filter Parameters

<table>
<thead>
<tr>
<th>Variable</th>
<th>Skyler Min</th>
<th>Skyler Max</th>
<th>UMaXX Min</th>
<th>UMaXX Max</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Z_h$</td>
<td>20 dBZ</td>
<td>60 dBZ</td>
<td>20 dBZ</td>
<td>60 dBZ</td>
</tr>
<tr>
<td>$Z_{dr}$</td>
<td>-5 dB</td>
<td>5 dB</td>
<td>-5 dB</td>
<td>5 dB</td>
</tr>
<tr>
<td>$\rho_{hv}$</td>
<td>0.8</td>
<td>1.0</td>
<td>0.5</td>
<td>1.0</td>
</tr>
<tr>
<td>SNR</td>
<td>10 dB</td>
<td>–</td>
<td>–</td>
<td>–</td>
</tr>
</tbody>
</table>

5.3.1 Side-by-Side PPI

Several different types of plots were created for the purpose of analyzing and comparing the radar datasets. The simplest of these is a side-by-side comparison of Plan Position Indicator (PPI) plots. These plots provide a qualitative overview of the data points being compared. To create them, the chosen Skyler file is paired with a UMaXX file with matching elevation and a timestamp difference within a specified interval. In this case, a time interval of 10 sec was chosen. Each weather product is plotted in this side-by-side manner. Figure 5.9 shows uncalibrated Skyler data beside UMaXX data before the DRaCX filter is applied. Figure 5.10 shows same datafiles after Skyler calibration and the DRaCX filter has been applied. Note that all plots now show identical range bins and noise has been excluded.

Once the PPI plots have been filtered by DRaCX in this manner, the UMaXX data products can be directly subtracted from the Skyler data products at every range bin. If UMaXX is considered the ‘truth’ value, then this difference represents the calibration error of the Skyler radar at each point. In this comparison, velocity acts as a sort of control, since it is unaffected by the partial calibration and is least impacted by factors such as weather type, radar hardware, or scan angle. Figure 5.11 shows this error for both uncalibrated and calibrated Skyler data. However, because polarimetric corrections are small, it is difficult to quantitatively evaluate the calibration using a single PPI plot. Nonetheless, this demonstrates the ability of the DRaCX filter to directly compare the two radar datasets. Therefore, applying this method to a broader statistical analysis is required.
Figure 5.9. Side-by-side PPI comparison of uncalibrated Skyler and UMaXX data collected on August 21, 2019. Displayed is $Z_h$, $Z_{dr}$, $v_d$, $\rho_{hv}$, and $\phi_{dp}$.

5.3.2 Average Scan Pattern

In comparing the datasets of the two radars to each other, it is useful to characterize the scan pattern over the field of view. Thus, average scan pattern plots are created and discussed here. To do this, all valid datapoints from the all-inclusive
Figure 5.10. Side-by-side PPI comparison of calibrated and DRaCX filtered Skyler and UMaXX data collected on August 21, 2019. Displayed is $Z_h$, $Z_{dr}$, $v_d$, $\rho_{hv}$, and $\phi_{dp}$.

dataset (four days combined), are averaged in time and range. To be specific, all data is filtered using the DRaCX method described for the PPI scans above such that only weather data collected by both radars within 30 seconds of each other is included. For this analysis, the DRaCX filter is relaxed to the parameters shown in Table 5.4.
Figure 5.11. PPI of difference between DRaCX filtered Skyler and UMaXX data collected on August 21, 2019. Displayed is $Z_h$, $Z_{dr}$, $v_d$, $\rho_{hv}$, and $\phi_{dp}$. Uncalibrated data is shown on the left, while calibrated is shown on the right.

Next, for each scan angle (azimuth and elevation), data was averaged for all time and across all ranges. Therefore, both Skyler and UMaXX are averaged over the same range and time intervals.
Table 5.4. Relaxed DRaCX Filter Parameters

<table>
<thead>
<tr>
<th>Variable</th>
<th>Skyler Min</th>
<th>Skyler Max</th>
<th>UMaXX Min</th>
<th>UMaXX Max</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Z_h$</td>
<td>10 dBZ</td>
<td>70 dBZ</td>
<td>10 dBZ</td>
<td>70 dBZ</td>
</tr>
<tr>
<td>$Z_{dr}$</td>
<td>-5 dB</td>
<td>5 dB</td>
<td>-5 dB</td>
<td>5 dB</td>
</tr>
<tr>
<td>$\rho_{hv}$</td>
<td>0.8</td>
<td>1.0</td>
<td>0.5</td>
<td>1.0</td>
</tr>
<tr>
<td>SNR</td>
<td>0 dB</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

Figure 5.12. Averaged uncalibrated Skyler scan pattern for inclusive dataset.

The result is then plotted as averaged weather product versus radar scan angle. This scan pattern is shown for uncalibrated and calibrated Skyler data (Figures 5.12 and 5.13) and for UMaXX data (Figure 5.14). The difference in averaged scan pattern between radars is also shown by subtracting the UMaXX average from the Skyler average in Figure 5.15.

In Figure 5.12, averaged $Z_h$ and $Z_v$ are seen to range from around 23 to 33 dBZ, with lower values appearing at higher elevations. Fewer heavy rain range bins are present at high elevations. $Z_{dr}$ bias of about ±2 dB are visible in a pattern similar to Figure 5.2. For velocity, it is worth noting that this weather is moving faster on
Figure 5.13. Averaged calibrated Skyler scan pattern for inclusive dataset.

Figure 5.14. Averaged UMaXX scan pattern for inclusive dataset.
Figure 5.15. Averaged difference in scan pattern for inclusive dataset. UMaXX data was subtracted (in dB) from calibrated Skyler data.

the right side of the scan. $\rho_{hv}$ is consistently around 0.9, however it is lower than UMaXX. This is likely due to Skyler employing ATAR polarization, in which the time offset between the H and V channels causes a decrease in their correlation. $\phi_{dp}$ shows extreme bias similar to Figure 5.2.

After partial calibration in Figure 5.13, velocity and $\rho_{hv}$ remain unchanged. $Z_h$ and $Z_v$ show slight variation, mainly at higher elevations. $Z_{dr}$ is far more uniform, with the exception of the top corners of the scan range, farthest from the principal planes and where scan angle bias is greatest.

Figure 5.14 is considered to be truth values of averaged weather, assuming perfect calibration of UMaXX. Thus Figure 5.15 shows the remaining averaged error in Skyler data after calibration. Remaining reflectivity error is small, but appears to follow a slowly varying pattern similar to the UMaXX radome effects discussed earlier, implying that these errors may be due to imperfect UMaXX calibration and not errors in
Skyler’s polarimetric bias correction. $Z_{dr}$ is fairly uniform across the scan range, with remaining variation aligning well with expectations based on the partial correction approach. However, values seem consistently lower than UMaXX by approximately 1 dB. This will be further discussed later. Velocity varies by around 1 m/s on the right side, where averaged velocities are higher. $\rho_{hv}$ error is also highest on the right, where weather is moving faster. This supports the theory that lower $\rho_{hv}$ values are due to ATAR of Skyler versus STSR of UMaXX. $\phi_{dp}$ shows only slight remain error at the edges. These seem to be areas of heavy weather.

5.3.3 Scatter Histograms

Finally, a 2D scatter point density histogram of Skyler versus UMaXX radar data is used to visualize correlation between the two radars for each weather product. These plot a single weather product, with the UMaXX value on the x-axis, the Skyler value on the y-axis, and the color bar showing the distribution density of the 2D histogram. The grid size density for each weather product was chosen to be 0.5 dB for $Z_h$, 0.1 dB for $Z_{dr}$, 0.5 m/s for $v_d$, and 1.0 deg for $\phi_{dp}$.

A regression linear fit is applied to each plot using the Least Absolute Deviations (LAD) method. This minimizes the function $\sum_{i=1}^{n} |y_i - \hat{y}_i|$ where $\hat{y}_i = b_0 + b_1 x_{i1} + b_2 x_{i2} + ... + b_k x_{ik}$ which is the linear fit. This is used instead of the more traditional Ordinary Least Squares (OLS), as the latter is more easily skewed by outliers. The LAD fit is shown in color, while a one-to-one relationship is shown in black for reference. Statical parameters shown are the Regression Coefficient (RC), Intercept Point (IP), and Correlation Coefficient (CC). Tables 5.5, 5.6, 5.7, and 5.8 also summarize these statistical results. The correlation between the two radars (CC) gives a quantitative measure of the relationship between the two radars and by extension, the effectiveness of the phased-array calibration. However, the fit line itself (RC and IP) may not be as useful. A linear fit of this type assumes no error in x-axis data, averaging only
Figure 5.16. Scatter plot with linear fit of uncalibrated Skyler vs. UMaXX with 10 sec offset for inclusive dataset. $Z_h$ is with SNR > 0 dB, while $Z_{dr}$, $v_d$, and $\phi_{dp}$ are with SNR > 30 dB.

along the y-axis. As we expect error in both Skyler and UMaXX data, the linear fit line will be skewed. For completeness, the fit line will still be included, but a better evaluation of the data is performed by comparing the histogram data cloud to the 1:1 black line.

In the following scatter plots, elevation ranges from 4 to 30 degrees. In addition, the noise filter threshold for plots of $Z_{dr}$, $v_d$, and $\phi_{dp}$ has been increased to SNR > 30 dB, while for plots of reflectivity the threshold is SNR > 0 dB, in order to compare data of interest for each variable. A detailed explanation of why these specific parameters were chosen can be found in section 5.4 below.
Figure 5.17. Scatter plot with linear fit of calibrated Skyler vs. UMaXX with 10 sec offset for inclusive dataset. $Z_h$ is with SNR > 0 dB, while $Z_{dr}$, $v_d$, and $\phi_{dp}$ are with SNR > 30 dB. In $Z_{dr}$, dashed black line shows 1:1 offset by -1 dB in Skyler.

Table 5.5. Reflectivity Linear Fit Summary

<table>
<thead>
<tr>
<th>Dataset</th>
<th>Calibrated</th>
<th>Offset</th>
<th>RC</th>
<th>IP</th>
<th>CC</th>
</tr>
</thead>
<tbody>
<tr>
<td>All-inclusive</td>
<td>No Cal</td>
<td>10 s</td>
<td>0.66</td>
<td>9.07 dB</td>
<td>0.75</td>
</tr>
<tr>
<td>...</td>
<td>Cal</td>
<td>10 s</td>
<td>0.65</td>
<td>10.29 dB</td>
<td>0.78</td>
</tr>
<tr>
<td>...</td>
<td>Cal</td>
<td>60-90 s</td>
<td>0.52</td>
<td>14.14 dB</td>
<td>0.65</td>
</tr>
</tbody>
</table>

Table 5.6. Differential Reflectivity Linear Fit Summary

<table>
<thead>
<tr>
<th>Dataset</th>
<th>Calibrated</th>
<th>Offset</th>
<th>RC</th>
<th>IP</th>
<th>CC</th>
</tr>
</thead>
<tbody>
<tr>
<td>All-inclusive</td>
<td>No Cal</td>
<td>10 s</td>
<td>0.38</td>
<td>-0.32 dB</td>
<td>0.24</td>
</tr>
<tr>
<td>...</td>
<td>Cal</td>
<td>10 s</td>
<td>0.46</td>
<td>-0.23 dB</td>
<td>0.32</td>
</tr>
<tr>
<td>...</td>
<td>Cal</td>
<td>60-90 s</td>
<td>0.22</td>
<td>0.03 dB</td>
<td>0.17</td>
</tr>
</tbody>
</table>
Figure 5.18. Scatter plot with linear fit of calibrated Skyler vs. UMaXX with 60-90 sec offset for inclusive dataset. $Z_h$ is with SNR $> 0$ dB, while $Z_{dr}$, $v_d$, and $\phi_{dp}$ are with SNR $> 30$ dB.

Table 5.7. Velocity Linear Fit Summary

<table>
<thead>
<tr>
<th>Dataset</th>
<th>Calibrated</th>
<th>Offset</th>
<th>RC</th>
<th>IP</th>
<th>CC</th>
</tr>
</thead>
<tbody>
<tr>
<td>All-inclusive</td>
<td>No Cal</td>
<td>10 s</td>
<td>0.89</td>
<td>-0.03 m/s</td>
<td>0.90</td>
</tr>
<tr>
<td>...</td>
<td>Cal</td>
<td>10 s</td>
<td>0.88</td>
<td>-0.02 m/s</td>
<td>0.90</td>
</tr>
<tr>
<td>...</td>
<td>Cal</td>
<td>60-90 s</td>
<td>0.79</td>
<td>0.38 m/s</td>
<td>0.79</td>
</tr>
</tbody>
</table>

Table 5.8. Differential Phase Linear Fit Summary

<table>
<thead>
<tr>
<th>Dataset</th>
<th>Calibrated</th>
<th>Offset</th>
<th>RC</th>
<th>IP</th>
<th>CC</th>
</tr>
</thead>
<tbody>
<tr>
<td>All-inclusive</td>
<td>No Cal</td>
<td>10 s</td>
<td>0.90</td>
<td>94.14 deg</td>
<td>0.49</td>
</tr>
<tr>
<td>...</td>
<td>Cal</td>
<td>10 s</td>
<td>0.83</td>
<td>2.18 deg</td>
<td>0.74</td>
</tr>
<tr>
<td>...</td>
<td>Cal</td>
<td>60-90 s</td>
<td>0.29</td>
<td>5.67 deg</td>
<td>0.42</td>
</tr>
</tbody>
</table>
Figure 5.16 compares uncalibrated Skyler data to UMaXX data, providing a reference for the calibration to improve upon. Figure 5.17 shows the same data after applying the polarimetric calibration to Skyler. Each weather variable shown in these two figures is discussed here. As mentioned above, the red fit line is not expected to match the black 1:1 line perfectly, even for well calibrated Skyler data. Any residual error in UMaXX data will skew a linear fit of this type, which only accounts for error along the y-axis. Therefore, the two factors used in evaluating the calibration will be the correlation between the two radars (CC) and a comparison of the histogram data cloud to the 1:1 black line.

Reflectivity appears fairly noisy, as there are over ten million datapoints being plotted. However, the vast majority of these points are concentrated between 20 and 40 dBZ and follow the 1:1 black line with decent correlation in that region. Applying calibration improves CC and decreases spread slightly. Additional findings while comparing reflectivity are discussed in section 5.4. Velocity data follows close to the 1:1 line and has consistently high correlation, verifying that both radars are indeed observing the same weather targets and setting a ‘baseline control’ for the other variables. Without calibration, Skyler differential phase has a large offset (IP), low correlation (CC) and a wide spread on the Skyler axis (y-axis). Applying calibration drastically improves all three of these features, and while datapoints are concentrated at zero $\phi_{dp}$, a trend can be seen following the 1:1 black line.

Differential reflectivity is weakly correlated when plotted and compared in this way. Several factors could be contributing to this:

1. Smaller sample size. $Z_{dr}$ will only return high values where very large hydrometeors are present. Such data makes up small percentage of typical weather data, with most data having $Z_{dr}$ values close to zero.

2. The very small range of $Z_{dr}$ values ($\pm 5$ dB) means noise in the data has a much larger impact, resulting in low correlation.
3. Skyler's use of ATAR adds a time delay between H and V channels, reducing their correlation compared to UMaXX's STSR polarization. This effect is clearly seen when comparing $\rho_{hv}$ of both radars, but would also affect $Z_{dr}$ and $\phi_{dp}$ as well.

In the $Z_{dr}$ plot in Figure 5.17, the data does not closely match the 1:1 solid black line. However, if an additional 1:1 line offset by -1 dB in Skyler is shown (dashed black line), the data follows this line well. This agrees with the results from the averaged scan pattern in Figure 5.15. Since this 1 dB offset in $Z_{dr}$ appears in all data at all scan angles, it is not related to the polarimetric calibration of Skyler. It is not apparent which of the two radars is contributing to this offset.

In Figure 5.18, it can be seen that increasing the allowed time offset between Skyler and UMaXX radar scans from ten seconds to 60-90 seconds results in a drastic decrease in CC for all variables. This is because the target weather is dynamic and decorrelations over time. This agrees well with the results of a similar study done by Asai et. al. in Tokyo, Japan [42]. This result demonstrates the advantage of rapid scan updates from phased-array weather radar. It also seems likely that time offset between radars is still the largest source of error in scatter plots. However, reducing offset below ten seconds provides too few datapoints for meaningful analysis.

One additional analysis which can be done is a comparison of reflectivity versus differential reflectivity for each radar. This type of comparison was developed by H. Al-Sakka et. al. [4] to define membership functions which classify categories of precipitation, such as rain, snow, ice, or hail. Each of these precipitation types has its own profile when plotting $Z_h$ vs. $Z_{dr}$, which can be used to distinguish between them (Figure 5.19).

Figure 5.20 shows scatter histograms of $Z_h$ vs. $Z_{dr}$ for Skyler and UMaXX using the all-inclusive four day dataset. Data is limited to 4 deg. elevation, and a polynomial fit trend line is shown. It is immediately apparent that the UMaXX data is much
Figure 5.19. Reflectivity vs differential reflectivity at X-Band (figure from Al-Sakka et. al. [4]): Shows $Z_h$ vs. $Z_{dr}$ profile for various forms of precipitation.

Figure 5.20. Reflectivity vs differential reflectivity at 4 deg. elevation. Skyler data is shown on the left, UMaXX data on the right. The red line shows a polynomial fit of the data.

more spread out in $Z_{dr}$, possibly due to residual effects of non-uniformity of the UMaXX radome. Secondly, the red trend line shows that on average, the UMaXX $Z_{dr}$ is around 1 dB lower than that of Skyler. Comparing these plots to the rain profile in Figure 5.19, the Skyler data appears to fit the profile more closely, implying that the $Z_{dr}$ offset is due to a miscalibration of UMaXX when correcting for system and radome effects.
5.4 Comparison Challenges

5.4.1 Noise at High Reflectivity

When comparing Skyler and UMaXX reflectivity ($Z_h$) directly through a scatter point density histogram, the data appears very ‘noisy’ for high values of $Z_h$. That is, there are many datapoints where high $Z_h$ (40-60 dBZ) Skyler measurements correspond to low $Z_h$ (10-30 dBZ) UMaXX measurements, and vice-versa. These appear as clusters of points in the top left and bottom right of the plot respectively. The source of these ‘noisy’ regions was investigated and the following causes discovered:

**Ground Clutter** Regions of high Skyler $Z_h$ but low UMaXX $Z_h$ were caused by ground clutter contamination of Skyler data. Nearly all contamination occurs at 2 degrees or below. Excluding the 0 and 2 degree elevation scans removes this noisy region.

This makes sense because ground clutter would appear very bright and highly correlated. UMaXX, which has been running continuously for several years now, employs a complex clutter classifier trained on large quantities of pre-classified data. Skyler’s clutter filter is far simpler due to time and data limitations. While implementing a more robust clutter classifier for Skyler would be ideal, for the purposes of this research, it makes sense to simply exclude the contaminated regions. Thus, all scatter plots and analysis are performed on elevations ranging from 4 to 30 degrees.

**Rain Attenuation** Regions of low Skyler $Z_h$ but high UMaXX $Z_h$ were caused by attenuation of Skyler through heavy rain. Skyler cannot penetrate as far through heavy rain as UMaXX, as can be clearly seen in PPI plots. Since $\phi_{dp}$ also increases through heavy rain, it can be used to isolate this phenomenon. Limiting data to where $-15 \text{ deg} < \phi_{dp} < 15 \text{ deg}$ eliminated this noise on the scatter plot.
Figure 5.21. Comparison of scatter plot contaminates in $Z_h$. Top left: $Z_h$ comparison with both contaminates present. Top right: $Z_h$ with $\phi_{dp}$ limited to $\pm15$ deg. Bottom left: $Z_h$ with both contaminates removed. Bottom right: $Z_h$ with both contaminates removed.

While Skyler and UMaXX both attenuate through heavy rain, Skyler has a much lower peak transmit power and uses a linear chirp rather than a short pulse. It also has a slightly wider beamwidth. The means UMaXX will still receive a strong signal behind heavy rain, while Skyler may be reaching the limit of its detection sensitivity. Excluding data with high $\phi_{dp}$ values would reduce this issue. However, since weather containing high $Z_{dr}$ and $\phi_{dp}$ values is interest to this comparison, scatter plots and analysis include all $\phi_{dp}$ values.

Figure 5.21 demonstrates the effects of these contaminates and their removal. The top left scatter plot shows the $Z_h$ comparison with both contaminates present.
Top right is the same data with El = 2 degrees removed, eliminating ground clutter contamination. Bottom left shows $Z_h$ when $\phi_{dp}$ is limited to ±15 deg, reducing attenuation effects. Finally, the bottom right shows $Z_h$ when both contaminates are removed, greatly reducing spread and increasing correlation between radars.

Figure 5.22 shows an example of attenuation contamination and its removal from a PPI perspective. Calibrated Skyler data is shown in the left column, while UMaXX data is in the right column. The top row is unfiltered reflectivity. The second row is filtered using DRaCX as described in section 5.3.1. The third row adds the additional filter of $|\phi_{dp}| < 15$ deg. The fourth row shows the difference between Skyler and UMaXX reflectivity, with the $\phi_{dp}$ filter on the right, and without it on the left. The final row shows unfiltered $\phi_{dp}$ for reference.

From the PPI view it becomes clear that the region of highest discrepancy between radars corresponds to the region looking through the storm, which is also where $\phi_{dp}$ is highest. Applying the filter based on $\phi_{dp}$ accurately masks this region.

For an example of ground clutter contamination, refer to Figure 5.9 which shows a large amount of ground clutter visible in the unfiltered PPI at 2 degrees elevation. While most of this clutter is removed by the filter applied in Figure 5.10, a small amount still remains.

### 5.4.2 $Z_{dr}$ and $\phi_{dp}$ Noise

Another challenge in comparing radar datasets arises when examining $Z_{dr}$ and $\phi_{dp}$. These weather products are typically close to zero for most weather data, only increasing in the presence of heavy rain. Thus, if all data is included, the $Z_{dr}$ and $\phi_{dp}$ plots will be overwhelmingly dominated by values near zero and noise. Such data can be characterized by low SNR and prevents accurate correlation being visible in the scatter plots. Therefore, if $Z_{dr}$ and $\phi_{dp}$ plots are specifically filtered to only include
Figure 5.22. PPI of attenuation contamination taken on August 28, 2019. First row from top: Unfiltered reflectivity. Second row: Filtered using DRaCX. Third row: additional $|\phi_{dp}| < 15$ deg filter. Fourth row left: Skyler and UMaXX difference, without $\phi_{dp}$ filter. Fourth row right: as left, with $\phi_{dp}$ filter. Bottom row: Unfiltered $\phi_{dp}$.

Data points where SNR is high, then non-zero values of $Z_{dr}$ and $\phi_{dp}$ become dominate and a useful comparison can be made.
Figure 5.23. Scatter plots of $\phi_{dp}$ for varying noise filter thresholds. Top left: SNR > 0 dB. Top right: SNR > 10 dB. Bottom left: SNR > 20 dB. Bottom right: SNR > 30 dB.

Figure 5.23 shows $\phi_{dp}$ plotted for varying filter threshold of SNR. As SNR filter increases, the data centered at zero degrees becomes less prominent and noise drastically decreases, allowing the weather data of interest to become visible. The number of data points being plotted also greatly decreases, while correlation between radars increases. This trend also holds for $Z_{dr}$ and velocity. Therefore, in all scatter plots, the noise filter threshold for plots of $Z_{dr}$, velocity and $\phi_{dp}$ has been increased to SNR > 30 dB, while for plots of reflectivity the threshold is left at SNR > 0 dB to show the full range of reflectivity.
5.4.3 Linear Depolarization Ratio

Originally, Linear Depolarization Ratio (LDR) data was to be included in the statistical analysis. However, this was made difficult due to two main factors. The first being that the UMaXX radar typically operates in STSR mode, in which LDR cannot be measured. In order to measure LDR, one channel (V) must be switched to dump to a load on transmit, and thus all other polarimetric measurements are not possible. Therefore, to collect LDR data on UMaXX, \(Z_{dr}\), \(\phi_{dp}\), and \(\rho_{hv}\) must be sacrificed for that full dataset. Since \(Z_{dr}\), \(\phi_{dp}\), and \(\rho_{hv}\) are more commonly used in hydrometeor measurement and classification, we prioritized calibration of these instead. In other words, there are only a few UMaXX datasets containing LDR data.

Secondly, while the Skyler radar can measure LDR using its ATAR mode of HH, HV, VV, VH, doing so increases the decorrelation time between HH and VV pulses, while also coupling the velocity and correlation measurements. However, the main issue with LDR measurements is due to the low power (and thus low sensitivity) of the Skyler radar. Skyler can either collect data with high sensitivity using LFM chirp mode, but is blind at close range, or collect low sensitivity data with single pulse mode. Obtaining LDR data requires high sensitivity at close range, since the cross-pol returns are so low. All Skyler LDR data is therefore extremely noisy and was deemed unusable in this research.

5.4.4 Other Challenges

When starting this project, the best approach to take was not readily apparent. Several routes were investigated a fair amount before being dropped for an alternative. For example, while attempting to characterize the \(G\) matrix of Skyler, many types of measurements were made including near-field chamber measurements, quasi-vertical measurements, drone flight calibration, and mobile radar observations. Ultimately, averaged light rain calibration performed in-situ was deemed the best option.
The research also had to adapt on the theory side as well. Target dependent terms were originally to be included in corrections. Wet radome corrections were also to be included. Both of these were removed due to their complexity and likelihood to introduce greater measurement errors. As previously discussed, measurement and comparison of LDR was originally planned and then dropped.

Data collection had its challenges as well. A large amount of Skyler data was collected in short pulse mode rather than chirp. This was to be used for close range, uniform rain comparison, but SNR is so low in this mode that it was unusable except to calibrate $G$. A very large dataset collected on 10/31/19 was corrupted and rendered completely unusable due to a software bug in Skyler's datalogger. This bug occurs intermittently when beginning transmission. Another dataset collected on 8/19/19 was lost due to a networking issue where UMaXX’s datalogger would crash from too much traffic on the tower’s network.

Hardware failures while the testbed was installed on the tower presented the biggest challenge, as someone would have to climb up the 70-foot ladder to make repairs. At one point the reference clock inside Skyler failed while it was installed on the tower and had to be replaced. Later, water began collecting inside the rear exhaust fan, so drainage holes had to be drilled. Most memorably, a mouse once chewed through the tower’s fiberoptic cable, causing the entire testbed of both Skyler and UMaXX to be disconnected from the network until the cable could be replaced.

While ultimately these challenges do not contribute directly to the main goal of this research, facing and overcoming each one of them has provided valuable experience and insight which may be of use in the future.
CHAPTER 6
CONCLUSIONS

6.1 Summary
Phased-array weather radar have great potential as the next upgrade to major weather radar networks such as NEXRAD, providing faster update times and scan flexibility over current reflector dish radar. However, phased-array radar need polarimetric calibration and are known to have more bias sources than dish radar. Prior works have described these polarimetric bias sources theoretically, but experimental data collected by a polarimetric phased-array weather radar is still limited. Additionally, no standard method of calibration for polarimetric bias exists for phased-arrays, though some have been proposed.

Therefore, this research provides a fully operational X-Band, weather radar polarimetric testbed. The testbed is capable of performing experiments on a phased-array weather radar, while evaluating polarimetric calibration through simultaneous operation of a co-located dish radar. A detailed description of the components of both the Skyler phased-array radar and the UMass Experimental X-Band (UMaXX) radar is provided. Radar installation and data collection procedures are described as well.

In addition, this research proposes a novel method of phased-array polarimetric calibration which improves on prior work by decomposing bias sources into independent linear terms. This modular approach is designed to be practical, easy to implement, and adaptable to any radar system. Bias modules are: main beam gain and relative phase ($G$), radiated dipole polarizations ($D$), and array tilt and roll pro-
jection ($T$). Also, weather variable corrections are separated into linear and non-linear terms.

Finally, this research delivers a detailed demonstration of both the calibration method and the weather radar polarimetric testbed. Through implementation of the testbed, the calibration method’s performance is evaluated using a dual-polarization phased-array radar by direct comparison of weather observations to a co-located mechanically-scanned reference radar.

The results of this evaluation confirm that Skyler and UMaXX are able make qualitatively similar observations of the same weather targets at nearly the same time. Increasing the time offset between observations decreases the correlation between radars in every case. The polarimetric calibration improves Skyler’s correlation with UMaXX in all cases. However, the partial correction appears insufficient to calibrate all variables over the entire scan range. In addition, this evaluation provides insight into the effects of ground clutter, rain attenuation, and noise on this sort of comparison.

6.2 Conclusions

As stated in section 1.1.4, the main objective of this research is to study phased-array weather radar polarimetry through the implementation and operation of a radar testbed. Specific deliverables are:

1. To design and construct a weather radar polarimetric testbed comprised of a dual-polarization phased-array radar and a co-located mechanically-scanned reference radar.

2. To develop a practical calibration model specifically for polarimetric phased-array weather radar using antenna theory and various prior research works.
3. To evaluate the calibration model’s performance through direct comparison of weather observations using the weather radar polarimetric testbed.

These three deliverables were each addressed in Chapters 3, 4 and 5 respectively.

6.2.1 Deliverable 1 (Chapter 3)

The weather radar testbed was successfully built and operated using both the Skyler and UMaXX radars. It provides a means to test polarimetric calibration of a phased-array radar using real weather data and comparison to a reference dish radar. Such a testbed is valuable because phased-array weather radar data is still very limited. In addition to its application in this research, the testbed serves a secondary purpose as well, as UMaXX provides continuous weather radar coverage of the local region, while Skyler can be transferred to a mobile platform for field deployments.

6.2.2 Deliverable 2 (Chapter 4)

A novel calibration model was successfully created and implemented. This approach is valuable because it breaks down a complex process into separate parts and then provides a simple, adaptable solution to each. The approach improves on prior works because it: can incorporate both theoretical corrections and existing calibration techniques, allows correction of arbitrary tilt and roll, and can be easily adapted to many types of phased-array radars. It is also easier to study and understand polarimetric bias sources when they are presented in this way.

It is worth noting that the evaluation performed on the radar testbed was merely one possible implementation of this model. In this case, $G$ was measured experimentally via in-situ light rain, while patch radiators were used for $D$, and $T$ was measured using a tilt/roll sensor mounted to the radar box. The same model could be applied to a different phased-array using different measurement techniques. Therefore, the
results of the testbed comparison data reflect the quality of this implementation of
the model, not the value of the model itself.

6.2.3 Deliverable 3 (Chapter 5)

The calibration model was successfully implemented on the weather radar polari-
metric testbed. A direct comparison of the weather observations of the two radars
was also successfully performed using the DRaCX filter. This was used to present an
evaluation of the calibration model’s performance. The results of this evaluation are
discussed here.

Based on the side-by-side PPI plots, Skyler and UMaXX can make qualitatively
similar observations. Data is filtered such that both radars are observing the same
weather at nearly the same time using the DRaCX filter. Based on the scatter plots,
increasing the time offset threshold decreased the correlation between radars in every
case. This also supports the assumption that the testbed is successfully observing
the similar targets with both radars. However, remaining time offset between radars
may still be a leading cause of error, due to rapid decorrelation of weather data over
time.

When examining the scatter plots, in all cases the polarimetric calibration of
Skyler improved its correlation with UMaXX compared to the uncalibrated data.
This would indicate that the calibration is effective. Calibrated data also generally
followed near the 1:1 trend line. However, these scatter plots were still fairly noisy even
after calibration. It is difficult to say whether this is due to remaining uncalibrated
polarimetric biases or due to other errors in radar measurements and processing of
both radars. It seems likely that residual error associated with non-uniformity of the
UMaXX radome is a main contributor to the remaining bias.

The averaged scan pattern plots show much clearer discrepancies between radars.
They indicate that the implemented partial correction is sufficient to calibrate Skyler
over most, but not all of the scan range. This agrees with the theoretical model, which predicts the partial linear correction to have significant bias remaining at the extremes of the scan range. Regardless, the calibration resulted in an overall improvement to weather measurements.

In the analysis, Skyler $Z_{dr}$ measurements were seen to be offset from UMaXX by about -1 dB. This offset was independent of scan angle, and thus is not related to the polarimetric calibration of Skyler. Based on the $Z_h$ vs. $Z_{dr}$ analysis, it seems most likely due to an error introduced in calibration of UMaXX $Z_{dr}$.

In addition, this evaluation did provide insight into the effects ground clutter, rain attenuation, and noise can have on this sort of comparison. New approaches were needed to adapt, which were detailed in section 5.4. This is also of value, as it will allow future experiments and evaluations to improve.

6.3 Future Work

The main deliverables of this research allow for a broad range of potential future research. A few main examples are discussed here.

1. Make the UMass radar testbed available as a research tool.

Now that the UMass radar testbed is built and operational, it is valuable tool that could be made available to future research projects. As more calibration techniques are described and refined, the testbed could be used to experimentally verify and compare them. Additionally, both Skyler and UMaXX are fully functional X-Band weather radar individually. Either one could be used for local or even mobile experiments.

2. Using this polarimetric calibration approach on other radar systems.

It could be valuable to experimentally verify the performance of the bias decomposition approach on other radar systems. Since the main advantage of
the method is its adaptability and ease of implementation, it could in theory be applied to any phased-array system. Specifically, the model could easily be adapted to the newer Raytheon Skyler radars, which use dipole radiators instead of patches. The approach could also be applied to radars with various frequency bands, radiating element types (patch, dipole, slot), array sizes and geometries (linear, planar, cylindrical), and polarization schemes (ATAR, ATSR, STSR).

3. Perform further experiments related to this research.

Due to the large scope of this research, many aspects could not be fully explored. For example, wet radome corrections, element cross-pol radiation, and LDR measurements are all accounted for in the calibration model but were not included in the experimental evaluation. Implementing a more advanced clutter filter might improve the results presented here. A full correction including target dependent terms could be implemented and evaluated. It also might be interesting to experimentally compare individual corrections of each bias source.

4. Run a long-term continuous experiment.

In this research, Skyler could only be run periodically because of limited data storage and processing capabilities, and temperamental software. It also could only be deployed seasonally since it was mounted outside the radar dome and had no protection against the colder, harsher winter weather. With more time to plan out and set up a real time data collecting and processing network, a long-term experiment could be run using the testbed. This could include a self-updating version of the calibration method to study the evolution or degradation of the calibration over time.
APPENDIX A
FULL RADAR DATA LIST

Table A.1. UMaXX Tower Data List

<table>
<thead>
<tr>
<th>Date</th>
<th>Time Start</th>
<th>Time End</th>
<th>Scan</th>
<th>Pulse</th>
<th>Pol Mode</th>
<th>PRF</th>
</tr>
</thead>
<tbody>
<tr>
<td>10/11/18</td>
<td>13:00:14 UTC</td>
<td>20:59:52 UTC</td>
<td>Mode 1</td>
<td>1 us</td>
<td>HH+VV</td>
<td>Dual 2400 / 1600 Hz</td>
</tr>
<tr>
<td>10/27/18</td>
<td>12:44:38 UTC</td>
<td>16:47:55 UTC</td>
<td>Mode 2</td>
<td>1 us</td>
<td>HH+VV</td>
<td>Dual 2400 / 1600 Hz</td>
</tr>
<tr>
<td>11/03/18</td>
<td>03:50:07 UTC</td>
<td>04:40:08 UTC</td>
<td>Mode 2</td>
<td>1 us</td>
<td>HH+VV</td>
<td>Dual 2400 / 1600 Hz</td>
</tr>
<tr>
<td>11/09/18</td>
<td>19:19:01 UTC</td>
<td>03:11:29 UTC</td>
<td>Mode 2</td>
<td>1 us</td>
<td>HH+HV</td>
<td>Dual 2400 / 1600 Hz</td>
</tr>
<tr>
<td>11/13/18</td>
<td>14:22:22 UTC</td>
<td>14:59:53 UTC</td>
<td>Mode 2</td>
<td>1 us</td>
<td>HH+VV</td>
<td>Dual 2400 / 1600 Hz</td>
</tr>
<tr>
<td>08/13/19</td>
<td>14:25:40 UTC</td>
<td>15:01:04 UTC</td>
<td>Mode 2</td>
<td>1 us</td>
<td>HH+VV</td>
<td>Dual 2400 / 1600 Hz</td>
</tr>
<tr>
<td>08/19/19</td>
<td>01:00:20 UTC</td>
<td>16:00:04 UTC</td>
<td>Mode 3</td>
<td>1 us</td>
<td>HH+VV</td>
<td>Dual 2400 / 1600 Hz</td>
</tr>
<tr>
<td>08/21/19</td>
<td>23:00:25 UTC</td>
<td>00:20:47 UTC</td>
<td>Mode 2</td>
<td>1 us</td>
<td>HH+VV</td>
<td>Dual 2400 / 1600 Hz</td>
</tr>
<tr>
<td>08/28/19</td>
<td>20:12:05 UTC</td>
<td>02:57:43 UTC</td>
<td>Mode 4</td>
<td>1 us</td>
<td>HH+VV</td>
<td>Dual 2400 / 1600 Hz</td>
</tr>
<tr>
<td>09/02/19</td>
<td>22:28:02 UTC</td>
<td>23:17:14 UTC</td>
<td>Mode 4</td>
<td>1 us</td>
<td>HH+VV</td>
<td>Dual 2400 / 1600 Hz</td>
</tr>
<tr>
<td>09/04/19</td>
<td>16:00:03 UTC</td>
<td>20:00:06 UTC</td>
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<td>1 us</td>
<td>HH+VV</td>
<td>Dual 2400 / 1600 Hz</td>
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<td>17:00:02 UTC</td>
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<td>HH+VV</td>
<td>Dual 2400 / 1600 Hz</td>
</tr>
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<td>09/24/19</td>
<td>18:00:06 UTC</td>
<td>20:00:07 UTC</td>
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<td>1 us</td>
<td>HH+VV</td>
<td>Dual 2400 / 1600 Hz</td>
</tr>
<tr>
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<td>17:39:56 UTC</td>
<td>20:04:09 UTC</td>
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<td>1 us</td>
<td>HH+VV</td>
<td>Dual 2400 / 1600 Hz</td>
</tr>
<tr>
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<td>20:00:18 UTC</td>
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<td>HH+VV</td>
<td>Dual 2400 / 1600 Hz</td>
</tr>
<tr>
<td>10/16/19</td>
<td>21:20:19 UTC</td>
<td>02:40:08 UTC</td>
<td>Mode 4</td>
<td>1 us</td>
<td>HH+VV</td>
<td>Dual 2400 / 1600 Hz</td>
</tr>
<tr>
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<td>14:00:29 UTC</td>
<td>14:42:30 UTC</td>
<td>Mode 3</td>
<td>1 us</td>
<td>HH+VV</td>
<td>Dual 2400 / 1600 Hz</td>
</tr>
<tr>
<td>10/31/19</td>
<td>04:17:10 UTC</td>
<td>23:59:59 UTC</td>
<td>Mode 4</td>
<td>1 us</td>
<td>HH+VV</td>
<td>Dual 2400 / 1600 Hz</td>
</tr>
<tr>
<td>11/01/19</td>
<td>00:00:00 UTC</td>
<td>07:59:51 UTC</td>
<td>Mode 4</td>
<td>1 us</td>
<td>HH+VV</td>
<td>Dual 2400 / 1600 Hz</td>
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</table>

Table A.2. UMaXX Scan Modes

<table>
<thead>
<tr>
<th>Scan Mode</th>
<th>Update Time</th>
<th>Azimuth</th>
<th>Elevations</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mode 1</td>
<td>2 min</td>
<td>0 to 360</td>
<td>2, 4, 6, 8</td>
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<tr>
<td>Mode 2</td>
<td>2:43 min</td>
<td>-45 to 45</td>
<td>0, 2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24, 26, 28, 30</td>
</tr>
<tr>
<td>Mode 3</td>
<td>1 min</td>
<td>0 to 360</td>
<td>1, 2, 4, 6, 2</td>
</tr>
<tr>
<td>Mode 4</td>
<td>3:08 min</td>
<td>-45 to 45</td>
<td>0, 2 (360 Az), 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24, 26, 28, 30</td>
</tr>
<tr>
<td>Date</td>
<td>Time Start</td>
<td>Time End</td>
<td>Mode</td>
</tr>
<tr>
<td>------------</td>
<td>-------------</td>
<td>-------------</td>
<td>------</td>
</tr>
<tr>
<td>10/11/18</td>
<td>16:22:04 UTC</td>
<td>20:51:44 UTC</td>
<td>Chirp</td>
</tr>
<tr>
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<td>12:48:22 UTC</td>
<td>18:11:22 UTC</td>
<td>Chirp</td>
</tr>
<tr>
<td>11/03/18</td>
<td>02:16:31 UTC</td>
<td>04:38:51 UTC</td>
<td>Chirp</td>
</tr>
<tr>
<td>11/09/18</td>
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<td>03:06:07 UTC</td>
<td>Chirp</td>
</tr>
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<td>11/13/18</td>
<td>13:57:58 UTC</td>
<td>14:02:34 UTC</td>
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</tr>
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</tr>
<tr>
<td>08/13/19</td>
<td>14:41:26 UTC</td>
<td>15:22:47 UTC</td>
<td>Chirp</td>
</tr>
<tr>
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<td>15:47:29 UTC</td>
<td>16:43:58 UTC</td>
<td>Pulse</td>
</tr>
<tr>
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<td>17:13:09 UTC</td>
<td>Chirp</td>
</tr>
<tr>
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<td>01:36:18 UTC</td>
<td>02:43:23 UTC</td>
<td>Chirp</td>
</tr>
<tr>
<td>08/19/19</td>
<td>14:49:05 UTC</td>
<td>15:08:05 UTC</td>
<td>Chirp</td>
</tr>
<tr>
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<td>16:17:17 UTC</td>
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<td>Chirp</td>
</tr>
<tr>
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<td>17:22:18 UTC</td>
<td>17:36:12 UTC</td>
<td>Pulse</td>
</tr>
<tr>
<td>08/21/19</td>
<td>22:42:14 UTC</td>
<td>22:59:37 UTC</td>
<td>Chirp</td>
</tr>
<tr>
<td>08/21/19</td>
<td>23:08:29 UTC</td>
<td>00:18:19 UTC</td>
<td>Chirp</td>
</tr>
<tr>
<td>08/28/19</td>
<td>19:14:33 UTC</td>
<td>23:36:56 UTC</td>
<td>Chirp</td>
</tr>
<tr>
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<td>23:37:56 UTC</td>
<td>00:06:16 UTC</td>
<td>Pulse</td>
</tr>
<tr>
<td>08/29/19</td>
<td>00:06:41 UTC</td>
<td>00:30:59 UTC</td>
<td>Chirp</td>
</tr>
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<td>08/29/19</td>
<td>01:32:49 UTC</td>
<td>02:08:29 UTC</td>
<td>Pulse</td>
</tr>
<tr>
<td>09/02/19</td>
<td>17:52:20 UTC</td>
<td>23:01:08 UTC</td>
<td>Chirp</td>
</tr>
<tr>
<td>09/04/19</td>
<td>16:15:47 UTC</td>
<td>19:05:42 UTC</td>
<td>Chirp</td>
</tr>
<tr>
<td>09/04/19</td>
<td>19:45:49 UTC</td>
<td>20:54:21 UTC</td>
<td>Pulse</td>
</tr>
<tr>
<td>09/12/19</td>
<td>14:19:29 UTC</td>
<td>14:27:53 UTC</td>
<td>Chirp</td>
</tr>
<tr>
<td>09/12/19</td>
<td>14:28:31 UTC</td>
<td>14:51:31 UTC</td>
<td>Chirp</td>
</tr>
<tr>
<td>09/12/19</td>
<td>14:52:46 UTC</td>
<td>15:20:34 UTC</td>
<td>Pulse</td>
</tr>
<tr>
<td>09/12/19</td>
<td>15:21:03 UTC</td>
<td>16:49:45 UTC</td>
<td>Chirp</td>
</tr>
<tr>
<td>09/24/19</td>
<td>18:41:08 UTC</td>
<td>19:34:55 UTC</td>
<td>Chirp</td>
</tr>
<tr>
<td>09/24/19</td>
<td>19:35:49 UTC</td>
<td>19:42:25 UTC</td>
<td>Chirp</td>
</tr>
<tr>
<td>09/26/19</td>
<td>17:38:09 UTC</td>
<td>18:17:32 UTC</td>
<td>Pulse</td>
</tr>
<tr>
<td>09/26/19</td>
<td>18:17:55 UTC</td>
<td>18:39:45 UTC</td>
<td>Chirp</td>
</tr>
<tr>
<td>09/26/19</td>
<td>18:40:01 UTC</td>
<td>20:02:31 UTC</td>
<td>Pulse</td>
</tr>
<tr>
<td>09/26/19</td>
<td>21:18:28 UTC</td>
<td>22:07:28 UTC</td>
<td>Chirp</td>
</tr>
<tr>
<td>10/07/19</td>
<td>14:24:38 UTC</td>
<td>18:21:50 UTC</td>
<td>Chirp</td>
</tr>
<tr>
<td>10/27/19</td>
<td>14:05:59 UTC</td>
<td>15:12:58 UTC</td>
<td>Pulse</td>
</tr>
<tr>
<td>10/31/19</td>
<td>14:27:52 UTC</td>
<td>07:57:00 UTC</td>
<td>Chirp</td>
</tr>
</tbody>
</table>
### Table A.4. Skyler Mobile Data List

<table>
<thead>
<tr>
<th>Date</th>
<th>Time Start</th>
<th>Time End</th>
<th>Location</th>
</tr>
</thead>
<tbody>
<tr>
<td>06/02/18</td>
<td>00:28:22 UTC</td>
<td>00:54:30 UTC</td>
<td>Kent, NE</td>
</tr>
<tr>
<td>06/09/18</td>
<td>14:24:16 UTC</td>
<td>21:47:58 UTC</td>
<td>Lincoln, IA</td>
</tr>
<tr>
<td>06/11/18</td>
<td>20:15:53 UTC</td>
<td>23:52:04 UTC</td>
<td>Rockford, NE</td>
</tr>
<tr>
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<td>01:15:03 UTC</td>
<td>23:59:59 UTC</td>
<td>Dawson, NE</td>
</tr>
<tr>
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<td>00:00:15 UTC</td>
<td>02:00:24 UTC</td>
<td>Rosston, OK</td>
</tr>
<tr>
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<td>02:02:02 UTC</td>
<td>03:16:37 UTC</td>
<td>Fairview, OK</td>
</tr>
<tr>
<td>06/17/19</td>
<td>23:11:04 UTC</td>
<td>23:26:40 UTC</td>
<td>Lamar, CO</td>
</tr>
<tr>
<td>06/18/19</td>
<td>19:08:33 UTC</td>
<td>20:49:21 UTC</td>
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</tr>
<tr>
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<td>23:48:58 UTC</td>
<td>Campbell, TX</td>
</tr>
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<td>01:59:56 UTC</td>
<td>02:16:20 UTC</td>
<td>Firstview, CO</td>
</tr>
<tr>
<td>06/22/19</td>
<td>21:08:14 UTC</td>
<td>22:23:54 UTC</td>
<td>Wilson, KS</td>
</tr>
</tbody>
</table>
APPENDIX B

CROSS-POLARIZED RADIATION

The $G$ matrix presented earlier assumed ideal elements producing no cross-polar radiation. Cross-polar radiation from imperfect elements can be treated directly in $G$ if it can be assumed that the source of the cross-polar radiation is that from a rotated dipole of the same type as that producing the co-polar radiation, and not from some other equivalent source. For example, a microstrip patch excited in horizontal polarization is represented by an equivalent $z'$-directed magnetic dipole. Cross-polar radiation from this source that can be represented in $G$ would be that due to an equivalent $y'$-directed magnetic dipole. With this assumption, the product $DG$ is given by

$$DG = \begin{bmatrix}
g_{hh} + g_{vh} \frac{\sin \phi' \cos \theta'}{\sqrt{\sin^2 \phi' \cos^2 \theta' + \cos^2 \phi'}} & g_{hv} + g_{vv} \frac{\sin \phi' \cos \theta'}{\sqrt{\sin^2 \phi' \cos^2 \theta' + \cos^2 \phi'}} \\
g_{vh} \frac{-\cos \phi'}{\sqrt{\sin^2 \phi' \cos^2 \theta' + \cos^2 \phi'}} & g_{vv} \frac{-\cos \phi'}{\sqrt{\sin^2 \phi' \cos^2 \theta' + \cos^2 \phi'}}
\end{bmatrix}. \quad (B.1)$$

This representation describes cross-polarization observed at the aperture resulting in cross-polarized far-fields that are not orthogonal to the co-polarized fields except along the principal planes. This representation is correct to the extent that the cross-polarized radiation comes from the assumed source. If instead the cross-polarized radiation is due to spurious radiation from another source, such as feedlines or other structures on or near the antenna, then the assumed projection may be wrong.

Alternatively, one could consider the cross-polarized radiation in the far-field. Cross-polarization unit vectors can be defined for the horizontal and vertically oriented dipole radiators, and these unit vectors are orthogonal to their co-polarized...
counterparts. However, the equivalent source producing these cross-polarized fields, are of opposite types as described in [27]. That is, the equivalent source of the cross-polar fields of a $z'$-directed magnetic current source is a $z'$-directed electric current source. This cannot be treated directly in $G$ above since it must be multiplied by a different $D$. One can, however, treat them separately by putting the off-diagonal terms of $G$ in a separate matrix, say $G^x$, multiplying by an appropriate $D^x$, and then adding the results. With this approach, the $D^x$ matrices are defined as in (4.19) and (4.20) but of opposite source type:

$$D^x_m = D_e, \quad D^x_e = D_m. \quad (B.2)$$

Using this formulation for a microstrip patch radiator we obtain

$$DG + D^xG^x = \begin{bmatrix}
g_{hh} & -\cos \phi' \\
g_{eh} & g_{he} \sqrt{\frac{\sin^2 \phi' \cos^2 \phi' + \cos^2 \phi'}{\sin^2 \phi' \cos^2 \phi' + \cos^2 \phi'}} \\
g_{hv} & \frac{-\sin \phi' \cos \phi'}{\sqrt{\sin^2 \phi' \cos^2 \phi' + \cos^2 \phi'}} \\
g_{vu} & g_{uv} \sqrt{\frac{\sin^2 \phi' \cos^2 \phi' + \cos^2 \phi'}{\sin^2 \phi' \cos^2 \phi' + \cos^2 \phi'}}
\end{bmatrix}, \quad (B.3)$$

which is seen to be different from the earlier case. While the projections are different, it is important to remember that the definitions of $g_{hv}$ and $g_{eh}$ are also different in the two cases. In the former case they are defined in terms of orthogonal but otherwise identical sources at the aperture, whereas in the second case, they are defined by orthogonal polarizations in the far-field.

Choosing the appropriate representation depends upon the application and the method of characterizing cross-polar radiation. The former approach more easily represents what might be measured in a near-field chamber. Although the latter approach requires a slight modification to the original equations, it can be easily constructed from the far field pattern of a single element. To the extent that the latter approach relies upon truly cross-polarized fields at the target location, it seems to be the appropriate definition. That is, field observations of non-depolarizing scatterers
with imperfect elements would result in a cross-polar response consistent with the latter definition.

While this treatment has been included here for completeness, we do not consider cross-polarized radiation in our subsequent analysis. The elements used in our case exhibit low cross-polar radiation such that it could not be measured with sufficient signal-to-noise ratio from field measurements.
APPENDIX C

WET RADOME EFFECTS

Because of its potential impact on measurements, we also consider the possibility of a wet radome covering the face of the PPAR. This is necessary since we use field measurements of precipitation in close proximity to the radar which implies the radome is also likely to be wet.

At X-band, a film of water on the radome can yield significant losses [43]. Salazar et al. [44] estimated transmission losses for planar radomes both including and lacking hydrophobic coatings. We first consider the latter case, where the water forms a continuous film over the surface rather than beading. This yields the worst-case attenuation.

The effect of a wet radome may be considered as a modification to the $G$ matrix, where additional attenuation and relative phase are imparted to the radar signal as a function of scan angle due to the water film on the radome. For a flat, rectangular radome tilted from the horizontal, and assuming a uniform, laminar water film, the thickness of the film is found using [44]:

$$d_w = \left( \frac{3\mu_kWR}{g\tan \theta_p} \right)^{\frac{1}{3}}$$

(C.1)

where $\mu_k$ is the kinematic viscosity of water, $W$ is the length of the square radome, $g$ is gravitational acceleration, $R$ is the rain rate in meters per second, and $\theta_p$ is the tilt angle of the radome relative to the horizontal plane. The radome on the Skyler
Figure C.1. Radome water film thickness vs rain rate for two array tilt angles.

The prototype is a thin, coated-fabric, so we consider only the effect of the water film. It imparts a loss given by

\[ L = \left| \frac{(1 - \Gamma^2)e^{-j\theta_w}}{1 - \Gamma^2e^{-j2\theta_w}} \right|^2 \]  

(C.2)

where \( \Gamma \) is the air-water reflection coefficient and \( \theta_w \) is the electrical thickness of the film given by \( k_w d_w \) where \( k_w \) is the wavenumber in the water. Because the radome is in very close proximity to the radiating elements (a few cm), we use the polarization basis of the elements themselves, and consider the effect of scan angle on the transmission through the wet radome. In general, the plane wave radiated by the array face will impinge on the radome with both parallel (TM) and perpendicular (TE) polarization components relative to the plane of incidence. The plane of incidence is the plane containing the radiation direction \( \hat{k} \), and the unit vector normal to the array face, \( \hat{x}' \).

Figure C.1 shows the estimated film thickness from (C.1) as a function of rain rate for two tilt angles: one corresponding to the typical deployment (\( \theta_p = 75^\circ \)), and one corresponding to the angle when the array is stowed for transport (\( \theta_p = 6^\circ \)), in
which case the array points nearly vertically. We observe the thickness of a water film in the latter case is substantially greater than when deployed.

Figure C.2 shows the impact of the water film alone on the H and V polarization channel measurements assuming $g_{hh} = g_{vv} = 1$. That is, it may be interpreted as the contribution to $G$ due to the presence of water on the radome. Shown are the two-way transmission losses for H and V polarization, their ratio (indicative of, but not identical to, $Z_{dr}$ bias), and their differential phase. These were calculated assuming a rain rate of 1 mm hr$^{-1}$, temperature of 17°C, frequency of 9.6 GHz, and a dielectric constant for pure water of $\epsilon_w = 60.3 - j33.1$ [45]. As can be seen, the impact of a thin film of water on the radome can impart an attenuation exceeding 2 dB and varying by a dB or more with scan angle. This suggests rather substantial attenuation and differential phase as a function of rain rate. However, the assumption of a uniform laminar film tends to overestimate attenuation in many cases [43, 44], so this is a pessimistic estimate of loss.

For radomes with hydrophobic coatings, water will bead on the surface rather than forming a continuous film, and the distribution of drops on the surface will impart substantially less attenuation. Obtaining an accurate expression for the loss requires knowledge of the drop size distribution (DSD) of the rain, properties of the hydrophobic surface, the tilt angle of the radome, and the rain duration. Salazar et al [44] and Mancini et al [46] showed measured and modeled attenuations at X-band and S-band respectively. To speed calculation, they employed a mixing model to arrive at an effective dielectric constant for the layer of droplets covering the radome, where the key parameter was the overall fraction of the droplet-layer volume consisting of water. The effective dielectric constant of the droplet layer was given by the Maxwell-Garnett equation

$$\epsilon_{eff} = \frac{2\delta(\epsilon_r - 1) + \epsilon_r + 2}{2 + \epsilon_r - \delta(\epsilon_r - 1)}.$$  \hspace{1cm} (C.3)
Figure C.2. Impact of water film with scan angle on 2-way power and relative phase. Clockwise from upper left H-polarization, V-polarization, differential phase (H-V), and differential power (H/V).

where $\epsilon_r$ is the relative permittivity of water, and $\delta$ is the fractional volume of water contained in the droplet layer. The droplet layer thickness was determined by the maximum droplet size in the DSD and surface properties (drops exceeding a given size will roll off). While this fraction will vary with rain rate, DSD, rain duration, and surface properties, numerical simulations for a coated fabric tilted 15° from the vertical, as used by the Skyler prototype, show that it is generally a small fraction of the order of 10%. Shown in Figure C.3 is the resulting attenuation assuming a volume fraction of 10% and a droplet layer thickness of 5 mm on a teflon material. In this case the attenuation is only a fraction of a decibel though still significant enough to compromise differential reflectivity measurements near the scan edges. Although the layer thickness is much greater than the continuous film case, the effective dielectric constant is much lower than that of pure water. We do note that for the Maxwell-Garnett relation to be valid it is necessary that the droplet size be a small fraction of
Figure C.3. Impact of water droplets with scan angle on 2-way power and relative phase. Clockwise from upper left H-polarization, V-polarization, differential phase (H-V), and differential power (H/V).

the wavelength in the effective medium \(d \lesssim \lambda_{\text{eff}}/10\) [47]. While this size criterion is satisfied for most droplets at X-band, it is not satisfied for the largest droplets. This result is more representative of the loss properties of the Skyler prototype radome under wetting. Nonetheless, the field measurements of \(G\) may be assumed to be influenced by the presence of water on the radome.


