A survey of college board entrance examinations in elementary algebra from 1921-1941.

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A SURVEY OF COLLEGE BOARD ENTRANCE EXAMINATIONS IN ELEMENTARY ALGEBRA FROM 1921-1941

BY

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INTRODUCTION

Every applicant for admission to a college must have a high mark in algebra along with other subjects or is required to pass an entrance examination. For this reason College Board Entrance Examinations in Elementary Algebra are given at least once a year in many sections of United States and other countries.

The total registration of those taking College Board Entrance Examinations in 1940 was 23,057 of which over 2,000 were in Elementary Algebra. The number of colleges, universities, and scientific schools holding membership in the Board was 44 in 1940 but the Board examined candidates for admission to 185 educational institutions.

At one time examinations were held in many different high schools, private schools, or academies on the date appointed by the Board. At the present time centers are established in certain localities. In 1940 there were 41 such centers in Massachusetts, those in this vicinity being in Deerfield, Mount Hermon, Easthampton, Northampton, and Springfield.

Factors responsible for the Board's steady growth in recent years are the need of more and more colleges for some highly reliable and independent measure of their
candidates' abilities and attainments and the flexibility of its program. "An examination in mathematics of the kind set by the Board, is not a mere aggregate of random questions on the subject matter. It is a carefully planned structure, designated to test various phases of the candidate's reaction to the instruction that he has received. It is made up of questions of different types, serving different purposes, and there is no reason to expect that one group of questions, taken by itself, will give a test similar in character to another or to the entire examination." (1)

The writer has a personal interest in this problem, The Study of College Board Entrance Examinations in Elementary Algebra, because as a prospective teacher of mathematics this is a problem that will be met in the classroom of the high school. More and more students desire to go to college and since colleges have increased their entrance requirements more high school students are required to take college entrance examinations. In some high schools the mathematics teacher holds special classes for those students who are interested in some particular school of higher learning. It may be necessary for me to teach similar classes, therefore the writer has a desire to know what types of exercises are

(1) "The Work of the College Entrance Examination Board 1901-1925." p. 193
used most frequently, for an illustration, will the student be asked to solve an example in factoring, an equation or a problem? Furthermore a successful teacher needs to know what different types of examples to stress in preparation for coming examinations.

From the pupils' point of view this study of the College Entrance Examinations will be valuable in preparing them for their coming examination in Algebra. It will give them practice in the kinds of exercises they will ordinarily meet. It will stimulate confidence when they come up against similar writings in the examination. Finally, they can see for themselves if they are concentrating on the proper things and are putting their time where it is needed.

This study would be useful to any teacher of mathematics as the writer has included charts, tables, and graphs of all problems in mathematics in the College Board Entrance Examinations in Elementary Algebra in the last twenty years. By an inspection of the charts one can see that an example in simplifying was given eighteen years out of the twenty and might be expected to be given this year, while a true and false question was asked only twice in twenty years.

Since the College Board aims to make their examinations comprehensive tests of the student's knowledge, a study of such tests over a period of years would give
the teacher of algebra a broader view of his subject with special stress on the important phases of it. Such a study should give a clearer idea of algebra step by step - from the very simplest example to the more difficult. It also stimulates in the writer a determination to teach the fundamentals clearly from the ground up with special stress on the child's thinking the problem through to its conclusion and then bringing his intelligence to bear on the results.

After studying this problem the writer feels that she can now approach with some degree of confidence the matter of preparing students for a College Board Entrance Examination in Algebra. She will be in a better position to know just what ground to cover.
SCOPe AND PROCEDURE

SCOPe — This study is a survey of College Board Entrance Examinations in Elementary algebra to quadratics as they have been given each June during the years 1921 to 1940 inclusive. Copies of these are included in the appendix.

PROBLEM — The general procedure followed in this study was as follows:

(a) Data was collected—a copy of each examination in Mathematics Al—Algebra to quadratics as found in copies of College Board Entrance Examination books obtained at the public libraries.

(b) A list was made of the different exercises, equations, problems, simplifications, evaluations, and corresponding examples found in several elementary high school algebra texts.

(c) The examples in the College Board Entrance Examinations were classified into their general fields and checked with the titles of chapters of the algebra books. A graph was drawn to show how many times the same general example appeared each year.

(d) The general fields were subdivided into specific classes, for example, the broad field of factoring can be broken up into many different cases such
as taking out a common monomial, grouping, a trinomial that is a perfect square, difference of two squares, a cube, or by two or three cases combined. A chart has been included under each subdivision.

(e) Key words to aid the pupil in his study were found such as "check your answer", "factor completely", and "carry out to hundredths place."

(f) Finally all these special features were drawn together in a final summary regarding the outcome of the study.

Further information regarding specific details of procedure may subsequently be found in the discussion.
AIMS IN THE STUDY OF ALGEBRA

Mathematics is a kind of language -- a "divine shorthand" as one enthusiast expressed -- the most precise and abridged yet evolved, and truly international in scope. It furnishes the most accurate and adequate view of infinity to be found in any subject and is fast becoming essential to the study of economics and the calculations of modern business. Winger said, The study of mathematics fosters careful, accurate, sustained thinking, stimulating the while thinking itself. It strengthens the reason, develops the power of generalization, cultivates the imagination, and brings one face to face with chaste but naked truth."

The National Committee on Mathematical Requirements attempted to lay down some general principles which should govern the aims of mathematical instruction. These general aims were described as (a) practical, (b) disciplinary, and (c) cultural. "The primary purposes of the teaching of mathematics should be to develop those powers of understanding and of analyzing relations of quantity and of space which are necessary to an insight into and control over our environment and to an appreciation

of the progress of civilization in its various aspects, and to develop those habits of thought and of action which will make these powers effective in the life of the individual. It is further asserted that 'drill in algebraic manipulation should be limited to those processes and to the degree of complexity required for a thorough understanding of principles and for probable applications either in common life or in subsequent courses which a substantial proportion of the pupils will take.' (2)

The human mind has never invented a labor-saving machine equal to algebra. Comte said that "algebra has for its object the resolution of equation; taking this expression in its full logical meaning which signifies the transformation of implicit functions into equivalent explicit ones. In the same way arithmetic may be defined as destined to the determination of the value of functions. Thus 'Algebra is the calculus of Functions, and Arithmetic the Calculus of Values.'" (3)

Whitehead said, "That laws of algebra though suggested by arithmetic, do not depend on it. They depend entirely on the conventions by which it is stated that certain modes of grouping the symbols are to be con-


(3) Moritz, Robert Edouard, "Memorabilia Mathematica or The Philomath Quotation-Book." p. 277
sidered as identical. This assigns certain properties to the marks which form the symbols of algebra. The laws regulating the manipulation of algebraic symbols are identical with those of arithmetic. It follows that no algebraic theorem can ever contradict any result which could be arrived at by arithmetic; for the reasoning in both cases merely applies the same general laws to different classes of things. If an algebraic theorem can be interpreted in arithmetic, the corresponding arithmetical theorem is therefore true." (4)

An algebra test to be of value must contain the following:

(a) Exercises of sufficiently wide ranges of difficulty as a test to measure the varying degrees of ability.

(b) Various types of problem under each different topics.

(c) Exercises which the ordinary pupil can work.

(d) Examples which cover adequately the field of algebra and hence a fair instrument of measurement.

(e) Clear cut examples which fall definitely into particular groups, so that they serve to measure the student's different abilities. In this way the teach-

(4) Moritz, Robert Edouard, op. cit., p. 276
Reformers in the field of mathematics have come under the influence of a revision of the theory of mental discipline and a change in the conception of the cultural values of traditional curricular subjects and are thus working along rational lines. They are evolving a new type of training which is free from the rigidity of the traditional system, and which correlates the various branches of mathematics.

According to Charles A. Stone the aims of progressive makers in formulating the mathematics programs are as follows:

1. To select material which is worth while in itself and of educational value.

2. To select material which satisfies the real needs and the life and studies of the adolescent boy or girl, and must lie within their experiences. All material not adapted to their mental development and capacities is to be rejected. It should contribute to the pupils' liberal education and not be entirely vocational. It should contribute to his mental training by developing mathematical methods of thought, by teaching him to think accurately and quantitatively and by helping him to acquire effective habits of study as applied in mathematical situations.

3. To bring in fundamental elements of voca-
tional mathematics and those of future mathematical courses.

4. To arrange for tryout courses which will reveal to the pupil his interest, aptitudes, and abilities and assist him to decide whether or not he should continue to study mathematics.

5. To obtain equality of opportunity by providing relative amounts of work to take care of individual differences.

He further states that algebra should be introduced in the seventh year as a natural means of expressing facts about numbers. One cannot proceed very far in the study of mathematics without encountering the need for algebraic symbolism and formulas, for ability to solve equations and to evaluate algebraic expressions. When the child is able to appreciate the convenience of symbolic language in stating the conditions of a problem, algebra will have a significant meaning for him.” (5)

(5) Stone, Charles A., "An Approach to The Solution of the Problem Which The Traditional Mathematics Program Presents." pp. 450-1
In my study of a survey of College Board Entrance Examinations in Elementary Algebra, the following textbooks were examined:

1. "Durell's School Algebra" by Fletcher Durell,
2. "Algebra" by W. Longley and H. B. Marsh,
3. "New School Algebra" by George A. Wentworth.

These books were divided into the following general fields:

(a) Algebraic Symbols, Definitions, Notations, and Literal Expressions,

(b) Positive and Negative Numbers,

(c) Addition and Subtraction,

(d) Multiplication and Division,

(e) Factors,

(f) Common Factors and Multiplies,

(g) Fractions,

(h) Equations,

(i) Graphs,

(j) Problems,

(k) Logarithms.


DISCUSSION OF DATA

**TABLE I**

Distribution of the different General exercises in Elementary Algebra in the College Board entrance Examinations 1921-1941.

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</table>

- Evaluation
- Factoring
- Equations
- Simplification
- Problems
- Logarithms
- Graphs
- True and False Questions
- Total
(1) GENERAL FIELD -- Table I shows the distribution of the different exercises in the College Entrance Examinations as found over a period of 20 years. The divisions are:

(a) Evaluation or find the value of,
(b) Factoring,
(c) Equations,
(d) Simplifications,
(e) Problems,
(f) Logarithms,
(g) Graphs,
(h) True and false questions.

The total is given for each group. From the table one can see that at least one example in factoring was given every year and some years as many as five different exercises in factoring were given.

GRAPHS 1, 2, 3, 4, 5, 6, and 7 represent the same facts as Table I but in different way.

TABLE II

Distribution of Evaluation in the College Board Entrance Examinations 1921-1941.

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</table>
GRAPH 1. Showing the Distribution of Evaluation in the College Board Entrance Examinations 1921-1941.

GRAPH 2. Showing the Distribution of Factoring in the College Board Entrance Examinations 1921-1941.
GRAPH 3. Showing the Distribution of Equations in the College Board Entrance Examinations 1921-1941.

GRAPH 4. Showing the Distribution of Simplification in the College Board Entrance Examinations 1921-1941.
GRAPH 5. Showing the Distribution of Problems in the College Board Entrance Examinations 1921-1941.


GRAPH 7. Showing the Distribution of Graphs in the College Board Entrance Examinations 1921-1941.
(2) EVALUATION; find the value of, or examples on numbers -- Table II shows in more details the distribution of the exercises in the College Board Entrance Examinations which have been placed under this group evaluation.

(a) Examples dealing with radicals, e.g.
\[ \sqrt{32} + \sqrt{\frac{1}{4}} + \sqrt{2/9} \]

(b) Given the formula or examples and substitute values for the unknown letters, e.g. given the formula
\[ s = \frac{n}{2} \left[ 2a - (n - 1) d \right] \]
to find \( d \) when
\[ s = 15 \frac{1}{2}, \quad a = 1/3, \quad \text{and} \quad n = 6. \]
Another example if
\[ a = \frac{3}{10} \quad \text{and} \quad b = 0.4, \]
what is the value of \( 8a^2 - 10ab \)?

(c) Substitute in radicals, e.g. if \( x = 4 \), find the value of
\[ 2 \sqrt{9/4} - \frac{3}{\sqrt{-2}} / x \]

(d) Check your answers or results after simplifying, substitute knowns for the unknowns and also in the original expression.

**TABLE III**

Distribution of the Field Radicals Under Evaluation.

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Table III shows subdividing the general field radicals into still minor groups as:

(a) Whole numbers and fractions, e.g. reduce the following expression to its simplest form
\[ \frac{\sqrt[4]{5}}{3} + \sqrt[4]{45} - 3\sqrt{3.2} \]

(b) Rationalize the denominator and find its value to the nearest hundredth:
\[ \frac{8 + 3\sqrt{7}}{3 + \sqrt{7}} \]

(c) Simplify and find to the nearest tenth or hundredth, e.g. \(6\sqrt{3/2} - 24 + 3\sqrt{2/3}\).

(d) Multiplying of parenthesis with numbers unradical or compound radicals, e.g.
\( (3\sqrt{2} - \sqrt{3})(\sqrt{2} - 3\sqrt{3}) \).

(3) Factoring — Table IV shows the distribution of the general field factoring into its subdivisions as found in the College Board Entrance Examinations from 1921 to 1940 inclusive. Factoring is one of the basic examples of algebra, for without knowing how to factor a student can not advance very far in mathematics. Many problems are solved quickly by being able to factor. Factoring is an algebraic expression for finding two or more terms which when multiplied together will give the original expression as the product. Not all polynomials are factors as a prime expression.

The factoring of a polynomial may be divided into a number of different cases, some of which are listed below:
TABLE IV
Distribution of the General Field Factoring

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Factoring included in other problems: | | | | | | | | | | | | | | | | | | | | | | |
Case I — When the terms have a common monomial factor, e.g. 4ax - x².

Case II — When the terms of the polynomial can be grouped to show a common compound, e.g. x³ - x² - x + 1, or (x - 4)² - 5(x - 4).

Case II (a) — After changing the sign proceeding a parenthesis, e.g. 3a(x - 2y) - 4b(2y - x).

Case III — When a trinomial is a perfect square, e.g. 4a² - 4ab + b².

Case IV Part I — When a binomial is the difference of two perfect squares, e.g. x² - a².

Case IV Part II — When one or both squares are compound, e.g. (x - y)² - z².

Case IV Part III — When by properly grouping the expression may be written as the difference of two squares, e.g. 2ab - b² - a² + c².

Case IV Part IV — When a trinomial has the form of x⁴ + x²y² + y⁴, e.g. a⁴ - 13a² + 56.

Case V — When a trinomial has the form x + ax + b, e.g. a² - 2a - 24, and 20 - x - x².

Case VI — When a trinomial has the form of ax + bx + c, e.g. 2x² - xy - 3y² and 6x² - 7xy - 20y².

Case VII — When a binomial is the sum or differences of two cubes, e.g. x³ + 81, or x⁶ - 27y³.

Examples with two or more cases combined, as Case I and Case III, e.g. 2ax² - 4axy + 2ay².
(4) **LINEAR EQUATION** -- An equation is a statement of the equality of two algebraic expressions. A linear equation is one which, when reduced to its simplest form, contains only the first power of the unknown letter or letters.

**TABLE V**

Distribution of linear equations in the College Board entrance Examinations in Elementary Algebra 1921-1941.

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Table V shows the distribution of linear equations into the four major fields:

(a) Simple numerical,
(b) Fractional,
(c) Simultaneous,
(d) Radicals used in equation, e.g.

\[ 1 + \sqrt{x^2 - 5} = x. \]

Table VI shows the distribution of the exercises classed under simple numerical equations:

(a) Simple, e.g. 3x = 6.
(b) Decimals, e.g. 0.2a = 6.
(c) Literal equation, one in which some or all of the known quantities are represented by letters, generally the first of the alphabet, e.g. \(4x - 3(x - 4a) = x - 2(x - 2a)\).

**TABLE VI**

Distribution of Simple Numerical Linear Equations in the College Board Entrance Examinations in Elementary Algebra 1921-1941.

| 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 | 31 | 32 | 33 | 34 | 35 | 36 | 37 | 38 | 39 | 40 |
|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|
| Simple Decimals | | | | | | | | | | | | | | | | | | | |
| Literal | | | | | | | | | | | | | | | | | | | |

**TABLE VII**

Distribution of Fractional Linear Equations in the College Board Entrance Examinations in Elementary Algebra 1921-1941.

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Table VII shows the distribution of the field of fractional equations, ones in which an equation contains fractions. Groups are:

(a) Constant denominators, e.g.
\[ \frac{7x - 5}{6} = \frac{5(x + 1)}{4} + \frac{1}{3} = \frac{x - 7}{2} \]

(b) The unknown in the denominator, e.g.
\[ \frac{x - 3}{x - 2} = \frac{x - 7}{x - 3} \]

(c) Unknown in the denominator but necessary to change a set of signs before finding the least common denominator, e.g.
\[ \frac{4}{x + 1} + \frac{x}{1 - x} = \frac{1}{x - 1} + \frac{x^2 - 3}{1 - x^2} \]

(d) Literal fractional equations, e.g.
solve the following equation for r: \( d^3 = \frac{5r + 1}{m} \).

TABLE VIII

Distribution of Simultaneous Linear Equations in the College Board Entrance Examinations 1921-1941.

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Table VIII shows the distribution in the field of simultaneous equations over the 20 years of the College Board Entrance Examinations in Elementary Algebra. Simultaneous equations are equations which are a set or system of equations in which more than one unknown quantity is used, and the same symbols stand for the same unknown. The minor groups are:

(a) Simple, e.g. \[ 2x - y = 4, \quad 2x + 3y = 12. \]

(b) Decimals and fractions, e.g.
\[ x - \frac{y - 2}{3} = 0, \]
\[ 0.2x + 0.3y = 5. \]

(c) Literal, e.g. \[ x + y = a + b, \]
\[ \frac{x - a}{b} = \frac{y - b}{a}. \]

(5) SIMPLIFICATION — The reducing of an algebraic expression from the complex to the simple.

Table IX shows the distribution of the field simplification into the following groups:

(a) Two or three types in one, e.g.
\[ 1 - \frac{1}{1 + a} = a. \]

(b) Radical, fractional exponents, and negatives, e.g. Free the following expression from negative and fractional exponents, and from radicals: \( \left( \sqrt[3]{5} \ x^{\frac{7}{4}} y^{-\frac{5}{3}} \right)^{2} \), Reduce to simplest form: \[ \frac{a^{\frac{3}{4}} b^{\frac{1}{2}}}{b^{\frac{1}{2}} a}. \]

(c) Simple multiplication, find the product, and combine terms, e.g. 1a) \[ 4x^{2} - x (2 - x), \]
2^a) Multiply \((4a^2 - a^3x)(-2a^3x^{r-1})\), 2^a) find the product of \((1 - a)(a - 1)\).

**Table II**

Distribution of Simplification in the College Board entrance examinations in elementary algebra 1921-1941.

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(d) Multiplying squares, two terms, and a minus proceeding a parenthesis, e.g. \((a - b)^2 - 2(a^2 - ab - b^2)\).

(e) Change a set of signs, combine, and cancel, e.g.

\[
\frac{6a}{4 - 5a} + \frac{8}{5a - 4}.
\]

Table I shows the distribution of the exercises which have been placed under the field of two or three types in one example:
### TABLE I

**Distribution on Exercises of two or three types of simplification in the College Board Entrance Examinations in Elementary Algebra 1921-1941.**

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<td>Minus preceding a parenthesis</td>
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<td>Cancellation of terms</td>
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<td>Change signs and continue</td>
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(a) Find the least common denominator, multiply, and combine, e.g. \( \frac{a}{m} + \frac{m}{a-m} \times \frac{m-a}{a^2+m^2} \).

(b) Division (invert the divisor), e.g. \( \frac{b}{a} (a+b) \left( \frac{x}{a} - \frac{x}{b} \right) \div \left( \frac{a}{u} - \frac{b}{u} \right) \).

(c) Factoring in the exercise, e.g. \( \frac{1/5 + 1/2a}{4a - 25/a} \).

(d) Minus preceding a parenthesis, e.g. \( \frac{a^2}{a+b} - \frac{b^2}{a-b} - (a-b) \).
(e) Cancellation of terms.
(f) Change signs and continue.

(6) **PROBLEMS** -- Some good suggestions for solving a problem are first read the problem carefully so that you will understand just what it means. Then determine what quantity you will represent by an unknown letter. As a rule it is best to let the smallest or simplest quantity be represented by the unknown letter. Determine what quantities are equal to each other and state this fact in the form of an equation. Solve the equation, and then check the results in the statement of the problem.

Table XI shows the distribution of the different kinds of problems into the following groups:

(a) Mixture per cent problems -- e.g. A dealer has two kinds of tea worth 60 cents and 70 cents a pound, respectively. How many pounds of each must be taken to make 130 pounds worth 66 cents a pound?

(b) Ratio problems -- e.g. The ages of two sisters are in the ratio 5/3. Four years from now their ages will be in the ratio 7/5. Find their ages now.

(c) Literal problems -- e.g. A man walks $d$ miles to a certain place at the rate of $m$ miles an hour. He returns over the same road at the rate of $(m + 1)$ miles an hour. Express the total time which he has walked. What has been his average rate?
Table II

<table>
<thead>
<tr>
<th>Table XI</th>
<th>Distribution of problems in the College Board Entrance Examinations in Elementary Algebra 1921-1941.</th>
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<tbody>
<tr>
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<td>21 22 23 24 26 27 28 29 30 31 32 33 34 35 36 37 38 39 40</td>
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<tr>
<td>Mixture percent</td>
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<td>Ratio</td>
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<td>Time, rate, distance</td>
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<td>Investment, interest, or salary</td>
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<tr>
<td>Number, coins and pupils</td>
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<td>Time, working together</td>
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</table>

(d) Rate, time, distance travel problems -- e.g.

Four hours after a battle cruiser left port, two airplanes started in pursuit. The first plane overtook the cruiser in 1 one hour. The second plane, flying with a speed 25 miles an hour less than the first, overtook the cruiser in 1 hour and 20 minutes. Find the speed of the cruiser.

(e) Investment, interest, or salary problems -- e.g.

A certain sum of money was loaned at simple in-
terest. At the end of 10 years the total interest was $540 less than the amount of the loan. At the end of 20 years the total interest was $120 greater than the amount of the loan. Find the rate of interest.

(f) Number problems, coin, and pupils in a classroom problems — e.g. 1a) Members of the Athletic Association paid 15 cents admission to a contest, and non-members paid 25 cents admission. There were 278 paid admissions and the total receipts were $49.90. How many non-members attended the contest? 2a) There were two-thirds as many women as there were men on a train. At the next station 6 men and 8 women got off the train and 9 men and 2 women got on the train. How many men and how many women were on the train at first?

(g) Time problems — all working together — e.g.
A coal company can fill a certain order from one mine in 3 weeks and from a second mine in 5 weeks. How many weeks would be required to fill the order if both mines are used?

Table XII shows the distribution of problems classified under the field of literal problems:

(a) Cost, interest, and money problems — e.g.
The superintendent of a co-operative society wishes to sell a number of goods costing $a$ dollars at such a price that, after deducting $r$ per cent of the selling price the society will receive the cost price. How much should he charge?
TABLE XII

Distribution of literal problems in the College Board Entrance Examinations in Elementary Algebra 1921-1941.

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(b) Time to do a piece of work problem -- e.g. If a man can do a piece of work in \(d\) days, what part of the work can he do in \(n\) days?

(c) Time, rate, distance problems -- e.g. If a man travels \(d\) miles in \(s\) hours, how far can he go in \(t\) hours.

(7) LOGARITHMS -- Logarithms are exponents. The logarithms of any number referred to a given number as a base is the power to which the given base must be raised to produce the given number. Any positive number except one may be taken as the base. The base commonly used is ten and all numbers are expressed either accurately or approximately as powers of ten.

Table XIII shows the distribution of the problems dealing with logarithms and degrees found on the
College Board Entrance Examinations in Elementary Algebra over the twenty years.

**TABLE XIII**

Distribution of Degree and Logarithms Problems in the College Board Entrance Examinations in Elementary Algebra 1921-1941.

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(a) Right triangle problems -- e.g. 1

1. Find the angles of a right triangle if its hypotenuse is 20 inches long and one side is 8 inches long.
2. In the right triangle ABC, \( \angle C = 90^\circ \), \( \angle A = 40^\circ 25' \), \( b = 12 \). Find the value of \( a \) and \( c \).

(b) Angle of elevation problems -- e.g. 1

1. What is the height of a tree if its shadow is 80 feet long when the angle of elevation of the sun is 32°?
2. What is the angle of elevation of the sun when a vertical flagpole casts a shadow two-thirds of its own length?

(c) Examples to find from the tables on sin or cos -- e.g. \( \cos 57^\circ 32' \)?

(8) KEY WORDS to aid the pupil in his study were found:
(a) Free the following expression from negative and fractional exponents, and from radicals.

(b) Derive a formula.

(c) Find the value to the nearest tenth or nearest hundredth.

(d) Reduce to simplest form.

(e) Rationalize the denominator, e.g. \( \frac{3}{5 + 3\sqrt{7}} \).

(f) Check by substituting known for the unknowns in the original expression and then in your answer, e.g.

Simplify: \( \frac{2x^2 - 18}{x^2 + 4x - 21} - \frac{x + \frac{1}{3}}{x - \frac{1}{3}} \left( 3 - \frac{x + 27}{x + 7} \right) \).

Check by letting \( x = 5 \) in the original expression and in your result.

(g) Factor completely -- e.g. \( 15a^2 - 2a^2x^2 - a^2x^4 \).
CONCLUSION

After examining the College Board Entrance examinations, one finds the following common exercises to be stressed in preparing one for coming examinations in Elementary Algebra:

(a) Evaluation.
   (1) Radicals.
   (2) Substitute in formula and example values for the unknown.

(b) Factoring.
   (1) Case I -- a common term.
   (2) Case IV Part I -- binomial the difference of two perfect squares.
   (3) Case V -- trinomial the form of $x^2 + ax + b$.

(c) Linear equations.
   (1) Fractional equations.
   (2) Simultaneous linear equations.

(d) Simplification.
   (1) Find the least common denominator.
   (2) Factoring.
   (3) Cancellation.

(e) Problems.
   (1) Literal.
(2) Time, rate, and distance.

(3) Number problems.

(f) Logarithms, exercises dealing with angles, and problems including right triangles were not given previous to the year 1924, but one of each has been included every year since.

(g) Every year after 1926 there was an exercise on graphs. The student was usually required to plot two equations on the same axes and find the point where they cross.

(h) There seems to be a trend to give many more exercises than formerly and to cover a wider range.

(i) The later tests include more interesting problems dealing with such concrete facts as the airplane and the modern factory bringing material up-to-date.

The following common exercises need only to be normally covered in reviewing for examinations.

(a) Evaluation.

(1) Substitute in radicals.

(2) Rationalize the denominator.

(3) Find to the nearest tenth or hundredth.

(b) Factoring.

(1) Case III -- trinomial a perfect square.
(b) Case IV Part II -- one or both squares are compound.

(c) Case VII -- binomial has the sum and or differences of two cubes.

(c) Simple numerical linear equations.

(d) Problems.

(1) Ratio.

(e) A true and false question appeared on the examination in 1925 and twice in 1933.
APPLICATION

This study was an attempt to make a survey of College Board Entrance Examinations in Elementary Algebra from 1921 to 1940 inclusive. Such a study will aid the following:

(a) The writer, who can use the results of the study in her classroom.
(b) The pupil, who is required to take College Board Entrance Examinations in Algebra.
(c) Any teacher of mathematics, who is preparing students for College Board Examinations.
(d) Those who find it necessary to make out similar tests.

The writer has included detailed graphs and tables of the general fields and subdivisions of the major fields. Copies of the twenty examinations have been included in the appendix.

It is the hope of the author that the tables and graphs, and conclusions developed in this study may also be of help to others.
1. Factor:
   (a) $20 - x - x^2$,
   (b) $3x^3 - 11$,
   (c) $a^2 - 13a^2 + 36$.

2. Solve the simultaneous equations:
   \begin{align*}
   2x - 5y - 10 &= 0, \\
   9x + 5y - 14.5 &= 0.
   \end{align*}

3. Simplify:
   \[
   \left( m + \frac{p}{m - p} \right) \div \left( m - \frac{(m - p) p^2}{m^2 - p^2} \right)
   \]

4. Free the following expression from negative and fractional exponents, and from radicals:
   \[
   \left( \sqrt[5]{5} \times \frac{1}{2} \times \sqrt[3]{\frac{2}{3}} \right)^{1/2}
   \]

5. The federal income tax on incomes between $12,000 and $14,000 was in 1919 as follows: first, a uniform tax of $190 on all such incomes; secondly, an additional tax of 5 per cent on the excess of such an income over $12,000. Write down a formula expressing the total tax, $T$, which a man must pay, whose income, $x$, lay between the foregoing limits.

6. A workman, wishing to explode a blast of powder, set the fuse to cause the explosion to take place in 30 seconds. He ran back at the rate of eight yards per second. How far had he run when he heard the explosion, if sound travels at the rate of 1,080 feet per second.
1. Factor:
   (a) \(2x^2 - 12x + 18\),
   (b) \(a^2 + 12a - 49 + 36\),
   (c) \(x^3 - x^2 - x + 1\).

2. Simplify:
   (a) \(\frac{a}{x} + \frac{a}{n} \div \frac{a - \frac{a}{n}}{a + \frac{a}{n}}\)
   (b) \(\sqrt{32} + \sqrt{\frac{1}{3}} + \sqrt{\frac{2}{9}}\).

3. Solve the following equation for \(x\):
   \[\frac{2x - 3}{a + x} - \frac{a - 3}{x} = 0\]

4. Given the formulas:
   \[L = ar^{n-1}\]
   \[S = \frac{rL}{r - 1}\]
   eliminate \(a\) and derive a formula for \(L\).

5. If I lend a certain sum of money for a certain time at \(6\%\) (at simple interest), the interest exceeds the loan by \$160; but if I lend it at \(4\%\) for one-half that time, the loan exceeds the interest by \$480. What is the sum?

6. A passenger boat makes the trip from New York to Boston by way of the Cape Cod Canal, which is 11 miles long, in 15 and one-third hours. A freighter, makes the same trip in 39 hours. The average rate of the freighter, when both boats are outside the canal, is half that of the passenger boat; and its average rate, when both boats are inside the canal, is two-thirds that of the passenger boat. How far is it from New York to Boston by way of the canal?
Monday, June 19
9:30 a.m. 2 hours

1. Factor:
   (a) $3a^2 - 30ab + 75b^2$
   (b) $61x^4 - 49y^4$
   (c) $4x^2 + 12x + 9 - x^2$

2. Given the formula:
   $$S = \frac{n}{2} \left[ 2a - (n-1)d \right]$$
   a) Find the term $d$ in terms of the other letters;
   b) Find $d$ when $s = 15 \frac{1}{2}$, $a = \frac{1}{3}$, $n = 6$.

3. Simplify:
   $$\frac{b}{a} \left( a + b \right) \left( \frac{x}{a} - \frac{x}{b} \right) + \left( a - \frac{b^2}{a} \right)$$

4. Solve the simultaneous equations:
   $$\begin{align*}
x - \frac{y - 2}{3} &= 0 \\
0.2x + 0.3y &= 5
\end{align*}$$

5. Find to the nearest hundredth, the value of:
   $$\sqrt{189} - 8 \frac{2}{3} - 12 \sqrt{7/3}$$

6. A recent advertisement of the telephone company makes the following statement: "The United States, with only one-sixteenth of the population of the world, has two-thirds of the world's telephones." Find the ratio of the number of telephone owners out of every thousand persons in the United States to the number of telephone owners out of every thousand persons in the rest of the world.
Old and New Requirements

Sunday, June 16 9:30 a.m. 2 hours

1. Factor:
   (a) \(4x^2 - 0.25\),
   (b) \(6x^2 - 24x - 24\),
   (c) \(3n(x - 2y) - 4b(2y - x)\).

2. Simplify:
   \[\frac{2ax}{x^2 - a^2} + \frac{3n}{a + x} + \frac{m - n}{a - x} \times \frac{a^2 - b^2}{2n}\]

3. Reduce to simplest form:
   (a) \(\frac{-\frac{3}{2} + \frac{1}{2}}{b - \frac{1}{2}} \div \frac{1}{\sqrt{2}}\)

4. Old Requirement:
   The superintendent of a co-operative society wishes to sell to a number goods costing \(a\) dollars at such a price that after deducting \(\frac{x}{r}\) per cent of the selling price, the society will receive the cost price. How much should be charged?

5. New Requirement:
   (a) The height of the ridge pole of a house is 8 feet above the eaves, and the distance between the eaves is 50 feet. What angle does the roof make with the horizontal?
   (b) The hypotenuse of a right triangle is 13 feet, and one of the angles is 30° 43′ (30.73). Find the lengths of the longer of the other sides.

6. Solve for \(x\) and \(y\):
   \[x + y = a + b, \quad b - \frac{y - a}{x - b} = y - b\]

7. The distance from \(A\) to \(B\) is 100 miles. A train, going from \(A\) to \(B\), meets with an accident 25 miles from \(B\), and its speed for the rest of the trip is thus reduced by one-half. It arrives at \(B\) an hour late. What is its usual rate from \(A\) to \(B\)? Show how you derive your answer.
Mathematics 10—Algebra to Generalize

Old and New Requirements

Monday, June 15

9:30 a.m.; 2 hours

1. a) What are the factors of \( 21 - ax - a^2 \)?
   b) What factors of \( x^6 - 27y^3 \)? One factor: \( x^4 - 10x^2y \)?
   c) Simplify: \( 3x^2 - x( -6 + 2x) - x \).

2. Copy the statements below and write after each one "yes" if the statement is true, and "no" if it is false.
   a) \( \frac{a}{b} + \frac{c}{d} = \frac{a + c}{b + d} \)
   b) \( \sqrt{a/y} = \frac{\sqrt{a}}{\sqrt{y}} \)
   c) If \( y = \frac{2 + \sqrt{3}}{3} \), circle smaller or a given larger.
   d) \( (a + b)^2 = a^2 + b^2 \).

3. Simplify:
   a) \((4x^2 + x^3)( -2x + x^2)^{-1}\).
   b) \((x^3y\frac{1}{2})^{-\frac{1}{3}}\)
   c) \(\frac{\sqrt{3}/5}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}\)

   d) Rationalize the denominator of \(\frac{3 + 3\sqrt{7}}{3 + \sqrt{7}}\)

   Now: c) If \( a = 1 \) and \( b = 0.4 \), what is the value of \( 3a^2 - 10ab \)?
   d) Given \( \sqrt{2} = 1.414 \), find the value of \( \sqrt{50} \).

4. Old: Solve for \( x \) and \( y \): \( \frac{2}{3}x - \frac{3}{4}y = 10 \)
   \( 0.6x + 0.7y = -2 \).

   Now: a) Given the formula \( s = \frac{1}{2}vt \), find \( t \) in terms of \( v \) and \( s \). Substitute this value of \( t \) into the formula \( a = \frac{1}{2}at^2 \), and hence derive a formula for \( a \) in terms of \( v \) and \( s \).
5. \text{Solve for } x: \quad \frac{x - 3}{6} - \frac{2x - 1}{12} - \frac{3 - 10}{x - 3} = 0

Find a 10\text{ ft} ladder leaning against a stone wall just reaching the top of the wall and making an angle of 64^\circ \text{ with the level ground. How wide is the wall?}

6. There were two-thirds as many women as there were men on the train. At the next station 6 men and 8 women got off the train, and 9 men and 2 women got on. There were then twice as many men as women on the train. How many men and how many women were on the train at first?
1. a) Simplify \(4x^2 - x(2 - x)\).
   
b) If \(2 + r = \frac{1}{2} r\), what does \(r\) equal?
   
c) Simplify: \((3 a^{-1} b^{2} c^{-3}) (-2 a^{4} b^{3} c^{2})\).
   
d) Factor: \(4x^2 - 16 - 12xy + 9y^2\).

2. Simplify: \(\frac{2x^2 - 18}{x^2 + 4x - 21} - \frac{x - \frac{1}{2}}{x - 3}\left(3 - \frac{x + 27}{x + 7}\right)\)

   Check by letting \(x = 5\) in the original expression and in your result.

3. a) Simplify \(6\sqrt{3/2} - \sqrt{24} + 3\sqrt{3/3}\) and find the value of the result to the nearest hundredth.
   
b) In the right triangle \(ABC\), \(C = 90^\circ\), \(A = 40^\circ 25'\), \(b = 12\). Find the value of \(a\) and \(c\).

4. It is given that \(C = \frac{E}{R + \frac{P}{n}}\)
   
a) What is the effect of \(C\) if \(R\) increases?
   
b) What is the effect of \(C\) if \(n\) increases?
   
c) Find \(r\) in terms of the other letters.
   
d) Find \(r\) when \(E = 18.3\), \(C = 6\), \(R = 3\), and \(n = 4\).

5. If in the formula \(s = vt - \frac{1}{2}at^2\), it is known that \(v = 6\) and \(a = 2\), plot the graph of the resulting equation for values of \(t\) from \(t = 0\) to \(t = 6\).

6. On an algebra test 39 more pupils passed than failed. On the next test, 7 who had passed the first test failed, while one-third of those who failed the first test passed the second. As a result, 31 more passed the second test than failed it. What was the record of passing and failing on the first test?
1. a) If \( a = 2 \), find the value of \( \frac{2}{3} (a + 4) - \frac{5a}{3} \).

b) Simplify: \( \frac{a}{a-b} + \frac{b}{b-c} \).

c) Factor: \( a^4 - b^2/16 \).

d) Factor: \( x^2y - 5xy + 14y \).

2. Simplify: \( \frac{a^2}{a+b} - \frac{b^2}{a-b} - (a-b) \).

Check your result by using the values \( a = 3, \ b = 2 \).

3. Given the formulas \( v^2 = 2gh \) and \( v = gt \), eliminate \( v \) and solve the resulting equation for \( t \). Take \( g = 32 \) and find the value of \( t \) when \( h = 100 \).

4. Plot the graph of \( y = x^3 - 6x^2 + 9x + 1 \), using for \( x \) only the values 0, 1, 2, 3, 4. From the graph estimate the values of \( x \) when \( y = 2 \).

5. a) Find the angles of a right triangle if its hypotenuse is 20 inches long and one side is 8 inches long.

b) If \( x = 4 \), find the value of \( 2 \sqrt{\frac{1}{4} - \sqrt{\frac{3}{x}}} \).

6. A, B, and C have equal incomes from investments. A has invested \( \$2,000 \) less than B and \( \$2,500 \) more than C. The rate of interest received by A is 1 per cent more than that received by B, and 2 per cent less than that received by C. How much has A invested?
Mathematics \(\text{A1}--\text{Algebra to Quadratics}\)

Monday, June 18

9:30 a.m. 2 hours

1. a) Factor: \(a^4 - (5a + 6)^2\).

b) Simplify: \(\frac{1/5 + 1/2a}{4n - 25/a}\).

c) Simplify: \(9x^3 y^2 z \sqrt{x^6} = y^4 z^2\).

2. Solve \(\frac{4}{x + 1} + \frac{x}{1 - x} = \frac{1}{x - 1} + \frac{x^2 - 3}{1 - x^2}\).

3. a) Simplify: \(2 \sqrt{5/3} - \sqrt{60}\).

Find the value of the result to the nearest tenth.

b) A man walked 1,000 yards up a slope which makes an angle of \(20^\circ 15'\) with the horizontal plane. How high was he then above the horizontal plane from which he started?

4. A dealer has two kinds of tea worth 60 cents and 70 cents a pound, respectively. How many pounds of each must be taken to make a mixture of 130 pounds worth 66 cents a pound?

5. Plot the graph of the equation \(y = 2x^2 - 12x + 20\) for values of \(x\) from 0 to 6 inclusive, and estimate from your graph the values of \(x\) for which \(y = 6\frac{3}{5}\).

6. A boat starts in 12 minutes and the landing is 1 mile away.
If you walk 4 miles an hour and you run 8 miles an hour, how many minutes may you walk and how many must you run in order to reach the landing just in time to board the boat?
Mathematics Al--Algebra to Quadratics

Monday, June 17

9:30 a.m. 2 hours

Part I  (counts 40%)

1. Factor:  \( x^2 - x - 6 \).

2. Factor:  \( x^3 - ax \).

3. Factor:  \( (x + 4)^2 - 5(x + 4) \).

4. Solve the simultaneous equations:  \( x + y = 16 \),  
   \( x - y = 4 \).

5. If \( a = \frac{1}{2} \), find the value of \( (4 + \frac{1}{2}) - (8a - 3) \).

6. Simplify:  \( x(x - 2) + x(3 - x) \).

7. Simplify:  \( \frac{2x}{x - y} + \frac{2y}{y - x} \).

8. Simplify:  \( \sqrt{2}/3 + \sqrt{2}/2 \).

9. Simplify:  \( \frac{1}{x + 1} - \frac{3}{x} \).

10. Solve the equation:  \( \frac{x - 1}{3} = \frac{2x + 3}{4} \).

11. If the cost of setting the type for leaflet is $12 and the
cost of paper and printing is 4 cents per copy, write a formula
which will give in dollars the total cost, \( T \), of \( n \) copies.

12. The hypotenuse of a right triangle is 10 inches long and one
angle is 40°. Find the length of the side opposite the given angle.

Part II  (counts 60%)

13. A coal company can fill a certain order from one mine in 3
weeks and from a second mine in 5 weeks. How many weeks would be
required to fill the order if both mines are used?

14. Temperatures from noon until midnight were recorded as follows:
12 noon  2 p.m.  4 p.m.  6 p.m.  8 p.m.  10 p.m.  12 p.m.

Represent the data by a graph, from the graph estimate the
temperature at 5 p.m.
15. Solve the equation: \[ \frac{1}{x+1} - \frac{1}{1-x} = \frac{4}{2x+1}. \]

16. A man had $15,000. He invested a part of his money in bonds paying a fixed rate of interest, and the remainder in a business enterprise. During the first year the business paid interest at a rate which was 1% less than that of the bonds, and the man's total income was $810. During the second year the business paid interest at a rate which was 1% more than that of the bonds, and total incomes was $990. How much money was invested in bonds?
Mathematics A1—Algebra to Quadratics

Monday, June 16

9:30 a.m. 2 hours

Part I

(This part of the examination will count 40 per cent. No credit will be given in Part I for answers which are only partially correct.)

1. Simplify $(x - y)^2 - (x - 3y)(x + y) - 4y^2$.

2. If $mx - y = -b$, find the numerical value of $y$ when $x = 1/7$, $b = -5$, $m = 49$.

3. Factor $7a^4b^2 - 7a^2b^2$.

4. Factor $x^2 + x - 2$.

5. Factor $x^2(x - 3) + 3 - x$.

6. Simplify $\frac{2a + b}{a + b} - 1$.

7. If $a = 3$, $b = 2$, $c = 32$, find the value of $\sqrt{\frac{a^2b^2}{c}}$.

8. If $y = kx$ where $k$ is constant, and $y = 2$ when $x = 3$, find $y$ when $x = 4$.

9. Solve the equation $1 + \sqrt{x^2 - 5} = x$.

10. Solve the simultaneous equations $2x - y = 4$, $2x + 3y = 12$.

11. If a man can do a piece of work in $d$ days, what part of the work can he do in $n$ day?

12. A ladder 30 feet long leaning against a vertical wall makes an angle of $20^\circ$ with the wall. How far is the foot of the ladder from the wall?

Part II (counts 60%)

13. A train running between 2 towns arrives at its destination 10 minutes late when it runs 48 miles per hour and 16 minutes late when it runs 45 miles per hour. Find the distance between the towns and the schedule time of the journey.
14. a) Plot the graph of the equation \( y = 5x - x^2 \).

b) From the graph estimate the value of \( x \) for which \( y = 5 \).

c) From the graph state the value of \( x \) for which \( y \) is greater than 6.

15. Solve the equation \[ \frac{x + 6}{x - 3} + \frac{x}{5} = \frac{x(x + 3)}{5(x - 3)} - \frac{3}{3 - x} \].

16. A librarian saved one-third of his salary for each of two years and took a year off at half-pay. At the end of the third year he had spent all of his money, including the interest for 1 year at 5% on the savings of the first year, and had a debt of £450. What was his salary in this third year if he spent twice as much as in each of the two preceding years.
Part I

(This part of the examination will count 40 per cent. No credit will be given in Part I for answers which are only partially correct. Candidates who at the end of the first hour have not completed Part I should proceed to Part II, returning to Part I later if time permits.)

1. Simplify \((a - b)^2 - 2(a^2 - ab - b^2)\).

2. Factor \(k^2 - 2k - 48\).

3. If a man travels \(d\) miles in \(a\) hours, how far can he go in \(t\) hours?

4. Simplify \(\frac{a - \frac{a}{b}}{1 - \frac{1}{b}}\).

5. Factor \(61a^m - a^m\).

6. In the formula \(v = \frac{1}{3} \pi r^2 h\), find \(h\) when \(v = 96.2\), \(r = 3\), \(\pi = 3.14\).

7. Simplify \(\frac{a + 2b}{a - b} + \frac{2a + b}{b - a}\).

8. Solve the simultaneous equations: \(x + y = 8\), \(2x - y = 7\).

9. Simplify \(\sqrt{1 - \left(\frac{1}{3}\right)^2} - \frac{2}{3} \sqrt{3}\).

10. Find the value of \((3 - \sqrt{5})^2 + \sqrt{180}\).

11. Solve the equation \(5 - 3/4 (3 - y) = 0\).

12. A alone can shovel a sidewalk in 30 minutes, and B alone in 20 minutes. In how many minutes can both together shovel it?

Part II

(This part of the examination will count 60 per cent.)

13. a) Plot the graph of \(y = x^2 - 5\).

b) Plot the graph of \(2x - 3y + 1 = 0\), using the same axes as in (a).

c) Estimate from the figure the values of \(x\) and \(y\) which satisfy both equations.
14. a) Given the right triangle ABC, with the right angle at C, AB = 70 feet, and angle A = 66° 22'. Find the length of the side BC.

b) What is the angle of elevation of the sun when a vertical flagpole casts a shadow two-thirds of its own length?

15. When $7300 is invested, part of it at 5 per cent and the remainder at 6 per cent, the yearly income is $34 greater than if it had all been invested at 5 per cent. How much is invested at each rate?

16. A man can row 11 miles downstream in the time it takes him to row 7 miles against the stream. He rows downstream for 3 hours, then turns and rows back for 3 hours, but finds that he is still 5 miles from his starting-place. How fast does the stream flow? What is the man's rate in still water?
Part I

(This part of the examination will count 40 per cent. No credit will be given in Part I for answers which are only partially correct.)

1. Factor \( a^3 + 5a^2 - 6a \).

2. Factor \( 2x(1-x) + 3(x-1) \).

3. Simplify \( 1 - \frac{1}{1 + a} - a \).

4. Simplify \( \frac{2x}{4 - x^2} + \frac{2}{x + 2} \).

5. If \( n \) pounds of tea cost \( x \) dimes, how many cents does one pound cost?

6. A ladder 20 feet long, leaning against a wall, touches the wall at a point 16.5 feet above the ground. What is the angle between the ladder and the wall?

7. If \( L = a + (n - 1)d \), find the value of \( L \) when \( a = 3 \) and \( 1/6 \), \( n = 48 \), \( d = -1/3 \).

8. Solve the following equation for \( r \): \( d^3 = \frac{5r + 1}{m} \).

9. Simplify \( 2(a - b)^2 - 3(a + b)(a - b) \).

10. If \( y \) varies inversely as \( x \), and \( y = 2 \) when \( x = \frac{1}{2} \), find \( x \) when \( y = 1/2 \).

11. Simplify \( \frac{2 + x + 2}{2x - 1} - \frac{x - 1}{x - 1} \).

12. Find the value of \( 2\sqrt{4/x^2} - \sqrt{3/x} \) if \( x = 9 \).

Part II (counts 60 per cent)

13. Solve the following equation for \( x \):

\[
\frac{x(x + a)}{b(x - a)} + \frac{x + 2a}{a - x} = \frac{x}{b} - \frac{b}{x - a}
\]
14. a) Plot the graph of the equation \( d = 3t^2 - t \) from \( t = 0 \) to \( t = 8 \).

b) Estimate from the graph the value of \( \frac{\sqrt{d}}{2} \) when \( t = 4 \frac{1}{2} \).

c) Estimate from the graph the values of \( t \) when \( d = 10 \).

15. A goldsmith has two alloys of gold, the first being 80 per cent pure gold, the second 50 per cent pure gold. How many ounces of each must he take to make 75 ounces of an alloy which shall be 72 per cent pure gold?

16. Between town A and town B there is a hilly road with no level stretches. In going from A to B, the road runs up hill for 22\( \frac{1}{2} \) miles and down hill for 11\( \frac{3}{4} \) miles. A man drives from A to B over this road in one hour, and then returns to A over the same road in 52\( \frac{3}{4} \) minutes. When going up hill he travels at a constant rate; when going down hill he travels at another constant rate. Find these rates.
Mathematics Al—Algebra to Quadratics

Monday, June 19

9:30 a.m. 2 hours

Part I

(This part of the examination will count 40 per cent. No credit will be given in Part I for answers which are only partially correct.)

1. Solve the equation \(0.02 = 5.5\).

2. Solve the equation \(2m - \frac{2m + 3}{2} = 3\).

3. Simplify \(a - 2 - \frac{a - a}{a - 1}\).

4. Simplify \(\frac{x^3 - 2x^2 - 3x + 6}{x - 2}\).

5. Simplify \(\frac{6a}{4 - 3a} + \frac{3}{3a - 4}\).

6. Factor completely \(3x^3y - 18x^2y + 27xy\).

7. Simplify \(\frac{x + y}{x - 2y} \times \frac{x^2 - 4y^2}{x^2} \div \frac{x + 2y}{x + y}\).

8. If \(S = 2\pi r^2 - 2\pi rh\), find the value of \(h\) in terms of the other letters.

9. Simplify \(\sqrt{2/3} + \sqrt{3/2}\).

10. Write a formula for the cost, \(c\), of a telegram containing \(k\) words if it costs \(m\) cents for 12 words and \(s\) cents for each additional word. Assume that \(k\) is greater than 12.

Part II

11. a) Plot the graph of the equation \(y = x^2 + 3x - 1\).

b) Draw the line \(y = 8\), using the same axes as in (a).

c) Estimate from the figure the values of \(x\) at the points where the graphs intersect.
12. a) Solve the simultaneous equations:
   \[7x + 6y = 14,\]
   \[5x - 2y = 8.\]

   b) At a time when the angle of elevation of the sun is 41° 33', a tower casts a shadow 82.5 feet long. What is the height of the tower?

13. How many quarts of pure water must be added to 12 quarts of an acid solution that contains 15 per cent acid to make a solution that contains 10 per cent acid?

14. At an election A and B were candidates for office. A received 120 more votes than B and was elected. If one-eighth of those who voted for A had voted for B, the other votes remaining unchanged, B would have received 710 more votes than A. How many votes were cast?
Part I

(No credit will be given in Part I for answers which are only partially correct.)

1. Factor \(4a^2 - 4ab + b^2\).

2. Evaluate the expression \(\frac{3a - b}{2a + 1} - \frac{3c - 2b}{c + 2a}\) when \(a = 2\), \(b = 3\), \(c = 1\).

3. Solve for \(x\): \(\frac{a - x}{a + x} = \frac{a}{a - x}\).

4. In the formula \(V = \frac{1}{2} \pi a^2h\), if \(a\) remains unchanged in value, doubling \(h\) will have what effect on the value of \(V\)?

5. Solve for \(n\): \(S = v + \frac{1}{2} a \left(2n - 10\right)\).

6. Simplify \(a^2 - 12ab + 36 b^2\).

7. Simplify \(\frac{b^2 - 2abc + x^2}{2abc - b^2 - x^2}\).

8. Find the product \((1 - a)(a - 1)\).

9. Solve for \(y\): \(2.2y = 7.5 - 3.8y\).

10. What is the angle of elevation of the top of a tower, which is 102.4 feet high, at a horizontal distance of 40 feet from the foot of the tower?

Part II

11. a) Plot the graph of the equation \(S = \frac{1}{2}t^2\) from \(t = -4\) to \(t = 4\).

b) Estimate from the graph the values of \(t\) when \(S = 3.5\).

12. There are two numbers such that if the first is increased by 1 and the second diminished by 1, their product is diminished by 4. If the first is diminished by 1 and the second increased by 2, their product is increased by 16. Find the numbers.
13. The value of 92 coins consisting of nickels and quarters is $15.30. Find the number of coins of each kind.

14. If a man walks from P to Q at an average rate of 3 miles per hour and returns at an average rate of 4 miles per hour, he takes 5 minutes longer than when he goes from P to Q and back at an average rate of 3 1/2 miles per hour. Find the number of miles from P to Q.
Monday, June 17

9:30 a.m.  2 hours

1. a) Factor completely  
   \[ 2ax^3 - 2ax^2 - 12ax. \]

   b) Simplify  
   \[ \frac{a}{a - 2} + \frac{2}{2 - a} + 2. \]

   c) Factor  
   \[ 2x^2 - xy - 3y^2. \]

2. a) Solve the equation  
   \[ \frac{3x - 4}{2} - (x - 1) = 5. \]

   b) Simplify  
   \[ \frac{a}{x} + \frac{b}{y} + \frac{c}{xy}. \]

   c) If \( m = 9 \), find the value of  
   \[ \sqrt{\frac{m - 1}{2}} - \sqrt{\frac{1}{m + 7}}. \]

3. How many pounds of coffee worth 50 cents a pound must be added to 10 pounds of coffee worth 30 cents a pound to make a mixture worth 42 cents a pound?

4. Solve the following system of equations:  
   \[ 2x + 3y = -1/6, \]
   \[ 3x - 5y = -1/2. \]

5. a) Using the value \( \sqrt{2} = 1.414 \), find the value of  
   \[ 5 \sqrt{6} - \sqrt{50}. \]

   b) The hypotenuse of a right triangle is 30 inches long and one angle is 65°. Find the length of the shortest side.

6. a) Plot the graph of the equation \( xy = 12 \) for values of \( x \) from 1 to 8, inclusive.

   b) Plot the graph of the equation \( y = 3x - x^2 \), using the same axes and the same values of \( x \) as in (a).

   c) Estimate from the figure pairs of values of \( x \) and \( y \) which satisfy both equations.

7. A trip of 2,000 miles may be made partly by train and partly by airplane. If the train is used for 600 miles, the trip requires 27 hours and 20 minutes. If the train is used for 900 miles, the trip requires 31 hours. Find the average speed of the train and of the airplane.

8. A man deposited a part of his capital in a saving bank, which paid interest at the rate of \( 3\frac{1}{2} \) per cent, and invested the remainder in bonds, which paid interest at the rate of \( 3\frac{1}{2} \). Her received twice as much income from the bonds as from the savings bank. If the same amount of capital had been invested in a business, which paid interest at the rate of 5 per cent, the annual income would have been increased by 185. How much capital did the man have?
Tuesday, June 16

1. a) Factor \( x^2y - 5xy^2 - 6y \).

   b) Factor \( (a - b)x - cy + by \).

   c) Given the formula \( S = \frac{xL - a}{r - 1} \); find \( a \) if \( S = 252 \), \( r = 2 \), \( L = 123 \).

   d) Two books cost \( b \) dollars. The first book cost \( r \) cents more than the second. Express in cents the cost of each book.

2. a) Solve the following system of equations:

\[
\begin{align*}
x - 3 &= -2, \\
\frac{5y}{x + 7y} &= 6.
\end{align*}
\]

   b) Given \( A = P - \frac{Prt}{100} \); find \( P \) in terms of \( A, r, \) and \( t \).

3. Solve the equation \( \frac{x - 3}{x + 4} = \frac{x - 9}{x + 3} \).

4. A man 230 feet from the foot of a tower finds that the angle of elevation of the top of the tower is \( 38° 45' \). Find the height of the tower to the nearest foot.

5. Members of the Athletic Association paid 15 cents admission to a contest, and non-members paid 25 cents admission. There 278 paid admissions and the total receipts were \$48.90. How many non-members attended the contest?

6. a) Plot the graph of the equation \( y + 5 = x^2 \).

   b) Plot the graph of the equation \( 4x - 3y = 0 \) on the same axes as in (a).

   c) Estimate from the figure the values of \( x \) and \( y \) which satisfy both equations.

7. A company that has failed in business is able to pay 24 cents on the dollar, but had the company been able to collect a certain debt of \$600, it could have paid 28 cents on the dollar. How much did the company owe at the time of the failure?

8. The cost of publication of each copy of a certain magazine is \( 6 \frac{1}{2} \) cents. It sells to dealers for 6 cents, and the amount received for advertising is 10 per cent of the amount received for all magazines issued beyond 10,000. Find the least amount of numbers of magazines which can be issued without loss.
1. a) Factor completely $15a^2 - 2a^2x - a^2x^2$.
   b) Factor $x^2 - y^2 - (x - 5)(x - y)$.
   c) Simplify $\frac{a + x}{a - x} - \frac{a}{x - a}$.

2. a) Find the value of the following expression when $x = \frac{1}{L - b}$:
    
    $\frac{1}{x} + \frac{2}{b} - \frac{b}{x} - 1$

   b) Solve for $t$: $V = \frac{st + 2}{b}$.

   c) Find the value of the following expression when $x = \frac{\sqrt[3]{2 - x}}{x}$.

3. In 1 hour A can walk 1 mile farther than B. They start at the same place and, after walking in opposite directions for 5 hours, are 40 miles apart. How many miles can A walk in 1 hour?

4. Solve the simultaneous equations:
   
   $2x + y = 4/3$
   
   $\frac{x}{2} + \frac{y}{2} = \frac{1}{3}$.

5. a) Plot the graph of the equation $y = x^2 - 2x$.
   b) Plot the graph of the equation $x = y^2 - 2y$, using the same axes as in (a).
   c) Estimate from the figure all pairs of values of $x$ and $y$ which satisfy both equations.

6. a) A flagpole 20 feet long stands upright on top of a building which is 100 feet high. From a certain point on the ground, the angle of elevation of the bottom of the flagpole is $45^\circ$. What is the angle of elevation of the top of the flagpole?
   b) Find the value of $(3\sqrt{2} - \sqrt{3})(\sqrt{2} - 3\sqrt{3})$.

7. When working together, A and B can finish a piece of work in $2n$ days. They worked together for $n$ days when A stopped and B finished the job alone in $p$ days. How long would it take A to do all the work alone?

8. A charitable society has enough money to feed the residents of an institution for a certain number of days. If 40 of the residents are transferred to another city, the money will last 40 days longer, but if 30 additional residents are admitted to the institution, the money will last 20 days less. How many residents has the institution?
Tuesday, June 21

1. Factor \( \sqrt{a^2 - 16b^2} \).

2. Factor \( 3m^2x^2 - 18m^2x - 27m^2 \).

3. Solve the following formula for \( h \) in terms of the other letters:
   \[ S = 2\pi r (r + h) \].

4. Give four integral values for \( k \) each of which will make it possible to factor \( x^2 - kx - 6 \). Write the factors in each case.

5. A man walks \( k \) miles to a certain place at the rate of \( m \) miles an hour. He returns over the same road at the rate of \( (m - 1) \) miles an hour. Express the total time which he has walked. What has been his average rate?

6. The cost of 4 yards of velvet and 3 yards of silk is \$45.60; the cost of 3 yards of the same velvet and 5 and \( \frac{3}{4} \) yards of the same silk is \$45.40. What is the price per yard of the velvet and of the silk?

7. Test whether the following statements are true or false. Show all your work.
   a) When \( x = -1 \), \( x^2 + 2x + 3x + 2 = 0 \).
   b) When \( x = 2 - 2\sqrt{3} \), \( x^2 - 4x - 8 = 0 \).
   c) If \( \sqrt{5} = \frac{4}{9} \) and \( a = 16 \), then \( b = 9 \).
   d) If \( y \) varies as \( x \) and \( y = 3 \) when \( x = 2 \), then \( y = 4.02 \) when \( x = 2.68 \).

8. A tree casts a shadow 15 feet 9 inches long at the same time that a nearby post 6 feet high casts a shadow 3 feet 4 inches long. Find the height of the tree and the angle of elevation of the sun at the time of the observation.

9. Plot the graph of each of the following equations on the same axes:
   \[ y = x^2 - 2x - 3 \]
   \[ 4x + 4y = 3 \]

10. For what values of \( x \) if any, is each of the following statements true?
    a) \( \frac{x + 10}{x^2 + 4x - 5} - \frac{3}{x - 1} = \frac{5}{x + 5} \).
11. A merchant buys tea for 40 cents and some for 75 cents a pound. He sells a mixture of the teas for 66 cents a pound and gains 10 cents per pound. In what proportion does he mix them?

12. There are two roads between A and B. Two men drive from A to B in the same length of time but by different roads. On the return journey each takes the road he did not take in going to B, and one man takes 19 minutes longer than the other to reach A. If the respective rates of the two men are uniformly 15 and 20 miles an hour, find the lengths of the two roads between A and B.
Mathematics Al--Algebra to Quadratics

Tuesday, June 20

1. Factor \(2ax^2 - 16ay^2\).

2. Factor \(6x^2 - 7xy - 20y^2\).

3. Factor \(m^4n^2 - 3m^2n^3 - 10mn^4\).

4. Factor \(c^2 + 9c - 36\).

5. Factor \(x^2(x - b) + a^2(b - x)\).

6. Find the value of the following expression when \(y = 3/4\): \(\frac{4 + 2y}{5 - 4y^2}\).

7. Find the value of the following expression when \(x = 1/a\): \(\frac{1 - ax^2}{x} + \frac{ax - 1}{a} + 2 - a\).

8. Simplify \((1/a - 1/b)(1 - b/a)\left(\frac{n^2}{a-b}\right)\).

9. Solve the following formula for \(h\): \(S = 2\pi r^2 + 2\pi rh\).

10. Solve for \(t\): \(S = \frac{a}{b - 2t} + 5\).

11. Reduce the following expression to its simplest form: \(\sqrt{14/5} + \sqrt{45} - 3\sqrt{32}\).

12. Solve the system of equations:
   \[
   \begin{align*}
   \frac{x}{2} + 2y &= \frac{4}{3} \\
   \frac{5x}{6} - \frac{y}{3} &= -\frac{5}{6}
   \end{align*}
   \]

13. a) Plot the graph of the equation \(y = 7 - x\).
    b) Plot the graph of the equation \(y = 5x - x^2\), using the same axes as in a).
    c) Estimate from the figure all pairs of values of \(x\) and \(y\) which satisfy both equations.

14. A company owns three factories which manufacture the same product. When each factory works at full capacity, a certain order can be filled by the first factory alone in 3 weeks, by the second factory alone in 4 weeks, and by the third factory alone in 6 weeks. How many weeks will be required to fill the order if all three factories are used, but each factory works at only one-third of its full capacity?
15. Four hours after a battle cruiser left port, two airplanes started in pursuit. The first plane overtook the cruiser in 1 hour. The second plane, flying with a speed 25 miles an hour less than the first, overtook the cruiser in 1 hour and 20 minutes. Find the speed of the cruiser.

16. A certain sum of money was loaned at simple interest. At the end of 10 years the total interest was $540 less than the amount of the loan. At the end of 20 years the total interest was $120 greater than the amount of the loan. Find the rate of interest.
1. If \( r = \frac{1}{2} \), find the value of \((r + 1) - \left(1 - \frac{1}{r}\right)\).

2. If \( x = \frac{1}{2} \) and \( y = 1/3 \), find the value of \( \sqrt{3x} - \sqrt{3 + 3y} \).

3. Simplify \( 3x^2 - x(3x - 2) \).

4. Simplify \( \frac{3}{a - b} + \frac{1}{b - a} \).

5. Simplify \( \frac{x}{a^2 - x^2} + \frac{1}{x + a} \).

6. Simplify \( \sqrt{3/5} + \sqrt{5/3} \).

7. Factor \( 16x^2 - 9y^2 \).

8. Factor \( x^2 - 2x - 3 \).

9. Factor \( ax - ay + bx - by \).

10. Factor \( 6x^2 - 7x - 3 \).

11. Factor \( 2ab - b^2 - a^2 + c^2 \).

12. Solve the equation \( \frac{x - 2}{3} = \frac{3x + 1}{2} \).

13. Find from your tables \( \cos 57^\circ 32' \).

14. What is the height of a tree if its shadow is 30 feet long when the angle of elevation of the sun is \(32^\circ\)?

15. Find the angles of a right triangle if its hypotenuse is 40 inches long and one side is 10 inches long.

16. A man working alone can paint his garage in 6 hours. The man and his son, working together, can paint it in 4 hours. How long will it take the son alone to do the job?

17. The ages of two sisters are in the ratio 5/3. Four years from now their ages will be in the ratio 7/5. Find their ages now.

18. Solve the following system of equations for \( x \) and \( y \).

\[
\begin{align*}
2ax + by & = m, \\
ax + 2by & = n.
\end{align*}
\]
19. a) Plot the graph of the equation \( x + y = 2 \).

b) Plot the graph of the equation \( 8y = x^3 \), using the same axes as in (a).

c) Estimate from the figure all pairs of values of \( x \) and \( y \) which satisfy both equations.

20. A merchant arranges his selling prices in such a way that, after deducting \( r \) per cent of the selling price for expenses, he will make a profit equal to \( p \) per cent of the cost price. What should be the selling price of an article which cost \( c \) dollars?
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