TEV-SCALE LEPTON NUMBER VIOLATION: $0\nu\beta\beta$ DECAY, THE ORIGIN OF MATTER, AND ENERGY FRONTIER PROBES

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TEV-SCALE LEPTON NUMBER VIOLATION: $0\nu\beta\beta$
DECAY, THE ORIGIN OF MATTER, AND ENERGY
FRONTIER PROBES

A Dissertation Presented
by
SEBASTIÁN URRUTIA QUIROGA

Submitted to the Graduate School of the
University of Massachusetts Amherst in partial fulfillment
of the requirements for the degree of
DOCTOR OF PHILOSOPHY

September 2023
Department of Physics
TEV-SCALE LEPTON NUMBER VIOLATION: $0\nu\beta\beta$
DECAY, THE ORIGIN OF MATTER, AND ENERGY 
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SEBASTIÁN URRUTIA QUIROGA

Approved as to style and content by:

Michael J. Ramsey-Musolf, Chair

Jordy De Vries, Member

Rafael Coelho Lopes de Sá, Member

Gregory Grason, Member

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Department of Physics
DECÁLOGO DEL MAESTRO

Ama, si no puedes amar mucho, no enseñes a niños.
Simplifica, saber es simplificar sin restar esencia.
Insiste, repite como la naturaleza repite las especies, hasta alcanzar la perfección.
Enseña, con intención de hermosura, porque la hermosura es madre.
Maestro, sé fervoroso. Para encender lámparas has de llevar fuego en el corazón.
Vivifica tu clase. Cada lección ha de ser viva como un ser.
Cultívate, para dar, hay que tener mucho.
Acuérdate de que tu oficio no es mercancía sino que es servicio divino.
Antes de dictar tu lección cotidiana, mira a tu corazón y ve si está puro.
Piensa en que Dios te ha puesto a crear el mundo del mañana.

TEACHER’S DEICALOGUE

Love, if you cannot love much, do not teach children.
Simplify, to know is to simplify without losing essence.
Insist, repeat as nature repeats species, until perfection is achieved.
Teach, with the intention of beauty, for beauty is the mother.
Teacher, be fervent. To ignite lamps, you must carry fire in your heart.
Enliven your class. Each lesson must be as alive as a being.
Cultivate yourself, to give, one must have much.
Remember that your profession is not a commodity, but divine service.
Before delivering your daily lesson, look into your heart and see if it is pure.
Think that God has placed you to create the world of tomorrow.

Gabriela Mistral*

* Lucila Godoy Alcayaga, better known as Gabriela Mistral, was born on April 7, 1889, in Vicuña, Chile, and passed away on January 10, 1957, in New York, USA. A distinguished Chilean poet and educator, she emerged as a key figure in both Chilean and Latin American literature. In 1945, she became the first Latin American author to be awarded the Nobel Prize in Literature.
ACKNOWLEDGMENTS

First, I must thank my family for their love and support. I would not have made it this far without them. Specifically, I thank Daniela, Spin ☁, and Memé, Mamá, and Daniela in Chile for their unconditional love and support when things got intense. I must also thank my friends, old and new, who have helped keep me sane during the last five years, especially during this pandemic. I cannot express my gratitude enough to my friend Samyukta Krishnamurthy, who has been a constant source of motivation, laughter, and friendship throughout my time as a graduate student. Thank you for being my rock at UMass.

I am also thankful to those in the High Energy Physics community with whom I have not yet worked directly but who have nonetheless encouraged and supported me along the way. Special thanks also to my collaborators Julia Harz, Tianyang Shen, Gang Li, Michael Graesser, Giovanna Cottin, Juan Carlos Vasquez, Supriya Senapati, and Dyson Kennedy, without whom none of this would have been possible. I want to extend my gratitude to the High Energy Theory Group at William & Mary, especially Marc Sher, for their warm and welcoming host for the past year and a half.

Special thanks to my committee members, Prof. Jordy de Vries, Prof. Rafael Coelho Lopes de Sá, and Prof. Gregory Grason, for their advice and support these past several years. Their suggestions and comments have made this work possible. I would like to extend my heartfelt thanks to the community in the Physics Department, who have provided me with invaluable support and guidance over the past few years. I am especially grateful for the wisdom and expertise generously shared
by Katie Bryant, Brokk Toggerson, Narayanan Menon, Benjamin Davidovitch, and Heath Hatch.

I would like to express my deepest gratitude to the ACFI Theory Group, including both current and former members, for their invaluable knowledge, insights, and friendship. They have been an endless source of inspiration and support throughout my academic journey. Additionally, I would like to thank Prof. Michael J. Ramsey-Musolf for his exceptional mentorship, guidance, and advocacy for diversity, equity, inclusion, and accessibility in physics. As a role model and a pillar of support, he has made an indelible impact on my academic and personal growth. I feel incredibly fortunate to have been a part of his group and to have benefited from his wisdom and expertise.
ABSTRACT

TEV-SCALE LEPTON NUMBER VIOLATION: $0\nu\beta\beta$ DECAY, THE ORIGIN OF MATTER, AND ENERGY FRONTIER PROBES

SEPTEMBER 2023

SEBASTIÁN URRUTIA QUIROGA

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Directed by: Professor Michael J. Ramsey-Musolf

Lepton number violation (LNV) offers promising theoretical pathways to several unresolved problems in particle and nuclear physics and unveils a diverse range of phenomenology across different energy scales. TeV-scale LNV is especially relevant for both its experimental accessibility and its broad-ranging impact, making it a key area of interest for both theoretical and experimental physicists. In this thesis, we explore three distinct scenarios within the LNV research landscape.

Our first analysis concerns the implications of TeV-scale LNV effects in thermal leptogenesis and its complementary sensitivity in neutrinoless double beta ($0\nu\beta\beta$) decay and collider experiments. We employed a simplified model to scrutinize the parameter space where standard thermal leptogenesis is rendered unviable, overcoming the limitations of previous effective field theory (EFT) approaches. We found that the new particle spectrum can have a decisive impact on the relative sensitivity of the $0\nu\beta\beta$ decay and collider probes.
The second scenario we examined involved a class of models that, at low energies, give rise to dimension-9 LNV vector operators, typically referred to as chirally suppressed in terms of their contribution to $0\nu\beta\beta$ decay. Utilizing a simplified model that generates those types of operators, we studied and compared the sensitivities of $0\nu\beta\beta$ decay experiments and LHC searches. The searches for $0\nu\beta\beta$ decay, which are here diluted by the chiral suppression of the vector operators, are found to be less constraining than the LHC searches, whose reach is increased by the assumed kinematic accessibility of the mediator particles.

Lastly, in the third scenario, we conducted a thorough study of the connection of the $0\nu\beta\beta$ decay process with lepton-number-conserving (LNC) processes in the type-II seesaw scenario of the left-right symmetric model. Through the use of an EFT approach, we improved the sensitivity in $0\nu\beta\beta$-decay experiments compared to previous studies. The results underscored that the LNC searches or measurements at high- and low-energy experiments, which are complementary to the $0\nu\beta\beta$-decay searches, are able to assess the type-II seesaw mechanism and sensitive to a right-handed doubly-charged scalar with mass up to $\sim 5$ TeV.

Through these studies, we have demonstrated the potential of new physics at the intersection of the energy, intensity, and cosmic frontiers in addressing significant queries in particle and nuclear physics.
# TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>ACKNOWLEDGMENTS</td>
<td>v</td>
</tr>
<tr>
<td>ABSTRACT</td>
<td>vii</td>
</tr>
<tr>
<td>LIST OF TABLES</td>
<td>xii</td>
</tr>
<tr>
<td>LIST OF FIGURES</td>
<td>xiv</td>
</tr>
<tr>
<td>CHAPTER</td>
<td></td>
</tr>
<tr>
<td>INTRODUCTION</td>
<td>1</td>
</tr>
<tr>
<td>1. THE STANDARD MODEL IN A NUTSHELL</td>
<td>6</td>
</tr>
<tr>
<td>1.1 Introduction</td>
<td>6</td>
</tr>
<tr>
<td>1.2 Lagrangian of the Standard Model</td>
<td>7</td>
</tr>
<tr>
<td>1.3 Electroweak Symmetry Breaking: the Higgs Mechanism</td>
<td>9</td>
</tr>
<tr>
<td>1.4 Theoretical Challenges</td>
<td>13</td>
</tr>
<tr>
<td>1.4.1 Neutrino masses and lepton number violation</td>
<td>13</td>
</tr>
<tr>
<td>1.4.2 Baryon Asymmetry of the Universe</td>
<td>19</td>
</tr>
<tr>
<td>2. THEORETICAL AND COMPUTATIONAL ANALYSIS TOOLS</td>
<td>27</td>
</tr>
<tr>
<td>2.1 Introduction</td>
<td>27</td>
</tr>
<tr>
<td>2.2 Thermal leptogenesis</td>
<td>28</td>
</tr>
<tr>
<td>2.2.1 CP violation</td>
<td>29</td>
</tr>
<tr>
<td>2.2.2 Boltzmann equation</td>
<td>30</td>
</tr>
<tr>
<td>2.2.2.1 Decay, Scattering, and Washout terms</td>
<td>34</td>
</tr>
<tr>
<td>2.2.2.2 Subtracting the real intermediate states (RIS)</td>
<td>37</td>
</tr>
<tr>
<td>2.2.2.3 Dealing with infrared (IR) divergences</td>
<td>40</td>
</tr>
</tbody>
</table>
2.3 Computer tools for Collider simulations
2.3.1 Background for same-sign dilepton searches
2.3.2 Monte Carlo simulations
2.3.3 Machine Learning for particle physics

2.4 Neutrinoless Double Beta Decay
2.4.1 Effective Operators
2.4.1.1 Power counting and chiral suppression

3. TEV-SCALE LEPTON NUMBER VIOLATION: CONNECTING LEPTOGENESIS, NEUTRINOLESS DOUBLE BETA DECAY, AND COLLIDERS
3.1 Introduction and Motivation
3.2 A simplified model for TeV-scale LNV
3.3 Leptogenesis
3.4 Collider study
3.4.1 Event generation and classification
3.4.2 Signal generation and phenomenology
3.4.3 Background generation and validation
3.5 $0\nu\beta\beta$ Decay
3.6 Combined results and discussion
3.7 Summary and Remarks of the Chapter

4. UNCOVERING A CHIRALLY SUPPRESSED MECHANISM OF NEUTRINOLESS DOUBLE BETA DECAY WITH LHC SEARCHES
4.1 Introduction and Motivation
4.2 Simplified Model
4.3 $0\nu\beta\beta$ Decay
4.3.1 Dimension-9 operators
4.3.2 Neutrino masses: dimension-9 operators
4.3.3 Dimension-7 operator
4.3.4 Neutrino masses: dimension-7 operators
4.4 Collider searches at the LHC
4.4.1 Leptoquark searches
4.4.2 Same-sign dilepton plus dijet search
### LIST OF TABLES

<table>
<thead>
<tr>
<th>Table</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.1</td>
<td>Quantum numbers of elementary fields in the Standard Model.</td>
</tr>
<tr>
<td>1.2</td>
<td>Best fit of neutrino oscillation data extracted from Ref. [60]. NH (IH) refers to the normal (inverted) hierarchy in neutrino masses. Notice that $\Delta m_{31}^2 &gt; (&lt;) 0$ in the normal (inverted) hierarchy.</td>
</tr>
<tr>
<td>2.1</td>
<td>Power-counting estimates of the contribution of low-energy dimension-9 operators to the amplitudes in Eq. (2.61), in terms of the Higgs vev $v$ and expansion parameter $\epsilon_\chi \equiv m_\pi / \Lambda_\chi$, where $\Lambda_\chi \sim m_N \sim 1 \text{ GeV}$. The electron mass and energies are taken to scale as $E_1 \sim E_2 \sim m_e \sim \Lambda_\chi \epsilon_\chi^3$. This table assumes the NMEs follow the $\chi$EFT power counting. $C_{\text{vector}}^{(9)}$ indicates any of the vector operators in Eq. (2.54). Note that to estimate the overall scaling of the amplitudes, one needs to take into account that, up to insertions of dimensionless couplings, the Wilson coefficients scale as follows: $C_{1L, 4L, 5L}^{(9)} = \mathcal{O}(v^3 / \Lambda^3) \text{ or } \mathcal{O}(v^5 / \Lambda^5)$ (depending on the underlying model) and $C_i^{(9)} = \mathcal{O}(v^5 / \Lambda^5)$ for the remaining dimension-9 operators. Extracted and adapted from Ref. [134].</td>
</tr>
<tr>
<td>3.1</td>
<td>Kinematic classification of production and successive decays involved in diagram (b), Fig. 3.8, in our simplified model.</td>
</tr>
<tr>
<td>3.2</td>
<td>Numerical example of the phenomenological behavior of the signal cross section at $\sqrt{s} = 14$ TeV. We took $m_F = 1$ TeV, $g_{L1} = 0.1$, and $g_{Q1} = 0.01$ in both scenarios. The results were obtained from MADGRAPH [107].</td>
</tr>
<tr>
<td>3.3</td>
<td>Validation of the charge misidentification implementation. The ATLAS results extracted from Fig. 2 in Ref. [119], and our results make use of the probability density defined in Ref. [174].</td>
</tr>
</tbody>
</table>
3.4 Individual contributions to the cross-section for the different types of background for the LHC at 14 TeV. $\sigma_{\text{before}}$ corresponds to the cross-section before the signal selection, $\sigma_{\text{after}}$ corresponds to the cross-section after the signal selection, and $\sigma_{\text{RNN}}$ is the cross-section tagged as background events by the RNN. The last row shows the total background cross-section. .......................... 87

3.5 Individual contributions to the cross-section for the different types of background for the FCC-hh at 100 TeV. The definitions of $\sigma_{\text{before}}$, $\sigma_{\text{after}}$, and $\sigma_{\text{RNN}}$ are equivalent as in Table 3.4. The last row shows the total background cross-section. ......................... 88

4.1 The cascade decays of leptoquarks in the presence of $\Psi$. ................. 127

4.2 The benchmark points that we choose for the collider and $0\nu\beta\beta$-decay studies. ................................................................. 130

5.1 Different choices for the mass of right-handed gauge boson $M_{W_R}$ and the $W_L - W_R$ mixing parameter $\xi$ used in our analysis. ............... 148
# LIST OF FIGURES

<table>
<thead>
<tr>
<th>Figure</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.1</td>
<td>Illustration of the scalar potential $V(H)$ for the symmetric (left panel) and the broken (right panel) electroweak-symmetric phase, depending on the sign of the mass parameter $\mu^2_H$.</td>
</tr>
<tr>
<td>1.2</td>
<td>Pictorial representation of the effective operator $\mathcal{O}<em>{B+L}$. $L_e, L</em>\mu$, and $L_\tau$ correspond to the first, second, and third generations of lepton doublets. $Q_{ud}, Q_{cs}$, and $Q_{tb}$ correspond to the first, second, and third generations of quark doublets.</td>
</tr>
<tr>
<td>2.1</td>
<td>Feynman diagrams contributing to the $CP$ asymmetry parameter $\epsilon$. The internal lepton $L$ and Higgs $H$ are on-shell.</td>
</tr>
<tr>
<td>2.2</td>
<td>Feynman diagrams contributing to standard, thermal leptogenesis. $U$ and $D$ are up- and down-quarks, $V$ is a $SU(2)_L \times U(1)_Y$ gauge boson, $L$ is a lepton, $H$ is a Higgs boson, and $N_1$ is the lightest right-handed neutrino.</td>
</tr>
<tr>
<td>2.3</td>
<td>Example of a $t\bar{t}$ event with possible multiple misreconstructed objects. Adapted from Refs. [105,106].</td>
</tr>
<tr>
<td>2.4</td>
<td>Illustration of the Monte Carlo event simulation. The forward direction is typically understood as the simulation stage, while the inverse direction corresponds to the analysis and unfolding stage. Adapted from Refs. [113,114].</td>
</tr>
<tr>
<td>2.5</td>
<td>Data-driven charge misidentification probability. Plot (a) shows the charge misidentification probability component as a function of $p_T$, and plot (b) shows the component as a function of $</td>
</tr>
<tr>
<td>2.6</td>
<td>Black Box diagram relating the Majorana nature of neutrinos with $0\nu\beta\beta$ decay. Adapted from Ref. [125].</td>
</tr>
</tbody>
</table>
2.7 Different contributions to $0 \nu \beta \beta$ decay: A light neutrino exchange between two point-like vertices is classified as "long-range". Contributions mediated by heavy particles are classified as "short-range". Adapted from Ref. [130].

2.8 The two basic tree-level topologies realizing a dimension-9 $0 \nu \beta \beta$ operator. External lines are fermions; internal lines can be fermions (solid) or scalars, or vectors (dashed). Extracted from Ref. [130].

2.9 A schematic overview of the EFT approach to evaluate the $0 \nu \beta \beta$-decay amplitude starting from high-scale $\Delta L = 2$ dynamics. Adapted from Ref. [134].

2.10 Different contributions to two-nucleon operators for $0 \nu \beta \beta$ decay relevant to the heavy particle exchange mechanism of dimension-9 LNV operators. Adapted from Refs. [131,134].

3.1 One-loop contribution to the Weinberg operator [153] induced by the interactions in Eq. 3.2. Note that the magnitude of this contribution is proportional to the coupling $\lambda_{HS}$ that does not enter the amplitudes for $0 \nu \beta \beta$-decay or same-sign dilepton plus di-jet production in $pp$ collisions.

3.2 The realization of the $0 \nu \beta \beta$-decay dim-9 operator induced by the interactions in Eq. 3.2. The scalar $S$ transforms as $(1, 2, 1)$ under $SU(3)_C \times SU(2)_L \times U(1)_Y$, and the Majorana fermion $F$ transforms as an SM gauge singlet.

3.3 Relevant lepton-number violating processes arising from our extended Lagrangian that contribute to the washout additionally to the usual washout processes which occur within the standard thermal leptogenesis scenario including right-handed neutrinos.

3.4 Evolution of the thermal masses. Due to a negligible difference between the thermal masses of $Q$, $u$, and $d$, we only show $m_Q$. In this regime, the thermal masses are basically independent of the initial $m_X(T = 0)$ value. This is in contrast to the thermal mass of the right-handed neutrino whose thermal correction leaves the large $m_{N_0} = 10^{10}$ GeV unchanged until small values of $z < 10^{-2}$. 

xv
3.5 Evolution of the yield of the right-handed neutrino $Y_N$ (blue solid line) and the yield of the baryon asymmetry for the standard scenario $Y_B^{[\text{std}]}$ (green dashed line) and including our new contributions $Y_B$ (orange solid line) for the weak (left panels) and strong (right panels) washout regime and two different example values for $g_L = g_Q = \{10^{-3}, 10^{-6}\}$. The equilibrium abundance $Y_N^{(eq)}$ is given as a gray dashed line. The red dotted line indicates $z_{eq}$. .................................................. 75

3.6 Evolution of the different contributions relevant for the yield of the baryon asymmetry $Y_B$ (orange solid line) for the weak (left panels) and strong (right panels) washout regime and two different example values for $g_L = g_Q = \{10^{-3}, 10^{-6}\}$. The green dotted lines show when $m_H(z) = m_N(z) + m_L(z)$ and $m_N(z) = m_H(z) + m_L(z)$, respectively. The blue dotted line shows when $m_S(z) = m_F(z) + m_L(z)$. .............................................. 76

3.7 $g_L-g_Q$–plane for $\epsilon = 10^{-6}$ (left panel) and $\epsilon = 1$ (right panel), with $m_{N_0} = 10^{10}$ GeV and $m_F = 1$ TeV. The red areas indicate the couplings that lead to a too-strong washout and don’t result in the observed baryon asymmetry for the different mass hierarchies shown in the panels. Considering the weak ($K = 10^{-2}$) or strong ($K = 10^{2}$) washout regime does not affect the plot visibly. Notice how the extension of the viable thermal leptogenesis region depends on the new particle spectrum—a feature not readily seen within the previously used pure EFT approach. ......................... 79

3.8 Feynman diagrams of the two-jet, same-sign dilepton signal ($pp \to e^\pm e^\pm jj$) in our simplified model. Diagram (a) matches the $0\nu\beta\beta$-decay diagram in Fig. 3.2. ...................... 80

3.9 Comparison between the CMS results in Ref. [102] and our implementation of the FakeSim method proposed in Ref. [115] for a particular choice of its parameters, as mentioned in Note 5. Distributions of the kinematic variables $H_T$ and $E_T^{\text{miss}}$ are shown in the left and right panels, respectively. ......................... 86

3.10 Contribution of dim-9 LNV $\pi\pi ee$ interaction to $0\nu\beta\beta$ decay at tree level ................................................................. 91
3.11 Interplay between leptogenesis, collider searches, and $0\nu\beta\beta$-decay experiments for $m_F = 1$ TeV, and $m_S/m_F = \{0.5, 0.99, 1.5\}$. As in Fig. 3.7, we present the nonviable leptogenesis region in red; in dark green, we show the $0\nu\beta\beta$-decay exclusion from KamLAND-Zen (dotted line) and future tonne-scale experiments (dashed line); the collider LNV (same sign dilepton plus dijet) exclusion is shown in blue for the LHC at 14 TeV with integrated luminosities of 100 fb$^{-1}$ (dotted line) and 3 ab$^{-1}$ (dashed line), and the FCC-hh at 100 TeV with 30 ab$^{-1}$ (dash-dotted line); and, in dark gray, we present the LHC di-jet exclusion, as discussed in Section 3.4.

3.12 Complementarity between collider searches and $0\nu\beta\beta$-decay experiments for $g_L = g_Q$ and $m_S/m_F = \{0.5, 0.99, 1.5\}$. Comparison with an EFT analysis for the scale of new physics $\Lambda = (m_S^4 m_F)^{1/5}$ and the effective coupling $g_{\text{eff}} = g_L = g_Q$. In dark green, we show the $0\nu\beta\beta$-decay exclusion from KamLAND-Zen (dotted line) and future tonne-scale experiments (dashed line). The collider LNV exclusion is shown in blue for the LHC at 14 TeV with integrated luminosities of 100 fb$^{-1}$ (dotted line) and 3 ab$^{-1}$ (dashed line), and the FCC-hh at 100 TeV with 30 ab$^{-1}$ (dash-dotted line). The EFT benchmark point is shown as a red star, where $g_{\text{eff}} = 1$ is assumed for the discussion.

3.13 Complementarity between the next generation of colliders and $0\nu\beta\beta$-decay experiments for $g_L = g_Q$ and $m_S/m_F = \{0.5, 0.99, 1.5\}$. The definitions of $\Lambda$ and $g_{\text{eff}}$ are the same as in Fig. 3.12. In dark green, we show the $0\nu\beta\beta$-decay exclusion from future tonne-scale experiments, and the reach of the FCC-hh LNV searches at 100 TeV with 30 ab$^{-1}$ is shown in blue.

4.1 Quark-level Feynman diagrams that induce $\Delta L = 2$ vector $0\nu\beta\beta$-decay operators at low energies.

4.2 Hadron-level Feynman diagrams for $0\nu\beta\beta$ decay induced by the LNV operators $\pi NNee$ (left) and $NNNNee$ (right), which are denoted by the black squares.
4.3 Half-lives of $0\nu\beta\beta$ decay as a function of the effective LNV scale $
abla \equiv \Lambda/|C_{Qa1} + C_{Qd1}|^{1/5}$. The solid black curve is obtained with $g_{\nu}^{\pi N} = g_{\tau}^{\pi N} = 1$ and $g_{\nu}^{NN} = g_{\tau}^{NN} = 0$. The dotted (dashed) black curve is obtained with the same settings of $g_{\nu}^{\pi N}$, $g_{\tau}^{\pi N}$ but with $g_{\nu}^{NN} = g_{\tau}^{NN} = 1/2 (-1/2)$. The solid red and dashed red lines correspond to the constraints on the half-life in KamLAND-Zen and future ton-scale experiments for $^{136}$Xe.

4.4 The LNV vector operators contribute to neutrino masses at three-loop or higher order and are completely negligible. Shown here is a typical three-loop Feynman diagram contributing to the neutrino mass. The red dot depicts the dimension-9 operators in Eq. (4.34).

4.5 An illustrative LNV Feynman diagram of the UV theory that contributes to the neutrino mass. The heavy degrees of freedom are highlighted in red.

4.6 Quark-level Feynman diagram that induce $\Delta L = 2$ dimension-7 $0\nu\beta\beta$-decay operator $O_{Leu\bar{d}H}$ at low energies.

4.7 Hadron-level Feynman diagram for $0\nu\beta\beta$ decay, induced by the LNV operator $np\bar{e}\nu^c$ denoted by the black square. The other vertex denotes the single $\beta$ decay operator from SM interactions.

4.8 Half-lives of $0\nu\beta\beta$ decay as a function of the effective LNV scale $
abla \equiv \Lambda/|C_{Leu\bar{d}H}|^{1/3}$. The solid red and dashed red lines correspond to the constraints on the half-life in the KamLAND-Zen and future ton-scale experiments for $^{136}$Xe.

4.9 Representative Feynman diagrams for the parton-level pair production of leptoquarks ($R^{\pm Q}$). Here, $g$ denotes a gluon, $q$ denotes a quark, and the electric charge $Q = 2/3, 1/3$.

4.10 Feynman diagram for the cascade decays of leptoquarks. The labels $P_1, \ldots, P_7$ denote the possible particles in the chain, which are specified in Table 4.1.

4.11 Feynman diagram for the parton-level process $\bar{u}d \rightarrow e^- \Psi^0$ in SS-1.
4.12 The current and projected sensitivities in the plane of $y_{qd} - y_{e\Psi}$ for BP1 and BP2. The red regions are excluded at 90% C.L. by the KamLAND-Zen (solid curve) and ton-scale (dashed curve) experiments. The blue regions are excluded at 95% C.L. by the dijet searches with the integrated luminosities of 139 fb$^{-1}$ (solid curve) and 3 ab$^{-1}$ (dashed curve). The green regions are excluded at 95% C.L. by the SSDL searches with the integrated luminosities of 36.1 fb$^{-1}$ (solid curve) and 3 ab$^{-1}$ (dashed curve). The green dot-dashed curves correspond to the 5$\sigma$ discovery potential at the HL-LHC. The product of leptoquark couplings $|\lambda_{ed}\lambda_{u\Psi}| = 1$ is assumed. $y_{e\Psi}$ and $y_{qd}$ denote the magnitudes of the couplings with the absolute value symbols omitted. 

4.13 Same as Fig. 4.12 but for BP3 and BP4.

5.1 Constraints on the $W_R$ mass and the mixing parameter $\xi$. The black dots correspond to different benchmark choices: (I) $M_{W_R} = 7$ TeV, $\xi = 0.35$, (II) $M_{W_R} = 15$ TeV, $\xi = 0.75$, and (III) $M_{W_R} = 25$ TeV, $\xi = 0.8$. 

5.2 Feynman diagrams contributing to $0\nu\beta\beta$ decay in the LRSM. The cross vertices ($\times$) in between $W_L - W_R$ propagators denote the left-right mixing. In the limit of type-II dominance, there is no mixing between left-handed $\nu$ and right-handed $N$ neutrinos.

5.3 Doubly-charged scalar contributions to the PV Møller scattering. Notice that each vertex violates lepton number by two units.

5.4 Parameter scan of $0\nu\beta\beta$-decay constraints on the right-handed doubly-charged scalar mass $M^{\pm\pm}_{\delta_R}$ and the Yukawa coupling $|(f_{R\delta})_{ee}|$ in the parity-violating mLRSM with type-II seesaw dominance in the normal hierarchy with $M_{W_R} = 7$ TeV and $\xi = 0$ (left) and $\xi = 0.35$ (right). The right-handed neutrino mass $M_{N_R}$ is also displayed as a secondary vertical axis. All the red points (regions above or between the solid red lines) are excluded by the most recent KamLAND-Zen results [10]. The solid red lines correspond to the exclusion limits calculated using an analytical estimation (see text for discussion). The dashed magenta lines correspond to the same exclusion limits using the prospect by ton-scale experiments [267].

xix
5.5 Same as in Fig. 5.4, parameter scan of $0\nu\beta\beta$-decay constraints on the right-handed doubly-charged scalar mass $M^{\delta^{\pm\pm}}_R$ and the Yukawa coupling $|(f_R)_{ee}|$ in the parity-violating mLRSM with type-II seesaw dominance in the normal hierarchy with $M_{W^R} = 15$ TeV and $\xi = 0$ (left) and $\xi = 0.75$ (right). 

5.6 Same as in Fig. 5.4, parameter scan of $0\nu\beta\beta$-decay constraints on the right-handed doubly-charged scalar mass $M^{\delta^{\pm\pm}}_R$ and the Yukawa coupling $|(f_R)_{ee}|$ in the parity-violating mLRSM with type-II seesaw dominance in the normal hierarchy with $M_{W^R} = 25$ TeV and $\xi = 0$ (left) and $\xi = 0.8$ (right).

5.7 Inverse $0\nu\beta\beta$-decay half-life ($T_{1/2}^{0\nu}$)$^{-1}$ as a function of the Yukawa coupling $|(f_R)_{ee}|$ for $W_L - W_R$ mixing parameters $\xi = 0$ (left) and $\xi = 0.35$ (right). We fix the mass of the doubly-charged scalar at $M^{\delta^{\pm\pm}}_R = 100$ GeV. The three curves in the figure show the results for $M_{W^R} = 7$ TeV (blue), $M_{W^R} = 15$ TeV (yellow), and $M_{W^R} = 25$ TeV (green). The inverse half-life considering only the light left-handed neutrino contributions is shown in red. The latest KamLAND-Zen exclusion limits are represented by the grey shaded area, with the future ton-scale experiment bounds indicated for comparison.

5.8 Combined sensitivities to the right-handed doubly-charged scalar mass $M^{\delta^{\pm\pm}}_R$ and the Yukawa coupling $|(f_R)_{ee}|$ in $0\nu\beta\beta$-decay, MOLLER, and collider experiments in the parity-violating mLRSM with type-II seesaw dominance. The normal hierarchy is assumed. Benchmark values of the parameters $M_{W^R} = 7$ TeV and $\xi = 0$ (left), $\xi = 0.35$ (right) are chosen. The right-handed neutrino mass $M_{N^R}$ is also displayed as a secondary vertical axis. The MOLLER prospect [266] is shown as a dashed blue line, and $0\nu\beta\beta$ decay limits from current and future experiments are shown as solid red and dashed magenta lines, respectively. The figure also displays the direct searches at the LHC (solid gray), pair production prospect at the FCC-hh (dashed gray), and Bhabha scattering limits at LEP (solid orange) and CEPC prospect (dashed green). The $0\nu\beta\beta$-decay constraint by the KamLAND-Zen experiment from Ref. [231] (dash-dotted black) is also included for comparison.
5.9 Same as in Fig. 5.8, combined constraints from LNC and $0\nu\beta\beta$-decay experiments on the right-handed doubly-charged scalar mass $M_{\delta^{\pm\pm}}$ and the Yukawa coupling $|\langle f_{R} \rangle_{ee}|$ in the parity-violating mLRSM with type-II seesaw dominance in the normal hierarchy with $M_{WR} = 15$ TeV and $\xi = 0$ (left) and $\xi = 0.75$ (right). 165

5.10 Same as in Fig. 5.8, combined constraints from LNC and $0\nu\beta\beta$-decay experiments on the right-handed doubly-charged scalar mass $M_{\delta^{\pm\pm}}$ and the Yukawa coupling $|\langle f_{R} \rangle_{ee}|$ in the parity-violating mLRSM with type-II seesaw dominance in the normal hierarchy with $M_{WR} = 25$ TeV and $\xi = 0$ (left) and $\xi = 0.8$ (right). 166

B.1 A simple 3-depth decision tree example, in which each note represents a cut based on the input parameters ($p_{Tj}$, $p_{Te}$, and $E_{T}$). “UC” standards of “unclassified” in the picture. Notice this is only an illustration of a decision tree, and it is not reflecting the true classification result of our collider study. 176

B.2 Visual representation of a standard RNN in folded version (left) and unfolded version (right). The sequence $\{x_1, x_2, x_3, \ldots, x_t\}$ represents the input, $\{y_1, y_2, y_3, \ldots, y_t\}$ represents the predicted output, and $\{h_0, h_1, h_2, \ldots, h_t\}$ holds the information from the previous input. The graph illustrates that, at any given time, $t$, the current layer will be updated with respect to a new input. 180

B.3 Visual representation of the Recurrent Neural Network (RNN) used in our study. The kinematic properties of jets, electrons, and missing energy are analyzed by independent Gated Recurrent Units (GRUs). The outcome is merged into a sequential neural network that produces a decision score $d$ between 0 and 1. 181

C.1 For scalar operators, the amplitude for $nn \rightarrow ppee$ receives an additional contribution from the Feynman diagram shown here, which must be combined with those shown in Fig. 4.2. 183
INTRODUCTION

With the current knowledge of the Standard Model (SM) of particle physics, the lepton number appears to be an accidentally conserved quantum number at the classical level. However, many beyond the Standard Model (BSM) scenarios contain lepton number violating (LNV) interactions. Consider, for instance, a minimal extension of the SM with right-handed neutrinos. A lepton-number-conserving Dirac mass term would lead to massive neutrinos, as required by neutrino oscillations. However, unless one explicitly requires lepton number conservation, one may also include an LNV right-handed neutrino mass term. Diagonalization of the full mass matrix for the neutral leptons implies that light neutrinos are also Majorana particles. For sufficiently heavy right-handed neutrinos, the Dirac mass Yukawa couplings may be as large as $\mathcal{O}(1)$ while accommodating the scale of light neutrino masses implied by neutrino oscillations and cosmological neutrino mass bounds. The light neutrino interactions then inherit the LNV properties of Majorana neutrinos as a consequence of this well-known seesaw neutrino mass mechanism [1–6].

At the same time, LNV interactions can play an important role in generating the baryon asymmetry of the Universe (BAU). Explaining why the Universe contains more baryonic matter than antimatter is a forefront challenge for high-energy and nuclear physics. Sakharov observed it [7] nearly half a century ago that explaining this asymmetry requires three ingredients in the physics of the early Universe: (i) baryon number ($B$) violation, (ii) $C$ and $CP$ violation and (iii) an out-of-equilibrium condition (or $CPT$ violation). As these conditions are not sufficiently satisfied within the SM, the observed BAU clearly requires BSM physics. Due to the SM ($B+L$)-violating
electroweak sphalerons being active above the electroweak scale, one possibility is to generate first a lepton asymmetry which then gets translated into a baryon asymmetry; baryogenesis via leptogenesis [8]. Hence, the observation of LNV interactions can have far-reaching consequences. It could not only give indications on a Majorana contribution to neutrino masses but could also have implications for the validity of different leptogenesis scenarios.

If LNV interactions exist in nature, then it is natural to explore the possible associated mass scale, $\Lambda$. In the context of standard thermal leptogenesis and the simplest scenario with right-handed neutrinos, the consistency with the phenomenology of light neutrinos implies $\Lambda \gtrsim 10^9$ GeV [9]. Direct experimental access to new particles and LNV interactions at these scales is clearly unfeasible, although indirect indications could be seen via searches for neutrinoless double beta ($0\nu\beta\beta$) decay. Searches to date have yielded null results, with the strongest limit in the current half-life of $^{136}$Xe having been set by the KamLAND-Zen experiment [10]. The next generation of “ton-scale” $0\nu\beta\beta$ decay searches aim to increase this sensitivity [11–16]. In the “standard mechanism”, wherein the underlying process involves the exchange of three light Majorana neutrinos, the observation of a signal would point to a high scale for $\Lambda$ associated with standard thermal leptogenesis and the conventional seesaw mechanism.

This dissertation explores the possibility of the existence of LNV interactions at multiple scales below the high scale required by standard thermal leptogenesis. Specifically, we investigate the hypothesis that additional LNV interactions could exist with an associated energy scale as low as $\mathcal{O}(\text{TeV})$. In particular, LNV interactions at this scale could contribute directly to $0\nu\beta\beta$ decay at an observable level, independent of the contribution of the light neutrino spectrum. However, discovering LNV in $0\nu\beta\beta$
decay alone would not provide information about the underlying mechanism. In con- 
trast, observing an LNV signal in high-energy $pp$ collisions, such as pairs of same-sign 
leptons and an associated dijet pair, would indicate LNV at the TeV scale. This raises 
questions about the potential implications for leptogenesis and the complementarity 
between low- and high-energy experiments to discover the underlying mechanism of 
LNV.

In this dissertation, we consider the opportunity to exploit TeV-scale LNV at the 
interface between the Energy, Intensity, and Cosmic frontiers. The Energy frontier in- 
volves experiments conducted at high-energy particle accelerators, such as the Large 
Hadron Collider (LHC) at CERN or future experiments. By colliding particles at 
extremely high energies, physicists aim to discover new particles, interactions, and 
BSM phenomena. The Intensity frontier focuses on experiments designed to be sensi- 
tive to small discrepancies from SM predictions or detect phenomena that are either 
heavily suppressed or forbidden by SM symmetries. These experiments can involve 
intense particle beams or large samples and require monitoring for background and 
systematic effects. The Cosmic frontier involves the study of the Universe on the 
largest scales, using astronomical observations and astrophysical measurements to in- 
vestigate fundamental questions of physics.

The first two chapters of this dissertation are dedicated to presenting a brief re- 
view of the SM and the necessity of beyond the standard model physics to address 
two particular problems (Chapter 1) and to introduce the theory and analysis tools 
used for the completion of this thesis (Chapter 2).

Chapter 3 presents a scenario in which we explore how TeV-scale LNV interactions 
affect the standard thermal leptogenesis paradigm. We employ a simplified model to
overcome the limitations of previous effective field theory (EFT) approaches, where the new degrees of freedom are integrated out, demonstrating the crucial role of particle spectrum in the early Universe dynamics. Our Fig. 3.7 shows how the extension of the viable thermal leptogenesis region depends on the new particle spectrum. Furthermore, we study the complementary sensitivity in $0\nu\beta\beta$ decay and collider experiments in light of different mass hierarchies, as shown in Fig. 3.12.

Chapter 4 examines the details of the $0\nu\beta\beta$-decay calculations. Dimension-9 LNV operators are classified as scalar and vector operators, depending on the Lorentz structure of the electron bilinear. With no underlying theory, the contribution to $0\nu\beta\beta$ decay from the vector operators is chirally suppressed compared to the scalar contributions. Utilizing a simplified model that generates those types of operators, we show that the naïve estimations for the suppression factor do not consider the magnitude of the ratio between nuclear matrix elements (NME). We also study and compare the sensitivities of $0\nu\beta\beta$-decay experiments and LHC searches, presenting the results in Figs. 4.12 and 4.13.

It is important to highlight that we will employ a simplified model approach as a middle ground between the use of EFT and a UV complete theory that admits specificity yet sacrifices generality. Simplified models have been used extensively in other contexts, such as dark matter studies [17–19], electroweak phase transition dynamics [20–22], and collider phenomenology [23–25], to mention a few examples. Our particular simplified model choices are intended to highlight the connection between $0\nu\beta\beta$ decay and collider phenomenology in a spirit of broadness and generality. Nevertheless, in our investigations, we intend to utilize a twofold methodology that encompasses both the application of simplified models and the adoption of approaches
based on complete UV theories.

Chapter 5 presents a detailed study of the connection of the decay process $0\nu\beta\beta$ with lepton-number-conserving (LNC) processes, such as parity-violating electron scattering, in the type II seesaw scenario of the left-right symmetric model. We improve the previous results using chiral EFT to compute the doubly-charged scalar and right-handed neutrino contributions to the $0\nu\beta\beta$-decay half-life, as shown in Fig. 5.8. We study the effects of nonzero left-right mixing for different values of $M_{W_R}$ to highlight the interplay between LNC searches or measurements at high- and low-energy experiments and $0\nu\beta\beta$ decay.

Lastly, we conclude by summarizing the main findings of this dissertation and addressing future lines of work.
CHAPTER 1
THE STANDARD MODEL IN A NUTSHELL

1.1 Introduction

The Standard Model of particle physics [26–28] is a theoretical framework that describes the behavior of all known subatomic particles and their interactions. It is based on the Yang-Mills theory [29] and the electroweak symmetry breaking mechanism [30–32], and is considered one of the most successful and well-tested theories in physics. The discovery of the $W^\pm$ and $Z$ bosons at CERN in 1983 [33–36], the top quark at Fermilab in 1995 [37,38], and the Higgs boson at the Large Hadron Collider (LHC) in 2012 [39,40] marked significant achievements for particle physics research.

The particle content of the SM includes 12 elementary fermions, which are grouped into three generations. They are also divided into two types: leptons and quarks. Leptons include the electron, muon, and tau and their corresponding neutrinos, which do not participate in the strong interaction. On the other hand, quarks participate in strong interactions and are the building blocks of protons and neutrons. The model also incorporates 12 gauge bosons, responsible for mediating the fundamental interactions of nature, including the photon for electromagnetic interactions, the $W^\pm$ and $Z$ bosons for weak interactions, and gluons for strong interactions. Additionally, the Higgs boson, a scalar field that gives particles mass, is included in the SM.

The SM is built on the $G_{SM} = SU(3)_C \times SU(2)_L \times U(1)_Y$ gauge group, which describes the strong and electroweak interactions. This introductory chapter will
provide an overview of the SM of particle physics, including its main components, particle content, and important predictions. It will also briefly mention some of the limitations of the SM that motivate the development of this thesis.

1.2 Lagrangian of the Standard Model

The SM includes three generations of left-handed and right-handed leptons and quarks. They are denoted as:

\[
L_\alpha = \begin{pmatrix} \nu_{L\alpha} \\ \ell_{L\alpha} \end{pmatrix}, \quad Q_\alpha = \begin{pmatrix} u_{L\alpha} \\ d_{L\alpha} \end{pmatrix}, \quad \ell_{R\alpha}, \quad u_{R\alpha}, \quad d_{R\alpha},
\]

(1.1)

where \( L \) and \( Q \) are \( SU(2)_L \) doublets formed by left-handed lepton and quark fields, respectively. The chirality is defined by the projection operators, \( \psi_{L/R} \equiv P_{L/R} \psi \), with \( P_{L/R} \equiv (1 \mp \gamma_5)/2 \) and \( \psi \) denoting a fermion field. Every left-handed charged lepton has a right-handed counterpart (\( \ell_{R\alpha}, u_{R\alpha}, \text{and} \ d_{R\alpha} \)) that transforms as singlets under the \( SU(2)_L \) gauge group. The index \( \alpha = 1, 2, 3 \) accounts for the three generations of leptons and quarks.

To provide a mechanism to explain how fermions and gauge bosons acquire masses, the SM introduces a scalar field \( H \), the so-called "Higgs boson." It transforms as a singlet under \( SU(3)_C \) and a complex doublet under \( SU(2)_L \), and it has hypercharge \( Y_H = 1 \). In the fundamental representation of \( SU(2)_L \), it is represented as:

\[
H = \begin{pmatrix} H^+ \\ H^0 \end{pmatrix}.
\]

(1.2)

The gauge invariance of \( G_{SM} \) introduces 12 gauge fields, one for each group generator. They are denoted as \( G^a_\mu \ (a = 1, \ldots, 8) \) for \( SU(3)_C \), \( W^i_\mu \ (i = 1, 2, 3) \) for \( SU(2)_L \).
and $B_\mu$ for $U(1)_Y$. It also requires the introduction of the covariant derivative $D_\mu$

$$D_\mu \Psi \equiv \left( \partial_\mu - ig_s \frac{\lambda^a}{2} G^a_\mu - ig_2 \frac{\tau^i}{2} W^i_\mu - ig_1 Y_\Psi B_\mu \right) \Psi , \quad (1.3)$$

where $\Psi$ denotes either a fermion or a scalar field, $\lambda^a$ ($a = 1, \ldots, 8$) are the Gell-Mann matrices, $\tau^i$ ($i = 1, 2, 3$) are the Pauli matrices, and $g_s$, $g_1$, and $g_2$ are the coupling constants for each group. The hypercharge of the corresponding field $Y_\Psi$ is defined in Table 1.1. It is worth pointing out that singlet representations (1) in Table 1.1 imply the absence of the corresponding gauge field(s) in Eq. (1.3).

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<th></th>
<th>Leptons</th>
<th>Quarks</th>
<th>Scalar</th>
</tr>
</thead>
<tbody>
<tr>
<td>$SU(3)_C$</td>
<td>$1$</td>
<td>$3$</td>
<td>$1$</td>
</tr>
<tr>
<td>$SU(2)_L$</td>
<td>$2$</td>
<td>$1$</td>
<td>$1$</td>
</tr>
<tr>
<td>$U(1)_Y$</td>
<td>$-1/2$</td>
<td>$1/6$</td>
<td>$1/2$</td>
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Table 1.1: Quantum numbers of elementary fields in the Standard Model.

Finally, we can construct a gauge, Lorentz invariant Lagrangian\(^1\) that looks as:

$$\mathcal{L}_{\text{SM}} = \mathcal{L}_{\text{gauge}} + \mathcal{L}_{\text{fermion}} + \mathcal{L}_{\text{scalar}} + \mathcal{L}_{\text{Yukawa}} , \quad (1.4)$$

$$\mathcal{L}_{\text{gauge}} = -\frac{1}{4} G^a_{\mu\nu} G^{a\mu\nu} - \frac{1}{4} W^i_{\mu\nu} W^{i\mu\nu} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu} , \quad (1.5)$$

$$\mathcal{L}_{\text{fermion}} = \bar{\psi}_\alpha i \gamma_\mu \gamma^\mu \psi_\alpha , \quad (1.6)$$

$$\{ \psi = L, Q, e_R, u_R, d_R \}$$

$$\mathcal{L}_{\text{scalar}} = (D_\mu H)^\dagger (D^\mu H) - V(H) , \quad (1.7)$$

$$V(H) = \mu_H^2 H^\dagger H + \frac{\lambda_H}{4} (H^\dagger H)^2 , \quad (1.8)$$

\(^1\)It is also renormalizable. For a short but inspiring review on this topic, see Ref. [41].
\[ \mathcal{L}_{\text{Yukawa}} = - Y_\ell^{\alpha\beta} \bar{\ell}_\alpha H R_\beta - Y_u^{\alpha\beta} \bar{Q}_\alpha \tilde{H} u_{R\beta} - Y_d^{\alpha\beta} \bar{Q}_\alpha H d_{R\beta} + \text{H.c.}, \]  

(1.9)

with gauge indices \( a = 1, \ldots, 8 \) and \( i = 1, 2, 3 \), and flavor indices \( \alpha, \beta = 1, 2, 3 \).

The abbreviation “H.c.” stands for Hermitian conjugate. Here, \( \tilde{H} \equiv i\tau^2 H^* \) and \( B_{\mu\nu} \), \( W^i_{\mu\nu} \), and \( G^a_{\mu\nu} \) are the field strength tensors.

1.3 Electroweak Symmetry Breaking: the Higgs Mechanism

Another consequence of gauge invariance is the massless condition for gauge bosons and fermions, as noticed in Eqs. (1.5) and (1.6), respectively. Nevertheless, all the experimental evidence [42] suggests that all the elementary particles, with the exception of the photon (and gluons), are massive. The Higgs (Brout-Englert) mechanism provides a theoretical explanation of how gauge bosons and fermions acquire their masses by spontaneously breaking the electroweak (EW) \( SU(2)_L \times U(1)_Y \) symmetry.

To explain the nature of spontaneous symmetry breaking, let us analyze the scalar potential in Eq. (1.8). It has two parameters \( \mu_H^2 \) and \( \lambda_H \), usually referred to as the mass parameter and the self-coupling, respectively. The potential stability immediately sets the constraint \( \lambda_H > 0 \). The sign of the mass parameter, however, determines the nature of the minimum (vacuum) of the potential:

- If \( \mu_H^2 > 0 \), then the potential has one trivial minimum at \( \langle H \rangle = (0 \ 0)^T \), as depicted in Fig. 1.1a.
- If \( \mu_H^2 < 0 \), then the potential has a degenerate minimum at \( \langle H^\dagger H \rangle = v^2 / 2 \), where

\[
v = \sqrt{-\mu_H^2 / \lambda_H / 4}, \tag{1.10}\]


is the vacuum expectation value of the Higgs field. This scenario is depicted in Fig. 1.1b.

![Figure 1.1: Illustration of the scalar potential $V(H)$ for the symmetric (left panel) and the broken (right panel) electroweak-symmetric phase, depending on the sign of the mass parameter $\mu_H^2$.](image)

The rotational symmetry of the minimum depicted in Fig. 1.1b reflects the fact that, before EW symmetry breaking, the Higgs potential is invariant under $U(1)_Y$. When a specific vacuum is selected, the so-called spontaneous symmetry-breaking (SSB) process, the potential’s minimum breaks the $SU(2)_L \times U(1)_Y$ symmetry, but a residual $U(1)_{em}$ symmetry remains [43]. We will see that this phenomenon is responsible for the photon to be massless.

After the SSB, we can parametrize the Higgs field fluctuations around the vacuum as

$$H = \begin{pmatrix} G^+ \\ \frac{1}{\sqrt{2}}(v + h + iG^0) \end{pmatrix},$$

(1.11)
where $G^+, G^0$, and $h$ are dynamical degrees of freedom with zero vev. From the kinetic term in Eq. (1.7), we notice that the use of the parametrization in Eq. (1.11) will induce mass terms for the gauge bosons:

- Charged sector.

$$|D_\mu H|^2 \supseteq \frac{g_2^2 v^2}{8} \left[ (W^1_\mu)^2 + (W^2_\mu)^2 \right],$$

$$\equiv m_W^2 W^+_\mu W^-_\mu,$$

(1.12)

(1.13)

where $m_W = \frac{g_2 v}{2}$ and $W^\pm_\mu = \frac{1}{\sqrt{2}}(W^1_\mu \mp i W^2_\mu)$.

- Neutral sector.

$$|D_\mu H|^2 \supseteq \frac{v^2}{8} \left( g_1 B_\mu - g_2 W^3_\mu \right)^2,$$

$$= \frac{1}{2} \begin{pmatrix} B_\mu & W^3_\mu \end{pmatrix} \begin{pmatrix} \frac{g_1^2 v^2}{4} & -\frac{g_1 g_2 v^2}{2} \\ -\frac{g_1 g_2 v^2}{2} & \frac{g_2^2 v^2}{4} \end{pmatrix} \begin{pmatrix} B^\mu \\ W^3_\mu \end{pmatrix},$$

$$\equiv \frac{1}{2} \begin{pmatrix} B_\mu & W^3_\mu \end{pmatrix} M^2 \begin{pmatrix} B^\mu \\ W^3_\mu \end{pmatrix}.$$

(1.14)

(1.15)

(1.16)

The presence of the mixing term $B_\mu W^3_\mu$ indicates that the physical fields are linear combinations of $B_\mu$ and $W^3_\mu$. We can diagonalize the mass matrix $M$ with a $2 \times 2$ rotation matrix $R_w$ parametrized by an angle $\theta_W$, and define mass eigenstates $A_\mu$ and $Z_\mu$ as:

$$\begin{pmatrix} A_\mu \\ Z_\mu \end{pmatrix} \equiv R_w \begin{pmatrix} B_\mu \\ W^3_\mu \end{pmatrix} = \begin{pmatrix} c_w B_\mu + s_w W^3_\mu \\ -s_w B_\mu + c_w W^3_\mu \end{pmatrix},$$

(1.17)
where \( c_w = \cos \theta_W \) and \( s_w = \sin \theta_W \). The mixing angle \( \theta_W \) (also known as the weak mixing angle or the Weinberg angle) satisfies

\[
\begin{align*}
\cos \theta_W &= \frac{g_2}{\sqrt{g_1^2 + g_2^2}}, & \sin \theta_W &= \frac{g_1}{\sqrt{g_1^2 + g_2^2}}, & \tan \theta_W &= \frac{g_1}{g_2}.
\end{align*}
\]  

(1.18)

The diagonalized mass matrix reads

\[
M_d^2 \equiv R_w M^2 R_w^T = \begin{pmatrix}
0 & 0 & 0 \\
0 & \left( g_1^2 + g_2^2 \right) v^2 / 4 & m_{\gamma}^2 \\
0 & m_{\gamma}^2 & m_Z^2
\end{pmatrix},
\]  

(1.19)

where \( m_\gamma = 0 \) and \( m_Z = \sqrt{g_1^2 + g_2^2} v^2 / 2 \). We explicitly see here that, after SSB, the \( U(1)_{em} \) gauge field \( A_\mu \) remains massless, and it is identified with the photon.

The \( U(1)_{em} \) coupling constant is defined as the electric charge

\[
 e = g_2 \sin \theta_W = g_1 \cos \theta_W.
\]  

(1.20)

Similarly, Eq. (1.9) gives us the different mass terms for each fermion:

\[
L_{\text{Yukawa}} \ni Y^{\alpha \beta}_{\ell} \sqrt{2} \bar{\ell}_{\alpha \beta} R_{\beta} + Y^{\alpha \beta}_u \sqrt{2} \bar{u}_{\alpha \beta} L_{\alpha} + Y^{\alpha \beta}_d \sqrt{2} \bar{d}_{\alpha \beta} L_{\beta} + H.c.,
\]  

(1.21)

where we immediately read the fermion mass matrices:

\[
m_\ell^{\alpha \beta} \equiv \frac{Y^{\alpha \beta}_{\ell}}{\sqrt{2}} v, \quad m_u^{\alpha \beta} \equiv \frac{Y^{\alpha \beta}_u}{\sqrt{2}} v, \quad m_d^{\alpha \beta} \equiv \frac{Y^{\alpha \beta}_d}{\sqrt{2}} v.
\]  

(1.22)

Notice that only left-handed neutrinos \( \nu_L \) and right-handed anti-neutrinos \( \bar{\nu}_R \) are observed in nature. This fact was confirmed by Goldhaber et al. by measuring photon polarization and using conservation of angular momentum [44]. The most important consequence of this fact is, therefore, that neutrinos are strictly massless in the SM.
1.4 Theoretical Challenges

The SM has been incredibly successful in describing the fundamental particles, and interactions in nature [45–47]. Since its development in the 1970s, it has been tested and confirmed by numerous experiments, leading to discoveries such as the Higgs boson in 2012. Its successes have included predicting the existence and properties of particles such as the $W^\pm$ and $Z$ bosons and accurately describing the behavior of strong and weak interactions.

However, the Standard Model also faces some significant theoretical challenges that it cannot currently explain. In this thesis, we will explore them, with a particular focus on the issues of neutrino masses and baryon asymmetry. We will examine the current state of knowledge in these areas, the latest experimental results, and some novel theoretical models we have proposed to address these challenges.

1.4.1 Neutrino masses and lepton number violation

In the late 1960s, Raymond Davis and John Bahcall designed and built an experiment to test the Standard Solar Model introduced by Bahcall. The experiment took place in the Homestake Gold Mine in Lead, South Dakota, and used Chlorine to measure the electron neutrino flux from Sun. The experimental result showed a third of the predicted flux [48], giving origin to the so-called Solar neutrino problem. Solar neutrino deficit was confirmed by GALLEX/GNO [49, 50] and SAGE [51]. Besides, the disappearance of atmospheric muon-neutrinos was also measured by the Super-Kamiokande experiment [52, 53].

Theorists had been thinking about the possibility of neutrino mixing in analogy with the same phenomenon in the quark sector and the CKM matrix:
• 1957 - Bruno Pontecorvo, wondering if other particles could experiment analogous behavior to $K^0 \leftrightarrow \bar{K}^0$ oscillations, proposed the idea of neutrino/anti-neutrino oscillations [54].

• 1962 - Maki, Nakagawa, and Sakata (in the context of a model for nucleons) proposed that the weak neutrinos known at the time were superpositions of “true” neutrinos with definite masses and that this could lead to transitions between the different weak neutrino states [55].

• 1967 - Pontecorvo considered the effects of all different types of oscillations in light of what was then known and pointed out –before any results from the Davis experiment were known– that the rate in that experiment could be reduced by a factor of two compared with predictions [56].

The final piece of the neutrino puzzle was provided by a combination of several independent experiments. The Super-Kamiokande experiment reported evidence for the oscillation of atmospheric neutrinos, while the Sudbury Neutrino Observatory confirmed flavor change in solar neutrinos by measuring the ratio between charged-current and neutral-current fluxes [57, 58]. Additionally, the KamLAND experiment observed neutrino oscillations with reactor anti-neutrinos [59]. Together, these experiments provided conclusive evidence for the phenomenon of neutrino oscillation, which had eluded scientists for decades.

The unavoidable conclusion from all the current data is that neutrinos oscillate from one flavor to another, meaning that they are massive particles. The status of neutrino oscillations, including results from different experiments, can be found in Ref. [60] where the authors perform a global fit of the oscillation parameters consistent with neutrino mixing, and their results are shown in Table 1.2. And even further, the latest results from KATRIN [61] derive an upper limit of 0.8 eV on the absolute
mass scale of neutrinos, giving the tightest results to date.

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<tbody>
<tr>
<td>$\Delta m^2_{21}$</td>
<td>$7.50^{+0.22}_{-0.20}$</td>
<td>$7.50^{+0.22}_{-0.20}$</td>
</tr>
<tr>
<td>$</td>
<td>\Delta m^2_{31}</td>
<td>$</td>
</tr>
<tr>
<td>$\sin^2 \theta_{12}$</td>
<td>$0.318^{+0.016}_{-0.016}$</td>
<td>$0.318^{+0.016}_{-0.016}$</td>
</tr>
<tr>
<td>$\sin^2 \theta_{23}$</td>
<td>$0.574^{+0.014}_{-0.014}$</td>
<td>$0.578^{+0.010}_{-0.017}$</td>
</tr>
<tr>
<td>$\sin^2 \theta_{13}$</td>
<td>$0.02200^{+0.000069}_{-0.000062}$</td>
<td>$0.02225^{+0.000064}_{-0.000070}$</td>
</tr>
<tr>
<td>$\delta/\pi$</td>
<td>$1.08^{+0.13}_{-0.12}$</td>
<td>$1.58^{+0.15}_{-0.16}$</td>
</tr>
</tbody>
</table>

Table 1.2: Best fit of neutrino oscillation data extracted from Ref. [60]. NH (IH) refers to the normal (inverted) hierarchy in neutrino masses. Notice that $\Delta m^2_{31} > (<) 0$ in the normal (inverted) hierarchy.

For a fermion field, Lorentz invariance allows two types of mass terms: $\overline{\Psi} \Psi$ and $\overline{\Psi} \Psi = \Psi^T C^{-1} \Psi$ [62], where the superscript $c$ denotes charge conjugation and $C$ is the charge conjugation matrix. Under a $U(1)$ symmetry transformation ($\Psi \rightarrow e^{i\alpha} \Psi$), the first term is invariant, whereas the second one is not. The first mass term is called Dirac mass, and the second one is Majorana. Charged fermions (leptons and quarks) must be Dirac-like particles because they have $Q|q\rangle \neq 0$ and $Q|\ell\rangle \neq 0$, so the $U(1)$ symmetry is, in fact, $U(1)_{\text{em}}$. The same conclusion does not apply to neutrinos since $Q|\nu\rangle = 0$, and they can be Dirac fermions (i.e., $\nu \neq \nu^c$) if there is an exact $U(1)$ symmetry, the lepton number. Otherwise, neutrinos are Majorana fermions satisfying $\nu = \nu^c$. 
A Dirac mass term

If we expand the SM by adding a right-handed neutrino $\nu_R$, we could write a Lagrangian for the neutrino interacting with the Higgs boson, such that a neutrino mass could be generated:

$$\mathcal{L}_\text{Yukawa}^\nu = -Y_\nu \bar{L} H \nu_R + \text{H.c.},$$

(1.23)

where flavor indices are omitted for the sake of the discussion. After the electroweak symmetry breaking (EWSB), a Dirac mass is generated for neutrinos:

$$m_D = \frac{Y_\nu v}{\sqrt{2}}.$$  

(1.24)

The problem with this procedure is that, for a purely Dirac mass term, we need a Yukawa coupling in the order of $10^{-12}$ to be consistent with observations, which is perhaps hard to explain\(^2\). Since we need to add a neutral gauge singlet, no symmetry precludes us from writing down also a Majorana mass term. By including a new energy scale, it is possible to give an alternative explanation to neutrino masses due to lepton number violation (LNV).

A Majorana mass term

Our previous discussion stated the crucial fact that a Majorana mass term implies LNV. This accidental symmetry is conserved at tree-level in the Standard Model, which suggests the existence of new physics beyond the SM (BSM). It was first noted by Weinberg [63] that one can add to the SM an effective dimension-5 operator

\(^2\)Consider the case of the electron’s Yukawa coupling, $Y_{\ell e} \sim 10^{-6}$. 

16
such that, after EWSB, the non-zero vacuum expectation value generates a Majorana mass term for neutrinos:

\[ m_\nu = \frac{C_{\text{eff}} v^2}{\Lambda}. \]  

(1.26)

It is important to highlight the fact that, in contrast to the Dirac masses of the charged fermions, the neutrino mass is proportional to \( v^2 \) instead of being linear in \( v \). Also, Eq. (1.26) is consistent with current neutrino oscillation phenomena for \( C_{\text{eff}} \sim 1 \) and \( \Lambda \lesssim 10^{13} \) GeV.

An important feature of the \( d = 5 \) Weinberg operator in (1.25) is that it violates the lepton number by two units, i.e., \( \Delta L = 2 \) in contrast to the SM Lagrangian, which presents the lepton number conservation. Unfortunately, Eq. (1.25) presents the same limitations as any other effective operator since we are not able to answer some key questions about them. For instance,

<table>
<thead>
<tr>
<th>Key questions for effective operators</th>
</tr>
</thead>
<tbody>
<tr>
<td>Q1. What mechanism gives rise to it, meaning a UV-complete theory?</td>
</tr>
<tr>
<td>Q2. What is its associated mass scale ( \Lambda )?</td>
</tr>
<tr>
<td>Q3. Does the operator have a flavor structure?</td>
</tr>
</tbody>
</table>

There exist some scenarios where the effective operator in (1.25) arises only radiatively [64,65], but it is more common to assume that it is induced at tree-level by the exchange of heavy particles with masses \( M_{\text{heavy}} \sim \Lambda \), where \( \Lambda \) is the energy scale of new physics violating lepton number. A few examples are discussed below:
There could be “heavy” right-handed neutrinos, a scheme known as Type-I seesaw mechanism [1–6]

\[ \mathcal{L}_{\text{Yukawa}}^{\text{Type-I}} = -Y_N^{\alpha i} \bar{L}_\alpha \tilde{H} N_i + \text{H.c.}, \]  

(1.27)

where \( N_i \) are heavy, right-handed Majorana neutrinos; after integrating them out, we obtain the effective operator in Eq. (1.25) with \( \Lambda \sim M_N \), the mass of the right-handed neutrinos.

An alternative scenario, the so-called Type-II seesaw mechanism, introduces a complex \( SU(2)_L \) scalar triplet [66–71]

\[ \mathcal{L}_{\text{Yukawa}}^{\text{Type-II}} = -Y_\Delta^{\alpha\beta} L_\alpha^T C i\sigma^2 \Delta_L L_\beta + \text{H.c.}, \]  

(1.28)

where \( \Delta_L \) is a complex \( SU(2)_L \) scalar triplet whose neutral component acquires a non-zero vev after EWSB.

In addition to the SM field content, the Type-III seesaw mechanism [72, 73] consists of \( SU(2)_L \) fermion triplets with zero hypercharge,

\[ \mathcal{L}_{\text{Yukawa}}^{\text{Type-III}} = -Y_\Sigma^{\alpha i} \bar{L}_\alpha \Sigma_{R_i}^c i\sigma^2 H^* + \text{H.c.}, \]  

(1.29)

where \( \Sigma_{R_i}^c \) denotes the charge-conjugate of \( \Sigma_{R_i} \), \( i.e. \Sigma_{R_i}^c = C(\Sigma_{R_i})^T \) with \( C \) being the charge-conjugation matrix.

There are other widely used mechanisms, like inverse [74, 75] and linear [76, 77] seesaw models, or radiative neutrino mass models (see Ref. [78] for a comprehensive review).
1.4.2 Baryon Asymmetry of the Universe

One of the most important open questions in physics is closely related to our daily life. Even though, since the introduction of anti-matter in the context of the Dirac equation, it seems that matter and anti-matter behave similarly [79], we still ignore why our universe contains more baryonic matter than anti-matter. The observed baryon asymmetry of the universe (BAU) is usually quantified in terms of the baryon-to-photon number density based on the PLANCK 2018 data [80,81]

\[ \eta_B^{\text{obs}} = \frac{n_B}{n_\gamma} = (6.12 \pm 0.04) \times 10^{-10}, \]  

(1.30)

or the abundance, normalized to the entropy density \(s\),

\[ Y_B^{\text{obs}} = \frac{n_B}{s} = (8.71 \pm 0.06) \times 10^{-11}. \]  

(1.31)

It was first noticed by Andrei Sakharov\(^3\) in 1967 that three conditions are required to explain the non-vanishing net baryon abundance [7]:

**Sakharov conditions**

1. Baryon number \(B\) violation.
2. \(C\) and \(CP\) violation.
3. Either out-of-equilibrium conditions or \(CPT\) violation.

In principle, the SM contains some of these three ingredients. However, the effects of \(CP\) violation are too weak [82], and the Higgs mass excludes any electroweak phase transition in a pure SM context [83], so out-of-equilibrium conditions never materialized in this scenario. These facts provide evidence for physics beyond the SM [84].

\(^3\)Andrei Sakharov won the Nobel Peace Prize in 1975 for his work in human rights advocacy, disarmament, and peace.
In this dissertation, we focus our attention on the mechanism of leptogenesis (LG) to explain the BAU [8]. For the standard leptogenesis scenario, at least two heavy right-handed neutrinos $N_i$ are introduced, describing out-of-equilibrium dynamics. They generate the $CP$ asymmetry via the one-loop decay of the lightest right-handed neutrino [85, 86]. We consider the broadly studied situation [87–89] where the lepton asymmetry is produced in a single flavor, the neutrino masses are hierarchical ($M_1 \ll M_2, M_3$), and the decays of the two heavier neutrinos ($N_2$ and $N_3$) are neglected.

The produced lepton asymmetry is then rapidly converted into baryon asymmetry by $(B + L)$-violating sphaleron interactions [90]. As we mentioned before, there are some constraints in this standard scenario in order to ensure its viability. More precisely, a high-scale LNV is required, with $M_1 \gtrsim 10^9$ GeV [9].

**Sphaleron interactions**

Starting from a baryon symmetric universe ($B = 0$), we need $B$-violating processes to end up with a universe where $B \neq 0$. In the SM, these types of interactions are given by non-perturbative solutions of the electroweak field equations. While $(B - L)$ is conserved, $(B + L)$ is anomalous at transitions between different $SU(2)_L$ vacua [91]. Changes in $B$ and $L$ are related to topological effects (i.e., the variations in the Chern-Simons number $\Delta N_{cs} = \pm 1, \pm 2, \ldots$) such that

$$\Delta B = \Delta L = N_f \Delta N_{cs}.$$  \hspace{1cm} (1.32)

Here $N_f$ is the number of generations. In the SM, $\Delta B = \Delta L = \pm 3n \ (n \in \mathbb{N})$, so the smallest transition is $\Delta B = \Delta L = \pm 3$ [84].
The $SU(2)_L$ vacuum transitions (the so-called instantons) lead to an effective, lowest-order operator

$$\mathcal{O}_{B+L} = \prod_{i=1,2,3} (Q_i^r, Q_i^g, Q_i^b, L_i),$$  

(1.33)

which gives 12-fermion interactions depicted in Fig. 1.2, *e.g.*, 

$$\bar{\nu} + d + \tau \rightarrow d + 2s + 2b + t + \nu_e + \nu_\mu + \nu_\tau,$$  

(1.34a)

$$e + \mu + \tau \rightarrow 3u + 3d + 3c.$$  

(1.34b)

Figure 1.2: Pictorial representation of the effective operator $\mathcal{O}_{B+L}$. $L_e, L_\mu,$ and $L_\tau$ correspond to the first, second, and third generations of lepton doublets. $Q_{ud}, Q_{cs},$ and $Q_{tb}$ correspond to the first, second, and third generations of quark doublets.

At $T = 0$, the transition rate between different vacua is exponentially suppressed; thus, it is negligible [91]. However, the presence of a thermal bath makes the transition possible not by tunneling between vacua but through thermal fluctuations over the barrier separating these minima. When the temperature is greater than the height of the barrier, these $(B + L)$-violating processes (so-called sphalerons) are no longer suppressed by the Boltzmann factor and can occur at a significant rate.
To relate lepton number asymmetry to baryon number asymmetry, we consider that each quark, lepton, and Higgs field (generically, $\Psi_i$) in the weakly coupled plasma with temperature $T$ and volume $V$ is assigned a chemical potential $\mu_i$ [89,91]. There are $5N_f + 1$ chemical potentials in the SM with one Higgs doublet and $N_f$ generations of fermions. The asymmetry in particle/antiparticle number densities is given by the derivative of the grand potential $\Omega(\{\mu_i\}, T)$,

$$n_i - \bar{n}_i = -\frac{\partial \Omega(\{\mu_i\}, T)}{\partial \mu_i},$$  \hspace{1cm} (1.35)

where the grand potential is expressed in terms of the grand partition function $Z(\{\mu_i\}, T, V)$,

$$Z(\{\mu_i\}, T, V) = \text{Tr} \left\{ \exp \left[ -\beta \left( \mathcal{H} - \sum_i \mu_i Q_i \right) \right] \right\},$$  \hspace{1cm} (1.36a)

$$\Omega(\{\mu_i\}, T) = -\frac{T}{V} \ln \left[ Z(\{\mu_i\}, T, V) \right],$$  \hspace{1cm} (1.36b)

where $\beta = 1/T$, $\mathcal{H}$ is the Hamiltonian operator, and $Q_i$ is the charge operator for the corresponding field. For a non-interacting gas of massless particles, assuming $\beta \mu_i \ll 1$, Eq. (1.35) becomes

$$n_i - \bar{n}_i \approx n_i \left\{ \begin{array}{ll} \frac{1}{6} g_i T^3 & (\text{fermions}), \\ \beta \mu_i & (\text{bosons}), \\ 2\beta \mu_i & \end{array} \right. ,$$  \hspace{1cm} (1.37)

where $g_i$ is the number of internal degrees of freedom of the corresponding field. In the high-temperature plasma, some processes give rise to constraints among various chemical potentials in thermal equilibrium, for example, [89,91]

1. The effective operator $\mathcal{O}_{B+L}$ induced by the EW sphalerons imposes the relation

$$\sum_i (3\mu_q_i + \mu_{\ell_i}) = 0 .$$  \hspace{1cm} (1.38)
2. The QCD sphaleron processes\(^4\) leading to interactions between left- and right-handed quarks when in equilibrium, impose

\[\sum_i (2\mu_{q_i} - \mu_{u_i} - \mu_{d_i}) = 0.\] (1.39)

3. The condition of the plasma having zero net hypercharge at all temperatures gives

\[\sum_i \left(\mu_{q_i} + 2\mu_{u_i} - \mu_{d_i} - \mu_{\ell_i} - \mu_{e_i} + \frac{2}{N_f} \mu_h\right) = 0.\] (1.40)

4. The Yukawa interactions between the Higgs doublet left- and right-handed fermions add three extra relations,

\[\mu_{q_i} - \mu_h - \mu_{d_j} = 0,\] (1.41a)
\[\mu_{q_i} + \mu_h - \mu_{u_j} = 0,\] (1.41b)
\[\mu_{\ell_i} - \mu_h - \mu_{e_j} = 0.\] (1.41c)

The mentioned relationships are valid if the associated interactions achieve thermal equilibrium. For Baryogenesis, the temperature range of interest is between 100 GeV and \(10^{12}\) GeV [84]. Within this range, gauge interactions typically reach equilibrium. Conversely, Yukawa interactions reach equilibrium within a narrower temperature range, which is dependent on the Yukawa coupling strength. However, we will disregard this complexity in our following discussion about leptogenesis and comment on the impact of including these effects in the next subsection.

\(^4\)These interactions are described by the operator \(\mathcal{O}_{\text{QCD}} = \prod_i (Q_i Q_i^c u_{R_i}^c d_{R_i}^c).\)
From Eq. (1.37), we can define the baryon number density

\[ n_B \equiv n_b - \bar{n}_b = \frac{1}{6} g_b T^2 B, \quad (1.42) \]

where the baryon number can be expanded in terms of the chemical potentials,

\[ B = \sum_i (2\mu_{q_i} + \mu_{u_i} + \mu_{d_i}). \quad (1.43) \]

Analogously for the lepton number density,

\[ n_L \equiv n_\ell - \bar{n}_\ell = \frac{1}{6} g_\ell T^2 L, \quad (1.44a) \]
\[ L = \sum_i (2\mu_{\ell_i} + \mu_{e_i}). \quad (1.44b) \]

Consider the case where all Yukawa interactions are in equilibrium and assume equilibrium among different generations, i.e., \( \mu_{\ell_i} \equiv \mu_\ell, \mu_{q_i} \equiv \mu_q, \mu_{u_i} \equiv \mu_u, \) and \( \mu_{d_i} \equiv \mu_d \). Therefore, all the chemical potentials can then be expressed in terms of \( \mu_\ell \) and the corresponding \( B \) and \( L \) asymmetries are

\[ B = -\left( \frac{4N_f}{3} \right) \mu_\ell, \quad (1.45a) \]
\[ L = \left( \frac{14N_f^2 + 9N_f}{6N_f + 3} \right) \mu_\ell. \quad (1.45b) \]

We conclude that \( B, L, \) and \( (B - L) \) are related by:

\[ B = c_s (B - L), \quad L = (c_s - 1)(B - L), \quad (1.46) \]

where the constant \( c_s \) is given by [91]
\[ c_s = \frac{8N_f + 4}{22N_f + 13} . \quad (1.47) \]

For three generations of fermions, \( N_f = 3 \), we have

\[ c_s = \frac{28}{79} \approx 0.35443 , \quad (1.48a) \]
\[ (c_s - 1) = -\frac{51}{79} \approx -0.64557 . \quad (1.48b) \]

For models with \( N_H \) Higgs doublets, the parameter \( c_s \) is given by,

\[ c_s = \frac{8N_f + 4N_H}{22N_f + 13N_H} . \quad (1.49) \]

Consequently, the \((B + L)\)-violating processes allow us to convert \( L \) asymmetry into \( B \) asymmetry. Because the \((B - L)\) current is anomaly-free, the value of the \((B - L)\) asymmetry at a time \( t = t_f \), after leptogenesis is completed, determines the value of the baryon asymmetry today \( (t = t_0) \),

\[ B(t_0) = c_s (B - L)(t_f) . \quad (1.50) \]

**Spectator processes**

During leptogenesis, the densities of various particle species can change due to multiple phenomena. Some of them, such as the decay of heavy neutrinos or other LNV interactions that washout the lepton asymmetry, happen at a pace similar to the Universe’s expansion rate. Thus, they require proper description through Boltzmann equations, as it will be discussed in Section 2.2.

On the other hand, some \((B - L)\)-conserving processes can occur quite rapidly, depending on the temperature, and result in specific relations between the chemical potentials of different particle species, as shown in Eqs. (1.38)–(1.41). These are
denoted as “spectator processes” as they do not change the lepton number directly; instead, they change the lepton and Higgs doublets densities on which the rates of the washout processes depend. The topic of spectator processes and their impacts on leptogenesis was first discussed in Ref. [92], and a comprehensive analysis of the numerical importance of each spectator process is available in Ref. [93].

The rate at which the Universe expands depends on the temperature differently than the rates of various particle interactions. As the temperature decreases, more interactions begin to occur faster than the Universe’s expansion rate and reach equilibrium. Therefore, the equilibrium conditions evolve along with the temperature changes [89,93]. For example, at temperatures above $10^{13}$ TeV, only gauge and top-Yukawa interactions are in equilibrium, yielding the relation $c_s = -1$. On the other hand, the strong sphalerons reach equilibrium at $T \sim 10^{13}$ TeV; while the relation $c_s = -1$ also holds for this case, the Higgs number asymmetry is reduced by a factor of $21/23$, which enhances the washout effects. The quantitative significance of the different spectator processes and their equilibrium temperatures can be found in Refs. [89,93].
CHAPTER 2
THEORETICAL AND COMPUTATIONAL ANALYSIS TOOLS

2.1 Introduction

Particle and nuclear physics are fascinating and intricate fields of study that seek to uncover the Universe’s fundamental building blocks. This complexity requires using both theoretical and computational tools for analysis, as these two aspects often complement and support each other.

Theoretical frameworks provide a robust foundation for developing hypotheses and predictions, which are then tested using experimental data. These theories are continuously refined and reevaluated as new information becomes available, driving the progression of our understanding of the underlying physics. Computational tools, on the other hand, play a crucial role in facilitating the analysis of large datasets generated by particle detectors in accelerator experiments or solving complex differential equations numerically.

In this chapter, we will present transversal concepts and techniques to the entire dissertation. This chapter will serve as the basis for the more specialized topics covered in subsequent chapters by providing a comprehensive overview of the fundamental principles and computational methodologies.

As we delve into this chapter, it should be noted that a significant portion of the content is devoted to reviewing established work in the field, providing a founda-
tion for the following explorations. However, amidst this review, we also introduce original research and unique methodologies developed specifically for this work. Our contributions include developing and customizing several key tools designed to address specific challenges in our study. We have carefully tailored these tools to ensure accurate and efficient analysis, pushing beyond conventional methods to pioneer new approaches to these complex problems.

2.2 Thermal leptogenesis

One of the most popular explanations for the observed baryon asymmetry is baryogenesis via leptogenesis [8]. In this mechanism, a lepton asymmetry is generated via the \((B - L)\)-violating decays of right-handed heavy neutrinos. Due to the interference of the tree-level and one-loop contribution to the decay [85,86], and the presence of at least two right-handed neutrinos, a net \(CP\) asymmetry can occur. When the decays fall out of equilibrium during the cooling of the Universe, a final lepton asymmetry is generated. If this happens before the electroweak phase transition, the Standard Model electroweak sphaleron processes can transfer this lepton asymmetry into a baryon asymmetry. In the standard leptogenesis scenario, the same interactions that induce the right-handed neutrino decay also cause \(\Delta L = 1\) and \(\Delta L = 2\) scattering processes that can destroy this asymmetry, the so-called “washout” processes again. If the latter effects are too strong, the generated asymmetry can be again destroyed.

We consider the broadly studied situation [87–89] where the lepton asymmetry is produced in a single flavor, the right-handed neutrino masses are hierarchical \((M_1 \ll M_2, M_3)\), and the production of the two heavier neutrinos \((N_2\) and \(N_3)\) are neglected. The interaction part of the Lagrangian is given by:

\[
\mathcal{L}_{\text{int}} = Y_{\nu i} \bar{L}(i\tau^2)H^* N_i - \frac{M_i}{2} \bar{N}_i^c N_i + \text{H.c.},
\]

(2.1)
where $L = (\nu_L, e_L)^T$ refers to the lepton first generation. Notice that the interactions of $N_1$ violate lepton number regardless of their assignments. If $L(N_1) = 1$, then the mass term in Eq. (2.1) violates lepton number by two units ($\Delta L = 2$); if $L(N_1) = 0$, then the Yukawa term violates it by one unit ($\Delta L = 1$). An immediate consequence of this is the LNV decay of $N_1$.

### 2.2.1 $CP$ violation

Since $N_1$ can decay to both $N_1 \to LH$ and $N_1 \to \overline{LH}^*$, a net lepton asymmetry will be produced if there is an asymmetry in the rates. The asymmetry in the decay of $N_1$ is defined by

$$\epsilon \equiv \frac{\Gamma(N_1 \to LH) - \Gamma(N_1 \to \overline{LH}^*)}{\Gamma(N_1 \to LH) + \Gamma(N_1 \to \overline{LH}^*)}, \quad (2.2)$$

where, by definition, $|\epsilon| \leq 1$. The $CP$ asymmetry parameter $\epsilon$ arises from the interference of the tree-level and the one-loop diagrams depicted in Fig. 2.1.

The analytical expression for $\epsilon$ can be decomposed into two contributions, the vertex correction $\epsilon_V$ in Fig. 2.1 (middle) and the wave function piece $\epsilon_W$ in Fig. 2.1 (right), and is given by [94,95]

$$\epsilon = \epsilon_V + \epsilon_W, \quad (2.3a)$$

$$\epsilon_V = \frac{1}{8\pi} \sum_{k \neq 1} I_{k1} g_V(x_k), \quad (2.3b)$$
\[ \epsilon_W = -\frac{1}{8\pi} \sum_{k \neq 1} I_{k1} g_W(x_k), \quad (2.3c) \]

where we have defined
\[ I_{k1} \equiv \frac{\text{Im} \left[ (Y^\dagger_\nu Y_\nu)_{k1}^2 \right]}{(Y^\dagger_\nu Y_\nu)_{11}} \quad (2.4) \]

\[ x_j \equiv M_j^2/M_1^2, \]

and
\[ g_V(x) = \sqrt{x} \left[ 1 - (1 + x) \ln \left( \frac{1 + x}{x} \right) \right], \quad (2.5a) \]
\[ g_W(x) = -\frac{\sqrt{x}}{1-x}. \quad (2.5b) \]

Notice that, for \( x \gg 1 \),
\[ g_V(x) - g_W(x) \longrightarrow -\frac{3}{2\sqrt{x}} - \frac{5}{6x^{3/2}} + \ldots \quad (2.6) \]

In the hierarchical limit \( (M_1 \ll M_2, M_3) \),
\[ \epsilon \simeq \frac{3}{16\pi} \frac{1}{(Y^\dagger_\nu Y_\nu)_{11}} \sum_{k \neq 1} \text{Im} \left[ (Y^\dagger_\nu Y_\nu)_{k1}^2 \right] \frac{M_1}{M_k}. \quad (2.7) \]

### 2.2.2 Boltzmann equation

In out-of-equilibrium transport phenomena, the Boltzmann equation formalizes the statement that the rate of change in the abundance of a given particle is the difference between the rates for producing and annihilating that species [96]:

\[ \mathcal{L}[f_X] = \mathcal{C}[f_X], \quad (2.8) \]

where \( \mathcal{L} \) is the Liouville operator (variation with respect to proper time), \( \mathcal{C} \) is the collision term (net change due to scattering in/out, decays/inverse decays, etc.), and
$f_X$ is the distribution function of a given particle $X$.

After introducing the number density

$$n_X = g_X \int \frac{d^3p}{(2\pi)^3} f_X,$$  \hspace{1cm} (2.9)

where $g_X$ is the degeneracy factor of $X$ counting for the internal degrees of freedom, we can integrate Eq. (2.8) over the phase space. The left-hand side (LHS) becomes [86,87]

$$\frac{g_X}{(2\pi)^3} \int \frac{d^3p}{2E_X} \tilde{C}[f_X] = \dot{n}_X + 3H n_X,$$  \hspace{1cm} (2.10)

where $H$ is the Hubble expansion rate, and the right-hand side (RHS) of Eq. (2.8) can be written as

$$\frac{g_X}{(2\pi)^3} \int \frac{d^3p}{2E_X} C[f_X] = - \int d\Pi_X \cdots d\Pi_i d\Pi_j (2\pi)^4 \delta^4(p_X + p_a + \cdots - p_i - p_j - \cdots)$$

$$\times \left\{ |\mathcal{M}|^2_{Xa\rightarrow ij... f_X} f_a \cdots (1 \pm f_i) (1 \pm f_j) \cdots$$

$$- |\mathcal{M}|^2_{ij...\rightarrow Xa... f_if_j} f_a \cdots (1 \pm f_X) (1 \pm f_a) \cdots \right\},$$  \hspace{1cm} (2.11)

where $d\Pi_X = d^3p_X/(2\pi)^3 2E_X$ and $|\mathcal{M}|^2$ is the squared transition amplitude summed over initial and final polarizations. Henceforth, we neglected Pauli blocking or Bose enhancement, which is to a good accuracy valid ($\langle E \rangle \approx 3T$ such that $(1 \pm f_X) \approx 1$ [97]). Then, the RHS of Eq. (2.8) becomes

$$\frac{g_X}{(2\pi)^3} \int \frac{d^3p}{2E_X} C[f_X] = - \sum_{a,i,j,...} [Xa \cdots \leftrightarrow ij \cdots]$$  \hspace{1cm} (2.12)

where we have defined
\[ [Xa \cdots \leftrightarrow ij \cdots] \equiv \frac{n_X n_a \cdots}{n_X^{(eq)} n_a^{(eq)} \cdots} \gamma^{(eq)}(Xa \cdots \rightarrow ij \cdots) - \frac{n_i n_j \cdots}{n_i^{(eq)} n_j^{(eq)} \cdots} \gamma^{(eq)}(ij \cdots \rightarrow Xa \cdots), \quad (2.13) \]

with \( n_X^{(eq)} \) the equilibrium number density of the particle \( X \), and \( \gamma^{(eq)} \) as the space-time density of scatterings/decays, in thermal equilibrium, of the various interactions in which an \( X \)-particle takes part:

\[
\gamma^{(eq)}(Xa \rightarrow ij) = \int d\Pi_X d\Pi_a d\Pi_i d\Pi_j \]
\[
(2\pi)^4 \delta^{(4)}(p_N + p_a - p_i - p_j)f_N f_a |M|^2. \quad (2.14)
\]

By neglecting \( CP \)-violating effects, the inverse processes have the same reaction densities [87].

Consequently, the integrated Boltzmann equation is given by:

\[
\dot{n}_X + 3H n_X = - \sum_{a,i,j,\ldots} [Xa \cdots \leftrightarrow ij \cdots]. \quad (2.15)
\]

From a cosmological point of view, the Hubble expansion rate can be written as

\[
H = \frac{\dot{a}}{a}, \quad (2.16)
\]

where \( a(t) \) is the expansion parameter. Hence, the term \( 3H \) in Eq. (2.15) is taking into account the dilution due to the expansion of the universe. It is convenient to reabsorb it by normalizing the number density \( n_X \) to the entropy density \( s \). If we introduce the abundance \( Y_X \equiv n_X/s \), then the LHS of Eq. (2.15) can be written as

\[
\dot{n}_X + 3H n_X = \dot{Y}_X s + (\dot{s} + 3H s)Y_X = \dot{Y}_X s + \left( \dot{s} + \frac{3s \dot{a}}{a} \right) Y_X. \quad (2.17)
\]

Assuming an adiabatic and isentropic expansion,
\[ 0 = \frac{d}{dt} \left( s a^3 \right) = \dot{s} a^3 + 3 s a^2 \dot{a} = a^3 \left( \dot{s} + \frac{3s \dot{a}}{a} \right), \tag{2.18} \]

in light of which Eq. (2.17) becomes:

\[ \dot{n}_X + 3H n_X = s \dot{Y}_X. \tag{2.19} \]

Additionally, during radiation era, \( a \propto t^{1/2} \) and \( H \propto t^{-1} \). Let us introduce the dimensionless quantity \( z \equiv M_1/T \), where \( M_1 \) is the mass of the lightest right-handed neutrino. Since \( H \propto T^2 \) as well, both time \( t \) and this dimensionless variable \( z \) are related to each other (\( dt \propto z \, dz \)). Therefore, we can study the evolution of the abundance \( Y_X = n_X/s \) as a function of \( z \) instead of time \( t \) [87]:

\[ Y_N \equiv \frac{n_{N1}}{s}, \tag{2.20a} \]

\[ zHs \frac{dY_N}{dz} = - \sum_{a,i,j,...} \left[ N_1 a \leftrightarrow ij \cdots \right], \tag{2.20b} \]

where \( Y_N \) is the abundance of the lightest right-handed neutrino. The Hubble expansion rate, as a function of \( z \), is simply

\[ H(z) = \sqrt{\frac{8\pi^3 g_* M_1^2}{90}} \frac{1}{M_{Pl} z^2} \simeq 1.66 \sqrt{\frac{g_* M_1^2}{M_{Pl} z^2}}, \tag{2.21} \]

where \( g_* = g_{SM} = 106.75 \) is the total number of degrees of freedom, and \( M_{Pl} = 1.22 \times 10^{19} \) GeV is the Planck mass.

For decays, Eq. (2.14) reduces to

\[ \gamma^{(eq)}(N_1 \rightarrow ij \cdots) = \gamma^{(eq)}(ij \cdots \rightarrow N_1) = n_{N1}^{(eq)} K_1(z) \frac{K_2(z)}{\Gamma_D}, \tag{2.22} \]

with
\[ n_{N1}^{(eq)} = \frac{3\zeta(3)gNT^3}{8\pi^2}z^2K_2(z), \quad (2.23) \]

where \( K_\ell(z) \) is the modified Bessel function of the second kind of order \( \ell \) and \( \Gamma_D \) is the total decay width of the lightest right-handed neutrino. At zero temperature or \( z \to \infty \),

\[ \Gamma_D = \Gamma_D(z \to \infty) = g_w \frac{|Y_{\nu 1}|^2M_1}{16\pi^2}. \quad (2.24) \]

The factor \( g_w = 2 \) serves as a reminder of the two fundamental \( SU(2)_L \) degrees of freedom in the isospin doublet. It is customary to define the \textit{standard washout parameter} \[8,88,98]\]

\[ K \equiv \frac{\Gamma_D(z \to \infty)}{H(z = 1)}, \quad (2.25) \]

with \( H(z) \) given in Eq. (2.21), that distinguishes the two parametric regimes of weak (\( K \ll 1 \)) and strong (\( K \gg 1 \)) washout.

On the other hand, the \( 2 \leftrightarrow 2 \) scattering term is given by

\[ \gamma^{(eq)}(ij \leftrightarrow ab) = \frac{T}{64\pi^4} \int_{s_{\text{min}}}^{\infty} ds \sqrt{s} \tilde{\sigma}(s) K_1 \left( \frac{\sqrt{s}}{T} \right) \quad (2.26) \]

where \( \tilde{\sigma} \equiv 2s \lambda[1, m_i^2/s, m_j^2/s] \sigma \) is the \textit{reduced cross-section}, \( \sigma \) is the total cross-section summed over initial and final quantum numbers, \( \lambda[a, b, c] \equiv (a - b - c)^2 - 4bc \) is the Källen function, and \( s_{\text{min}} = \max\{(m_i + m_j)^2, (m_a + m_b)^2\} \).

2.2.2.1 Decay, Scattering, and Washout terms

By neglecting sphalerons, the relevant LNV processes for thermal leptogenesis are shown in Figure 2.2. Notice that diagrams (a), (b), and (c) correspond to the \( CP \) asymmetry discussed in Section 2.2.1. These processes can be organized as follows:
Figure 2.2: Feynman diagrams contributing to standard, thermal leptogenesis. $U$ and $D$ are up- and down-quarks, $V$ is a $SU(2)_L \times U(1)_Y$ gauge boson, $L$ is a lepton, $H$ is a Higgs boson, and $N_1$ is the lightest right-handed neutrino.
• **Decay diagrams.** These $\Delta L = 1$ diagrams are denoted by

$$D \equiv [N_1 \leftrightarrow LH]. \quad (2.27)$$

• **Scattering diagrams with $\Delta L = 1$.** Involving the exchange of either a lepton $L$ or a Higgs $H$ in the $s$-channel

$$S_s = H_s + V_s, \quad (2.28a)$$

$$H_s \equiv [LN_1 \leftrightarrow UD], \quad (2.28b)$$

$$V_s \equiv [LN_1 \leftrightarrow H^*V], \quad (2.28c)$$

or in the $t$-channel

$$S_t = H_t + V_t, \quad (2.29a)$$

$$2H_t \equiv [N_1 \bar{U} \leftrightarrow D\bar{L}] + [N_1 \bar{D} \leftrightarrow U\bar{L}], \quad (2.29b)$$

$$2V_t \equiv [N_1 H \leftrightarrow V\bar{L}] + [N_1 V \leftrightarrow H^*\bar{L}]. \quad (2.29c)$$

• **Scattering diagrams with $\Delta L = 2$.** Involving the exchange of a right-handed neutrino $N_1$ in the $s$-channel

$$N_s \equiv [LH \leftrightarrow \bar{L}H^*], \quad (2.30)$$

or in the $t$-channel

$$N_t \equiv [LL \leftrightarrow H^*H^*]. \quad (2.31)$$
We assume that $N_1$, $L$, and $L$ can be out of thermal equilibrium, while $Y_X = Y_X^{(eq)}$ for $X = \{H, U, D, V\}$. Then, the set of Boltzmann equations for $N_1$, $L$, and $L$ are:

\[
\begin{align*}
    zHs \frac{Y_N}{dz} &= -D - \overline{D} - S_s - \overline{S}_s - S_t - \overline{S}_t, \quad (2.32) \\
    zHs \frac{Y_L}{dz} &= D - N_s - N_t - S_s - \overline{S}_t, \quad (2.33) \\
    zHs \frac{Y_{\overline{L}}}{dz} &= \overline{D} + N_s - N_t - \overline{S}_s + S_t. \quad (2.34)
\end{align*}
\]

Here, the bar denotes the exchange of particles by antiparticles in the previous definitions. Let us assume that the $CP$-asymmetry parameter satisfies $\epsilon \ll 1$. By the $CPT$ theorem, we have the following relations between the different decay rates:

\[
\begin{align*}
    \Gamma(N_1 \to LH) &= \Gamma(\overline{L}H^* \to N_1) = \frac{(1 + \epsilon)\Gamma_D}{2}, \quad (2.35a) \\
    \Gamma(N_1 \to \overline{L}H^*) &= \Gamma(LH \to N_1) = \frac{(1 - \epsilon)\Gamma_D}{2}, \quad (2.35b)
\end{align*}
\]

where $\Gamma_N$ is given in Eq. (2.24). Combining all of these results, we can write the decay contributions in Eq. (2.27) as:

\[
\begin{align*}
    D &= \frac{\gamma_D}{2} \left[ (1 + \epsilon) \frac{Y_N}{Y_N^{(eq)}} - (1 - \epsilon) \frac{Y_L}{Y_L^{(eq)}} \right], \quad (2.36a) \\
    \overline{D} &= \frac{\gamma_D}{2} \left[ (1 - \epsilon) \frac{Y_N}{Y_N^{(eq)}} - (1 + \epsilon) \frac{Y_L}{Y_L^{(eq)}} \right], \quad (2.36b)
\end{align*}
\]

where we have defined $\gamma^{(eq)}(N_1 \to LH) + \gamma^{(eq)}(N_1 \to \overline{L}H^*) \equiv \gamma_D$ for simplicity.

### 2.2.2.2 Subtracting the real intermediate states (RIS)

At leading order in the couplings, $2 \leftrightarrow 2$ scatterings must be calculated at tree-level ($CP$-conserving). Notice that in Eq. (2.30), the $LH \leftrightarrow \overline{L}H^*$ scattering cross-section in $N_s$ is mediated by an exchange of $N_1$ in the $s$–channel. If that particle is
on-shell, then the very same contribution is already considered into the BEs by the successive decay/inverse decay $LH \leftrightarrow N_1 \leftrightarrow \bar{L}H^*$. Consequently, it is necessary to subtract the real intermediate states to avoid double-counting.

Let us define $\gamma_{N_s}$ as the space-time density term corresponding to the $LH \leftrightarrow \bar{L}H^*$ scattering process. Therefore, its on-shell version will be

$$\gamma_{N_s}^{\text{on-shell}}(LH \rightarrow \bar{L}H^*) = \gamma^{(\text{eq})}(LH \rightarrow N_1) \ Br(N_1 \rightarrow \bar{L}H^*) = (1 - \epsilon)^2 \gamma_D \frac{\gamma_D}{4}.$$  \hspace{1cm} (2.37)

Consequently, and using the CTP theorem, we define the RIS density terms as

$$\gamma^{(\text{eq})}(LH \rightarrow \bar{L}H^*) \equiv \gamma_{N_s}(LH \rightarrow \bar{L}H^*) - \gamma_{N_s}^{\text{on-shell}}(LH \rightarrow \bar{L}H^*),$$

$$= \gamma_{N_s} - (1 - \epsilon)^2 \gamma_D \frac{\gamma_D}{4}, \hspace{1cm} (2.38a)$$

$$\gamma^{(\text{eq})}(\bar{L}H^* \rightarrow LH) \equiv \gamma_{N_s}(\bar{L}H^* \rightarrow LH) - \gamma_{N_s}^{\text{on-shell}}(\bar{L}H^* \rightarrow LH),$$

$$= \gamma_{N_s} - (1 + \epsilon)^2 \gamma_D \frac{\gamma_D}{4}, \hspace{1cm} (2.38b)$$

so then

$$N_s = \frac{Y_L}{Y_L^{(\text{eq})}} \gamma^{(\text{eq})}(LH \rightarrow \bar{L}H^*) - \frac{Y_{\bar{L}}}{Y_{\bar{L}}^{(\text{eq})}} \gamma^{(\text{eq})}(\bar{L}H^* \rightarrow LH),$$

$$= \frac{Y_{\Delta L}}{Y_L^{(\text{eq})}} \left( \gamma_{N_s} - \frac{\gamma_D}{4} \right) + \epsilon \gamma_D + O(\epsilon^2), \hspace{1cm} (2.39)$$

having defined the lepton asymmetry $Y_{\Delta L} \equiv Y_L - Y_{\bar{L}}$ and used the fact that $Y_L^{(\text{eq})} = Y_{\bar{L}}^{(\text{eq})}$ and $Y_L + Y_{\bar{L}} = 2Y_L^{(\text{eq})} + O(\epsilon)$. Finally, the resultant set of equations is

$$z H s \frac{dY_N}{dz} = - \left( \frac{Y_N}{Y_N^{(\text{eq})}} - 1 \right) \left( \gamma_D + 2\gamma_{S_s} + 4\gamma_{S_t} \right), \hspace{1cm} (2.40a)$$
\[ zHs \frac{dY_{\Delta L}}{dz} = \gamma_D \left[ \epsilon \left( \frac{Y_N}{Y_N^{\text{(eq)}}} - 1 \right) - \frac{Y_{\Delta L}}{Y_L^{\text{(eq)}}} \right] - \frac{Y_{\Delta L}}{Y_L^{\text{(eq)}}} \left( 2\gamma_{\text{sub}} + 2\gamma_{St} + \gamma_{Ss} \frac{Y_N}{Y_N^{\text{(eq)}}} \right), \]

\[ = \epsilon \gamma_D \left( \frac{Y_N}{Y_N^{\text{(eq)}}} - 1 \right) - \frac{Y_{\Delta L}}{Y_L^{\text{(eq)}}} \left( 2\gamma_{N} + 2\gamma_{St} + \gamma_{Ss} \frac{Y_N}{Y_N^{\text{(eq)}}} \right), \quad (2.40b) \]

where \( \gamma_{\text{sub}} \equiv \gamma_N + \gamma_{ns} - \gamma_D / 4 \) and \( \gamma_N = \gamma_{N_t} + \gamma_{N_s} \). The factors of 2 correspond to the inclusion of anti-particles and the \( u \)-channel scattering amplitudes. After rearranging Eq. (2.40a), we define the decay rate \( D(z) \) as

\[ D(z) \equiv \frac{\gamma_D}{zH(z)n_{N1}^{\text{(eq)}}} = \frac{1}{zH(z)} \frac{K_1(z)}{K_2(z)} \Gamma_D, \quad (2.41) \]

the \( \Delta L = 1 \) scattering rates in Eqs. (2.40a) and (2.40b), \( S(z) = 2S_s(z) + 4S_t(z) \), with

\[ S_{s/t}(z) = \frac{\gamma_{S_{s/t}}}{zH(z)n_{N1}^{\text{(eq)}}} = \frac{1}{zH(z)} \frac{M_1}{48\pi^2 \zeta(3) K_2(z)} I_{S_{s/t}}(z) \quad (2.42) \]

and the \( \Delta L = 2 \) scattering rates in Eq. (2.40b), \( N(z) = N_s(z) + N_t(z) \), with with

\[ N_{s/t}(z) = \frac{\gamma_{N_{s/t}}}{zH(z)n_{N1}^{\text{(eq)}}} = \frac{1}{zH(z)} \frac{M_1}{48\pi^2 \zeta(3) K_2(z)} I_{N_{s/t}}(z), \quad (2.43) \]

where we have introduced the dimensionless version of the integral in Eq. (2.26),

\[ I_{S_{s/t},N_{s/t}}(z) \equiv \int_{x_{\text{min}}}^{\infty} dx \sqrt{x} \hat{\sigma}(s_{s/t},n_{s/t})(x) K_1(\sqrt{x} z), \quad x \equiv \frac{s}{M_1^2}. \quad (2.44) \]

If we finally introduce the washout term \( W(z) \),

\[ W(z) = 2 \left[ N(z) + S_t(z) \right] \left( \frac{Y_N^{\text{(eq)}}}{Y_L^{\text{(eq)}}} \right) + S_s(z) \left( \frac{Y_N}{Y_L^{\text{(eq)}}} \right), \quad (2.45) \]

we can rewrite the set of Boltzmann equations written in its typical form (e.g., Refs. [87–89]):
The connection between the baryon asymmetry $Y_B$ and the lepton asymmetry $Y_{\Delta L}$ is provided by the $(B - L)$-conserving EW sphaleron and given in Eq. (1.46)

$$Y_B = \left( \frac{c_s}{c_s - 1} \right) Y_{\Delta L}.$$

### 2.2.2.3 Dealing with infrared (IR) divergences

Some of the diagrams in Fig. 2.2 present a challenge beyond the overcounting we faced in the previous part. For example, let us consider diagram (n); by calculating the correspondent cross-section, we notice an infrared (IR) divergence that artificially enhances that particular process. These soft and collinear IR divergences are a common feature in diagrams with massless gauge bosons in the external states. One possible way to address this difficulty is to introduce a regulator [99] by adding small thermal masses to these massless particles. A more rigorous treatment to cancel the divergences requires the inclusion of 1-loop diagrams to the three-body decay/inverse decay amplitudes [87], according to the KNL theorem [100, 101]. In Ref. [87], the authors perform a detailed analysis of IR divergences, and they conclude that it is safe to neglect the $\Delta L = 1$ diagrams involving a gauge boson $V$ after the proper divergences cancellation. We will follow this approach and drop the contributions previously described.

### 2.3 Computer tools for Collider simulations

Since the lepton number is not a gauge but an accidental symmetry, there is no fundamental reason it must be conserved. As we will describe in the next chapters, there are well-motivated attempts to look for LNV at different energy scales.
Although low-energy experiments can provide some constraints, such as probes for Charged Lepton Flavor Violation (CLFV) (for example, $\mu \rightarrow eee$, $\mu \rightarrow e\gamma$, or $\mu \rightarrow e$ conversion), they do not directly test lepton number. Particle colliders, on the other hand, offer a direct way to test the lepton number. One approach that has garnered attention involves looking for specific final states in $pp$ collisions. A common feature among these different searches is using a final state with two same-sign leptons plus two jets ($pp \rightarrow jj\ell^\pm\ell^\pm$), as in Refs. [102,103], for instance. The current status of LNV searches does not show significant deviations from the SM, so a better understanding of both signal and background is needed.

2.3.1 Background for same-sign dilepton searches

Backgrounds in the same-sign dilepton final state can be divided into three categories [102–104], where the latter two are depicted in Fig. 2.3.

- **SM processes with same-sign dileptons**, including diboson production (considering $W^\pm$, $Z$, $H$, and prompt $\gamma$), single boson production in association with a $t\bar{t}$ pair, and “rare” processes (e.g., $t\bar{t}\ell\ell$ and double-parton scattering).

- **Charge misidentification** from events with opposite-sign isolated leptons in which the charge of an electron is misidentified, mostly due to severe *bremsstrahlung* in the tracker material. Simulation-based studies conclude that the electron charge misidentification probability is $O(10^{-4} - 10^{-1})$, while for muons is negligible.

- **Jet-fake leptons** from heavy-flavor decays, where hadrons are misidentified as leptons, or electrons from unidentified conversions of photons in jets. Depend-

\footnote{This range can be obtained from Fig. 2.5.}
2.3.2 Monte Carlo simulations

To perform our collider phenomenology study, we need to generate events for both the signal and background using Monte Carlo (MC) simulation. In the case of SM processes with same-sign leptons, we rely on MadGraph 5 aMC@NLO [107] to generate the parton level events, PYTHIA8 [108] for parton showering and hadronization (including jet matching to avoid overcounting), and Delphes3 [109] for fast detector simulation. The flow chart of the event simulation is depicted in Fig. 2.4.

A similar process is needed to study the BSM events, but a new step is required. After choosing a model to study, the correspondent Lagrangian needs to be translated into a series of files to be interpreted by Madgraph. These files contain information about parameters, particles, and interactions present in the model. Currently, the two
Figure 2.4: Illustration of the Monte Carlo event simulation. The forward direction is
typically understood as the simulation stage, while the inverse direction corresponds
to the analysis and unfolding stage. Adapted from Refs. [113,114].

most popular software to perform this task are FEYNRULES [110] and SARAH [111].

Finally, we need to process the data generated in the simulation. For particle physics,
there is a data analysis framework having implemented Lorentz transformations, 4-
vector properties, and particle classes called ROOT [112].

Due to the difficulty of precisely simulating jet fakes, we follow a simple approach
taking into advantage the relation between the originating jet and the fake lepton
[115]. This method is based on two functions:

- A **mistag efficiency**, \( \epsilon_{j \rightarrow \ell} = \epsilon_{j \rightarrow \ell}(j_{\text{orig}}) \), representing the probability that a par-
ticular jet \( j_{\text{orig}} \) is mistagged as a lepton. In a general case, this efficiency is a
function of the jet’s transverse momentum:

\[
\epsilon_{j \rightarrow \ell}(p_T j) = \epsilon_{200} \left[ 1 - (1 - r_{10}) \left( \frac{200 - p_T j / \text{GeV}}{200 - 10} \right) \right], \tag{2.48}
\]

where

\[
\epsilon_{200} \equiv \epsilon_{j \rightarrow \ell}(200 \text{ GeV}), \tag{2.49a}
\]
\[
r_{10} \equiv \frac{\epsilon_{j \rightarrow \ell}(10 \text{ GeV})}{\epsilon_{j \rightarrow \ell}(200 \text{ GeV})}. \tag{2.49b}
\]
• A transfer function, $T_{j \rightarrow e} = T_{j \rightarrow e}(j_{\text{orig}}, e_{\text{fake}})$, which represents a probability distribution function for the properties of the originating jet $j_{\text{orig}}$ to be inherited to the fake electron $e_{\text{fake}}$. The transfer function is assumed as a truncated Gaussian distribution with mean $\mu$ and standard deviation $\sigma$, normalized to unit-area in the interval $\alpha \in (0, 1)$:

$$T_{j \rightarrow e}(\alpha) = \frac{1}{\sqrt{2\pi}\sigma N} \exp \left( -\frac{(\alpha - \mu)^2}{2\sigma^2} \right),$$

where

$$\alpha \equiv 1 - \frac{pT_{e_{\text{fake}}}}{pT_{j_{\text{orig}}}},$$

$$N \equiv \frac{1}{2} \left[ \text{Erf} \left( \frac{1 - \mu}{\sqrt{2}\sigma} \right) + \text{Erf} \left( \frac{\mu}{\sqrt{2}\sigma} \right) \right].$$

Using real data or simulations from ATLAS or CMS Collaborations (e.g., Refs. [102,103]), it is possible to fit the set of parameters $\{\epsilon_{200}, r_{10}, \mu, \sigma\}$ to find consistent results for different theoretical studies, as it was shown in Refs. [116–118]. We have implemented this method directly in DELPHES and generated the $t\bar{t}$ and $W^\pm + 3j$ backgrounds.

Finally, to estimate the charge misidentification background, we introduce a misidentification probability for electrons from current LNV ATLAS searches [119], while for muons, we assume no misidentification at all [102]. The electron charge misidentification probability is modeled as a separable function of electron’s $p_T$ and $\eta$, $P(p_T, \eta) = \sigma(p_T)f(\eta)$, and its data-driven values are shown in Fig. 2.5. We follow a common approach in collider phenomenological studies [120,121] to incorporate this probability as a weight in opposite-sign generated events: $w = P_1(1-P_2) + P_2(1-P_1)$, where $P_i$ is the misidentification probability of the $i^{th}$ electron. Therefore, we consider
that an $e^+e^-$ event could give rise to either $e^+e^+$ or $e^-e^-$ events.

### 2.3.3 Machine Learning for particle physics

Particle physics has increasingly turned to Machine Learning (ML) due to its potential to transform data analysis and simulations. As of May 16, 2023, the *HEPML-Living Review*\(^2\) that provides a comprehensive list of citations for developing and applying ML approaches to experimental, phenomenological, or theoretical analyses, has 788 papers. In fact, the training and testing of a neural network serve as a cornerstone of this dissertation, which will be detailed in Chapter 3. Machine Learning’s ability to learn from and make predictions based on data makes it an invaluable tool in the analysis and interpretation of the complex and vast datasets typically associated with particle physics experiments. Furthermore, Machine Learning’s contribution is

not limited to data analysis. It also plays a significant role in enhancing the precision of particle simulations. We will dedicate this section to discussing how ML assists the simulation chain, facilitating both comprehensive analyses and precise simulations.

In Ref. [114], the author summarizes some of the most evident characteristics of high-energy research that make ML the appropriate approach:

- **Data volume.** Both instrumentalists and theorists generate large amounts of data, both labeled (at the simulation level) and unlabeled (at the detector level); ML algorithms are trained and tested using large datasets.

- **Complexity.** Some of the data structures involved in different analyses are high-dimensional and highly correlated; ML is expressive (the ability to represent a wide range of functions or relations) and interpretable (how easily the behaviors and predictions of a model can be understood).

- **Signal detection.** Typical analyses involve rare and elusive signals among large backgrounds; ML has high accuracy and sensitivity.

- **Computing Budget.** The combined processes of simulation and analysis are computationally expensive; ML is comparatively faster.

- **Increasing interest.** More than 150 paper/year; ML is fun!³

Cutting-edge ML techniques hold significant potential for enhancing all aspects of event generation. By increasing resource efficiency, they pave the way for more versatile and accurate predictions. ML algorithms like neural networks have found applications in improving the modules of classic event generators, such as phase space

³Direct quote from Ref. [114].
sampling, evaluation of scattering amplitudes, calculation of loop integrals, and sim-
ulation of parton showers [113]. They have also been used in the determination of
parton distribution functions. Some ML algorithms can even be trained to perform
end-to-end event generation at the parton level, with or without considering detector
effects [113,114].

ML is also assisting in inverting the simulation chain, going beyond merely enhanc-
ing traditional signal/background discrimination methods like cut-based analysis or
Boosted Decision Trees (BDTs). State-of-the-art ML architectures like U-net, graph
neural networks (GNNs), and DeepSets are now being used in ML-based particle re-
construction, providing an alternative to the traditional particle flow approach [113].

Efforts are being made to implement ML-unfolding of detector effects and poste-
rior unfolding at the parton level from both classifier and density-based approaches
[113,114]. Inference of parameters and the Matrix Element Method (MEM), used
for estimating fundamental physics parameters from individual events, are now get-
ning assistance from ML architectures like Invertible Neural Networks (INNs) and
Bayesian neural networks [113,114].

2.4 Neutrinoless Double Beta Decay

Double beta ($\beta\beta$) decay is a nuclear transition from an initial nucleus with mass
and atomic numbers ($A, Z$) to a final nucleus with ($A, Z + 2$) [122]. The first observed
decay mode was the two-neutrino double-beta ($2\nu\beta\beta$) decay, where two electrons and
two anti-neutrinos are emitted [123],

\begin{equation}
(A, Z) \rightarrow (A, Z + 2) + 2e^- + 2\bar{\nu}_e + Q_{\beta\beta},
\end{equation}
where $Q_{\beta\beta}$ is the energy released during the transition. This decay is consistent with the SM predictions, with a typical half-life of $> 10^{19}$ yr. There is a theoretical process very similar to $2\nu\beta\beta$ decay, but in this transition, no neutrinos are emitted:

$$(A, Z) \rightarrow (A, Z + 2) + 2e^{-} + Q_{\beta\beta}. \quad (2.53)$$

This SM-forbidden mode is known as neutrinoless double beta ($0\nu\beta\beta$) decay. Its importance relies on the fact that it is a sensitive probe for Majorana neutrino masses [124] since it violates lepton number by two units, $\Delta L = 2$, which is incompatible with a Dirac-like nature of neutrinos. The Schechter-Valle theorem (also known as Black Box theorem) [125] states that the observation of $0\nu\beta\beta$ decay proves that neutrinos are Majorana particles. Graphically the theorem can be depicted as shown in Figure 2.6. For a version of the Schechter-Valle theorem including lepton flavor violation, see Ref. [126].

If $0\nu\beta\beta$ decay is observed, then neutrinos have a Majorana mass term at least at the 4-loop level. However, the previous statement does not preclude the existence of a lower loop-level leading contribution. For example, as we mentioned before, the Type-I seesaw model provides a tree-level mass term. Moreover, the Black Box induced Majorana mass [127] provides a correction $\delta m_{\nu} = \mathcal{O}(10^{-28}$ eV), a contribution too small to be compatible with neutrino oscillation experiments (see Table 1.2).
Contributions to $0\nu\beta\beta$ decay can be separated into two categories, depending on the mass of the mediator particle. Fig. 2.7 shows the two possibilities: short-range and long-range [128–130]. We will focus our attention on the short-range interactions since they can be accessible for both the LHC and the next generation of $0\nu\beta\beta$-decay experiments. These contributions correspond to all processes where no light neutrinos are exchanged. They can be understood, from the perspective of Effective Field Theory (EFT), as dimension-9 effective vertices $O_{d=9} \propto \bar{u} d \bar{d} e \bar{e} e^c$. At tree-level, there are only two possible topologies [130] for these kinds of operators, which are shown in Fig. 2.8.

For topology I, the internal particles between vertices $v_1 - v_2$ and $v_3 - v_4$ can be either scalars or vectors, while the particle between vertices $v_2 - v_3$ must be a fermion. On the other hand, for topology II, all the internal particles must be either scalars or vectors. For a systematic decomposition and study of these different operators and topologies, see Ref. [130].
Figure 2.8: The two basic tree-level topologies realizing a dimension-9 $0\nu\beta\beta$ operator. External lines are fermions; internal lines can be fermions (solid) or scalars, or vectors (dashed). Extracted from Ref. [130].

### 2.4.1 Effective Operators

As we stressed in the previous sections, the fundamental mechanism giving birth to LNV at an energy scale $\Lambda$ is unknown, if there even is one. In order to address this difficulty, the Effective Field Theory (EFT) approach presents several advantages. The first application of EFT in the context of $0\nu\beta\beta$ decay calculations was presented in Ref. [131], where the basic framework was established and largely developed in Refs. [132–136]. This method is depicted in Fig. 2.9, and the main ideas are listed below:

- We assume that LNV occurs at an energy scale $\Lambda \gg m_W$. After integrating out the heavy, beyond SM fields, there is a set of gauge-invariant $\Delta L = 2$ operators, starting with the $d = 5$ operator in Eq. (1.25). In this dissertation, we mostly study gauge-invariant $\Delta L = 2$ operators in the Standard Model Effective Field Theory (SMEFT) with dimensions seven and nine.

- At scales below $m_W$, after electroweak-symmetry breaking, the Higgs field obtains its vacuum expectation value (vev), and heavy particles ($t$, $h$, $Z$, and $W^\pm$) are integrated out of the theory; we match our operators to a low-energy SMEFT. We evolve these operators to the GeV scale by considering the one-loop QCD renormalization of such operators.
• At the GeV scale, quark operators are matched onto Chiral Perturbation Theory (χPT) [137,138]. χPT is the effective theory of QCD which describes interactions at low energy in terms of baryons, mesons, photons, and leptons. Hadronic or lattice QCD inputs in the form of low energy constants (LECs) are required.

• At the $\mathcal{O}(10-100 \text{ MeV})$ scale, we combine all the possible interactions (short- and long-range) to form the two-nucleon $nn \rightarrow ppee$ transition operator in Chiral Effective Field Theory (χEFT), to be used in many-body nuclear calculations.

• We finally evaluate the $0\nu\beta\beta$-decay transition operators between the ground states of the relevant nuclei. The nuclear many-body matrix elements, together with the phase space factors and short-distance Wilson coefficients, combine to give the $0^+ \rightarrow 0^+$ decay rate.

Below the electroweak scale, the set of $SU(3)_C \times U(1)_{em}$ invariant four-quark two-lepton operators can be written as [131,132,134]

$$\mathcal{L}_{\Delta L=2}^{(9)} = \frac{1}{v^5} \sum_i \left[ (C^\alpha_i e_R \bar{C} e_R^T + C^\alpha_i e_L \bar{C} e_L^T) O_i + C^\alpha_i e_R \bar{e} \gamma_5 C e_T O_i^\mu \right],$$

(2.54)

where $O_i$ and $O_i^\mu$ are four-quark operators that are Lorentz scalars and vectors, respectively. Both the scalar and vector operators have been discussed in Refs. [131, 132,134] and can be written as

\begin{align}
O_1 &= \bar{q}_L^\alpha \gamma_\mu \tau^+ q_L^\alpha \bar{q}_L^\beta \gamma_\mu \tau^+ q_L^\beta, & O'_1 &= \bar{q}_R^\alpha \gamma_\mu \tau^+ q_R^\alpha \bar{q}_R^\beta \gamma_\mu \tau^+ q_R^\beta, \\
O_2 &= \bar{q}_R^\alpha \gamma_\mu \tau^+ q_L^\alpha \bar{q}_R^\beta \gamma_\mu \tau^+ q_L^\beta, & O'_2 &= \bar{q}_L^\alpha \gamma_\mu \tau^+ q_R^\alpha \bar{q}_L^\beta \gamma_\mu \tau^+ q_R^\beta, \\
O_3 &= \bar{q}_R^\alpha \gamma_\mu \tau^+ q_R^\beta \bar{q}_R^\beta \gamma_\mu \tau^+ q_R^\beta, & O'_3 &= \bar{q}_L^\alpha \gamma_\mu \tau^+ q_R^\alpha \bar{q}_L^\beta \gamma_\mu \tau^+ q_R^\beta, \\
O_4 &= \bar{q}_L^\alpha \gamma_\mu \tau^+ q_L^\beta \bar{q}_R^\beta \gamma_\mu \tau^+ q_R^\beta, & O'_4 &= \bar{q}_L^\alpha \gamma_\mu \tau^+ q_L^\alpha \bar{q}_R^\beta \gamma_\mu \tau^+ q_R^\beta, \\
O_5 &= \bar{q}_L^\alpha \gamma_\mu \tau^+ q_L^\beta \bar{q}_R^\beta \gamma_\mu \tau^+ q_R^\alpha.
\end{align}

(2.55)
Figure 2.9: A schematic overview of the EFT approach to evaluate the $0\nu\beta\beta$-decay amplitude starting from high-scale $\Delta L = 2$ dynamics. Adapted from Ref. [134].
\( O_\mu^6 = (\bar{q}_L \tau^+ \gamma^\mu q_L) (\bar{q}_L \tau^+ q_R) , \quad O_\mu' = (\bar{q}_R \tau^+ \gamma^\mu q_R) (\bar{q}_R \tau^+ q_L) , \quad \) (2.56a)

\( O_\mu^7 = (\bar{q}_L \tau a \gamma^\mu q_L) (\bar{q}_L \tau q_L a \tau) + \gamma^\mu q_L \), \quad O_\mu' = (\bar{q}_R \tau a \gamma^\mu q_R) (\bar{q}_R \tau a \gamma^\mu q_R) + \gamma^\mu q_R , \quad \) (2.56b)

\( O_\mu^8 = (\bar{q}_L \tau^+ \gamma^\mu q_L) (\bar{q}_L \tau^+ q_R) + q_L , \quad O_\mu' = (\bar{q}_R \tau^+ \gamma^\mu q_R) (\bar{q}_R \tau^+ q_L) + q_R , \quad \) (2.56c)

\( O_\mu^9 = (\bar{q}_L t a \tau \gamma^\mu q_L) (\bar{q}_L t a \tau^+ q_R) + q_L , \quad O_\mu' = (\bar{q}_R t a \tau \gamma^\mu q_R) (\bar{q}_R t a \tau^+ q_L) + q_R , \quad \) (2.56d)

where \( q_{L,R} = (u, d)_{L,R} \) are left- and right-handed isospinors, \( \tau^\pm = (\tau_1 \pm i \tau_2)/2 \) with \( \tau_i \) the Pauli matrices, and \( \alpha, \beta \) are color indices. In Eq. (2.55), the \( O_i' \) operators are related to the \( O_i \) by parity. In Eq. (2.56), the second column of operators is also related to the first column by a parity transformation.

As we stated before, after integrating out the heavy SM fields, the next step is to match the four-quark components of the operators in Eqs. (2.55) and (2.56) onto \( \chi \)PT. As a perturbation theory, we organize the power counting using the expansion parameter \( \epsilon_\chi \equiv p/\Lambda_\chi \), where \( p \sim m_\pi \) is the typical momentum transfer and \( \Lambda_\chi \approx 1 \text{ GeV} \) is the chiral symmetry-breaking scale. This corresponds to the ratio between soft scales \( Q \) and hard scales [139].

The basic idea is to enumerate all potential operators that exhibit the same chiral symmetry properties under \( SU(2)_L \times SU(2)_R \) as the quark-level operators. Each of these operators is associated with an unknown coefficient that characterizes the non-perturbative aspect of QCD. These low-energy constants (LECs) can be approximated using naïve dimensional analysis (NDA) [131,140], a power-counting scheme to keep track of the scaling factors\(^4\)

\(^4\)They can also be derived from experimental data or existing lattice QCD computations.
\[ \mathcal{L}_{NDA} = F_\pi^2 \Lambda_\chi^2 \left( \frac{\bar{N}N}{\Lambda_\chi F_\pi^2} \right)^a \left( \frac{\partial^\mu}{\Lambda_\chi} \right)^b \left( \frac{\pi}{F_\pi} \right)^c, \]  

where \( \Lambda_\chi \equiv 4\pi F_\pi \) and \( F_\pi \) is the pion decay constant. To account for the lepton bilinears in the operators, which can be constructed to depend on the electron mass and/or the outgoing electron momenta, we assign \( Q \sim m_e \sim m_\pi \epsilon^2 \) \cite{133} to incorporate this new scale into the \( \chi PT \) power counting.

In Refs. \cite{131,134}, the authors characterize each of the contributions to the chiral Lagrangians in terms of the number of pion and nucleon fields, as shown in Fig. 2.10. For example, the scalar operators \( O_1-O_5 \) in Eq. (2.55) generate the \( \pi\pi ee \), \( \pi NN ee \), and the \( NNNN ee \) LNV couplings, and the mesonic chiral Lagrangian for them is \cite{134}

\[ \mathcal{L}_{\pi\pi}^{\text{scalar}} = \frac{F_0^4}{4} \left[ \frac{5}{3} g_{11}^{\pi\pi} C_{1L}^{(9)} L^{21 \mu} L_{21 \mu} + \left( g_{22}^{\pi\pi} C_{2L}^{(9)} + g_{33}^{\pi\pi} C_{3L}^{(9)} \right) \text{Tr} \left\{ U^{\tau+} U^{\tau+} \right\} 
\]

\[ + \left( g_{44}^{\pi\pi} C_{4L}^{(9)} + g_{55}^{\pi\pi} C_{5L}^{(9)} \right) \text{Tr} \left\{ U^{\tau+} U^{\tau+} \right\} \right] \frac{e_L^C e_L^T}{\epsilon^5} + \left( L \leftrightarrow R \right) + \ldots, \]  

(2.58)
where $U = u^2 = \exp(i \pi \cdot \tau / F_0)$ is the matrix of pseudo-Goldstone boson fields, $F_0$ is the pion decay constant in the chiral limit, and $L^\mu_{k\ell} = i (UD^\mu U^\dagger)_{k\ell}$. We adopt $F_\pi = 92.2 \text{ MeV}$ for the physical pion decay constant. As highlighted in Ref. [134], the estimation of the LECs using NDA is expected to be $g^{\pi\pi}_{2,3,4,5} = \mathcal{O}(\Lambda^2)$, while $g^{\pi\pi}_1 = \mathcal{O}(1)$.

Notice that, as firstly discussed in Ref. [131], the operators $O_{2,3,4,5}$ induce leading order (LO), non-derivative $\pi\pi ee$ operators. In contrast, the first $\pi\pi ee$ operators induced by $O_1$ contain next-to-next-to-leading order (NNLO), two derivatives operators; therefore, they are relatively suppressed [134].

The rest of the $\pi NN ee$ and $NNNN ee$ terms for dimension-9 scalar operators, as well as vector operators, are carefully discussed in Ref. [134]; we present a summary of the results in the next paragraphs.

The $\pi NN ee$ operators are only relevant for the $O_1$ operator and can be written as

$$\mathcal{L}^{\text{scalar}}_{\pi NN} = g_A g_{1N} C^{(9)}_{1L} F_0^2 \left[ N S^\mu u_{\tau+} u_N \mathrm{Tr} \left\{ u_\mu u_{\tau+} u^\dagger \right\} \right] \frac{\bar{e}_L C\bar{e}_L^T}{v^5} + (L \leftrightarrow R)$$

$$= \sqrt{2} g_A g_{1N} C^{(9)}_{1L} F_0 \left[ \bar{b} S \cdot (\partial \pi^-) n \right] \frac{\bar{e}_L C\bar{e}_L^T}{v^5} + (L \leftrightarrow R) + \ldots , \quad (2.59)$$

where $u_\mu = u^\dagger L_\mu u = i \left[ u (\partial_\mu - i r_\mu) u^\dagger - u^\dagger (\partial_\mu - i l_\mu) u \right]$, $g_A \simeq 1.27$, and $N = (p, n)^T$ are the nucleon fields characterized by the nucleon spin $S^\mu$ and the nucleon velocity $v^\mu$. In the nucleon restframe we have $S^\alpha = (0, \sigma / 2)$ and $v^\mu = (1, 0)$. While unknown, the LEC $g^{\pi N}_1$ is expected to be $\mathcal{O}(1)$. 
Similarly, an NDA-based power counting suggests that the $NNNNe$ interactions are only relevant for the $O_1$ operator, and they would be as relevant as the $\pi\pi ee$ and the $\pi NNee$ interactions [131]. Nonetheless, as highlighted by Ref. [141], the $nn \rightarrow ppee$ scattering amplitude has a logarithmic UV divergence that can be absorbed by promoting the $NNNNe$ interactions stemming from the $O_{2,3,4,5}$ scalar operators to leading order. Consequently, the relevant $NNNNe$ interactions can be written as

$$L_{NNN}^{\text{scalar}} = g_{1}^{NN} C_{1L}^{(9)} (\bar{N} u^\dagger \tau^+ uN)(\bar{N} u^\dagger \tau^+ uN) \frac{\bar{e}_L C e_L^T}{v^5}$$

$$+ \left( g_{2}^{NN} C_{2L}^{(9)} + g_{3}^{NN} C_{3L}^{(9)} \right) (\bar{N} u^\dagger \tau^+ u^\dagger N)(\bar{N} u^\dagger \tau^+ u^\dagger N) \frac{\bar{e}_L C e_L^T}{v^5}$$

$$+ \left( g_{4}^{NN} C_{4L}^{(9)} + g_{5}^{NN} C_{5L}^{(9)} \right) (\bar{N} u^\dagger \tau^+ u^\dagger N)(\bar{N} u^\dagger \tau^+ u^\dagger N) \frac{\bar{e}_L C e_L^T}{v^5} + (L \leftrightarrow R)$$

$$= \left( g_{1}^{NN} C_{1L}^{(9)} + g_{2}^{NN} C_{2L}^{(9)} + g_{3}^{NN} C_{3L}^{(9)} + g_{4}^{NN} C_{4L}^{(9)} + g_{5}^{NN} C_{5L}^{(9)} \right) (\bar{p}n)(\bar{p}n) \frac{\bar{e}_L C e_L^T}{v^5}$$

$$+ (L \leftrightarrow R) + \ldots , \quad (2.60)$$

where $g_{i}^{NN} = O(1)$ in the Weinberg power counting, but need to be promoted to $O((4\pi)^2)$ in the case of $O_{2,3,4,5}$.

For the $O_{1,2,3}$ operators, which are related by parity to $O_{1,2,3}$, the $\pi\pi ee$, $\pi NNee$, and $NNNNe$ Lagrangians have the same form (with $C_{1,2,3(L,R)}^{(9)} \rightarrow C_{1,2,3(L,R)}^{(9)}$) after expanding the meson matrices $u$ (and $U$) in Eqs. (2.58), (2.59), and (2.60) to two, one, and zero pions, respectively.

As pointed out initially in the seminal work in Ref. [131] and later in Ref. [132], the vector operators induce mesonic interactions ($\pi\pi ee$) involving a derivative on the pion fields, which give rise to contributions proportional to the electron mass. Conversely, $\pi NNee$ and $NNNNe$ interactions are proportional to the pion momentum. Therefore, they are larger than the purely mesonic contributions by a factor of $1/\epsilon_\chi$. 

56
Now, we can obtain an expression for the inverse half-life for $0^+ \rightarrow 0^+$ transitions. Let us define the $0\nu\beta\beta$-decay amplitude [134],

\[
A = \frac{g_A^2 G^2_{F} m_e}{\pi R_A} \left[ A_\nu \bar{u}(k_1) P_R C \bar{u}^T(k_2) + A_R \bar{u}(k_1) P_L C \bar{u}^T(k_2) \right. \\
+ A_E \bar{u}(k_1) \gamma_0 C \bar{u}^T(k_2) \left( \frac{E_1 - E_2}{m_e} \right) \\
+ A_{m_e} \bar{u}(k_1) C \bar{u}^T(k_2) + A_M \bar{u}(k_1) \gamma_0 \gamma_5 C \bar{u}^T(k_2) \right],
\] (2.61)

where $E_{1,2}$ ($k_{1,2}$) are the energies (momenta) of the outgoing electrons. An overall factor from the various sub-amplitudes $A_i$ was extracted, in particular the ratio of $m_e/R_A$, where $m_e$ is the electron mass and $R_A = 1.2 A^{1/3} \text{fm}$ in terms of $A$, the number of nucleons of the final nucleus. These sub-amplitudes depend on the Wilson coefficients of the $\Delta L = 2$ operators, on hadronic matrix elements and nuclear matrix elements, and are provided in Refs. [133,134].

Finally, by using the decomposition of the amplitude in Eq. (2.61), we obtain the Master Formula for the $0\nu\beta\beta$-decay rate [134]:

\[
(T_{1/2}^{0\nu})^{-1} = g_A^4 \left[ G_{01} \left( |A_\nu|^2 + |A_R|^2 \right) - 2(G_{01} - G_{04}) \text{Re}\{A_\nu A_R^*\} + 4G_{02} |A_E|^2 \\
+ 2G_{04} \left( |A_{m_e}|^2 + \text{Re}\{A_{m_e}^* (A_\nu + A_R)\} \right) \right. \\
- 2G_{03} \text{Re}\{(A_\nu + A_R) A_E^* + 2A_{m_e} A_E^*\} \\
+ G_{09} |A_M|^2 + G_{06} \text{Re}\{(A_\nu - A_R) A_M^*\} \right],
\] (2.62)

where $G_{0i}$ are phase space factors. Explicit examples of the use of this so-called Master Formula can be found in Refs. [134,142] (for the minimal left-right symmetric model) and Appendix A (for the simplified model presented in Chapter 3).
2.4.1.1 Power counting and chiral suppression

To finish this section, we will expand on the basics of the formalism described above. The classification of the different operators in Eq. (2.54) relies on our ability to organize the effective lepton-hadron operators in an expansion in powers of $\epsilon_\chi \equiv p/\Lambda_\chi$, where $p \sim m_\pi$ is the typical momentum transfer and $\Lambda_\chi \simeq 1\text{ GeV}$ is the chiral symmetry-breaking scale.

The chiral order of the $\Delta L = 2$ operators can be found in Refs. [131,134], and it is summarized in Table II and Table 3, respectively. We will show an explicit example for the scalar operators, except operators $O_1$ and $O'_1$ which give a chirally suppressed contribution proportional to $\partial_\mu \pi \partial^\mu \pi ee$. We follow the discussion in Refs. [134,143].

The contributions of dimension-9 scalar operators to the $0\nu\beta\beta$-decay amplitude is given by

$$A_{\text{scalar}} = \frac{g^2 A_G^2 m_e}{\pi R_A} \left[ A_\nu \bar{u}(k_1) P_R C \bar{u}^\top(k_2) + A_R \bar{u}(k_1) P_L C \bar{u}^\top(k_2) \right],$$

(2.63)

where the reduced amplitudes $A_\nu$ and $A_R$ are

$$A_\nu(R) = \frac{m^2_N}{m_e v} \left[ -\frac{1}{2m^2_N} C^{(9)}_{\pi \pi \nu L(R)} M_{PS,\text{sd}} + \frac{m^2_\pi}{2m^2_N} C^{(9)}_{\pi \nu \nu L(R)} M_{P,\text{sd}} - \frac{2}{g^2_\nu m^2_N} C^{(9)}_{\pi \nu \nu \nu L(R)} M_{F,\text{sd}} \right],$$

(2.64)

where $C^{(9)}_{\pi \pi \nu L(R)}$, $C^{(9)}_{\pi \nu \nu L(R)}$, and $C^{(9)}_{\pi \nu \nu \nu L(R)}$ are linear in the Wilson coefficients $C_i$ of the dimension-9 scalar operators, and in the LECs. In Eq. 2.64, we explicitly see that the purely mesonic contribution is $O(\epsilon^0_\chi)$, while the $\pi NN ee$ and $NNNNNe$ contributions are $O(\epsilon^2_\chi)$ and they are chirally suppressed. For the LO $\pi \pi ee$ contribution, the power-counting estimate of the contribution of each reduced amplitude is given

58
by $m_e A_{e(R)} \sim \Lambda^2 / v$. We can iterate this process for all the dimension-9 operators to complete the power-counting estimates shown in Table 2.1.

To appreciate the utility of this naïve power counting, let us compare the LNV vector operator with the scalar operators in terms of their relative contribution to the $0\nu\beta\beta$-decay amplitude. In the absence of any underlying model and consequently no linear ordering of the magnitudes of the Wilson coefficients of these operators, the vector operators are quantitatively smaller than the scalar operators in $0\nu\beta\beta$ decay due to the suppression of the $0\nu\beta\beta$-decay amplitude by a factor of $O(\epsilon^{-2}) \sim 60$. To see this, notice that

1. The purely mesonic scalar operators generate the $\pi\pi ee$ interaction at LO order, and they are $O(\epsilon^0_\chi)$ in power counting, according to Table 2.1.

2. The vector operators are mapped onto the interactions $\pi NN ee$ and $NNNN ee$, and they are $O(\epsilon^2_\chi)$ in power counting, according to Table 2.1. Consequently, these contributions are suppressed in chiral power counting.

We will see in Chapter 4 that this naïve estimation requires considering the size of the NMEs, and the suppression is not as large as expected by using only chiral power counting.
Table 2.1: Power-counting estimates of the contribution of low-energy dimension-9 operators to the amplitudes in Eq. (2.61), in terms of the Higgs vev $v$ and expansion parameter $\epsilon_\chi \equiv m_\pi / \Lambda_\chi$, where $\Lambda_\chi \sim m_N \sim 1$ GeV. The electron mass and energies are taken to scale as $E_1 \sim E_2 \sim m_e \sim \Lambda_\chi \epsilon_\chi^3$. This table assumes the NMEs follow the $\chi$EFT power counting. $C^{(9)}_{\text{vector}}$ indicates any of the vector operators in Eq. (2.54). Note that to estimate the overall scaling of the amplitudes, one needs to take into account that, up to insertions of dimensionless couplings, the Wilson coefficients scale as follows: $C^{(9)}_{1L, 4L, 5L} = \mathcal{O}(v^3 / \Lambda^3)$ or $\mathcal{O}(v^5 / \Lambda^5)$ (depending on the underlying model) and $C^{(9)}_i = \mathcal{O}(v^5 / \Lambda^5)$ for the remaining dimension-9 operators. Extracted and adapted from Ref. [134].
3.1 Introduction and Motivation

As stated in the Introduction, the presence of LNV interactions in nature allows us to explore its possible associated mass scale, Λ. In the absence of further experimental information, it is interesting to ask whether LNV interactions may live at multiple scales, including not only the high scale required by standard thermal leptogenesis but also at lower scales. In this Chapter, we investigate the possibility that additional LNV interactions may exist with the associated Λ as low as $O(\text{TeV})$. LNV interactions at this scale could contribute directly to $0\nu\beta\beta$ decay at an observable level, regardless of the contribution from the light neutrino spectrum. On the other hand, observation of an LNV signal in high energy $pp$ collisions, such as pairs of same sign leptons and an associated di-jet pair, would point to LNV at the TeV scale. One would naturally wonder about the resulting implications for leptogenesis.

In the subsequent sections, we explore these implications in detail using a concrete, simplified model for illustration. This setup allows us to (a) identify the concrete parameter space still viable for leptogenesis, (b) find the direct comparison of the reach of $0\nu\beta\beta$-decay experiments and collider experiments, including their future prospects, and (c) account for detailed early universe dynamics. Our study builds on earlier analyses that were performed in an effective field theory (EFT) context [144–147]. For sufficiently heavy Λ, one may integrate the new degrees of freedom, yielding a set of
non-renormalizable LNV interactions built from SM fields only. These operators have odd mass dimension $d$, starting with the well-known “Weinberg operator” ($d = 5$) that gives rise to light neutrino Majorana masses. Importantly, the EFT analyses imply that in the context of TeV scale LNV, observation of a signal in either $0\nu\beta\beta$ decay and/or $pp$ collisions could be fatal for the viability of standard thermal leptogenesis. The reason is that the observation of LNV interactions based on higher dimensional operators would imply a strong washout of a pre-existing lepton-asymmetry that was potentially generated at a high scale.

EFT analyses, however, are limited in their validity and lack information on concrete values for couplings and masses of the new physics (NP) involved. The importance of including RGE running of the Wilson coefficients from the TeV scale to the GeV scale and the inclusion of background in the collider analysis are key aspects that were previously not considered. Hence, we confront in this work the previous EFT results with a simplified model. Doing so allows us to investigate in detail the interplay of leptogenesis with LNV searches. The choice of simplification in the model is intended to highlight the connection between $0\nu\beta\beta$ decay and collider phenomenology in light of the viability of thermal leptogenesis in a spirit of broadness and generality. In the same logic, in the context of analyzing $0\nu\beta\beta$ decay, the simplified model of Refs. [148, 149] gives rise to the leading-order (LO) long-range pion-exchange amplitude expected to have the maximal impact on the $0\nu\beta\beta$-decay rate. Among the subset of simplified models that share this feature (see Refs. [131,132] and references therein), the one that we adopt here minimally extends the SM in terms of particles and interactions.

We include the new interactions explicitly in the leptogenesis Boltzmann equations with thermal mass effects taken into account, allowing us to analyze in detail
the dependence of the baryon asymmetry of the Universe on the masses and couplings associated with the new particles and their interactions. From the Boltzmann equation solutions, we identify the regions of the model mass and coupling parameter space for which the TeV scale LNV interactions would render unviable standard thermal leptogenesis (also assuming the presence of the heavy right-handed neutrino as described above). We then utilize state-of-the-art hadronic and nuclear physics methods relevant to $0\nu\beta\beta$ decay and machine learning techniques for collider LNV searches to delineate the sensitivity of these probes to the leptogenesis unviable parameter space.

This Chapter is based on Ref. [150].

3.2 A simplified model for TeV-scale LNV

We consider the standard thermal leptogenesis scenario introduced and described in Section 2.2. Henceforth, we will refer to the lightest right-handed neutrino $N_1$ simply as $N$, dropping the flavor subscript. The interaction Lagrangian in Eq. (2.1) is rewritten as

$$\mathcal{L} = y_N \bar{L}(i\tau^2)H^*N - \frac{m_N}{2}N^c\tilde{N}N + \text{H.c.},$$

(3.1)

where $y_N \equiv Y_{\ell_1}$ and $m_N \equiv M_1$ in Eq. 2.1. While assuming that a lepton-asymmetry might have been generated via the decay of right-handed neutrinos at a high scale, we want to investigate the impact of additional LNV interactions at the TeV scale. For these purposes, we adopt a simplified model framework that has been previously used to explore the $0\nu\beta\beta$ decay and collider interplay [148,149]. This particular model represents a possible realization of the dim-9 effective operator studied in Refs. [131,132,144] as we will discuss in more detail later.
As discussed in Refs. [148, 149], one possible minimal model that gives rise to the LO $\pi\pi ee$ interactions responsible for $0\nu\beta\beta$ decay\(^1\) (see Section 3.5 for a detailed discussion) includes a scalar $S$ transforming as $(1, 2, 1)$ under $SU(3)_C \times SU(2)_L \times U(1)_Y$ and a Majorana fermion $F$ that transforms as an SM gauge singlet\(^2\). The Lagrangian reads

\[
\mathcal{L} = g_Q \overline{Q} S d_R + g_L \overline{L} (i \tau^2) S^* F - m_S^2 S^\dagger S - \frac{m_F}{2} \overline{F} c F + \lambda_{HS} (S^\dagger H)^2 + \text{H.c.} + \ldots, \tag{3.2}
\]

where $Q = (u_L, d_L)^T$ and $q_R = (u_R, d_R)^T$ are the left-handed and right-handed quark isospinors, respectively. In a full, UV-complete theory such as RPV SUSY [152], $S$ and $F$ are identified as the slepton and the lightest neutralino fields, respectively. The ellipsis in Eq. (3.2) indicates other possible terms such as $S H^3$ and $S^3 H$. For simplicity, we will omit those terms and also assume that the heavy neutrino $N$ will not interact with the new fields introduced. It is important to notice that our simplified model assumes $\langle S \rangle = 0$ at tree-level. Although the scalar potential is, in principle, arbitrary and a positive $S$-mass term could accommodate a zero VEV, these assumptions are made for the sake of simplicity since their effects are not necessarily related to our focus on the $0\nu\beta\beta$ decay/collider interface.

Besides generating small Majorana neutrino masses via the Type-I seesaw mechanism induced by the right-handed neutrino $N$, additional contributions can be generated at one-loop level via the interactions in the Lagrangian (3.2), as shown in Fig. 3.1. The magnitude of the contribution to the neutrino mass matrix is controlled by the coupling $\lambda_{HS}$ and can be estimated as

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\(^1\)As discussed in Refs. [131, 132, 134, 151], several quark-lepton effective operators can give rise to different interactions in the chiral Lagrangian. See Ref. [143] for an example of a simplified model mapping onto $\pi NNee$ interactions.

\(^2\)Note, we use the convention $Y = Q_{EM} - T_3$. 
Figure 3.1: One-loop contribution to the Weinberg operator [153] induced by the interactions in Eq. 3.2. Note that the magnitude of this contribution is proportional to the coupling $\lambda_{HS}$ that does not enter the amplitudes for $0\nu\beta\beta$-decay or same-sign dilepton plus di-jet production in $pp$ collisions.

$$m_\nu \sim \frac{\lambda_{HS} (g_L \langle H \rangle)^2}{\Lambda} \frac{1}{16\pi^2}.$$  \hspace{1cm} (3.3)

If $S$ acquires a non-zero VEV, the mixing between $S$ and $H$ will modify the estimation in Eq. (3.3). We will not address this possibility here since our focus is on illustrating the interplay between colliders and $0\nu\beta\beta$-decay experiments.

For low energy $0\nu\beta\beta$-decay process, the heavy particles in the Lagrangian (3.2) can be integrated out, yielding the effective dim-9 LNV interaction:

$$\mathcal{L}_{\text{eff}, \text{LNV}} = C_1 \frac{\Lambda^5}{\Lambda^5} O_1 + \text{H.c.,} \quad O_1 = \overline{Q} \tau^+ d \overline{Q} \tau^+ d \overline{L} L^c.$$  \hspace{1cm} (3.4)

We can match

$$C_1 = g_L^2 g_Q^2 \quad \text{and} \quad \Lambda^5 = m_S^4 m_F .$$  \hspace{1cm} (3.5)

Interestingly, this demonstrates that TeV-scale masses for $m_S$ and $m_F$ are not in conflict with constraints from neutrino masses, (3.3), as in such a model realization, the contribution to $0\nu\beta\beta$ decay (depending on $g_L, g_Q$ only) is independent of lowest order contribution to the neutrino mass (depending on $\lambda_{HS}$). In the following, we will study the impact of the new interactions in (3.2) on the baryon asymmetry generated from the heavy right-handed neutrinos and their detection possibilities at colliders.
3.2 The realization of the $0\nu\beta\beta$-decay dim-9 operator induced by the interactions in Eq. 3.2. The scalar $S$ transforms as $(1, 2, 1)$ under $SU(3)_C \times SU(2)_L \times U(1)_Y$, and the Majorana fermion $F$ transforms as an SM gauge singlet.

3.3 Leptogenesis

In the standard leptogenesis scenario, if the washout processes are too strong, the generated asymmetry can be whipped out. In Ref. [154], the authors show that the observation of a generic $\Delta L = 2$ LNV signal at the LHC or in $0\nu\beta\beta$-decay experiments (via an operator of a dimension seven or higher) would directly imply a significant washout rate and hence would render the asymmetry generation insufficient [144, 145]. While this interplay has been previously described in an effective field theory approach only, we want to investigate this within our simplified model set-up as described in Section 3.2. To this end, we analyze the potential to generate the observed baryon asymmetry within our simplified model, which consists of the SM extended by a right-handed neutrino (standard leptogenesis scenario) and an additional new physics contribution as defined in Eq. (3.2) leading to additional LNV washout processes experimentally accessible at the TeV scale.

Hereby, we indicate the contributions that arise from Eq. (3.2), with a tilde ($\tilde{\cdot}$) and the contributions arising from the standard thermal leptogenesis Lagrangian,
Eq. (3.1), without any additional marker (see Fig. 3.3). We will use the notation introduced in Section 2.2.2.1.

- Decays and inverse decays ($\Delta L = 1$):

$$\tilde{D} \equiv [F \leftrightarrow S^\pm L^\pm] .$$ (3.6)

- Scattering processes ($\Delta L = 1$), with the subscript indicating exchange via $s$- or $t$-channel:

$$\tilde{S}_s = [LF \leftrightarrow UD] , \quad 2\tilde{S}_t = [F\bar{U} \leftrightarrow DL] + [F\bar{D} \leftrightarrow UL] .$$ (3.7)

- Scattering processes ($\Delta L = 2$) with the Majorana fermion $F$ as mediator:

$$\tilde{N}_s \equiv [LS \leftrightarrow \bar{L}\bar{S}] \quad \text{and} \quad \tilde{N}_t \equiv [LL \leftrightarrow \bar{S}\bar{S}] .$$ (3.8)

As discussed in Section 2.2.2.3, our analysis generally neglects the scattering processes with gauge bosons in order to avoid an unphysical logarithmic enhancement of $g^2 \ln(gT/m_N)$ that would not occur in a full thermal calculation at NLO. In principle, IR divergences would arise for a soft gauge boson. If regularized with a thermal mass, a logarithmic dependence ($g^2 \ln(gT/m_N)$) would remain. As was demonstrated in [155] for the standard leptogenesis scenario with right-handed neutrinos, this logarithmic dependence would cancel when including the corresponding thermal distribution of the gauge bosons in a thermal plasma. This cancellation would, for instance, naturally occur in a calculation using the Closed Time Path formalism [156]. As the scattering processes can be generally seen as a higher-order correction to the corresponding decays, it was recommended by [155] not to take these problematic processes into account, as a regularization with a thermal mass would lead to bigger
Figure 3.3: Relevant lepton-number violating processes arising from our extended Lagrangian that contribute to the washout additionally to the usual washout processes which occur within the standard thermal leptogenesis scenario including right-handed neutrinos.

uncertainties than when directly neglecting those. As a similar behavior is expected for the gauge scattering in our extended model (the dominant washout contribution arises from the inverse decay, as will be discussed later in more detail), we neglect these processes also in our washout calculation. Following the same argumentation, we also neglect the scattering with light quarks that would give rise to IR divergences in the standard leptogenesis scenario. As IR divergences do not appear in the quark scattering in our extended model set-up due to the massive scalar $S$ in the propagator (cp. Figs. 3.3a–3.3c), we systematically include these scattering processes in our washout calculation.

For describing the evolution of the lepton asymmetry, we include all the outlined processes, Eqs. (3.6)–(3.8) and derive the Boltzmann equations for the yield of the right-handed neutrino $Y_N = n_N/s$ and the $(B - L)$ asymmetry $Y_{B-L}$. Notice that
the yield of the \((B - L)\) asymmetry \(Y_{B-L}\) is explicitly related with the yield of the baryon asymmetry \(Y_B\) by [91]

\[
Y_B = \frac{28}{79} Y_{B-L}.
\] (3.9)

We assume that all SM particles are in thermal equilibrium at all times. Due to its fast gauge interactions, we can assume equilibrium also for the new, heavy particle \(S\). The new particle \(F\), however, is not strictly in equilibrium during all of the relevant time, but we have checked that the assumption of equilibrium will not affect the evolution of \(Y_{B-L}\) while having a significant advantage with respect to the computing time. Furthermore, \(D\), \(W\), and \(S\) indicate the relevant (inverse) decay, washout, and scattering processes, including both the usual standard leptogenesis interactions as well as the ones arising from our new Lagrangian in Eq. (3.2).

In the following, we will study both the strong \((K = 10^2)\) and weak \((K = 10^{-2})\) washout scenario. As the new contribution in our model, Eq. (3.2) is not directly interacting with the sterile neutrino sector, the Boltzmann equation for the change of the heavy right-handed neutrino number density remains unaltered with respect to the standard leptogenesis case in Eq. (2.46a). In contrast, the Boltzmann equation describing the \(Y_{B-L}\) asymmetry (or lepton asymmetry in Eq. (2.46b), since \(Y_{B-L} \approx -Y_{\Delta L}\)) evolution has to be adjusted to incorporate the new processes involved. However, as we do not assume any other \(CP\)-violating source than the heavy right-handed neutrino decay, the decay rate \(D(z)\) in Eq. (2.41) remains unaltered. For the washout we have to consider both the standard washout contributions and our new interactions such that we adjust the washout term \(W\) as follows

\[
W^{\text{tot}}(z) = W(z, Y_N) + \tilde{W}(z).
\] (3.10)
The contribution $W(z, Y_N)$ corresponds to the expression of the standard case

$$W(z, Y_N) = 2\left[N(z) + S_t(z)\right] \left(\frac{Y_N}{Y_{B-L}^{(eq)}}\right) + S_s(z) \left(\frac{Y_N}{Y_{B-L}^{(eq)}}\right), \quad (3.11)$$

with the $\Delta L = 1$ scattering rates $S_{s/t}(z)$ as defined in Eq. (2.42) and the $\Delta L = 2$ scattering rate $N(z)$ defined in Eq. (2.43). Note that, as discussed in Section 2.2.2.2, the inclusion of real intermediate states (RIS) in the $N_s$ scattering can lead to double counting with respect to the decays of the heavy right-handed neutrinos. Hence, as suggested in [155], we do not include the decay term in the washout contribution in order to prevent double counting.

The new contributions to the washout can be expressed as

$$\tilde{W}(z) = \left[\frac{1}{2} \tilde{D}(z) + 2\tilde{N}(z) + 2\tilde{S}_t(z) + \tilde{S}_s(z)\right] \left(\frac{Y_N^{(eq)}}{Y_{B-L}^{(eq)}}\right), \quad (3.12)$$

where the decay rate of the heavy particles $F$ or $S$, depending on their mass hierarchy, is given by

$$\tilde{D}(z) = \frac{\gamma_D}{zH(z)n_N^{(eq)}} = \frac{1}{zH(z)} f_{F,S} \left(\frac{m_{F,S}}{m_N}\right)^2 K_1(z m_{F,S}/m_N) K_2(z) \Gamma_{F,S}. \quad (3.13)$$

Here, $m_N$ differs from $m_{N0}$ when thermal effects are included. Comparing this expression with the decay rate of the heavy right-handed neutrino in Eq. (2.41), we notice some small adjustments. First, we rescaled the argument of the Bessel function by $(m_{F,S}/m_N)$. Secondly, we accounted for the equilibrium number density of $F$ or $S$ in $\gamma_D$. Due to the following relation of the equilibrium number densities

$$n_F^{(eq)}(z) = n_N^{(eq)}(z) \left(\frac{m_F}{m_N}\right)^2 \frac{K_2(z m_F/m_N)}{K_2(z)}, \quad (3.14a)$$
\begin{equation}
n_{S}^{(eq)}(z) = n_{N}^{(eq)}(z) \frac{2}{3} \left( \frac{m_{S}}{m_{N}} \right)^{2} \frac{K_{2}(z m_{S}/m_{N})}{K_{2}(z)},
\end{equation}

we also have to rescale our expression by \( f_{F,S}^{2}(m_{F,S}/m_{N}) \) with \( f_{F} = 1 \) or \( f_{S} = 2/3 \) due to the different number of degrees of freedom of \( F, S \) with respect to \( N \). Similarly, we can proceed with the \( \Delta L = 1, 2 \) scattering rates in Eqs. (2.42) and (2.43) to adjust for the new physics contributions,

\begin{align}
\tilde{S}_{s/t}(z) &= \frac{\gamma_{\tilde{S}_{s/t}}}{z H(z) n_{N}^{(eq)}} = \frac{1}{z H(z) 48 \pi^{2} \zeta(3) K_{2}(z)} \left( \frac{m_{F}}{m_{N}} \right)^{3} I_{\tilde{S}_{s/t}}(z), \\
\tilde{N}_{s/t}(z) &= \frac{\gamma_{\tilde{N}}}{z H(z) n_{N}^{(eq)}} = \frac{1}{z H(z) 48 \pi^{2} \zeta(3) K_{2}(z)} \left( \frac{m_{F}}{m_{N}} \right)^{3} I_{\tilde{N}}(z),
\end{align}

with

\begin{equation}
I_{(\tilde{S}_{s/t},\tilde{N}_{s/t})}(z) = \int_{y_{\text{min}}}^{\infty} dy \sqrt{y} \tilde{\sigma}_{(\tilde{S}_{s/t},\tilde{N}_{s/t})}(y) K_{1}(\sqrt{y} z m_{F}/m_{N}).
\end{equation}

For convenience we have defined \( y = \sqrt{s}/m_{F}^{2} \). Note that, in contrast to the standard leptogenesis scenario, we do not include the RIS subtraction when the hierarchy \( m_{S} > m_{F} \) holds (see Fig. 3.4). Hence, no double counting occurs for \( z \lesssim 10^{6} \) as the contribution \( \tilde{D} \) in Eq. 3.12 includes only the inverse decays \( FL \rightarrow S \) and not \( SL \rightarrow F \). Only the latter one would lead to a double counting as it is already accounted for in the scattering processes \( \tilde{N}_{s} \) when the \( F \) in the propagator is produced resonantly (cp. Fig. 3.3d). For later times, RIS subtraction is once again performed, and Eq. 3.12 loses its inverse decay term (cf. Eq. 3.11).

In our set-up, we consider a heavy right-handed neutrino with \( m_{N0} = 10^{10} \text{GeV} \) around which scale the generation of the asymmetry takes place roughly. At such high temperatures, masses can receive sizeable thermal corrections, which could even...
lead to an altered mass hierarchy with respect to $T = 0$. Hence, we consider thermal masses in our evolution of the neutrino and lepton number density:

$$m_N^2(T) = m_{N0}^2 + \frac{1}{8} y_N^2 T^2$$  \hspace{1cm} (3.17)

$$m_F^2(T) = m_{F0}^2 + \frac{1}{8} g_L^2 T^2$$  \hspace{1cm} (3.18)

$$m_S^2(T) = m_{S0}^2 + \left[ \frac{3}{16} g_2^2 + \frac{1}{16} g_Y^2 + \frac{1}{12} g_Q^2 + \frac{1}{12} g_L^2 \right] T^2$$  \hspace{1cm} (3.19)

$$m_H^2(T) = \left[ \frac{3}{16} g_2^2 + \frac{1}{16} g_Y^2 + \frac{1}{4} y_t^2 + \frac{1}{2} \lambda_H \right] T^2$$  \hspace{1cm} (3.20)

$$m_Q^2(T) = \left[ \frac{1}{16} g_3^2 + \frac{1}{18} g_Y^2 + \frac{1}{8} y_t^2 \right] T^2$$  \hspace{1cm} (3.21)

$$m_d^2(T) = \left[ \frac{1}{16} g_3^2 + \frac{1}{72} g_Y^2 + \frac{1}{8} y_b^2 + \frac{1}{8} g_Q^2 \right] T^2$$  \hspace{1cm} (3.22)

$$m_u^2(T) = \left[ \frac{3}{32} g_2^2 + \frac{1}{32} g_Y^2 + \frac{1}{16} g_L^2 \right] T^2 ,$$  \hspace{1cm} (3.23)

where we can express $T$ as $T = m_{N0}/z$. The definitions of $m_{F0}$ and $m_{S0}$ are analogous to $m_{N0}$. Here, we neglect contributions from the right-handed neutrino to the standard model masses, as those will only become relevant for $T > m_N$. In this regime, however, contributions are negligible with respect to the ones originating from the SM itself due to a comparably small coupling ($y_N^{\text{strong}} = 0.02$, $y_N^{\text{weak}} = 0.0002$). Hence they can be safely neglected. We show the evolution of the thermal masses in Fig. 3.4, where we have chosen $K = 10^2$ such that $y_N = 0.02$ and $g_L = g_Q = 10^{-2}$. Relative to the heavy neutrino mass at zero temperature, the thermal corrections have almost no impact on $m_N(T)$, except for $z > 10^{-2}$. This is in contrast to the evolution of the other particle masses. For instance, even when choosing at zero temperature $m_{F0} > m_{S0}$, the thermal corrections grow faster for $S$ than $F$ such that in the relevant temperature regime, the hierarchy of the particles changes (e.g., at $z = 1$, $m_S > m_F$). Another interesting feature happens for the mass hierarchy of the Higgs boson and
the right-handed neutrino. For \( z \gtrsim 0.6 \), the Higgs boson becomes heavier than the right-handed neutrino such that at higher temperatures, the decay \( H \rightarrow NL \) opens up. We account for this effect by adapting \( \Gamma_D \) in Eq. (2.41) to be \( \Gamma_D = \Gamma(N \rightarrow LH) \) for \( m_N(T) > m_H(T) \) and \( \Gamma_D = \Gamma(H \rightarrow LN) \) for \( m_N(T) < m_H(T) \). Moreover, the evolution of both the right-handed neutrino \( Y_N \) and the baryon asymmetry \( Y_B \) yields depend on the inclusion of thermal masses; our results in Fig. 3.5 differ from previous zero-temperature calculations (cf. Ref. [88]) not only in thermal history but also in the final value of the asymmetry.

In order to study the impact of the additional contributions of our model, we choose \( \epsilon = 10^{-6} \) and \( m_{N_0} = 10^{10} \) GeV. We show the Boltzmann evolution for the
yield of the right-handed neutrino $Y_N$ (blue solid line) and the yield of the baryon asymmetry in Fig. 3.5. We compare the $Y_B$ evolution of the standard scenario without (green dashed line) and with our new contributions (orange solid line). The evolution in the weak washout (left panels) and strong washout (right panels) regime is shown for two different example values $g_L = g_Q = \{10^{-3}, 10^{-6}\}$. Generally, we observe that the equilibrium yield of the right-handed neutrino $Y_N^{(\text{eq})}$ is reached much faster in the strong washout regime due to the larger decay rate (cp. Fig. 3.6, green solid line). Additionally, we present the evolution of the different, relevant contributions in Fig. 3.6.

Scenario I ($g_L = g_Q = 10^{-3}$). As naively expected, for relatively large couplings, the largest effect of the new TeV-scale LNV washout terms can be observed. Comparing Figs. 3.5a and 3.6a (weak washout), we see that the constant behavior of $Y_B$ at around $z \approx 0.6$ is caused by the dip that the decay rate $D$ receives due to the closing of $N \rightarrow HL$ and the opening of $H \rightarrow NL$ for small $z$. Even though the washout originating from $\tilde{W}$ is stronger than the standard contribution $W$, it has no visible impact on the evolution of $Y_B$ (this picture changes for couplings of $O(10^{-1})$). Around $z > 7$, when the $W$ contribution decreases strongly, the $\tilde{W}$ contribution remains constant and leads to a strong washout such that $Y_B$ falls below the observed value for the baryon asymmetry in contrast to the standard leptogenesis scenario.

A comparable situation is found for the strong washout regime (Figs. 3.5b and 3.6b). Due to the larger coupling, $y_N$, the washout originating from the standard leptogenesis scenario $W$ is now dominant $z < 10$, see Fig. 3.6b. Hence, the behavior of the standard leptogenesis scenario is followed longer up to larger $z$. However, when $W$ decreases significantly while $\tilde{W}$ remains constant, $Y_B$ gets again fully washout out.
Figure 3.5: Evolution of the yield of the right-handed neutrino $Y_N$ (blue solid line) and the yield of the baryon asymmetry for the standard scenario $Y_B^{[\text{std}]}$ (green dashed line) and including our new contributions $Y_B$ (orange solid line) for the weak (left panels) and strong (right panels) washout regime and two different example values for $g_L = g_Q = \{10^{-3}, 10^{-6}\}$. The equilibrium abundance $Y_N^{(\text{eq})}$ is given as a gray dashed line. The red dotted line indicates $z_{\text{eq}}$. 

75
Figure 3.6: Evolution of the different contributions relevant for the yield of the baryon asymmetry $Y_B$ (orange solid line) for the weak (left panels) and strong (right panels) washout regime and two different example values for $g_L = g_Q = \{10^{-3}, 10^{-6}\}$. The green dotted lines show when $m_H(z) = m_N(z) + m_L(z)$ and $m_N(z) = m_H(z) + m_L(z)$, respectively. The blue dotted line shows when $m_S(z) = m_F(z) + m_L(z)$.
The $T$-independent washout term $\widetilde{W}$ for $z < 10^6$ in Fig. 3.6 can be understood as follows. The dominant contribution to $\widetilde{W}$ is given by the inverse decay $\tilde{D}$ involving $F \leftrightarrow SL$ or $S \leftrightarrow FL$. This expression is a function of the decaying particle’s mass over temperature ($m_{F,S}/T$) and the right-handed neutrino mass ($m_N$). As shown in Fig. 3.4, for the relevant temperature range, both $m_F(T)$ and $m_S(T)$ are linear in temperature$^3$ and $m_N(T) \approx m_{N0}$. Consequently, all quantities involved are effectively independent of the temperature.

The magnitude of $\widetilde{W}(z < 10^6)$ can be naively estimated from a power counting on the lepton number violating couplings, generically referred to as $g$. Since this washout term is dominated by the inverse decay, and this process involves one lepton number violating vertex, then $\widetilde{W} = \mathcal{O}(g^2)$. It is important to highlight the fact that, in a zero temperature approximation, the most relevant contribution to $\widetilde{W}^{T=0}$ is given by the scattering terms. Since these terms involve two lepton number violating vertices, it follows that $\widetilde{W}^{T=0} = \mathcal{O}(g^4)$. Therefore, the inclusion of thermal effects is necessary to avoid an artificial suppression of the washout contribution coming from our simplified model. For lower temperatures, around $z = 10^6$, the $\tilde{D}$ contribution drops due to the mass hierarchy inversion, $m_F(T) \approx m_S(T)$, leading to the corresponding drop in $\widetilde{W}$.

**Scenario II ($g_L = g_Q = 10^{-6}$).** For smaller couplings, the washout contribution $\widetilde{W}$ is much smaller (see Figs. 3.6c and 3.6d) and only at later times dominant in comparison to the conventional washout processes. Hence, the baryon asymmetry $Y_B$ drops below the observed baryon asymmetry also at later times for both the strong and weak washout scenario (see Figs. 3.5c and 3.5d).

---

$^3$The proportionality constant can be obtained from Eqs. (3.18) and (3.19).
Finally, we compare the final yield of the baryons asymmetry with the experimentally observed value. The results are summarized in Fig. 3.7, choosing $m_{N_0} = 10^{10}$ GeV. We show in red the parameter space that cannot account for the observed baryon asymmetry for the strong and weak washout scenario for two choices of the $CP$-asymmetry parameter, the usual choice $\epsilon = 10^{-6}$ and the maximal possible $CP$-asymmetry $\epsilon = 1$. We observe that standard thermal leptogenesis is rendered unviable for lepton-number violating couplings $g_L$ of the order of $10^{-6}$ for $m_S \neq m_F$ and $10^{-4}$ for $m_S \sim m_F$, respectively. It is important to highlight that the viable thermal leptogenesis region’s extension relies heavily on the new particle spectrum, a characteristic not easily discernible when employing the previously used pure EFT approach. This demonstrates the value of our chosen methodology in revealing crucial information that might otherwise be obscured. The presence of a difference in two orders of magnitude significantly impacts the exploration of the viable region through long-lived particle searches. This substantial variation emphasizes the importance of accurately characterizing the particle spectrum, as it can greatly influence our understanding of the thermal history and collider phenomenology in leptogenesis studies. Moreover, a discovery of lepton-number violating new physics at collider or $0\nu\beta\beta$-decay experiments in this parameter range would have far-reaching consequences on the validity of standard thermal leptogenesis.

### 3.4 Collider study

The current experimental literature presents different results of various searches for LNV signals, ranging from specific decay modes of new particles (e.g., searches for $H^{\pm\pm} \to \ell^\pm\ell^\pm$ [119, 157]) to comprehensive studies of BSM theories (e.g., Left-Right symmetric models [158] or $R$-parity violating SUSY [159]), including the connection with $CP$-violating effects at the LHC (e.g., rare $W^\pm$ decays [160]). The current status
Figure 3.7: $g_L$-$g_Q$–plane for $\epsilon = 10^{-6}$ (left panel) and $\epsilon = 1$ (right panel), with $m_{N0} = 10^{10}$ GeV and $m_F = 1$ TeV. The red areas indicate the couplings that lead to a too-strong washout and don’t result in the observed baryon asymmetry for the different mass hierarchies shown in the panels. Considering the weak ($K = 10^{-2}$) or strong ($K = 10^{2}$) washout regime does not affect the plot visibly. Notice how the extension of the viable thermal leptogenesis region depends on the new particle spectrum —a feature not readily seen within the previously used pure EFT approach.
of those searches shows no evidence for significant deviations from the SM [102,104].

The goal of our work is to study the interplay and complementarity between collider phenomenology and $0\nu\beta\beta$-decay experiments. Since the latter one involves both electrons and quarks—at a fundamental level—, our analysis is focused on studying the production of two same-sign electrons and two jets in a proton-proton collider, namely $pp \rightarrow jje^\pm e^\pm$. Our simplified model allows two topologies associated with the signal, as shown in Fig. 3.8.

Figure 3.8: Feynman diagrams of the two-jet, same-sign dilepton signal ($pp \rightarrow e^\pm e^\pm jj$) in our simplified model. Diagram (a) matches the $0\nu\beta\beta$-decay diagram in Fig. 3.2.

In principle, some direct searches might also restrict our model. The first term in Eq. 3.2 makes this model sensitive to current experimental limits from di-jet resonant production. The ATLAS collaboration has recently published searches for low-mass [161] and high-mass [162] resonances in the mass distributions of two jets. Reinterpreting those results, specifically the generic Gaussian-shaped distributions, we find that values for $g_Q \gtrsim 0.1$ are roughly excluded for our parameter region of interest. The second term in Eq. 3.2 shows a potential sensitivity to single lepton plus missing $E_T$ searches [163] if $F$ decays outside the detector. However, based on our estimations for the decay length [164], we conclude that $F$ will decay promptly.
3.4.1 Event generation and classification

To perform our collider study, we have implemented the model (3.2) using the pipeline described in Section 2.3.2. For both signal and background, we impose a set of basic selection cuts \((p_T^j, p_T^\ell > 20 \text{ GeV}, |\eta_j| < 2.8, |\eta_\ell| < 2.5)\) at the generator level [148], and a pre-selection rule \((N_j \geq 2, N_{\ell \pm \ell \pm} \geq 2)\) at the classification level.

Our analysis is focused on the potential reach of the LHC at 14 TeV, in addition to the hypothetical FCC-hh [165] and SppC [166] at 100 TeV. The Delphes software package [109] incorporates the configuration card for ATLAS and CMS detectors at the LHC, while the equivalent one for the FCC-hh baseline detector is available online\(^4\) and a detailed description of the implementation is available in Ref. [167].

The event classification is based on a custom-made Recurrent Neural Network (RNN) inspired by previous experiences [168] in the context of event discrimination using Deep Learning. An RNN allows us to classify events with unordered, variable length inputs, such as the number of jets or electrons [169]. Our implementation uses the kinematic properties of jets, electrons, and missing \(E_T\) to differentiate signal-like and background-like events. A detailed description of our topology is given in Appendix B, where we have also included a brief introduction to RNNs.

Additionally, we also implemented a Boosted Decision Tree (BDT) to compare these two machine learning approaches’ results. We found that both the RNN and the BDT implementations presented a similar performance, consistent with previous

comparisons available in the literature [170–172]. As both implementations have similar performance, our choice of using an RNN is based on its ease of use since it offers a little more flexibility than a BDT.

3.4.2 Signal generation and phenomenology

The cross-section associated with a $t$-channel production in Fig. 3.8, $\sigma_{[t]}$, has a coupling dependence given by $(g_L g_Q)^4$ and it is relatively insensitive of the mass hierarchy between $F$ and $S^\pm$. Additionally, the $s$-channel cross-section in Fig. 3.8, $\sigma_{[s]}$, always involves the production of an on-shell particle. These two facts make the latest one to dominate the cross-section over the first one, $\sigma(pp \rightarrow jj e^\pm e^\pm) \approx \sigma_{[s]}(pp \rightarrow jj e^\pm e^\pm)$. Consequently, the behavior of diagram (b) gives us insights into the total cross-section.

To understand the coupling dependence of $\sigma_{[s]}$, notice that the physical processes vary depending on the mass hierarchy due to kinematic constraints as summarized in Table 3.1. Each sub-process corresponds to the successive production or decay chains of the signal.

<table>
<thead>
<tr>
<th>Case</th>
<th>Mass hierarchy</th>
<th>Process</th>
</tr>
</thead>
<tbody>
<tr>
<td>C1</td>
<td>$m_S &lt; m_F$</td>
<td>$pp \rightarrow e^\pm F$, $F \rightarrow e^\pm S^\pm$, $S^\pm \rightarrow jj$</td>
</tr>
<tr>
<td>C2</td>
<td>$m_S = m_F$</td>
<td>$pp \rightarrow e^\pm F$, $F \rightarrow e^\pm jj$</td>
</tr>
<tr>
<td>C3</td>
<td>$m_S &gt; m_F$</td>
<td>$pp \rightarrow S^\pm$, $S^\pm \rightarrow e^\pm F$, $F \rightarrow e^\pm jj$</td>
</tr>
</tbody>
</table>

Table 3.1: Kinematic classification of production and successive decays involved in diagram (b), Fig. 3.8, in our simplified model.

If we use the narrow-width approximation, it is possible to decompose the different sub-processes in Table 3.1 in the following manner:
C1. When $F$ is heavier than $S^\pm$, the cross-section corresponds to the production of an on-shell $F$ in addition to an electron --with $\sigma_{[s]}(pp \to e^\pm F) \propto (g_L g_Q)^2$-- followed by two cascade decays. These decay modes have branching ratios that are coupling independent since those are the only ones kinematically allowed:

$$\sigma_{[s]}(pp \to e^\pm e^\pm jj) = \sigma_{[s]}(pp \to e^\pm F) \times \text{Br}(F \to e^\pm S^\pm) \times \text{Br}(S^\pm \to jj) \propto (g_L g_Q)^2 \times \frac{2}{3}.$$

(3.25)

C2. In the same fashion, if both $S^\pm$ and $F$ have equal masses, then the cross-section also corresponds to the production of an on-shell $F$ accompanied by an electron, followed by the decay of $F$ into a pair of jets and an electron. This decay, again, is the only one possible --so it has a branching ratio equal to one-- and it is mediated by an off-shell $S^\pm$ propagator:

$$\sigma_{[s]}(pp \to e^\pm e^\pm jj) = \sigma_{[s]}(pp \to e^\pm F) \times \text{Br}(F \to e^\pm jj) \propto (g_L g_Q)^2 \times \frac{2}{3}.$$

(3.26)

C3. Finally, the case where $m_S > m_F$ provides a more subtle dependence. The full cross-section can be thought of as the on-shell production of $S^\pm$ --with $\sigma_{[s]}(pp \to S^\pm) \propto g_Q^2$-- followed by two successive decays. In this regime, $S^\pm$ is allowed to decay into two jets or a pair $e^\pm F$. The branching ratio is a function of the two couplings, as shown in Eq. 3.27:

$$\sigma_{[s]}(pp \to e^\pm e^\pm jj) = \sigma_{[s]}(pp \to S^\pm) \times \text{Br}(S^\pm \to e^\pm F) \times \text{Br}(F \to e^\pm jj) \propto (g_Q)^2 \times \frac{2}{3}.$$

(3.27)

where $\kappa$ depends on the ratio between the two masses satisfying $\kappa > 1$ for $m_S > m_F$. It is worthwhile to highlight two limiting cases:
− If \( g_Q \gg g_L \), then \( \text{Br}(S^\pm \to e^\pm F) \propto (g_L/g_Q)^2 \) and the cross section scales with \( g_L^2 \).

− If \( g_Q \ll g_L \), then the \( \text{Br}(S^\pm \to e^\pm F) \approx 1 \) and the cross section scales with \( g_Q^2 \).

A key difference between the three cases is the magnitude of the respective cross-sections. To illustrate, consider the production cross sections in cases C1–C2 and C3, *i.e.*, \( \sigma[s](pp \to e^\pm F) \) and \( \sigma[s](pp \to S^\pm) \), respectively. In the first one, notice that the momentum transfer along the \( s \)-channel needed to produce an on-shell \( F \) implies a suppression since the particle in the propagator is off-shell. However, the production in the latest one directly creates an on-shell \( S^\pm \), so no suppression is applied. The different order in the couplings reinforces the difference between the magnitudes of the cross sections for the same set of parameters, as we illustrate in Table 3.2.

<table>
<thead>
<tr>
<th>Production mode</th>
<th>( m_S = 0.5 \text{ TeV} )</th>
<th>( m_S = 2 \text{ TeV} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \sigma(pp \to jje^\pm e^\pm) )</td>
<td>( 1.199 \times 10^{-8} ) pb</td>
<td>( 8.874 \times 10^{-5} ) pb</td>
</tr>
<tr>
<td>( \sigma[s](pp \to jje^\pm e^\pm) )</td>
<td>( 1.195 \times 10^{-8} ) pb</td>
<td>( 8.874 \times 10^{-5} ) pb</td>
</tr>
</tbody>
</table>

Table 3.2: Numerical example of the phenomenological behavior of the signal cross section at \( \sqrt{s} = 14 \text{ TeV} \). We took \( m_F = 1 \text{ TeV}, g_L = 0.1, \) and \( g_Q = 0.01 \) in both scenarios. The results were obtained from MADGRAPH [107].

### 3.4.3 Background generation and validation

The three background categories \( s \) in the same-sign dilepton final state were introduced in Section 2.3.2. For our analysis, we study the effects of the dominant contributions: EW diboson processes, charge misidentification involving \( \gamma/Z^* \), and jet-fakes produced by \( tt \) and \( W^\pm + 3j \) processes [102,148]. The diboson simulation
involved the generation of $W^\pm W^\pm$, $W^\pm Z$, and $ZZ$ events plus jets, and their respective leptonic decays, as implemented in Ref. [148].

Due to the difficulty of precisely simulating jet-fakes, we implement the “FakeSim” method proposed in Ref. [115] as an additional module in Delphes. This data-driven approach takes into account the relation between the originating jet and the fake lepton, as discussed in Section 2.3.2. It is based on two functions$^5$:

1. A mistag efficiency, $\epsilon_{j\rightarrow\ell}(p_T^j)$, representing the probability that a particular jet $j$ is mistagged as a lepton $\ell$.

2. A transfer function, $T_{j\rightarrow\ell}$, modeling the probability distribution function that maps $p_T^j$ into the fake $p_T^\ell$.

In Fig. 3.9, we compare representative CMS results (digitized from Fig. 3 in Ref. [102]) with our result obtained using the FakeSim method. As can be seen from the plot, we reproduce the overall behavior after a parameter fitting. It is possible to optimize the choice of the FakeSim parameters if, for instance, flavor effects are included by introducing flavor-dependent mistag efficiencies [173].

To estimate the charge misidentification background, we introduce a misidentification probability for electrons from current doubly charged Higgs boson searches by the ATLAS collaboration [119]. The electron charge misidentification probability is modeled as a separable function of electron’s $p_T$ and $\eta$, $P(p_T,\eta) = \sigma(p_T)f(\eta)$, and its data-driven values are extracted from Fig. 3 in Ref. [119]. We follow a common approach in collider phenomenological studies [120, 121] to incorporate this probability as a weight in opposite-sign generated events, as detailed in Section 7.1.3 in

$^5$In the FakeSim method, these functions are parameterized by four quantities, namely $\{\epsilon_{200}, r_{10}, \mu, \sigma\}$. See Ref. [115] for additional details.
Figure 3.9: Comparison between the CMS results in Ref. [102] and our implementation of the FakeSim method proposed in Ref. [115] for a particular choice of its parameters, as mentioned in Note 5. Distributions of the kinematic variables $H_T$ and $E_{T}^{\text{miss}}$ are shown in the left and right panels, respectively.

Ref. [174]. In Table 3.3, we compare our background estimation with the ATLAS results extracted from Fig. 2 in Ref. [119] for a $Z \rightarrow ee$ peak data sample. The ratio of same-charge/opposite-charge events for ATLAS is 0.93%, and we obtained 0.70%.

<table>
<thead>
<tr>
<th>Number of events</th>
<th>Opposite-charge (OC)</th>
<th>Same-charge (SC)</th>
<th>SC/OC ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>ATLAS</td>
<td>$1.23 \times 10^7$</td>
<td>$1.14 \times 10^5$</td>
<td>0.93%</td>
</tr>
<tr>
<td>Our estimation</td>
<td>$9.41 \times 10^6$</td>
<td>$6.59 \times 10^4$</td>
<td>0.70%</td>
</tr>
</tbody>
</table>

Table 3.3: Validation of the charge misidentification implementation. The ATLAS results extracted from Fig. 2 in Ref. [119], and our results make use of the probability density defined in Ref. [174].

Notice that the two data-driven methods previously described were validated using LHC data. Table 3.4 presents the background cross section for the three classes, detailing the effects of the signal selection rule ($N_j \geq 2$, $N_{\ell^\pm \ell^\pm} \geq 2$) and the back-
ground discrimination by using the RNN. The first column shows the cross section before the signal selection ($\sigma_{\text{before}}$), as given directly by Madgraph. The second column shows the cross-section after applying the signal selection rule ($\sigma_{\text{after}}$), where the number of events is initially reduced. Finally, the third column shows the background cross section classified by the RNN ($\sigma_{\text{RNN}}$), and we notice the background reduction provided by our machine learning implementation. Since there are no estimations for these types of backgrounds for a future 100 TeV hadron collider, we use the same set of parameters and functions varying only the energy of the center of mass. Table 3.5 presents the results of our estimation.

$$
\begin{array}{cccc}
\text{Background type} & \sigma_{\text{before}} \text{ (pb)} & \sigma_{\text{after}} \text{ (pb)} & \sigma_{\text{RNN}} \text{ (pb)} \\
\hline
\text{Diboson} & WW & 3.28 \times 10^{-3} & 6.40 \times 10^{-4} & 6.87 \times 10^{-5} \\
 & WZ & 2.59 \times 10^{-2} & 6.65 \times 10^{-3} & 2.10 \times 10^{-4} \\
 & ZZ & 1.32 \times 10^{-3} & 5.62 \times 10^{-4} & 1.14 \times 10^{-5} \\
\hline
\text{Jet-fake} & W + 3j & 1.79 \times 10^{-1} & 4.34 \times 10^{-2} & 1.78 \times 10^{-4} \\
 & tt & 9.11 \times 10^{-2} & 2.64 \times 10^{-2} & 6.10 \times 10^{-5} \\
\hline
\text{Charge misidentification} & t\bar{t} & 3.33 \times 10^{-2} & 1.54 \times 10^{-2} & 4.45 \times 10^{-4} \\
 & Z/\gamma^* & 2.54 \times 10^{-1} & 1.37 \times 10^{-1} & 4.89 \times 10^{-3} \\
\end{array}
$$

Table 3.4: Individual contributions to the cross-section for the different types of background for the LHC at 14 TeV. $\sigma_{\text{before}}$ corresponds to the cross-section before the signal selection, $\sigma_{\text{after}}$ corresponds to the cross-section after the signal selection, and $\sigma_{\text{RNN}}$ is the cross-section tagged as background events by the RNN. The last row shows the total background cross-section.

### 3.5 $0\nu\beta\beta$ Decay

The results from searches for $0\nu\beta\beta$ decay place complementary constraints on the model parameters in a manner that can complement the collider search results. However, in order to account for the different scales, we need to evolve the operator
Table 3.5: Individual contributions to the cross-section for the different types of background for the FCC-hh at 100 TeV. The definitions of $\sigma_{\text{before}}$, $\sigma_{\text{after}}$, and $\sigma_{\text{RNN}}$ are equivalent as in Table 3.4. The last row shows the total background cross-section.

$\mathcal{O}_1$ in Eq. (3.4) from the TeV scale to the GeV scale using renormalization group running. Hereby, operators receive QCD and electroweak corrections and mix with other operators. The RGE evolution was studied in Ref. [148], with the dominant QCD corrections considered. The relevant part of $\mathcal{O}_1$ contributing to $0\nu\beta\beta$ decay is:

$$\mathcal{L}^{\text{eff}}_{\text{LNV}} = \frac{C_{\text{eff}}}{2\Lambda^5} \left( \mathcal{O}_{2}^{++} - \mathcal{O}_{2}^{-+} \right) \bar{e}_{L} e_{R}^c + \text{H.c.} , \quad (3.28)$$

where $e_{R}^c \equiv (e_{L})^c$, $C_{\text{eff}} \approx C_1(1 \text{GeV}) = 0.092C_1(\Lambda)$ [148], and the operator $\mathcal{O}_2$ being [131]:

$$\mathcal{O}_{2}^{ab} = (\bar{q}_R \tau^a q)(\bar{q}_R \tau^b q) \pm (\bar{Q}_R \tau^a q_R)(\bar{Q}_R \tau^b q_R) . \quad (3.29)$$

When $a = b$, the operator with subscript $+(-)$ are even (odd) eigenstates of parity. As the hadronic (four-quark) part of the operator carries no dependence on the lepton kinematics, the matrix element for the process factorizes into a leptonic and hadronic part. The computation of the former is straightforward. For the latter,
we first match the four-quark operator $O_{2+}^{++} - O_{2-}^{++}$ onto hadronic degrees of freedom most appropriate for computation of the nuclear transition matrix element, following the effective field theory (EFT) approach delineated in Refs. [131, 134]. The leading order contribution to the nuclear matrix element (NME) arises from the pion-exchange amplitude of Fig. 3.10, where the LNV $\pi\pi ee$ interaction emerges from matching $O_{2+}^{++} \bar{e}_L e_R^c$ onto the two pion-two electron operator in Eq. 3.28:

$$C_{\text{eff}} \Lambda_H^2 F_\pi^2 \langle \pi^- \pi^- | \bar{e}_L e_R^c + \text{H.c.} \rangle, \quad (3.30)$$

where $F_\pi = 92.2 \pm 0.2$ MeV is the pion decay constant [175], and $\Lambda_H$ is a mass scale associated with the hadronic matrix element (HME) of the four quark operator $O_{2+}^{++}$,

$$\Lambda_H^2 F_\pi^2 = \frac{1}{2} \langle 0 | (O_{2+}^{++} - O_{2-}^{++}) | \pi^- \pi^- \rangle. \quad (3.31)$$

In the earlier work of Ref. [148] $\Lambda_H^2$ was obtained using factorization/vacuum saturation to estimate the HME, yielding $\Lambda_H^2 = m_{\pi}^4/(m_u + m_d)^2 \approx -7.5$ GeV$^2$ for $m_{\pi} = 139$ MeV and $m_u + m_d = 7$ MeV. Subsequently, the authors of Ref. [151] noted that one may relate $O_{2+}^{++}$ to analogous $\Delta S \neq 0$ four-quark operators using $SU(3)$ flavor symmetry. Consequently, one may exploit flavor $SU(3)$ to obtain estimates of $\Lambda_H^2$ from the corresponding strangeness changing $K^0 \rightarrow \bar{K}^0$ and $K \rightarrow \pi\pi$ matrix elements. The result yields $\Lambda_H^2 = -(3.16 \pm 0.7)$ GeV$^2$ at the matching scale $\mu = 3$ GeV. The Cal-Lat collaboration performed a direct computation of the matrix element in Eq. (3.31), obtaining $\Lambda_H^2 = -(2.15 \pm 0.36)$ GeV$^2$ at $\mu = 3$ GeV in the RI/SMOM scheme [176]. In what follows, we will adopt the Cal-Lat value.

When used to evaluate the amplitude in Fig. 3.10, the interaction in Eq. (3.30) yields an effective two nucleon-two electron operator whose nuclear matrix elements
(NMEs) may be evaluated using state-of-the-art many-body methods. The resulting expression for the decay rate is

\[ \frac{1}{T_{1/2}^{0\nu}} = \left[ G_{0\nu} \times (1 \text{ TeV})^2 \right] \left( \frac{\Lambda_H}{\text{TeV}} \right)^4 \left( \frac{1}{144} \right) \]

\[ \times \left( \frac{v}{\text{TeV}} \right)^8 \left( \frac{1}{\cos \theta_C} \right)^4 \left| M_0 \right|^2 \left[ \frac{C_{\text{eff}}^2}{(\Lambda/\text{TeV})^{10}} \right] , \]

\[ G_{0\nu} = (G_F \cos \theta_C g_A)^4 \left( \frac{\hbar c}{R} \right)^2 \left( \frac{1}{32\pi^2 h \ln 2} \right) I(E_{\beta\beta}) , \]

(3.32a)

(3.32b)

with \( \theta_C \) being the Cabibbo angle, \( I(E_{\beta\beta}) \) the electron phase space integral

\[ \int_{E_{\beta\beta} - m_e}^{E_{\beta\beta} - m_e} dE_1 F(Z + 2, E_1)F(Z + 2, E_2)p_1E_1p_2E_2 , \]

(3.33)

\( E_2 = E_{\beta\beta} - E_1 \), and \( F(Z + 2, E_{1,2}) \) are factors that account for distortion of the electron wave functions in the field of the final state nucleus. The NME is given by

\[ M_0 = \langle \Psi_f | \sum_{i\neq j} \frac{R}{\rho_{ij}} \left[ F_1 \sigma_i \cdot \sigma_j + F_2 T_{ij} \right] \tau_i^+ \tau_j^+ | \Psi_i \rangle , \]

(3.34)

where \( T_{ij} = 3\sigma_i \cdot \hat{\rho}_{ij} \sigma_j - \sigma_i \cdot \sigma_j \), \( R = r_0 A^{1/3} \), \( \rho_{ij} \) is the separation between nucleons \( i \) and \( j \), and the functions \( F_{1,2}(|\rho_{ij}|) \) are given in Ref. [131]. Note that we have normalized the rate to the conventionally-used factor \( G_{0\nu} \) that contains quantities associated with the SM weak interaction, even though the LNV mechanism here involves no SM gauge bosons. Note also that Eq. (3.32a) corrects two errors in the corresponding expression in Ref. [148]: (a) the inclusion here of a factor of \( g_A^4 \) and (b) an additional overall factor of \( 1/8 \). The latter arises from a factor of \( 1/2 \) due to the presence of two electrons in the final state and a factor of \( (1/2)^2 \) that one must include to avoid double counting in the NME since the sum runs over \( i \neq j \) rather than \( i < j \).
In the analysis of Ref. [148], a value of $M_0 = -1.99$ for the transition $^{76}\text{Ge} \rightarrow ^{76}\text{Se}$ was adopted from the quasiparticle random phase approximation (QRPA) computation of Ref. [177]. Here, we use the results of a more recent proton-neutron (pn) QRPA computation of Ref. [178]. The resulting value for $M_0(^{76}\text{Ge} \rightarrow ^{76}\text{Se}) = -4.74$. At present, the most sensitive limit on the half-life has been obtained using $^{136}\text{Xe}$, for which the matrix element in Ref. [178] is $M_0(^{136}\text{Xe} \rightarrow ^{136}\text{Ba}) = -2.63$. Both pnQRPA values assume no “quenching” of $g_A$.

It is important to emphasize that the calculated NMEs exhibit considerable theoretical uncertainties. The earlier work of Ref. [148] accounted for the combined effect of these uncertainties as well as those in HMEs by varying the value of $M_0$ by a factor of two. The subsequent chiral $SU(3)$ and lattice computations of $\langle 0|O_{2^+}^{++}|\pi^-\pi^-\rangle$ have reduced the hadronic uncertainty to the $\mathcal{O}(10\%)$ level. In the case of the NME, however, it has been realized that in the context of few-nucleon effective field theory, consistent renormalization requires the presence of a contact interaction in addition to the long-range two-pion exchange amplitude [141]. The corresponding operator coefficient and nuclear matrix element are presently unknown. We thus retain a factor of two uncertainty in the NME to account for both the bona fide nuclear many-body uncertainties as well as the effect of the “counterterm” contribution.
Figure 3.11: Interplay between leptogenesis, collider searches, and $0\nu\beta\beta$-decay experiments for $m_F = 1$ TeV, and $m_S/m_F = \{0.5, 0.99, 1.5\}$. As in Fig. 3.7, we present the nonviable leptogenesis region in red; in dark green, we show the $0\nu\beta\beta$-decay exclusion from KamLAND-Zen (dotted line) and future tonne-scale experiments (dashed line); the collider LNV (same sign dilepton plus dijet) exclusion is shown in blue for the LHC at 14 TeV with integrated luminosities of 100 fb$^{-1}$ (dotted line) and 3 ab$^{-1}$ (dashed line), and the FCC-hh at 100 TeV with 30 ab$^{-1}$ (dash-dotted line); and, in dark gray, we present the LHC di-jet exclusion, as discussed in Section 3.4.

### 3.6 Combined results and discussion

In the following, we combine our results from the previous sections in order to investigate the reach and interplay of collider and $0\nu\beta\beta$-decay experiments and the implications of a possible discovery for the generation of the baryon asymmetry.

In Fig. 3.7, we illustrated the parameter space (red) that was found to lead to a baryon asymmetry smaller than the observed one because of a too-strong washout due to the new TeV-scale interactions. Fixing the masses of the new particles $F$ and $S$ around the TeV scale, we can read off the couplings $g_L$ and $g_Q$, for which we would expect a strong washout that would prevent a large enough baryon asymmetry. Hence, a discovery of a process within this parameter space would preclude the viability of the standard thermal leptogenesis scenario.
As depicted in Fig. 3.2, the new interactions in the Lagrangian, Eq. (3.2), can lead to \(0\nu\beta\beta\) decay. We show in green in Fig. 3.11 the region currently excluded region by KamLAND-Zen (green dotted line), but also the future reach of tonne-scale experiments (green dashed line). With this region lying in the red area, we can conclude that observation of \(0\nu\beta\beta\) decay realized by a dim-9 operator with new physics at the TeV scale would rule out the standard leptogenesis paradigm. As demonstrated in Fig. 3.7 (right), this conclusion remains valid even for the most optimistic choice of maximal \(CP\) asymmetry.

With \(0\nu\beta\beta\) decay being a low energy process, it is not sensitive to the mass hierarchy \(m_S/m_F\) and explicit couplings \(g_L\) and \(g_Q\) of the new degrees of freedom at the TeV scale, but only to the effective coupling \(C_1\) and scale \(\Lambda\), as stated in Eq. (3.5). Therefore, the decreased reach from left to right in Fig. 3.11, is caused by the increase in \(\Lambda\), leading to a suppression of the process.

High-energy collider experiments, however, can resolve the TeV-mass scale and hence depend on the mass hierarchy of the new particles \(F\) and \(S\). We show the current 14 TeV collider limits with 100 fb\(^{-1}\) integrated luminosity for the different mass hierarchies (blue dotted line), as well as for future 3 ab\(^{-1}\) integrated luminosity (blue dashed line) and the FCC-hh with 100 TeV and 30 ab\(^{-1}\) (blue dashed-dotted line). As discussed in detail in Section 3.4, the mass hierarchy is crucial for the reach of the collider searches. For \(m_S > m_F\), \(S\) can be produced resonantly followed by subsequent decays into a signature of two same-sign electrons and two jets, leading to the strongest constraints (see the dominant \(s\)-channel process depicted in Fig. 3.8b). We also reiterate that for \(g_Q > g_L\) the limits are mainly restricted by \(g_L^2\) and independent of \(g_Q\), while for \(g_L > g_Q\), the limits are mainly constrained by \(g_Q^2\) (and not \(g_L\)). For \(m_S \lesssim m_F\), in contrast, the collider constraints are much weaker, as \(S\) can
be produced only off-shell. If we compare the current and future collider sensitivities (blue) to the red region for all three mass hierarchies, we see that observation of two same-sign electrons and two jets would similarly rule out the standard thermal leptogenesis scenario for weak and strong washout.

While a same sign di-electron plus di-jet signature directly points to an LNV process, direct searches for the particles $F$ and $S$ also constrain the model parameter space. We show the limits from dijet resonant searches in gray. While covering already the full collider reach for $m_S \lesssim m_F$, they are less sensitive for $m_S > m_F$. The interplay of the di-jet, same sign di-electron, and $0\nu\beta\beta$-decay sensitivities in Fig. 3.11 illustrates the complementarity of these probes. Drawing on all of them will be important if a non-zero LNV signal is seen at either low or high energies.

As apparent from Fig. 3.11, the relative reaches of $0\nu\beta\beta$-decay experiments and collider LNV searches depend decisively on the new particle spectrum. This emphasizes the importance of pursuing and combining the low- and high-energy frontiers in order to cover wide ranges of the parameter space but also to identify, in case of observation, the underlying new physics. For instance, for $m_S \gtrsim m_F$, the observation of $0\nu\beta\beta$ decay would imply an observable signal at the LHC or would point towards another underlying $0\nu\beta\beta$-decay mechanism.

It is interesting to compare our simplified model results with those obtained using the EFT framework. To that end, we show in Fig. 3.12 the effective coupling $g_{\text{eff}} = g_L = g_Q = C_1^{1/4}$ versus the scale of new physics $\Lambda = (m_S^4 m_F)^{1/5}$ for the different mass hierarchies shown in the previous figures. Note that we fix the absolute masses $m_F, m_S$ only indirectly via the scale $\Lambda$. We indicate the limit on the scale of new physics when naively assuming $g_{\text{eff}} = g_L = g_Q = 1$ with a red star. Comparing the
Figure 3.12: Complementarity between collider searches and $0\nu\beta\beta$-decay experiments for $g_L = g_Q$ and $m_S/m_F = \{0.5, 0.99, 1.5\}$. Comparison with an EFT analysis for the scale of new physics $\Lambda = (m_S^4 m_F)^{1/5}$ and the effective coupling $g_{\text{eff}} = g_L = g_Q$. In dark green, we show the $0\nu\beta\beta$-decay exclusion from KamLAND-Zen (dotted line) and future tonne-scale experiments (dashed line). The collider LNV exclusion is shown in blue for the LHC at 14 TeV with integrated luminosities of 100 fb$^{-1}$ (dotted line) and 3 ab$^{-1}$ (dashed line), and the FCC-hh at 100 TeV with 30 ab$^{-1}$ (dash-dotted line). The EFT benchmark point is shown as a red star, where $g_{\text{eff}} = 1$ is assumed for the discussion.

In three different panels, it becomes again obvious that whether $0\nu\beta\beta$ decay or collider searches are more sensitive crucially depends on the relative mass hierarchy. As $0\nu\beta\beta$ decay cannot resolve the heavy new physics, combining both experimental approaches is crucial.

In Fig. 3.13, we finally show a similar set of plots, in which we allow for a larger new physics scale $\Lambda$, demonstrating the future reach of a tonne-scale $0\nu\beta\beta$-decay experiment and the FCC-hh. While $0\nu\beta\beta$-decay experiments will reach a sensitivity of $\Lambda \approx 10$ TeV, the FCC-hh will reach between $\Lambda \approx 5$ TeV and $\Lambda \approx 15$ TeV, depending on the mass hierarchy.

These results demonstrate that observing an LNV signal at $0\nu\beta\beta$ decay or collider experiments can potentially exclude the standard thermal leptogenesis scenario. Given their complementary experimental reach, combining the high- and the low-
energy frontiers is crucial to probe the mechanism behind the baryon asymmetry generation.

3.7 Summary and Remarks of the Chapter

The observation of LNV would have profound implications for our understanding of nature, from the origin of neutrino masses to the matter-antimatter asymmetry of the Universe. In this work, we studied the interplay between three different physical aspects linked by LNV at the TeV scale: $0\nu\beta\beta$ decay, collider phenomenology, and thermal leptogenesis. Previous studies have been performed [144, 145] from an effective field theory (EFT) standpoint, showing the complementarity between current and future experimental results and their potential to falsify the standard thermal leptogenesis mechanism as an explanation for the origin of matter. Since these first analyses have not yet considered the latest methods, such as the inclusion of background in the collider analysis or the RGE running of the Wilson coefficients across the different energy scales, we extended previous work and studied the impact on the resulting phenomenology. Moreover, we specify a concrete, simplified model that...
allows us to study aspects that are not possible to capture by a pure EFT approach, such as different mass hierarchies of the new physics involved. By addressing the key questions with a simplified model using state-of-the-art techniques, we also establish a theoretical and computational setup for any other model to perform similar studies like this one.

To this end, we considered the SM extended by at least two right-handed neutrinos leading to neutrino masses via the Type-I seesaw mechanism and additionally a scalar doublet $S$ and a Majorana singlet $F$, both having masses around the TeV-scale. This model was introduced in Ref. [148], where the interplay between collider searches at the LHC and $0\nu\beta\beta$-decay experiments was studied. Besides a detailed analysis of the implications of a possible observation of a $\Delta L = 2$ lepton-number violating signal for the validity of standard thermal leptogenesis, we extended previous works by using the latest hadronic and nuclear matrix elements, improving on the derivation of the $0\nu\beta\beta$-decay half-life, and updating the corresponding predictions.

We also revisited the collider study in Ref. [148], where a prompt signature of two electrons plus jets in the final state at the LHC was analyzed. There, the different backgrounds (generated using standard MC techniques with misidentification and mistagging probabilities) were differentiated from the signal using a cut-based analysis. Our collider study differs from that of Ref. [148] in both generation and analysis of events. Firstly, we extended the $\Delta L = 2$ lepton-number violating prompt signature by considering both $e^-e^-$ and $e^+e^+$ in the final state. The background contributions were improved by the implementation of data-driven methods to emulate the effects of misidentification and mistagging. Finally, the event classification was based on cutting-edge machine learning (ML) algorithms, specifically boosted decision trees and neural networks. Our experience with ML techniques resonates
with current literature in terms of versatility and implementation time. With these techniques, we identified the already excluded region from the latest LHC runs as well as the future exclusion potential of the high-luminosity LHC or a hypothetical 100 TeV $pp$ collider. For comparison with the reach of $0\nu\beta\beta$ decay, we identified three scenarios with different mass hierarchies between the new particles $S$ and $F$. We demonstrate that collider searches are more sensitive to heavier $S$ due to an enhancement via on-shell production.

To study the effect of these new TeV-scale interactions in the context of thermal leptogenesis, we have implemented a set of Boltzmann equations, including the usual expressions involving right-handed neutrinos [87, 88]. In order to account for the washout processes arising from the new interactions of our model, we extended this set-up by the corresponding rates of the relevant LNV terms. We have studied the implications of these new interactions for the weak and strong washout regimes. For both of them, we have found that for couplings larger than $g_L \approx \mathcal{O}(10^{-6})$, an observation of a TeV-scale interaction implies a fast enough washout of any asymmetry previously generated by the out-of-equilibrium decay of right-handed neutrinos at a high scale, as shown in Fig. 3.7.

Based on our analysis, we could demonstrate that the discovery of LNV via the observation of $0\nu\beta\beta$ decay could preclude the viability of standard thermal leptogenesis if TeV-scale LNV interactions dominate the decay process. Our results are generally consistent with previous EFT estimates [144,145,179]. However, in order to confirm that the dominant contribution arises from a dim-9 contribution, additional information is needed. Different ideas for identifying the underlying mechanism exist, such as comparison of results from different isotopes [180–182], observation of a discrepancy with the sum of neutrino masses determined by cosmology [183], a deviation
in meson decays [146,147], or signals of LNV TeV-scale new physics from pp collider searches. Besides being able to possibly confirm the underlying new physics, the observation of an LNV signal at the Large Hadron Collider and/or a future 100 TeV pp collider would independently render standard thermal leptogenesis invalid. We could demonstrate that the relative potential of $0\nu\beta\beta$ decay and collider experiments to falsify the standard thermal leptogenesis scenario depends decisively on the new particle spectrum. We would like to stress that the observation of such an experimental signature would not necessarily be in conflict with the scale of light neutrino masses implied by neutrino oscillation experiments as well as cosmological and astrophysical neutrino mass probes. Our analysis also brings to light the opportunities offered by the relative smallness of LNV couplings, essential for a viable leptogenesis scenario. This aspect particularly opens a door for collider signatures of long-lived particles (LLP). New strategies to seize these opportunities are already being developed and will be further explored in our future work.

While our analysis was focused on the generation of a lepton asymmetry at a high scale via the decay of right-handed neutrinos, the general implications can be, in principle, transferred to similar mechanisms such as other high-scale leptogenesis scenarios. Specific models, however, might escape the general implications, e.g., scenarios with a dark sector featuring a global $U(1)_X$ symmetry [184]. Therefore, in order to conclusively falsify models, a dedicated analysis should be performed. Moreover, while we concentrated in our analysis on the electron sector only, there is the caveat that a lepton asymmetry was generated in another (decoupled) flavor sector. In order to address this point, we plan to extend our work by studying flavor effects, which open up interesting links to new collider signatures and low-energy observables.
As the discovery of a TeV-scale LNV signal at $0\nu\beta\beta$-decay experiments or current and future colliders will have far-reaching consequences on the validity of standard thermal leptogenesis, such searches are of high relevance in the quest for new physics and, in particular, for the origin of the baryon asymmetry of our Universe.
CHAPTER 4
UNCOVERING A CHIRALLY SUPPRESSED MECHANISM OF NEUTRINOLESS DOUBLE BETA DECAY WITH LHC SEARCHES

4.1 Introduction and Motivation

The systematic application of chiral perturbation theory (\(\chi\)PT) and chiral effective field theory (\(\chi\)EFT) to \(0\nu\beta\beta\) decay was first pioneered by Ref. [131], which also pointed out general the dominance of pionic contributions for several dimension-9 operators.\(^1\) In that work, the classification of possible \(\Delta L = 2\) LNV operators at the dimension-9 level, including only quark and lepton fields, that contribute to \(0\nu\beta\beta\) decay was performed using Weinberg power counting [186, 187]. In particular, the quark-lepton operators are organized according to the power counting of the mapped hadron-lepton operators in \(\chi\)EFT. These \(0\nu\beta\beta\)-decay operators do not necessarily generate the observed neutrino masses and could give sizable contributions to \(0\nu\beta\beta\) decay [188].

In this Chapter, we will study the phenomenology of TeV-scale LNV arising from certain dimension-9 vector operators discussed below, using a simplified model that provides an ultra-violet (UV) completion for these operators and that is \(SU(2)_L \times U(1)_Y\) gauge invariant. Importantly, at low energies, the leading contribution of the dimension-9 operators to \(0\nu\beta\beta\) decay occurs at next-to-next-to-leading order (N\(^2\)LO)

\(^1\)In the case of specific models (R-parity-violating supersymmetry) the importance of pion interactions was first noticed in Ref. [185].
in Weinberg’s power counting, in striking contrast to most other dimension-9 LNV operators (so-called “scalar” operators) that can occur at leading order (LO). Moreover, a renormalization group analysis of the vector operators implies that their treatment according to Weinberg power counting is robust [134, 189]; the promotion of higher order contact operators to LO that enters the scalar channel (as well as the conventional light Majorana neutrino exchange mechanism) [141, 189–191] does not occur in this case.

While current and future ton-scale $0\nu\beta\beta$-decay experiments are insensitive to the details of the underlying LNV mechanism because they are only sensitive to the decay rate and not, for example, angular or energy correlations between the two outgoing electrons, LHC searches have a greater potential to uncover it, if the associated mass scale is at the TeV. The reason is that if the particles that generate these operators have masses close to the TeV mass scale, they could be produced on-shell at the LHC, greatly increasing their production cross-section. In the case of dimension-9 LNV scalar operators, this potential has already been demonstrated [148, 150, 192].

Turning to $0\nu\beta\beta$ decay, because these vector operators contribute to the amplitude at $N^2$LO, their contributions to the $0\nu\beta\beta$-decay rate are naively suppressed by a factor of $(\Lambda_\chi/m_\pi)^4 \sim 2 \times 10^3$, with $\Lambda_\chi \simeq 1$ GeV a typical hadronic scale and $m_\pi$ the pion mass. As a result of this suppression – which is purely a consequence of low-energy hadronic physics – and the increase of production cross section with on-shell particles, LHC searches are relatively stronger at constraining these operators than direct $0\nu\beta\beta$-decay limits. We emphasize that in the case of dimension-9 LNV vector operators, the chiral suppression of the $0\nu\beta\beta$-decay rate will further promote the sensitivity of LHC searches in comparison with $0\nu\beta\beta$-decay searches. In contrast, the perturbative QCD renormalization group evolution of the vector operators is found to
cause their Wilson coefficients at the hadronic scale to be slightly enhanced compared
to the TeV scale, partly offsetting the previous effect.

We derive the sensitivities of $0\nu\beta\beta$ decay for the KamLAND-Zen [10] and future
ton-scale experiments and investigate the current and projected sensitivities to TeV-
scale LNV at the LHC in our simplified model. Our results clearly illustrate the
complementary role of these experiments in probing the TeV-scale physics described
here.

This Chapter is based on Ref. [143].

4.2 Simplified Model

To see how the $0\nu\beta\beta$ decay and collider probes interplay with each other in the
tests of TeV-scale LNV, we will work in the context of simplified models. The system-
atic decomposition of $0\nu\beta\beta$-decay operators with the possible realizations is sketched
in Refs. [130, 193]. We will particularly pay attention to the UV realization of a
class of $0\nu\beta\beta$-decay operators, which are suppressed at hadron-level in chiral power
counting but are potentially accessible at colliders and next-generation $0\nu\beta\beta$-decay
experiments and give small contributions to the neutrino masses.

Consider the following model with a scalar field $S \in (1, 2, 1/2)$, a leptoquark field
$R \in (3, 2, 1/6)$, and a Dirac fermion field $\Psi \in (1, 2, -1/2)$, where $(X, Y, Z)$ corre-
sponds to the representations under the $SU(3)_C$, $SU(2)_L$, and $U(1)_Y$ gauge groups,
respectively. They interact with the SM fields via

\[
\mathcal{L}_{\text{int}} = y_{qd} \bar{Q} d_R + y_{qu} \bar{u}_R S^T \epsilon Q + y_{e\Psi} \bar{e}_R S^T \Psi_L \\
+ \lambda_{ed} \bar{L} e^{*} d_R + \lambda_{u\Psi} \bar{u}_R u_R + \lambda_{d\Psi} \bar{d}_L \Psi_L d_R
\]
+ \gamma C\bar{u}^T}

where $Q = (u_L, d_L)^T$, $L = (\nu e_L, e_L)^T$, $\epsilon = i\sigma^2$, $\psi_{L/R} \equiv P_{L/R}\psi$, $u_R^c = P_L C\bar{u}^T$ with $P_{L/R} \equiv (1 \mp \gamma_5)/2$, $\psi$ denoting the fermion fields and $C$ being the usual charge conjugation matrix. The fields $S$ and $\Psi_L$ have the same quantum numbers as the SM Higgs and lepton doublet fields, respectively. A mass term $\bar{\Psi}_R L$ can be removed by a field redefinition of $\Psi_L$ and $L$. Following Ref. [194], we assume for simplicity that, at tree-level, $S$ has no vacuum expectation value, and so, at this order in perturbation theory, all of its components are physical.

With three generations of SM fermions, there is no reason a priori for the Yukawa couplings of $S$ to quarks and charged leptons to be aligned with the SM Yukawa couplings. Tree-level exchange of $S$ generically leads to dangerous flavor-changing neutral current processes and charged lepton number violating processes. We will not address this problem in this Chapter since we focus on illustrating the interplay between the LHC and $0\nu\beta\beta$-decay experiments.

This model violates overall lepton number whenever $\lambda_{ed}\lambda_{u\Psi} \neq 0$ or $\lambda_{d\Psi}\lambda_{u\Psi} \neq 0$. To see that, consider a fictitious lepton-number-like $U(1)_L$, under which the fields are charged, and the Yukawa couplings are treated as spurions and also charged for making the Lagrangian invariant. Thus the lepton numbers $q(L) = q(e) = q(\Psi) = 1$, $q(S) = 0$, and $q(R) = r$ can be arbitrary. Then the Yukawa couplings necessarily have a charge such that $q(\lambda_{ed}) = q(\lambda_{d\Psi}) = 1 + r$ and $q(\lambda_{u\Psi}) = 1 - r$, implying $q(\lambda_{ed}\lambda_{u\Psi}) = q(\lambda_{d\Psi}\lambda_{u\Psi}) = 2$. In Eq. (4.1), the term $\lambda_{d\Psi}\epsilon L^\dagger d_R$ does not contribute to $0\nu\beta\beta$ decay at tree-level and is not considered further.
To conclude this section, I will provide the decay widths of the new particles. These formulas will be useful in Section 4.4 for conducting the collider analysis of the model. For brevity, all of the couplings are assumed to be real, and the step functions $\theta(m_X - m_Y - \ldots)$ for the decay processes $X \to Y + \ldots$ are omitted.

- The two-body decay widths of $\Psi^0$ and $S^+$ are

$$
\Gamma(S^+ \to \Psi^0 e^+) = \frac{y_{e\Psi}^2 (m_S^2 - m_{\Psi}^2)^2}{16\pi m_S^3}, \quad (4.2)
$$

$$
\Gamma(\Psi^0 \to S^+ e^-) = \frac{y_{e\Psi}^2 (m_{\Psi}^2 - m_S^2)^2}{32\pi m_{\Psi}^3}, \quad (4.3)
$$

$$
\Gamma(S^+ \to \bar{d}u) = \frac{3}{16\pi} m_S (y_{qu}^2 + y_{qd}^2), \quad (4.4)
$$

$$
\Gamma(\Psi^0 \to R^{2/3} \bar{u}) = \Gamma(\Psi^- \to R^{-1/3} \bar{u}) = \frac{\lambda_{e\Psi}^2 (m_{\Psi}^2 - m_R^2)^2}{16\pi m_R^3}. \quad (4.5)
$$

- The two-body decay widths of $\Psi^-$ and $S^0$ are

$$
\Gamma(S^0 \to \bar{d}d) = \frac{3}{16\pi} m_S y_{qd}^2, \quad (4.6)
$$

$$
\Gamma(S^0 \to \bar{u}u) = \frac{3}{16\pi} m_S y_{qu}^2, \quad (4.7)
$$

$$
\Gamma(\Psi^- \to S^0 e^-) = \frac{y_{e\Psi}^2 (m_{\Psi}^2 - m_S^2)^2}{64\pi m_{\Psi}^3}, \quad (4.8)
$$

$$
\Gamma(S^0 \to \Psi^+ e^-) = \frac{y_{e\Psi}^2 (m_S^2 - m_{\Psi}^2)^2}{32\pi m_S^3}, \quad (4.9)
$$

$$
\Gamma(S^0 \to \Psi^- e^+) = \frac{y_{e\Psi}^2 (m_S^2 - m_{\Psi}^2)^2}{32\pi m_S^3}. \quad (4.10)
$$

- The two-body decay widths of $R$ are

$$
\Gamma(R^{2/3} \to e^+ d) = \Gamma(R^{-1/3} \to \bar{\nu} d) = \frac{\lambda_{e\Psi}^2 m_R}{16\pi}, \quad (4.11)
$$

$$
\Gamma(R^{2/3} \to \Psi^0 u) = \Gamma(R^{-1/3} \to \Psi^- u) = \frac{\lambda_{e\Psi}^2 (m_R^2 - m_{\Psi}^2)^2}{16\pi m_R^3}. \quad (4.12)
$$
Figure 4.1: Quark-level Feynman diagrams that induce $\Delta L = 2$ vector $0\nu\beta\beta$-decay operators at low energies.

- The three-body decay widths of $\Psi^0$ are computed numerically using MadGraph5 [107] and checked with CalcHEP [195]. For the sake of convenience, we provide the decay widths in the limit of $m_{\Psi} \ll m_S, m_R$

$$\Gamma(\Psi^0 \to e^- u \bar{d}) = \frac{1}{2048\pi^3} \frac{m_{\Psi}^5}{m_S^4 m_R^4} y_{e\Psi}^2 (y_{qu}^2 + y_{qd}^2),$$  \hspace{1cm} (4.13)

$$\Gamma(\Psi^0 \to e^+ u \bar{d}) = \frac{1}{2048\pi^3} \frac{m_{\Psi}^5}{m_R^4} \lambda_{ed}^2 \lambda_{\Psi}^2.$$  \hspace{1cm} (4.14)

Notice that, since the lepton number of $\Psi^0$ is +1, the decay mode $\Psi^0 \to e^- u \bar{d}$ conserves lepton number whereas $\Psi^0 \to e^+ u \bar{d}$ violates it.

4.3 $0\nu\beta\beta$ Decay

At low energies, this model generates dimension-9 and dimension-7 operators that contribute to $0\nu\beta\beta$ decay. In each of these cases, the amplitude for $0\nu\beta\beta$ decay is suppressed, either by $m_n^2$ in the case of the dimension-9 vector operators or in the case of the dimension-7 operator, by the electron mass or energies of the outgoing electrons. We next discuss these in turn.

4.3.1 Dimension-9 operators

At the quark level, the interactions in this model lead to two $0\nu\beta\beta$-decay contributions shown in Fig. 4.1, arising from the interactions in Eq. (4.1) proportional to
and $y_{qd}$, respectively. After integrating out the $S$, $R$, and $\Psi$ fields, the effective
interactions take the forms of

\begin{equation}
(\bar{u}_L d_R)(\bar{e}_R u_R^c)(\bar{e}_L d_R), \quad (\bar{u}_R d_L)(\bar{e}_R u_R^c)(\bar{e}_L d_R), \quad (4.15)
\end{equation}

with $u_R^c \equiv (u_R)^c$. Using the Fierz identity, we obtain

\begin{equation}
(\bar{e}_R u_R^c)(\bar{e}_L d_R) = \frac{1}{2}(\bar{u}_R \gamma^\mu d_R)(\bar{e}_R \gamma_\mu e_L^c). \quad (4.16)
\end{equation}

In the approach of the SM effective field theory (SMEFT), the effective interactions
after integrating out the heavy fields are [131–136]

\begin{equation}
\mathcal{L}_{\text{SMEFT}}^{(9)} = \frac{1}{\Lambda^5}(C_{Qu1}O_{Qu1} + C_{Qd1}O_{Qd1}) + \text{H.c.}, \quad (4.17)
\end{equation}

where $\Lambda$ is the LNV scale, and the dimension-9 quark-lepton vector operators are
expressed as

\begin{align}
O_{Qu1} &= (\bar{u}_R Q)(\bar{u}_R \gamma_\mu d_R)(\bar{e}_R \gamma^\mu L^c), \quad (4.18a) \\
O_{Qd1} &= \epsilon_{ij}(\bar{Q}^j d_R)(\bar{u}_R \gamma_\mu d_R)(\bar{e}_R \gamma^\mu L^j). \quad (4.18b)
\end{align}

The Wilson coefficients are given by

\begin{align}
\frac{C_{Qu1}}{\Lambda^5} &= \frac{y_e \lambda_e \lambda_\Psi}{2m_S^2m_R^2m_\Psi} y_{qu}, \quad (4.19a) \\
\frac{C_{Qd1}}{\Lambda^5} &= \frac{y_e \lambda_e \lambda_\Psi}{2m_S^2m_R^2m_\Psi} y_{qd}. \quad (4.19b)
\end{align}
where \( m_R, m_S, \) and \( m_\Psi \) are physical masses of \( R, S \) and \( \Psi \), respectively.

Note that under the fictitious \( U(1)_L \) introduced earlier, the charge of these Wilson coefficients is exactly canceled by the charge of the effective operators \( O_{Qu1} \) or \( O_{Qd1} \), so that the above \( \mathcal{L}^{(9)}_{\text{SMEFT}} \) is invariant. Hereafter we assume without loss of generality that all of the couplings are real.

We evolve the operators \( O_{Qu1} \) and \( O_{Qd1} \) down to the EW scale, which mix with the color octet vector operators \( O_{Qu2} \) and \( O_{Qd2} \), respectively, written as

\[
O_{Qu2} = (\bar{u}_R t^a Q)(\bar{u}_R t^a \gamma_\mu d_R)(\bar{e}_R \gamma^\mu L^c), \tag{4.20a}
\]
\[
O_{Qd2} = \epsilon_{ij} (\bar{Q}^i t^a d_R)(\bar{u}_R t^a \gamma_\mu d_R)(\bar{e}_R \gamma^\mu L^c). \tag{4.20b}
\]

The corresponding Wilson coefficients \( C_{Qu2} \) and \( C_{Qd2} \) are normalized in a similar way as \( C_{Qu1} \) and \( C_{Qd1} \) in the Lagrangian. The one-loop QCD RG equation for the Wilson coefficients \( C_{Qu1} \) and \( C_{Qu2} \) is [134],

\[
\frac{d}{d \ln \mu} \begin{pmatrix} C_{Qu1} \\ C_{Qu2} \end{pmatrix} = \frac{\alpha_s}{4\pi} \begin{pmatrix} -40/9 \\ 4/3 \\ 80/27 \\ 46/9 \end{pmatrix} \begin{pmatrix} C_{Qu1} \\ C_{Qu2} \end{pmatrix}, \tag{4.21}
\]

where \( \alpha_s \) is the strong coupling. The evolution of the Wilson coefficients \( C_{Qd1} \) and \( C_{Qd2} \) is governed by the RG equation of the same form with the substitution: \( C_{Qu1} \to C_{Qd1} \) and \( C_{Qu2} \to C_{Qd2} \).

Below the EW scale, the effective interactions are written in the low-energy effective field theory (LEFT) [131,132,134,196]

\[
\mathcal{L}_{\text{LEFT}}^{(9)} = \frac{1}{v^5} \sum_{i=6}^{9} C_i O_i^{\mu\nu} \bar{e}_5 \gamma_\mu \gamma_\nu e^c + \text{H.c.}, \tag{4.22}
\]
where the vacuum expectation value \( v = 246 \) GeV, and the vector operators are given by

\[
O_6^{\mu'} = (\bar{q}_R \tau^+ \gamma^\mu q_R)(\bar{q}_R \tau^+ q_L), \quad (4.23a)
\]
\[
O_7^{\mu'} = (\bar{q}_R t^a \tau^+ \gamma^\mu q_R)(\bar{q}_R t^a \tau^+ q_L), \quad (4.23b)
\]
\[
O_8^{\mu'} = (\bar{q}_R \tau^+ \gamma^\mu q_R)(\bar{q}_L \tau^+ q_R), \quad (4.23c)
\]
\[
O_9^{\mu'} = (\bar{q}_R t^a \tau^+ \gamma^\mu q_R)(\bar{q}_L t^a \tau^+ q_R), \quad (4.23d)
\]

where \( \tau^+ = (\tau^1 + i \tau^2)/2 \) with \( \tau^1 \) and \( \tau^2 \) being the Pauli matrices, and \( q_{L,R} = (u_{L,R}, d_{L,R})^T \) denote the left- and right-handed quark isospin doublets. For the color octet operators \( O_7^{\mu'} \) and \( O_9^{\mu'} \), \( t^a = \lambda^a/2, a = 1, \ldots, 8, \text{Tr}[t^at^b] = \delta^{ab}/2, \lambda^a \) denote the \( SU(3)_C \) Gell-Mann matrices in the fundamental representation, and the summation over the index \( a \) is assumed.

The matching conditions at the electroweak scale \( m_W = 80.4 \) GeV are

\[
C'_6(7)(m_W) = \frac{v^5}{2\Lambda^5} C_{Qu1(2)}(m_W), \quad (4.24a)
\]
\[
C'_8(9)(m_W) = \frac{v^5}{2\Lambda^5} C_{Qd1(2)}(m_W). \quad (4.24b)
\]

The RGEs for the Wilson coefficients in the LEFT take the same form in Eq. (4.21) with the substitution: \( C_{Qu1} \rightarrow C'_6(C'_7), C_{Qu2} \rightarrow C'_8(C'_9); \) and with the change in the \( \beta \)-function of \( \alpha_s \). Choosing a typical high scale \(^2 \Lambda = 2 \) TeV, the Wilson coefficients \( C'_{6,7,8,9} \) at the scale \( m_0 = 2 \) GeV, are given by

---

\(^2\)We neglect the difference if \( m_{S,R,\Psi} \) deviate from 2 TeV, which is a minor effect.
Figure 4.2: Hadron-level Feynman diagrams for $0\nu\beta\beta$ decay induced by the LNV operators $\pi NNee$ (left) and $NNNNee$ (right), which are denoted by the black squares.

$\begin{pmatrix} C'_{6(8)}(m_0) \\ C'_{7(9)}(m_0) \end{pmatrix} = \frac{Z}{2} \begin{pmatrix} v^5 \\ \Lambda^5 \end{pmatrix} \begin{pmatrix} C_{Qu1}(Qd1)(\Lambda) \\ C_{Qu2}(Qd2)(\Lambda) \end{pmatrix}, \quad (4.25)$

with

$Z = \begin{pmatrix} 1.43 & -0.23 \\ -0.10 & 0.68 \end{pmatrix}. \quad (4.26)$

The matrix $Z$ gives the one-loop running from the scale $\mu = \Lambda$, where $C_{Qu1}(\Lambda)$ and $C_{Qd1}(\Lambda)$ are given in Eq. (4.19) and $C_{Qu2}(\Lambda) = C_{Qd2}(\Lambda) = 0$, to the scale $\mu = m_0$.

In chiral perturbation theory, the quark-lepton operators are mapped onto hadron-lepton operators defined below the scale, $\Lambda_\chi \sim 1\,\text{GeV}$, with the non-perturbative QCD dynamics encoded by low-energy constants (LECs) [137, 138].

In several ways, the behavior of the vector operators in chiral perturbation theory is in striking contrast to that of scalar operators. For vector operators, their lowest order local interaction with pions is of the form $(\pi \partial_{\mu}\pi)\bar{e}\gamma^\mu\gamma_5 e^c$ which, after using the pion equations of motion, is $\pi\pi ee$, but importantly suppressed by a factor of $m_e$ and negligible [131, 132] – for further details see Appendix C. Unlike scalar operators, vector operators do not induce any operators at LO in the chiral power counting.
The leading contribution of the vector operators $O_{6,7,8,9}^{\mu\nu,\bar{c}}$ to the $0\nu\beta\beta$ decay amplitude instead arises from their mapping onto the hadronic operators $\pi N N N e e$ and $NN N N N e e$ [131, 132, 134] which are at NLO and $N^2$LO in Weinberg’s power counting [186, 187], respectively. While requiring that the amplitude is regulator independent implies that the LECs for these two operators are related, in the case of vector operators, no “promotion” of the $NN N N N e e$ operators to lower chiral order is needed, and at least for these vector operators, Weinberg’s power counting appears correct [134, 189]. In the case of vector operators, then, the $\pi N N e e$ and $NN N N N e e$ operators make comparable contributions to the $0\nu\beta\beta$-decay amplitude. The $0\nu\beta\beta$-decay rate, however, has a sizable uncertainty due to the unknown values of these LECs, as is illustrated in Fig. 4.3.

In heavy-baryon chiral perturbation theory [197], the effective Lagrangian for the leading hadron-level LNV interactions $\pi N N e e$ and $NN N N N e e$ is given by [134]

$$L_{eff}^{\text{vector}} = L_{\pi N}^{\text{vector}} + L_{NN}^{\text{vector}} ,$$

$$L_{\pi N}^{\text{vector}} = \frac{1}{v^5} \sqrt{2} g_A F_\pi \vec{p} S^\alpha \cdot \partial_\alpha \vec{n} \left[ g_{\pi N}^{\pi N} C_V^{(9)} (\vec{n}) + \tilde{g}_{\pi N}^{\pi N} \tilde{C}_V^{(9)} \right] \times \nu^\mu \bar{e} \gamma_\mu \gamma_5 e^c ,$$

$$L_{NN}^{\text{vector}} = \frac{1}{v^5} \langle \vec{p} n \rangle \langle \vec{p} n \rangle \left[ g_6^{NN} C_V^{(9)} (\vec{n}) + g_7^{NN} \tilde{C}_V^{(9)} \right] \times \nu^\mu \bar{e} \gamma_\mu \gamma_5 e^c ,$$

where $F_\pi = 91.2$ MeV, $g_A = 1.27$, and $S^\alpha$ and $\nu^\mu$ are the nucleon spin and velocity. The coefficients $C_V^{(9)}$ and $\tilde{C}_V^{(9)}$ are given by

$$C_V^{(9)} = C_6^{(9)} (m_0) + C_8^{(9)} (m_0) ,$$

$$\tilde{C}_V^{(9)} = C_7^{(9)} (m_0) + C_9^{(9)} (m_0) .$$

The LECs $g_{\pi N}^{\pi N}, \tilde{g}_{\pi N}^{\pi N}, g_6^{NN}, g_7^{NN} = O(1)$ in the naïve dimensional analysis (NDA) [140, 198], and little is known about them. Inspecting the numerical solution to the
RG Eqs. (4.25) and (4.26), one finds $\mathcal{C}_V^{(9)} = 1.43 v^5 / (2\Lambda^5) [C_{Qu1}(\Lambda) + C_{Qd1}(\Lambda)]$ – so that this Wilson coefficient has an $O(1)$ enhancement at low scales \(^3\) – and a small non-vanishing $\tilde{\mathcal{C}}_V^{(9)}$ has been generated.

In Fig. 4.2, we show the Feynman diagrams for the $0\nu\beta\beta$ decay at the hadron-level, that is the transition $nn \to ppee$, which are induced by the NLO operators $\pi NNee$ and $N^2LO$ operators $NNNNee$. In chiral power counting, the transition amplitudes of these two diagrams are at the same order. For the vector operators consider here, the amplitude for nuclear $0\nu\beta\beta$ decay $0^+ \to 0^+$ is [134]

$$A_{\text{vector}} = \frac{g_A^2 G_F^2 m_e}{\pi R_A} \mathcal{A}_M \bar{u}(k_1) \gamma_0 \gamma_5 C \bar{u}^T(k_2), \quad (4.29)$$

where $k_1$ and $k_2$ are the momenta of the emitted electrons, $G_F$ is the Fermi coupling constant, the radius $R_A = 1.2 A^{1/3}$ fm with $A$ the number of nucleons of the nucleus, and the reduced amplitude is given by

$$\mathcal{A}_M = \frac{m_e^2}{m_e v} \left[ \frac{1}{2} \left( g_V^{\pi N} C_V^{(9)} + g_V^{\pi N} \tilde{C}_V^{(9)} \right) M_{P,\text{sd}} - \frac{2}{g_A^2} \left( g_6^{NN} C_V^{(9)} + g_7^{NN} \tilde{C}_V^{(9)} \right) M_{F,\text{sd}} \right], \quad (4.30)$$

with $M_{P,\text{sd}} \equiv (M_{GT,\text{sd}} + M_{T,\text{sd}})$, where the basic NMEs $M_{GT,\text{sd}}^{AP} = -2.80$, $M_{T,\text{sd}}^{AP} = -0.92$ and $M_{F,\text{sd}} = -1.53$ for $^{136}\text{Xe}$ calculated using the quasi-particle random phase approximation (QRPA) [178]. The reader is referred to Ref. [133, 134] for definitions of these basic NMEs. The inverse half-life of the $0\nu\beta\beta$ decay is expressed as

$$(T_{1/2}^{0\nu})^{-1} = g_A^4 G_{09}^2 |\mathcal{A}_M|^2, \quad (4.31)$$

where the phase space factor $G_{09} = 2.8 \times 10^{-14}\text{yr}^{-1}$ for $^{136}\text{Xe}$ [199], following a trivial rescaling as discussed in [133, 134]; see also Ref. [200, 201]. Three-body nu-

\(^3\)Note that the factor of $v^5/(2\Lambda^5)$ comes from the definitions of Wilson coefficients.
cleon interactions such as $NNNNNNe$ contributing to $0^+ \to 0^+$ are expected to contribute at higher order in the Weinberg power counting and are not considered here.

As discussed above, the vector operators $O^{\mu}_{6,7,8,9} \bar{e} \gamma_{\mu} \gamma_5 e$ are mapped onto the operators $\pi NNee$ and $NNNNee$, whose contributions are suppressed in chiral power counting. This can be seen in Eq. (4.30) with the amplitude $\mathcal{A}_M$ being proportional to $m_\pi^2$, while amplitudes for the LO hadron-lepton operators are proportional to $\Lambda_\chi^2$ [131, 132, 134]. An explicit UV completion of these scalar quark-lepton operators can be found in Ref. [149].

As an aside, one may na"ively expect that in the absence of any underlying model and consequently no linear ordering of the magnitudes of the Wilson coefficients of these operators, the vector operators are quantitatively less important than scalar operators in $0\nu\beta\beta$ decay due to the suppression of the $0\nu\beta\beta$-decay amplitude by a factor of $(\Lambda_\chi/m_\pi)^2 \sim 60$. However, this level of suppression may not be realized in practice. To illustrate, consider a comparison with the $0\nu\beta\beta$-decay amplitude ($\mathcal{A}_{\text{scalar}}$) induced by scalar operators. It turns out that this suppression is sensitive to the size of certain unknown LECs as well as NMEs. To see why, first consider the limit where the unknown LECs for operators $NNNNee$ contributing to $\mathcal{A}_{\text{scalar}}$ follow the Weinberg power counting expectation of $O(1)$, then

$$\frac{\mathcal{A}_{\text{vector}}}{\mathcal{A}_{\text{scalar}}} \sim \frac{m_\pi^2}{m_N^2} \frac{M_{P,sd}}{M_{P,sd}}$$

(4.32)

where $m_N \sim \Lambda_\chi$ is the nucleon mass and $\mathcal{A}_{\text{scalar}}$ arises from the LO pion-exchange interaction. Here $M_{P,sd}$ is an NME that appears in the expression for $\mathcal{A}_{\text{scalar}}$; more details on each can be found in Appendix C. As shown there, the ratio of NMEs is about $6 - 8$ across various isotopes and methods for estimating their values. Thus,
the naïvely expected suppression may be too severe by nearly an order of magnitude. In addition, recent work suggests that the LECs for the $NNNNee$ arising from the scalar operators may be considerably larger than $\mathcal{O}(1)$ \cite{134,141,189–191} – for more details the reader is referred to Appendix C. The $NNNNee$ contribution to $A_{\text{scalar}}$ involves yet another NME, further clouding the estimated magnitude of the full scalar amplitude.

Substituting the values of the phase space factor, NMEs, and the other constants into Eq. (4.31), we obtain

$$
(T_{1/2}^{0\nu})^{-1} \simeq 6.0 \times 10^{-22} \left[ (0.98g_V^{\pi N} - 9_6^{NN}) - (0.069\tilde{g}_V^{\pi N} - 0.070g_7^{NN}) \right]^2 \\
\times |y_{e\Psi}(y_{qu} + y_{qd})\lambda_{ed}\lambda_{u\Psi}|^2 \times \left( \frac{\text{TeV}^5}{m_S^2 m_R^2 m_{\Psi}} \right)^2 \text{yr}^{-1}, \quad (4.33)
$$

where the scale of 1 TeV normalizes all of the masses.

The most stringent $0\nu\beta\beta$-decay limit comes from the experiment KamLAND-Zen \cite{10}: $T_{1/2}^{0\nu} > 2.3 \times 10^{26}$ yr at 90\% confidence level (C.L.). The constraint from the final results of the GERDA experiment \cite{202} is slightly weaker. There exist a number of planned experiments at the ton-scale \cite{11–16} that aim to improve the half-life sensitivity by about 2 orders of magnitude, reaching $T_{1/2}^{0\nu} > 10^{28}$ yr at 90\% C.L. From Eq. (4.33), we thus infer that future ton-scale $0\nu\beta\beta$-decay experiments are able to probe TeV-scale LNV with the masses $m_S = m_R = m_{\Psi} = 1$ TeV and the couplings $|y_{e\Psi}(y_{qu} + y_{qd})\lambda_{ed}\lambda_{u\Psi}| \gtrsim (3 - 9) \times 10^{-3}$ for $g_V^{\pi N} = \tilde{g}_V^{\pi N} = 1$ and $g_6^{NN} = g_7^{NN} = \pm 1/2$.

In Fig. 4.3, we show for three sets of assumed values for these low-energy constants, the half-lives of $0\nu\beta\beta$ decay for $^{136}$Xe as a function of the LNV scale. Clearly, we can see that typically, the LNV scale up to around 4.5 – 6 TeV for the Wilson coefficients $C_{Qu1} + C_{Qd1} = 1$ is in the reach of next-generation ton-scale $0\nu\beta\beta$-decay experiments.
Figure 4.3: Half-lives of $0\nu\beta\beta$ decay as a function of the effective LNV scale $\Lambda \equiv \Lambda/|C_{Q1} + C_{Q4}|^{1/5}$. The solid black curve is obtained with $g_{\nu}^{\pi N} = \tilde{g}_{\nu}^{\pi N} = 1$ and $g_{6}^{NN} = g_{7}^{NN} = 0$. The dotted (dashed) black curve is obtained with the same settings of $g_{\nu}^{\pi N}$, $\tilde{g}_{\nu}^{\pi N}$ but with $g_{6}^{NN} = g_{7}^{NN} = 1/2 (-1/2)$. The solid red and dashed red lines correspond to the constraints on the half-life in KamLAND-Zen and future ton-scale experiments for $^{136}$Xe.

The same figure also illustrates that the sensitivity of the inferred LNV scale to these LECs is sizable.

### 4.3.2 Neutrino masses: dimension-9 operators

Before closing this section, it is interesting to discuss the connection in this model between $0\nu\beta\beta$ decay and neutrino masses. On general grounds, the Schechter-Valle theorem [125] implies that the observation of $0\nu\beta\beta$ decay implies the existence of a light neutrino Majorana mass term. For the model of interest here, the $0\nu\beta\beta$-decay operators can induce Majorana neutrino masses at the loop level. It is instructive first to consider this connection after integrating out the heavy degrees of freedom.

Retaining only the light degrees of freedom, we obtain the loop-induced Majorana mass from diagrams such as Fig. 4.4. The red dot in Fig. 4.4 depicts the effective dot in the form of

$$
\frac{C_{Q1}}{\Lambda^5}(\bar{u}_R u_L)(\bar{u}_R \gamma_\mu d_R)(\bar{\epsilon}_R \gamma^\mu \nu_L), \quad \frac{C_{Q4}}{\Lambda^5}(\bar{d}_L d_R)(\bar{u}_R \gamma_\mu d_R)(\bar{\epsilon}_R \gamma^\mu \nu_L). 
$$

(4.34)
Figure 4.4: The LNV vector operators contribute to neutrino masses at three-loop or higher order and are completely negligible. Shown here is a typical three-loop Feynman diagram contributing to the neutrino mass. The red dot depicts the dimension-9 operators in Eq. (4.34).

In the SMEFT, they arise from the \( \nu \)-component of the \( SU(2)_L \times U(1)_Y \) invariant dimension-9 effective operators in Eq. (4.18). The contribution of these operators to the neutrino mass is highly suppressed by three insertions of the light quark masses and one of the electron mass, as well as by the loop factors, and is estimated as

\[
m_\nu \sim \frac{m_d m_u m_e \Lambda_{UV}^2}{(16\pi^2)^3} \frac{\Lambda^5}{\Lambda} \left( m_u C_{Qu1} + m_d C_{Qu2} \right),
\]

\[
\simeq 10^{-24} \text{ MeV} \left( \frac{m_{u,d}^3 m_e}{\text{MeV}^4} \right) \left( \frac{\Lambda_{UV}}{1 \text{ TeV}} \right)^2 \left( \frac{1 \text{ TeV}}{\Lambda} \right)^5,
\]

which is completely negligible compared to the actual neutrino masses. Here \( \Lambda_{UV} \) is an UV cutoff, with \( \Lambda_{UV} \sim \Lambda \). Two powers of \( \Lambda_{UV} \) arise from the quadratic divergence in the “bubble” loop of light quarks appearing below the “horizontal line” in Fig. 4.4. (In dimensional regularization, two powers of the light quark masses would appear instead.) In obtaining this estimate, we have defined \( \Lambda^5 \equiv (m_S^2 m_R^2 m_\Psi) \) and set equal to one the product of Yukawa couplings appearing in the Wilson coefficients \( C_{Qu1} \) and \( C_{Qd1} \): \( \frac{1}{2} y_e \lambda_{ed} \lambda_{u} y_{qu} \equiv \frac{1}{2} y_e \lambda_{ed} \lambda_{u} y_{qd} \equiv 1 \). The simplified model presented here cannot generate the observed neutrino masses, which, therefore, must arise from

116
In the simplified model, where all the fields are not integrated out, a neutrino mass is first generated at the three-loop order, as shown, for example, by the Feynman diagram given in Fig. 4.5. If $S$ acquires a vacuum expectation value (vev) $\langle S \rangle$, then a neutrino mass is generated at the two-loop order, as can be seen from a straightforward inspection of Fig. 4.5. With $\Lambda_{\text{UV}} \sim \Lambda$, the estimate for the neutrino mass from such a two-loop diagram is larger by a factor of $16\pi^2\langle S \rangle/m_d$ compared to the estimate from the three-loop diagram. In this case, the neutrino mass is still suppressed far below its actual value. If a vev for $S$ does not occur at tree-level, at one-loop a term $S^\dagger H$ in the Lagrangian is generated, which leads to $\langle S \rangle \sim m_d \Lambda_{\text{UV}}^2/(16\pi^2m_S^2)$. With $\langle S \rangle$ of this size, then the direct three-loop diagram in Fig. 4.5 and two-loop $\langle S \rangle$ induced neutrino masses are parametrically the same sizes.

We end by returning to the Schechter-Valle “black box” theorem [125]. It is noted that the contribution to Majorana neutrino masses arising from an insertion of the $0\nu\beta\beta$ decay “black-box operator” occurs at the four-loop level. The induced
shift in the Majorana neutrino mass, derived using the experimental limit, is roughly $2 \times 10^{-28}$ eV [127]. Here we have updated the numerical results of Ref. [127] to reflect the more recent KamLAND-Zen limit [10]. This value is much smaller than the direct contribution to neutrino masses arising in the simplified model considered here.

### 4.3.3 Dimension-7 operator

Here we consider the effects of the interaction

$$y_{e\Psi}^\prime \bar{\Psi}_L H e_R$$ (4.36)

on $0\nu\beta\beta$ decay. This interaction appears in the last line of the Lagrangian in Eq. (4.1) and is allowed by all symmetries. It does not contribute to the LNV dimension-9 operators, but its presence does generate an LNV dimension-7 operator above the electroweak scale. The Yukawa coupling $y_{e\Psi}^\prime$ has zero lepton number.

To see that it cannot be forbidden by any symmetry, note that the model already has Yukawa couplings of $S$ to the SM quarks. Under any discrete symmetry, $S$ and $H$ must have the same charge, which means we cannot distinguish $S$ from $H$. Thus if we have $\bar{\Psi}_L S e_R$ in the Lagrangian, we cannot forbid $\bar{\Psi}_L H e_R$. Any $Z_N$ discrete symmetry cannot forbid this operator.
Integrating out $S$ and $R$ at tree-level gives the following new contribution to the
effective theory below that mass scale,

$$\mathcal{L}_{\text{SMEFT}}^{(7)} = \frac{1}{\Lambda^3} C_{\text{Leud}H} O_{\text{Leud}H} + \text{H.c.}, \quad (4.37)$$

where the dimension-7 operator is given by

$$O_{\text{Leud}H} = (\bar{u}_R \gamma_\mu d_R)(\bar{e}_R \gamma^\mu L^c) H^*,$$  

with a Wilson coefficient

$$\frac{C_{\text{Leud}H}}{\Lambda^3} = \frac{1}{2} \frac{y_{e\Psi}^l \lambda_{ed} \lambda_{u\Psi}}{m_{R}^2 m_{\Psi}}. \quad (4.39)$$

The Feynman diagram generating this effective interaction is shown in Fig. 4.6. Note that this Wilson coefficient depends on the same combination of Yukawa couplings ($\lambda_{ed} \lambda_{u\Psi}$) that appears in the Wilson coefficients of the dimension-9 operators. Of course, here $y_{e\Psi}^l$ also occurs, which does not appear in the dimension-9 Wilson coefficients.

The operator $O_{\text{Leud}H}$ is proportional to a current of quarks, so the QCD RG evolution of this operator is trivial. After the electroweak symmetry breaking, the Higgs doublet $H$ develops the vev $\langle H \rangle = v/\sqrt{2}$, and the following effective Lagrangian in the LEFT is obtained

$$\mathcal{L}_{\text{LEFT}}^{(6)} = \frac{1}{v^2} C_{\text{VR}}^{(6)} O_{\text{VR}}^{(6)} + \text{H.c.}, \quad (4.40)$$

where the dimension-6 operator is
Figure 4.7: Hadron-level Feynman diagram for $0\nu\beta\beta$ decay, induced by the LNV operator $np\bar{e}\nu^c$ denoted by the black square. The other vertex denotes the single $\beta$ decay operator from SM interactions.

\[
O_{VR}^{(6)} = (\bar{u}_R \gamma_{\mu} d_R)(\bar{e}_R \gamma_{\mu} \nu^c).
\] (4.41)

and the Wilson coefficient is given by

\[
\frac{1}{\nu^3} C_{VR}^{(6)} = \frac{1}{\sqrt{2} \Lambda^3} C_{LeudH}^{re}.
\] (4.42)

Due to the presence of the neutrino, this interaction makes a long-distance contribution to $0\nu\beta\beta$ decay as shown in Fig. 4.7.

Below the scalar $\Lambda_{\chi} \sim 1$ GeV, the quark-lepton operators are mapped onto hadron-lepton operators. Up to NLO in chiral expansion, the resulting one-body current for single $\beta$ decay was shown explicitly in Ref. [133].

At LO in the Weinberg power counting, the dimension-6 operator $O_{VR}^{(6)}$ makes a vanishing contribution to $0\nu\beta\beta$ decay. The first non-vanishing contribution is obtained by expanding the one-body vector and axial currents to NLO [133]. The contribution of the dimension-6 operator $O_{VR}^{(6)}$ to the amplitude for $0^+ \rightarrow 0^+$ is given by [133, 134]
\[ \mathcal{A}_{VR} = \frac{g_A^2 G^2_F m_e}{\pi R_A} \left[ A_E \bar{u}(k_1) \gamma_0 C \bar{u}^T(k_2) \frac{E_1 - E_2}{m_e} + A_{m.e} \bar{u}(k_1) C \bar{u}^T(k_2) \right], \]  

(4.43)

where \( E_1 \) and \( E_2 \) are the energies of the emitted electrons, and the reduced amplitudes are

\[ A_E = C^{(6)}_{VR} M_{E,R}, \quad A_{m.e} = C^{(6)}_{VR} M_{m.e,R}, \]  

(4.44)

and the NMEs

\[ M_{E,R} = -\frac{1}{3} \left[ \frac{g_V^2}{g_A^2} M_F - \frac{1}{3} (2M_{GT}^{AA} + M_T^{AA}) \right], \]  

(4.45a)

\[ M_{m.e,R} = \frac{1}{6} \left[ \frac{g_V^2}{g_A^2} M_F + \frac{1}{3} (M_{GT}^{AA} - 4M_T^{AA}) \right] \]

\[ + 3(M_{GT}^{AP} + M_{GT}^{PP} + M_T^{AP} + M_T^{PP}) \]. \]  

(4.45b)

Here, \( g_V = 1 \), and the six long-range NMEs are given by \( M_F = -0.89, M_{GT}^{AA} = 3.16, M_{GT}^{AP} = -1.19, M_{GT}^{PP} = 0.39, M_T^{AP} = -0.28, \) and \( M_T^{PP} = 0.09 \) for \( ^{136}\text{Xe} \) calculated using the QRPA method [178]. The NME \( M_T^{AA} \) cannot be extracted from current calculations of light and heavy neutrino Majorana mass contributions to \( 0\nu\beta\beta \) decays of current experimental interest. Here we set it to zero as it is expected to be small compared to the other NMEs.\(^4\) With these as input, \( M_{E,R} = 0.89 \) and \( M_{m.e,R} = -0.41 \), again for \( ^{136}\text{Xe} \).

The amplitude is proportional to the outgoing electron energies or the electron mass and, therefore, highly suppressed. In the Weinberg power counting, three-body

\(^4\) In the case of light nuclei, Variational Monte Carlo (VMC) calculations for \(-M_T^{AA} \simeq 0.06 - 0.25\) are obtained in [203]. Among \( \Delta I = 2 \) nuclear isospin transitions, the largest value found is \(-M_T^{AA} \simeq 0.15\).
nucleon interactions and loops of pions are expected to make comparable contributions. As a result, the limit and projection obtained below should be viewed as an order-of-magnitude estimate [133].

The inverse half-life is given by [133]

\[
(T_{1/2}^{0\nu})^{-1} = g_A^4 \left[ 4G_{02}|A_E|^2 - 4G_{03}\text{Re}(A_{m_e}A_E) + 2G_{04}|A_{m_e}|^2 \right], \tag{4.46}
\]

with the phase space factors \(G_{02} = 3.2 \times 10^{-14}\text{yr}^{-1}\), \(G_{03} = 0.86 \times 10^{-14}\text{yr}^{-1}\), and \(G_{04} = 1.2 \times 10^{-14}\text{yr}^{-1}\) for \(^{136}\text{Xe}\). Substituting for \(C_{\text{VR}}^{(6)}\), the NMEs, and the phase space factors give

\[
(T_{1/2}^{0\nu})^{-1} = 8.6 \times 10^{-18}|y'_{e\Psi}\lambda_{ed}\lambda_{w\Psi}|^2 \times \left(\frac{\text{TeV}^3}{m_R^2m_{\Psi}}\right)^2 \text{yr}^{-1}. \tag{4.47}
\]

The Kamland-Zen limit of \(T_{1/2}^{0\nu} > 2.3 \times 10^{26}\text{yr}\) gives a constraint of \(|y'_{e\Psi}\lambda_{ed}\lambda_{w\Psi}| < 3.3 \times 10^{-5}\) for \(m_R = m_{\Psi} = 1\text{TeV}\). Setting the Yukawa couplings equal to one, this bound translates into a bound on \(\Lambda \equiv (m_R^2m_{\Psi})^{1/3} \gtrsim 40\text{TeV}\). Current and projected
limits are shown in Fig. 4.8, where we can see that future ton-scale experiments are sensitive to the LNV scale up to 90 TeV for the Wilson coefficient $C_{Leu} = 1$.

One can see that $0\nu\beta\beta$ decay is more sensitive to the dimension-7 operator $O_{Leu}$ compared to the dimension-9 vector operators. Following Refs. [133,134], this can be understood in chiral power counting. Denoting $\epsilon_\chi = q/\Lambda_\chi$ with $q \sim m_\pi$ the typical momentum transfer of $0\nu\beta\beta$ decay, then the typical outgoing electron energies scale as $E \sim \text{MeV} \sim \Lambda_\chi \epsilon_\chi^3$. Since the reduced amplitudes $m_e A_E$ and $m_e A_{m_e}$ induced by $O^{(6)}_{\text{VR}}$ are proportional to the electron energies, they scale as $\Lambda_\chi \epsilon_\chi^3$. In contrast, $m_e A_M$ induced by the dimension-9 vector operators scales as $\Lambda_\chi^2 \epsilon_\chi^2/v$. Although all of them are chirally suppressed, $A_M$ is smaller than $A_E$ and $A_{m_e}$ by a factor of $\Lambda_\chi/(v \epsilon_\chi) \simeq 0.03$.

The existence of the Yukawa interaction $y' e\Psi L e_R$ would also contribute to the deviation from unitarity of the Pontecorvo-Maki-Nakagawa-Sakata matrix and mass mixing of charged leptons with more phenomenological implications and constraints. However, since the LHC searches considered in the next section are insensitive to this interaction, we simply assume that it is suppressed, which might be realized in UV theories, and focus on the interplay of chirally suppressed $0\nu\beta\beta$ decay induced by the dimension-9 vector operators and LHC searches.

### 4.3.4 Neutrino masses: dimension-7 operators

This discussion closes with some comments about neutrino masses. Like the dimension-9 operator previously discussed in Section 4.3.1, the dimension-7 operator Eq. (4.37) also generates neutrino masses at higher loop order. In particular, a neutrino mass is first generated at two loops and is finite. It is highly suppressed due to one electron mass and two light quark mass insertions, and one estimates that
which is completely negligible. Consequently, the observed values of the neutrino masses do not constrain the size of this Wilson coefficient.

### 4.4 Collider searches at the LHC

We will investigate the complementary test of TeV-scale LNV in our simplified model using LHC searches. The process with a pair of first-generation leptons with the same charge \((e^\pm e^\pm)\) and at least two jets would provide a clear sign of LNV at the LHC. Searches for this same-sign dilepton plus dijet signal have thus far yielded null results, leading to constraints on different models for TeV-scale LNV. We analyze the corresponding implications for our simplified model below. Additionally, searches for leptoquarks as well as those for dijet resonances, which do not rely on LNV signatures, can also be used to extend the sensitivities to the masses and couplings of new particles in our simplified model.

In this section, we will reinterpret the existing searches, which are performed at the 13 TeV LHC with integrated luminosities of \(~ 40 \text{ fb}^{-1} \sim 140 \text{ fb}^{-1}\), in terms of the corresponding LNV and lepton number conserving processes generated in our model, then make projections for the high-luminosity LHC (HL-LHC) with the integrated luminosity of \(3 \text{ ab}^{-1}\). In all of our projections for exclusion and discovery at the HL-LHC, we make the strong assumption that the selection cuts and efficiencies remain unchanged from those in existing searches. The reaches presented here might be improved with the optimization of cuts.
Note that both $y_{qu}$ and $y_{qd}$ terms in Eq. (4.1) can contribute to the scalar production at the LHC, which do not interfere. For the charged scalar $S^\pm$, their contributions to the production cross section are the same if $y_{qu} = y_{qd}$. For the neutral scalar $S^0$, however, the contribution to the production cross section from the $y_{qu}$ term is larger than that from the $y_{qd}$ term since $u$-quark parton distribution function (PDF) is typically about two times larger than $d$-quark PDF for the Bjorken $x \sim m_S/\sqrt{s}$, which is around 0.1 – 0.2. Here, $\sqrt{s} = 13$ TeV is the center-of-mass energy of the LHC. Consequently, the constraints from the dijet search in the case of non-vanishing $y_{qd}$ are relatively weaker than that for non-vanishing $y_{qu}$.

Besides, different from the $0\nu\beta\beta$-decay half-life that depends on $(y_{qu} + y_{qd})^2$, there is no interference between the contributions from the $y_{qu}$ and $y_{qd}$ terms to the production cross sections of $pp \rightarrow S^\pm, S^0$. Hereafter, to simplify the presentation of our analysis, we assume $y_{qu} = 0$ and $y_{qd} \neq 0$ and study the interplay of LHC searches and $0\nu\beta\beta$ decay. Our conclusions would be qualitatively similar if $y_{qd} = 0$ and $y_{qu} \neq 0$ were assumed instead.

4.4.1 Leptoquark searches

Direct search for pairs of first-generation leptoquarks by the ATLAS collaboration at the 13 TeV LHC with an integrated luminosity of 139 fb$^{-1}$ has excluded the leptoquark mass region below 1.8 TeV [204]. To make a conservative projection for the future sensitivity to leptoquarks, we assume that both the observed upper limit on the number of signal events and the number of the background events in Ref. [204] scale with the integrated luminosity and obtain that the lower limit on the leptoquark mass at the LHC is about 2 TeV with an integrated luminosity of 1 ab$^{-1}$. These constraints become weaker if the decay branching ratio of leptoquark into electron $e^+/e^−$ and quark is less than 1.
Figure 4.9: Representative Feynman diagrams for the parton-level pair production of leptoquarks \((R^{\pm Q})\). Here, \(g\) denotes a gluon, \(q\) denotes a quark, and the electric charge \(Q = 2/3, 1/3\).

Figure 4.10: Feynman diagram for the cascade decays of leptoquarks. The labels \(P_1, \ldots, P_7\) denote the possible particles in the chain, which are specified in Table 4.1.

In the analysis of the leptoquark search \([204]\), a pair of opposite-sign electrons \(e^+e^-\) and at least two jets are required in the final state. In addition, the pairs of electron and jet closest in the invariant mass must satisfy \(m_{ej}^{\text{asym}} \equiv (m_{ej}^{\text{max}} - m_{ej}^{\text{min}})/(m_{ej}^{\text{max}} + m_{ej}^{\text{min}}) < 0.2\), where \(m_{ej}^{\text{max}}\) and \(m_{ej}^{\text{min}}\) denote the larger and smaller of the two electron-jet invariant masses. The upper limits on \(\sigma \times B \times A\) for each leptoquark mass are derived by performing fits to the distribution of \(m_{ej}^{\text{mean}}\), which is defined as \(m_{ej}^{\text{mean}} \equiv (m_{ej}^{\text{max}} + m_{ej}^{\text{min}})/2\). Here, \(\sigma\) denotes the leptoquark pair production cross section, \(B\) is the product of the decay branching ratios of intermediate particles, and \(A\) describes the overall acceptance.

The signal process \(pp \rightarrow R^{2/3} R^{-2/3}, R^{2/3} \rightarrow e^d, R^{-2/3} \rightarrow e^{-\bar{d}}\) has the same acceptance \(A\) as Ref. \([204]\). Thus the upper limit on \(\sigma \times B\) reported in Ref. \([204]\) can be directly applied to this signal process. However, this conclusion is not true.
for other signal processes due to different cut acceptances. To be more specific, in our simplified model, the leptoquark $R$ can also have a cascade decay if $m_R > m_\Psi$.

In Figs. 4.9 and 4.10, we show schematic Feynman diagrams for the pair production and cascade decays of leptoquarks.

In fact, for the other leptoquark pair production processes such as $pp \to R^{2/3}R^{-2/3}$, $R^{-2/3} \to e^- \bar{d}$, $R^{2/3} \to \Psi^0 u$, $\Psi^0 \to e^+ \bar{u}d$, and $pp \to R^{+1/3}R^{-1/3}$, $R^{+1/3} \to \Psi^+ \bar{u}$, $R^{-1/3} \to \Psi^- u$, $\Psi^\pm \to e^\pm \bar{d}d$, after imposing the cut $m_{ej}^\text{asym} < 0.2$ the efficiencies and the values of $m_{ej}^\text{mean}$ are smaller since there are more jets in the final state. This leads to a smaller overall acceptance $A$. Finally, it is noted that in our model, the processes $pp \to e^+ \Psi^0$, $\Psi^0 \to e^- u\bar{d}$ and $pp \to e^- \Psi^0$, $\Psi^0 \to e^+ d\bar{u}$ also contribute to our reinterpretation of the leptoquark search [204], but their contributions are substantially reduced by the selection cut on $m_{ej}^\text{asym}$.

After taking into account all of the signal processes with the possible decays of $R$ and deducing the corresponding acceptance, the lower bound on the leptoquark mass is expected to be smaller than that in Ref. [204]. Instead of reanalyzing the fits for all of these signals, we will assume conservatively the leptoquark mass $m_R = 2 \text{TeV}$.

The leptoquark pair production $pp \to R^{+1/3}R^{-1/3}$ can also give signatures of jets + MET (monojet), jets + $e^\pm$ + MET, where the missing energy (MET) comes from

<table>
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<tr>
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<tr>
<td>$R^{2/3}$</td>
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</table>

Table 4.1: The cascade decays of leptoquarks in the presence of $\Psi$. 
neutrino(s) in $R^{+1/3} \rightarrow d\bar{\nu}$ and/or $R^{-1/3} \rightarrow d\bar{\nu}$ and $e^\pm$ come from the cascade decay of $R^{\pm1/3}$. However, due to the low trigger efficiencies of these signals, the constraints are expected to be very weak and will not be considered hereafter.

Finally, it is worth noting that other searches for first-generation leptoquark may give constraints comparable to the pair production with the assumption that both the leptoquark coupling $\lambda_{ed}$ and decay branching ratio are equal to 1. In Ref. [205], a search for the single production of leptoquark was performed by the CMS Collaboration at the 8 TeV LHC with an integrated luminosity of 19.6 fb$^{-1}$. The recast and projection of this search with the same selection cuts being imposed [206] show that $m_R < 1.4$ TeV could be excluded at the 13 TeV LHC with an integrated luminosity of 36 fb$^{-1}$. For comparison, the search for leptoquark pair production has excluded $m_R < 1.435$ TeV by the CMS Collaboration with an integrated luminosity of 35.9 fb$^{-1}$ [207]. Ref. [208] finds that $m_R < 3.6$ TeV would be able to be excluded at the HL-LHC with an integrated luminosity of 3 ab$^{-1}$ with the boosted decision tree method being used in the analysis. It was found in Refs. [206, 209] that other constraints on $m_R$ can be obtained in the searches for single resonant production and Drell-Yan production, which could exclude $m_R < 2.5$ TeV and $m_R < 3.8$ TeV at the 13 TeV LHC with the integrated luminosities of 36 fb$^{-1}$ and 139 fb$^{-1}$, respectively, and $m_R < 4.2$ TeV with the integrated luminosity of 3 ab$^{-1}$ in the former process.

One might thus ask if $m_R = 2$ TeV that we choose for our benchmarks satisfies the current constraints in these leptoquark searches. It is important to note that the cross-sections of (1) the single production of leptoquark and (2) the single resonant production of leptoquark are proportional to $|\lambda_{ed}|^2$, and the cross-section of (3) the Drell-Yan production mediated by the exchange of leptoquark is proportional to $|\lambda_{ed}|^4$. By decreasing $\lambda_{ed}$, these constraints will be significantly released. With integrated
luminosities of 36 fb$^{-1}$, 36 fb$^{-1}$ and 139 fb$^{-1}$ at the 13 TeV LHC, $m_R < 0.9$ TeV, 1.8 TeV, 1.5 TeV with $\lambda_{ed} = 0.6$, 0.4, 0.6 for the processes (1), (2), and (3), respectively [209]. On the other hand, the impact on the mass reach for the pair production of leptoquarks is much smaller.

### 4.4.2 Same-sign dilepton plus dijet search

In order to directly test TeV-scale LNV at the LHC, we study the same-sign dilepton plus dijet (SSDL) processes. In our simplified model, lepton number is violated if $\lambda_{ed} \lambda_{u} \Psi \neq 0$, which translates into (1) if $\bar{\Psi}^0 \rightarrow e^- \bar{d} u$ or its charge conjugate occurs since the lepton number of $\Psi^0$ is equal to 1; (2) if $R^{-2/3} \rightarrow e^- \bar{d}$ and $R^{+2/3} \rightarrow \Psi^0 u$ or their charge conjugates occur simultaneously. To this end, we will consider the following signal processes (with processes having charge-conjugate final states not shown explicitly):

- **SS-1**: $pp \rightarrow e^- \bar{\Psi}^0$, $\bar{\Psi}^0 \rightarrow e^- \bar{d} u$. It proceeds through an s-channel production of $S^-$, followed by an LNV (cascade or three-body) decay of $\bar{\Psi}^0$.

- **SS-2**: $pp \rightarrow R^{+2/3} R^{-2/3}$, $R^{-2/3} \rightarrow e^- \bar{d}$, $R^{+2/3} \rightarrow \Psi^0 u$, $\Psi^0 \rightarrow e^- \bar{d} u$ if $m_R > m_{\Psi}$. Here the decay of $\Psi^0$ is lepton number conserving.

- **SS-3**: $pp \rightarrow R^{+2/3} R^{-2/3}$, $R^{-2/3} \rightarrow \bar{\Psi}^0 \bar{u}$, $R^{+2/3} \rightarrow \Psi^0 u$, $\Psi^0 \rightarrow e^- \bar{d} u$ and $\bar{\Psi}^0 \rightarrow e^- \bar{d} u$ if $m_R > m_{\Psi}$. Here the decay of $\Psi^0$ is lepton number conserving, and the decay of $\bar{\Psi}^0$ is lepton number violating.

The parton-level processes $\bar{u}d \rightarrow e^- \bar{\Psi}^0$ in SS-1 and $gg, q\bar{q} \rightarrow R^{+2/3} R^{-2/3}$ in SS-2 and SS-3 are shown in Fig. 4.11 and Fig. 4.9, respectively. The decays of $\bar{\Psi}^0$, $R^{+2/3}$ and their anti-particles can be read off in Fig. 4.10 and Table 4.1.

Searches for the right-handed $W^\pm_R$ boson and heavy neutrino at the 13 TeV LHC with the integrated luminosities of 36.1 fb$^{-1}$ [210], and 35.9 fb$^{-1}$ [211] have been
Figure 4.11: Feynman diagram for the parton-level process $\bar{u}d \to e^-\bar{\Psi}^0$ in SS-1.

performed by the ATLAS and CMS Collaborations, respectively. The ATLAS analysis [210] is divided into the same-sign and opposite-sign channels with $e^\pm e^\pm$ and $e^+e^-$, respectively. In comparison with the SSDL search, the opposite-sign dilepton search [210] suffers from much larger SM backgrounds and is always less sensitive to the couplings $y_{e\Psi}$ and $y_{qd}$ unless the magnitudes of the couplings satisfy $|\lambda_{ed,\psi}| \ll |y_{e\Psi,qd}|$. In the CMS analysis [211], there is no charge requirement on electrons, so its sensitivity cannot compete with that of the ATLAS analysis.

<table>
<thead>
<tr>
<th>Benchmark Points</th>
<th>$m_{\Psi}$</th>
<th>$m_S$</th>
<th>$m_R$</th>
<th>$\sigma_{S^\pm}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>BP1</td>
<td>1.0 TeV</td>
<td>2.0 TeV</td>
<td>2.0 TeV</td>
<td>3.2$y^2_{qd}$ pb</td>
</tr>
<tr>
<td>BP2</td>
<td>1.9 TeV</td>
<td>2.0 TeV</td>
<td>2.0 TeV</td>
<td>3.2$y^2_{qd}$ pb</td>
</tr>
<tr>
<td>BP3</td>
<td>1.0 TeV</td>
<td>4.5 TeV</td>
<td>2.0 TeV</td>
<td>6.4$y^2_{qd}$ fb</td>
</tr>
<tr>
<td>BP4</td>
<td>3.0 TeV</td>
<td>3.5 TeV</td>
<td>2.0 TeV</td>
<td>78$y^2_{qd}$ fb</td>
</tr>
</tbody>
</table>

Table 4.2: The benchmark points that we choose for the collider and $0\nu\beta\beta$-decay studies.

In Table 4.2, we show four benchmark points for $m_R = 2$ TeV and different $m_S$ and $m_{\Psi}$ assuming $m_S > m_{\Psi}$ to ensure that the cross-section of the SSDL signal is sizable enough. The cross section of $pp \to R^{+2/3}R^{-2/3}$ is 0.0155 fb at the 13 TeV LHC for the leptoquark mass $m_R = 2$ TeV [212], while that of $pp \to S^\pm$ depends on the value of $y_{qd}$, as shown in the last column. After taking into account the decay branching ratios, we find that SS-2 and SS-3 always give negligible contributions
compared to SS-1 unless the magnitudes of the couplings satisfy $|\lambda_{ed,u}\Psi| \ll |y_{e\Psi,qd}|$ or $|\lambda_{ed,u}\Psi| \sim |y_{e\Psi,qd}| \ll 1$.

If $|\lambda_{ed,u}\Psi| \ll |y_{e\Psi,qd}|$, the branching ratio of the lepton number violating decay $\bar{\Psi}^0 \rightarrow e^- \bar{d}u$ is proportional to $\lambda^4/y^4 \ll 1$, where $\lambda \sim \lambda_{ed,u}\Psi$ and $y \sim y_{qd,e}\Psi$, so that the cross sections of SS-1 and SS-3 are suppressed by the decay branching ratios compared to SS-2. If $|\lambda_{ed,u}\Psi| \sim |y_{e\Psi,qd}| \ll 1$, the cross section of SS-1 is suppressed by small $|y_{qd}|^2$, while those of SS-2 and SS-3 are not suppressed. In either case, the contribution of SS-2 and SS-3 to the SSDL signal can be comparable to that of SS-1. Nevertheless, in these two limits, the sensitivity of the SSDL signal is very suppressed due to the small signal cross-section. We have checked that for the benchmark points in Table 4.2, there is no interplay of the SSDL search and $0\nu\beta\beta$ decay\(^5\). Thus, we will only consider SS-1 and recast the SSDL search by the ATLAS Collaboration [210].

The following selection criteria (SR-ee cuts) are applied in Ref. [210]. A pair of same-sign electrons with the transverse momenta $p_T > 30$ GeV and the pseudorapidity $|\eta| < 2.47$ are selected. All selected events contain at least two jets with $p_T > 100$ GeV and $|\eta| < 2.0$. The invariant masses of two electrons ($m_{ee}$) and two jets ($m_{jj}$) satisfy $m_{ee} > 400$ GeV and $m_{jj} > 110$ GeV. The scalar sum of $p_T$ of electrons and the two most energetic jets ($H_T$) is larger than 400 GeV.

In the recast, we simulate the signal process SS-1 and obtain the selection efficiencies passing the SR-ee cuts. Owing to the same selection criteria, the SM

\(^5\)For illustration, we assume $y_{qд} = y_{e\Psi}/4 = 0.1$, which satisfies the existing dijet constraint as we shall see, and set $\lambda_{ed} = \lambda_{u\Psi} \equiv \lambda$. For BP2, the cross-section of SS-2 is comparable to that of SS-1 if $\lambda = 0.025$. Compared to the case of $\lambda = 1$, however, the total cross-section of the SSDL signal is smaller by an order of $10^3$, and the $0\nu\beta\beta$-decay rate is suppressed by an order of $10^6$. The situation is similar for BP1 and BP3.
backgrounds are the same as those modeled in Ref. [210]. Thus we can easily take the number of SM background events from Ref. [210] for the integrated luminosity of 36.1 fb$^{-1}$, which is 11.2 with the **SR-ee** cuts being imposed. For a larger integrated luminosity $L$, the number of SM background events is assumed to scale with the integrated luminosity, that is $11.2 \times \frac{L}{36.1}$ fb$^{-1}$.

We use the simulation pipeline described in Section 2.3. The selection efficiencies of signals events in SS-1 passing **SR-ee** cuts are 0.30, 0.10, 0.27, and 0.38 for the benchmark points BP1, BP2, BP3, and BP4, respectively\(^6\). Note that the selection efficiency for BP2 is smaller than those for the other benchmark points since the energy of electron produced in association with $\Psi$ is limited by the mass difference between $m_S$ and $m_\Psi$.

After imposing **SR-ee** cuts, a likelihood fit of the $H_T$ distribution was performed in Ref. [210] to derive the observed limits since signals tend to have larger $H_T$ compared to the SM backgrounds. We generate $H_T$ distributions of the signal SS-1 for four benchmark points, which are compared with the SM background $H_T$ distribution in Ref. [210], and find that the cut $H_T > 1.25$ TeV can efficiently separate the signals from the SM backgrounds. The overall selection efficiencies are 0.26, 0.09, 0.27, and 0.38 for BP1, BP2, BP3, and BP4, respectively – which are more-or-less the same as those for $H_T > 400$ GeV – and only $\sim 1$ background event remains with the integrated luminosity of 36.1 fb$^{-1}$. While the sensitivity of the SSDL search is improved, we will only impose the **SR-ee** cuts to avoid overestimating the sensitivity.

We evaluate the exclusion limit at 95% C.L. using the asymptotic formula [213]

\(^6\)They are validated against the efficiencies reported by the ATLAS Collaboration [210] in the context of right-handed $W^\pm_R$ boson and heavy neutrino.
\[ Z_{\text{excl}} \equiv \sqrt{2[s - b \ln ((s + b)/b)]} = 1.96 , \]  

where \( s \) and \( b \) are the numbers of signal and background events after passing the selection cuts, respectively. Besides the expected exclusion limit, we also consider the 5\( \sigma \) discovery potential for the SSDL search, which is obtained with \(^\text{[213]}\)

\[ Z_{\text{disc}} \equiv \sqrt{2[(s + b) \ln ((s + b)/b) - s]} = 5 . \] 

As mentioned earlier, to assess future prospects at the HL-LHC, we assume that the selection cuts and efficiencies remain unchanged. The sensitivities to the couplings in Eq. (4.1) will be discussed in Section 4.5.

### 4.4.3 Dijet resonance search

The mass of the scalar \( S \) and its couplings are constrained by the dijet resonance searches at the 13 TeV LHC with the integrated luminosities of 139 fb\(^{-1}\) \(^\text{[214]}\), and 137 fb\(^{-1}\) \(^\text{[215]}\) by the ATLAS and CMS Collaborations, respectively. In both searches, the upper limits on the product of signal cross-section and acceptance, the latter of which depends on the transverse momenta and pseudo-rapidities of jets, for resonances in some benchmark models were obtained. A simple scaling was used in Ref. \(^\text{[216]}\) to reinterpret the results \(^\text{[214, 215]}\) in the benchmark of new gauge boson \( W' \) to other mediators. We have verified that the reinterpretation in Ref. \(^\text{[216]}\) works well for Ref. \(^\text{[215]}\) by the CMS Collaboration. Specifically, we generate the signal \( pp \to S^\pm \) and assume that \( S^\pm \) only decays into \( u\bar{d} \) or \( d\bar{u} \), and set \( \sigma(pp \to S^\pm) \times A \) to be equal to the limits observed by the CMS Collaboration given in Ref. \(^\text{[215]}\), in which the acceptance of \( A = 0.5 \) for a scalar is declared. The upper limit on the coupling of \( S^\pm \) to the quarks we obtain agrees well with that in Ref. \(^\text{[216]}\). However, the acceptance for a scalar was not reported by the ATLAS Collaboration in Ref. \(^\text{[214]}\). Using the reinterpretation results given in Ref. \(^\text{[216]}\), we find that the acceptance for the
scalar in the ATLAS analysis [214] is about $A = 0.60$, 0.89, and 0.55 for $m_S = 2$ TeV, 3.5 TeV, and 4.5 TeV, respectively.

In Ref. [214], the 95% C.L. observed upper limits on $\sigma_{\text{dijet}} \times A$ are obtained after performing the likelihood fit of the dijet invariant mass distribution, where $\sigma_{\text{dijet}}$ denotes the signal cross-section. For $m_S = 2$ TeV, 3.5 TeV, and 4.5 TeV, we obtain that the upper limits on $\sigma_{\text{dijet}}$ are 38.3 fb, 5.3 fb and 6.3 fb, respectively. The projection of the upper limits is derived using Eq. (4.49), assuming that the numbers of signal and background events both scale with the integrated luminosity.

The total dijet cross section is given by

$$
\sigma_{\text{dijet}} = \sigma_{S^+} \text{Br}(S^+ \rightarrow \bar{d}u) + \sigma_{S^-} \text{Br}(S^- \rightarrow \bar{u}d) + \sigma_{S^0} \text{Br}(S^0 \rightarrow \bar{d}d), \quad (4.51)
$$

within our signal model, where the contribution of the neutral scalar $S^0$ is also included. Here $\sigma_{S^\pm}$ and $\sigma_{S^0}$ are the production cross sections of $S^\pm$ and $S^0$, respectively, and “Br” denotes the decay branching ratio. By requiring the dijet cross section $\sigma_{\text{dijet}}$ to be smaller than its observed upper limit for a given resonance mass [214], we obtain the exclusion limits of the couplings $y_{qd}$ and $y_{e\Psi}$ at 95% C.L., which will be presented in Section 4.5.

### 4.5 Results and discussion

In Section 4.3 and Section 4.4, we have investigated the $0\nu\beta\beta$ decay and LHC searches in our simplified model, respectively. Our interest is the regime where both sets of searches provide complementary and/or overlapping sensitivities. Thus, we consider new particle masses that allow for potentially observable effects at the LHC. Specifically, we assume that the masses $m_S > m_{\Psi}$ and $m_R = 2$ TeV, with benchmark values of $m_{\Psi}$ and $m_S$ shown in Table 4.2. From the expected sensitivity to $0\nu\beta\beta$-
decay rate at future ton-scale experiments, $|y_{e\psi}y_{qd}\lambda_{ed}\lambda_{u\psi}| \gtrsim \mathcal{O}(10^{-3})$ is required with a sizable uncertainty due to unknown values of LECs for vector operators. For illustration, we assume the LECs $g_{V}^{\pi N} = \tilde{g}_{V}^{\pi N} = 1$ and $g_{6}^{NN} = g_{7}^{NN} = 0$, and the couplings $|\lambda_{ed}\lambda_{u\psi}| = 1$. For a larger (smaller) $|\lambda_{ed}\lambda_{u\psi}|$, the sensitivity of $0\nu\beta\beta$ decay to the couplings $y_{qd}$ and $y_{e\psi}$ is stronger (weaker), while the LHC sensitivities are less affected. Besides, the sensitivity of $0\nu\beta\beta$ decay becomes stronger (weaker) for negative (positive) values of $g_{6,7}^{NN}$. In order to differentiate the couplings $\lambda_{ed}$ and $\lambda_{u\psi}$, a more detailed analysis of leptoquark searches in our simplified model is needed. As discussed in Section 4.4, the single production, single resonant production of the leptoquark, and the Drell-Yan production are sensitive to the coupling $\lambda_{ed}$.

We show the sensitivities in the $0\nu\beta\beta$ decay, SSDL search, and dijet search in Fig. 4.12 and Fig. 4.13. The current and projected exclusion limits are depicted in solid and dashed curves, respectively. The red regions are excluded by the $0\nu\beta\beta$-decay experiment KamLAND-Zen and future ton-scale experiments at 90% C.L. The current constraints from the SSDL search (green) and dijet search (blue) are obtained with the integrated luminosities of 36.1 fb$^{-1}$ and 139 fb$^{-1}$ at the 13 TeV LHC, respectively, while the future projections for the HL-LHC are made with the integrated luminosity of 3 ab$^{-1}$. We also show the $5\sigma$ discovery potential of the SSDL search at the HL-LHC in a green dot-dashed curve.

Both the SSDL and dijet searches are sensitive to the scalar mass $m_{S}$. For relatively small $m_{S}$, the cross-section of $pp \rightarrow S^{\pm}$ is sizable for BP1 and BP2. From the left panel of Fig. 4.12, the accessible region of $y_{qd}$ and $y_{e\psi}$ (the absolute value symbols are omitted) at future ton-scale $0\nu\beta\beta$-experiments has already been excluded by

\footnote{We will not consider the case $|\lambda_{ed,u\psi}| \ll |y_{e\psi,qd}|$, which the SSDL and $0\nu\beta\beta$-decay searches are insensitive to.}
Figure 4.12: The current and projected sensitivities in the plane of $y_{qd} - y_{e\Psi}$ for BP1 and BP2. The red regions are excluded at 90% C.L. by the KamLAND-Zen (solid curve) and ton-scale (dashed curve) experiments. The blue regions are excluded at 95% C.L. by the dijet searches with the integrated luminosities of 139 fb$^{-1}$ (solid curve) and 3 ab$^{-1}$ (dashed curve). The green regions are excluded at 95% C.L. by the SSDL searches with the integrated luminosities of 36.1 fb$^{-1}$ (solid curve) and 3 ab$^{-1}$ (dashed curve). The green dot-dashed curves correspond to the 5σ discovery potential at the HL-LHC. The product of leptoquark couplings $|\lambda_{ed}\lambda_{u\Psi}| = 1$ is assumed. $y_{e\Psi}$ and $y_{qd}$ denote the magnitudes of the couplings with the absolute value symbols omitted.
the existing SSDL and dijet searches for BP1. In such a scenario, there is no signal expected in $0\nu\beta\beta$-decay experiments barring extremely small $y_{qd}$.

The $0\nu\beta\beta$-decay and LHC experiments are complementary to the other benchmark points. For BP2, $m_\Psi$ is close to $m_S$, so that the decay branching ratio of $S^\pm \to e^\pm \Psi$ is largely suppressed, and the selection efficiency in the SSDL search is smaller compared to BP1. In the right panel of Fig. 4.12, we can see that the combination of the existing SSDL and dijet searches gives stronger constraints compared to the $0\nu\beta\beta$-decay search at KamLAND-Zen. For $y_{qd} \lesssim 0.1$, which is currently allowed, it is possible to observe a TeV-scale LNV signal in a large portion of the region $y_{e\Psi} \gtrsim 0.25$ in both future $0\nu\beta\beta$ decay and SSDL searches. If, however, no signal is observed, most of the region $y_{e\Psi} \gtrsim 0.15$ can be excluded.

In Fig. 4.13, larger values of $m_S$ are considered. The cross-sections of $pp \to S^\pm$ for BP3 and BP4 are smaller than those for BP1 and BP2 by $2-3$ orders of magnitude, as seen in Table 4.2. For BP3, the sensitivities to the couplings $y_{qd}$ and $y_{e\Psi}$ in KamLAND-Zen and future ton-scale $0\nu\beta\beta$-decay experiments are always better than
those of the current and projected SSDL searches, respectively, which is shown in the left panel. In addition, the constraint from the existing dijet search is much weaker than that for BP1 and BP2. For future prospects, the dijet search can probe a large portion of the region $y_{qd} \gtrsim 0.4$. One can observe TeV-scale LNV signal in both future SSDL and $0\nu\beta\beta$-decay searches for $y_{e\Psi} \gtrsim 0.35$ and $y_{qd} \gtrsim 0.15$, which is indicated by the $5\sigma$ contour. If a signal of LNV is only observed in ton-scale $0\nu\beta\beta$-decay experiments, it implies that either $y_{e\Psi} \lesssim 0.2$ or $y_{qd} \lesssim 0.1$, which could be distinguished using the dijet searches.

For BP1, BP2, and BP3, we have assumed that the Dirac fermion mass $m_\Psi$ is below the leptoquark mass $m_R$. For BP4, an alternative scenario $m_\Psi > m_R$ is considered, and $m_S$ is slightly larger than $m_\Psi$. In comparison with BP1, the sensitivity of SSDL search is reduced due to the smaller production cross section and decay branching ratio of $S^\pm$, although the decay branching ratio of $\Psi$ increases. Besides, the sensitivity of $0\nu\beta\beta$ decay also degrades since the decay rate is proportional to $1/(m_S^4 m_R^4 m_\Psi^2)$. In the right panel of Fig. 4.13, we can see that the projected sensitivity of SSDL search at the HL-LHC is better than that of $0\nu\beta\beta$ decay in ton-scale experiments in contrast to BP3, after taking into account the constraint from the dijet search. In analogy with BP2, it is very promising to observe TeV-scale LNV for $y_{e\Psi} \gtrsim 0.3$. We also note that BP2 and BP4 might be distinguished with differential distributions, such as $p_T$ of electrons or jets and $H_T$.

It is worth noting a few caveats about our projections: if the mass of the new particles is sufficiently large, the LHC loses sensitivity, and a future $pp$ collider may be needed; we assumed the same set of selection cuts and efficiencies that occurs at the time of this writing, in LHC collaboration analyses of 13 TeV LHC data; and we did not include any effects due to pile-up or pile-up subtraction. Our results indi-
cate that the HL-LHC has promising prospects for uncovering the chirally-suppressed mechanism with TeV-scale LNV, which is complementary to the ongoing $0\nu\beta\beta$-decay experiments. Our results suggest a more detailed study, including a more realistic assessment of backgrounds and optimizing the reach of the HL-LHC for TeV-scale LNV, is warranted.

4.6 Summary and Remarks of the Chapter

In this Chapter, we have studied $\Delta L = 2$ LNV interactions, whose contributions to $0\nu\beta\beta$ decay are chirally suppressed in a simplified model. While the $0\nu\beta\beta$ decay is insensitive to the details of the underlying mechanism, the LHC can provide complementary tests in the same-sign dilepton plus dijet search channel and, by using dijet and leptoquark searches, further distinguish different scenarios of TeV-scale LNV in our simplified model. We have calculated the half-life of $0\nu\beta\beta$ decay and investigated the current and projected sensitivities of the LHC searches.

In three of the benchmark points we examined (BP1, BP2, and BP4), we find that due to the on-shell production of a $2 - 3.5$ TeV charged scalar ($S^\pm$) in the $s$-channel at the LHC, the sensitivities of SSDL searches are better than those of $0\nu\beta\beta$-decay searches. Dijet searches can place severe constraints, and in particular, a large portion of parameter space accessible to future SSDL and $0\nu\beta\beta$-decay searches has been excluded by the existing dijet search. However, for a fourth benchmark point (BP3) in which the scalar particle has a mass of 4.5 TeV, we find that $0\nu\beta\beta$-decay searches have better sensitivity compared to the SSDL search, and the constraint from dijet search is much weaker. In all of the scenarios that we consider, most of the region within reach of the $0\nu\beta\beta$-decay search at future ton-scale $0\nu\beta\beta$-decay experiments can be covered at the $5\sigma$ discovery level in the SSDL search at the HL-LHC.
From the benchmark studies, we could obtain some general results about the future $0\nu\beta\beta$ decay and LHC searches for TeV-scale LNV, which at low energies generates dimension-9 vector operators:

- If one observes an LNV signal in the SSDL search but not in $0\nu\beta\beta$ decay, it implies that either the coupling of scalar to quark or lepton is extremely small (e.g., BP1) or the masses of new Dirac fermion and scalar are close (e.g., BP2 and BP4).

- If one observes an LNV signal in $0\nu\beta\beta$ decay but not in the SSDL search, however, there might exist a heavy scalar with its coupling to quarks or leptons is small (e.g., BP3).

- If LNV signals are observed in both $0\nu\beta\beta$ decay and SSDL search, either the production or decay of the scalar at the LHC is suppressed (e.g., BP2, BP3, and BP4).

- If there is no LNV signal observed, then LNV may have other origins or the lepton number may be conserved at the classical (Lagrangian) level.

In the case of LNV signal(s) being observed, depending on whether we observe a signal in a dijet search, we know whether the coupling of scalar to quarks is large or not.

We have focused on the potential of the LHC to search for the particles and interactions in the simplified model that underlie the dimension-9 LNV operators. It would be interesting to extend our analysis of LHC searches. It would also be interesting to revisit the issues explored here in a model that has acceptable levels of flavor-changing neutral current and charged lepton number violating processes.
CHAPTER 5
REVISITING NEUTRINOLESS DOUBLE BETA DECAY
IN THE TYPE-II SEESAW MECHANISM AND ITS
CONNECTION WITH LEPTON NUMBER CONSERVING
PROCESSES

5.1 Introduction and Motivation

One of the key open problems for particle physics is the origin of neutrino masses. If neutrinos are Majorana fermions, the mass term violates the lepton number by two units $\Delta L = 2$. The simplest realizations of the LNV Weinberg operator [153] at dimension-5 are type-I [1–5], type-II [66–71], and type-III [72,73] seesaw mechanisms at tree level. In order to understand how neutrinos masses are generated, it is essential to determine the Majorana nature of the neutrino, which can be undoubtedly assessed in $0\nu\beta\beta$ decay process $(A, Z) \rightarrow (A, Z + 2) + e^- + e^-$ [125].

There have been extensive studies of $0\nu\beta\beta$ decay beyond the standard mechanism with the exchange of light Majorana neutrinos [130, 131, 133, 134, 134, 143, 148–150, 185,192,217–222]. In the EFT framework, possible $\Delta L = 2$ LNV sources of $0\nu\beta\beta$ decay can be systematically described [131,133,134]. A recent review of this approach can be found in Ref. [223]. Other searches in the high-energy and low-energy (or high-precision) frontiers might also be used to test lepton number violation, which have been studied in UV complete models [131,134,218,219,221,222] and simplified models [130,143,148–150,192,220]. For example, at the LHC, searching for the same-sign charged electron pairs [224] provides a unique test of TeV-scale LNV, which is
complementary to the $0\nu\beta\beta$-decay searches in next-generation experiments [148,150].

Besides the LNV searches, new physics entering the $0\nu\beta\beta$ decay might also manifest in lepton-number-conserving (LNC) processes, such as direct searches for new particles or precision tests of SM interactions. In the approach of Standard Model (SM) EFT, it is also shown recently that dimension-6 LNC operators can help to discriminate the type-I, -II, and -III seesaw mechanisms [225].

$0\nu\beta\beta$ decay in the minimal type-I seesaw models and the minimal left-right symmetric model (mLRSM) [5,69] has drawn lots of attention because they provide simple explanations of the observed neutrino masses. It turned out that a detection of $0\nu\beta\beta$ decay in the former requires that the active-sterile neutrino mixing is extraordinarily small [226–228], which might fade the merit of type-I seesaw mechanism. If only one scalar triplet coupled to the left-handed leptons is introduced as in the canonical type-II seesaw mechanism, the contribution to $0\nu\beta\beta$ decay from the exchange of doubly-charged scalar [229] is negligible due to the severe constraints from charged lepton flavor violation [230]. In the mLRSM, where both type-I and type-II seesaw mechanisms are achievable, one can study different scenarios.

In particular, Ref. [231] found that contribution to $0\nu\beta\beta$ decay from the exchange of doubly-charged scalar in the LRSM may be substantial depending on whether the left-right symmetry is imposed. Moreover, Ref. [231] showed that the low-energy precision tests of parity violation in Møller scattering play an important role in the assessment of such a scenario. Besides the contribution from doubly-charged scalar, Ref. [221] obtained that contribution from the exchange of both left- and right-handed $W^\pm$ bosons, which leads to the long-range pion exchange at hadronic level [131], would
significantly increase the $0\nu\beta\beta$ decay rate.

In this study, we will revisit the connection between $0\nu\beta\beta$-decay search and LNC searches and reevaluate the half-life of $0\nu\beta\beta$ decay in the type-II seesaw scenario of the LRSM with the left-right symmetry not being imposed. Compared to Ref. [231], we pay attention to the type-II seesaw scenario and improve the earlier analysis in the way that: (1) the mixing between left- and right-handed $W$-bosons allowed by electroweak precision data is considered; (2) contributions to $0\nu\beta\beta$ decay from the exchange of right-handed neutrinos and doubly-charged scalar are included and evaluated using the EFT; (3) the region of right-handed neutrino mass as light as about 10 MeV is explored. We find that the EFT description of $0\nu\beta\beta$ decay gives a significant improvement in its sensitivity compared to the previous study, especially for a non-zero mixing between left- and right-handed $W$-bosons parameter. The interplay between $0\nu\beta\beta$ decay and the LNC processes plays an important role in assessing the type-II seesaw mechanism of generating neutrino masses.

This Chapter is based on Ref. [232].

### 5.2 Minimal Left-Right Symmetric Model

The minimal left-right symmetric model (mLRSM) [5, 69, 233–236] extends the SM gauge group to $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$, where $B$ and $L$ denote the SM abelian baryon and lepton quantum numbers. Henceforth, we denote the field representation under the $SU(2)_{R,L}$ and $U(1)_{B-L}$ groups by $(X_R, Y_L, Z_{B-L})$. The scalar sector consists of one bidoublet $\Phi \in (2, 2^*, 0)$,

$$\Phi = \begin{pmatrix} \phi_1^0 & \phi_2^+ \\ \phi_1^- & \phi_2^0 \end{pmatrix}, \quad (5.1)$$
and two scalar triplets $\Delta_L \in (1, 3, 2), \Delta_R \in (3, 1, 2)$

$$\Delta_{L,R} = \begin{pmatrix} \delta^+_{L,R}/\sqrt{2} & \delta^+_{L,R} \\ \delta^0_{L,R} & -\delta^+_{L,R}/\sqrt{2} \end{pmatrix}.$$  \tag{5.2}

The extended gauge group is broken after the neutral scalars acquire a vev

$$\langle \Delta_R \rangle = \begin{pmatrix} 0 & 0 \\ v_R/\sqrt{2} & 0 \end{pmatrix}, \quad \langle \Delta_L \rangle = \begin{pmatrix} 0 & 0 \\ v_L e^{i\theta_L}/\sqrt{2} & 0 \end{pmatrix},$$

$$\langle \Phi \rangle = \begin{pmatrix} \kappa/\sqrt{2} & 0 \\ 0 & \kappa'/e^{i\alpha}/\sqrt{2} \end{pmatrix},$$  \tag{5.3}

where all parameters are real [237]. The right-handed scalar triplet vev, $v_R$, is taken to lie well above the electroweak scale $v = 246$ GeV. The bidoublet vevs, $\kappa$ and $\kappa'$, satisfy $\sqrt{\kappa^2 + \kappa'^2} = v$. The phases $\alpha$ and $\theta_L$ induce CP violation and will be set to zero ($\alpha, \theta_L = 0$) for the purposes of this dissertation.

Right-handed (RH) neutrinos appear in the doublet $L_R = (N, \ell^R)\,^T$ and are thus charged under $SU(2)_R$ and couple to right-handed gauge bosons. It also couples to the scalar sector via the Yukawa interactions

$$\mathcal{L}_Y \ni - \bar{Q}_L (\Gamma_3 \Phi + \bar{\Gamma}_3 \tilde{\Phi}) Q_R - \bar{L}_L (\Gamma_1 \Phi + \bar{\Gamma}_1 \tilde{\Phi}) L_R$$

$$- \bar{L}_L \tau_2 \Delta_L f_L L_L - \bar{L}_R \tau_2 \Delta_R f_R L_R + \text{H.c.},$$  \tag{5.4}

where $\tilde{\Phi} = \tau_2 \Phi^* \tau_2$, $\tau_2$ is the Pauli matrix, $\Psi_{c,L,R} = P_{R,L} \Psi^c$ with $\Psi^c = C \Psi^T$, $C$ is the charge conjugation matrix satisfying $C = -C^{-1} = -C^\dagger = -C^T$, and $P_{R,L} = (1 \pm \gamma_5)/2$ are the left- and right-handed projectors.
After the electroweak symmetry breaking (EWSB), the leptonic Yukawa interactions in Eq. (5.4) give rise to neutrino masses through the type-I [1–5] and type-II [66–71] seesaw mechanisms in terms of the symmetric 6 × 6 matrix [238,239]

\[
M_n \equiv \begin{pmatrix}
M_L & M_D \\
M^T_D & M_R
\end{pmatrix},
\]

where the Dirac and Majorana mass matrices are given by

\[
M_D = \left(\kappa \Gamma_1 + \kappa' \Gamma_1^T\right)/\sqrt{2}
\]

and

\[
M_{L,R} = \sqrt{2} f_{L,R} v_{L,R},
\]

respectively. We diagonalize this neutrino mass matrix by using a 6 × 6 unitary matrix \(U\),

\[
m_\nu = \text{diag}(m_1, m_2, \ldots, m_6) = U^T M_n^\dagger U,
\]

and define mass eigenstate \(\tilde{\nu} = (\nu_1, \ldots, \nu_6)^T \equiv \tilde{\nu}_m + \tilde{\nu}_m^c\) via

\[
\tilde{\nu}_m = U^\dagger \begin{pmatrix}
\nu_L \\
N^c
\end{pmatrix}.
\]

The neutrinos \(\nu_{1,2,3}\) are active neutrinos, while \(\nu_{4,5,6}\) are sterile neutrinos. We can decompose the unitary matrix \(U\) as [240]

\[
U = U_1 U_2,
\]

with

\[
U_1 = \begin{pmatrix}
1 & R \\
-R^\dagger & 1
\end{pmatrix}, \quad U_2 = \begin{pmatrix}
U_{\text{PMNS}} & 0 \\
0 & U_R
\end{pmatrix},
\]

where \(U_{\text{PMNS}}\) is the usual 3 × 3 neutrino-mixing matrix, i.e. Pontecorvo-Maki-Nakagawa-Sakata (PMNS) matrix, \(U_R\) is an additional 3 × 3 matrix and the matrix
\( R = M_D M_R^{-1} \) gives rise to the active-sterile mixing. This particular parametrization allows us to explicitly show the seesaw mechanism by noticing that [240]

\[
U^\dagger_1 M_n U^*_1 = \begin{pmatrix} M_\nu & 0 \\ 0 & M_N \end{pmatrix},
\]

with

\[
M_\nu = M_L - M_D M_R^{-1} M_D^T,
\]

\[
M_N = M_R.
\]

This block-diagonal matrix is then diagonalized by the action of \( U_2 \),

\[
U^\dagger_2 \begin{pmatrix} M_\nu & 0 \\ 0 & M_N \end{pmatrix} U^*_2 = \begin{pmatrix} \widehat{M}_\nu & 0 \\ 0 & \widehat{M}_N \end{pmatrix},
\]

where \( \widehat{M}_\nu \equiv \text{diag}(m_1, m_2, m_3) \) and \( \widehat{M}_N \equiv \text{diag}(m_4, m_5, m_6) \).

Since the focus of this study is on the type-II mechanism, we will assume here that the type-I seesaw contribution is small, i.e., the mLRS is in the type-II dominance regime for neutrino mass generation. Then, \( M_D = 0 \) and the neutrino mass matrix in Eq. (5.5) becomes

\[
M_n = \begin{pmatrix} M_L & 0 \\ 0 & M_R \end{pmatrix}.
\]

From Eqs. (5.11a), (5.11b) and (5.12), we obtain

\[
U^\dagger_{\text{PMNS}} M_L U^*_{\text{PMNS}} = \widehat{M}_\nu,
\]

\[
U^\dagger_R M_R U^*_R = \widehat{M}_N.
\]
For the purposes of our work, the charged electroweak gauge bosons are the relevant fields. After the breaking of the $SU(2)_{L,R}$ gauge symmetries, we define the mass eigenstates $W_{1,2}^\pm$ as

$$
\begin{pmatrix}
W_L^+ \\
W_R^+
\end{pmatrix} = 
\begin{pmatrix}
\cos \zeta & -\sin \zeta \\
\sin \zeta & \cos \zeta
\end{pmatrix}
\begin{pmatrix}
W_1^+ \\
W_2^+ 
\end{pmatrix},
$$

where $\tan \zeta = \kappa \kappa'/v_R^2$. The charged gauge boson masses are

$$
M_{W_1} \simeq M_{W_L} = \frac{g_L v}{2} \equiv M_W, 
$$

$$
M_{W_2} \simeq M_{W_R} = \frac{g_R v_R}{\sqrt{2}},
$$

where $g_{L,R}$ are the gauge couplings of $SU(2)_{L,R}$. For convenience, we introduce

$$
\lambda = \frac{M_{W_1}^2}{M_{W_2}^2}, \quad \tan \beta = \frac{\kappa'}{\kappa} \equiv \xi.
$$

Notice that if the left-right symmetry is imposed, then $g_L = g_R$ equal to the SM $SU(2)_L$ gauge coupling $g$ and $M_W/M_{W_R} \simeq v/(\sqrt{2}v_R) \ll 1$.

5.3 Current Experimental Constraints at High Energies

The mass of the right-handed gauge boson $W_R$ is excluded up to $5$ TeV by direct searches performed by the ATLAS [210, 241] and CMS [211] collaborations at the LHC for $W_R$ decaying into sterile neutrinos. The CMS Collaboration also obtained a new lower limit excluding masses below $5.4$ TeV by using the full LHC Run-2 data of $138 \text{ fb}^{-1}$ [242]. The left-right mixing angle is also bounded ($\zeta \leq 1.25 \times 10^{-3}$) by requiring the CKM matrix unitarity [221, 243], but this constraint is weaker than those from direct searches for $W_R$ [142, 210, 211, 241].
<table>
<thead>
<tr>
<th>$M_{W_R}$</th>
<th>7 TeV</th>
<th>15 TeV</th>
<th>25 TeV</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\xi$</td>
<td>0</td>
<td>0.35</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 5.1: Different choices for the mass of right-handed gauge boson $M_{W_R}$ and the $W_L - W_R$ mixing parameter $\xi$ used in our analysis.

Ref. [222] shows that the ratio of vevs $\xi \equiv \tan \beta$ in Eq. (5.17) is constrained by the $\rho$ parameter [244],

$$\rho = 1 + \left[ -\sin^2(2\beta) + \frac{(1 - \tan^2 \theta_W)^2}{2} \right] \lambda, \quad (5.18)$$

where $\theta_W$ is the weak mixing angle. Notice that large $\xi$ values are allowed by the $\rho$ parameter for heavy $W_R$. Additionally, $\xi < 0.8$ is required to ensure the validity of perturbative expansions in the mLRSM scalar sector [245]. Both constraints are depicted in Fig. 5.1. In our analysis, we will choose different benchmark values for $\xi$ depending on the $W_R$ mass value, as shown in Table 5.1.

The mass of the doubly-charged scalar is excluded up to 870 GeV by direct searches performed by the ATLAS [246,247] and CMS [248] collaborations at the LHC. A future high-energy $pp$ collider like the Super Proton-Proton Collider (SPPC) [166] or the Future Circular Collider (FCC-hh) [249], operating at a center-of-mass energy of 100 TeV, is expected to probe the doubly-charged scalar to higher masses. Pair-production could occur via the Drell-Yan process, and the reach for the doubly-charged scalar mass could extend up to 5.07 TeV (2.51 TeV) for $M_{W_R} = 7$ TeV ($M_{W_R} = 15, 25$ TeV) with an integrated luminosity of 30 ab$^{-1}$ [250].

Besides direct searches, the Bhabha scattering process $e^+e^- \rightarrow e^+e^-$ places an indirect constraint on the mass of the heavy doubly-charged scalar. Ref. [231] obtained $M_{\delta_R^{\pm}/(f_R)_{ee}} > 1.5$ TeV based on the Large Electron-Positron (LEP) collider
data [251], The authors also rescale this limit for a future high-energy lepton collider like the Circular Electron Positron Collider (CEPC) an integrated luminosity of 1 ab\(^{-1}\) [252]. The reach at the CEPC is estimated to be 30 times stronger than that at the LEP.

### 5.4 Current Experimental Constraints and Future Sensitivities at Low Energies

In this section, we will investigate the sensitivities to the type-II seesaw scenario of the mLRS in low-energy processes. Ref. [221] has noticed that the constraints on the mass of \(W_R\) from \(K\) and \(B\) meson mixing measurements [253–255] are weaker than that from direct searches at the LHC.

Searches for charged lepton flavor violation (CLFV), such as \(\mu \rightarrow e\gamma\), \(\mu \rightarrow eee\), and \(\mu \rightarrow e\) conversion in nuclei, have set very stringent constraints on BSM physics [256,257]. Considering only the left-handed doubly-charged scalar contributions, the
lower bound is $M_{k_{L}} \pm \sqrt{(f_{L}^*)_{ee}(f_{L})_{ee}}^{-1/2} > 208 \text{ TeV}$ [231] from $\mu \rightarrow eee$ searches. A similar constraint is expected for the right-handed doubly-charged scalar. However, as shown in Ref. [231], such a bound does not apply if the left-right symmetry is not assumed in the Yukawa sector, namely $f_{R} \neq f_{L}$. The latter can be realized in, for example, the mLRS model with $D$-parity breaking [231, 258]. Hence, we will concentrate on the flavor-conserving processes at low energies.

5.4.1 Neutrinoless double beta decay

Since the LRSM provides a simple mechanism to explain the origin of neutrino masses in Eqs. (5.11a), it is natural to understand why it has been extensively studied in the context of $0\nu\beta\beta$ decay —see, for instance, Refs. [5, 69, 142, 219, 221, 231, 259–261]. In most of these works, the focus has been put on the right-handed neutrino exchange contribution to the $0\nu\beta\beta$-decay amplitude and the implications for collider searches. Our interest is to complement these studies by analyzing the interplay with another low-energy, high-intensity experiment, such as the PV Møller scattering. In Ref. [231], the authors studied the effects of the doubly-charged scalar contribution to $0\nu\beta\beta$ decay, but they argue, based on some estimates, that the right-handed neutrino contribution is safely negligible in their entire parameter space. We will show, using the recent advances in Effective Field Theory (EFT) [131, 133, 134, 262], that both contributions are sizeable and their relative magnitude depends on the parameter region.

In the EFT approach, the heavy degrees of freedom are integrated out at the right-handed scale ($M_{W_{R}}$) and matched to all effective operators that are invariant under the SM gauge group containing only SM fields, the so-called Standard Model EFT (SMEFT). The higher-dimensional operators then evolve to the electroweak scale ($M_{W}$), where the remaining heavy fields are integrated out again. At even lower energies, around $\Lambda_{\chi} \sim \text{GeV}$, the resulting EFT operators are mapped onto hadronic
lepton-number-violating operators by the use of chiral EFT, the low-energy EFT of QCD.

The type-II seesaw relation in Eq. (5.11b) directly relates the mass of right-handed neutrinos with the magnitude of the Yukawa coupling, $(\widehat{M}_N)_{ii} \sim (f_R)_{ii} v_R (i = 1, 2, 3)$. Exploring very small couplings would imply very light right-handed neutrinos, and masses $(\widehat{M}_N)_{ii} \lesssim \Lambda_\chi (i = 1, 2, 3)$ would require keeping these fields as dynamical degrees of freedom. We adopted the approach developed in Refs. [142, 262] for light sterile neutrinos to handle this scenario. There, the authors match the high-energy lepton-number-violating Lagrangian to the neutrino-extended Standard Model EFT ($\nu$SMEFT) [263, 264], and now both SM fields, as well as singlet $\nu_R$ fields, are part of the higher-dimensional operators. Using interpolation functions, they show that the use of $\nu$SMEFT is accurate even if the right-handed neutrino is heavy enough to be integrated out, comparing their results with SMEFT.

We will briefly describe the EFT formulae here and refer the reader to Refs. [131, 133, 134, 142, 262] for further technical details.

The Feynman diagrams for the contributions to $0\nu\beta\beta$ decay in type-II dominance scenario are shown in panels (a)-(d) of Fig. 5.2. Besides, we also consider the contributions from right-handed doubly-charged scalar $\delta_{R}^{--}$ in panels (e) and (f). The inverse half-life can be expressed as

$$
(T_{1/2}^{0\nu})^{-1} = g_A^4 \left[ G_{01} (|A_L|^2 + |A_R|^2) - 2(G_{01} - G_{04}) \text{Re}\{A_L^* A_R\} \right]
$$

(5.19)

where the coupling constant $g_A = 1.271$, and $G_{01}$, $G_{04}$ denote the phase-space factors [134]. The amplitudes $A_{L,R}$ are defined as

151
Figure 5.2: Feynman diagrams contributing to $0\nu\beta\beta$ decay in the LRSM. The cross vertices (×) in between $W_L - W_R$ propagators denote the left-right mixing. In the limit of type-II dominance, there is no mixing between left-handed $\nu$ and right-handed $N$ neutrinos.
\[ A_L = \sum_{i=1}^{3} A_L(m_i), \quad (5.20a) \]
\[ A_R = \sum_{i=4}^{6} A_R(m_i) + A_R(M_{\delta_R^{\pm\pm}}). \quad (5.20b) \]

where \( m_i \) are the active/sterile neutrino masses in Eq. (5.6). The subamplitudes \( A_{L,R}(m_i) \) are given by [142,262]

\[ A_L(m_i) = -\frac{m_i}{4m_e}[M_V(m_i) + M_A(m_i)] \left( C_{VLL}^{(6)} \right)_{ei}^2, \quad (5.21a) \]
\[ A_R(m_i) = -\frac{m_i}{4m_e} \left\{ M_V(m_i) \left( C_{VRR}^{(6)} + C_{VLR}^{(6)} \right)_{ei}^2 + M_A(m_i) \left( C_{VRR}^{(6)} - C_{VLR}^{(6)} \right)_{ei}^2 \right\}, \quad (5.21b) \]

where \( m_e \) is the electron mass and \( M_{A,V} \) are nuclear matrix elements (NMEs). Notice that the left- and right-handed contributions are proportional to the light-neutrino and the right-handed neutrino masses, respectively. The Wilson coefficients in the mass basis at the scale \( \mu = M_W \) are

\[ C_{VLL}(M_W) = -2V_{ud}PU, \quad (5.22a) \]
\[ C_{VLR}(M_W) = V_{ud} \left( v^2 C_{L}^{(6)}(M_{W_R}) \right) P_s U^*, \quad (5.22b) \]
\[ C_{VRR}(M_W) = \left( v^2 C_{R}^{(6)}(M_{W_R}) \right) P_s U^*. \quad (5.22c) \]

The Wilson coefficients \( C_{L,R}^{(6)}(M_{W_R}) \) are obtained by integrating out the \( W_R \) boson, and they are

\[ C_R^{(6)}(M_{W_R}) = -\frac{1}{v_R^2} V_{ud}, \quad (5.23a) \]
\[ C_L^{(6)}(M_{W_R}) = 2\frac{\xi}{1 + \xi^2} \frac{C_R^{(6)}(M_{W_R})}{V_{ud}^R}. \quad (5.23b) \]
The projectors in the flavor basis $P$ and $P_s$ are

$$P = (\mathcal{I}_{3 \times 3} \ 0_{3 \times 3}) , \quad P_s = (0_{3 \times 3} \ \mathcal{I}_{3 \times 3}) . \quad (5.24)$$

On the other hand, the subamplitude

$$A_R(M_{\delta_R^{\pm \pm}}) = \frac{m_N^2}{m_e v} M_R^{(9)}$$

in the EFT approach with dimension-9 effective operators [131] are explicitly given in Ref. [134]. Here, $m_N \sim 1$ GeV is the nucleon mass. For conciseness, we will only repeat the Wilson coefficients of the relevant dimension-9 effective operators [142] as follows:

$$C_{1R}^{(9)'}(M_{\delta_R^{\pm \pm}}) = -\frac{v^5}{M_{\delta_R^{\pm \pm}}}(P_s U^*)_{ei} , \quad (5.26a)$$

$$C_{1R}^{(9)}(M_W) = 4 V_{ud} V_{ud}^* \left( \frac{\xi}{1 + \xi^2} \right) C_{1R}^{(9)'}(M_{\delta_R^{\pm \pm}}) , \quad (5.26b)$$

$$C_{4R}^{(9)}(M_W) = 4 V_{ud} V_{ud}^* \left( \frac{\xi}{1 + \xi^2} \right) C_{1R}^{(9)'}(M_{\delta_R^{\pm \pm}}) . \quad (5.26c)$$

The Wilson coefficients $C_{1R}^{(9)'}(M_{\delta_R^{\pm \pm}})$ and $C_{1R,4R}^{(9)}(M_W)$ are defined at different scales. The QCD renormalization-group-evolution equations of them and the other Wilson coefficients above are given in Refs. [134, 142]. Note that the contributions of right-handed neutrinos with masses $(\hat{M}_N)_{ii} > \Lambda_X$ ($i = 1, 2, 3$) can also be integrated out, rendering dimension-9 effective operators [221]. The resulting contributions to $0\nu\beta\beta$ decay have also been included in Eq. (5.21a) in the framework recently developed in Refs. [142, 262].

### 5.4.2 Parity-violating Møller scattering

Parity-violating asymmetries in polarized electron scattering, such as Møller scattering [265], can constrain the 4-electron effective vertex in a model-independent way.
In particular, the MOLLER Collaboration proposes to measure the parity-violating (PV) asymmetry in the scattering of polarized electrons off unpolarized electrons using the 11 GeV beam in Hall A at Jefferson Lab to measure the electron’s weak charge $Q_{eW}$ to an overall accuracy of 2.4% [266]. The MOLLER sensitivity can be expressed as the following lower bound

$$\Lambda \frac{1}{\sqrt{|g_{RR}^2 - g_{LL}^2|}} \approx 7.5 \text{ TeV},$$

(5.27)

where $g_{LL,RR}$ are the coupling constants of the new vector and axial-vector interactions between left-handed and right-handed electrons, respectively, and $\Lambda$ is the mass scale of the new physics [266]. This constraint can be interpreted as a bound on the ratio of the mass of the doubly-charged scalar to its Yukawa coupling with electrons.

In the LRSM, the leptonic Yukawa interactions in Eq. (5.4) allow for both the left- and the right-handed doubly-charged scalars to contribute to the PV Møller scattering. This contribution consists of an $s$-channel exchange depicted in Fig. 5.3. The corresponding PV effective Lagrangian can be written as [231]

$$\mathcal{L}_{\text{eff}}^{\text{PV}} = \frac{|(f_L)_{ee}|^2}{2M_\delta^2} (\bar{e}_L \gamma^\mu e_L)(\bar{e}_L \gamma^\mu e_L) + (L \leftrightarrow R).$$

(5.28)

However, due to the severe constraint on $\delta_{R}^{\pm \pm}$ from the CLFV searches as discussed above, only the implications of $\delta_{R}^{\pm \pm}$ exchange in light of the MOLLER prospect will be considered.

In Eq. (5.28), we can directly match the corresponding expressions in Eq. (5.27), where the effective mass scale is $\Lambda = M_\delta^{\pm \pm}$ and the effective couplings are $|g_{RR}|^2 = |(f_R)_{ee}|^2/2$ and $g_{LL} = 0$. Therefore, the PV precision measurement would constraint
Figure 5.3: Doubly-charged scalar contributions to the PV Møller scattering. Notice that each vertex violates lepton number by two units.

the ratio between the right-handed doubly-charged scalar mass $M_{\delta_R^{\pm\pm}}$ and its Yukawa coupling to electrons $(f_R)_{ee}$ to $|266|

\[
\frac{M_{\delta_R^{\pm\pm}}}{|(f_R)_{ee}|} \gtrsim 5.3 \text{ TeV} \quad (5.29)
\]

at 1σ C.L. Notice that Ref. [231] reports this value at the 95% C.L. and the bound is modified to 3.7 TeV.

5.5 Results and Discussion

We present here the experimental constraints on a parity-violating mLRSM with type-II seesaw dominance, specifically under complementary results from 0νββ-decay and PV Møller scattering experiments. Under this regime, the $6 \times 6$ block-diagonal matrix in Eq. (5.10) allows us to independently diagonalize the $3 \times 3$ mass matrices $M_\nu$ and $M_N$ corresponding to active and sterile neutrinos, respectively. Notice the later ones could have masses of the order of the right-handed scale but can also be significantly lower.

To establish a concrete $f_R$-dependence between the related observables (i.e., 0νββ-decay half-life and PV asymmetry), we will focus our study on the effects of one sterile, right-handed neutrino with mass

\[
M_{N_R} \equiv m_A = \sqrt{2} v_R (f_R)_{ee}, \quad (5.30)
\]
as shown in Eq. (5.11a). On the other hand, the masses of the active neutrinos are completely determined once the lightest active neutrino mass is chosen and the hierarchy is specified. In the normal hierarchy (NH), $m_1 < m_2 < m_3$,

$$m_1 = m_{\nu_{\text{min}}}, \quad (5.31a)$$

$$m_2 = (m_1^2 + \Delta m_{21}^2)^{1/2}, \quad (5.31b)$$

$$m_3 = (m_1^2 + \Delta m_{31}^2)^{1/2}, \quad (5.31c)$$

while in the inverted hierarchy (IH), $m_3 < m_1 < m_2$,

$$m_3 = m_{\nu_{\text{min}}}, \quad (5.32a)$$

$$m_1 = (m_3^2 - \Delta m_{31}^2)^{1/2}, \quad (5.32b)$$

$$m_2 = (m_3^2 + \Delta m_{21}^2 - \Delta m_{31}^2)^{1/2}, \quad (5.32c)$$

where the squared mass difference $\Delta m_{ij}^2 \equiv m_i^2 - m_j^2$, and $m_{\nu_{\text{min}}}$ denotes the lightest active neutrino mass. The PMNS matrix is defined in the usual way

$$U_{\text{PMNS}} = \begin{pmatrix}
1 & 0 & 0 \\
0 & c_{23} & s_{23} \\
0 & -s_{23} & c_{23}
\end{pmatrix}
\begin{pmatrix}
c_{13} & 0 & s_{13}e^{-i\delta} \\
0 & 1 & 0 \\
-s_{13}e^{i\delta} & 0 & c_{13}
\end{pmatrix}
\begin{pmatrix}
c_{12} & s_{12} & 0 \\
-s_{12} & c_{12} & 0 \\
0 & 0 & 1
\end{pmatrix}
\begin{pmatrix}
1 & 0 & 0 \\
0 & e^{i\lambda_1} & 0 \\
0 & 0 & e^{i\lambda_2}
\end{pmatrix}, \quad (5.33)$$

where $\lambda_{1,2}$ are the Majorana phases, $c_{ij} \equiv \cos \theta_{ij}$, and $s_{ij} \equiv \sin \theta_{ij}$. The squared mass differences and mixing angles extracted from current neutrino oscillation experiments are shown in Table 1.2.
We will start by showing the $0\nu\beta\beta$-decay constraints on the parity-violating mLRSM with type-II seesaw dominance. For neutrino oscillation parameters, we adopt their central values in Table 1.2 except for the Majorana phases $\lambda_{1,2}$, whose effects are marginalized in calculating the $0\nu\beta\beta$-decay rate. Additionally, the lightest active neutrino mass is randomly generated with the upper bound $m_{\nu_{\text{min}}} < 0.05\,\text{eV}$. Once the neutrino oscillation parameters are fixed, the inverse half-life in Eq. (5.19) depends only on a few parameters:

**P1.** The mass of the doubly-charged scalar $M_{\delta R}^{\pm\pm}$ and its Yukawa coupling to electrons $(f_R)_{ee}$.

**P2.** The mass of right-handed boson $M_{W_R}$ and the $W_L - W_R$ mixing parameter $\xi$.

Notice that the right-handed neutrino mass $M_{N_R}$ is completely determined by these parameters by combining Eqs. (5.16b) and (5.30). Since PV Möller scattering is only sensitive to the parameters in **P1**, (cf. Eq. (5.29) and subsequent discussion), we choose different benchmark scenarios for the $W_R$ mass and the $W_L - W_R$ mixing parameter, as shown in Table 5.1, and the results of a parameter scan over the doubly-charged mass and its Yukawa coupling are shown in Figs. 5.4, 5.5, and 5.6. The values of the left-right mixing parameter $\xi$ were selected to be consistent with the constraints depicted in Fig. 5.1. The figure highlights the impact of $W_R$ mass on the upper bound of $\xi$ as determined by the limits discussed in Section 5.3.

We use the most recent KamLAND-Zen results [10] to distinguish the excluded parameter regions, shown as red points. Solid red lines bound these regions, and they can be analytically understood by estimating the parameter dependence of the main contribution in each case. We also replicate boundary estimation using the ton-scale prospects [267]; these results are shown as dashed magenta lines.
Figure 5.4: Parameter scan of $0\nu\beta\beta$-decay constraints on the right-handed doubly-charged scalar mass $M_{\delta^{++}}$ and the Yukawa coupling $|(f_R)_{ee}|$ in the parity-violating mLRS with type-II seesaw dominance in the normal hierarchy with $M_{W_R} = 7$ TeV and $\xi = 0$ (left) and $\xi = 0.35$ (right). The right-handed neutrino mass $M_{N_R}$ is also displayed as a secondary vertical axis. All the red points (regions above or between the solid red lines) are excluded by the most recent KamLAND-Zen results [10]. The solid red lines correspond to the exclusion limits calculated using an analytical estimation (see text for discussion). The dashed magenta lines correspond to the same exclusion limits using the prospect by ton-scale experiments [267].

Figs. 5.4 and 5.5 show two separate regions that are consistent with the observed $0\nu\beta\beta$-decay constraints. The top region accommodates large Yukawa couplings and heavy doubly-charged scalars, while the lower region is independent of the doubly-charged scalar mass and is consistent with small couplings. The gap between these regions narrows with increasing $W_R$ boson mass and widens with non-zero $W_L - W_R$ mixing, and disappears completely for $M_{W_R} = 25$ TeV in Fig. 5.6.

To better understand these results, we will study the $0\nu\beta\beta$-decay inverse half-life and determine which contributions are dominant based on the magnitude of the Yukawa couplings for a fixed $W_R$ and $\delta_R^-$ mass. Fig. 5.7 displays the inverse $0\nu\beta\beta$-decay half-life as a function of the Yukawa coupling $|(f_R)_{ee}|$ in both the left and right panels for different values of $W_L - W_R$ mixing. We keep the mass of the doubly-charged scalar fixed at $M_{\delta^{++}} = 100$ GeV for the sake of the discussion. The curves in the figure show the results for three $W_R$ benchmark masses. To show inverse half-life
Figure 5.5: Same as in Fig. 5.4, parameter scan of $0\nu\beta\beta$-decay constraints on the right-handed doubly-charged scalar mass $M_{\delta^{\pm\pm}}$ and the Yukawa coupling $|\langle f_R \rangle_{ee}|$ in the parity-violating mLRSM with type-II seesaw dominance in the normal hierarchy with $M_{W_R} = 15$ TeV and $\xi = 0$ (left) and $\xi = 0.75$ (right).

Figure 5.6: Same as in Fig. 5.4, parameter scan of $0\nu\beta\beta$-decay constraints on the right-handed doubly-charged scalar mass $M_{\delta^{\pm\pm}}$ and the Yukawa coupling $|\langle f_R \rangle_{ee}|$ in the parity-violating mLRSM with type-II seesaw dominance in the normal hierarchy with $M_{W_R} = 25$ TeV and $\xi = 0$ (left) and $\xi = 0.8$ (right).
considering only the light left-handed neutrino contributions, a red dashed curve is added. The latest KamLAND-Zen exclusion limits are represented by the grey shaded area, with the future ton-scale experiment bounds indicated for comparison.

By comparing the three mass benchmark curves, we can observe the impact of increasing the magnitude of $M_{WR}$ on the curves, regardless of the mixing parameter. Heavier $W_R$ makes the curve flatter and closer to the lower limit of the pure light left-handed neutrino contributions. In other words, increasing $M_{WR}$ enlarges the parameter region that is consistent with $0\nu\beta\beta$-decay limits. For instance, comparing the 7-TeV (blue) and the 25-TeV (green) curves in the left panel of Fig. 5.7, we can see that the 7-TeV curve has two disjoint regions that are consistent with $0\nu\beta\beta$-decay experiments for $|(f_R)_{ee}| \lesssim 10^{-7}$ and $|(f_R)_{ee}| \sim 10^{-3}$. Meanwhile, the 25-TeV curve has one large allowed parameter region for $|(f_R)_{ee}| \lesssim 1$. These two results are also visible in the left panels of Figs. 5.4 and 5.6.

The specific shape of the curve over the coupling range can be determined by analyzing the different contributions to the amplitude in Eq. (5.21b) and identifying the ranges where the individual contributions shown in Fig. 5.2 dominate. There are three regimes to be considered:

- **Doubly-charged scalar dominance.** For large couplings, the doubly-charged scalar contribution to the amplitude dominates the inverse half-life in Eq. (5.19). By analyzing diagrams (e) and (f) in Fig. 5.2, we notice that the inverse half-life resulting after integrating out both the $W_R$ and $\delta^{--}_R$ will be proportional to

$$
(T_{1/2}^{0\nu})^{-1} \propto \left| p_\Delta(\xi) \frac{(f_R)_{ee}}{M_2^{\delta^\pm R}} \frac{\nu_R}{M_{WR}} \right|^2 = \frac{p_\Delta^4(\xi)}{M_2^4 M_{WR}^8} |(f_R)_{ee}|^2,
$$

(5.34)
where \( p_\Delta(\xi) \) is a linear combination of \( \{1, \xi/(1 + \xi^2), \xi^2/(1 + \xi^2)^2\} \), taking care of the \( W_L - W_R \) mixing. The magnitude of the coefficients depends on the phase-space factor, NMEs and the value of the low-energy constants taken from Refs. [142, 262]. This contribution is responsible for the diagonal boundary in Figs. 5.4, 5.5, and 5.6. It is worth noting that this region of the parameter space aligns with the findings in Ref. [231], which focused only on the effects of the doubly-charged scalar as new physics contributions to \( 0\nu\beta\beta \) decay.

- **Heavy right-handed neutrino dominance.** For moderate couplings, the contribution from right-handed neutrinos dominates the \( 0\nu\beta\beta \)-decay amplitude, and the right-handed neutrino mass is large enough to be integrated out. From diagrams (b), (c), and (d) in Fig. 5.2, we estimate the inverse half-life as

\[
(T_{1/2}^{0\nu})^{-1} \propto \left| p_{N,h}(\xi) \frac{1}{M_{W_R}} \frac{1}{M_{M_R}^4} \right|^2 = \frac{p_{N,h}^2(\xi)}{M_{W_R}^{10}} \frac{1}{|f_{ee}|^2}, \tag{5.35}
\]

where we have used the seesaw relation in Eq. (5.30) and, similarly, \( p_{N,h}(\xi) \) is a linear combination of \( \{1, \xi/(1 + \xi^2), \xi^2/(1 + \xi^2)^2\} \), taking care of the \( W_L - W_R \) mixing. This contribution is independent of \( M_{\delta^{\pm}} \) and denoted by the upper horizontal boundary in Figs. 5.4 and 5.5.

- **Light right-handed neutrino dominance.** The small-coupling region corresponds to the scenario where the contribution from right-handed neutrinos still dominates the \( 0\nu\beta\beta \)-decay amplitude, but the right-handed neutrino mass is small enough to keep them as a dynamical degree of freedom. By analyzing diagrams (b), (c), and (d) in Fig. 5.2, we can estimate the inverse as

\[
(T_{1/2}^{0\nu})^{-1} \propto \left| p_{N,l}(\xi) \frac{M_{N_R}}{\langle q \rangle^2} \frac{1}{M_{W_R}^4} \right|^2 = \frac{p_{N,l}^2(\xi)}{\langle q \rangle^4 M_{W_R}^{10}} \frac{1}{|f_{ee}|^2}, \tag{5.36}
\]
Figure 5.7: Inverse $0\nu\beta\beta$-decay half-life $(T^{0\nu}_{1/2})^{-1}$ as a function of the Yukawa coupling $|(f_R)_{ee}|$ for $W_L - W_R$ mixing parameters $\xi = 0$ (left) and $\xi = 0.35$ (right). We fix the mass of the doubly-charged scalar at $M_{\delta^{\pm\pm}} = 100$ GeV. The three curves in the figure show the results for $M_{WR} = 7$ TeV (blue), $M_{WR} = 15$ TeV (yellow), and $M_{WR} = 25$ TeV (green). The inverse half-life considering only the light left-handed neutrino contributions is shown in red. The latest KamLAND-Zen exclusion limits are represented by the grey shaded area, with the future ton-scale experiment bounds indicated for comparison.

where $\langle q \rangle$ denotes the averaged momentum transfer (about $100 - 200$ MeV), and $p_{N,l}(\xi)$ is a linear combination of $\{1, \xi/(1+\xi^2), \xi^2/(1+\xi^2)^2\}$ that takes into account the $W_L - W_R$ mixing. This contribution is responsible for the lower horizontal boundary in Figs. 5.4 and 5.5.

Now, the effects of a non-zero left-right mixing parameter ($\xi \neq 0$) can be understood as follows: as observed in the right panel of Fig. 5.7, a non-zero mixing modifies the relative size of the dominance regime regions by extending the region where the light right-handed neutrino description applies and shrinking the region where the heavy right-handed neutrino description applies. This can be visualized as a translation of the diagonal and upper horizontal boundaries, if present, in Figs. 5.4, 5.5, and 5.6 towards the right and upwards, respectively.

To illustrate the complementary roles of $0\nu\beta\beta$ decay and the LNC processes, we present results from current constraints and future prospects at high and low energies.
described in Section 5.4. The experimental limits on $M_{\delta_R^{\pm\pm}}$ and $|(f_R)_{ee}|$ are shown in Figs. 5.8, 5.9, and 5.10 for two mixing parameter values, and include collider searches (LEP, CEPC, LHC, FCC-hh), MOLLER sensitivity, and $0\nu\beta\beta$-decay sensitivities from current and future experiments. The $0\nu\beta\beta$-decay limits are based on the analytical expressions in Eqs. (5.34), (5.35), and (5.36). We add an auxiliary vertical axis to show the right-handed neutrino mass $M_{N_R}$ using the seesaw relation in Eq. (5.30).

For comparison, we have included the $0\nu\beta\beta$-decay constraint for the KamLAND-Zen experiment from Ref. [231], which is reproduced for different values of $M_{W_R}$. Note that Ref. [231] assumed a fixed value of $M_{W_R} = 3.3$ TeV, which has been excluded by the LHC searches at Run-2 phase [242,268,269]. In order to show the improvement owing to the EFT description of $0\nu\beta\beta$ decay, the contribution of right-handed neutrino and the left-right mixing are not considered in the rescaled result of Ref. [231] as depicted in the dot-dashed black curve in Fig.5.8.

We emphasize key features of the results:

- The use of an EFT approach provides a significant improvement in the $0\nu\beta\beta$-decay calculations and their corresponding sensitivity, as we explicitly see in the left panel of Fig. 5.8. The results in the left panels of Figs. 5.9 and 5.10 are even more suggestive since the limit using the previous calculation method described in Ref. [231] is out of the parameter space.

- The $0\nu\beta\beta$ decay and PV Møller scattering processes constrain $|(f_R)_{ee}|/M_{\delta_R^{\pm\pm}}^2$ and $|(f_R)_{ee}|/M_{\delta_R^{\pm\pm}}$, respectively. Their interplay is influenced by the value of $W_L - W_R$ mixing, as demonstrated in Figs. 5.8, 5.9, and 5.10. When the mixing is zero ($\xi = 0$, left panels), the exclusion limits from the MOLLER experiment are consistently stronger than both current and future $0\nu\beta\beta$-decay experiments,
Figure 5.8: Combined sensitivities to the right-handed doubly-charged scalar mass $M_{R^{±±}}$ and the Yukawa coupling $|\langle f_R \rangle_{ee}|$ in $0\nu\beta\beta$-decay, MOLLER, and collider experiments in the parity-violating mLRSM with type-II seesaw dominance. The normal hierarchy is assumed. Benchmark values of the parameters $M_{W_R} = 7\text{ TeV}$ and $\xi = 0$ (left), $\xi = 0.35$ (right) are chosen. The right-handed neutrino mass $M_{N_R}$ is also displayed as a secondary vertical axis. The MOLLER prospect [266] is shown as a dashed blue line, and $0\nu\beta\beta$ decay limits from current and future experiments are shown as solid red and dashed magenta lines, respectively. The figure also displays the direct searches at the LHC (solid gray), pair production prospect at the FCC-hh (dashed gray), and Bhabha scattering limits at LEP (solid orange) and CEPC prospect (dashed green). The $0\nu\beta\beta$-decay constraint by the KamLAND-Zen experiment from Ref. [231] (dash-dotted black) is also included for comparison.

Figure 5.9: Same as in Fig. 5.8, combined constraints from LNC and $0\nu\beta\beta$-decay experiments on the right-handed doubly-charged scalar mass $M_{R^{±±}}$ and the Yukawa coupling $|\langle f_R \rangle_{ee}|$ in the parity-violating mLRSM with type-II seesaw dominance in the normal hierarchy with $M_{W_R} = 15\text{ TeV}$ and $\xi = 0$ (left) and $\xi = 0.75$ (right).
Figure 5.10: Same as in Fig. 5.8, combined constraints from LNC and 0νββ-decay experiments on the right-handed doubly-charged scalar mass $M_{\delta_{11}^{\pm \pm}}$ and the Yukawa coupling $|\langle f_R \rangle_{ee}|$ in the parity-violating mLRSM with type-II seesaw dominance in the normal hierarchy with $M_{W_R} = 25$ TeV and $\xi = 0$ (left) and $\xi = 0.8$ (right).

taking into account the direct search constraints from Section 5.4. Conversely, for non-zero mixing ($\xi \neq 0$, right panels), these two types of experiments complement each other.

- The MOLLER experiment may hold important implications for the understanding of 0νββ-decay experiments. As seen in Fig. 5.9, a non-zero result in MOLLER with a corresponding observation in 0νββ-decay experiments for $M_{\delta_{11}^{\pm \pm}} \lesssim 5$ TeV would imply a non-zero $W_L - W_R$ mixing value (right panel) over zero mixing (left panel). In essence, the interplay between these two experiments may shed light on the underlying mechanism behind 0νββ decay, should future searches yield a positive result.

- Searches and measurements at high-energy colliders complement those at low-energy experiments. Direct searches for right-handed doubly-charged scalar $\delta_{11}^{\pm \pm}$ at hadron colliders are sensitive to $M_{\delta_{11}^{\pm \pm}} \lesssim 5$ TeV irrespective of the Yukawa coupling $|\langle f_R \rangle_{ee}|$. Measurements of the Bhabha scattering at lepton colliders would be able to exclude a large portion of parameter space that is allowed by the 0νββ-decay and MOLLER experiments.
5.6 Summary and Remarks of the Chapter

Exploring the different mechanisms responsible for generating non-zero light neutrino masses is one of the most urgent matters in particle physics; as of April 4th, 2023, a search for “neutrino mass” on the INSPIRE database reveals a total of 40,130 articles. This work revisits the connection of $0 \nu \beta \beta$ decay with lepton-number-conserving (LNC) processes in the type-II seesaw scenario of the left-right symmetric model. Although $0 \nu \beta \beta$-decay experiments can determine the Majorana nature of neutrinos associated with lepton number violation $\Delta L = 2$, LNC processes at high and low energies are complementary in constraining the key parameters.

In the approach of effective field theory, we recalculate the half-life of $0 \nu \beta \beta$-decay, including contributions from the right-handed doubly charged scalar, right-handed neutrino, and active neutrinos, and consider the cases for a non-zero left-right mixing parameter. Our result shows a significant improvement of the sensitivity in $0 \nu \beta \beta$-decay experiments compared to a previous study.

By considering the allowed values of right-handed gauge boson mass $M_{W_R}$ and left-right mixing parameter by direct searches at the LHC and electroweak precision data, we compare the sensitivities to the right-handed doubly-charged scalar mass $M_{\delta^{\pm\pm}_R}$ and the Yukawa coupling $| (f_R)_{ee} |$ in the $0 \nu \beta \beta$-decay experiments, MOLLER, and collider experiments.

We show that the left-right mixing affects the interplay of $0 \nu \beta \beta$ decay and parity-violating Møller scattering processes. If the left-right mixing parameter $\xi$ is zero, the sensitivity in MOLLER prospect is consistently stronger than that in future tonne-scale $0 \nu \beta \beta$-decay experiments. For a non-zero $\xi$ parameter, the MOLLER sensitivity is complementary to the sensitivity of tonne-scale experiments. The studies of direct
searches and precision measurements at colliders also play an important role and could probe the parameter region beyond the reaches of low-energy experiments.
CONCLUSION

The Standard Model of particle physics has achieved remarkable success in describing the elementary particles and fundamental interactions that govern the Universe, as briefly described in Chapter 1. It has withstood rigorous experimental testing and provided a solid foundation for understanding the microscopic world. However, despite its tremendous accomplishments, the SM leaves many critical questions unanswered. For instance, it cannot explain the observed neutrino masses, as the model originally predicted neutrinos to be massless. Additionally, the SM does not account for the baryon asymmetry of the Universe, which is the observed imbalance between matter and antimatter. These open questions, among others, highlight the limitations of the Standard Model and suggest that new physics lies beyond its reach, encouraging scientists to persist in their exploration of particle physics’ boundaries, striving to achieve a thorough understanding of the cosmos.

The aim of this dissertation was to contribute to the progress of theoretical physics by exploring the interface between the Energy, Intensity, and Cosmic frontiers. Delving into the fundamental laws that govern the Universe and their diverse applications is at the heart of high energy and nuclear physics, and cosmology. As the complexity of the problems addressed in these fields grows, there is an increasing need for more sophisticated tools encompassing both computational and theoretical advancements. These tools are essential for the thorough analysis of data collected from experiments and observations, as well as for making accurate predictions based on existing and emerging models. In light of this, the dissertation has presented and elaborated on
the relevant tools, both computational and theoretical, required for the studies carried out within its scope in Chapter 2. The development and refinement of such tools will play a crucial role in our ability to explore the intricacies of the Universe further and uncover the underlying principles that dictate its behavior.

In this dissertation, we have examined the implications of LNV at the TeV scale for leptogenesis, $0\nu\beta\beta$ decay, and collider phenomenology. By investigating these distinct yet interconnected phenomena, we emphasize the importance of synergy between the Energy, Intensity, and Cosmic frontiers in enhancing our understanding of the underlying physical mechanisms. Each frontier provides unique insights and perspectives, and their interplay can effectively complement and refine our knowledge, setting the path for a more comprehensive understanding of the fundamental laws of Nature. We have conducted an in-depth exploration of various aspects related to TeV-scale LNV interactions. In Chapter 3, we discussed their impact on the standard thermal leptogenesis paradigm in the context of a simplified model and emphasized the significance of the particle spectrum in associated phenomena. In Chapter 4, we illustrated how a chirally suppressed mechanism could enhance the sensitivity of collider searches using a more sophisticated simplified model. Finally, in Chapter 5, we investigate the complementarity between parity-violating scattering and $0\nu\beta\beta$-decay experiments within the framework of the minimal left-right symmetric model, highlighting the role of the left-right mixing. These comprehensive analyses serve to illuminate the diverse implications of LNV at the TeV scale and further emphasize the importance of the complementarity between the three frontiers.

While this dissertation has provided meaningful contributions and advancements to our understanding of TeV-scale LNV, it has also unveiled new layers of complexity and opened up fresh lines of research. As often happens in the journey of a Ph.D. stu-
dent, each answer we have found has sparked new questions and highlighted intriguing challenges that remain to be addressed. Let us reflect on some of these queries that our work has brought to the forefront related to the addition of flavor effects.

In the context of thermal leptogenesis, our previous assumption of flavor universality of the couplings implies that the washout of the lepton number asymmetry for any generation will imply a washout for all three flavors. However, there could exist a hierarchy of couplings associated with different lepton generations. It may be the case that the first-generation couplings are sufficiently large to generate $0\nu\beta\beta$ decay and collider signatures, implying a washout of the first-generation lepton number asymmetry. Nevertheless, the second and third-generation couplings may be considerably smaller, so they could avoid washing out the asymmetries in these generations. In this case, the exploration of the viability of a flavored leptogenesis scenario is mandatory, wherein the lepton asymmetry must be computed generation-by-generation before determining the total baryon asymmetry. We are exploring the use of the Closed Time Path (CTP) or Kadanoff-Baym formalism [270,271] to compute the lepton asymmetries. The CTP formulation allows us to track the evolution of flavor off-diagonal correlations that are not explicitly included in the Boltzmann framework. Additionally, it automatically ensures the RIS subtraction with no extra effort.

The addition of flavor effects is also crucial to elucidate the strength of our conclusions about the collider sensitivity since we focused only on the leptons of the first generation. For example, by adding muons and/or taus, the backgrounds are quite different; charge misidentification is highly suppressed when the final state involves muons, and taus require a non-prompt, hadronic background. It is possible to extend the scope of low-energy probes by studying the interplay and complementarity with future charged lepton flavor violation (CLFV) experiments, such as $\mu \to e\gamma$ and
\( \mu \rightarrow 3e \) at the Paul Scherrer Institute (PSI) and the searches for \( \mu \rightarrow e \) conversion in nuclei at Fermilab and J-Parc. Additionally, The relative smallness of LNV couplings needed for a viable leptogenesis scenario opens a door for collider signatures of long-lived particles (LLP), and new strategies are already being developed [272].

In conclusion, this dissertation contributes a meaningful piece to the collective endeavors of the scientific community in pursuit of BSM phenomena. As we continue our quest for new physics, the author’s passion for physics remains unwavering, as his commitment to this long journey has just begun.
APPENDIX A

AN EXPLICIT EXAMPLE FOR THE $0\nu\beta\beta$-DECAY MASTER FORMULA

The simplified model introduced in Chapter 3 includes two new fields: a scalar doublet $S$ and a Majorana singlet fermion $F$. As a reminder of Eq. (3.2), the interaction Lagrangian relevant for $0\nu\beta\beta$ decay reads

$$\mathcal{L}_{\text{int}} = g_Q \overline{Q} S d_R + g_L \overline{L} (i\tau^2) S^* F + \text{H.c.}, \quad (A.1)$$

where $Q = (u_L, d_L)^T$ and $q_R = (u_R, d_R)^T$ are the left-handed and right-handed quark isospinors, respectively. The Feynman diagram depicting this model’s contribution to $0\nu\beta\beta$ decay is shown in Fig. 3.2. At the dimension-9 level, after integrating out both $F$ and $S^\pm$ fields, the $0\nu\beta\beta$-decay effective Lagrangian can be written as

$$\mathcal{L}^{(9)}_{\Delta L=2} = \frac{1}{2} \frac{g_Q^2 g_L^2}{m_S^4 m_F} (\overline{u}_L d_R)(\overline{u}_L d_R)(\overline{e}_L e^c_L) = \frac{1}{v^5} C^{(9)\nu}_{2L} \overline{e}_L C \overline{e}_L^T O_2', \quad (A.2)$$

with

$$C^{(9)\nu}_{2L} = \left( \frac{v^5}{m_S^4 m_F} \right) \left( \frac{g_Q^2 g_L^2}{2} \right), \quad (A.3)$$

where the Wilson coefficients and operator basis were defined in Eqs. (2.54) and (2.55). For simplicity, we neglect the RGE effects discussed in Ref. [134]. The next step is to match the quark-level theory onto the chiral Lagrangian. The scalar
operator $O'_2$ generates the $\pi\pi ee$ and $NNN\pi ee$ LNV couplings shown in Fig. 2.10. The relevant combinations of scalar couplings are defined as

\begin{equation}
C^{(9)}_{\pi\pi L} = g^{\pi\pi}_2 C^{(9)'}_{2L}, \tag{A.4a}
\end{equation}

\begin{equation}
C^{(9)}_{NN L} = g^{NN}_2 C^{(9)'}_{2L}, \tag{A.4b}
\end{equation}

where $g^{\pi\pi}_2 = 2.0 \pm 0.2 \text{GeV}^2$ and $g^{NN}_2 = \mathcal{O}(4\pi)^2$ are LECs encoding hadronic matrix elements. Now, the corresponding sub-amplitude in Eq. (2.61) is given by

\begin{equation}
\mathcal{A}_\nu = \frac{m_N^2}{m_e v} \mathcal{M}^{(9)}_\nu, \tag{A.5}
\end{equation}

with

\begin{equation}
\mathcal{M}^{(9)}_\nu = -\frac{1}{2m_N^2} C^{(9)}_{\pi\pi L} \left( \frac{1}{2} M^{AP}_{GT, sd} + M^{PP}_{GT, sd} + \frac{1}{2} M^{AP}_{T, sd} + M^{PP}_{T, sd} \right) - \frac{2}{g^2 \pi m_N^2} C^{(9)}_{NN L} M_{F, sd}. \tag{A.6}
\end{equation}

Finally, as indicated in Eq. (2.62), the $0\nu\beta\beta$-decay inverse half-life is given by

\begin{equation}
(T_{1/2}^{0\nu})^{-1} = g_A^4 G_{01} |\mathcal{A}_\nu|^2. \tag{A.7}
\end{equation}
Differentiating the signal ($S$) from the background ($B$) is a typical classification problem that can be solved using machine learning techniques. Given an ensemble of observables $X$, for each collider event, one can train a model $M$ to separate signal events from background events with high accuracy. In this paper, we primarily use a recurrent neural network (RNN) to train the classification and a boosted decision tree (BDT) to cross-check the performance of our discriminant.

**B.1 Boosted Decision Tree (BDT)**

A decision tree is a set of criteria in a tree-based structure that recursively splits the events into two groups. Following the simplified diagrammatic representation shown in Fig. B.1, one can start with a set of unclassified events. At each node, the criterion is defined such that “background-like” events are removed, and this continues until the signal events are efficiently separated from the background. An ensemble algorithm such as boosting can be applied to this decision tree to improve the classification further, and this forms the BDT.

At each node split, the $S$ and $B$ separation can be improved further by using certain criteria such as the Gini index and entropy factor. These criteria are defined such that minimizing them at each node increases the purity of the $S$ and $B$ data sets, hence maximizing the discriminating power. The detailed mathematical definition of
the Gini index and entropy could be found in any machine learning textbook.

The decision tree method is powerful but can be easily over-fitted, i.e., a tiny change in the input data set may result in large differences and inconsistencies in the classification results. To avoid this, a set of BDTs can be trained in a sequence such that the successive tree is created to minimize the error of the previous tree. Several sets of these can be trained, and the final classification result is determined by the majority vote from all the BDTs. In this study, we use the AdaBoost\(^1\) (Adaptive Boost) to test a small part of the parameter space of our simplified model.

---

\(^1\)AdaBoost is a method implemented by Toolkit for Multivariate Data Analysis (TMVA), a built-in package in ROOT [273].
The details of the algorithm are described as follows. The $i$-th tree trained is called $T^i$, $W^i$ is its voting weight, and $w^i_j$ is the weight of the $j$-th sample in the $i$-th tree:

- The first tree ($T^1$) is trained such that all the samples have the same weight. Either the Gini or the entropy criteria can be used for this training. For each sample, the tree predicts if the event is “signal-like” or “background-like”. The discrete output for each event is as follows:

$$y^i_j = \begin{cases} 
+1 & \text{for signal-like events}, \\
-1 & \text{for background-like events}.
\end{cases}$$

- Each tree, $T^i$, has the voting weight $W^i$ which is defined as:

$$W^i = \frac{1}{2} \log \left( \frac{1 - \text{err}}{\text{err}} \right)$$  \hspace{1cm} (B.1)

with \text{err} being the ratio of the misclassified samples to the total samples. A sample is misclassified if its prediction $y^i_j$ is different than the truth value $\hat{y}$, where $\hat{y}$ is 1 for signal events and -1 for background events. The voting weight is higher for trees with better classification. A small shift inside the logarithm will be added in practice to prevent infinity when the error is 1 or 0.

- Subsequent trees ($T^2$, $T^3$, \ldots, $T^N$) are generated. For each tree, $T^i$, that is trained, every sample is re-weighted depending on how it was classified. If the classification is correct, the sample weight is given as $w^i_j = w^{i-1}_j e^{-W^{i-1}}$. If the classification is incorrect, then the sample weight is given by $w^i_j = w^{i-1}_j e^{W^{i-1}}$. After the training and the re-weighting, all weights ($w^i_j$) will be normalized, meaning for each tree, the weights ($w^i_j$) will sum to 1.
In each of the training, when certain samples are misidentified by the tree, they would be emphasized in the next tree due to the re-weighting. This is because misidentified samples from the previous tree would have a higher weight in the next training, forcing the Gini or entropy criteria to classify them correctly.

The process of tree training will end when a previously determined total number of trees $N$ has been reached.

The final classification result, $y_j$ for the $j$-th sample is

$$y_j = \frac{1}{N} \sum_{i=1}^{N} y_j^i W^i,$$  \hspace{1cm} (B.2)

where $W^i$ is the voting weight of the $i$-th tree and $N$ is the total number of trees that are trained.

This BDT method is a powerful algorithm for signal-background classification however is very time-consuming. This is because each tree must be generated sequentially, and a completely new series of trees must be trained with new parameter choices for the simplified model. Given that we are interested in several different choices of masses and couplings in the model, we implement a different method described in the next section. This method allows us to build a classification model for the whole parameter space efficiently.

### B.2 Recurrent Neural Network (RNN)

A neural network (NN) is a deep learning model comprising a series of linear and non-linear transformations. The goal is to find an optimal set of parameters that transform a set of initial inputs to approximate the target. The idea is that this predictive model is made of connected units or nodes that mimic the neurons in the brain. The network consists of a series of layers that “learn” to classify the events
as signals or backgrounds through transformations. The type of the layer and the number of layers are defined by the network topology and are optimized to improve the classification. When we provide the network with a set of inputs, \( x \), it passes through these layers, undergoing transformations, one after the other. The network then outputs its set of predictions \( y \).

To improve the NN, one can define a loss function \( \mathcal{L}(y, \hat{y}) \), where \( \hat{y} \) is the truth value and \( y \) is the model output. As the NN becomes a better classifier, the predictions \( y \) will get closer to the truth value \( \hat{y} \) hence reducing the loss. Hence one can optimize the parameters inside the NN by minimizing the loss function.

Usually, the inputs of a NN have fixed lengths. However, our simulations contain events with different numbers of particles, and the inputs do not have the same length. We can use a recurrent neural network (RNN) as our deep learning tool as it allows inputs of variable lengths using Gated Recurrent Units\(^2\) (GRUs) \([275, 276]\). A standard RNN has the property that the structure of the hidden layers will be updated when new inputs are provided, and it also has the ability to “remember” parts of the previous input for optimized classification. This is depicted in Fig. B.2.

As stated before, we use this RNN to separate signal and background events. Kinematic properties of jets, electrons, and missing \( E_T \) are used as inputs to three independent, recurrent networks. These are initially trained in parallel and then merged together into a fully connected sequential neural network. The described topology is depicted in Fig. B.3. Based on the input variables, the network assigns a

\(^2\)When the RNNs get very deep, they may tend to suffer from two major weaknesses - divergent or vanishing gradients during the minimization of the loss function. In simple words, the network has difficulties in “learning” from inputs far away in the sequence, and it makes predictions based mostly on the most recent ones. A typical manner to address this problem is by using a GRU \([274]\).
Figure B.2: Visual representation of a standard RNN in folded version (left) and unfolded version (right). The sequence \( \{x_1, x_2, x_3, \ldots, x_t\} \) represents the input, \( \{y_1, y_2, y_3, \ldots, y_t\} \) represents the predicted output, and \( \{h_0, h_1, h_2, \ldots, h_t\} \) holds the information from the previous input. The graph illustrates that, at any given time, \( t \), the current layer will be updated with respect to a new input.

decision score \((d)\) to every event. This score goes from \( d = 0 \), indicating a perfectly-background-like event, indicating perfectly-background-like event, to \( d = 1 \), indicating a perfectly-signal-like event. We then determine a cutoff \( d^* \) such that all events satisfying \( d > d^* \) are classified as signal \( S \), and the rest is background, maximizing the signal significance \( S/\sqrt{S+B} \).
Figure B.3: Visual representation of the Recurrent Neural Network (RNN) used in our study. The kinematic properties of jets, electrons, and missing energy are analyzed by independent Gated Recurrent Units (GRUs). The outcome is merged into a sequential neural network that produces a decision score \( d \) between 0 and 1.
APPENDIX C

HADRQN-LEVEL 0νββ-DECAY AMPLITUDES: COMPARING DIMENSION-9 LNV VECTOR AND SCALAR OPERATORS

Below the GeV scale, dimension-9 LNV quark-lepton operators induce local LNV hadron-lepton interactions. These can be described using chiral perturbation theory, which is an effective theory with an expansion parameter \( \epsilon = p/\Lambda \). For dimension-9 LNV operators, their leading interactions with hadrons are through induced local \( \pi \pi ee \), \( \pi NNe \), and \( NNNNee \) operators. For the sake of brevity, we will not give the explicit forms of the dimension-9 LNV operators. For further details, the reader is referred once again to Ref. [134].

For all dimension-9 vector operators \( O^\mu_{6,7,8,9} \) and \( O^\mu_{6,7,8,9} \), no sizable non-derivative \( \pi \pi ee \) operators are generated at LO [131] or through \( N^2 \)LO [132]. The LNV operators induced by the vector operators that involve two \( \pi \)'s are of the form \( (\pi \partial^\mu \pi)\bar{e}\gamma^\mu\gamma_5 e^c \), and \( (\partial^\nu \pi)(\partial_\nu \partial^\mu \pi)\bar{e}\gamma^\mu\gamma_5 e^c \), at LO and \( N^2 \)LO respectively. The point here is that the pion equations of motion can be used to show that all of these local operators are proportional to the non-derivative operator \( \pi \pi ee \) with a coefficient proportional to the electron mass \( m_e \) and, therefore, negligible at this level.

In the Weinberg power counting, then, the leading contributions of vector operators to the amplitude arise from \( \pi NNee \) and \( NNNNee \) operators, which both contribute at \( \mathcal{O}(\epsilon^2) \) to the \( nn \to ppee \) amplitude.
Naïvely then, the transition amplitude arising from dimension-9 vector operators is suppressed by $\mathcal{O}(\epsilon_\chi^2)$ compared to that of scalar operators that arise from $O_{2,3,4,5}$ and $O_{2,3}'$ which contribute at $\mathcal{O}(\epsilon_\chi^0)$ to the amplitude.\footnote{In this Appendix we do not consider the scalar operators $O_1$ and $O_1'$ which give a chirally suppressed contribution to $0\nu\beta\beta$ decay.} While we find this expectation to be true generally, it depends on an interplay between NMEs and the size of unknown LECs that occurs, in the case of scalar operators, in the expression for the amplitude. The point of this Appendix is to expand on this statement.

Going into further detail, the reason dimension-9 scalar operators $O_{2,3,4,5}$ and $O_{2,3}'$ contribute at $\mathcal{O}(\epsilon_\chi^0)$ to the amplitude is because they induce non-derivative $\pi\pi ee$ operators that are unsuppressed by $m_e$ or any chiral power. In this case, the total amplitude is given by summing the Feynman diagrams in Fig. C.1, together with Fig. 4.2. For these scalar operators, in the Weinberg power counting, the second diagram in Fig. 4.2 is naively suppressed to the diagram in Fig. C.1 by $\epsilon_\chi^2$. But there is a further subtlety: the contribution of the local $\pi\pi ee$ operator to the total amplitude, obtained by solving the Lippman-Schwinger equation for the strong $NN$ potential, is actually UV divergent. Requiring that the total amplitude is independent of the regulator necessitates promoting the local $NNNNee$ operator from $N^2$LO to LO, which would violate Weinberg’s power counting. The RG equation relating the LECs

---

**Figure C.1**: For scalar operators, the amplitude for $nn \to ppee$ receives an additional contribution from the Feynman diagram shown here, which must be combined with those shown in Fig. 4.2.
of the $\pi\pi ee$ and $NNN eee$ operators suggests that the LEC of the latter is actually larger by a factor of $(4\pi)^2$ compared to NDA. So that in the chiral power counting, the diagram in Fig. C.1 and the second diagram in Fig. 4.2 contribute at the same order [134, 141, 189]. This feature will be important to the discussion that follows.

To see this explicitly, the contributions of dimension-9 scalar operators\(^2\) to the amplitude for nuclear $0\nu\beta\beta$ decay $0^+ \to 0^+$ is given by [134]

\[
A_{\text{scalar}} = \frac{g_A^2 G_F m_e}{\pi R_A} \left[ A_{\nu}(k_1) P_R C u^T(k_2) + A_R(\bar{u}(k_1) P_L C \bar{u}(k_2) \right], \tag{C.1}
\]

where the reduced amplitudes $A_{\nu}$ and $A_R$ are

\[
A_{\nu}(A_R) = \frac{m_N^2}{m_e v} \left[ -\frac{1}{2m_N^2} C_{\pi\pi L(R)}^{(9)} M_{PS, sd} + \frac{m_N^2}{2m_N^2} C_{\pi N L(R)}^{(9)} M_{P, sd} \right.
\]

\[
\left. - \frac{2}{g_A^2 m_N^2} C_{NNL(R)}^{(9)} M_{F, sd} \right], \tag{C.2}
\]

where $C_{\pi\pi L(R)}^{(9)}$, $C_{\pi N L(R)}^{(9)}$, and $C_{NNL(R)}^{(9)}$ are linear in the Wilson coefficients $C_i$ of the dimension-9 scalar operators, and in LECs. Their detailed expressions do not matter for this discussion but can be found in Ref. [134]. Each of these is multiplied by the NMEs

\[
M_{PS, sd} \equiv \frac{1}{2} M_{GT, sd}^{AP} + M_{GT, sd}^{PP} + \frac{1}{2} M_{T, sd}^{AP} + M_{T, sd}^{PP}, \tag{C.3a}
\]

\[
M_{P, sd} \equiv M_{GT, sd}^{AP} + M_{T, sd}^{AP}, \tag{C.3b}
\]

which are linear combinations of short-distance Gamow-Teller and tensor nuclear matrix elements and by a short-distance Fermi nuclear matrix element $M_{F, sd}$, respectively. Expressions for these matrix elements in terms of “neutrino potentials” can

---

\(^2\)But note the previous footnote 1.
Also be found in Ref. [134].

Next, first consider the value of $M_{PS,sd}$. For $^{136}$Xe, $M_{GT,sd}^{AP} = -2.80$, $M_{GT,sd}^{PP} = 1.06$, $M_{T,sd}^{AP} = -0.92$ and $M_{T,sd}^{PP} = 0.36$, calculated using the quasi-particle random phase approximation (QRPA) [178]. While the first three NMEs are $O(1)$, a partial cancellation occurs in the summation: $M_{PS,sd} = -0.44$. A similar pattern of partial cancellation across other isotopes $^{76}$Ge, $^{82}$Se, and $^{130}$Te, whether evaluated by QRPA [178], shell models [277], or interacting boson models (IBM) [278,279] is seen to occur:

$$
\begin{array}{c|cccc}
M_{PS,sd} & ^{76}\text{Ge} & ^{82}\text{Se} & ^{130}\text{Te} & ^{136}\text{Xe} \\
\hline
\text{QRPA [178]} & -0.79 & -0.575 & -0.78 & -0.44 \\
\text{Shell [277]} & -0.315 & -0.28 & -0.32 & -0.25 \\
\text{IBM [278]} & -0.37 & \\
\end{array}
\tag{C.4}
$$

For vector operators, the NME for the left diagram appearing in Fig. 4.2 is given by $M_{P,sd}$, and is currently

$$
\begin{array}{c|cccc}
M_{P,sd} & ^{76}\text{Ge} & ^{82}\text{Se} & ^{130}\text{Te} & ^{136}\text{Xe} \\
\hline
\text{QRPA [178]} & -6.2 & -4.47 & -6.4 & -3.7 \\
\text{Shell [277]} & -2.3 & -2.1 & -2.4 & -1.9 \\
\text{IBM [278]} & -2.34 & \\
\end{array}
\tag{C.5}
$$

Let us now consider each of the three terms in Eq. (C.2) in turn. The first one arises from induced $\pi\pi\ell\ell$ interactions. Specifically, $C_{\pi\pi\ell\ell}^{(9)}(R)$ depends on the Wilson coefficients $C_i$ and the LECs $g_{\ell=2,3,4,5}^{\pi\pi}$, the latter of which are known from chiral $SU(3)$ [151] and lattice QCD [176] to be $\sim$ few $\times$ GeV$^2$, in agreement with naïve dimensional analysis (NDA). Thus all else being equal, the size of the quantity $C_{\pi\pi\ell\ell}^{(9)}(R)/m_N^2$ is just given by the size of the Wilson coefficients $C_i$ appearing in the
expressions for $C^{(9)}_{\pi\pi L(R)}$.

For the second term, $C^{(9)}_{\pi\pi NL(R)}$ depends on $g_1^{\pi N}$ and $g_1^{\pi \pi}$, which at this chiral order only occurs for $O_1$ and $O'_1$. Since we are not considering $O_1$ and $O'_1$ in this Appendix, we can neglect the second term in our comparison of the scalar operators $O_{2,3,4,5}$ and $O'_{2,3}$ to the vector operators.

The last term in Eq. (C.2) arises from induced 4-nucleon operators ($NNNNee$) and is from the diagram on the right in Fig. 4.2. It is naïvely suppressed compared to the first term, depending as it does on the explicit factor of $m_\pi^2/m_N^2$. However, the prefactor $C^{(9)}_{NNL(R)}$ depends on the LECs $g_i^{NN}$, and RG evolution suggests $g_i^{NN} = O( (4\pi)^2 )$ [134, 189]. If these LECs are that large, then the last term cannot be neglected.

But first, suppose these LECs are much smaller than $O( (4\pi)^2 )$. Then the ratio of amplitudes induced by vector operators to scalar operators is, all else being equal,

$$\frac{A_{\text{vector}}}{A_{\text{scalar}}} \sim \frac{m_\pi^2}{m_N^2} \frac{M_{P,\text{sd}}}{M_{PS,\text{sd}}}.$$  \hspace{1cm} (C.6)

The ratio of NMEs appearing above is currently

<table>
<thead>
<tr>
<th>$M_{P,\text{sd}}/M_{PS,\text{sd}}$</th>
<th>$^{76}\text{Ge}$</th>
<th>$^{82}\text{Se}$</th>
<th>$^{130}\text{Te}$</th>
<th>$^{136}\text{Xe}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>QRPA</td>
<td>7.8</td>
<td>7.8</td>
<td>8.3</td>
<td>8.5</td>
</tr>
<tr>
<td>Shell</td>
<td>7.3</td>
<td>7.6</td>
<td>7.6</td>
<td>7.8</td>
</tr>
<tr>
<td>IBM</td>
<td>6.3</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

and exhibits remarkable stability across isotopes and methods for estimating the NMEs. In particular, this ratio is $O(6 - 8)$, and is driven by the “small” values of the
NME appearing in $A_{\text{scalar}}$, rather than a suppression of the NME appearing in $A_{\text{vector}}$.

In other words, under the assumption that the contribution of the right diagram in Fig. 4.2 to $A_{\text{scalar}}$ is $O(m_\pi^2/m_N^2) \simeq 1/60$, the amplitude $A_{\text{vector}}$ is only suppressed compared to $A_{\text{scalar}}$ by an amount $\simeq (6 - 8)/60$, rather than $1/60$.

However, this conclusion is sensitive to the unknown values of the LECs $g_{iNN}^*$ contributing to $A_{\text{scalar}}$, as we now explain. The reason is that in the expression for $A_{\text{scalar}}$, the contribution of the 4-nucleon operator is weighted by the NME $M_{F,sd}$, which are not small:

$$ M_{F,sd} | \quad ^{76}\text{Ge} \quad ^{82}\text{Se} \quad ^{130}\text{Te} \quad ^{136}\text{Xe} $$

<table>
<thead>
<tr>
<th>QRPA [178]</th>
<th>-3.46</th>
<th>-2.53</th>
<th>-2.97</th>
<th>-1.53</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shell [277]</td>
<td>-1.46</td>
<td>-1.37</td>
<td>-1.61</td>
<td>-1.28</td>
</tr>
<tr>
<td>IBM [278]</td>
<td>-1.1</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

In the extreme situation that the LECs $g_{2,3,4,5}^{NN} \simeq O((4\pi)^2)$, the relevant comparison of NME’s is instead to $M_{P,sd}/M_{F,sd}$

$$ M_{P,sd}/M_{F,sd} | \quad ^{76}\text{Ge} \quad ^{82}\text{Se} \quad ^{130}\text{Te} \quad ^{136}\text{Xe} $$

<table>
<thead>
<tr>
<th>QRPA [178]</th>
<th>1.8</th>
<th>1.8</th>
<th>2.2</th>
<th>2.4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shell [277]</td>
<td>1.6</td>
<td>1.5</td>
<td>1.5</td>
<td>1.5</td>
</tr>
<tr>
<td>IBM [278]</td>
<td>2.1</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

which varies on the $O(1.5 - 2.5)$.

Of course, the enhancement of the LECs appearing in the $0\nu\beta\beta$-decay amplitude induced by the scalar operators $O_{2,3,4,5}$ and $O_{2,3}'$ may not be as large as $(4\pi)^2$. In that case, the comparison between the $0\nu\beta\beta$-decay amplitude induced by vector and scalar operators lies somewhere between the two extremes described here. Reducing the uncertainty on the sizes of the LECs is greatly needed.

187


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