The idea of progress in mathematics education: an historical study.

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THE IDEA OF PROGRESS IN MATHEMATICS EDUCATION:
AN HISTORICAL STUDY
A Dissertation Presented
By
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AN HISTORICAL STUDY

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ABSTRACT

The Idea of Progress in Mathematics Education: An Historical Study

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It is axiomatic that the fundamental beliefs of a society influence the structure of education which that society provides; one of these, the idea of Progress, was central in the development of American society. The idea of Progress is retraced within the context of the developments in mathematics and in mathematics education. An effort to determine the specific influence which the idea of Progress had on the developments of these two areas was made in order to answer the following problem: Did the idea of Progress which developed during the eighteenth and nineteenth centuries influence mathematics curriculum development in public education?

This study was conducted under the following assumptions: There have been changes in the idea of Progress in western civilization, and that these changes are manifested in educational theories which have been proposed, and that these changes may be observed within the mathematics curri-
This study was limited to the idea of Progress in the West; special attention was given to materials in mathematics education between the years 1920 and 1977. All of the work was directed toward the goal of obtaining historical evidence for the purpose of investigating the stated question.

The work is divided into five chapters: the first presents the general procedures utilized; the second summarizes the historical developments from Ancient Greece through the early twentieth century; the third chapter concerns itself exclusively with developments during the twentieth century; the fourth chapter considers more contemporary issues, namely the reactions to the innovations in mathematics education which occurred during the sixties; the fifth chapter presents, in summary form, the findings and implications of this study.

The principal findings were: Prior to the eighteenth century four major concepts dominated the intellectual ambience so as to preclude the development of the idea of Progress prior to that period; an interest in mathematics developed as mathematics played a role in transforming life, and that this transformation was intimately connected to the development of the idea of Progress; the influence of the idea of Progress in mathematics education is not apparent until the late nineteenth century when two competing forms of the idea became apparent; these forms can be detected
in the late twentieth century in the UICSM and in the SMSG curriculum projects; in Europe the curriculum developments of the fifties and sixties were written from a socialist standpoint but were criticized on liberalist grounds, while in the United States the curriculum developments were based on liberalist grounds but criticized from a socialist standpoint.
<table>
<thead>
<tr>
<th>Chapter</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Introduction</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>An Overview of Developments</td>
<td>9</td>
</tr>
<tr>
<td>3</td>
<td>New Directions and Techniques in 20th Century America</td>
<td>94</td>
</tr>
<tr>
<td>4</td>
<td>Reactions and Criticisms</td>
<td>130</td>
</tr>
<tr>
<td>5</td>
<td>Summary, Findings, Implications, and Recommendations</td>
<td>145</td>
</tr>
</tbody>
</table>
CHAPTER I

INTRODUCTION

To a thoughtful person, the identification of the fundamental beliefs which support his activities is extremely important. This search for identification is equally important to a society, especially when it develops a program of education. Indeed, Israel Scheffler has stated that this identification is one of two focal points in the development of a curriculum. He stated:

Mastery of truths has to do with getting the appropriate beliefs; acquisition of methods and operations involves getting the right skills. For each subject, there are characteristic and peculiar truths as well as distinctive and appropriate skills. To find these and to state them is to produce a curriculum. What could be more familiar—or more misguided?\(^1\)

Within the context of mathematics education this same thesis has been held by many eminent mathematicians and mathematics educators. For example, the noted French mathematician, René Thom, has stated: "In fact, whether one


wishes it or not, all mathematical pedagogy, even if scarcely coherent, rests on a philosophy of mathematics."² That is to say that mathematics education rests upon fundamental beliefs concerning mathematics. Similarly, if needs are recognized to depend upon fundamental beliefs, the British mathematics educator, H. B. Griffiths' statement that, "...our problem as professional teachers of mathematics is to be able sensibly to link the growth of the mathematical organism with the changing needs of our Society,"³ can be recognized to be a reiteration of the same idea. The Russian historian of mathematics, A. P. Yushkevich, has stated that the subject of mathematics itself is "a form of social consciousness."⁴ This would seem to indicate that a philosophy of mathematics ultimately rests upon social consciousness, that is, upon knowledge of the fundamental beliefs of a society.

As has been noted, the fundamental beliefs of a society influence the very structure of education which that society provides for its youths, and moreover, influence mathematics itself; hence all the more so do these beliefs influence mathematics education.


Progress is the central idea which inspired our society according to Octavio Paz. Paz came to this conclusion after careful study of the revolutionary works produced by the anthropologist Clause Levi-Strauss.

The idea of Progress within western civilization is rather recent; this idea first developed during the eighteenth and nineteenth centuries. The historical development of this idea appears in Chapter II of this work.

The influence of the idea of Progress upon education was studied by Christopher Clarke, and the results of his findings were presented in his doctoral dissertation. In that work, Clarke showed that the idea of Progress influenced the origins and curriculum theories of three principle educational philosophies of the late nineteenth and early twentieth centuries; Experimentalism, Essentialism, and Reconstructionism. Clarke stated within his recommendations that research should be conducted in the area of Process-Structure philosophy, as expressed by Alfred North Whitehead and Jerome Bruner, to determine whether this philosophy continued the tradition of the idea of Progress

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in American curriculum theory.\(^7\)

Griffiths and Howson have stated that "educational research has not greatly influenced curriculum planners. It is relevant then to look at some of the questions which one would like educational researchers to answer--questions which lie at the heart of any attempt to devise a mathematical curriculum."\(^8\) These questions are to be found, as noted earlier by Scheffler, in the appropriate beliefs of the society, which according to Paz, are to be found in the idea of Progress. This idea of Progress did have an influence, as Clarke noted, within certain educational curriculum theories. The connection of mathematics and of mathematics education with these fundamental beliefs of society has also been indicated.

**Statement of the Problem**

Did the idea of Progress which developed during the eighteenth and nineteenth centuries influence mathematics curriculum development in public education?

**Subproblems**

1. If the idea of Progress did influence mathematics curricula, how was this influence manifested?

\(^7\)Clarke, p. 175.

2. If the idea of Progress did not influence the mathematics curriculum of any given educational theory, what were the reasons for this lack of influence?

3. Are the social goals, the points which society should strive for as they are related to the idea of Progress, attainable through the mathematics curriculum?

4. What was and what is now considered Progress in Mathematics Education, and is this concept related to the concept of progress as defined by the leading philosophers?

Importance of the Problem

1. It is clear that there is an "absence of a science of learning, and thus, a fortiori, of a science of mathematical education..." In a recent work, Hans Freudenthal, trusting historical analogies, has stated that he believes that the first subject area to develop a science of education probably will be mathematics. If such a science develops, it is important that the use to which this science is put aids society to progress; it is important that aims be identified and recognized. It is in society's best interest to determine what constitutes progress in this field before such a science develops. This historical study should help to place these matters into proper perspective.

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9H. B. Griffiths and A. G. Howson, p. 81.

2. Critics of the curriculum changes which occurred in mathematics education during the late fifties and sixties of this century have claimed that these changes did not constitute progress in mathematics education. It is important to identify what constitutes progress so that curriculum workers may be able to operate more effectively. An historical study should help to gain some insight on this topic.

Assumptions

1. There have been changes in the idea of Progress in western civilization, and that these changes are manifested in the educational theories which have been proposed.

2. That changes in metaphysics have influenced curricula changes, and that these changes may be observed within the mathematics curricula.

Limitations

This study is limited to the idea of Progress in the West. An historical perspective of educational philosophy and of metaphysics is presented, particularly within the framework of American public education; focal attention is given to underlying ideas for curricula of mathematics in American public education. Special attention is given to materials developed between the years 1920 and 1977. All of the work is directed toward the goal of obtaining his-
torical evidence for the purpose of investigating the questions stated in the Problem and the related subproblems.

**Procedures**

1. A search for related works was conducted in Dissertation Abstracts International, Educational Resources Information Center (ERIC) Documents. A computer search was conducted in files suggested by the librarian-computer-file specialist for related works.

2. A search was conducted of the card catalog for primary and secondary works related to nineteenth and twentieth century philosophical development as well as works concerned with mathematics education and philosophy of mathematics. The greatest emphasis was placed on primary works. Secondary works were used as a framework to support the major development.

3. A search was conducted of the primary and secondary works of (1) Plato, (2) Jean-Jacques Rousseau, (3) August Comte, (4) John Dewey, (5) Alfred North Whitehead, (6) Rene Thom, (7) Morris Kline, (8) major curricular materials, namely: Cambridge Conference on Mathematics Education, School Mathematics Study Group (SMSG), University of Illinois Committee on School Mathematics (UICSM), and the National Advisory Committee on Mathematics Education (NACOM) Report. These works were used to determine how Progress, particularly in mathematics education, has been defined and demonstrat-
ed. Particular attention was paid to both internal and external criticism of the material reviewed.

**Organization of the Study**

The work appears in five major chapters. The first introduces the general procedures which were used and serves as introduction to the work.

The second chapter contains a sketch of the historical development of the social goals and the place of the study of mathematics and of philosophy from the beginnings of Western Civilization in Greece, through the Middle Ages and the Renaissance. More detailed treatment is given to the important period of the Enlightenment through the early twentieth century.

The third chapter concerns itself exclusively with developments during the twentieth century. Here particular attention was given to the developments during the Dewey period and to the curricular changes of the fifties and sixties; the latter part of the chapter deals with the philosophy of Whitehead and its influence upon more recent developments; this includes material from the Cambridge Conference, SMSG and UICSM.

The next chapter is concerned with more contemporary issues, namely the reactions to recent innovations within its historical frame.

The final chapter presents, in summary form, the find-
ings, implications and recommendations for further study which have resulted from this study.
CHAPTER II

AN OVERVIEW OF DEVELOPMENTS

This historical overview is intended principally to provide a view of the development of one idea, the idea of Progress, along with the shared influence of this idea in philosophy, mathematics, and mathematics education. Its completeness is presented as a sketch limning the major features of these developments; the image which results will, as with any sketch, probably suggest different closures within intended bounds.

The term "Progress" with a capital "P" indicates a unique point of view on change, a point of view which represents a theory of the historical process which developed during the eighteenth century. The term "progress" with a lower case "p" indicates movement or development in some general sense without a theoretical framework. The distinction which is made here is similar to the distinction between Republican, which indicates a particular system of beliefs on political and economic matters, and republican, which indicates only a political system of indirect representation of the people.¹

There are two major orders of ideas. Some ideas are held to be good or bad and depend upon the human will for their realization or repression; some of these, such as liberty, slavery, toleration, equality of opportunity, socialism, and democracy are subjects for consideration in ethics; they are accepted or rejected as a matter of moral judgment. There is another order of ideas which does not depend upon human will; these are ideas which are judged to be true or false; some of these, such as Fate, Providence or personal immortality are "accepted or rejected not because they are believed to be useful or injurious, but because they are believed to be true or false." These are the ideas which form the foundations of philosophies; they are the unquestioned axioms of civilizations which permeate the climate of thought:

The idea of the progress of humanity is an idea of this kind, and it is important to be quite clear on the point. We now take it so much for granted, we are so conscious of constantly progressing in knowledge, arts, organizing capacity, utilities of all sorts, that it is easy to look upon Progress as an aim, like liberty or a world-federation, which it only depends on our own efforts and good-will to achieve. But though all increases of power and knowledge depend on human effort, the idea of Progress of humanity, from which all these particular progresses derive their value, raises a definite question of fact which man's wishes or labours cannot affect any more than his wishes or labours can prolong life beyond the grave.3


3 Bury, pp. 1-2.
In its most simplistic form, the idea of Progress meant "that civilization has moved, is moving, and will move in a desirable direction." The idea of Progress therefore requires a judgment of the past and a prophecy for the future. It requires the view that the historical process be, in nature, free of an external will, otherwise the idea of Progress would lapse into the idea of Providence. Furthermore, the strongly Newtonian concept of the linear progression of time is a necessary condition for the idea of Progress; judgments are made with respect to linear progressions of historical developments and this thrust is extrapolated into an indefinite future.

**Ancient Greece**

The concept of Progress was unknown to the ancient Greeks. So much of Western Civilization traces its roots to the civilization of ancient Greece that this statement appears at first startling. Many of the antecedents for the formation of this concept were not present. Bury\(^4\), in this study of the idea of Progress, offered the following reasons:

(a) Their recorded history did not extend over a sufficiently long period of time for any judgment to be made concerning a trend toward desirable goals.

\(^4\)Bury, p. 2.

\(^5\)Bury, pp. 7-11 passim.
(b) The science and mathematics which they developed did not transform conditions of life. There were no series of impressive scientific discoveries which would indicate a potentially indefinite increase in knowledge or a mastery of the forces of nature.

(c) They had a profound veneration of antiquity. Their concept of change meant corruption and disaster. (Cf. Politics, ii, 5 Aristotle).

In addition to these reasons, there is the additional fact that the prevalent concept of time for the Greeks was considerably different from the modern view of time as a linear continuum indefinitely extendable. For these people time was cyclical and history repetitive.

The Greeks rendered physical time by the word chronos. It is determined as a parameter of the spatial kinematic motion of the planets (Timaeus, 37E). Aristotle refers to the elliptic motion of the celestial bodies as 'eternal' (Physics IV, 221b, 3-4), but in this sense 'eternal' means periodicity, recurrence, or perhaps a 'perpetual present'.

Delvaille, in his earlier study of the idea of Progress, appeared to hedge on this point when he stated, "Voilà pourquoi il est légitime de compter Platon parmi les théoriciens du Progrès," but he followed this statement immediate-
ly with a qualifying phrase which tempered this Gallic overstatement. He stated, "Mais il faut préciser dans quel sens Platon parle du progrès," which placed his thinking more in concord with Bury's ideas on the matter.

This study nevertheless began with ancient Greece because, as Morris Kline stated, "In the history of civilization the Greeks are preeminent, and in the history of mathematics the Greeks are the supreme event."9

One reason for this preeminence in the history of civilization was the fact that their thinking represented a new direction from 'primitive' thought, a direction which was to be taken by Western Civilization.

In the Ionian school of philosophy, rational thought was emerging from the mythological dream-world. It was the beginning of the great adventure: the Promethean quest for natural explanations and rational causes, which, within the next two thousand years, would transform the species more radically than the previous two hundred thousand had done. 10

Foremost among the Ionians was Pythagoras of Samos, a philosopher about whom it is said his "influence on the ideas, and thereby on the destiny, of the human race was probably greater than that of any single man before or

7Delvaille, p. 233.

8Bury, pp. 9-11 passim.


after him."

For Pythagoras, the world could best be explained by mathematics in the form of arithmetic and geometry; these formed the basis of his ontology; they were the realities of permanence within a fluid world. As Koestler explained:

Numbers are eternal while everything else is perishable; they are of the nature not of matter, but of mind... The ecstatic contemplation of geometrical forms and mathematical laws is therefore the most effective means of purging the soul of earthly passion, and the principal link between man and divinity.  

Mathematics, then, became the unifying principle of explanation; here are found the roots of its importance in Western thought. This importance led to a great deal of interest in determining relationships which existed within arithmetic and geometry. These newly discovered relationships took on mystical meanings and eventually led to the foundation of a secret society for the study of mathematics referred to as the Pythagorean Brotherhood. "The Pythagoreans pursued mathematics as a kind of religious contemplation, as a way to approach the eternal Truth." Mathematics was to be the principle of synthesis for reality.

Arithmetic meant the arithmetic of rational numbers, and this exclusive concern with rational numbers and geometry

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11 Koestler, p. 25.

12 Koestler, p. 28.

held within itself the seeds of destruction of the Brotherhood. When incommensurable quantities were discovered in the study of geometry, this development appeared to signal a flaw in the very foundation of this philosophy which was to explain all of existence. This discovery was the cause of grave consequence to the Brotherhood.

It is said that the Pythagoreans kept the discovery of irrational numbers...a secret, and that Hippasos, the disciple who let the scandal leak out, was put to death.\textsuperscript{14}

The next attempt at a synthesis of reality was made by Plato and by members of his Academy. The influence of Plato's philosophy, which had its roots in the mathematical thinking of the Pythagoreans, as well as the thinking of "primitive" societies, had been extensive. In fact Professor A. N. Whitehead once remarked that "the safest general characterization of the European philosophical tradition is that it consists in a series of footnotes to Plato."\textsuperscript{15}

Professor Eliade, after extensive study of "primitive" cultures, was moved to make the following conclusion:

Hence it could be said that this 'primitive' ontology has a Platonic structure; and in that case Plato could be regarded as the outstanding philosopher of 'primitive mentality,' that is, as the thinker who succeeded in giving philosophical currency and validity to the modes of life and behavior of archaic humanity.\textsuperscript{16}

\textsuperscript{14} Koestler, p. 40.
\textsuperscript{15} Koestler, p. 52.
The roots of Plato's metaphysics can be observed in the thinking of mythic man for whom objects or acts became real and meaningful only insofar as they participated in some reality which transcended them.

Among countless stones, one stone becomes sacred... The object appears as the receptacle of an exterior force that differentiates it from its milieu and gives it meaning and value. This force may reside in the substance of the object or in its form;...It resists time; its reality is coupled with perenniality.17

As indicated earlier, Plato's view of the changes in the world were, however, far removed from the concept of Progress. "'Change' for Plato was virtually synonymous with degeneration."18 History was for him a tale "of descent and devolution--as opposed to evolution by ascent."18

For Plato the World of Reality was the World of Ideas "which consists only of perfect Forms or Ideas...the World of Appearances...is a shadow and copy of the former..."18 It was natural then that the preeminence of the forms of geometry should continue to be considered important for study.

Within his philosophy a pedagogy of mathematics was indicated.

Plato...opposed scientific research by experimental and mechanical methods, teaching that truth should be

17 Eliade, p. 34.
18 Koestler, p. 55.
sought in the study of pure geometry.¹⁹

This thinking was derived from his mentor Socrates and is well illustrated in a sequence found in the Meno in which the following account appeared:

All that we call learning thus consists in recollecting that which we knew before birth. To prove this position, Socrates cross-examines a slave of Meno. This slave has never been taught mathematics, but in answer to a series of leading questions propounded by Socrates, he evolves a mathematical theorem, viz, that a square, to be doubled the area of another square, must be described on the diagonal...of that square...The slave thus exhibits a knowledge of geometry, and has never learned it in this life; the inference is that he acquired the knowledge in the antenatal state when he was not yet a man.²⁰

Another important pedagogical praxis of this period which was important to mathematical study was the oral tradition of instruction. Reading proofs of geometric propositions transcribed from Greek texts is a difficult and tortuous task today because these proofs were intended for oral presentation where students could follow diagrams pointed to by the teachers; in written form these proofs are difficult to comprehend. As Professor Van der Waerden stated, "The proofs are logically sound, but they are not suggestive. One feels caught as in a logical mousetrap, but one fails


to see the guiding line of thought.\textsuperscript{21}

Although Greece played a preeminent role in the development of Western civilization, and although an intellectual perspective was formed which provided the genesis for developments in the visual arts, in music, in science and mathematics of Western civilization, the idea of Progress was not present. Not only was the idea of Progress not developed, but the very possibility of its development did not exist, as has been noted.

\textbf{Ancient Rome}

It was after the surrender by the Greek general Pyrrhus of the city of Tarentine in 272 B.C. that Rome ascended to dominance in Western civilization when it governed the territory from the Arno and the Rubicon southward. At that time, "Roman education was a practical utilitarian affair taught by the father of the family."\textsuperscript{22}

Although defeated in military conquests, the Greeks were nevertheless victorious in extending their influence westward by capturing the minds and imagination of Rome's youth through education. Although some conservatives, notably Cato, opposed this captivation of Roman society—a society distinguished by its military and political


\textsuperscript{22}Frederick M. Wheelock, Quintilian as Educator (New York: Twayne Publishers, 1974), p. 9.
...in republican Rome there was an ever increasing need of rhetorical and oratorical power in public life: in the public assemblies and the Senate, in elections and deliberations, in the courts, and in the administration of Rome's growing imperial power. Accordingly, the already well-developed skill of the Greeks in this field was eagerly adopted by the Romans.  

All learning was directed almost exclusively toward the development of skilled oratory; this thrust even included the learning of mathematics. This fact was well illustrated in the educational theory of Quintilian which was written around 90 A.D. In the following passage the word "geometry" refers to both arithmetic and Euclidean geometry. Quintilian stated,

As regards geometry, it is granted that portions of this science are of value for the instruction of children: for admittedly it exercises their minds, sharpens their wits and generated quickness of perception. But it is considered that the value of geometry resides in the process of learning, and not as with other sciences in the knowledge thus acquired. Such is the general opinion...Geometry arrives at its conclusions from definite premises, and by arguing from what is certain proves what was previously uncertain. Is not this just what we do in speaking?  

For the Romans, then, the study of mathematics was not an endeavor to be undertaken for its own sake; as a result,

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23 Wheelock, p. 12.

24 Wheelock, pp. 70-71.
almost no mathematics was developed by the Romans. Professor Kline stated that "Roman mathematics hardly warrants mention." This lack of interest in mathematics was surprisingly displayed even by professional people. It was said that "the architect Vitruvius, who loves to display his learning in various fields, says next to nothing about mathematics."  

The concept of Progress did not develop during this period; the temper of the times made the possibility of the development of this idea perhaps less viable, since there were still deep and abiding interests in mystical philosophies which emphasized a cyclic and repetitive view of history. Although great advances had been made in the political and economic spheres, the concept of Progress failed to develop.

It might be thought that the establishment of Roman rule and order in a large part of the known world, and the civilizing of barbarian peoples, could not fail to have opened to the imagination of some of those who reflected on it in the days of Virgil or of Seneca, a vista into the future. But there was no change in the conditions of life likely to suggest a brighter view of human existence. With the loss of freedom pessimism increased, and the Greek philosophies of resignation were needed more than ever. Those whom they could not satisfy turned their thoughts to new mystical philosophies and religions, which were

25 Kline, p. 178.

26 Van der Waerden, p. 276.
little interested in the earthly destinies of human society.\(^{27}\)

However, two factors which later helped to form the idea of Progress emerged during the period of the Roman Empire. One had taken early root from seminal ideas of the Greeks; this was the concept of ecumenism, the concept that all men are brothers, or as the Stoics had said, "...man's true country is not his own particular city, but the ecumene."\(^{28}\)

This idea, which in the Roman Empire and in the Middle Ages took the form of a universal State and a universal Church, passed afterwards into the conception of the intercohesion of peoples as contributors to a common pool of civilization—a principle which, when the idea of Progress at last made its appearance in the world, was to be one of the elements in its growth.\(^{29}\)

The other factor germinated later; this second factor which helped to form the idea of Progress made its appearance in the later days of the Roman Empire with the introduction of Christianity. The linear concept of time is traceable to Christian ideas derivative of the Hebraic concept of the Messiah.

A transition was made at some point in the history of ancient Israel from a cultic re-enactment to the sense of history. Entered into the time of history, an event can never be re-enacted; for the recurrent cycle of nature does not control the event, but the event

\(^{27}\)Bury, p. 20.

\(^{28}\)Bury, p. 23.

\(^{29}\)Bury, p. 24.
controls history. The event brings with it a certain
deadline, a once-for-all deadline... The major doctrine
instead of re-enactment is 'rememberance,' a major word
theme in Deuteronomy.30

...the Bible articulated a view of time which demand-
ed affirmation and realization of the possibilities of
life through time rather than by cultic destruction of
time in favor of eternity... To achieve this process,
time was conceived as linear, something which proceeded
in linear advance. Each day of life was one day closer
to its fulfillment, and one day further from its crea-
tion.31

The Middle Ages

The philosophy of St. Augustine provides a focus for
the early Middle Ages. He was born in A.D. 354 in the town
of Tagaste in the Roman province of Numidia; he received a
traditional Roman education first provided at home by his
mother Monica, a Christian, then through more formal train-
ing in grammar and rhetoric. After his studies he began
a search for meaning in life, of which he later wrote in his
Confessions; in the course of this search he read and later
translated works of Plato with which he was much impressed.

There are those who find in this last year of
spiritual searching two separate conversions, the first
to Platonism and the second to Christianity.32

The influence of Platonism upon St. Augustine and in

31Yaker, p. 32.
32George Howie, Educational Theory and Practice in
St. Augustine (New York: Teachers College Press, 1969),
p. 15.
turn the influence of St. Augustine's teachings among early Christians is a key to understanding the development of Christian philosophy.

Early Christian thinkers certainly recognized in Platonism a certain kinship, even if more or less remote, with their own Weltanschauung and, though Aristotle was later to become the philosopher par excellence of Scholasticism, Augustinianism stands rather in the line of the Platonic tradition.33

Because of this tremendous influence upon Christian thinking, it is important to note St. Augustine's ideas concerning education. For St. Augustine, "the proper goal of all educational effort is the understanding of the nature of God..."34

In his quest for an understanding of God, St. Augustine believed that the study of numbers, by which term he understood arithmetic and geometry, would be an aid to understanding. In his words:

When such a person inquires into divine matters, it is not in vain; for such matters are now not only to be believed but also to be contemplated, understood and retained. But whoever is still a slave to his desires and sighs after the things which are transient will make all the mistakes which it is possible to make... But it will all be learned quite easily by the man who understands simple and intelligible numbers.35

This thought was consonant with the training which St.


35 Howie, St. Augustine on Education, p. 260.
Augustine had received and the sentiments of Quintillian on this subject which were cited above.

A more direct influence of Greek mathematical thinking can be traced to Boethius a century later. Boethius had translated many of the Greek texts into Latin; these texts were to become the source material for the study of the seven liberal arts in the course of the first millennium of Christianity.  

The strong influence of the axiomatic method upon Boethius can be observed in the opening paragraph of one of his Theological Tractates, where the following appeared:

Therefore, as is the practice in the mathematical and other sciences, I have set down terms and rules according to which I shall develop all that follows.  

This sentence is then followed by nine statements which are presented as axiomatic and from which his proposed proof proceeds.

Towards the end of the Roman Empire, while the Empire was being beseiged by incursions of barbarian tribes and while it was also in the midst of political and economic disintegration, the study of mathematics was continued in the monasteries of Europe. This study was given for the preservation of existing mathematical knowledge, not for


37 Boethius, Theological Tractates, cited in Wm. Bryar and G. Stengren, pp. 105-106.
active search into its meaning and its extension. It is in this sense that Otto Neugebauer states:

For the history of mathematics and astronomy the traditional division of political history into Antiquity and Middle Ages is of no significance. In mathematical astronomy ancient methods prevailed until Newton and his contemporaries opened a fundamentally new age by the introduction of dynamics into the discussion of astronomical phenomena. 38

Nevertheless, it should be recognized that mathematics was a relatively important subject of instruction in the monastery schools. 39 This fact can be illustrated from the works of Cassiodorus, a former Roman senator who retired in his old age to the monastery in Calabria where he wrote assiduously both on Christian and secular subjects. In one of his books, On the Art and Disciplines of Liberal Letters, he recalled the tradition of the seven liberal arts 40 which formed the curriculum of secular study in the Middle Ages. Cassiodorus' influence in education during the Middle Ages was very strong. In fact H. G. Wells 41 stated:

Probably his influence was even greater than that of Saint Benedict in making monasticism into a powerful instrument for the restoration of social order in the Western World.

The Liberal Arts curriculum consisted of two levels of education; the first part, called the Trivium, consisted of Grammar, Logic and Rhetoric; the second part, the Quadrivium, consisted of Geometry, Arithmetic, Astronomy and Music. Although it appears from this listing that mathematics had a major role within the curriculum, most treatises written during this period appear to dwell upon the first part of the curriculum, the Trivium; it is conjectured that this may have been the result of the influence of Roman education, which stressed oratory as its major goal.

How little mathematics actually survived during this period can be assessed from the writings of Alcuin of York, a leading figure in monastic education during the eighth century. His treatise *De Cursu Et Saltu Lunae Ac Bissextio* demonstrates that ancient methods for calculating the changes of the moon, a necessity in monasteries for the determination of Easter, had been transmitted through the centuries. However, his more purely mathematical work, *The Propositions of Alcuin*, demonstrates a lack of understanding of mathematics at a fundamental level.

The propositiones consist in the main of very simple exercises, all solved by painfully rudimentary methods. Not one of them exhibits an apprehension on Alcuin's part of any mathematical idea or formula. Forty-five of the fifty-three propositions may, by courtesy, be styled exercises in reckoning.42

42West, p. 109.
Further insights into the state of mathematical understanding during this period can be obtained from the writings of one of Alcuin's students, Rabanus, who in his book, *Computus* (On Reckoning), went through an explanation of counting by means of an elaborate finger bending system; he was able to count to ten thousand in this manner, and by further placing the hands on various parts of the body he was able to count to one million. At the end of the book he outlined the Roman system of weights and measures, but had no system of fractions; he "...like Alcuin, has no notion of what a fraction is."\(^43\)

The method of teaching used by Alcuin was catechetical; questions and answers provided by the teacher.\(^44\) The teaching of mathematics was reduced to an exercise in rote memory with no attempt made at understanding.

Around 1100 A.D. the structure of learning was beginning to change; increased trade and travel by Europeans brought them into contact with Arab, Byzantine and Near-East civilizations, which introduced Europeans both to ancient Greek philosophy and to the mathematics which the Arabs had both developed and preserved.\(^45\) These new con-

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\(^{43}\) West, p. 153.

\(^{44}\) West, p. 45.

\(^{45}\) Kline, p. 205.
tacts led to a revival of learning which historians now call the renaissance of the twelfth century. 46

Indeed, the eleventh through the thirteenth centuries—especially between 1050 and 1300—may be thought of as a renaissance of medieval learning, the intellectual, literary and esthetic flowing of the Middle Ages... stimulated largely by the influx of Greco-Muslim scholarship in an ever increasing number of translations... Moreover, this passion for Greco-Muslim learning in a sense characterized the very nature of this early renaissance and gave this renaissance its intellectual-educational scope and direction. 47

As a result of this revival, the traditional liberal arts curriculum was reconsidered, especially as it related to the study of mathematics. New ideas were infused into the curriculum and included algebra, Euclidean geometry, trigonometry and the introduction of the Hindu-Arabic numeration system. 48 The Hindu-Arabic numeration system had been introduced in translations by Adelard of Bath, but characteristic of the influence which commerce exerted to effect this renaissance, the Hindu-Arabic system was more widely used in commercial spheres after its introduction in the book Liber Abaci by Leonardo of Pisa (Fibonacci), the son of a merchant. 49

48 Nakosteen, pp. 190-91.
The seed of a new spirit was also planted through Muslim contacts; this was the spirit of experimentation, the spirit of observation and inquiry. It was a spirit which

...the Muslims had adapted from their Greek and Hellenistic teachers and transmitted to the Latin world without fully understanding or exploiting its far reaching possibilities and promises themselves.50

This experimental spirit was to be the major force for the introduction of a new era. Among those who were most inspired by this new spirit were Robert Grosseteste and his student Roger Bacon. Delvaille found the thinking of Roger Bacon so permeated by this new spirit that he stated,

...nous assistons a l'éveil de l'esprit scientifique avec Roger Bacon; ce penseur a, sur le Progrès, des vues ou l'on peut plus que pressentir l'esprit moderne.51

Bury took a more negative stand on this matter; he stated,

Thus Friar Bacon's theories of scientific reform so far from amounting to an anticipation of the idea of Progress, illustrates how impossible it was that this idea could appear in the Middle Ages. The whole spirit of medieval Christianity excluded it. The conceptions which were entertained of the working of divine Providence...had the same effect as the Greek theories of the nature of change and of recurring cycles of the world. Or rather, they had a more powerful effect, because they were not reasoned conclusions, but dogmas guaranteed by divine authority.52

50 Nakosteen, p. 187.

51 Delvaille, p. 97. Translation provided at the end of Chapter II.

52 Bury, pp. 28-29.
These two views can be reconciled, to some degree, if the idea of divine Providence is looked upon as the idea of Progress which has been embodied, a view which was taken by Octavio Paz.\(^\text{53}\)

One notable figure in the development of mathematics during this period was Nicole Oresme, Bishop of Lisieux and a teacher in the Parisian College of Navarre. He has been reputed to have made contributions to at least two major developments; they are the concept of function and the concept of coordinate geometry.\(^\text{54}\)

Mathematics was probably studied only at universities except for practical applications, which were learned during apprenticeship in a Guild.

That part of mathematics now found in arithmetic books, so far as it then existed, was regarded as a practical art, useful chiefly to tradesmen.\(^\text{55}\)

For pre-university level education, the Middle Ages continued the rhetorical instruction which had constituted Roman education.\(^\text{56}\) This was probably due to the fact that

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\(^\text{54}\) Kline, pp. 210-11.


\(^\text{56}\) Adamson, p. 194.
most of the formal teaching was carried on by clergymen.\textsuperscript{57}

The rise of the medieval university during the twelfth century was a continuation of the monastic tradition. The curriculum consisted, in large measure, of scholastic philosophy and theology or other professional studies in jurisprudence and medicine. The quality of teaching at these universities was reported by the scholar of medieval universities, Hastings Rashdall, to have been poor. He stated:

In the higher faculties, too, we have encountered a tendency on the part of the doctors to evade the obligation of teaching without surrendering its emoluments, while the real teaching devolved upon half-trained bachelors...in medieval times students were more anxious to learn than teachers were to teach.\textsuperscript{58}

\textbf{The Renaissance (1400-1600)}

The period of the Renaissance was a period of critical thinking concerning the political, social, economic and educational institutions of the European countries. Continued increase of trade and the growth of manufacturing were forces which helped to change the old order. Better manuscripts of ancient Greek texts became available for translation and study. Printing by means of movable type, which was invented by Johann Gutenberg about 1450, allowed for wide distribution of these works in the vernacular as well as

\textsuperscript{57}James A. Johnson, A Brief History of Student Teaching (DeKalb, Ill.: Creative Educational Materials, 1968), p. 2.

\textsuperscript{58}Hastings Rashdall, The Universities of Europe in the Middle Ages, ed. F. Powicke, and A. Emden (1895; London: Oxford Univ. Press, 1936), III, p. 452.
in Latin.

Among the works in mathematics which appeared were the first printed edition of Euclid's *Elements* in a Latin translation by Johannes Campanus, the first four books of Apollonius' *Conic Sections*, Diophantus' *Arithmetica* and the works of Pappus.\(^{59}\)

The trigonometric works which appeared were motivated by the study of astronomy and the needs of navigation. Corrections to extant Latin translations of Ptolemy's *Almagest* were started by George Peurbach (1423-61) of Vienna and continued by his student Johannes Muller\(^{60}\) (1436-76) (or Johannes Montaregio),\(^{61}\) known as Regiomontanus. In the sixteenth century, trigonometry began to break away from astronomy and acquired status as a branch of mathematics.\(^{62}\)

The decimal fractions were introduced by Simon Stevin (1548-1620) in his pamphlet *La Disme*, which was published in 1585.

In science, the works of Copernicus and Galileo Galilei were causes of a revolution involving the perspective of man's place in the universe. The increased interest in

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\(^{59}\)Kline, p. 219

\(^{60}\)Kline, p. 238.


\(^{62}\)Kline, p. 240.
empiricism which had been generated in the later medieval period now began to bring into question many of the previously uncritically accepted verities.

The dominant philosophy which re-emerged during the Renaissance was Platonism and Pythagoreanism.63 From this arose a revival of the basic creed that "Nature is rational, simple and orderly, and it acts in accordance with immutable laws."64 The theological explanations of the Middle Ages were re-interpreted to mean that God's design was to be known through mathematics. It was in this sense that Professor Kline stated that the Renaissance scientist was a theologian and that the search for mathematical laws of nature were acts of devotion.65

In the area of education, the universities played only a minor role in the new developments of the Renaissance because these institutions were dominated by theologians who considered experimentation unnecessary.66 Nevertheless, great changes occurred in education at lower levels during this period principally through efforts at universal education in the vernacular; mathematics was instrumental in


64 Kline, p. 218.

65 Kline, p. 219.

66 Kline, p. 221.
this effort.

The first book on arithmetic to be published in English was written by Robert Recorde between 1540 and 1542. This publication was prompted by mercantile interest in the subject. These same interests promoted the establishment of general education, as the following indicates:

The origin of the elementary school as such is to be found in the demand made by commerce and industry for junior clerks and for workmen who could read and write the vernacular and, in few instances, make out or at least understand a bill.

Prior to this period, pre-university education had as its aim to promote the interest of the Church either through tuition-free song-schools for the development of local choirs or through grammar schools for the preparation of future priests and leaders in Latin and Greek grammar. In Protestant areas of Germany, the control of such schools, today's German gymnasium, was held by municipal governments.

Such schools (burgher schools) were wholly controlled and supported by the secular authorities, and in the content of the school-work better represented the economic interests and demands of the citizens.

Ultimately, however, these interests and demands do not

67 Struik, p. 4.
68 Adamson, p. 207.
69 Adamson, pp. 207-8.
appear to have been very different from those of the church-controlled Latin grammar school. A curriculum consisting of ten classes, devised by John Sturm (1507-89) for the gymnasium of the city of Strassburg, had as its ideal the development of students fluent in Latin.

As a result of this intensive devotion to the one ideal [Latin rhetoric], Sturm's curriculum excluded all other subjects: even mathematics was given only a formal recognition, in the statement that arithmetic and astrology were to be studied in the later years, practically as a portion of university work. But it appears from accounts of the actual work of the school, that no time was found for carrying out even these meager provisions.\footnote{Monroe, p. 62.}

Similarly, in English schools, the following account provides some insight into mathematic education in pre-university schools:

In fact, Brinsley, a school master, complains that boys are sometimes so ignorant, even of the very figures, that they can 'hardly tell the number of pages, sections, chapters, or other divisions in their books to find what they should.' However it is not his business to teach them so inferior a subject...\footnote{J. H. Brown, Elizabethan Schooldays (1912; rpt. London: Oxford Univ. Press, 1933), pp. 86-7.}

In England, children started school at the age of six or seven and were generally expected to know how to read and write English upon entry; they were expected to be ready for university admission at the age of fifteen or sixteen.\footnote{Brown, p. 44.} Hence, some of the best educated students by the age of
sixteen knew almost no mathematics.

The method of teaching generally appears to have made use of the brighter students to teach, as the following account demonstrates:

Each class with Platter (a student at Sturm's school) was organized into groups of ten—decurions—partially under charge of the brightest boys...Here, again, the practice was similar to that at Strassburg, at Goldberg, and in the Jesuit schools.74

The Jesuits, formally the Society of Jesus, had been organized by Ignatius Loyola in 1534 to counteract the Protestant Reformation. Part of their activities included education of young adults by rules formally called the Ratio Studiorum, which included the earliest reference to organized training of a professional nature for teachers, such as supervised student teaching.75

Although this had been a period of tremendous progress in almost all fields of knowledge, the conditions for the development of the idea of Progress were not fulfilled. The thought that nature could exist without divine intervention as well as with independent laws of action was so weak that Professor Kline was prompted, as was mentioned earlier, to identify the Renaissance scientist with theologians. Another reason was the reinforcing view from Plato

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74 Monroe, p. 68.
75 Johnson, pp. 5-6.
that any change could be identified with corruption.\textsuperscript{76} It was also an easy inference from the perspective of the Renaissance; great works in philosophy, astronomy, physics, medicine and mathematics recently translated from the Greek dominated the intellectual work. What more natural inference could be made in light of these awe inspiring findings from the past than that civilization had followed a path of degeneration. Although there were rebels of the "tyranny of antiquity"\textsuperscript{77} such as Copernicus, who undermined the authority of Ptolemy, and Vesalius, who undermined the prestige of Galen, their influence did not constitute open rebellion to authority. As Professor Charles Singer stated, "The recovery of the Greek texts, even if it enlarged the mental horizon, chained men's minds more closely than ever to the past."\textsuperscript{78}

Although Delvaille shows himself to be a firm believer in the idea of Progress in this synopsis of the situation, it does provide a good view of the situation.

C'est l'époque de la Renaissance que l'on peut considérer comme caractéristique d'un progrès qui s'est accompli dans le monde, dans les idées, dans les institutions...si l'on n'a pas élaboré de nombreuse

\textsuperscript{76}Bury, p. 32.

\textsuperscript{77}Bury, p. 33.

théories du Progrès, on l'a, en partie, réalisé.\textsuperscript{79}

Bury\textsuperscript{80} pointed out in his study that there were two major factors which prepared the way for the possibility of the idea of Progress to develop. These were: (a) self-confidence was restored to human reason, and (b) life on this planet was recognized as possessing a value independent of hopes and fears for life beyond the grave.

The word "progress" itself began to take on the figurative meaning of "advance, growth, development"\textsuperscript{81} around the beginning of the seventeenth century (1603) in contrast to its original concrete usage to indicate a journey as, for example, its use in the title of John Bunyan's work, Pilgrims Progress.

The Intellectual Revolution of the 17th Century

The seventeenth century was a century of unprecedented growth in mathematics, growth not only in the sense of increased activity, but also growth of new fields; a growth which eclipsed the mathematics produced by ten centuries of Greek inspired mathematics.

The variety of new fields opened up in this brief period is impressive. The rise of algebra as a science

\textsuperscript{79}Delvaille, p. 120. Translation provided at the end of Chapter II.

\textsuperscript{80}Bury, pp. 29-30.

(because the use of literal coefficients permitted a measure of proof) as well as the vast expansion of its methods and theory, the beginnings of projective geometry and the theory of probability, analytic geometry, the function concept, and above all the calculus were major innovations, each destined to dwarf the one extensive accomplishment of the Greeks—Euclidean geometry.\textsuperscript{82}

It was also a century of conflict, an intellectual conflict between the tradition of the ancients and the new scientific thinking which had germinated in the previous century. This was a conflict of great consequence, as Delvaille stated:

\begin{quote}
Disons, pour conclure, que la Querelle des anciens et des modernes a une portée plus grande que celle d'une simple dispute des littérateurs.\textsuperscript{83}
\end{quote}

One of the major champions for this new scientific thinking was Sir Francis Bacon, about whom it was said, "...bears the same relationship to the movement under discussion as, for example, Karl Marx bears to the development of Communism..."\textsuperscript{84} His was a vision for the future. He attacked the spirit of reverence for the teachings of the ancients and strongly advocated that learning be based on observation. His position was diametrically opposed to Platonic epistemology where reality resides in Ideas:

\begin{quote}
\textsuperscript{82}Kline, p. 391.
\textsuperscript{83}Delvaille, p. 209. Translation provided at the end of Chapter II.
\end{quote}
Men must begin, he says, from the very foundations and effect a total reconstruction of all sciences. Especially must the mind be freed from every opinion or theory, and proceed straight to nature. He exorts men 'with unpossessed minds' and 'with minds washed clean from opinion to study it [nature] in purity and integrity.' Much, he claims, may be expected of men of well-purged minds, who apply themselves directly to experience and particulars.85

Francis Bacon's views on mathematics were influenced by Platonist ideas since he viewed mathematics as part of metaphysics86 and so mathematics was somewhat suspect within his scheme of empirical observation of nature. This position can be discerned even in his evaluation of the Copernican system. As Richard Jones stated:

The Copernican system he regarded as only a contrivance by which celestial phenomena could be explained mathematically, and not as necessarily being true in nature. In this case, as in others, he objects to a system which depends more upon reason than upon physical data.87

The roots for this particular view of mathematics appeared as early as the eleventh century within the Arabic tradition and may have influenced Bacon. Roshdi Rashed, in a study of Algebra in the eleventh century stated,

[Les algebristes arabes] ont tous admis que l'unité de l'objet algébrique est fondée dans la généralité des opérations et non plus dans celle des êtres mathématiques,

85 Jones, p. 49.
87 Jones, p. 51.
On the other hand, Descartes, started from the same basic assumption, namely, the assumption that the mind should be swept clean of all opinion, but arrived at a different conclusion concerning mathematics. He sought structure in nature: he started with the simple principle, "Cogito, ergo sum," upon which, by means of reasoning and clear ideas, he sought to construct a sound edifice. Unlike Bacon he was not wary of the structures of mathematics, but rather embraced these within his philosophy. Descartes stated:

I was not much troubled to inquire where I needed to begin, for I already knew that it was by the things which are simplest and easiest to know, and considering that among all those who have previously sought truth in the sciences, the mathematicians alone could find any demonstrations (that is to say, any certain and evident reasons) I did not doubt that it was by the same methods that they have carried out their investigations.

Descartes' views were more universally adopted, and it was during this period that mathematics began to lose its separate identity, so closely did it become allied to science. Professor Kline stated:

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89 Jones, p. 49.

The boundary between mathematics and science became blurred. Paradoxically, as science began to rely more and more upon mathematics to produce its physical conclusions, mathematics began to rely more and more upon scientific results to justify its own procedures.\(^91\)

Mathematical and scientific developments during the latter part of the seventeenth century were dominated principally by the ideas of two men, Gottfried Leibnitz and Isaac Newton. Their divergent views of metaphysics were a subject of debate during the period of the Enlightenment.

Leibnitz's rationalism attached itself to a unified universal conception of metaphysics.\(^92\) As Paul Hazard eloquently stated:

Leibnitz symbolized the counter-offensive of metaphysics. His part it was to remind us that when every conceivable malediction had been heaped upon it, it was to metaphysics we had to go if we would learn aught of the mystery of Being, and lay bare the ultimate secret.\(^93\)

His idea of the basic continuity of nature which can be studied through the structures of mathematics led him to state "Cum Deus calculat, fit mundus", i.e., As God calculates so the world is made,\(^94\) strengthening the meta-

\(^91\)Kline, p. 395.


\(^93\)Hazard, p. 304.

\(^94\)Kline, p. 219.
phor of God the mechanical clockmaker which Nicole Oresme had first introduced 300 years before.\textsuperscript{95} It also led him to search for the laws governing this continuous structure of nature, a search which led to his discovery of the calculus, along with Newton. Metaphysics was not of great concern on the part of Newton; for him the quest for knowledge became a quest for relationships and not one for ultimate realities. As Newton stated at the beginning of the Principia, "For I here design only to give a mathematical notion of these forces, without considering their physical causes and seats."\textsuperscript{96}

Ambitious as his goal was, to detach himself from metaphysics by postulating the "absolutes" of his system, this goal was not entirely realized. As E. W. Strong pointed out:

> When Newton postulates absolute, true, and mathematical space, time, and motion, he introduces principles which are not evinced by experiments. Such principles, so far as Newton believes they express a real order of nature, are thereby metaphysical in the sense of being unverified assumptions. They are constructs...not themselves inductively derived and thereby empirically grounded.\textsuperscript{97}

Although the axiomatic foundations for the new mathematics, which was being developed or created, was lacking,

\textsuperscript{95}Lynn White, Jr., "Science and the Sense of Self," Daedalus, Spring 1978, p. 58.

\textsuperscript{96}Cited in Kline, p. 335.

and although the philosophic grounds were in dispute, a spirit of pragmatism justified it. Its utility had allowed such understanding in mechanics that other scholars tried to apply mathematics to other areas of life. This desire to find mathematical relationships in all areas of life became so pervasive that at the end of the century, books with titles such as the following were being published: Political Arithmetic by Sir William Perry (1690) and Mathematical Principles of Christian Theology by John Craig (1699). This search for the rational order was to be singularly dominant during the following century.

In the midst of this intellectual activity very few changes occurred in the educational institutions. Academies of science were formed to provide a place of discussion and dissemination of these new ideas. John Webster, in his study of the status of education, stated:

It is true that the English Universities became scientific centers in the decade of the 1650's, but it ...maintained the authority of the ancients in academic lecturing. Surely it was widely held that the universities were hopelessly conservative. Bacon had stated that in the universities 'everything is found adverse to the progress of science'...

The institutions that a modern person would expect to play the major role in the creation and dissemination of knowledge—the universities—were ineffective.

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98 Delvaille, p. 221.

They were conservative and dogmatic, controlled by the official religions of the respective countries and very slow to incorporate new knowledge.\textsuperscript{100}

At the pre-university level the situation does not appear to have changed from the previous century except perhaps in schools within commercial districts where demand existed for skills in arithmetic;\textsuperscript{101} in general, arithmetic was considered a subject for private lessons perhaps in the manner that piano lessons are considered today. The Utrecht instructions for country schools (1654) states only that "they \[school teachers\] shall also teach the youth to understand numbers of the chapters of the Holy Scriptures and the Psalms."\textsuperscript{102} However, teachers who were sent out by the West India Company to New Netherland were prescribed to teach "reading, writing, ciphering, and arithmetic."\textsuperscript{103} Kilpatrick stated that, "while the outlying Dutch villages, except commercially minded Albany, offer only reading and writing, New Amsterdam (later New York), so far as we can say, always included arithmetic in its curriculum."\textsuperscript{104}

\textsuperscript{100}Kline, p. 397.


\textsuperscript{102}Kilpatrick, p. 33.

\textsuperscript{103}Kilpatrick, p. 34.

\textsuperscript{104}Kilpatrick, p. 221.
There were the stirrings of new thinking in education. Comenius was one of the first educational thinkers to legislate especially for young children. Throughout the Middle Ages and the Renaissance periods, it was generally taken for granted that the child was a *hominculus* (a little man), hence education had aimed at assisting him to mimic adult ways—he studied the same subjects as his elders; he learned to read and spell Latin words, and formal grammar was forced on him by memory. 105

Kilpatrick claimed that the first schools which can truly be called public schools were established in the United States.

That the Dutch schools of America are properly called public seems unquestionable. They were open to all the children, were controlled by the duly constituted civil authorities, and were both housed and supported by the public moneys. 106

Concern for the professional preparation of teachers was voiced, and in France the first normal school was established by Jean Baptiste de la Salle at Rheims in 1685 for the preparation of men in the order he founded, the Brothers of the Christian Schools. 107 It was to be, however, over a century before state supported teacher training schools

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107 Johnson, pp. 9-10.
were established.  

The roots of the idea of Progress were beginning to take hold; it was implicit in the very title of Sir Bacon's book *Novum Organum*.

The belief that there is no closed door to human progress, that learning can grow and man's condition improve, if not infinitely, at least to some far-off felicity, together with a feeling of obligation to assist the process, was responsible for much of the enthusiasm of the reformers. Like Bacon they turned from the past and faced the future.  

The very word progress was beginning to change in meaning. Prior to about 1603 the word indicated a journey (e.g., the title of the book *Pilgrim's Progress*), but now took on the meaning of "advance, growth, development".  

Delvaille stated that, "Pour eux, Progrès signifiera le perfectionnement dont l'intelligence est susceptible dans ses acquisitions... la séparation d'avec les anciens."  

Professor Bury stated that three conditions are necessary for the development of the idea of Progress and that these were fulfilled in the seventeenth century.  

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108 Johnson, p. 17.  
111 Delvaille, p. 151. Translation provided at the end of Chapter II.  
112 Bury, p. 66.
These are:

Firstly, the idea that the Greeks and Romans had achieved a level of civilization which posterity could never hope to reach had to be destroyed. This had been accomplished as has been shown.

Secondly, the value of mundane life and of the subservience of knowledge to human needs had to be acknowledged. The secular spirit of the Renaissance and the works of Bacon accomplished this. It is the root of modern utilitarianism.

Thirdly, science had to be placed on an acceptable foundation. The principle of the uniformity and invariability of the laws of nature as established by Descartes provided this foundation.

The development of the idea of Progress was important for the development of science since belief in Progress is one of the characteristics of scientific thought, according to a study of the sociological roots of science by Edgar Zilsel. He elaborated in his study on this idea where he stated that the idea of Progress involves:

(1) the insight that scientific knowledge is brought about step by step through contributions of generations of explorers building upon and gradually amending the findings of their predecessors;
(2) the belief that this process is never completed;
(3) the conviction that contribution to this development, either for its own sake or for the public benefit, constitutes the very aim of the true scientist.113

The last point which Zilsel made concerning the idea of Progress was further emphasised by Jones, who brought to light the association of social needs and of humanitarianism with the science of this period; it was emphasized that the needs of the poor were one of the first to be provided for. 114 There may have been close connections between these expressed needs of the poor and the emergent concern for public education. Jones, paraphrasing Hartlib, stated that, "The insight into social needs...is clearly borne out by the many instrumentalities devised by our world to meet these needs." 115

The Age of Enlightenment (The Eighteenth Century)

The success of Newton's search for the rational order of the universe spurred investigations into almost every area of life. This was a time when no problem seemed insoluble, when it seemed that all of nature could be organized and controlled; it was the age of discovery by reason born out of Descartes' skepticism: an age where skepticism is viewed as a virtue of intelligence. The following entry appears in l'Encyclopédie under the heading "critique" which is signed by Marmontel:

La crédulité est le partage des ignorants; l'incredulité décidée, celui des demi-savants; la doute

114 Jones, p. 151.

115 Jones, p. 151.
methodique, celui des sages.\textsuperscript{116}

Since they wanted to take nothing on faith, all of existence came under renewed examination; as Henry Steele Commager, in the opening pages of his recent book concerned with the Enlightenment, stated:

What remarkable men they were...There was a prodigality about them; they recognized no bounds to their curiosity, no barriers to their thought, no limits to their activities or, for that matter, to their authority.\textsuperscript{117}

The same thought was presented by Hazard in the following statement:

They were men of mettle and they dared high things. They were obsessed to a degree which we of today can hardly imagine by the great, fundamental problems of life.\textsuperscript{118}

What was the major inspiration for their "obsession"? Their faith had changed drastically from that of Saint Augustine; they were putatively men of no faith, but their faith, although now well articulated at the beginning of the century, lay in the idea of Progress as Bury clearly indicated, "Belief in it [Progress] is an act of faith."\textsuperscript{119} Further, this faith in Progress was the major inspiration

\textsuperscript{116} Albert Soboul, ed., l'Encyclopédie; textes choisis (Paris: Éditions Sociales, 1962). Translation provided at end of Chapter II.

\textsuperscript{117} Henry Steele Commager, The Empire of Reason (Garden City, N.Y.: Anchor Press, 1977), p. 3.


\textsuperscript{119} Bury, p. 4.
of the eighteenth century, according to Delvaille, who wrote the following:

L'idée de Progrès sera et l'inspiratrice du XVIIIe siècle, et le résultat de son activité. Elle résumerai tous ses efforts; elle sera le centre autour duquel se livrera la grande bataille d'ou sortira la société moderne.120

It was during the eighteenth century that theoretical considerations of this increasingly important idea began to take form in the writings principally of the Abbé de Saint-Pierre, Fontenelle, Montesquieu, Voltaire, Turgot, Chastellux, Condorcet and Saint-Simon. The detailed delineations of the theoretical foundations for the idea of Progress during this century form the major portion of the studies presented by Bury121 and by Delvaille.122 As the implications of the idea of Progress were explored, and as these implications were accepted as endemic to the human race, the idea of Progress lost its purely theoretical status while deep sentiments for reform of existing social and political institutions were voiced. Delvaille expressed this fact when he stated, "Pour eux, Progrès ne signifiera pas seulement: marche en avant, mais surtout: réformes pour l'avenir."123

120 Delvaille, p. 233. Translation provided at the end of Chapter II.
121 Bury, op. cit.
122 Delvaille, op. cit.
123 Delvaille, p. 233. Translation provided at the end of Chapter II.
The scientific view applied itself to that measure of past and future, time. Saffin stated,

Towards the end of the seventeenth century and at the beginning of the eighteenth, the hitherto unimportant concept of time impinged itself on human consciousness, and the chain of being had to be adjusted to include the link of progress. 124

Although the theoreticians of the idea of Progress were Europeans, the response for reform had its realization in North America rather than on the European continent, according to Henry Steele Commager, who devoted the entire text of his recent study on the period of the Enlightenment to this thesis. In the "Preface" to this work he wrote:

The thesis of this book can be stated quite simply: The Old World imagined, invented, and formulated the Enlightenment, the New World--certainly the Anglo-American part of it--realized it and fulfilled it. 125

Education was one of the areas of social reforms which was considered to be of great importance to many of the philosophes; in fact Peter Gay referred to the enlightenment as "in essence pedagogic." 126

They found the guarantee they required, not in an induction from the past experience of the race, but in an a priori theory: the indefinite malleability of


125 Commager, op. cit., p. xi.

human nature by education and institutions.\footnote{Bury, op. cit., p. 165.}

Helvetius proposed, in \textit{De l'esprit} (1758), that intellectual and moral inequalities between men arise entirely from differences in education and social circumstances.\footnote{Bury, op. cit., p. 166.} Although recognized as the source of the present day controversy referred to as the nature-nurture controversy, this thesis was influential in opening thought to the possibility of general public education for the purpose of achieving progress of mankind. This was an idea which had been foreshadowed by Comenius, as noted above. In 1763 there appeared a pamphlet entitled \textit{De l'Education Publique}, anonymously written, but generally ascribed to Diderot,\footnote{Barnard, op. cit., p. 232.} which promotes the thesis of general education. This thesis was not welcomed by all philosophes, some of whom saw in such a proposal a possible deterioration of the societal support provided by an uneducated working-class.

Voltaire in a letter to the author of the \textit{Essai} (La Chalotais) thanks him for having forbidden studies to the working-classes.\footnote{Barnard, op. cit., p. 235.}

This was a sentiment less snobbish than it at first
appears today. A general sense of despair existed toward "the general wretchedness, illiteracy, and brutishness of the poor, which appeared by and large incurable." The strong sense of hierarchy within society was generally accepted; it was axiomatic, accepted without analysis nor apology.

This was the condition the educator confronted: the vulgar were, and would doubtless forever be, prey to passion and superstition; reason was beyond them. Accordingly, Locke confined his educational program—the study of Latin and other ornaments of gracious civilization—to gentlemen, and recommended that the children of the poor be sent to special 'working schools' where they would learn 'spinning or knitting, or some other part of woolen manufacture' and such edifying matters as 'some sense of religion'.

The proponents of general public education countered these objections with the concept of the possible emergence of a meritocricy, "an aristocracy of education emerging from a democracy of opportunity." The potential for actualization of these ideas in America existed principally because there was no established aristocracy.

The rift which existed in Europe, the rift between theory and practice and to some extent the difference, according to Commager, between the Enlightenment in Europe and America, was eloquently presented by Peter Gay in the

131 Gay, op. cit., p. 517.
132 Gay, op. cit., p. 518.
133 Gay, p. 520.
following account:

Progress itself called for new victims, and the very improvements that lightened the burdens of many intensified the sufferings of others: for the majority (to speak with Samuel Johnson) the eighteenth century remained a time in which there was little to be enjoyed and much to be endured. The new style of thought was in the main reserved to the well-born, the articulate, and the lucky: the rural and the urban masses had little share in the new dispensation.\textsuperscript{134}

Although severely criticized, almost no change occurred during this period within the educational establishment. A basic conservatism was evident especially in schools. Peter Gay stated that throughout the Enlightenment, in the midst of tremendous changes in the intellectual perspectives which were occurring, "It was not schools that changed, but the times."\textsuperscript{135} This observation of fundamental conservatism was also noted by Rashdall, the scholar of medieval universities, in a reflection he shared while comparing education during the fourteenth and eighteenth centuries; he wrote:

It is surprising how little the intellectual superiority of the eighteenth century over the fourteenth impressed itself upon the course of ordinary school and university education,...; but our intellectual advance since the medieval period had less to do with the improvement in the substance of the method of education than the academic world complacently imagined.

\textsuperscript{134} Gay, p. 4.

\textsuperscript{135} Gay, p. 504.
It was in the main what he picked up out of school and lecture-room that differentiated the educated man of the eighteenth century from the educated man of the fourteenth.136 (Italics mine.)

The subjects generally taught at the high-school level, as well as some suggestions for change, can be observed in l'Encyclopédie under the entry "collège" signed by the mathematician, D'Alembert. l'Encyclopédie for men of the eighteenth century has been likened to the cathedral for men of the Middle Ages as the collective chef-d'œuvres of the vision of the ages.137 D'Alembert stated that there were five major areas of education; they are: humanities, rhetoric, philosophy, morals and religion. Later, he made a plea for greater concern to the living languages with a concomitant decreased emphasis on Latin and Greek; along with changes he sought are the early introduction of geometry.138

The introduction of competitive examinations within the educational establishment was one innovation of the eighteenth century which was to have social implications in later periods. This introduction appeared to be a result

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of admiration for the Chinese civil service examination used in the Mandrinate. 139

Among those who did call for a radical change in education was Jean-Jacques Rousseau. His was a call for change not in the direction indicated by the theorists of the idea of Progress, not in the direction of an increased scientific approach to what was taught; it was a reaction to the education of the preciosity of eighteenth century man; it was a call in the direction of a return to nature, a direction which directly confronted the concept of Progress. One of his principal works on education, Emile, did not begin with inductive arguments based on observations; it was principally written with a deductive construction based upon the stoic premise of nature's basic goodness. 140 His thoughts on social evolution were antithetical to the idea of Progress; Benjamin Schwartz recognized a retrogression in his analysis of Rousseau when he stated, "Rousseau himself acknowledges a kind of cumulate progress of the 'science and arts' which makes man's situation progressively worse." 141


Bury appears to concur in this judgment, and yet recognizes Rousseau as a major influence upon those who actively sought the promises of the idea of Progress. He stated:

...the most powerful single force for the Revolution of 1789 was Rousseau, who, though he denied Progress and blasphemed civilization, had promulgated the doctrine of the sovereignty of the people, giving it an attractive appearance of mathematical precision; and to this doctrine the revolutionaries attached their optimistic hopes.\footnote{142}{Bury, p. 204.}

Because of this influence which Rousseau exerted, Delvaille, in his study, was moved to place Rousseau among the devoted adherents of the idea, despite Rousseau's own declamations of Progress. He stated:

Il y a entre ces deux hommes [Voltaire et Rousseau] une différence de caractère, de jugement d'aspirations; l'un était épris de modernisme, l'autre, au moins en apparence, semblait tourner le dos a son siècle et souhaiter un recul; ils ont cependant, l'un et l'autre, prépare l'époque moderne. Voltaire et Rousseau doivent être considérés comme des partisans du Progrès;...\footnote{143}{Delvaille, p. 386. Translation provided at the end of Chapter II.} (Italics mine.)

Rousseau's call for reform, though often cast as a call to primitivism, was not feral at base; it was more a call for attention to the sensate life of children and an abatement or total abandonment of preciosity in learning. Rousseau has been interpreted as saying that,

...the physical sciences, like mathematics, physics, and astronomy, are human contrivances which, if solidly
grounded on the pure experience of the senses, extend their range and protect them against the errors of imagination.144

Another indication of Rousseau's thought, one which Barnard characterized as "most important," can be found in his definition of Nature:

Nature as defined by him is 'primitive dispositions, including our sensations and feelings of pleasure and pain, together with the judgments founded on these.'145

The belief in the mathematical construct of the universe, the legacy of Newton and Leibnitz, lost its theological hue among mathematicians and scientists of the eighteenth century. Increased successes in mathematical methods applied to nature strengthened the bond of mathematics with the sciences. The problems of the foundations of mathematics were not considered or were considered to be of little importance; the fact that mathematics continued to predict nature, the fact that it was successful, was all that mattered. Increasingly difficult mathematics arose and yielded to solution. Kline referred to the eighteenth century as the "century of the ingenious."146 In reference to their lack of concern for the foundations of their mathematical work, Kline stated that, "They were so intoxicated with their

145 Barnard, p. 256.
146 Kline, p. 614.
physical successes that most often they were indifferent to the missing rigor."147 One problem in geometry, which proved to have great consequences later, was the "parallel axiom" problem. Euclid's axiom on parallel lines did not appear to be as intuitive as his other axioms; the problem was either to replace it with a more intuitively acceptable one or to prove it from the other axioms. Some notable work on this problem was done by G. Saccheri (1667-1733) and by G. Klugel (1739-1812). D'Alembert said of this problem that it was "the scandal of the elements of geometry."148 Although it was considered a "scandal", it was not considered a major crisis. In fact, "There was a feeling that algebra is to be preferred for discovery, geometry for demonstration."149 Demonstration here had a pejorative tone in its use; it slowed down the progress of new knowledge, the discovery of new laws of nature. In fact, ...

...the complaint in the earlier eighteenth century in France was that traditional geometry was too concerned with logical niceties at the expense of practical motivation.149

The arena for change during the Enlightenment, as noted above, was America. Meaningful change was more apparent in

147 Kline, p. 618.

148 Kline, p. 867.

the political life of the new nation than in the educational life of its children. The influence of mathematics on the intellectual life of the leaders of America was apparent in the very form of the Declaration of Independence, which begins by listing its axioms or its "self-evident" truths. The youthful United States was the political legacy and promise of the Enlightenment. The new nation was not concerned so much with theories as with practical devices to implement these theories. The Puritans had brought with them strong religious sentiments which permeated all their social institutions; their English background also had ensured similarities to educational institutions of that country.

The purpose of the colonial schools in America, like their counterparts in Europe, was based in the church and religion...

...The real concern of the colonists was that all children learn to read in order to be able to read the catechism and the Bible and know the will of God. 150

Three conceptions of education emerged in early American education. These were: (a) the parochial schools of Protestant Pennsylvania and Catholic Maryland; (b) public education intended for orphans and poor under public care, and (c) Latin grammar schools and private schools for children of higher classes. 151


151 Paulos, p. 13.
In the previous century the concept of free public education had been given form by the Puritans, but the relationship with religion was intended to be the major influence upon curriculum.  

Mathematics education was not considered beyond its usefulness in the practical applications such as the mechanical arts or navigation. For the ordinary citizen this amounted to a bare knowledge of counting. The Puritan, John Wallis, had stated in 1697 that,

Mathematics...were scarce looked upon as academical studies, but rather mechanical; as the business of traders, merchants, seamen, carpenters, surveyors of lands, or the like ...  

Epistemologically, the Puritans differed sharply from other sects such as Calvinism and Anglicanism in their use of rationalism in religion; they "thought that rationalism was subordinated to faith and empiricism." It was this attachment to empiricism, which their faith fostered by demanding detailed, systematic, methodic and diligent analysis of their conversion experiences to determine the validity of such experiences, which made Puritanism less dogmatic and

152 Paulos, p. 16.


154 Greaves, p. 5.

155 Greaves, p. 64.
more open to advances in science. Richard Greaves stated that,

If he was to make progress in the arts and sciences the Puritan had to divest himself of the deadly assumption that he already possessed the truth, and that the task at hand was confined to its explanations.\textsuperscript{156}

The key to his liberation was his spirit of empiricism, which led him to applications of mathematics. The practical applications gained in importance in the United States during the eighteenth century, so much so that by 1789, both Massachusetts and New Hampshire required arithmetic in their schools.\textsuperscript{157}

Emphasis in the schools was literary, providing a common education in reading, writing, and the catechism. They were more institutions of indoctrination than institutions where individual thought was fostered. This seemed reasonable in view of the structure of the society where consensus was always sought. Indeed, even the institution of the "town meeting" in New England, often thought of as a unique forum for free expression, was really meant to achieve consensus.

Differences were generally called unhappy, and the primacy of peace rendered discord, argument, and parties unacceptable. The meetings were not presented with a choice of competing interests or opinions. They met to reassert the unity of townsmen...Neither the defense of

\textsuperscript{156} Greaves, p. 39.

private interests nor the projection of personal ideas was welcome.158

The influence of English traditions on American educational institutions began to wane during the eighteenth century so that by 1750, Paulos stated, "...there was a marked abandonment of English ideas, schools and methods of instructions."159 After the War for Independence the idea of public supported education suffered a decline of interest except for a few national leaders, notably among them John Adams, who voiced the opinion that public education should be supported and that instead of serving religion this education should be given to instill citizenship and a feeling of nationalism. This movement away from religious purposes in education was connected with a utilitarian spirit which was dominant at the end of the century. Robert Middlekauff stated:

...America was young and still to be built; and moderns dedicated to the vision of a new, unique America could not contain their impatience with anything that did not contribute to the building.

...The trouble with classical learning, they liked to say, was that it was useful only to the clergy. And moderns tended to be contemptuous of the clergy, if not actually anticlerical.160

159 Paulos, p. 17.
The idea of public education did not gain much support during this period. Paulos stated that "in fact, there was no real educational consciousness in America before 1820--only five states had good provisions for schooling."\(^{161}\)

The training of teachers was non-existent since teaching was not looked upon as a permanent profession.

The best teachers in those days were college students or college graduates who engaged in teaching as a stepping-stone to something better.\(^ {162}\)

This evaluation of the general quality of education was presented ninety years after the turn of the century, in 1890, by Florian Cajori, an historian of mathematics and mathematics education. He further stated that although the best teachers were the young college students, mentioned above, they were not the representative ones. He wrote,

The representative school-masters of by-gone times were the itinerant school-masters. They were mostly foreigners. Their qualifications seemed to be the inability to earn anything in any other way. They were generally without families and had no fixed residence; they kept school first in one place and then in another, and wandered about homeless. Many were given to drinking and gambling. As a class, their knowledge was limited to the merest elements.\(^ {163}\)

Since college students were trained in the classics, and since their own mathematical preparation was weak, the

\(^{161}\)Paulos, p. 18.


\(^{163}\)Cajori, p. 52.
situation concerning mathematics education merely reinforced itself. For example, the following is an account of the mathematics required for entrance into Harvard:

In 1802 the standard for admission to Harvard College was raised. In mathematics a knowledge of Arithmetic to the 'Rule of Three' was required. Thus, in 1803, for the first time had it become necessary, according to regulations, for a boy to know something about arithmetic before he could enter Harvard. We surmise, however, that the requirements in arithmetic were very light, for we know from the diary of a student in the Freshman class in 1807 that arithmetic continued to be taught during the first year at college.  

The Nineteenth Century—Age of Progress

Prior to the nineteenth century, the idea of Progress had often been discussed among intellectuals, as has already been noted above. Discussions had taken place concerning the possibility or impossibility of, among others, human progress, intellectual progress, social progress, educational progress, political progress, and moral progress. Progress in each case had meant, generally, the inevitable betterment of mankind in those areas. But what was to be the ultimate extrapolation of better was never made clear; how was this system of betterment working again never was clearly stated.

Hitherto it [Progress] had been a vague optimistic doctrine which encouraged the idealism of reformers and revolutionaries, but could not guide them. It had waited like a handmaid on the abstractions of Nature and Reason; it hardly realized an independent life.

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164 Cajori, p. 60.
The time had come for systematic attempts to probe its meaning and definitely to ascertain the direction in which humanity is moving.\footnote{165}{Bury, p. 278.}

This systematic attempt to probe its meaning took two distinct routes: one based on the ideas of Karl Marx, the other based on the ideas of Charles Darwin; one of constructive idealists and socialists, the other corresponding to the political theory of liberalism.\footnote{166}{Bury, p. 236.}

Since the theory of Progress as developed by Marx had less influence in America, no further discussion of this route will be traced in this study other than to note that within Marx's theory, Progress is given a terminus; goals of development are achieved by arrangement of society and "there is no stimulus to cause further changes, and the dynamic character of history disappears."\footnote{167}{Bury, p. 236.}

In the development influenced by the dynamism of evolution is to be found the theory of Progress which Americans accepted. As Arthur Ekirch pointed out:

In America the successful development of a seemingly limitless expanse of land and resources made the people peculiarly susceptible to a belief in so dynamic an idea as that of progress..., the impact of the American environment made it inevitable that the American people should interpret the idea of progress in
the light of their own interests and experience.168

The theory of evolution was seen by the theorists of the idea of Progress as the mechanism by which Progress was achieved. The search for this mechanism had continued from the eighteenth century so that by mid-nineteenth century, when the Origin of Species was published, several modifications had become accepted. One of these, a reaction to the failure of the French Revolution to fulfill its visionary hopes, had been proposed by Saint-Simon and accepted by Comte, both of whom were deeply concerned with the idea of Progress.

The eighteenth century thinkers had left Progress a mere hypothesis based on a very insufficient induction; their successors sought to lift it to the rank of a scientific hypothesis, by discovering a social law as valid as the physical law of gravitation. This was the object both of Saint-Simon and of Comte!169

One of these new ideas was the basic inequality of men. "Complete equality is absurd; inequality, based on merit, is reasonable and necessary."170 Bury stated that the fullest explanation of Saint-Simon's doctrine was given by one of his disciples, Bazard:

The human race is conceived as a collective being which unfolds its nature in the course of generations,


169 Bury, p. 284.

170 Bury, p. 286.
according to a law—the law of Progress—which may be called a physiological law of the human species...\(^{171}\)

The vision of the progress of mankind was made organic by Saint-Simon in contrast to the more myopic views hitherto expressed. Frank Manuel in his study stated:

Saint-Simonian man had infinite potentialities in all directions; he could at one and the same time progress in power over nature, in expansive feeling, and in the endless accumulation of knowledge.

...the good social structure should be organismic, a harmony of complex, and different, parts. The organic society, in contrast to the atomist egalitarian society which functions like inanimate clockwork, then cries out for a 'vitalist' force...\(^{172}\)

The vitalist force, the soul of humanity as postulated by the Saint-Simonians, was love, a conclusion in keeping with the spirit of Romanticism of the period.

Saint-Simon's most notable successor was Auguste Comte. Bury said of him, "Auguste Comte did more than any preceding thinker to establish the idea of Progress as a luminary which could not escape men's vision."\(^{173}\)

Auguste Comte's life was one of polarities between deep sentiments and rigid intellectual precision. His early writings indicate that he "taught his contemporaries...that all they knew and needed to know was the exact re-

\(^{171}\)Bury, p. 287.


\(^{173}\)Bury, p. 290.
lations between the physical objects to be found in the physical world."\(^{174}\) A Platonic love affair with a married woman, who died within a year of his acquaintance with her, had a profound effect on his life. He was "transfigured by his glimpse of a new world of feeling."\(^{175}\)

He applied his analytic mind in the same direction as Saint-Simon, but in a manner so successful that Bury stated:

The brilliant suggestions of Saint-Simon, the writings of Bazard and Enfantin, the vagaries of Fourier, might be dismissed as curious rather than serious propositions, but the massive system wrought out by Comte's speculative genius--his organic scheme of human knowledge, his elaborate analysis of history, his new science of sociology--was a great fact with which European thought was forced to reckon.\(^{176}\)

Similar praise for Comte's accomplishments are given by Manuel, who stated:

Comte raised the idea of progress to a new level when, in addition to technological, scientific, intellectual, and moral achievement which his predecessors had recognized, he envisaged a progressive growth of consciousness.\(^{177}\)

The mental strain under which Comte existed and the mental imbalance from which he suffered are eloquently stated by Manuel. He wrote:


\(^{175}\)Barzun, p. 355.

\(^{176}\)Bury, p. 290.

\(^{177}\)Manuel, p. 111.
The French idea of perfectibility received its final nineteenth-century shape from Auguste Comte, one of the most tragic figures in the history of thought—a psychotic genius who built a grand architectonic structure around the idea of progress during his lucid intervals between bouts with the demons of insanity. His is by far the most magnificent creation of the French school, its climax...\textsuperscript{178}

In view of the citation made concerning the influence which Helvetius had exerted on the thinking towards education in the course of the previous century, the following observation made by Bury is presented.

Of particular importance was the great fallacy, which Helvetius propagated that human intellects are equal....For it supported the dogmas of popular sovrance and social equality, and justified the principle of the right of private judgement.

These three principles—popular sovranity, equality, and what he calls the right of free examination—are in Comte's eyes vicious and anarchical.\textsuperscript{179}

Charles Darwin's *Origin of Species* appeared in 1859. To the theoretician of the idea of Progress this publication provided the mechanism by which Progress operated. As Bury pointed out, since the process of evolution is "a neutral scientific conception, compatible either with optimism or pessimism,"\textsuperscript{180} two connections had to be made: one, that social life obeys the same general laws of evolution as nature, and secondly that the process involves the movement

\textsuperscript{178} Manuel, p. 109.

\textsuperscript{179} Bury, p. 297.

\textsuperscript{180} Bury, p. 335.
of man in a desirable direction, that is, an increase in happiness. This connection was made by Darwin himself, who concluded his treatise with the following:

And as natural selection works solely by and for the good of each being, all corporeal and mental environments will tend to progress towards perfection.\(^\text{181}\)

By the seventies and eighties of the nineteenth century the idea of Progress was becoming an article of faith and so by 1920 or so Bury stated that indefinite Progress was generally assumed as an axiom.\(^\text{182}\)

America was ripe for the acceptance of the idea of Progress, as noted above, but the idea is more difficult to follow since, as Ekirch pointed out:

...the idea of progress was sufficiently vague in its meaning and congenial in its portent to be susceptible of use in justifying even contradictory tendencies in the age of which it was a dominant, if not the most widely cherished, idea.\(^\text{183}\)

Not only was the idea of Progress used to justify contradictory tendencies, but the two separate routes which the idea of Progress had taken, namely, socialism and liberalism, were enmeshed within American rhetoric.\(^\text{184}\) One innovation to the idea of Progress accepted in America is presented in the study by Ekirch, who stated:


\(^{182}\) Bury, pp. 346-348.

\(^{183}\) Ekirch, p. 37.

\(^{184}\) Ekirch, passim.
However, in the United States....the American people felt that, although progress was indeed certain, it could nevertheless be impeded or accelerated by human will and effort.  

This fact is further illustrated by the following lines from the 'hymn' of the Franklin Club, an English club formed to honor Benjamin Franklin:

Degraded churls, why listless stand?  
The work of progress needs a hand.

Except for experiments in Utopian societies, the dominant strain of the idea of Progress in America was liberalism. This dominant strain received a severe indictment by the noted scholar of American studies, Henry Alonzo Myers. He wrote:

The gospel of progress is a booby trap, a delayed-action bomb, in the long run more dangerous to democratic society than the unadorned doctrine of the survival of the fittest.

...Not by chance, but by a tragic reversal of intention, the nineteenth-century faith in progress, popularized by Hegelianism, Darwinism, and Marxism, has been a principal cause of war and disaster.

...The idea of progress is a baited hook. The bait is the promise of unearned happiness, of a world freed by magic from hardship and suffering; the hook is the doctrine of inequality.

The doctrine of Progress was used in the nineteenth century to justify slavery, the activities of the robber

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185 Ekirch, p. 12.


188 Ekirch, p. 232.
barons, oppression and near extinction of the native Indian population, among other causes. Ekirch, on the other hand, stated that such rationalizations were not dominant. He said:

With the presence of a vast reservoir of lands in the West, and with the tremendous power made available through technological advancement, there seemed to be every reason for faith in the physical progress of America, and the idea was therefore most widely considered in its material, not its philosophic, aspects.

Within the context of a society where the idea of Progress permeated almost all thinking, education was sure to take a central role; this is illustrated in the following:

For those intellectual or religious persons who caviled at the ruthlessness of American expansion and exploitation, its defenders could point to the evidence of cultural and educational improvement.

It is significant that the major innovations in education appeared during the latter part of the century, after the acceptance of the idea of Progress. Mid-nineteenth century America had experienced drastic changes: the industrialization changed people's lives; large urban centers developed; massive immigrations of Europeans occurred; and the Civil War was fought. In the early part of the century, education

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189 Myers, p. 116.


191 Ekirch, p. 267.

192 Ekirch, p. 267.
had changed little from the eighteenth century. The moni-
torial system\textsuperscript{193} or 'pupil-teacher-center' system of educa-
tion was introduced although, as has been shown, this was
really an old idea. The extent of its use, however, is not
historically clear. In mathematics education, the early part
of the century was referred to by Florian Cajori as the
period of English influence. The curriculum was principally
determined by legislation or textbook writers,\textsuperscript{194} and the
textbook writers of mathematics were strongly influenced by
English texts.\textsuperscript{195} The quality of these texts was severely
criticized by Cajori, who wrote:

The arithmetics of this time were little more than
Pandora's boxes of ill-formed rules to be committed to
memory. Reasoning was exiled from the realm of arithme-
tic, and memory was made to rule supreme. A science
chiefly intended to cultivate the understanding was
offered to the exercise merely of memory.\textsuperscript{196}

The reason offered by Cajori for this state of affairs
was that arithmetic had been diverted for commercial pur-
poses alone. He concluded with a colorful Biblical analogy:

Thus the sins of the early English pedagogues were
visited upon the children in England and America unto
the third and fourth generation.\textsuperscript{197}

Even in college texts, although proofs were offered,
they were written into footnotes, where "They could be

\textsuperscript{193} Johnson, pp. 19-21.
\textsuperscript{194} Paulos, p. 89.
\textsuperscript{195} Cajori, p. 45.
\textsuperscript{196} Cajori, p. 49.
\textsuperscript{197} Cajori, p. 50.
taken or omitted by teachers and pupils at pleasure, and were generally omitted."^{198}

Mathematics was studied only where it had applications; the developments taking place on the continent did not have an impact in the United States. This is brought out by the following anecdote reported by Cajori:

About 1816 a friend of Comte in this country warned that French mathematician and philosopher against the purely practical spirit that prevailed in this new country, and against coming here, by saying: 'If Lagrange were to come to the United States he could only earn his livelihood by turning surveyor.'^{199}

The idea of Progress does not appear to have been consciously a part of mathematics education during this period. Its influence is unclear unless one adopts the perspective of the French observer of democracy in America, Alexis de Tocqueville, who observed that this idea exercised an influence, "even on men who, living entirely for the purposes of action and not of thought, seem to conform their actions to it, without knowing anything about it."^{200}

In the latter part of the nineteenth century, as the idea of Progress in its Darwinian form gained ascendancy, great changes occurred in American education. The noted historian of Progress during this period, Arthur Ekirch, stated that:

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^{198} Cajori, p. 56.
^{199} Cajori, p. 94.
^{200} Alexis de Tocqueville, Democracy in America cited in Ekirch, p. 23.
No better example of the practical effect of the idea of progress could be found than in this tremendous growth of American education. In a democratic, progressive nation an educated populace was considered to be of vital importance.  

An educated populace had always been advocated as a necessity for the republic in order to secure the proper selection of its leaders; the real debate was centered about the question of educating everyone or developing an educated elite class within the republic. Thomas Jefferson, one of the strong proponents for the position of public education, has said, "There are two subjects, indeed, which I shall claim a right to as long as I breathe, the public education and the subdivision of counties into wards. I consider the continuance of republican government as absolutely hanging on these two hooks." Education was considered as basic to progress:

Education, they agreed [early formulators of educational plans], lies at the root of progress. Without schooling, the citizen's well-being suffers, and so does that of the nation. Education, they went on, is not only a prerequisite of the general welfare but also a child's natural and inalienable right. To ensure the possibility of its attainment, the republic needs to invest itself with a new education, one which will not only forward universal enlightenment and progress, but also stimulate leadership.

In his study of the meaning of Progress in progressive

201 Ekirch, p. 195.
education, Lawrence Thomas also noted a change in the concept of Progress during the mid-century period. He saw the change as one principally of form of expression; prior to mid-nineteenth century the forms used to express the idea of Progress were still "classical forms of changeless patterns." He perceived the work of Darwin as casting out these forms, and the idea of Progress, as developed by Dewey, was signified by the following:

1) The novel is genuinely new, not merely the revelation of an antecedently complete and perfected reality.
2) The proper ends of life do not exist timelessly, waiting to be discovered by a priesthood, but must be constructed out of the present circumstances by everyone involved.
3) All values and ends have an instrumental quality, and no end is fixed or final.
4) Progress is contingent, having no general formula, and no one can be sure that this dynamic world is tending in any certain or pre-established direction.
5) In short, progress is experimental, improving the adaptation of human living to changing conditions.

Various factors converged on education during the mid-century period. Industrialization helped to produce large urban centers where education was looked upon as an extension of the factory, so that urban school structures even resembled factories, and by the end of the century the following was considered reasonable:

205 Thomas, p. 388.
Our schools are, in a sense, factories in which the raw materials (children) are to be shaped and fashioned into products to meet various demands of life.\textsuperscript{206}

This statement was made by an extensively published author on education, Ellwood P. Cubberley. His writing also reflected the racist attitude, mentioned above, which developed during this period of massive immigration;\textsuperscript{207} over one million immigrants entered the United States between 1845 and 1865. For another example of this influence, a naturalized citizen, Carl Schurz, in a speech entitled "True Americanism" stated that the Anglo-Saxon spirit had been the locomotive of progress and that the locomotive needed a train to follow it; these were the other races which needed to be pulled along in the path of progress.\textsuperscript{208}

These forces led to a reconsideration of the goals of public education to include a socializing factor, that of Americanization, a term which appears to have been used as loosely as that of progress. There were forces, from early in the century, to continue public support of sectarian schooling. As early as 1827 Massachusetts put an end to the practice of buying sectarian texts at taxpayers' expense. When Horace Mann, the founder of the modern concept of the high school,


\textsuperscript{207}Paulos, p. 32.

\textsuperscript{208}Ekirch, p. 93.
ordained in 1834 that the Bible was to be read without comment in the public schools, he was denounced as Anti-christ by the churches.\textsuperscript{209}

This tremendous growth, occurring in the midst of the scientific era, not surprisingly led to the development of research in education. Research methods in education were not well received by the educational community since the results of the first published research appeared as a series of exposés in a popular magazine, the \textit{Forum}. This original research and the articles based on it were the work of Joseph Mayer Rice, the editor of \textit{Forum} and a physician who had studied pedagogy for two years.\textsuperscript{210}

These exposes brought another problem to the surface. The tremendous growth which the schools experienced had been accomplished through political means; school systems were highly politicized; appointments were made not necessarily on the basis of merit or qualifications, but by political connections—"teaching and administrative positions were bought and sold."\textsuperscript{211}

Horace Mann, the champion of public education, had stated as early as 1848 that education should be the power

\textsuperscript{209} \textit{Myer}, p. 196.
\textsuperscript{210} \textit{Paulos}, p. 62.
\textsuperscript{211} \textit{Paulos}, pp. 34-35.
behind the upward-mobility of American citizens. He saw education as "the balance wheel of the social machinery.... it prevents being poor." Measures of progress were identified with measures of intelligence as early as 1835. In that year George Bancroft, an historian, gave an address at Williams College in which he stated:

The exact measure of the progress of civilization is the degree in which the intelligence of the common mind has prevailed over wealth and brute force; in other words, the measure of the progress of civilization is the progress of the people.  

It appears, from Bancroft's statement, that if an exact measure of "intelligence of the common mind" were available, this would produce an exact measure of progress.

The feasibility of obtaining such a measure was studied during the latter part of the nineteenth century and the early part of the twentieth century. It is somewhat enigmatic that the general proliferation within American society of the idea of widespread testing for the purpose of obtaining a social hierarchy, at least within schools, was occurring during the same period that the Chinese examination system was being abolished. An idea which first entered Western civilization from China was beginning to wax in America at the very time it was waning at its source. The following had already been acknowledged to exist in China:

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212 Ekirch, p. 196.

213 Ekirch, p. 262.
(a) The supplying of candidates for recommendation and examination became more or less the privilege of certain influential families, and the original purpose of the system, to provide able men from a wide range of the population, was lost.  

(b) In theory anyone, with a few exceptions, had the right to take the examinations, but in practice as a rule only someone who had grown up in an environment of scholarly tradition or came from a family that could support him while he acquired the necessary literary education could hope to compete. 

(c) A defect was inherent in the nature of the examination system...This weakness was the one-sidedness of the course of instruction (Bildungsgang) required by the examination, which in the end inevitably had a petrifying effect on the whole intellectual life of the people. Instead of wise and morally outstanding representatives of government authority the system supplied incompetent officials ignorant of the ways of the world; instead of an intellectual aristocracy (it supplied) a class of arrogant narrow-minded literati. 

(d) During a reform movement, the newly established schools could not flourish....because the promising young men were primarily concerned with preparation for the examinations, the only road to social prestige, material profit, and political influence. 

Another influence on education during the latter part of the century was the introduction in America of the works of European psychologists and educators which had been developed in the early part of the century; namely the works of men such as Froebel, Pestalozzi and Herbart, men who


215 Franke, p. 7.


217 Franke, p. 41.
followed in the tradition of Rousseau. One of the leaders in American education to promote this vision of education was John Dewey. Within the context of this study on the influence of the idea of Progress, John Dewey represents a unique synthesis. Dewey had little interest in intellectual pursuits until his junior year at the University of Vermont.

Then, of a sudden, in a physiology course, a book by Thomas Henry Huxley set him afire. The blunt materialism of Darwin's magnificent contemporary ignited him. He had always been certain, as an earnest Christian, that man's life on earth was shaped by moral will; never, in any event, had the thought assailed him, as the iconoclastic Huxley now asserted, that life's determining forces were unalterably material.218

This caused great stress in his life and induced him to scholarly pursuit of a reconciliation of the difference. Under the guidance of George Sylvester Morris at the Johns Hopkins University he was led to the philosophic idealism of Hegel wherein matter is considered illusory, and "...life is the never-ending upward struggle toward God's Universal Mind."219

Dewey later went to the University of Chicago, where he taught Hegelian philosophy; again he encountered a change of attitude. As Meyer put it:

218Meyer, p. 258.

219Meyer, p. 259.
For all the tug of the Hegelian postulates, everywhere there was the inescapable countertug of the Middle West. A land brimming with the juices of life... During Dewey's sojourn, Middle Western America skidded into hard times, and for every fat magnifico it hatched, there were legions of life's losers drifting in helplessness and despair...As a result, Dewey's intellectual concern now veered toward the social problems which raged and rioted all around.220

These formative factors are clearly visible in Dewey's philosophy of education. For example, in a criticism of the social structure then existing in schools, he wrote:

Indeed, almost the only measure for success is a competitive one, in the bad sense of that term—a comparison of results in the recitation or in the examination to see which child has succeeded in getting ahead of others in storing up, in accumulating, the maximum of information. So thoroughly is this the prevailing atmosphere that for one child to help another in his task has become a school crime.221

In his consideration of various theories of education, he gave recognition to Hegel for making a contribution to Rousseau's theory. He wrote:

The recognition...that great historic institutions are active factors in the intellectual nurture of mind was a great contribution to educational philosophy. It indicated a genuine advance beyond Rousseau, who had marred his assertion that education must be a natural development and not something forced or grafted upon individuals from without, by the notion that social conditions are not natural.222

220 Meyer, p. 259.


Mathematics was one of the subjects which received close attention by Dewey in his rejection of faculty-psychology. He wrote:

Mathematics is said to have, for example, disciplinary value in habituating the pupil to accuracy of statement and closeness of reasoning;...But clearly mathematics does not accomplish such results, because it is endowed with miraculous potencies called values; it has these values if and when it accomplishes these results, and not otherwise...But unfortunately, the tendency is to treat the statement as indicating powers inherently residing in the subject, whether they operate or not, and thus to give it a rigid justification. If they do not operate, the blame is put not on the subject as taught, but on the indifference and recalcitrancy of pupils.223

What John Dewey appears to have done with the idea of Progress is to place less emphasis on the Darwinian concept of the "survival of the fittest" and place more direct and intensive focus on the reality of inherent and necessary change. He wrote:

In its contrast with the ideas both of unfolding of latent powers from within, and of formation from without, whether by physical nature or by the cultural products of the past, the idea of growth results in the conception that education is a constant reorganizing or reconstructing of experience.

...We thus reach a technical definition of education; it is that reconstruction or reorganization of experience which adds to the meaning of experience, and which increases ability to direct the course of subsequent experience.224

Since education as defined by Dewey is so value laden, mathematics, among all other subjects which are taught, thus

received a value judgment in terms of the individual.

The general development of mathematics education during the latter part of the nineteenth century was perceived by Florian Cajori to have been greatly influenced by French writers. Writing in 1890, during a period where the textbook defined much of the curriculum, he stated that, "The improvements in mathematical text-books and reforms in mathematical instruction were due to French influences."  

Cajori had harsh comments to make relative to mathematical education during this period:

There have been improvements in the methods of instruction, but not so extensive as might be wished. Traditional methods have long had almost full sway. The mathematical teaching has been bad.

...The trouble has been, all along, not so much in the lack of ability in students, as in the wretched character of the mathematical instruction.

Cajori proceeded then to list some of the specific changes in mathematical instruction which appeared to him to be hopeful signs of change. He stated:

The great desideratum in our preparatory schools and colleges has been less memorizing, less cramming, more thorough training in the fundamental branches, more object teaching, more drill, more frequent and well-guided inquiries, greater freedom from formalism, a stronger spirit of free inquiry.

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225 Paulos, p. 89.
226 Cajori, p. 99.
227 Cajori, p. 100.
Although the theoretical and philosophical outlooks had changed greatly during this period, from the perspective of the classroom as presented by Cajori, little had changed.

In the study of mathematics itself, the nineteenth century was a century of fundamental changes where the very epistemological foundations of mathematics were weakened. It was also a century of great production, contrary to the dire predictions which had been expressed by men such as Lagrange, d'Alembert and Euler during the previous century, and illustrated by the following statement made in 1810 by Delambre in his report to the Institut de France:

> It would be difficult and rash to analyse the chances which the future offers to the advancement of mathematics; in almost all its branches one is blocked by insurmountable difficulties; perfection of detail seems to be the only thing which remains to be done. All these difficulties appear to announce that the power of our analysis is practically exhausted."\(^\text{229}\)

Delambre's predictions for the future of mathematics proved to be unnecessarily pessimistic. In fact, mathematics blossomed into new areas of study and expanded more than in the eighteenth century.\(^\text{230}\) The extent of this development was apparent from its bulk alone; there are over four hundred pages devoted to an outline of these developments in Morris Kline's account of the history of mathematics.\(^\text{231}\) Two of the major developments of this period were the es-

\(^{229}\)Jean-Baptiste Delambre cited in Kline, p. 623.

\(^{230}\)Kline, p. 624.

\(^{231}\)Kline, pp. 626-1039.
tablishment of rigor in analysis and the developments which took place in geometry.

The movement to shore up the foundations of the calculus was initiated by Bolzano and Cauchy. During the eighteenth century, as has been noted, little concern had been expressed for the foundations of analysis since so many answers to the riddles of nature were being successfully answered; the assumption was that proper foundations must exist for the subject to be so fruitful. The problems in the foundations of analysis were so successfully solved, in the minds of the mathematicians during the nineteenth century, that Poincaré, in his address to the Second International Congress on Mathematics in 1900, stated:

Now in analysis today, if we care to take pains to be rigorous, there are only syllogisms or appeals to the intuition of pure number that can possibly deceive us. One may say today that absolute rigor has been attained.

The other development of importance, relative to this study, was the re-emergence of interest in geometry. The developments in this area were "revolutionary", and "amidst all the complex technical creations of the nineteenth century the most profound one..." It was pro-

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232 Kline, p. 1025.
233 Henri Poincaré cited in Kline, p. 1025.
234 Kline, p. 1023.
235 Kline, p. 861.
found because it affected the very basic interpretation of geometry as the expression of "real" space. From a philosophical point of view it had been assumed that Euclidean geometry was one of the Kantian a priori given for knowledge. As a result of the work in this area of mathematics, Gauss stated:

I am becoming more and more convinced that the (physical) necessity of our (Euclidean) geometry cannot be proved, at least not by human reason nor for human reason. Perhaps in another life we will be able to obtain insight into the nature of space, which is now unattainable. Until then we must place geometry not in a class with arithmetic, which is purely a priori, but with mechanics.\(^{236}\)

These developments led to a break with metaphysics; the independent existence of mathematics on the axiomatic foundations of arithmetic or of geometry severed the ties to traditional epistemology. "The recourse to metaphysics was no longer needed. As Lord Kelvin put it, 'Mathematics is the only good metaphysics.'\(^{237}\) Mathematics became a language unto itself; it had obtained a very precise syntax, but its semantic value was lost.\(^{238}\) In the eighteenth century,

\[^{236}\text{Kline, p. } 872.\]
\[^{237}\text{Kline, p. } 1026.\]
\[^{238}\text{Semantic is used here to mean "the totality of considerations concerning those concepts which, roughly speaking, express certain connections between the expressions of a language and the objects and states of affairs referred to by these expressions." A. Tarski, Logic, Semantics, Metamathematics (Oxford, Claredon Press, 1956), p. 401.}\]
...there was every expectation that man's preferred method of communication would become ever more abstract in the future until he talked nothing but pure mathematics.239

At the end of the nineteenth century, if this perception had been held, then language would have degenerated to an extreme of solecism, given the current perception of mathematics. Concern for this state of affairs expressed itself in more thorough investigations, during the twentieth century, into the foundations of mathematics.

No evidence could be found, in this study, that any of these developments in mathematics had any effect upon mathematics education in the United States at any level below graduate school level.

The idea of Progress was very much an idea which had great impact within the society, but its ill-defined character caused confusion to someone seeking a firm definition of the concept; its use rather is traced to the various aspects which arose under the aegis of Progress. The use of the term by someone such as Dewey was far different from its use by someone such as Calhoun. The need for mass public education in a democratic society was affirmed and great steps were taken to assure its implementation: for example, the Morrill Land Grant College Act of 1862 was a unique achievement within any society.

239 Manuel, p. 104.
The role played by mathematics in education was either that of a technician's tool or that of an esoteric subject taught to "special" students who were bound for college; it was part of their entry card to college. Although strains of the thinking which was to dominate in the twentieth century can clearly be discerned, the dominant themes were control, advancement, and change under the general term of Progress.
Author's Translations

6Delvaille. Thus is it legitimate to number Plato among the theoreticians of Progress. But is it necessary to delimit the manner in which Plato speaks of progress.

51Delvaille. ...the dawn of the scientific spirit is evident in Roger Bacon; on the idea of Progress this thinker has expressed views wherein more than a trace of the modern spirit is evident.

79Delvaille. Progress was achieved in the world, in the realm of ideas and in institutions during the Renaissance, a period characterized by this idea...Although no theories of Progress were developed, Progress was at least partially achieved.

83Delvaille. In conclusion, it can be stated that the quarrel of the ancients and moderns had an importance far exceeding a simple debate among literati.

88Roshdi Rashed. [Arabian algebraists] all agree that algebra rests on the abstract quality of the operations involved (the structure of the operations) and not upon mathematical entities or beings which are indistinguishably segments or numbers.

111Delvaille. For them Progress signifies the perfection which intelligence is capable of attaining (on its own)...a separation with the ancients.

115Sobul. Belief is the lot of the ignorant; resolute disbelief, that of the pseudosophisticate; methodical doubt, that of wise men.

120Jules Delvaille. Progress was both the motive of inspiration and the result of the eighteenth century. It sums up the century; it was the center of the arena for the battle from which modern society emerged victorious.

123Delvaille. Progress was for them not merely a procession forward but, above all, a reformation for the future.
Delvaille. Between these two men [Voltaire and Rousseau] there are differences in character, opinions and hopes; one was infatuated with modernism, the other, at least in appearance, seemed to turn his back to his contemporaries and hoped for a return of the old days; nevertheless each played a part in preparing for the modern era. Both Voltaire and Rousseau must be considered as partisans of Progress.
CHAPTER III

NEW DIRECTIONS AND TECHNIQUES IN 20TH CENTURY AMERICA

The development of the idea of Progress continued through the first half of the twentieth century with the infusion of new ideas, although some return of the cyclic concept of history emerged in the writings principally of Nietzsche and Spengler.

Manuel subdivided the many forms of the idea of Progress during the twentieth century into three types:

...first, those who are loyally bound to the French concept of perfectibility in any one of its eighteenth- or nineteenth-century forms; second, a new school of philosophical life-scientists writing cosmic history; and last, the contemporary Marxists.1

Representative of the philosophical life-scientists to which Manuel referred is the Jesuit paleontologist Teilhard de Chardin. His development of the idea of Progress was an extension of Darwin's evolutionary concept wherein a direction to the evolutionary process was posited. This direction, that of growth in consciousness, is one which has been mentioned in Chapter II of this work as being first stated by Comte. Manuel stated:

These imaginative life-scientists assert that a new spirituality is about to possess mankind and become a permanent acquisition of the species, that we are about to ascend to a higher stage in the autonomous unfolding

of the irreversible world-historical process.\(^2\)

This theory of the idea of Progress was readily held by many in America, especially by the youth of America during the decade preceding this writing. There was great concern for consciousness raising groups and liberation movements which captured the imagination of many Americans; for example, one of the "best-sellers" of this period outlined three evolutionary levels of consciousness.\(^3\) In the light of the historical datum on America during the twentieth century this phenomenon was difficult for Manuel to understand, since in his 1965 book, he exhibited a strain of incredulity in the following:

In the midst of the horrors of the twentieth-century world they believe that we are actually witnessing the initial breakthrough.\(^4\)

In an article written in the early sixties, Sir Julian Huxley expressed this new view of Progress in the following words:

...anything that we can properly call progress or advance in evolution becomes increasingly dependent on the advance of mind...This is most obvious in regard to rational awareness, to intellectual and scientific knowledge and its organization, but it also applies to

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\(^2\) Manuel, p. 146.


\(^4\) Manuel, p. 150.
ethical and aesthetic and other non-rational awareness.\textsuperscript{5}

Huxley proceeded to show that this called for a new meta-physics, one which is more in keeping perhaps with that developed by Leibnitz or Whitehead; he stated:

...our new awareness has shown that we can no longer think in terms of dualism of soul and body, of mind and matter: we need a new monistic approach, and therefore a new terminology. We are neither mind nor bodies, but a combination of the two: we are mind-bodies.

This evolutionary approach clearly implies the rejection of the objective existence of all abstract concepts or ideas.\textsuperscript{6}

This thesis perhaps allows a rational explanation for the interest of Americans, during the last decade, in Oriental philosophies, and for the recent publication, by a physicist, of the book The Tao of Physics: An Exploration of the Parallels Between Modern Physics and Eastern Mysticism.\textsuperscript{7} It was in physics, and especially in the works of Albert Einstein, that a crisis concerning the basic rationality of nature had its beginning. Proponents of the idea of Progress had pointed to the control man exercised over nature through the empirical rational knowledge obtained in science as a proof of its fact, but work in elementary particle physics had convinced physicists of the basic and fundamental un-


\textsuperscript{6} Huxley, p. 878.

\textsuperscript{7} Fritjof Capra, The Tao of Physics (Boulder: Shambala, 1975).
predictability of nature. Probabilities were the most firm statements which could be asserted; deterministic models were useful only in predicting gross behavior and were useless when trying to study nature at a more fundamental level. Some of the early scientists themselves turned to Oriental philosophies for possible alternative explanations; Niels Bohr even chose the yin-yang symbol for his coat-of-arms when he was knighted, an affirmation of his indebtedness to Eastern thought. For example, Niels Bohr has written that,

Isolated material particles are abstractions, their properties being definable and observable only through their interaction with other systems.8

He further acknowledged a need for fundamental revisions in Western epistemological thinking when he stated:

For a parallel to atomic theory...we must turn to those kinds of epistemological problems with which already thinkers like the Buddha and Lao Tzu have been confronted, when trying to harmonize our position as spectators and actors in the great drama of existence.9

Contact with Oriental philosophy had never been so meaningful to Western philosophy until this period; philosophers such as Heraclitus and Leibnitz had expressed philosophies of nature which reflected Oriental modes of thought, but their impact was minimal on this matter within western civilization. One of the major twentieth century writers of a metaphysics who attempted to incorporate the major

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findings of modern physics was Alfred North Whitehead. Whitehead had been one of the principal leaders, in collaboration with Bertrand Russell, in the study of the foundations of mathematics; he had also worked extensively in modern physics.

His metaphysics is one of dynamics; he eschewed such static concepts as "being", "substance", and "essence" and advocated the use of the dynamic concepts of "becoming", "process" and "evolution" to explain reality.

To look at bits of matter as the fundamental unit of reality is illusory; for Whitehead these fundamental pieces are "moments of experience" or what he terms "actual occasions" and "eternal objects". One advantage of this "experience" model is to emphasize the basic interrelatedness of reality; moments of experience cannot be thought of in isolation; relation to the world around them is necessary.

In order to show how these actual entities relate to one another and develop or evolve, Whitehead is forced to invent a new vocabulary. Among some of the terms he introduces are positive and negative prehension, conceptual prehension, and nexus.

Eternal objects are the pure possibilities, somewhat like Plato's forms except that they have a relatedness to other eternal objects to achieve an organismic condition; they have no existence apart from the actualities in which they are manifested and are merely possibilities available
for actualization.

Prehensions are the concrete facts of relatedness; in a sense they are "feelings". As an actual occasion emerges, it "feels" all the data available to it in its universe, and these are its prehensions. The objects of consciousness, e.g. the human body or a stone, constitute an interrelated group of actual occasions called a nexus. A nexus is any set of actual occasions experienced as related to each other.

For Whitehead nothing is experienced alone, neither is there a separate knower and thing known, but rather he is led to an epistemology of interaction between what is usually referred to as knower and the object of knowledge within the same reality. A new actual occasion is "formed" by its prehensions of the past and is therefore its own unique synthesis of its past, perishing in its turn so that the process continues. In Whitehead's phrase, "The many become one and are increased by one." 10

The view that change is the basic reality influences Whitehead's thoughts concerning social institutions and social theories; he wrote:

Our sociological theories, our political philosophy, our practical maxims of business, our political economy, and our doctrines of education, are derived from an unbroken tradition of great thinkers and of practical examples, from the age of Plato in the fifth century before Christ to the end of the last century. The whole of this tradition is warped by the vicious assumption that each generation will substantially live amid

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the conditions governing the lives of its fathers and will transmit those conditions to mould with equal force the lives of its children. We are living in the first period of human history for which this assumption is false.11

The further implication for education is that, "...our training must prepare individuals to face a novelty of conditions."12 He observed the changes which had taken place within his lifetime, changes in institutions, changes in politics, and above all changes caused through scientific thinking as indicative of changes to come in education:

Any serious fundamental change in the intellectual outlook of human society must necessarily be followed by an educational revolution. It may be delayed for a generation by vested interests or by the passionate attachment of some leaders....Education which is not modern shares the fate of all organic things which are kept too long.13

Since "Each individual embodies an adventure of existence,"14 Whitehead urged that education evoke initiative and end by encouraging it.15 He also urged teachers, at least university professors, to exhibit themselves in their true character,

...that is, as an ignorant man thinking actively

12 Whitehead, Adventures of Ideas, p. 118.
utilizing his small share of knowledge. In a sense, knowledge shrinks as wisdom grows; for details are swallowed up in principles.\(^\text{16}\)

He has also expressed himself concerning the place of abstraction in education; he is clear to point out that the true practice of education,

...must start from the particular fact, concrete and definite for individual apprehension, and must gradually evolve towards the general idea. The devil to be avoided is the cramming of general statements which have no reference to individual personal experiences.\(^\text{17}\)

Whitehead identified what he termed as some of the "worst defects" in education in the following:

It is the assumption, unconscious and uncriticized, that logical simplicity can be identified with priority in the process constituting an experient occasion.

...This identification of priority in logic with priority in practice has vitiated thought and procedure from the first discovery of mathematics and logic by the Greeks. For example, some of the worst defects in educational procedure have been due to it.\(^\text{18}\)

For Whitehead education must be both useful and directed at the present. The present is "holy ground; for it is the past, and it is the future."\(^\text{19}\) Consistent with his metaphysics of space-time he reiterates this theme; "the present holds within itself the complete sum of existence, backwards

and forwards, that whole amplitude of time, which is eternity." 20

Whitehead, in collaboration with Bertrand Russell, was a founder of one of the principal schools of philosophy of mathematics which developed in the twentieth century, the logistic school. This was an outgrowth of the interest generated from the "embarrassment" of the eighteenth century which developed to become the "crisis" of the nineteenth century mentioned earlier.

Within mathematics of the twentieth century Kline stated that, "By far the most profound activity...has been the research on the foundations." 21 These researches helped to establish several schools of thought concerning the foundations of Mathematics. The principal schools are generally referred to generically by the following terms: Logicism, Intuitionism, and Formalism. 22 A more extended list was given by Klenk 23 which includes, in addition, Platonism, Empiricism, Conventionalism. These six major schools will be briefly sketched in the following few paragraphs.


The philosopher Wittgenstein rejected total acceptance of any one school of thought, yet he perceived some truth in each; his personal philosophy of mathematics is a synthesis or banyan tree view of mathematics wherein its roots are anchored in many places.24

Platonism and Intuitionism define mathematics in terms of an ontology, whereas the others maintain that mathematics needs no ontology. Platonists hold that the study of mathematics is the study of the "eternal forms" which have an independent existence in the world of ideas; Intuitionists hold that "mathematics is the study of a certain sort of mental entity...."25

In each of these philosophies of mathematics, the act of doing mathematics is an act of discovery. The purpose of mathematics is to probe nature in such a way as to discover the nature and relationship of these mathematical entities.

The other philosophies of mathematics lead to a quite different perspective concerning the activity of mathematics. Logicism holds that "the concepts of mathematics can be derived from logical concepts through explicit definitions."26

24 Klenk, passim.
25 Klenk, p. 18.
26 Kuyk, p. 102.
Since logic is a priori, its derivative mathematics must also necessarily be a priori, and hence mathematics reduces to logic; it is a piece of machinery for the mind and has no independent ontological existence. In the teaching of mathematics, its implication is that precise definitions and strict application of formal logic are what should be taught.

Closely allied to logicism is formalism, which some see as an outgrowth of logicism.

Logicism tried to meet it [to put entire mathematic on a certain basis] by a further modeling of arithmetic on logic. However, this went aground on account of the necessary acceptance of axioms which were more mathematical than logical in nature.27

Formalism tried to overcome the ontological and epistemological difficulties essentially by denying this existence. One of the leaders of the formalist school was Hilbert.

Instead of rejecting everything in mathematics that cannot claim real meaning found on intuitive evidence (construction), Hilbert proposes that we rid the whole body of mathematics of all meaning whatsoever. All mathematical statements are to be reduced to mere formulas....27

Work in this area advanced to a point where Ackerman and Von Newmann proved the consistency 'arithmetics' under certain conditions, but as Weyl states,

A gap remained which seemed harmless at the time.... Then came a catastrophe: assuming that consistency is established, K. Gödel showed how to construct arithme-

27 Kuyk, pp. 123-124.
tical propositions which are evidently true and yet reducible (sic) within the formalism.

...Obviously completeness of a formalism in the absolute sense in which Hilbert had envisaged it was now out of the question. 28

These difficulties dealt a severe blow to the Formalist school: "few logicians or mathematicians now maintain that formal logical systems can serve as the foundations of mathematics." 29

Empiricism holds that "mathematical propositions are just high level generalizations about the empirical world." 30 Mathematics is looked upon as an abstract physics; under this philosophy mathematics is considered a language, the language of nature.

Conventionalism's main position is "the belief that the theorems of mathematics are merely an analytic consequence of the way the concepts of logic and mathematics are defined." 31 Within this philosophy "it is the meanings of the logical signs which determine that one proposition shall follow from another; the power of logical compulsion is vested in meaning." 32

29 Klenk, p. 35.
30 Klenk, p. 7.
31 Klenk, p. 41.
32 Klenk, p. 7.
During an historical period when Progress in the form of control over nature was dominant, a sense of loss of control concerning the very foundations of both physics and mathematics was being experienced. In his recent work Clyde Hopkins stated, after a review of the foundations crisis, that "The ontological status of these abstract mathematical entities and the epistemological grounds of our knowledge of them are open questions." If progress is associated with growth, then mathematics experienced tremendous progress during this period. This growth was so great that Koppelman attempted a taxonomy of this progress in order to construct a "guide for working out specific cases." These methods of growth are identified as: ordering, transplantation, fission, fusion, conceptual generalization, and pure invention.

The well-known mathematician, Jean Dieudonné, appeared to identify progress in mathematics with this growth; he even denied that the controversy concerning the foundations of mathematics was a crisis. He stated:

There is no crisis in mathematics. Mathematics has never been as prosperous as it has been in the last ten years. Never before had we proved so many new and

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33 Clyde Hopkins, "A Discussion of Some Consequences of Contemporary Mathematics for Philosophy: A Historical Approach," 1976 Diss. The Pennsylvania State University, p. 188.
35 Koppelman, pp. 457-460.
powerful theorems.  

The concern for foundations is given some support by Kreisel, who stated:

Just as we do not see the external world differently after we are told a new fundamental theory of matter, so we do not think of familiar mathematical objects such as the integers or ordered pairs differently after foundational analysis. But occasionally, and this is typical of the relation between practice and foundational theory, theory will change an area of practice out of all recognition.  

Whitehead's reactions to the events surrounding the investigations of the foundations of mathematics were more pluralistic.

Thus the palmary instances of human certainty, Logic and Mathematics, have given way under the scrutiny of two thousand years. To-day we have less apparent ground for certainty than had Plato and Aristotle. The natural rebound from this conclusion is skepticism.

...But it is fatal to dismiss antagonistic doctrines, supported by any body of evidence, as simply wrong. Inconsistent truths—that is, truths in conformity to some evidence—are seed beds of suggestiveness. The progress which they suggest lies at the very root of knowledge.  

These developments within mathematics had no perceptible effect on developments of mathematics education. Indeed, mathematics education found itself on the defensive by 1936, a fact which is patently shown by the keynote article written for the Yearbook of the National Council of


Teachers of Mathematics, entitled "Attacks on Mathematics and How to Meet Them." Some of the reasons for this development can be traced to changes in attitudes towards the curriculum in general and rather conservative positions taken by mathematics teachers relative to these changes.

Prior to the turn of the century, a few changes had occurred in mathematics education, as has already been noted. "The first experimental schools to deviate from the subject-discipline design were founded by John Dewey in 1894 and J. L. Meriam in 1904." These experimental curricula centered around the needs of the student, as outlined by Dewey's philosophy of education, rather than subject matter. That is, subject matter had to be justified according to the needs of the student, rather than artificially imposed to exercise the mind. The research work of E. L. Thorndike and R. S. Woodworth, published in 1901, had been accepted as demonstration "that the idea of mental discipline was scientifically untenable." The teaching of mathematics on the basis that it provided training for mental faculties of logic and clear thinking could no longer be justified.

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John Dewey's philosophy of education did not receive widespread attention until William Kilpatrick translated this philosophy into a more concrete method. The year of this publication was the year of the Armistice of World War I, 1918, the year which Paulos identified as the one in which curriculum as a field emerged. The nation looked to education as a leader in social change:

In the 1920's the concern had been to make the curriculum socially useful, to make it fit a new industrial society, to prepare students on the basis of scientifically identified needs of adults.

Under these pressures mathematics education was on the defensive.

Not only had educators like Kilpatrick questioned the aims and objectives of mathematics education, but also the student population was offering evidence to mathematics educators of dissatisfaction with the nature of the curriculum. The response of mathematics educators included both a negative defensive attitude, indicating bewilderment on the part of some, and on the part of others some very positive forward-looking steps in the direction of curricular change.

It appears that mathematics educators looked upon their subject as having a fixed, immutable content and form; little

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42 Paulos, p. 408.
44 Paulos, p. 203.
of the idea of Progress appears to have influenced their perception of their subject matter.

Although there was no such thing as a universally accepted elementary program in the United States by 1927, reading and arithmetic were almost always taught in all schools. 46

Mathematics played a major role in college preparatory curricula although such curricula constituted only one-third of all curricula. The College Entrance Examination Board, founded in 1900, was exerting more influence on high school mathematics curricula. In the 1923 Report of the National Committee on Mathematical Requirements the following appeared:

College entrance examinations exert in many schools ...... an influence which is very far-reaching.

... The reason for the almost controlling influence of entrance examinations in the schools... is readily recognized. Schools sending students to such colleges for men as Harvard, Yale, and Princeton...... or to any institution where examinations form the only or prevailing mode of admission, inevitably direct their instruction toward the entrance examination. This remains true even if only a small percentage of the class intends to take these examinations, the point being that the success of a teacher is often measured by the success of his or her students in these examinations. 47

Although finding fault with the examinations, this committee recommended the traditional subject matter for grades

46 Paulos, p. 254.


48 Bidwell and Clason, p. 389.
ten, eleven and twelve and also concluded that "there is no conflict between the needs of those pupils who ultimately go to college and those who do not." On the whole, a rather conservative position was maintained in mathematics education; the years following the report were years of declining enrollments in mathematics.

It appears that there was a general conservative posture throughout education, for as late as 1937 the following was reported:

On the whole, however, present practice was just a superficial attack on the whole problem of revision. Basically it assumed that traditional content had value and clung to a logical organization of subject matter set-out-to-be-learned.

One of the reasons offered for the failure of the progressive education movement is presented by Ullich, who stated:

...did these teachers themselves--and here is the most salient reason for the progressive failure--have clear aims and policies? They rightly rejected rigid indoctrination in the name of freedom and wished to leave the dusty groves of obsolete traditions. But few people dare abandon known strongholds, even though they may be crumbling, without seeing new signposts for thought and action. The only ones, whether willing or coerced, who acquired a clear goal, were the Russian educators.

In connection with the idea of Progress, it has already been mentioned that Dewey's idea of Progress was not the individualistic idea of Progress generally accepted in the

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49 Osborne and Crosswhite, p. 209.

50 Paulos, p. 268.

United States, but was closer to Marxian concepts. Ullich points out, however, that there was a strong strain of individualism within Western progressivism which Russian educators detected through early experimentation with American progressive forms of education. This is brought out by Kandel:

In Soviet Russia the efforts in education were directed to making a complete break with anything bourgeois until it was found, after fifteen years of trial to see "how it worked when tried," that the innovations had failed to cultivate the desirable objects of social allegiance. Discipline and examinations were restored....

In America the leaders of progressive education also sought social change or social progress, but saw the existing schools as blocks to this progress. Kilpatrick stated that "This right of parents or other grown-ups to determine what children shall think must be essentially revised." The test of survival should be placed on existing thought and value, he continues:

Our duty is so to prepare the rising generation to think that they can and will think for themselves, even ultimately, if they so decide, to the point of revising or rejecting what we now think. Our chosen beliefs will have to stand this ordeal. If they are worthy to survive, the probabilities are that they will stand this test. If they cannot stand this test, the probabilities are that they ought not to survive.

The implications of these ideas for mathematics are

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54 Kilpatrick, p. 60.
stated by Kilpatrick in the following:

Rid the schools of dead stuff...For most pupils, Latin can and should follow Greek into the discard. Likewise with most of mathematics for most pupils.55

It appears that few of these innovations were put extensively into practice. One of the reasons was the influence of colleges upon the high school curriculum, as Paulos reported:

In the course of a two-day convention of the Progressive Education Association [1930], nearly every suggestion made was rejected for fear it would risk students' chances of being admitted to college. If a student did not follow the pattern of subjects and units prescribed by colleges, he risked being rejected admission to a higher institution.56

Another reason, for mathematics education, may have been a lack of interest on the part of college mathematicians to move in this direction. DeVault and Weaver stated that:

This period [1923-1952] probably represents a low point in the involvement of mathematicians in the elementary school program....Insofar as the discipline of mathematics was represented, it seemed to be so in the determination of the nature of mathematics not so much as a tool subject but rather as a part of the cultural heritage which required educational attention in its own right, not necessarily because it was useful in other endeavors.57

The mathematics education community maintained a conservative position, important in the selection process for college work, confident of its testing procedures. The

55 Kilpatrick, p. 111.
56 Paulos, p. 275.
57 DeVault and Weaver, p. 132.
following is from the NCTM Yearbook for 1940:

Mathematics is often described as a "hard" subject. ...While these features of mathematics cannot be denied, it is also true that each step forward in the subject is, as a rule, a very simple one.

...Early in each year the mathematical maturity of each pupil should be determined. In case the required information is not available from reports, inventory tests may be needed to determine the amount of ground that may be covered during the semester,...

...pupils should be made to realize that each day's work counts toward success or failure.

...The slow learner profits by at least the same degree of motivation, of cultural enrichment and interest, as do other pupils. But interest is primarily a means of stimulating effort, not a substitute for effort.\(^{58}\)

The perception of the primary purpose of evaluations are also presented in this Yearbook in a manner which appears to be consistent with selectivity.

Primary Purpose. Many teachers would assert that the chief purpose of testing is to provide a basis for assigning marks....But tests are given for many other purposes, among which are the following: to maintain standards, to select and reject pupils, \([\text{italics mine}]\) to discover strengths and weaknesses of individual pupils or of the class as a whole, to provide a powerful incentive to study, to furnish a convenient method of instruction, to stimulate or even enforce improvement of teaching, to afford a basis for the appraisal of teachers and departments, to serve as a basis for accrediting schools and colleges, to furnish data for educational guidance, to accumulate materials for research.\(^{59}\)

It is clear from this that the thinking concerning evaluation by testing was concerned most with the selection of the "best" students for admission to the "best" colleges.


\(^{59}\) Reeve, p. 164.
Rating and ranking of both individuals and schools were of primary interest. The report continues:

Thus it is possible to distinguish at least two types of purposes which differ in point of view. The first is primarily concerned with evaluation of the educational status and progress of individual pupils; the second is primarily concerned with evaluation of the school as an institution. Both types are important, but it is quite obvious that evaluation of the second type depends upon evidence obtained from evaluation of the first type.\textsuperscript{60}

The Second World War forced the attention of America in the unique direction of winning the war. Paulos stated that:

During these years \([1941-1945]\) the number of students attending high school dropped, and teachers entered the armed forces or took better-paying war jobs. Universities kept functioning through military research projects and training programs. Teachers and students engaged in war bond drives; building funds and general revenues for education were greatly decreased.\textsuperscript{61}

During the post-war period, America reassessed its needs; technology had developed and had been used during the war period at an expanded rate; this was the period of the emergence of America as a technological society. One of these reports on manpower needs, the Steelan Report, indicated that functional competence in mathematics, for some at least, "was not sufficient to meet the demands of a postwar, scientifically oriented society."\textsuperscript{62} Manpower

\textsuperscript{60} Reeve, p. 164.

\textsuperscript{61} Paulos, p. 324.

shortages were shown to exist in government, engineering, skilled trades and almost every facet of the working world. It is reported that some of this shortage was due to the fact that "the type of mathematical training necessary to cope with the newer applications was not being provided in the schools."63

By the 1950's America was again in a period of prosperity.

In the 1950's people were enjoying a standard of living far beyond any they had previously known. Family income had more than tripled since 1918.64

Persuasions, aimed at youth to continue their education, were framed in monetary terms, with slogans such as, "a college education is worth $100,000."65 Much less publicized was the research result which showed that:

The most important factor determining the level of education achieved by the head of a household in the USA was shown by the survey to be not IQ nor ability, nor the income of parents, but the educational attainment of his father.66

Mathematics continued to be one of the major determinants for college entry. In a study, conducted in 1953, of

63 Jones and Coxford, p. 69.
64 Paulos, p. 330.
66 Griffiths and Howson, p. 75.
college entrance requirements at 138 liberal arts colleges, it was determined that administrators considered student preparation in English and mathematics and, only to a lesser extent, in foreign languages, science and social studies.  

The influence of Senator Joseph McCarthy was one of the reasons offered by the British scholars, Griffiths and Howson, for the lack of programs concerned with the education of "underpriviledged children". Programs wherein the socialist form of the idea of Progress would be a generative idea were not prominent. There was an extreme fear that such programs would be labeled "Communist" and un-American.

On October 4, 1957, an event of tremendous national consequence occurred; the Russians placed the first satellite, Sputnik I, in orbit. This was viewed as a stunning blow to American scientific prestige; it weakened confidence in American engineering superiority and in its educational system. The impact on education is given by the following:

Since education produces scientists and technologists, it was argued that funds must be pumped into education, especially to improve its scientific and mathematical quality. The 'hawks' of Congress were quick to produce cash--education for them was now 'Defense'--and educators were naturally quick to express their arguments for education in the hawks' terms, even though their own aims may have been entirely different. Certainly, past

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67 Paulos, p. 363.

68 Griffiths and Howson, p. 138.
history did not encourage them to argue on liberal grounds.\textsuperscript{69}

Prior to the Sputnik launch, attempts at reforms in mathematics education based upon scientific-manpower needs had begun at the University of Illinois. This work gained national attention through publication of articles by Henderson and Dickman\textsuperscript{70} and by Edwards, Jones and Meserve.\textsuperscript{71} The studies at the University of Illinois were the result of an unusual cooperative effort of various schools within the University, coordinated by Max Beberman, which was named the University of Illinois Committee on School Mathematics (UICSM).

In December 1951 the Colleges of Education, Engineering, and Liberal Arts and Sciences established the University of Illinois Committee on School Mathematics to investigate problems concerning the content and teaching of high school mathematics in grades 9-12.\textsuperscript{72}

The national spirit of competition with the Russians, prior to Sputnik, is apparent in the opening paragraph of a UICSM Project Staff report issued in 1956:

\begin{itemize}
  \item \textsuperscript{69} Griffiths and Howson, p. 138.
  \item \textsuperscript{70} Kenneth B. Henderson and Kern Dickman, "Minimum Mathematical Needs of Prospective Students in a College of Engineering," The Mathematics Teacher, XLV (February, 1952), pp. 89-93.
\end{itemize}
The American school patron has been bombarded with statistics which purport to show that America's production of scientifically trained personnel is lagging far behind America's needs for such personnel and certainly far behind the production of technically trained people in the Soviet Union.\textsuperscript{73}

The materials produced by the UICSM Project were directed for use by the general population of students preparing for college; its intent was not restricted to "high ability" students. Its aim was,

...to search for ways of helping the high schools in the state of Illinois, particularly the small high schools, to develop a mathematics curriculum which would be more effective in preparing students to meet the new requirements for entrance--four units of high school mathematics--into the College of Engineering.\textsuperscript{73} (Italics mine.)

The intent of the directors of the UICSM Project appears to have been motivated by the socialist form of the idea of Progress, although the framework in which the Project was presented clearly places its work within the national spirit of competition and survival vis-à-vis the Soviet Union, a conclusion which gives greater credence to Griffiths and Howson's statement of footnote 69.

Given the stated intent of the UICSM, it is not surprising that the mathematics content of the school material did not change considerably from the traditional high school mathematics curriculum. An analysis of UICSM High School Mathematics Units 1-4 presented by an NCTM Committee reports:

The UICSM materials are not greatly different from

\textsuperscript{73} UICSM Project Staff, "The University of Illinois School Mathematics Program," in Bidwell and Clason, p. 655.
conventional elementary algebra in content, but the approach and techniques are substantially different.\textsuperscript{74}

Thus UICSM appears to have been meeting the recommendations of the 1955 Commission on Mathematics of the College Entrance Examination Board:

The Commission found that much of the traditional curriculum was both necessary and desirable in the modern world, but other parts of it were useless vestiges of a bygone era. In addition, the Commission noted that the spirit in which mathematics was being taught in most high schools left a great deal to be desired in the understanding of mathematics being developed and in the attitudes toward mathematics being instilled in the minds of the students.\textsuperscript{75}

After 1957, the situation of curriculum development changed radically with unprecedented massive funding by the federal government. The major curricular undertaking in mathematics education was the School Mathematics Study Group (SMSG).

Between 1957 and 1965, there was a definite renewed interest in seeing the schools as purveyors of knowledge. Americans wanted the schools to produce scientists, mathematicians, and scholars.\textsuperscript{76}

This production was considered to be a matter of national survival. Admiral Rickover, a long-standing critic of education, stated "The mood in America has changed. Our technological supremacy has been called in question and we know


\textsuperscript{76} Paulos, p. 397.
we have to deal with a formidable competitor." 77 These factors forced attention on the "elite" of the schools:

Studies indicated that most curriculum changes, for the first time in fifty years, were in the area of the academics. There was a renewed emphasis upon obtaining subject matter knowledge and upon serving the gifted student. 78

The sensitivity to survival fears is forcefully utilized in the following excerpt from the Report of the Secondary-School Curriculum Committee of NCTM issued in 1959.

The men of this planet are also alarmingly fierce. Indeed we find it expedient within the confines of our own quiet society to make perhaps one in every one hundred able-bodied males a policeman. Possibly even more alarming is the fact that the advanced and powerful nations of the earth have within our life-span demonstrated that their capacities for cruelty and horror are the equal of anything in history. So if some new group surges into the lead, it will not be astonishing if it happens to obliterate us in the process. 79

This same report also indicated the need for selectivity of the fittest for mathematics, as well as the degree of monetary concurrence which the Federal government gave these decisions as the following shows:

No stronger testimony in support of the proposals of this report can be found than that implied in the provisions of the National Defense Education Act of 1958....

Sixty million dollars has been authorized for the

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78 Paulos, p. 432.
next four years for testing pupils and guiding the able ones toward college. Also 28 million dollars has been authorized for institutes to improve the qualifications of counselors who guide the able youth toward maximum development.

The provision of such vast sums of money is striking indication of the national concern over the need for a greatly improved program of mathematics in the schools of our nation.

It was under these conditions, conditions which are clear expressions of the Darwinian form of the idea of Progress, that SMSG emerged; its development was an outgrowth of several conferences, as the following sketch indicates.

Four months after the Sputnik launch, February 21, 1958, the National Science Foundation sponsored a conference of mathematicians in Chicago to address these needs, as the following indicates:

The purpose of the conference, called the Chicago Conference on Research Potential and Training, was to survey the problem of supply and demand with respect to research mathematicians. 81

Adequate early schooling was identified by the Chicago Conference as the long-range solution to the problem of the shortage of adequately trained persons; concentrating effort on the training of mathematical personnel in the colleges would constitute only a short-range solution. 82 The Chicago Conference adopted a resolution which suggested that the

81 Wooton, pp. 9-10.
82 Wooton, p. 10.
president of the American Mathematical Society consult with presidents of the Mathematical Association of America and of the National Council of Teachers of Mathematics to appoint a committee of mathematicians to seek funds towards the resolution of the problem of the existing state of school mathematics curriculum.  

This resulted in the Cambridge Conference, not to be confused with other Cambridge Conferences of 1963 and 1966, where research mathematicians conferred with physicists involved with the Physical Science Study Committee, a group which had more experience with national curricular revision, to begin the work outlined at the Chicago Conference. Everything was done with great haste and with a sense of urgency; the Cambridge Conference started on February 28, 1958, just one week after the Chicago Conference, and on March 1, 1958, the Conference made the following specific recommendations:

1. to hold a four- or five-week writing session the next summer to prepare a detailed syllabus for a model secondary-mathematics curriculum beginning with the seventh grade, and
2. to arrange for the preparation and publication of monographs on topics in mathematics of interest and value to secondary school students.  

A month later, April 3, 1958, Edward G. Begle was appointed to direct the work which was based at Yale Univer-

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83 Wooton, p. 10.

84 Wooton, p. 11.
sity, since it had earlier indicated a willingness to take institutional leadership of the project. The name School Mathematics Study Group was also adopted at that time. One month later, May 7, 1958, the group was awarded a grant of $100,000, which it had sought from the National Science Foundation, to carry out its charge. About two months later, June 23, 1958, the first writing groups began their work. The predominant philosophy of mathematics which appears in the work of each of the working groups, although not directly stated, appears to be the formalist/logicist philosophy, for indeed even the didactics appears to have taken this approach. For example, the junior high school working group stated:

The group accepted the hypothesis that at this level experience with and appreciation of abstract concepts, the role of definition, the development of precise vocabulary and thought, experimentation and proof, were essential and appropriate.

Although the material which they developed was tested at centers where the "I.Q. of most students was over 100" there were reports that "certain chapters have been difficult for the slower students."

The group working on a first course in algebra were

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85 Wooton, p. 13.
87 Mayor, p. 7.
similarly motivated:

The order of the material in the first five chapters is dictated by some rather strong logical considerations which are worth remembering.\textsuperscript{88}

which is followed later by,

The first theorem in the course occurs in Chapter 6 and is the uniqueness of the additive inverse. We felt the need for this theorem very strongly, but we also found it a difficult theorem to begin with. It is really hard to get across to students what you are fussing about.\textsuperscript{89}

One group, the group working on geometry, seemed to be concerned with the possibility of working in too narrow a constraint of rigor.

It may be true, in some philosophical sense, that there is such a thing as absolute deductive rigor. But absolute rigor is not a part of the working equipment of most professional mathematicians.\textsuperscript{90}

Further in this same article:

Here it appears that the question of rigor is being over-emphasized. Euclid was very rigorous indeed, by high school standards.\textsuperscript{91}

The only group to state a method of teaching was the Writing Team for Grades 4, 5, and 6; they stated that, "Emphasis was placed on the \textbf{guided discovery} method of teaching


\textsuperscript{89} Pollak, p. 18.


\textsuperscript{91} Moise, p. 30.
and test material for the pupil was so written."\(^{92}\)

Attention has been focused in this study on two curricula projects, UICSM and SMSG, the former because "The precedents set by UICSM became a pattern for later projects"\(^{93}\) in general administrative patterns, and the latter because it was the most influential of all curricula projects given its genesis from the joint recommendations of the three leading national groups concerned with mathematics or mathematics education.

Many curricula projects developed under these social and political pressures, implemented with the help of unprecedented federal funding in this area.

During the late 1950's, National Science Foundation allocations to course content improvement increased tenfold, and the U.S. Office of Education appropriations to research nearly tripled. The National Defense Education Act of 1958, as well as other bills, provided additional resources for the national curriculum projects. By 1971, over one hundred million dollars had been used in curriculum development in mathematics and science alone.\(^{94}\)

The "gifted" student was given special attention:

...Our country's immediate problem is one of spotting these students [gifted], helping them to develop interest in mathematics, and planning a mathematics program that will be effective in producing the compe-

\(^{92}\) "History and Philosophy of the Writing Team for Grades 4, 5, and 6" Philosophies and Procedures of SMSG Writing Teams (n.p.: SMSG, 1965), p. 56.

\(^{93}\) Jones and Coxford, p. 73.

\(^{94}\) Paulos, p. 416.
tence desired. 95

In a later publication, jointly published by NEA and NCTM, the opening sentence acknowledged the problem of catering to the "gifted" student within our society; it stated, "We must find a way to cultivate the ideal of excellence while retaining the moral values of equality." 96

The work of the curriculum projects was considered to be revolutionary, 97 and indeed it was a rupture from its traditional development.

One of the primary forces, from a pedagogic point of view, to teach from the view of mathematical structures came from the work of the psychologist Jerome Bruner.

The tendency in all recent curriculum reform has been for the scholars to attempt to organize the program around what Jerome Bruner, in his Process of Education, has termed the "structure" of the subject. 98

Jerome Bruner had worked closely with the mathematics educator Zoltan Dienes and used instructional examples from


98 Osborne and Crosswhite, p. 288.
From an epistemological point of view, Bruner comes closest to that of Plato, as exemplified in the selection from *Meno* mentioned in Chapter II, because for Bruner, ...it is rarely something outside the learner that is discovered. Instead the discovery involves an internal reorganization of previously known ideas in order to establish a better fit between those ideas.\(^{100}\)

Although the work of Bruner is alluded to, no curriculum project utilized his ideas in a systematic way. "Some projects such as SMSG very deliberately espoused no doctrinaire methodology."\(^{101}\)

In conclusion, the idea of Progress is much in evidence during the development of the "new" mathematics curricula of the 50's and 60's, but in conflicting forms which had been generated after mid-nineteenth century. In the UICSM Project the socialistic form of the idea of Progress is apparent in the work, if not in the justification of the work. In the SMSG Project, the liberalist form of the idea of Progress is patently present both in the work of this group as well as in the justification of the work. Later forms of the idea of Progress were not observed in any of these projects.

The SMSG Program was dominated by the perspective of

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\(^{100}\)Shulman, p. 35.

\(^{101}\)DeVault and Weaver, p. 143.
mathematics from the formalist/logicist philosophy; most writing groups adopted a presentation of mathematics which clearly reflects this frame of reference. The use of "structure" was dominant from this point of view, and the appearance of Bruner's work merely reinforced rather than motivated this attitude.

Whitehead's ideas on education and on metaphysics, although he had been a leading researcher in the formalist school, had no apparent influence on the curriculum writers of this period.

Although born out of a larger political conflict between America and the Soviet Union, mathematics education played an important role in America's rise from a sense of defeat, but Progress, according to the Soviet physicist Andrei Sakharov, cannot be judged in such survival terms; in his book, Progress, Coexistence and Intellectual Freedom: Sakharov dismisses the notion of a necessary struggle to the finish between East and West, and foresees an eventual convergence of the socialist and capitalist worlds, an immense program of Soviet-American assistance to the underdeveloped countries, and the creation of a world government.102

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CHAPTER IV

Reactions and Criticisms

Reactions and criticisms to the new mathematics curricula took many forms. One of the leading critics was Morris Kline; he had sought reforms in mathematics education before the Sputnik launch had caused national attention to be focused on the problem.\(^1\) He perceived the direction which the reforms were taking as one which was adding new defects, and in addition, was not solving the old problems. He stated:

We have shown that the traditional curriculum is defective in a number of respects and that the new mathematics curriculum certainly does not remedy the defects of the traditional curriculum. In addition, it introduces new defects. Put roughly for the moment, the direction should be diametrically opposite to that taken by the new mathematics.\(^2\)

This general assessment was also voiced by seventy-five mathematicians who signed a memorandum entitled "On the Mathematics Curriculum of the High School," published in 1962, which stated that a more favorable climate existed for improvements in mathematics education, but:


It would, however, be a tragedy if the curriculum reform should be misdirected and the golden opportunity wasted. There are, unfortunately, factors and forces in the current scene which may lead us astray.3

Specific areas of the new curricula which were criticized can be classified in eight general areas. One of the most generally criticized aspects of the new programs was the emphasis which was placed upon formalism and upon the logical development of mathematics with a consequent loss of intuitive presentations of mathematics.

Morris Kline devoted two chapters of his book, *Why Johnny Can't Add*, to these topics, in which he stated:

> The logical approach is it is claimed ... also the pedagogical approach and the panacea for the difficulties students have had in learning mathematics.4

Every curriculum claims to teach the student how to discover results for himself. Discovery, contrasted with and opposed to the passive and uncritical acceptance of finished or polished statements of theorems and proofs, amounts to the creation or re-creation of mathematics by the student, possible with the guidance of the teacher....5

Kline drew some of his arguments from historical considerations of the development of mathematics as the following illustrate:

> ...during the centuries in which the major branches of mathematics were built up there was no logical development for most of it. Apparently the intuitions of great

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men are more powerful than their logic.\(^6\)

There is not much doubt that the difficulties the great mathematicians encountered are precisely the stumbling blocks that students experience and that no attempt to smother these difficulties with logical verbiage can succeed.\(^7\)

If mathematical education of the traditional type has suffered from the martinetts who imposed rote learning, the newer education will suffer more horribly from the rigor-mongers.\(^8\)

Such historical references were also used to criticize the stated or implied pedagogy of these curricula; he stated:

The rigorous development of a branch of mathematics is often so artificial that it is meaningless.\(^9\)

The great mathematicians who have taken an interest in pedagogy always stressed that strict logical presentations are entirely subordinate to the substance, which is learned intuitively. Rigor may save mathematics, but it will surely lose the pupils.\(^10\)

The purpose of this axiomatic structure was to satisfy the needs of professional mathematicians who insist on deductive structure, but it was never intended as a pedagogical approach.\(^11\)

Kline also criticized potential misunderstandings of mathematics which students could obtain; he wrote:

Many teachers, having delivered a series of such theorems and proofs, walk out of their classrooms very satisfied with themselves. But the students are not

\(^6\)Kline, Why Johnny Can't Add, p. 47.

\(^7\)Kline, Why Johnny Can't Add, p. 49.

\(^8\)Kline, Why Johnny Can't Add, p. 72.

\(^9\)Kline, Why Johnny Can't Add, p. 68.

\(^10\)Kline, Why Johnny Can't Add, p. 71.

\(^11\)Kline, Why Johnny Can't Add, pp. 50-51.
satisfied. They have not understood what was going on and all they can do is memorize what they heard.12

The insistence on a logical approach also deceives the student in another way. He is led to believe that mathematics is created by geniuses who start with axioms and reason directly from the axioms to the theorems.... It is intellectually dishonest to teach the deductive approach as though the results were acquired by logic.13

Many of these same developments in mathematics education occurred in France where, in addition, there had been the strong, indirect influence from a group of mathematicians, the Bourbakists, who had produced a codification of mathematics for university study and for reference; this work was written entirely in the axiomatic-logicist format.

One of the leading critics of curricula changes which occurred there was the mathematician René Thom. In a reaction to the application of formalist methods at the pre-university level, Thom stated:

In the early development of a child, explicit and deductive learning play absolutely no part: when learning to walk, it would be more of a hindrance than a help to understand the anatomy of the leg....14

The real problem which confronts mathematics teaching is not that of rigor, but the problem of the development of 'meaning' of the 'existence' of mathematical objects.15 (Original italics.)

In Great Britain, where a stronger traditional tie of

12Kline, Why Johnny Can't Add, p. 53.
13Kline, Why Johnny Can't Add, pp. 54-55.
mathematics with applications had existed, a vituperative attack was made on curricula reforms by J. M. Hammersley. He stated in his caustically titled article:

There is also a belief in some quarters that mathematics is a dendritic discipline: that you must proceed sequentially from the root to the branches... Even if such a doctrine is met on the score of logic (and this is arguable), it is didactically bad.\(^\text{16}\)

...one of the richest sources of mathematical discovery lies in the very imprecision of one's ideas and in ambiguity and confusion over notation.\(^\text{17}\)

Structure is the skeleton which ossifies a subject for all except a few academics. It codifies, and creates a museum piece for appreciation but not for touching or for use.\(^\text{18}\)

The important thing was to gain early familiarity with the mathematical language: to learn, as it were, French as a child and prattle about childish things in French rather than struggle with it at a mature age...\(^\text{19}\)

Max Beberman, the founder of the UICSM Project, added his caution towards a tendency in this direction; he stated:

...there is the terrible hazard of thinking that any approach which emphasizes logical explanations leads to understanding...\(^\text{20}\)


\(^{17}\)Hammersley, p. 71.

\(^{18}\)Hammersley, p. 72.

\(^{19}\)Hammersley, p. 72.

Another mathematician who had worked extensively in mathematics education, H. Griffiths, reminded his readers that, "real mathematics is created by argument and discussion, shunning dogmatic, authoritarian assertion."\(^{21}\) Adding to this list is C. V. Newsom, who cited Felix Klein in the following:

Now, however, we are learning that good mathematicians had too free a hand in the development of the programs. The words of Felix Klein, wise mathematician and pedagogue, were ignored; he wrote: 'The presentation (of mathematics) in the schools should be psychological and not systematic. The teacher, so the speak, must be a diplomat. He must take account of the psychic processes in the boy in order to grip his interest; and he will succeed only if he presents things in a form intuitively comprehensible...'\(^{22}\)

The historian of mathematics, Kenneth O. May, in a review essay of a Russian history of mathematics book, stated:

Indeed the recent passion for proof and introspective obsession with foundations has led to mathematics being taught in a way that falsifies its nature.\(^{23}\)

Another large area of criticism with the new curricula centered about its lack of attention to applications of mathematics; they argued that mathematics was taught too much as an isolated subject divorced from its roots. The following citations support this position. For example, Kline stated:


\(^{22}\) C. V. Newsom, "The Image of the Mathematician," AMM, October (1972) 880.

Mathematics is not an isolated, self-sufficient body of knowledge. It exists primarily to help man understand and master the physical, the economic and the social worlds. It serves ends and purposes.24

By neglecting motivation and application the pedagogues have caused mathematics education to suffer... Some of the poorest teaching of mathematics is traceable to teachers treating the subject as though it had no connection with anything beyond its technical confines. What is especially grievous, then, about the teaching of mathematics, traditional or new, is not that the teachers do not know what they are teaching but that they do not know and so cannot show pupils why mathematics is vital.25

Lucien Kinney struck the same note when he wrote his observations in the following:

Emphasis on the structure of mathematics, however, has led to de-emphasis on applications. Such applications as occur are largely from the physical sciences...26

Andrew Gleason, one of the mathematicians who had been a member of the steering committee for the 1963 Cambridge Conference on School Mathematics, wrote an acknowledgement, five years after that conference, of the close relationship between science and mathematics. He stated:

Modern science is inextricably bound up with mathematics, while mathematics owes many of its fundamental ideas to science. Certainly neither subject would have advanced to its present state without the other.27

24Kline, Why Johnny Can't Add, p. 177.
25Kline, Why Johnny Can't Add, p. 183.
In the early years of the curriculum projects, Vincent Glennon had voiced concern about the possible misdirection which such reforms could take. In an editorial preface to an issue of Educational Leadership, devoted to the changes which were taking place in mathematics education, he wrote the following forecast of potential difficulty:

Finally, in the school program that places an undue emphasis on arithmetic as a science of numbers, or pure game, unrelated to life, the program will quickly lose the apparent initial interest in abstract mathematics and become a sterile subject.

...the one emphasis that is having the greatest impact on the school mathematics program today is the science of number, of pure game, point of view.28

The original impetus given to the reforms of the sixties had been America's apparent technical lag with Soviet Russia, and the consequent need for better training of Americans in mathematics; this need was apparently not fulfilled. In a 1972 article, Newsom wrote the following:

Only a few weeks ago, the head of a large research laboratory expressed his dismay that so few people who had specialized in mathematics had any serious background in a physical science, in economics, in business, or in any other area where mathematics has become important. Then he said, 'We are living in an interdisciplinary world. Too many mathematicians have separated themselves from that world.'29

Further criticism of the mathematics curricula reforms centered about the specialized language which had been in-

29Newson, p. 880.
roduced into the new curricula. One critic, Kline, devoted an entire chapter of the previously mentioned book to this area alone. He stated there, for example, that:

In keeping with their aim to secure precision, the modern texts define carefully every concept that is used. The consequence is an immense amount of terminology... An actual count of the number of terms introduced in... algebra and geometry shows that several hundred terms are introduced in each one. Of course students are expected to learn and use these terms.30

Terminology, particularly pretentious terminology, is no substitute for substance. In view of the emphasis on terminology the reformers evidently believe that giving names to things automatically confers power over them.31

Hammersley concurred with these sentiments; he stated in his article that "A serious weakness in modern mathematics is its pre-occupation with mathematical jargon which foster the patter mentioned above [in this article]."32

Rene Thom's criticism of this matter is more philosophical; his comments are addressed in particular to the language of set theory, as the following illustrates:

Everything considered, the excessive optimism bred by the use of set theory symbols has its roots in a philosophical error.... Modern protagonists of set theory should realize that this theory is insufficient to account for even the most elementary deductive steps of ordinary thought. An example follows illustrating a mix of semantic fields from set theory.

30 Kline, Why Johnny Can't Add, p. 80.
31 Kline, Why Johnny Can't Add, p. 83.
32 Hammersley, p. 67.
One of the fundamental constraints imposed by accurate thought is precisely the avoidance of mixing distinct semantic fields. This mixing has a name—delirium.33

Both Hammersley and Kline proposed that many of the excesses could have been avoided had teachers of mathematics critically questioned the proposals of the mathematicians. Kline stated:

One might think that the pedagogical weakness of the college professors would be offset by the high school teachers and the education professors. Surely the latter two groups should know what can be taught to elementary and high school students and what might motivate these young people. But the high school teachers and the professors of education were overawed by the mathematician....It seems fair to say on the basis of what transpired that in every curriculum group the college professors dominated the educators and the schoolteachers. The educators bowed down to the idols, not knowing that most had feet of clay.34

Hammersley made a similar observation in his article; he wrote:

...some schoolteachers accept modern mathematics for the schools, not for any inherent merits which they themselves perceive in it, but because they have been led to understand that it is highly regarded by university mathematicians who (so the argument runs) ought to know best.35

From the perspective of the idea of Progress, Hammersley and Thom appear to have adhered to the liberalist form of this idea. Hammersley in fact directly criticizes the

34 Kline, Why Johnny Can't Add, p. 163.
35 Hammersley, p. 66.
new mathematics curriculum for holding to a socialist form of the idea of Progress; he stated that, "the traditional syllabus demands greater achievement as far as examination standards go." 36 He then offered the suggestion that this may have been done so as not to frighten teachers from the new syllabus. His position was further clarified when he stated:

But it may also be the case that the proponents of modern mathematics or new syllabuses, in trying to realize a numerate democracy, have had to lower standards and dilute the mathematical content to meet the necessarily slighter attainments of their enlarged clientele. 37

This comment was directed toward British reforms, but because of the observations which were made in Chapter III of this study, they would hardly be appropriately applied to the SMSG Program; by extension, however, they might have reflected Hammersley's views toward the UICSM Project.

Rene Thom also appears to have held to a liberalist form of the idea of Progress; he stated the following:

Some affirm that the use of set theory permits the entire renovation of mathematics teaching and that, thanks to this change, the average student will be able to achieve mastery of the curriculum. Needless to say, this is pure illusion....As soon as one comes face to face with real mathematics (i.e., real numbers, geometry,

36 Hammersley, p. 67.
37 Hammersley, p. 67.
functions), one redisCOVERs that there is no royal road and that only a minority of students are capable of fully understanding the material.\textsuperscript{38} (Italics mine.)

Although this was neither a very strong nor clear statement of belief in a liberalist form of the idea of Progress, his position in this regard is more clearly discernable in the following "goals of teaching", which Thom listed:

Some still persist in thinking that, in one form or another, one of the goals of teaching is selection, that is to say, determining the aptitudes of each student and developing them to the maximum, with particular emphasis on the gifted student....None of this is conceivable within a framework of "useful" studies, where all the elements, included because of their technical utility, are dogmatically taught and where scholarly excellence is defined as exact and rapid memorization of given material. Only those topics which have a quality of "play" have educational value...\textsuperscript{39}

Both Hammersley and Thom were Europeans reacting to curricular changes in Great Britain and France respectively, countries where education has traditionally been elitist. It appears that these reactions were to perceptions of reform movements motivated by a spirit similar to the motivating spirit of the UICSM Project rather than that of the SMSG Project.

Reactions from American critics were just the opposite from the ones noted above. American critics attacked elitist tendencies in curricula projects; they did so from

\textsuperscript{38}Thom, p. 689.

\textsuperscript{39}Thom, p. 696.
a spirit consistent with the socialist form of the idea of Progress. For example, Morris Kline wrote:

Before we can consider the approach and contents of a suitable elementary and high school curriculum, we must consider the objectives or goals of these stages of education. On the elementary school level there can be no consideration of preparation for college. Only a small percentage of these students will go to college. Even on the high school level, from which about fifty percent of present day graduates enter college, the students are still ignorant of the nature and importance of the various subjects they are asked to take. For many subjects, including mathematics (beyond arithmetic) the high school offering is an introduction. Moreover, very few of the college-bound students will specialize in mathematics....Put negatively, there should be no attempt to train professionals in mathematics and little concern for what future study in mathematics may require.\(^\text{40}\)

In the following passage, where he evaluated the anti-social motives of some mathematicians, Kline made his position clearer:

The creative mathematician may derive some emotional values such as satisfaction of the ego, pride in achievement, and glory--values none too noble in any case--but the student cannot derive even these values from the study of the subject....Intellectual challenge may arouse some people, but one could hardly refute those who would maintain that the challenge of building a more humane society and securing honest leaders are more important.\(^\text{41}\)

These are fairly clear statements indicative of a belief in the socialist form of the idea of Progress; their spirit is indicative of a critical attitude toward the liberalist form of the idea of Progress which motivated some of

\(^\text{40}\)Kline, Why Johnny Can't Add, pp. 173-174.

\(^\text{41}\)Kline, Why Johnny Can't Add, p. 180.
the reforms.

The memorandum mentioned earlier, which had been signed by seventy-five mathematicians, also indicated this same frame of mind. Under the first guideline, which they considered to be fundamental, is listed the following:

The mathematics curriculum of the high school should provide for the needs of all students: ...While providing for the other students the curriculum can also offer the most essential materials to the future mathematicians. Yet to offer such subjects to all students as could interest only the small minority of prospective mathematicians is wasteful and amounts to ignoring the needs...of society as a whole.\(^4\)

Andrew Gleason, in a report on the Cambridge Conference of 1963, appears to have agreed with this position. He wrote:

We agreed that education in science and mathematics was not to be thought of in elitist terms. We were not trying to "beef-up" the curriculum in an effort to see how fast we could force-feed our scientifically talented youngsters. Quite the contrary...it is essential to make them meaningful to every school child.\(^3\)

The Report of the National Advisory Committee on Mathematical Education (NACOME Report) entitled "Overview and Analysis of School Mathematics Grades K-12", acknowledged that one of the criticisms of the new curricula was that it had failed to meet the needs for basic mathematical literacy of average and low ability students."\(^4\) A change

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\(^4\)Cited in Kline, Why Johnny Can't Add, p. 136.

\(^3\)Gleason, p. 119.

in national priorities was cited as a possible reason for adopting a different attitude. The report stated:

Despite legitimately changing national priorities, where humanities or sociological concerns supercede the technological, mathematics remains an ever-growing requisite for...the society we have become.45

These new demands of society were reflected throughout the report in such matters as concern for women and for minorities in mathematics, in concern for the affective domain in mathematics education and in concern for the uses and abuses of testing procedures. The report appears to have been written in a spirit consistent with a moderate belief in the socialist form of the idea of Progress, a spirit which was perhaps reflective of the changed national priorities which the report itself noted.

In conclusion, it has become clear that a belief in the socialist form of the idea of Progress was held by most American critics of mathematics curriculum revisions, whereas the liberalist form of the idea was held by European critics; each was reacting to a program written in the opposite spirit. The idea of Progress, although not always explicitly stated, permeated the substance of formed curricula as well as the motives of its critics. In none of the material which was studied was there evidence of other forms of the idea of Progress than those forms which had been developed by the mid-nineteenth century.

45 NACOME Report, p. 141.
CHAPTER V

Summary, Findings, Implications and Recommendations

Summary

This study was undertaken to determine whether the idea of Progress which developed during the eighteenth and nineteenth centuries influenced mathematics curriculum development in public education. Further effort was made to explore the following related sub-problems: (1) If such an influence existed, how was this influence manifested? (2) If such an influence did not exist, then why not? (3) Were social goals, as they related to the idea of Progress, attained through the mathematics curriculum? (4) In what manner was Progress in mathematics education defined?

This study was limited to the history of the development of the idea of Progress in Western thought and to the influence this idea had in mathematics education, especially in public education, in the United States. The writer assumed: (1) that there had been changes in the idea of Progress in western civilization, and that these changes were manifested in the mathematics curricula of public schools, (2) that changes in metaphysical theories had influenced curricula changes and that such changes could be observed within mathematics curricula.
1. Prior to the eighteenth and nineteenth centuries, the intellectual ambiance was dominated by four major concepts: (1) a veneration of the past; and (2) the consequent identification of change with degeneration; (3) the belief that superior intelligent forces, God or gods, controlled daily human activity; (4) time was cyclic. These four concepts precluded the development of the idea of Progress.

The science and mathematics which had developed did not transform life, since these developments were considered as philosophical speculations to provide a synthesis for reality. Mathematics was used principally for calendrical determination of special dates or merely for purposes of counting. Most educated people were totally ignorant of mathematics beyond an ability to recognize numerals.

An emphasis upon deductive logic, a result of Greek developments in axiomatic geometry, was a mode of thought which persisted through the centuries, principally in theological and legalistic studies, but not in mathematics, its root source.

2. The rise of scientific thought, brought about by the re-introduction of Greek texts from Arabic sources, led to the development of the idea of Progress.

A greater interest in mathematics and science developed, but it was not until the dominant philosophical perspectives were challenged by such men as Francis Bacon,
Rene Descartes, Gottfried Leibnitz, and Isaac Newton that barriers to further extension of knowledge in these fields was possible. The availability of Greek texts in mathematics served, at first, only to chain men's minds closer to the past. These challenges eventually led to a decreased interest in metaphysics for some of the new scientist-philosophers. Pre-university education, through the eighteenth and nineteenth centuries, became more structured and focused on literary studies; little attention was given to mathematics and science beyond its usefulness in practical applications in mechanical arts and commerce.

3. It was not until the late nineteenth century that the idea of Progress which developed during the eighteenth and nineteenth century had any influence on mathematics education in the United States. At that time, conflicting forces were evident in the curricula, each displaying one of the two principal forms of the idea of Progress which developed in mid-nineteenth century. The liberalist form of the idea of Progress was dominant during the early part of the twentieth century; competitive examinations in mathematics, indirectly based on Chinese models, were introduced for selection of students. John Dewey represented a countervailing force to these pressures. It is interesting to note that, although the Chinese examination system was idiosyncratic, many factors which led to its abolition in China during the same period that America was beginning
to adopt competitive examinations, surfaced in America during the twentieth century.

4. Neither the epistemological nor the educational works of Whitehead were influential in the developments of mathematics education reform during the fifties and sixties.

The process-psychology developed by Jerome Bruner was less influential in this movement to teach mathematical structures; the real motive force for this approach to mathematics education was the dominant formalism-logicism philosophy of mathematics accepted by many mathematicians who worked in these curricular projects. The UICSM Project was motivated by the socialist form of the idea of Progress, although the justification for its work was framed in the liberalist form of the idea. The SMSG Project was thoroughly infused with the spirit of the liberalist form of the idea of Progress.

5. The major American critics of the mathematics curriculum projects of the fifties and sixties held a belief in the socialist form of the idea of Progress and reacted to programs written in the liberalist spirit of the idea.

Although the socialist form of the idea of Progress was held by American critics, the liberalist form of the idea was held by most European critics; each was reacting to programs written in the opposite spirit.
Implications

The forces present in an accepted social philosophy directly influence educational philosophy and ultimately these forces influence the curriculum of mathematics education. As immune to such philosophies as many may believe mathematics education to be, historical facts provide evidence to the contrary. Mathematics education continues to play an important social role, whether it wants the burden of such a role or not. The result of what happens in a mathematics classroom in our society very often determines the future of many humans; the implications for careful consideration of the mathematics curriculum and thoughtful control of the purposes to which mathematics education is directed are patent and important, as this final quotation clearly illustrates:

Mathematics now plays an important role in the process of determining who will get what, in part because it is considered to be especially difficult, especially objective, and especially useful to contemporary man. Tests of mathematical proficiency are regarded as a fair and efficient means of eliminating large numbers of superfluous aspirants to choice degrees. Unfortunately, in the process of testing larger and larger numbers of terrified candidates for success, we are telling our students that performance is the name of the game. If they emerge from their courses with any interest in mathematics at all, it will not be a thoughtful interest. It is bad form to ask what it all means. Since reality is so elusive, models are the order of the day, and truth is relative to the model, a kind of super-chess. ¹

Recommendations

1. It is important that the social implications of widespread testing in mathematics be studied. The uses and abuses of testing, within a model of social philosophy, have important implications.

2. In light of the apparent lack of assertiveness on the part of mathematics educators when confronted by subject-matter experts during the curriculum reforms of the fifties and sixties, it is important that a study to determine the reasons for this apparent lack of assertiveness be conducted. Society is beginning to rely more heavily on expert opinion to direct itself, hence it is important to study factors which may be generally operative in such situations where two groups of experts meet, especially in cases where each group may hold to different social philosophies.

3. The factors which influenced mathematics education, both during the eighteenth century and during the twentieth century, had a connection with Oriental modes of thought. With increasing attention being given in America to Eastern modes of thought, it is important to determine what the implications of such a mixture of mind-sets are for education.
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