An assessment of the cognitive development of concept of angle in children at the third-grade and fourth-grade levels.

Sara H. Robinson
University of Massachusetts Amherst

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AN ASSESSMENT OF THE COGNITIVE DEVELOPMENT OF CONCEPT OF ANGLE IN CHILDREN AT THE THIRD-GRADE AND FOURTH-GRADE LEVELS

A Dissertation Presented
By
Sara Hudson LaClaire Robinson

Submitted to the Graduate School of the University of Massachusetts in partial fulfillment of the requirements for the degree of

DOCTOR OF EDUCATION

May 1986

School Of Education
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DEDICATION

To my family: Donald, my husband; Gayle, Paul, and Tina, my children, I dedicate this study. Their love made it all possible.

"Love...... endureth all things."

I Cor. 13:7
ACKNOWLEDGMENT

I would like to thank all who faithfully gave me support and courage to pursue my study to completion. It was the friendly encouragement from friends as well as the guidance from professionals that strengthened me to meet my goals and bring my research to fulfillment.

I would like to express special thanks to my chairman, Professor William J. Masalski. He was a source of continual support and direction throughout the project. His suggestions and advice encouraged my endeavors as well as adding sustenance to my study. I am truly grateful for his time, interest, and contributions that helped me bring this project to completion.

I would also like to express my thanks to my committee members, Professor Richard D. Konicek and Professor George H. Knightly. Dr. Konicek was always a source of strength and new insights. His support and enthusiasm provided energy for the study. Dr. Knightly's expertise in mathematics proved invaluable. My thanks is not only for their given support, but also for their encouragement to pursue the project in further research.

My study would have been impossible except for the willing and cooperative students that served as my subjects. To them I am extremely grateful.

I wish to express my thanks to others at the Bement School who saw me through this project. To past Headmaster John N. Butler, who saw the beginnings, and to present Headmaster Peter P. Drake, who witnessed the completion, I express my thanks.
My greatest indebtedness and appreciation are extended to my family who through their love allowed me the opportunity to embark on such a project. My husband, Donald, so very faithfully upheld me throughout the process and even to the end sacrificed hours for proofreading and documentation. Tina, who lovingly allowed me time, is glad Mommy is finished. To Paul and Gayle, who lovingly helped with household duties and patiently waited for their times, I express my love and thanks. I thank my parents for their care and understanding that made the process possible. To my family I give my love and thanks!
ABSTRACT

An Assessment of the Cognitive Development of Concept of Angle in Children at the Third-Grade and Fourth-Grade Levels

May 1986

Sara H. Robinson, B.A., University of Massachusetts
M.A., University of Massachusetts
Ed.D., University of Massachusetts
Directed by: Professor William J. Masalski

Observation and assessment was made of the cognitive development of concept of angle in children at the third-grade and fourth-grade levels. A clinical interview approach was used to probe and elicit cognitive thought and assess the developmental level of understanding of each student. Two sessions were conducted at least four months apart. Fourteen students participated in the first session and a subset of ten students participated in the second session.

Each session used three methods for observing and assessing each student's cognitive development of concept of angle. The three methods were designed by the researcher incorporating three different modes of presenting angles: visual perception, manipulative construction, Logo computer language programming. Observation was made as to the effect length of arms of an angle and/or rotation of an angle had on correct assessment of angle measurement. The subjects were scored according to their responses on each of the methods. A total score was derived by summing the scores on the three methods. Using
the total scores, cognitive developmental levels of concept of angle were established and named "Robinson" Levels of Development.

"Robinson" levels were compared with Piagetian developmental levels for concept of angle. Piagetian levels correspond to age development. "Robinson" levels do not correspond to age development. There was no significant correlation for "Robinson" levels of development with age. A significant correlation was found for "Robinson" levels of development with the total mathematics achievement scores as determined by a given standardized achievement test. The researcher concluded that total mathematic achievement scores are a good indicator for identifying students' developmental levels using "Robinson" developmental levels. Age of students did not clearly indicate students' developmental levels.

The "Robinson" cognitive developmental levels of concept of angle show promise of providing a useful means of identifying a student's developmental level. Once the developmental level is determined appropriate curriculum at the individual developmental level can be provided.
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CHAPTER I

INTRODUCTION

General Introductory Remarks

The focus of this study was to observe and assess the cognitive development of concept of angle in children at the third-grade and fourth-grade levels (8-10 years old). The study used a three task method approach to classify cognitive development of concept of angle. Performance on each of the three task methods involving angles determined each child's developmental level. The levels determined by the study were compared to Piagetian levels that Piaget and his colleagues established and assessed according to ages of children. The levels determined by this study were correlated with the students' ages and with the students' total math achievement scores as determined by the Stanford Achievement Tests (see Chapter IV), using Kendall Correlation Coefficients. Tests of independence using Cochran Q Test were used to establish relationships between test methods to determine if the three test methods designed by the researcher were testing for the same concept. The researcher then projected recommendations for the curriculum for the study of angles for third and fourth graders and cited areas for future research.
Statement of the Problem

The issue at hand is to understand cognitive development of concept of angle and to assess the appropriateness of material published for the study of angles in the curriculum for third and fourth graders. Recently published texts from several publishing companies include angle as part of the curriculum for these grades. The review of a representative sample of textbooks (Heath; Houghton Mifflin; Silver Burdett; Scott, Foresman; Addison-Wesley) showed use of a deductive approach. In these texts a definition is sometimes given and specific examples are given to be identified as angles. Details of material presented in these texts are presented in Chapter II.

In general, the texts use a lecture method to tell the students what an angle is rather than methods of discovery. Here the question of appropriateness of material and methodology again arises. The study addresses this question by assessing the cognitive development of concept of angle at the third- and fourth-grade level and by making application for the curriculum.

It is also important to the study to understand the involvement of Logo in the cognitive development of concept of angle. The Logo computer language involves the use of angles and angle measurements. A triangle called the "turtle" pivots according to left or right turn commands. The command input is followed by a number corresponding to measurement in degrees. The child's conceptualization of the turtle's movement has not yet been determined (The Journal for Research in Mathematics Education, 1984). Piaget attests that once concepts of angle are developed, angle measurement is readily attained
(Piaget, and Inhelder, 1967). Therefore, it is important to understand the child's cognitive development of concept of angle in order to determine the child's readiness for angle measurement.

Textbook series and Logo instruction show that children at the third- and fourth-grade level are being instructed in the study of angles. The appropriateness of angle instruction is dependent on the conceptualization of angle. If the cognitive development of concept of angle is unknown, then a child's readiness for instruction is also unknown. It may be that present instruction and lesson activity are not only inappropriate but inadequate to prepare the student for the understanding of angles.

Statement of Purpose and Rationale

The purpose of this study was to investigate and evaluate the cognitive development of concept of angle found in children at the third- and fourth-grade level (8-10 years old). Children are being instructed in the study of angles at least as early as the third-grade level. The material and instruction as presented in the third- and fourth-grade textbooks are limited in scope. The effect Logo has on the understanding of angles is not determined.

There is a need to recognize the cognitive development of concept of angle at this level so that instruction in angles is appropriate to the child's readiness. The intent of this study is to contribute information about children's
cognitive development of concept of angle so that appropriate information is
presented at the appropriate level.

Interpreting research done by Piaget and his colleagues, Richard
Copeland reported ages at which mathematical concepts develop (Copeland,
1984). The development of these concepts have important implications for
teaching methodology (Ibid.). It is implied that these reported mathematical
concepts should be developed by the child in order for instruction in that area to
be effective. However, these reports do not involve the study of angles.

In *The Child's Conception of Space* by Piaget and Inhelder (1967), the
authors report the finding of research using similar triangles to show
"conservation of angles", more frequently referred to as "equality of angles" (see
Chapter II). Piagetian research used triangles to observe children's
developments in assessing similarity of triangles. The first method used
inscribed triangles and the second method used cut out cardboard triangles. To
determine the similarity of triangles using the two methods, the first method
used parallelism of the sides to determine if two triangles were similar. The
second method used equality of the angles by superimposing the angles of the
cardboard triangle (Piaget and Inhelder, 1967). Both of Piaget's methods
produced similar results (see Chapter II). Piaget concluded that a concept of
angle cannot precede that of parallelism (Ibid.). Piaget noted that at ages 7-8
years spacial perception slowly improves and continues to do so until maturity
(Ibid.). Piagetian research found that the similarity of triangles is not based on
parallelism but on equality of angles (Ibid.). Piaget noted a psychological
development from the straight line to parallels to equality of angles to similar
triangles (Ibid.). Piaget related ages of children to each stage of development.
Of particular note are the children beginning at age 6 years to 7 years, 6 months (stage IIIB) who compare angle but with no degree of accuracy (Ibid.). The children in the next stage (IIIA), ages 7 to 9 years, spontaneously superimposed the triangles and discovered the measure of the angles to be equal and independent of the length of the sides of the cardboard triangles. Determining the similarity of triangles on the basis of equality of angles was not mastered until the age beginning at 9 years (stage IIIB) (Ibid.).

Piaget's developmental levels determined by methods using triangles suggest the ages 8 to 10 years for investigating the concept of angle. The presented study focused on the angle isolated from any geometric shape to determine cognitive development of concept of angle.
**Definition of Terms**

Angle: An angle is the union of two rays which have the same endpoint, but do not lie on the same line (Moise, 1963). A pictorial representation of an angle is shown in Figure 1.

![Figure 1: A Pictorial Representation of an Angle](Moise, 1963)

Length of the arms: The independent lengths of two segments which have a common endpoint and form a pictorial representation of an angle is the length of the arms.

Concept of angle: The comprehensive understanding of the definition of angle, thus realizing that the orientation of an angle and/or length of the arms of an angle do not affect the angle.

Measurement of angle: To each angle there corresponds exactly one real number, called its measure (Keedy, 1981).

Congruent angles: Angles with equal measure are congruent.
Approach and Organization

An exploratory clinical interview approach was used to: (1) investigate the cognitive development of concept of angle of third- and fourth-grade students; (2) assess the thought process and; (3) identify developmental levels. The researcher interviewed on a one-to-one basis five third-grade and nine fourth-grade students, totaling fourteen subjects.

The visual representations of angles used segments in order to depict angles as viewed in reality. Because the edge of objects, such as desks, roof lines, letter constructions, and clock hands have an end, the terminology "length of the arms" is used to label this visual representation.

Three task methods were used. Method I used five visual comparisons of paired angles. The paired angles displayed various combinations of angles. Card 1 showed congruent angles in mirror image with the length of the arms shorter on one of the angle representations. Card 2 showed non-equivalent angles, oriented in the same direction, and with length of the arms equivalent in length. Card 3 showed right angles with differing orientation. Card 4 showed acute congruent angles with differing orientation and with varying length of the arms. Card 5 showed obtuse congruent angles with differing orientation and with varying length of the arms. Method II used the construction of angles by manipulating chenille wires. The subjects were shown a pair of right angles with varying length of the arms, followed by probing questions and the assignment for the subjects to construct angles with three other chenille wires,
each varying in length. Method III used Logo. One at a time, three angles were displayed on the computer monitor. The subjects were asked to command the "turtle" to draw their own angles of the same measurement elsewhere on the screen. Chapter III provides specific design and procedures. At least four weeks after the first set of three methods were used to assess the cognitive development of concept of angle, a second session using three methods similar in tasks was used in order to provide a test for reliability.

Contributions to Education

The intent of this study is to make a purposeful contribution to the curriculum and methodology for the teaching of angles within the realm of education. A student's developmental level of concept of angle needs to be assessed and understood before educational objectives concerning angle can be rightfully expected. This study provides an assessment tool and the established developmental levels provide a guide for determining the appropriate expectations for the study of angles at the third- and fourth-grade levels. Computations with Logo can have better constructive use if the child's cognitive development of concept of angle is known. The study uses the established developmental levels to make curriculum and methodological recommendations for education.
Limitations

The small and exclusive population sampling was a limitation to the study. Because of the interview process a limited number of subjects was used. Because of the school's admission policy, the students have an I.Q. of 100 or above. Therefore, the results of the study cannot be generalized to a wide population even within the tested age group of 8 to 10 year olds. However, the study indicates strong trends and provides a tool for assessment of cognitive development of concept of angle and establishes developmental levels as a reference in determining appropriate educational expectations.

This study was not a test of measurement of angle or measurement of the length of the arms. Logo involves these measurements, but cognitive development, not accuracy of measurement, was the concentration of the study. The other methods used did not call for exact degrees or lengths, only perceptions of varying degrees and lengths were probed for assessment purposes.

This study was limited to angles in two dimensional space. Projection of an angle was limited to that plane.
In assessing the cognitive development of concept of angle, an exploratory clinical interview approach was used to probe and provide data for identifying developmental levels in children 8 to 10 years old. The third and fourth graders at Bement School in Deerfield, Massachusetts were interviewed. The interview process took place within the established definitions and parameters. The limitations of the study are recognized and taken into consideration.

Chapter II reviews the literature that gives support to the study and that identifies the need for such a study. Chapter III gives the design and procedure for the clinical interviews of the study. The three methods are detailed as to their respective tasks. Perception, manual construction, and Logo computer programming are the general categories of the three methods. Chapter IV follows the three methods as a means of organization. Responses made by the subjects on each of the tasks within the methods were reported as exemplary remarks and the responses were identified as to the level of cognitive development of concept of angle. The chapter reports the results of the study. The results were analyzed and an assessment as to developmental level was made. Developmental levels were statistically correlated with students' ages and students' total mathematics scores determined by the Stanford Achievements Tests to determine significances. The chapter reports the reliability as determined using data from the first and second sessions.
Chapter V relates and compares the developmental levels of Piagetian research to developmental levels established in this study. Chapter VI makes recommendations for education and identifies areas for future research that would enhance the study of angles. Chapter VII summarizes the study.
CHAPTER II

LITERATURE

Introduction

The researcher used three main areas of concentration for searching literature pertinent to the study. They were Piagetian resources, Logo based resources, and mathematical textbooks for the third- and fourth-grade levels. Studies concerning angles done by Piaget and his colleagues is primary to the presented study. Other studies involving angles are limited. The researcher did an ERIC search and reviewed abstracts and indexed dissertations presented to American universities from 1861-1984. The search revealed one study relating to the presented study. Logo based resources were searched and, again, limited study has been done with respect to angles. Search in this area revealed the need for studies to be done that assess the claims made by that programming language. Textbooks and resource materials for the third- and fourth-grade levels are sources that were reviewed and are cited in terms of their approach and methodology in studying angles.
Jean Piaget and his colleagues observed children in measurement settings over an extended period of time. Piaget formulated developmental stages of the child from his results. Richard Copeland and other educators have interpreted these developmental stages, making implications for education (Copeland, 1984).

Piaget's studies of children led him to formulate four major psychological stages of development (Ginsburg and Opper, 1969). The Sensorimotor Stage takes place generally from birth to 2 years. In this stage the child learns direct interaction with the environment. The Pre-operational Stage takes place generally from 1 1/2 years to 7 years. The child is now able to use symbolism. Copeland notes that "for some mathematical concepts children do not leave the preoperational stage until 9 or 10 years" (Copeland, 1984).

This observation shows how general the ages are in each stage of development, as well as the developmental lag that mathematical concepts may have. The Concrete Operational Stage takes place generally from 7 to 12 years. In this stage logical thought takes place and is based on physical manipulation of objects. It is at this stage that conservation of measurements shows development. The Formal Operations Stage takes place generally from 11 years to adult. At this stage the child is able to reason with symbols rather than rely on objects in the physical world as a basis for thinking (Ginsburg and Opper, 1969).

Piaget further details these four developmental stages and classifies stages and substages labeled by Roman numerals and letters. It is the finding
relating to these stages and substages that have greatest relevancy to this study.

In *The Child's Conception of Space* by Piaget and Inhelder (1967), researchers used similar triangles to investigate conservation of angles. Piaget notes that from the geometrical standpoint the group of similarities ensures the conservation of angles. Piaget found conservation of angles as an offshoot of conservation of parallels. Since his studies showed that the child keeps sides parallel when using the rhombus, Piaget next studied how the child perceived the similarity of inscribed triangles using parallelism of their sides. He then studied how the child perceived the angles as being equal (Piaget and Inhelder, 1967). The term "conservation of angles" is then the full conception of the ideas of equality between angles. The actual term, however, is used infrequently and the concept is more frequently referred to as "equality of angles."

Piaget's research used triangles to observe children's developments in assessing equality of angles. His first method used inscribed triangles and his second method used cut out cardboard triangles. To determine the similarity of triangles of the two methods, the first method used parallelism of the sides to determine if two triangles were similar. The second used equality of the angle measurements by superimposing the angles of the cardboard triangles (Piaget and Inhelder, 1967).

Piaget's Stage I is the development of children less than 4 years old and no questions were asked at this level since children cannot even draw a simple geometric figure (Ibid.). Using Method I, Piaget found that at Stage II the child showed no thinking in terms of angles. Stage IIA showed that no parallelism
existed and Stage IIB showed a beginning of parallelism (Ibid.). At Stage IIIA
the children established parallelism between sides of triangles but angles were
not referred to explicitly, in fact, only two children spoke of angles on their own
accord (Ibid.). At Stage IIIB the children showed a beginning of elicitations
showing relationships of measurements (Ibid.). In Piaget's Stage IV Formal
Operations take place.

The ages of the children questioned correspond to the substages as follows:

IIA: from 4 years to 5 years up to 6 years, 6 months
IIB: 6 years to 7 years, 6 months
IIIA: 7 years to 9 years
IIIB: 9 years to 9 years, 6 months

(Ibid.)

Piaget's Method II, using cardboard triangles to show comparison of
similar triangles, produced similar results. The children in Stage IIA compared
absolute dimensions of the cardboard triangles in determining similarity. The
height and length of sides were the criteria with no attention to angles. The
children in Stage IIB compared angle but with no degree of accuracy (Ibid.).
The children in Stage IIIA spontaneously superimposed the triangles and
discovered the measure of the angles may be equal, independent of the lengths
of the sides of the cardboard triangles, but determining the similarity of triangles
on the basis of equality of angles was not mastered until Stage IIIB (Ibid.).
With this study of triangles and previous spacial studies, Piaget concluded that a concept of angle cannot precede that of parallelism (Ibid.). He noted that at ages 7 to 8 years spacial perception slowly improved and continued to do so until maturity (Ibid.).

Piaget found from the responses of children in Stage III that there is a psychological development showing an established conservation of parallel lines to assess similarity of triangles which is followed by an attention to angles that firmly establishes whether or not triangles are similar. Thus the similarity of triangles is not based on parallelism but equality of angles (Ibid.). Piaget noted this procession as a psychological development from the straight line to parallels to equality of angles and then to similar triangles (Ibid.).

The presented study focused on the angle itself separate from a geometric shape. When figures of angles are part of the protocol of this study, they are drawn without the direction of the ray. Angles as viewed in reality have truncated arms and the protocol is intended to simulate that aspect of the real world that children view. It is important to note that the age level of the subjects (8 to 10 years) corresponds to Stage III, the stage in which the subjects gave angles more careful consideration.

Another related Piagetian study is reported in The Child's Conception of Geometry by Piaget, Inhelder, and Szeminska. In this study the children were asked to reproduce the illustration shown in Figure 2.
Responses by the subjects showed that accurate reproduction was achieved at Stage IIIB but not before (Piaget, Inhelder, and Szeminska, 1960). Prior to Stage IIIB perception was fairly accurate where an attempt was made to equate angles without measuring them. In the earlier stages the attempt to reproduce the figure was made by taking linear measurements and creating congruent triangles, for the figure does not present itself as a system of angles (Ibid.). A perpendicular measurement to help reproduce the figure is not predominantly used until Stage IV (Ibid.).

Jane Rowland did a study in 1972 to relate developmental level and achievement in learning angle measure. She used Piagetian established tasks with fourth, fifth, and sixth graders. Typical behavior for levels of angle conception without a reference to age was used. She compared these developmental levels with her study and found a significant concurrence of the tasks in order ranking the subjects (Rowland, 1972). She also found positive correlation between developmental levels and angle measure test scores. Her study noted the scant amount of available research that related to the study of
angles, and she recognized the need for more research to contribute to curriculum development and methodology in enhancing the concept of angle. A direction she recommended for research is the "development of an instrument for determining a child's angle conception" (Rowland, 1972). The presented study involved that scope of study.

Richard Copeland recognized that Piaget's stages of development have important implications for teaching methodology (Copeland, 1984). The child should not be expected to perform a task before the concept concerning that task is attained. Learning begins with physical action on an object and develops through the psychological stages as established by Piaget (Ibid.).

The development of spacial ideas proceeds at two different levels-- 1) perception, 2) intellect. The two levels develop on their own and produce conflict until an equilibrium is met (Ibid.). Linear measurement is an example. In order to measure, a change of position must take place (whether it be the eye or a measuring instrument) (Ibid.). The change of position must be related to a reference system and an invariant must be recognized. Copeland states it well, "Fundamental to and a prerequisite for understanding measurement is conservation of distance and length." (Ibid.)

In the same vein, understanding children's cognitive developmental level of concept of angle is educationally efficient for developing curriculum. This study provides an assessment tool for determining the cognitive development by presenting angles in rotational change and by changing the length of the arms of an angle.
This study used Logo tasks as the third method to assess the cognitive development of concept of angle. Perceptions of angles and the use of angle measurement were observed and recorded by the researcher. The researcher recorded Logo commands as used by the subject and used a tape recorder to record all conversation. Logo language commands for the pivoting turtle requires an input of degree measurement. Piaget attested the ready attainment of measurement once concepts are developed (Piaget and Inhelder, 1967). The researcher observed and assessed the use of the number representing degree measurement as used by the subject in determining conceptualization of angle. One of the educational objectives for Logo is to explore angles and engage the student in abstract thinking by relating ideas to concrete experiences (Carter, 1983). The National Council of Teachers of Mathematics recognizes the struggles that currently exist to identify the appropriate roles that Logo plays in the development of students' understanding of mathematical concepts. For this reason the Research Advisory Committee feels a need for research emphasis on the issue of the relationship between the computer language students use and concept attainment abilities. The Committee stated that claims have been made for the value of Logo in developing such abilities but research offers only a few definitive findings (Journal for Research in Mathematics Education, 1984).

The researcher did an ERIC search, researched educational journals, and reviewed abstracts and indexed dissertations presented to American
universities from 1861 to 1984. References as to the role Logo plays in the conceptualization of angle is indeed extremely limited.

Books and articles written on Logo concentrate on the instruction of Logo or the philosophy of Logo. Logo provides an environment for learning how to learn. It approaches learning from top to bottom in that it does not take the form of specific directions, but takes the form of suggestions that help the learner learn how to formulate and evaluate problems (Streibel, 1983). Papert in his book, Mindstorms, presented Logo in a heuristic perspective. Learning is under the control of the programmer. Logo provides the tool for the learner to express abstract ideas in a concrete format (Papert, 1980). Instructional material dictate angle degrees, often presented in the form of geometric shapes or by telling the programmer to "do this", followed by a turn command and number signifying degree of angle measurement. One author approached angles by asking the learner to "imagine pivoting on the heel of your foot." Then he proceeded to instruct angle degrees for geometric shapes (Anieta, 1985).

Indeed, research in determining the relationship between Logo and the development of concepts, specifically the concept of angle, is lacking. Some research examining the potential of Logo as a mathematical tool was conducted in London. Richard Noss did a study and wrote a dissertation on the effect of Logo in various areas of mathematics. Within the scope of his study he hypothesized that Logo activities lead to an enhancement of children's understanding of angles (Noss, 1985). Noss conducted his study with school children between the ages of 8 and 10 years (second-grade through fourth-grade). A geometric pencil-and-paper test comprised of eight questions relating to angle were given to two groups of children. There was a subset of 84
children having had about 75 minutes a week of Logo classes and there was a subset of 92 children having had no Logo classes.

Noss observed that "angles posed a much greater problem than lengths." (The other geometric aspect that Noss studied.) He found that the orientation of the angle was confusing for many children, particularly the younger ones, and the Logo turtle served as an introduction to the concept of angle (Ibid.). In testing for concept of angle in terms of perceived effects due to Logo work, Noss found it necessary to devise his own test items to evaluate aspects of the concept of angle. He tested for right angle conservation, acute angle conservation (using triangles and paired angles), and size assessment (big, small) of external angles. Noss varied the "length of arms" of the angles as a "distractor." He used this visual representation in questions involving right angle conservation and involving one pair of acute angles. For the right angle conservation items, Noss found that the effect of Logo programming was strongest in the youngest children. Therefore, he concluded that Logo was an effective means of introducing concept of right angle.

The angle conservation test items included angles in triangles and straightforward presentations of equal or unequal angles. The comparison of the two unequal angles was displayed by angles whose arm lengths were inversely related to the angle measurement. Neither of the triangle test items showed significant variation between the two groups. Also there was no significant variation shown for the presentation of equal angles. There was a significant variation in favor of the Logo group on the test item of the unequal angles. Noss claimed that the Logo work enhanced the concept of angle over those who had been introduced to angle by conventional means without
exposure to Logo (Noss, 1985). Noss supposed that those who answered that test item incorrectly confused the angle measurement with the length of its arms.

Noss concluded that the finding from his study indicate that Logo based turtle geometric activities enhance the concept of angle (Noss, 1985). Noss' conclusions are suggestive without strong definitive findings, but his study is indicative of the emphasis in research that is needed in order to identify the role Logo plays in understanding mathematical concepts, particularly the concept of angle. Intuitively, a teacher involved with Noss' study stated that because of Logo, "Concept of angle is understood so much better than if I had stood at the blackboard with a protractor because the children are making the angles themselves for a specific purpose." (Noss, 1985) Noss viewed the importance of Logo as an entry route rather than a "cul-de-sac" (Noss 1984).

Textbook Methodology Reviewed

The researcher selected and reviewed textbooks that are promoted and widely used in her geographical area. Third- and fourth-grade level texts published by Heath; Houghton Mifflin; Silver Burdett; Scott, Foresman; and Addison-Wesley were reviewed. The researcher found that these texts use a deductive approach. In these texts a definition may be given and specific examples are given to be identified as angles. The texts take on a lecture approach rather than that of discovery.
A fourth-grade mathematical text (Scott, Foresman, and Co., 1978) instructs the teacher to take two pairs of scissors that are of different sizes and open them to an angle of equal measurement. The teacher's edition instructs the teacher to "explain that the angles formed by both pair of scissors are equal even though the lengths of the sides of the scissors are not the same." The text asks the teacher to draw an angle on the board and extend the sides of the angle and then to "ask if this is the same angle as before." The teacher is asked to "emphasize that the length of the sides do not affect the sides of the angles" (Bolster et al, 1978). In a subsequent Scott, Foresman fourth-grade edition (1985), less emphasis is placed on the actual teacher instruction expected, but the teacher is asked to stress that the lengths of the sides have nothing to do with the size of the angle. Also, the teacher is asked to "watch for students who do not recognize right angles shown in different positions" (Ibid.).

These texts are asking the teacher to instruct the student in concepts related to angle. The students are given exercises to meet the objectives of naming angles and identifying right angles. The fourth-grade text (Scott, Foresman, and Co., 1985) defines the angle as two rays with the same endpoint. This is one step beyond the third-grade text of the same series which does not define angle for the student but does use pictures depicting angles including right angles for identification by the student. Thus the lesson tries to develop a feeling for an angle (Ibid.).

In the Silver Burdett series (Vogeli et al, 1981), the third-grade curriculum's objective is to have the student recognize angles including right angles. Angle is not defined and infinite length of the rays is not stressed at this grade level. All figures of angles use segments intersecting at an endpoint.
The lessons of the fourth-grade text build upon these concepts to include 1) definition of an angle as two rays with a common endpoint; 2) figures of angles using the infinite length of the rays; 3) identification of acute, right, obtuse angles, and congruent angles. Congruent angles are identified as having the same size and shape (Ibid.).

In the Houghton Mifflin Mathematics series (Duncan et al, 1983), the curriculum at the third-grade level includes geometry but the study of angle is not an objective. At the fourth-grade level angle is defined and the objective is to identify right, acute, and obtuse angles.

In the Addison-Wesley Mathematics series (Eicholz et al, 1985) the third-grade curriculum focuses on identity of angles including right angles. Angle is not defined for the student, but the teacher is instructed to teach angles with drawings of angles whose rays point in different directions and to explain that "the arrows indicate that the sides of an angle extend infinitely in the directions indicated" (Ibid.). The fourth-grade curriculum builds upon these same concepts and defines angle. Congruent angles are included in the topic "Congruent Figures" (Ibid.).

The scope and sequence of instruction in concepts of angle for the third- and fourth-grade level as presented in the cited elementary textbook series indicate that three out of five series teach the subject of angle beginning at the third-grade level. The fourth-grade level builds upon these concepts. Table 1 shows the scope and sequence of these studies for the cited mathematics series. In general the primary objectives of these texts are to identify angles that are pictured in real life objects or to identify figures of rays, some intersecting and others not intersecting. At most two workbook pages in each text provide
these lessons. The concepts of angle involving length of sides of angles and rotation of angles are left for the teacher to "stress."

<table>
<thead>
<tr>
<th>Publisher</th>
<th>Third Grade</th>
<th>Fourth Grade</th>
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<tbody>
<tr>
<td></td>
<td>Angle Defined</td>
<td>Angle Identified</td>
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<tr>
<td>Scott-Foresman</td>
<td>N</td>
<td>Y</td>
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<tr>
<td>Silver Burdett</td>
<td>N</td>
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<td>Houghton Mifflin</td>
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<td>Heath</td>
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<tr>
<td>Addison Wesley</td>
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N=No \ Y=Yes

Table 1. Scope and Sequence of Study of Angle
The School Mathematics Study Group (SMSG) at Yale included angle instruction in sample text materials designed to improve mathematics teaching in elementary schools (Rowland, 1972). The depth of instruction is greater in the SMSG material than in the published textbooks. SMSG's objectives are to develop the understanding that an angle is a set of points that include the following conditions:

1. It is the union of two rays with a common endpoint.
2. The rays do not lie on the same line.
3. The common endpoint of the rays is called the vertex. It is also the only member of the intersection of the rays.
4. Each ray is called a side of the angle.

(Mathematics for the Elementary School, 1963)

The SMSG study put an emphasis on "parts of angles." Students are asked to elicit representations of parts of angles such as desk corners, scissors, word constructions, letters of alphabets, clocks. The students are expected to explain why these are only "parts of angles." The expected answer is that "rays extend without ending and the edges of the object end" (Ibid.).

Suggested teaching procedures include having figures of angles in various position so that pupils do not get the idea that one of the rays of an angle must be parallel to the bottom of the paper (Ibid.).
Summary

Piagetian research of similar triangles and reproduction of a geometric figure provides age corresponding stages of development. The intent of the presented study was to focus specifically on the angle itself and investigate closely and assess the cognitive development of concept of angle of the 8 to 10 year old child, the age bracket that corresponds most closely to the development of thinking in terms of angles according to Piagetian research.

Limited Logo studies suggested that concept of angle is enhanced by Logo activities. Logo instructional material and textbook methodology presume a development of concept of angle. Textbooks use a deductive approach to instruct a concept of angle. The protocol of this study was to identify cognitive developmental levels of concept of angle and to make educational recommendations.
CHAPTER III

DESIGN AND PROCEDURE

Introduction

In order to achieve the purpose of the study which is to investigate and assess the development of concept of angle found in children at the third- and fourth-grade level (8-10 years old), clinical interviews using three main task methods were used to explore each student's cognitive development. Method I involved visual perception of paired drawings of angles. Method II involved construction and ordering of angles. Method III involved Logo language, using the computer to command the turtle to pivot and to command the drawing of the arms of an angle. The clinical interview questions probed and elicited cognitive thought and level of understanding. A second testing situation, at least four weeks after the initial testing situation, was given to provide data to test for reliability.
**Population**

The subjects were third- and fourth-grade students at The Bement School in Deerfield, Massachusetts. Fourteen students were interviewed for session 1 and a subset of ten students were interviewed for session 2. The subset was comprised of ten students due to the time limitation of the school calendar. One student (DS) did not complete session 2 because he ran out of time and the researcher was unable to reschedule his return. The researcher interviewed each subject in a clinical setting on a one-to-one basis. The researcher used a tape recorder to record the conversations between the researcher and the subjects. Because of the population size and the limited sampling of a population, there are limitations to the study. These limitations need to be taken into account when reviewing the results of the study. The results cannot be generalized, therefore, to a wide population.

**Interview Sessions**

Each session, which lasted 40 to 50 minutes, began with: "What do you think an angle is?" If the student's answer was descriptive of an angle, the researcher said, "Give me some examples of angles." If the student's reply indicated that the student was uncertain of what an angle is, the researcher referred to Figure 3 to stimulate thought with figures of an angle, a squiggle, a
circle, and a blob. The researcher then asked, "Which figure shows an angle?" If the student responded correctly, the testing proceeded. If the student did not respond correctly, the angle was identified for the student and testing proceeded. The purpose of the initial question was to establish a frame of reference for the subject and allow insight into the subject's initial perception of angle for the interviewer. If Figure 3 was needed to clarify a frame of reference for angle, then the subject was not totally overwhelmed by the interviewer's expectations for identifying an angle and the interviewer knew the minimal base from which the subject was drawing an identification of angle. Therefore, the use of Figure 3 allowed the interviewer to proceed with the interview process even if the subject showed no interpretation of an angle.

Session 1

The protocol of the three methods of session 1 follows. The purposeful intent of each of the tasks incorporated in the three methods is identified in Chapter IV along with representative remarks by the subjects.

Method I. Each of five cards (Figure 4) was presented one at a time to the subject. Exemplary questions were: "Are the angle measurements on this card the same or different?" "Why do you think that?" Further probing questions were asked to accomplish the intent. (See Appendix A.)

Method II. The student was shown two representations of angles each made from chenille wire. (Figure 5) Exemplary questions were: "What do you think of these angle measurements?" "Are they the same measurement or is one smaller or larger than the other?" "Why do you think that?" Further probing questions were asked to accomplish intent. (See Appendix A.) The student
was given three straight pieces of chenille wire of varied lengths and was asked to make an angle with each. The subject was then asked to order the five representations of angles from smallest to largest and to explain the ordering.

Method III. Three pre-programmed figures of angles in Logo were individually presented to the student. The researcher knew from experience with the subjects their knowledge of the basic Logo commands (FD, BK, RT, LT). The subjects had Logo exposure for at least four months, 30 minutes per week. The three pre-programmed figures of angles in Logo were presented one at a time to the student. The turtle was pre-positioned to the right of the figure in the "home" position and the student was asked to make another angle about the same measurement elsewhere on the screen. (Figure 6) An angle measurement within 5 degrees of the pre-programmed angle was considered acceptable. The researcher focused on position of the angle, rotation of the angle, and length of the arms of the angle. Appropriate questions were asked to probe the subject of his/her conceptions. (See Appendix A.)

Session 2

Session 2 was given at least four weeks later to a subset of the subjects. Ten students participated in session 2.

Method I. Each of two cards (Figure 7) was presented one at a time to the subject. Exemplary questions were: "Tell me about the angles. Do they have the same measurement or different measurements. Is one larger or smaller than the other?" The student was probed to explain the answer(s) given. (See Appendix C.)
Method II. The student was shown an angle representation made from a chenille wire and was given two other wires, one short in length (red), the other longer in length (green). Exemplary questions are: "Choose one of the wires and make an angle measurement greater than the one shown to you." If the student chose the longer wire, the researcher asked, "Can you make an angle measurement greater than the one shown to you with the red (shorter) wire?" Probing continued until the researcher was certain of the subject's cognitive process, indicating whether or not the shorter wire was reserved for small angle measurements or whether it could indeed be used to make an angle of greater measurement than the one presented to the subject. (See Appendix C.)

Method III. One pre-programmed figure of an angle in Logo was presented to the student. Two tasks were asked concerning the angle representation. First, the student was asked to make another angle representation about the same measurement elsewhere on the screen. Secondly, the student was asked to make an angle representation about one-half the measurement of the researcher's angle representation. Appropriate probing questions were asked in regard to the student's angles. (See Appendix C.)

Treatment of the Data

The recorded responses from each of the subjects of both sessions were analyzed. The subjects were scored according to the responses to the tasks of
the methods. The obtained score was then used as the criteria for identifying the subjects' developmental levels. After developmental levels were established for each of the subjects, correlations with the subject's age and total mathematics achievement score were calculated. The developmental levels determined from the study were compared to Piagetian developmental levels for concept of angle. The data provided a basis on which to make recommendations for education in the study of angle.

Representative audio-taped interviews were transcribed and reported in the appendix.
Figure 3: Illustrations Used to Stimulate Thought of Angles
Figure 4: Five Cards Presented for Angle Testing
Figure 5: Two Angles Made From Chenille Wire
Figure 6: Three Logo Pre-Programmed Angle Figures

Presentation 1

Presentation 2

Presentation 3
Figure 7: Two Cards Presented for Angle Testing
CHAPTER IV

THE METHODS

Introduction

The cognitive development of concept of angle in third and fourth graders (8 - 10 years old) was observed by means of the three methods previously described. The performance observed on each method was scored and the scores determined the subject's developmental level. In this chapter, the data are analyzed according to the three methods and the results of the three methods are compared.

Technique and General Results of Method I

Method I was designed by the researcher to observe how visual representations of angles were perceived by the subjects. Five cards, each with a pair of drawn angles on them (Figure 4), were presented to the subject one at a time. The first card displayed congruent angles in mirror position. One angle representation had arms considerably longer than the arms of the other angle pictured. The second card pictured angles of different measurements, but the
length of the arms on both angles were equivalent in length. The third card pictured two right angles with arms of equal length, but one angle was oriented to a varying position by 45 degrees. The fourth card pictured two acute and congruent angles, but the angles varied both in orientation and in the lengths of the arms that represented the angle. The fifth card pictured two obtuse angles that were congruent, but varied both in orientation and in the lengths of the arms that made the angle.

The cards used in Method I were presented one at a time to each child and the responses made were scored according to the effect or lack of effect the length of the arms and/or the orientation of the angle had on the angle measurement. By this standard the child's developmental level was determined for Method I. The developmental levels are reported in Chapter V. A representative transcript of the clinical interview is included in the Appendix. Quotations which demonstrate various levels of responses from the subjects are included in reporting general results.

The aim of the first card was to present equality of angle measurement with angle representation varying in length of the arms, and the angles oriented to the same base line, but facing each other. Of the fourteen subjects interviewed, seven were misled by the lengths of the arms of the angles. Those that were misled viewed the angle with the longer arms as being "bigger", although the angle measurements were equal. The other seven subjects ignored the length of the arms and correctly assessed the angle measurements as being the "same." None of the fourteen subjects were misled by the orientation of the angles.
The ages of the children that were misled by the length of the sides ranged from 8 years, 7 months to 10 years, 1 month. The ages of the other seven children ranged from 8 years, 11 months to 10 years, 4 months. The overlap does not give a clear indication of development according to ages with respect to the first card.

The second card's aim was to present an inequality of angles and an equality in the lengths of the arms of the angles. Four of the fourteen subjects responded that the length of the arms had an effect on the angle, but were not committed to saying the angle measurements were equal on this card. The ages of the children that responded such ranged from 8 years, 7 months to 10 years, 1 month. The ages of the children that responded correctly to the angle measurements, without any regard to the length of the arms, ranged from 9 years, 4 months to 10 years, 1 month. The overlap again does not give a clear indication of development according to age.

The third card's aim was to maintain a consistency of angle measurement and length of arms and to vary the rotation of the angle. Four of the fourteen subjects responded that the "turned" angle (angle not oriented to the bottom of the card and located on the right side of the card) was different. SG (10 years, 1 month old -- designated by 10;1 and used accordingly hereinafter) responded, "If the right angle is moved it would be the same angle." The ages of the children that responded to the rotation ranged from 8 years, 8 months to 10 years, 1 month. The ages of the children that responded correctly to the angle measurements ranged from 8 years, 1 month to 10 years, 4 months. The overlap again does not give a clear indication of development according to age.
The fourth card's aim was to present congruent, acute angles that varied in rotation and length of arms. Eight of the fourteen subjects were misled by the lengths of the arms, one of whom also responded that rotation made a difference. Six subjects showed absolute conviction that the angle measurements were equal. The ages of the children that were misled by the visual representation ranged from 8 years, 7 months to 10 years, 4 months. The ages of the children that were confident of the equality of angles ranged from 9 years, 4 months to 9 years, 8 months. It is interesting to note that this age bracket is nestled in the age bracket of children that were misled by the visual representation.

The fifth card's aim was to present congruent, obtuse angles that varied in rotation and length of arms. Again, eight of the subjects were misled by the lengths of the arms, two of whom also viewed the rotation as affecting the angle measurement. On this task SG (10;1) responded that "position doesn't matter," although his response to the third card indicated that the rotation of the angle did have an effect. With respect to the length of the arms, SG responded, "If the longer sides were shortened to get the same length, they (angles) would be the same." SL (8;7) responded that the angles would be the "same if you cut off the lines." MR (9;4), on the other hand, responded, "Length of lines doesn't make any difference -- it's the direction the lines point that makes the angle." The ages of the children that were misled by the length of the arms and/or rotation ranged from 8 years, 7 months to 10 years, 1 month. The children that responded confidently as to the equality of angle measurement ranged in ages from 9 years, 4 months to 10 years, 4 months. The overlap does not give a clear indication of development according to age.
Technique and General Results of Method II

Method II was designed by the researcher to observe the development of concept of angle by using the construction of angles by means of chenille wires. The subject was given two angles made from wires and was asked to compare them (Figure 5). In the second task of this method, the subject was given three straight wires and was asked to make an angle with each. The subject was then asked to order the five angle measurements from smallest to largest.

The responses on the two tasks of Method II were scored according to the effect or lack of effect the length of the arms and/or rotation of the angle had on the angle measurement. The scores determined the student's developmental level on Method II.

The first task's aim was to present the subject with two angles made from varying lengths of wires. The subject was asked to compare the angle measurements. Of the fourteen subjects, six were misled by the length of the arms made by the wire. DS (8;1) responded that the researcher's gray angle, as opposed to the researcher's black angle, was "bigger because it has more wire." Some of the subjects superimposed the angles or placed one angle inside the other. However, this manipulation did not insure correct evaluation. TR (8;11) superimposed the wires given her and determined that the length of arms on one angle was longer than the length of arms on the other angle. She responded that the angles are the "same but not the same." TR was given credit
for recognizing equality of angle measurement, but her response showed that she had a concern for the length of the arms in comparing the angles. Eight of the subjects made correct comparisons on the first task. The rotation of the angles seemed to have no effect. The ages of the subjects that were misled by the length of the arms ranged from 8 years, 8 months to 10 years, 1 month. The ages of the subjects that made correct comparisons ranged from 9 years, 4 months to 9 years, 8 months. The latter group is nestled in the former group in respect to age.

The second task's aim was to allow the subjects to create their own angles from varying lengths of chenille wire and show comparison of the angles by ordering the angles from smallest to largest measurements. Ordering of the angle measurements did not necessarily correspond to the responses made to the first task in this method. One subject ordered the angles correctly without being misled by length of arms or rotation of angles, whereas he had been misled on the first task by length of arms. Three other subjects, who were not misled by the length of arms on the first task, were misled by the length of the arms that the wires made on the second task. The other ten subjects' responses were consistent to both tasks in Method II. Five of those ten were misled by the length of the arms, whereas five of the ten ordered the angle measurements correctly, ignoring the length of the arms. SL (8;7) when asked to compare her brown angle with the researcher's gray angle responded, "It's hard to tell...measuring the sides will tell you which is bigger." DS (8;1) responded, "The gray is bigger because it has more wire." PG (9;8) compared two angles of the same measurement and responded, "They are alike, except longer." The responses SL and DS made show misleading by the length of the
arms. PG's response shows a correct assessment of the angles, but not without some concern for the length of the arms.

**Technique and General Results of Method III**

Method III was designed by the researcher to observe children's conceptions of angles using Logo. Logo graphics necessitate a command followed by a number to represent length units or degree measurements. One at a time, the subject was presented with an angle measurement representation created by a pre-programmed Logo picture (Figure 6). The subject was asked to make the same angle measurement elsewhere on the screen, using the Logo commands which the subject already knew. Completing a task, the subject was asked to compare the angle measurement of his/her angle with that of the pre-programmed angle. As part of the clinical interviewing process, the interviewer on occasion would extend one arm of the subject's picture representation by using an additional FORWARD command and then asked the subject to assess the angle measurements.

On the three Logo tasks, the subjects made a pictorial representation of the one presented on the screen. On an individual basis, each subject was evaluated using his/her picture representation, the thought process, and the responses made. These inputs were used to score the subject according to the effect or lack of effect the length of the arms and/or rotation of the angle had on
the angle measurement. The scores determined the student's developmental level on Method III.

On the first Logo task, the subjects were presented with a 90 degree angle, rotated 10 degrees to the right. The upright arm had a length of FORWARD 60 and the other arm a length of FORWARD 30. (FORWARD is a Logo command that moves the "turtle" the designated number of units forward in accordance with the directional heading of the "turtle." Hereinafter, the distance the "turtle" moved will be referred to as "units.") Of the fourteen subjects, eight responded to the length of the arms as having an effect on the angle measurement. Some of the children used their fingers as a tool for measuring and comparing the length of the arms on their angle to the length of the arms on the presented angle. For these subjects there was extensive concentration on making the length of the arms equivalent in linear measurement. TR (8;11) oriented her angle the same as the pre-programmed representation. She turned in her seat to simulate the needed "turtle" turn. Having oriented the "turtle" 15 degrees to the right, she then made the length of the arms on her angle representation exactly the length of the arms on the pre-programmed representation. Even though TR's angle measurement was less than the measurement of the pre-programmed representation, she evaluated the angle measurements to be the same. The interviewer extended the lower arm, 30 units, and asked, "What do you think about the angle measurement now?" TR responded, "It looks different now...looks fatter." KB (8;8) made an angle representation 10 degrees less than the pictured representation and made the length of the arms the exact measurement of the pictured representation. In making the second arm of that angle, KB commanded a
length of 40, then erased 10, ending with a length of 30 for that arm. She assessed the angle measurement of her angle as being the same as the angle measurement of the researcher's angle. The interviewer then lengthened the second arm and asked, "What do you think about the angle measurement now?" KB responded that her angle measurement was now "bigger."

Six of the fourteen subjects showed that the length of the arms had no effect on the angle measurement. None of the subjects showed that rotation of the angles had any effect on the angle measurement. MR (9;4) recognized the angle measurement as 90 degrees, and he made the same angle measurement with no rotation. His length of the arms varied slightly from the pre-programmed representation. In assessing his angle measurement with the presented angle measurement, MR responded, "Same angle, not same length of sides." CP (9;4) made the same representation as the pre-programmed picture. The interviewer added length to an arm and asked, "What do you think about the angle measurement now?" CP responded with absolute confidence that the angle measurement was still the same.

The second Logo task presented an angle measurement of 30 degrees; each arm had a length of 32. The angle representation was positioned with the vertex pointing toward the bottom of the screen. Five of the fourteen subjects showed that the length of the arms affected the angle measurement. TV (9;9) made an angle measurement 5 degrees less than the pictured representation with length of the arms slightly shorter (30 units) than the pre-programmed picture (32 units). The orientation of the angles was the same. In assessing the angle measurements, TV responded, "This line seems to be the same but didn't go over enough, so yours (pre-programmed picture) is bigger." The interviewer
extended the second arm and asked, "What do you think about the angle measurement now?" TV's new assessment was that this made "it bigger." TV was using both angle measurement and length of the arms to come to a conclusion. When the length of the arms were nearly equal in length in both representations, TV concentrated more on the angle measurement. When one of the arms was extended in length, TV turned his concentration to the length of the arms and thus made his assessment incorrectly. SG (10;1) made an angle measurement 60 degrees greater than the pre-programmed angle measurement, made the length of the arms longer by 8, and oriented the angle the same as the pre-programmed representation. When asked to assess the angle measurement, SG responded, "It's hard to say." SG was misled by the length of the arms. TM (10;4) made an angle measurement 50 degrees greater than the pre-programmed representation. He made the length of the arms shorter by 6 on one side and shorter by 10 on the other side. He oriented the angle the same as the pre-programmed angle. TM assessed his angle representation with the pre-programmed angle representation and responded, "Same shape but yours (pre-programmed angle) is larger ... same if you take yours (pre-programmed angle) and spread the lines apart a little more. It would be longer in length but shorter in width(?)". TM showed confusion as to how to assess the angle measurement. He was concerned about the spread of the arms, but could not distinguish the effect the spread of the arms had from the effect the length of the arms had.

On the second Logo task, nine of the fourteen subjects ignored the length of the arms in making correct assessment of the angle measurements. MR (9;4) made an angle measurement 10 degrees greater than the pre-programmed
representation and made the length of the arms 30 (2 shorter than the preprogrammed representation). The orientation of the angles was the same. MR referred to his pictured angle as "imperfect" because he assessed his as being "larger." The interviewer extended the length of one of the arms on MR's angle representation, but MR insisted that the angle measurement remained unchanged. DS (8;11) made an angle measurement 5 degrees greater than the pre-programmed representation and made the length of the arms 30. DS assessed his angle as being "wider" or "bigger." Further probing showed that he was confident that the length of the arms had no effect on the angle measurement.

On the third Logo task, the subjects were presented with an obtuse angle of 110 degrees. One arm of the angle was 20 units the other arm was 53 units. The angle representation was oriented so the longer arm was parallel to the bottom of the screen. Of the fourteen subjects, the same five subjects that responded that the length of the arms affected the angle measurement on the second task, responded accordingly on the third task. TV (9;9) made an angle representation with an angle measurement of 10 degrees greater than the pre-programmed representation. One arm of TV's angle had the same length measurement as the pre-programmed arms, and the other arm of TV's angle was 7 longer. TV assessed the angles and responded, "My angle is longer that way (length of the second arm) and curved too much." The interviewer shortened one arm of TV's angle representation. Then TV compared this angle measurement with the newly represented angle measurement and responded that the shortening of the arm "made the angle (measurement) smaller." TM (10;4) made an angle measurement 10 degrees smaller than the angle
measurement on the pre-programmed representation. The length of one arm on TM's representation was the same as the pre-programmed representation. The other arm on TM's angle was 3 shorter than the pre-programmed representation. TM assessed the angle measurements as being "exactly alike because same length of each one." ND (9;6) made extensive effort to orient his angle representation in the same direction as the pre-programmed representation. He used his fingers extensively as a measuring tool to make the length of the arms the same length as in the pre-programmed representation. ND's angle measurement was 5 degrees less than the pre-programmed representation and the length of one arm was the same length as the pre-programmed representation, and the other arm was 3 shorter. ND assessed the angle measurements as being equal, but when questioned and probed, ND wavered between length of the arms having an effect on the angle measurement and not having any effect. In conclusion, he rationalized that the longer arms show more space so the angle measurement is bigger. DS (8;11) made an angle representation with an angle measurement 13 degrees less than the pre-programmed representation. He made one of the arms the same length as in the pre-programmed representation and made the other arm 3 units longer. DS gave no concern to the length of the arms when he assessed his angle representation against the pre-programmed representation. He assessed the pre-programmed angle measurement as bigger because "mine (arm) goes up straighter and down more; so yours (angle measurement) is a little bigger." DS was explaining in his own way that the angle measurement was "bigger" because the spread of the arms was greater. The length of the arms had no effect on the angle measurement.
These cited subjects exemplified the developmental levels of concepts of angle as are described in Chapter V. There is the case where the length of the arms misled the subject's correct assessment of the angle measurement; there is the case where the subject wavers between a correct assessment and being misled; there is the case where the subject is confident of the correct assessment without any misleadings.

**Comparison of Results of the Methods**

For the three methods, rotation of the angle representation was not a primary indicator in determining the subject's cognitive development of concept of angle. The third, fourth, and fifth cards of Method I were the only tasks in which rotation misled any of the subjects. These three tasks misled four of the fourteen subjects. TR (8;11) was misled on all of these three tasks. The third card, showing two 90 degree angles represented by arms of equal length with one angle representation rotated 45 degrees, misled TR to conclude that the angle measurements were not equal. On the fourth and fifth cards, TR concluded that rotation and length of the arms made a difference in correctly assessing the angles. Three other subjects were misled by rotation on the third card. One of these subjects misled by the third card was also misled by rotation on the fifth card. PG (9;8) assessed the right angles on the third card correctly, and he assessed the equal angle measurement on the fifth card as being "different a little, in a way, because of the way they are facing." The other
subjects were not misled by rotation of angles, but some stated that it would be easier to compare the angles if they were "turned the same way."

On Method II and III, the subjects had control of angle rotation. When the subjects ordered the wires in Method II, eight of the fourteen subjects oriented their angle representations in a uniform manner, either with one arm parallel to the table edge or with respect to the vertices pointing in the same direction. The same eight subjects that arranged their wires in a uniform manner oriented their Logo angle representations in the same direction as the pre-programmed representation. Five of the fourteen subjects who arranged their wire representations in a non-uniform manner also oriented some or all of their Logo angle representations differently from the orientation of the pre-programmed representations. One subject arranged the wire angles in a non-uniform manner and oriented the Logo angles in the same direction as the pre-programmed representations. The question arises whether the orientation of the angles was a convenience for comparison of angle measurement or whether it was a discriminating variable. A relationship of the perception of angle orientation to developmental level is presented in Chapter V.

For the three methods, length of the arms of the angle representations was the primary factor in determining the subject's cognitive development of concept of angle. Ten of the fourteen subjects were misled by the length of the arms on at least one of the methods. There was no consistency among these subjects as to what task(s) were most misleading. In the cases of SL (8;7), KB (8;8), and DS (8;11), they were not misled by the length of the arms on the Logo tasks of Method III. However, the length of the arms on most tasks of Methods I and II were misleading to them. The Logo command of LT (LEFT TURN) and RT
(RIGHT TURN) determined fixed angle measurements for them and the angle measurements were independent of any FD (FORWARD) or BK (BACKWARD) commands. MR (9;4) was misled by the length of the arms on the wire tasks of Method II, and he was not misled on Method I and III. Using the shortest wire MR made the largest angle measurement of the five constructed angles, but ordered it as the smallest. (Figure 8) When probed, MR responded, "Red (shortest wire, largest angle measurement) could be the largest." However, he continued to show confusion by the length of the arms. CM (9;6) was misled by the length of the arms on Method I and correct assessments were made on Method II and III. When probed as to how he would prove his assessment of the angle measurements, CM responded that he would measure the length of the arms with a centimeter ruler. TM (10;4) was misled by the length of the arms on both Method II and III, but not on Method I. ND (9;6) was misled by the length of the arms on Method III only. ND used his fingers to measure the length of the arms on the pre-programmed representation. When probed he responded that "longer arms make more space and the angle measurement is bigger."
Three of the fourteen subjects were misled by the length of the arms on all three methods. Ten of the fourteen subjects were misled by length of the arms on at least one of the methods. Four of the fourteen subjects were misled by rotation of angles on Method I. A relationship of the perception of the length of the arms and rotation of angles to developmental level is presented in Chapter V.

Figure 8: MR's (9;4) Response to Ordering of Wires in Method II

<table>
<thead>
<tr>
<th>(red)</th>
<th>(brown)</th>
<th>(purple)</th>
<th>(black)</th>
<th>(gray)</th>
</tr>
</thead>
</table>
| Smallest Angle Measurement to Largest Angle Measurement

"same angle measurement"
Analysis of Data

The three methods were designed to provide criteria for determining cognitive developmental levels of concept of angle. The scope of the study limits the generalization of the established developmental levels. Therefore, let it be noted that the analyzed data cannot be generalized to a wide population because of the population specifications of the study. The established developmental levels, however, provide a reference for identifying a student's concept of angle. The three methods together provided a better indicator for establishing developmental levels than any one method because the subjects were able to express themselves in three modes: perception, manipulative construction, and Logo programming. The three methods provided observational tools by which the researcher assessed and scored responses of the subjects and identified developmental levels. The subjects' responses were evaluated as to whether or not the length of the arms of an angle and/or rotation of an angle had any effect on the students' ability to access angle measurement correctly.

Each subject was scored according to his/her response to each of the tasks of the three methods. If the subject responded correctly, indicating that he/she had an understanding of concept of angle, the subject's understanding of angle was designated by the number "1." Level 1 indicates that neither length of the arms nor rotation of angle affected the subject's perception of angle measurement. If the subject responded incorrectly, indicating that he/she was misled by thinking that the length of the arms and/or rotation of the angles
affected angle measurement, the subject's understanding of angle was scored by the number 0. Level 0 indicates that length of the arms and/or rotation of angle affected the subject's perception of angle measurement.

The 1 or 0 for each method was determined by the responses on the respective tasks. On the tasks of Method I, the subject received a 1 if the subject correctly responded to three or more of the five tasks by indicating that neither length of the arms nor rotation of angles had any effect on angle measurement. The subject received a 0 if incorrect responses were made to more than two tasks. On the tasks of Method II, the subject received a 1 or 0 according to the response on the second task since the first task was incorporated into the second task. The 1 designates a correct response in that neither the length of the arms nor rotation of angles had any effect on angle measurement. The 0 designates an incorrect response in that length of the arms and/or rotation had an effect on the assessment of angle measurement. On the tasks of Method III, the subject received a 1 if responses to two or more of the three tasks showed that neither length of the arms nor rotation of angles had any effect on angle measurement. Otherwise the subject received a 0.

Table 2 shows the scoring of each subject in each of the three methods and gives the subject's total score for all three methods.
<table>
<thead>
<tr>
<th>Subject by Total Standing</th>
<th>Method 1</th>
<th>Method 2</th>
<th>Method 3</th>
<th>Total of Methods</th>
</tr>
</thead>
<tbody>
<tr>
<td>SG</td>
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<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>TR</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>TV</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>KB</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
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<tr>
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<td>0</td>
<td>1</td>
</tr>
<tr>
<td>DS</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>CM</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
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<td>PG</td>
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<td>3</td>
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<tr>
<td>SD</td>
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<td>1</td>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>CP</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>3</td>
</tr>
</tbody>
</table>

Table 2. Levels of Methods by Subject--Session 1
Definition of Developmental Levels

The subjects' total scores for all three methods were then used as the criteria for determining the developmental levels. "Robinson" levels of development were established. Because of the findings of the study, the researcher determined and named the levels of development on behalf of the researcher. The levels are designated by 0, 1, 2, 3. Each developmental level signifies the total of the evaluations on each of the three methods and is recorded in column five of Table 2. Zero is the developmental level defined as having a sum of zero (0) for the three methods, indicating the subject was misled by length of the arms and/or rotation of angles on all three methods. One is the developmental level defined as having a sum of one (1) for the three methods, indicating that the subject was not misled by length of the arms or rotation of angles on one method. Two is the developmental level defined as having a sum of two (2) for the three methods, indicating that the subject was not misled by length of the arms or rotation of angles on two methods. Three is the development level defined as having the sum of three (3) for the three methods, indicating that the subject was not misled by length of the arms or rotation of angles on any of the three methods. Figure 9 shows the number of students at each stage of the developmental levels.
Stage 0 of the developmental levels is termed as the "starting" level of development of concept of angle. At this level the subject's assessment of angle measurement was repeatedly misled by length of the arms and/or rotation of angles. Figure 10 shows that the length of the arms misled the subjects more than the rotation of the angles.
Figure 10. Subjects Misled by Length of Arms and/or Rotation
of Angle According to Methods
The "starting" level shows the ability to make an assessment and rationalize a judgment of angle measurement, but inaccurately. At the "starting" level the student was able to recognize and identify angles, but could not make correct assessments in comparing angle measurements. Judging from the responses and actions, the subjects apparently used linear measurement in determining angle measurement. Therefore, the subjects were probably misled by the length of the arms of an angle, thinking that a greater space created by longer arms of an angle gives greater measurement to that angle. The subjects, apparently thinking that the orientation of angles altered the angle measurement, were misled by rotation of angles, in some instances.

Stage 1 of the developmental levels is termed as the "competing" level of development of concept of angle. The "competing" level shows the ability to make an assessment and rationalize a judgment of angle measurement with minimal accuracy. The subjects at this level were misled by length of the arms in two of the three methods, making correct angle measurement assessments in one of the three methods. At this level of development the subject's thinking was at a competitive level being swayed heavily by misleading input, but showing the ability to overcome erroneous persuasion to make correct assessment in some instances. The responses at this level show that there is mental struggling taking place.

Stage 2 of the developmental levels is termed the "placing" level of development of concept of angle. The "placing" level shows the ability to make an assessment and rationalize a judgment of angle measurement with near consistent accuracy. The subjects at this level were misled by length of the arms of an angle. Rotation of angles was no longer a concern for this level of
development. At this level the subjects were close to a full understanding of concept of angle, but had not completely reached the developmental level at which they were not misled by the length of the arms of an angle.

Stage 3 of the developmental levels is termed the "finishing" level of development of concept of angle. The "finishing" level shows the ability to make an assessment and rationalize a judgment of angle measurement with consistent accuracy. The subjects were confident that neither length of the arms nor rotation of angles affected angle measurement. At this level the subjects showed a developed concept of angle. The term "finishing" implies an ongoing completion. Having attained the concept of angle, the student is at the developmental level to use the concept in achieving academic knowledge in respect to angle. It is at this developmental level instruction involving angles can be effectively presented.

Developmental Levels and Corresponding Ages

The researcher compared the developmental levels of the subjects with their ages. Table 3 relates the corresponding ages to each of the subjects and Table 4 presents the age range for each of the four developmental levels. The levels of development do not follow any consecutive age pattern. The earlier stages 0 and 1 encompass greater age ranges (8 years, 11 months to 10 years, 1 month and 8 years, 7 months to 10 years, 4 months, respectively) than the later stages 2 and 3 encompass (9 years, 4 months to 9 years, 6 months and 9
years, 4 months to 9 years, 8 months, respectively). The researcher concludes that the "Robinson" developmental levels do not correspond to age development. The researcher correlated developmental levels with age and the calculation is reported in the next section of this chapter. In Chapter V, the researcher compared the findings of Piaget with the findings of this study. Piaget's developmental levels correspond to age development.

**Developmental Levels and Total Math Achievement Scores**

The researcher compared the developmental levels of the subjects with their total math achievement scores. Within the classroom setting, each subject performed on a Stanford Achievement Test in May 1985. The total mathematics score, comprised of individual scores on mathematics concepts, mathematics computation, and mathematic application, was used as the assessment of the subject's mathematical ability and achievement. The total mathematics scores are recorded in school grades by year and month in accordance with the student's achievement level.

Inherent to the Stanford Achievement Test, itself, the total mathematics scores are limited in scope. Within each of the three testing sections of the test, there were a limited number of geometric questions. Because geometric questions characteristically relate more heavily to angles than computational questions, a test with greater emphasis on geometry type questions would most likely give scores that are better predictors of the students' developmental
levels. The Stanford Achievement Test was administered in accordance with the school's testing policy, and it provides an available achievement score that is typical of achievement scores accessible to educators.

Table 3 relates the corresponding total mathematics scores to each of the subjects, and Table 5 presents the total mathematics score range for each of the four developmental levels. There is an overlapping of test scores within each of the earlier and later stages, but a distinct difference in scores separates the earlier stages from the later stages. Stage 0 ranges from 5 years, 8 months to 5 years, 9 months in grade scored achievement levels; stage 1 ranges from 5 years, 2 months to 5 years, 6 months; stage 2 ranges from 8 years, 9 months to 9 years, 2 months; stage 3 ranges from 6 years, 9 months to 9 years, 5+ months. The researcher correlated "Robinson" developmental levels with total mathematics achievement scores. The calculation is reported in the next section of this chapter.
<table>
<thead>
<tr>
<th>Subject by Level</th>
<th>Age (Year-Month)</th>
<th>Total Math Score Grade (Year-Month)</th>
<th>Developmental Level</th>
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<tbody>
<tr>
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<tr>
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<tr>
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Table 3. Subject’s Age, Total Math Score and Developmental Level
### Table 4. Developmental Levels with Corresponding Ages

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<th>Developmental Level Stage</th>
<th>Age Range (Year-Month)</th>
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<tr>
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<td>(9-4) to (9-6)</td>
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<tr>
<td>3</td>
<td>(9-4) to (9-8)</td>
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</tbody>
</table>

### Table 5. Developmental Levels with Total Math Scores

<table>
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<tr>
<th>Developmental Level Stage</th>
<th>Total Math Score Range (Grade Year-Grade Month)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>(5-8) to (5-9)</td>
</tr>
<tr>
<td>1</td>
<td>(5-2) to (5-6)</td>
</tr>
<tr>
<td>2</td>
<td>(8-9) to (9-2)</td>
</tr>
<tr>
<td>3</td>
<td>(6-9) to (9-5+)</td>
</tr>
</tbody>
</table>
Statistics of Data

Data from the study were statistically analyzed in the following areas.

Methods

Three test methods were used on each of two sessions. The Cochran Q Test was used to determine if a difference exists between the test methods on each of the sessions. If no significant difference exists then the researcher can conclude from the study that a close association exists among the test methods. Therefore, each method tested for the same understanding concerning concept of angle. The null hypothesis is that no difference exists between the test methods. If the significance is greater than or equal to .05, the researcher accepts the hypothesis. If the significance is less than .05, the researcher rejects the hypothesis. For session 1, the Cochran Q significance was found to be .368, level of significance greater than .05 (Table 6). Therefore, the null hypothesis was accepted at the .05 level of significance. The researcher concludes that there was commonality in the three test methods of each of the sessions in determining developmental levels for concept of angle.

For session 2, the Cochran Q Test showed an indifference, in that the scores on the test methods were identical. Because one subject did not complete the testing session due to a time limitation, the correlation is calculated using nine subjects (Table 7). Therefore, the researcher concluded that there was no difference in the test methods in session 2. Each of the test
methods determined the same result in determining the developmental levels for concept of angle. Therefore, the levels are considered equivalent for the three task methods of session 2.

Correlation of Level with Age

The level as determined by the combination of the subject's score on each of the three test methods of session 1 was compared with the age of the student using Kendall Correlation Coefficients. The null hypothesis is that there is no difference between developmental level and age. The correlation coefficient was .1129 which is not significant at the .05 level (Table 8). Therefore, the researcher concluded there is no significant correlation of level with age for this study. The researcher rejected the null hypothesis.

Correlation of Level with Mathematics Achievement Score

The level as determined by the summation of the subject's score on each of the three test methods of session 1 was correlated with the subject's total mathematics score on the Stanford Achievement Test, taken in May, 1985. The Kendall Correlation Coefficients were used. The null hypothesis is that no difference exists between developmental level and mathematics achievement score. The correlation coefficient was .4796 which is significant at the .05 level; actual significance level is .017 (see Table 8). Therefore, the researcher concluded that a significant correlation exists between developmental level and total mathematics score. The researcher accepted the null hypothesis.
Test of Reliability

A second test session was given at least four weeks after the first test session to test for reliability. Ten students of the third and fourth grade level (8-10 years) participated in the second session. A subset of ten students from the fourteen was used because of the time restriction due to the school calendar. Clinical interviews, using three task methods, were used to explore each student's cognitive development. Method I involved the perception mode; Method II involved manipulative construction; Method III involved Logo programming. The interview sessions took place on a one-to-one basis with the researcher. A tape recorder was used to record all conversation between the subject and the researcher. Each session lasted 40 to 50 minutes. Task methods and sample questions are presented in the appendix. Table 9 shows the determined developmental levels on session 2.

The researcher calculated the correlation between the levels determined on session 1 with the levels determined on session 2. Corresponding predictability was made. If the correlation between the sessions is 1, perfect prediction is possible.

The calculated correlation coefficient between the levels of the two sessions is 0.91 ($r=15/16.4=+.91$), showing a high degree of relation between the sessions. This represents an 82 percent association ($r$ squared). The researcher can state that 82 percent is known of what would have to be known to make a perfect prediction from one session to the other. There is an unexplained variance of 18 percent. The prediction is high for the two testing
sessions, but it must be emphasized that the population is very small and does not provide a good measure that can be generalized to another population. However, the correlation measure provides a high reliability for using the test sessions as good predictors for assessing concept of angle.
Table 6. Comparisons of Test Methods--Session 1

<table>
<thead>
<tr>
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<tr>
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<td>8</td>
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\[\text{df}=2 \quad \text{Significance}=0.368\]

Table 7. Comparisons of Test Methods--Session 2

<table>
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<tr>
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</thead>
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</tbody>
</table>

\[\text{df}=2 \quad \text{Significance}=1\]

Table 8. Kendall Correlation Coefficients

<table>
<thead>
<tr>
<th>Factors Compared (N=14)</th>
<th>Significance</th>
<th>Correlation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Level With Age</td>
<td>0.307</td>
<td>0.1129</td>
</tr>
<tr>
<td>Level With Math Achievement Score</td>
<td>0.017</td>
<td>0.4796</td>
</tr>
<tr>
<td>Subject by Total Standing</td>
<td>Method 1</td>
<td>Method 2</td>
</tr>
<tr>
<td>--------------------------</td>
<td>---------</td>
<td>---------</td>
</tr>
<tr>
<td>SG</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>TV</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>DS</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>TM</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>KB</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>SL</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>CP</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>JR</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>PG</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>ND</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

Table 9. Levels of Methods by Subject--Session 2
Comparison of Piagetian and Robinson Levels of Development

In accomplishing the purpose of this study, namely to observe and assess cognitive development of concept of angle in third and fourth graders, it is important to relate the finding of Piagetian research to the findings of the presented study. Table 10 presents the behavior typical of each developmental level of concept of angle according to Piagetian research as described in Chapter II. The table shows a direct correlation between substage level progression and age growth. Beginning at 4 years and progressing to 6 years, 6 months, level IIA shows an inability to use any measurements and shows no understanding of angles on any of the three Piagetian tasks presented. Beginning at age 6 years and progressing to 7 years, 6 months, level IIB shows an insight into concept of angle without any accuracy on the three Piagetian tasks. Beginning at age 7 years and progressing to 9 years, level IIIA shows accurate comparison and reproductions of triangles and linear paired angles. Beginning at 9 years, 6 months, level IV shows attainment of proportionality in dimensional
relations. It is at this level and age that a concept of angle is fully developed according to the Piagetian levels of development.

In observing the cognitive development of concept of angle in third and fourth graders, this study established the "Robinson" levels of development. The correlation of level with mathematics achievement scores is significant, whereas the correlation of level with age is not. Because the mathematics scores do not progress linearly with the established levels, it is important to emphasize only the behavior typical of each developmental level of concept of angle. Table 11 shows the "Robinson" levels of development. Level 0, the "starting" level, shows a reliance on linear measurement of the arms of angle to assess angle measurement and on the same orientation of the angles. This level shows a cognitive development dependent on inaccurate means of assessing angle measurement. Level 1, the "competing" level, shows a continued reliance on linear measurement of arms of angles, but the rotation of angles is no longer an influencing factor in assessing angle measurement. This level shows a cognitive development that most often implements misleading input, but not always, indicating a cognitive struggle is taking place. Level 2, the "placing" level, shows that most often angle measurements are assessed correctly without regard to length of arms of the angle or rotation of the angle. This level shows a cognitive development that approaches a full understanding of angle measurement, but is not yet complete. Level 3, the "finishing" level, shows accurate assessment of angle measurement on all the presented tasks. This level shows a developed concept of angle. It is at this developmental level that the student can achieve in studies related to angles.
Keeping in mind that Piagetian tasks implemented triangles and linear paired angles and this study's tasks implemented isolated angles defined by segments, a comparison of Piagetian levels of development with "Robinson" levels of development is made. Behavior typical of "Robinson's" level 0 is pre-behavioral of Piagetian level IIIA. Superimposing was used at level 0 as it is in level IIIA, but behavior at level 0 relies heavily on the length of the arms in assessing angle measurement. Behaviors of "Robinson" level 1 and Piagetian level IIIA are closely aligned in that they show insight of angle measurement. Behaviors of "Robinson" level 2 and Piagetian IIIB are closely aligned in that assessments of angles are more systematically and frequently correctly made. "Robinson" level 3 and Piagetian IV both show attainment of the concept of angle.

Comparison of the ages at each of these paired levels fits a general pattern, but is not as precisely defined as the ages corresponding to the Piagetian levels themselves. The beginning age of "Robinson" level 0 (8 years, 11 months) fits within the Piagetian age for level IIIA. The ending age (10 years, 1 month) of the "Robinson" level 0, however, far exceeds the parameters of the Piagetian levels cited. The ages of "Robinson" level 1 (8 years, 7 months to 10 years, 4 months) are advanced of the age of level IIIA (7 years to 9 years). Ages at the other comparable levels are compatible, however.

According to Piagetian levels of development, one can conclude that a student has a developed concept of angle by 9 years, 6 months. According to "Robinson" levels of development, a student of 10 years, 1 month, might well be in the "starting" or level 0 of development. Whereas, another student of 9 years, 4 months has a developed concept of angle. Thus this study focuses on
prediction of cognitive development of concept of angle either by means of an interview testing situation with angles or use of mathematics achievement test scores. Ages eight to ten years were used only as a general parameter on which to focus for appropriate age testing.
## Behavior Typical of Each Developmental Level of Concept of Angle

<table>
<thead>
<tr>
<th>Substage Level</th>
<th>Age</th>
<th>Draw Pairs of Inscribed Triangles</th>
<th>Sort Cut-out Cardboard Triangles</th>
<th>Replicate the following Figure:</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>prior 4 yrs.</td>
<td>Questions not appropriate to this age level.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>IIA</td>
<td>4 - 6;6 yrs.</td>
<td>makes perceptual comparisons which lead to constant error</td>
<td>compares absolute dimensions in determining similarity of triangles; uses height and length of sides as the criteria with no attention to angles.</td>
<td>sketches angles; no use of measures</td>
</tr>
<tr>
<td>IIB</td>
<td>6-7;6 yrs.</td>
<td>has intuitive idea of parallelism</td>
<td>compares angle but with no degree of accuracy</td>
<td>uses linear measurement; visually estimates slope of segment; uses no angle measurement</td>
</tr>
<tr>
<td>IIIA</td>
<td>7-9 yrs.</td>
<td>uses parallelism without reference to angles</td>
<td>superposes the triangles and discovers the measure of the angles may be equal, independent of the length of the sides of the triangle</td>
<td>makes linear measurements of segments and slope; does not use a system of angles; shows insight into parallelism</td>
</tr>
<tr>
<td>IIIB</td>
<td>9-9;6 yrs.</td>
<td>uses comparisons to take account of parallelism and elementary dimensional relations</td>
<td>uses equality of angles and parallel sides to determine similarities</td>
<td>is able to accurately reproduce the figure:</td>
</tr>
<tr>
<td>IV</td>
<td>&gt;9;6 yrs.</td>
<td>This level of development shows attainment of &quot;true proportionality&quot;.</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

= Piagetian Geometry Test Item (Figure 2)

Table 10. Piagetian Levels of Development
<table>
<thead>
<tr>
<th>Level</th>
<th>Age Range</th>
<th>Math Score Range</th>
<th>M1 Perceptive Assessment</th>
<th>M2 Wire Construction</th>
<th>M3 Logo Reproduction</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 &quot;starting&quot;</td>
<td>8;11-10;1</td>
<td>5.8-5.9</td>
<td>uses linear measurement to assess angle measurement; indicates rotation of angle affects angle measurement</td>
<td>uses linear measurement; superposes angles, concentrating on length of arms; orientes angles the same</td>
<td>uses linear measurement to insure equal angle measurement; maintains slope by preserving a given direction of arms; keeps orientation the same</td>
</tr>
<tr>
<td>1 &quot;competing&quot;</td>
<td>8;7-10;4</td>
<td>5.2-5.6</td>
<td>shows insight of angle measurement, but relies on linear measurement; rotation of angles not an influencing factor</td>
<td>uses linear measurement as a strong influence in superposing; uses varying orientation of angles</td>
<td>shows insight of angle measurement; indicates linear measurement not a strong influencing factor; indicates rotation does not affect angle measurement</td>
</tr>
<tr>
<td>2 &quot;placing&quot;</td>
<td>9;4-9;6</td>
<td>8.9-9.2</td>
<td>perceives angle measurement most often with no regard to length of arms</td>
<td>assesses angle measurement correctly by superposing angles</td>
<td>creates angle measurement most often with no regard to length of arms</td>
</tr>
<tr>
<td>3 &quot;finishing&quot;</td>
<td>9;4-9;8</td>
<td>6.9-9.5</td>
<td>perceives correct angle measurement</td>
<td>assesses angles correctly by superposing</td>
<td>confidently and accurately creates equal angle measurement</td>
</tr>
</tbody>
</table>

Table 11. Robinson Levels of Development
CHAPTER VI

RECOMMENDATIONS

Recommendations for Education

In light of this study, recommendations for education are made.

Assessment of Concept of Angle

There is a need to assess the student's concept of angle before a student is instructed in angle related studies. Before a student is expected to answer angle related problems, the student needs a developed concept of angle. To determine a student's cognitive development of concept of angle, an assessment such as the one used in this study should be conducted. The "Robinson" developmental levels provide a reference for evaluating a student's responses in angle related studies.

Use of Mathematics Achievement Scores

Within the tested age group of eight to ten year olds, mathematics achievement scores are a significant predictor of development of concept of angle. If a student (eight to ten years) is a high mathematics achiever then development of concept of angle can be predicted to be at a later level of development. In the opinion of the researcher, the score from a test that had the
greatest number of geometric questions would be the best indicator of the student's developmental level of concept of angle.

**Discriminate Use of Angle Presentations in Mathematic Textbooks**

The presentations on angles in the mathematics textbooks at the third and fourth grade level should be used discriminately. As reported in Chapter II, several textbooks at this age level show use of a deductive approach. The textbooks use a lecture methodology of instruction as if dictating a developed concept of angle.

**Curriculum Construction to Align with Developmental Levels**

Assuming assessment, as recommended previously, has been made and the student has been identified with one of the cognitive developmental levels of concept of angle as established in this study, curriculum for the study of angles should be planned accordingly.

The curriculum involving the study of angles should spiral at the individual level, providing a continuously growing environment for the student. As levels of concept of angle are developing, the curriculum should be ready to provide the needed tools on which to build. The curriculum needs to provide activities that promote exploration and discovery. Teachers need to be guides and challengers, not lecturers, that assess developmental levels and teach accordingly. The students need to have the opportunity to exchange ideas among themselves, to share, stimulate, and learn together in a socially interactive way.
Stage 0. At this level activities that enhance exploration and discovery need to invest strongly in body movement and manipulative materials. Curriculum suggestions are as follows:

- students make large and small shapes and objects such as letters, clocks, books, using such tools as geo-rules, sticks, clay, yarn, wire
- students employ body movements to go short distances and long distances in the same direction then repeat the same movements in a different direction
- students use a large plastic triangle in a sandbox to simulate length and turns of Logo turtle, making different size designs, then write steps to make a chosen design, do it, change direction, do it again
- students use carpenter tools to make various size frames
- students use Logo to explore "turtle" turns, creating simple pictures of objects like teepees, tables, chairs, then create smaller ones, creating some that fall down
- encourage students to try ideas and see what happens.

Stage 1. Activities that enhance exploration and discovery begin to make a stronger transitory path from manipulatives to written or verbal words. Suggestions for curriculum at this level are as follows:

- students identify angles in the real world and describe them in words as the student reproduces the shape using manipulatives, pencil and paper, body movements
students cut and sew a quilt square, using a pattern such as Lincoln Logs, making small and large squares

students play a game of prediction using Logo, turning turtle and moving forward to get to designated spot (spots can be strategically placed for varied length of arms in creating equal angle measurements)

students pace out a big box and a small box, marking the floor with chalk or tape, noting turns and forward movements, then creating the shapes on the screen in Logo (perhaps REPEAT command can be introduced at this point)

-encourage students to try ideas and see what happens.

Stage 2. Activities that enhance exploration and discovery take on more complex thinking at this level. Curriculum suggestions are as follows:

students pace out a circle, smallest possible then larger, marking it with chalk or tape, allowing students to experience and discover movements from beginning to ending point with same heading, relating in words what occurred

students try creating a circle in Logo

students use pattern blocks to create designs and patterns, transferring ideas to Logo

-encourage students to try ideas and see what happens.

Stage 3. Activities that enhance exploration and discovery are an open arena for the students at this "finishing" level of development. Again, the teacher
should challenge and guide the student to investigate the world of angles.

Suggestions for curriculum are as follows:

- using Logo, guide students to "explore" the degree number to get the turtle to turn the number of degrees in a circle, rather than "telling" 360 degrees
- guide students to explore and discover the drawing of a circle, arcs, flowers, using repeat commands
- guide students to explore degree turns for regular geometric shapes, discovering how many total degrees turned to return to starting location with same heading
- introduce protractors to draw different size circles, creating designs, shapes, tesselations
- students should create complex Logo designs, using variable inputs
- students should take on any challenge with angle and angle measurements, manipulative, concrete, or abstract
- encourage students to try ideas and see what happens.

Develop a Spiraling Curriculum

The study provides a means for establishing a curriculum for the individual at the third- and fourth-grade level, using the established developmental levels as a guide. The curriculum for the study of angle should be considered in a broader spectrum, incorporating kindergarten through high
school. The approaches should always allow for exploration and discovery. The inductive approach allows the teacher to introduce a new aspect of angle, then guide the students in making discoveries that enhance their concept of angle. In allowing for individual styles of learning, a deductive approach is another approach, but it too, should allow for exploration and discovery. In the deductive approach the teacher tells the student what the inductive approach student is learning for him/herself. It is up to the deductive approach student to understand how the information was derived. This is too often overlooked and the given information is taken for granted. In general the student should explore, using concrete materials first to gain a base for understanding, then make a transition slowly to abstract thinking.

**Recommendations for Future Research**

There is little available research on a child's concept of angle. This study skims the surface of a deep and vast area of study that is needed. Future research to enhance understanding of concept of angle is recommended.

Valid tests for assessing concepts of angle are needed. The researcher designed her own methods for assessing concept of angles. Richard Noss, in his study of the effect that Logo has on a student's ability to understand angle, designed his own questions (Noss, 1985). Interestingly, Noss' paired angles on his paper-and-pencil test were comparable to the researcher's paired angles of Method I. Jane Rowland's study also lacked for a test to indicate a student's understanding of angle (Rowland, 1972). The researcher assessed her own methods as a good test design. The researcher suggests that considerable
attention be given to the protocol of the interviewing procedure. If the study is replicated, the protocol of possible answers given by subjects should be coordinated with further probing questions that are used with consistency throughout the interviews. Within this protocol of answers and follow-up probing questions, the interviewer should carefully select language in accordance with the child's level of understanding.

This study was limited by the population sampling. The results of a replication of this study using a larger and less restrictive sampling of population could provide better generalization. The researcher suggests that a replication of the study be extended to include second graders and fifth graders. Such a study could offer credibility to the established "Robinson" developmental levels as well as provide a broader base for determining individual developmental levels.

The computer language Logo is a relatively new and popular tool for exploring graphics. Many claims have been made for the value of Logo in the classroom, but little research has been done to validate these claims. A study of Logo as an instructional tool and its effect on concept of angle would be a valuable study to better establish Logo in the classroom curriculum.

A study assessing and correlating developmental levels of concept of angle with other variables than were correlated in this study would provide a wider base of predictors for assessing a student's cognitive development of concept of angle. Other suggested variables are reading level, I.Q., and hemispheric tendency of the brain. Correlating this tendency would give an indication as to the abilities of the spacial thinkers and the logical sequence
thinkers in respect to concept of angle. Perhaps one tendency is favored over the other when understanding angles.

A replication of this study or any part of it for any of the above reasons would offer substance to understanding the concept of angles. The three methods used provided scores for assessment in three modes: visual perception, manipulative construction, and Logo. Statistically the methods shared commonality and the methods taken collectively provided a comprehensive evaluation for each of the students. The protocol in each of the methods could be better refined. In some ways session 2 did that. Three paired angles represented on cards would seem adequate, representing right angles, obtuse angles, and acute angles. The protocol of Method II and III that asked the subject to produce an angle measurement smaller or greater than the presented one, more readily revealed the effect or lack of effect the length of the arms of an angle had on the angle measurement.

The researcher recommends a longitudinal study to follow the developmental levels of concept of angle in students from the kindergarten level through higher mathematics. Such a study would provide insight into the student's ongoing development in this area and provide a criteria by which to assess the curriculum with respect to the degree geometric instruction, specifically the study of angles, is taught and the effects it has on the student's understanding in mathematics that involves dimensions in space. Such a study should also take into consideration the student's style of learning, showing spacial or sequential tendencies.
CHAPTER VII

SUMMARY

The study focused on observing and assessing the cognitive development of concept of angle of third and fourth graders. The study revealed that research is lacking in this area. The two testing sessions of this study were conducted by the researcher using a clinical interview approach. Three task methods developed by the researcher were used in each of the two sessions. The task methods used visual perceptions, manipulative construction, and the Logo computer language to assess the subjects' cognitive developmental levels of concept of angle.

The researcher formulated her own developmental levels from the observations and named them "Robinson" Levels of Development. The researcher found that the levels significantly correlated with total mathematics achievement scores. There was no significant correlation of levels with age.

The "Robinson" Levels of Development were compared with Piagetian developmental levels. The Piagetian developmental levels were established by drawing pairs of inscribed triangles, sorting cut-out triangles, and reproducing a linear pair of angles. Piagetian levels correspond with age development. Both studies show that developmental levels of concept of angle are most evident between eight and ten years of age.
The study showed that total mathematics achievement scores are a better predictor of concept of angle than age. Future research should be done to establish other predictors for assessing students' developmental levels of concept of angle.

The researcher found a developmental progression of levels that she termed: "starting", "competing", "placing", and "finishing". The "starting" level shows the length of arms of an angle and rotation of angles misled the student to make incorrect assessment of angle measurement. The "competing" level shows an insight of angle measurement, but the student is still misled by the length of arms of an angle to make incorrect assessment of angle measurement. At this level, however, the student is no longer misled by the rotation of angles in correctly assessing angle measurement. The "placing" level shows correct assessment of angle measurement with infrequent regard to length of arms of an angle. The "finishing" level shows correct assessment of angle measurement without any consideration given to the length of the arms. This progression of development is important to education in that it provides a reference for identifying a student's cognitive developmental level for concept of angle so that appropriate and adequate experiences can be best provided to the student.

The study emphasized the need for the students' assessment of concept of angle before formal educational instruction is presented. Educational curriculums need to provide appropriate material and methods of instruction that allows the students to explore and discover with angles.
APPENDIX A

Transcription of Recorded Interview- Session 1

Method I - Five Sets of Cards

Interviewer: What do you think an angle is?

ND (9;6): Well...it's kind of like a point with degrees - with wide degrees and short degrees.

I: OK. Give me some examples of angles - maybe some you can see in this room - some examples showing angles.

ND: OK. That big book.

I: The book up there - okay. How is that showing angles?

ND: One side and the other side join together.

I: OK. Sounds good to me. All right, now, I'm going to show you some cards - five cards. Here are the two angles on the first card. Look at those and tell me if the angles on this card have the same measurement or different measurements - or which angle might be greater than another angle. OK?

ND: Yup. (Extended pause).

I: What do you think? What about those angles on the card?

ND: I don't know.

I: What about the measure of those angles. Just tell me what you think. (Long pause). If you're worried about an answer that is right or wrong, it's not that I'm looking for an answer right or wrong. I'm looking for what you're thinking. If you look at these and I say "are these angles of a different or the same measure?", tell me what you think.

ND: Yes or no?

I: Sure - "yes" or "no".

ND: This one (right) is different from that one a little bit.

I: All right. This one on the right is different from that one. Why?
ND: Because this one is a smaller angle and this one is a larger angle.

I: OK. And why do you think this one is a smaller angle than this one.

ND: Because it just looks smaller.

I: OK. This one looks smaller. I have a friend I showed these cards to and that's what she said. She said this one's smaller than that one because the lines are longer on this one. What do you think about what she said?

ND: (Long pause) I don't know.

I: If you had to prove this one was smaller than that one, how might you prove it?

ND: Well...I'd prove it because there's more space inbetween this one a little bit. That's it!

I: All right. Here's card two and the angles on card two. What can you tell me about these angles. Are they the same or different? Is one larger or smaller?

ND: (Quickly). They're different. This one is larger and this one is smaller.

I: All right. And why do you say this one over here on the left is larger?

ND: Because it comes together sort of like a "V" and there's more space inbetween than this one.

I: OK. Let's go on to the next card - card three. Tell me about these angles. Are the angle measurements the same or different?

ND: They're different a lot. This one is bigger than that one.

I: OK. The one on the left is bigger than the one on the right. OK. And why do you think it is bigger.

ND: (Pause). 'Cause there's more space inbetween than this one. (Long pause). These are more slanted.

I: The lines on the angle are more slanted? Just talk what you're thinking.

ND: OK. This one is bigger because it looks bigger and it has more white in the middle and more space.

I: OK - that's what I want. Just talk what you think. OK. Here's card number four. What about these angles - the measurement of these angles - are they the same or different?
I: OK. So you’re saying that the sides are longer and one is shorter. (Pause). My friend told me because the lines are longer on this one, the angle is bigger. Would you agree with my friend?

ND: No!

I: You would disagree with her?

ND: Mmm.

I: Okay and for what reason?

ND: Well, it doesn’t matter how much the other lines go. It just measures how long the angle is.

I: All right. What do you mean by the angle?

ND: Where they cross and how wide it is.

I: So what did you say about the measurement of these two angles?

ND: Well...the angle part is mostly the same but the lines that are connecting them are different.

I: All right - good. Let’s go on to card five.

ND: OK.

I: What can you tell me about these angle measurements on card five?

ND: Well...they look the same and...umm...the angle part - where it comes together - but this one has bigger lines.

I: OK. So what about the angle measurement?

ND: So I’d say they’re both about the same except for the line part.

I: Is that "line part" going to change the angle or leave it the same?

ND: It’s going to leave it the same.

I: OK. Is that it? Do you have anything more to say about those two angles?
Method II: Wires

I: All right, I'll put the card away and get out some chenille wire. I'm going to show you a couple of angles and you tell me which one of those angles is greater in measurement.

ND: I think this one.

I: You think the red one is. And why do you think the red one is greater?

ND: Because...the space in the middle looks bigger.

I: OK. And if you had to prove that, can you show me anyway that the space in the middle is bigger?

ND: Well, if you put them together I think this would be a little bit bigger.

I: All right. Why don't you go ahead and do that.

ND: Uh...oh.

I: OK. What can you say about these angles? Do you want to change your mind, or do you want to say anything more, or keep your same decision?

ND: I think they're both the same.

I: All right. Now I'm going to give you pieces of wire and I want you to make your own angles. Here are three different wires. Make an angle out of each of these.

(Time passes).

I: OK. Now I want you to take all five angles and order them from smallest to largest, beginning on your left.

(Time passes).

I: I notice you're putting some on top of the other.

(Time passes).

ND: There.

I: Good. Now, which one is your smallest angle?
ND: This one.

I: All right, and the one on your left is largest?

ND: Yup. This one's the largest.

I: All right. So you're saying that this green one is smaller than the grey one?

ND: Yup.

I: And this red one's smaller than the green one, and the purple is smaller than the red one?

ND: Yup.

I: When you were putting them on top of each other, what did you discover?

ND: That...umm...some angles are bigger than others.

I: All right. It's interesting how you have all the points of the angles pointing toward each other. Did you design that for any special reason?

ND: Yeah. Because it shows a little better how the angles are bigger and smaller.

I: All right. And how do you think it shows it?

ND: Well...they're like in it overall, and you can kind of see they're bigger.

I: OK. Fine. All right, let's move over to the computer.

Method III: Logo Programming

I: OK, N. Here you have my angle made on the computer and you have the turtle elsewhere. Using Logo commands that you know, make another angle of the same measure somewhere else on the screen. (Figure 6., picture 1)

ND: OK...the same measure?

I: Right.

ND: (Makes angle). Uh oh.
Figure 11: ND (9;6) Angle Representation

I: What do you mean "uh oh"?
ND: Oh, no...I guess that's pretty good.
ND: This is a little longer.
I: OK. That's a little longer...are you satisfied to leave it a little longer?
ND: No.
I: No? You're changing it...
ND: (Interrupts). That's good...that's good.
I: You're changing...why do you say that's good now?
ND: Well... because it doesn't matter if this is longer to the angle part.
I: No...really? I have another friend who told me if you got a lot of space between the two sides of the angle you're going to have a bigger angle. What do you think?
ND: I think (pause)...I think...I'm going to change this.
I: OK. What do you think then. Why do you want to change it?
ND: Because...mmm...(pause).
I: Do you think my friend is right?
ND: Which one?
I: The one who said if there is a lot of space between the two sides of an angle then you’re going to have a real big angle - instead if there is a little bit of space.

ND: Yeah. (Begins to change length of arm).

I: So you think my friend's right and you think you need to make that space smaller?

ND: OK. How do I make it smaller?

I: Do you want to make the turtle go back down the line? Use BK. If you want to erase it change PC to black.

ND: OK! (Erases part of segment).

I: Do you feel better about that?

ND: Yeah.

I: OK. So if my line were shorter here would we have different angles, do you think?

ND: No.

I: Why don't you think we'd have different angles?

ND: I think it doesn't matter.

I: You think what doesn't matter?

ND: If that's shorter.

I: So if I made my line shorter then you would have to change your angle, you think?

ND: I think so.

I: What would you do to change yours?

ND: Well, I'd just make that shorter.

I: Make that line shorter for yours, too?

ND: Yeah.

I: All right. Does everything else look okay? Anything else you wish you could change?
ND: This could be straighter.

I: OK. Your line and the angle could be straighter? And how would you straighten it?

ND: You would probably straighten it by making this all black and starting over again. Turn turtle to left some and draw a new line. (ND points with finger).

I: Let's call the next angle I have for you. OK. Here's my new angle. Again, make the same angle somewhere else on the screen. (Figure 6., picture 2)

ND: OK.

Figure 12: ND (9;6) Logo Picture:

ND: There.

I: Are you pleased with that?

ND: Yeah.

I: OK. What can you say about your angle and my angle.

ND: Well, I think this one...looks...mmm...I think the angles are pretty much similar.

I: OK. I'm going to change your angle (picture) a little bit by having your turtle go forward another FD 50. And then I'm going to have it go back down the total length 80 and then copy the same angle (LT 35) and draw another line FD 80. (Interviewer computes while talking through commands.)
Figure 13: I's Logo picture:

I: Now what can you say about the new angle - the alteration I made to your angle?

ND: You made the lines that connect the angle bigger.

I: Yes, I did. Now what have I done for that angle?

ND: Umm...you made a bigger angle.

I: I made a bigger angle? (Pause - no response). That's what my friend told me and so I'll tell my friend you agree with her.

ND: OK.

I: OK. Anything else about my angle or your angle you used to have?

ND: Umm...now this one is bigger than that one.

I: OK. So if I extend those lines some more - suppose another FD 30 - what would you say now?

ND: Let me see - okay - I think (pause) I think if this...I think you made the angle a little bigger...mmm...let me see...I think you have to do both sides to make it bigger.

I: I see. So if I extended the right hand side of this angle the same distance then you would say the new angle is...?

ND: Bigger.

I: OK. Let's go on to my third and last one. Again, make the same angle that I have elsewhere on the screen.

ND: (Works hard at getting incline of first segment correct.) There. That looks good.
I: You're satisfied with that, are you?

ND: Yup. (Continues to compute picture.) OK.

I: You like that?

ND: This could be longer.

I: The line could be longer?

ND: Yup. (Works to make line longer - measures with fingers and makes line longer.) That's good!

I: OK. Now you like that, right? Tell me - you were really satisfied with the length of the lines when you first made them, then you changed them until you got them just to your satisfaction.

ND: Yup.

I: Tell me why you think that was important?

ND: Well, just because it's better.

I: It looks better?

ND: Mmm.

I: How about the angle measurement?

ND: Well, I guess for that, too.

I: So you're saying that if the length of the lines hadn't been to your satisfaction, the angle wouldn't be to your satisfaction, either?

ND: No, not really. I guess it would just be the same.

I: What would be the same?

ND: The angle. (Pause.) Uh oh. I can't make up my mind. Does it matter if...

I: Let me draw an angle (picture) and see if you can make up your mind.

ND: OK.
I: (Makes angle picture as talking through the commands.) FD 15, keep T 75 the same, FD. Now compare my new angle with your angle. My lines are a lot shorter than yours. What are you going to say?

Figure 14: ND (9;6) Logo Picture

ND: I guess they're both the same. They are the same angle but the lines that connect the angle are different. It's the same angle.

I: All right. So it's the same angle. So going back to yours, you really wanted those lines to be precise - a precise length.

ND: But now...it doesn't matter.

I: What makes you feel so confident now?

ND: Because...I realize the angle part is just the part where the two lines meet, not where they go out - not where they spread.

I: So it doesn't matter about the length of the lines?

ND: No.

I: Not really? My friend says it does. 'Cause my friend says, like in his angle I made here, there really isn't much space between those two lines, so this should be a smaller angle.

ND: (Interrupts). Oh! Now I get it. Yeah, it does matter...if the lines...'cause the smaller angles and the bigger angles...more space.

I: So you like what my friend thinks?
ND: Yeah!

I: So the lines really do have a difference because...?

ND: Yeah. Because there's more space and traps more space in the middle.

I: So, it really was important to get those lines the right length, you think?

ND: Yeah.

I: Now what do you say about my angle?

ND: It's not as big as that one.

I: You're sure about that?

ND: Yeah.

I: Thanks. I hope you enjoyed this.

ND: Yup.
Session 2 began by the researcher remarking that the student would be doing some activities similar to the activities the first time we met (Session 1). There would be angles on cards to look at, making of angles with wire, and Logo programming.

**Method 1:** The researcher presented the student with two cards, one at a time. The student was asked to look at the angles and tell if the angle measurements were different or the same. If different, which angle measurement is larger (or smaller).

![Figure 16: Angles Presented on Cards Session 2](image)
Method II: The student was shown an angle representation made from a chenille wire and was given two other wires, one short in length, the other longer in length. The student was asked to choose one of the wires and make an angle measurement greater than the one shown. If the longer wire was chosen, the researcher probed to see if the student could make an angle measurement greater than the one shown with the shorter wire. If the shorter wire was chosen, the researcher probed to see if the student could make an angle measurement smaller than the one shown with the longer wire.

Method III: The student was presented with one pre-programmed representation of an angle in Logo. The student was asked a) to make an angle measurement the same as the researcher's angle measurement elsewhere on the screen and b) to make an angle measurement one-half the researcher's angle measurement elsewhere on the screen.
APPENDIX C

Transcription of Recorded Interview - Session 2

**Method I - Two Sets of Cards**

Interviewer: Tell me if you think the angle measurement is the same, or one is greater than or less than the other.

JR (9;8): The first one (on the left) is wider.

I: The one on the left is wider so that makes the angle measurement...what?

JR: Well, larger.

I: OK. Now, the one here on the right has some longer sides and my friend says that makes the angle measurement greater.

JR: It's not!

I: It's not? OK. Now, you would say the one on the left is greater?

JR: Yup.

I: Can you tell me how much greater it is?

JR: Umm...5 degrees maybe.

I: OK. Good (Pause). Here is card 2. Again, tell me about these angle measurements. Is one greater than the other; is one smaller than the other; are they the same angle measurement? What do you think?

JR: Mmm...they're the same.

I: OK. The same. What makes it look like they're the same to you?

JR: Because it looks like it's the same amount of degree, too.

I: OK. Fine - on to the wires.

**Method II: Wires**

I: (Angle made of chenille wire is presented). Now, choose a wire and make an angle measurement greater than mine.
JR: OK. (Makes angle). There.

\[\text{(red wire)}\]

Figure 19: JR (9;8) Representation of Angle Using Red Chenille Wire

I: OK. You chose the red one, and how do you know that's greater than mine?
JR: Because when I put them together, both sides don't touch all the way.
I: OK. Now take the other wire (longer one) and make an angle smaller than the one I made.
JR: (Makes angle).

\[\text{(green wire)}\]

Figure 20: JR (9;8) Representation of Angle Using Green Wire

I: OK. What makes that green angle smaller than mine?
JR: Because there's less amount of degrees in between.
I: OK. Good. So, if I were to put them in order from smallest to largest...
JR: (Puts wires in order - green, grey, red).
I: Good.

**Method III**: Logo Programming

I: OK, J, make an angle with the same angle measurement as mine somewhere else on the screen.

I: What do you think?

JR: I think they're the same
I: OK. Because mine sits and yours stands up - doesn't that make any difference?

JR: Nope!

I: You're sure?

JR: Positive!

I: Positive. Now, J, elsewhere on the screen make an angle about one-half the measurement of my angle.

JR: Mmm.

Figure 23: JR (9;8) First Logo Picture of One-Half Researcher's Angle

JR: Mine's less than half.

I: Less than half?


Figure 24: JR (9;8) Second Logo Picture of One-Half Researcher's Angle
I: Show me where one-half would be with your finger.

JR: About like that.

I: What makes it less than half?

JR: It looks more like a straight line than it should.

I: So it should look less like a straight line?

JR: Yeah.

I: OK. Your sides of the angle are the shortest. Does that help?

JR: It doesn't matter.

I: OK. That's it. Excellent. Thanks!
APPENDIX D

Consent Form

I, Sara Robinson, am conducting a study to observe and assess the cognitive development of concept of angle of third and fourth graders. The primary use of the information gathered in the study will be used to write a thesis to be presented at the University of Massachusetts. The obtained knowledge may also lead to further relevant publications, presentations, and future studies. In all written materials and oral presentations in which I may use information from the study, I will not use names of the participants. Transcripts will use initials of names of the participants.

Your child is being asked to participate in this study. Your child will meet with me on a 1-1 basis for approximately 40 minutes. During this time your child will participate in three types of task methods concerning angles. The first task method uses visual perception of angles, the second uses manipulation of chenille wire to construct angles, and the third uses the Logo language on the computer. A repeat session will be given four or more weeks later. All sessions will take place at Bement School at a time that is convenient to those involved.

The sessions will be audio-taped and later transcribed. The information gathered will be analyzed to assess developmental levels of third and fourth graders in respect to understanding angles. In no way does the response of the individual affect his/her academic standing.

All third and fourth graders at Bement School are being asked to participate in this study. Participation or non-participation of the student will not affect the standing of the student in any way. While consenting at this time to participate in this study, you are free to withdraw consent and to discontinue participation in the research sessions at any time without prejudice to the student.

In signing this form you are agreeing to your child's participation and are agreeing to the use of the information obtained as previously stated.

Please feel free to contact me as to any questions you might have. Thank you for your kind consideration in this matter.

I, ____________________________, have read the above statement and agree to my child's participation as stated above. Child's name:

Signature of participant's parent: ________________________________

Signature of participant: ________________________________

Date: ________________________________

Interviewer: ________________________________
BIBLIOGRAPHY


Robinson, Sara, "Conversion of Angle: A Pilot Study", paper presented at University of Massachusetts, Amherst, April, 1982.


