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EXPLORING THE POTENTIAL OF KNOWLEDGE ENGINEERING AND HYPERCARD FOR ENHANCING TEACHING AND LEARNING IN MATHEMATICS

A Dissertation Presented

by

DONNA E. LALONDE

Submitted to the Graduate School of the
University of Massachusetts in partial fulfillment
of the requirements for the degree

DOCTOR OF EDUCATION

September 1991

School of Education

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Throughout my doctoral program, I have benefitted from working with Dr. Richard Clark and Dr. Robert Maloy. As the Chair of my committee, Dr. Clark has been a tireless advocate for clarity and quality in this work. I have benefitted greatly from his insights. Dr. Robert Maloy influenced my thinking about knowledge engineering by providing a phenomenological perspective. He is a role model because of the thoughtfulness with which he accomplishes any task. My outside member, Dr. Franklin Wattenberg contributed to my thinking about the teaching of mathematics. His suggestions helped to strengthen the arguments made in this dissertation.

Finally, Emma and Larry have supported me throughout this process. Emma's good humor and positive outlook allow me to believe that the work is worth doing. Larry's unconditional support made it possible to surmount most obstacles.

ABSTRACT

EXPLORING THE POTENTIAL OF KNOWLEDGE ENGINEERING AND HYPERCARD FOR ENHANCING THE TEACHING AND LEARNING IN MATHEMATICS

SEPTEMBER, 1991

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This study adapted the knowledge engineering process from expert systems research and used it to acquire the combined knowledge of a mathematics student and a mathematics teacher. The knowledge base acquired was used to inform the design of a hypercard learning environment dealing with linear and quadratic functions.

The researcher, who is also a mathematics teacher, acted as both knowledge engineer and expert. In the role of knowledge engineer, she conducted sixteen sessions with a student-expert. The purpose of the knowledge engineering sessions was to acquire an explicit representation of the student's expertise. The student's expertise was her view of mathematical concepts as she understood them. The teacher also made explicit her understanding of the same mathematical concepts discussed by the student. A graphical representation of the knowledge of both student and teacher was developed. This knowledge base informed the design of a hypercard learning environment on functions.

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CHAPTER 1

INTRODUCTION AND CONTEXT

"For want of a nail, the shoe was lost;
For want of a shoe, the horse was lost;
For want of horse; the rider was lost;
For want of rider; the battle was lost;
For want of a battle; the kingdom was lost!"
[Gleick, 1987, p. 23]

1.1 Introduction

This dissertation describes the process used to design and implement an interdisciplinary hypercard learning environment for improving mathematics education. Aspects of the knowledge engineering process were used to develop a knowledge base which represented the combined expertise of a student and a teacher. The commercially available hypercard application was used to represent this knowledge of linear and quadratic functions and graphs in precalculus mathematics and introductory science. The central purpose, of this research, was to document the process of constructing a knowledge-based system which:

a. is informed by knowledge engineering sessions with a student-expert and a teacher-expert;

b. represents heuristic and published knowledge;

c. responds to the National Council of Teachers of Mathematics guidelines; and

d. employs technology which is widely accessible to teachers at all levels.

To accomplish these goals, the researcher worked with a college student who had recently completed the second semester of a two semester precalculus course at a four year public University.

The role of the student was to help elucidate the differences between a teacher's mathematical knowledge and the student's mathematical knowledge.

1.2 Researcher's Starting Conditions

Viewed as a dynamical system, the research presented in this dissertation is partly explained through a discussion of the initial or starting conditions. The bit of folklore introducing this chapter was also used by James Gleick to begin a discussion of the Butterfly Effect. In the study of dynamical systems, the Butterfly Effect is used to explain the system's dependence on initial conditions [Gleick, 1987]. In the fable, the initial conditions, a description of the system at its conception, is simply the lack of a nail. The dynamical system in this project and its initial conditions are more difficult to describe because it is in large part a result of the my evolution as an educator. Thus, some knowledge of the me and an understanding of this work, in the context of the current research on the impact of technology on education, is required.

The system is interdisciplinary because my approach to the study of mathematics and science has been to make connections. As a Master's project in theoretical Biochemistry, I completed calculations which supported the study of photosynthetic systems. Since completing this work, my focus has been teaching chemistry and mathematics at the college level and working in secondary mathematics and science teacher education with an emphasis on technology in teacher education. These foci represent starting

conditions which influence my desire to evolve a system toward an increased understanding of appropriate representations of mathematical knowledge, narrowing the expert-novice gap. In addition, the development of this system involves an effort to deal with more global issues of technology and its place in education. Questions related to this topic are the basis of the other set of initial conditions which influenced this work.

As an instructor of precalculus mathematics at the University of Massachusetts at Amherst, the only technology I use in my classes is an overhead projector. An informal survey of my colleagues shows that I am not unique; the most sophisticated technology used is a piece of chalk or a transparency pen and overhead projector.. There are notable exceptions. A special calculus sequence, "Calculus in Context", requires some of the homework assignments be done using the computer. Special sections of courses, for example "Differential Equations", require extensive microcomputer laboratory work using commercially available software packages like "Derive". This is juxtaposed with my role as co-instructor of a course for first-year secondary school teachers entitled "Impact of Computers on Schools and Society" which encourages the teachers to integrate technology into their teaching.

1.3 Technology and the Curriculum

Why use technology to facilitate learning in the curriculum? Because for the most part we have concentrated on the trivial answers to this question, we have not been able to encourage large scale changes in the curriculum.

Trivial answers imply that real math and science are only possible with computers. Learning to evaluate an integral no longer is necessary since numerous software packages (and even calculators) exist to do it for you. Students today are a part of a video generation, and they won't be motivated to learn in a technologically "dull" environment.

Trivial answers are the wrong justification because they do not focus our attention on what we can accomplish with computers, which is different from the traditional paper and pencil approach. A stronger justification for the redesign of the curriculum to include computers is that the technology allows exploration of exciting open-ended problems from subjects like chaos or evolution. Problems, particularly those involving interesting mathematics, are often not as easily accessible without the use of computers. Part of the motivation for undertaking the work described in the following chapters was to push beyond the trivial reasons for including technology in the curriculum.

As a researcher, educator, and citizen, I was heavily influenced, in pursuing this study, by my continuing effort to become comfortable with the educational and societal consequences of an impending Information Age. I agree with B. A. Sheil who wrote: "most previous major technological innovations, like the introduction of the automobile, although they may be profound changes in artifacts, did not require a change in any basic patterns of thinking" [Sheil, 1988, p. 86]. As I developed the present work, it was with the goal of contributing to the development of a new set of thinking skills which will be required of all citizens. A goal of this work was to stretch the boundaries of mathematics education to encompass education for life in the 21st Century.

The driving force in curriculum development should not be the training of a productive work force; however, we cannot be so removed from our students that we not pay attention to the real concern of being able to find jobs. As we think about teaching, we have to understand how technology is changing the work environment. Shoshana Zuboff reports in her book In the Age of the Smart Machine that "the new technology signals the transposition of work activities to the abstract domain of information; toil no longer implies physical depletion. "Work" becomes the manipulation of symbols, and when this occurs, the nature of skill is redefined." Zuboff relates peoples' description of work in factories and other settings where smart machines controlled production processes: "Accomplishing work came to depend more upon thinking about and responding to an electronically presented symbolic medium than upon acting out know how derived from sentient experiences" [Zuboff, 1988, p. 95]. Given this new definition of the nature of work, survival in our society depends on having the new skills necessary to do work. This is motivation for developing learning environments which foster the development of this new set of "intellective skills". Putting Zuboff's notion of "intellective skills" in the domain of the classroom, consider the words of one English educator, William Costanzo: "As teachers we ought to think about the ways our students may be influenced by these machines - not just as vehicles of information, but also and primarily as models of how to see and think, to read and write and reason" [Costanzo, 1988, p. 28].

Richard Dawkins in his book the Blind Watchmaker also makes the argument that technology changes the way in which we think. He writes "the computer can be powerful friend to the imagination" [Dawkins, 1987, p. 74].

Dawkins and Zuboff are exploring related areas, but Dawkins's framing of the implications of the technology is less centered on work. I recognize the power of the workplace to influence educational reform but am more comfortable with Dawkins's focus on imagination, almost a plea for life long learning. It is this sense that technology will foster the development of new insights and be one part of the starting conditions for this work.

Whatever the prevailing view, technology is changing the way people think. For the benefit of our students, educators need to grapple with these changes even if doing this forces us into new territories. As Costanzo writes: "I venture this far into the language of computer science and cognition because, I think, our profession has not gone far enough to understand the new technology and what its doing to our language" [Costanzo, 1988, p. 32].

1.4 Artificial Intelligence

One of the most engaging areas of research is the nature of human intelligence. Questions considered under this rubric include: What is the nature of memory, how do we solve problems, and what is learning? Clearly these questions cut across traditional boundaries and are investigated by cognitive science, psychology, philosophy, and artificial intelligence. Education should not be left out of this list. Regularly educators struggle with the issues of human intelligence; they are, but could more directly be, contributing to the discourse in the research community.

The artificial intelligence community has contributed enormously to our understanding of human intelligence. Contributions from the community will continue to be significant. This is, in part, due to a shift in focus of some members of the artificial intelligence community away from the purely computational. Luc Steels, for example, writes:

> "Textbooks talk about different computational formalisms such as rules, frames, and knowledge programming. They assume that knowledge can be translated more or less directly into computational structures from observations of the expert's problem solving or from verbal reports about this knowledge. It is true that at some point in the process of developing a working application, we have to face decisions on which implementation medium to use; however the computational answer is only partly satisfactory. The gap between the implementation level and the knowledge and problem solving that we observe in the human expert is too wide" [Steels, 1990, p.29].

As the artificial intelligence community becomes more interested in narrowing this gap by probing human intelligence, teachers need to be aware of this research. Teachers need mechanisms by which they can learn about research and contribute actively to research projects. This means being familiar with the language and substance of existing research. As I started work on the present project, I viewed it as a potential mechanism to introduce teachers to the field of Artificial Intelligence and encourage them to begin asking questions and recording their findings. If there is going to be a partnership between the AI and teaching communities, the teachers cannot be viewed as receptacles for information. Their roles must involve more active participation in all aspects of the research process.

1.5 Teachers and Technology

Introducing technology into the curriculum can either be done for reasons supported by pedagogy and sociology or, because in the search for the "quick fix", technology is the current candidate. The introduction and the curriculum developed will obviously be more powerful if it is the former reasons which motivate us to change. Whatever the motivation, some modification of curriculum seems likely. It is important to raise some of the issues which must be dealt with as we modify what is taught. This provides yet another initial condition, a sensitivity to the culture of schools and the need for the teacher to play a central role if effective change is to occur.

Power On!, the study completed by the Office of Technology Assessment, showed that access to technology is not sufficient to generate enthusiasm for redoing the curriculum [U.S. Congressional Office of Technology Assessment, 1988]. Sarason describes the culture of schools:

> "The dominant impression one gains is that school personnel believe that there is a system, that it is run by somebody or bodies in some central place, that it tends to operate as *a* never ending source of obstacles to those within the system, that a major goal of the individual is to protect against the baleful influences of the system, and that any one individual has and can have no effect on the system qua system" [Sarason, 1982, p. 163].

Teachers will use what they understand and feel invested in, so it is critical to provide the teacher with ample opportunity to contribute to the design process. A computer scientist would not consider building a medical expert system without medical practitioners as contributors to the design process. It only makes sense that software, to be used in the schools, be designed by a team including teachers.

The National Council of Teachers of Mathematics has concluded that it is critical for educators both in methods and domain courses to model good teaching behavior for their prepracticum students. The way in which mathematics is taught, in large part, determines success or failure on the part of the student and establishes positive or negative feelings about the subject. This is not meant to take discussions of mathematics education to the popular psychology level. It does mean that if our concerns about the mathematics competency of students in the United States are genuine, we have to be willing to consider that attitudes developed at an early age have a significant impact on the students carrying on in mathematics.

The current work fits within the general rubric of teaching and learning in mathematics; technology and artificial intelligence provide the context. The teachers' responsibilities need to include active involvement in the development of new technology, planning, implementation, and evaluation of new technology in the classroom, and contributions to the research effort endeavoring to understand human thinking and learning. As Dede states: "Using cognition enhancers {e.g. hypermedia}, however, requires more than learning how to activate the machines and issue commands; the style of working must change" [Dede, 1989, p. 24]. The same could be said of the style of teaching. This project explores one area where it is possible to involve teachers actively in a research agenda while supporting necessary changes in style.

One avenue toward better teaching is to study the differences between experts and novices to elucidate the differences in order to facilitate the development of expertise. Teachers engage in this exploration on an almost

daily basis as they encourage their students to develop some new skill or to discover a new concept. Teachers have not been encouraged to frame their activities as research questions, and they have not been provided with a mechanism to organize their data.

Rather than attempting to establish a distinct set of technological skills appropriate for all teachers, this work will explore one mechanism for linking exploration of technology to research on teaching. Through the use of applications like hypercard, teachers will be able to explore issues of teaching and technology.

There is some historical support for the proposition that developing knowledge-based systems is a productive learning experience. This work is in part about discussing a new conceptualization of the teacher's role, that of the teacher as knowledge engineer. Allowing teachers to see themselves in this role moves us in the direction of including teachers as part of the research team involved in the development of intelligent learning environments. It introduces the vocabulary and some of the ideas without being overwhelming. Working in the Hypercard environment focuses the work at a level of technology which is accessible to most teachers. It encourages the teachers to generate their own applications based in large part on their experiences. This serves to validate their experience as well as to ensure that the software reflects their perspective which will contribute to it being utilized. It also encourages the teachers to begin developing the intellective skills necessary for survival in the information age. One of the most effective ways for students to develop these skills is to have role models to follow.

1.6 Overview of the Study

This introduction has focused on the initial conditions, that is my preconcerns. In the next chapter, I will review the literature which supports this research. Descriptions of the knowledge engineering process as it followed in the development of expert systems and of the modified knowledge engineering process used in this research process followed during the course of the research and the results are contained in Chapter Three. A summary of the knowledge engineering sessions conducted during the course of this research and a knowledge representation developed based on these sessions are also presented in chapter three. An introduction to hypercard and a description of the hypercard application implemented as a result of this work are found in Chapter Four. Design decisions made as a consequence of the knowledge engineering sessions are presented in Chapter Four. Conclusions and implications for future work are discussed in Chapter Five.

Since the remainder of this document concentrates on what this work is, it is important to be explicit about what this work is not. This work used software applications and hardware currently available in most secondary schools. This equipment is not necessarily state of the art, but it is appropriate to the current culture of most schools. This work was not exhaustive in its investigation of the understandings of the student, as it involved only one student and one teacher.

CHAPTER 2

REVIEW OF THE LITERATURE

2.1 Introduction

The major challenge of this project is to explore the implications of the knowledge engineering process for mathematics education. The context for this work is the design and implementation of a hypercard application which represents the views of two participants, both expert, of selected topics in precalculus and their relationship to the natural sciences. One expert, a teacher and knowledge engineer, has subject area expertise; the other, a precalculus student provides the perspective of the learner. This work is informed by literature in three areas: (1) expert systems; (2) hypertext and hypermedia applications; and (3) mathematics education.

2.2 Expert Systems

To provide a foundation for this work, I will describe an expert system and briefly discuss some of the most significant work in expert systems research. This will be followed by a survey of the more promising applications of expert systems to education. The section will conclude with a discussion of current research on knowledge acquisition and knowledge representation which are relevant to this research project.

An expert system is a computer program, capable of solving problems which requires significant knowledge of the problem domain. A typical expert system is composed of three parts - the knowledge base which is the explicit representation of expertise in a particular area, the inference engine, which provides direction to the system as to how use the knowledge in the

knowledge base to accomplish a specific agenda, and the user interface, which allows for human machine interaction [Buchanan and Shortliffe, 1984].

Since the mid-sixties research in the area of expert systems has contributed to the field of Artificial Intelligence. Historically, most notable are DENDRAL, a system which assists chemists in the interpretation of mass spectroscopy data, MACYSMA, a system which solves problems in differential and integral calculus, and MYCIN, a system which diagnoses and suggests treatment for infectious blood diseases [Hayes-Roth, Waterman, and Lenat, 1983]. Although expert systems research is only a small part of the field of artificial intelligence, the early work in expert systems provided the foundation for other research areas. Of particular importance is the work on Intelligent Tutoring Systems (ITS) which directed the attention of the research community to issues of tutoring and the components of an effective tutor namely, the teacher, the student, and the domain expert [Clancey, 1987-a]. More recent work on the development of Intelligent Learning Environments extends the ITS research and keeps active the investigation into learning [White and Frederiksen, 1990]. Researchers in this area while working to develop computer systems address issues which grapple with the nature of knowledge. For example, there is Anderson and co-workers ACT* theory of cognition [Anderson, Boyle, Corbett, and Lewis, 1990], and the work by Laird, Newell and colleagues to build a system capable of general intelligent behavior (SOAR) [Laird, Newell, and Rosenbloom, 1987]. Steels and co-workers are studying the "components" of expertise which has the potential to contribute to the teacher education community's understanding of what constitutes expertise in teaching [Steels, 1990].

In the previous paragraph some of the implications of expert systems and related research for the education community were presented. It is worth recognizing the interesting work which has been accomplished. Knox-Quinn has reported on a successful summer course during which junior high students used expert system shells (the shells consist of the user interface and an inference engine; the designer constructs the knowledge base) to build their own expert systems. Engaging in this activity allowed the students to gain insight into their decision making process [Knox-Quinn, 1988]. In the work reported by Knox-Quinn the students did not design systems in a traditional academic area; Morelli reports on a summer workshop during which junior high school students built an expert system in the domain of botany [Morelli, 1990]. Trollip and Lippert report on a college course for education students where the class project was to build an expert system. The course was on Intelligent CAI so the students developed expert systems which would be useful to CAI developers. This work supports the idea that building expert systems is a productive endeavor [Trollip and Lippert, 1987]. The Trainee Teacher Support System (TTSS) is an expert system designed to provide advise novice teachers on classroom practice. TTSS is an example of research which joins education and artificial intelligence [Wood, 1988].

The expert systems literature describes in great detail the work involved in the development of a system [Hayes-Roth, et. al, 1983]. Of particular importance in this work is the phase of expert system development referred to as knowledge engineering. During this process a knowledge engineer, typically a person with computer science background, and a domain expert, work together to enable the knowledge engineer to formulate

the expert's knowledge in machine usable form. This machine usable form is the knowledge base of the expert system.

Construction of the knowledge base or knowledge acquisition is time consuming and is identified as the rate limiting step. Much current research is involved with methods of facilitating this process. Of interest are efforts to design knowledge acquisition tools which are "aware" of and capitalize on the expert's representation of knowledge [Gruber and Cohen, 1987]. Although not of direct relevance to this project, it supports the proposition that it is beneficial to focus on better understanding of the expert's representation. Of more direct importance to this project is work which provides specific techniques for knowledge acquisition. Davies and Hakiel have written a general article which outlines in some detail the steps of the knowledge acquisition process with advice on how to accomplish the task [Davies and Hakiel, 1988].

Clancey has written extensively about the role of the knowledge engineer. He supports the concept that the knowledge engineer is an excellent model for a good student because it is critical for the knowledge engineer to be able to ask good questions [Clancey, 1987-b]. Therefore by studying the knowledge engineering process, we will learn something about asking good questions. The process as well as the product has value.

In addition to providing the system with expertise, studying the knowledge base provides insights into the nature of the domain and supports the transition from expert to novice. Clancey has detailed an argument for viewing the knowledge base as a qualitative model of the domain [Clancey, 1989]. This is important because often new teachers and students lack a model of the domain they are attempting to understand. Without an

appropriate model, it is difficult to be successful in learning. In addition, as Kolodner asserts: "the evolution from novice to expert requires introspection and examination of the knowledge used in solving problems" [Kolodner, 1984, p. 96]. Part of this evolution is comes from experience which Kolodner claims "turns unrelated facts into expert knowledge" [Kolodner, 1984, p. 96]. She continues: "It implies that even if a novice and an expert had the same semantic knowledge (i.e. knew the same facts), the expert's experience would have allowed him to build up better episodic definitions of how to use it" [Kolodner, 1984, p. 87]. Part of the work described in this document involves exploring computer based applications which might support the introspection necessary for the development of expertise. There are examples of this introspection in the teacher education community. A number of researchers have documented improvements in the teaching environment when teachers consciously monitor and reflect on their behavior [Kounin, 1970, Good and Brophy, 1984, and Canning, 1991]. One method of reflection proposed by Mannin and Payne is the Cognitive Self-Direction Methodology for Teacher Education. A significant part of this methodology is "self-talk" where master teachers record and transcribe their ideas and responses to various classroom occurrences [Manning and Payne, 1989].

Previous work has demonstrated the potential for knowledge based systems. It is clear that the knowledge engineering process and the process of determining an appropriate representation for knowledge has the potential to inform the research in numerous domains. As Gammack and Anderson write: "Knowledge is not simply a static organization of facts, but must also acknowledge the dynamic context in which it is applied" [Gammack and Anderson, 1990, p. 19]. This is a powerful statement when considered from

the perspective of an educator and a student. In addition to providing our students with both surface and deep knowledge we have to think about the framework or context in which the knowledge is applied.

Consideration of the nature of knowledge raises perplexing questions. Of particular interest is casting the question in terms of the Heisenberg Uncertainty Principle: "In classical physics we do not have to take into account the fact that in answering the question-doing an experiment-we alter the state of the object. We can ignore the interaction of the apparatus and the object under investigation. For quantum objects like electrons this is no longer the case. The very act of observation changes the state of the electron" [Pagels, 1982, p. 74]. Clancey has begun exploring the implications of the "Uncertainty Principle" view of knowledge for artificial intelligence [Clancey, in press]. This current project will explore the implications for mathematics education.

2.3 Hypermedia

Hypermedia has already had an impact on education. Numerous successful projects involving hypermedia exist including:

> The Perseus Project, a collaborative effort, has the goal of implementing a hypermedia system to assist in the study of Greek Civilization [Crane and Mylonas, 1988];
>
> Particles and interactions HyperCard Software, developed by scientists and secondary school educators at Lawrence Berkeley Laboratory [Fundamental Particles and Interactions Chart Committee, 1989];
>
> The Enriched Learning and Information Environments (ELIE), an effort by researchers at Indiana University, to design hypermedia learning environments which will contribute to workplace and university productivity [Schwen, et. al., 1990];

Exploring the theory of intertextuality using a PC based hypertext system which allowed students to generate their own applications to support the study of Milton's Paradise Lost [Havholm and Stewart, 1990]; and

Neuro Syllabus a project at the University of Arizona which uses hypercard to allow students to explore the material covered in a course, supporting user addition of information [Louie and Rubeck, 1989].

These applications as well as the hypertext literature support the work of this project by demonstrating the applicability of HyperCard and other hypermedia systems as appropriate development tools for educational applications [e.g. Freidler and Shabo, 1989 and Raker, 1989]. In addition to these examples of hypermedia, teacher educators at Vanderbilt University are using hypermedia in their elementary teacher education program. Goldman and Barron write: "We believe that hypermedia technology has the potential for creating a new type of teacher education program-one that moves traditional college and university courses away from a teacher directed lecture format and into a problem solving/analytical mode" [Goldman and Barron, 1990, p. 29].

A discussion of scale is important to this project since many of the successfully hypermedia projects involved significant commitment of resources. It is important to point to successful examples which used limited resources. The work of Havholm and Stewart is relevant. This work involved incorporating the development of hypertext applications into the required work of an undergraduate literary theory course. The applications were developed by the literature students and three student programmers working with the class. The instructors report on the the results: "Finally, however crude or partial the models, their examination as a kind of deductive "result" of the theory made it possible for students to be unusually clear about the

powers and limitations of the theories themselves [Havholm and Stewart, 1990, p. 48].

The knowledge base segment of the expert systems literature focuses our attention on the need to make knowledge accessible for exploration and introspection if expertise is to evolve. The opportunity to browse a hypercard environment promotes this exploration and introspection. Traversing a network, constructed out of an expert's experience, will encourage the novice's evolution to expert. Research supports the proposal that hypercard is a good tool for developing knowledge-based systems and learning environments [Harris and Cady, 1988, Marchionini, 1988, Tsai, 1988-89]. As Evans writes: "Hypercard can be ideal for implementing several types of knowledge-based applications. Many learning, reference, and diagnostic systems already have been created using only the simple associative links that HyperCard provides. Interestingly, this form of knowledge representation is so intuitive that many of the domain experts who authored these stacks did not realize they were actually creating knowledge-based systems" [Evans, 1990, p. 317].

A new technology which combines hypertext with expert systems is Expertext. Barlow and co-workers describe their view of expertext systems as systems which will promote a "sharing of intelligence between the user and the expertext system' [Barlow, et al., 1989, p. 117]. This linking of expert systems with hypertext is interesting because it has the potential to address some concerns about the navigability of large hypertext systems. Endeavors to develop intelligent hypertext systems will present additional opportunities to address the questions of the nature of knowledge.

2.4 Mathematics Education

"Mathematicians have a naive idea of pedagogy. They believe that if they state a series of concepts, theorems, and proofs correctly and clearly, and with plenty of symbols, they must necessarily be understood. This like an American speaking English loudly to a Russian who does not know English, in the belief that his increased volume will ensure understanding [Kline, 1977, p. 117]. Kline was considering the question "why the professors can't teach?" points out the problems in undergraduate mathematics education. The National Council of Teachers of Mathematics (NCTM) have issued curriculum guidelines for teaching secondary mathematics which address similar problems to those seen by Kline in undergraduate education. According to the NCTM, all mathematics curriculum should demonstrate mathematics as problem solving, mathematics as reasoning, mathematics as communication, and mathematical connections. In addition the curriculum should promote an understanding of the historical and cultural context of the material [NCTM, 1989]. This project is an attempt to integrate Kline's and the NCTM's recommendations into knowledge-based learning environments.

Cognitive studies in mathematics education is an active area of investigation. A comprehensive review of this literature is not appropriate, but it is informative to summarize the widely accepted modes of inquiry. A number of researchers have examined areas in an effort to both characterize "mal-rules" and to contribute an understanding of learning is accomplished in mathematics [e.g. Matz, 1983, Brown and VanLehn, 1980, and Payne and Squibb, 1990]. Another area of inquiry is to concentrate on problem solving [e.g. Reed, Dempster, and Ettinger, 1985, Riley, Greeno, and Heller, 1983, and Sweller and Cooper, 1985]. Finally Perkins and Simmons have

attempted to characterize misunderstandings that are both within a particular domain and interdisciplinary between science, math, and computer programming [Perkins and Simmons, 1988].

The expert systems and mathematics education literature encourages the study of expert novice differences [e.g. Larkin, et al., 1980 and Leinhardt, 1989]. Much work has been done on these differences as they appear in the domain of mathematics, specifically in regard to algebra, of which functions and graphs are a part. Wenger states: "students can perform the required symbolic manipulations correctly, but they have difficulty knowing which approach to select. This observation suggests that students' difficulties result not so much from the content of their mathematical knowledge but from its organization" [Wenger, 1987, p. 220]. Wenger feels textbooks that often present topics as discrete units contribute this deficiency in organization [Wenger, 1987]. A strength of the hypercard system is that it links concepts together which support the development of an appropriate mathematical organization.

Kaput argues for the development of environments which encourage connections between representations. He states: "Ongoing ETC work suggests that appropriate experience in multiple, linked representation environment may provide webs of referential meaning missing from much of school mathematics and may also generate the cognitive control structures required to traverse these webs and tap the real power of mathematics as a personal intellectual resource" [Kaput, 1989, p. 180]. This project is responsive to Kaput's recommendations. It will provide the opportunity for the novice to explore the representations of an expert thereby supporting the development of expertise. By taking advantage of the inherent hypercard

structure to link various representations, it will encourage the user to develop a sense of meaning not just of symbol.

As stated previously, this project will implement an environment to allow the user to realize the connections between topics in mathematics and the sciences. Senk reviews the literature which supports the need for this type of tool:

> Weiss (1987) reports that secondary mathematics teachers generally have little formal coursework in applications of mathematics. Furthermore, the University of Chicago School Mathematics Project (Usiskin, 1986/1987) has consistently found that, although mathematics teachers are generally willing and interested in using realistic applications, few know where to find examples within the grasp of secondary school students (Hedges, Stodolsky, Mathison, and Flores, 1986) [Senk, 1989, p. 216].

Kline supports the need for interdisciplinary work at the undergraduate level: "What should a college course in mathematics for liberal arts students offer? The answer is contained in the question. The liberal arts values of mathematics are to be found primarily in what mathematics contributes to other branches of our culture" [Kline, 1977, p.129].

The use of technology in the teaching and learning of mathematics is an area of inquiry related to this work. Some examples have been previously discussed, and it is worth citing others which are of interest. *Word Problem Assistant* (WPA) developed by Thompson encourages the student to pay attention to relationships between parameters involved in the problem not on formulae [Thompson, 1989]. Cornu and Dubinsky propose educational software development based on a cognitive theory. In their view "mathematical knowledge consists of constructing objects and processes and the methods of construction, called *reflective abstraction*, include:

interiorization, coordination, encapsulation, reversal, and generalization" [Cornu and Dubinsky, 1989, p. 75]. They are using a programming language called ISTEL to assist their students in reflective abstraction [Cornu and Dubinsky, 1989].

Although there are many research areas in mathematics education, it is clear that the nature of knowledge in mathematics is a priority [e.g. Davis, 1989, Larkin, 1989, and Lampert, 1990]. This research project will contribute to the discourse in this area.

CHAPTER 3

DESIGN AND CONDUCT OF THE KNOWLEDGE ENGINEERING COMPONENT OF THE STUDY: ACQUIRING THE KNOWLEDGE BASE

3.1 Introduction

This chapter describes both the design and the actual process followed in constructing the knowledge-base from which the hypercard application described in Chapter Four was formulated. This chapter is organized in two major sections.

Section 3.2 focuses on design, starting with the research from expert systems and, particularly knowledge engineering. It then presents a rationale for hypercard as the selected tool for building the implementation, and for functions and their graphs as the content to be explored. It includes a description and rationale for the "novice-expert" employed in the study, and the planned roles of both the novice-expert and the knowledge engineer-researcher. This section concludes with the projected (and actual) plan of knowledge engineering sessions held.

Section 3.3 reports the process of, and insights gained through, the knowledge engineering sessions and concludes with a graphical representation of the knowledge-base. Rather than reporting interactions sequentially for each of the sixteen sessions, the researcher reports results topically, emphasizing those interactions which underscored expert-novice and knowledge engineer-expert differences and which particularly informed the knowledge-base representation.

3.2 The Design

3.2.1 Expert Systems and Knowledge Engineering

The foundation for this work comes from expert systems research, particularly the process of knowledge engineering. In conceptualizing this work, I planned to act as knowledge engineer with a student as expert and use these interactions to design and build a hypercard application. Essentially I planned to follow a standard knowledge engineering program which I will describe here.

The knowledge acquisition process or the "mapping of expertise" to the expert system progresses through five major stages. The first stage entails the identification of the problem the expert system will be expected to solve. During the first stage the domain of the expert system is established, making it possible to identify human experts. For example the domain of MYCIN, one of the first expert systems, is the diagnosis and treatment of infectious blood diseases; DENDRAL, a project begun at Stanford in 1965, is used to assist chemist in the interpretation of mass spectroscopy data. Experts for the former, thus, were physicians; Implicit in identifying the problem is assuring that the problem is significant enough to warrant the effort of development and implementation.

Stage two is the conceptualization, which involves making explicit the knowledge identified in the first stage. It is during this stage that the knowledge engineer works with the human expert or experts to develop the knowledge base of the expert system. Using the DENDRAL project as an example, chemists who were expert in the field of mass spectroscopy would be interviewed by the knowledge engineer. Both published and heuristic knowledge would be acquired. The knowledge engineer would engage the

experts with the goal of ascertaining the knowledge which distinguished them in their field.

During the third stage, formalization, a structure is determined for the knowledge. Early systems were rule based. Formalization involved transforming the expertise into if-then rules which would be manipulated by the system's inference engine to solve problems. Other formalizations include frames and schemes. The formalization focuses the attention of the researchers on the nature of knowledge, and this is currently an important research topic [Clancey, in press].

Having accomplished these three stages, a prototype system is implemented and tested during the fourth and fifth stages [Buchanan, et. al.., 1983, pp. 140-147]. Testing provides the opportunity for the expert and perhaps other appropriately identified individuals to evaluate the system for validity and usefulness of the expertise as ii is represented. It is worth noting that in expert systems development, as with any development process, the stages are not discrete. Rather they form a set of interacting activities which result in the completed product. For example, it may be the case that parts of stage one and stage two occur simultaneously or that as a result of discussions nominally in stage two, stage one is modified or it may be that parts of implementation and testing occur while stage two is still in process.

In the present project, some of the decisions normally made during stage three were made prior to beginning the knowledge acquisition process [Gruber and Cohen, 1987]. Because a goal of this project was to investigate the applicability of hypercard, I began the process knowing that the knowledge would be represented in a hypercard system. The formalization that would take place during stage three would involve designing cards and

constructing the hypercard links to represent the knowledge obtained during stage two. Although I would not be making the traditional decisions about representations, I would still have to grapple with the critical issue in expert system design, that the system representation match the expert's representation. This is important because research has shown that a correspondence between the system's representation and the expert's facilitates the knowledge acquisition process. Part of this work was to explore the extent to which hypercard would support the development of an application which did match the expert's representation thus making it an appropriate tool for learning environments.

3.2.2 Hypercard

The choice of hypercard as the environment for the implementation of this work was motivated by the current research involving hypercard systems and my observations that this environment was useful, based on having worked with it in other contexts. My experiences included working on a hypercard application to help undergraduate tutors of language minority and culturally diverse secondary school students. The TEAMS tutoring program at the University of Massachusetts places undergraduates as tutors in local elementary and secondary schools and alternative education programs. I worked on a hypercard application which would provide information about the schools and strategies for tutoring. The positive reception of the tutors to the prototype was encouraging and strengthened my inclination to use hypercard for this project. The tutors liked the prototype because it accurately represents knowledge they have and find useful. In addition to my positive experience with hypercard, there are the successful applications in the

education community which I discussed in Chapter 2. Thus, I adopted as a working theory that the education community was beginning to recognize the usefulness of the hypercard environment.

The potential for a general acceptance of the application by the education community is an essential part of this current work. The many examples of software, foisted upon the community as "save the world" stuff, which are now collecting dust in classrooms across the country, made me carefully consider my choice of application. Since an essential part of my theory asserts that teachers must be comfortable using the software, and they must feel that meaningful support is available, I wanted my implementation to be consistent this theory.

Secondly, the implications from much of the hypercard research is that the hypercard environment stimulates learning. This is another significant component of my theory. Essentially any hypercard application is a graph so even tentative explorations result in the user making connections between pieces of information. With every use of a system, the potential exists for making new connections or for fortifying existing connections.

Finally, there is a practical reason for using hypercard as an environment. It is readily available and relatively straightforward to use. The education community has been forced to deal with applications that were either prohibitively expensive or prohibitively difficult to learn to use. As with most computer applications, levels of sophistication will vary, and some users will be able to build amazing applications while others may not vary from the simple model. Our goal is not to train computer scientists, but to develop a technological way of thinking (a "high tech state of mind") [Quinn Patton, 1987, p. 15].

As I stated in the introduction, the emphasis should be on how technology will change the way we teach, not just to automate our curriculum.

3.2.3 The Domain

The first stage of the knowledge engineering process was the identification of the problem that the expert would be expected to solve. This came directly from my experience as a teacher. For the past four years, I have been teaching precalculus mathematics to undergraduates at the University of Massachusetts. Prior to this I taught chemistry and mathematics at a community college. I have always been interested in the applications of mathematics to science. My dissatisfaction with the way much of precalculus is taught and the recent report from the National Council of Teachers of Mathematics on the teaching of mathematics provided the basis for stage one [NCTM, 1989].

It obvious from the performance of my students and other students that they are not able to transfer their knowledge from math class to chemistry or physics class. As a chemistry teacher, I often felt if the students could do the mathematics, teaching chemistry would be easy. In fact one of the reasons I am now teaching mathematics is to gain some insight into mathematics education and to simultaneously improve the teaching of mathematics and the teaching of science. For the present study, I wanted to work on a topic in Precalculus which I felt was critical to success in science and a topic which was essential to further success in mathematics. The topic of functions fit both these criteria, so it was selected as the problem area. Secondarily, I felt work in this area would help my expert, Beth, in future math and science courses, and I wanted work on this project to be beneficial in this way.

The study of functions is usually the way a calculus course is introduced. It was not my goal to attempt to construct a calculus course, so the final part of stage one was to narrow the topic sufficiently to ensure a reasonable chance of success. I finally decided I would concentrate on linear and quadratic functions at the level of sophistication of precalculus students. The topic was of interest to me, and it was one appropriate to the level of my expert.

3.2.4 Identifying the Expert

Crucial to the development process of an expert system is identifying a human expert or experts in a particular domain. The human expert will work with the knowledge engineer to construct the knowledge base. The project will depend on the knowledge engineer and the expert being able to sustain a productive working relationship. With this awareness, I asked Beth to work on this project. Beth had been a student in one of my classes during the Fall 1989 semester and the Spring 1990 semester. Both semesters the classes were large lectures with approximately 200 students. In this environment the student must show a great deal of initiative to establish a relationship with an instructor. Early during the Fall Semester, Beth started making regular appointments to work with me outside of class. She is a "nontraditional" student in that she is 28 years old and works full-time while attending school part-time. She is pursuing a degree program in environmental science, a field different from her current work which is occupational therapy. She obtained an Associates Degree in Occupational Therapy. Beth reports being successful in mathematics in high school, but she did not take courses beyond the minimum requirements. All new students at the University of

Massachusetts take a placement exam and performance on this exam determines the level of mathematics into which the student will be placed. Beth began with the slowest paced Precalculus class. She is a diligent student, and she earned an A both semesters.

Since Beth and I had worked together for two semesters prior to beginning this project, we had established a good relationship. We were comfortable with each other, and it was easy for her to discuss what she did not understand. Beth is a highly motivated person; she is thoughtful about how her learning is progressing and works at articulating any difficulties she is encountering. These are essential qualities for a project like this. In addition our relationship was substantial enough to sustain the hard work necessary to complete the project. Of course, the other expert participant in the study was me.

3.2.5 Conceptualization

Having accomplished the two identification parts of the process, the second stage, the conceptualization stage, was started. In my mind this is both the most difficult and the most interesting. It is during this stage that the teacher will learn a great deal about her knowledge and about how students understand a particular topic. In beginning this stage, I was inclined to divide the topic into several parts based on my knowledge of the domain and on how I had previously taught the topic. In this way the process differed from a typical knowledge engineering session. The knowledge engineer (KE) is not usually an expert in the domain although the KE may have some familiarity with the domain. This difference is important because it points to the dual role the teacher plays in the development process. The teacher is both KE and expert.

3.2.6 Meeting Plans

At the beginning of this work, I planned our meetings to be about 2.5 hours in duration, and I expected we would have 15-20 sessions. This could be altered if problems arose during our discussions. We would work as long as was necessary to address any problems. More sessions could be scheduled if the project demanded them. Although knowledge engineering sessions are often videotaped or audiotaped, I decided not to record our sessions. Two factors influenced my decision: I did not want to encumber the process since I was developing a model which could be easily implemented by teachers, and I wanted to avoid any discomfort taping might cause. I felt that I could keep adequate written records; the experience of the initial sessions supported this feeling.

I planned to begin our sessions by suggesting a topic and asking Beth to talk about her understanding and knowledge of a particular topic. I would also ask her to write down how she thought about a particular topic. My decision to begin this way came from the substantial literature exploring expert/novice differences and expert systems. For example we began our first session with the word "function". I asked Beth to talk about what she connects to this mathematical concept and to write down examples.

From my previous work with Beth, I was aware that she had practiced the different types of problems a great deal, so I didn't anticipate she would have difficulties with the arithmetic. Therefore, I wanted to focus our sessions on identifying the source of her difficulties, and what could eventually be a part of the application to support overcoming the difficulties.

Although much about Beth is atypical, my expectation was that much of my work with her would be generalizable to many other students.

Stage three is the formulation stage. In the traditional expert system development this would involve constructing if-then rules compatible with the inference engine or developing appropriate frames or schemes. Since I had already decided to use hypercard, I was viewing formulation in terms of designing the system's cards and buttons. An important part of the formulation process would be deciding which cards to link together and how much information to include in each card. For this work, the formulation process would also address issues of how the text should be displayed. I planned to work on this part of the project independently from Beth. My work between meetings would involve making my knowledge explicit, integrating my knowledge with Beth's, and planning for the next sessions.

The final stages of expert system development involve building a prototype system and testing the prototype. I planned to have Beth experiment with each piece of the prototype system and give me feedback as she used the system. This would allow me to make changes, as appropriate, and have Beth continue to test the hypercard implementation.

3.2.7 The Design Process

In this part of the discussion I will move from the theoretical to the design level. This is the most concrete and will include a description of the design process and implications of the process for my teaching and thinking about mathematics.

The initial phase of this work followed the fairly traditional format of any expert system development. As I indicated in the beginning of this chapter,

the knowledge engineer, having identified an expert, would develop a plan of action for the first interaction with the expert. It should be noted, that even at this early stage this work differs from a traditional expert systems project. The knowledge engineer would not, in all likelihood, have interacted with the expert before. In addition the knowledge engineer is not expected to contribute domain expertise to the project. For the present work it was not necessary, as part of the planning, to consider how to establish a working relationship, and it was important to be aware that during the sessions with Beth my role was primarily that of knowledge engineer, not expert. As important as our previous working relationship was to establishing a productive environment for this project, I was aware that I needed to carefully monitor my responses to avoid making assumptions about Beth's knowledge based on past interactions. My ability to be the knowledge engineer, building a system, would be central to the success of this project because it would allow me to remove myself from the role of teacher and take on a new role. In fact, this new role provided me with many insights which I will discuss throughout the next section.

Initially I viewed the project as having three major phases, each dealing with a major topic: phase one, definition of functions; phase two, linear functions; and phase three, quadratic functions. As knowledge engineer and expert, I had decided that an integral part of my mathematical knowledge base was the connections I made to the uses of mathematics in other domains. In the beginning of the project, I was not able to be completely explicit about this aspect of my knowledge, but I began the project aware that "applications" would be an essential part of the system. Initially applications were the fourth phase; however, it became clear that integrating

applications into the other phases was more appropriate. This approach was a more accurate representation of my knowledge base. Based on my teaching experiences, I felt the majority of the sessions would be devoted to quadratic functions and linear functions. Table 1 gives an overview of the sixteen knowledge engineering sessions.

In preparation for each session, I made some notes about the topics we would be discussing. Much of what I wrote down was similar to material I would present in an introductory class on each of the topics. For example, included in the initial material on functions was a working definition, several of the more generally used representations, and several applications of the concept in the physical sciences. In part, my motivation for beginning this way was to provide me with some means of comparing and evaluating my understanding of the topic with Beth's.

Table 1 Overview of Knowledge Engineering Sessions

DATE	DURATION	TOPIC
July 6, 1990	2 hours	discussion of the project; definition of functions; introduction to Macintosh computers
July 13, 1990	2.5 hours	linear functions; graphing linear equations
July 20, 1990	1 hour	graphing and graphs
July 27, 1990	2 hours	applications of linear functions; discussion of unfamiliar terminology (from applications); interaction with the system
August 10, 1990	2.5 hours	connecting math to other subjects; focus on Beth's tendency to have a narrow piecemeal view of each topic
August 22, 1990	2 hours	quadratic functions
Sept. 16, 1990	4 hours	graphing
Sept. 23, 19901	3 hours	word problems - applications
October 6, 1990	2 hours	applications dealing with quadratic functions
October 21, 1990	3 hours	interaction with system; behavior of functions
Nov. 4, 1990	3 hours	exploring issues of context
Nov. 11, 1990	3 hours	applications
Nov. 18, 1990	2 hours	connecting math to other disciplines
Dec. 2, 1990	3 hours	graphing functions; problem solving
Dec. 9, 1990	2 hours	connecting to other math classes
Jan. 8, 1991	3 hours	review of system; wrap-up

For each new topic we began our sessions with my suggesting a topic and Beth talking about her understanding and knowledge of the particular topic. As I noted earlier, Beth had developed significant skill with the algebraic manipulations involved in most problems, and she was certainly capable of doing the arithmetic associated with these problems so we did not concentrate on this aspect of the problem solution.

3.3 Knowledge Engineering Sessions

In preparation for a knowledge engineering session the knowledge engineer would probably develop a script which contained useful questions to be asked of the expert. It usually facilitates the process if the knowledge engineer is familiar with the language used by the expert and is able to ask the questions using the appropriate "buzz words" of the domain. To prepare for the first session with Beth I made the decision to use our first meeting to explore the general concept of functions and developed questions which I felt would allow us to explore her knowledge in this area. I wanted to determine the following:

a. was she able to define a function,

b. was she able to define terms like domain and range,

c. was she able to describe several representations of functions, and

d. did she use examples in her discussion?

These questions provided the boundaries for the territory that I wanted to cover. In this case the issue of using the language of the expert was very

significant. I did not want to use language that was unfamiliar to Beth or that would be suggestive of the way she should respond. As the sessions progressed it became clear I would have to suggest a direction for us to proceed, or we would be stalled. Before doing this I waited to establish that Beth was not able to contribute anything further on a particular topic. My cues for this were comments like "it makes me mad I don't remember this stuff" or "I can't think of anything else."

We met for our first meeting and throughout the summer in my office in the School of Education a place were Beth and I had spent time together throughout the previous semester. The office has a table which allowed us to spread papers out and work without feeling confined. Beth came to the meetings from work so we took time to talk about how work was going, the summer weather, general conversation to establish a comfortable environment before beginning the knowledge engineering sessions.

To begin our first meeting, I explained the project and expressed my appreciation for her willingness to participate. After about fifteen minutes we were ready to begin work. We started the first session with the word "function" which I wrote on a piece of paper. I asked Beth to talk about the concept of function in mathematics. She struggled to formulate a definition of a function and was not able to state a definition. She did remember the notation f(x) was associated with functions because she said "I think it has to do with f(x) = something". She gave a number of examples like f(x) = 3x + 2; all her examples involved linear functions.

She remembered that the vertical line test existed: "I think there is a vertical line test". Most introductory discussions of functions present a tool for recognizing the graph of a function referred to as the "vertical line test".

A function assigns each member of the domain to one and only one member in the range therefore a vertical line drawn through any x in the domain of a graph representing a function must intersect the graph at one and only value of y. Beth, like most students, did not have difficulty applying this straightforward test to successfully distinguish graphs representing functions from graphs representing relations. I sketched several graphs, and she identified the graphs of functions. I probed to determine if looking at the graphs helped Beth to construct a definition. Studying the graphs representing functions and those representing relations was not enough for Beth to be able to develop a definition of a function.

Since she had suggested that functions have "something to do with f(x)", I asked her to evaluate several functions. We used her examples, and I suggested let x = 3. She described evaluation as replacing the x in the equation with a 3 and carrying out the appropriate operations, i.e. multiply by 3 and add 2 to the product. As she completed the calculation she asked "is this correct?", and I confirmed that it was.

At this point it seemed clear that Beth did not remember the formal definition of a function. As final confirmation of this, I asked her if the terms "domain" and "range" were familiar. Her response was "I think they're connected, but I can't remember". She followed this with "It makes me mad that I can't remember this". I did not want her to feel frustrated, so I tried to reassure her that she would remember as we proceeded.

To end this part of the first session, I presented a definition of a function which stated that a function is a one to one correspondence between the members of one set, the domain, and the members of another set, the range. I related the vertical line test to the definition. She seemed to be able

to make connections between the special nature of a function as conveyed by the vertical line test. I introduced some other commonly used representations for functions and we talked about how the representations were all connected. Her comment was "Oh, yeah, I see now, but I didn't think about it before."

We finished the first session with an introduction to the Macintosh computer. Beth's experience with computers had been limited to word processing, so I wanted her to have a chance to play with the Macintosh. I had obtained some shareware from the Boston Computer Society which I thought Beth would enjoy. The particular shareware is a hypercard application which "finger spells" words and phrases which the user inputs. Using this application gave Beth a brief introduction to hypercard and initial experience with the mouse.

After only one session, we were both aware that knowledge engineering sessions are hard work. It is easy to get discouraged because the pace can be excruciatingly slow and the session can be made up of many false starts. One session was also enough to fortify the inclination that my knowledge about mathematics education would be enhanced and strengthened from these sessions. The most general example of this is the effect that our meetings had on my teaching. Beth and I began meeting in July, so we had been meeting on an almost weekly basis for two months prior to the beginning of the Fall 1990 semester. During the Fall semester I taught three classes, one large lecture section of the one semester Precalculus and two sections of Essential Algebra. The summer's meetings with Beth made me less likely to make assumptions about the connections students were able to make. I was much more careful to be explicit about connections

which are critical for complete understanding of a topic. This perspective was particularly beneficial in the large lecture. In a class of over 200 people, it is easy to become removed from the students, but mindful of the expertise contributed by Beth, I was more careful to view the class from the learner's side of the overhead. In fact throughout the Fall and Spring semesters, the knowledge engineering sessions influenced my teaching and events from my classes influenced questions I would ask during the knowledge engineering sessions. A symbiotic relationship quickly developed.

Following the traditional knowledge engineering scheme, part of my job as knowledge engineer involved transforming the expertise shared during each session into a useful representation for the system. As I have preciously discussed, the selection of hypercard as the tool in large part determined the how knowledge would be represented. I wanted to capitalize on the ability to link pieces of information to alleviate confusion and misconceptions about topics. To accomplish this I paid careful attention to the questions which Beth asked throughout the process. I used her questions as indications of what links needed to be built into the system. For example, we were discussing the idea of "domain convention" to identify the domain of a function. The domain convention states that if the domain of the function is not explicitly stated then the domain is all real numbers for which the function is defined and is real. Beth wanted assurance on what constitutes the set of real numbers. In response to this I linked explanatory text under the term "real numbers". In addition, during the Fall semester when we discussed functions in my Precalculus Class, I was careful to be explicit about the set of real numbers.

As I thought about what should be included in this first section, of the hypercard application, which I have classified as the introduction, I was inclined to include representations which are widely accepted. For example, a figure showing the domain of a function as a collection of numbers and the range as a collection of numbers with each number in the domain connected to a number in the range is commonly used. Figure 1 illustrates this representation. I included this representation in my application since a part of learning mathematics is becoming comfortable with numerous representations. In addition, one goal of this work was to provide an environment which encouraged people to make connections and part of making connections is familiarity with the various representations for the same concept. The hypercard system allowed me to explicitly link the two most often used representations of functions, mapping of one set to another and f(x). An early indication that even successful students were not making connections between the various representations and the definition was Beth's difficulty formulating a definition of a function.. Although she was not able to define a function, she was able to easily evaluate, for example, f(2) given that $f(x) = 3x + 2$.

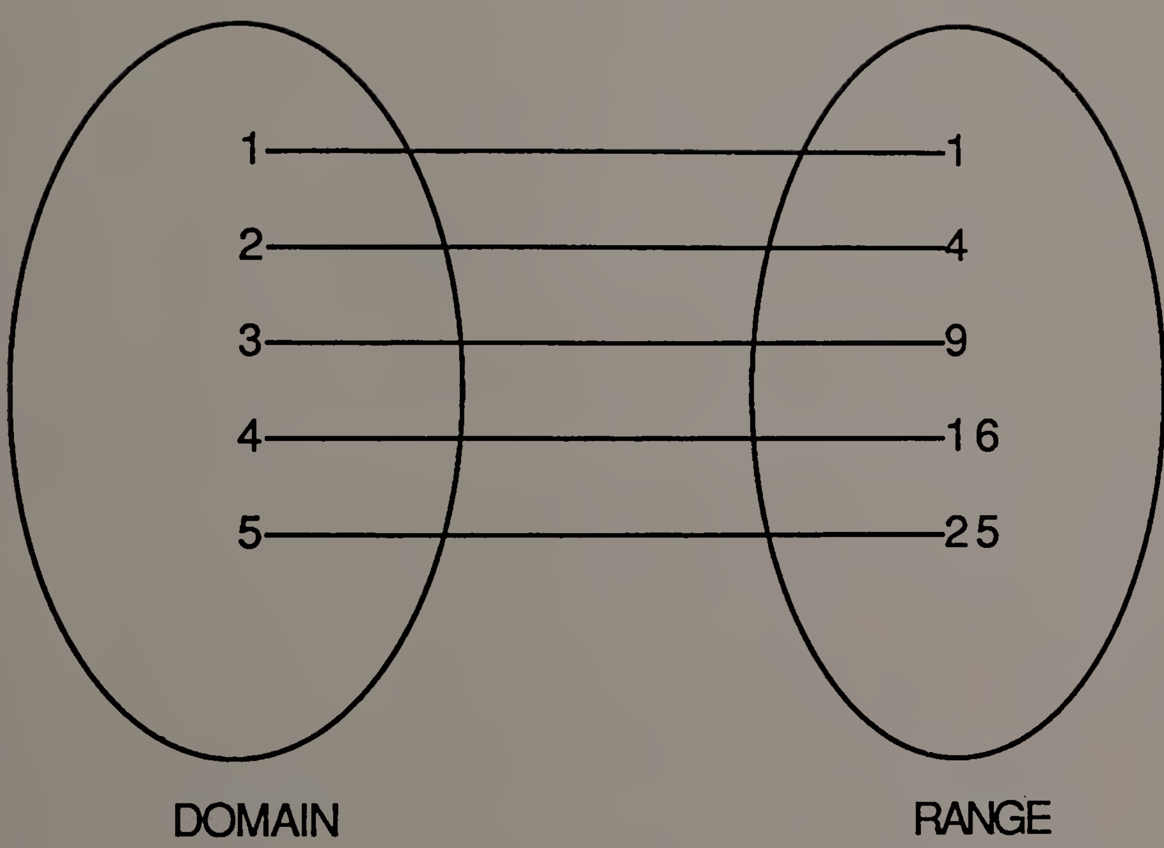


Figure 1 A Commonly Used Representation of a Function

The knowledge engineering sessions with Beth were not the only sessions informing the design of the system. I was attempting to integrate her expertise and my expertise. Her difficulty with the definition of functions forced me think about my own knowledge. In part this was easy. I was more familiar with the terms and the symbols so the formal aspects of stating the definition were less intimidating for me. There was a more significant difference, however; I could think of a number of examples from physical systems which illustrate the dependence which is the critical component of the definition of a function. Domain and range were not abstract labels for sets of unconnected elements. The domain was the set of concentrations of a certain amino acid in a solution, and the range was the absorbance of the solutions as predicted by Beer's Law. Beth lacked this sense of context so I needed to include examples which would help provide this for her.

In preparation for our second knowledge engineering session, I decided to concentrate on applications which were linear functions and would, I thought, address the issue of providing a context. Two main concerns guided my choice of examples. The first was an issue of terminology; I did not want to confuse Beth with jargon from other domains. I tried to think of examples which would not be trivial but would would not require a side trip into the study of the particular domain. The second concern was centered around the existence of misconceptions in science [e.g. Clement, 1983]. I did not want to include an example which would support an erroneous view of the world. I decided on an example from chemistry involving the Ideal Gas Law and one from forensic science involving predicting the height of a woman. In addition to deciding on these examples, I thought about the concepts and notation which we would need to

cover in our discussion of linear functions. I planned to deal extensively with slope, intercepts and the slope intercept form of the equation of a line.

Our work on linear functions was introduced in much the same manner as was our work on functions. I wrote down an example of a linear equation, $y = 3x + 2$ and asked Beth what she thought about in relation to this equation. She recognized that it was a linear equation so I asked her to sketch the graph of this equation. She constructed the graph by building a chart of ordered pairs which she plotted on an rectangular coordinate system. She asked for confirmation that this was an appropriate approach. As she was plotting points, I noted that faced with the same task, I would have sketched the graphs using the information presented in the equation. In this form the y-intercept and the slope is easily identified, and this is sufficient information to make a sketch. In the situation where a function is described by an equation, the equation, for me, identifies general characteristics of the behavior of the function and therefore the shape of the graph. Beth did not have this same view of the function. She needed to build the sketch ordered pair by ordered pair. Although we both could sketch the graph, my view was more global and Beth's more narrow.

Continuing with the same example, I asked Beth about the concept of slope. She immediately told me how to calculate slope. Asking for some encouragement she selected two points on the line and calculated the slope. She commented that she remembered in the slope intercept form of the equation the coefficient of the "x" term was the slope. Beth's approach was correct but emphasized the specifics of this example. As I thought about slope, what was important to me was the concept of change. Viewed this way; slope communicates about the behavior of a linear function. The slope

of a line, being positive or negative, can be used to distinguish linear functions.

One of the points that Beth plotted was the y-intercept. She did not seem to attach any particular significance to this point so I asked if she knew what it was called. She correctly identified it as the y-intercept. When I think about using linear functions to describe a physical system, the intercept is an important part of the description. Back to the amino acid example, if the concentration of amino acid is zero then absorbance is zero. For this system a function with y-intercept of zero is required. This is a "context" which is part of my knowledge and not part of Beth's knowledge.

Our third knowledge engineering session was brief and spent reviewing graphing. We talked about coordinate systems and reviewed the terminology used to describe graphs like quadrant. We looked at graphing a continuous line as compared to graphing points. Most of this meeting centered on surface knowledge, and there were no major discrepancies revealed between Beth's and my knowledge.

Part of our fourth session involved Beth working with the hypercard system. Following the knowledge engineering scheme, the expert would have an opportunity to experiment with a prototype of the system providing feedback on the integrity of the expertise. Traditionally this testing of a prototype would occur after the knowledge engineer has had sufficient time to develop the knowledge base and integrate into a system with some form of an user interface and an inference engine. This part of the process can be difficult because the expert is confronted with an explicit representation of her expertise. In the case of this work, I did not have to worry about the development of an inference engine and the interface was somewhat

determined by hypercard and the system I had available. I did have to discuss my representation of her expertise with the expert. Realistically, I could not expect Beth to evaluate the system in terms its potential impact on learning. She did evaluate text in terms of her ability to read and understand their content. Her overall reaction to the first part of the system was positive. Even on the small scale, she commented that it was helpful to be able to move around and yet to easily be able to read the definition of functions again.

We were able to use the system as the catalyst for our continued discussion of linear functions. In addition to the chemistry and the forensic science example, I was considering adding an example involving the Celsius and Fahrenheit temperature scales. My first concern, especially in terms of the chemistry example, was one of language. I asked Beth if this example was too difficult. Her comment was that seeing even the unfamiliar applications was beneficial because it forced her to make connections and provided a broader perspective of the mathematics. She said: "f(x) is still my favorite but it is good to see other examples". Beth enjoyed the forensic science example, describing it as fun.

During our fifth and sixth sessions aspects of previous discussions emerged again and new insights developed. Since success in science and mathematics centers on problem solving, I asked Beth how she approached problems. She commented that she worried about getting the "correct answer". Describing test situations she said: "sometimes my mind goes blank". When she sketched the graph of the line, her approach was point by point. She did not seem to be thinking about the overall behavior. This was reiterated as she discussed a very narrow view of problem solving.

Our next sessions dealt with quadratic functions. We started our discussion of quadratic functions with the function $f(x) = x^2$ and Beth's first comment was that she remembered the graph representing this function would be a parabola. Beth commented: "there is another type of parabola, one that opens down; isn't there?". I confirmed that there was and asked if she could give an example of the equation. Even though she had all the pieces: the equation should involve an x^2 term and f(x) should be negative, she was not able to generate the equation. This indicated to me that she was not making a connection between the equation, one representation of the function, and the graph, another representation of the function. We looked at another quadratic function, $f(x) = x^2 + 4x - 3$. In Beth's precalculus class, we had used the technique of "completing the square" to determine the vertex of a parabola. Beth approached this example by attempting to complete the square to find the vertex of the parabola to begin to construct the graph. She found the vertex so I asked her what her next step would be. She said: "to make the graph I would construct a table of values". I encouraged her to do this and she picked several points and began to generate the table. She did not immediately choose 0 as either an x or y coordinate so I asked about intercepts. This was enough prodding, and she correctly determined the x-intercepts (with the help of a calculator) and the y-intercept.

Beth had remembered that there were two "types" of parabolas, but she did not attach as much significance to this as I did. As was the case during our discussions of linear functions with positive and negative slope, she, again, did not attempt to make explicit a classification of quadratic functions.

We spent two sessions dealing with word problems and applications involving quadratic functions. For example, in introductory physics a significant amount of time is spent studying classical mechanics. One of the topics and associated class of problems which are discussed has to do with parabolic motion. The problems require students to manipulate the equation $s = \frac{-1}{2} at^2 + v_0 t$ where s, is vertical distance of some object from a starting point (usually a ball and the ground), t is time and v_0 is initial velocity and a is the acceleration due to gravity. If this equation is graphed on a rectangular coordinate system, the graph is a parabola opening down. A typical question is: how long does it take for the ball to return to the ground? To answer this question involves determining the x intercepts of this graph. Beth had difficulty with this question even though she could determine the x intercepts of similar graphs. She could carry out the manipulations, but her manipulations were not connected to any theory and could not be evaluated in terms of the conditions set by the problem. From questions I am often asked in class like "how do you know where it crosses the axis", some students do not know what calculations are relevant to finding the intercepts.

Another part of this class of problems deals with determining the maximum height reached by the projectile and the time at which it reaches this maximum. The solution to this problem, for me, is trivial because my knowledge of quadratic functions includes the information of a maximum or minimum point. This critical point is the vertex of the parabola described by the equation representing the function. Beth was not able to make this connection. Although she was familiar with parabolas, she did not immediately recognize the vertex as the solution. Beth does not have context

for her knowledge; she lacks a history which would facilitate the development of a problem solving strategy that would make use of the available knowledge. Again comparing Beth's knowledge to mine, the discrepancies seemed to be explained by the connections I make and my more global view.

We spent the next several sessions graphing and solving word problems. The sessions were important in part for the opportunity to interact for uninterrupted period of times and for the opportunity to further elucidate how context for knowledge is communicated. Knowledge is dynamic and for me to be able to construct a representation of Beth's knowledge and my knowledge we need multiple opportunities to work together. Watching Beth solve problems even when she obtained the correct answer provided insight into her knowledge base. She repeatedly demonstrated a narrow view of the problem. Each step was a separate piece not generally connected to a broader problem solving scheme.

At this point in the process, we had covered the topics which would be represented in the system. Our last sessions were less directed. They were opportunities to try out some examples (based on Beth's reaction I decided not to include several of these examples and to focus on the examples already a part of the system), to explore further the connection of mathematics to other disciplines, to talk about how math classes connected (since Beth was currently taking Calculus 127, the first semester of a calculus for course for the life and social sciences), to extend our work to more complicated functions, and to review the system.

When I first conceptualized this work the focus was on the implementation. The early insights gained shifted the focus to the process and what could be contributed to mathematics education by viewing a novice

mathematics student as expert. In this case, the expertise which was to be acquired was significant because it would make explicit what was not known to the student. Although not the traditional casting, the novice mathematics student is expert on what he or she does not know. Beth was not the only expert; the role of knowledge engineer was extended to include work with myself as expert. In the case of self as expert, the meaning of expert is a more traditional one. Based on study in the field and experience, I had attained a level of competence ability which usually characterizes an "expert". Over the course of the project, the insights about mathematics and mathematics education were significant enough to change the focus of the work to exploring the knowledge engineering process.

In the preceding discussion, I have reported my use of the knowledge engineering process with a novice mathematics student and myself. It, therefore, seems appropriate to summarize this chapter using the expected end product of the knowledge engineering session, an explicit representation of the knowledge-base.

3.3.1 Representation of the Knowledge Base

A representation of the knowledge base constructed as a result of our knowledge engineering sessions is presented in Figure 2. Figure 2 is a graph. The choice of representation is important and warrants explanation. There are two major reasons for this choice of representation: (1) as will be discussed in the next chapter, the computer based implementation of this work is a hypercard application and a graph is an appropriate representation of a hypercard application; (2) one of the most important results that emerged from this work is the idea of viewing knowledge in a context; this

representation, at least in part, allows for this view. The graph is composed of vertices and edges. The vertices are mathematical information and the edges, the connections, represent the context. Viewed as a graph, it is a representation of knowledge.

The vertices and the edges are not equivalent. In an effort to show both Beth's knowledge and mine, I have adopted a notation which is summarized on the figure. Beth and I shared some information and given our different levels it was not surprising that some information was more equally shared than other information. For example, I could give an explicit definition of a function and Beth could not. I classified this as "weakly shared" information. She had learned "the vertical line" test so she was able to differentiate between a graph representing a function and a graph representing a relation. She was also able to give some examples of functions, but she had difficulty formulating a definition. For these reasons, I made the definition vertex, weakly shared. In contrast, she had no difficulty with evaluation, when asked directly: if $f(x) = 3x + 2$, what does $f(2) = ?$ This showed an ability to perform the manipulation without an understanding of the meaning of the manipulation. Thus, in terms of the representation, the evaluating vertex is strongly shared but the connections between definition and representation and between representation an equation are weak. Her ability to evaluate functions did deteriorate as the functions became more complicated. She was much more comfortable with linear functions than with quadratic functions. She graphically represented a linear function with more confidence than exhibited with quadratic functions. Although, she did present a local view, she graphed by plotting points. A typical comment was: "I know its a line, but I'll plot points to see what it looks like." This was in

contrast to my more global approach. Except for attention to intercepts, I sketched the line as a "whole".

In the case of quadratic functions, she had more difficulty both in terms of the manipulations involved in evaluation and the connection to the graph. As is shown in Figure 2, there are vertices which are labelled as "not shared". These vertices represent information which I had and Beth did not. An important vertex is labelled "dependence". Viewing functions as a means of describing dependence in nature is important in understanding the applications of mathematics to the natural sciences.

One of the problems Beth found difficult involved projectile motion. In part, I believe her difficulty is explained by the missing vertices of dependence and those labeled maximum and minimum and the edges connecting these vertices. Because Beth lacks the context for her information, she has a difficult time solving problems which involve quadratic functions.

Other vertices represented as "not shared" are set theory, elements, applications, inverse, and direct. At Beth's level, one would not expect a sophisticated understanding of set theory, but some sense of a function being the the connection between two collections of objects is important. Without this information and information of dependence, it will be difficult for Beth to recognize a function outside of the mathematics class. If in an application problem she does not recognize the functional relationship, she will not be able to apply the skills she does have.

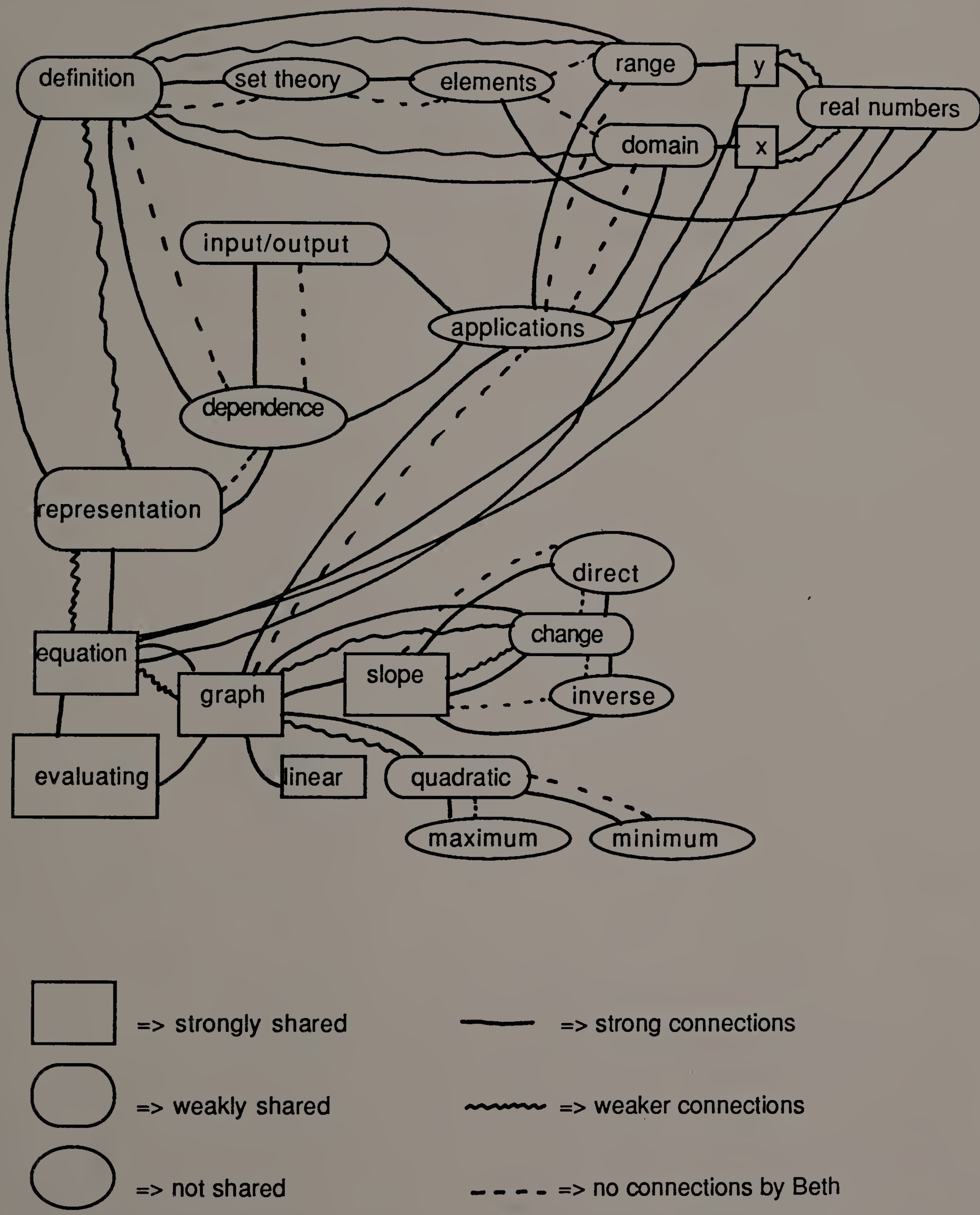


Figure 2 A Graphical Representation of the Experts' Knowledge Base

In summary, Figure 2 is derived from the application of the knowledge engineering process and seems a reasonable description of both experts' knowledge bases. The process which led to Figure 2 also informed the implementation which is described in the following Chapter.

CHAPTER 4

HYPERCARD APPLICATION

4.1 Introduction

In this chapter I will describe the hypercard application which was constructed based on the work presented in Chapter 3. This chapter is organized in three parts. The first part of the chapter is a description of hypercard and the programming language hypertalk including an explanation of the means of representing information. The second part of this chapter is an overview of the prototype system concentrating on the domain content which is presented. Specific details of the system and design decisions which were made, based on the knowledge engineering sessions, constitutes the final part of this chapter.

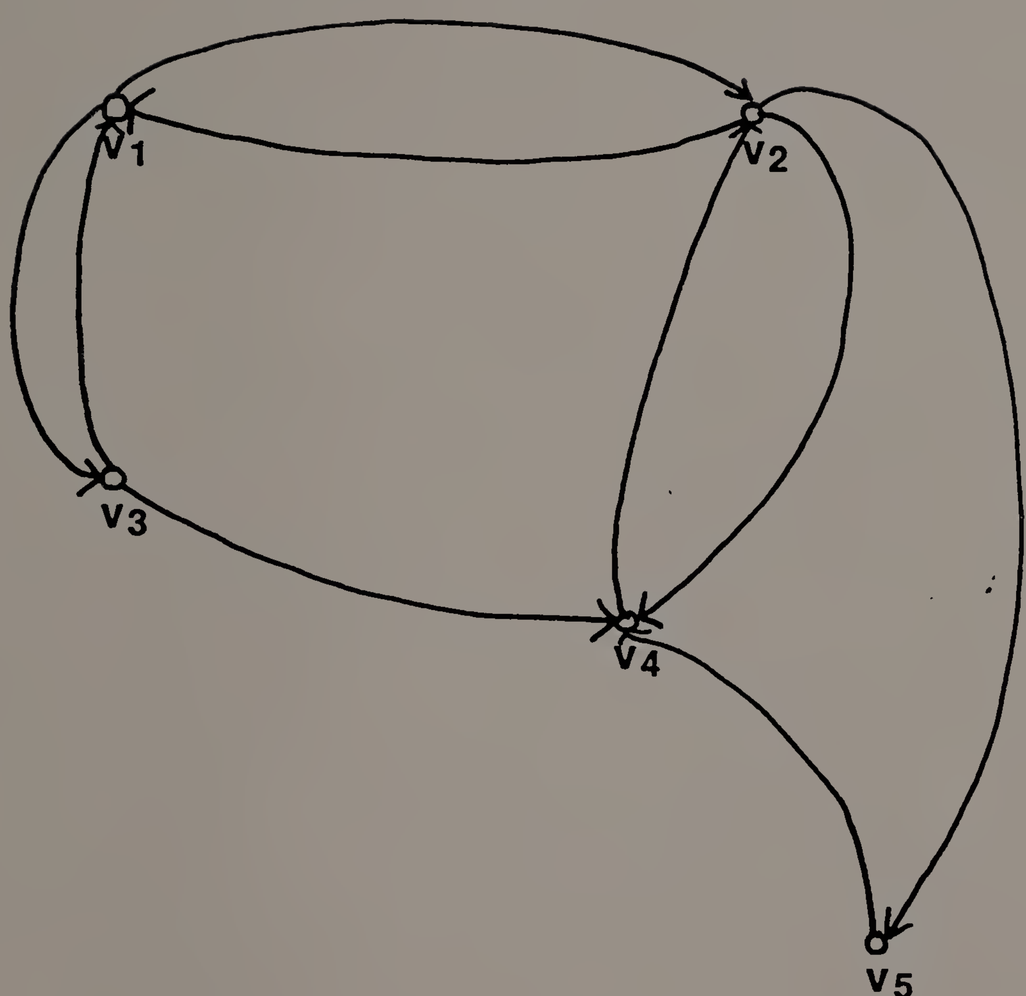
4.2 A View of Hypercard

Numerous descriptions of hypercard exist throughout a growing body of literature. For the purpose of this work, hypercard can be viewed as a computer application which allows the user to construct a directed graph (or digraph). A definition of a directed graph is:

> a finite nonempty set V together with an irreflexive relation relation R on V [Chartrand, 1977, p. 16].

A graph is often represented by a picture. As shown in Figure 3 the set V is the set of vertices and the vertices are connected by directed edges or arcs determined by the relation R. When used in mathematical modelling, the vertices of a graph often represent a state of the system and the directed edges the connections between states. In the hypercard application, the designer builds hypercard stacks. The stacks are a collection of electronic index cards.

$V = \{v_1, v_2, v_3, v_4, v_5\}$



$R = \{(v_1, v_2), (v_1, v_3), (v_2, v_1), (v_2, v_4), (v_2, v_5), (v_3, v_1), (v_3, v_4), (v_4, v_2), (v_5, v_4)\}$

Figure 3 A Directed Graph with Five Vertices

Figure 4 shows an example of the starting form. The cards are the vertices of the graph. As in a graph, the cards are connected, or in the terminology of hypercard "linked". The links are the directed edges of the graph. Consider Figure 3 again. Select a vertex and traverse the graph by travelling along the directed edges. This is analogous to what is accomplished in an hypercard system. In the hypercard system rather than using your finger or some other object and tracing the directed edges, a mouse would be used to allow navigation of the system.

Linking is accomplished by placing buttons on the the cards. Figure 5 shows an example of card with three buttons. Using the mouse to select a particular button and clicking on the selected button results in moving between cards, the electronic version of a directed edge. There are two primary means of attaching a particular action to a button. A "hard link" utilizes the button info screen and a system provided "link to" command to connect a button to a particular card. The other method requires scripting. A script is the instructions which result in a specific action when a button is "clicked". This is represented in Figure 6 and Figure 7.

The developer is provided with a variety of tools to display information on the cards. In addition, other applications like MacDraw can be used to design graphics, and the graphics can be imported into the hypercard application. Although information can be placed directly on the cards, it is possible to divide the cards into distinct parts called fields. There are several types of fields available; examples are shown in Figure 8. Following the graph theoretical description, fields are classified as vertices.

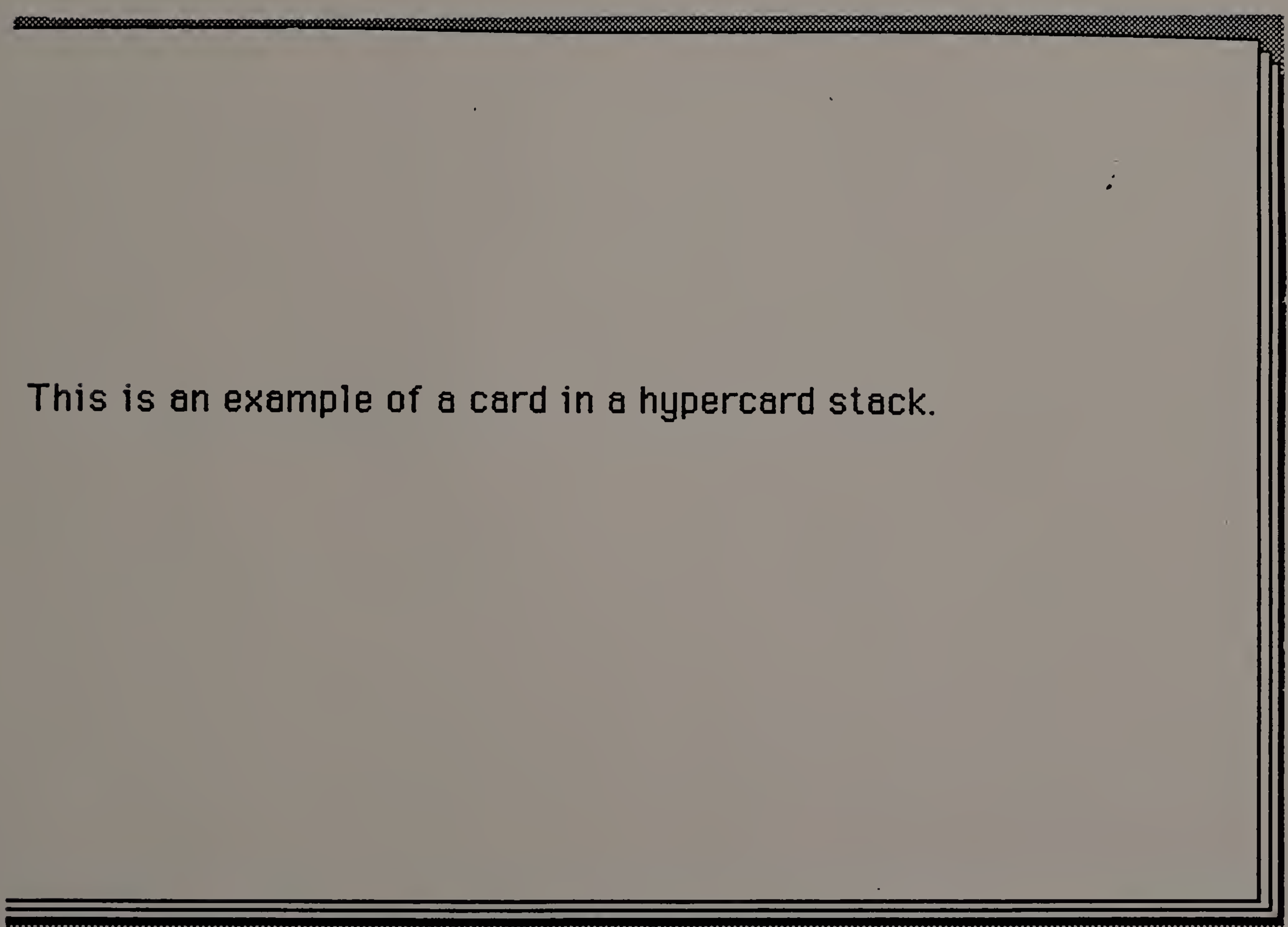


Figure 4 A Blank Card the Template for Hypercard

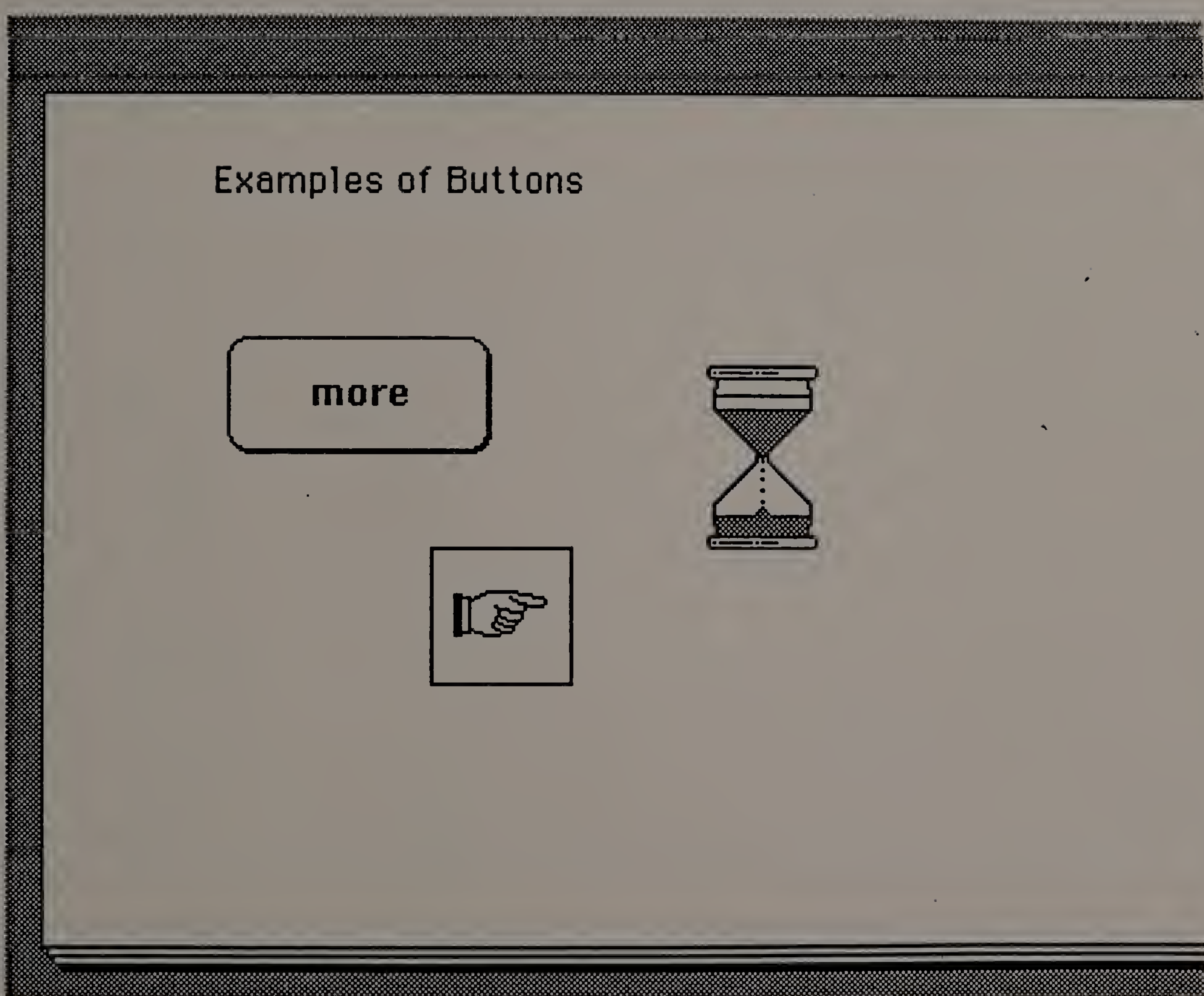


Figure 5 Examples of Buttons Used in a Hypercard Application

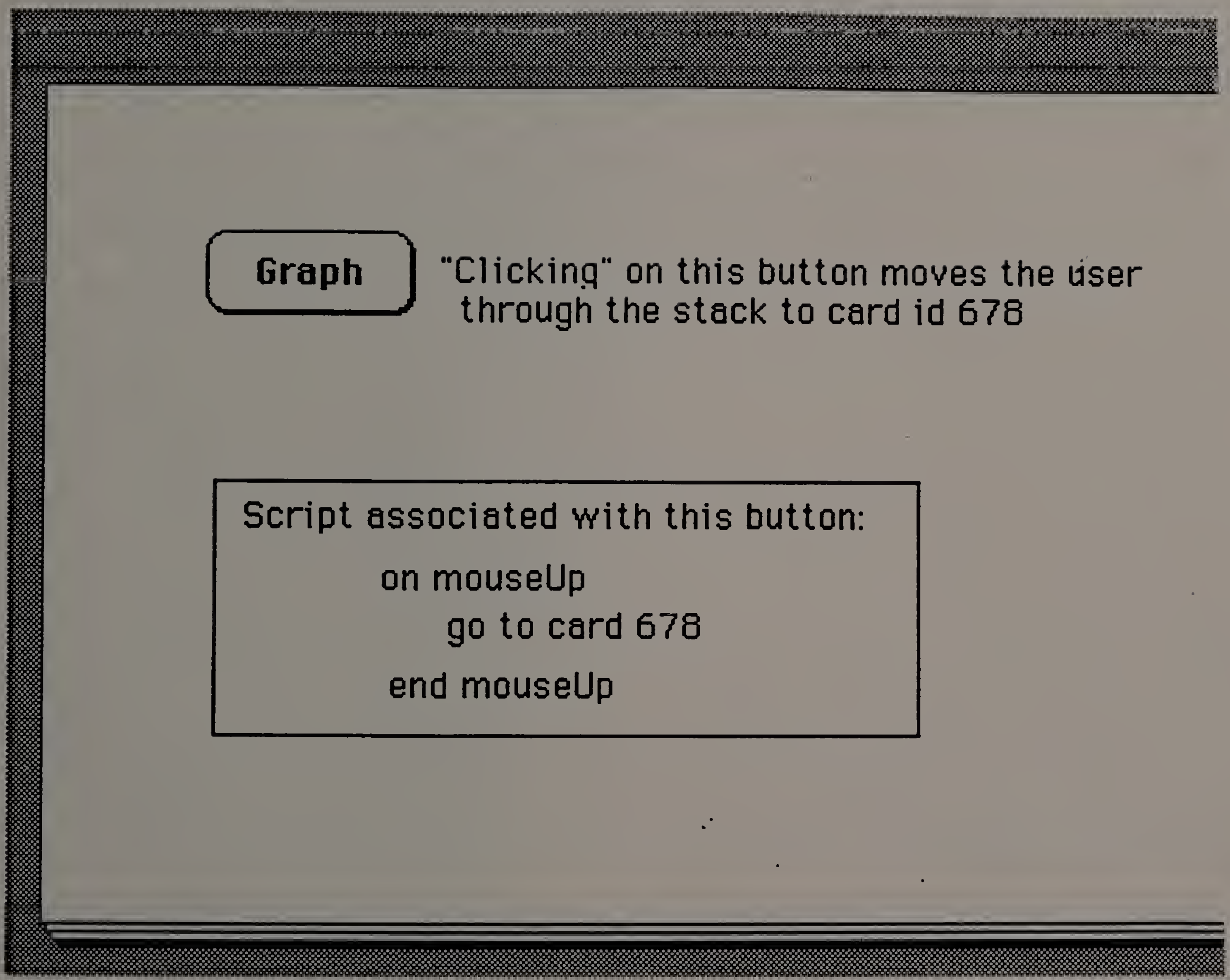


Figure 6 A Button and the Script Associated with this Button

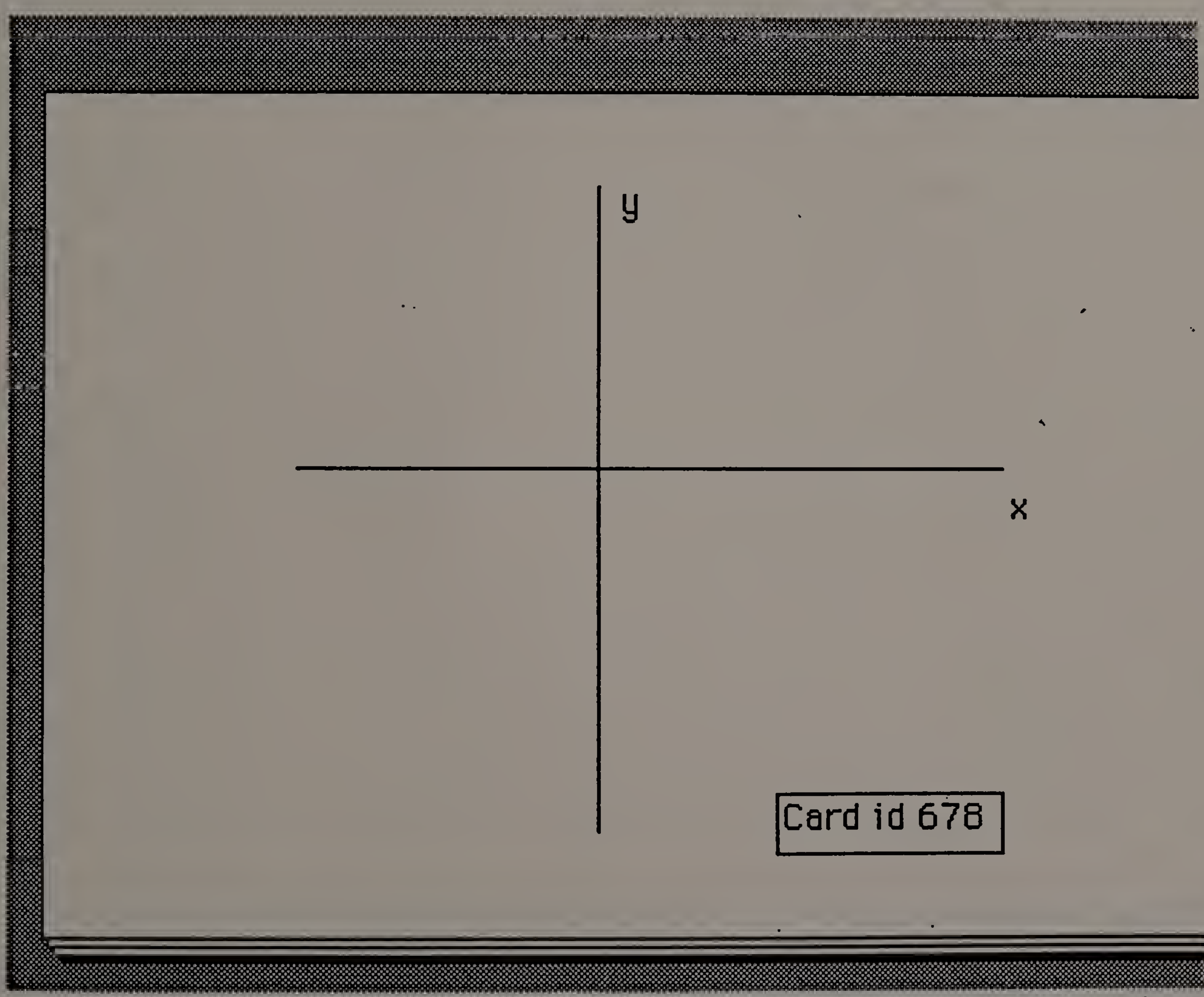


Figure 7 The Action Which Results from the Button in Figure 6

Figure 8 Field Types Available in Hypercard

This classification is made because the fields are used to represent knowledge. Buttons are also used to control access to the fields. An example of this process is given in Figure 9 and Figure 10.

The scripts are written in hypertalk, the programming language associated with hypercard. Hypertalk is a structured language with much in common with more traditional languages like Pascal or Basic. The unique aspect of hypertalk is that it was designed to work with the hypercard environment. In this regard it has as objects cards and buttons.

4.3 Domain Knowledge

As discussed in Chapter 3 the major concepts which were covered in this system were linear and quadratic functions and their graphs. The concept of functions may be a part of the 5th or 6th grade mathematics curriculum and is likely to be discussed at least at a surface level in Algebra II. Any precalculus or college algebra course will devote significant time to the study of functions and related topics because these topics are essential for Calculus and higher level mathematics.

My own teaching experience and a survey of the most widely used precalculus (or college algebra) texts established the surface knowledge to be included. In presenting any topic at the introductory level, I am aware that the presentation must be formulated so that it is simultaneously mathematical correct and understandable.

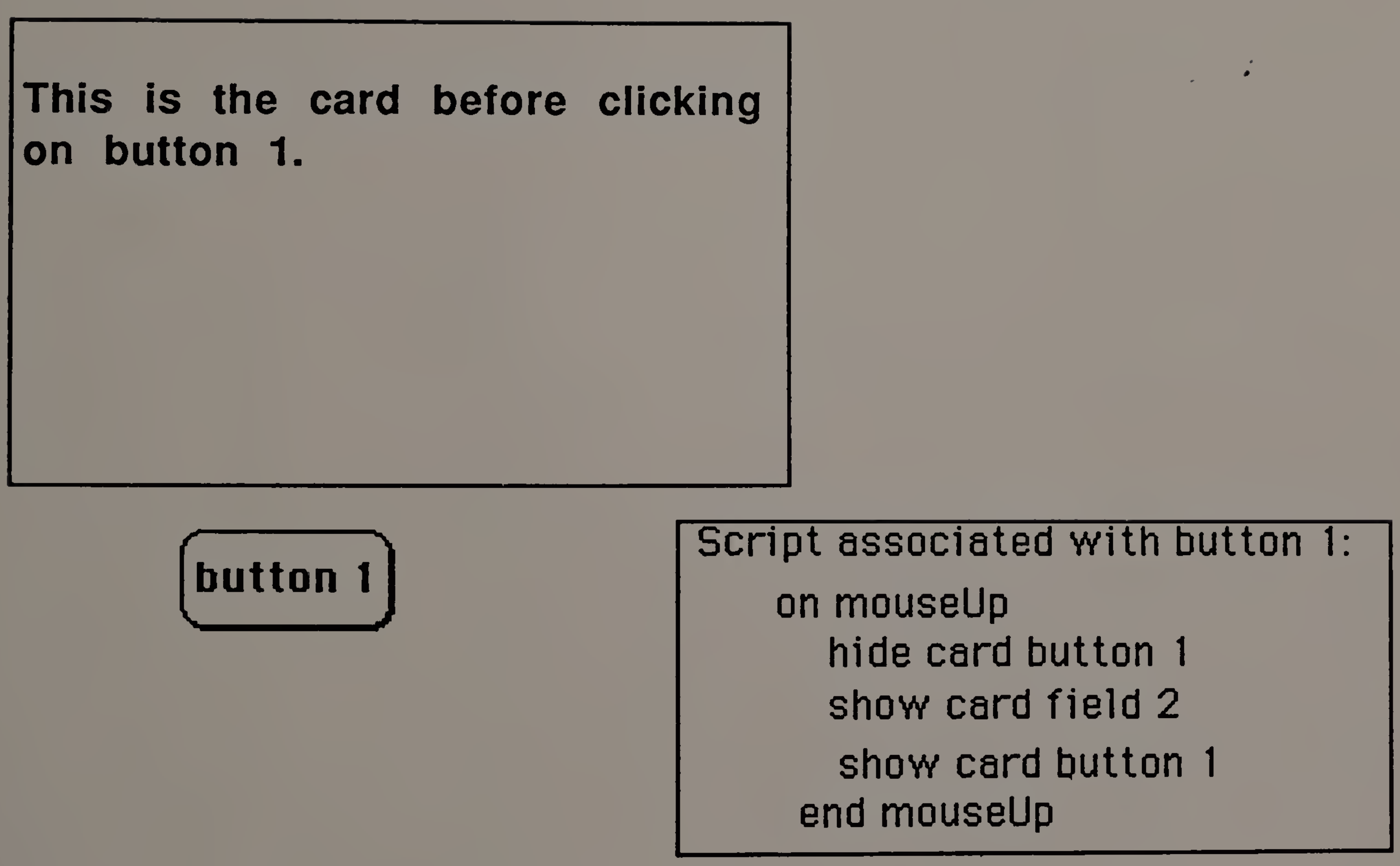
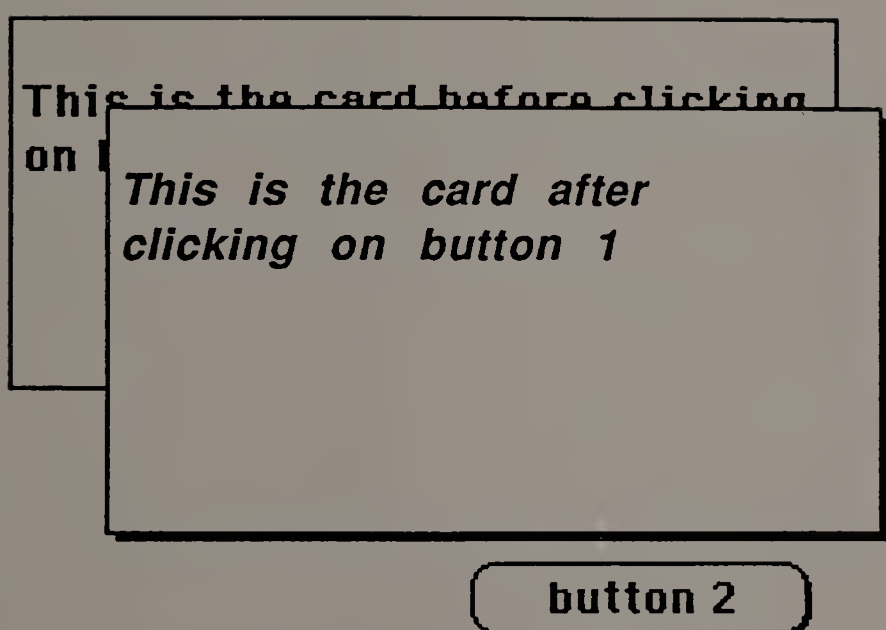


Figure 9 A Card and Associated Button Action




```
Script associated with button 2:
        on mouseUp
           hide card field 2
           hide card button 2
           show card button 1
        end mouseUp
```

Figure 10 Card after Button 1 is "Clicked"

A mathematician might give as a definition of a function:

> A function is a relation R such that, for every x in the dom(R) and every y and z in codom(R),
>
> xRy $\wedge$ => y = z [Foulis and Munem, 1988, p. 116].

This definition although mathematically correct would not communicate much to the typical precalculus of college algebra student. A definition, more appropriate for the college algebra or precalculus student is:

> A function f from set D to set E is a correspondence that assigns to each element x of D a unique element y of E [Swokowski, 1986, p. 128].

This definition will be more meaningful and is sufficiently rigorous for a student at the introductory level. It is essential that the initial presentation of concepts provide the learner with a foundation which will support the application of the concepts in other domains and will prepare the intellectual path to further work in mathematics. In determining what to include and how to present mathematical knowledge in this particular application consideration was given to these questions:

a) is the presentation mathematical correct and will it support further study both in mathematics and other domains?

b) is the presentation at a level which is understandable to the majority of students?

c) for any given concept have the most widely used definitions, notation, representations been used?

These questions served as guidelines to me in my role as expert in this process. Beth's contributions as expert influenced the answers.

The other topics represented in the system are linear functions, quadratic functions, and graphical representation of these classes of

functions. In addition to the concepts, since the successful student must be familiar with commonly used notation related to functions, the notation associated with functions was presented. Topics related to the study of functions and their graphs, for example definition of real numbers, slope, rectangular coordinate systems, and intercepts were also included.

4.4 Description of the Prototype System

The description of this system and the design decisions made as the system was being constructed is essentially a discussion of the knowledge of two experts, a teacher and a student. There were, however, design decisions made based on information reported in the hypercard literature and on existing hypercard stacks like the Interactive Particle Physics Stack. For the most part, these resources influenced card design and overall system organization. Since these issues are relevant to the system as a whole, I will begin with a discussion of them and conclude with a discussion of the more specific issues.

One of the goals of this application was to present an environment with very few rules with the view that increased benefit to the user would result from uninhibited exploration. To this end the user is introduced to the application by a card which explains the use of the use of buttons and the minor number of conventions which will be followed throughout the system. Since exploration is accomplished primarily with buttons, an understanding of buttons and their use is an essential skill.

A concern reported in the hypercard literature is that large systems will not be effective because it is to difficult to maneuver within the system. The user gets lost and frustrated and no learning occurs. The Interactive Particle

Physics Stack deals with this problem by including a system overview which can be accessed at any time. This overview represents the various topics covered in the system. Although the system of this current work is not large, an overview card was developed and included. The overview card consists of six buttons labelled: introduction to functions, linear functions, applications involving linear functions, quadratic functions applications involving quadratic functions, and bibliography. At any time, a user can move to this card by using the navigate button which is a part of each card.

From the overview card a user moves throughout the system by selecting the button labelled with the desired topic. When a particular topic is selected, the user moves to the introductory card of the particular topic area. This card is not meant to be a map of the system so it does not show all the connections. It is a representation of the major concepts of the system, and an anchor for the system users.

Each major section is introduced with cover card containing a definition button. The first section is functions. This section is intended to provide the user with a definition of functions and an introduction to some of the most important conventions. The definition is given with the words "function", correspondence and sets highlighted. If the user chooses the word "function" he/she discovers that the word was first used by Leibinz in 1687. Selecting the word "correspondence" relates the idea of correspondence to different temperature scales, using the Celsius and Fahrenheit scales to show the correspondence between 0 Celsius and 32 degrees Fahrenheit as the normal freezing point of water. Choosing the word "sets" provides a brief definition of a set.

The definition card also has a button labelled "examples". The first example shows one of the more traditional representations of functions and introduces the concept of real functions as being those where the domain is the set of real numbers or some subset of the set of real numbers. The term real numbers is highlighted and at the first level several examples of elements of the set of real numbers are provided. If this is not sufficient another level can be accessed via the button labelled "more". At this next level an overview of the various sets of numbers is depicted. This overview shows the set of complex numbers and the subsets of this set. Additional information about any of these sets is obtained by selecting the set of interest. The information provided is a combination of examples and some history of the numbers.

The next card in this introductory portion deals with notation which is commonly used to refer to functions. A key word on this card is "notation". By choosing the "notation", the user is provided with some history associated with the use of various representations used to identify functions. There is also an examples button on this card which again allows access to the other representations for functions.

The next topic area which is dealt with is linear functions. This area is also introduced with a cover card which is followed by definition card. Since the graphical representation of functions is important both to this system and to mathematics and the sciences, graphing is introduced in connection with the definition of a linear function. The next card deals with the concept of evaluating a function. The function f with $f(x) = 2x + 2$ is used as the example. Using the introduced idea of mapping elements in the domain to elements in the domain, the method of evaluating the function is given. To reinforce the

difference between f and f(x), the f is also given as the set of ordered pairs consisting of an element in the domain and the corresponding element in the range.

An understanding of the concepts which follow the introduction to linear functions depends on knowledge of the rectangular coordinate system and graphing lines, so a card dealing with the critical aspects of graphing lines is included. Represented on the card is a rectangular coordinate system. The line y = x is graphed with the points (4, 4) and (-4, -4) labelled. The x-axis and y-axis are labelled with buttons when selected give more information about the axis and the notation used. There are three other buttons on this card; selecting any of these buttons presents the user with more information about the terms. The buttons are "more info", "slope" and "y-intercept".

The first level of the "more info" button gives a brief summary of the coordinate system. If this is not sufficient there is another level. At the second level the user is given more detail. The four quadrants of the rectangular coordinate system are labelled. In addition four points are graphed showing the sign of both the x and y coordinate in each of the quadrants.

The "slope" button also divided the information about slope into two levels. The first level provides a brief definition of slope. Since there is some skill involved in calculating slopes, the next level asks the user to calculate slopes for various lines. It is easy to make arithmetic errors, so the user is cautioned about this.

The final button on this card is the "y-intercept" button. The concept of intercepts is important not only to the discussion of linear functions but to quadratic functions and to interpreting graphs in the solution of problems in other domains. This presentation is also divided into two levels with the brief discussion being linked to other contexts.

A series of cards develops the idea of generating the graph of a function. The previously introduced function f such that $f(x) = 2x + 2$ is used in this part. As the user proceeds through this section more details are added to the graph. Selected points on the graph are related to the previous representation of this function. This series of cards is followed a graph of another linear function. This function was also previously introduced; it is the function with $f(x) = 9/5x + 32$. In the introductory portion of the system, the example of Celsius and Fahrenheit temperature scales was used to explain the idea of correspondence as it relates to functions. This example is presented again in the context of linear functions. On this card it is presented as an example of an application which can further explored by choosing the application button.

The application button is linked to a card which explores methods for ascertaining information about a physical system. The methods are graphical and algebraic. There are numerous examples from chemistry and physics where the experimenter is required to determine some quantity from a graphical representation of data. The example given in the system is based on Beer's law which allows for the determination of the concentration of a solute based on spectroscopic data. The same information can be determined using the algebraic method, and this connection between equation and graph is represented on this card.

Two other examples of linear functions are presented. The first is from forensic science. Based on substantial data, forensic scientist have developed a function which relates the length of a human female's humerus to her height. In this function, the domain is bone lengths reported in centimeters and the range is heights reported in centimeters. In addition to presenting the function, the user is given the opportunity to test her skill as a forensic scientist and calculate the height of a woman given the length of the humerus.

The final example in the linear functions section deals with ideal gas law from chemistry. For this example it was necessary to provide some information about the chemistry involved. The first card summarizes the important terms in the ideal gas law. The following cards explore the correspondence between volume and temperature. Under conditions were the ideal gas law is a reasonable model, the volume of a gas can be expressed as a linear function of temperature, and a constant (assuming pressure and amount of gas are kept constant). This example is a more complicated linear function but relates the concept of linear functions to a physical system.

As with the previous sections, the quadratic functions section begins with a cover card which connects to the definition. The introduction of quadratic or second degree functions also requires an explanation of the term degree and the introduction of exponential notation. The first card in this section presents this introductory material.

The format of the first card provides the definition of second degree functions with the words "degree" and "notation x^2" highlighted. Choosing "degree" provides the user with a more elaborate definition of degree and

relates this concept back to linear or first degree functions. The other highlighted term deals with exponential notation. At this level, the use of exponents is not a new concept. However, the format used in the system may be unfamiliar. The explanation provided deals with the format and also gives a brief history of exponential notation.

The critical part of this section is the discussion of graphs which represent quadratic functions. Graphs of quadratic functions are parabolas opening up or down. The first function considered is the function f, such that $f(x) = x^2$. On this graph, the points (-4, 16) and (4, 16) are highlighted. If either of these points is selected, the user is return to the linear functions section and the graph of $f(x) = x$. On this graph the points (4, 4) and (-4, 4) are highlighted. In addition to connecting this section to the linear functions section, there is a button labelled "domain convention" which reminds the user that the domain convention applies for second degree functions. An important feature of the graphs representing second degree function is the existence of a maximum or minimum point. On this card, the user is made aware that functions of this type have minima. The next card in this section reviews the previous discussion but in the context of the second degree function such that $f(x) = -x^2$. Functions of this type are essential for understanding classical dynamics. The connection to physics is made through this card. The example used to establish a physical context describes the the height (measured from the ground) of a baseball as it relates to time.

The final part of the quadratic section deals with another application involving second degree functions. The application domain is biology and relates the birth of bear cubs in New England to time with 1980 being the

starting point. The function is more complicated but the same interpretation as was accomplished with linear functions is possible. In this case, the question of maximum number of births is presented. As with linear functions the graphical method and the algebraic method of arriving at the answer are both presented. The user may return to the linear functions section which dealt with a graphical method and an algebraic method of answering questions concerning linear functions.

The final section of the system is a bibliography. The bibliography provides a user with sources of information for the topics dealt with in this system. The references in the bibliography are also useful for further exploration of other topics in mathematics and deeper investigation of functions.

4.5 Design Decisions Based on Knowledge Engineering Sessions

The traditional approach to presenting the concept of functions is to state the definition and provide some examples of functions. The examples are usually abstract in the sense that the elements of the domain and range are not related to events or quantities from the student's experience. It became clear from work with Beth that establishing this context was critical both to the initial understanding of the mathematical concepts and to the application of the concepts outside of the mathematics course. Recall that Beth was not able to define functions, she had not connected the concept of domain and range to any other knowledge. At another point, she was able to find the x-intercepts of a parabola but could not use this knowledge to answer a physics problem which required finding the x-intercepts. Each mathematical concept was introduced with a definition utilizing the hypertext

capabilities to layer additional information with the critical terms and parts of the definition. For example, in the definition card the word "correspondence" is hooked to further information. In the case of this application, commonly used representations which indicate correspondence are presented. This is included because it allows unfamiliar concepts to be investigated. A user thus has the opportunity to explore the meaning of an unfamiliar term. As an additional potential benefit, hypertext enabled me to draw attention to terms which might be familiar because they are used in common speech and therefore might be misunderstood because they have very specific meanings in this mathematical context. For example, the dictionary defines correspondence as "the agreement of things with one another". The mathematical definition of correspondence is a mapping of members of one set to members of another set; in a mathematical context, the definition is more specific. The term is perhaps familiar enough that a student would encounter the word and not be motivated to check his or her understanding yet in a hypercard system it stands out providing the impetus to investigate.

The capability to utilize hypertext to provoke additional investigation of familiar terms was one part of the strategy to make knowledge of the concept of function more shared so linking explanatory text under familiar terms like correspondence would challenge the user's view of the terms. In addition to this strategy, I included several commonly used representations for functions emphasizing on that these were all representations of the same concept. It was clear from our discussions that Beth was very focused on the rule $f(x)$ = something as the important component of what constitutes a function. Including the other representations was an effort to broaden her sense of the concept. It is easy for the beginning mathematics student to get caught up in

the mechanics and lose sight of theory. Beth exhibited this by concentrating on the evaluation of functions not on formulating a working definition.

Related to her inability to articulate a definition of a function, she did not distinguish between f and f(x). I adopted several features in response to this problem. When presenting the notation associated with functions, I included a historical perspective to enforce the idea that notation was developed and is distinct from the concept. As part of the introductory card, I included the button with f and f(x) which reinforced the distinction.

As a result of the sessions with Beth, several parts were included in this introduction section. She wanted confirmation of what constituted the set of real numbers, so this was included. This is important because the applications encountered are most likely to involve functions over the reals.

One theme which was repeated during almost every interaction with Beth was her inability to connect the mathematics to other areas. She expressed concern that she would learn a body of knowledge during a semester and not retain the knowledge. Her description of her learning comes from her world as an occupational therapist; she described herself as having "splinter skills". The implication of this was she was able to bring together enough of the concepts to solve some problems, but she was not confident in her ability to explain the theory underlying her solution or her long term ability to solve problems. This is not a trivial issue to respond to; however, one part of the response I believe is to frame the mathematical theory in a context outside of mathematics. One of the dangers of selecting examples is to obfuscate the concepts in some new domain's complicated terminology. The initial example of temperature scale was chosen because it

did not require significant domain knowledge to understand. It was also an example which could be referenced in the linear functions part of the system.

In this system the discussion of linear functions is tied to the graphical representation of linear functions. Two factors influenced the development of the graphing portion of the system. It was important to provide an overview of the terminology used in describing graphs and to represent the rectangular coordinate system. In addition, making the connection between the graphical representation of functions and the previously presented representation was essential. Initially Beth viewed the representations as distinct, and this makes it less likely she would be able to appropriately apply her mathematical knowledge in another setting.

A significant portion of our discussions of graphing concerned the concept of slope. Several ideas emerged which influenced the system. First, it was worth reviewing the definition of slope; Beth found it useful to be able to confirm knowledge even that in which she had confidence. As I have indicated she constructed a table of values when asked to sketch a graph. It was clear that she did not associate with a positive slope a line where as x increases y increases. This global view is important so I made an explicit statement of the implication of positive slope and referred back to the temperature example to connect the idea of positive slope to a physical example. Success in science is in part determined by the ability to recognize whether or not a graph or an equation is a reasonable description of the physical system. This example and the connections made were designed to encourage the development of this ability.

My discussion of slope with Beth influenced my design of the presentation of y-intercept in the implementation. By linking the discussion of

intercept to a system with which Beth was familiar, I was attempting to encourage her to expand her view of intercept. The goal was to present the mathematics but to present multiple scenarios of the concepts, so the user would develop a broader perspective of the concepts.

In selecting the applications to be included in the system, my initial choices involved concepts which would be familiar to a most students. Since one of the goals of the project was to make connections, I included an example from forensic science and a chemistry example. Beth viewed the forensic science example as fun and was intrigued by the possibility that she new some "useful" mathematics. In our discussion of the Ideal Gas Law example, she indicated it was very helpful in expanding her sense of the applicability of the mathematics she was learning.

Much of what was included in the quadratic functions section of the system was influenced by my previous work with Beth. Many of the weaknesses in Beth's knowledge base were confirmed in our discussions of quadratic functions. Since new notation was required, the introductory card presented the exponential notation, and the concept of degree. To encourage the integration of this knowledge into the already existing scheme, I related the definition of degree back to linear functions.

Graphing quadratic functions was more difficult for Beth in part because the functions are described by more complicated equations. I linked the the points (0, 0), (4, 16), and (-4, 16) on the graph of $f(x) = x^2$ to the points (0, 0), (4, 4) and (-4, 4) on the graph of $f(x) = x$ so Beth would view graphing this functions in terms of previous knowledge. When asked to graph this function, she was not confident generating a table of values and plotting points was an appropriate way to proceed. By linking these points, I wanted

to reinforce that knowledge learned in one context was applicable in other contexts. Related to this was the idea of viewing the graph more globally. Graphing these three points is enough to show that the graphs are different, and this is important in terms of developing a global view of the behavior of functions. It is also a significant component of the ability to apply mathematical knowledge to the solution of problems in science and other domains. Clearly a quadratic function is not appropriate to describe the correspondence between Celsius and Fahrenheit temperature scales, and it is important to cultivate an awareness of this. This theme was continued throughout the section on quadratic functions by connecting parts of the graphs of quadratic functions to parts of the graphs of linear functions.

Beth was able to carry out the computation necessary to determine the vertex of a parabola, but she did not associate any particular significance with this point. Recognizing that the existence of a maximum or minimum point is important in describing the behavior of a function and in the solution of many problems, I emphasized this aspect of the quadratic functions.

Given the emphasis that is placed on classical dynamics in any introductory physics class, it seemed useful to include as an example the function which describes the position of a classical projectile, near the surface of the earth, as a function of time. The other example is from biology and relates the number of bears cubs born as a function of time with the starting time being 1980. As with the examples included in the linear functions section, the functions included in this section were designed to be relevant in terms of further study and also interesting.

The applications presented another opportunity to encourage thinking about what characteristics are important in the mathematical description of a

physical state or behavior. For example in the case of the projectile, a linear function would not be adequate but a quadratic function is. Again, this type of analysis is essential and develops by making connections.

In this chapter I have presented a description of the system which was developed as a result of applying the techniques of knowledge engineer with the student as one expert and the teacher as a second expert. The design decisions which were made based on this expertise. As described in the preceding section, the system is an explicit representation of this combined expertise.

CHAPTER 5

SUMMARY AND IMPLICATIONS

5.1 Introduction

A friend who runs a learning center for adults in a Basic Education Program recently commented that if we provide people with a supportive learning environment and throughout the learning process ask what they are learning, they will articulate what they don't know or seem not to understand. A student in my precalculus class talked about being in the Chemistry Department Resource Center when the professor of the large lecture course she was taking sat down and asked what she was working on. The student was having difficulty with a problem so she began explaining to her professor how she was approaching the solution of the problem. The professor was intrigued and surprised at her approach. He commented that he would never have suspected that way of thinking about the particular problem.

This research concerns asking students about their understanding of concepts and their approaches to problems, and developing an explicit representation of the student knowledge obtained, to improve teaching. It is about seeing the learner as an expert just as the teacher is an expert. The learner is an expert in the world of her own experience. Seeing students as experts transforms teaching.

In the first part of this chapter I will summarize the major findings of this work. In the next section I will discuss extensions of the system that was developed as part of this work. I will conclude with a discussion of the implications of this research for teaching and teacher education.

5.2 Summary of Findings

Based on the substantial experience of the artificial intelligence community in building expert systems, this research project involved using the knowledge engineering process to investigate a teacher's and a student's knowledge of functions and their graphs. Building an expert system requires identifying an expert in the domain of the system. A knowledge engineer works with the expert to construct the knowledge base of the system. The present study required the teacher to take on the role of knowledge engineer with a student as expert. The expertise the student provides is what she doesn't know. In addition the teacher is knowledge engineer with herself as expert. The outcome is to develop an explicit representation of the teacher's knowledge in a framework which makes it accessible to the student. Although this process does not demand building a knowledge base for a computer based system, this work explored the feasibility of developing a hypercard system based on the knowledge engineering sessions.

During the course of this project, the researcher conducted sixteen knowledge engineering sessions with a student and, based on these sessions, developed a hypercard system dealing with linear and quadratic functions and selected applications in the natural sciences. Hypercard was chosen as the environment for the implementation for several reasons: the implications from much of the hypercard research is that the hypercard environment is one that stimulates learning; it is an environment which is accessible and gaining acceptance in the education community; and it is relatively straightforward to use.

As the expert systems literature confirms, knowledge engineering is difficult work. This was affirmed in the course of this project. The process was facilitated by the relationship which existed between the researcher and the student. The commitment of the student to the project was an important aspect of its success. It is often difficult for people to share expertise; so willingness on the part of the expert to work at this process is essential to productive knowledge engineering sessions.

In addition to my role as knowledge engineer, I was also an expert and in both these roles I had the opportunity to view my knowledge of mathematics from a fresh perspective. As I was cataloging Beth's responses to questions, I was also cataloging my own. After teaching an introductory course for a number of years, it is easy to become removed from the mathematical significance and beauty of the ideas presented. During this work, as an expert, I was able to appreciate and renew my enthusiasm for the ideas discussed in the precalculus course. As a knowledge engineer, I was relieved of the burden of having to answer the questions and provided with the opportunity to ponder the questions. In fact as this project progressed, I realized that the role of teacher and knowledge engineer are not mutually exclusive.

Once adopted the role of knowledge engineer is not easily relinquished. Over the course of the Fall and Spring semesters, my work on this project influenced my interactions with students in my classes. Students who came to my office for extra help were more likely to find themselves involved in quasi knowledge engineering sessions. Questions used with Beth, like what are you thinking about here? or why did you approach the problem this way? became a more prevalent part of my tutoring.

As was anticipated, differences existed between my knowledge of functions and graphs and Beth's knowledge of the domain. These differences were both in terms of knowledge of particular concepts and the connections between knowledge units. As a result of the knowledge engineering sessions, I was able to construct a graphical representation of my knowledge and Beth"s knowledge. This representation, presented in Chapter 3, showed the differences which existed in our knowledge bases. Identification of these differences informed the design of the hypercard system. Multiple representations of concepts were presented and links were made between applications of the concepts and the concepts.

In a classroom setting, when I use examples to introduce a particular concept, my implicit goal is to help students make connections that ultimately provide the context for knowledge. Beth and I discussed her view of examples, and how she uses examples in her learning. What emerged from these discussions is that her approach is to try to learn techniques applied to the specific examples and, when encountering a new problem she tries to fit the problem into a familiar situation. She admits that she sometimes does not understand the motivation for a particular approach. Beth's comments are not surprising, and from my experience she is not alone in this approach to learning mathematics. Based on the knowledge engineering sessions with Beth, I realize that as mathematics educators we have to be more explicit about the connections we want our students to make. Students lack a history which would facilitate the development of a broader view of a particular example.

The knowledge engineering sessions informed the content of the hypercard application, and my goal was to minimize the need for computer expertise informed the technical design decisions. Through use of the hypercard objects of cards, fields, and buttons, I was able to build a modest system which explored linear and quadratic functions. In the system the mathematical concepts were placed in a broader context linking them to examples from the natural sciences. For example, the ideal gas law from chemistry was used to connect the concept of linear functions to an scientific application. Throughout the application the power of hypertext was utilized to provide multiple levels of explanation and encourage exploration of the system.

Hypercard was an appropriate and promising application for representing the knowledge base developed during this work. The most significant aspect of the application is the ease with which hypertext is created. Since one of the goals of this project was to minimize the requirements in terms of programming expertise, I used the simplest structures for creating hypertext and these were sufficient to accomplish the desired outcome. Although I am not a novice programmer, in my judgement it is possible to acclimate quickly to the hypercard environment and quickly begin productive system development.

As stated previously, it is critical for students to view knowledge in multiple contexts so they are able to integrate new knowledge into a broader framework which supports their knowledge of mathematics and its applications. Hypercard is particularly well suited for allowing the linking of

multiple examples and thereby establishing a variety of contexts for knowledge. Since exploration is in some part determined by the user, it provides the opportunity to establish a variety of contexts just by choosing different routes through the system.

5.3 Potential Additions to the System

One of the most interesting aspects of the computer based system is the flexibility that it provides in terms of curriculum and presentation of topics. A hypercard system can remain a work in progress, and this may encourage thinking about the concepts and relationships. To capitalize on this aspect of the system and to support reflection, a useful modification of the system would be to build in the potential for users to leave notes at various points in the system. I envision the content of the notes ranging from comments on the usefulness of a particular part of the system, to suggestions for improvements, additional applications, and comments on each users' view of the knowledge. For example in the section on linear functions, user contributed information might be various formalizations of the definition of functions.

The historical component of the system might be expanded and a multicultural component might be added. Since context plays such a central role in this work, such additions to the learning environment are important just to establish another connection for the mathematical knowledge. A secondary benefit of providing a cultural context is that in so doing differences in learning styles based on culture may be addressed.

The system could be expanded in terms of breadth of material so it could be used on a regular basis in a College Algebra or Precalculus class. This could be beneficial for a number of reasons. Integrating the modified system into a class would facilitate evaluation of the system in terms of its contributions to learning. Secondly, it would provide the opportunity to consider this system and other examples of technology as an essential part of the class rather than an ancillary to be used only casually. Finally, it could serve as a catalyst for cooperative learning between students and teacher if all viewed themselves as contributors to the class knowledge base.

5.4 Implications for Mathematics Education

To conclude this work, I would like to revisit the National Council of Teachers of Mathematics view of mathematics curriculum. The organization states that all mathematics curriculum should demonstrate mathematics as problem solving, mathematics as reasoning, mathematics as communication, and mathematical connections [NCTM, 1989]. I believe that the knowledge engineering process as a mode of inquiry and the construction of knowledge based learning environments can contribute to satisfying the mathematics education community's need to meet and perhaps surpass the standards.

A substantial part of this work has been about connections. The goal of the knowledge engineering process is to represent the knowledge base of an expert or experts in a machine usable form. Researchers in knowledge engineering and related fields report that an important component of expertise is the ability to construct a framework for knowledge units which relates pieces of information and allows for the development of a deeper knowledge. A substantial part of the knowledge engineering process is

about making these connections explicit so the integration of aspects of the knowledge engineering process into mathematics education will assist in meeting the goal of encouraging the the development of connections. This work has shown that one of the strengths of the hypercard environment is the ability to link concepts to each other.

Knowledge engineering encourages the teacher to be explicit not only about the existence of connections but also to consider the strength of the connections. Engaging students in knowledge engineering sessions will be useful not only in terms of curriculum modification but may provide new ways to think about evaluation. A recent study reported in the Journal of Chemical Education reported that students who were able to successfully complete a written test on chemical equilibria expressed a number of misconceptions when interviewed and asked qualitative questions about the same subject. The interviews revealed that the students were not able to make connections and, in fact, that their written solutions, although getting them correct answers were based on incorrect beliefs [Bergquist and Heikkinen, 1990]. This suggest that our focus on quantitative evaluation is a disservice to students. We need to explore other means of evaluation and some part of the knowledge engineering process may be useful as an evaluation tool.

Although computer based learning environments are not the only route to exploring mathematics as the language of the natural sciences, this work has indicated they are one route. Studying a variety of applications simultaneous with the study of mathematical concepts supports the formation of the view that mathematics offers an efficacious representation for many principles of the natural sciences. Investigating a variety of applications is

one means of developing an understanding of how mathematics communicates scientific knowledge.

The stages of the knowledge engineering process are similar to the steps often suggested for problem solving. A knowledge engineer is required to solve the very difficult problem of obtaining and representing the knowledge of an expert in a useable form. Taking on the role of knowledge engineer may augment the teacher's problem solving skills. If the teacher views himself as a better problem solver, he will be more comfortable and more likely to model appropriate problem solving behavior for his students. A significant part of effective education is modelling for our students sound learning techniques.

Recently researchers have reported that increasing teachers' knowledge about their students' knowledge and problem solving abilities would encourage the development of curriculum which would promote learning and problem solving [Carpenter, et. al., 1989]. Central to the knowledge engineering process is expert's knowledge and problem solving heuristics. When the expert is a student and the knowledge engineer is a teacher, the teacher will become familiar with the student's knowledge and problem solving abilities.

Finally, we can consider the potential for the process of knowledge engineering and designing computer based learning environments to affect teachers' and students' views of mathematics as reasoning. This work has attempted to show that the knowledge engineering process, coupled with the design of a hypercard environment, will help students and teachers construct a broad context for their mathematical knowledge. For the teacher, mathematical reasoning is required to judge the appropriateness of particular

applications and representations. As was suggested in the discussion of problem solving, it is important that the teacher model desired behavior for her students. For the student, participation in a knowledge engineering session requires him to be explicit about his knowledge and heuristics which forces an evaluation of mathematical reasoning used in problem solving. In addition the student who navigates a hypercard environment like the one presented in this work will be exploring the mathematical reasoning used to build the system.

5.5 Implications for Teacher Education

In the last part of this discussion I will focus on the implications of this work for teacher education. A recent report from the National Center for Research on Teacher Learning suggests an agenda for research on learning to teach based on three hypothesis. I believe that this current work addresses some of the issues raised in the report and is a rich mode of inquiry for continued contribution. This part of the discussion will be organized around the three hypothesis presented in the report.

The first hypothesis addresses teachers' experience-limited beliefs about teaching, subject matter, and diverse learners:

> "We hypothesize that, in order for teachers to alter these resilient beliefs, they must be introduced to an idea that is plausibly better and must be provoked to question their own experiences and to question the beliefs that are founded in those experiences" [Kennedy, 1991, p. 21].

My own experiences throughout this project validate this hypothesis. When I began this project, I thought of myself as a good teacher. I was willing to work hard. I was flexible. I had empathy with students. My subject area

knowledge, enthusiasm for learning, and commitment to the educational process were all strong. But even good teachers can become complacent. Most good teachers cannot remain complacent, and in search of a means of removing the complacency they often leave teaching or commit their psychic energies elsewhere. This project has provided me with the opportunity to realize that teaching introductory classes is a rich environment for sophisticated thought about mathematics. As I tried to catalogue my knowledge, I had new insights into the nature of mathematics.

As I have emphasized throughout this discussion, the focus of the knowledge engineering process is on the explicit representation of knowledge. Taken seriously it is impossible to engage in this process and not be reflective about one's subject area knowledge. This work goes beyond simple reflection because the representation of the knowledge base is the expected product. The knowledge engineering community is engaged in research about the appropriateness of current representations, but this does not negate the benefit of attempting to make knowledge explicit even if the representation is flawed.

Viewing the student as expert and endeavoring to construct an explicit representation of student's expertise focuses the attention on the learner in an unique way. Although it is unrealistic to expect a transformation in the teacher solely as a result of taking on the knowledge engineering role, providing the teacher with a new window through which to look at learning will challenge existing beliefs. This is not suggest that there are ten easy steps to knowledge engineering and the reform of teacher education. The knowledge engineering process is difficult and often is not successful.

However, it is concerned with the issues of knowledge and beliefs and provides another mode of inquiry.

In the realm of teacher education potential extensions of this work include expanding the number of participants. I envision a workshop where teachers are trained in the knowledge engineering process and work with learners to construct a computer based learning environment. The teachers would be asked to keep track of the evolution of their thinking about the subject matter and the nature of learners. If expertise, as Kolodner suggests, evolves based on introspection, this process is supportive of that evolution. In addition, working toward the goal of building a system provides the teachers and the learners the chance to develop a broader view of technology. By engaging in this process they would begin to develop the "intellective" skills essential for the 21st century. Finally constructing a computer based learning environment makes sharing of the work a more likely possibility.

The second hypothesis proposes:

> "We hypothesize that, in order to enhance their subject matter knowledge, teachers need to encounter substantive ideas within the context of the domain as a whole and need to learn substantive ideas by participating in worthwhile academic tasks. We also hypothesize that, in order to learn how to connect subject matter to diverse learners, teachers need to learn about diversity in its cultural and community contexts" [Kennedy, 1991, p. 22].

It is difficult for me to think of more substantive ideas than those considered in a discussion of knowledge representation. A strength of this current work is that ideas are discussed in a context of building a product which is immediately useful. It is this union of theoretical and practical which will be

appealing to both the novice and experienced teacher faced with realities of school culture. In other parts of this discussion, I have already mentioned that a strength of the hypercard environment is that it encourages individualized exploration of topics. The issues of learner differences are complicated, and a hypertext based system cannot respond to all the issues. It is one area with some demonstrated success and some potential for further work.

Initially I had planned to establish a cultural context for the topics presented in this current work. I believe the system would be strengthened if it included an expanded historical component and current examples of the contributions to mathematics from diverse cultural groups. As the work progressed, issues of process became the more central focus, but strengthening the cultural context is an important area of future system modification. A useful starting point for developing cultural awareness may be acquiring the knowledge necessary to construct a cultural component of a learning environment.

Central to the knowledge acquisition process is the knowledge engineer's understanding of the context or framework the expert has formed for knowledge. In the learner's case culture is an important aspect of knowledge. It would be difficult to engage an expert in knowledge engineering sessions and not learn something about the influence of culture. If the development of cultural awareness is an established goal, the sessions would be even more informative.

The third and final hypothesis addresses teacher reflection:

> "We hypothesize that teacher learning can best occur when teachers have opportunities to stop action so that slower and more detailed deliberation is possible, opportunities to see explicit connections between relevant concepts and criteria and teaching situations, and opportunities to see connections between relevant concepts and criteria and their own behavior" [Kennedy, 1991, p. 22].

I have reported that one of the benefits of this work was the opportunity for me to change roles from teacher to knowledge engineer. Not only was I able to engage in "slower and more detailed deliberation"; I was also able to remove, for the most part, the expectation of providing the answers. As a knowledge engineer I was focused on acquiring Beth's knowledge; these sessions removed the influences of getting the homework done or preparing for the test. Time spent representing my own knowledge also supported this "more detailed deliberation". Again, I was not the teacher planning a lesson, constructing a test, or developing a problem set; I was attempting to represent my knowledge base.

Because of my work on this project, I was less likely to make assumptions about connections students were able to make. In my classes during the Fall and Spring semester, I was much more careful to be explicit about connections which I viewed as critical for understanding a topic. This work reinforced for me that establishing a framework for knowledge is as important as providing the knowledge units. In this work I considered a small part of the domain. Exploration of other parts of the domain would contribute further to addressing the concerns expressed in this final hypothesis.

5.6 Conclusion

> "A common experience, when some colleague would try to explain some piece of mathematics to me, would be that I should listen attentively, but almost totally uncomprehending of the logical connections between one set of words and the next. However, some guessed image would form in my mind as to the ideas that he was trying to convey-formed entirely on my own terms and seemingly with very little connection with the mental images that had been the basis of my colleague's own understanding-and I would reply. Rather to my astonishment, my own remarks would usually be accepted as appropriate, and the conversation would proceed to and fro in this way. It would be clear, at the end of it, that some genuine and positive communication had taken place. Yet the actual sentences that each one of us would utter seemed only very infrequently to be actually understood! [Penrose, 1989, p. 427]

In the above passage the mathematical physicist Roger Penrose describes from his experience how mathematical knowledge is communicated. He could be describing knowledge engineering sessions. Although knowledge engineering is about "genuine and positive communication", it is also about developing an explicit representation of the knowledge which has been communicated. As an individual teacher I benefitted from the communication. It provided me with an opportunity to view a student's knowledge and my own from a fresh perspective. As a teacher interested in contributing to the improvement of the teaching and learning of mathematics, I benefitted from the opportunity to develop an explicit representation of a student's knowledge and my knowledge. Before beginning this work, I considered myself a good teacher in part because I believed that I used

examples effectively. As a knowledge engineer acquiring expertise I became more aware of the need to weave the examples into the existing knowledge. Context is more than a "real world situation". It is about connections which are continually being changed, often made stronger but sometimes made weaker.

Knowledge engineering is also about collaboration. Although, under the best of circumstances, teacher-student interactions are about collaboration, this project allowed me to work with a student in a new environment. We shared our expertise. We were able to focus our attention on knowledge and not on the chapters to be covered or the next test.

The personal rewards of this research encouraged me to explore other avenues for the using the knowledge engineering process. For example, I integrated some aspects of the process into my tutoring sessions with students, and I encouraged colleagues to think more explicitly about their students' expertise and the ways to tap this expertise.

The tough questions remain. We will continue to debate the nature of knowledge and issues of teaching and teacher education. I believe the best conclusion for this project is that it was part of the debate and an interesting, challenging, and motivating experience. I want to continue to contribute to the debate.

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