CHOICE BEHAVIOR AND REWARD STRUCTURE: A TEST OF THREE MATHEMATICAL MODELS

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Choice Behavior and Reward Structure: A Test of Three Mathematical Models

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Introduction

The Estes and Straughan (1954) two choice, non-contingent probability learning situation involves the presentation of a signal (e.g., light, buzzer) to which S responds by predicting one of two mutually exclusive and exhaustive events. The prediction of the more frequent event on trial n, $E_{1,n}$, is designated $A_{1,n}$, and the prediction of the less frequent event on trial n, $E_{2,n}$, is designated $A_{2,n}$. This learning situation is described as "non-contingent" because the occurrence of either event on trial n is independent of the response, i.e.,

$$P(E_{i,n}) = P(E_{i,n}|A_{j,n}), \quad i,j = 1,2.$$

The events E_1 and E_2 occur with probabilities π and $1-\pi$, respectively.

One purpose of the present research is to consider several models which predict behavior in the situation just described. These models will be examined for their ability to account for the choice behavior of Ss who have received extended training on one of two reinforcement schedules (levels of π) and one of two payoff conditions. In particular, these models will be evaluated for their ability to describe the mean and variability of response probabilities, as well as several response probabilities conditional on previous sequences of events and responses.

In this section, the experimentation in non-contingent two-choice behavior of human Ss will be summarized and the import of the findings from these studies on the development of mathematical psychology will be considered. This section will conclude with a discussion and evaluation of several models which yield predictions of response probability, with specific emphasis on those that yield predictions of trial to trial changes in performance.

Previous experimentation

The relationship between theory and experiment is nicely demonstrated in the area of mathematical learning theory. Many early and several later studies in non-contingent two-choice probability learning (see Appendix A) involved Ss trained on one of several levels of π for less than 300 trials with no incentive other than the knowledge of the correctness of their predictions. In these studies the probability of predicting a given event appeared to stabilize (reach an asymptote) at the probability that the event occurred. This phenomenon has been called "matching" and is designated by $\lim_{n\to\infty} P(A_{i,n}) = \pi_i$.

Several subsequent non-payoff studies (see Appendix A) in which Ss were trained with more than 300 trials have evidenced "overshooting", i.e., an asymptotic level of responding exceeding matching. This finding brings into question the validity of models for choice behavior which pre-

dict matching asymptotes. In defense of these models, Estes (1962) has suggested that either: (a) After a long series of trials, variables which do not represent the effects of reinforcing operations begin to come into play; or (b) the quantitative assumptions concerning the effects of reinforcement and non-reinforcement must be modified. If overshooting results from additional processes becoming effective later in the experimental session, then those models which predict a matching asymptote may tentatively be judged appropriate to describe choice behavior when the effectiveness of these additional processes is experimentally prevented. If, on the contrary, overshooting represents the ultimate effect of the conditioning process, then those models which predict matching must be judged inappropriate to describe choice behavior. A second purpose of the present research will be to consider the evidence which bears on the validity of these two alternatives.

Another finding of major theoretical importance observed in experiments on choice behavior was the decreasing probability of predicting an event as the number of consecutive occurrences of that event increased. This phenomenon is called the negative recency effect, or the gambler's fallacy (see Appendix A, Table 2). The observation of this phenomenon brings into question the validity of those models for choice behavior (e.g., Bush & Mosteller, 1955; Estes, 1959; Suppes & Atkinson, 1960; and Myers & Atkinson, 1964) which

identify the event following a response as a reinforcer, i.e., that which increases response strength. That negative recency effects show evidence of diminishing with an increasing number of trials (Anderson, 1960; Anderson & Whalen, 1960; Edwards, 1961; Edwards & Tannenbaum, 1961; Lindman & Edwards, 1961; Derks, 1962; Derks, 1963; Jones & Myers, 1965), suggests that this identification may become valid later in learning and that these models will then adequately describe choice behavior. The present study will provide further data on this question and will permit an examination of the adequacy of several models to describe choice behavior pre-asymptotically where negative recency effects are most often observed.

Models for choice behavior

We now turn to a consideration of several models for choice behavior. Those of particular interest predict overshooting and describe response probability as a function of the outcomes of previous trials. Several static models have been proposed, e.g., Expected Value Matching Generalization (Edwards, 1956), Relative Expected Loss Minimization Rule (Edwards, 1956), Expected Gain (Taub & Myers, 1961), Maximization of Expected Utility (Siegel, 1959; Radlow & Siegel, 1960; and Siegel, 1961) and the Scanning Model (Estes, 1962); these are of less interest because they do not contribute to an understanding of the learning process and their asymptotic goodness-of-fit is no

better than that provided by stochastic[1] models. Further, with the exception of the Siegel et al. model, they do not adequately describe the effects of payoff. We will consider two types of stochastic models for describing choice behavior which are able to predict overshooting: (1) Linear and (2) Finite state Markovian with restricted transitions (Estes, 1959, p. 11).

Linear models. Following Bush & Mosteller (1955) there are two types of linear models which are applicable for the prediction of choice behavior in the situation described above: (a) Experimenter-Controlled; and (b) Experimenter-Subject-Controlled.

In each of these types of models, Bush and Mosteller (1955) describe response probability on trial *n* in terms of the application of different operators to response probability on trial *n*-1. They use an operator to mean an instruction applied conditionally to response probability, either increasing it, leaving it unchanged, or decreasing it.

For the Experimenter-Controlled equal α case (Bush & Mosteller, 1955, p. 280), the probability of predicting an event on trial *n* increases if that event occurred on trial *n*-1, i.e., reinforcement is assumed to be independent of the *S*'s response on trial *n*-1. This model will not be of further interest, however, as it can only predict asymptotic matching.

For the Experimenter-Subject-Controlled case (Bush &

Mosteller, 1955), change in response probability is described by four operators. The operator applied is dependent on the response and event outcome which occurred on trial $\underline{n}$-1. This set of operators, given by the linear theorem (Anderson, 1959), appears in Eq. 1 in Bush and Mosteller (1955) notation.

$$Q_{jk}P_x(A_{1,n-1}) = P_x(A_{1,n}|E_{k,n-1}A_{j,n-1}) = \alpha_{jk}P_x(A_{1,n-1}) + (1-\alpha_{jk})\lambda_{jk} \tag{1}$$

In Eq. 1, $P_x(A_{1,n}|E_{k,n-1}A_{j,n-1})$ is the probability of an A_1 response for organism $\underline{x}$ given event E_k and response A_j on trial $\underline{n}$-1, $P_x(A_{1,n-1})$ is the probability of an A_1 response for organism $\underline{x}$ on trial $\underline{n}$-1, and k=1 or 2 depending on which event occurred on trial $\underline{n}$-1, and j=1 or 2 depending on what response was made. The parameter α_{jk} is an operator parameter which measures the effectiveness of the outcome of the previous trial on response probability; and λ_{jk} is the limiting value to which response probability would go with repeated application of operator Q_{jk}. Depending on the restrictions that are placed on the parameters α_{jk} and λ_{jk}, several variations of this model result. The two which seem most applicable to prediction of choice behavior and are able to predict overshooting are the: (a) Reinforcement-extinction (R-E) model; and (b) secondary reinforcement (SR) model.

The Bush and Mosteller (1955) R-E model assumes that the probability of predicting an event E_i, i=1,2 on trial

$\underline{n}$ increases whenever that event occurs on trial $\underline{n}$-1, but that there is a differential increase depending on whether the event was predicted correctly (e.g., $E_{1,n-1}A_{1,n-1}$) or incorrectly (e.g., $E_{1,n-1}A_{2,n-1}$). The operators for this R-E model (Eq. 2) are obtained from the linear theorem (Eq. 1) with the following restrictions:

$$\alpha_{11} = \alpha_{22} = \alpha_1 \qquad \lambda_{11} = \lambda_{21} = 1$$
$$\alpha_{12} = \alpha_{21} = \alpha_2 \qquad \lambda_{12} = \lambda_{22} = 0$$

We have

$$(2)\quad \begin{aligned} P_x(A_{1,n}|E_{1,n-1}A_{1,n-1}) &= \alpha_1 P_x(A_{1,n-1}) + (1-\alpha_1) \\ P_x(A_{1,n}|E_{2,n-1}A_{1,n-1}) &= \alpha_2 P_x(A_{1,n-1}) \\ P_x(A_{1,n}|E_{1,n-1}A_{2,n-1}) &= \alpha_2 P_x(A_{1,n-1}) + (1-\alpha_2) \\ P_x(A_{1,n}|E_{2,n-1}A_{2,n-1}) &= \alpha_1 P_x(A_{1,n-1}) \end{aligned}$$

These operators characterize a R-E model because the occurrence of an event on each trial strengthens (reinforces) one response and weakens (tends to extinguish) the other response.

Averaging over organisms and events we have

$$(3)\quad \begin{aligned} P(A_{1,n}) = {} & V_{2,n-1}[\alpha_1(2\pi-1)-\alpha_2(2\pi-1)] + \\ & V_{1,n-1}[2\pi\alpha_2-\alpha_1(2\pi-1)] + \pi(1-\alpha_2) \end{aligned}$$

where $V_{2,n-1}$ is the second raw moment of the distribution of response probabilities, and $V_{1,n-1}$ is the average probability of an A_1 response on trial $\underline{n}$-1.

If there is a reason to believe that some stimuli associated with reward ($A_{1,n-1}E_{1,n-1}$) occur with non-reward

$(A_{1,n-1}E_{2,n-1})$, the presense of these stimuli on non-rewarded trials will reinforce the response which was made. These reinforcing stimuli are called secondary reinforcers and the phenomenon secondary reinforcement. If the following restrictions are placed on the parameters of the linear theorem (Eq. 1), the set of operators obtained (Eq. 4) constitutes the SR model described by Bush and Mosteller (1955).

$$\alpha_{11} = \alpha_{22} = \alpha_1 \qquad \lambda_{11} = \lambda_{12} = 1$$
$$\alpha_{12} = \alpha_{21} = \alpha_2 \qquad \lambda_{22} = \lambda_{21} = 0\ ;$$

$$(4)\quad \begin{aligned} P_x(A_{1,n}|E_{1,n-1}A_{1,n-1}) &= \alpha_1 P_x(A_{1,n-1}) + (1-\alpha_1) \\ P_x(A_{1,n}|E_{2,n-1}A_{1,n-1}) &= \alpha_2 P_x(A_{1,n-1}) + (1-\alpha_2) \\ P_x(A_{1,n}|E_{1,n-1}A_{2,n-1}) &= \alpha_2 P_x(A_{1,n-1}) \\ P_x(A_{1,n}|E_{2,n-1}A_{2,n-1}) &= \alpha_1 P_x(A_{1,n-1}) \end{aligned}$$

Averaging over organisms and events we have

$$(5)\quad \begin{aligned} P(A_{1,n}) = {} & V_{2,n-1}[\alpha_1(2\pi-1)-\alpha_2(2\pi-1)] + \\ & V_{1,n-1}[1-\alpha_1(2\pi-1)-\alpha_2(2\pi-1)] \end{aligned}$$

<u>Finite state Markov models</u>. Using the concepts of stimulus-response psychology, several models for the prediction of choice-behavior have been formulated which view learning as a stochastic process described by a discrete parameter (trials) Markov chain (Parzen, 1962). The models with which we will be concerned have been described by Estes (1959, p. 11) as having restricted transitions, i.e., the changes in response probability from one trial to the

next may only increase or decrease by a constant amount or remain unchanged.

Among the first of these models was the pattern model (Estes, 1959; Suppes & Atkinson, 1960). In this model the number of effective stimulus elements is either assigned e.g., one, two - or may be treated as a parameter to be estimated from the data. The states have been identified with the responses of predicting an event. Response probability is considered a function of the number of elements conditioned to predicting the more frequent event. Although it is not an essential restriction, it has generally been assumed that exactly one element (pattern) is sampled from the set of N elements on each trial and with probability $\underline{c}$, the sampled element transits to the state of conditioning predicting the reinforcing event. The response made on each trial is that to which the sampled element is conditioned. As these N-element models have a matching asymptote for any value of N, they will be of no further interest.

In order to account for the overshooting frequently observed with extended training or payoff (see Appendix A), Atkinson (1961) formulated a "strength of conditioning" model in which the number states of conditioning was to be estimated from the data. This model assumes that associated with each stimulus element is an integer $\underline{j}$ which varies from $\underline{0}$ to $\underline{s}$. The transition of the element(s) from state $\underline{0}$ to $\underline{s}$ indicates an increase in the strength of conditioning and

results in an increase in the probability of the response predicting the more frequent event. The conditioning state of the sampled element increases by one (toward state s) with probability θ when an E_1 occurs, and decreases by one with probability θ when an E_2 occurs. On any trial the probability of an A_1 response is given by j/s where j is the ordinal number of the state in which the sampled element lies and s denotes the maximum value of conditioning strength. The predicted asymptotic level of responding is given by Eq. 6.

$$(6) \qquad \begin{aligned} P(A_{1,\infty}) &= \frac{s(1-a) - a(1-a^s)}{s(1-a)(1-a^{s+1})}, & a \neq \tfrac{1}{2} \\ P(A_{1,\infty}) &= \tfrac{1}{2}, & a = \tfrac{1}{2} \end{aligned}$$

where $a = (1-\pi)|\pi$.

More recently, Atkinson (1962) and Myers and Atkinson (1964) have presented an extension of the pattern model which allows learning to occur with correct as well as incorrect predictions. This has been named the weak-strong (W-S) conditioning model. These models were derived from stimulus sampling concepts to provide a framework within which the effects of incentive and motivational variables could be analyzed. As the Atkinson (1962) and Myers and Atkinson (1964) models are very similar, only the more recent model will be discussed in detail.

It is convenient to first consider the axioms of this model, a branch diagram illustrating the transitions among

the four states, and then a transition matrix.

Sampling Axioms.

S_1. The stimulus situation associated with the onset of each trial is represented by a set of N stimulus elements. On each trial exactly one stimulus is randomly sampled from this set.

S_2. Given the set of stimulus elements available for sampling on a trial, the probability of sampling a given element is independent of trial number and the preceding pattern of events.

Conditioning Axioms.

C_1. On every trial each stimulus element is conditioned to exactly one response. Further, the element is either weakly or strongly conditioned to that response. (The strong conditioning state for A_i response is denoted S_i, the weak state by W_i).

C_2. If a stimulus is sampled on a trial and is strongly conditioned to the A_i response, then (a) the stimulus remains strongly conditioned to the A_i response if that response is reinforced; and (b) with probability δ the stimulus becomes weakly conditioned to the A_i response if some other response is reinforced.

C_3. If a stimulus is sampled on a trial and is weakly conditioned to the A_i response then (a) with probability μ the stimulus becomes strongly conditioned to the A_i response if that response is reinforced and (b) with probability δ the stimulus becomes weakly conditioned to the A_j ($i \neq j$) response if A_j is reinforced.

C_4. Stimulus elements that are not sampled on a trial do not change their conditioning state.

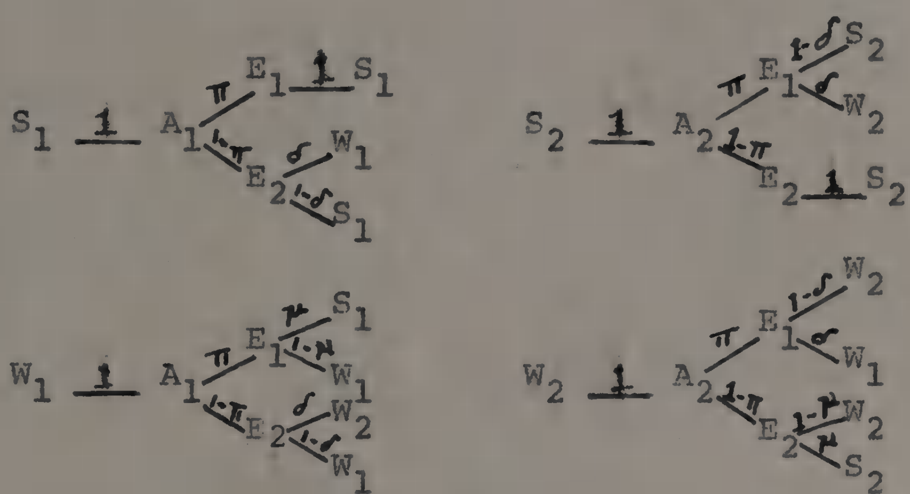
Response axiom.

R_1. If the sampled element is conditioned to the A_i res-

ponse either weakly or strongly, then that response will occur with probability 1.

The branch diagram below illustrates the transitionings among conditioning states for the subset of trials on which an element is sampled.

(7)



From these branch diagrams we may form a matrix of transitions among states.

(8)
$$\begin{array}{c c}
 & \begin{array}{cccc} S_1 & W_1 & W_2 & S_2 \end{array} \\
\begin{array}{c} S_1 \\ W_1 \\ W_2 \\ S_2 \end{array} &
\left[\begin{array}{cccc}
1-\delta(1-\pi) & \delta(1-\pi) & 0 & 0 \\
\mu\pi & 1-\mu\pi-\delta(1-\pi) & \delta(1-\pi) & 0 \\
0 & \delta\pi & 1-\delta\pi-\mu(1-\pi) & \mu(1-\pi) \\
0 & 0 & \delta\pi & 1-\delta\pi
\end{array}\right]
\end{array}$$

For simplicity, states may be numbered as follows: $S_1=1$, $W_1=2$, $W_2=3$, and $S_2=4$. Since the four state Markov chain defined by the transition matrix above is irreducible and aperiodic, the quantity u_j exists, where

(9)
$$u_j = \lim_{m\to\infty} P_{ij}^{(m)}$$

The u_j's may be computed by

(10) $$\mu_j = \frac{D_j}{\sum_i D_i}$$

where for the two-choice non-contingent situation

(11) $$D_1 = \pi^3 \qquad D_3 = (1-\pi)^2\pi\phi$$
$$D_2 = (1-\pi)\pi^2\phi \qquad D_4 = (1-\pi)^3$$

and $$\phi = \frac{\delta}{\mu}$$

Atkinson (1962) has shown that at asymptote, the probability of an A_1 response is a simple function of the u_j's. Specifically

(12) $$\lim_{n\to\infty} P(A_{1,n}) = P(A_1) = u_1 + u_2$$

For the non-contingent choice situation this is equal to:

(13) $$P(A_1) = \frac{\pi^3 + \pi^2(1-\pi)\phi}{\pi^3 + (1-\pi)^3 + \pi(1-\pi)\phi}$$

where $0 < \mu \leq 1$ and $0 \leq \delta \leq 1$ and $P(A_1)$ has bounds

(14) $$\pi \leq P(A_1) < \frac{\pi^3}{\pi^3 + (1-\pi)^3}$$

Evaluation of models.

We will now consider the models described in the preceding section with regard to their mathematical tractability, goodness-of-fit and their psychological meaningfulness. By mathematical tractability of a model we will mean the ease with which different predictive statistics may be derived and the ease of obtaining estimates for the theo-

retical parameters of which behavior is assumed a function. The goodness-of-fit of a model refers to the extent of correspondence between experimentally observed and predicted statistics. By psychological meaningfulness of a model we will refer to the extent that the parameters used to describe choice behavior vary with the manipulation of the experimental variables with which they are identified, and remain invariant with the manipulation of other variables.

Linear models. In only three studies (Bush & Wilson, 1956; Bogartz, 1965; Myers, Suydam, & Heuckeroth, 1965) have the R-E and SR models been applied to two-choice behavior, and in one (Myers, Suydam, & Heuckeroth, 1965) the application has been to primarily non-symmetric payoff[2] data. Bush and Wilson (1956) tested the SR model using data from 22 paradise fish, each of which was trained with a $\pi = .75$ for 140 trials. They used a Monte-Carlo technique (Bush & Mosteller, 1955, p. 129) to obtain predictions of choice behavior from theoretical Ss (stat-fish). Only those protocols showing a distribution of successes comparable to the experimental Ss were used to obtain predictions of the learning curve, the mean and standard deviation of the number of successes and runs of successes. Examination of the learning curve for their experimental Ss suggests that performance had not stabilized. Although no goodness-of-fit statistics are reported, inspection reveals that this model predicts a level of learning, and a mean and standard deviation of success-runs greater than found

for the experimental Ss. With regard to parameter identification, Bush and Wilson (1956) found, as they expected, the parameter identified with primary reward (α_1) was more reinforcing than the parameter identified with secondary reward (α_2).

Bogartz (1965) has applied the R-E model to predict the asymptotic response probability of choice data collected with preschool children run under a .5 or .8 level of π. Marked deviations in fit were attributed in part to the fact that the data showed evidence of alternation patterns and position preferences.

Identifying the conditioning rate parameters (α_1 and α_2) of the R-E and SR models, and δ_1 and δ_2 of the W-S model, with the size of the regret (Savage, 1957) associated with each trial outcome, Myers, Suydam, and Heuckeroth (1965) have tested the ability of these models to describe the asymptotic first-order conditional probabilities of the form described on p. 35 . for data collected by Myers and Suydam (1964). These data were for human Ss trained with either a .6 or .8 level of π for 300 trials with symmetric or non-symmetric payoff. Parameters identified with the largest regret size were shown to have the greatest reinforcing effect for each model. Examining the goodness-of-fit of these models suggested that the W-S model resulted in a better fit than the R-E model which, in turn, was better than the SR model; however, it was concluded that

any claims of superiority must await tests with other data sets and other statistics. Further, since most of the data were obtained under non-symmetric payoff conditions, the relevance of these results for the symmetric payoff case must be considered as suggestive at best.

The paucity of work with the linear models is presumably due to the difficulty of estimating the second raw moment, V_2. Bush and Mosteller (1955) have pointed out that the problem in using expressions involving the second moment arises because each moment is a function of the next higher moment and no closed expression can be obtained. Several authors (Bush & Mosteller, 1955; Mosteller & Tatsuoka, 1960) have suggested techniques by which the predicted asymptotic response probability may be approximated. However, the techniques are not useful for obtaining estimates of the rate parameters and moments which are required for a fine grain analysis of the data. Bush and Wilson (1956) comment on a technique of estimating these rate parameters and moments using the first three moments of the observed distribution of successes in each block of 10 trials with formulas for moments of the p-value distribution (Bush & Mosteller, 1955, p. 98). Anderson (1959) has suggested obtaining estimates of the rate parameters and moments by equating the observed first-order joint probabilities of the form $P(A_{i,n}E_{j,n-1}A_{k,n-1})$, $i,j,k = 1,2$, to the prediction equations and solving appropriate expressions simultaneously. As there was no known reason to believe the Bush and Wilson (1956) technique

was better, a method similar to that suggested by Anderson (1959) was employed in the present experiment for convenience (see also Bogartz, 1965; Myers, Suydam, & Heuckeroth, 1965). Specifically, parameter estimates were obtained in the present experiment by minimizing the sum of squared deviations of pooled observed and predicted second-order joint frequencies (see Appendix D for the function minimized). The estimation algorithm is briefly described in Appendix E.

Finite state Markov models. While the Atkinson (1961) *s* state model has been able to successfully predict asymptotic response probability of choice behavior under payoff, the goodness-of-fit is poorer than found using the Myers and Atkinson (1964) W-S model (Myers[3], 1965). While *s* has been shown to increase with payoff in a symmetric payoff situation (Atkinson, 1961), no clear identification exists for θ. Further, the parameter identification becomes even more questionable when non-symmetric payoff is used.

The W-S model (Myers & Atkinson, 1964) seems somewhat more promising with respect to parameter identification. Applying the W-S model to symmetric and non-symmetric payoff data (Myers & Suydam, 1964) and using a regret (Savage, 1957) identification, Myers, Suydam, and Heuckeroth (1965) have reported that parameters identified with increasing regret size showed an increasing reinforcing effect. In several other studies (e.g., Calfee, 1963; Myers & Atkinson, 1964; Suydam et al., 1964; Jones[4], 1965) a reward-non-reward identification has been used, i.e., δ has been

identified with the effects of loss (non-reward), μ with the effects of gain (reward), and N with the number of stimulus elements. According to these identifications, Myers and Atkinson (1964) have suggested that δ and μ appear to be an increasing function of the amount of payoff, and that a decrease in N with increased payoff suggests that Ss attend to fewer cues as motivation is increased. In each of these studies, except Jones[4], N was found to decrease with increases in payoff. The findings with regard to δ and μ are not clear. Sometimes δ and μ increase with payoff (Myers & Atkinson, 1964; Suydam et al., 1964, 1¢ and 10¢ aware groups), and in other cases (Jones[4]; Suydam et al., 1964) these parameters do not vary consistently with payoff.

The mathematical tractability of the W-S model is evidenced by the wealth of statistics which have been already derived (see Appendix C, Part A.1-5). Theoretical expressions are available for marginal response probabilities, for response probabilities conditional on previous responses and events, and for standard deviations of marginal response probabilities (Myers & Atkinson, 1964; Heuckeroth & Myers, 1965).

With regard to fits to data, several authors (Myers & Atkinson, 1964; Suydam et al., 1964; Myers, Suydam, & Heuckeroth, 1965; and Jones[4]) present evidence for the ability of the W-S model to describe several asymptotic

statistics for data collected from human Ss. Calfee (1963) has also demonstrated the model's applicability to animal data. He ran two groups of rats in a non-contingent two-choice problem for 3680 trials with a .65 and .8 level of π, respectively. Excellent fits were obtained for both marginal and conditional asymptotic statistics. Heuckeroth and Myers (1965) have provided a preliminary test of the ability of the W-S model to predict asymptotic standard deviation of response proportions[5], the learning curve, and pre-asymptotic first-order conditional probabilities using data published by Suppes and Atkinson (1960). Their findings indicate that predicted asymptotic standard deviations are slightly lower than observed, especially for the non-payoff group, and that using parameter estimates taken at asymptote yields a predicted learning curve whose rate is somewhat higher than observed.

The large deviations between observed and predicted statistics, e.g., some of the asymptotic second-order conditional statistics, could have occurred because the sampling assumptions of the model have been violated. Myers and Atkinson (1964) have further suggested that these findings might be attributed to the unreliability of these statistics, or that the model may require some modification to adequately describe these statistics. These authors have suggested that additional experimentation involving more trials and subjects was required to decide whether these deviations in

fit were attributable to unreliability of the estimates. The present research will provide more data so that a further evaluation may be made.

Myers and Atkinson (1964) have generalized the W-S model so that a stimulus element can be in any one of *k*-stages of conditioning to a response, e.g., with $k=3$ an element could be in a weak, intermediate, or strong state of conditioning. When several versions of the *k*-stage model ($k=2,3,4,5,10$) were used to describe the first-order conditional probabilities for several sets of human choice data, the best fits were usually obtained with the two-stage (W-S) model (Myers & Atkinson, 1964). We will, therefore, restrict our study to this two-stage model in the present experiment.

Summary of purpose

Evidence from a large body of literature (Appendix A) has shown that the choice behavior of human *S*s trained for a large number of trials or with payoff overshoots the marginal event probability. One purpose of the present research is to compare the ability of two types of mathematical models formulated within the framework of statistical learning theory to describe the choice behavior of *S*s trained under conditions expected to lead to overshooting. The types of models with which we are concerned are the infinite state (linear) and finite state Markovian models. In our selection of the specific models for study, it was first

required that they be able to predict overshooting. The experimenter-controlled equal α (Bush & Mosteller, 1955) and N-element pattern models (Estes, 1959; Suppes & Atkinson, 1960) were rejected because they failed to satisfy this requirement. It was also required that the models selected provide evidence of being able to describe choice data, i.e., show goodness-of-fit, have appropriate parameter identification, and be mathematically tractable. The finite state Markov model described by Myers and Atkinson (1964) -- the W-S model -- was judged to satisfy these requirements most admirably. Except for the criterion of mathematical tractability, two linear models (R-E and SR) also show promise of being able to describe choice data. Consequently, these three models are compared on a large array of statistics for data collected under several experimental conditions. Examining the goodness-of-fit of several models on several statistics and determining whether parameters vary in a predictable manner with the manipulation of variables identified with the parameters and otherwise remain invariant, provides evidence regarding the validity of our hypotheses (as expressed in the assumptions of the models) about the learning process.

The early coincidental finding of observed and predicted asymptotic matching has led at least one writer (Estes, 1962) to question whether the occurrence of overshooting is due to the introduction of variables not manipulated by the

experimenter as training progresses. An additional purpose of the present research is to attempt to evaluate whether additional processes are becoming effective as the experimental session progresses, or whether the course of learning represents the ultimate effect of the conditioning process.

Method

Subjects: The subjects for this experiment were 80 male and 80 female University of Massachusetts undergraduates taking a course in Introductory Psychology.

Apparatus: Each *S* sat before a black panel (15½ x 10 inches) upon which two 1" diameter green jewel lights were mounted. These lights served as the reinforcing events. Directly below each light was a toggle switch with which *S* indicated his event-prediction. A buzzer, sounded for ½ second at the beginning of each trial, signaled *S* to make his prediction. Three seconds later one of the event lights came on for 1 second. Three and one-half seconds later the buzzer signaled the beginning of the next trial. The predictions of each *S* were recorded on an Esterline Angus Operations Recorder. The sequence of events presented to each *S* was controlled by a Western Union tape transmitter. In each experimental session, data was collected from 1 to 4 *Ss* as scheduling permitted.

Sequences: For each level of π (.6 and .8), ten sequences of 600 events each were constructed so that the number of runs of each event was approximately at its expected value (Nicks, 1959), and the proportion of E_1's did not deviate by more than 10% from π in each 50-trial block. Table 1 shows the distribution of run lengths for each reinforcement schedule.

Procedure: Eight groups of 20 Ss representing all combinations of π (.6 and .8), payoff (gains and losses of 0¢ and 1¢), and sex each made 600 binary predictions. In each payoff-sex group for a given level of π, data was collected from two males and two females with one of the ten sequences of events. The Ss in the payoff groups were given several sheets with "+" and "-" columns to keep track of their correct and incorrect predictions, respectively. The Ss in the non-payoff groups made their predictions under the usual no-feedback condition. All Ss received instructions that the experiment was concerned with the study of the processes by which people make decisions (see Appendix B), and that their task was to make as many correct predictions as possible. Ss in the payoff groups were told, in addition, they would win or lose 1¢ on each trial depending on the correctness of their predictions. All Ss were told that the sequence of events was strictly random and that attempting to find a pattern would probably hurt their scores.

Table 1. Distribution of Run Lengths for Each Reinforcement Schedule

Run Length	$\pi = .6$		$\pi = .8$	
	E_1	E_2	E_1	E_2
1	57	85	17	77
2	35	35	14	15
3	21	15	12	3
4	12	6	9	1
5	7	2	8	
6	4	1	7	
7	4		7	
8	2		6	
9	1		3	
10	1		3	
11			3	
12			3	
13			1	
14			1	
15			1	
16			1	
Total runs	144	144	96	96
Total trials	600		600	

Results

The results of the present experiment will be presented in three sections: 1) Marginal probabilities; 2) Run data; and 3) Application of models to data.

Marginal probabilities

A preliminary analysis of variance invölving payoff, π, sex, and event sequences within π was performed on the total number of A_1 predictions for each S. The results of this analysis indicated that all sources of variance involving sequences were non-significant. Consequently, a second analysis of variance (Table 2) involving payoff, π, and sex was performed on the number of A_1 responses in successive blocks of 50 trials. This analysis (Table 2) and Figure 1 indicate that Ss trained with payoff attain significantly higher levels of performance ($P<.001$) than Ss trained with no payoff, and Ss trained under a .8 level of π predict the E_1 event more frequently ($P<.001$) than those trained with .6 level of π. No interaction between these two factors is present. Further, the rate of learning is significantly greater with payoff ($P<.01$) than with no payoff, and with a .8 level of π ($P<.001$) than with a .6 level of π. The significant triple interaction, P x π x T, ($P<.005$) indicates that the effect of incentive (payoff) increases as the experimental session progresses for the .6 group, but shows an initial increase followed by a decrease for the .8

group.

From Figure 1 we see that the level of performance, $P(A_1)$, in each experimental group appears to overshoot π. A series of simple t tests performed for each block of 50 trials showed that all groups do significantly exceed a level of responding described as matching: s) The .6-0¢ group overshoots π only in the last block of 50 trials ($P<.05$); b) the .6-1¢ group overshoots π after 450 trials ($P<.01$); c) the .8-0¢ group overshoots π after 200 trials ($P<.01$); and d) the .8-1¢ group overshoots π after only 100 trials ($P<.01$).

Further inspection of Figure 1 suggests that the curves for the .6 groups have not leveled off toward an asymptote. When separate analyses of variance were performed over the last two blocks of 50 trials for each of these groups, this impression was borne-out only for the .6-1¢ group. Consequently, it may be questionable to use asymptotic equations (Appendix C) for parameter estimation and for the prediction of asymptotic statistics of the data for this group. However, the use of these equations for this group might be justified by the fact that other authors (e.g., Myers & Atkinson, 1964; Suydam et al., 1964; and Myers, Suydam & Heuckeroth, 1965) have applied these equations successfully in their parameter estimation and in the prediction of several statistics for data collected from Ss trained for less trials than used in the present experiment, i.e., for data which is also pre-asymptotic.

Table 2. Analysis of Variance of the Number of A_1 Responses In Each Block of 50 Trials

Source of Variance	df	SS	MS	F
Between Ss	159	131,680.90		
Payoff (P)	1	3,572.60	3,572.60	12.38***
Reinforcement Schedule(π)	1	83,437.50	83,437.50	289.11***
Sex	1	297.00	297.00	1.03
P x π	1	64.80	64.80	<1
P x Sex	1	160.30	160.30	<1
π x Sex	1	219.20	219.20	<1
P x π x Sex	1	62.80	62.80	<1
Ss/P X π x Sex	152	43,866.70	288.60	
Within Ss	1760	61,541.30		
Trial Blocks (T)	11	25,199.80	2,290.89	119.97***
P x T	11	501.80	45.62	2.39*
π x T	11	2,899.20	263.56	13.80***
Sex x T	11	99.90	9.08	<1
P x π x T	11	530.80	48.25	2.53*
P x Sex x T	11	104.10	9.46	<1
π x Sex x T	11	110.40	10.04	<1
P x π x Sex x T	11	167.30	15.21	<1
Ss x T/P x π X Sex	1672	31,928.00	19.10	
Total	1919	193,222.20		

*P<.01

**P<.005

***P<.001

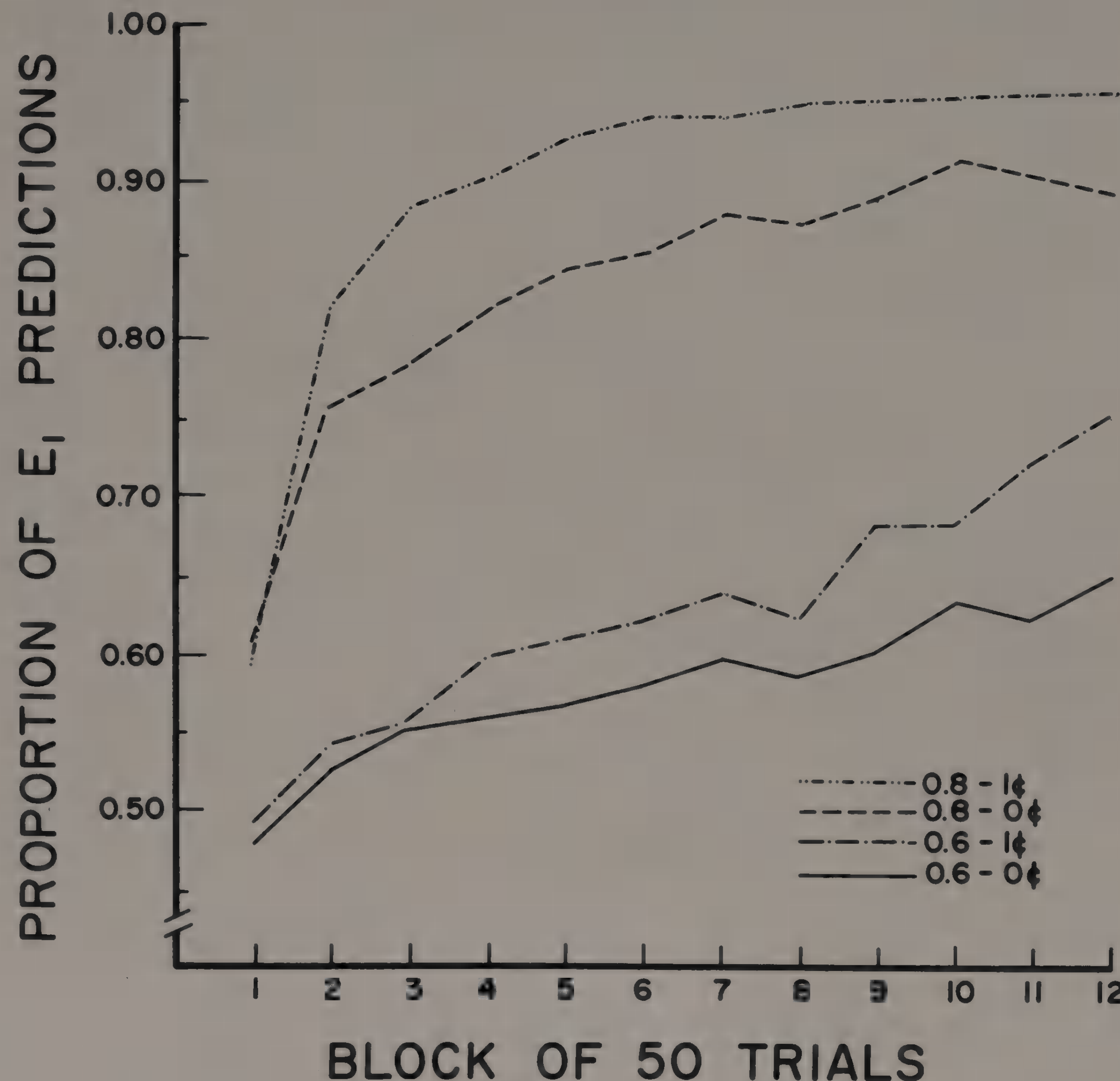


Fig. 1. Learning curves for each experimental group.

Run data

To evaluate the effects of previous events throughout the course of learning, the proportion of A_1 responses in each block of 100 trials was plotted following runs of 1 to 9 E_1's and 1 to 5 E_2's for the .6 groups (Figures 2 & 3); and following runs of 1 to 10 E_1's and 1 to 3 E_2's for the .8 groups (Figures 4 & 5).

As used here, a run of $\underline{j}$ E_i's, against which response probability on trial $\underline{n}$ is conditionalized, refers to a $\underline{j}$+1 tuple of the form $E_{i,n-1}E_{i,n-2}\cdots E_{i,n-j}E_{k,n-j-1}$, $i,k=1,2$, $i\neq k$, $j=1,\ldots$. The response probability conditionalized against such a $\underline{j}$+1 tuple is called a $\underline{j}+1^{st}$ order E_i reinforcement-run statistic.

In each of these figures (Figures 2-5), several changes in recency effects over trial blocks can be noted:

a) The progressive elevation and flattening of the E_1 run curves during the later blocks indicates that negative recency decreases over trials. By the third block of 100 trials these curves show little evidence of negative recency for the .8 groups, however, this effect is still present in the last block of 100 trials for the .6 groups.

b) Except for the .6-0¢ group, the probability of an A_1 response decreases when conditionalized against longer runs of E_2 events, i.e., positive recency is exhibited. Inspection of the E_2 run curves suggests that positive recency occurs earlier in learning and is more salient for the .8

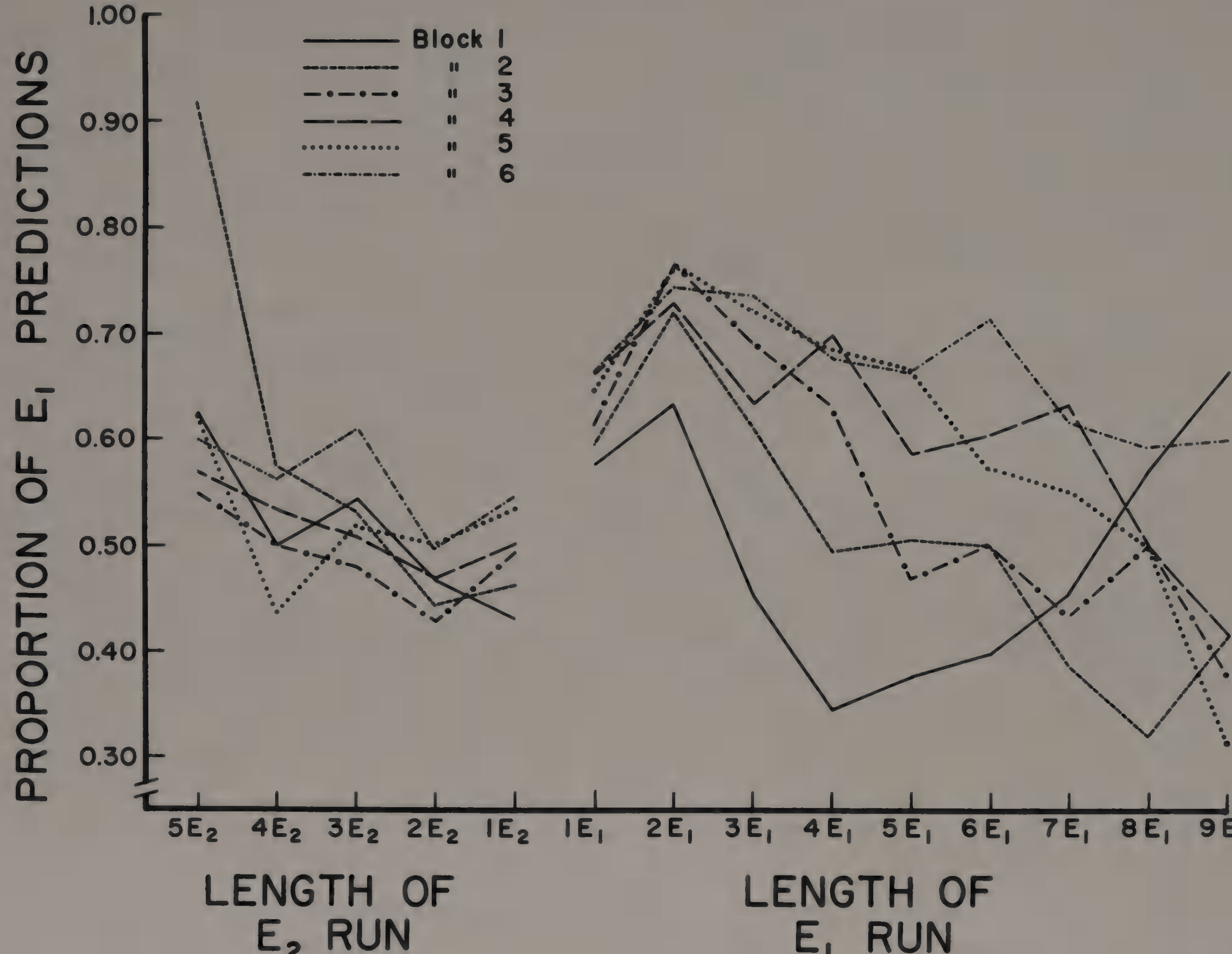


Fig 2. Run curves for successive blocks of 100 trials for the .6-0¢ group.

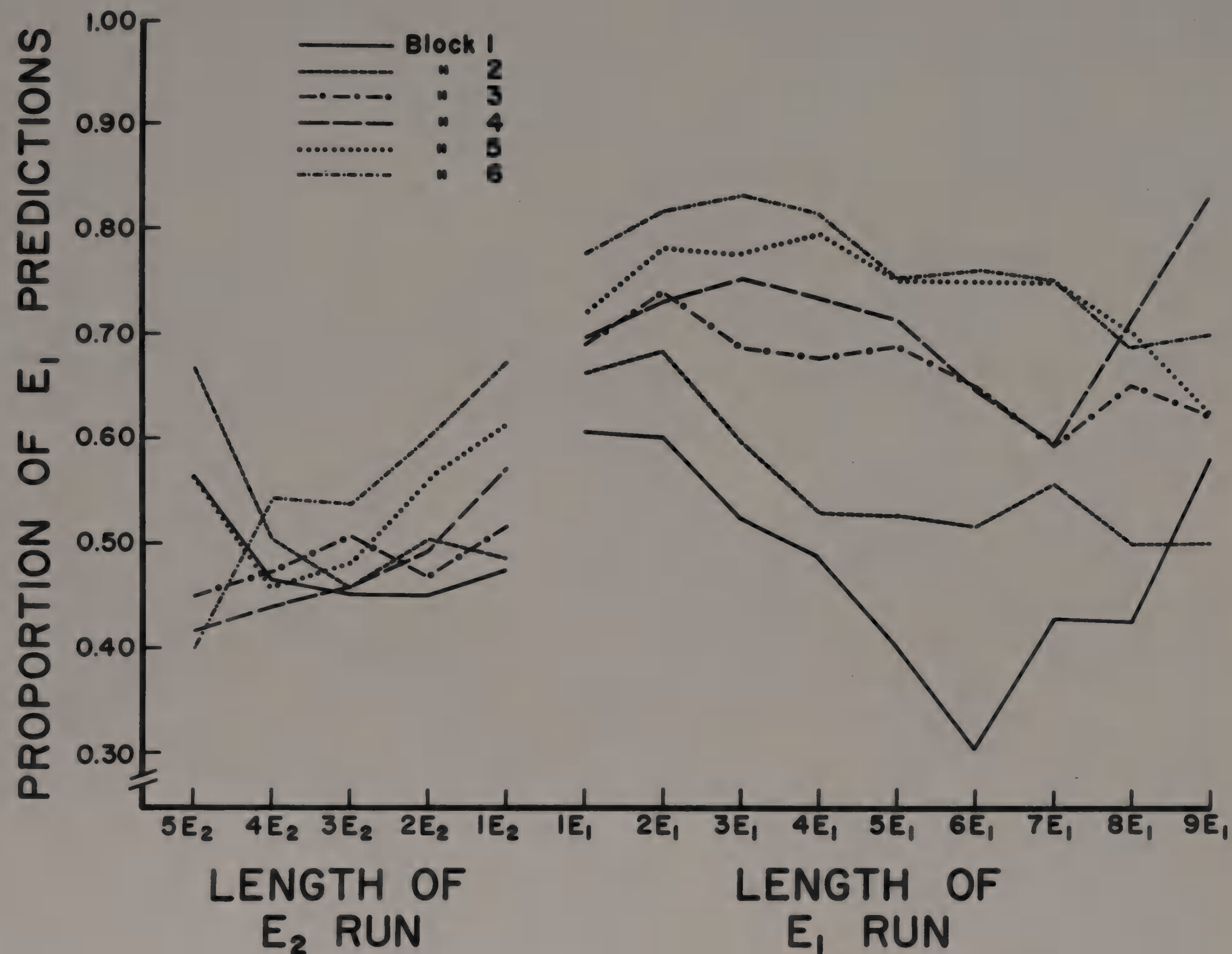


Fig. 3. Run curves for successive blocks of 100 trials for the .6-1¢ group.

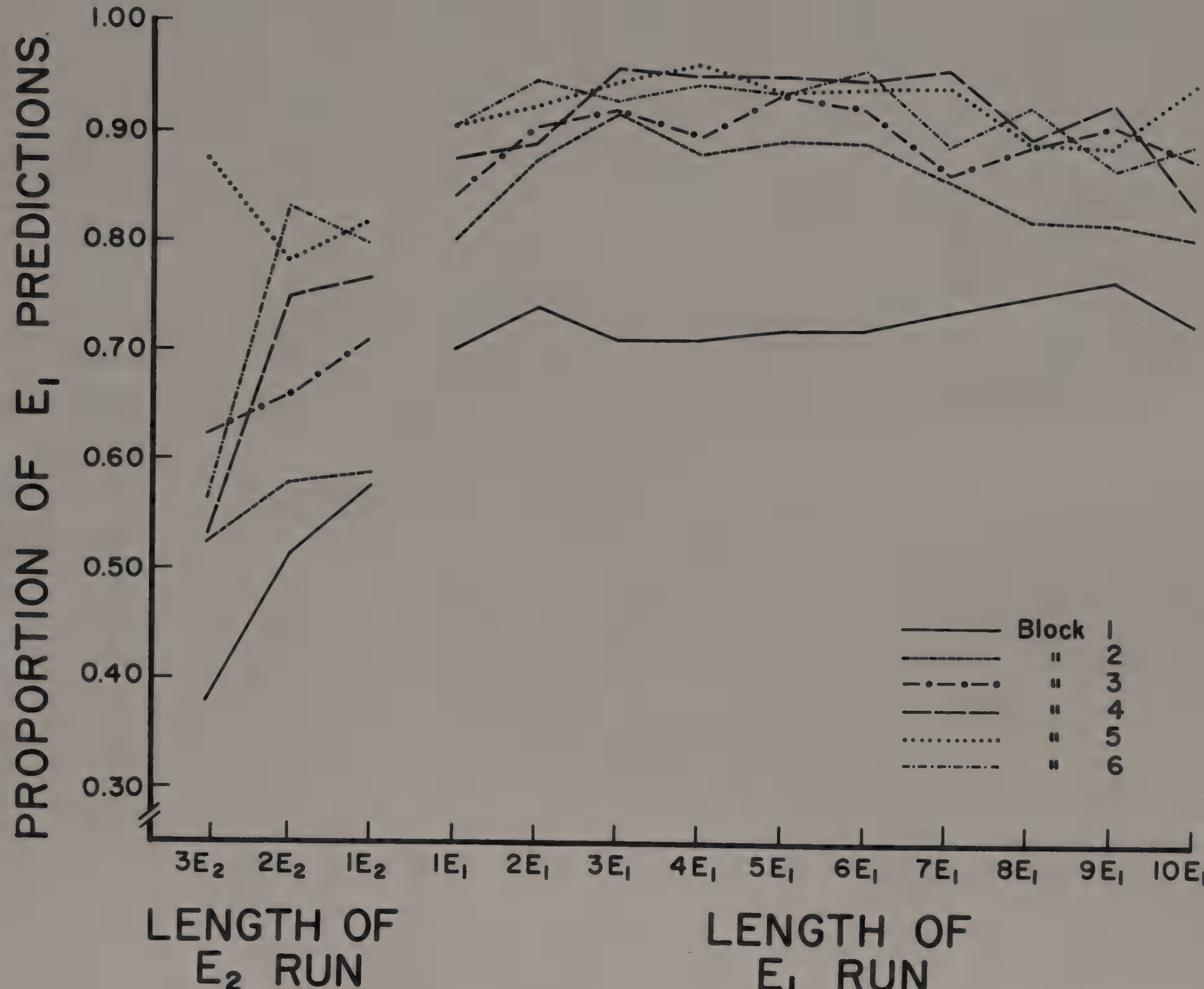


Fig. 4. Run curves for successive blocks of 100 trials for the .8-0¢ group.

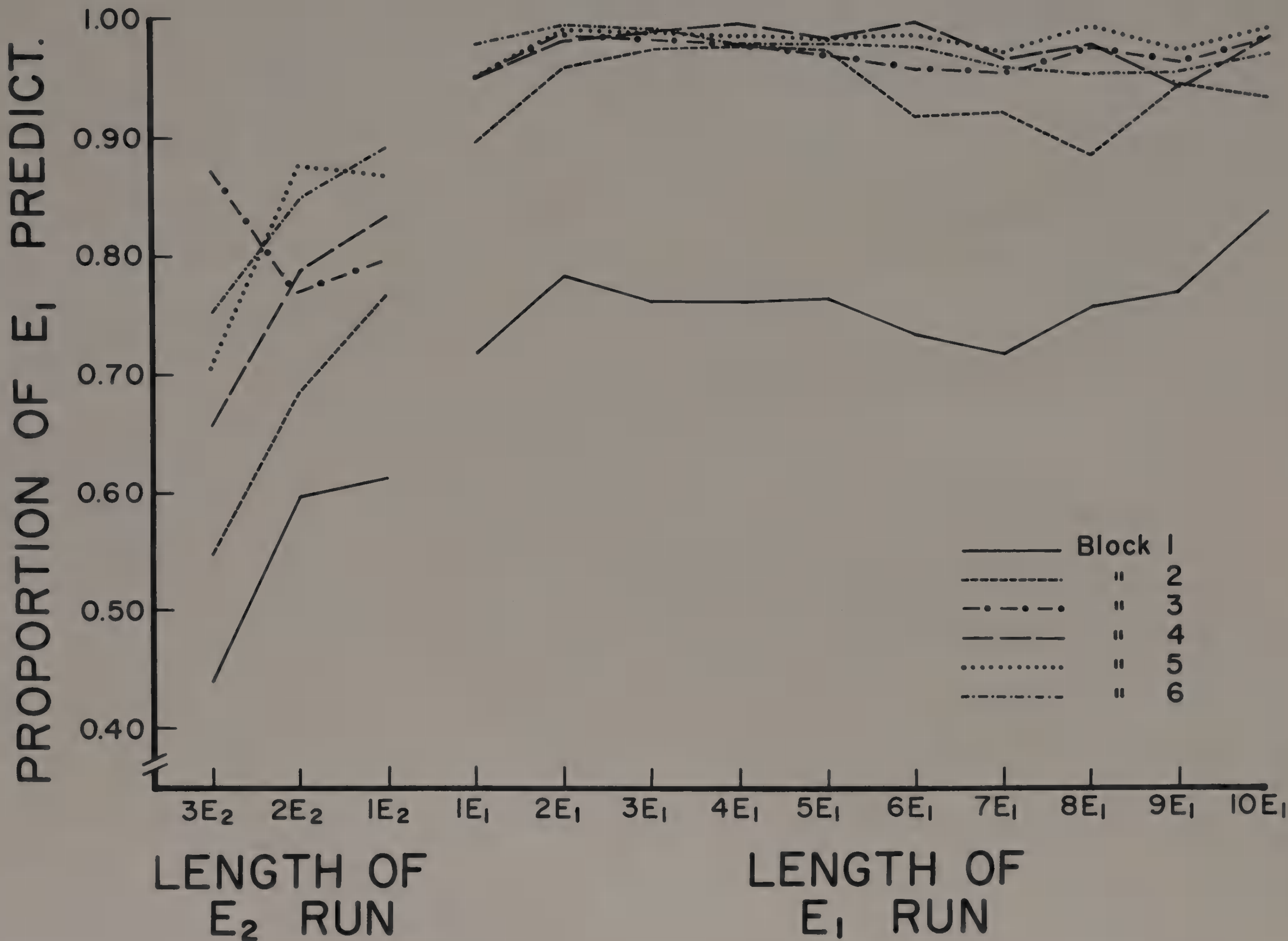


Fig. 5. Run curves for successive blocks of 100 trials for the .8-1¢ group.

groups than for the .6 groups. Further, this effect appears to increase for the .6-1¢ and .8-0¢ groups, but decrease for the .8-1¢ group as later trial blocks are approached.

Recency effects also appear to be a function of payoff, but to a lesser extent than the effects of π or trials. Specifically:

a) The E_1 run curves show less negative recency with payoff than with no payoff;

b) The E_2 run curves, especially for the .6 group, show more positive recency with payoff than without payoff. This effect is most noticeable within the first 200 trials for the .8 groups.

Application of models to data

In this section we will consider the ability of two linear models (Bush & Mosteller, 1955) and a finite state Markov model (Myers & Atkinson, 1964) to describe a wide array of statistics from the data obtained in this experiment. The size of the trial block used to compute each observed and predicted statistic was selected with two considerations in mind:

a) The observed statistic should be reliably estimated; and b) enough data points should be provided so that the ability of the models to describe the entire course of learning could be evaluated. In view of these considerations, the observed and predicted first-order conditional probabilities of the form $P(A_{1,n}|E_{j,n-1}A_{k,n-1})$, j,k=1,2 and marginal

response probabilities, $P(A_{1,n})$, were obtained for each block of 50 trials (Tables 3a-d, 4a-d, and 7a-b, respectively). With the same considerations in mind, the second-order conditional probabilities [e.g., $P(A_{1,n}|E_{2,n-1}A_{1,n-1}E_{1,n-2}A_{1,n-2} \cap E_{2,n-1}A_{2,n-1}E_{2,n-2}A_{2,n-2})$], reinforcement-run statistics (R-R-S), $P(A_{1,n}|E_{i,n-1}E_{i,n-2}\cdots E_{i,n-j}E_{k,n-j-1})$, $i,k=1,2$, $i \neq k$, $j=1,\ldots$, and standard deviation of response proportions were obtained for the last block of 100 trials (Tables 5a-d, 6a-b, and 8, respectively).

Generally, there are two types of entries in Tables 3a-d, 4a-d, 5a-d, and 6a-b: a) Observed and predicted conditional probabilities; and b) the frequency of the events against which response probability is conditionalized, i.e., $N(\cdot|\ldots)$. Where not indicated otherwise, all entries in these tables should be regarded as probabilities. The observed probabilities in Table 7a-b are based on 2000 observations each. Tables 3d and 4d also contain the parameter estimates obtained from the data in the last block of 50 trials. The parameter estimates obtained from the data in the last block of 100 trials appear in Table 5a-d.

While the choice of the statistic used in parameter estimation was somewhat arbitrary, it was considered desirable that: a) The same statistic should be used in estimation for each model; b) there should be more independent equations than parameters; and c) after a large number of iterations of the algorithm (see Appendix E), the parameters estimated

should be relatively invariant with additional iterations. With these considerations in mind, the parameter estimates used in the present experiment were obtained by pooling specific observed second-order joint probabilities and minimizing a least squares function (see Appendix D). Those statistics for which predictions were made in successive blocks of 50 trials were obtained by minimizing this function using the second-order joint counts from the last block of 50 trials; those statistics tested during the last 100 trials were obtained by minimizing this same function, but using the second-order joint counts from the last 100 trials. Such a procedure was expected to yield a reasonable test of each model's ability to describe the statistics in the last block of trials at the very least. As the use of asymptotic equations (Appendix C) in estimation, strictly speaking, is appropriate only with asymptotic data, it was considered more desirable to evaluate the ability of each model to describe the pre-asymptotic statistics with near asymptotic estimates, rather than possibly introduce some bias in the parameter estimates and therefore the predictions by using these equations for estimation in the pre-asymptotic blocks.

All predicted statistics for the W-S model and the second-order sequential statistics for the linear models were obtained with the equations in Appendix C. All remaining predicted statistics for the linear models were obtained by a Monte-Carlo technique. This technique in-

volved inserting estimated values of α_1 and α_2 in the unaveraged linear operators (Eqs. 2&4) and using a set of random numbers with a uniform probability distribution to generate the theoretical protocols (stat-rats). These protocols were then analyzed in the same manner as the experimental data. As the predicted statistics were relatively invariant when 120 or 200 stat-rats were computed, 200 stat-rats were generated for each experimental condition.

The statistic used to evaluate the goodness-of-fit of these models is the absolute average deviation (AAD) of observed and predicted statistics weighted by the frequency of the events against which the A_1 response is conditionalized (see Appendix F). This statistic has been computed for each model for each experimental group for the first-order conditional probabilities (Tables 3e & 4e) during each block of 50 trials, the second-order conditional probabilities and the R-R-S's during the last 100 trials (Tables 5a-d and 6a-c, respectively), and the entire learning curve (Table 7a-b). The results of these tests of the models are described below:

<u>First-order conditional probabilities</u>. Response probabilities conditionalized against the response-event combinations of the immediately preceding trial are presented in Tables 3a-d and 4a-d. The W-S and R-E models are able to describe these statistics consistently and clearly better

Table 3a. Observed and Predicted First-Order Conditional Probability of the Form $P(A_1|E_1A_1)$ and Marginal Count $N(\cdot|E_1A_1)$ for Each .6 Experimental Group

BLOCK	.6-0¢ $N(\cdot\|E_1A_1)$	.6-0¢ OBSERVED	.6-0¢ PREDICTED W-S	.6-0¢ PREDICTED R-E	.6-0¢ PREDICTED SR	.6-1¢ $N(\cdot\|E_1A_1)$	.6-1¢ OBSERVED	.6-1¢ PREDICTED W-S	.6-1¢ PREDICTED R-E	.6-1¢ PREDICTED SR
1	553	.517	.774	.699	.951	574	.580	.547	.788	.963
2	595	.645	.799	.699	.999	585	.699	.573	.736	.999
3	656	.665	.800	.707	1.000	648	.702	.591	.790	1.000
4	648	.667	.800	.703	1.000	693	.785	.608	.779	1.000
5	647	.705	.800	.686	1.000	684	.797	.623	.762	.999
6	734	.726	.800	.739	1.000	758	.813	.636	.787	1.000
7	681	.717	.800	.707	1.000	732	.806	.649	.767	1.000
8	672	,766	.800	.691	1.000	697	.834	.661	.754	1.000
9	738	.741	.800	.736	1.000	820	.859	.671	.800	1.000
10	746	.791	.800	.713	1.000	804	.848	.681	.776	1.000
11	737	.752	.800	.738	1.000	856	.864	.690	.791	1.000
12	844	.790	.800	.751	1.000	959	.864	.699	.811	1.000

Table 3b. Observed and Predicted First-Order Conditional Probability of the Form $P(A_1|E_2A_1)$ and Marginal Count $N(\cdot|E_2A_1)$ for Each .6 Experimental Group

BLOCK	.6-0¢					.6-1¢				
	$N(\cdot\|E_2A_1)$	OBSERVED	PREDICTED W-S	PREDICTED R-E	PREDICTED SR	$N(\cdot\|E_2A_1)$	OBSERVED	PREDICTED W-S	PREDICTED R-E	PREDICTED SR
1	387	.525	.607	.505	.945	379	.525	.545	.587	.943
2	442	.498	.645	.470	.999	479	.530	.573	.551	.999
3	431	.483	.646	.520	1.000	438	.546	.591	.589	1.000
4	453	.543	.646	.515	1.000	481	.597	.607	.592	1.000
5	466	.543	.646	.487	1.000	511	.611	.622	.559	1.000
6	402	.537	.646	.519	1.000	464	.597	.636	.603	1.000
7	490	.578	.646	.495	1.000	522	.669	.648	.575	1.000
8	481	.576	.646	.488	1.000	531	.669	.660	.545	1.000
9	442	.609	.646	.533	1.000	525	.678	.671	.617	1.000
10	502	.612	.646	.539	.999	533	.700	.681	.602	1.000
11	489	.661	.646	.537	1.000	566	.693	.690	.621	1.000
12	435	.611	.646	.540	.999	519	.723	.698	.637	1.000

Table 3c. Observed and Predicted First-Order Conditional Probability of the Form $P(A_1|E_1A_2)$ and Marginal Count $N(\cdot|E_1A_2)$ for Each .6 Experimental Group

BLOCK	.6-0¢					.6-1¢				
	OBSERVED		PREDICTED			OBSERVED		PREDICTED		
	$N(\cdot\|E_1A_2)$		W-S	R-E	SR	$N(\cdot\|E_1A_2)$		W-S	R-E	SR
1	631	.420	.558	.626	.118	610	.416	.493	.606	.117
2	505	.550	.559	.607	.004	515	.513	.490	.564	.002
3	528	.542	.559	.635	.000	536	.453	.490	.601	.001
4	520	.525	.559	.622	.001	475	.507	.490	.603	.000
5	485	.559	.559	.639	.000	448	.538	.491	.591	.001
6	498	.520	.559	.652	.000	474	.502	.491	.598	.000
7	431	.575	.559	.602	.000	380	.568	.492	.589	.001
8	452	.529	.559	.602	.000	427	.482	.492	.553	.000
9	486	.529	.559	.654	.000	404	.520	.493	.615	.001
10	422	.581	.559	.643	.001	364	.552	.493	.603	.001
11	459	.556	.559	.634	.001	340	.597	.494	.623	.000
12	452	.553	.559	.670	.001	337	.656	.495	.655	.000

Table 3d. Observed and Predicted First-Order Conditional Probability of the Form $P(A_1|E_2A_2)$ and Marginal Count $N(\cdot|E_2A_2)$ for Each .6 Experimental Group*

BLOCK	.6-0¢					.6-1¢				
	OBSERVED		PREDICTED			OBSERVED		PREDICTED		
	$N(\cdot\|E_2A_2)$		W-S	R-E	SR	$N(\cdot\|E_2A_2)$		W-S	R-E	SR
1	389	.468	.365	.415	.083	397	.441	.489	.374	.049
2	418	.371	.369	.413	.000	381	.357	.489	.366	.003
3	345	.423	.370	.432	.000	338	.420	.489	.372	.000
4	339	.416	.370	.445	.000	311	.334	.489	.386	.000
5	362	.381	.370	.416	.000	317	.334	.489	.371	.000
6	326	.402	.370	.441	.000	264	.318	.490	.402	.000
7	358	.444	.370	.415	.000	326	.319	.490	.370	.000
8	355	.346	.370	.392	.000	305	.289	.491	.332	.000
9	294	.374	.370	.441	.000	211	.360	.491	.410	.000
10	290	.369	.370	.455	.000	259	.320	.492	.415	.001
11	275	.345	.370	.414	.000	198	.404	.493	.402	.000
12	229	.397	.370	.461	.000	145	.400	.494	.425	.000
$\delta(\alpha_1)$			.539	.795	.582			.058	.712	.546
$\mu(\alpha_2)$			.181	.794	.836			1.000	.853	.936
$N(V_1)$			2.325	.694	.637			3.474	.812	.741

*Parameter estimates were obtained by minimizing a least-squares function (see Appendix D) which involved pooled second-order joint-probability equations (Appendix C) and the corresponding pooled counts in the last 50 trials.

Table 3e. The Absolute Average Deviation (AAD) of Observed and Predicted First-Order Conditional Probabilities $[P(A_1|E_jA_k),\ j,k=1,2]$ for Each .6 Experimental Group

	.6-0¢			.6-1¢		
BLOCK	W-S	R-E	SR	W-S	R-E	SR
1	.1536	.1320	.3790	.0472	.1454	.3655
2	.0824	.0461	.4403	.0797	.0313	.4070
3	.0953	.0491	.4464	.0688	.0874	.3962
4	.0848	.0493	.4268	.0938	.0346	.3510
5	.0581	.0458	.4148	.0994	.0436	.3489
6	.0651	.0486	.3964	.1034	.0458	.3321
7	.0632	.0351	.4117	.1074	.0520	.3238
8	.0400	.0725	.3689	.0973	.0839	.2990
9	.0387	.0601	.3730	.1000	.0658	.2911
10	.0171	.0744	.3582	.1074	.0782	.2885
11	.0260	.0640	.3564	.1038	.0575	.2924
12	.0164	.0672	.3501	.1220	.0508	.2821
All blocks	.0617	.0620	.3935	.0942	.0647	.3315

Table 4a. Observed and Predicted First-Order Conditional Probability of the Form $P(A_1|E_1A_1)$ and Marginal Count $N(\cdot|E_1A_1)$ for Each .8 Experimental Group

BLOCK	.8-0¢ OBSERVED $N(\cdot\|E_1A_1)$		.8-0¢ PREDICTED W-S	R-E	SR	.8-1¢ OBSERVED $N(\cdot\|E_1A_1)$		.8-1¢ PREDICTED W-S	R-E	SR
1	952	.715	.896	.938	.953	930	.705	.934	.976	.979
2	1175	.868	.933	.953	1.000	1278	.891	.972	.988	1.000
3	1207	.882	.933	.950	1.000	1375	.944	.972	.986	1.000
4	1264	.911	.933	.949	1.000	1391	.964	.972	.987	1.000
5	1310	.927	.933	.958	1.000	1434	.980	.972	.991	1.000
6	1378	.915	.933	.960	1.000	1519	.975	.972	.989	1.000
7	1347	.930	.933	.956	1.000	1456	.984	.972	.987	1.000
8	1431	.942	.933	.962	1.000	1546	.983	.972	.990	1.000
9	1438	.943	.933	.964	1.000	1544	.984	.972	.991	1.000
10	1406	.941	.933	.953	1.000	1461	.982	.972	.988	1.000
11	1418	.941	.933	.951	1.000	1500	.983	.972	.989	1.000
12	1406	.947	.933	.961	.999	1498	.982	.972	.989	1.000

Table 4b. Observed and Predicted First-Order Conditional Probability of the Form $P(A_1|E_2A_1)$ and Marginal Count $N(\cdot|E_2A_1)$ for Each .8 Experimental Group

BLOCK	.8-0¢					.8-1¢				
	$N(\cdot\|E_2A_1)$	OBSERVED	PREDICTED W-S	PREDICTED R-E	PREDICTED SR	$N(\cdot\|E_2A_1)$	OBSERVED	PREDICTED W-S	PREDICTED R-E	PREDICTED SR
1	236	.606	.732	.786	.930	238	.576	.861	.909	.953
2	306	.644	.800	.826	1.000	330	.706	.919	.928	1.000
3	330	.609	.800	.817	1.000	358	.788	.919	.911	.999
4	343	.650	.800	.814	1.000	378	.783	.919	.919	1.000
5	360	.731	.800	.832	1.000	392	.798	.919	.919	1.000
6	303	.766	.800	.841	1.000	325	.843	.919	.939	1.000
7	381	.829	.800	.825	1.000	395	.851	.919	.927	1.000
8	291	.753	.800	.842	1.000	316	.839	.919	.934	1.000
9	309	.786	.800	.827	.999	324	.883	.919	.935	1.000
10	389	.866	.800	.824	1.000	408	.873	.919	.923	1.000
11	358	.835	.800	.842	1.000	377	.891	.919	.932	1.000
12	348	.802	.800	.839	.999	375	.896	.919	.926	1.000

Table 4c. Observed and Predicted First-Order Conditional Probability of the Form $P(A_1|E_1A_2)$ and Marginal Count $N(\cdot|E_1A_2)$ for Each .8 Experimental Group

BLOCK	.8-0¢					.8-1¢				
	$N(\cdot\|E_1A_2)$	OBSERVED	PREDICTED W-S	PREDICTED R-E	PREDICTED SR	$N(\cdot\|E_1A_2)$	OBSERVED	PREDICTED W-S	PREDICTED R-E	PREDICTED SR
1	608	.482	.826	.735	.506	630	.473	.756	.596	.296
2	393	.618	.847	.772	.000	290	.731	.778	.768	.000
3	341	.692	.848	.740	.000	173	.809	.778	.681	.005
4	280	.750	.848	.787	.013	153	.771	.778	.742	.002
5	230	.713	.848	.808	.000	106	.896	.778	.757	.002
6	238	.714	.848	.852	.000	97	.804	.778	.874	.002
7	193	.751	.848	.759	.013	84	.810	.778	.755	.002
8	193	.720	.848	.808	.024	78	.923	.778	.745	.000
9	178	.725	.848	.781	.035	72	.819	.778	.827	.000
10	134	.851	.848	.779	.012	79	.911	.778	.760	.000
11	150	.787	.848	.825	.013	68	.882	.778	.791	.000
12	158	.759	.848	.834	.062	66	.818	.778	.690	.000

Table 4d. Observed and Predicted First-Order Conditional Probability of the Form $P(A_1|E_2A_2)$ and Marginal Count $N(\cdot|E_2A_2)$ for Each .8 Experimental Group*

BLOCK	.8-0¢ OBSERVED $N(\cdot\|E_2A_2)$		.8-0¢ PREDICTED W-S	R-E	SR	.8-1¢ OBSERVED $N(\cdot\|E_2A_2)$		.8-1¢ PREDICTED W-S	R-E	SR
1	164	.451	.544	.365	.363	162	.500	.508	.272	.146
2	86	.349	.565	.479	.000	62	.452	.523	.371	.000
3	82	.402	.565	.434	.000	54	.278	.523	.248	.009
4	73	.329	.565	.403	.000	38	.368	.523	.292	.000
5	60	.450	.565	.484	.000	28	.357	.523	.328	.000
6	41	.341	.565	.514	.000	19	.474	.523	.444	.000
7	39	.385	.565	.359	.000	25	.360	.523	.395	.000
8	45	.400	.565	.422	.000	20	.450	.523	.333	.000
9	35	.629	.565	.516	.000	20	.400	.523	.250	.000
10	31	.677	.565	.479	.000	12	.583	.523	.258	.000
11	34	.647	.565	.532	.000	15	.400	.523	.217	.000
12	48	.521	.565	.513	.040	21	.524	.523	.333	.000
$\delta(\alpha_1)$			.829	.586	.688			.754	.435	.548
$\mu(\alpha_2)$			.268	.886	1.000			1.000	.951	1.000
$N(V_1)$			2.712	.886	.882			2.225	.943	.943

*Parameter estimates were obtained by minimizing a least-squares function (see Appendix D) which involved pooled second-order joint-probability equations (Appendix C) and the corresponding pooled counts in the last 50 trials.

Table 4e. The Absolute Average Deviation (AAD) of Observed and Predicted First-Order Conditional Probabilities [$P(A_1|E_jA_k)$, j,k=1,2] for Each .8 Experimental Group

BLOCK	.8-0¢ W-S	.8-0¢ R-E	.8-0¢ SR	.8-1¢ W-S	.8-1¢ R-E	.8-1¢ SR
1	.2174	.2156	.1693	.2348	.2273	.2618
2	.1186	.1159	.2740	.0977	.1085	.2429
3	.0973	.0863	.2755	.0531	.0641	.1563
4	.0630	.0610	.2359	.0354	.0463	.1345
5	.0363	.0517	.1959	.0386	.0403	.1086
6	.0388	.0636	.1898	.0167	.0305	.0897
7	.0207	.0198	.1615	.0263	.0202	.0810
8	.0300	.0370	.1568	.0278	.0296	.0811
9	.0218	.0290	.1492	.0181	.0161	.0663
10	.0208	.0251	.1369	.0225	.0234	.0806
11	.0187	.0131	.1429	.0181	.0173	.0679
12	.0188	.0226	.1401	.0134	.0175	.0668
All blocks	.0585	.0617	.1856	.0502	.0534	.1198

Table 5a. Observed and Predicted Second-Order Sequential Statistics Conditionalized Against a Pooled Probability Space for the .6-0¢ Group*

TRIAL N-2 N-1 A E A E	OBSERVED $N[\cdot \mid {EAEA \atop EAEA}]$	OBSERVED $P[A_1 \mid {EAEA \atop EAEA}]$	PREDICTED W-S $P[A_1 \mid {EAEA \atop EAEA}]$	PREDICTED R-E $P[A_1 \mid {EAEA \atop EAEA}]$	PREDICTED SR $P[A_1 \mid {EAEA \atop EAEA}]$
1 1 1 1 / 2 2 1 2	827	.810	.779	.770	.800
1 1 1 2 / 2 2 2 2	580	.566	.635	.635	.661
1 2 1 1 / 2 1 1 1	704	.754	.754	.714	.757
1 2 1 2 / 2 1 1 2	391	.668	.618	.535	.669
1 2 2 1 / 2 1 2 2	376	.521	.476	.539	.498
1 2 2 2 / 2 2 2 1	313	.495	.429	.244	.387
2 2 1 1 / 1 1 2 1	371	.544	.625	.579	.654
1 1 2 2 / 2 1 2 1	358	.453	.516	.557	.486
$\delta(\alpha_1)$			.472	.832	.596
$\mu(\alpha_2)$			.132	.815	.813
$N(V_1)$			2.622	.676	.630
V_2				.536	.368
V_3				.471	.190
AAD			.0405	.0737	.0411

*See Table 6b Footnote.

Table 5b. Observed and Predicted Second-Order Sequential Statistics Conditionalized Against a Pooled Probability Space for the .6-1¢ Group*

TRIAL N-2 N-1 A E A E	OBSERVED $N[\cdot \mid {EAEA \atop EAEA}]$	OBSERVED $P[A_1 \mid {EAEA \atop EAEA}]$	PREDICTED W-S $P[A_1 \mid {EAEA \atop EAEA}]$	PREDICTED R-E $P[A_1 \mid {EAEA \atop EAEA}]$	PREDICTED SR $P[A_1 \mid {EAEA \atop EAEA}]$
1 1 1 1 2 2 1 2	1027	.868	.866	.851	.876
1 1 1 2 2 2 2 2	661	.667	.766	.714	.748
1 2 1 1 2 1 1 1	754	.855	.818	.809	.844
1 2 1 2 2 1 1 2	433	.704	.663	.619	.726
1 2 2 1 2 1 2 2	294	.639	.495	.617	.610
1 2 2 2 2 2 2 1	244	.508	.368	.206	.391
2 2 1 1 1 1 2 1	250	.636	.610	.719	.765
1 1 2 2 2 1 2 1	257	.553	.527	.590	.564
$\delta(\alpha_1)$			.672	.702	.552
$\mu(\alpha_2)$			.999	.849	.974
$N(V_1)$			2.188	.784	.718
V_2				.658	.516
V_3				.577	.369
AAD			.0517	.0588	.0387

*See Table 6b Footnote.

Table 5c. Observed and Predicted Second-Order Sequential Statistics Conditionalized Against a Pooled Probability Space for the .8-0¢ Group*

TRIAL N-2 N-1 A E A E	OBSERVED $N[\cdot \mid {EAEA \atop EAEA}]$	 $P[A_1 \mid {EAEA \atop EAEA}]$	PREDICTED W-S $P[A_1 \mid {EAEA \atop EAEA}]$	 R-E $P[A_1 \mid {EAEA \atop EAEA}]$	 SR $P[A_1 \mid {EAEA \atop EAEA}]$
1 1 1 1 2 2 1 2	2107	.949	.951	.956	.954
1 1 1 2 2 2 2 2	562	.813	.852	.842	.929
1 2 1 1 2 1 1 1	675	.933	.883	.902	.914
1 2 1 2 2 1 1 2	145	.834	.740	.762	.879
1 2 2 1 2 1 2 2	124	.774	.814	.785	.704
1 2 2 2 2 2 2 1	55	.636	.660	.286	.517
2 2 1 1 1 1 2 1	169	.817	.835	.888	.818
1 1 2 2 2 1 2 1	83	.651	.736	.638	.624
$\delta(\alpha_1)$			.818	.608	.708
$\mu(\alpha_2)$			.294	.908	1.000
$N(V_1)$			3.097	.889	.887
V_2				.797	.802
V_3				.716	.736
AAD			.0229	.0245	.0288

*See Table 6b Footnote.

Table 5d. Observed and Predicted Second-Order Sequential Statistics Conditionalized Against a Pooled Probability Space for the .8-1¢ Group*

TRIAL N-2 N-1 A E A E	OBSERVED $N[\cdot \mid {EAEA \atop EAEA}]$	 $P[A_1 \mid {EAEA \atop EAEA}]$	PREDICTED W-S $P[A_1 \mid {EAEA \atop EAEA}]$	 R-E $P[A_1 \mid {EAEA \atop EAEA}]$	 SR $P[A_1 \mid {EAEA \atop EAEA}]$
1 1 1 1 2 2 1 2	2320	.984	.983	.989	.988
1 1 1 2 2 2 2 2	629	.893	.952	.922	.970
1 2 1 1 2 1 1 1	663	.979	.939	.963	.968
1 2 1 2 2 1 1 2	126	.905	.793	.861	.935
1 2 2 1 2 1 2 2	67	.880	.787	.876	.744
1 2 2 2 2 2 2 1	32	.562	.513	.227	.393
2 2 1 1 1 1 2 1	57	.877	.752	.929	.878
1 1 2 2 2 1 2 1	26	.615	.700	.610	.624
$\delta(\alpha_1)$			.764	.394	.486
$\mu(\alpha_2)$			1.000	.951	1.000
$N(V_1)$			2.226	.943	.942
V_2				.889	.895
V_3				.839	.855
AAD			.0248	.0153	.0213

*See Table 6b Footnote.

Table 6a. Observed and Predicted Second, Third, and Fourth-Order E_1 Reinforcement-Run Statistics for All Experimental Groups*

GROUP	OBSERVED		PREDICTED			OBSERVED		PREDICTED			OBSERVED		PREDICTED		
	N(†)		W-S	R-E	SR	N(††)		W-S	R-E	SR	N(†††)		W-S	R-E	SR
.6-0¢	924	.662	.627	.594	.574	608	.748	.684	.671	.587	364	.736	.732	.732	.570
AAD			.035	.068	.088			.064	.077	.161			.004	.004	.166
.6-1¢	924	.777	.686	.811	.946	608	.827	,738	.857	.949	364	.832	.787	.892	.951
AAD			.091	.034	.169			.089	.030	.122			.045	.060	.119
.8-0¢	656	.905	.850	.734	.850	584	.945	.886	.799	.867	448	.929	.914	.846	.851
AAD			.055	.171	.055			.059	.146	.078			.015	.083	.078
.8-1¢	656	.977	.922	.959	.926	584	.981	.942	.971	.929	448	.989	.958	.982	.933
AAD			.055	.018	.051			.039	.010	.052			.031	.007	.056

† $N(\cdot|E_1E_2)$
†† $N(\cdot|E_1E_1E_2)$
††† $N(\cdot|E_1E_1E_1E_2)$
*See Table 6b Footnote.

Table 6b. Observed and Predicted Second, Third, and Fourth-Order E_2 Reinforcement-Run Statistics for All Experimental Groups*

GROUP	OBSERVED		PREDICTED			OBSERVED		PREDICTED			OBSERVED		PREDICTED		
	N(†)		W-S	R-E	SR	N(††)		W-S	R-E	SR	N(†††)		W-S	R-E	SR
.6-0¢	920	.549	.609	.583	.572	300	.497	.531	.467	.529	108	.611	.461	.400	.509
AAD			.060	.034	.023			.034	.030	.032			.150	.211	.102
.6-1¢	920	.673	.703	.772	.946	300	.600	.613	.650	.932	108	.537	.511	.475	.900
AAD			.030	.099	.273			.013	.050	.332			.026	.062	.363
.8-0¢	652	.798	.829	.720	.846	112	.830	.720	.583	.823	16	.563	.610	.472	.813
AAD			.031	.078	.048			.110	.247	.007			.047	.091	.250
.8-1¢	652	.890	.929	.932	.926	112	.848	.805	.845	.907	16	.750	.657	.725	.900
AAD			.039	.042	.036			.043	.003	.059			.093	.025	.150

† $N(\cdot|E_2E_1)$
†† $N(\cdot|E_2E_2E_1)$
††† $N(\cdot|E_2E_2E_2E_1)$

*Parameter estimates were obtained by minimizing a least-squares function (see Appendix D) which involved pooled second-order joint-probability equations (Appendix C) and the corresponding pooled counts in the last 100 trials.

Table 6c. The Absolute Average Deviation of Observed and Predicted Second, Third, and Fourth-Order E_1, E_2, and E_1+E_2 Reinforcement-Run Statistics for Each Experimental Group

GROUP	AVERAGE DEVIATION OF E_1 RUN STATISTICS			AVERAGE DEVIATION OF E_2 RUN STATISTICS			AVERAGE DEVIATION OF E_1+E_2 RUN STATISTICS		
	W-S	R-E	SR	W-S	R-E	SR	W-S	R-E	SR
.6-0¢	.0383	.0586	.1264	.0614	.0475	.0315	.0479	.0540	.0873
.6-1¢	.0815	.0377	.1443	.0258	.0849	.2936	.0586	.0572	.2058
.8-0¢	.0458	.1390	.0691	.0427	.1025	.0463	.0448	.1275	.0619
.8-1¢	.0431	.0123	.0527	.0407	.0361	.0416	.0423	.0198	.0492

Table 7a. Observed and Predicted Marginal Response Probability [$P(A_1)$] For Each .6 Experimental Group*

BLOCK	.6-0¢ OBSERVED	.6-0¢ PREDICTED W-S	.6-0¢ PREDICTED R-E	.6-0¢ PREDICTED SR	.6-1¢ OBSERVED	.6-1¢ PREDICTED W-S	.6-1¢ PREDICTED R-E	.6-1¢ PREDICTED SR
1	.479	.621	.588	.656	.490	.520	.637	.647
2	.528	.648	.564	.693	.543	.534	.579	.678
3	.552	.648	.600	.695	.554	.545	.636	.680
4	.558	.648	.596	.695	.598	.555	.635	.680
5	.568	.648	.578	.695	.612	.565	.606	.680
6	.582	.648	.624	.695	.622	.574	.648	.676
7	.600	.648	.576	.695	.641	.583	.610	.675
8	.588	.648	.562	.695	.627	.591	.576	.675
9	.602	.648	.630	.695	.688	.599	.668	.675
10	.637	.648	.614	.695	.684	.607	.643	.674
11	.626	.648	.618	.695	.724	.614	.662	.670
12	.652	.648	.650	.695	.754	.621	.696	.670
AAD		.0654	.0328	.1106		.0574	.0498	.0721

*See Table 3d Footnote.

Table 7b. Observed and Predicted Marginal Response Probability [P(A_1)] For Each .8 Experimental Group*

BLOCK	.8-0¢ OBSERVED	.8-0¢ PREDICTED W-S	.8-0¢ PREDICTED R-E	.8-0¢ PREDICTED SR	.8-1¢ OBSERVED	.8-1¢ PREDICTED W-S	.8-1¢ PREDICTED R-E	.8-1¢ PREDICTED SR
1	.608	.847	.867	.885	.597	.897	.919	.882
2	.758	.894	.907	.990	.822	.950	.966	.945
3	.784	.894	.896	.990	.885	.950	.950	.945
4	.820	.894	.899	.990	.904	.950	.958	.945
5	.850	.894	.915	.990	.930	.950	.966	.945
6	.856	.894	.927	.990	.940	.950	.977	.945
7	.881	.894	.902	.990	.942	.950	.963	.945
8	.878	.894	.925	.990	.950	.950	.972	.945
9	.893	.894	.924	.990	.954	.950	.974	.945
10	.916	.894	.906	.990	.954	.950	.963	.945
11	.906	.894	.917	.990	.958	.950	.969	.945
12	.896	.894	.925	.989	.956	.950	.963	.945
AAD		.0589	.0737	.1440		.0499	.0623	.0482

*See Table 3d Footnote.

Table 8. Observed and Predicted Standard-Deviation of Response Proportions for All Experimental Groups*

GROUP	OBSERVED	PREDICTED		
		W-S	R-E	SR
.6-0¢	.161	.112	.060	.496
.6-1¢	.152	.130	.047	.218
.8-0¢	.082	.049	.100	.357
.8-1¢	.077	.044	.044	.255

*See Table 6b Footnote.

than the SR model for all experimental groups, especially the .8 groups. The differences in fit of the W-S and R-E models is noticeably different only for the .6-1¢ group, where the R-E model is better able to describe these statistics. With regard to the ability of the R-E and W-S models to describe these statistics over the course of learning, changes in the AAD statistic indicate that there is generally a gradual improvement in fit as the later trial blocks are approached. Two exceptions to this generalization can be noted: a) The fits of the W-S model for the .6-1¢ group show a gradual deterioration as asymptotic blocks are approached; and b) the fit of the R-E model in block 8 for the .6 groups is somewhat worse than it is for several preceding blocks.

Second-order conditional probabilities. Response probabilities conditionalized against the response-event combinations of the two immediately preceding trials are presented in Table 5a-d. Each model is able to describe these

statistics [e.g., $P(A_{1,n}|E_{1,n-1}A_{1,n-1}E_{1,n-2}A_{1,n-2} \cap E_{2,n-1} A_{1,n-1}E_{2,n-2}A_{2,n-2})$] quite well for each experimental group, especially the .8 groups. The most noticeable deviation is found with the fit of the R-E model to the .6-0¢ data.

Reinforcement-run statistics, R-R-S. Response probabilities conditionalized against the preceding run of events are presented in Table 6a-b. While there are specific variations, the average deviation for the E_1 R-R-Ss or the E_1 and E_2 R-R-Ss combined is smallest for the W-S model for the no-payoff groups. Similarly, the R-E model shows smallest deviations for the payoff group. The SR model is able to describe the E_2 R-R-Ss quite well under all experimental conditions, except for the .6-1¢ group. The W-S model describes these statistics best for this group. All models are able to describe the E_2 R-R-Ss with about the same precision for the .8-1¢ group. The most noticeable deviations in fit are for the SR model with the .6 groups (especially the .6-1¢ group) and the R-E model for the .8-0¢ group. There are two additional points of interest to note: a) The fit of the fourth-order E_1 R-R-S for the W-S model are generally better than the fit to these third-order statistics; and b) the predicted E_1 R-R-Ss are generally smaller than observed in the data. The only exception noted is for the predictions of the .6-1¢ group with the linear models.

Learning curve. Marginal response probabilities are presented in Table 7a-b. Generally, each of these models is

able to describe the learning curves with about the same accuracy. The most noticeable deviations are for the predictions of the SR model for the no-payoff groups. Except for the predictions of the .6-1¢ group, the learning curves predicted by each model show a rate of learning greater than observed in the early blocks; and further, the predictions of the W-S and SR models stabilize more rapidly than the observed probabilities. The predictions for the .6-1¢ groups by each model in the last block of 50 trials are markedly below the observed value; and further, the learning curves predicted by the W-S and R-E models continue to rise and show no evidence of stabilizing.

Standard-deviation of response proportions (Table 8). Except for the predictions of the R-E model for the .8-0¢ group, the W-S and R-E models predict a lower level of asymptotic variability than observed in the data. The predictions of the SR model are markedly greater than the observed value for each experimental group. The W-S model shows the best fit for each .6 group; the R-E model describes this statistic as well as the W-S model for the .8-1¢ group, and noticeably better for the .8-0¢ group.

Discussion

In this section we will first consider the experimental findings in terms of the previous review of literature. In particular, we will be concerned with the effects that π and payoff have on marginal response probability and the rate of learning. The run data will also be of interest, with particular emphasis on the effects of π, payoff, and trials have on the occurrence of negative recency. This will be followed by an evaluation of the models with regard to goodness-of-fit and psychological meaningfulness, and some comments regarding deviations in fit, extended training, and pre-experimental response tendencies. Finally, we will conclude this section with an attempt to evaluate whether overshooting in later trial blocks is a function of the introduction of variables not specially manipulated by the experimenter.

Experimental effects

Marginal probabilities. The results of the present experiment and those studies cited in Appendix A, Table 1, are in general agreement in showing that higher levels of π lead to higher levels of event prediction, and that the introduction of an incentive leads to overshooting. The present results also indicate that the rate of learning is a function of the level of π and payoff condition. These findings indicate that: a) The more frequently a response is reinforced, the higher the rate and level of performance; and

b) the introduction of an incentive leads to a higher rate of learning and level of performance than when no incentive is used.

The rates of learning for the two .6 groups in the present experiment were slower than those observed by most other investigators (Appendix A) under identical π and payoff levels. However, the rates of learning found for Edwards' (1961) .6-0¢ group and a .6-1¢ "unaware" group by Suydam et al. (1964) are similar to those found in the present experiment. These slower rates of learning observed for .6 groups in the present experiment may have been caused by placing too much emphasis in the instructions upon the mechanics of responding (Appendix B) rather than the task to be learned.

The rates of learning for the .8 groups in the present experiment are similar to those reported by Myers et al. (1963), slightly exceed those reported by Anderson and Whalen (1960), and are somewhat lower than those reported by Edwards (1956), Taub and Myers (1961), and Friedman et al. (1963). The differences in rates of learning noted among these studies are probably a reflection of procedural differences, e.g., the number of trials over which the event sequences were randomized, instructions, or the number of levels of π on which Ss were trained.

In Figure 1 the amount of overshooting found for the .6-1¢ and .8-0¢ groups was larger than found in studies previously reported in the literature (Appendix A). This large

amount of overshooting could be due, in part, to the fact that the sequence of events used in the present experiment was randomized over a larger block of trials than did most of the studies cited in Appendix A. Randomizing events over increasingly larger blocks of trials generally results in an increase in the mean and variability of run length. Gambino and Myers (1965) have reported that when a .5 level of π is used, increasing the variability of the run length leads to an increase in perseverative behavior, i.e., predicting the event which has just occurred. If this increase in perseverative behavior with increased variability of run length proves to hold with the systematic manipulation of the run structure at other levels of π, the extent of overshooting found in the present experiment would be consistent with such a finding. That such a relationship may hold is evidenced by the fact that Jones and Myers (1965) reported greater overshooting when events were randomized over long rather than short blocks of trials, i.e., where the variability of run length was greater.

Run data. The results of the present experiment are consistent with the findings of several authors (Nicks, 1959; Atkinson, Sommer, & Sternman, 1960; Anderson & Whalen, 1960; Edwards, 1961; Craig & Myers, 1963; Jones & Myers, 1965), in showing less negative recency, the greater the level of π. These findings are consistent with a suggestion made by Restle (1961) that Ss respond to runs of reinforcing

events of different lengths as differential cues. For example, experiencing $\underline{k}$ E_i events in a sequence with few long runs (such as are usually found in event sequences for a .6 level of π) would be more likely to serve as a cue to predict E_j, $i \neq j$, whereas a sequence with more long runs (such as are usually found in event sequences for a .8 level of π) would be more likely to serve as a cue to predict the same event.

Another finding frequently reported in the literature is the decreasing or unlearning of the negative recency effect as the training session progresses (e.g., Anderson, 1960; Anderson & Whalen, 1960; Edwards, 1961; Edwards & Tannenbaum, 1961; Lindman & Edwards, 1961; Derks, 1962; Derks, 1963; Jones & Myers, 1965). Except for the E_2 run statistics of the .6-0¢ group, this result is also obtained in the present experiment. The decrease in negative recency as training progresses could be considered a consequence of two processes: a) The extinction of pre-experimental response tendencies (Estes, 1962), e.g., looking for patterns in the event sequence; and b) the acquisition of response tendencies which are dependent upon the reinforcing properties of the sequence.

It was also found in the present experiment that negative recency appears to be less when payoff is used than under no-payoff conditions. Atkinson, Sommer, & Sternman (1960) and Derks (1962) have reported a similar effect. Since the addition of an incentive to the experimental situa-

tion does not increase the amount of information in the event sequence, the effect of introducing an incentive may be to increase the _Ss'_ attentiveness to the reinforcing properties of the event sequence, and thereby increase the saliency of past events in the _Ss'_ memories.

Evaluation of models — goodness-of-fit

One way of evaluating the ability of a model to describe data is to examine its goodness-of-fit, i.e., the extent of correspondence between observed and predicted statistics.

W-S model

First-order conditional and marginal response probabilities. In testing this model, we have seen that, except for the .6-1¢ group, when parameters are estimated with data from the last block of 50 trials, the fit to the first-order conditional probabilities and the learning curve improves as the asymptotic blocks are approached (Tables 3e, 4e, & 7a-b). In the early blocks, the predicted statistics level off toward an asymptote too quickly, a result consistent with that reported by Heuckeroth and Myers (1965) for the Suppes and Atkinson (1960) data. The replicability of this finding suggests that when the parameters of the W-S model are estimated with data at one level of learning, generally they will not be appropriate to describe performance at other points in the learning process, i.e., the model's assumption of parameter invariance over trial blocks requires revision.

The AAD statistic (Appendix F) for the first-order conditional probabilities (Tables 3e & 4e) and marginal response probability (Table 7a-b) for all but the .6-1¢ experimental group shows deviations of less than 2% in the last block of 50 trials. The size of these deviations compares favorably with the fits to these statistics from several other human choice studies (e.g., Myers & Atkinson, 1964; Suydam et al., 1964), and from one experiment with rats (Calfee, 1963). Considering that slightly different functions were being minimized to obtain parameter estimates in these studies (see Myers & Atkinson, 1964) compared to that used in the present experiment (see Appendix D), the ability of the W-S model to describe these statistics asymptotically seems well established.

The marked deviations in the fit to the asymptotic first-order conditional probabilities and marginal response probabilities for the .6-1¢ group are puzzling when viewed against the good fits to these statistics for the other experimental groups and the fits reported in other choice studies. These results could possibly be due to the fact that the estimated value of δ (Table 3d) for this group is much lower than that found for other sets of data obtained from human Ss. There may be at least three reasons such a deviant value was obtained: a) The function being minimized (Appendix D) has either failed to iterate to the minimum value, or it has reached a relative rather than an absolute

minimum; b) the use of asymptotic equations (Appendix C) in estimation when the observed data is clearly not asymptotic (see p. 27) may result in parameter estimates lacking in mearningfulness; and c) the model may not be appropriate to describe choice data when the observed marginal response probability is outside or near the asymptotic bounds predicted by the model (Calfee, 1963, pp. 95-96). Which of these alternative explanations, if any, is appropriate, may be decided: a) After the function used in parameter estimation in the present experiment is investigated; b) an estimation procedure more appropriate to pre-asymptotic data is used; and c) the data is tested with a model whose upper asymptotic bound is greater than the W-S's, e.g., the Myers and Atkinson (1964) three-stage model. Some evidence for the validity of the third alternative is found when we note that the observed marginal response probability in block 12 (Table 7a) for the .6-1¢ group is within 2% of the W-S upper asymptotic bound. It should also be noted that the estimated value of δ for this group (Table 3d) is comparable to those reported by Calfee (1963, Tables 8b, 10a-b, 12a-b) for groups whose marginal response probability exceeded the upper asymptotic bound for the W-S model.

<u>Second-order conditional probabilities</u>. The deviations in fit for the .6-0¢ and .6-1¢ groups in the present experiment on these statistics is 4.1% and 5.2%, respectively. This compares quite favorably to the 5.6% de-

viation for one group reported by Calfee (1963, Table 13b, p. 87) using a .65 level of π. For other .65 groups reported by Calfee (1963), the fits are considerably worse. These groups, however, also have a marginal response probability which exceeds the predicted upper asymptotic bound for the W-S model. The deviations in fit for the .8-0¢ and .8-1¢ groups in the present experiment show deviations of 2.3% and 2.5%, respectively. These fits compare favorably with those reported by Calfee (1963) for .8-incentive data, and by Myers and Atkinson (1964) with .8-0¢ data collected by Friedman et al. (1963). These results demonstrate the ability of the W-S model to describe response probability conditionalized against the responses and events on the preceding two trials with about the same precision across several sets of data.

Earlier it was pointed out that Myers and Atkinson (1964) suggested that poor fits to the second-order conditional probabilities may be attributed either to the unreliability of these statistics, or that the model may require revision to handle these statistics. If the poor fits were due to unreliability, then we would expect the fit to improve when the observed statistic is based on a larger number of observations. With over twelve times as many observations as used in the present experiment, however, Calfee (1963) reports deviations two to five times as large. Further, with almost twice as many observations as used in the

present experiment, the fit to the Friedman et al. (1963) data (see Myers & Atkinson, 1964) is about the same as found for our .8-0¢ group. The results of this evaluation imply that the model will require revision to handle these statistics.

Reinforcement-run statistics (R-R-S). Calfee (1963) has obtained excellent fits (usually less than 1% deviation) to the third and fourth-order R-R-S using the Myers and Atkinson (1964) three-stage conditioning model with rats. The fit of the W-S model to the third-order E_1 R-R-S for data collected by Jones and Myers (1965), however, shows deviations of about 10% for the .6 groups and 4.5% for the .75 groups. The deviations reported for the present data (Table 6a-c) are somewhat greater than those reported by Calfee (1963), but somewhat less than those found with data collected by Jones and Myers (1965).

Calfee (1963) may have obtained better fits to the R-R-Ss than reported in the present experiment, or for data collected by Jones and Myers (1965), because the identification of an event following a response as a reinforcer may have been more appropriate for rats than human Ss. Alternatively, these differences might be attributed to the fact that he used more event sequences than Jones and Myers (1965) or than were used in the present experiment. When more event sequences are used, the actual probability of an E_1 event on any trial more closely approximates the average event prob-

ability, π, and consequently the prediction equations (Appendix C, part A.3) are more appropriate (see Anderson, 1964, p. 136). It is also possible that the good fits reported by Calfee (1963) were obtained because his observed and predicted R-R-Ss were obtained after more training had been given than was used in the present experiment or by Jones and Myers (1965). An increase in the number of training trials makes it more probable that pre-experimental response tendencies like negative recency will have extinguished, and that the choice behavior more nearly resembles that described by the model.

The better fits to these statistics found in the present experiment compared with those obtained with the data collected by Jones and Myers (1965) may be similarly explained. Specifically, the observed and predicted R-R-Ss in the present experiment were obtained from a later block of trials, more event sequences were used, and each of these sequences was randomized over a larger block of trials than reported by Jones and Myers (1965).

From inspection of Table 6a we note that the E_1 R-R-Ss predicted by the W-S model for all experimental groups in the present experiment are lower than observed, i.e., more positive recency exists in the data than predicted by the model, at least when choice behavior is conditionalized on up to the preceding four events. A similar finding was also noted for most of the third-order E_1 R-R-Ss predicted with

this model using data collected by Jones and Myers (1965), and to a lesser extent for the E_1 R-R-Ss predicted by Calfee (1963) using the Myers and Atkinson (1964) three-stage model. The replicability of this finding across several data sets and under different experimental conditions suggests that the W-S model will require revision to handle these statistics.

Standard-deviation of response proportions. The predicted asymptotic standard deviations of response probabilities found for the W-S model in the present study were smaller than the observed values (Table 8), especially with no payoff. These results are consistent with those reported by Heuckeroth and Myers (1965) for the Suppes and Atkinson (1960) data. While the fits to this statistic were reasonably good, that these predictions were consistently below the observed values for all experimental groups suggests that the model may require revision, e.g., introducing a neutral state of conditioning into the model might cause an increase in the predicted variability. Alternatively, introducing some form of diversion into the experimental situation might reduce the *Ss*' utility for varying their responses (Siegel, 1964), and thereby reduce the observed variability in the data.

Linear models

First-order conditional probabilities (Tables 3a-e, 4a-e). We have seen that the ability of the SR model to

describe the first-order conditional probabilities is quite poor for each experimental group, and except for the .8-1¢ group, shows little evidence of improving over blocks of trials. The R-E model, however, is able to predict these statistics quire well, especially for the .8 groups. Further, the fit for the R-E model improves rapidly through the first seven trial blocks and then shows some deterioration followed by a more gradual improvement.

It is noted (Table 3d) that the estimated values of α_1 and α_2 for the R-E model with .6-0¢ data are essentially equal (a deviation of .001). Therefore, it is of interest to compare the predictions of the R-E model for this group in the present experiment with the predictions of the equal α model (Bush & Mosteller, 1955) for comparable groups reported elsewhere in the literature. Suppes and Atkinson (1960) have applied the equal α model to predict the asymptotic first-order conditional probabilities for their .6-0¢ group (Group Z). Their fit (1-2% deviation) is much better than reported for the .6-0¢ group in the present experiment (6-7% deviation).

The generally good fits for the R-E model found in the present experiment are consistent with the good asymptotic fit to these statistics reported by Myers, Suydam, and Heuckeroth (1965); however, the SR model shows markedly poorer fits. The poorer fit to these statistics for the SR model in the present experiment could be attributed to dif-

ferences in the experimental situation to which the model was being applied, or to the fact that these statistics were predicted with equations in Appendix C by Myers, Suydam, and Heuckeroth (1965), but with a Monte-Carlo technique in the present experiment. Since the parameter estimation techniques used in the present experiment and by Myers, Suydam, and Heuckeroth (1965) are similar, evidence of the importance of the prediction method was found by comparing the asymptotic fit to these statistics with the present experimental data when the equations in Appendix C were used, and the Monte-Carlo technique was used. The results of this comparison showed quite clearly for each experimental group that Monte-Carlo predictions result in poorer fits, especially for the SR model. In view of the comments by Sternberg (1963) suggesting that the goodness-of-fit of a model appears to be a function of the statistic used in parameter estimation, the results of this comparison also makes it reasonable to suggest that the most appropriate statistic to use in parameter estimation for a model may depend upon the prediction technique. An orthogonal comparison using different statistics in estimation, different models, and the two prediction techniques is required to evaluate this suggestion.

<u>Second-order conditional probabilities</u> (Table 5a-d). These statistics have not been previously tested with the R-E or SR models. The fits obtained for these models in the

present experiment are quite good, especially for the .8 groups (Table 5c-d).

Reinforcement-run statistics (R-R-S) (Table 6a-c). The fits to these statistics for the R-E model are good for all experimental groups except the .8-0¢ group. Similarly, the SR model is able to predict these statistics reasonably well for all experimental groups except the .6-1¢ group. Except for the .6-1¢ group, both models predict less positive recency than exists in the E_1 R-R-S data. Only the SR model shows a clear tendency to predict less positive recency than found in the E_2 R-R-S data. That the E_2 R-R-Ss predicted with the SR model tend to be greater than the observed values is consistent with the run statistics predicted with the SR model for paradise fish (Bush & Wilson, 1956).

Learning curve (Table 7a-b). It has been noted that the predicted learning curves for the two linear models for each experimental group show an initial rate of learning faster than observed. The predicted learning curve reported by Bush & Wilson (1956) for the SR model was higher than the observed curve, but increased at about the same rate. The fits to the learning curves for each model, for all but the .6-1¢ experimental group, improve as asymptotic blocks are approached. In fact, except for the .6-1¢ group, the fits of the R-E model in the last block of trials in the present experiment are quite good, a result inconsistent with that reported by Bogartz (1965). For the .6-1¢ group, both linear

models, as well as the W-S model, predict an asymptotic level of responding notably lower than observed.

The estimated value of α_2 for the SR model is 1.00 for each .8 group (Tables 4d, 5c-d). Bush and Mosteller (1955) have applied the identity operator ($\alpha_2 = 1$) model to data collected at a .75 level of π by Brunswick (1939) for rats and to unpublished data for human Ss collected by Goodnow in 1951 (see Bush & Mosteller, 1955, p. 294). The predicted learning curves show deviations of 3-4% and 6-7%, respectively. These predictions compare well with those found for the SR model for the .8-1¢ group in the present experiment.

It has already been noted that the estimated values of α_1 and α_2 for the .6-0¢ group with the R-E model are essentially equal. Bush and Mosteller (1955) have applied the equal α model to data collected with $\pi = .66$ by Neimark (1951). Her predicted learning curve shows an average deviation of 3-4%, a result consistent with that found with the R-E model for the .6-0¢ group in the present experiment. Suppes and Atkinson (1960) have also applied the equal α model to predict the learning curve for their .6-0¢ group (Group Z). Although the predicted learning curve has a matching asymptote, the rate of learning in the early blocks is too high, a result consistent with that found for each model in the present experiment.

<u>Standard-deviation of response probability</u> (Table 8). The predictions of this statistic by the R-E model are

quite good for the .8 groups, and except for the .8-0¢ group, are lower than the observed values. The predictions of the SR model are consistently and usually markedly higher than the observed values, a finding consistent with that reported by Bush and Wilson (1956).

Evaluation of models — psychological meaningfulness

Each of the models we are considering contains parameters identified with the effects of reward and non-reward or secondary reinforcement. Another way to evaluate these models is to see whether the estimated values of these parameters vary with the manipulations of the experimental variables with which the parameters are identified, and remain invariant with the manipulation of other variables. It has been suggested that the parameters estimated with the data in the last 50 trials for the .6-1¢ group (Table 3d) for the W-S model may be inappropriate because the observed marginal response probability is close to the upper asymptotic bound for this model. As the parameter estimates based on the data from the last 100 trials, in addition to being more reliable, do not show evidence of this possible inappropriateness, the present evaluation will be made with the more reliable estimates (Tables 9, 11, & 12).

W-S model

Myers and Atkinson (1964) have suggested that it might be appropriate to assume the effects of reward (μ) and non-reward (δ) increase, and the number of stimulus elements (N)

decrease with increases in payoff. We will use these identifications in the evaluation of the parameter identification and invariance for this model.

From Table 9 below we see, in agreement with these identifications, that the effects of reward (μ) increases and the number of stimulus elements (N) decreases with the introduction of payoff. The effect of non-reward (δ), however,

Table 9. Parameter Estimates for the W-S Model for Each Experimental Group

		π					
		.6			.8		
		δ	μ	N	δ	μ	N
Payoff	0¢	.472	.132	2.622	.818	.294	3.097
	1¢	.672	.999	2.188	.764	1.000	2.226

increases with the introduction of payoff only for the .6 group; the effect of non-reward decreases somewhat with the introduction of payoff for the .8 group. Parameter estimates for μ and N appear invariant over levels of π for the payoff groups, but increase with π under no payoff. The effect of non-reward (δ) also increases with π. At each level of π, the effects of reward changes more rapidly than the effects of non-reward.

The parameter estimates for the W-S model in the present experiment generally show the same ordinal relationship as reported for several sets of data by Myers and Atkinson (1964); however, the size of the estimates for each parameter varies considerably. Further, as already noted in the

introduction, there are cases (Jones[4]; Suydam et al., 1964) where not even the ordinal relationship found among parameter estimates by Myers and Atkinson (1964) holds. Such findings are clearly disturbing if we want a model which can predict as well as describe data.

Attributing the variation in parameter estimates among different studies to sampling variability of the parameters, serves as a partial explanation at best. It seem that a more important source of variability may arise from the fact that the parameter estimates reported elsewhere (Myers & Atkinson, 1964; Suydam et al., 1964; Jones[4]) and in the present experiment were estimated from data which represent different points in the learning process. If the effectiveness of the experimental variables with which the parameters are identified do change throughout the course of learning, such changes should be reflected by the failure to find parameter invariance over trial blocks. When parameter estimates for the W-S model were examined for each experimental group for successive blocks of 50 trials, invariance of parameters was not found. Therefore, it was also appropriate in evaluating the psychological meaningfulness of the W-S model, to compare the estimates reported for different sets of data with those obtained from comparable blocks of trials in the present experiment (Table 10). The W-S model parameters were estimated with data from trials 161-240 for the Suppes and Atkinson (1960) data, trials 193-288 for the

Friedman et al. (1963) data, and from trials 251-300 for the Suydam et al. (1964) and Jones[4] data.

Table 10. A Comparison of the Parameter Estimates Reported for the W-S Model in Several Human Choice Studies with Those Obtained from Comparable Blocks of Trials in the Present Experiment

	Experimental Groups								
	.6-0¢			.6-1¢			.8-0¢		
Experiment	δ	μ	N	δ	μ	N	δ	μ	N
Suppes & Atkinson (1960)*	.700	.020	3.90						
Present Experiment	.460	.001	3.92						
Friedman et al. (1963)*							.500	.030	2.44
Present Experiment							.650	.001	2.56
Suydam et al. (1964)	.686	.002	4.26	.627	.029	2.13			
Jones[4]	.539	.465	2.28						
Present Experiment	.510	.001	2.87	.440	.050	2.02			

*Parameter estimates reported by Myers and Atkinson (1964)

We see from Table 10 that when parameter estimates are obtained from comparable points in the learning process, in all but one case, the effects of reward (μ) and the number of stimulus elements (N) appear invariant, and in only one case does the effect of non-reward (δ) appear invariant. This invariance suggests that a stimulus sampling model like the W-S may be developed which will be able to predict as

well as describe choice data. The variability in the parameters which is still present might be attributed to differences in the function being minimized in the parameter estimation (see Myers & Atkinson, 1964, and Appendix D), the difference in the effects of reinforcement, especially of non-reward (δ), when the event sequences are randomized over different size trial blocks, or variation in the instructions among these studies (see Table 10) as well as sampling variability of the parameters.

SR model

In evaluating the psychological meaningfulness of the SR model, we will assume that: a) The effectiveness of primary reward is greater than secondary reward (see Bush & Wilson, 1956); b) the effectiveness of primary reward increases with the introduction of payoff (see Myers & Atkinson, 1964); and c) according to a line of reasoning developed below, the effectiveness of secondary reward decreases with the introduction of positive reinforcement (gain) for correct responding and negative reinforcement (loss) for incorrect responding.

We recall that the SR model assumes that if stimuli associated with reward also occur with non-reward, secondary reinforcement can occur, but does not necessarily occur. It seems reasonable to assume that the reinforcing effects of gain and loss have been well established in most Ss prior to entering the two-choice probability learning situation. With these strong reinforcers present initially, there seems

little reason to expect that the stimuli from the event lights or the prediction response would acquire secondary reinforcing properties greater than the effects of gain and loss, i.e., the introduction of gain and loss for correct and incorrect predictions, respectively, should reduce the effectiveness of secondary reward.

From Table 11 below, we see, in agreement with the assumed identifications, that: a) The effect of primary reward is greater than secondary reward (see also Bush & Wilson, 1956; Myers, Suydam, & Heuckeroth, 1965); b) the effect of primary reward increases with the introduction of payoff; and c) the effect of secondary reward decreases with the introduction of payoff. There is, however, some dependence of the parameters on π. Specifically, the effect of primary reward appears to increase with π when payoff is used, and decreases with π when no payoff is used. The effect of secondary reward decreases with π.

Table 11. Parameter Estimates for the SR Model for Each Experimental Group

		π			
		.6		.8	
		α_1	α_2	α_1	α_2
payoff	0¢	.596	.813	.708	1.000
	1¢	.552	.974	.486	1.000

<u>R-E model</u>

In evaluating the psychological meaningfulness of the R-E model, we will assume the identifications suggested by

Myers and Atkinson (1964) for μ and δ are also appropriate for α_1 and α_2, respectively, i.e., the effects of reward and non-reward increase with the introduction of payoff.

From Table 12 below, we see, in agreement with these identifications, the effect of reward increases with the introduction of payoff, however, the effect of non-reward decreases. Further, the parameters do not appear to be invariant over levels of π. Specifically, the effects of reward increase more rapidly, as a function of π and payoff, than the effects of non-reward decrease.

Table 12. Parameter Estimates for the R-E Model for Each Experimental Group

		π = .6		π = .8	
		α_1	α_2	α_1	α_2
payoff	0¢	.832	.815	.608	.908
	1¢	.702	.849	.394	.951

Evaluation — summary

From the evaluation of these models with regard to goodness-of-fit, it is clear that the SR model generally does not describe the data as well as the W-S and R-E models, although the parameter identification of the SR model was quite good. In view of the differences in fit of the SR model in the present experiment and by Myers, Suydam, and Heuckeroth (1965), and the fact that this discrepancy might be attributed to differences in the experimental situation or the statistic used in parameter estimation, it would be premature to reject this model at the present time. The parameter identi-

fication for the W-S model is somewhat better than for the R-E model; however, the deviations in fit for these two models are quite similar. While the slight differences in these two models (W-S and R-E) in the present experiment do not warrant rejecting either model, it appears that the modifications which might lead to an improvement in fit for the R-E model can come primarily from adding parameters or modifying the experimental situation so that the choice behavior corresponds with the predictions of the model. From the point of view of obtaining a psychologically meaningful and mathematically tractable theory, the modification of a stimulus sampling model, like the W-S model, seems more fruitful than modification of the linear models. In addition to adding parameters and modifying the experimental situation, one could, for example, systematically vary the axioms regarding the transition of stimulus elements to adjacent states of conditioning and examine the changes in the ability of the model to describe the statistics of the data. Alternatively, the states of the model could be varied, e.g., adding a neutral state of conditioning to the W-S model might lead to an increase in predicted variability.

Deviation in fit — extended training and pre-experimental response tendencies

Most models used to describe choice behavior identify the event following a response as a reinforcer, i.e., they predict positive recency. One of the most frequently cited

reasons for deviations in fit for a model is the occurrence of pre-experimental response tendencies like negative recency (e.g., Estes, 1964; Anderson, 1964). Estes (1964) has commented that until Ss learn about the properties of the sequence of events, his responses will be determined by generalizations from other situations. Estes has further suggested that giving Ss extended practice should extinguish these tendencies so that the choice behavior more nearly resembles that predicted by the model (generally stimulus sampling models). Anderson (1964) has argued that while extended practice may allow these pre-experimental response tendencies to extinguish, different practice regimes will train in different sorts of behavior, each of which will require a different model.

The good-fits of the W-S model to several asymptotic statistics reported in the present experiment, by Calfee (1963), by Myers and Atkinson (1964) for the Friedman et al. (1963) data, and by Friedman et al. (1963) with the equal π model lend support to Estes' suggestion regarding the effects that extended practice should have on choice data.

We note that the Ss trained by Calfee (1963) and Friedman et al. (1963) received extended practice on more than one level of π, while Ss in the present experiment received extended training on a single level of π, i.e., different practice regimes were used in these studies. As the W-S model has been able to describe the data from each of these

studies quite well, it seems that Anderson's (1964) suggestion that a different model will be needed to describe choice behavior after different practice regimes have been used needs to be qualified.

Overshooting—the result of reinforcing operations ?

It has been pointed out earlier in this paper that overshooting, i.e., an asymptotic level of responding exceeding matching, occurs in several studies (see Appendix A, Table 1) in which more than 300 trials are used and some incentive in addition to observing the correctness of one's predictions is introduced. This finding is not consistent with the asymptotic matching prediction derived from several models (e.g., Estes, 1959; Suppes & Atkinson, 1960). Estes (1962) has suggested that if the occurrence of overshooting is due to the introduction of variables not controlled by the experimenter and as a function of the number of trials, then these models would be appropriate to describe choice behavior when the effectiveness of these variables is experimentally controlled. If, on the contrary, overshooting represents the ultimate effect of the conditioning process, then the quantitative assumptions regarding the effects of reward and non-reward (as described by these models) must be modified.

If additional variables are becoming effective as a function of extended training, they could be evidenced by salient changes in the observed statistics or the goodness-

of-fit of different statistics over trial blocks.

From inspection of Figure 1 we see that the observed learning curves for the .6-0¢, .6-1¢, and .8-0¢ groups in the present experiment take a definite upward turn following 400 trials. Regarding the goodness-of-fit, it was noted that the predicted marginal response probability for the .6-1¢ group by each model was markedly lower than observed in the last block of trials. Further, it was noted that the predictions of the first-order conditional probabilities by the R-E model show a sudden deterioration in fit after the seventh block (Tables 3e, 4e), especially for the .6 groups.

It should be pointed out that while these findings are consistent with the suggestion that variables not controlled by the experimenter are entering as the experimental session progresses, it is also possible, as suggested earlier, that these salient changes are attributable to changes in the effectiveness over trial blocks of the experimentally manipulated variables. As the most obvious of the uncontrolled variables that could enter into the learning process with extended training are fatigue and boredom, a goal of future research might well be to provide Ss with extended training but attempt to control the influence of these variables, e.g., Ss could be run with periodic resting sessions, the required prediction response could be made variable from trial to trial, or the intertrial interval might be lengthened with the time between trials being spent in unrelated tasks of varying degrees of interest.

Summary

The Ss (80 males, 80 females) in the present experiment each made 600 predictions in the Estes and Straughan (1954) two-choice noncontingent probability learning situation under one of two levels of π (.6, .8) and payoff (0¢, 1¢). Ss trained with a .8 level of π and payoff attained significantly higher levels and rates of learning than Ss trained with a .6 level of π and no payoff, respectively. No sex differences were found. Each π-payoff group evidenced overshooting, although at different points in training. An examination of response probability conditionalized against the preceding run of events indicated that negative recency decreases with increases in training, π, and the introduction of payoff.

These data were used to evaluate the ability of three mathematical models (Bush & Mosteller, 1955 -- R-E and SR linear models; and the Myers & Atkinson, 1964 -- W-S Markov model) to describe the mean and variability of response probabilities, and several response probabilities conditionalized against previous sequences of events and responses. Generally, the W-S and R-E models are able to describe the data better than the SR model, although the parameter identification for the W-S and SR models is more appropriate than for the R-E model. While the differences in fit for the W-S and R-E models are quite small (frequently less than 1%), the W-S model shows a slight superiority to the R-E model in most cases. The deviations in fit noted

for all of these models in predicting the course of learning appears to result from the fact that the rate of learning in the predicted curves is too high, and especially for the W-S and SR models, the curves level off toward an asymptote too quickly.

The poor fits for the SR model could have resulted from choice of an inappropriate statistic for parameter estimation, consequently, it would be premature to reject this model at the present time. While the differences in the ability of the R-E and W-S models to describe the data in the present experiment are too small to warrant rejection of either model, it seems that a psychologically more meaningful and mathematically more tractable theory for predicting as well as describing choice behavior will be obtained through modification of a stimulus sampling model like the W-S model.

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Appendix A

Table 1 is presented primarily to cite findings of several studies which bear upon the probability matching hypothesis, with special regard to the effects of extended training and payoff.

Table 2 is presented to summarize those studies which have investigated negative recency and the variables which influence its occurrence, e.g., amount of training, π, instructions, payoff, and method of sequence randomization.

Table 1. Studies in Two-Choice Non-Contingent Probability Learning For Human Ss With Symmetric Payoff (Asymptotic Results)

Experiment College Ss	Ss	π	Procedure	Trials	Results
Humphreys (1939)	78	1.00 .50	NP; Predict whether a light would come on	24	matching
Grant, Hake & Hornseth (1951)	185	.50 .75 1.00	NP; Predict whether a light would come on	60	matching
Jarvik (1951)	78	.60 .67 .75	NP; Predict whether E would say "check" or "plus"; with feedback	87	no group exceeded matching
Hake & Hyman (1953)	40	.50 .75	NP; Predict whether a horizontal or vertical array of lights would appear; event conditional probability was varied, analysis based on last 120 trials	240	marginal & conditional probability matching
Burke, Estes, & Hellyer (1954)	72	.90	NP; signal to predict was varied (different patterns of lights)	120	matching

Table 1. (cont.)

Experiment College Ss	Ss	π	Procedure	Trials	Results
Estes & Straughan (1954)	48	.50 .70 .85	NP; analysis based on last 40 trials	120	each group approximated matching
Goodnow (1955a)	68	.50 .70 .90	Predictions were made under problem solving vs. gambling (risk = 1¢) orientation; "orientations" were orthogonal to level of π; analysis based on last 20 trials	120	.50 groups did not differ .70 problem solving group did not exceed matching all other groups evidenced overshooting
Goodnow & Postman (1955)	48	.50 .60 .70 .80 .90 1.00	Predictions were made under problem solving orientation	80	matching
Edwards (1956)	24	.50 .60 .70 .80	Each S made predictions on each level of π with a reward of 5¢ for each correct prediction; Ss made predictions with π=.5 in alternate sessions	150 per session	Performance under each schedule (except .50) evidenced overshooting

Table 1. (cont.)

Experiment College Ss	Ss	π	Procedure	Trials	Results
Neimark (1956)	40	.66	NP; analysis based on last 20 trials	100	matching
Toda (1956)	20	.75	four groups: a) Paid 150 yen before experiment, NP b) Paid 150 yen after experiment, NP c) Reward (+3 yen) for each correct prediction d) Risk ($\pm$ 10 yen) analysis based on last 20 trials	100	reward and risk groups evidenced overshooting; other groups matched
Estes (1957)	2	10 different	Each S made predictions on each level of π; π shifted every 25 trials; each level was used eight times	2000	Performance under each level of π evidenced matching
Gardner (1957)	48	.60 .70	NP; analysis based on last 165 trials	450	matching

Table 1. (cont.)

Experiment College Ss	Ss	π	Procedure	Trials	Results
Cotton & Rechtshaffen (1958)	48	.60 .70	NP; analysis based on last 165 trials	450	.60 group matched .70 group showed some evidence of overshooting
Engler (1958)	280	.50 .75	NP; event conditional probability was varied	240	marginal probability matching; some indication of conditional probability matching
Galanter & Smith (1958)	24	.75 Exp. I	Ss told to play for chips of no value; during trials 101-300 blocks of 5 trials were interspersed for which S was required to make the same prediction ("special trials")	300	"special trials" evidenced overshooting; non-"special trials" evidenced matching
		.75 Exp. II	Ss received either small or large payoff depending on performance in each block of 10 trials, analysis based on last 50 trials	300	low payoff Ss matched; high payoff Ss evidenced overshooting

Table 1. (cont.)

Experiment College Ss	Ss	π	Procedure	Trials	Results
La Berge (1959)	192	.50 .90	Four groups of Ss were run for either 0, 20, 60, or 200 trials on a .50 level of π then shifted to a .90 level of π	250	matching
Siegel & Goldstein (1959)	36	.75	Three groups: a) Predict for NP b) Predict for reward (+5¢) c) Predict under risk (±5¢)	100	NP group performed below matching; reward and risk groups evidenced overshooting, especially the risk group
Morse & Runquist (1960)	32		Experimental S predicted a binary sequence in which the sequence of events was clearly unscheduled by E. The same Ss then predicted whether a light would come on for a series of trials. Control Ss predicted only whether a light would come; event sequence used in light prediction was that generated by experimental Ss in first part of session	200	When the event sequence appeared to be generated by E (light prediction), all Ss matched; where event sequence appeared unscheduled by E, overshooting was evidenced

Table 1. (cont.)

Experiment College Ss	Ss	π	Procedure	Trials	Results
Radlow & Siegel (1960)	40	.70	Two groups: a) NP b) risk (±5¢)	200	only risk group evidenced overshooting
Suppes & Atkinson (1960)	90	.60	Three groups: a) NP b) risk (±5¢) c) risk (±10¢) analysis based on last 80 trials	240	NP group matched; risk groups evidenced overshooting, especially the high risk group
Myers, Reilly, & Taub (1961)	24	.50 .70 .90	Ss at each level of π were trained with risk (±1 chip)	150	.70 and .90 groups evidenced overshooting
Siegel & Abelson (1961)	60	.65 .70 .75	Ss at each level of π were trained with risk (±5¢); analysis based on last 20 trials	300	All groups evidenced overshooting
Siegel (1961)		.75	Two groups: a) control b) experimental group made predictions in a situation which called for variation in the motor response. Both groups trained under NP	240	Only the experimental group evidenced overshooting

Table 1. (cont.)

Experiment College Ss	Ss	π	Procedure	Trials	Results
Taub & Myers (1961)	[illegible]	.60 .80	Both groups trained under risk (±1¢)	100	Both groups showed evidence of overshooting
Cole (1962)		.67	NP; analysis based on last 500 trials	1000	overshooting
Derks (1962)	80 Exp. I	.75	Six groups: a) control b) outcome delay 15 sec. c) ITI 15 sec. d) speed set [Ss instructed to respond quickly] e) reward (+5¢) f) risk (±5¢)	250	Speed set group matched; all remaining groups evidenced overshooting to about same extent
McCracken, Osterhout, & Voss (1962)	30 Exp. I	.70	Three groups: a) conversational - Ss engaged in conversation with E using event stimuli (nonsense syllables) b) prediction - Ss told to try hard to consider each prediction independently of previous predictions c) prediction - Ss told to consider experiment as a general problem analysis based on last 40 trials of non-transfer Ss	280	group a) showed chance performance; groups b) and c) showed evidence of overshooting, especially group c)

Table 1. (cont.)

Experiment College Ss	Ss	π	Procedure	Trials	Results
McCracken et al. (cont.)	90 Exp. II	.70	Three groups: a) Prediction - set to consider experiment as a general problem, but to avoid a trial by trial basis of responding b) Prediction - Ss required to make predictions for blocks of trials rather than individual trials c) Prediction - Ss told to try to select response most likely correct	140	All groups evidenced overshooting, moreso than group b) and c) in Exp. I
Nies (1962)	192	.70	Event sequence appeared unscheduled by E for each of 4 experimental groups differing with respect to amount of information about event probability a) standard NP prediction b) pattern group - Ss told to expect patterns c) ratio group - Ss told event probability d) ratio-explanation group-Ss told event probability and what he could expect in a sequence of events	250	Groups a) and b) matched; groups c) and d) evidenced overshooting

Table 1. (cont.)

Experiment College Ss	Ss	π	Procedure	Trials	Results
Friedman et al. (1963)	80	.80	Ss run in three sessions under several levels of π for short blocks of trials; a large amount of training was given on a .80 level of π during the third session; analysis based on last 96 trials	288	matching
Myers, Fort, Katz, & Suydam (1963)	176	.60 .70 .80	Predictions were made under three payoff conditions: a) NP b) risk (±1¢) c) risk (±10¢) payoff conditions were orthogonal to level of π; analysis based on last 100 trials	400	All but .60-0¢ group evidenced overshooting, especially the payoff groups; only for .60 groups performance between the payoff groups differs greatly
Suydam et al. (1964)	192	.60 Exp. Ia	Ss were trained with NP, with and without feedback, or under one of two risk conditions: ±1¢ or ±10¢. Each group was run with one of two "sets": aware or unaware i.e., Ss knew or did not know that other incentive conditions existed; analysis based on last 50 trials	300	Ss trained under each aware set evidenced overshooting; only NP group with feedback in unaware group evidenced overshooting

Table 1. (cont.)

Experiment College Ss	Ss	π	Procedure	Trials	Results
Suydam et al. (cont.)	48 Exp. Ib	.60	Ss were run with a risk of ±1¢ or ±10¢ in the aware condition; analysis based on last 50 trials	300	Both groups evidenced overshooting, but show no consistent differences
	16 Exp. II	.60	Ss were run individually in a risk ±1¢ or ±10¢ condition, unaware condition; analysis based on last 50 trials	500	Both groups evidenced overshooting, but show no consistent differences
Non-College Adults					
Neimark & Shuford (1959)	270 AF men	.67	Two groups: a) predicted which of two letters would appear on card b) predicted and estimated frequency of a designated event; analysis based on last 30 trials	300	Combined data evidences matching; estimating frequency of an event seemed to increase predictions of that event
Nicks (1959)	360 AF men	.50 .67 .75	NP; Ss told to work for points, but no feedback; predicted which color light would come on; analysis based on last 288 trials	380	all groups matched

Table 1. (cont.)

Experiment Non-College Adults	Ss	π	Procedure	Trials	Results
Anderson & Whalen (1960)	54 student nurses	.50 .65 .80	NP; Ss predicted which of two events would occur and to indicate confidence of their prediction: analysis based on last 100 trials	350	All groups evidenced matching
Edwards (1961)	120 AF men	.50 .60 .70	NP; analysis based on last 80 trials	1000	.60 and .70 groups evidenced overshooting
Edwards & Tannenbaum (1961)	54	.60	Three levels risk (0¢, ±5¢, ±10¢) were combined factorially with method of sequence generation (sequences appeared unscheduled by E or not). No groups received feedback. Two additional groups: a) standard prediction situation with risk (±10¢) feedback trial by trial; b) standard prediction with problem solving orientation	750	All groups evidenced overshooting; 0¢ and 5¢ risk groups do not differ, but 10¢ risk groups show greatest overshooting
Gardner & Forsythe (1961)	192 soldiers	.60 .70	NP; Ss predicted whether a card would be blank or con- a variable number of letters; analysis based on last 140 trials	420	Each group evidenced overshooting

Table 1. (cont.)

Experiment Non-College Adults	Ss	π	Procedure	Trials	Results
Jones (1961)	48 AF wives	.60	NP; Ss predicted whether a particular event would occur or one of another four would occur; one group made 10 predictions per minute, a second group made 5; analysis based on last 60 trials	200	both groups approximated matching
Children					
Messick & Solley (1957)	7	1.00 .90 .75 .60	Exp. I: event stimuli were figures differing in size Exp. II: event stimuli were "happy" vs "sad" faces Exp. III: candy reward given for each correct guess	200	matching in first two experiments; older Ss evidenced overshooting in experiment three
Atkinson, Sommer, & Sternman (1960)	63	.50 .75	Amount of reinforcement manipulated by proportion of trials on which two E_1 events occurred	400	Group for which probability of two events was high evidenced overshooting; other groups matched

Table 1. (cont.)

Experiment Children	Ss	π	Procedure	Trials	Results
Jones & Liverant (1960)	80	.70 .90	Nursery and elementary school children predicted which response would result in a reward (redeemable chips); analysis based on last block of 20 trials	100	Younger children evidenced overshooting
Kessen & Kessen (1961)	36	.50 .70	NP; events predicted were figures painted on playing cards; two age graded groups	100	No group exceeded matching
Brackbill, Kappy, & Starr (1962)	48	.75	Second graders predicted whether a card would have a picture of a cat or dog; different groups received 0, 1, 3, or 5 marbles which were exchangeable for a toy	200	Reward groups evidenced overshooting; differences between reward and non-reward groups greater than differences among reward groups alone
Siegel & Andrews (1962)	24	.75	Two groups of preschool children trained with or without reward (predict which container over a button or over a reward, respectively); analysis based on last 20 trials	100	Reward groups evidenced overshooting

Table 1. (cont.)

Experiment Children	Ss	π	Procedure	Trials	Results
Craig & Myers (1963)	90	.60 .80	Kindergarten, fourth and eighth grade children predicted which of two lights would go on; NP; analysis based on last 40 trials	200	Fourth and eighth graders matched; kindergarten children were below matching

Note: The following abbreviations are used in this table: NP (no payoff); ITI (inter trial interval); AF (Air Force).

Table 2. Studies in Two-Choice Probability Learning in Which the Negative Recency Phenomenon Has Been Investigated

Experiment College Ss	Ss	π	Procedure	Trials	Results
Jarvik (1951)	78	.60 .67 .75	NP; Ss kept written record of predictions, sequences formed by randomizing within small blocks - few long runs	87	Prediction of E_1 decreases after a run of two E_1's
Goodnow (1955b)	5 Min. per group	.50 .70 .90	Predictions were made under problem solving vs gambling (risk ±1¢) orientation; "orientation" were orthogonal to level of π; analysis based on last 100 trials, sequences were randomized in 10 trial blocks	120	Negative recency evidenced in risk groups by tendency of Ss to repeat unsuccessful predictions on the preceding trial; Ss trained under a problem solving orientation do not show this tendency
Feldman (1959)	43	.71	Predictions were made by pressing a telegraph key; sequence randomized over entire set of trials and consequently contained long runs	200	No evidence of gambler's fallacy
Anderson (1960)	200	.50	Ss predicted sequences in which ($P(E_{1,n}\|E_{1,n-1})$ varied in .1 to .9 and $P(E_{1,n})$ did not vary more than 10% in each block of 50 trials; analysis based on last 200 trials	300	For groups where $P(E_{1,n}\|E_{1,n-1})$ is .1-.5 or .7 $P(A_{i,n}\|E_{i,n-1}E_{i,n-2}) > P(A_{i,n}\|E_{i,n-1}E_{i,n-2}E_{i,n-3})$; for group where $P(E_{1,n}\|E_{1,n-1}) = .6$,

Table 2. (cont.)

Experiment College Ss	Ss	π	Procedure	Trials	Results
Anderson (cont.)					$P(A_{i,n} \mid E_{i,n-1}E_{i,n-2}E_{i,n-3}) > P(A_{i,n} \mid E_{i,n-1}E_{i,n-2}E_{i,n-3}E_{i,n-4})$; groups where $P(E_{1,n} \mid E_{1,n-1}) =$ .8 or .9 show no negative recency; evidence is presented showing negative recency decreases over trial blocks for groups where $P(E_{1,n} \mid E_{1,n-1}) =$ .4-.9
Lindman & Edwards (1961)	25	.50	Ss predicted whether a red or green card would appear on each trial; sequence formed according to Nicks (1959) procedure over entire sequence	200	Evidence for decrease in negative recency from first to second 100 trial block
Derks (1962)	80 Exp. I	.75	Six groups: a) control; b) out-come delay 15 seconds; c) ITI 15 sec.; d) speed set; e) reward (+5¢); f) risk (±5¢). Sequences randomized over the whole series using a random number table	250	All groups evidence negative recency; $P(A_1)$ conditionalized on preceding run of events shows a decrease following $2E_1$'s, $4E_1$'s, $1E_1$, $2E_1$'s, 2 or 3 E_1's and $3E_1$'s for groups a-f, respectively

Table 2. (cont.)

Experiment College Ss	Ss	π	Procedure	Trials	Results
Derks (cont.)	10 Exp. II	.75	A control group made predictions for an extended number of trials; analysis based on successive blocks of 250 trials	1000	$P(A_1)$ conditionalized against preceding run of events shows a decrease after $2E_1$'s in first 250 trials, flattens in second block of 250 trials, and remains essentially constant during last two blocks
McCracken, Osterhout, Voss (1962)	120 Exps. I&II	.70	See Table 1 for group's description	140	Only groups b) and c) in Exp. I evidenced negative recency
Derks (1963)	60	.75	Three groups made predictions for sequences with different run structure a) short runs - all runs of length less than 5 b) long runs - all runs of length 2-11 c) extra-long runs - median run length of 6	300	Short run group evidenced negative recency in each 100 trial block; long run groups showed positive recency after first 100 trials
Gambino & Myers (1965)	144	.50	Ss predicted whether E would call out a "1" or a "2". Ss partitioned into one of 6 groups where sequences differed in mean run length (low, high) and variability of length of runs (low, medium, high)	540	Major conclusion: Ss trained with a sequence where variability of run length is high exhibit less negative recency

Table 2. (cont.)

Experiment College Ss	Ss	π	Procedure	Trials	Results
Jones & Myers (1965)	192	.60 .75	Levels of π were combined factorially with risk (0¢, ±5¢, ±25¢) and the number of trials over which the event occurrences were randomized (20 and 300); run curves presented for each 100 trials; data for all payoff groups pooled	300	When randomization is over block of 20 trials, negative recency is evidenced and increases over trial blocks; the converse is true for groups trained under event sequences randomized over 300 trials; more negative recency is exhibited under .60 than the .75 level of π.
Non-College Ss					
Nicks (1959)	360 AF men	.50 .67 .75	Sequences were constructed for four of five groups so that over the entire sequence the number of runs of each length came out at its expected value (a CR sequence); for one group (.50 level of π) a sequence containing very short and very long runs was used (NR sequence)	380	For Ss trained with a CR sequence negative recency was exhibited following the second E_i, $i=1,2$ in a run; run curves flatten more for higher levels of π; Ss trained with the NR sequence evidenced negative recency after one E_1 and two E_2's.

Table 2. (cont.)

Experimental Non-College Ss	Ss	π	Procedure	Trials	Results
Anderson & Whalen (1960)	54 student nurses	.50 .65 .80	Sequences were random, subject event frequency requirement in the first 50 trials and 100 trial blocks thereafter; in each 100 trial block (beginning with 51-150) five subsequences of E_1's for each level of π were selected for run analysis	350	$P(A_1)$ conditionalized on more events in a run decreases as run length increases; run curves flatten and decline later in E_1 run the greater the level of π, and the later the trial block for which the run curve is constructed.
Atkinson, Sommer, & Sternman (1960)	63	.50 .75	Amount of reinforcement manipulated by proportion of trials on which two E_1 events occurred; 5th and 6th graders; two sequences of 200 trials each for each reinforcement condition	400	Negative recency occurs later with higher level of π: after two E_1's for the .50 group and after four and five E_1's for the .75 groups; highest reinforcement group showed negative recency later in run of E_1 events
Edwards (1961)	120 AF men	.50 .60 .70	Sequences were formed according to Nicks (1959) and from a random number table in accordance with level of π; negative recency for different levels of π was based on last 480 trials	1000	No negative recency was observed in last 600 trials for any level of π; run curves for .50 group show a progressive flattening and peaking later in the run as training continues

Table 2. (cont.)

Experiment Non-College Ss	Ss	π	Procedure	Trials	Results
Craig & Myers (1963)	90	.60 .80	Kindergarten, fourth and eighth grade children predicted which of two lights would go on; sequences formed according to Nicks (1959)	200	Eighth grade children evidence negative recency after $4E_1$'s at both levels of π; fourth grade children evidence some behavior for .60 group, .80 fourth grade children evidence negative recency after $5E_1$'s; kindergarten children tend to alternate
Bogartz (1965)	24	.50 .80	Ss were 4-5½ year olds Made predictions for marbles which were redeemable for a toy, sequences randomized over blocks of 60 trials except for first 30	210	neither group showed appreciable negative recency

Note: The following abbreviations are used in this table: NP (no payoff); ITI (inter trial interval); AF (Air Force); CR (constrained randomization); NR (non-random)

Appendix B

Instructions

Part A.

The purpose of this experiment is to investigate the processes which determine how people make decisions. The board in front of you contains two large green lights. Directly beneath each light is a switch. You may ignore the little orange lights as they will not be used. On each trial a buzzer will sound and then one of the two green lights will go on. You are to predict which light will come on. When the buzzer sounds, your task is to pull the switch beneath the light you think will come on. (The method of operating the switches was demonstrated to each S.) When one of the lights comes on, push the switch back to its starting position even if you wish to predict the same light will go "on" on the next trial. It is important that the switch remain on until the light comes on for recording purposes. Therefore you may use the coming on of a light as the signal to push the switch back to its starting position. You should make your predictions so as to obtain the best possible score, however, you are cautioned against looking for patterns in the lights. The sequence determining which light will come on was generated in a strictly random fashion, and attempting to find a pattern will probably hurt your score. (non-payoff Ss only) — Are there any questions?

Part B.

The following instructions were then read to Ss in the payoff groups:

For each correct prediction you make, you will receive 1¢ and for each incorrect prediction, you will lose 1¢. To help us keep track of your wins and losses, you have been provided with several sheets of paper. Please place your name on the top of each sheet. (Pause) You will note that one column has "+"s and the other has "-"s. On every trial in which you make a correct prediction, place a check after the last check in the "+" column. On those trials in which you make an incorrect prediction, place a check after the last check in the "-" column. In other words, there should be no blank spaces between any two successive checks in a column. (A hypothetical sequence of wins and losses on five successive trials was presented to each S to be sure he understood the method of recording.) At the end of the experiment, you will be paid 1¢ for the difference between the number of checks in the "+" and "-" columns. -- Are there any questions?

Appendix C

<u>Part</u> <u>A</u>.1. Listed below is a summary of the techniques described by Heuckeroth and Myers (1965) for computing the first-order joint probabilities of the form $P(A_{1,n}E_{j,n-1}A_{k,n-1})$, $j,k=1,2$ for any trial, *n*

$$P(A_1E_1A_1) = \pi E\left[\left(\frac{K_{1,2,n-1} + K_{1,1,n-1}}{N}\right)^2\right]$$

$$P(A_1E_2A_1) = (1-\pi)\left[E\left(\frac{K_{1,2,n-1} + K_{1,1,n-1}}{N}\right)^2 - \frac{\delta E}{N}\left(\frac{K_{1,1,n-1}}{N}\right)\right]$$

$$P(A_1E_1A_2) = \pi\left[E\left(\frac{K_{1,2,n-1} + K_{1,1,n-1}}{N}\right) - E\left(\frac{K_{1,2,n-1} + K_{1,1,n-1}}{N}\right)^2 + \frac{\delta E}{N}\left(\frac{K_{2,1,n-1}}{N}\right)\right]$$

$$P(A_1E_2A_2) = (1-\pi)\left[E\left(\frac{K_{1,2,n-1} + K_{1,1,n-1}}{N}\right) - E\left(\frac{K_{1,2,n-1} + K_{1,1,n-1}}{N}\right)^2\right]$$

where $K_{i,j,n-1}$, $i,j=1,2$ is a random variable, the number of stimulus elements in state j of conditioning to response A_i on trial *n*-1, and *E* is the expectation operator. The expectation of each of these random variables on trial *n*-1 is obtained through a series of recursions on trial *n*-2 (see

Heuckeroth & Myers, 1965, for mathematical development). To obtain a solution for these joint probabilities, the recursive relationships involving 14 expectations are required. These relationships are expressed in the matrix multiplication

$$V'_n = V'_1(T')^{n-1}$$

where

$$V'_n = \Big[E\left(\frac{K_{1,2,n}}{N}\right) \quad E\left(\frac{K_{1,1,n}}{N}\right) \quad E\left(\frac{K_{2,1,n}}{N}\right) \quad E\left(\frac{K_{2,2,n}}{N}\right)$$

$$E\left(\frac{K_{1,2,n}}{N}\right)^2 \quad E\left(\frac{K_{1,1,n}}{N}\right)^2 \quad E\left(\frac{K_{2,1,n}}{N}\right)^2 \quad E\left(\frac{K_{2,2,n}}{N}\right)^2$$

$$E\left(\frac{K_{1,2,n}}{N}\right)\left(\frac{K_{1,1,n}}{N}\right) \quad E\left(\frac{K_{1,2,n}}{N}\right)\left(\frac{K_{2,1,n}}{N}\right)$$

$$E\left(\frac{K_{1,2,n}}{N}\right)\left(\frac{K_{2,2,n}}{N}\right) \quad E\left(\frac{K_{1,1,n}}{N}\right)\left(\frac{K_{2,1,n}}{N}\right)$$

$$E\left(\frac{K_{1,1,n}}{N}\right)\left(\frac{K_{2,2,n}}{N}\right) \quad E\left(\frac{K_{2,1,n}}{N}\right)\left(\frac{K_{2,2,n}}{N}\right) \Big]$$

All but the following entries in the V'_1 vector are defined as 0:

$$E\left(\frac{K_{1,1,1}}{N}\right) = E\left(\frac{K_{2,1,1}}{N}\right) = .5$$

$$E\left(\frac{K_{1,1,1}}{N}\right)^2 = E\left(\frac{K_{2,1,1}}{N}\right)^2 = E\left(\frac{K_{1,1,1}}{N}\right)\left(\frac{K_{2,1,1}}{N}\right) = .25$$

and

$$T' = \begin{bmatrix}
1-a' & a' & 0 & 0 & a'/N & a'/N & 0 & 0 & -a'/N & 0 & 0 & 0 & 0 & 0 \\
b & 1-a'-b & a' & 0 & b/N & (a'+b)/N & a'/N & 0 & -b/N & 0 & 0 & a-a'/N & 0 & 0 \\
0 & a & 1-a-b' & b' & 0 & a/N & (a+b')/N & b'/N & 0 & 0 & 0 & -a/N & 0 & b'/N \\
0 & 0 & a & 1-a & 0 & 0 & a/N & a/N & 0 & 0 & 0 & 0 & 0 & -a/N \\
0 & 0 & 0 & 0 & 1-2a' & 0 & 0 & 0 & a' & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1-2a'-2b & 0 & 0 & b & 0 & 0 & a'-a & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1-2a-2b' & 0 & 0 & 0 & 0 & a & 0 & b' \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1-2a & 0 & 0 & 0 & 0 & 0 & a \\
0 & 0 & 0 & 0 & 2b & 2a' & 0 & 0 & 1-2a'-b & a' & 0 & -a & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & a & 1-a-a'-b' & b' & a' & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & a & 1-a'-a & 0 & a' & 0 \\
0 & 0 & 0 & 0 & 0 & 2a & 2a' & 0 & 0 & b & 0 & 1-a'-2a-b'-b & b' & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & b & 0 & 1-a-a'-b & a' \\
0 & 0 & 0 & 0 & 0 & 0 & 2a & 2b' & 0 & 0 & 0 & 0 & a & 1-2a-b'
\end{bmatrix}$$

<u>Part</u> <u>A</u>.2. Listed below are the expressions for the asymptotic joint probabilities of the form $P(A_{1,n}E_{j,n-1}A_{k,n-1}E_{l,n-2}A_{m,n-2})$ for the weak-strong model $(j,k,l,m = 1,2)$. The conditional statistics may be obtained by noting that in the noncontingent case

$$P(A_i|E_jA_kE_lA_m) = \frac{P(A_iE_jA_kE_lA_m)}{P(E_j)Pr(E_l)P(A_k|E_lA_m)P(A_m)}$$

where $P(E_1) = \pi$ and $P(E_2) = (1-\pi)$.

$$P(A_1E_1A_1E_1A_1) = \frac{\pi^2}{N^2}\,[A+3(N-1)A^2 + (N-1)(N-2)A^3]$$

$$P(A_1E_1A_1E_1A_2) = \frac{\pi^2}{N^2}\,[u_3\delta + (N-1)A(B+2u_3\delta) + (N-1)(N-2)A^2B]$$

$$P(A_1E_1A_1E_2A_1) = \frac{\pi(1-\pi)}{N^2}\,[C+(N-1)A(A+2C)+(N-1)(N-2)A^3]$$

$$P(A_1E_1A_1E_2A_2) = \frac{\pi(1-\pi)}{N^2}\,\{(N-1)AB[1+(N-2)A]\}$$

$$P(A_1E_1A_2E_1A_1) = \frac{\pi^2}{N^2}\,\{(N-1)A[u_3\delta+B+(N-2)AB]\}$$

$$P(A_1E_1A_2E_2A_1) = \frac{\pi(1-\pi)}{N^2}\,\{u_2\delta^2+(N-1)[A\delta(u_2+u_3)+BC]+(N-1)(N-2)A^2B\}$$

$$P(A_1E_1A_2E_1A_2) = \frac{\pi^2}{N^2}\,\{u_3\delta(1-\delta)+u_4\delta^2+(N-1)[2Bu_3\delta+AD] + (N-1)(N-2)AB^2\}$$

$$P(A_1E_1A_2E_2A_2) = \frac{\pi(1-\pi)}{N^2}\,[u_3\delta(1-\mu)+(N-1)B(u_3\delta+A)+(N-1)(N-2)AB^2]$$

$$P(A_1E_2A_1E_1A_1) = \frac{\pi(1-\pi)}{N^2} \{u_1+u_2[1-\delta(1-\mu)]+(N-1)A(2A+C) + (N-1)(N-2)A^3\}$$

$$P(A_1E_2A_1E_1A_2) = \frac{\pi(1-\pi)}{N^2} [u_3\delta(1-\delta)+(N-1)(BC+2Au_3\delta)+(N-1)(N-2)A^2B]$$

$$P(A_1E_2A_1E_2A_1) = \frac{(1-\pi)^2}{N^2} [u_1(1-\delta^2)+u_2(1-\delta)^2+3(N-1)AC+(N-1)(N-2)A^3]$$

$$P(A_1E_2A_1E_2A_2) = \frac{(1-\pi)^2}{N^2} (N-1)B[C+(N-2)A^2]$$

$$P(A_1E_2A_2E_1A_1) = \frac{\pi(1-\pi)}{N^2} (N-1)AB[1+(N-2)A]$$

$$P(A_1E_2A_2E_1A_2) = \frac{\pi(1-\pi)}{N^2} (N-1)[Bu_3\delta+AD+(N-2)AB^2]$$

$$P(A_1E_2A_2E_2A_1) = \frac{(1-\pi)^2}{N^2} (N-1)[BC+Au_2\delta+(N-2)A^2B]$$

$$P(A_1E_2A_2E_2A_2) = \frac{(1-\pi)^2}{N^2} (N-1)AB[1+(N-2)B]$$

where

$$A = u_1 + u_2$$
$$B = u_3 + u_4$$
$$C = u_1 + u_2(1-\delta)$$
$$D = u_3(1-\delta) + u_4$$

and the μ_j, $j=1,4$, have been defined in Eqs. 9-11.

Part A.3. Listed below are the E_1 and E_2 second, third, and fourth-order R-R-S.

a. Second-order E_i R-R-S

$$P(A_i|E_iE_j) = \frac{1}{N^2}\Big[S_i + (1-\delta)W_i + [\delta W_i + (1-\mu)W_j]\delta + (N-1)[2(S_i+W_i) + \delta(W_j-W_i)] + (N-1)^2[S_i+W_i]\Big]$$

b. Third-order E_i R-R-S

$$P(A_i|E_iE_iE_j) =$$

$$\frac{1}{N^3}\Big[S_i + W_i[1-\delta(1-\delta)^2] + W_j[(1-\mu)\{1-(1-\delta)^2\} + \mu\delta^2]$$
$$+ S_j\delta^2 + (N-1)[S_i+W_i+W_j\{1-(1-\delta)^2\} + S_j\delta^2 +$$
$$2\{(S_i+W_i) - W_i\delta(1-\delta) + \delta(1-\mu)W_j\}]$$
$$+ (N-1)^2[3(S_i+W_i)+\delta(2W_j-W_i)] + (N-1)^3[S_i+W_i]\Big]$$

c. Fourth-order E_i R-R-S

$$P(A_i|E_iE_iE_iE_j) =$$

$$\frac{1}{N^4}\Big[S_i + W_i[1-\delta(1-\delta)^3] + W_j[(1-\mu)\{1-(1-\delta)^3\}$$
$$+ \mu\{\delta-\delta(1-\delta)^2 + \delta^2(1-\delta)\}] + S_j[\delta\{1-(1-\delta)^2\}+\delta^2(1-\delta)]$$
$$+ (N-1)[S_i+W_i+W_j\{1-(1-\delta)^3\}+S_j\{\delta\{1-(1-\delta)^2\}+\delta^2(1-\delta)\}]$$
$$+ 3(N-1)[S_i+W_i-W_i\delta(1-\delta)^2+W_j\{(1-\mu)\{1-(1-\delta)^2\}+\mu\delta^2\}$$
$$+S_j\delta^2]$$
$$+ 3(N-1)^2[2(S_i+W_i)-W_i\{\delta(1-\delta)\}+W_j\{(1-\mu)\delta+1-(1-\delta)^2\}$$
$$+S_j\delta^2]$$
$$+ (N-1)^3[4(S_i+W_i)+\delta(3W_j-W_i)] + (N-1)^4[S_i+W_i]\Big]$$

where $i,j = 1,2 \; i \neq j$ and

$$S_1 = \mu_1$$

$$S_2 = \mu_4$$

$$W_1 = \mu_2$$

$$W_2 = \mu_3$$

and $\mu_j, j = 1,4$ have been defined in equations 9-11.

Part A.4. The method of calculating the asymptotic variance of response proportions is obtained from the calculation:

$$\mathrm{var}(S_k) = K\{P(A_{1,\infty}) - [P(A_{1,\infty})]^2\} +$$

$$2 \lim_{n\to\infty} \sum_{i=1}^{j-1} \sum_{j=2}^{k} P(A_{1,n+j}A_{1,n+i}) -$$

$$K(K-1)[P(A_{1,\infty})]^2$$

where: a) S_k is the sum of K trials

b) $P(A_{1,\infty})$ is defined by Eq. 13,

and c) $\lim_{n\to\infty} \sum_{i=1}^{j-1} \sum_{j=2}^{k} P(A_{1,n+j}A_{1,n+i})$ is defined by the vector-matrix multiplication $V''_{n+k} = V''_{n+1}(T'')^{k-1}$

where 1) $V''_{n+k} = [P(S_{1,n+k}S_{1,n})\ P(W_{1,n+k}S_{1,n})\ P(W_{2,n+k}S_{1,n})$
$P(S_{2,n+k}S_{1,n})\ P(S_{1,n+k}W_{1,n})\ P(W_{1,n+k}W_{1,n})$
$P(W_{2,n+k}W_{1,n})\ P(S_{2,n+k}W_{1,n})]$

and 2) V''_{n+1} vector has the elements

a) $\lim_{n\to\infty} P(S_{1,n+1}S_{1,n}) = \mu_1\left[\frac{N-1}{N}\mu_1 + \frac{1-(1-\pi)\delta}{N}\right]$

b) $\lim_{n\to\infty} P(W_{1,n+1}S_{1,n}) = \mu_1\left[\frac{N-1}{N}\mu_2 + \frac{(1-\pi)\delta}{N}\right]$

c) $\lim_{n\to\infty} P(W_{2,n+1}S_{1,n}) = \mu_1\left[\frac{N-1}{N}\mu_3\right]$

d) $\lim_{n\to\infty} P(S_{2,n+1}S_{1,n}) = \mu_1\left[\frac{N-1}{N}\mu_4\right]$

e) $\lim_{n\to\infty} P(S_{1,n+1}W_{1,n}) = \mu_2\left[\frac{N-1}{N}\mu_1 + \frac{\pi\mu}{N}\right]$

f) $\lim\limits_{n\to\infty} P(W_{1,n+1}W_{1,n}) = \mu_2\left[\frac{N-1}{N}\mu_2 + \frac{1-\pi\mu-(1-\pi)\delta}{N}\right]$

g) $\lim\limits_{n\to\infty} P(W_{2,n+1}W_{1,n}) = \mu_2\left[\frac{N-1}{N}\mu_3 + \frac{(1-\pi)\delta}{N}\right]$

h) $\lim\limits_{n\to\infty} P(S_{2,n+1}W_{1,n}) = \mu_2\left[\frac{N-1}{N}\mu_4\right]$

where μ_j, j= 1,4 is defined in Eqs. 9-11,

and 3) matrix T" is defined below

$$\begin{bmatrix} 1-a' & a' & 0 & 0 & 0 & 0 & 0 & 0 \\ b & 1-a'-b & a' & 0 & 0 & 0 & 0 & 0 \\ 0 & a & 1-a-b' & b' & 0 & 0 & 0 & 0 \\ 0 & 0 & a & 1-a & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1-a' & a' & 0 & 0 \\ 0 & 0 & 0 & 0 & b & 1-a-b' & a' & 0 \\ 0 & 0 & 0 & 0 & 0 & a & 1-a-b' & b' \\ 0 & 0 & 0 & 0 & 0 & 0 & a & 1-a \end{bmatrix}$$

where (a) $a = \frac{\pi\delta}{N}$

(b) $a' = \frac{(1-\pi)\delta}{N}$

(c) $b = \frac{\pi\mu}{N}$

(d) $b' = \frac{\mu(1-\pi)}{N}$

Part A.5. The learning curve is obtained by the matrix vector solution $V_n = V_1T^{n-1}$

where

a) $V_n = [P(S_{1,n})\ P(W_{1,n})\ P(W_{2,n})\ P(S_{2,n})]$

and (1) $P(S_{1,n})$ and $P(W_{1,n})$ are the probabilities a stimulus element on trial n is sampled from the strongest or weakest state of conditioning to an A_1 response, respectively.

(2) $P(S_{2,n})$ and $P(W_{2,n})$ are similarly defined for an A_2 response.

and where

b) T =

$$
\begin{bmatrix}
1-\frac{(1-\pi)\delta}{N} & \frac{(1-\pi)\delta}{N} & 0 & 0 \\
\frac{\pi\mu}{N} & 1-\frac{\pi\mu+(1-\pi)\delta}{N} & \frac{(1-\pi)\delta}{N} & 0 \\
0 & \frac{\pi\delta}{N} & 1-\frac{(1-\pi)\mu+\pi\delta}{N} & \frac{(1-\pi)\mu}{N} \\
0 & 0 & \frac{\pi\delta}{N} & 1-\frac{\pi\delta}{N}
\end{bmatrix}
$$

c) $V_1 = [0\quad .5\quad .5\quad 0]$

Part B.1. Listed below are the expressions for the asymptotic joint probabilities of the form $P(A_{1,n}E_{j,n-1}A_{k,n-1})$ for the Bush and Mosteller (1955) Reinforcement Extinction Model (j,k=1,2) averaged over organisms and events. The conditional statistics may be obtained by noting that in the non-contingent case

$$P(A_1|E_jA_k) = \frac{P(A_1E_jA_k)}{P(E_j)P(A_k)}$$

$$P(A_1E_1A_1) = \pi\{\alpha_1V_2 + [1-\alpha_1]V_1\}$$

$$P(A_1E_1A_2) = \pi\{1 - \alpha_2 - V_1[1-2\alpha_2] - V_2[\alpha_2]\}$$

$$P(A_1E_2A_1) = (1-\pi)\{\alpha_2V_2\}$$

$$P(A_1E_2A_2) = (1-\pi)\{\alpha_1[V_1-V_2]\}$$

Part B.2. Listed below are the expressions for the asymptotic joint probabilities of the form $P(A_{1,n}E_{j,n-1}A_{k,n-1}E_{l,n-2}A_{m,n-2})$ for the Bush and Mosteller (1955) Reinforcement Extinction Linear Model ($j,k,l,m = 1,2$) averaged over organisms and events. The conditional statistics may be obtained by noting that in the non-contingent case

$$P(A_1|E_jA_kE_lA_m) = \frac{P(A_1E_jA_kE_lA_m)}{P(E_j)P(A_k|E_lA_m)P(E_l)P(A_m)}$$

where $P(E_1) = \pi$ and $P(E_2) = 1-\pi$

$$P(A_1E_1A_1E_1A_1) = \pi^2\{V_1[1-\alpha_1-\alpha_1{}^2+\alpha_1{}^3] + V_2[\alpha_1(1+\alpha_1-2\alpha_1{}^2)] + V_3[\alpha_1{}^3]\}$$

$$P(A_1E_1A_1E_1A_2) = \pi^2\{1-\alpha_2(1+\alpha_1-\alpha_1\alpha_2) - V_1[1-\alpha_2(2+2\alpha_1-3\alpha_1\alpha_2)] - V_2[\alpha_2(1+\alpha_1-3\alpha_1\alpha_2)] - V_3[\alpha_1\alpha_2{}^2]\}$$

$$P(A_1E_1A_1E_2A_1) = \pi(1-\pi)\{V_2[\alpha_2(1-\alpha_1)] + V_3[\alpha_1\alpha_2{}^2]\}$$

$$P(A_1E_1A_1E_2A_2) = \pi(1-\pi)\{V_1[\alpha_1(1-\alpha_1)] - V_2[\alpha_1(1-\alpha_1-\alpha_1{}^2)] - V_3[\alpha_1{}^3]\}$$

$$P(A_1E_1A_2E_1A_1) = \pi^2\{V_1[\alpha_1(1-\alpha_1\alpha_2)] - V_2[\alpha_1(1-2\alpha_1\alpha_2)] - V_3[\alpha_1{}^2\alpha_2]\}$$

$$P(A_1E_1A_2E_1A_2) = \pi^2\{\alpha_2(1-\alpha_2{}^2) - V_1[\alpha_2(2-3\alpha_2{}^2)] + V_2[\alpha_2(1-3\alpha_2{}^2)] + V_3[\alpha_2{}^3]\}$$

$$P(A_1E_1A_2E_2A_1) = \pi(1-\pi)\{V_1[1-\alpha_2] - V_2[\alpha_2(1-2\alpha_2)] - V_3[\alpha_2{}^3]\}$$

$$P(A_1E_1A_2E_2A_2) = \pi(1-\pi)\{1-\alpha_2-V_1[1+\alpha_1-\alpha_2(1+2\alpha_1)] + V_2[\alpha_1(1-2\alpha_2-\alpha_1\alpha_2)] + V_3[\alpha_1{}^2\alpha_2]\}$$

$$P(A_1E_2A_1E_1A_1) = \pi(1-\pi)\{V_1[\alpha_2(\alpha_1-1)^2] + V_2[2\alpha_1\alpha_2(1-\alpha_1)] + V_3[\alpha_1{}^2\alpha_2]\}$$

$$P(A_1E_2A_1E_1A_2) = \pi(1-\pi)\{\alpha_2(\alpha_2-1)^2 - V_1[\alpha_2(1-4\alpha_2+3\alpha_2{}^2)] - V_2[\alpha_2{}^2(2-3\alpha_2)] - V_3[\alpha_2{}^3]\}$$

$$P(A_1E_2A_1E_2A_1) = (1-\pi)^2\{\alpha_2{}^3V_3\}$$

$$P(A_1E_2A_1E_2A_2) = (1-\pi)^2\{\alpha_1{}^2\alpha_2(V_2-V_3)\}$$

$$P(A_1E_2A_2E_1A_1) = \pi(1-\pi)\{V_1[\alpha_1{}^2(1-\alpha_1)] - V_2[\alpha_1{}^2(1-2\alpha_1) - V_3[\alpha_1{}^3]\}$$

$$P(A_1E_2A_2E_1A_2) = \pi(1-\pi)\{\alpha_1\alpha_2(1-\alpha_2) - V_1[\alpha_1\alpha_2(2-3\alpha_2)] + V_2[\alpha_1\alpha_2(1-3\alpha_2)] + V_3[\alpha_1\alpha_2{}^2]\}$$

$$P(A_1E_2A_2E_2A_1) = (1-\pi)^2\{\alpha_1\alpha_2[V_2-\alpha_2V_3]\}$$

$$P(A_1E_2A_2E_2A_2) = (1-\pi)^2\{V_1[\alpha_1{}^2] - V_2[\alpha_1{}^2(1+\alpha_1)] + V_3[\alpha_1{}^3]\}$$

Part C. 1. Listed below are the expressions for the asymptotic joint probabilities of the form $P(A_{1,n}E_{j,n-1}A_{k,n-1})$ for the Bush and Mosteller (1955) Secondary Reinforcement Model (j,k=1,2) averaged over organisms and events. The conditional statistics may be obtained by noting that in the non-contingent case

$$P(A_1|E_jA_k) = \frac{P(A_1E_jA_k)}{P(E_j)P(A_k)}$$

$$P(A_1E_1A_1) = \pi\{\alpha_1V_2 + (1-\alpha_1)V_1\}$$

$$P(A_1E_1A_2) = \pi\{\alpha_2(V_1-V_2)\}$$

$$P(A_1E_2A_1) = (1-\pi)\{\alpha_2V_2 + (1-\alpha_2)V_1\}$$

$$P(A_1E_2A_2) = (1-\pi)\{\alpha_1(V_1-V_2)\}$$

Part C.2. Listed below are the expressions for the asymptotic joint probabilities of the form $P(A_{1,n}E_{j,n-1}A_{k,n-1}E_{l,n-2}A_{m,n-2})$ for the Bush and Mosteller (1955) Secondary Reinforcement Linear Model $(j,k,l,m=1,2)$ averaged over organisms and events. The conditionalsstatistics may be obtained by noting that in the non-contingent case

$$P(A_1|E_jA_kE_lA_m) = \frac{P(A_1E_jA_kE_lA_m)}{P(E_j)P(A_k|E_lA_m)P(E_l)P(A_m)}$$

$$P(A_1E_1A_1E_1A_1) = \pi^2\{V_1[1-\alpha_1-\alpha_1{}^2+\alpha_1{}^3] + V_2[\alpha_1(1+\alpha_1-2\alpha_1{}^2)] + V_3[\alpha_1{}^3]\}$$

$$P(A_1E_1A_1E_1A_2) = \pi^2\{V_1[\alpha_2(1-\alpha_1)] - V_2[\alpha_2(1-\alpha_1-\alpha_1\alpha_2)] - V_3[\alpha_1\alpha_2{}^2]\}$$

$$P(A_1E_1A_1E_2A_1) = \pi(1-\pi)\{V_1[1-\alpha_2-\alpha_1\alpha_2(1-\alpha_2)] + V_2[\alpha_2(1+\alpha_1-2\alpha_1\alpha_2)] + V_3[\alpha_1\alpha_2{}^2]\}$$

$$P(A_1E_1A_1E_2A_2) = \pi(1-\pi)\{V_1[\alpha_1(1-\alpha_1)] - V_2[\alpha_1(1-\alpha_1-\alpha_1{}^2)] - V_3[\alpha_1{}^3]\}$$

$$P(A_1E_1A_2E_1A_1) = \pi^2\{V_1[\alpha_1\alpha_2(1-\alpha_1)] - V_2[\alpha_1\alpha_2(1-2\alpha_1)] - V_3[\alpha_1{}^2\alpha_2]\}$$

$$P(A_1E_1A_2E_1A_2) = \pi^2\{V_1[\alpha_2{}^2] - V_2[\alpha_2{}^2(1+\alpha_2)] + V_3[\alpha_2{}^3]\}$$

$$P(A_1E_1A_2E_2A_1) = \pi(1-\pi)\{V_1[\alpha_2{}^2(1-\alpha_2)] - V_2[\alpha_2{}^2(1-2\alpha_2)] - V_3[\alpha_2{}^3]\}$$

$$P(A_1E_1A_2E_2A_2) = \pi(1-\pi)\{V_1[\alpha_1\alpha_2] - V_2[\alpha_1\alpha_2(1+\alpha_1)] + V_3[\alpha_1{}^2\alpha_2]\}$$

$$P(A_1E_2A_1E_1A_1) = \pi(1-\pi)\{V_1[1-\alpha_1(1+\alpha_2-\alpha_1\alpha_2)] + V_2[\alpha_1(1+\alpha_2-2\alpha_1\alpha_2)] + V_3[\alpha_1{}^2\alpha_2]\}$$

$$P(A_1E_2A_1E_1A_2) = \pi(1-\pi)\{V_1[\alpha_2(1-\alpha_2)] - V_2[\alpha_2(1-\alpha_2-\alpha_2{}^2)] - V_3[\alpha_2{}^3]\}$$

$$P(A_1E_2A_1E_2A_1) = (1-\pi)^2\{V_1[1-\alpha_2(1+\alpha_2-\alpha_2{}^2)] + V_2[\alpha_2(1+\alpha_2-2\alpha_2{}^2)] + V_3[\alpha_2{}^3]\}$$

$$P(A_1E_2A_1E_2A_2) = (1-\pi)^2\{V_1[\alpha_1(1-\alpha_2)] - V_2[\alpha_1(1-\alpha_2-\alpha_1\alpha_2)] - V_3[\alpha_1{}^2\alpha_2]\}$$

$$P(A_1E_2A_2E_1A_1) = \pi(1-\pi)\{V_1[\alpha_1{}^2(1-\alpha_1)] - V_2[\alpha_1{}^2(1-2\alpha_1)] - V_3[\alpha_1{}^3]\}$$

$$P(A_1E_2A_2E_1A_2) = \pi(1-\pi)\{V_1[\alpha_1\alpha_2] - V_2[\alpha_1\alpha_2(1+\alpha_2)] + V_3[\alpha_1\alpha_2{}^2]\}$$

$$P(A_1E_2A_2E_2A_1) = (1-\pi)^2\{V_1[\alpha_1\alpha_2(1-\alpha_2)] - V_2[\alpha_1\alpha_2(1-2\alpha_2)] - V_3[\alpha_1\alpha_2{}^2]\}$$

$$P(A_1E_2A_2E_2A_2) = (1-\pi)^2\{V_1[\alpha_1{}^2] - V_2[\alpha_1{}^2(1+\alpha_1)] + V_3[\alpha_1{}^3]\}$$

Appendix D

The following least-squares function was minimized to obtain all parameter estimates

$$N \sum_{i=1}^{2} \left[\begin{bmatrix} P(A_iE_1A_1E_1A_1) + \\ P(A_iE_2A_1E_2A_2) \end{bmatrix} - \begin{bmatrix} \hat{P}(A_iE_1A_1E_1A_1) + \\ \hat{P}(A_iE_2A_1E_2A_2) \end{bmatrix} \right]^2 +$$

$$N \sum_{i=1}^{2} \left[\begin{bmatrix} P(A_iE_2A_1E_1A_1) + \\ P(A_iE_2A_2E_2A_2) \end{bmatrix} - \begin{bmatrix} \hat{P}(A_iE_2A_1E_1A_1) + \\ \hat{P}(A_iE_2A_2E_2A_2) \end{bmatrix} \right]^2 +$$

$$N \sum_{i=1}^{2} \left[\begin{bmatrix} P(A_iE_1A_1E_2A_1) + \\ P(A_iE_1A_1E_1A_2) \end{bmatrix} - \begin{bmatrix} \hat{P}(A_iE_1A_1E_2A_1) + \\ \hat{P}(A_iE_1A_1E_1A_2) \end{bmatrix} \right]^2 +$$

$$N \sum_{i=1}^{2} \left[\begin{bmatrix} P(A_iE_2A_1E_2A_1) + \\ P(A_iE_2A_1E_1A_2) \end{bmatrix} - \begin{bmatrix} \hat{P}(A_iE_2A_1E_2A_1) + \\ \hat{P}(A_iE_2A_1E_1A_2) \end{bmatrix} \right]^2 +$$

$$N \sum_{i=1}^{2} \left[\begin{bmatrix} P(A_iE_1A_2E_2A_1) + \\ P(A_iE_2A_2E_1A_2) \end{bmatrix} - \begin{bmatrix} \hat{P}(A_iE_1A_2E_2A_1) + \\ \hat{P}(A_iE_2A_2E_1A_2) \end{bmatrix} \right]^2 +$$

$$N \sum_{i=1}^{2} \left[\begin{bmatrix} P(A_iE_2A_2E_2A_1) + \\ P(A_iE_1A_2E_2A_2) \end{bmatrix} - \begin{bmatrix} \hat{P}(A_iE_2A_2E_2A_1) + \\ \hat{P}(A_iE_1A_2E_2A_2) \end{bmatrix} \right]^2 +$$

$$N \sum_{i=1}^{2} \left[\begin{bmatrix} P(A_iE_1A_1E_2A_2) + \\ P(A_iE_1A_2E_1A_1) \end{bmatrix} - \begin{bmatrix} \hat{P}(A_iE_1A_1E_2A_2) + \\ \hat{P}(A_iE_1A_2E_1A_1) \end{bmatrix} \right]^2 +$$

$$N \sum_{i=1}^{2} \left[\begin{bmatrix} P(A_iE_2A_2E_1A_1) + \\ P(A_iE_1A_2E_1A_2) \end{bmatrix} - \begin{bmatrix} \hat{P}(A_iE_2A_2E_1A_1) + \\ \hat{P}(A_iE_1A_2E_1A_2) \end{bmatrix} \right]^2$$

where $N = \sum_{j=1}^{2} \sum_{k=1}^{2} \sum_{l=1}^{2} \sum_{m=1}^{2} M_{jklm}$ and M_{jklm} is defined as the number of occurrences of E_jA_k on trial $\underline{n}$-1 and E_lA_m on trial

n-2 over a block of trials of arbitrary size; and $P(A_i E_j A_k E_l A_m)$ and $\hat{P}(A_i E_j A_k E_l A_m)$, $(i,j,k,l,m = 1,2)$ are the observed and predicted second-order joint probabilities, respectively.

Appendix E

The algorithm used in parameter estimation has been described in great detail by Wood (1962) and consequently will be considered only briefly here. The function being minimized (e.g., see Appendix D) is computed for a point in the parameter space. The value of the function resulting from use of this set of coordinate (parameter values) is called a <u>base point</u>. Following calculation of the base point, a series of <u>exploratory moves</u> is made to determine the direction in which the minimum value of this function lies. These moves can be characterized by the computation of the function after individually increasing or decreasing each parameter from its value at the base point by a prescribed <u>step size</u>. If the function decreases with a move in one direction, that value of the parameter is retained and the next is varied; if the function increases with a given move, the reverse move is made. If the function increases with both moves, the step size for <u>that</u> parameter may be decreased. A move which decreases the size of the function is termed a <u>success</u> and the step size for <u>that</u> parameter is increased. The set of parameters obtained from the exploratory moves becomes a new base point. These exploratory moves are followed by a <u>pattern move</u>, i.e., each parameter is shifted away from this new base point by an amount equal to the difference between the old and new base point values, and in the same direction as the successful exploratory moves.

If the pattern move is a success, the current parameter values become the new base point from which a new set of exploratory moves is made. If the pattern move failed, the old base point is retained, the step size is reduced, and a new set of exploratory moves begins. This sequence of steps can be repeated until the parameters or the function being minimized is as stable as desired.

Appendix F

Listed below are the absolute average deviation (AAD) statistics used to evaluate the fits of:

Part A. First-order conditional probabilities (Tables 3e & 4e)

1. For each block

$$AAD = \frac{\sum_{j=1}^{2} \sum_{k=1}^{2} N_{ijk} \left| P_i(A_1|E_jA_k) - \hat{P}_i(A_1|E_jA_k) \right|}{\sum_{j=1}^{2} \sum_{k=1}^{2} N_{ijk}}, \quad i = 1,12$$

2. Over all blocks

$$AAD = \frac{\sum_{i=1}^{12} \sum_{j=1}^{2} \sum_{k=1}^{2} N_{ijk} \left| P_i(A_1|E_jA_k) - \hat{P}_i(A_1|E_jA_k) \right|}{\sum_{i=1}^{12} \sum_{j=1}^{2} \sum_{k=1}^{2} N_{ijk}}$$

where N_{ijk} is the number of E_jA_k events in the $i^{\underline{th}}$ block of 50 trials; and $P_i(A_1|E_jA_k)$ and $\hat{P}_i(A_1|E_jA_k)$ are the observed and predicted first-order conditional probabilities for the $i^{\underline{th}}$ block of 50 trials, respectively.

<u>Part B</u>. <u>Second-order conditional probabilities</u> (Tables 5a-d)

$$
\mathrm{AAD} = \frac{
\begin{aligned}
&\left[N_{1111} + N_{2122}\right] \left| P\left(A_1 \middle| \begin{matrix} E_1A_1E_1A_1 \\ E_2A_1E_2A_2 \end{matrix}\right) - \hat{P}\left(A_1 \middle| \begin{matrix} E_1A_1E_1A_1 \\ E_2A_1E_2A_2 \end{matrix}\right) \right| + \\
&\left[N_{2111} + N_{2222}\right] \left| P\left(A_1 \middle| \begin{matrix} E_2A_1E_1A_1 \\ E_2A_2E_2A_2 \end{matrix}\right) - \hat{P}\left(A_1 \middle| \begin{matrix} E_2A_1E_1A_1 \\ E_2A_2E_2A_2 \end{matrix}\right) \right| + \\
&\left[N_{1121} + N_{1112}\right] \left| P\left(A_1 \middle| \begin{matrix} E_1A_1E_2A_1 \\ E_1A_1E_1A_2 \end{matrix}\right) - \hat{P}\left(A_1 \middle| \begin{matrix} E_1A_1E_2A_1 \\ E_1A_1E_1A_2 \end{matrix}\right) \right| + \\
&\left[N_{2121} + N_{2112}\right] \left| P\left(A_1 \middle| \begin{matrix} E_2A_1E_2A_1 \\ E_2A_1E_1A_2 \end{matrix}\right) - \hat{P}\left(A_1 \middle| \begin{matrix} E_2A_1E_2A_1 \\ E_2A_1E_1A_2 \end{matrix}\right) \right| + \\
&\left[N_{1221} + N_{2212}\right] \left| P\left(A_1 \middle| \begin{matrix} E_1A_2E_2A_1 \\ E_2A_2E_1A_2 \end{matrix}\right) - \hat{P}\left(A_1 \middle| \begin{matrix} E_1A_2E_2A_1 \\ E_2A_2E_1A_2 \end{matrix}\right) \right| + \\
&\left[N_{2221} + N_{1222}\right] \left| P\left(A_1 \middle| \begin{matrix} E_2A_2E_2A_1 \\ E_1A_2E_2A_2 \end{matrix}\right) - \hat{P}\left(A_1 \middle| \begin{matrix} E_2A_2E_2A_1 \\ E_1A_2E_2A_2 \end{matrix}\right) \right| + \\
&\left[N_{1122} + N_{1211}\right] \left| P\left(A_1 \middle| \begin{matrix} E_1A_1E_2A_2 \\ E_1A_2E_1A_1 \end{matrix}\right) - \hat{P}\left(A_1 \middle| \begin{matrix} E_1A_1E_2A_2 \\ E_1A_2E_1A_1 \end{matrix}\right) \right| + \\
&\left[N_{2211} + N_{1212}\right] \left| P\left(A_1 \middle| \begin{matrix} E_2A_2E_1A_1 \\ E_1A_2E_1A_2 \end{matrix}\right) - \hat{P}\left(A_1 \middle| \begin{matrix} E_2A_2E_1A_1 \\ E_1A_2E_1A_2 \end{matrix}\right) \right|
\end{aligned}
}{
\sum_{j=1}^{2} \sum_{k=1}^{2} \sum_{l=1}^{2} \sum_{m=1}^{2} N_{jklm}
}
$$

where N_{jklm} is the number of occurrences of E_jA_k on trial <u>n</u>-1 and E_lA_m on trial <u>n</u>-2 in the last block of 100 trials; and e.g.,

$$
P\left(A_1 \middle| \begin{matrix} E_1A_1E_1A_1 \\ E_2A_1E_2A_2 \end{matrix}\right) \quad \text{and} \quad \hat{P}\left(A_1 \middle| \begin{matrix} E_1A_1E_1A_1 \\ E_2A_1E_2A_2 \end{matrix}\right)
$$

are the observed and predicted second-order conditional probabilities, respectively; where response probability is conditionalized against a pooled probability space.

Part C. Reinforcement-run statistics (R-R-S) (Table 6c)

1. Individual statistics

$$AAD = \left| P(A_1|E_i \ldots E_j) - \hat{P}(A_1|E_i \ldots E_j) \right|, \; i,j = 1,2, \; i \neq j$$

2. All E_i R-R-S

$$AAD = \frac{\left[\begin{array}{l} N(\cdot \mid E_iE_j) \left| [P(A_i|E_iE_j) - \hat{P}(A_1|E_iE_j)] \right| + \\ N(\cdot|E_iE_iE_j) \left| [P(A_1|E_iE_iE_j) - \hat{P}(A_1|E_iE_iE_j)] \right| + \\ N(\cdot|E_iE_iE_iE_j) \left| [P(A_1|E_iE_iE_iE_j) - \hat{P}(A_1|E_iE_iE_iE_j)] \right| \end{array}\right]}{[N(\cdot|E_iE_j) + N(\cdot|E_iE_iE_j) + N(\cdot|E_iE_iE_iE_j)]}, \; i,j=1,2, i \neq j$$

3. E_1 and E_2 R-R-S

$$AAD = \frac{\sum_{i=1}^{2} \sum_{\substack{j=1 \\ i \neq j}}^{2} \left[\begin{array}{l} N(\cdot|E_iE_j) \left| [P(A_1|E_iE_j) - \hat{P}(A_1|E_iE_j)] \right| + \\ N(\cdot|E_iE_iE_j) \left| [P(A_1|E_iE_iE_j) - \hat{P}(A_1|E_iE_iE_j)] \right| + \\ N(\cdot|E_iE_iE_iE_j) \left| [P(A_1|E_iE_iE_iE_j) - \hat{P}(A_1|E_iE_iE_iE_j)] \right| \end{array}\right]}{\{\sum_{i=1}^{2} \sum_{\substack{j=1 \\ i \neq j}}^{2} [N(\cdot|E_iE_j) + N(\cdot|E_iE_iE_j) + N(\cdot|E_iE_iE_iE_j)]\}}$$

where $P(A_1|E_i \ldots E_j)$'s and $\hat{P}(A_1|E_i \ldots E_j)$'s are the observed and predicted R-R-Ss, respectively; and $N(\cdot|E_i \ldots E_j)$'s are the denominators for the observed R-R-Ss in the last block of 100 trials. Each of these values, $N(\cdot|E_i \ldots E_j)$, $P(A_1|E_i \ldots E_j)$, and $\hat{P}(A_1|E_i \ldots E_j)$, appear in Table 6a-b.

Part D. Learning curve (Table 7a-b)

$$AAD = \frac{\sum_{b=1}^{12} \left| [P(A_{1,b}) - \hat{P}(A_{1,b})] \right|}{12}$$

where $P(A_{1,b})$ and $\hat{P}(A_{1,b})$ are the observed and predicted marginal response probability for block b, respectively.

Footnotes

1. We will refer to a model as "stochastic" if it yields predictions of trial to trial changes in performance.

2. For Ss trained under symmetric-payoff, the outcome of each correct prediction, i.e., $A_{1,n}E_{1,n}$ or $A_{2,n}E_{2,n}$, results in the same payoff, and the outcome of each incorrect prediction, i.e., $A_{1,n}E_{2,n}$ or $A_{2,n}E_{1,n}$, results in the same loss. When the payoff and loss for each correct and incorrect prediction, respectively, are not equal, but rather a function of the particular response-event outcome, payoff is said to be non-symmetric.

3. Myers, Personal Communication.

4. Jones, Personal Communication.

5. The observed standard deviation of response probabilities in Table 8 was computed with the following formula:

$$\sqrt{\frac{\sum_{i=1}^{40} P_{ij}^{2} - \frac{\left(\sum_{i=1}^{40} P_{ij}\right)^{2}}{40}}{39}}\,, \quad j=1,4$$

where P_{ij} is the proportion of A_1 responses for the i^{th} S in the j^{th} experimental group for trials 501-600.

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