UMass Physics 131 Unit 3

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UNIT 3 OVERVIEW

UMASS AMHERST Instructor's Notes

This overview is also available as a video **here (https://www.youtube.com/watch?v=rzGwxTZtnrA)** .

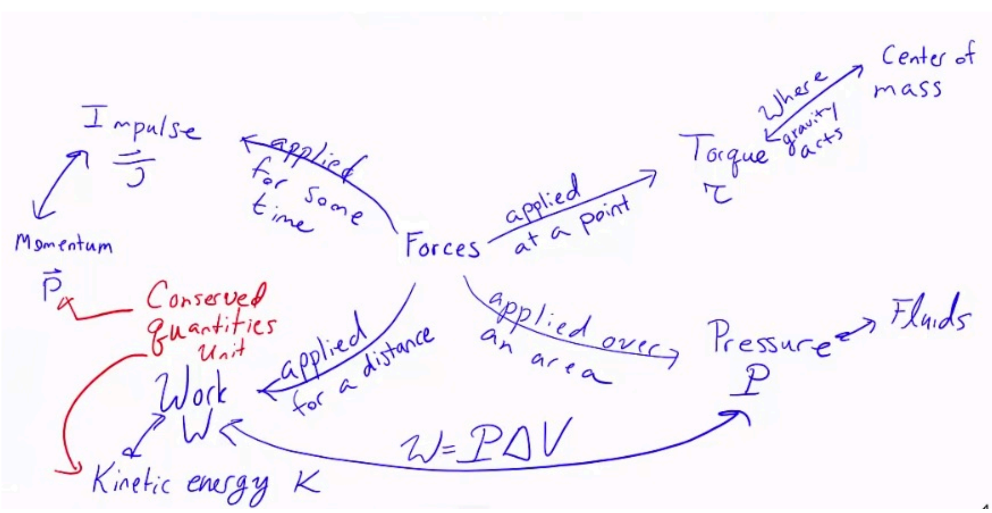
What is the meaning of the title of this unit? Well in our last unit, we introduced the ideas of forces and Newton's laws, and while forces are sufficient to determine the acceleration, there are clearly other quantities that are interesting if we stop and think about it. For example, it is easier to open a door by pushing on the door on the far side from the hinge where the knob is located than by pushing on the door near the hinge. If you've never really thought about this while opening a door, I encourage you to go try it. Similarly, if I apply a force to a ball for a short time, I get a different result than if I apply that same force for a long amount of time. If I apply the force for a long amount of time, the final velocity of the ball will be larger. For these reasons, we will be exploring in this unit forces in conjunction with other quantities. All our principles from the last unit still work, and many of these situations could be analyzed solely within the context of Newton's laws if you need it to. However, the new ideas we're going to introduce in this unit are often simpler to think about and to work with. However, with all the new concepts, choosing which concept to apply in each situation becomes its own unique challenge.

So, let's do a quick overview of the different concepts we're going to talk about in this unit. The first unit we will discuss is torque, and this is the fact that where forces are applied can matter. So, this goes back to the door; applying a force near the hinge results in a different experience than applying a force far away from the hinge at the knob. The next quantity is impulse; how long we apply a force also matters. We will also introduce the idea of pressure. Pressure is the fact the area over which the force is applied can matter. This quantity is particularly relevant when we discuss fluids, which, remember, include both gases and liquids. The final quantity we will discuss in this unit is work. Work is the fact that the distance over which the forces applied matters. If I apply a force over a long distance, I can get a different result than if I apply that same force over a short distance. Work can also be expressed in terms of pressure to talk about the work done on, or by, fluids.

There are some new symbols to learn when discussing these quantities. Torque is represented by the τ. Impulse is represented by $\vec{J}$; note that it is a vector quantity. Pressure, represented by the capital P, and work, I will use the capital W. Along the way in discussing these concepts, we will meet some other important ideas.

When we discuss torque, it will be important to introduce the idea of center of gravity. Torque is when you're interested about where the forces are being applied. If you're interested in where the forces are being applied, then you need to think about where does the force of gravity act. This is the idea behind center of gravity. When we discuss impulse, we will introduce the quantity momentum which uses a lowercase p, and you can see is also a vector. Momentum is a quantity connected to impulse, which we will revisit in greater detail in our unit on conserved quantities. The final quantity we will introduce in this unit is the quantity of kinetic energy represented by a capital K. Kinetic energy is connected to work, and again, we will revisit this quantity in our unit on conserved quantities.

Because of all the different quantities we're introducing in this unit, a nice way to organize them might be map.



So, we can think of the idea of forces from the last unit. So, a force applied at a point takes us to a torque, which we represent by the Greek τ, and the idea of torque is going to be connected to the idea of center of mass, as the center of mass dictates where

gravity acts. We can also talk about forces being applied over an area, and this brings us to the idea of pressure, which again we represent by a capital P, and pressure will connect to our study of fluids. We can talk about forces being applied for some amount of time, and this brings us to the idea of impulse, J, which is connected to the idea of momentum as we'll see in this prep, which is represented by the lowercase p. And we can talk about forces being applied for a distance, bringing us to work, W, which is related to the idea of kinetic energy, K, and both will be related to our conserved quantities unit. Also, during this prep, you will see how work connects to the idea of pressure as work being pressure times the change in volume. This idea of a map to help you sort of organize all the information is a great study tool when studying physics, and I encourage you to maybe build your own using this as a core as you go through this unit.

1 IMPULSE

1.1 Linear Momentum and Force

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Your Quiz will Cover
- Calculate the momentum for any object
- Recall that momentum is a vector
- From the change in momentum, compute the average force

Linear Momentum

The scientific definition of linear momentum is consistent with most people's intuitive understanding of momentum: a large, fast-moving object has greater momentum than a smaller, slower object. **Linear momentum** is defined as the product of a system's mass multiplied by its velocity. In symbols, linear momentum is expressed as

$$\mathbf{p} = m\mathbf{v}. \tag{1.1}$$

Momentum is directly proportional to the object's mass and also its velocity. Thus the greater an object's mass or the greater its velocity, the greater its momentum. Momentum $\mathbf{p}$ is a vector having the same direction as the velocity $\mathbf{v}$. The SI unit for momentum is $\mathrm{kg \cdot m/s}$.

Linear Momentum

Linear momentum is defined as the product of a system's mass multiplied by its velocity:

$$\mathbf{p} = m\mathbf{v}. \tag{1.2}$$

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The main focus for this section is the definition of momentum above, as well as the calculation of momentum. The following is a good example of what we expect you to be able to do with regards to the calculation. Also, pay attention to the discussion in the example as well, as it talks about how both mass and velocity can affect momentum.

Example 1.1 Calculating Momentum: A Football Player and a Football

(a) Calculate the momentum of a 110-kg football player running at 8.00 m/s. (b) Compare the player's momentum with the momentum of a hard-thrown 0.410-kg football that has a speed of 25.0 m/s.

Strategy

No information is given regarding direction, and so we can calculate only the magnitude of the momentum, p. (As usual, a symbol that is in italics is a magnitude, whereas one that is italicized, boldfaced, and has an arrow is a vector.) In both parts of this example, the magnitude of momentum can be calculated directly from the definition of momentum given in the equation, which becomes

$$p = mv \tag{1.3}$$

when only magnitudes are considered.

Solution for (a)

To determine the momentum of the player, substitute the known values for the player's mass and speed into the equation.

$$p_{\text{player}} = (110\ \mathrm{kg})(8.00\ \mathrm{m/s}) = 880\ \mathrm{kg \cdot m/s} \tag{1.4}$$

Solution for (b)

To determine the momentum of the ball, substitute the known values for the ball's mass and speed into the equation.

$$p_{\text{ball}} = (0.410\text{ kg})(25.0\text{ m/s}) = 10.3\text{ kg}\cdot\text{m/s} \tag{1.5}$$

The ratio of the player's momentum to that of the ball is

$$\frac{p_{\text{player}}}{p_{\text{ball}}} = \frac{880}{10.3} = 85.9. \tag{1.6}$$

Discussion

Although the ball has greater velocity, the player has a much greater mass. Thus the momentum of the player is much greater than the momentum of the football, as you might guess. As a result, the player's motion is only slightly affected if he catches the ball. We shall quantify what happens in such collisions in terms of momentum in later sections.

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The next part will tie into the next section, impulse, so be sure to pay attention to this part here as well.

Momentum and Newton's Second Law

The importance of momentum, unlike the importance of energy, was recognized early in the development of classical physics. Momentum was deemed so important that it was called the "quantity of motion." Newton actually stated his **second law of motion** in terms of momentum: The net external force equals the change in momentum of a system divided by the time over which it changes. Using symbols, this law is

$$\mathbf{F}_{\text{net}} = \frac{\Delta\mathbf{p}}{\Delta t}, \tag{1.7}$$

where $\mathbf{F}_{\text{net}}$ is the net external force, $\Delta\mathbf{p}$ is the change in momentum, and Δt is the change in time.

Newton's Second Law of Motion in Terms of Momentum

The net external force equals the change in momentum of a system divided by the time over which it changes.

$$\mathbf{F}_{\text{net}} = \frac{\Delta\mathbf{p}}{\Delta t} \tag{1.8}$$

Making Connections: Force and Momentum

Force and momentum are intimately related. Force acting over time can change momentum, and Newton's second law of motion, can be stated in its most broadly applicable form in terms of momentum. Momentum continues to be a key concept in the study of atomic and subatomic particles in quantum mechanics.

This statement of Newton's second law of motion includes the more familiar $\mathbf{F}_{\text{net}} = m\mathbf{a}$ as a special case. We can derive this form as follows. First, note that the change in momentum $\Delta\mathbf{p}$ is given by

$$\Delta\mathbf{p} = \Delta(m\mathbf{v}). \tag{1.9}$$

If the mass of the system is constant, then

$$\Delta(m\mathbf{v}) = m\Delta\mathbf{v}. \tag{1.10}$$

So that for constant mass, Newton's second law of motion becomes

$$\mathbf{F}_{\text{net}} = \frac{\Delta\mathbf{p}}{\Delta t} = \frac{m\Delta\mathbf{v}}{\Delta t}. \tag{1.11}$$

Because $\frac{\Delta\mathbf{v}}{\Delta t} = \mathbf{a}$, we get the familiar equation

$$\mathbf{F}_{\text{net}} = m\mathbf{a} \tag{1.12}$$

when the mass of the system is constant.

Newton's second law of motion stated in terms of momentum is more generally applicable because it can be applied to systems where the mass is changing, such as rockets, as well as to systems of constant mass. We will consider systems with varying

mass in some detail; however, the relationship between momentum and force remains useful when mass is constant, such as in the following example.

Example 1.2 Calculating Force: Venus Williams' Racquet

During the 2007 French Open, Venus Williams hit the fastest recorded serve in a premier women's match, reaching a speed of 58 m/s (209 km/h). What is the average force exerted on the 0.057-kg tennis ball by Venus Williams' racquet, assuming that the ball's speed just after impact is 58 m/s, that the initial horizontal component of the velocity before impact is negligible, and that the ball remained in contact with the racquet for 5.0 ms (milliseconds)?

Strategy

This problem involves only one dimension because the ball starts from having no horizontal velocity component before impact. Newton's second law stated in terms of momentum is then written as

$$\mathbf{F}_{\text{net}} = \frac{\Delta \mathbf{p}}{\Delta t}. \tag{1.13}$$

As noted above, when mass is constant, the change in momentum is given by

$$\Delta p = m\Delta v = m(v_{\text{f}} - v_{\text{i}}). \tag{1.14}$$

In this example, the velocity just after impact and the change in time are given; thus, once Δp is calculated, $F_{\text{net}} = \frac{\Delta p}{\Delta t}$ can be used to find the force.

Solution

To determine the change in momentum, substitute the values for the initial and final velocities into the equation above.

$$\begin{aligned} \Delta p &= m(v_{\text{f}} - v_{\text{i}}) \\ &= (0.057\text{ kg})(58\text{ m/s} - 0\text{ m/s}) \\ &= 3.306\text{ kg}\cdot\text{m/s} \approx 3.3\text{ kg}\cdot\text{m/s} \end{aligned} \tag{1.15}$$

Now the magnitude of the net external force can determined by using $F_{\text{net}} = \frac{\Delta p}{\Delta t}$:

$$\begin{aligned} F_{\text{net}} &= \frac{\Delta p}{\Delta t} = \frac{3.306\text{ kg}\cdot\text{m/s}}{5.0\times10^{-3}\text{ s}} \\ &= 661\text{ N} \approx 660\text{ N}, \end{aligned} \tag{1.16}$$

where we have retained only two significant figures in the final step.

Discussion

This quantity was the average force exerted by Venus Williams' racquet on the tennis ball during its brief impact (note that the ball also experienced the 0.56-N force of gravity, but that force was not due to the racquet). This problem could also be solved by first finding the acceleration and then using $F_{\text{net}} = ma$, but one additional step would be required compared with the strategy used in this example.

Section Summary

- Linear momentum (*momentum* for brevity) is defined as the product of a system's mass multiplied by its velocity.
- In symbols, linear momentum $\mathbf{p}$ is defined to be

$$\mathbf{p} = m\mathbf{v}, \tag{1.17}$$

  where m is the mass of the system and $\mathbf{v}$ is its velocity.
- The SI unit for momentum is $\text{kg}\cdot\text{m/s}$.
- Newton's second law of motion in terms of momentum states that the net external force equals the change in momentum of a system divided by the time over which it changes.
- In symbols, Newton's second law of motion is defined to be

$$\mathbf{F}_{\text{net}} = \frac{\Delta \mathbf{p}}{\Delta t}, \tag{1.18}$$

  $\mathbf{F}_{\text{net}}$ is the net external force, $\Delta\mathbf{p}$ is the change in momentum, and Δt is the change time.

Conceptual Questions

Exercise 1.1

An object that has a small mass and an object that has a large mass have the same momentum. Which object has the largest kinetic energy?

Exercise 1.2

An object that has a small mass and an object that has a large mass have the same kinetic energy. Which mass has the largest momentum?

Exercise 1.3

Professional Application

Football coaches advise players to block, hit, and tackle with their feet on the ground rather than by leaping through the air. Using the concepts of momentum, work, and energy, explain how a football player can be more effective with his feet on the ground.

Exercise 1.4

How can a small force impart the same momentum to an object as a large force?

Problems & Exercises

Exercise 1.5

(a) Calculate the momentum of a 2000-kg elephant charging a hunter at a speed of 7.50 m/s. (b) Compare the elephant's momentum with the momentum of a 0.0400-kg tranquilizer dart fired at a speed of 600 m/s. (c) What is the momentum of the 90.0-kg hunter running at 7.40 m/s after missing the elephant?

Solution

(a) $1.50\times10^{4}\text{ kg}\cdot\text{m/s}$

(b) 625 to 1

(c) $6.66\times10^{2}\text{ kg}\cdot\text{m/s}$

Exercise 1.6

(a) What is the mass of a large ship that has a momentum of $1.60\times10^{9}\text{ kg}\cdot\text{m/s}$, when the ship is moving at a speed of 48.0 km/h? (b) Compare the ship's momentum to the momentum of a 1100-kg artillery shell fired at a speed of 1200 m/s.

Exercise 1.7

(a) At what speed would a $2.00\times10^{4}\text{-kg}$ airplane have to fly to have a momentum of $1.60\times10^{9}\text{ kg}\cdot\text{m/s}$ (the same as the ship's momentum in the problem above)? (b) What is the plane's momentum when it is taking off at a speed of 60.0 m/s? (c) If the ship is an aircraft carrier that launches these airplanes with a catapult, discuss the implications of your answer to (b) as it relates to recoil effects of the catapult on the ship.

Solution

(a) $8.00\times10^{4}\text{ m/s}$

(b) $1.20\times10^{6}\ \text{kg}\cdot\text{m/s}$

(c) Because the momentum of the airplane is 3 orders of magnitude smaller than of the ship, the ship will not recoil very much. The recoil would be $-0.0100\ \text{m/s}$, which is probably not noticeable.

Exercise 1.8

(a) What is the momentum of a garbage truck that is $1.20\times10^{4}\ \text{kg}$ and is moving at $10.0\ \text{m/s}$? (b) At what speed would an 8.00-kg trash can have the same momentum as the truck?

Exercise 1.9

A runaway train car that has a mass of 15,000 kg travels at a speed of $5.4\ \text{m/s}$ down a track. Compute the time required for a force of 1500 N to bring the car to rest.

Solution
54 s

Exercise 1.10

The mass of Earth is $5.972\times10^{24}\ \text{kg}$ and its orbital radius is an average of $1.496\times10^{11}\ \text{m}$. Calculate its linear momentum.

1.2 Impulse

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Your Quiz will Cover
- From an impulse, compute the change in momentum
- Identify which aspect of an F(t) graph represents impulse
- Compute the net change in momentum of an object using an F(t) graph

The effect of a force on an object depends on how long it acts, as well as how great the force is. In **m42156 (https://legacy.cnx.org/content/m42156/latest/#fs-id1356444)** , a very large force acting for a short time had a great effect on the momentum of the tennis ball. A small force could cause the same **change in momentum**, but it would have to act for a much longer time. For example, if the ball were thrown upward, the gravitational force (which is much smaller than the tennis racquet's force) would eventually reverse the momentum of the ball. Quantitatively, the effect we are talking about is the change in momentum $\Delta\mathbf{p}$.

By rearranging the equation $\mathbf{F}_{\text{net}} = \dfrac{\Delta\mathbf{p}}{\Delta t}$ to be

$$\Delta\mathbf{p} = \mathbf{F}_{\text{net}}\Delta t, \tag{1.19}$$

we can see how the change in momentum equals the average net external force multiplied by the time this force acts. The quantity $\mathbf{F}_{\text{net}}\Delta t$ is given the name **impulse**. Impulse is the same as the change in momentum.

Impulse: Change in Momentum

Change in momentum equals the average net external force multiplied by the time this force acts.

$$\Delta\mathbf{p} = \mathbf{F}_{\text{net}}\Delta t \tag{1.20}$$

The quantity $\mathbf{F}_{\text{net}}\Delta t$ is given the name impulse.

There are many ways in which an understanding of impulse can save lives, or at least limbs. The dashboard padding in a

car, and certainly the airbags, allow the net force on the occupants in the car to act over a much longer time when there is a sudden stop. The momentum change is the same for an occupant, whether an air bag is deployed or not, but the force (to bring the occupant to a stop) will be much less if it acts over a larger time. Cars today have many plastic components. One advantage of plastics is their lighter weight, which results in better gas mileage. Another advantage is that a car will crumple in a collision, especially in the event of a head-on collision. A longer collision time means the force on the car will be less. Deaths during car races decreased dramatically when the rigid frames of racing cars were replaced with parts that could crumple or collapse in the event of an accident.

Bones in a body will fracture if the force on them is too large. If you jump onto the floor from a table, the force on your legs can be immense if you land stiff-legged on a hard surface. Rolling on the ground after jumping from the table, or landing with a parachute, extends the time over which the force (on you from the ground) acts.

Example 1.3 Calculating Magnitudes of Impulses: Two Billiard Balls Striking a Rigid Wall

Two identical billiard balls strike a rigid wall with the same speed, and are reflected without any change of speed. The first ball strikes perpendicular to the wall. The second ball strikes the wall at an angle of 30° from the perpendicular, and bounces off at an angle of 30° from perpendicular to the wall.

(a) Determine the direction of the force on the wall due to each ball.

(b) Calculate the ratio of the magnitudes of impulses on the two balls by the wall.

Strategy for (a)

In order to determine the force on the wall, consider the force on the ball due to the wall using Newton’s second law and then apply Newton’s third law to determine the direction. Assume the x-axis to be normal to the wall and to be positive in the initial direction of motion. Choose the y-axis to be along the wall in the plane of the second ball’s motion. The momentum direction and the velocity direction are the same.

Solution for (a)

The first ball bounces directly into the wall and exerts a force on it in the $+x$ direction. Therefore the wall exerts a force on the ball in the $-x$ direction. The second ball continues with the same momentum component in the y direction, but reverses its x-component of momentum, as seen by sketching a diagram of the angles involved and keeping in mind the proportionality between velocity and momentum.

These changes mean the change in momentum for both balls is in the $-x$ direction, so the force of the wall on each ball is along the $-x$ direction.

Strategy for (b)

Calculate the change in momentum for each ball, which is equal to the impulse imparted to the ball.

Solution for (b)

Let u be the speed of each ball before and after collision with the wall, and m the mass of each ball. Choose the x-axis and y-axis as previously described, and consider the change in momentum of the first ball which strikes perpendicular to the wall.

$$p_{\text{xi}} = mu;\ p_{\text{yi}} = 0 \tag{1.21}$$

$$p_{\text{xf}} = -mu;\ p_{\text{yf}} = 0 \tag{1.22}$$

Impulse is the change in momentum vector. Therefore the x-component of impulse is equal to $-2mu$ and the y-component of impulse is equal to zero.

Now consider the change in momentum of the second ball.

$$p_{\text{xi}} = mu\cos 30^{\circ};\ p_{\text{yi}} = -mu\sin 30^{\circ} \tag{1.23}$$

$$p_{\text{xf}} = -mu\cos 30^{\circ};\ p_{\text{yf}} = -mu\sin 30^{\circ} \tag{1.24}$$

It should be noted here that while p_{x} changes sign after the collision, p_{y} does not. Therefore the x-component of impulse is equal to $-2mu\cos 30^{\circ}$ and the y-component of impulse is equal to zero.

The ratio of the magnitudes of the impulse imparted to the balls is

$$\frac{2mu}{2mu\cos 30^{\circ}} = \frac{2}{\sqrt{3}} = 1.155. \tag{1.25}$$

Discussion

The direction of impulse and force is the same as in the case of (a); it is normal to the wall and along the negative x -direction. Making use of Newton's third law, the force on the wall due to each ball is normal to the wall along the positive x -direction.

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Pay attention to the example above, as it provides a good example of how impulse and momentum are connected, as well as working with momentum and impulse as vector quantities. Also, this next line brings up an important idea to impulse. Most of the time, the force is not constant over time, however, you can think of the average force instead.

Our definition of impulse includes an assumption that the force is constant over the time interval Δt . *Forces are usually not constant*. Forces vary considerably even during the brief time intervals considered. It is, however, possible to find an average effective force F_{eff} that produces the same result as the corresponding time-varying force. **Figure 1.1** shows a graph of what an actual force looks like as a function of time for a ball bouncing off the floor. The area under the curve has units of momentum and is equal to the impulse or change in momentum between times t_1 and t_2 . That area is equal to the area inside the rectangle bounded by F_{eff} , t_1 , and t_2 . Thus the impulses and their effects are the same for both the actual and effective forces.

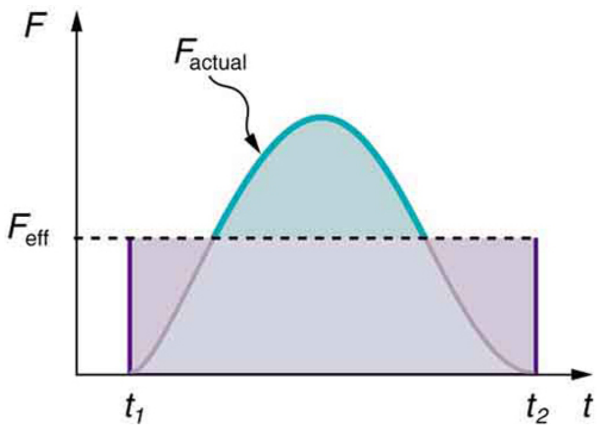

Figure 1.1 A graph of force versus time with time along the x -axis and force along the y -axis for an actual force and an equivalent effective force. The areas under the two curves are equal.

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To get impulse from a graph, you can take the area under the curve, since impulse is force times time. The average force here is the force that makes a rectangle with the same area as the area under the curve, which would also result in the same impulse.

Making Connections: Take-Home Investigation—Hand Movement and Impulse

Try catching a ball while "giving" with the ball, pulling your hands toward your body. Then, try catching a ball while keeping your hands still. Hit water in a tub with your full palm. After the water has settled, hit the water again by diving your hand with your fingers first into the water. (Your full palm represents a swimmer doing a belly flop and your diving hand represents a swimmer doing a dive.) Explain what happens in each case and why. Which orientations would you advise people to avoid and why?

Making Connections: Constant Force and Constant Acceleration

The assumption of a constant force in the definition of impulse is analogous to the assumption of a constant acceleration in

kinematics. In both cases, nature is adequately described without the use of calculus.

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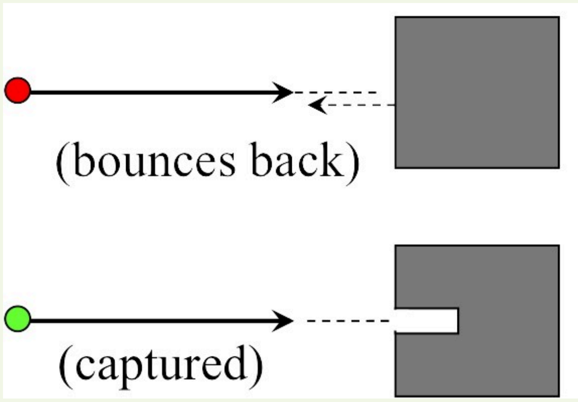
Here's an additional example of interest. This example is from:
umdberg / Example: The impulse-momentum theorem. Available at: **http://umdberg.pbworks.com/w/page/98831066/Example%3A%20The%20impulse-momentum%20theorem. (http://umdberg.pbworks.com/w/page/98831066/Example%3A%20The%20impulse-momentum%20theorem)** (Accessed: 24th July 2017)

Example 1.4 UMDBerg / Example: The impulse-momentum theorem

A box rests on an air table and can slide freely without friction. If a small frictionless puck is slid towards the box consider two situations: it bounces straight back with about the same velocity or it is captured. If the interaction times between the box and the puck are the same, which puck exerts a greater force on the box?



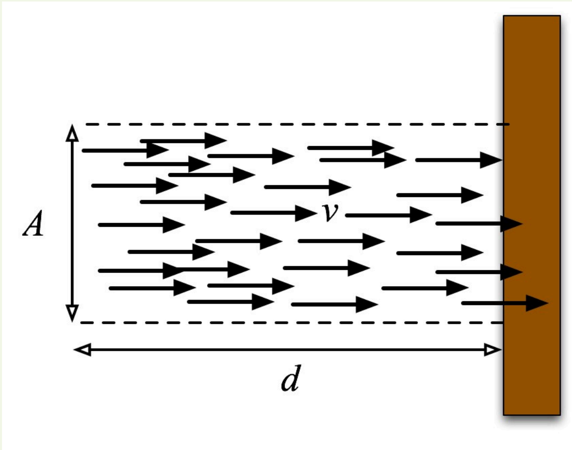
This is a rather trivial problem and doesn't seem very interest. We'll see however, in the next problem that it has interesting implications.

This is a qualitative problem, but we can still use an equation to solve it: the Impulse-momentum theorem. The change in momentum of the puck is equal to the impulse it receives from the box. The box seems much bigger than the puck so lets ignore the box's motion at first. If the puck has mass m and velocity v in the captured case the magnitude of the change in momentum is about mv -- it goes from mv to 0. In the bounces back case, the momentum of the puck goes from mv to $-mv$, so the magnitude of the change is $2mv$. (Remember that momentum is a vector quantity. It first decrease to 0, then decreases even further to negative values.) So the impulse received by the puck from the box is twice as big in the bounces back case as in the captured case. If the interaction times are the same (given) then the force the box exerts in the bounces back case is twice as big as in the captured case.

But that's the force of the box on the puck. What about the force of the puck on the box? Of course these two forces are related by **Newton's 3rd law (http://umdberg.pbworks.com/w/page/68390209/Newton's%203rd%20law%20(2013))** : In any interaction, the force that two objects exert on each other is equal and opposite. So if we know the force the box exerts on the puck, we know the force the puck exerts on the box.

This trivial case can be imbedded in a much more interesting case: molecules hitting a wall. Again, we will take only a simple case -- a stream of molecules in a vacuum. But we will see later that the same reasoning will allow us to understand how a gas exerts pressure and to extract the physical meaning of the ideal gas law in terms of molecules.

Suppose of stream of gas having cross sectional area A is traveling in a vacuum and is directed at a wall. If the density of molecules in the gas is n (number of molecules per cubic meter) and they are traveling with a speed v, what will be the average force that the molecules exert on the wall if (a) they stick to the wall, and (b) they bounce off the wall with the same speed they hit the wall with?



As is typical in any problem, there are assumptions hidden in the way the problem is stated and we have to figure out how to treat it. The wall is being bombarded by lots of little molecules. Each one that hits it will exert a sudden quick force on the wall and then so will the next, and the next, etc. So there will be lots of tiny little forces that vary quickly. The problem can't mean for us to calculate those -- there isn't enough information about the wall molecule interaction. But the fact that the problem uses a macroscopic word ("wall") and a microscopic word ("molecule") suggests that we might make some reasonable approximations.

So let's assume that we have lots of molecules in the gas and that they are moving fast. The word "average" suggests that we shouldn't focus on the individual fluctuations of the force but rather on the result of lots of molecules. Since "wall" implies much, much bigger than a molecule, let's assume that the wall doesn't move significantly when a molecule hits it. (A typical molecule has a mass on the order of 10^{-26} kg and a wall might have a mass of a few kgs.)

Each molecule that hits the wall changes its momentum. To get a force, we might use the impulse-momentum theorem. But that gives the force the wall exerts on the molecule. We want the force the molecule exerts on the wall! Of course these two forces are related by **Newton's 3rd law (http://umdberg.pbworks.com/w/page/68390209/Newton's%203rd%20law%20(2013))** : In any interaction, the force that two objects exert on each other is equal and opposite. So if we know the force the wall exerts on the molecule, we know the force the molecule exerts on the wall. Since the times during the interaction are equal, the impulse that the wall gives to the molecule must be equal and opposite to the impulse that the molecule gives to the wall.

This also resolves the time issue. On a time scale natural for the wall, lots of molecules will hit it. The impulse momentum theorem tells us the amount of impulse the wall must provide to a bunch of molecules in a certain time interval, Δ*t*. This will then tell us the amount of impulse the molecules provide to the wall in that time. Since we are told what happens to the velocities of the molecules, we can figure out their momentum change. Then we can calculate the average force the molecules exert on the wall.

$$\left\langle \vec{F}_{wall \to molecules} \right\rangle \Delta t = \Delta \vec{p}_{molecules}$$

$$\left\langle \vec{F}_{molecules \to wall} \right\rangle = \left\langle \vec{F}_{wall \to molecules} \right\rangle = \frac{\Delta \vec{p}_{molecules}}{\Delta t}$$

This is a rather standard way to use the Impulse-Momentum theorem. If you know the momentum change in a time interval, you can infer the impulse and therefore something about the average forces during that interval.

Let's see how that works here. Consider case (a): the molecules stick to the wall. In that case, the molecule initially had momentum mv and after the collision it basically stops. (Assuming that the wall doesn't recoil significantly. This of course depends on our assumptions about how big the wall is and how big the stream of gas is.) This means each molecule changes its momentum by an amount mv: from mv to 0.

Now let's consider a time interval in which many molecules will hit the wall. In a time interval, Δ*t,* how many will hit? To get this, look at the figure above. In a time interval, Δ*t*, a molecule will move a distance $d = v\Delta t$. If we take our distance *d* in the figure to be $v\Delta t$ then all the molecules in there will hit the wall and stick. How many is that? Well, we know the density and the volume of molecules hitting the wall is A x $d = Av\Delta t$. So the total number, *N*, hitting the wall in that time is the density times the volume or

N = number hitting the wall in time $\Delta t = nAv\Delta t$

So since each molecule changes its momentum by mv, the total change in momentum of the molecules in that time is Nmv, which gives a force

$$\left\langle \vec{F}_{molecules \to wall} \right\rangle = \left\langle \vec{F}_{wall \to molecules} \right\rangle = \frac{\Delta \vec{p}_{molecules}}{\Delta t} = \frac{mv(nAv\Delta t)}{\Delta t} = mnAv^2$$

For case (b), if each molecule bounces back with the same speed as it entered it changes its momentum from *mv* to -*mv*, a total change of 2*mv*. Therefore, the result will be twice as big as if the molecule stuck to the wall.

Joe Redish 8/6/15

Section Summary

- Impulse, or change in momentum, equals the average net external force multiplied by the time this force acts:

$$\Delta \mathbf{p} = \mathbf{F}_{\text{net}} \Delta t. \quad (1.26)$$

- Forces are usually not constant over a period of time.

Conceptual Questions

Exercise 1.11

Professional Application

Explain in terms of impulse how padding reduces forces in a collision. State this in terms of a real example, such as the advantages of a carpeted vs. tile floor for a day care center.

Exercise 1.12

While jumping on a trampoline, sometimes you land on your back and other times on your feet. In which case can you reach a greater height and why?

Exercise 1.13

Professional Application

Tennis racquets have “sweet spots.” If the ball hits a sweet spot then the player's arm is not jarred as much as it would be otherwise. Explain why this is the case.

Problems & Exercises

Exercise 1.14

A bullet is accelerated down the barrel of a gun by hot gases produced in the combustion of gun powder. What is the average force exerted on a 0.0300-kg bullet to accelerate it to a speed of 600 m/s in a time of 2.00 ms (milliseconds)?

Solution

$9.00 \times 10^3 \text{ N}$

Exercise 1.15

Professional Application

A car moving at 10 m/s crashes into a tree and stops in 0.26 s. Calculate the force the seat belt exerts on a passenger in the car to bring him to a halt. The mass of the passenger is 70 kg.

Exercise 1.16

A person slaps her leg with her hand, bringing her hand to rest in 2.50 milliseconds from an initial speed of 4.00 m/s. (a) What is the average force exerted on the leg, taking the effective mass of the hand and forearm to be 1.50 kg? (b) Would the force be any different if the woman clapped her hands together at the same speed and brought them to rest in the same time? Explain why or why not.

Solution

a) 2.40×10^{3} N toward the leg

b) The force on each hand would have the same magnitude as that found in part (a) (but in opposite directions by Newton's third law) because the change in momentum and the time interval are the same.

Exercise 1.17

Professional Application

A professional boxer hits his opponent with a 1000-N horizontal blow that lasts for 0.150 s. (a) Calculate the impulse imparted by this blow. (b) What is the opponent's final velocity, if his mass is 105 kg and he is motionless in midair when struck near his center of mass? (c) Calculate the recoil velocity of the opponent's 10.0-kg head if hit in this manner, assuming the head does not initially transfer significant momentum to the boxer's body. (d) Discuss the implications of your answers for parts (b) and (c).

Exercise 1.18

Professional Application

Suppose a child drives a bumper car head on into the side rail, which exerts a force of 4000 N on the car for 0.200 s. (a) What impulse is imparted by this force? (b) Find the final velocity of the bumper car if its initial velocity was 2.80 m/s and the car plus driver have a mass of 200 kg. You may neglect friction between the car and floor.

Solution

a) $800\text{ kg}\cdot\text{m/s}$ away from the wall

b) 1.20 m/s away from the wall

Exercise 1.19

Professional Application

One hazard of space travel is debris left by previous missions. There are several thousand objects orbiting Earth that are large enough to be detected by radar, but there are far greater numbers of very small objects, such as flakes of paint. Calculate the force exerted by a 0.100-mg chip of paint that strikes a spacecraft window at a relative speed of $4.00\times10^{3}\text{ m/s}$, given the collision lasts $6.00\times10^{-8}\text{ s}$.

Exercise 1.20

Professional Application

A 75.0-kg person is riding in a car moving at 20.0 m/s when the car runs into a bridge abutment. (a) Calculate the average force on the person if he is stopped by a padded dashboard that compresses an average of 1.00 cm. (b) Calculate the average force on the person if he is stopped by an air bag that compresses an average of 15.0 cm.

Solution

(a) 1.50×10^{6} N away from the dashboard

(b) 1.00×10^{5} N away from the dashboard

Exercise 1.21

Professional Application

Military rifles have a mechanism for reducing the recoil forces of the gun on the person firing it. An internal part recoils over a relatively large distance and is stopped by damping mechanisms in the gun. The larger distance reduces the average force needed to stop the internal part. (a) Calculate the recoil velocity of a 1.00-kg plunger that directly interacts with a 0.0200-kg bullet fired at 600 m/s from the gun. (b) If this part is stopped over a distance of 20.0 cm, what average force is exerted upon it by the gun? (c) Compare this to the force exerted on the gun if the bullet is accelerated to its velocity in 10.0 ms (milliseconds).

Exercise 1.22

A cruise ship with a mass of 1.00×10^7 kg strikes a pier at a speed of 0.750 m/s. It comes to rest 6.00 m later, damaging the ship, the pier, and the tugboat captain's finances. Calculate the average force exerted on the pier using the concept of impulse. (Hint: First calculate the time it took to bring the ship to rest.)

Solution

4.69×10^5 N in the boat's original direction of motion

Exercise 1.23

Calculate the final speed of a 110-kg rugby player who is initially running at 8.00 m/s but collides head-on with a padded goalpost and experiences a backward force of 1.76×10^4 N for 5.50×10^{-2} s.

Exercise 1.24

Water from a fire hose is directed horizontally against a wall at a rate of 50.0 kg/s and a speed of 42.0 m/s. Calculate the magnitude of the force exerted on the wall, assuming the water's horizontal momentum is reduced to zero.

Solution

2.10×10^3 N away from the wall

Exercise 1.25

A 0.450-kg hammer is moving horizontally at 7.00 m/s when it strikes a nail and comes to rest after driving the nail 1.00 cm into a board. (a) Calculate the duration of the impact. (b) What was the average force exerted on the nail?

Exercise 1.26

Starting with the definitions of momentum and kinetic energy, derive an equation for the kinetic energy of a particle expressed as a function of its momentum.

Solution

$$\begin{aligned} &\mathbf{p} = m\mathbf{v} \Rightarrow p^2 = m^2v^2 \Rightarrow \frac{p^2}{m} = mv^2 \\ &\Rightarrow \frac{p^2}{2m} = \frac{1}{2}mv^2 = KE \\ &KE = \frac{p^2}{2m} \end{aligned} \tag{1.27}$$

Exercise 1.27

A ball with an initial velocity of 10 m/s moves at an angle 60° above the $+x$-direction. The ball hits a vertical wall and bounces off so that it is moving 60° above the $-x$-direction with the same speed. What is the impulse delivered by the wall?

Exercise 1.28

When serving a tennis ball, a player hits the ball when its velocity is zero (at the highest point of a vertical toss). The racquet exerts a force of 540 N on the ball for 5.00 ms, giving it a final velocity of 45.0 m/s. Using these data, find the mass of the ball.

Solution

60.0 g

Exercise 1.29

A punter drops a ball from rest vertically 1 meter down onto his foot. The ball leaves the foot with a speed of 18 m/s at an

angle 55° above the horizontal. What is the impulse delivered by the foot (magnitude and direction)?

Glossary

change in momentum: the difference between the final and initial momentum; the mass times the change in velocity

impulse: the average net external force times the time it acts; equal to the change in momentum

linear momentum: the product of mass and velocity

second law of motion: physical law that states that the net external force equals the change in momentum of a system divided by the time over which it changes

2 CONSERVATION OF MOMENTUM

2.1 Introduction

This chapter covers conservation of momentum. However, this is not something we will be covering in this course, so reading this chapter is not required, and has been included for your reference.

2.2 Conservation of Momentum

Momentum is an important quantity because it is conserved. Yet it was not conserved in the examples in **Impulse (https://legacy.cnx.org/content/m42159/latest/)** and **Linear Momentum and Force (https://legacy.cnx.org/content/m42156/latest/)** , where large changes in momentum were produced by forces acting on the system of interest. Under what circumstances is momentum conserved?

The answer to this question entails considering a sufficiently large system. It is always possible to find a larger system in which total momentum is constant, even if momentum changes for components of the system. If a football player runs into the goalpost in the end zone, there will be a force on him that causes him to bounce backward. However, the Earth also recoils —conserving momentum—because of the force applied to it through the goalpost. Because Earth is many orders of magnitude more massive than the player, its recoil is immeasurably small and can be neglected in any practical sense, but it is real nevertheless.

Consider what happens if the masses of two colliding objects are more similar than the masses of a football player and Earth—for example, one car bumping into another, as shown in **Figure 2.1**. Both cars are coasting in the same direction when the lead car (labeled m_2) is bumped by the trailing car (labeled m_1). The only unbalanced force on each car is the force of the collision. (Assume that the effects due to friction are negligible.) Car 1 slows down as a result of the collision, losing some momentum, while car 2 speeds up and gains some momentum. We shall now show that the total momentum of the two-car system remains constant.

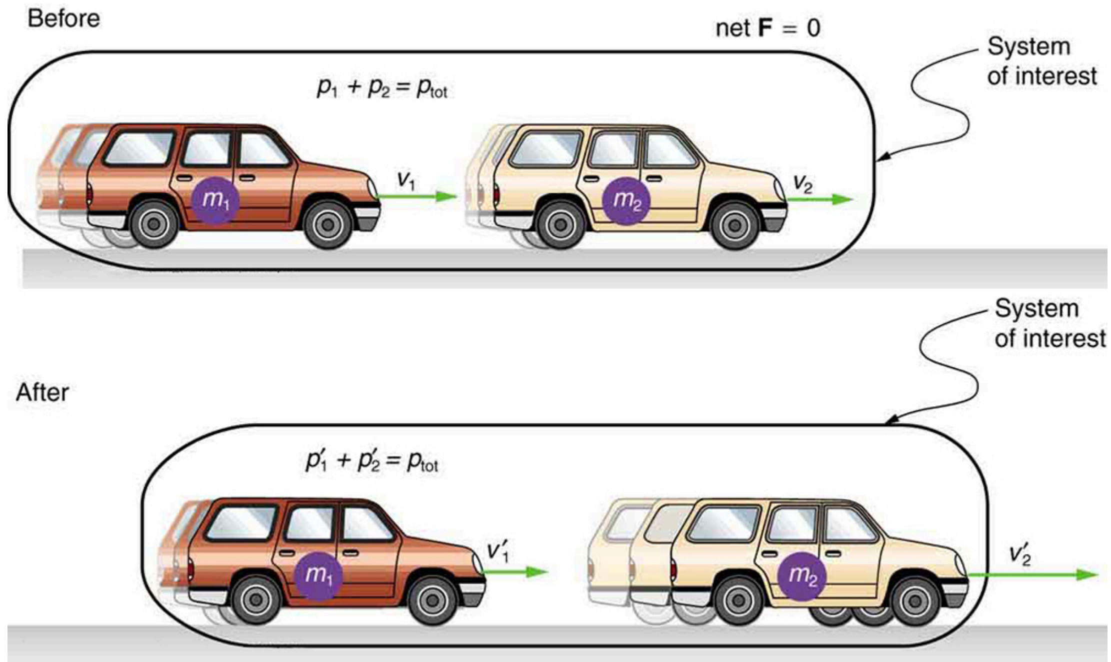


Figure 2.1 A car of mass m_1 moving with a velocity of v_1 bumps into another car of mass m_2 and velocity v_2 that it is following. As a result, the first car slows down to a velocity of v'_1 and the second speeds up to a velocity of v'_2 . The momentum of each car is changed, but the total momentum p_{tot} of the two cars is the same before and after the collision (if you assume friction is negligible).

Using the definition of impulse, the change in momentum of car 1 is given by

$$\Delta p_1 = F_1 \Delta t, \tag{2.1}$$

where F_1 is the force on car 1 due to car 2, and Δt is the time the force acts (the duration of the collision). Intuitively, it seems obvious that the collision time is the same for both cars, but it is only true for objects traveling at ordinary speeds. This assumption must be modified for objects travelling near the speed of light, without affecting the result that momentum is conserved.

Similarly, the change in momentum of car 2 is

$$\Delta p_2 = F_2 \Delta t, \tag{2.2}$$

where F_2 is the force on car 2 due to car 1, and we assume the duration of the collision Δt is the same for both cars. We know from Newton's third law that $F_2 = -F_1$, and so

$$\Delta p_2 = -F_1 \Delta t = -\Delta p_1. \tag{2.3}$$

Thus, the changes in momentum are equal and opposite, and

$$\Delta p_1 + \Delta p_2 = 0. \tag{2.4}$$

Because the changes in momentum add to zero, the total momentum of the two-car system is constant. That is,

$$p_1 + p_2 = \text{constant}, \tag{2.5}$$

$$p_1 + p_2 = p'_1 + p'_2, \tag{2.6}$$

where p'_1 and p'_2 are the momenta of cars 1 and 2 after the collision. (We often use primes to denote the final state.)

This result—that momentum is conserved—has validity far beyond the preceding one-dimensional case. It can be similarly shown that total momentum is conserved for any isolated system, with any number of objects in it. In equation form, the **conservation of momentum principle** for an isolated system is written

$$\mathbf{p}_{\text{tot}} = \text{constant}, \tag{2.7}$$

or

$$\mathbf{p}_{\text{tot}} = \mathbf{p}'_{\text{tot}}, \tag{2.8}$$

where $\mathbf{p}_{\text{tot}}$ is the total momentum (the sum of the momenta of the individual objects in the system) and $\mathbf{p}'_{\text{tot}}$ is the total momentum some time later. (The total momentum can be shown to be the momentum of the center of mass of the system.) An **isolated system** is defined to be one for which the net external force is zero $(\mathbf{F}_{\text{net}} = 0)$.

Conservation of Momentum Principle

$$\begin{aligned} \mathbf{p}_{\text{tot}} &= \text{constant} \\ \mathbf{p}_{\text{tot}} &= \mathbf{p}'_{\text{tot}} \quad \text{(isolated system)} \end{aligned} \tag{2.9}$$

Isolated System

An isolated system is defined to be one for which the net external force is zero $(\mathbf{F}_{\text{net}} = 0)$.

Perhaps an easier way to see that momentum is conserved for an isolated system is to consider Newton's second law in terms of momentum, $\mathbf{F}_{\text{net}} = \dfrac{\Delta \mathbf{p}_{\text{tot}}}{\Delta t}$. For an isolated system, $(\mathbf{F}_{\text{net}} = 0)$; thus, $\Delta \mathbf{p}_{\text{tot}} = 0$, and $\mathbf{p}_{\text{tot}}$ is constant.

We have noted that the three length dimensions in nature—x, y, and z—are independent, and it is interesting to note that momentum can be conserved in different ways along each dimension. For example, during projectile motion and where air resistance is negligible, momentum is conserved in the horizontal direction because horizontal forces are zero and momentum is unchanged. But along the vertical direction, the net vertical force is not zero and the momentum of the projectile is not conserved. (See **Figure 2.2**.) However, if the momentum of the projectile-Earth system is considered in the vertical direction, we find that the total momentum is conserved.

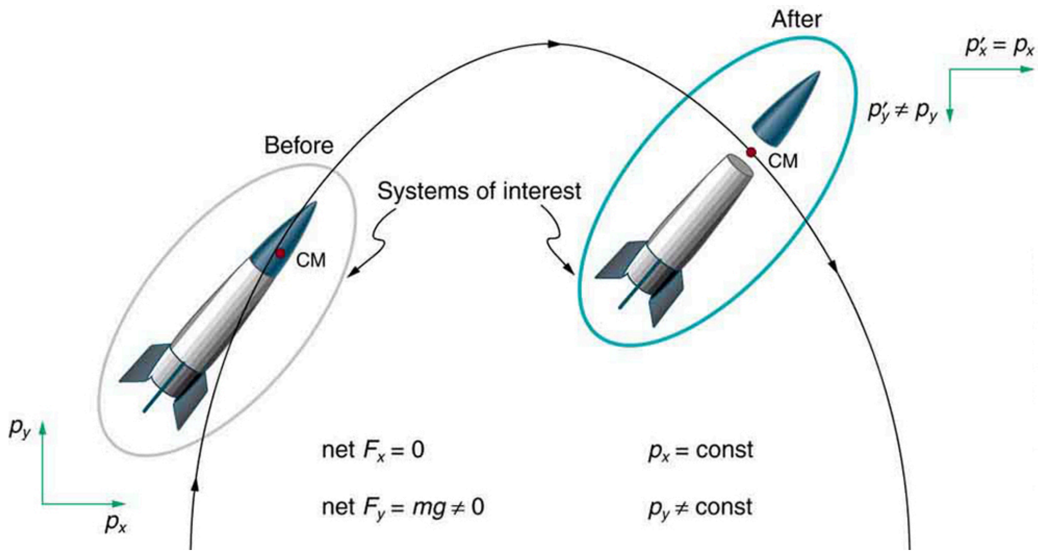


Figure 2.2 The horizontal component of a projectile's momentum is conserved if air resistance is negligible, even in this case where a space probe separates. The forces causing the separation are internal to the system, so that the net external horizontal force $F_{x-\text{net}}$ is still zero. The vertical component of the momentum is not conserved, because the net vertical force $F_{y-\text{net}}$ is not zero. In the vertical direction, the space probe-Earth system needs to be considered and we find that the total momentum is conserved. The center of mass of the space probe takes the same path it would if the separation did not occur.

The conservation of momentum principle can be applied to systems as different as a comet striking Earth and a gas containing huge numbers of atoms and molecules. Conservation of momentum is violated only when the net external force is not zero. But another larger system can always be considered in which momentum is conserved by simply including the source of the external force. For example, in the collision of two cars considered above, the two-car system conserves momentum while each one-car system does not.

Making Connections: Take-Home Investigation—Drop of Tennis Ball and a Basketball

Hold a tennis ball side by side and in contact with a basketball. Drop the balls together. (Be careful!) What happens? Explain your observations. Now hold the tennis ball above and in contact with the basketball. What happened? Explain your observations. What do you think will happen if the basketball ball is held above and in contact with the tennis ball?

Making Connections: Take-Home Investigation—Two Tennis Balls in a Ballistic Trajectory

Tie two tennis balls together with a string about a foot long. Hold one ball and let the other hang down and throw it in a ballistic trajectory. Explain your observations. Now mark the center of the string with bright ink or attach a brightly colored sticker to it and throw again. What happened? Explain your observations.

Some aquatic animals such as jellyfish move around based on the principles of conservation of momentum. A jellyfish fills its umbrella section with water and then pushes the water out resulting in motion in the opposite direction to that of the jet of water. Squids propel themselves in a similar manner but, in contrast with jellyfish, are able to control the direction in which they move by aiming their nozzle forward or backward. Typical squids can move at speeds of 8 to 12 km/h.

The ballistocardiograph (BCG) was a diagnostic tool used in the second half of the 20th century to study the strength of the heart. About once a second, your heart beats, forcing blood into the aorta. A force in the opposite direction is exerted on the rest of your body (recall Newton's third law). A ballistocardiograph is a device that can measure this reaction force. This measurement is done by using a sensor (resting on the person) or by using a moving table suspended from the ceiling. This technique can gather information on the strength of the heart beat and the volume of blood passing from the heart. However, the electrocardiogram (ECG or EKG) and the echocardiogram (cardiac ECHO or ECHO; a technique that uses ultrasound to see an image of the heart) are more widely used in the practice of cardiology.

Making Connections: Conservation of Momentum and Collision

Conservation of momentum is quite useful in describing collisions. Momentum is crucial to our understanding of atomic and subatomic particles because much of what we know about these particles comes from collision experiments.

Subatomic Collisions and Momentum

The conservation of momentum principle not only applies to the macroscopic objects, it is also essential to our explorations of atomic and subatomic particles. Giant machines hurl subatomic particles at one another, and researchers evaluate the results by assuming conservation of momentum (among other things).

On the small scale, we find that particles and their properties are invisible to the naked eye but can be measured with our instruments, and models of these subatomic particles can be constructed to describe the results. Momentum is found to be a property of all subatomic particles including massless particles such as photons that compose light. Momentum being a property of particles hints that momentum may have an identity beyond the description of an object's mass multiplied by the object's velocity. Indeed, momentum relates to wave properties and plays a fundamental role in what measurements are taken and how we take these measurements. Furthermore, we find that the conservation of momentum principle is valid when considering systems of particles. We use this principle to analyze the masses and other properties of previously undetected particles, such as the nucleus of an atom and the existence of quarks that make up particles of nuclei. Figure 2.3 below illustrates how a particle scattering backward from another implies that its target is massive and dense. Experiments seeking evidence that **quarks** make up protons (one type of particle that makes up nuclei) scattered high-energy electrons off of protons (nuclei of hydrogen atoms). Electrons occasionally scattered straight backward in a manner that implied a very small and very dense particle makes up the proton—this observation is considered nearly direct evidence of quarks. The analysis was based partly on the same conservation of momentum principle that works so well on the large scale.

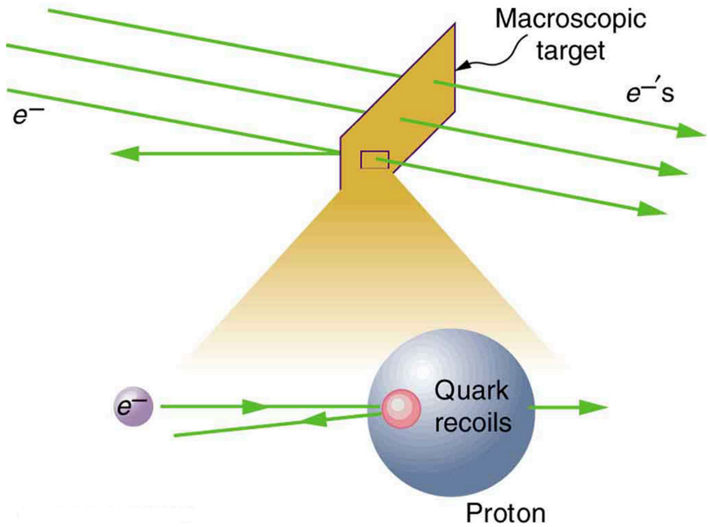


Figure 2.3 A subatomic particle scatters straight backward from a target particle. In experiments seeking evidence for quarks, electrons were observed to occasionally scatter straight backward from a proton.

Section Summary

- The conservation of momentum principle is written

$$\mathbf{p}_{\text{tot}} = \text{constant} \tag{2.10}$$

  or

$$\mathbf{p}_{\text{tot}} = \mathbf{p}'_{\text{tot}} \quad \text{(isolated system)}, \tag{2.11}$$

  $\mathbf{p}_{\text{tot}}$ is the initial total momentum and $\mathbf{p}'_{\text{tot}}$ is the total momentum some time later.
- An isolated system is defined to be one for which the net external force is zero $(\mathbf{F}_{\text{net}} = 0)$.
- During projectile motion and where air resistance is negligible, momentum is conserved in the horizontal direction because horizontal forces are zero.
- Conservation of momentum applies only when the net external force is zero.
- The conservation of momentum principle is valid when considering systems of particles.

Conceptual Questions

Exercise 2.1

Professional Application

If you dive into water, you reach greater depths than if you do a belly flop. Explain this difference in depth using the concept of conservation of energy. Explain this difference in depth using what you have learned in this chapter.

Exercise 2.2

Under what circumstances is momentum conserved?

Exercise 2.3

Can momentum be conserved for a system if there are external forces acting on the system? If so, under what conditions? If not, why not?

Exercise 2.4

Momentum for a system can be conserved in one direction while not being conserved in another. What is the angle between the directions? Give an example.

Exercise 2.5

Professional Application

Explain in terms of momentum and Newton’s laws how a car’s air resistance is due in part to the fact that it pushes air in its direction of motion.

Exercise 2.6

Can objects in a system have momentum while the momentum of the system is zero? Explain your answer.

Exercise 2.7

Must the total energy of a system be conserved whenever its momentum is conserved? Explain why or why not.

Problems & Exercises

Exercise 2.8

Professional Application

Train cars are coupled together by being bumped into one another. Suppose two loaded train cars are moving toward one another, the first having a mass of 150,000 kg and a velocity of 0.300 m/s, and the second having a mass of 110,000 kg and a velocity of -0.120 m/s. (The minus indicates direction of motion.) What is their final velocity?

Solution
0.122 m/s

Exercise 2.9

Suppose a clay model of a koala bear has a mass of 0.200 kg and slides on ice at a speed of 0.750 m/s. It runs into another clay model, which is initially motionless and has a mass of 0.350 kg. Both being soft clay, they naturally stick together. What is their final velocity?

Exercise 2.10

Professional Application

Consider the following question: *A car moving at 10 m/s crashes into a tree and stops in 0.26 s. Calculate the force the seatbelt exerts on a passenger in the car to bring him to a halt. The mass of the passenger is 70 kg.* Would the answer to this question be different if the car with the 70-kg passenger had collided with a car that has a mass equal to and is traveling in the opposite direction and at the same speed? Explain your answer.

Solution
In a collision with an identical car, momentum is conserved. Afterwards $v_{\text{f}} = 0$ for both cars. The change in momentum will be the same as in the crash with the tree. However, the force on the body is not determined since the time is not known. A padded stop will reduce injurious force on body.

Exercise 2.11

What is the velocity of a 900-kg car initially moving at 30.0 m/s, just after it hits a 150-kg deer initially running at 12.0 m/s in the same direction? Assume the deer remains on the car.

Exercise 2.12

A 1.80-kg falcon catches a 0.650-kg dove from behind in midair. What is their velocity after impact if the falcon's velocity is initially 28.0 m/s and the dove's velocity is 7.00 m/s in the same direction?

Solution
22.4 m/s in the same direction as the original motion

2.3 Elastic Collisions in One Dimension

Let us consider various types of two-object collisions. These collisions are the easiest to analyze, and they illustrate many of the physical principles involved in collisions. The conservation of momentum principle is very useful here, and it can be used whenever the net external force on a system is zero.

We start with the elastic collision of two objects moving along the same line—a one-dimensional problem. An **elastic collision** is one that also conserves internal kinetic energy. **Internal kinetic energy** is the sum of the kinetic energies of the objects in the system. **Figure 2.4** illustrates an elastic collision in which internal kinetic energy and momentum are conserved.

Truly elastic collisions can only be achieved with subatomic particles, such as electrons striking nuclei. Macroscopic collisions can be very nearly, but not quite, elastic—some kinetic energy is always converted into other forms of energy such as heat transfer due to friction and sound. One macroscopic collision that is nearly elastic is that of two steel blocks on ice. Another nearly elastic collision is that between two carts with spring bumpers on an air track. Icy surfaces and air tracks are nearly frictionless, more readily allowing nearly elastic collisions on them.

Elastic Collision

An **elastic collision** is one that conserves internal kinetic energy.

Internal Kinetic Energy

Internal kinetic energy is the sum of the kinetic energies of the objects in the system.

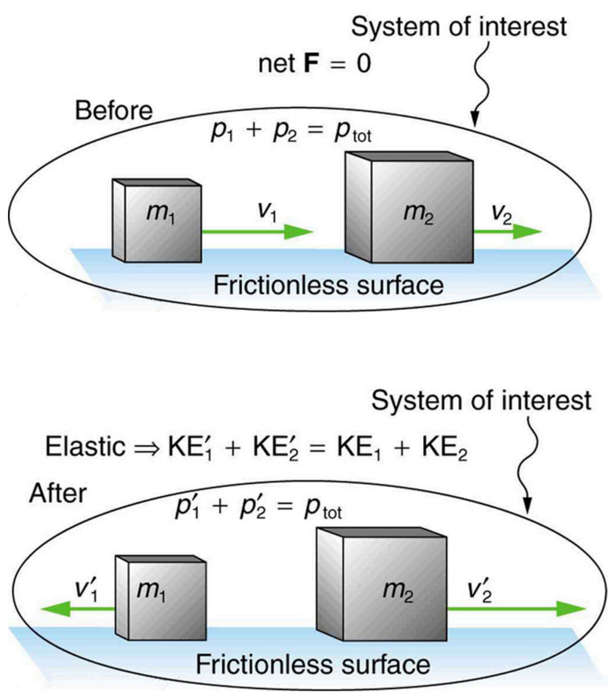


Figure 2.4 An elastic one-dimensional two-object collision. Momentum and internal kinetic energy are conserved.

Now, to solve problems involving one-dimensional elastic collisions between two objects we can use the equations for

conservation of momentum and conservation of internal kinetic energy. First, the equation for conservation of momentum for two objects in a one-dimensional collision is

$$p_1 + p_2 = p'_1 + p'_2 \quad (F_{\text{net}} = 0) \tag{2.12}$$

or

$$m_1 v_1 + m_2 v_2 = m_1 v'_1 + m_2 v'_2 \quad (F_{\text{net}} = 0), \tag{2.13}$$

where the primes (') indicate values after the collision. By definition, an elastic collision conserves internal kinetic energy, and so the sum of kinetic energies before the collision equals the sum after the collision. Thus,

$$\frac{1}{2}m_1 v_1{}^2 + \frac{1}{2}m_2 v_2{}^2 = \frac{1}{2}m_1 v'_1{}^2 + \frac{1}{2}m_2 v'_2{}^2 \quad \text{(two-object elastic collision)} \tag{2.14}$$

expresses the equation for conservation of internal kinetic energy in a one-dimensional collision.

Example 2.1 Calculating Velocities Following an Elastic Collision

Calculate the velocities of two objects following an elastic collision, given that

$$m_1 = 0.500\text{ kg},\ m_2 = 3.50\text{ kg},\ v_1 = 4.00\text{ m/s, and } v_2 = 0. \tag{2.15}$$

Strategy and Concept

First, visualize what the initial conditions mean—a small object strikes a larger object that is initially at rest. This situation is slightly simpler than the situation shown in **Figure 2.4** where both objects are initially moving. We are asked to find two unknowns (the final velocities v'_1 and v'_2). To find two unknowns, we must use two independent equations. Because this collision is elastic, we can use the above two equations. Both can be simplified by the fact that object 2 is initially at rest, and thus $v_2 = 0$. Once we simplify these equations, we combine them algebraically to solve for the unknowns.

Solution

For this problem, note that $v_2 = 0$ and use conservation of momentum. Thus,

$$p_1 = p'_1 + p'_2 \tag{2.16}$$

or

$$m_1 v_1 = m_1 v'_1 + m_2 v'_2. \tag{2.17}$$

Using conservation of internal kinetic energy and that $v_2 = 0$,

$$\frac{1}{2}m_1 v_1{}^2 = \frac{1}{2}m_1 v'_1{}^2 + \frac{1}{2}m_2 v'_2{}^2. \tag{2.18}$$

Solving the first equation (momentum equation) for v'_2, we obtain

$$v'_2 = \frac{m_1}{m_2}(v_1 - v'_1). \tag{2.19}$$

Substituting this expression into the second equation (internal kinetic energy equation) eliminates the variable v'_2, leaving only v'_1 as an unknown (the algebra is left as an exercise for the reader). There are two solutions to any quadratic equation; in this example, they are

$$v'_1 = 4.00\text{ m/s} \tag{2.20}$$

and

$$v'_1 = -3.00\text{ m/s}. \tag{2.21}$$

As noted when quadratic equations were encountered in earlier chapters, both solutions may or may not be meaningful. In this case, the first solution is the same as the initial condition. The first solution thus represents the situation before the collision and is discarded. The second solution $(v'_1 = -3.00\text{ m/s})$ is negative, meaning that the first object bounces backward. When this negative value of v'_1 is used to find the velocity of the second object after the collision, we get

$$v'_2 = \frac{m_1}{m_2}(v_1 - v'_1) = \frac{0.500\text{ kg}}{3.50\text{ kg}}[4.00 - (-3.00)]\text{ m/s} \tag{2.22}$$

or

$$v'_2 = 1.00\text{ m/s}. \tag{2.23}$$

Discussion

The result of this example is intuitively reasonable. A small object strikes a larger one at rest and bounces backward. The larger one is knocked forward, but with a low speed. (This is like a compact car bouncing backward off a full-size SUV that is initially at rest.) As a check, try calculating the internal kinetic energy before and after the collision. You will see that the internal kinetic energy is unchanged at 4.00 J. Also check the total momentum before and after the collision; you will find it, too, is unchanged.

The equations for conservation of momentum and internal kinetic energy as written above can be used to describe any one-dimensional elastic collision of two objects. These equations can be extended to more objects if needed.

Making Connections: Take-Home Investigation—Ice Cubes and Elastic Collision

Find a few ice cubes which are about the same size and a smooth kitchen tabletop or a table with a glass top. Place the ice cubes on the surface several centimeters away from each other. Flick one ice cube toward a stationary ice cube and observe the path and velocities of the ice cubes after the collision. Try to avoid edge-on collisions and collisions with rotating ice cubes. Have you created approximately elastic collisions? Explain the speeds and directions of the ice cubes using momentum.

PhET Explorations: Collision Lab

Investigate collisions on an air hockey table. Set up your own experiments: vary the number of discs, masses and initial conditions. Is momentum conserved? Is kinetic energy conserved? Vary the elasticity and see what happens.

Figure 2.5 Collision Lab (http://cnx.org/content/m42163/1.3/collision-lab_en.jar)

Section Summary

- An elastic collision is one that conserves internal kinetic energy.
- Conservation of kinetic energy and momentum together allow the final velocities to be calculated in terms of initial velocities and masses in one dimensional two-body collisions.

Conceptual Questions

Exercise 2.13

What is an elastic collision?

Problems & Exercises

Exercise 2.14

Two identical objects (such as billiard balls) have a one-dimensional collision in which one is initially motionless. After the collision, the moving object is stationary and the other moves with the same speed as the other originally had. Show that both momentum and kinetic energy are conserved.

Exercise 2.15

Professional Application

Two manned satellites approach one another at a relative speed of 0.250 m/s, intending to dock. The first has a mass of $4.00\times10^3\text{ kg}$, and the second a mass of $7.50\times10^3\text{ kg}$. If the two satellites collide elastically rather than dock, what is their final relative velocity?

Solution
0.250 m/s

Exercise 2.16

A 70.0-kg ice hockey goalie, originally at rest, catches a 0.150-kg hockey puck slapped at him at a velocity of 35.0 m/s. Suppose the goalie and the ice puck have an elastic collision and the puck is reflected back in the direction from which it came. What would their final velocities be in this case?

2.4 Inelastic Collisions in One Dimension

We have seen that in an elastic collision, internal kinetic energy is conserved. An **inelastic collision** is one in which the internal kinetic energy changes (it is not conserved). This lack of conservation means that the forces between colliding objects may remove or add internal kinetic energy. Work done by internal forces may change the forms of energy within a system. For inelastic collisions, such as when colliding objects stick together, this internal work may transform some internal kinetic energy into heat transfer. Or it may convert stored energy into internal kinetic energy, such as when exploding bolts separate a satellite from its launch vehicle.

Inelastic Collision

An inelastic collision is one in which the internal kinetic energy changes (it is not conserved).

Figure 2.6 shows an example of an inelastic collision. Two objects that have equal masses head toward one another at equal speeds and then stick together. Their total internal kinetic energy is initially $\frac{1}{2}mv^2 + \frac{1}{2}mv^2 = mv^2$. The two objects come to rest after sticking together, conserving momentum. But the internal kinetic energy is zero after the collision. A collision in which the objects stick together is sometimes called a **perfectly inelastic collision** because it reduces internal kinetic energy more than does any other type of inelastic collision. In fact, such a collision reduces internal kinetic energy to the minimum it can have while still conserving momentum.

Perfectly Inelastic Collision

A collision in which the objects stick together is sometimes called "perfectly inelastic."

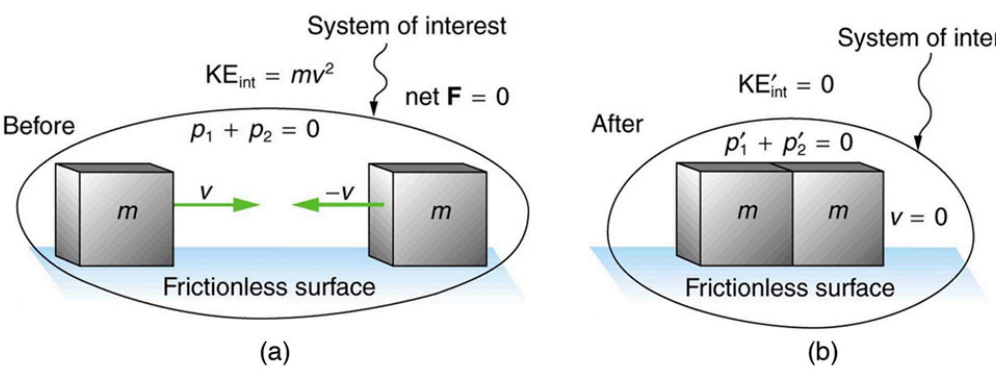


Figure 2.6 An inelastic one-dimensional two-object collision. Momentum is conserved, but internal kinetic energy is not conserved. (a) Two objects of equal mass initially head directly toward one another at the same speed. (b) The objects stick together (a perfectly inelastic collision), and so their final velocity is zero. The internal kinetic energy of the system changes in any inelastic collision and is reduced to zero in this example.

Example 2.2 Calculating Velocity and Change in Kinetic Energy: Inelastic Collision of a Puck and a Goalie

(a) Find the recoil velocity of a 70.0-kg ice hockey goalie, originally at rest, who catches a 0.150-kg hockey puck slapped at him at a velocity of 35.0 m/s. (b) How much kinetic energy is lost during the collision? Assume friction between the ice and the puck-goalie system is negligible. (See **Figure 2.7**)

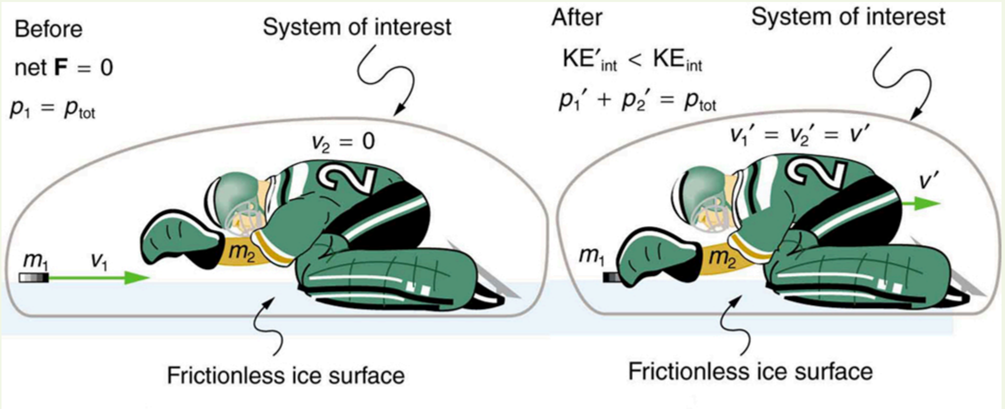


Figure 2.7 An ice hockey goalie catches a hockey puck and recoils backward. The initial kinetic energy of the puck is almost entirely converted to thermal energy and sound in this inelastic collision.

Strategy

Momentum is conserved because the net external force on the puck-goalie system is zero. We can thus use conservation of momentum to find the final velocity of the puck and goalie system. Note that the initial velocity of the goalie is zero and that the final velocity of the puck and goalie are the same. Once the final velocity is found, the kinetic energies can be calculated before and after the collision and compared as requested.

Solution for (a)

Momentum is conserved because the net external force on the puck-goalie system is zero.

Conservation of momentum is

$$p_1 + p_2 = p'_1 + p'_2 \tag{2.24}$$

or

$$m_1 v_1 + m_2 v_2 = m_1 v'_1 + m_2 v'_2. \tag{2.25}$$

Because the goalie is initially at rest, we know $v_2 = 0$. Because the goalie catches the puck, the final velocities are equal, or $v'_1 = v'_2 = v'$. Thus, the conservation of momentum equation simplifies to

$$m_1 v_1 = (m_1 + m_2)v'. \tag{2.26}$$

Solving for v' yields

$$v' = \frac{m_1}{m_1 + m_2} v_1. \tag{2.27}$$

Entering known values in this equation, we get

$$v' = \left(\frac{0.150 \text{ kg}}{70.0 \text{ kg} + 0.150 \text{ kg}}\right)(35.0 \text{ m/s}) = 7.48\times10^{-2} \text{ m/s}. \tag{2.28}$$

Discussion for (a)

This recoil velocity is small and in the same direction as the puck's original velocity, as we might expect.

Solution for (b)

Before the collision, the internal kinetic energy KE_{int} of the system is that of the hockey puck, because the goalie is initially at rest. Therefore, KE_{int} is initially

$$\begin{aligned} \text{KE}_{\text{int}} &= \frac{1}{2}mv^2 = \frac{1}{2}(0.150 \text{ kg})(35.0 \text{ m/s})^2 \\ &= 91.9 \text{ J}. \end{aligned} \tag{2.29}$$

After the collision, the internal kinetic energy is

$$\begin{aligned} \text{KE}'_{\text{int}} &= \frac{1}{2}(m + M)v^2 = \frac{1}{2}(70.15 \text{ kg})\left(7.48\times10^{-2} \text{ m/s}\right)^2 \\ &= 0.196 \text{ J}. \end{aligned} \tag{2.30}$$

The change in internal kinetic energy is thus

$$\begin{aligned} \mathrm{KE'}_{\mathrm{int}} - \mathrm{KE}_{\mathrm{int}} &= 0.196\ \mathrm{J} - 91.9\ \mathrm{J} \\ &= -91.7\ \mathrm{J} \end{aligned} \tag{2.31}$$

where the minus sign indicates that the energy was lost.

Discussion for (b)

Nearly all of the initial internal kinetic energy is lost in this perfectly inelastic collision. $\mathrm{KE}_{\mathrm{int}}$ is mostly converted to thermal energy and sound.

During some collisions, the objects do not stick together and less of the internal kinetic energy is removed—such as happens in most automobile accidents. Alternatively, stored energy may be converted into internal kinetic energy during a collision. **Figure 2.8** shows a one-dimensional example in which two carts on an air track collide, releasing potential energy from a compressed spring. **Example 2.3** deals with data from such a collision.

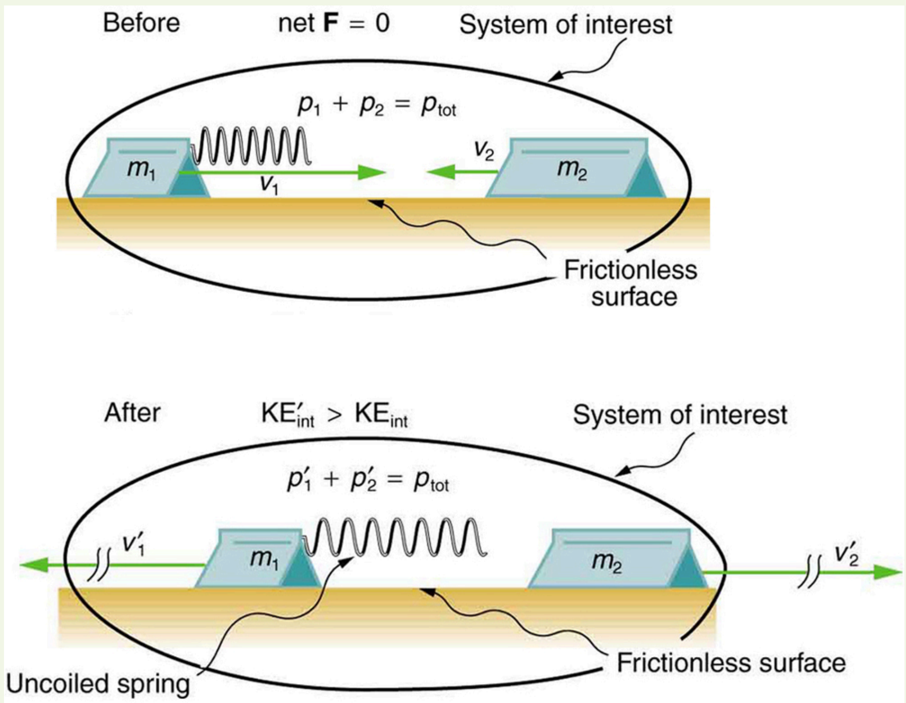


Figure 2.8 An air track is nearly frictionless, so that momentum is conserved. Motion is one-dimensional. In this collision, examined in **Example 2.3**, the potential energy of a compressed spring is released during the collision and is converted to internal kinetic energy.

Collisions are particularly important in sports and the sporting and leisure industry utilizes elastic and inelastic collisions. Let us look briefly at tennis. Recall that in a collision, it is momentum and not force that is important. So, a heavier tennis racquet will have the advantage over a lighter one. This conclusion also holds true for other sports—a lightweight bat (such as a softball bat) cannot hit a hardball very far.

The location of the impact of the tennis ball on the racquet is also important, as is the part of the stroke during which the impact occurs. A smooth motion results in the maximizing of the velocity of the ball after impact and reduces sports injuries such as tennis elbow. A tennis player tries to hit the ball on the "sweet spot" on the racquet, where the vibration and impact are minimized and the ball is able to be given more velocity. Sports science and technologies also use physics concepts such as momentum and rotational motion and vibrations.

Take-Home Experiment—Bouncing of Tennis Ball

1. Find a racquet (a tennis, badminton, or other racquet will do). Place the racquet on the floor and stand on the handle. Drop a tennis ball on the strings from a measured height. Measure how high the ball bounces. Now ask a friend to hold the racquet firmly by the handle and drop a tennis ball from the same measured height above the racquet. Measure how high the ball bounces and observe what happens to your friend's hand during the collision. Explain your observations and measurements.
2. The coefficient of restitution (c) is a measure of the elasticity of a collision between a ball and an object, and is defined as the ratio of the speeds after and before the collision. A perfectly elastic collision has a c of 1. For a ball bouncing off the floor (or a racquet on the floor), c can be shown to be $c = (h/H)^{1/2}$ where h is the height to which the ball bounces and H is the height from which the ball is dropped. Determine c for the cases in Part 1 and

for the case of a tennis ball bouncing off a concrete or wooden floor ($c = 0.85$ for new tennis balls used on a tennis court).

Example 2.3 Calculating Final Velocity and Energy Release: Two Carts Collide

In the collision pictured in **Figure 2.8**, two carts collide inelastically. Cart 1 (denoted m_1 carries a spring which is initially compressed. During the collision, the spring releases its potential energy and converts it to internal kinetic energy. The mass of cart 1 and the spring is 0.350 kg, and the cart and the spring together have an initial velocity of 2.00 m/s. Cart 2 (denoted m_2 in **Figure 2.8**) has a mass of 0.500 kg and an initial velocity of -0.500 m/s. After the collision, cart 1 is observed to recoil with a velocity of -4.00 m/s. (a) What is the final velocity of cart 2? (b) How much energy was released by the spring (assuming all of it was converted into internal kinetic energy)?

Strategy

We can use conservation of momentum to find the final velocity of cart 2, because $F_{\text{net}} = 0$ (the track is frictionless and the force of the spring is internal). Once this velocity is determined, we can compare the internal kinetic energy before and after the collision to see how much energy was released by the spring.

Solution for (a)

As before, the equation for conservation of momentum in a two-object system is

$$m_1 v_1 + m_2 v_2 = m_1 v'_1 + m_2 v'_2 . \tag{2.32}$$

The only unknown in this equation is v'_2. Solving for v'_2 and substituting known values into the previous equation yields

$$\begin{aligned} v'_2 &= \frac{m_1 v_1 + m_2 v_2 - m_1 v'_1}{m_2} \\ &= \frac{(0.350 \text{ kg})(2.00 \text{ m/s}) + (0.500 \text{ kg})(-0.500 \text{ m/s})}{0.500 \text{ kg}} - \frac{(0.350 \text{ kg})(-4.00 \text{ m/s})}{0.500 \text{ kg}} \\ &= 3.70 \text{ m/s}. \end{aligned} \tag{2.33}$$

Solution for (b)

The internal kinetic energy before the collision is

$$\begin{aligned} \text{KE}_{\text{int}} &= \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 \\ &= \frac{1}{2}(0.350 \text{ kg})(2.00 \text{ m/s})^2 + \frac{1}{2}(0.500 \text{ kg})(-0.500 \text{ m/s})^2 \\ &= 0.763 \text{ J}. \end{aligned} \tag{2.34}$$

After the collision, the internal kinetic energy is

$$\begin{aligned} \text{KE}'_{\text{int}} &= \frac{1}{2} m_1 v'^2_1 + \frac{1}{2} m_2 v'^2_2 \\ &= \frac{1}{2}(0.350 \text{ kg})(\text{-}4.00 \text{ m/s})^2 + \frac{1}{2}(0.500 \text{ kg})(3.70 \text{ m/s})^2 \\ &= 6.22 \text{ J}. \end{aligned} \tag{2.35}$$

The change in internal kinetic energy is thus

$$\begin{aligned} \text{KE}'_{\text{int}} - \text{KE}_{\text{int}} &= 6.22 \text{ J} - 0.763 \text{ J} \\ &= 5.46 \text{ J}. \end{aligned} \tag{2.36}$$

Discussion

The final velocity of cart 2 is large and positive, meaning that it is moving to the right after the collision. The internal kinetic energy in this collision increases by 5.46 J. That energy was released by the spring.

Section Summary

- An inelastic collision is one in which the internal kinetic energy changes (it is not conserved).
- A collision in which the objects stick together is sometimes called perfectly inelastic because it reduces internal kinetic energy more than does any other type of inelastic collision.
- Sports science and technologies also use physics concepts such as momentum and rotational motion and vibrations.

Conceptual Questions

Exercise 2.17

What is an inelastic collision? What is a perfectly inelastic collision?

Exercise 2.18

Mixed-pair ice skaters performing in a show are standing motionless at arms length just before starting a routine. They reach out, clasp hands, and pull themselves together by only using their arms. Assuming there is no friction between the blades of their skates and the ice, what is their velocity after their bodies meet?

Exercise 2.19

A small pickup truck that has a camper shell slowly coasts toward a red light with negligible friction. Two dogs in the back of the truck are moving and making various inelastic collisions with each other and the walls. What is the effect of the dogs on the motion of the center of mass of the system (truck plus entire load)? What is their effect on the motion of the truck?

Problems & Exercises

Exercise 2.20

A 0.240-kg billiard ball that is moving at 3.00 m/s strikes the bumper of a pool table and bounces straight back at 2.40 m/s (80% of its original speed). The collision lasts 0.0150 s. (a) Calculate the average force exerted on the ball by the bumper. (b) How much kinetic energy in joules is lost during the collision? (c) What percent of the original energy is left?

Solution

(a) 86.4 N perpendicularly away from the bumper

(b) 0.389 J

(c) 64.0%

Exercise 2.21

During an ice show, a 60.0-kg skater leaps into the air and is caught by an initially stationary 75.0-kg skater. (a) What is their final velocity assuming negligible friction and that the 60.0-kg skater's original horizontal velocity is 4.00 m/s? (b) How much kinetic energy is lost?

Exercise 2.22

Professional Application

Using mass and speed data from **m42156 (https://legacy.cnx.org/content/m42156/latest/#fs-id1356444)** and assuming that the football player catches the ball with his feet off the ground with both of them moving horizontally, calculate: (a) the final velocity if the ball and player are going in the same direction and (b) the loss of kinetic energy in this case. (c) Repeat parts (a) and (b) for the situation in which the ball and the player are going in opposite directions. Might the loss of kinetic energy be related to how much it hurts to catch the pass?

Solution

(a) 8.06 m/s

(b) -56.0 J

(c)(i) 7.88 m/s; (ii) -223 J

Exercise 2.23

A battleship that is $6.00\times10^{7}\ \text{kg}$ and is originally at rest fires a 1100-kg artillery shell horizontally with a velocity of 575 m/s. (a) If the shell is fired straight aft (toward the rear of the ship), there will be negligible friction opposing the ship's recoil. Calculate its recoil velocity. (b) Calculate the increase in internal kinetic energy (that is, for the ship and the shell). This energy is less than the energy released by the gun powder—significant heat transfer occurs.

Exercise 2.24

Professional Application

Two manned satellites approaching one another, at a relative speed of 0.250 m/s, intending to dock. The first has a mass of $4.00\times10^{3}\ \text{kg}$, and the second a mass of $7.50\times10^{3}\ \text{kg}$. (a) Calculate the final velocity (after docking) by using the frame of reference in which the first satellite was originally at rest. (b) What is the loss of kinetic energy in this inelastic collision? (c) Repeat both parts by using the frame of reference in which the second satellite was originally at rest. Explain why the change in velocity is different in the two frames, whereas the change in kinetic energy is the same in both.

Solution

(a) 0.163 m/s in the direction of motion of the more massive satellite

(b) 81.6 J

(c) $8.70\times10^{-2}\ \text{m/s}$ in the direction of motion of the less massive satellite, 81.5 J. Because there are no external forces, the velocity of the center of mass of the two-satellite system is unchanged by the collision. The two velocities calculated above are the velocity of the center of mass in each of the two different individual reference frames. The loss in KE is the same in both reference frames because the KE lost to internal forces (heat, friction, etc.) is the same regardless of the coordinate system chosen.

Exercise 2.25

Professional Application

A 30,000-kg freight car is coasting at 0.850 m/s with negligible friction under a hopper that dumps 110,000 kg of scrap metal into it. (a) What is the final velocity of the loaded freight car? (b) How much kinetic energy is lost?

Exercise 2.26

Professional Application

Space probes may be separated from their launchers by exploding bolts. (They bolt away from one another.) Suppose a 4800-kg satellite uses this method to separate from the 1500-kg remains of its launcher, and that 5000 J of kinetic energy is supplied to the two parts. What are their subsequent velocities using the frame of reference in which they were at rest before separation?

Solution

0.704 m/s

–2.25 m/s

Exercise 2.27

A 0.0250-kg bullet is accelerated from rest to a speed of 550 m/s in a 3.00-kg rifle. The pain of the rifle's kick is much worse if you hold the gun loosely a few centimeters from your shoulder rather than holding it tightly against your shoulder. (a) Calculate the recoil velocity of the rifle if it is held loosely away from the shoulder. (b) How much kinetic energy does the rifle gain? (c) What is the recoil velocity if the rifle is held tightly against the shoulder, making the effective mass 28.0 kg? (d) How much kinetic energy is transferred to the rifle-shoulder combination? The pain is related to the amount of kinetic energy, which is significantly less in this latter situation. (e) Calculate the momentum of a 110-kg football player running at 8.00 m/s. Compare the player's momentum with the momentum of a hard-thrown 0.410-kg football that has a speed of 25.0 m/s. Discuss its relationship to this problem.

Solution

(a) 4.58 m/s away from the bullet

(b) 31.5 J

(c) –0.491 m/s

(d) 3.38 J

Exercise 2.28

Professional Application

One of the waste products of a nuclear reactor is plutonium-239 $\left({}^{239}\mathrm{Pu}\right)$. This nucleus is radioactive and decays by splitting into a helium-4 nucleus and a uranium-235 nucleus $\left({}^{4}\mathrm{He} + {}^{235}\mathrm{U}\right)$, the latter of which is also radioactive and will itself decay some time later. The energy emitted in the plutonium decay is $8.40\times10^{-13}\ \mathrm{J}$ and is entirely converted to kinetic energy of the helium and uranium nuclei. The mass of the helium nucleus is $6.68\times10^{-27}\ \mathrm{kg}$, while that of the uranium is $3.92\times10^{-25}\ \mathrm{kg}$ (note that the ratio of the masses is 4 to 235). (a) Calculate the velocities of the two nuclei, assuming the plutonium nucleus is originally at rest. (b) How much kinetic energy does each nucleus carry away? Note that the data given here are accurate to three digits only.

Exercise 2.29

Professional Application

The Moon's craters are remnants of meteorite collisions. Suppose a fairly large asteroid that has a mass of $5.00\times10^{12}\ \mathrm{kg}$ (about a kilometer across) strikes the Moon at a speed of 15.0 km/s. (a) At what speed does the Moon recoil after the perfectly inelastic collision (the mass of the Moon is $7.36\times10^{22}\ \mathrm{kg}$) ? (b) How much kinetic energy is lost in the collision? Such an event may have been observed by medieval English monks who reported observing a red glow and subsequent haze about the Moon. (c) In October 2009, NASA crashed a rocket into the Moon, and analyzed the plume produced by the impact. (Significant amounts of water were detected.) Answer part (a) and (b) for this real-life experiment. The mass of the rocket was 2000 kg and its speed upon impact was 9000 km/h. How does the plume produced alter these results?

Solution

(a) $1.02\times10^{-6}\ \mathrm{m/s}$

(b) $\mathbf{5.63\times10^{20}\ J}$ (almost all KE lost)

(c) Recoil speed is $6.79\times10^{-17}\ \mathrm{m/s}$, energy lost is $6.25\times10^{9}\ \mathrm{J}$. The plume will not affect the momentum result because the plume is still part of the Moon system. The plume may affect the kinetic energy result because a significant part of the initial kinetic energy may be transferred to the kinetic energy of the plume particles.

Exercise 2.30

Professional Application

Two football players collide head-on in midair while trying to catch a thrown football. The first player is 95.0 kg and has an initial velocity of 6.00 m/s, while the second player is 115 kg and has an initial velocity of –3.50 m/s. What is their velocity just after impact if they cling together?

Exercise 2.31

What is the speed of a garbage truck that is $1.20\times10^{4}\ \mathrm{kg}$ and is initially moving at 25.0 m/s just after it hits and adheres to a trash can that is 80.0 kg and is initially at rest?

Solution

24.8 m/s

Exercise 2.32

During a circus act, an elderly performer thrills the crowd by catching a cannon ball shot at him. The cannon ball has a mass of 10.0 kg and the horizontal component of its velocity is 8.00 m/s when the 65.0-kg performer catches it. If the performer is on nearly frictionless roller skates, what is his recoil velocity?

Exercise 2.33

(a) During an ice skating performance, an initially motionless 80.0-kg clown throws a fake barbell away. The clown's ice skates allow her to recoil frictionlessly. If the clown recoils with a velocity of 0.500 m/s and the barbell is thrown with a

velocity of 10.0 m/s, what is the mass of the barbell? (b) How much kinetic energy is gained by this maneuver? (c) Where does the kinetic energy come from?

Solution

(a) 4.00 kg

(b) 210 J

(c) The clown does work to throw the barbell, so the kinetic energy comes from the muscles of the clown. The muscles convert the chemical potential energy of ATP into kinetic energy.

2.5 Collisions of Point Masses in Two Dimensions

In the previous two sections, we considered only one-dimensional collisions; during such collisions, the incoming and outgoing velocities are all along the same line. But what about collisions, such as those between billiard balls, in which objects scatter to the side? These are two-dimensional collisions, and we shall see that their study is an extension of the one-dimensional analysis already presented. The approach taken (similar to the approach in discussing two-dimensional kinematics and dynamics) is to choose a convenient coordinate system and resolve the motion into components along perpendicular axes. Resolving the motion yields a pair of one-dimensional problems to be solved simultaneously.

One complication arising in two-dimensional collisions is that the objects might rotate before or after their collision. For example, if two ice skaters hook arms as they pass by one another, they will spin in circles. We will not consider such rotation until later, and so for now we arrange things so that no rotation is possible. To avoid rotation, we consider only the scattering of **point masses**—that is, structureless particles that cannot rotate or spin.

We start by assuming that $\mathbf{F}_{\text{net}} = 0$, so that momentum $\mathbf{p}$ is conserved. The simplest collision is one in which one of the particles is initially at rest. (See **Figure 2.9**.) The best choice for a coordinate system is one with an axis parallel to the velocity of the incoming particle, as shown in **Figure 2.9**. Because momentum is conserved, the components of momentum along the x-and y-axes $(p_x \text{ and } p_y)$ will also be conserved, but with the chosen coordinate system, p_y is initially zero and p_x is the momentum of the incoming particle. Both facts simplify the analysis. (Even with the simplifying assumptions of point masses, one particle initially at rest, and a convenient coordinate system, we still gain new insights into nature from the analysis of two-dimensional collisions.)

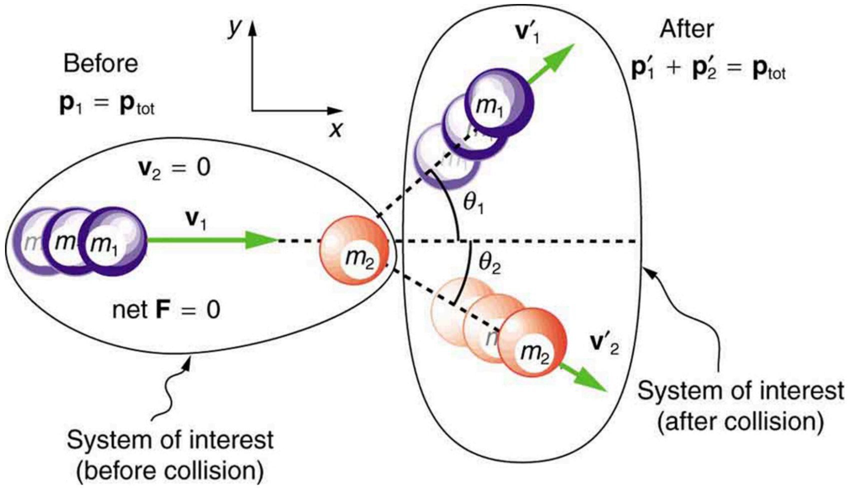


Figure 2.9 A two-dimensional collision with the coordinate system chosen so that m_2 is initially at rest and v_1 is parallel to the x-axis. This coordinate system is sometimes called the laboratory coordinate system, because many scattering experiments have a target that is stationary in the laboratory, while particles are scattered from it to determine the particles that make-up the target and how they are bound together. The particles may not be observed directly, but their initial and final velocities are.

Along the x-axis, the equation for conservation of momentum is

$$p_{1x} + p_{2x} = p'_{1x} + p'_{2x}. \tag{2.37}$$

Where the subscripts denote the particles and axes and the primes denote the situation after the collision. In terms of masses and velocities, this equation is

$$m_1 v_{1x} + m_2 v_{2x} = m_1 v'_{1x} + m_2 v'_{2x}. \tag{2.38}$$

But because particle 2 is initially at rest, this equation becomes

$$m_1 v_{1x} = m_1 v'_{1x} + m_2 v'_{2x}. \tag{2.39}$$

The components of the velocities along the x-axis have the form $v \cos \theta$. Because particle 1 initially moves along the x-axis, we find $v_{1x} = v_1$.

Conservation of momentum along the x-axis gives the following equation:

$$m_1 v_1 = m_1 v'_1 \cos \theta_1 + m_2 v'_2 \cos \theta_2, \tag{2.40}$$

where θ_1 and θ_2 are as shown in **Figure 2.9**.

Conservation of Momentum along the x-axis

$$m_1 v_1 = m_1 v'_1 \cos \theta_1 + m_2 v'_2 \cos \theta_2 \tag{2.41}$$

Along the y-axis, the equation for conservation of momentum is

$$p_{1y} + p_{2y} = p'_{1y} + p'_{2y} \tag{2.42}$$

or

$$m_1 v_{1y} + m_2 v_{2y} = m_1 v'_{1y} + m_2 v'_{2y}. \tag{2.43}$$

But v_{1y} is zero, because particle 1 initially moves along the x-axis. Because particle 2 is initially at rest, v_{2y} is also zero. The equation for conservation of momentum along the y-axis becomes

$$0 = m_1 v'_{1y} + m_2 v'_{2y}. \tag{2.44}$$

The components of the velocities along the y-axis have the form $v \sin \theta$.

Thus, conservation of momentum along the y-axis gives the following equation:

$$0 = m_1 v'_1 \sin \theta_1 + m_2 v'_2 \sin \theta_2. \tag{2.45}$$

Conservation of Momentum along the y-axis

$$0 = m_1 v'_1 \sin \theta_1 + m_2 v'_2 \sin \theta_2 \tag{2.46}$$

The equations of conservation of momentum along the x-axis and y-axis are very useful in analyzing two-dimensional collisions of particles, where one is originally stationary (a common laboratory situation). But two equations can only be used to find two unknowns, and so other data may be necessary when collision experiments are used to explore nature at the subatomic level.

Example 2.4 Determining the Final Velocity of an Unseen Object from the Scattering of Another Object

Suppose the following experiment is performed. A 0.250-kg object (m_1) is slid on a frictionless surface into a dark room, where it strikes an initially stationary object with mass of 0.400 kg (m_2). The 0.250-kg object emerges from the room at an angle of $45.0°$ with its incoming direction.

The speed of the 0.250-kg object is originally 2.00 m/s and is 1.50 m/s after the collision. Calculate the magnitude and direction of the velocity $(v'_2$ and $\theta_2)$ of the 0.400-kg object after the collision.

Strategy

Momentum is conserved because the surface is frictionless. The coordinate system shown in **Figure 2.10** is one in which m_2 is originally at rest and the initial velocity is parallel to the x-axis, so that conservation of momentum along the x- and y-axes is applicable.

Everything is known in these equations except v'_2 and θ_2, which are precisely the quantities we wish to find. We can find two unknowns because we have two independent equations: the equations describing the conservation of momentum in the x- and y-directions.

Solution

Solving $m_1 v_1 = m_1 v'_1 \cos\theta_1 + m_2 v'_2 \cos\theta_2$ for $v'_2 \cos\theta_2$ and $0 = m_1 v'_1 \sin\theta_1 + m_2 v'_2 \sin\theta_2$ for $v'_2 \sin\theta_2$ and taking the ratio yields an equation (in which θ2 is the only unknown quantity. Applying the identity $\left(\tan\theta = \frac{\sin\theta}{\cos\theta}\right)$, we obtain:

$$\tan\theta_2 = \frac{v'_1 \sin\theta_1}{v'_1 \cos\theta_1 - v_1}. \tag{2.47}$$

Entering known values into the previous equation gives

$$\tan\theta_2 = \frac{(1.50 \text{ m/s})(0.7071)}{(1.50 \text{ m/s})(0.7071) - 2.00 \text{ m/s}} = -1.129. \tag{2.48}$$

Thus,

$$\theta_2 = \tan^{-1}(-1.129) = 311.5^\circ \approx 312^\circ. \tag{2.49}$$

Angles are defined as positive in the counter clockwise direction, so this angle indicates that m_2 is scattered to the right in **Figure 2.10**, as expected (this angle is in the fourth quadrant). Either equation for the x- or y-axis can now be used to solve for v'_2, but the latter equation is easiest because it has fewer terms.

$$v'_2 = -\frac{m_1}{m_2} v'_1 \frac{\sin\theta_1}{\sin\theta_2} \tag{2.50}$$

Entering known values into this equation gives

$$v'_2 = -\left(\frac{0.250 \text{ kg}}{0.400 \text{ kg}}\right)(1.50 \text{ m/s})\left(\frac{0.7071}{-0.7485}\right). \tag{2.51}$$

Thus,

$$v'_2 = 0.886 \text{ m/s}. \tag{2.52}$$

Discussion

It is instructive to calculate the internal kinetic energy of this two-object system before and after the collision. (This calculation is left as an end-of-chapter problem.) If you do this calculation, you will find that the internal kinetic energy is less after the collision, and so the collision is inelastic. This type of result makes a physicist want to explore the system further.

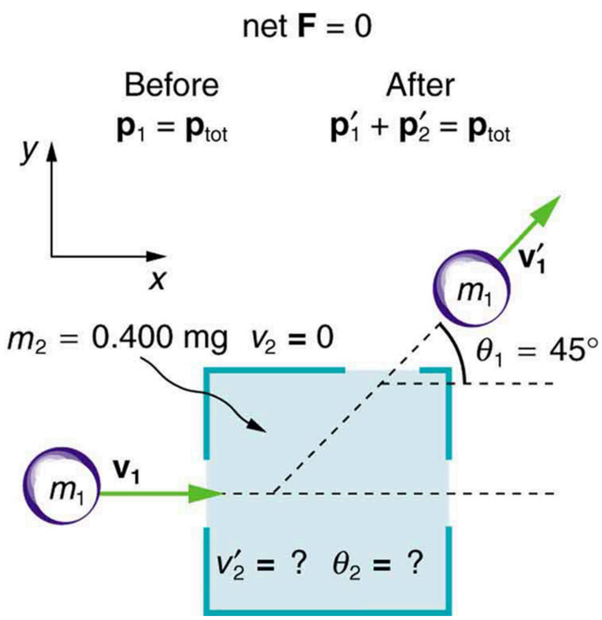


Figure 2.10 A collision taking place in a dark room is explored in **Example 2.4**. The incoming object m_1 is scattered by an initially stationary object. Only the stationary object's mass m_2 is known. By measuring the angle and speed at which m_1 emerges from the room, it is possible to calculate the magnitude and direction of the initially stationary object's velocity after the collision.

Elastic Collisions of Two Objects with Equal Mass

Some interesting situations arise when the two colliding objects have equal mass and the collision is elastic. This situation is nearly the case with colliding billiard balls, and precisely the case with some subatomic particle collisions. We can thus get a mental image of a collision of subatomic particles by thinking about billiards (or pool). (Refer to **Figure 2.9** for masses and angles.) First, an elastic collision conserves internal kinetic energy. Again, let us assume object 2 (m_2) is initially at rest. Then, the internal kinetic energy before and after the collision of two objects that have equal masses is

$$\frac{1}{2}mv_1{}^2 = \frac{1}{2}mv'_1{}^2 + \frac{1}{2}mv'_2{}^2. \tag{2.53}$$

Because the masses are equal, $m_1 = m_2 = m$. Algebraic manipulation (left to the reader) of conservation of momentum in the x- and y-directions can show that

$$\frac{1}{2}mv_1{}^2 = \frac{1}{2}mv'_1{}^2 + \frac{1}{2}mv'_2{}^2 + mv'_1v'_2 \cos(\theta_1 - \theta_2). \tag{2.54}$$

(Remember that θ_2 is negative here.) The two preceding equations can both be true only if

$$mv'_1v'_2 \cos(\theta_1 - \theta_2) = 0. \tag{2.55}$$

There are three ways that this term can be zero. They are

- $v'_1 = 0$: head-on collision; incoming ball stops
- $v'_2 = 0$: no collision; incoming ball continues unaffected
- $\cos(\theta_1 - \theta_2) = 0$: angle of separation $(\theta_1 - \theta_2)$ is $90°$ after the collision

All three of these ways are familiar occurrences in billiards and pool, although most of us try to avoid the second. If you play enough pool, you will notice that the angle between the balls is very close to $90°$ after the collision, although it will vary from this value if a great deal of spin is placed on the ball. (Large spin carries in extra energy and a quantity called *angular momentum*, which must also be conserved.) The assumption that the scattering of billiard balls is elastic is reasonable based on the correctness of the three results it produces. This assumption also implies that, to a good approximation, momentum is conserved for the two-ball system in billiards and pool. The problems below explore these and other characteristics of two-dimensional collisions.

Connections to Nuclear and Particle Physics

Two-dimensional collision experiments have revealed much of what we know about subatomic particles, as we shall see in **Medical Applications of Nuclear Physics (https://legacy.cnx.org/content/m42646/latest/)** and **Particle Physics (https://legacy.cnx.org/content/m42667/latest/)**. Ernest Rutherford, for example, discovered the nature of the atomic nucleus from such experiments.

Section Summary

- The approach to two-dimensional collisions is to choose a convenient coordinate system and break the motion into components along perpendicular axes. Choose a coordinate system with the x-axis parallel to the velocity of the incoming particle.
- Two-dimensional collisions of point masses where mass 2 is initially at rest conserve momentum along the initial direction of mass 1 (the x-axis), stated by $m_1v_1 = m_1v'_1 \cos\theta_1 + m_2v'_2 \cos\theta_2$ and along the direction perpendicular to the initial direction (the y-axis) stated by $0 = m_1v'_{1y} + m_2v'_{2y}$.
- The internal kinetic before and after the collision of two objects that have equal masses is
$$\frac{1}{2}mv_1{}^2 = \frac{1}{2}mv'_1{}^2 + \frac{1}{2}mv'_2{}^2 + mv'_1v'_2 \cos(\theta_1 - \theta_2). \tag{2.56}$$
- Point masses are structureless particles that cannot spin.

Conceptual Questions

Exercise 2.34

Figure 2.11 shows a cube at rest and a small object heading toward it. (a) Describe the directions (angle θ_1) at which the small object can emerge after colliding elastically with the cube. How does θ_1 depend on b, the so-called impact parameter? Ignore any effects that might be due to rotation after the collision, and assume that the cube is much more massive than the small object. (b) Answer the same questions if the small object instead collides with a massive sphere.

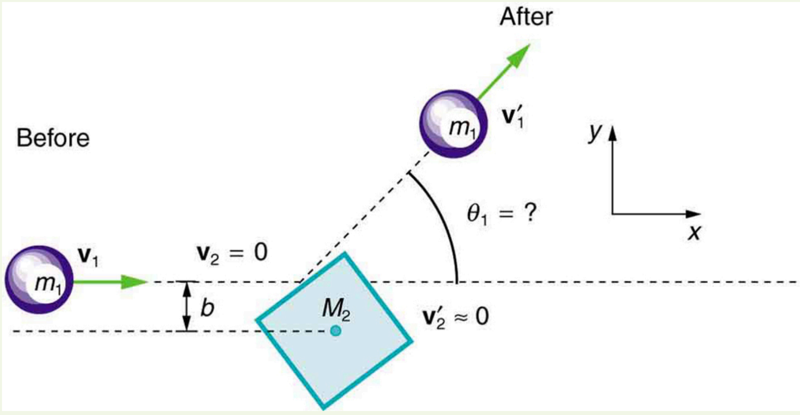


Figure 2.11 A small object approaches a collision with a much more massive cube, after which its velocity has the direction θ_1 . The angles at which the small object can be scattered are determined by the shape of the object it strikes and the impact parameter b .

Problems & Exercises

Exercise 2.35

Two identical pucks collide on an air hockey table. One puck was originally at rest. (a) If the incoming puck has a speed of 6.00 m/s and scatters to an angle of 30.0° ,what is the velocity (magnitude and direction) of the second puck? (You may use the result that $\theta_1 - \theta_2 = 90^\circ$ for elastic collisions of objects that have identical masses.) (b) Confirm that the collision is elastic.

Solution

(a) 3.00 m/s, 60° below x -axis

(b) Find speed of first puck after collision: $0 = mv'_1 \ \sin 30^\circ - mv'_2 \ \sin 60^\circ \Rightarrow v'_1 = v\prime_2 \frac{\sin 60^\circ}{\sin 30^\circ} = 5.196 \text{ m/s}$

Verify that ratio of initial to final KE equals one: $\left.\begin{array}{l} \text{KE} = \frac{1}{2}mv_1{}^2 = 18m \text{ J} \\ \text{KE} = \frac{1}{2}mv'_1{}^2 + \frac{1}{2}mv'_2{}^2 = 18m \text{ J} \end{array}\right\} \frac{\text{KE}}{\text{KE}'} = 1.00$

Exercise 2.36

Confirm that the results of the example **Example 2.4** do conserve momentum in both the x - and y -directions.

Exercise 2.37

A 3000-kg cannon is mounted so that it can recoil only in the horizontal direction. (a) Calculate its recoil velocity when it fires a 15.0-kg shell at 480 m/s at an angle of 20.0° above the horizontal. (b) What is the kinetic energy of the cannon? This energy is dissipated as heat transfer in shock absorbers that stop its recoil. (c) What happens to the vertical component of momentum that is imparted to the cannon when it is fired?

Solution

(a) -2.26 m/s

(b) $7.63 \times 10^3 \text{ J}$

(c) The ground will exert a normal force to oppose recoil of the cannon in the vertical direction. The momentum in the vertical direction is transferred to the earth. The energy is transferred into the ground, making a dent where the cannon is. After long barrages, cannon have erratic aim because the ground is full of divots.

Exercise 2.38

Professional Application

A 5.50-kg bowling ball moving at 9.00 m/s collides with a 0.850-kg bowling pin, which is scattered at an angle of 85.0° to the initial direction of the bowling ball and with a speed of 15.0 m/s. (a) Calculate the final velocity (magnitude and direction) of the bowling ball. (b) Is the collision elastic? (c) Linear kinetic energy is greater after the collision. Discuss how spin on the ball might be converted to linear kinetic energy in the collision.

Exercise 2.39

Professional Application

Ernest Rutherford (the first New Zealander to be awarded the Nobel Prize in Chemistry) demonstrated that nuclei were very small and dense by scattering helium-4 nuclei $\left({}^{4}\mathrm{He}\right)$ from gold-197 nuclei $\left({}^{197}\mathrm{Au}\right)$. The energy of the incoming helium nucleus was $8.00\times10^{-13}\ \mathrm{J}$, and the masses of the helium and gold nuclei were $6.68\times10^{-27}\ \mathrm{kg}$ and $3.29\times10^{-25}\ \mathrm{kg}$, respectively (note that their mass ratio is 4 to 197). (a) If a helium nucleus scatters to an angle of 120° during an elastic collision with a gold nucleus, calculate the helium nucleus's final speed and the final velocity (magnitude and direction) of the gold nucleus. (b) What is the final kinetic energy of the helium nucleus?

Solution

(a) $5.36\times10^{5}\ \mathrm{m/s}$ at -29.5°

(b) $7.52\times10^{-13}\ \mathrm{J}$

Exercise 2.40

Professional Application

Two cars collide at an icy intersection and stick together afterward. The first car has a mass of 1200 kg and is approaching at $8.00\ \mathrm{m/s}$ due south. The second car has a mass of 850 kg and is approaching at $17.0\ \mathrm{m/s}$ due west. (a) Calculate the final velocity (magnitude and direction) of the cars. (b) How much kinetic energy is lost in the collision? (This energy goes into deformation of the cars.) Note that because both cars have an initial velocity, you cannot use the equations for conservation of momentum along the x-axis and y-axis; instead, you must look for other simplifying aspects.

Exercise 2.41

Starting with equations $m_1 v_1 = m_1 v'_1 \cos\theta_1 + m_2 v'_2 \cos\theta_2$ and $0 = m_1 v'_1 \sin\theta_1 + m_2 v'_2 \sin\theta_2$ for conservation of momentum in the x- and y-directions and assuming that one object is originally stationary, prove that for an elastic collision of two objects of equal masses,

$$\frac{1}{2}mv_1^2 = \frac{1}{2}mv'^2_1 + \frac{1}{2}mv'^2_2 + mv'_1 v'_2 \cos(\theta_1 - \theta_2) \tag{2.57}$$

as discussed in the text.

Solution

We are given that $m_1 = m_2 \equiv m$. The given equations then become:

$$v_1 = v_1 \cos\theta_1 + v_2 \cos\theta_2 \tag{2.58}$$

and

$$0 = v'_1 \sin\theta_1 + v'_2 \sin\theta_2. \tag{2.59}$$

Square each equation to get

$$\begin{aligned} v_1^2 &= v'^2_1 \cos^2\theta_1 + v'^2_2 \cos^2\theta_2 + 2v'_1 v'_2 \cos\theta_1 \cos\theta_2 \\ 0 &= v'^2_1 \sin^2\theta_1 + v'^2_2 \sin^2\theta_2 + 2v'_1 v'_2 \sin\theta_1 \sin\theta_2. \end{aligned} \tag{2.60}$$

Add these two equations and simplify:

$$\begin{aligned} v_1^2 &= v'_1{}^2 + v'_2{}^2 + 2v'_1 v'_2(\cos\theta_1 \cos\theta_2 + \sin\theta_1 \sin\theta_2) \\ &= v'_1{}^2 + v'_2{}^2 + 2v'_1 v'_2\left[\frac{1}{2}\cos(\theta_1-\theta_2) + \frac{1}{2}\cos(\theta_1+\theta_2) + \frac{1}{2}\cos(\theta_1-\theta_2) - \frac{1}{2}\cos(\theta_1+\theta_2)\right] \\ &= v'_1{}^2 + v'_2{}^2 + 2v'_1 v'_2 \cos(\theta_1-\theta_2). \end{aligned} \tag{2.61}$$

Multiply the entire equation by $\frac{1}{2}m$ to recover the kinetic energy:

$$\frac{1}{2}mv_1^2 = \frac{1}{2}mv'_1{}^2 + \frac{1}{2}mv'_2{}^2 + mv'_1 v'_2 \cos(\theta_1-\theta_2) \tag{2.62}$$

Exercise 2.42

Integrated Concepts

A 90.0-kg ice hockey player hits a 0.150-kg puck, giving the puck a velocity of 45.0 m/s. If both are initially at rest and if the ice is frictionless, how far does the player recoil in the time it takes the puck to reach the goal 15.0 m away?

2.6 Introduction to Rocket Propulsion

Rockets range in size from fireworks so small that ordinary people use them to immense Saturn Vs that once propelled massive payloads toward the Moon. The propulsion of all rockets, jet engines, deflating balloons, and even squids and octopuses is explained by the same physical principle—Newton's third law of motion. Matter is forcefully ejected from a system, producing an equal and opposite reaction on what remains. Another common example is the recoil of a gun. The gun exerts a force on a bullet to accelerate it and consequently experiences an equal and opposite force, causing the gun's recoil or kick.

Making Connections: Take-Home Experiment—Propulsion of a Balloon

Hold a balloon and fill it with air. Then, let the balloon go. In which direction does the air come out of the balloon and in which direction does the balloon get propelled? If you fill the balloon with water and then let the balloon go, does the balloon's direction change? Explain your answer.

Figure 2.12 shows a rocket accelerating straight up. In part (a), the rocket has a mass m and a velocity v relative to Earth, and hence a momentum mv. In part (b), a time Δt has elapsed in which the rocket has ejected a mass Δm of hot gas at a velocity v_e relative to the rocket. The remainder of the mass $(m - \Delta m)$ now has a greater velocity $(v + \Delta v)$. The momentum of the entire system (rocket plus expelled gas) has actually decreased because the force of gravity has acted for a time Δt, producing a negative impulse $\Delta p = -mg\Delta t$. (Remember that impulse is the net external force on a system multiplied by the time it acts, and it equals the change in momentum of the system.) So, the center of mass of the system is in free fall but, by rapidly expelling mass, part of the system can accelerate upward. It is a commonly held misconception that the rocket exhaust pushes on the ground. If we consider thrust; that is, the force exerted on the rocket by the exhaust gases, then a rocket's thrust is greater in outer space than in the atmosphere or on the launch pad. In fact, gases are easier to expel into a vacuum.

By calculating the change in momentum for the entire system over Δt, and equating this change to the impulse, the following expression can be shown to be a good approximation for the acceleration of the rocket.

$$a = \frac{v_e}{m}\frac{\Delta m}{\Delta t} - g \tag{2.63}$$

"The rocket" is that part of the system remaining after the gas is ejected, and g is the acceleration due to gravity.

Acceleration of a Rocket

Acceleration of a rocket is

$$a = \frac{v_e}{m}\frac{\Delta m}{\Delta t} - g, \tag{2.64}$$

where a is the acceleration of the rocket, v_e is the escape velocity, m is the mass of the rocket, Δm is the mass of the ejected gas, and Δt is the time in which the gas is ejected.

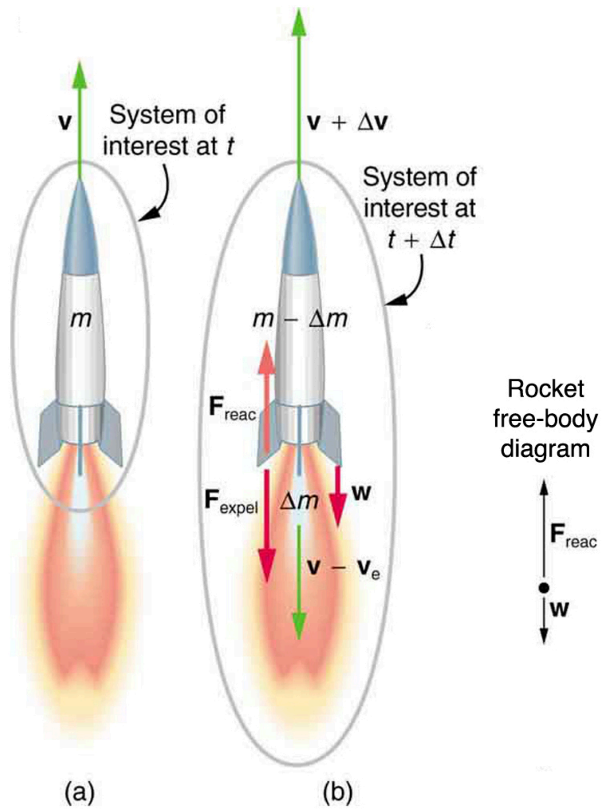


Figure 2.12 (a) This rocket has a mass m and an upward velocity v. The net external force on the system is $-mg$, if air resistance is neglected. (b) A time Δt later the system has two main parts, the ejected gas and the remainder of the rocket. The reaction force on the rocket is what overcomes the gravitational force and accelerates it upward.

A rocket's acceleration depends on three major factors, consistent with the equation for acceleration of a rocket **.** First, the greater the exhaust velocity of the gases relative to the rocket, v_e, the greater the acceleration is. The practical limit for v_e is about 2.5×10^3 m/s for conventional (non-nuclear) hot-gas propulsion systems. The second factor is the rate at which mass is ejected from the rocket. This is the factor $\Delta m / \Delta t$ in the equation. The quantity $(\Delta m / \Delta t)v_e$, with units of newtons, is called "thrust." The faster the rocket burns its fuel, the greater its thrust, and the greater its acceleration. The third factor is the mass m of the rocket. The smaller the mass is (all other factors being the same), the greater the acceleration. The rocket mass m decreases dramatically during flight because most of the rocket is fuel to begin with, so that acceleration increases continuously, reaching a maximum just before the fuel is exhausted.

Factors Affecting a Rocket's Acceleration

- The greater the exhaust velocity v_e of the gases relative to the rocket, the greater the acceleration.
- The faster the rocket burns its fuel, the greater its acceleration.
- The smaller the rocket's mass (all other factors being the same), the greater the acceleration.

Example 2.5 Calculating Acceleration: Initial Acceleration of a Moon Launch

A Saturn V's mass at liftoff was 2.80×10^6 kg, its fuel-burn rate was 1.40×10^4 kg/s, and the exhaust velocity was 2.40×10^3 m/s. Calculate its initial acceleration.

Strategy

This problem is a straightforward application of the expression for acceleration because a is the unknown and all of the terms on the right side of the equation are given.

Solution

Substituting the given values into the equation for acceleration yields

$$\begin{aligned} a &= \frac{v_e}{m}\frac{\Delta m}{\Delta t} - g \\ &= \frac{2.40\times10^3 \text{ m/s}}{2.80\times10^6 \text{ kg}}\left(1.40\times10^4 \text{ kg/s}\right) - 9.80 \text{ m/s}^2 \\ &= 2.20 \text{ m/s}^2. \end{aligned} \tag{2.65}$$

Discussion

This value is fairly small, even for an initial acceleration. The acceleration does increase steadily as the rocket burns fuel, because m decreases while v_e and $\frac{\Delta m}{\Delta t}$ remain constant. Knowing this acceleration and the mass of the rocket, you can show that the thrust of the engines was $3.36\times10^7 \text{ N}$.

To achieve the high speeds needed to hop continents, obtain orbit, or escape Earth's gravity altogether, the mass of the rocket other than fuel must be as small as possible. It can be shown that, in the absence of air resistance and neglecting gravity, the final velocity of a one-stage rocket initially at rest is

$$v = v_e \ln\frac{m_0}{m_r}, \tag{2.66}$$

where $\ln(m_0/m_r)$ is the natural logarithm of the ratio of the initial mass of the rocket (m_0) to what is left (m_r) after all of the fuel is exhausted. (Note that v is actually the change in velocity, so the equation can be used for any segment of the flight. If we start from rest, the change in velocity equals the final velocity.) For example, let us calculate the mass ratio needed to escape Earth's gravity starting from rest, given that the escape velocity from Earth is about $11.2\times10^3 \text{ m/s}$, and assuming an exhaust velocity $v_e = 2.5\times10^3 \text{ m/s}$.

$$\ln\frac{m_0}{m_r} = \frac{v}{v_e} = \frac{11.2\times10^3 \text{ m/s}}{2.5\times10^3 \text{ m/s}} = 4.48 \tag{2.67}$$

Solving for m_0/m_r gives

$$\frac{m_0}{m_r} = e^{4.48} = 88. \tag{2.68}$$

Thus, the mass of the rocket is

$$m_r = \frac{m_0}{88}. \tag{2.69}$$

This result means that only $1/88$ of the mass is left when the fuel is burnt, and $87/88$ of the initial mass was fuel. Expressed as percentages, 98.9% of the rocket is fuel, while payload, engines, fuel tanks, and other components make up only 1.10%. Taking air resistance and gravitational force into account, the mass m_r remaining can only be about $m_0/180$. It is difficult to build a rocket in which the fuel has a mass 180 times everything else. The solution is multistage rockets. Each stage only needs to achieve part of the final velocity and is discarded after it burns its fuel. The result is that each successive stage can have smaller engines and more payload relative to its fuel. Once out of the atmosphere, the ratio of payload to fuel becomes more favorable, too.

The space shuttle was an attempt at an economical vehicle with some reusable parts, such as the solid fuel boosters and the craft itself. (See **Figure 2.13**) The shuttle's need to be operated by humans, however, made it at least as costly for launching satellites as expendable, unmanned rockets. Ideally, the shuttle would only have been used when human activities were required for the success of a mission, such as the repair of the Hubble space telescope. Rockets with satellites can also be launched from airplanes. Using airplanes has the double advantage that the initial velocity is significantly above zero and a rocket can avoid most of the atmosphere's resistance.

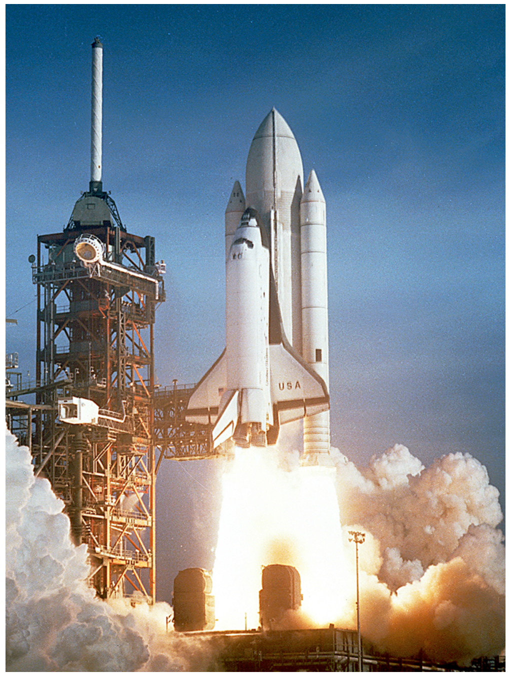


Figure 2.13 The space shuttle had a number of reusable parts. Solid fuel boosters on either side were recovered and refueled after each flight, and the entire orbiter returned to Earth for use in subsequent flights. The large liquid fuel tank was expended. The space shuttle was a complex assemblage of technologies, employing both solid and liquid fuel and pioneering ceramic tiles as reentry heat shields. As a result, it permitted multiple launches as opposed to single-use rockets. (credit: NASA)

PhET Explorations: Lunar Lander

Can you avoid the boulder field and land safely, just before your fuel runs out, as Neil Armstrong did in 1969? Our version of this classic video game accurately simulates the real motion of the lunar lander with the correct mass, thrust, fuel consumption rate, and lunar gravity. The real lunar lander is very hard to control.

PhET Interactive Simulation

Figure 2.14 Lunar Lander (http://legacy.cnx.org/content/m42166/1.6/lunar-lander_en.jar)

Section Summary

- Newton's third law of motion states that to every action, there is an equal and opposite reaction.
- Acceleration of a rocket is $a = \frac{v_e}{m}\frac{\Delta m}{\Delta t} - g$.
- A rocket's acceleration depends on three main factors. They are
  1. The greater the exhaust velocity of the gases, the greater the acceleration.
  2. The faster the rocket burns its fuel, the greater its acceleration.
  3. The smaller the rocket's mass, the greater the acceleration.

Conceptual Questions

Exercise 2.43

Professional Application

Suppose a fireworks shell explodes, breaking into three large pieces for which air resistance is negligible. How is the motion of the center of mass affected by the explosion? How would it be affected if the pieces experienced significantly more air resistance than the intact shell?

Exercise 2.44

Professional Application

During a visit to the International Space Station, an astronaut was positioned motionless in the center of the station, out of reach of any solid object on which he could exert a force. Suggest a method by which he could move himself away from this position, and explain the physics involved.

Exercise 2.45

Professional Application

It is possible for the velocity of a rocket to be greater than the exhaust velocity of the gases it ejects. When that is the case, the gas velocity and gas momentum are in the same direction as that of the rocket. How is the rocket still able to obtain thrust by ejecting the gases?

Problems & Exercises

Exercise 2.46

Professional Application

Antiballistic missiles (ABMs) are designed to have very large accelerations so that they may intercept fast-moving incoming missiles in the short time available. What is the takeoff acceleration of a 10,000-kg ABM that expels 196 kg of gas per second at an exhaust velocity of $2.50\times10^3\text{ m/s}$?

Solution

39.2 m/s^2

Exercise 2.47

Professional Application

What is the acceleration of a 5000-kg rocket taking off from the Moon, where the acceleration due to gravity is only 1.6 m/s^2, if the rocket expels 8.00 kg of gas per second at an exhaust velocity of $2.20\times10^3\text{ m/s}$?

Exercise 2.48

Professional Application

Calculate the increase in velocity of a 4000-kg space probe that expels 3500 kg of its mass at an exhaust velocity of $2.00\times10^3\text{ m/s}$. You may assume the gravitational force is negligible at the probe's location.

Solution

$4.16\times10^3\text{ m/s}$

Exercise 2.49

Professional Application

Ion-propulsion rockets have been proposed for use in space. They employ atomic ionization techniques and nuclear energy sources to produce extremely high exhaust velocities, perhaps as great as $8.00\times10^6\text{ m/s}$. These techniques allow a much more favorable payload-to-fuel ratio. To illustrate this fact: (a) Calculate the increase in velocity of a 20,000-kg space probe that expels only 40.0-kg of its mass at the given exhaust velocity. (b) These engines are usually designed to produce a very small thrust for a very long time—the type of engine that might be useful on a trip to the outer planets, for example. Calculate the acceleration of such an engine if it expels $4.50\times10^{-6}\text{ kg/s}$ at the given velocity, assuming the acceleration due to gravity is negligible.

Exercise 2.50

Derive the equation for the vertical acceleration of a rocket.

Solution

The force needed to give a small mass Δm an acceleration $a_{\Delta m}$ is $F = \Delta m a_{\Delta m}$. To accelerate this mass in the small time interval Δt at a speed v_e requires $v_e = a_{\Delta m}\Delta t$, so $F = v_e \frac{\Delta m}{\Delta t}$. By Newton's third law, this force is equal in magnitude to the thrust force acting on the rocket, so $F_{\text{thrust}} = v_e \frac{\Delta m}{\Delta t}$, where all quantities are positive. Applying Newton's second law to the rocket gives $F_{\text{thrust}} - mg = ma \Rightarrow a = \frac{v_e}{m}\frac{\Delta m}{\Delta t} - g$, where m is the mass of the rocket and unburnt fuel.

Exercise 2.51

Professional Application

(a) Calculate the maximum rate at which a rocket can expel gases if its acceleration cannot exceed seven times that of gravity. The mass of the rocket just as it runs out of fuel is 75,000-kg, and its exhaust velocity is 2.40×10^3 m/s. Assume that the acceleration of gravity is the same as on Earth's surface $\left(9.80\ \text{m/s}^2\right)$. (b) Why might it be necessary to limit the acceleration of a rocket?

Exercise 2.52

Given the following data for a fire extinguisher-toy wagon rocket experiment, calculate the average exhaust velocity of the gases expelled from the extinguisher. Starting from rest, the final velocity is 10.0 m/s. The total mass is initially 75.0 kg and is 70.0 kg after the extinguisher is fired.

Exercise 2.53

How much of a single-stage rocket that is 100,000 kg can be anything but fuel if the rocket is to have a final speed of 8.00 km/s, given that it expels gases at an exhaust velocity of 2.20×10^3 m/s?

Solution

2.63×10^3 kg

Exercise 2.54

Professional Application

(a) A 5.00-kg squid initially at rest ejects 0.250-kg of fluid with a velocity of 10.0 m/s. What is the recoil velocity of the squid if the ejection is done in 0.100 s and there is a 5.00-N frictional force opposing the squid's movement. (b) How much energy is lost to work done against friction?

Solution

(a) 0.421 m/s away from the ejected fluid.

(b) 0.237 J.

Exercise 2.55

Unreasonable Results

Squids have been reported to jump from the ocean and travel 30.0 m (measured horizontally) before re-entering the water. (a) Calculate the initial speed of the squid if it leaves the water at an angle of 20.0°, assuming negligible lift from the air and negligible air resistance. (b) The squid propels itself by squirting water. What fraction of its mass would it have to eject in order to achieve the speed found in the previous part? The water is ejected at 12.0 m/s; gravitational force and friction are neglected. (c) What is unreasonable about the results? (d) Which premise is unreasonable, or which premises are inconsistent?

Exercise 2.56

Construct Your Own Problem

Consider an astronaut in deep space cut free from her space ship and needing to get back to it. The astronaut has a few packages that she can throw away to move herself toward the ship. Construct a problem in which you calculate the time it takes her to get back by throwing all the packages at one time compared to throwing them one at a time. Among the things to be considered are the masses involved, the force she can exert on the packages through some distance, and the distance to the ship.

Exercise 2.57

Construct Your Own Problem

Consider an artillery projectile striking armor plating. Construct a problem in which you find the force exerted by the projectile on the plate. Among the things to be considered are the mass and speed of the projectile and the distance over which its speed is reduced. Your instructor may also wish for you to consider the relative merits of depleted uranium versus lead projectiles based on the greater density of uranium.

Glossary

conservation of momentum principle: when the net external force is zero, the total momentum of the system is conserved or constant

elastic collision: a collision that also conserves internal kinetic energy

inelastic collision: a collision in which internal kinetic energy is not conserved

internal kinetic energy: the sum of the kinetic energies of the objects in a system

isolated system: a system in which the net external force is zero

perfectly inelastic collision: a collision in which the colliding objects stick together

point masses: structureless particles with no rotation or spin

quark: fundamental constituent of matter and an elementary particle

3 INTRODUCTION TO TORQUE AND CENTER OF MASS

3.1 Torque and Center of Mass Introduction

In our study of physics so far, we have modeled everything as a dot for free-body diagrams. Where we applied our force to objects and how the object was shaped didn't matter. And, in a lot of cases, this is fine. When you push a box across the floor, pushing the box from upper part of one side is roughly the same as pushing the box from the lower part, which is also roughly the same as just pushing from the center of the side. However, there are also a lot of cases where the place that you push matters. In fact, there's one case that we encounter daily: doors.

If you can, try getting up and opening or closing a door by pushing it near the hinge. You might feel a bit silly, but you'll notice that you need considerably more force to move the door than if you pushed on it near end with the doorknob. Another exercise to try would be to try opening a door by pulling or pushing at an angle from the doorknob. You'll find that it's also harder to do that than just pushing or pulling straight onto the doorknob. In this chapter, we will be exploring this idea of where you push matters, which plays into two physics ideas: torque and center of mass.

3.2 Center of Mass

UMASS AMHERST Instructor's Notes

Your Quiz will Cover

- Given a series of masses, be able to calculate the position of the center of mass
- Construct a definition of center of mass that contains all of its attributes

What is Center of Mass?

Center of mass is an idea with multiple facets that are all related. The center of mass is the mass weighted average location of an object. Furthermore, the center of mass is the point at which gravity acts. When we get into our discussion of torque, we're going to be interested in the location at which each force is applied, which means we're going to need to figure out where the force of gravity acts. The force of gravity acts at the center of mass. So, the first part of the definition of center of mass is a mass weighted average location of an object. Another term for center of mass is center of gravity; there are some differences between the two, but for this course, these two terms will be synonymous.

Now, this is a lot going on, so, let's think about what a weighted average is first in a context with which you are probably more familiar with the idea. Think about your grades; not each assignment in a course counts the same. For example, in this class, the weights for the different assignments are provided in the table on the left.

Individual components (61% total)		Team components (39% of total)	
Online Homework	10%	Laboratory and other in-class activities	18%
iRATs	9% (50% of total RAT score)	tRATs	9% (50% of total RAT score)
Individual Exam I	14% (78% of total exam score)	Team Exam I	4% (22% of total exam score)
Individual Exam II	14%	Team Exam II	4%
Individual Exam III	14%	Team Exam III	4%

Individual components (61% total)		Team components (39% of total)	
Online Homework	95%	Laboratory and other in-class activities	90%
iRATs	75% (50% of total RAT score)	tRATs	95% (50% of total RAT score)
Individual Exam I	70%	Team Exam I	85%
Individual Exam II	70%	Team Exam II	85%
Individual Exam III	70%	Team Exam III	85%

Now, if you got the grades in the table on the right, what would your final grade be in this course? The weighted average is equal to the sum of all the values times its worth over the total, or

$$\sum \frac{Value * Worth}{Total}$$

Let's work this through for this example. So, in this example this hypothetical student achieved in 95, so that would be the value on the online homework, and the online homework is worth 10 out of a total of 100.

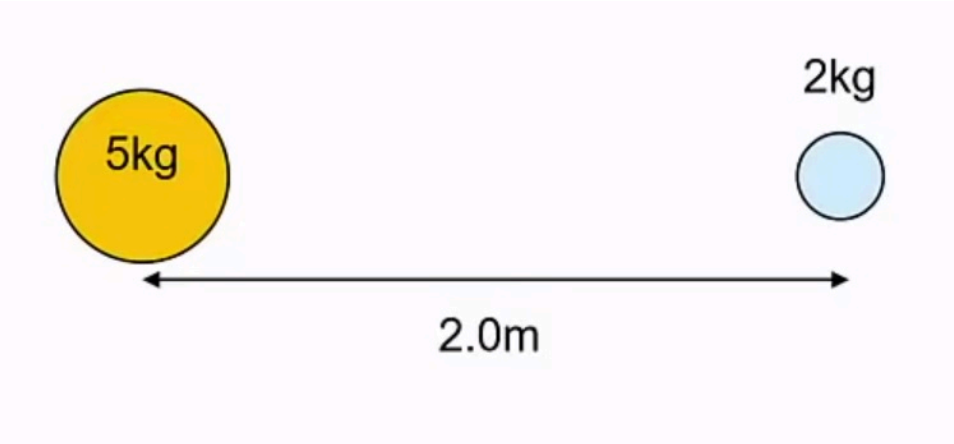
$$(95) * (\frac{10}{100})$$

Now I just repeat this for all the different categories, and then add them all up.

$$(95)\left(\frac{10}{100}\right)+(75)\left(\frac{9}{100}\right)+(70)\left(\frac{14}{100}\right)+(70)\left(\frac{14}{100}\right)+(70)\left(\frac{14}{100}\right)+(70)\left(\frac{14}{100}\right)+(90)\left(\frac{18}{100}\right)+(95)\left(\frac{9}{100}\right)+(85)\left(\frac{4}{100}\right)+(85)\left(\frac{4}{100}\right)+(85)\left(\frac{4}{100}\right)$$

If you do out this calculation, you get an 81.05, which if you go and look at the syllabus is a B+. This is a weighted average of scores for the course.

Center of mass is similarly a weighted average, only we're using mass to weight our average, and we're averaging position. So, let's look at this example of a 5-kilogram object and a 2-kilogram object, and let's calculate the center of mass for these two objects.



The first step is to establish a coordinate system. We're talking about positions I need a coordinate system, so I'm going to establish the positive x direction to be towards the right. Now we can apply the same idea with the grades here with the weight. We can set the position of the 5-kilogram to 0m, and the 2kg to 2m, and the weighted totals will be the position of each weight times its mass over the total mass, or:

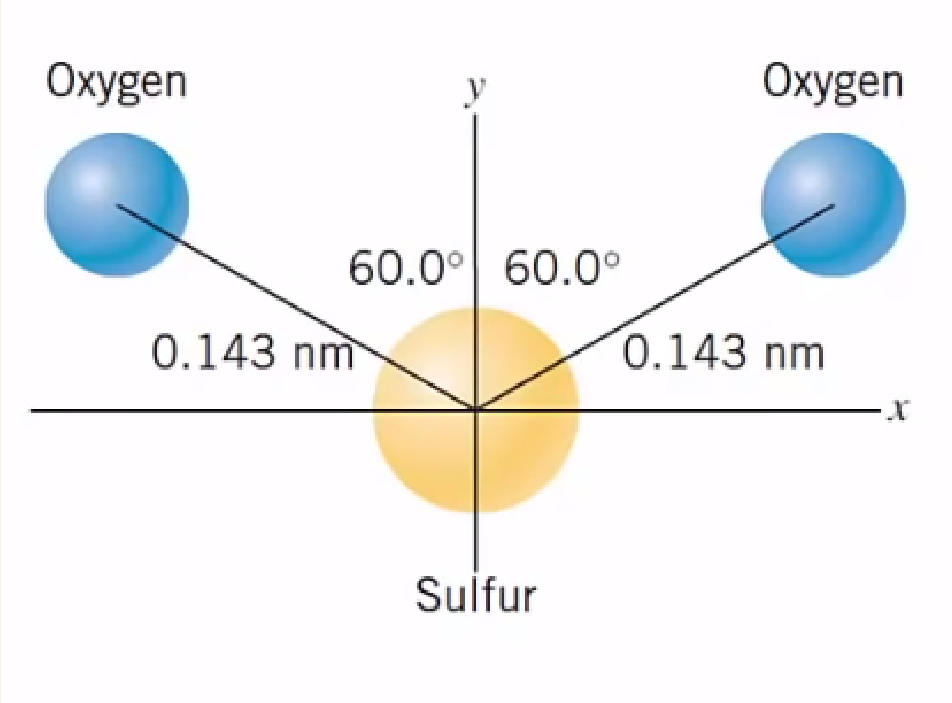
$$(0m)\left(\frac{5kg}{7kg}\right)+(2m)\left(\frac{2kg}{7kg}\right)$$

Notice that the kilograms units will cancel out, leaving it in meters. Since meters is what we're looking for, since we're looking for the position of the center of mass, this makes sense. If I plug this into a calculator, we can see that I get a numerical value of 0.57 meters, which means that the center of mass is somewhere here, much closer to the 5-kilogram object than to the 2-kilogram.

Now let's consider a 2-dimensional example.

Example 3.1

The drawing shows a sulfur dioxide molecule. The sulfur atom is twice as massive as the oxygen atom. Using this information and the data provided in the drawing, find the x and y coordinates of the center of mass of the sulfur dioxide molecule.



As usual, when working in two dimensions, we separate the x and y directions. There's no mass given in this problem, but we can just give the mass of the oxygen the variable M, which would make the mass of the sulfur atom 2M, since it's twice as massive. We also need to set up our coordinate system; let's have the positive x direction be towards the right, the positive y direction to be up, and we can set the sulfur atom to be at 0m in the x and 0m in the y.

Let's look at the sulfur atom to start. The set up for the center of mass contribution of the sulfur atom in the x-direction is:

$$x_{cm} = (x)\left(\frac{2M}{M_{tot}}\right)$$

M_{tot} will be the sum of the masses, which will be the mass of both oxygen atoms and the sulfur atom, or 4M. x is 0m, since we set the position of the sulfur atom to be at 0m. This 0m will make the whole value go to zero, so x_{cm} is 0 for the sulfur. Similarly, since the y position is also 0m, y_{cm} is also 0m.

Now let's look at the oxygen atoms. Again, we use the same setup, and we can substitute M_{tot} with 4M:

$$x_{cm} = (x)\left(\frac{M}{4M}\right)$$

The Ms cancel out here, giving us:

$$x_{cm} = (x)\left(\frac{1}{4}\right)$$

Doing a little trigonometry will give us our x-distance; making a triangle with the line going from the sulfur atom to the oxygen atom and the oxygen atom to the y-axis gives us an x distance sin(60)*0.153nm, or 0.124nm. For the oxygen atom on the left, this value is negative, since it's going left from the origin, and the one on the right is positive, since it's going right of the origin. Plugging these into the center of mass calculation gives us:

$$(0.124nm)\left(\frac{1}{4}\right) + (-0.124nm)\left(\frac{1}{4}\right)$$

Doing the calculation out gives us 0nm.

Alternatively, you can consider the symmetry of the problem. Since the masses are symmetrical both to the left and the right, we can conclude that the center of mass is going to fall along the y-axis at x=0nm. Noticing these symmetries can save you some time, and give you a way to check if your answer makes sense.

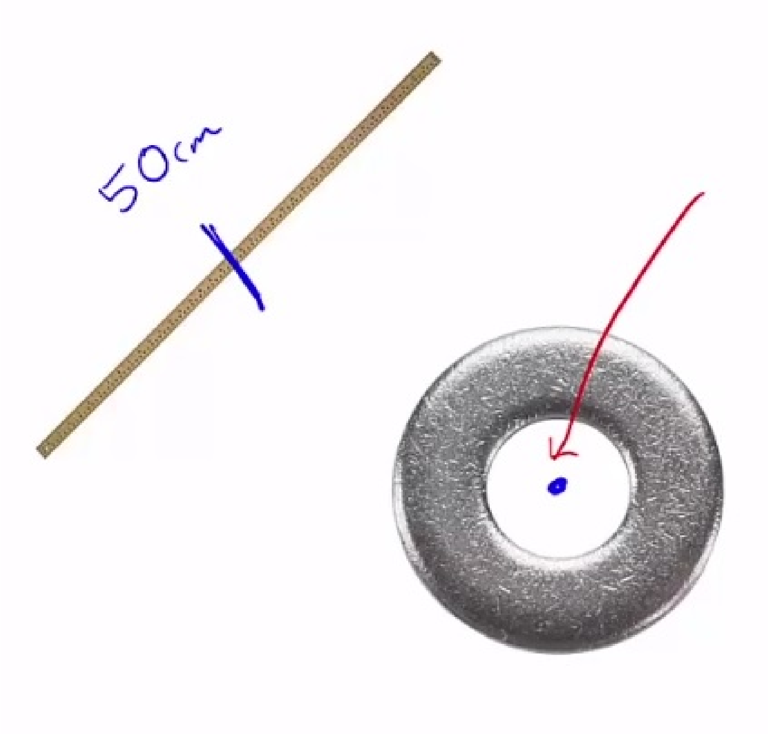
Moving on to the y-direction, we can do the same set up as the x-direction. Doing the trigonometry for the y position gives us cos(60)0.143nm, or 0.072nm. Since both oxygen atoms are above the origin, they are both positive distances away, so our calculation would be:

$$(0.072nm)\left(\frac{1}{4}\right) + (0.072nm)\left(\frac{1}{4}\right)$$

Note that unlike the x-components, the y-components do not cancel out. Doing the calculation gives us 0.036nm. We can write our final answer in coordinates as (0nm, 0.036nm).

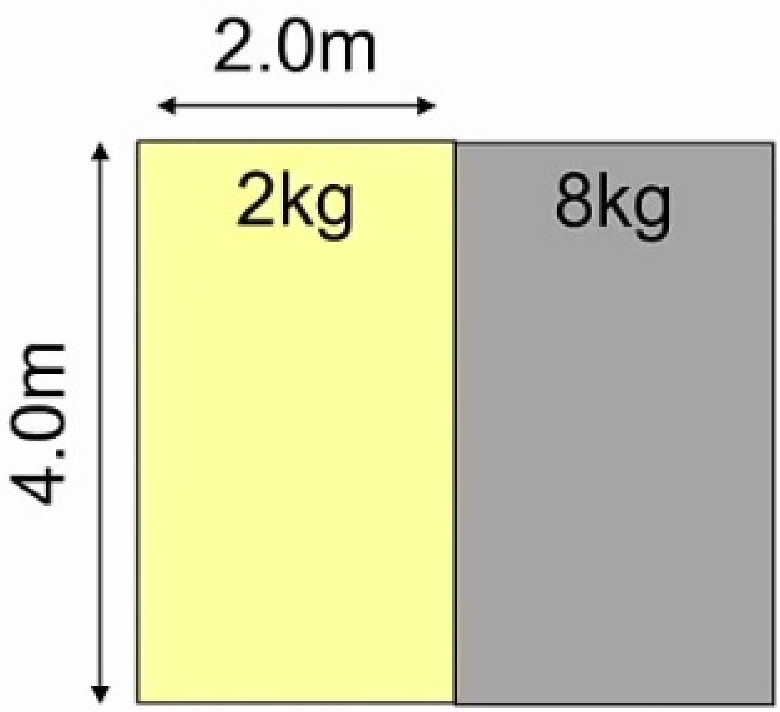
Non-point particles

But what about objects that aren't collections of point particles? Well, if the object is uniform, and this is a very important caveat, then the center of mass will be at the geometric center of the object. For example, if we have say a meter stick, then the center of mass will be at the 50-centimeter mark as that's the middle of the meter stick. If I have a metal washer, then the center of mass will be at the geometric center, right in the middle. You can notice that the center of mass does not need to be inside the body of the object. It can in fact be some random point in space. The center of mass of the washer is not within the metal, it's within the center of the hole.



But what about objects that are not uniform? Well, the answer in this case is divide the object up into uniform chunks, find the center of mass of each chunk, treat each chunk is a point particle located at the center of mass, and then calculate as usual.

So, here I have a nice example problem of a slab made of a light half and a heavy half, and let's think about trying to find the center of mass of this object. Thinking ahead, I expect it to be on this side of the middle, because this is the heavier side of the object. Furthermore, I can already say from the symmetry of the problem that the center of mass is going to lie along this line, right in the middle, vertically, of the object.



Now we follow the same procedure with point masses. First, we establish a coordinate system; we'll set the origin to be at the center of mass of the left chunk. This will simplify our calculation for reasons we'll explore in a bit. We'll also have to break down the problem into x and y coordinates. However, like earlier in this section, we can look at the symmetry of the problem to simplify it. You'll notice that the object is symmetrical in the y direction, and therefore we can assume that the center of mass will be at y=0m. Next, we set up our center of mass calculations for each chunk. For the left chunk, we have:

$$(0m) * (\frac{2kg}{2kg + 8kg})$$

Again, we take the position of the point mass and weight it with its mass over the total mass. Since the position is at 0m, however, we know it'll go to 0; this is why we set the origin to be at the center of mass of one of the objects. Moving on to the right chunk, we'll need to find the position of its center of mass. The center of mass of the object will be at its geometric center, and we can see that there's 1 meter on each side of the center of mass, and 2 meters separating both center of masses. So, we have:

$$(2m) * (\frac{8kg}{2kg + 8kg})$$

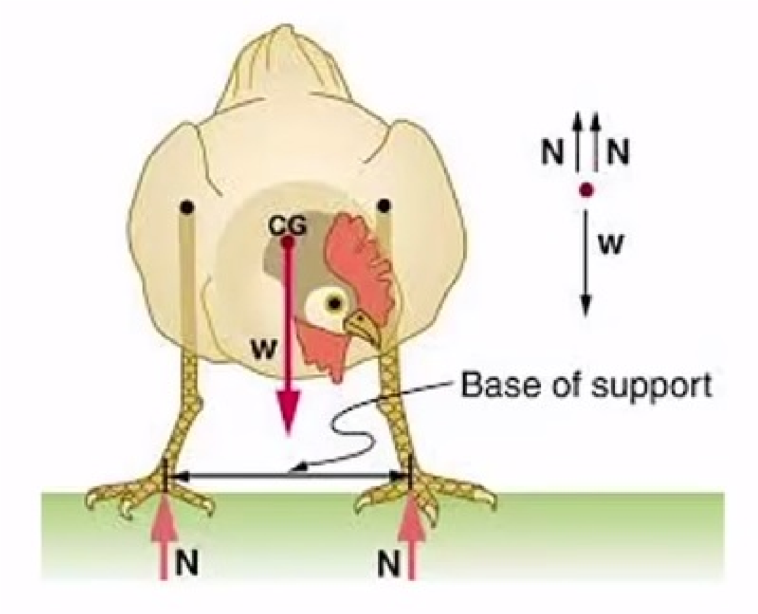
And the total weighted average of the center of mass will be:

$$x_{com} = 0 + (2m) * (\frac{8kg}{2kg + 8kg})$$

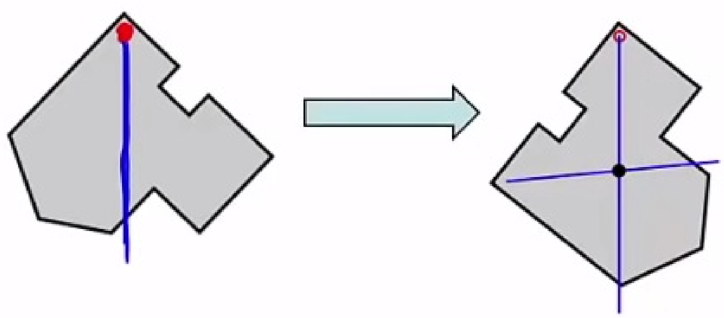
Doing the calculation, we get 1.6m, so the center of mass will be just to the left of the center of mass of the right chunk.

So, that's one part of the definition for center of mass, the mass weighted average position of the object. The center of mass is also the location where gravity can be said to act. As I said at the beginning of this video, when we discuss torque, we'll be interested in where each force acts. For example, when I open a door, I tend to apply the force at the knob in the door. Gravity acts at the center of mass.

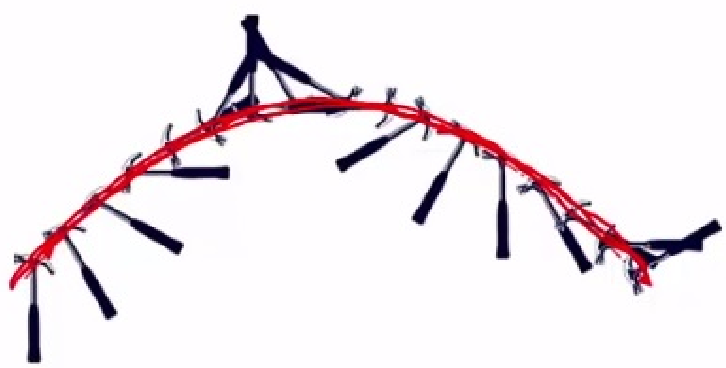
So, if we look at this chicken and we were to make a free body diagram, we would say there are two normal forces, one for each foot, and the weight force, but now we're going to start thinking about where each force is being applied. The two normal forces are being applied one on each foot, so they're being applied there, and the weight force is applied at the center of mass, or the center of gravity, remember that these are synonymous terms as far as this class is concerned, which is roughly in the middle of the chicken.



So, the normal forces get applied where the feet meet the ground and the weight force gets applied at the center of gravity, or the center of mass. So, this is the second part of the definition of center of mass. The center of mass is the point where gravity can be set to act. This part of the definition of center of mass has some consequences. The first piece is that if an object is suspended from a point, then the center of mass will be below the point from which it is hung.



So, let's say we have some oddly shaped object, and I suspend it from a point like here, and I draw a nice line hanging straight down. The center of mass is somewhere on that line. Now, if I take this same object and hang it from a different point, I know that the center of mass is somewhere on this line. Where the two lines cross will be the center of mass. This technique is useful for finding the center of mass of irregular objects. Another consequence of the definition of center of mass as being the point where gravity is said to act means that the center of mass is what follows the parabolic path that we know and love for objects and projectile motion. So, this hammer follows a rather complicated path, but the center of mass, which is closer to the head of the hammer, follows a nice parabolic path just like a ball would.



A final consequence of the center of mass being the point which gravity said to act deals with the balance of an object. Now, if

you hold an object under its center of mass, it will balance. So, for the hammer in the previous example, if I put my finger close to the head of the hammer, I can balance the hammer at that point. However, more complicated objects we're usually interested in not balancing on a single point, so we need to define a quantity known as base of support, and the base of support is the region where the object contacts the ground, plus the space in between.

So, for the example of our chicken the base of support is this area between the feet of the chicken, or if you were to look at a person in some rather fancy shoes, the base of support is everywhere the foot contacts the ground, so, this line here connected by a line, plus all the region in between. All of this is the base of support. if the center of mass of an object is above the base of support, then an object will balance. We will explore this in a laboratory activity in class in more tail and become more comfortable with this idea. Right now, what I want you to take away from it is the definition of base of support, and the fact that center of mass sort of has an aspect of its definition, which is that if the center of mass is over the base of support than an object or balance.

Summary

The center of mass in the center of gravity are, as far as this course is concerned, the same idea, so we might alternate between these two different terms, but as far as we're concerned, they're the same. And the definition of center of mass has many different aspects. The first aspect is that the center of mass is the mass weighted average position of an object, and that this point does not need to be inside the object itself, as we saw in the example of the washer. Second aspect of the center of mass is the center of mass is the point at which the force of gravity can be thought to act, and this aspect of the center of mass is definition has a couple of important consequences. The first is that, if an object is suspended, then the center of mass will be below this suspension point on a straight line. We can also say that the center of mass is what follows the parabolic path in projectile motion. A perhaps subtler consequence of this aspect of the center of mass' definition, as being the point at which gravity can be thought to act, is that an object will balance if its center of mass is over its base of support which again, base of support is defined as all of the points where an object meets the ground connected by straight lines, and all of the area inside.

3.3 Torque

UMASS AMHERST Instructor's Notes

Your Quiz will Cover

- Calculate the net torque exerted by any force on any object including sign
- Describe how to get the maximum torque on an object for a given force
- Contrast force and torque
- Identify the net torque and net force acting on an object at rest

This section is also available as a video lecture, available here:

- **Introduction to Torque (https://www.youtube.com/watch?v=gWHELBPNNKU)**
- **Torque Examples (https://www.youtube.com/watch?v=HjEojBTKveo)**

Before we go into torque, here's a little exercise to try out (yes, it involves doors). With a friend and a door, have your friend push the door open normally, by the doorknob. While they are pushing the door open, try and push back door close to the hinge (be careful not to apply too much force and slam the door into each other). Notice how much force you need to apply to resist the door opening. Now switch positions; notice how much force you need to apply to open the door.

This raises an interesting question; why does the person pushing near the hinge need a lot more force to overcome the force being applied near the doorknob? To understand what's happening here, we need to add another tool to our physics toolbox: torque.

Torque is the turning effect of a force. If a force would cause an object to rotate, like pushing on a door, that force is applying a torque. Mathematically, we represent torque as

$$\tau = rF_{\perp}$$

The r is the distance from the axis of rotation in which the force is being applied, and the F is the force perpendicular to the r. Notice that torque is a vector, and therefore has a direction. However, for torque, we don't apply the traditional up, down, left, right, and so on. Instead, we consider torque to be acting in two directions, clockwise and counterclockwise. A net torque of zero means that all the torque going counterclockwise is equal to the force going clockwise. The units of torque or Nm.

Let's go back to our door example, and try to figure out why the person pushing the door by the doorknob will generally beat out the person pushing near the hinge. If both pushers are pushing at relatively the same amount of force, we can say the force part of the torque is the same for both pushers. However, torque is both dependent on force and the distance from the axis of rotation, in this case the hinge. The person pushing closer to the hinge has a smaller r than that of the person pushing by the doorknob. Notice that in the equation for torque, if r increases, torque increases as well. Thus, we can see why the doorknob pusher has the advantage; their r is larger, so their torque will generally be larger as well.

The r in our equation above references an axis of rotation; what if the object is not rotating at all? This situation gives a freedom when working with problems. If the net torque is zero, then we can put the axis of rotation wherever we want along the axis of

(something something). For example, in the case with the door, if it's not rotating, then we can put the axis of rotation anywhere along the axis of the door; we could put it on the doorknob, the hinge, the middle of the door, or even miles away from the door, as long as the door is not rotating. A general rule of thumb is to put the axis of rotation on a point where a force is acting that we do not know and we're not looking for.

Example 3.2

The pedals of a bicycle rotate in a circle with a diameter of 40cm. What is the maximum force a 55-kg rider can apply by putting all her weight on one pedal?

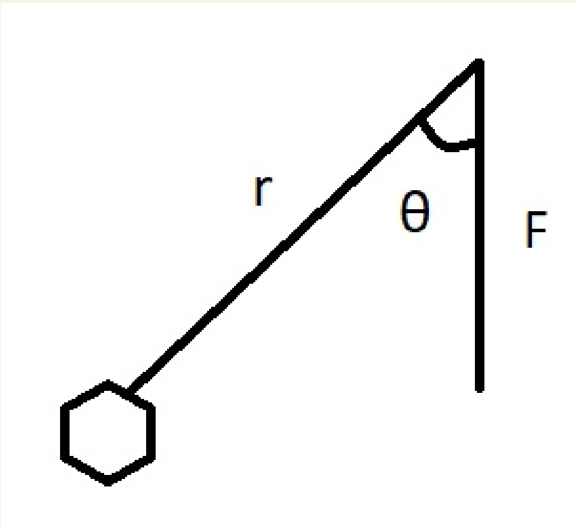
Let's start by considering the situation. The rider applies their force at the end of the pedal, and the pedal rotates at the center, so the distance away from the axis of rotation the force is being applied to should be the distance of the pedal from the center, or the radius of the circle the pedal makes. The diameter is 40 cm, so the radius is 20 cm, or 0.2 m. Next, if the maximum force the rider can apply is all their weight, then the maximum force is equal to the force of weight on the rider, mg. The mass of the rider is 55 kg, so the force is (55 kg)(9.81 kg/N), or 539.55 N. Now we can take our definition of torque and solve for these values, so: $\tau = (0.2m)(539.55N)$

Evaluating this gives us 107.91 Nm, which is the maximum torque.

Example 3.3

A bolt is screwed into a machine you are trying to disassemble, and it needs a torque of 20 Nm to unscrew. How much force do you need to apply to the end of a 30 cm wrench, at an angle of 30 degrees from the wrench, to unscrew the bolt?

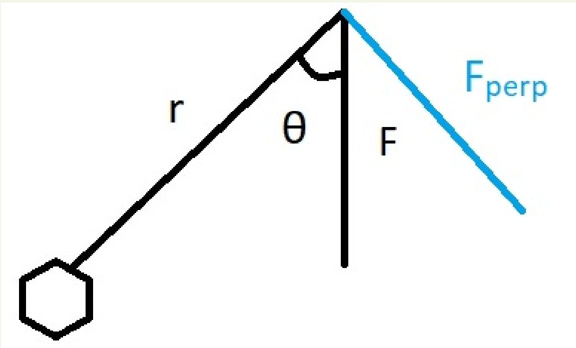
A good starting place with any problem is a simple drawing of the problem:



Here, r is the length of the wrench, F is the force applied, and θ is the angle the force makes with the wrench. Going back to our definition of torque,

$$\boldsymbol{\tau} = rF_{\perp}$$

we have the torque, which is the minimum torque needed to unscrew the bolt, and we have the distance from the axis of rotation to the force, which is the length of the screw. Since the force we're looking for is the force perpendicular to the lever arm, in this case the wrench, and so we need to find the perpendicular part of the force:



Since the perpendicular part of the force is perpendicular to the wrench, we know the angle it makes is 90 degrees, and so we know that the angle between the force and its perpendicular part is (90-30) degrees, or 60 degrees. $F_{\perp}$ is the adjacent to the force, which tells us that $F_{\perp}=F\cos(60)$. Now we have all the parts to solve for F. Substituting our values into the definition of torque:

$$20Nm = (.3m)(Fcos(60))$$

Solving for F brings us to:

$$F = \frac{20Nm}{.3m(\cos(60))}$$

Which gives us a force of 133.33 N, which is the force needed to unscrew the bolt.

4 WORK

4.1 Work: The Scientific Definition

What It Means to Do Work

The scientific definition of work differs in some ways from its everyday meaning. Certain things we think of as hard work, such as writing an exam or carrying a heavy load on level ground, are not work as defined by a scientist. The scientific definition of work reveals its relationship to energy—whenever work is done, energy is transferred.

For work, in the scientific sense, to be done, a force must be exerted and there must be displacement in the direction of the force.

Formally, the **work** done on a system by a constant force is defined to be *the product of the component of the force in the direction of motion times the distance through which the force acts*. For one-way motion in one dimension, this is expressed in equation form as

$$W = \ |\mathbf{F}|\,(\cos\theta)\,|\mathbf{d}|\,, \tag{4.1}$$

where W is work, $\mathbf{d}$ is the displacement of the system, and θ is the angle between the force vector $\mathbf{F}$ and the displacement vector $\mathbf{d}$, as in **Figure 4.1**. We can also write this as

$$W = Fd\cos\theta. \tag{4.2}$$

To find the work done on a system that undergoes motion that is not one-way or that is in two or three dimensions, we divide the motion into one-way one-dimensional segments and add up the work done over each segment.

What is Work?

The work done on a system by a constant force is *the product of the component of the force in the direction of motion times the distance through which the force acts*. For one-way motion in one dimension, this is expressed in equation form as

$$W = Fd\cos\theta, \tag{4.3}$$

where W is work, F is the magnitude of the force on the system, d is the magnitude of the displacement of the system, and θ is the angle between the force vector $\mathbf{F}$ and the displacement vector $\mathbf{d}$.

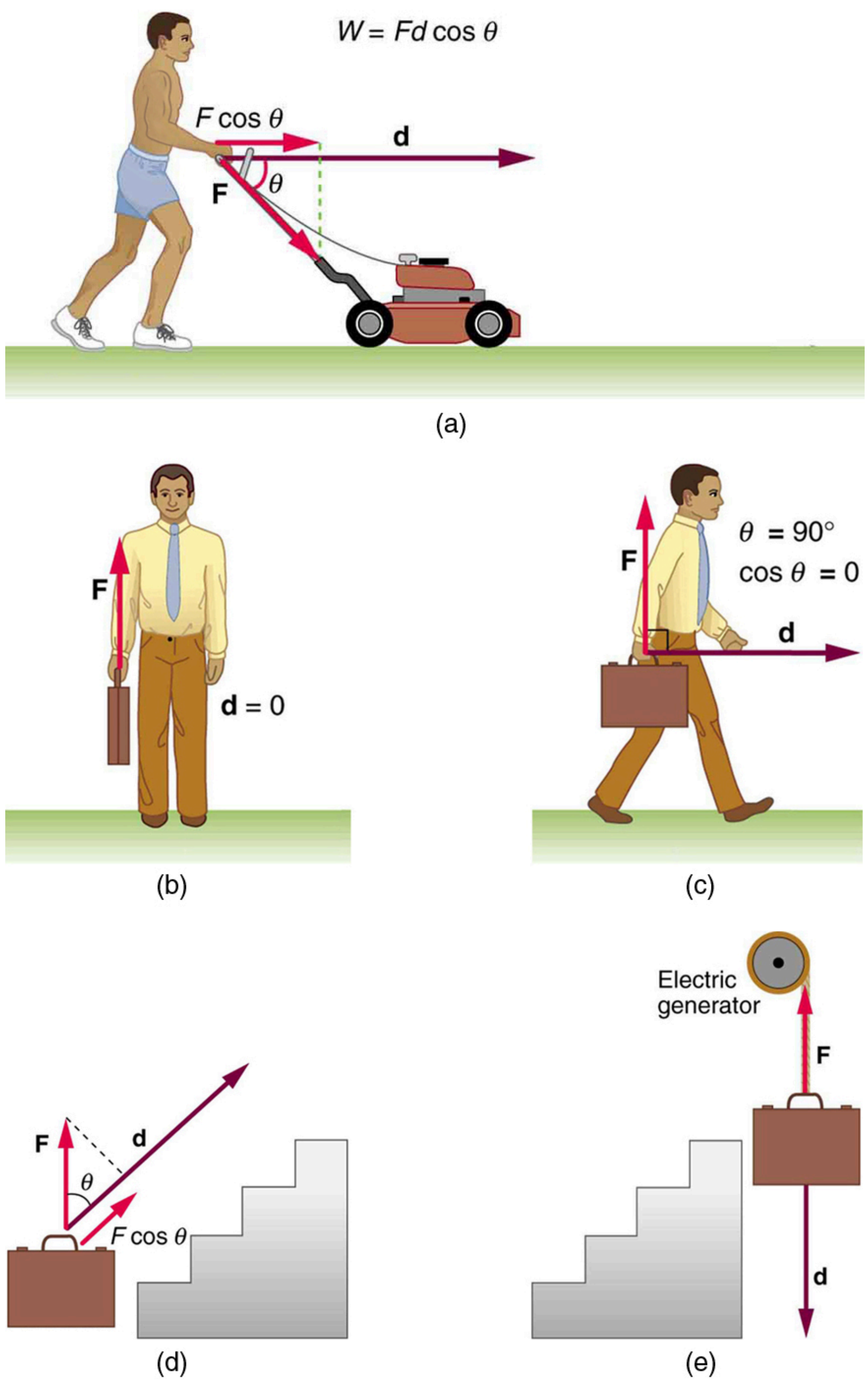


Figure 4.1 Examples of work. (a) The work done by the force $\mathbf{F}$ on this lawn mower is $Fd\cos\theta$. Note that $F\cos\theta$ is the component of the force in the direction of motion. (b) A person holding a briefcase does no work on it, because there is no displacement. No energy is transferred to or from the briefcase. (c) The person moving the briefcase horizontally at a constant speed does no work on it, and transfers no energy to it. (d) Work *is* done on the briefcase by carrying it up stairs at constant speed, because there is necessarily a component of force $\mathbf{F}$ in the direction of the motion. Energy is transferred to the briefcase and could in turn be used to do work. (e) When the briefcase is lowered, energy is transferred out of the briefcase and into an electric generator. Here the work done on the briefcase by the generator is negative, removing energy from the briefcase, because $\mathbf{F}$ and $\mathbf{d}$ are in opposite directions.

To examine what the definition of work means, let us consider the other situations shown in **Figure 4.1**. The person holding the briefcase in **Figure 4.1**(b) does no work, for example. Here $d = 0$, so $W = 0$. Why is it you get tired just holding a load? The answer is that your muscles are doing work against one another, *but they are doing no work on the system of interest* (the "briefcase-Earth system"—see **Gravitational Potential Energy (https://legacy.cnx.org/content/m42148/latest/)** for more details). There must be displacement for work to be done, and there must be a component of the force in the direction of the

motion. For example, the person carrying the briefcase on level ground in **Figure 4.1**(c) does no work on it, because the force is perpendicular to the motion. That is, $\cos 90^{\circ} = 0$, and so $W = 0$.

In contrast, when a force exerted on the system has a component in the direction of motion, such as in **Figure 4.1**(d), work *is* done—energy is transferred to the briefcase. Finally, in **Figure 4.1**(e), energy is transferred from the briefcase to a generator. There are two good ways to interpret this energy transfer. One interpretation is that the briefcase's weight does work on the generator, giving it energy. The other interpretation is that the generator does negative work on the briefcase, thus removing energy from it. The drawing shows the latter, with the force from the generator upward on the briefcase, and the displacement downward. This makes $\theta = 180^{\circ}$, and $\cos 180^{\circ} = -1$; therefore, W is negative.

Calculating Work

Work and energy have the same units. From the definition of work, we see that those units are force times distance. Thus, in SI units, work and energy are measured in **newton-meters**. A newton-meter is given the special name **joule** (J), and $1\text{ J} = 1\text{ N}\cdot\text{m} = 1\text{ kg}\cdot\text{m}^2/\text{s}^2$. One joule is not a large amount of energy; it would lift a small 100-gram apple a distance of about 1 meter.

Example 4.1 Calculating the Work You Do to Push a Lawn Mower Across a Large Lawn

How much work is done on the lawn mower by the person in **Figure 4.1**(a) if he exerts a constant force of 75.0 N at an angle 35° below the horizontal and pushes the mower 25.0 m on level ground? Convert the amount of work from joules to kilocalories and compare it with this person's average daily intake of $10,000\text{ kJ}$ (about 2400 kcal) of food energy. One *calorie* (1 cal) of heat is the amount required to warm 1 g of water by 1°C , and is equivalent to 4.184 J , while one *food calorie* (1 kcal) is equivalent to 4184 J .

Strategy

We can solve this problem by substituting the given values into the definition of work done on a system, stated in the equation $W = Fd\cos\theta$. The force, angle, and displacement are given, so that only the work W is unknown.

Solution

The equation for the work is

$$W = Fd\cos\theta. \tag{4.4}$$

Substituting the known values gives

$$\begin{aligned} W &= (75.0\text{ N})(25.0\text{ m})\cos(35.0^{\circ}) \\ &= 1536\text{ J} = 1.54\times10^{3}\text{ J}. \end{aligned} \tag{4.5}$$

Converting the work in joules to kilocalories yields $W = (1536\text{ J})(1\text{ kcal}/4184\text{ J}) = 0.367\text{ kcal}$. The ratio of the work done to the daily consumption is

$$\frac{W}{2400\text{ kcal}} = 1.53\times10^{-4}. \tag{4.6}$$

Discussion

This ratio is a tiny fraction of what the person consumes, but it is typical. Very little of the energy released in the consumption of food is used to do work. Even when we "work" all day long, less than 10% of our food energy intake is used to do work and more than 90% is converted to thermal energy or stored as chemical energy in fat.

Section Summary

- Work is the transfer of energy by a force acting on an object as it is displaced.
- The work W that a force $\mathbf{F}$ does on an object is the product of the magnitude F of the force, times the magnitude d of the displacement, times the cosine of the angle θ between them. In symbols,

$$W = Fd\cos\theta. \tag{4.7}$$

- The SI unit for work and energy is the joule (J), where $1\text{ J} = 1\text{ N}\cdot\text{m} = 1\text{ kg}\cdot\text{m}^2/\text{s}^2$.
- The work done by a force is zero if the displacement is either zero or perpendicular to the force.
- The work done is positive if the force and displacement have the same direction, and negative if they have opposite direction.

Conceptual Questions

Exercise 4.1

Give an example of something we think of as work in everyday circumstances that is not work in the scientific sense. Is energy transferred or changed in form in your example? If so, explain how this is accomplished without doing work.

Exercise 4.2

Give an example of a situation in which there is a force and a displacement, but the force does no work. Explain why it does no work.

Exercise 4.3

Describe a situation in which a force is exerted for a long time but does no work. Explain.

Problems & Exercises

Exercise 4.4

How much work does a supermarket checkout attendant do on a can of soup he pushes 0.600 m horizontally with a force of 5.00 N? Express your answer in joules and kilocalories.

Solution

$$3.00 \text{ J} = 7.17 \times 10^{-4} \text{ kcal} \tag{4.8}$$

Exercise 4.5

A 75.0-kg person climbs stairs, gaining 2.50 meters in height. Find the work done to accomplish this task.

Exercise 4.6

(a) Calculate the work done on a 1500-kg elevator car by its cable to lift it 40.0 m at constant speed, assuming friction averages 100 N. (b) What is the work done on the lift by the gravitational force in this process? (c) What is the total work done on the lift?

Solution

(a) $5.92 \times 10^5 \text{ J}$

(b) $-5.88 \times 10^5 \text{ J}$

(c) The net force is zero.

Exercise 4.7

Suppose a car travels 108 km at a speed of 30.0 m/s, and uses 2.0 gal of gasoline. Only 30% of the gasoline goes into useful work by the force that keeps the car moving at constant speed despite friction. (See **m42151 (https://legacy.cnx.org/content/m42151/latest/#import-auto-id2866785)** for the energy content of gasoline.) (a) What is the magnitude of the force exerted to keep the car moving at constant speed? (b) If the required force is directly proportional to speed, how many gallons will be used to drive 108 km at a speed of 28.0 m/s?

Exercise 4.8

Calculate the work done by an 85.0-kg man who pushes a crate 4.00 m up along a ramp that makes an angle of 20.0° with the horizontal. (See **Figure 4.2**.) He exerts a force of 500 N on the crate parallel to the ramp and moves at a constant speed. Be certain to include the work he does on the crate *and* on his body to get up the ramp.

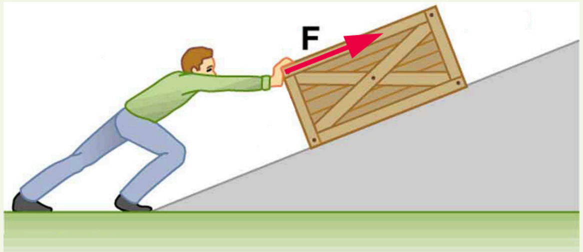


Figure 4.2 A man pushes a crate up a ramp.

Solution

$$3.14\times10^3\ \text{J} \tag{4.9}$$

Exercise 4.9

How much work is done by the boy pulling his sister 30.0 m in a wagon as shown in **Figure 4.3**? Assume no friction acts on the wagon.

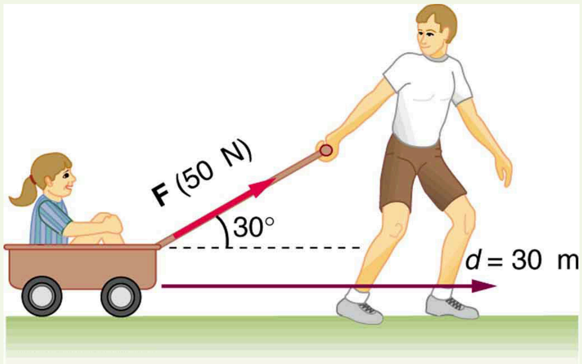


Figure 4.3 The boy does work on the system of the wagon and the child when he pulls them as shown.

Exercise 4.10

A shopper pushes a grocery cart 20.0 m at constant speed on level ground, against a 35.0 N frictional force. He pushes in a direction 25.0° below the horizontal. (a) What is the work done on the cart by friction? (b) What is the work done on the cart by the gravitational force? (c) What is the work done on the cart by the shopper? (d) Find the force the shopper exerts, using energy considerations. (e) What is the total work done on the cart?

Solution

(a) $-700\ \text{J}$

(b) 0

(c) 700 J

(d) 38.6 N

(e) 0

Exercise 4.11

Suppose the ski patrol lowers a rescue sled and victim, having a total mass of 90.0 kg, down a 60.0° slope at constant speed, as shown in **Figure 4.4**. The coefficient of friction between the sled and the snow is 0.100. (a) How much work is done by friction as the sled moves 30.0 m along the hill? (b) How much work is done by the rope on the sled in this distance? (c) What is the work done by the gravitational force on the sled? (d) What is the total work done?

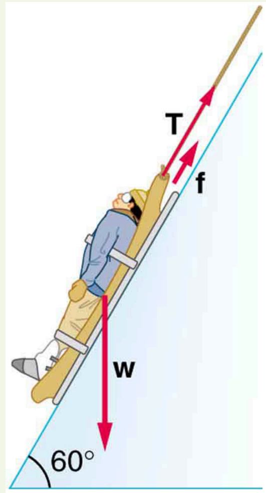


Figure 4.4 A rescue sled and victim are lowered down a steep slope.

4.2 Representing Work Graphically

Net Work and the Work-Energy Theorem

We know from the study of Newton's laws in **Dynamics: Force and Newton's Laws of Motion (https://legacy.cnx.org/content/m42129/latest/)** that net force causes acceleration. We will see in this section that work done by the net force gives a system energy of motion, and in the process we will also find an expression for the energy of motion.

Let us start by considering the total, or net, work done on a system. Net work is defined to be the sum of work done by all external forces—that is, **net work** is the work done by the net external force $\mathbf{F}_{\text{net}}$. In equation form, this is

$W_{\text{net}} = F_{\text{net}} d \cos\theta$ where θ is the angle between the force vector and the displacement vector.

Figure 4.5(a) shows a graph of force versus displacement for the component of the force in the direction of the displacement—that is, an $F \cos\theta$ vs. d graph. In this case, $F \cos\theta$ is constant. You can see that the area under the graph is $Fd \cos\theta$, or the work done. **Figure 4.5**(b) shows a more general process where the force varies. The area under the curve is divided into strips, each having an average force $(F \cos\theta)_{i(\text{ave})}$. The work done is $(F \cos\theta)_{i(\text{ave})} d_i$ for each strip, and the total work done is the sum of the W_i . Thus the total work done is the total area under the curve, a useful property to which we shall refer later.

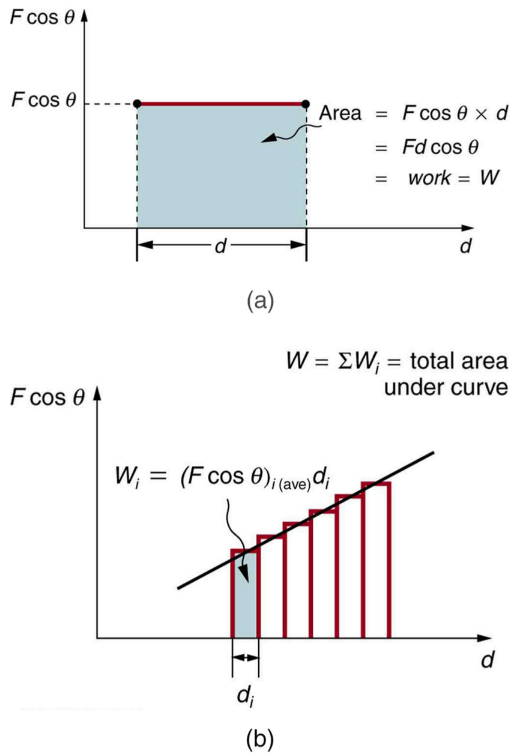


Figure 4.5 (a) A graph of $F \cos \theta$ vs. d, when $F \cos \theta$ is constant. The area under the curve represents the work done by the force. (b) A graph of $F \cos \theta$ vs. d in which the force varies. The work done for each interval is the area of each strip; thus, the total area under the curve equals the total work done.

4.3 Work in Terms of Pressure

UMASS AMHERST Instructor's Notes

This sections is also available as a video: **https://www.youtube.com/watch?v=7F4kS0tXkCU (https://www.youtube.com/watch?v=7F4kS0tXkCU)**

Let's explore another way to think about work. So, why do we need another way to think about work, why do we need another definition for this idea of work? Well the definition of work that we have, Fdcos(θ), works really well for objects like people and cars and blocks. However, in the life sciences and beginning in the next unit as well, we are often interested in the behavior of things that physicists call fluids, which means gases and liquids. For gases and liquids, you tend to not be interested in the force you apply to them but instead the pressure. To remind you, pressure is the force divided by the area, which means that it would have units of N/m^2. Some other common units of pressure that you might have seen are Pascals or atmospheres. Think about the air pressure in a tire. You might see that quoted as N/cm^2 or in atmospheres. Or, you might also think about the osmotic pressure of a fluid inside of a cell. It is therefore useful to be able to think about the work done on the fluid in terms of pressure.

How will we figure out the expression for work in terms of pressure? Well, we will do this simple example of a gas inside of a container with a piston.

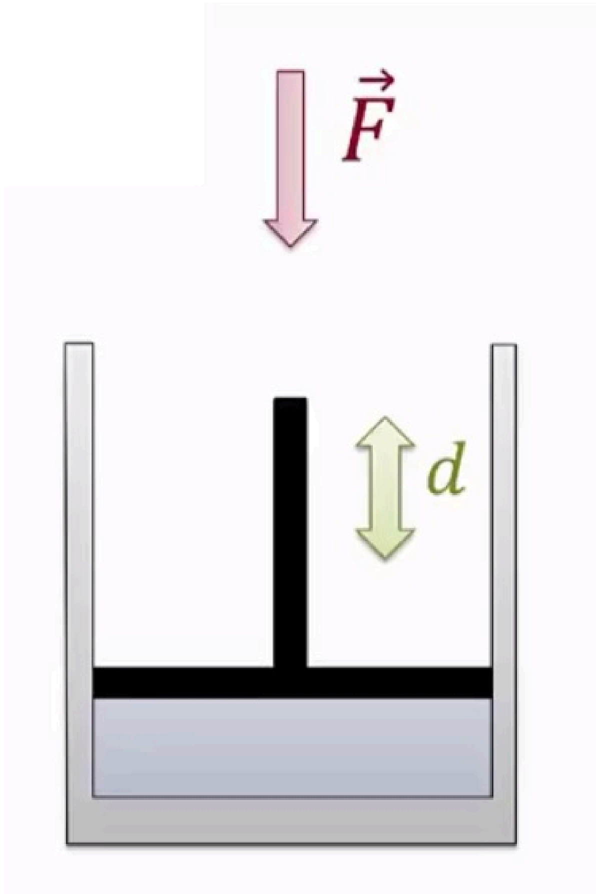


Figure 4.6 A gas being compressed by a piston. The piston does work on the gas equal to the force exerted on it times the distance it moves.

As we compress the gas, we apply a force on the piston for some distance. We are doing work on the gas, and since the force and the displacement are in the same direction, we know that that work is positive. Most the time in the life sciences in particular, we are interested in gases at what we call constant pressure, because organisms and chemical reactions are open to the air and therefore everything happens at a constant pressure of one at. In our example up to this point, the gas is going to increase in pressure as we compress. So, to solve that problem, let's puncture a little hole in the bottom, and let some gas escape to maintain constant pressure of the gas as we compress it.

How much work is done on the gas by our force in this admittedly very contrived case? Well, let's think about this. The gas has some amount of volume before we compress it. We can think about the area of the bottom of the container A, and the height h, and if you multiply these two quantities together, you'll get the volume, Ah. As we apply our force to the piston, compressing it and having some of the gas leak out to maintain constant pressure, the area stays the same, but the height shrinks, and it in shrinks by the exact amount of the displacement d that we compress the piston. So, we can say the change in height is equal to -d, where we have this negative sign because the height is getting smaller as the distance is getting larger.

The change in volume of the gas is then A, the area, which doesn't change, times the change in height or, -Ad. If we multiply this change in volume by the pressure of the gas inside, PΔV, and then we replace the pressure with this definition of force over area and volume AΔh, and then we use our relationship we've just discovered that the change in height is equal to negative the change in distance, then we see that the area's cancel out, and we are left with just a force times the distance. We're left with the amount of work that we did, Fd is the amount of work we did on this piston. The only difference is we get a negative sign.

Therefore, we can conclude that the work done on the gas is minus the pressure of the gas times the change in volume. We're getting this negative sign because if the force and the displacement are in the same direction, that means the volume of the gas is going to get smaller. So, a positive work on the gas will result in a smaller volume, a -ΔV.

In summary, for a fluid, i.e. a gas or liquid, at constant pressure, the work done by some external force on the fluid can be written in terms of the pressure of the fluid and the change in the volume of the fluid. Mathematically, we say that the work is equal to the -PΔV. Mathematically, we say that the W=-PΔV, and this negative sign results because positive work done on the gas will result in the gas compressing, and therefore shrinking volume. This concludes this video.

Glossary

energy: the ability to do work

joule: SI unit of work and energy, equal to one newton-meter

work: the transfer of energy by a force that causes an object to be displaced; the product of the component of the force in the direction of the displacement and the magnitude of the displacement

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