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Deriving Deadlines and Periods for Real-Time Update Transactions

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Abstract

Typically, temporal validity of real-time data is maintained by periodic update transactions. In this paper, we examine the problem of period and deadline assignment for these update transactions such that (1) the imposed workload is minimized. To this end, we propose a novel approach, named More-Less principle. By applying this principle, updates occur with a period which is more than the period obtained through traditional approaches but with a deadline which is less than the traditional period. We show that the More-Less principle is better than existing approaches in terms of schedulability and the imposed load. We examine the issue of determining the assignment order in which transactions must be considered for period and deadline assignment so that the resulting workloads can be minimized. To this end, the More-Less principle is first examined in a restricted case where the Shortest Validity First (SVF) order is shown to be an optimal solution. We then relax some of the restrictions and show that SVF is an approximate solution which results in workloads that are close to the optimal solution. Our analysis and experiments show that the More-Less principle is an effective design principle that can provide better schedulability and reduce update transaction workload while guaranteeing data validity constraints.

1 Introduction

A real-time database (RTDB) is composed of real-time objects which are updated by periodic sensor transactions. An object in the database models a real world entity, for example, the position of an aircraft. A real-time object is one whose state may become invalid with the passage of time. Associated with the state is a temporal validity interval. To monitor the states of objects faithfully, a real-time object must be refreshed by a sensor transaction before it becomes invalid, i.e., before its temporal validity interval expires. The actual length of the temporal validity interval of a real-time object is application dependent. Sensor transactions are generated by intelligent sensors which periodically sample the value of real-time objects. When sensor transactions arrive at RTDBs with sampled data values, their updates are installed and real-time data are refreshed. So one of the important design goals of RTDBs is to guarantee that temporal data remain fresh, i.e., they are always valid. Therefore, efficient design principles are desired to guarantee the freshness of temporal data in RTDBs while minimizing the workload resulting from periodic sensor transactions.

In this paper, we propose the More-Less principle, a design principle which maintains the freshness of temporal data while reducing the workload incurred by periodic sensor transactions. It is shown that the More-Less principle outperforms traditional approaches in terms of sensor transaction schedulability and imposed workload. Using the More-Less principle, transactions are considered in a given order and their periods and deadlines are assigned. So an important issue is to determine the order so that the imposed transaction workload can be minimized. It is demonstrated, through both analysis and experiments, that Shortest Validity First (SVF) is an efficient assignment order to minimize workload for update transactions.

This paper is organized as follows: Section 2 reviews traditional approaches and introduces the intuition underlying the More-Less principle. The More-Less principle is formally introduced in Section 3, and compared with a traditional approach. We also examine the issue of determining the assignment order. Specifically, we propose and analyze Shortest Validity First (SVF), an efficient transaction assignment order to minimize workload. An application of the More-Less principle is discussed in Section 4. Experimental results are presented in Section 5. We conclude the paper in Section 6.

2 Design Principles

In this section, traditional approaches for maintaining temporal validity, namely One-One and Half-Half principles, are reviewed, then the More-Less principle is introduced through an example. Formal definitions of some of the often-used symbols are given in Table 1.

We assume a simple execution semantics for periodic transactions: a transaction must be executed once every period. However, there is no guarantee on when an instance of
a periodic transaction is actually executed within a period.

2.1 One-One Principle

According to the first principle, the period and relative deadline\(^1\) of a sensor transaction have to be equal to the data validity interval. Because the separation of the execution of two consecutive instances of a transaction can be more than the validity interval, data can become invalid under the One-One principle. So this principle can not guarantee the freshness of temporal data in RTDBs.

Example 2.1: Consider Figure 1: A periodic sensor transaction \(\tau_i\) with deterministic execution time \(C_i\) refreshes temporal data \(X_i\) with validity interval \(\alpha_i\). The period \(P_i\) and relative deadline \(D_i\) of \(\tau_i\) are assigned the value \(\alpha_i\). Suppose \(J_{i,j}\) and \(J_{i,j+1}\) are two consecutive instances of sensor transaction \(\tau_i\). Transaction instance \(J_{i,j}\) samples data \(X_i\) with validity interval \([T, T + \alpha_i]\) at time \(T\), and \(J_{i,j+1}\) samples data \(X_i\) with validity interval \([T + \alpha_i, T + 2\alpha_i]\) at time \(T + \alpha_i\). From Figure 1, the actual arrival time of \(J_{i,j}\) and finishing time of \(J_{i,j+1}\) can be as close as \(2C_i\), and as far as \(2P_i\), i.e., \(2\alpha_i\) when the period of \(\tau_i\) is \(\alpha_i\). In the latter case, the validity of data \(X_i\) refreshed by \(J_{i,j}\) expires after time \(T + \alpha_i\). Since \(J_{i,j+1}\) can not refresh data \(X_i\) before time \(T + \alpha_i\), \(X_i\) is invalid from \(T + \alpha_i\) until it is refreshed by \(J_{i,j+1}\), just before the next deadline \(T + 2\alpha_i\).

2.2 Half-Half Principle

In order to guarantee the freshness of temporal data in RTDBs, the period and relative deadline of a sensor transaction are typically set to be less than or equal to one-half of the data validity interval \([10]\). In Figure 1, the farthest distance (based on the arrival time of a periodic transaction instance and the finishing time of its next instance) of two consecutive sensor transactions is \(2P_i\). If \(2P_i \leq \alpha_i\), then the freshness of temporal data \(X_i\) is guaranteed as long as instances of sensor transaction \(\tau_i\) does not miss their deadlines. Recent work \([6]\) on similarity-based load adjustment also adopts this principle to adjust periods of sensor transactions based on similarity bound.

Unfortunately, even though data freshness is guaranteed, this design principle at least doubles the sensor transaction workload in the RTDBs compared to the One-One principle. Next, we introduce a new principle which guarantees the freshness of temporal data but incurs much less workload compared to the Half-Half principle.

2.3 More-Less Principle: Intuition

The goal of the More-Less principle is to minimize sensor transaction workload while guaranteeing the freshness of temporal data in RTDBs. For simplicity of discussion, we assume that a sensor transaction is responsible for updating one temporal data item in the system. In More-Less, the period of a sensor transaction is assigned to be more than half of the validity interval of the temporal data updated by the transaction, while its corresponding relative deadline is assigned to be less than half of the validity interval of the same data. However, the sum of the period and relative deadline is always equal to the length of the validity interval of the data updated. Consider Figure 2. Let \(P_i > \frac{\alpha_i}{2}\), \(C_i \leq D_i < P_i\) where \(P_i + D_i = \alpha_i\). The farthest distance (based on the arrival time of a periodic transaction instance and the finishing time of its next instance) of two consecutive sensor transactions \(J_{ij}\) and \(J_{ij+1}\) is \(P_i + D_i\). In this case, the freshness of \(X_i\) can always be maintained if sensor transactions make their deadlines. Obviously, load incurred by sensor transaction \(\tau_i\) can be reduced if \(P_i\) is enlarged (which implies that \(D_i\) is shrunk). Therefore, we have the constraints \(C_i \leq D_i < P_i\) and \(P_i + D_i = \alpha_i\) which aim at minimizing the workload of periodic transaction \(\tau_i\).

---

\(^1\)The relative deadline of a transaction = transaction deadline - transaction arrival time.
Table 2. Comparison of three principles

<table>
<thead>
<tr>
<th>Principle</th>
<th>$D_i$</th>
<th>$P_i$</th>
<th>Utilization</th>
</tr>
</thead>
<tbody>
<tr>
<td>One-One</td>
<td>$\alpha_i$</td>
<td>$\alpha_i$</td>
<td>$U_i = \frac{\alpha_i}{2\alpha_i} = \frac{1}{2}$</td>
</tr>
<tr>
<td>Half-Half</td>
<td>$\frac{\alpha_i}{2}$</td>
<td>$\frac{\alpha_i}{2}$</td>
<td>$U_h = \frac{\frac{\alpha_i}{2}}{\frac{\alpha_i}{2} + \frac{\alpha_i}{2}} = \frac{1}{2}$</td>
</tr>
<tr>
<td>More-Less</td>
<td>$\frac{\alpha_i}{3}$</td>
<td>$\frac{\alpha_i}{3}$</td>
<td>$U_m = \frac{\frac{\alpha_i}{3}}{\frac{\alpha_i}{3} + \frac{\alpha_i}{3} + \frac{\alpha_i}{3}} = \frac{1}{3}$</td>
</tr>
</tbody>
</table>

Example 2.2: Suppose there is temporal data $X_t$ with validity interval $\alpha_t$ in a uniprocessor RTDB system. $\tau_t$ updates $X_t$ periodically. Our goal is to assign proper values to $P_i$ and $D_i$ given $C_i$ and $\alpha_t$ so that CPU utilization resulting from sensor transaction $\tau_t$ can be reduced. Suppose $C_i = \frac{1}{n}\alpha_t$, possible values of $P_i$, $D_i$ and corresponding CPU utilization according to the three different design principles are shown in Table 2.

Only Half-Half and More-Less can guarantee the freshness of temporal data $X_t$ if all the sensor transactions complete before their deadlines. We also notice that $U_h < U_m < U_i$. Intuitively, if $P_i = \frac{\alpha_t}{2\alpha_t}$, then $D_i = \frac{\alpha_t}{2\alpha_t}$, where $N > 2$. The freshness of temporal data in RTDBs is guaranteed if all sensor transactions complete before their deadlines. In such a case, we also notice that $U_m = \frac{\alpha_t}{(N+1)\alpha_t}$ and $U_h < U_m < U_i$. Theoretically, if $N \to \infty$, $U_m \to U_i$.

Unfortunately, how close $U_m$ can get to $U_i$ depends on $C_i$ since $D_i \geq C_i$ implies $\frac{\alpha_t}{N+1} \geq \frac{\alpha_t}{N}$. As $N$ increases, relative deadlines become shorter and sensor transactions are executed with more stringent time constraints.

Therefore, given a set of sensor transactions in RTDBs, we need to find periods and deadlines of update transactions based on the temporal validity intervals of data such that the workload of sensor transactions is minimized and the schedulability of the resulting sensor transactions is guaranteed. The More-Less principle achieves this, as shown in the next section.

3 More-Less: Analysis and Results

In this section, we formally introduce the More-Less principle with three constraints: Validity Constraint, Deadline Constraint and Schedulability Constraint. We then show that the schedulability of transactions and data freshness are guaranteed by More-Less. To understand the advantages of More-Less, we then compare More-Less with Half-Half and show that More-Less is superior to Half-Half in terms of schedulability and for minimizing CPU utilization. We show that assignment order, i.e., the order in which periods and deadlines are assigned has an important impact on schedulability and CPU utilization of solutions derived from More-Less. Therefore, to find an optimal assignment order for More-Less, we investigate the issues of assignment order with the aid of a concept named partitioning. We show that Shortest Validity First (SVF), an assignment order proposed in this paper, results in an optimal solution under certain restrictions. With the relaxation of some of the restrictions, it is proved that SVF produces an approximate solution within a certain bounded range of optimal solutions in general. SVF is shown to be a good heuristic solution in many applications, especially, where validity interval lengths are much larger than transaction computation times.

3.1 The Design Principle

From here on, $\mathcal{T} = \{\tau_i\}_{i=1}^m$ refers to a set of sensor transactions $\{\tau_1, \tau_2, ..., \tau_m\}$ and $\mathcal{X} = \{X_i\}_{i=1}^m$ refers to a set of temporal data $\{X_1, X_2, ..., X_m\}$ where $X_i$ ($1 \leq i \leq m$) is associated with a validity interval of length $\alpha_t$; transaction $\tau_i$ ($1 \leq i \leq m$) updates the corresponding data $X_i$. $C_i$, $D_i$ and $P_i$ ($1 \leq i \leq m$) denote execution time, relative deadline, and period of transaction $\tau_i$, respectively. Our goal is to determine $P_i$ and $D_i$ such that all the sensor transactions are schedulable and CPU utilization resulting from sensor transactions is minimized.

Although dynamic-priority scheduling is in general more effective than fixed-priority scheduling, they are also more difficult to implement and hence can incur higher system overhead than fixed-priority scheduling. Moreover, for many applications, it is possible to implement fixed-priority algorithms in the hardware level by the use of priority-interrupt mechanism. Thus, the overhead involved in scheduling tasks can be reduced to a minimal level [9]. Given this, we study fixed-priority scheduling algorithms in this paper.

For convenience, we use terms transaction and task interchangeably in this paper.

First, consider the longest response time for any instance of a periodic task $\tau_i$ where the response time is the difference between the transaction initiation time $(I_i + KP_i)$ and the transaction completion time where $I_i$ is the offset within the period.

Theorem 3.1: For a set of periodic tasks $\mathcal{T} = \{\tau_i\}_{i=1}^m$ with task initiation time $(I_i + KP_i$) ($K = 0, 1, 2, ..., m$), the longest response time for any instance of $\tau_i$ occurs for the first instance of $\tau_i$ when $I_1 = I_2 = ... = I_m = 0$.

For $I_i = 0$ ($1 \leq i \leq m$), the tasks are in phase because the first instances of all the tasks are initiated at the same time. It should be noted that we only discuss tasks in phase tasks in this paper. A time instant after which a task has the longest response time is called a critical instant, e.g., time 0 is a critical instant for all the tasks if those tasks are in phase.

Further, Leung and Whitehead [9] introduced a fixed-priority scheduling algorithm, deadline monotonic scheduling algorithm, in which task priorities are assigned inversely with respect to task deadlines, that is, $\tau_i$ has higher priority than $\tau_j$ if $D_i < D_j$ [9].

Theorem 3.2: For a set of periodic tasks $\mathcal{T} = \{\tau_i\}_{i=1}^m$ with $D_i \leq P_i$ ($1 \leq i \leq m$), the optimal fixed priority scheduling algorithm is the deadline monotonic scheduling
algorithm. A task set is schedulable by this algorithm if the first instance of each task after a critical instant meets its deadline.

Since the deadline monotonic algorithm is an optimal fixed priority scheduling algorithm for a set of tasks \( \{ \tau_i \}_{i=1}^m \) with \( D_i \leq P_i \ (1 \leq i \leq m) \), it is used to maintain the schedulability of periodic transactions in our approach.

The More-Less principle determines deadlines and periods of transactions such that the following three constraints are satisfied:

- **Validity Constraint:** \( P_i + D_i \leq \alpha_i \)
- **Deadline Constraint:** \( C_i \leq D_i \leq P_i \)
- **Schedulability Constraint:** Without loss of generality, assume that for \( i < j, \tau_i \rightarrow \tau_j \) (i.e., \( \tau_i \) precedes \( \tau_j \) when they are considered for deadline and period assignment\(^2\)). Because the transactions are scheduled by the deadline monotonic algorithm, the following inequality constraint must hold:

\[
\sum_{j=1}^i (n_{ij} \times C_j) \leq D_i \quad (1 \leq i \leq m),
\]

where \( n_{ij} \) denotes the number of times transaction \( \tau_j \) occurs before the first instance of \( \tau_i \) completes. Therefore, \( \sum_{j=1}^i (n_{ij} \times C_j) \) represents the response time of the first instance of \( \tau_i \). It is easy to see that for any \( i \), \( n_{ii} = 1 \).

The next theorem proves the correctness of the More-Less principle.

**Theorem 3.3:** Given a set of update transactions \( T = \{ \tau_i \}_{i=1}^m (m \geq 1) \) with CPU computation times \( \{ C_i \}_{i=1}^m \) and validity interval lengths \( \{ \alpha_i \}_{i=1}^m \) as well as an assignment order \( \tau_1 \rightarrow \tau_2 \rightarrow \ldots \rightarrow \tau_m \), the set of transactions is schedulable and data freshness is guaranteed.

**Proof:** We need to prove that the three constraints of the More-Less principle can guarantee the schedulability of transactions and freshness of data. Because of schedulability constraint, the first instance of every transaction can meet its deadline. Combined with the deadline constraint, it follows from Theorem 3.2 that the set of transactions can be scheduled by the deadline monotonic algorithm. Since transactions satisfy validity constraint, data freshness can also be guaranteed. Hence the set of transactions is schedulable and data freshness is guaranteed.

Given the More-Less principle, the optimization problem we need to solve is a non-linear programming problem: Determine \( D_i \) and \( P_i \) such that

\[
U_m = \sum_{i=1}^m \frac{C_i}{P_i}
\]

is minimized subject to the three constraints above.

From the three constraints underlying More-Less, we know that \( P_i \leq \alpha_i - \sum_{j=1}^i (n_{ij} \times C_j) \). Let \( P_i = \alpha_i - \sum_{j=1}^i (n_{ij} \times C_j) - \beta_i \) (\( \beta_i \geq 0 \)). Now we transform the problem to be an assignment order problem so that

\[
U_m = \sum_{i=1}^m \frac{C_i}{\alpha_i - \sum_{j=1}^i (n_{ij} \times C_j) - \beta_i}
\]

is minimized under the three constraints above.

**Example 3.1:** A set of transactions is given in Table 3 with transaction numbers, computation times, and validity interval lengths. Half-Half and More-Less are applied...
Example 3.1 shows that More-Less can have lower CPU utilization than Half-Half. Given any set of transactions, does More-Less produce better schedulability than Half-Half? This is answered in the affirmative next.

### 3.2 Comparison of More-Less and Half-Half

**Theorem 3.4:** If any set of update transactions $T = \{\tau_1\}_{i=1}^m$ ($m \geq 1$) can be scheduled to guarantee data freshness using any fixed priority scheduling algorithm based on periods derived from Half-Half, then it can also be scheduled by the deadline monotonic algorithm based on the More-Less principle.

**Proof:** If $m = 1$, it is trivial. Let us look at the case of $m > 1$. Without loss of generality, assume that transaction priorities are assigned in the order of $\tau_1 \rightarrow \tau_2 \rightarrow \ldots \rightarrow \tau_m$ by the Half-Half principle. Let us assume that the same priority order is retained by the More-Less principle. Let $D_i^H$ and $P_i^H$ denote the deadline and period of transaction $\tau_i$ in Half-Half, and $D_i^M$ and $P_i^M$ denote the deadline and period of transaction $\tau_i$ in More-Less, respectively. Also let $R_{ij}^H$ and $R_{ij}^M$ denote the response time of the $j$th instance of transaction $\tau_i$ in Half-Half and More-Less principle, respectively. We know that $D_i^H = D_i^H = \frac{t_i}{t_{ij}^H}$. Since the set of transactions can be scheduled by a fixed priority scheduling algorithm based on Half-Half, we will prove that $D_i^H \geq D_i^M$ and $P_i^H \leq P_i^M$. This can be proved by induction.

- In case of $m > 1$, we know that $C_i < D_i^H$. Otherwise, $C_i = D_i^H$ implies that $C_i = P_i^H$, i.e., $\tau_i$ would consume 100% CPU and other transactions would not be scheduled. Since $R_{i1}^H = C_i$ and $C_i < D_i^H$, we know that $R_{i1}^H < D_i^H$. Because $D_i^H = P_i^H = \frac{t_i}{t_{ij}^H}$, we have $P_i^H < \frac{t_i}{t_{ij}^H}$. Because $\tau_1$ has the highest priority in the task set, we have $R_{i1}^M = R_{i1}^M$. Hence $D_i^M = R_{i1}^M = C_i$, which is less than $\frac{t_i}{t_{ij}^M}$. According to More-Less, let $P_i^M = \alpha_i - D_i^M$, which implies $P_i^M > \frac{t_i}{t_{ij}^M}$.
- Assume that for all $1 \leq j < i - 1$, $D_j^M \leq D_j^H$ and $P_j^M \geq P_j^H$ hold. Then therefore we can conclude that $D_i^M \leq P_i^M$ ($1 \leq i \leq m$), and the first instance of $\tau_i$ ($1 \leq i \leq m$) can make its deadline. It directly follows from Theorem 3.2 that the set of transactions with deadlines and periods derived from More-Less can be scheduled by deadline monotonic.

From Theorem 3.4, if there is a feasible solution based on Half-Half for a set of transactions, there must be a feasible solution based on More-Less. However, the converse is not true. This is illustrated by Example 3.2.

**Example 3.2:** Transactions are listed in Table 4 with transaction numbers, computation times, and periods. Half-Half and More-Less are applied to the transaction set, and resulting deadlines and periods are shown in Table 4. It is clear from Table 4 that the transaction set resulting from Half-Half is non-schedulable because its CPU utilization is $\frac{4}{10} + \frac{4}{10} + \frac{5}{10} = 1.25 > 1.0$. However, transactions with periods resulting from More-Less is schedulable by assigning priorities $\tau_1 \rightarrow \tau_2 \rightarrow \tau_3 \rightarrow \tau_4$. Figure 3 shows that the first instance of every transaction in the set can meet its deadline, which indicates that the transaction set is schedulable according to Theorem 3.2. However, an assignment order $\tau_2 \rightarrow \tau_3 \rightarrow \tau_4 \rightarrow \tau_1$ under More-Less would not be able to produce a feasible solution. This indicates that assignment orders of transactions can significantly affect the schedulability of transactions.

**Table 3. Parameters and results for example 3.1**

<table>
<thead>
<tr>
<th>$i$</th>
<th>$C_i$</th>
<th>$\alpha_i$</th>
<th>More-Less</th>
<th>Half-Half</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>3</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>20</td>
<td>4</td>
<td>16</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$i$</th>
<th>$C_i$</th>
<th>$\alpha_i$</th>
<th>More-Less</th>
<th>Half-Half</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>4</td>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>5</td>
<td>2</td>
<td>3</td>
</tr>
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<td>8</td>
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<td>5</td>
</tr>
<tr>
<td>4</td>
<td>4</td>
<td>20</td>
<td>9</td>
<td>11</td>
</tr>
</tbody>
</table>

**Table 4. Parameters and results of example 3.2**

<table>
<thead>
<tr>
<th>$i$</th>
<th>$C_i$</th>
<th>$\alpha_i$</th>
<th>More-Less</th>
<th>Half-Half</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>4</td>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>2</td>
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<td>5</td>
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<td>5</td>
</tr>
<tr>
<td>4</td>
<td>4</td>
<td>20</td>
<td>9</td>
<td>11</td>
</tr>
</tbody>
</table>

**Figure 3. A solution produced by More-Less**

$R_{i1}^H = C_i + \sum_{j=1}^{i-1} (\frac{R_j^H}{P_j^H}) C_j > C_i + \sum_{j=1}^{i-1} (\frac{R_j^H}{P_j^H}) C_j$.

It is clear that $\exists(t \leq R_{i1}^H)$ such that $t = C_i + \sum_{j=1}^{i-1} (\frac{R_j^H}{P_j^H}) C_j$. Let $D_i^M = R_{i1}^M = t$. This implies that $D_i^M \leq R_{i1}^H$ because $t \leq R_{i1}^H$. Since $R_{i1}^H \leq D_i^H$, we have $D_i^M \leq D_i^H$, i.e., $D_i^M \leq \frac{t_i}{t_{ij}^H}$, which also implies $P_i^M \geq \frac{t_{ij}^H}{t_i}$ because $P_i^M = \alpha_i - D_i^M$. So $D_i^M \leq D_i^M$ and $P_i^M \leq P_i^M$ are true for $j = i$. In addition, if any set of transactions can be scheduled to guarantee data freshness by any fixed priority schedul-
ing algorithm based on Half-Half, there must be a solution based on More-Less with lower CPU utilization. It is clear from Theorem 3.4 that any conditions sufficient to guarantee the schedulability of a set of transactions using Half-Half must be sufficient to guarantee the schedulability using More-Less. The following lemma (see [14] for proof) gives a sufficient condition for schedulability of transactions based on Half-Half.

Lemma 3.1: Given any set of transactions $T = \{\tau_i\}_{i=1}^{m}$ $(m \geq 1)$, if

$$\sum_{j=1}^{k} \frac{\alpha_k}{\alpha_j} C_j \leq \frac{\alpha_k}{2} (1 \leq k \leq m)$$ (2)

holds, then the set of transactions are schedulable by More-Less.

It should be noted that Eq. 2 is only a sufficient condition for feasibility test of scheduling a set of transactions based on More-Less, it is not the necessary condition. However, Eq. 2 is both a necessary and sufficient condition of scheduling a set of transactions with fixed priority scheduling algorithms based on Half-Half, that is, if Eq. 2 does not hold, a set of transactions is not schedulable based on Half-Half. However, it may still be schedulable using More-Less. As illustrated in example 3.2, assignment orders in More-Less may have significant impact on the schedulability of transactions. How to choose an appropriate assignment order to determine deadlines and periods remains a problem. An optimal assignment order is desirable for More-Less to guarantee schedulability and minimize CPU utilization of transactions.

3.3 More-Less Principle: An Optimal Solution in a Restricted Case

As far as we know, there is no known solution to solve the previous non-linear programming problem corresponding to producing optimal periods and deadlines under More-Less. The complexity arises from not only the non-linearity, but also the permutation of $m$ transactions (i.e., the assignment order of the $m$ transactions), which is $O(m!)$. If we enumerate all the permutations of $m$ transactions to find the one with minimized CPU utilization, all $m!$ solutions have to be examined. It is obviously not efficient when the transaction set is large.

We now begin to examine the issue of finding optimal assignment order for More-Less. We first consider the problem with the following restriction:

Restriction (1): $\sum_{i=1}^{m} C_i \leq \min(\frac{\alpha_j}{T}) (1 \leq j \leq m)$.

Under this restriction, the first instance of all transactions can complete before half of the shortest validity interval. Given any assignment order of transactions, this implies that no higher priority transactions can recur before the first instance of the lowest priority transaction completes. Otherwise, suppose $J_{i2}$ (the 2nd instance of transaction $\tau_i$) $(1 \leq i \leq m)$ is the first recurring instance, and it occurs at time $t$ before the first instance of the lowest priority transaction completes. It implies that $t \geq P_i$. Because $P_i \geq \frac{C_i}{\alpha_i}$ according to More-Less, we have $t \geq \frac{C_i}{\alpha_i}$. Because not all the first instances from all transactions have completed yet, $t \leq \sum_{i=1}^{m} C_i$. Therefore we can conclude that $\frac{C_i}{\alpha_i} \leq \sum_{i=1}^{m} C_i$, which contradicts restriction (1). All the integers $n_{ij} (1 \leq i \leq m & 1 \leq j \leq i)$ in the schedulability constraint are reduced to 1 under such a restriction. Due to the short execution time of sensor transactions and relatively long validity interval length in many real applications (e.g., avionics system in [6], air traffic control, aircraft mission processor, and spacecraft control in [8]), restriction (1) is reasonable in many cases. We discuss relaxing this condition later in the paper. In the rest of Section 3.3, we assume that restriction (1) holds. In the rest of the paper, we also assume that transactions are ordered so that $\tau_i \rightarrow \tau_j$ for $i < j$ unless specified otherwise.

3.3.1 More-Less Principle: Optimal Assignment Order for Two Transactions

To motivate our approach to determining the ordering of transactions, we first study the characteristics of a set of two transactions: $\tau_1$ and $\tau_2$. The question we are trying to answer is, which one should precede the other? Two cases are examined:

1. $\tau_1 \tau_2$: $\tau_1 \rightarrow \tau_2$

$$\begin{align*}
P_1 &= \alpha_1 - C_1 \\
P_2 &= \alpha_2 - (C_1 + C_2)
\end{align*}$$ (3)

2. $\tau_2 \tau_1$: $\tau_2 \rightarrow \tau_1$

$$\begin{align*}
P_2 &= \alpha_2 - C_2 \\
P_1 &= \alpha_1 - (C_1 + C_2)
\end{align*}$$ (4)

In the above two cases, it should be noted that higher priority transaction only occurs once before the first instance of the lower priority transaction completes because restriction (1) holds. Let $U_{12}$ and $U_{21}$ denote the CPU utilization of transactions $\tau_1$ and $\tau_2$ in cases $\tau_1 \tau_2$ and $\tau_2 \tau_1$, respectively. Now we have

$$\begin{align*}
U_{12} &= \sum_{i=1}^{2} \frac{C_i}{P_i} = \frac{C_1}{\alpha_1 - C_1} + \frac{C_2}{\alpha_2 - (C_1 + C_2)} \\
U_{21} &= \sum_{i=1}^{2} \frac{C_i}{P_i} = \frac{C_2}{\alpha_2 - (C_1 + C_2)} + \frac{C_1}{\alpha_1 - (C_1 + C_2)}
\end{align*}$$ (5)

Without loss of generality, assume we want to show that $U_{12} \leq U_{21}$. That is

$$\frac{C_1}{\alpha_1 - C_1} + \frac{C_2}{\alpha_2 - (C_1 + C_2)} \leq \frac{C_2}{\alpha_2 - C_2} + \frac{C_1}{\alpha_1 - (C_1 + C_2)}$$ (6)

We now study the conditions that satisfy Eq. 6. Let $L_i$ denote the validity interval slack of transaction $\tau_i$, i.e., $L_i = \alpha_i - C_i$. Also let $\Delta\alpha_{ij} = \alpha_i - \alpha_j$, $\Delta L_{ij} = L_i -$
$L_j$, and $\Delta C_{ij} = C_i - C_j$. It is obvious that $\Delta \alpha_{ji} = -\Delta \alpha_{ij}$, $\Delta L_{ji} = -\Delta L_{ij}$, and $\Delta C_{ji} = -\Delta C_{ij}$. We now introduce the following theorem.

**Theorem 3.5:** If

1. $\frac{\alpha_i}{\alpha_j} \geq C_i + C_j$ and $\frac{\omega_i}{\omega_j} \geq C_i + C_j$ (i.e., restriction (1) holds),
2. $\Delta \alpha_{ji} \geq 0$ and $\Delta C_{ji} \leq 2\Delta \alpha_{ji}$, i.e., for any $\alpha_j \geq \alpha_i$,

then $U_{ij} \leq U_{ji}$.

Theorem 3.5, proved in [14], has the following properties under restriction (1): *stability, transitivity, and simplicity* as described below.

**Definition 3.1:** Stability: No matter how many transactions are assigned higher priority than two adjacent transactions $\tau_i$ and $\tau_j$ (i.e., no other transactions exist with priority between $\tau_i$ and $\tau_j$), the ordering of $\tau_i$ and $\tau_j$ is stable which means $U_{ij}$ is always less than $U_{ji}$.

**Property 1.** If transactions $\tau_i$ and $\tau_j$ satisfies the two conditions in Theorem 3.5, then the ordering of $\tau_i$ and $\tau_j$ is stable, i.e., $U_{ij} \leq U_{ji}$ always holds.

**Proof:** To prove that $U_{ij} \leq U_{ji}$ always holds, we need to prove that no matter how many transactions are assigned higher priority than $\tau_i$ and $\tau_j$, $U_{ij} \leq U_{ji}$ always holds.

Suppose $k$ transactions, $\tau_1, \tau_2, ..., \tau_k$, have been assigned higher priorities than $\tau_i$ and $\tau_j$. The sum of their computation times is $\sum_{i=1}^{k} C_i = C$. Now we want $U_{ij} \leq U_{ji}$ to hold, i.e., $\frac{\alpha_i}{\alpha_j} \leq \frac{C_j}{C_i} \leq \frac{\omega_i}{\omega_j}$. Let $\alpha'_i = C - \alpha_i$ and $\alpha'_j = \alpha_j - C$, thus

$$\Delta \alpha_{ji} = \alpha'_i - \alpha'_j = \alpha_i - \alpha_j = \Delta \alpha_{ij}. \quad (7)$$

We know that $\Delta C_{ji} \leq 2\Delta \alpha_{ji}$ if $C = 0$ because of restriction 2 in Theorem 3.5. Combined with Eq. 7, $\Delta C_{ji} \leq 2\Delta \alpha_{ji} \iff \Delta C_{ij} \leq 2\Delta \alpha_{ij}$, and this implies $\frac{\alpha'_i}{\alpha'_j} \leq \frac{C_j}{C_i} \leq \frac{\alpha'_j}{\alpha'_i}$ holds from Theorem 3.5. That is, $U_{ij} \leq U_{ji}$ holds.

**Definition 3.2:** Transitivity: If $\tau_i$ preceding $\tau_j$ results in lower CPU utilization for transactions $\tau_i$ and $\tau_j$ (i.e., $U_{ij} \leq U_{ji}$), and $\tau_j$ preceding $\tau_k$ results in lower CPU utilization for transactions $\tau_j$ and $\tau_k$ (i.e., $U_{jk} \leq U_{kj}$), then $\tau_i$ preceding $\tau_k$ results in lower CPU utilization for transactions $\tau_i$ and $\tau_k$ (i.e., $U_{ik} \leq U_{ki}$).

**Property 2.** Transactions satisfying conditions in Theorem 3.5 maintain transitivity.

**Proof:** Given transactions $\tau_i$, $\tau_j$ and $\tau_k$, suppose $\Delta C_{ij} \leq 2\Delta \alpha_{ji}$ and $\Delta C_{kj} \leq 2\Delta \alpha_{kj}$. Because $2\Delta \alpha_{kj} = 2\alpha_k - \alpha_i = 2(\alpha_k - \alpha_j) + 2(\alpha_j - \alpha_i) = 2\Delta \alpha_{kj} + 2\Delta \alpha_{ji} \geq \Delta C_{kj} + \Delta C_{ji} = \Delta C_{ki}$, we have $U_{ik} \leq U_{ki}$.

**Property 3.** Determining the conditions necessary from Theorem 3.5 for $U_{ij} \leq U_{ji}$ is computationally efficient because the computation of $\Delta \alpha_{ji}$ and $\Delta C_{ij}$ is simple.

**Discussion**

In Theorem 3.5, $\Delta \alpha_{ji} \geq 0$ and $\Delta C_{ji} \leq 2\Delta \alpha_{ji}$ include two cases:

1. $\Delta \alpha_{ji} \geq \Delta C_{ji}$, which implies $\alpha_i \leq \alpha_j$ and $\alpha_i - C_j \leq \alpha_j - C_i$, i.e., $\alpha_i$ and $\alpha_j - C_i$ order transactions in the same way.
2. $2\alpha_i \geq \Delta C_{ji}$, which implies $\alpha_i \leq \alpha_j$, $\alpha_j - C_i \leq \alpha_i - C_j$, and $\alpha_i - C_j \leq \alpha_j - C_i$, i.e., $\alpha_i$ and $\alpha_j - C_i$ do not order transactions in the same way.

$\tau_i$ preceding $\tau_j$ produces lower CPU utilization in the above two cases. Thus, $\alpha_i - C_i$ values of transactions may not produce the best assignment order. Said differently, the Least Slack First assignment algorithm may not produce the lowest utilization.

**3.3.2 More-Less Principle: Optimal Ordering of $m$ Transactions**

To generalize the comparison of two transactions, we need to examine a set of transactions $\{\tau_t\}_{t=1}^m$ with $m > 2$. We first introduce the second restriction in this paper.

**Restriction (2):**

$\alpha_1 \leq \alpha_2 \leq \cdots \leq \alpha_m$

$\Delta C_{i+1,i} \leq 2\Delta \alpha_{i+1,i} \quad (i = 1, 2, \ldots, m - 1)$

The next theorem proposes an optimal solution under restrictions (1) and (2).

**Theorem 3.6:** Given a set of transactions $T = \{\tau_t\}_{t=1}^m$, if restrictions (1) and (2) hold then an assignment order named *Shortest Validity First (SVF)*, which assigns orders to transactions in the *inverse* order of validity interval length and resolves ties in favor of a transaction with less *slack*\(^3\), results in the optimal CPU utilization among all possible assignment orders of the More-Less principle.

**Proof:** We need to prove that the transaction ordering scheme from SVF results in the lowest CPU utilization. From restriction (2) and Theorem 3.5, we know that $U_{i,i+1} \leq U_{i+1,i} \quad (1 \leq i \leq m - 1)$, and this is stable and transitive. Suppose there is an optimal assignment ordering $K$ resulting from an order different from SVF. But that order can always be achieved by a sequence of swapping of priorities of two adjacent transactions in our SVF scheme. From the *stability* and *transitivity* of Theorem 3.5, we know that every swap of orders of two adjacent transactions in the SVF scheme would result in higher CPU utilization. Thus order $K$ has higher CPU utilization than the SVF scheme. This contradicts the assumption that $K$ is optimal. Therefore we have proved that transaction ordering scheme based on SVF results in the optimal CPU utilization.

\(^3\)As in Table 1, slack $L_i$ for transaction $\tau_i$ is defined as $\alpha_i - C_i$. 

Definition 3.3: Given a set of transactions $T = \{\tau_i\}_{i=1}^m$ ($m \geq 1$), if a transaction $\tau_k$ ($1 \leq k \leq m$) is partitioned into $n$ ($n > 1$) independent subtransactions $\{\tau_{kj}\}_{j=1}^n$ with computation time $\{C_{kj}\}_{j=1}^n = \sum_{j=1}^n C_{kj}$, and validity interval length $\{\alpha_{kj}\}_{j=1}^n$ ($\alpha_{kj} = \alpha_k$), then the set of transactions $\{\tau_{kj}\}_{j=1}^n$ is a partition of $\tau_k$, and the resulting set of transactions $\{\tau_i\}_{i=1}^m \cup \{\tau_{kj}\}_{j=1}^n \cup \{\tau_{l}\}_{l=k+1}^m$ is a partition-transformed set of the original transaction set.

It should be noted that partition-transformation is transitive. For example, if transaction set $T_A$ is a partition-transformed set of transaction set $T_B$, and transaction set $T_C$ is a partition-transformed set of transaction set $T_B$, then transaction set $T_C$ is a partition-transformed set of transaction set $T_A$.

We now investigate the impact of partitioning on CPU utilization of optimal solutions of a transaction set. We want to understand whether partitioning transactions into smaller subtransactions with shorter computation times would produce optimal solutions with lower CPU utilization. The following theorem holds even when restriction (1) is not satisfied.

**Theorem 3.7**: Given any set of transactions $T = \{\tau_i\}_{i=1}^m$, a transaction $\tau_k$ ($1 \leq k \leq m$) can be partitioned into $n$ independent subtransactions $\{\tau_{kj}\}_{j=1}^n$ with $C_k = \sum_{j=1}^n C_{kj}$ and $\alpha_{kj} = \alpha_k$ ($1 \leq j \leq n$). Let the partition-transformed transaction set be $T'$. Then for any solution generated by More-Less, the optimal CPU utilization of $T'$ is less than the optimal CPU utilization of $T$.

**Proof**: For an optimal solution $S_{T'}$ of $T'$ generated by More-Less with assignment order $\tau_1 \rightarrow \tau_2 \rightarrow \ldots \rightarrow \tau_m$, if a transaction $\tau_i \in T$ ($1 \leq i \leq m$) can be partitioned into $n$ subtransactions $\tau_{k1}, \ldots, \tau_{kn}, \tau_{kn+1}, \ldots, \tau_{kn+m}$, $T'$ is transformed into a transaction set $T' = \{\tau_{i1}, \ldots, \tau_{i k1}, \ldots, \tau_{i kn}, \tau_{i kn+1}, \ldots, \tau_{i kn+m}\}$ where $\tau_{i j} = \tau_j (j \neq k)$ and $\tau_{ijk} = \tau_{ij}$ ($1 \leq k \leq n$). Based on More-Less, we can obtain a feasible solution $S_{T'}$ from $S_{T'}^\text{opt}$ immediately with

$$
\begin{align*}
D_j' &= D_j (j < k) \\
D_{ji}' &= D_j (j = k \& 1 \leq i \leq n - 1) \\
D_{ji}' &= D_j (j = k \& i = n) \\
D_j' &= D_j (j > k)
\end{align*}
$$

by assigning priorities in the order of $\tau_1 \rightarrow \ldots \rightarrow \tau_{k1} \rightarrow \ldots \rightarrow \tau_{kn} \rightarrow \tau_{kj1} \rightarrow \ldots \rightarrow \tau_{kn+m}$. Thus, we know that

$$
\begin{align*}
P_j' &= P_j (j < k) \\
P_{ji}' &= P_j (j = k \& 1 \leq i \leq n - 1) \\
P_{ji}' &= P_j (j = k \& i = n) \\
P_j' &= P_j (j > k)
\end{align*}
$$
We know that $T_P$ with above $\{P'_1\}$ and $\{P'_2\}$ can be scheduled because deadlines and periods are produced from a feasible solution, $S_{O^\text{opt}}$. Considering that

$$U_{T_P}^\text{opt} = \sum_{i=1}^{m} \frac{C_i}{F_i}, \quad (10)$$

and

$$U_{T_P} = \sum_{i=1}^{k} \frac{C_i}{F_i} + \sum_{i=k+1}^{n} \frac{C'_k}{F'_k} + \sum_{i=k+1}^{m} \frac{C_i}{F_i}, \quad (11)$$

we know that $U_{T_P}^\text{opt} > U_{T_P}$. Because $U_{T_P}^\text{opt}$, the optimal CPU utilization of $T_P$, is less than or equal to $U_{T_P}$, we can conclude that $U_{T_P}^\text{opt} > U_{T_P}$. This proves the theorem. □

Theorem 3.7 is important because it says that a partition-transformed set can have lower optimal CPU utilization than the optimal CPU utilization of its original transaction set. Theorem 3.7 can be applied repeatedly to every transaction in $T_C$. This generates a “finer” transaction set with even lower optimal CPU utilization. It is shown later in the paper that partitioning helps analyze More-Less.

Given a set of transactions $T$ which satisfies restriction (1) but does not satisfy restriction (2), we can partition transactions which violate restriction (2) into a set of subtransactions such that the partition-transformed transaction set $T_P$ satisfies restriction (2). The optimal CPU utilization of the partition-transformed transaction set ($U_{T_P}^\text{opt}$) can be obtained from Theorem 3.6, and this is less than the optimal CPU utilization of the original transaction set ($U_T^\text{opt}$) as per Theorem 3.7. Thus, for any given solution $S$ of $T$ and its CPU utilization $U_S$, $U_S - U_T^\text{opt} \leq U_S - U_{T_P}^\text{opt}$, because $U_S \geq U_T^\text{opt}$.

Definition 3.4: Partition/Merge: Given any set of transactions $T = \{T_1, T_2, \ldots, T_n\}$ with $\alpha_1 \leq \alpha_2 \leq \ldots \leq \alpha_m$, if restriction (1) holds but restriction (2) does not hold for $T$, we can reconstruct the transaction set by partitioning the computation time of transactions so that restriction (2) holds.

1. Partitioning of one transaction: If there is one transaction $T_k$ with $\alpha_k \leq \alpha_k$ and $C_k > C_k + \Delta C_k$, in which case restriction (2) does not hold, we can partition the computation time $C_k$ into $n$ (a positive integer) parts that satisfies $C_n \leq C_k$ and $\Delta C_k \leq \Delta C_k$ (which again implies $\Delta C_k < 2 \Delta C_k$). We can consider $\tau_k$ to consist of a set of $n$ subtransactions: $\tau_k = \{T_{k1}, T_{k2}, \ldots, T_{kn}\}$, in which $\alpha_{ki} = \alpha_k$ and $C_{ki} = \frac{C_k}{n}$ (1 ≤ $i$ ≤ $n$). We denote $\tau_k = \mathcal{P}(\tau_k)$. Let us substitute the set of transactions $\tau_{k1}, \tau_{k2}, \ldots, \tau_{kn}$ for transaction $T_k$ and form a new set of transactions $T_{\tau_k} = \{\tau_{i1}, \tau_{i2}, \ldots, \tau_{im}\}$. If we assign orders of transactions in $T_{\tau_k}$ according to SVF and derive periods based on Eq. 1, it is easy to see that $D_{ki} = D_k = 0$. Thus, $P_{ki} = P_k$.

2. Partitioning of more than one transaction: If there are multiple adjacent transactions that do not satisfy restriction (2), they are partitioned in the same way and the set of old transactions is transformed into a set of new transactions $T_P = \{T_1, T_2, \ldots, T_{m_p}\}$ ($m_p \geq m$). Transactions in $T_P$ now satisfy restriction (2), thus the optimal solution of transaction set $T_P$ can be achieved by applying theorem 3.6.

Merge (denoted as $P^{-1}$) is the inverse function of Partition. If $T_k = \mathcal{P}(\tau_k)$, then $\tau_k = \mathcal{P}^{-1}(T_k)$.

Let $U_T^\text{opt}$ and $U_{T_P}^\text{opt}$ denote the optimal solution of $T$ and $T_P$, respectively. It is obvious that

$$U_{T_P}^\text{opt} = \sum_{i=1, i \neq k}^{m} \frac{C_i}{F_i} + \sum_{j=1}^{n} \frac{C_j}{F''_j}. \quad (12)$$

As per Theorem 3.7, $U_{T_P}^\text{opt} \leq U_T^\text{opt}$. Applying Theorem 3.7 repeatedly to $T$, we know that the CPU utilization of the optimal solution of $T_P$, $U_{T_P}^\text{opt}$, satisfies

$$U_{T_P}^\text{opt} \leq U_{T_P}^\text{opt}. \quad (13)$$

Theorem 3.8: Given a set of transactions $T$ which satisfies restriction (1), let $U_T^\text{opt}$, $U_{T_P}^\text{opt}$, and $U_S$, denote the CPU utilization of an optimal solution of $T$, the optimal solution of $T_P$, and the approximate solution $S^*$ of $T$ derived from Shortest Validity First (SVF), respectively. The following inequality holds:

$$U_S^* \geq U_T^\text{opt} \geq U_{T_P}^\text{opt}. \quad (14)$$

Proof: $U_S^* \geq U_T^\text{opt}$ because $U_T^\text{opt}$ is the optimal CPU utilization of the same set of transactions. We know $U_T^\text{opt} \geq U_{T_P}^\text{opt}$ from Eq. 13. So the theorem follows. □

Definition 3.5: CPU utilization bound with respect to the optimal solution: Given a set of transactions $T = \{T_1, T_2, \ldots, T_m\}$ and its optimal CPU utilization $U_T^\text{opt}$, the CPU utilization bound of any solution $S$ with respect to its optimal solution, $B_S$, is defined as

$$B_S = U_S - U_T^\text{opt}, \quad (15)$$

where $U_S$ is the CPU utilization of solution $S$.

Theorem 3.9: Given a set of transactions $T = \{T_1, T_2, \ldots, T_m\}$ with $\alpha_1 \leq \alpha_2 \leq \ldots \leq \alpha_m$, suppose that $T$ satisfies restriction (1) but not restriction (2). $S^*$ is a solution from the SVF algorithm. Assume that $\Psi$ is a set of all subtasks in $T$ that are partitioned in a partition-transformation after which the resulting set of transactions $T_P$ satisfies restriction (2). The CPU utilization bound of $S^*$ with respect to the optimal solution of $T$, $B_{S^*}$, satisfies

$$B_{S^*} \leq 2 \sum_{k \in \Psi} \frac{C_k}{\alpha_k}. \quad (16)$$
Proof sketch: Let \(U_t^{opt}, U_t^{opt'}, T_t',\) and \(T_t\) denote the CPU utilization of an optimal solution of \(T_b\), the optimal solution of \(T_p\) partition-transformed from \(T_t\), and the solution \(S^{*}\) from the SVF algorithm, respectively. It follows from Theorem 3.8 that \(B_{S^{*}} = U_{S^{*}} - U_{T_t'}^{opt} \leq U_{S^{*}} - U_{T_t'}^{opt}.\) It is proved in [14] that \(U_{S^{*}} - U_{T_t'}^{opt} \leq 2 \sum_{k \in \Psi} (\frac{C_k}{\alpha_k})^2.\) □

Theorem 3.9 says that the CPU utilization from SVF is within \(2 \sum_{k \in \Psi} (\frac{C_k}{\alpha_k})^2\) of that of an optimal solution if restriction (1) holds. In many real applications, e.g., the avionics application [6] discussed later in the paper, sensor transaction computation time is in the range of milliseconds, validity interval length is in the range of hundreds of milliseconds and seconds. Thus \((\frac{C_k}{\alpha_k})^2\) for a sensor transaction \(T_k\) is about \(\frac{1}{100^3}\) to \(\frac{1}{100^6}\). The number of transactions which may belong to the transaction set \(\Psi\) is usually very limited. Therefore, this bound is actually very small and can be ignored in many situations, thus SVF becomes a near optimal solution.

The optimal solution for the general case of More-Less, i.e., when both Restrictions (1) and (2) are relaxed, is left as an open issue. However, as we shall show in Section 5, SVF is a good heuristic solution even in these situations.

4 More-Less Application: Similarity-Based Load Adjustment

In this section, we consider the similarity-based load adjustment [6] as an application of More-Less. The basic idea of similarity-based load adjustment is to skip the executions of transaction instances which produce similar outputs. The approach taken in [6] is to modify the execution frequencies of transactions such that only one instance of a transaction is executed for multiple periods. As a result, system workload is reduced. View r-serializability [6] is a criterion used to justify the correctness of transactions. Readers are referred to [4, 6] for details of similarity and view r-serializability.

In similarity-based load adjustment, a similarity bound is derived for each data object based on application semantics. Two write events of the same data objects are similar if their sampling times differ by an amount of time no greater than the similarity bound. In other words, write events on the same data object within similarity bound are interchangeable as input to a read without adverse effects. Therefore, some write or read events can be dropped in order to reduce system load without affecting data temporal correctness. Here, validity interval length is replaced by similarity bound to constrain the arrival time of a transaction instance and finishing time of its next instance.

Update and View principles are proposed in [6] to adjust the system load. Their update principle is based on the Half-Half principle. Based on More-Less, we derive new update and view principles to reduce the system load even further.

Suppose \(sb_j\) is the similarity bound for data object \(X_j\). Any two conflicting write events on \(X_j\) occur within \(sb_j\) are interchangeable as input to a read event due to similarity. Suppose \(P_j, P_j^H\) and \(P_j^M\) be the periods of transaction \(T_j\) before load adjustment, after load adjusted by Half-Half, and after load adjusted by More-Less, respectively. Let \(D_j^H\) and \(D_j^M\) be the deadlines of transaction \(T_j\) after load adjusted by the Half-Half and More-Less principle, respectively.

Update Principle: \(P_j^{M} + D_j^{M} \leq sb_j\)

In [6], the Half-Half principle is used to derive their update principle, which is \(2P_j^H \leq sb_j\). However, our update principle derived from More-Less is \(P_j^{M} + D_j^{M} \leq sb_j\). As shown in Figure 4, any read event will read from similar write events in both cases after load adjustment. In addition, because \(D_j^{M} \leq sb_j/2\), we know that \(P_j^{M} \geq sb_j/2 \geq P_j^H\), which reduces the system utilization factor for \(T_j\) by an amount of \(C_j^{M}/P_j^{M} - C_j^{H}/P_j^{H}\) compared to the previous update principle. Therefore, update principle derived from More-Less reduces load even further without sacrificing similarity-based data correctness.

View Principle: \(P_j^{M} + D_j^{M} + P_i^{M} \leq sb_j\)

Suppose transaction \(T_i\) with period \(P_i\) reads data object \(X_j\). Let \(P_i^H\) and \(P_i^{M}\) denote the period of transaction \(T_i\) adjusted by Half-Half and More-Less, respectively. View principle in [6] is defined as \(2P_i^H + P_i^H \leq sb_j\). In contrast, our view principle from More-Less is defined...
as $P_j^M + D_j^M + P_i^M \leq s b_j$. As shown in Figure 5, $P_j^M + D_j^M + P_i^M$ is the maximum temporal distance among the write events which might be read by instances of $\tau_i$ and their representatives before and after load adjustment. Therefore, the view principle derived from the More-Less principle can guarantee similarity-based data correctness.

The following example clearly indicates that update and view principles derived from More-Less can reduce system load more than update and view principles from Half-Half.

Example 4.1: We use an example in [6] to illustrate the effectiveness of the More-Less principle. Suppose there are two periodic transactions $\tau_1$ and $\tau_2$ in a single processor environment. Their computation times and periods are given in Table 8. $\tau_1$ periodically refreshes a data object $X$ and $\tau_2$ periodically reads the same data. The similarity bound $sb_X$ of $X$ is 22. According to update and view principles corresponding to the More-Less and Half-Half principles, the following inequalities must hold, respectively.

$$\begin{aligned}
P_1^M + D_1^M &< 22 \\
P_1^M + D_1^M + P_2^M &< 22 \\
2P_1^H &< 22 \\
2P_1^H + P_2^H &< 22
\end{aligned} \tag{17}$$

$$\begin{aligned}
P_1^M + D_1^M &< 22 \\
P_1^M + D_1^M + P_2^M &< 22 \\
2P_1^H &< 22 \\
2P_1^H + P_2^H &< 22
\end{aligned} \tag{18}$$

It is obvious that there are multiple solutions. Three different results after load adjustment are shown in Table 8. Let $U_H$ and $U_M$ denote the system CPU utilization after load adjustment based on the Half-Half and More-Less principles, respectively. In cases 1 and 2, $P_1^H = P_2^M = 5$ and $P_2^H = P_2^M = 10$, respectively. $U_H - U_M$, the difference in adjusted system load, is $\frac{1}{12}$ and $\frac{1}{18}$ in case 1 and 2, respectively. In case 3, $P_2^H = P_2^M = 15$, the system load adjusted from both principles are the same. This indicates that our update principle provides solutions with lower CPU utilization than the previous update principle.

Table 8. Parameters and results of example 4.1

<table>
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<tr>
<th>i</th>
<th>$C_i$</th>
<th>$P_i$</th>
<th>Case 1</th>
<th>Case 2</th>
<th>Case 3</th>
</tr>
</thead>
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<td>3</td>
<td>$P_1^m$</td>
<td>$D_1^m$</td>
<td>$P_1^m$</td>
</tr>
<tr>
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<td>2</td>
<td>5</td>
<td>12</td>
<td>3</td>
<td>6</td>
</tr>
</tbody>
</table>

5 Experiments

In this section, experimental results are presented to quantitatively show that More-Less produces solutions with better schedulability and lower CPU utilization than the Half-Half principle. A set of update sensor transactions is generated randomly: computation time of a sensor transaction is uniformly generated from 5 to 15 milliseconds, and validity interval length of an object is uniformly generated from 4000 to 8000 milliseconds. These values are similar to the values used in the experiments of [6] and data presented in the study of air traffic control system in [8].

The number of sensor transactions are varied to change the workload in the system. For each data point presented in a figure, the experiments are run multiple times so that CPU utilizations shown have relative half-widths about the mean of less than 5% at the 95% confidence interval.

The resulting CPU utilization generated from the One-One, Half-Half and More-Less with SVF ordering are presented in Figure 6. When the number of transactions is less than 200, the workload falls into the restricted case, i.e., restriction (1) is satisfied. This is because the sum of computation times of all the transactions is less than half of the minimum of all the validity interval lengths. It is observed that CPU utilization produced by More-Less is very close to that of One-One, and much less than that of the Half-Half principle. We would like to remind readers that One-One is used only as an artificial baseline – it does not guarantee the validity of temporal data. In this case, as we explained in Section 3.4, More-Less is very close to the optimal solution. This is clearly substantiated by the small difference in the CPU utilization between One-One and More-Less: CPU utilization of an optimal solution under More-Less should be between those for One-One and More-Less. When the number of transactions is more than 200, the workload falls into the general case because restriction (1) is not satisfied. In this case, we observe that CPU utilization of More-Less is still much less than that of Half-Half. However, the difference in CPU utilization of One-One and More-Less increases as system workload increases. The highest workload in our experiments is produced when the number of transactions is 375, and the corresponding CPU utilization under One-One, Half-Half and More-Less is about 65%, 130% and 92%, respectively. Half-Half can not produce a feasible solution when the number of transactions exceeds 300 because the corresponding CPU utilization exceeds 100%. But More-Less can still produce feasible solutions even when the number of transactions increases to 375.

In summary, when both the Half-Half and More-Less principles can be used to schedule a set of sensor update transactions, the More-Less principle can be used to produce solutions with much lower CPU utilization, thus more CPU capacity can be used by other transactions in the system. In addition, More-Less can be used to provide feasible solutions even when Half-Half can not be applied. In such situations, More-Less provides better schedulability.
Figure 6. CPU utilizations from three principles

6 Conclusions

Database systems in which time validity intervals are associated with the data are discussed in [13, 12, 6, 5, 4, 2]. Such systems introduce the need to maintain data temporal consistency in addition to logical consistency.

A design methodology for guaranteeing end-to-end requirements of real-time systems is presented in [2]. Their approach guarantees end-to-end propagation delay, temporal input-sampling correlation, and allowable separation times between updated output values. However, their solution is based on the assumption that all the periodic tasks have harmonic periods. However, we do not make the assumption that all the periods are harmonic.

The work presented in our paper is also related to the work of [6], but, as we showed, the schedulability of More-Less is better than Half-Half used in [6]. It is noted that More-Less guarantees a bound on the arrival time of a periodic transaction instance and the finishing time of the next instance. This is different from the distance constrained scheduling, a dynamic scheduling mechanism, which guarantees a bound of the finishing times of two consecutive instances of a task [3].

Very recently, we came across a paper by Burns and Davis [1] where SVF is proposed as a heuristic to determine optimal task assignment orders in the general case remains open.

References