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# THE WEAK OPE AND DIMENSION-EIGHT OPERATORS

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We discuss recent work which identifies a potential flaw in standard treatments of weak decay amplitudes, including that of $\epsilon'/\epsilon$. The point is that (contrary to conventional wisdom) dimension-eight operators contribute to weak amplitudes at order $G_F\alpha_s$ and without $1/M_W^2$ suppression. The effect of dimension-eight operators is estimated to be at the 100% level in a sum rule determination of the operator $\mathcal{Q}_7^{(6)}$ for $\mu = 1.5$ GeV, suggesting that presently available values of $\mu$ are too low to justify the neglect of these effects.

## 1 Motivation

### *1.1 Calculating Kaon Weak Amplitudes*

The modern approach to calculating a kaon weak nonleptonic amplitude $\mathcal{M}$ involves use of the operator product expansion,

$$\mathcal{M} = \sum_d \sum_i \mathcal{C}_i^{(d)}(\mu) \langle \mathcal{Q}_i^{(d)} \rangle_\mu \quad , \qquad (1)$$

in which the nonleptonic weak hamiltonian $\mathcal{H}_{\rm W}$ is expressed as a linear combination of local operators $\mathcal{Q}_i^{(d)}$. There is a sum over the dimensions (starting here at $d = 6$) of the local operators and a sum over all operators of a common dimension. In practice, the following hybrid methodology is employed:

1. The Wilson coefficients $\mathcal{C}_i^{(d)}(\mu)$ are calculated in $\overline{MS}$ renormalization.

2. The operator matrix elements $\langle \mathcal{Q}_i^{(d)} \rangle_\mu$ are calculated in cutoff renormalization at the energy scale $\mu$. The term 'cutoff' means specifically that $\mu$ serves as a 'separation scale' which distinguishes between short-distance and long-distance physics. Three different approaches falling into this category are quark models, $1/N_c$ expansion methods, and lattice-QCD evaluations.[a]

The reason for this hybrid approach is that it is not practical to carry out the (low energy) kaon matrix element evaluations with $\overline{MS}$ renormalization. Typical choices for the scale $\mu$ fall in the range $0.5 \le \mu(\text{GeV}) \le 3$, the lower part used in quark-model and $1/N_c$ evaluations and the upper part in lattice simulations.

The purpose of this talk is to describe some recent results:[1]

1. In a pure cutoff scheme, dimension-eight operators occur in the weak hamiltonian at order $G_F\alpha_s/\mu^2$, $\mu$ being the separation scale. This can be explicitly demonstrated (see Sect. 2) in a calculation involving a LR weak hamiltonian.

2. In dimensional regularization (DR), the $d = 8$ operators do *not* appear explicitly in the hamiltonian at order $G_F\alpha_s$. However, the use of a cutoff scheme for the calculation of the matrix elements of dimension-six operators requires a careful matching onto DR for which dimension-eight operators *do* play an important role.

These findings mean that hybrid evaluations, in the sense described above, of kaon matrix elements at low $\mu$ will contain (unwanted) contributions from dimension-eight operators. At the very least, this will introduce an uncertainty of unknown magnitude into the evaluation.

## 2 Cutoff Renormalization

[a] A list of references is given elsewhere.[1]

### 2.1 $\epsilon'/\epsilon$ in the Chiral Limit

The determination of $\epsilon'/\epsilon$ can be shown to depend upon the matrix elements $\langle(\pi\pi)_0|\mathcal{Q}_6^{(6)}|K\rangle$ and $\langle(\pi\pi)_2|\mathcal{Q}_8^{(6)}|K\rangle$.[2] In the chiral limit of vanishing light-quark mass, the latter matrix element (as well as that of operator $\mathcal{Q}_7^{(6)}$) can be inferred from certain vacuum expectation values, $\langle 0|\mathcal{O}_{1,8}^{(6)}|0\rangle \equiv \langle\mathcal{O}_{1,8}^{(6)}\rangle$, where $\mathcal{O}_{1,8}^{(6)}$ are dimension-six four-quark operators.[3] The use of soft-meson techniques to relate physical amplitudes to those in the world of zero light-quark mass is a well-known procedure of chiral dynamics.

### 2.2 Sum Rules for $\langle\mathcal{O}_{1,8}^{(6)}\rangle$

Numerical values for $\langle\mathcal{O}_{1,8}^{(6)}\rangle$ in cutoff renormalization can be obtained from the following sum rules,[3]

$$\frac{16\pi^2}{3}\langle\mathcal{O}_1^{(6)}\rangle_\mu^{(\text{c.o.})} = \int_0^\infty ds\ s^2 \ln\frac{s+\mu^2}{s}\Delta\rho$$
$$2\pi\langle\alpha_s\mathcal{O}_8^{(6)}\rangle_\mu^{(\text{c.o.})} = \int_0^\infty ds\ s^2 \frac{\mu^2}{s+\mu^2}\ \Delta\rho\ , \tag{2}$$

where $\Delta\rho(s)$ is the difference of vector and axialvector spectral functions, and $\Delta\Pi(Q^2)$ is the corresponding difference of isospin polarization functions ($\mathcal{I}m\ \Delta\Pi = \pi\Delta\rho$).

### 2.3 Physics of a LR Operator

One can probe the influence of $d = 8$ operators by considering the K-to-$\pi$ matrix element $\mathcal{M}(p)$,

$$\mathcal{M}(p) = \langle\pi^-(p)|\mathcal{H}_{\text{LR}}|K^-(p)\rangle\ , \tag{3}$$

where $\mathcal{H}_{\text{LR}}$ is a LR hamiltonian obtained by flipping the chirality of one of the quark pairs in the usual LL hamiltonian $\mathcal{H}_{\text{W}}$. The reason for defining such a LR operator is that, in leading chiral order, its K-to-$\pi$ matrix element is nonzero and yields information on $\langle\mathcal{O}_1^{(6)}\rangle$ and $\langle\mathcal{O}_8^{(6)}\rangle$.

To demonstrate this, we proceed to the chiral limit to find

$$\mathcal{M} \equiv \mathcal{M}(0) = \lim_{\text{p}=0}\mathcal{M}(p)$$
$$= \frac{3G_F M_W^2}{32\sqrt{2}\pi^2 F_\pi^2}\int_0^\infty dQ^2\ \frac{Q^4}{Q^2+M_W^2}\ \Delta\Pi\ . \tag{4}$$

This result is *exact* — it is not a consequence of any model. Information about $\langle\mathcal{O}_1^{(6)}\rangle$ and $\langle\mathcal{O}_8^{(6)}\rangle$ is obtained by performing an operator product expansion on $\Delta\Pi(Q^2)$. Working to first order in $\alpha_s$ we have

$$\mathcal{M} = \frac{G_F}{2\sqrt{2}F_\pi^2}\Bigg[\langle\mathcal{O}_1^{(6)}\rangle_\mu^{(\text{c.o.})}$$
$$+\frac{3}{8\pi}\ln\frac{M_W^2}{\mu^2}\langle\alpha_s\mathcal{O}_8^{(6)}\rangle_\mu + \frac{3}{16\pi^2}\frac{\mathcal{E}_\mu^{(8)}}{\mu^2} + \ldots\Bigg] \tag{5}$$

The three additive terms in Eq. (5) are proportional respectively to the quantities $\langle\mathcal{O}_1^{(6)}\rangle$, $\langle\mathcal{O}_8^{(6)}\rangle$ and $\mathcal{E}^{(8)}$. The last of these ($\mathcal{E}^{(8)}$) contains the effect of the $d = 8$ contributions.[b] For dimensional reasons, $\mathcal{E}^{(8)}$ must be accompanied by an inverse squared energy. This turns out to be the factor $\mu^{-2}$.

In Table 1 we display the numerical values (in units of $10^{-7}$ GeV$^2$) of the three terms of Eq. (5) for various choices of $\mu$. Observe for the lowest values that the dimension-eight term dominates the contribution from $\langle\mathcal{O}_1^{(6)}\rangle$. Only when one proceeds to a sufficiently large value like $\mu = 4$ GeV is the $d = 8$ influence suppressed.

## 3 Dimensional Regularization

Suppose one wishes to express the entire analysis in terms of $\overline{MS}$ quantities. To do so requires converting matrix elements in cutoff renormalization to those in $\overline{MS}$ renormalization. Recall, in dimensional regularization

[b] Although the $d = 8$ LL operators arising from $\mathcal{Q}_2^{(6)}$ have been determined[1], to our knowledge the individual $d = 8$ LR operators comprising $\mathcal{E}^{(8)}$ have not.

Table 1. Eq. (5) in units of $10^{-7}$ GeV$^2$.

| $\mu$ (GeV) | Term 1 | Term 2 | Term 3 |
|---|---|---|---|
| 1.0 | −0.12 | −3.84 | 0.64 |
| 1.5 | −0.28 | −3.49 | 0.30 |
| 2.0 | −0.44 | −3.24 | 0.17 |
| 4.0 | −0.89 | −2.63 | 0.04 |

one calculates in $d$ dimensions and for dimensional consistency introduces a scale $\mu_{\rm d.r.}$.

The dimensionally regularized matrix element for $\langle \mathcal{O}_1^{(6)} \rangle$ is found from the $d$-dimensional integral,[3]

$$\langle \mathcal{O}_1^{(6)} \rangle_\mu^{\rm (d.r.)} = \langle \mathcal{O}_1^{(6)} \rangle_\mu^{\rm (c.o.)} + \frac{d-1}{(4\pi)^{d/2}} \frac{\mu_{\rm d.r.}^{4-d}}{\Gamma\left(d/2\right)} \int_{\mu^2}^{\infty} dQ^2 \ Q^d \ \Delta\Pi(Q^2) \ . \quad (6)$$

The term in Eq. (6) containing the integral is proof that the dimensionally regularized matrix element $\langle \mathcal{O}_1^{(6)} \rangle_\mu^{\rm (d.r.)}$ will contain *short-distance* contributions. As written, this term becomes divergent for four dimensions and also is scheme-dependent. In the $\overline{\rm MS}$ approach, the divergent factor $2/\epsilon - \gamma + \ln(4\pi)$ is removed. The NDR scheme involves a certain procedure for treating chirality in $d$-dimensions. The final result is a relation (given here to $\mathcal{O}(\alpha_s)$) between the cutoff and $\overline{\rm MS}$-NDR matrix elements,

$$\langle \mathcal{O}_1^{(6)} \rangle_\mu^{(\overline{\rm MS}-{\rm NDR})} = \langle \mathcal{O}_1^{(6)} \rangle_\mu^{\rm (c.o.)} + \frac{3}{8\pi} \left[ \ln \frac{\mu_{\rm d.r.}^2}{\mu^2} - \frac{1}{6} \right] \langle \alpha_s \mathcal{O}_8^{(6)} \rangle_\mu + \frac{3}{16\pi^2} \cdot \frac{\mathcal{E}_\mu^{(8)}}{\mu^2} + \ldots \quad (7)$$

The effect of the $d = 8$ contribution to the weak OPE now appears in the $d = 6$ $\overline{\rm MS}$-NDR operator matrix element. Note also that the parameter $\mu_{\rm d.r.}$ is distinct from the separation scale $\mu$.

## 4 Evaluation of $B_{7,8}^{(3/2)}$

To suppress the effect of dimension-eight operators on the determinations of Eq. (2), one should evaluate the two sum rules for $\langle \mathcal{O}_{1,8} \rangle_\mu^{\rm (c.o.)}$ at a large value of $\mu$ (*e.g.* $\mu \geq$ 4 GeV) and then use renormalization group equations to run the matrix elements down to lower values of $\mu$ (*e.g.* $\mu = 2$ GeV).[4] Alternative approaches might involve the finite energy sum rule framework[5] or QCD-lattice simulations at sufficiently large $\mu$.

## 5 Concluding Remarks

This talk has dealt with an important aspect of calculating kaon weak matrix elements, the role of dimension-eight operators. In this regard, Eq. (7) is of special interest. It reveals that the relation between $\overline{\rm MS}$-NDR and cutoff matrix elements will involve not only mixing between operators of a given dimension but also mixing between operators of differing dimensions. The net result of our work is that existing work on $\epsilon'/\epsilon$ will be affected, especially for methods which take $\mu \leq 2$ GeV.

## Acknowledgments

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## References

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