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Phenomenological Issues in the Determination of $\Delta\Gamma_{\rm D}$

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Abstract

We consider the issue of determining the D^0-$\bar{D}^0$ width difference $\Delta\Gamma_{\rm D}$ experimentally. The current situation is reviewed and suggestions for further study are given. We propose a number of D^0 decay modes in addition to those studied in the recent E791, FOCUS and BELLE lifetime determination experiments. Then we address prospects for determining CF - CDS strong phase differences, like $\delta_{K\pi}$ which appears in the CLEO study of $D^0 \to K^+\pi^-$ transitions. We show how to extract $\delta_{K^*\pi}$ with CDS data and furthermore show when $D \to K_{\rm L}\pi$ data becomes available that $\delta_{K\pi}$ can also be obtained.

I. INTRODUCTION

In the Standard Model, effects of D^0-$\bar{D}^0$ mixing are much smaller than those in the kaon, B_d and B_s systems. However, charm-related experiments of increasing sensitivity have been carried out, leading to ever-improving bounds on the dimensionless mixing parameters $x_{\rm D} \equiv \Delta M_{\rm D}/\Gamma_{\rm D}$ and $y_{\rm D} \equiv \Delta\Gamma_{\rm D}/2\Gamma_{\rm D}$. Most recently, the E791, CLEO, FOCUS and BELLE collaborations have reported on attempts to detect mixing in the D-meson system. This has prompted discussion in the literature as to whether actual D-meson mixing (specifically a nonzero $\Delta\Gamma_{\rm D}$) is being seen for the first time. [1] Since a rigorous theoretical prediction for $\Delta\Gamma_{\rm D}$ is unlikely, experimental progress in this area is needed. In this paper, we discuss specific proposals for further work in lifetime difference measurements and in experimentally determining the strong phase δ (which occurs between Cabibbo-favored and Cabibbo-doubly-suppressed decays).

II. MEASUREMENTS OF LIFETIME DIFFERENCES

The E791 [2], FOCUS [3] and BELLE [4] experiments study the time dependence for $D^0(t) \to K^+K^-$ (CP = +1 final state) and $D^0(t) \to K\pi$ (CP-mixed final state) under the assumption that CP invariance is assumed. This is reasonable in view of both theoretical expectations based on Standard Model physics and also recent CLEO results (see Sect. III). If we adopt the convention that $\mathcal{CP}|D^0\rangle = +|\bar{D}^0\rangle$ and introduce the CP eigenstates

$$|D_{\frac{1}{2}}\rangle = \frac{1}{\sqrt{2}}\left[|D^0\rangle \pm |\bar{D}^0\rangle\right] \quad , \tag{1}$$

then $|D_1\rangle$ is CP-even and $|D_2\rangle$ is CP-odd. It follows from Eq. (1) that

$$|D^0(t)\rangle = \frac{1}{\sqrt{2}}\left[|D_1(t)\rangle + |D_2(t)\rangle\right] \quad , \tag{2}$$

where $|D_{1,2}\rangle$ evolve in time with distinct masses and decay widths,

$$|D_k(t)\rangle = e^{-iM_k t - \frac{1}{2}\Gamma_k t}|D_k\rangle \qquad (k = 1,2) \quad . \tag{3}$$

If the K^+K^- final state is overlapped with Eq. (2) only the $|D_1(t)\rangle$ part contributes, leading to the exponential decay equation

$$\Gamma_{K\bar{K}}(t) = A_{K\bar{K}} e^{-\Gamma_1 t} \quad . \tag{4}$$

For the $K\pi$ final state, we express the time evolution of D^0 as

$$|D^0(t)\rangle = f_+(t)|D^0\rangle + f_-(t)|\bar{D}^0\rangle \tag{5}$$

where

$$f_+(t) = \frac{1}{2}\sum_{k=1}^{2}\; e^{-iM_k t - \frac{1}{2}\Gamma_k t} \qquad f_-(t) = \frac{1}{2}\sum_{i=1}^{2}\,(-)^{i+1}\; e^{-iM_i t - \frac{1}{2}\Gamma_i t} \quad . \tag{6}$$

Then the above conditions $1, 2$ imply

$$\Gamma_{K^-\pi^++K^+\pi^-}(t) = A_{K\pi}\left[e^{-\Gamma_1 t} + e^{-\Gamma_2 t}\right] = 2A_{K\pi}e^{-(\Gamma_1+\Gamma_2)t/2}\cosh\left[(\Gamma_1 - \Gamma_2)\frac{t}{2}\right] \quad . \tag{7}$$

The experimental conditions are such that the cosh term in Eq. (7) is nearly unity. Thus the time dependence becomes exponential, allowing determination of $(\Gamma_1 + \Gamma_2)/2$. The E791, FOCUS and BELLE experiments measure the quantity $y_{\rm cp}$,

$$y_{\rm cp} \equiv \frac{\tau_{D^0\to K\pi}}{\tau_{D^0\to K^+K^-}} - 1 = \frac{\Gamma_1 - \Gamma_2}{\Gamma_1 + \Gamma_2} \tag{8}$$

and find

$$y_{\rm cp} = \begin{cases} (0.8 \pm 2.9 \pm 1.0)\% & \text{(E791)} \\ (3.42 \pm 1.39 \pm 0.74)\% & \text{(FOCUS)} \\ (1.0\ {}^{+3.8}_{-3.5}\ {}^{+1.1}_{-2.1})\% & \text{(BELLE)} \quad . \end{cases} \tag{9}$$

Due to its superior sensitivity the FOCUS determination dominates, the net result being a positive value for $y_{\rm cp}$ of several per cent at about the two standard deviation level.

A. Additional Decay Modes

We urge that additional lifetime studies on CP eigenstates of the neutral D be carried out. It is essential to improve the statistical data base and to acquire a sensitivity beyond the current 2σ level. Beyond that, there is still no experimental input on the pure $CP = -1$ lifetime. By using lifetimes obtained from pure $CP = \pm 1$ modes, one would be determining $\Delta\Gamma$ directly rather than comparing an average of $CP = \pm 1$ lifetimes with that of $CP = +1$.

There are a number of opportunities for further study, each final state occurring in D^0 decay being a potential candidate. We shall discuss just a limited number of these in the following, citing disadvantages as well as advantages. An important subset of our list has modes which contain a pair of mesons, each of which is self-conjugate under the CP operation. If each member of the pair has spin-zero, the orbital angular momentum is S-wave and the CP value of the two-particle state is simply the product of the individual CP values. If one meson has spin-zero and the other has spin-one, then conservation of angular momentum requires the particles to be in a P-wave. In this case, the CP value of the two-particle state becomes *minus* the product of the individual CP values.

Vertex identification is a key to a successful D^0 lifetime measurement. Starting from its (assumed known) production point, the D^0 will travel unobserved and ultimately decay into some final state particles. In the best case, all these are charged and the decay point becomes well determined. In the worst case, each primary decay product is neutral and, if unstable, decays itself into neutral particles. Then even with calorimetric information, attempting to fix a decay point is problematic. For reference, in the E791, FOCUS and BELLE experiments the detected modes ($K\pi$ and $K\bar{K}$) had just two particles (both charged) in each final state and the branching fractions were $\mathcal{B}_{K^-\pi^+} = (3.83 \pm 0.09) \cdot 10^{-2}$ and $\mathcal{B}_{K^-K^+} = (4.25 \pm 0.16) \cdot 10^{-3}$. [5]

Many D^0 decay modes contain neutral kaons in the final state. The neutral kaons will in turn decay as K_S or K_L mesons. For a lifetime determination measurement, a K_S mode is superior to a K_L mode because: (i) the K_S detection efficiency is rather larger than the K_L detection efficiency, so the statistics will be better for the former, (ii) the K_L decay occurs further from the D^0 decay vertex, so its background problem is more severe. Both of these considerations are inherent for any detector. However, since progress in dealing with K_L detection is anticipated the K_L modes should not be totally disregarded. To summarize, K_S detection is easier and can be done now whereas K_L is harder and may be done later, although not as well. Finally, we note that PDG listings give branching fractions for $D \to \bar{K}^0 X$ (X denotes other final state particles) rather than for $D \to K_{S,L}X$. It will suffice below to use the approximations

$$\Gamma_{D^0 \to K_S X} \simeq \Gamma_{D^0 \to K_L X} \simeq \frac{1}{2}\Gamma_{D^0 \to \bar{K}^0 X} \quad . \tag{10}$$

These relations are not exact because decay into K_S or K_L is subject to interference between Cabibbo favored (CF) and Cabibbo doubly suppressed (CDS) modes. [6] We discuss aspects of this interference in the next section.

Now we turn to the list of additional possible modes, partitioned according to the CP of the final states and presented as CP $= -1$, CP $= +1$ and CP-mixed.

Pure CP = −1 Modes

1. $K_S\phi$: Both the K_S and ϕ decay into charged final states, so this mode is an attractive one as regards particle detection. Since the $\phi \to K^+K^-$ transition is a strong decay, it occurs right at the D^0 decay vertex. Also, the ϕ has a narrow decay width. The branching fraction for this mode is acceptably large ($\mathcal{B}_{\phi K_S} = (4.3 \pm 0.5) \cdot 10^{-3}$).

2. $K_S\omega$: Although the branching fraction is respectable ($\mathcal{B}_{K_S\omega} = (1.05 \pm 0.2) \cdot 10^{-2}$), the ω decays predominantly via the three-body mode $\pi^+\pi^-\pi^0$ which renders it more difficult regarding identification of the decay vertex.

3. $K_S\rho^0$: In this case, the branching fraction is not unattractive ($\mathcal{B}_{K_S\rho^0} = (0.61 \pm 0.09) \cdot 10^{-2}$), and the $K_S\rho^0$ final state would decay into all charged particles. However, the larger width of the ρ^0 (compared to the ϕ) makes detection relatively more difficult.

4. $K_S\pi^0$: This mode has a reasonably large branching fraction $\mathcal{B}_{K_S\pi^0} = (1.06 \pm 0.11) \cdot 10^{-2}$). However, the presence of the π^0 hinders accurate vertex identification.

5. $K_S\eta$ and $K_S\eta'$: Both these modes are potentially interesting since the branching fractions are not highly suppressed ($\mathcal{B}_{K_S\eta} = (3.5 \pm 0.5) \cdot 10^{-3}$ and $\mathcal{B}_{K_S\eta'} = (8.5 \pm 1.3) \cdot 10^{-3}$). The problem of vertex ID for a final state η and η' would resemble that for a final state ω.

Pure CP = +1 Modes

1. $\pi^+\pi^-$: This mode provides a clean CP $= +1$ signal but has the disadvantage of a small branching fraction ($\mathcal{B}_{\pi^-\pi^+} = (1.52 \pm 0.09) \cdot 10^{-3}$), about three times less than K^+K^-. Also backgrounds could be a problem since since there are more $\pi^+\pi^-$ combinations in a typical event (although in the D^0 rest frame, the two pions emerge back-to-back with larger momenta than any other final state).

2. $K_{\rm L}\eta$ and $K_{\rm L}\eta'$: Although the branching fractions equal those for $K_{\rm S}\eta$ and $K_{\rm S}\eta'$, detection of the $K_{\rm L}$ presents difficulties, as discussed earlier.

3. $\pi^0\phi$: Particle ID is more of an issue than for the $K_{\rm S}\phi$ mode as the neutral π^0 decays via the chargeless two-photon mode. Even though the branching fraction here is comparatively large ($\mathcal{B}_{\pi^0 K_{\rm S}} = (1.05 \pm 0.11) \cdot 10^{-2}$), it is not sufficient to compensate for the detection problem. Moreover, other decay modes containing π^0's would be a source of background.

4. $K_{\rm S}f_0(980)$: This mode consists of a scalar-pseudoscalar pair in an S-wave, and has CP $= +1$ for $K_{\rm S}$ and CP $= +1$ for $f_0(980)$. Although the $K_{\rm S}$ and $f_0(980)$ each decay into charged particles, the branching ratio is small ($\mathcal{B}_{K_{\rm S}f_0(980)} = (2.9 \pm 0.8) \cdot 10^{-3}$). Similar comments apply to the $K_{\rm S}f_0(1370)$ final state and to the $K_{\rm S}f_2(1270)$ (except that here the final state is D-wave).

5. $\phi\rho^0$: This mode will have positive CP provided the ϕ and ρ^0 are in an S-wave of D-wave state. Both decay strongly into charged particles, so the decay point will have four emergent tracks. The branching fraction is rather small ($\mathcal{B}_{\phi\rho^0} = (6 \pm 3) \cdot 10^{-4}$).

6. $K_{\rm L}\pi^0$: This final state has the same branching fraction as $K_{\rm S}\pi^0$, but an even greater detection problem due to the $K_{\rm L}$. In practical terms, vertex identification would be an insurmountable obstacle.

Mixed CP $= \pm 1$ *Modes*

For definiteness consider a Dalitz plot analysis for the neutral ($Q = 0$) three-body state $(\pi\pi\bar{K})_{\rm Q=0}$. There will be resonance bands corresponding to the quasi two-body modes $(\rho\bar{K})_{\rm Q=0}$ and $(\pi\bar{K}^*)_{\rm Q=0}$. Although neither the ρ nor the $\bar{K}^*$ is a particularly narrow resonance, these decays are CKM dominant so the branching ratios are relatively large. Specific examples of mixed $CP = \pm 1$ modes are:

1. $\pi^+\bar{K}^{*-}$: This quasi two-particle state has a large branching fraction $\mathcal{B}_{\pi^+\bar{K}^{*-}} = (5.0 \pm 0.4) \cdot 10^{-2}$ and there are the two measureable $\bar{K}^*$ decay modes $K^{*-} \to \pi^0 K^-$ and $K^{*-} \to \pi^-\bar{K}^0$. The latter provides a rather clean three-body configuration, $\pi^+(\pi^- K_{\rm S})$ where the parentheses stress the K^{*-} parentage.

2. ρ^+K^-: The largest branching fraction among all quasi two-body final states for D^0 decay occurs here, $\mathcal{B}_{\rho^+K^-} = (10.8 \pm 0.9) \cdot 10^{-2}$. The ρ^+ decay proceeds through only the mode $\rho^+ \to \pi^+\pi^0$.

3. $\pi^0\bar{K}^{*0}$: The branching fraction $\mathcal{B}_{\pi^0\bar{K}^{*0}} = (3.1 \pm 0.4) \cdot 10^{-2}$ is relatively large. The associated three-body configurations will be $\pi^0(\pi^-\bar{K}^0)$ and the less useful $\pi^0(\pi^0 K^-)$.

III. MEASUREMENTS OF WRONG-SIGN D^0 TRANSITIONS

Another study which impacts on determining $\Delta\Gamma_{\rm D}$ is the CLEO experiment [7] which studies the decay rate for $D^0(t) \to K^+\pi^-$. This wrong-sign process can be produced both indirectly, from mixing followed by a CF decay, and directly, from CDS decay. The decay rate is given in the CP-invariant limit by

$$r(t) = e^{-t}\left[R_{\rm D} + \sqrt{R_{\rm D}}\, y' \cdot t + R_{\rm M} \cdot t^2\right] \quad . \tag{11}$$

The $R_{\rm D}$ term arises from CDS decay, the $R_{\rm M}$ term from mixing and the $\sqrt{R_{\rm D}}$ term from interference between the two. We also have the definitions

$$y' \equiv y\cos\delta - x\sin\delta\ , \qquad x' \equiv x\cos\delta + y\sin\delta \quad . \tag{12}$$

The parameter δ is the (strong-interaction) phase difference between between the CF and CDS amplitudes,

$$\delta \equiv \delta^{\rm (ch)}_{K\pi} \equiv \delta_{-+} - \delta_{+-} \quad , \tag{13}$$

where the phases δ_{-+} and δ_{+-} appear in the amplitudes

$$\mathcal{M}_{D^0\to K^-\pi^+} = |\mathcal{M}_{D^0\to K^-\pi^+}|\ e^{i\delta_{-+}} \qquad \mathcal{M}_{D^0\to K^+\pi^-} = |\mathcal{M}_{D^0\to K^+\pi^-}|\ e^{i\delta_{+-}} \quad . \tag{14}$$

Note that we sharpen the notation for δ ($\delta \to \delta^{\rm (ch)}_{K\pi}$) in Eq. (13) because we will encounter several analogous phases in our analysis.

The CP-invariant rate formula of Eq. (11) can be generalized to incorporate various sources of CP-violation (CPV), [8]

$$\begin{aligned} R_{\rm D} &\to R_{\rm D}\,(1 + A_{\rm D}) \qquad &&\text{(CDS)}\ , \\ y' &\to y'\,(1 + A_M/2) \qquad &&\text{(mixing)}\ , \\ \delta^{\rm (ch)}_{K\pi} &\to \delta^{\rm (ch)}_{K\pi} + \phi \qquad &&\text{(interference)}\ , \end{aligned} \tag{15}$$

where $A_{\rm D}$, A_M and ϕ parameterize the extent of CP violation. When the data is fit to include the effects of CP violation none is found,

$$A_{\rm M} = 0.23^{+.63}_{-.80} \pm .01\ , \qquad A_{\rm D} = -0.01^{+.16}_{-.17} \pm .01\ , \qquad \sin\phi = 0.00 \pm .60 \pm .01 \quad . \tag{16}$$

In the same fit one finds at 95% D.L.

$$x' = (0 \pm 1.5 \pm 0.2)\% \qquad \text{and} \qquad y' = (-2.5^{+1.4}_{-1.6} \pm 0.3)\% \tag{17}$$

or equivalently

$$|x'| < 2.9\% \qquad \text{and} \qquad -5.8\% < y' < 1.0\% \quad . \tag{18}$$

Given the present strength of the CLEO signals for x' and y', it is prudent to cite the results as bounds as in Eq. (18). One expects future experiments to reduce the statistical and systematic uncertainties. Even so, ignorance of the phase $\delta^{\rm (ch)}_{K\pi}$ will hamper efforts to compare the FOCUS/E791/BELLE results with those from CLEO.

A. On the Determination of y'

Can theory alone provide the value of $\delta_{K\pi}^{(\mathrm{ch})}$? Symmetry considerations are of only limited use. It is known that $\delta_{K\pi}^{(\mathrm{ch})}$ vanishes in the SU(3) invariant world [9,10], and this result has been recognized [11] in discussing aspects of the wrong-sign D^0 transitions. Thus, calculating the value of $\delta_{K\pi}^{(\mathrm{ch})}$ necessarily involves the physics of SU(3) breaking. Unfortunately, our limited understanding of physics in the charm region (especially the complicating effects of QCD) makes it difficult to perform reliable calculations. [12] It is, perhaps, not too surprising to find rather different statements in the literature about $\delta_{K\pi}^{(\mathrm{ch})}$ depending on the underlying approach. In one analysis [13], the findings of Ref. [14] and Ref. [15] are shown to imply rather small values for $\delta_{K\pi}^{(\mathrm{ch})}$, less than 15^o. However, the resonance model of Ref. [16] has considerably greater SU(3) breaking and obtains values as large as $\delta_{K\pi}^{(\mathrm{ch})} \sim 30^o$. The largest value cited for $\delta_{K\pi}^{(\mathrm{ch})}$ appears in Ref. [1] which shows that accepting the central values of the FOCUS and CLEO experiments leads to $\delta_{K\pi}^{(\mathrm{ch})}$ in the second quadrant. However, it has been argued [17] that within a reasonable range of SU(3)-breaking parameters it is not possible to arrive at very large values of $\delta_{K\pi}^{(\mathrm{ch})}$ (45^o or larger) of the type considered in Ref. [1].

In view of this state of affairs, it makes sense to explore what experiment can teach us.

B. Doing without K_{L} Data

Recalling our comments in the previous section on the relative measureability of $K_{\mathrm{S,L}}$ modes, we begin by assuming that only a data set *not* containing final state K_{L}'s is available. The inclusion of K_{L} data is covered later.

Our first conclusion concerns the CDS $D \to K\pi$ decays: $D^0 \to K^+\pi^-$, $D^+ \to K^+\pi^0$, $D^0 \to K^0\pi^0$ and $D^+ \to K^0\pi^+$. At present, only the first of these has been observed ($\mathcal{B}_{K^-\pi^+} = (1.46 \pm 0.30) \cdot 10^{-4}$). If only K_{S} data is used, then neither $D^0 \to K^0\pi^0$ nor $D^+ \to K^0\pi^+$ modes can be determined experimentally. This can be understood by considering CF and CDS transitions having a neutral kaon in the final state:

$D \to \bar{K}^0\pi$ CF Decays

Transition	Final State
$D^0 \to \bar{K}^0\pi^0$	$K_{\mathrm{S}}\pi^0$
$D^+ \to \bar{K}^0\pi^+$	$K_{\mathrm{S}}\pi^+$

$D \to K^0\pi$ CDS Decays

Transition	Final State
$D^0 \to K^0\pi^0$	$K_{\mathrm{S}}\pi^0$
$D^+ \to K^0\pi^+$	$K_{\mathrm{S}}\pi^+$

For both CDS transitions $D^0 \to K^0\pi^0$ and $D^+ \to K^0\pi^+$, the K^0 will decay via the same $K_{\mathrm{S}} \to \pi^+\pi^-$ mode as the CF transitions $D^0 \to \bar{K}^0\pi^0$ and $D^+ \to \bar{K}^0\pi^+$. Since the CF decays dominate, extracting information about CDS final states containing a K^0 from just K_{S} detection will be impossible. This negates performing a direct experimental measurement of $\delta_{K\pi}^{(\mathrm{ch})}$.

What is the situation for other possible final states like $K\rho$ or $K^*\pi$? Clearly, the same no-go result will hold for the $D \to K\rho$ decays. This leaves only the case of $D \to K^*\pi$. Since the K^* decays strongly into two different charge combinations of $K\pi$, each $D \to K^*\pi$ transition will have two final configurations. Continuing to assume that only the K_{S} mode in K^0 and $\bar{K}^0$ is observed, we obtain the following list:

$D \to \bar{K}^*\pi$ CF Decays

Transition	FS1	FS2
$D^0 \to K^{*-}\pi^+$	$(K^-\pi^0)\pi^+$	$(K_S\pi^-)\pi^+$
$D^0 \to \bar{K}^{*0}\pi^0$	$(K^-\pi^+)\pi^0$	$(K_S\pi^0)\pi^0$
$D^+ \to \bar{K}^{*0}\pi^+$	$(K^-\pi^+)\pi^+$	$(K_S\pi^0)\pi^+$

$D \to K^*\pi$ CDS Decays

Transition	FS1	FS2
$D^0 \to K^{*+}\pi^-$	$(K^+\pi^0)\pi^-$	$(K_S\pi^+)\pi^-$
$D^0 \to K^{*0}\pi^0$	$(K^+\pi^-)\pi^0$	$(K_S\pi^0)\pi^0$
$D^+ \to K^{*+}\pi^0$	$(K^+\pi^0)\pi^0$	$(K_S\pi^+)\pi^0$
$D^+ \to K^{*0}\pi^+$	$(K^+\pi^-)\pi^+$	$(K_S\pi^0)\pi^+$

Each $K\pi$ arising from K^* decay is enclosed in parentheses and $FS1$, $FS2$ are the two three-body final states per D decay. Each CDS transition with a K^{*0} in the final state has a configuration (FS2) identical to that of a CF transition with a $\bar{K}^{*0}$ in the final state. However, the other configurations (FS1) each contain a charged kaon and thus distinguish between CF and CDS decays.

Thus, all four $D \to K^*\pi$ CDS decays *can* be utilized. In those final states containing a K^{*+}, both configurations FS1 and FS2 will have a unique signature (it is, however, necessary to employ a Dalitz plot analysis to properly identify which '$K\pi$' composite is a product of K^* decay). For final states with a K^{*0}, there will be a reduction factor of 2/3 in the number of events since only the configuration FS1 can be used, *i.e.*

$$\Gamma_{D^0\to(K^+\pi^-)\pi^0} = \frac{2}{3}\,\Gamma_{D^0\to K^{*0}\pi^0} \qquad \text{and} \qquad \Gamma_{D^+\to(K^+\pi^-)\pi^+} = \frac{2}{3}\,\Gamma_{D^+\to K^{*0}\pi^+} \quad . \tag{19}$$

Thus we are led to analyze the phenomenology of $D \to K^*\pi$ transitions in both the CF and CDS sectors.

1. Cabibbo Favored ($\bar{K}^*\pi$) Decays

There are three Cabibbo favored (CF) $D \to \bar{K}^*\pi$ decays,

$$D^0 \to K^{*-}\pi^+ \ , \qquad D^0 \to \bar{K}^{*0}\pi^0 \ , \qquad D^+ \to \bar{K}^{*0}\pi^+ \quad . \tag{20}$$

These proceed through the QCD-corrected $\Delta S = \Delta C = \pm 1$ weak hamiltonian, which takes the form [18]

$$\mathcal{H}_{\rm W}^{\rm (CF)} = c_-\mathcal{H}_-^{\rm (CF)} + c_+\mathcal{H}_+^{\rm (CF)} \quad . \tag{21}$$

$\mathcal{H}_-^{\rm (CF)}$ and $\mathcal{H}_+^{\rm (CF)}$ transform alike under isospin, as the $I_3 = +1$ member of an isotriplet. Under SU(3), however, $\mathcal{H}_-^{\rm (CF)}$ belongs to $\mathbf{6}\oplus\mathbf{6}^*$ and $\mathcal{H}_+^{\rm (CF)}$ to $\mathbf{15}\oplus\mathbf{15}^*$. [19,20] The coefficients $c_\pm$ encode the short distance, perturbative part of the QCD corrections. At energy scale $M_{\rm W}$, $c_\pm$ have essentially equal magnitudes. As the energy scale is lowered, the coefficient c_- is enhanced whereas c_+ is suppressed.

Using just the isospin property of $\mathcal{H}_{\rm W}^{\rm (CF)}$, we express the above decay amplitudes as[1]

[1]Equivalent formulae can be written for $\bar{K}\pi$ and $\bar{K}\rho$.

$$
\begin{aligned}
\mathcal{M}_{K^{*-}\pi^+} &= A_1 e^{i\delta_1} + \frac{1}{2} A_3 e^{i\delta_3} \quad , \\
\mathcal{M}_{\bar{K}^{*0}\pi^0} &= -\frac{1}{\sqrt{2}} A_1 e^{i\delta_1} + \frac{1}{\sqrt{2}} A_3 e^{i\delta_3} \quad , \\
\mathcal{M}_{\bar{K}^{*0}\pi^+} &= \frac{3}{2} A_3 e^{i\delta_3} \quad ,
\end{aligned}
\tag{22}
$$

where the subscripts represent twice the isospin of the final state $\bar{K}^*\pi$ composites. Observe that these amplitudes obey the sextet-dominance constraints $\mathcal{M}_{\bar{K}^{*0}\pi^+} = 0$ and $\mathcal{M}_{K^{*-}\pi^+} = -\sqrt{2}\mathcal{M}_{\bar{K}^{*0}\pi^0}$. Upon either expanding the relation $0 = \langle \bar{K}^{*0}\pi^+ | [I_+, \mathcal{H}_{\rm W}^{\rm (CF)}] | D^0 \rangle$ or utilizing the amplitude relations of Eq. (22), one arrives at the isospin sum rule [21–24]

$$
\mathcal{M}_{K^{*-}\pi^+} + \sqrt{2}\mathcal{M}_{\bar{K}^{*0}\pi^0} - \mathcal{M}_{\bar{K}^{*0}\pi^+} = 0 \quad . \tag{23}
$$

Of interest to us here are the phase difference and amplitude ratio,

$$
\begin{aligned}
\cos(\delta_1 - \delta_3) &= \frac{\sqrt{2}}{4} \cdot \frac{3\Gamma_{K^{*-}\pi^+} + \Gamma_{\bar{K}^{*0}\pi^+} - 6\Gamma_{\bar{K}^{*0}\pi^0}}{\left[\Gamma_{\bar{K}^{*0}\pi^+}\left(3\Gamma_{K^{*-}\pi^+} + 3\Gamma_{\bar{K}^{*0}\pi^0} - \Gamma_{\bar{K}^{*0}\pi^+}\right)\right]^{1/2}} \quad , \\
\frac{A_3}{A_1} &= \left[\frac{2\Gamma_{\bar{K}^{*0}\pi^+}}{3\Gamma_{K^{*-}\pi^+} + 3\Gamma_{\bar{K}^{*0}\pi^0} - \Gamma_{\bar{K}^{*0}\pi^+}}\right]^{1/2} \quad .
\end{aligned}
\tag{24}
$$

The most recent data compilation [5] gives for the three CF modes $D \to \bar{K}^*\pi, \bar{K}\pi, \bar{K}\rho$,

Mode	$\delta_1 - \delta_3$	A_3/A_1
$\bar{K}^*\pi$	$103.9^o{}^{+17.2^o}_{-17.8^o}$	$0.25^{+0.02}_{-0.01}$
$\bar{K}\pi$	$90.2^o{}^{+7.1^o}_{-8.2^o}$	0.37 ± 0.03
$\bar{K}\rho$	$0.0^o \pm 44.9^o$	0.39 ± 0.10

The preceding equations can of course be used to obtain phase relations in addition to those in the above table, *e.g.* for the $\bar{K}\pi$ system,

$$
\delta_{K^-\pi^+} - \delta_3 = 79.5^o \ , \qquad \delta_{\bar{K}^0\pi^0} - \delta_3 = 110.3^o \quad , \tag{25}
$$

and so on. Staying temporarily with the $D \to \bar{K}\pi$ decays, let us compare physics of the real world with that of a world which is SU(3) symmetric and in which $c_+ = 0$:

	$\Gamma_{D^0\to\bar{K}^0\pi^0}/\Gamma_{D^0\to K^-\pi^+}$	$\Gamma_{D^+\to\bar{K}^0\pi^+}/\Gamma_{D^0\to K^-\pi^+}$
Hypothetical world	0.5	0.0
Real world	0.551 ± 0.006	0.296 ± 0.028

For these rates, at least, the agreement between the real world and the hypothetical world is not unreasonable. That the hypothetical world is, in some sense, nearby the real world will be useful later as a guiding principle in our study of the CDS amplitudes. This comparison between the real world and the hypothetical SU(3) world having $c_+ = 0$ explains the small observed values of A_3/A_1. In an SU(3) symmetric world, the limit $c_+ = 0$ would correspond to $A_3/A_1 = 0$ for the $D \to \bar{K}\pi$ amplitudes. Although the precise values of $c_\pm$ depend on the renormalization scheme (involving both the choice of operator basis and of renormalization scale μ), a typical numerical value is $c_+/c_- \simeq 0.5$ for the range $2.0 \geq \mu(\mathrm{GeV}) \geq 1.5$. [18] The short distance effects embodied in $c_\pm$ account for much of the suppression for A_3/A_1 observed in the above table, the rest arising from the operator matrix elements. Operator matrix elements play a much larger role in the kaon system ($\Delta I = 1/2$ rule) where QCD effects are more powerful.

2. Cabibbo Doubly Suppressed ($K^*\pi$) Decays

Corresponding to the three $D \to \bar{K}^*\pi$ CF decays of Eq. (20) are the following *four* $D \to K^*\pi$ decays in the Cabibbo doubly suppressed (CDS) sector,

$$D^0 \to K^{*+}\pi^- \ , \qquad D^+ \to K^{*0}\pi^+ \ , \qquad D^0 \to K^{*0}\pi^0 \ , \qquad D^+ \to K^{*+}\pi^0 \ . \tag{26}$$

The CDS weak hamiltonian has $\Delta S = -\Delta C = \pm 1$ and is written analogous to Eq. (21),

$$\mathcal{H}_{\mathrm{W}}^{(\mathrm{CDS})} = c_- \mathcal{H}_-^{(\mathrm{CDS})} + c_+ \mathcal{H}_+^{(\mathrm{CDS})} \ , \tag{27}$$

but now $\mathcal{H}_-^{(\mathrm{CDS})}$ and $\mathcal{H}_+^{(\mathrm{CDS})}$ behave differently under isospin, transforming respectively as an isosinglet and as the $I_3 = 0$ member of an isotriplet. Under SU(3), $\mathcal{H}_-^{(\mathrm{CDS})}$ (like $\mathcal{H}_-^{(\mathrm{CF})}$) transforms as a member of $\mathbf{6} \oplus \mathbf{6}^*$ and $\mathcal{H}_+^{(\mathrm{CDS})}$ (like $\mathcal{H}_+^{(\mathrm{CF})}$) transforms as a member of $\mathbf{15} \oplus \mathbf{15}^*$.

Performing isospin decompositions of the CDS decay amplitudes yields

$$\begin{aligned} \mathcal{M}_{K^{*+}\pi^-} &= \sqrt{2}\bar{A}_a e^{i\bar{\delta}_1} - \sqrt{2}\bar{A}_3 e^{i\bar{\delta}_3} \ , \qquad & \mathcal{M}_{K^{*0}\pi^+} &= \sqrt{2}\bar{A}_b e^{i\bar{\delta}_1} + \sqrt{2}\bar{A}_3 e^{i\bar{\delta}_3} \ , \\ \mathcal{M}_{K^{*0}\pi^0} &= -\bar{A}_a e^{i\bar{\delta}_1} - 2\bar{A}_3 e^{i\bar{\delta}_3} \ , \qquad & \mathcal{M}_{K^{*+}\pi^0} &= -\bar{A}_b e^{i\bar{\delta}_1} + 2\bar{A}_3 e^{i\bar{\delta}_3} \ , \end{aligned} \tag{28}$$

where the CDS isospin moduli and phases are labelled with super-bars. Corresponding to the above four decay CDS decay amplitudes are the four physical observables $\bar{A}_a$, $\bar{A}_b$, A_3 and $\bar{\delta}_1 - \bar{\delta}_3$. Two distinct $I = 1/2$ moduli ($\bar{A}_a$ and $\bar{A}_b$) occur because there are two independent sources of the $I = 1/2$ final state, the isoscalar $\mathcal{H}_-^{(\mathrm{CDS})}$ and the isovector $\mathcal{H}_+^{(\mathrm{CDS})}$,

$$\bar{A}_a \equiv \sqrt{3}A_1^{(-)} + A_1^{(+)} \qquad \text{and} \qquad \bar{A}_b \equiv \sqrt{3}A_1^{(-)} - A_1^{(+)} \ . \tag{29}$$

We will need to determine the phase $\bar{\Delta}$

$$\bar{\Delta} \ \equiv \ \bar{\delta}_1 - \bar{\delta}_3 \tag{30}$$

and the moduli ratios r, R,

$$r \equiv \frac{\bar{A}_3}{\bar{A}_a} \qquad \text{and} \qquad R \equiv \frac{\bar{A}_b}{\bar{A}_a} \ . \tag{31}$$

Since $\bar{A}_a$, $\bar{A}_b$ and $\bar{A}_3$ are moduli, we have $r > 0$ and $R > 0$. In addition, we note that $\bar{A}_3/\bar{A}_b = r/R$.

Taking the absolute square of each relation in Eq. (28) and forming ratios gives

$$\mathcal{R}_1 = \frac{2 + 2r^2 - 4r\cos\bar{\Delta}}{1 + 4r^2 + 4r\cos\bar{\Delta}} , \qquad \mathcal{R}_2 = \frac{2 + 2r^2 - 4r\cos\bar{\Delta}}{2R^2 + 2r^2 + 4rR\cos\bar{\Delta}} ,$$
$$\mathcal{R}_3 = \frac{2 + 2r^2 - 4r\cos\bar{\Delta}}{R^2 + 4r^2 - 4rR\cos\bar{\Delta}} , \tag{32}$$

where the $\{\mathcal{R}_k\}$ are the ratios of CDS decay rates,

$$\mathcal{R}_1 \equiv \frac{\Gamma_{D^0\to K^{*+}\pi^-}}{\Gamma_{D^0\to K^{*0}\pi^0}} , \qquad \mathcal{R}_2 \equiv \frac{\Gamma_{D^0\to K^{*+}\pi^-}}{\Gamma_{D^+\to K^{*0}\pi^+}} , \qquad \mathcal{R}_3 \equiv \frac{\Gamma_{D^0\to K^{*+}\pi^-}}{\Gamma_{D^+\to K^{*+}\pi^0}} . \tag{33}$$

By eliminating r and $\cos\bar{\Delta}$ from the relations in Eq. (32), one obtains a cubic equation in the variable R. However, there is an unphysical root $R = -1$, leaving the solution as a root of the quadratic equation

$$\mathcal{R}_2\mathcal{R}_3(2\mathcal{R}_1 - 1)R^2 + (2\mathcal{R}_2\mathcal{R}_3 + \mathcal{R}_1\mathcal{R}_3 - 2\mathcal{R}_1\mathcal{R}_2 - \mathcal{R}_1\mathcal{R}_2\mathcal{R}_3)\,R + \mathcal{R}_1\mathcal{R}_2 - 2\mathcal{R}_1\mathcal{R}_3 = 0 . \tag{34}$$

It turns out that to obtain the physical solution it is necessary to choose the square root of the discriminant as *positive*,

$$R = \frac{-b + \sqrt{b^2 - 4ac}}{2a} \tag{35}$$

where

$$\begin{aligned} a &= \mathcal{R}_2\mathcal{R}_3(2\mathcal{R}_1 - 1) , \\ b &= 2\mathcal{R}_2\mathcal{R}_3 + \mathcal{R}_1\mathcal{R}_3 - 2\mathcal{R}_1\mathcal{R}_2 - \mathcal{R}_1\mathcal{R}_2\mathcal{R}_3 \\ c &= \mathcal{R}_1\mathcal{R}_2 - 2\mathcal{R}_1\mathcal{R}_3 . \end{aligned} \tag{36}$$

To see why, let us consider a hypothetical world with $c_+ = 0$. Then since $\mathcal{H}_-^{(\mathrm{CDS})}$ is an isoscalar operator, it follows that

$$\bar{A}_3 = 0 \Longrightarrow r = 0 \qquad \text{and} \qquad \bar{A}_b = \bar{A}_a \Longrightarrow R = 1 . \tag{37}$$

As a consequence, we have

$$\mathcal{R}_1 = 2 , \qquad \mathcal{R}_2 = 1 , \qquad \mathcal{R}_3 = 2 , \tag{38}$$

from which the physical solution of Eq. (34) is identified. Returning to the real world, from R one obtains r

$$r = \left[\frac{(2 - \mathcal{R}_1)(1 + R\mathcal{R}_2) - 2(1 - \mathcal{R}_2R^2)(1 + \mathcal{R}_1)}{2(1 - \mathcal{R}_2)(1 + \mathcal{R}_1) - 2(1 - 2\mathcal{R}_1)(1 + R\mathcal{R}_2)}\right]^{1/2} \tag{39}$$

and lastly $\cos\bar{\Delta}$,

$$\cos\bar{\Delta} = \frac{2 - \mathcal{R}_1 + 2r^2(1 - 2\mathcal{R}_1)}{4r(1 + \mathcal{R}_1)} . \tag{40}$$

3. Determining the Phase $\delta^{(\rm ch)}_{K^*\pi}$

In the relation $\delta^{(\rm ch)}_{K^*\pi} \equiv \delta^{(K^*\pi)}_{-+} - \delta^{(K^*\pi)}_{+-}$, the CF phase $\delta^{(K^*\pi)}_{-+}$ cannot be determined from the relations in Eq. (22) because one cannot know the individual values of δ_1 and δ_3. However, the first relation in Eq. (22) does allow one to solve for $\delta^{(K^*\pi)}_{-+} - \delta_3$,

$$\delta^{(K^*\pi)}_{-+} - \delta_3 = tan^{-1}\left[\frac{\sin(\delta_1 - \delta_3)}{\cos(\delta_1 - \delta_3) + A_3/2A_1}\right] \quad . \tag{41}$$

Analogously, from the CDS relation in Eq. (28) we can solve for $\delta^{(K^*\pi)}_{+-} - \delta_3$,

$$\delta^{(K^*\pi)}_{+-} - \bar{\delta}_3 = \tan^{-1}\left[\frac{\sin\left(\bar{\delta}_1 - \bar{\delta}_3\right)}{\cos\left(\bar{\delta}_1 - \bar{\delta}_3\right) - \bar{A}_3/\bar{A}_a}\right] \quad , \tag{42}$$

where $\delta^{(K^*\pi)}_{+-}$ is the phase of the CDS $D^0 \to K^{*+}\pi^-$ amplitude.

Combining these two relations we find for $\delta^{(\rm ch)}_{K^*\pi}$

$$\begin{aligned} \delta^{(\rm ch)}_{K^*\pi} &\equiv \delta^{(K^*\pi)}_{-+} - \delta^{(K^*\pi)}_{+-} \\ &= \delta_3 - \bar{\delta}_3 + tan^{-1}\left[\frac{\sin(\delta_1 - \delta_3)}{\cos(\delta_1 - \delta_3) + A_3/2A_1}\right] - tan^{-1}\left[\frac{\sin\left(\bar{\delta}_1 - \bar{\delta}_3\right)}{\cos\left(\bar{\delta}_1 - \bar{\delta}_3\right) - \bar{A}_3/\bar{A}_a}\right] \quad . \end{aligned} \tag{43}$$

Under the assumptions that only $K_{\rm S}$ data is used and that isospin is a valid symmtery, we conclude that Eq. (43) will be the best one can do in a purely experimental determination of $\delta^{(\rm ch)}_{K^*\pi}$. An expression for $\delta^{(\rm ch)}_{K^*\pi}$ itself is obtained only via dropping the contribution $\delta_3 - \bar{\delta}_3$. One might argue that these phases occur in exotic channels and should be individually small. Unlike the case of the $D \to \bar{K}\pi$ amplitudes, there is no SU(3) prediction that $\delta^{(\rm ch)}_{K^*\pi} = 0$.

C. Including $K_{\rm L}$ Data

In the previous subsection, the avoidance of $K_{\rm L}$ data forced us to work with $D \to K^*\pi$ decays. The inclusion of $K_{\rm L}$ data allows us to return to the $D \to \bar{K}\pi$ and $D \to K\pi$ decays in the following. Each $D \to K_{\rm S,L}\pi$ mode will receive contributions from both CF and CDS sectors. Writing the transition amplitudes for $D^0 \to K_{\rm S,L}\pi^0$ and $D^+ \to K_{\rm S,L}\pi^+$ in a generic notation, we have

$$\begin{aligned} \mathcal{M}_{D\to K_{\rm S}\pi} &= \frac{1}{\sqrt{2}}\left[|\mathcal{M}_{\rm CF}|e^{i\delta_{\rm CF}} - |\mathcal{M}_{\rm CDS}|e^{i\delta_{\rm CDS}}\right] \quad , \\ \mathcal{M}_{D\to K_{\rm L}\pi} &= -\frac{1}{\sqrt{2}}\left[|\mathcal{M}_{\rm CF}|e^{i\delta_{\rm CF}} + |\mathcal{M}_{\rm CDS}|e^{i\delta_{\rm CDS}}\right] \quad . \end{aligned} \tag{44}$$

The corresponding decay widths will each contain three terms,

$$\begin{aligned} \Gamma_{D\to K_{\rm S}\pi} &= \frac{1}{2}\Gamma_{\rm CF} - \sqrt{\Gamma_{\rm CF}}\sqrt{\Gamma_{\rm CDS}}\cos(\delta_{\rm CF} - \delta_{\rm CDS}) + \frac{1}{2}\Gamma_{\rm CDS} \quad , \\ \Gamma_{D\to K_{\rm L}\pi} &= \frac{1}{2}\Gamma_{\rm CF} + \sqrt{\Gamma_{\rm CF}}\sqrt{\Gamma_{\rm CDS}}\cos(\delta_{\rm CF} - \delta_{\rm CDS}) + \frac{1}{2}\Gamma_{\rm CDS} \quad . \end{aligned} \tag{45}$$

To our knowledge it has been standard in the PDG data compilation for $D \to \bar{K}^0\pi$ to ignore all but the CF contribution by using the 'factor of two rule' in Eq. (10) to infer the CF $D \to \bar{K}^0 X$ branching fraction from that of $D \to K_{\rm S} X$. However, some account of sub-dominant terms is made in Ref. [25] by attributing to their neglect a source of 10% systematic error.

In terms of Cabibbo counting, the contributions on the right hand side of Eq. (45) go as $1 : \theta_c^2 : \theta_c^4$ or roughly $1 : 0.05 : 0.002$. Taking the sum of decay rates gives

$$\Gamma_{D\to K_{\rm S}\pi} + \Gamma_{D\to K_{\rm L}\pi} = \Gamma_{\rm CF} + \Gamma_{\rm CDS} \quad . \tag{46}$$

Since no existing facility can deliver 0.2% sensitivity, the $\Gamma_{\rm CDS}$ contribution to this equation is negligible and one arrives at the kind of relation given earlier in Eq. (10).

There *is*, however, the possibility of observing the $\mathcal{O}(\theta_c^2)$ interference term via the asymmetry measurement [6,26]

$$\mathcal{A} \equiv \frac{\Gamma_{D\to K_{\rm S}\pi} - \Gamma_{D\to K_{\rm L}\pi}}{\Gamma_{D\to K_{\rm S}\pi} + \Gamma_{D\to K_{\rm L}\pi}} \simeq -2\frac{\sqrt{\Gamma_{\rm CDS}}}{\sqrt{\Gamma_{\rm CF}}} \cos\left(\delta_{\rm CF} - \delta_{\rm CDS}\right) \quad , \tag{47}$$

or more specifically

$$\mathcal{A}_{00} = -2\sqrt{\frac{\Gamma_{D^0\to K^0\pi^0}}{\Gamma_{D^0\to \bar{K}^0\pi^0}}} \cos\left(\delta_{\bar{K}^0\pi^0} - \delta_{K^0\pi^0}\right) \ , \quad \mathcal{A}_{0+} = -2\sqrt{\frac{\Gamma_{D^+\to K^0\pi^+}}{\Gamma_{D^+\to \bar{K}^0\pi^+}}} \cos\left(\delta_{\bar{K}^0\pi^+} - \delta_{K^0\pi^+}\right) \ . \tag{48}$$

These asymmetries are $\mathcal{O}(\theta_c^2)$, so signals will occur at about the 5% level. Such measurements are difficult for existing B-factories but hopefully can be performed.

The detection of these asymmetries is clearly intriguing because they refer *directly* to $\delta_{\rm CF} - \delta_{\rm CDS}$. Although the phase differences $\delta_{\bar{K}^0\pi^0} - \delta_{K^0\pi^0}$ and $\delta_{\bar{K}^0\pi^+} - \delta_{K^0\pi^+}$ in Eq. (48) are for neutral modes (and not the charged case $\delta_{K^-\pi^+} - \delta_{K^+\pi^-}$) it is nonetheless valuable information. At a rigorous level, it follows from the positivity of decay widths that a negative (positive) asymmetry would correspond a phase difference in the first (second) quadrant. Beyond that one is forced into modelling $\Gamma_{\rm CDS}$. Since this contributes as a square root, the effect of model dependence is somewhat softened but still may be large. We note that $\delta_{\bar{K}^0\pi^0} - \delta_{K^0\pi^0} = 0$ in the SU(3) limit.

We conclude this section by considering how to implement a complete data set for the $D \to K\pi$ decays. It is understood from the preceding discussion that we organize the $K_{\rm S}$ and $K_{\rm L}$ final states into sums and differences. There will be a total of seven $D \to K\pi$ decays, of which three provide information on CF physics and four on CDS physics. Defining $\bar{\Gamma}_k \equiv p\bar{A}_k^2/(8\pi m_D^2)$ $(k = 1, 3)$, we have for CF-related decays

$$\begin{aligned}
\Gamma_{K^-\pi^+} &= \Gamma_1 + \sqrt{\Gamma_1\Gamma_3}\cos\left(\delta_1 - \delta_3\right) + \frac{1}{4}\Gamma_3 \\
\Gamma_{K_L\pi^0} + \Gamma_{K_S\pi^0} &= \frac{1}{2}\Gamma_1 - \sqrt{\Gamma_1\Gamma_3}\cos\left(\delta_1 - \delta_3\right) + \frac{1}{2}\Gamma_3 + \bar{\Gamma}_a + 4\sqrt{\bar{\Gamma}_a\bar{\Gamma}_3}\cos\left(\bar{\delta}_1 - \bar{\delta}_3\right) + 4\bar{\Gamma}_3 \\
&\simeq \frac{1}{2}\Gamma_1 - \sqrt{\Gamma_1\Gamma_3}\cos\left(\delta_1 - \delta_3\right) + \frac{1}{2}\Gamma_3 \\
\Gamma_{K_L\pi^+} + \Gamma_{K_S\pi^+} &= \frac{9}{4}\Gamma_3 + 2\bar{\Gamma}_b + 4\sqrt{\bar{\Gamma}_b\bar{\Gamma}_3}\cos\left(\bar{\delta}_1 - \bar{\delta}_3\right) + 2\bar{\Gamma}_3 \simeq \frac{9}{4}\Gamma_3 \quad .
\end{aligned} \tag{49}$$

In the latter two relations, we have made the approximation of discarding $\mathcal{O}(\theta_c^4)$ contributions (in accordance with the discussion around Eq. (46)). The approximate relations are seen to reproduce the content of Eq. (24). For the CDS-related decays we have

$$\begin{aligned}
\Gamma_{K^+\pi^-} &= 2\bar{\Gamma}_a - 4\sqrt{\bar{\Gamma}_a\bar{\Gamma}_3}\cos\left(\bar{\delta}_1 - \bar{\delta}_3\right) + 2\bar{\Gamma}_3 \\
\Gamma_{K^+\pi^0} &= \bar{\Gamma}_b - 4\sqrt{\bar{\Gamma}_b\bar{\Gamma}_3}\cos\left(\bar{\delta}_1 - \bar{\delta}_3\right) + 4\bar{\Gamma}_3 \\
\Gamma_{K_L\pi^0} - \Gamma_{K_S\pi^0} &= -\sqrt{2}\sqrt{\Gamma_1\bar{\Gamma}_a}\cos\left(\delta_1 - \bar{\delta}_1\right) - 2\sqrt{2}\sqrt{\Gamma_1\bar{\Gamma}_3}\cos\left(\delta_1 - \bar{\delta}_3\right) \\
&\quad +\sqrt{2}\sqrt{\Gamma_3\bar{\Gamma}_a}\cos\left(\delta_3 - \bar{\delta}_1\right) + 2\sqrt{2}\sqrt{\Gamma_3\bar{\Gamma}_3}\cos\left(\delta_3 - \bar{\delta}_3\right) \\
\Gamma_{K_L\pi^+} - \Gamma_{K_S\pi^+} &= 3\sqrt{2}\sqrt{\Gamma_3\bar{\Gamma}_b}\cos\left(\delta_3 - \bar{\delta}_1\right) + 3\sqrt{2}\sqrt{\Gamma_3\bar{\Gamma}_3}\cos\left(\delta_3 - \bar{\delta}_3\right) \quad ,
\end{aligned} \tag{50}$$

where $\bar{\Gamma}_k \equiv p\bar{A}_k^2/(8\pi m_D^2)$ $(k = a, b, 3)$. Each term in the first two relations is $\mathcal{O}(\theta_c^4)$ while each term in the latter two are $\mathcal{O}(\theta_c^2)$. As an aid to analyzing these equations, we propose a simplified scenario with $\delta_3 = \bar{\delta}_3 = 0$ and $\delta_1 = \pi/2$. The approximate equations which result are

$$\begin{aligned}
&\Gamma_{K^+\pi^-} \simeq 2\bar{\Gamma}_a - 4\sqrt{\bar{\Gamma}_a\bar{\Gamma}_3}\cos\bar{\delta}_1 + 2\bar{\Gamma}_3 \\
&\Gamma_{K^+\pi^0} \simeq \bar{\Gamma}_b - 4\sqrt{\bar{\Gamma}_b\bar{\Gamma}_3}\cos\bar{\delta}_1 + 4\bar{\Gamma}_3 \\
&\Gamma_{K_{\rm L}\pi^0} - \Gamma_{K_{\rm S}\pi^0} \simeq -\sqrt{2}\sqrt{\Gamma_1\bar{\Gamma}_a}\sin\bar{\delta}_1 + \sqrt{2}\sqrt{\Gamma_3\bar{\Gamma}_a}\cos\bar{\delta}_1 + 2\sqrt{2}\sqrt{\Gamma_3\bar{\Gamma}_3} \\
&\Gamma_{K_{\rm L}\pi^+} - \Gamma_{K_{\rm S}\pi^+} \simeq 3\sqrt{2}\sqrt{\Gamma_3\bar{\Gamma}_b}\cos\bar{\delta}_1 + 3\sqrt{2}\sqrt{\Gamma_3\bar{\Gamma}_3} \quad .
\end{aligned} \tag{51}$$

In general, one must solve numerically for the unknowns $\bar{\Gamma}_a$, $\bar{\Gamma}_b$, Γ_3 and $\sin\bar{\delta}_1$. Let us point out, however, the qualitative difference between the limiting cases $\bar{\delta}_1 \simeq \pi/2$ or $\bar{\delta}_1 \simeq 0$. To study this difference, it is *not* enough to measure just the $K^+\pi^-$ and $K^+\pi^0$ final states; the $K_{\rm L,S}\pi$ modes are required as well. It suffices to note here for $\bar{\delta}_1 \simeq \pi/2$ that $(\Gamma_{K_{\rm L}\pi^0} - \Gamma_{K_{\rm S}\pi^0})^2/\Gamma_{K^+\pi^-} \to \Gamma_1$ and $(\Gamma_{K_{\rm L}\pi^+} - \Gamma_{K_{\rm S}\pi^+})^2/\Gamma_{K^+\pi^0} \to \Gamma_3\bar{\Gamma}_3/\bar{\Gamma}_b$, whereas for $\bar{\delta}_1 \simeq 0$ both ratios become Γ_3. As $\bar{\delta}_1$ proceeds from $\pi/2$ to 0, the first ratio decreases but the second increases by almost an order of magnitude.

IV. CONCLUSIONS

The recent FOCUS experiment on $\Delta\Gamma_{\rm D}$ has yielded a signal at the several per cent level. By comparison, this experimental result is over an order-of-magnitude larger than the value $y_{\rm cp} \simeq 0.8 \cdot 10^{-3}$ obtained in a theoretical analysis [15] based on a sum over many D^0 decay modes. In this paper, we have avoided the temptation to provide a theoretical prediction of our own for $\Delta\Gamma_{\rm D}$. As stated earlier, we are not aware of any analytic approach in the charm region for which theoretical errors/uncertainties can be controlled. We therefore feel that whether or not the FOCUS result holds up over time is for future *experimental* work to decide.

At the very least, however, the E791, FOCUS, BELLE and CLEO studies serve to stimulate fresh thinking on a subject (D^0 mixing) that has long resisted progress. Our work in this paper has been to suggest further experimental work which would be of value:

1. We have described in Sect. II both positive and negative aspects of various D^0 decays beyond those used in the E791, FOCUS and BELLE experiments. In particular, we recommend that the $K_S\phi$, $K_S\omega$ and $K_S\rho^0$ modes be given serious attention. Each of these lies within the $CP = -1$ sector, which heretofore has only been probed indirectly via the mixed-CP case of the $D \to (K^-\pi^+ + K^+\pi^-)$ transition.

2. In Sect. III we divided our discussion of the strong phase $\delta \equiv \delta_{K\pi}^{(\mathrm{ch})}$ into two parts:

   Supposing that accurate data on K_L final states is not forthcoming, we concluded that it will not be possible to probe the phase $\delta_{K\pi}$ experimentally, but that the $\delta_{K^*\pi}$ decays *would* be accessible. Thus, we propose that branching fractions for the four CDS decays $D^0 \to K^{*+}\pi^-, K^{*0}\pi^0$ and $D^+ \to K^{*+}\pi^0, K^0\pi^{*+}$ be studied. At present, there is data only for the $D^+ \to K^{*0}\pi^+$ transition, with a stated uncertainty of about 44%. Although any CDS branching fraction will be very small, the availability of copious charm production at B-factories and hadron colliders allows for the study of this hidden corner of charm physics.[2]

   We explored the eventuality that accurate data on $K_L\pi$ final states will also be gathered. In principle, the asymmetries of Eq. (48) would provide direct examples of CF - CDS phase differences, but are hindered by the dependence on CDS branching fractions. A more ambitious program would be to collect the complete set of CDS $K\pi$ data displayed in Eq. (51). In principle, this would allow for a determination of $\delta_{+-}^{(K\pi)}$ like that given in Eq. (43) for $K^*\pi$. Finally, we note that although our approach in this paper has been limited to what can be learned from just decay rates, the study of Dalitz distributions in multibody final states offers a separate opportunity for attacking the '$\delta_{+-}^{(K\pi)}$ problem'.

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[2] After this paper was completed, an announcement appeared of a new CLEO measurement, $y_{CP} = -(1.1 \pm 2.5 \pm 1.4)\%$. [27] This is consistent with previous results (*cf* Eq. (9)). They also report a first measurement of the CDS mode $D^0 \to K^+\pi^-\pi^0$ which is a start of the exploration of the $K^*\pi$ CDS modes.

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