Charm Mixing - Theory

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I. INTRODUCTION

Given the forthcoming operation of the LHC, perhaps the dominant role of experimental flavor studies in particle physics will be supplanted by discoveries in the so-called new physics. Even if flavor physics faces an unsure future, all would acknowledge its remarkable recent progress via the observation of rare phenomena such as CP-violation in B-mesons or particle-antiparticle mixing for B_s and D^0 mesons. If new physics is indeed observed, experiments will be a asset in deciphering exotic LHC events.

My purpose in this talk is to describe two recent theoretical contributions to D^0 mixing [1, 2]. Both Ref. [1] and Ref. [2] should be considered in the context of the recent HFAG values [3].

\[
\begin{align*}
  x_D &\equiv \frac{\Delta M_D}{\Gamma_D} = (8.4^{+3.2}_{-5.3}) \times 10^{-3} \\
  y_D &\equiv \frac{\Delta \Gamma_D}{2 \Gamma_D} = (6.9 \pm 2.1) \times 10^{-3} .
\end{align*}
\]

In light of the Physical Review Letters criteria of 'observation' (> 5\sigma) or 'evidence' (3\sigma-to-5\sigma), we see that the above 2.4\sigma determination for x_D amounts to a 'measurement' (< 3\sigma). As such, we all await improvements in sensitivity for charm mixing.

The observed signal is seen to occur at about the 1% level. Whether or not this is the magnitude expected for the SM signal is a topic I will discuss shortly. At any rate, I wish to also consider the possibility of a NP component in D^0 mixing amplitude,

\[\mathcal{M}_{\text{mix}} = \mathcal{M}_{\text{SM}} + \mathcal{M}_{\text{NP}} .\]

The relative phase between \(\mathcal{M}_{\text{SM}}\) and \(\mathcal{M}_{\text{NP}}\) is not known. Thus, in our detailed study of various NP contributions to x_D in Ref. [2] we most often compared the NP predictions to \(\pm 1\sigma, \pm 2\sigma\) windows relative to the central x_D value of Eq. [1].

A. Operator Product Expansion (OPE) and Renormalization Group

An important technical aspect of Refs. [1, 2] is the process of relating an amplitude at some NP scale \(\mu = M\) to one at, say, the charm scale \(\mu = m_c\). This takes the form

\[\langle f | H_{\text{NP}} | i \rangle = G \sum_{i=1} C_i(\mu) \langle f | Q_i | i \rangle(\mu) ,\]

where the prefactor G has the dimension of inverse-squared mass, the \(C_i\) are dimensionless Wilson coefficients, and the \(Q_i\) are the effective operators. At the leading order of dimension six, it turns out that there are eight four-quark operators,

\[
\begin{align*}
  Q_1 &= (\bar{\pi} L \gamma_\mu c L) (\bar{\pi} L \gamma^\mu c L) , \\
  Q_2 &= (\bar{\pi} L \gamma_\mu c L) (\bar{\pi} R \gamma^\mu c R) , \\
  Q_3 &= (\bar{\pi} L c R) (\bar{\pi} R c L) , \\
  Q_4 &= (\bar{\pi} R c L) (\bar{\pi} R c L) , \\
  Q_5 &= (\bar{\pi} R \sigma_{\mu\nu} c L) (\bar{\pi} R \sigma_{\mu\nu} c L) , \\
  Q_6 &= (\bar{\pi} R \gamma_\mu c R) (\bar{\pi} R \gamma^\mu c R) , \\
  Q_7 &= (\bar{\pi} L c R) (\bar{\pi} L c R) , \\
  Q_8 &= (\bar{\pi} L \sigma_{\mu\nu} c R) (\bar{\pi} L \sigma_{\mu\nu} c R) .
\end{align*}
\]

Any given NP contribution will often involve several of these, but in all events never more than these eight. The evolution is determined by solving the RG equations obeyed by the Wilson coefficients,

\[\frac{d}{d \log \mu} \tilde{C}(\mu) = \hat{\gamma}^T \tilde{C}(\mu) ,\]

where \(\hat{\gamma}\) is the 8 \times 8 anomalous dimension matrix [4]. The output of this calculation is a set of RG factors \(r_1(\mu, M)\) which are expressed in terms of ratios of QCD fine structure constants evaluated at different scales, e.g. as with

\[
r_1(\mu, M) = \left( \frac{\alpha_s(M)}{\alpha_s(m_t)} \right)^{2/7} \left( \frac{\alpha_s(m_b)}{\alpha_s(m_t)} \right)^{6/25} \left( \frac{\alpha_s(m_b)}{\alpha_s(\mu)} \right)^{6/25} .
\]

B. Operator Matrix Elements

One needs ultimately to evaluate the \(D^0\)-to-\(\bar{D}^0\) matrix elements of the eight operators \(\{Q_i\}\). In general, eight non-perturbative parameters would need to be
II. STANDARD MODEL ANALYSIS

One can use quarks or hadrons as the basic degrees of freedom in carrying out the SM analysis of $D^0$ mixing. In principle, these should give the same result. However, as we shall see, rather different features appear in each description.

A. Quark-level Analysis

At leading order in the SM, the OPE for $D^0$ mixing consists of two dimension-six four-quark operators $\overline{D}_D^{(S)}$. The next order contains fifteen dimension-nine six-quark operators. For each increasing order in the OPE, there are still more local quark and gluon operators and the problem of determining operator matrix elements becomes ever more severe. For this reason, the dimension six sector has received by far the most attention.

The dimension six amplitude is depicted in Fig. 1. Since the $b$-quark is essentially decoupled due to the tiny $V_{ub}$ value, only the light $d, s$ quarks propagate in the loop. The Cabibbo dependence of this diagram, $\sin^2 \theta_c$, itself seems to suggest that the experimental signal (near the 0.01 level) is easily understood. But not so fast! For convenience, let us set $m_d = 0$. Then the only mass ratio that appears in the problem is

$$z \equiv \frac{m_s}{m_c} \approx 0.006.$$  \hfill (9)

Table IV examines one of the loop-functions for $\Delta \Gamma_D$ and shows the results of carrying out an expansion in powers of $z$. We see that the contributions of the individual intermediate states in the mixing diagram are not intrinsically small – in fact, they begin to contribute at $\mathcal{O}(z^0)$. However, flavor cancellations remove all contributions through $\mathcal{O}(z^2)$ for $\Delta \Gamma_D$, so the net result is $\mathcal{O}(z^3)$. Charm mixing clearly experiences a remarkable GIM suppression!

We understand the reason for this. $D^0$ mixing vanishes in the limit of exact SU(3) flavor symmetry. It is nonzero only because flavor SU(3) is broken, and indeed, $D^0$ mixing occurs at second order in SU(3) breaking [8]. A factor of $z$ will accompany each order of SU(3) breaking and the rate difference $y_D$ will experience an additional factor of $z$ due to helicity suppression.

Of course, this is just the leading order (LO) result in QCD, and we should consider the next-to-leading order result as well,

$$x_D = x_D^{(LO)} + x_D^{(NLO)},$$

$$y_D = y_D^{(LO)} + y_D^{(NLO)}.$$  \hfill (10)

This has been done in Ref. [9] and the results are summarized in Table IV which reveals that $y_D$ is given by $y_{D}^{(NLO)}$ to a reasonable approximation (due to the removal of helicity suppression by virtual gluons) whereas $x_D$ is greatly affected by destructive interference between $x_D^{(LO)}$ and $x_D^{(NLO)}$. The net effect is to render $y_D$ and $x_D$ of similar small magnitudes, at

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**TABLE I: Flavor cancellations in $\Delta \Gamma_D$.**

<table>
<thead>
<tr>
<th>Intermediate State</th>
<th>$\mathcal{O}(z^0)$</th>
<th>$\mathcal{O}(z^1)$</th>
<th>$\mathcal{O}(z^2)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$s\bar{s}$</td>
<td>1/2</td>
<td>$-3z$</td>
<td>$3z^2$</td>
</tr>
<tr>
<td>$d\bar{d}$</td>
<td>1/2</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$s\bar{d} + d\bar{s}$</td>
<td>-1</td>
<td>$3z$</td>
<td>$-3z^2$</td>
</tr>
<tr>
<td>Total</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>
least through this order of analysis, as compared to the experiment signal.

It is not inconceivable that the quark-level prediction of \( x_D \) and \( y_D \) just described might be considerably affected by a higher order in the OPE [3], which suffers less \( z \) suppression. Simple dimensional analysis [10] suggests the magnitudes \( x_D \sim y_D \sim 10^{-3} \) might be achievable, although order-of-magnitude cancellations or enhancements are possible.

### B. Hadron-level Analysis

Most of the work involving the hadron degree-of-freedom has been done on \( y_D \). One starts with the following general expression for \( \Delta \Gamma_D \),

\[
\Delta \Gamma_D = \frac{1}{M_D} \text{Im} \, I
\]

\[
I \equiv \langle D^0 | i \int d^4 x T \{ \mathcal{H}^{\Delta C=1}_w(x) \mathcal{H}^{\Delta C=1}_w(0) \} | D^0 \rangle .
\]

To utilize this relation, one inserts intermediate states between the \( | \Delta C | = 1 \) weak hamiltonian densities \( \mathcal{H}^{\Delta C=1}_w \). Although this can be done using either quark or hadron degrees of freedom, let us consider the latter here. Clearly, some knowledge of the matrix elements \( \langle \eta | \mathcal{H}^{\Delta C=1}_w | D^0 \rangle \) is required.

One approach is to model \( | \Delta C | = 1 \) decays theoretically and fit the various model parameters to charm decay data. Some time ago, \( \Delta \Gamma_D \) was determined in this manner and the result \( y_D \sim 10^{-3} \) was found [11]. This value is smaller than the recent BaBar and Belle central values.

Alternatively, one can arrange for charm decay data to play a somewhat different role. The earliest work in this regard focussed on the \( P^+ P^- = \pi^+ \pi^- , K^+ K^- , K^- \pi^+ , K^+ \pi^- \) states [12, 13]. In the flavor SU(3) limit, this subset of states gives zero contribution due to cancellations. But SU(3) breaking had already been known to be significant in individual charm decays. Since the study of charm decays in the 1980’s lacked an abundance of data, these references could only conclude that ‘\( y_D \) might be large’.

A modern version of this approach now exists, although the analysis takes an unexpected direction [3]. Since SU(3) breaking occurs at second order in \( D^0 \) mixing, let us hypothesize that the contribution of the \( P^+ P^- \) sector is in fact negligible due to flavor cancellations. Likewise for all other sectors whose decays are kinematically allowed. However, this cannot be true for four-pseudoscalars because decay into four-kaon states is kinematically forbidden. In Ref. [8] it is estimated that these ‘kinematically-challenged’ sectors can provide enough SU(3) violation to induce \( y_D \sim 10^{-2} \). I personally find such an argument to be an important advance in our understanding of the subject. At the same time, it is unfortunately more persuasive than compelling due to the uncontrollable uncertainties inherent in this line of reasoning.

To summarize, we have just described how the observed \( D^0 \) mixing signal could well arise from SM physics, but the associated numerical prediction is seen to be lacking in precision. This conceivably leaves room for some NP mechanism to co-contribute or even dominate the SM signal. In the following we consider in turn NP analyses of the width difference \( y_D \) and the mass difference \( x_D \).

### III. NP AND THE WIDTH DIFFERENCE

At first glance, it would appear unlikely that NP could affect \( y_D \) because the particles contributing to the loop amplitude of Fig. 1 must be on-shell. Since NP particles will be heavier than the charm mass, ‘there can be no NP contribution to \( y_D \’ \). Or so goes the argument.

However, as explained in Ref. [1], NP effects in \( \mathcal{H}^{\Delta C=1}_w \) can generally contribute to \( y_D \). In the loop amplitude of Fig. 1 the NP contribution (empirically small for \( \Delta C = -1 \) processes) arises from either of the two vertices. We represent the NP \( \Delta C = -1 \) hamiltonian as (indices \( i,j,k,\ell \) represent color),

\[
\mathcal{H}^{\Delta C=-1}_{NP} = \sum_{q,q'} D_{qq'} \left[ \overline{c}_1(\mu) O_1 + \overline{c}_2(\mu) O_2 \right] ,
\]

\[O_1 = \bar u_i \Gamma_{1} q_j \, \bar u_j \Gamma_2 c_i , \]

\[O_2 = \bar u_i \Gamma_{1} q_j \, \bar u_j \Gamma_2 c_j , \]

where \( D_{qq'} \) and the spin matrices \( \Gamma_{1,2} \) encode the NP model. \( \overline{c}_1(\mu) \) are Wilson coefficients evaluated at energy scale \( \mu \) and the flavor sums on \( q,q' \) extend over the \( d, s \) quarks.

This leads to a prediction for the NP contribution to \( y_D \). For a generic NP interaction, one finds (with the number of colors \( N_c = 3 \))

\[
y_D = - \frac{4 \sqrt{2} G_F}{M_D \Gamma_D} \sum_{q,q'} \mathbf{V}^\ast_{cq} \mathbf{V}_{aq} D_{qq'} (K_{i \delta i} \delta_{j \ell} + K_{2} \delta_{i \delta j} \delta_{k \ell}) \sum_{\alpha=1}^{5} I_\alpha (x,x') \langle D^0 \mid \mathcal{O}^{i j \ell} \mid D^0 \rangle , \]

where \( \{ K_{\alpha} \} \) are combinations of Wilson coefficients,

\[K_1 = (c_1 \overline{c}_1 N_c + (c_1 \overline{c}_2 + \overline{c}_1 c_2)) , \]

\[K_2 = c_2 \overline{c}_2 , \]
and the \(\mathcal{O}_{ijk\ell}^{(k)}\) are four-quark operators written down in Ref. \[1\]. Numerical results for some NP models are displayed in Table II.

<table>
<thead>
<tr>
<th>Model</th>
<th>(y_D)</th>
<th>Comment</th>
</tr>
</thead>
<tbody>
<tr>
<td>RPV-SUSY</td>
<td>(6 \cdot 10^{-6})</td>
<td>Squark Exchange</td>
</tr>
<tr>
<td></td>
<td>(-4 \cdot 10^{26})</td>
<td>Slepton Exchange</td>
</tr>
<tr>
<td>Left-right</td>
<td>(-5 \cdot 10^{-6})</td>
<td>‘Manifest’</td>
</tr>
<tr>
<td></td>
<td>(-8.8 \cdot 10^{-5})</td>
<td>‘Nonmanifest’</td>
</tr>
<tr>
<td>Multi-Higgs</td>
<td>(2 \cdot 10^{-10})</td>
<td>Charged Higgs</td>
</tr>
<tr>
<td>Extra Quarks</td>
<td>(10^{-8})</td>
<td>Not Little Higgs</td>
</tr>
</tbody>
</table>

One sees that the entries, aside from R-parity violating SUSY, produce small contributions. We emphasize, however, that Eq. \((12)\) and Eq. \((13)\) represent general formulae for the contribution of all NP models of \(|\Delta C| = 1\) interactions, encompassing those not included in Table III.

### IV. NP AND THE MASS DIFFERENCE

As the operation of the LHC looms near, the number of potentially viable NP models has never been greater. In this section, I will give an overview of Ref. \[2\], whose hallmark is the study of many \((21\) in all) NP models. Perhaps the best way to start is to consider the different ways that ‘extras’ can be added to the SM:

- Extra gauge bosons (LR models, etc)
- Extra scalars (multi-Higgs models, etc)
- Extra fermions (little Higgs models, etc)
- Extra dimensions (universal extra dims., etc)
- Extra global symmetries (SUSY, etc)

Although this approach does not provide a totally clean partition of NP models (e.g. obviously SUSY contains extra particles appearing in other categories), it proved useful to the authors of Ref. \[2\].

The broad menu of NP models which were analyzed is listed in Table IV. The extensive content of this list (e.g. there are four different SUSY realizations and three involving large extra dimension) indicates how rich the field of NP models has become. Of course, the subject of NP is by now fairly mature (in preparing this talk, I realized that my first paper on charged Higgs bosons \[13\] was written nearly 30 years ago!) and thus many models have been well exposed to the scrutiny of experiment. This would seem to imply that parameter spaces for the various models have shrunk so much that a measurement like \(D^0\) mixing would have little impact. In fact, in giving this talk in several venues I challenged each audience to predict how many of the 21 models considered here were constrained by the \(D^0\) mixing values or equivalently how many evaded constraint. Before answering this question, we consider a specific NP example in some detail.

Suppose a vector-like quark of charge \(Q = +2/3\) \[15\] is added to the SM. Recall that a vector-like quark is one whose electric charge is either \(Q = +2/3\) or \(Q = -1/3\) and which is an \(SU(2)_L\) singlet. Both choices of charge are actually well motivated, as such fermions appear explicitly in several NP models. For example, weak isosinglets with \(Q = -1/3\) appear in \(E_6\) GUTs \[16\] \[17\], with one for each of the three generations \((D, S, \text{and } B)\). Weak isosinglets with \(Q = +2/3\) occur in Little Higgs theories \[18\] \[19\] in which the Standard Model Higgs boson is a pseudo-Goldstone boson, and the heavy iso-singlet \(T\) quark cancels the quadratic divergences generated by the top quark in the mass of the Higgs boson. We restrict our attention here to the \(Q = +2/3\) case. Since the electroweak quantum number assignments are different than those for the SM fermions, flavor changing neutral current interactions will be generated in the left-handed up-quark sector. Thus, there will also be FCNC couplings with the \(Z^0\) boson \[17\]. These couplings contain a mixing parameter \(\lambda_{uc}\) which is con-

### Table IV: NP models studied in Ref. [2]

<table>
<thead>
<tr>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fourth Generation</td>
</tr>
<tr>
<td>(Q = -1/3) Singlet Quark</td>
</tr>
<tr>
<td>(Q = +2/3) Singlet Quark</td>
</tr>
<tr>
<td>Little Higgs</td>
</tr>
<tr>
<td>Generic Z'</td>
</tr>
<tr>
<td>Family Symmetries</td>
</tr>
<tr>
<td>Left-Right Symmetric</td>
</tr>
<tr>
<td>Alternate Left-Right Symmetric</td>
</tr>
<tr>
<td>Vector Leptoquark Bosons</td>
</tr>
<tr>
<td>Flavor Conserving Two-Higgs-Doublet</td>
</tr>
<tr>
<td>Flavor Changing Neutral Higgs</td>
</tr>
<tr>
<td>FC Neutral Higgs (Cheng-Sher ansatz)</td>
</tr>
<tr>
<td>Scalar Leptoquark Bosons</td>
</tr>
<tr>
<td>Higgsless</td>
</tr>
<tr>
<td>Universal Extra Dimensions</td>
</tr>
<tr>
<td>Split Fermion</td>
</tr>
<tr>
<td>Warped Geometries</td>
</tr>
<tr>
<td>Minimal Supersymmetric Standard</td>
</tr>
<tr>
<td>Supersymmetric Alignment</td>
</tr>
<tr>
<td>Supersymmetry with RPV</td>
</tr>
<tr>
<td>Split Supersymmetry</td>
</tr>
</tbody>
</table>
A tree-level contribution to $\Delta M$ is fully constrained by the earlier about how many models avoid being meaning-
for that. However, we can answer the question raised all 21 models of Table IV. One must refer to Ref. [2]
It is not possible here to summarize the results for in the form of graphs) like the one depicted in Fig. 3.

\[ \lambda_{uc} \equiv -(V_{ud}^* V_{cd} + V_{us}^* V_{cb} + V_{ub}^* V_{cb}) \]  \hspace{1cm} (15)

A tree-level contribution to $\Delta M_D$ is thus generated from $Z^0$-exchange (see Fig. 2). It is straightforward to calculate that

\[ x_D^{2/3} = \frac{G_F \lambda_{uc}^2}{\sqrt{2} M_D \Gamma_D} r_1(m_c, M_Z) |Q_1| |D^0| \]

\[ = \frac{2G_F f_D^2 M_D}{3\sqrt{2} \Gamma_D} B_D (\lambda_{uc})^2 r_1(m_c, M_Z) \]  \hspace{1cm} (16)

where we have made use of Eq. (4) and Eq. (7). The result is displayed in the graph of Fig. 3, which contrasts a $\pm \sigma$ window (dashed lines) about the HFAG central value with the NP prediction $x_D^{2/3}$ (solid line), as a function of the mixing parameter $\lambda_{uc}$. The bound on $\lambda_{uc}$ from $D^0$ mixing turns out to be roughly two orders of magnitude better than that from the CKM unitarity constraint.

Upon performing analogous analyses for the other NP models, we arrive at a set of constraints (mainly in the form of graphs) like the one depicted in Fig. 3. It is not possible here to summarize the results for all 21 models of Table IV. One must refer to Ref. [2] for that. However, we can answer the question raised earlier about how many models avoid being meaning-
fully constrained by the $D^0$ mixing data. The answer is just 4 of the 21 models, which came as a surprise to many. These 4 models are Split SUSY, Universal Extra Dimensions, LR Symmetric and Flavor-conserving Higgs Doublet.

It is of interest to briefly consider a few of these, in order to understand how the $D^0$ mixing constraints can be evaded:

1. Split SUSY [20]: This is a relatively new variant of SUSY (2003-4) in which SUSY breaks at a very large scale $M_S \gg 1000$ TeV. All scalars except the Higgs have mass $M \sim M_s$ whereas fermions have the usual weak-scale mass. It is known that large $D^0$ mixing in SUSY will involve squark amplitudes. But since squark masses in Split SUSY are huge, the mixing becomes suppressed.

2. Universal Extra Dimensions [21]: UED is a variant of the idea that TeV $-sized extra dimensions exist. There are no branes appearing in this approach, so all SM fields reside in the bulk and just one extra dimension is usually considered. Each SM field will have an infinity of KK excitations. It turns out that GIM cancellations suppress all but a few b-quark KK terms, but these are CKM inhibited.

So we see that suppressions can arise from more than one source and that the suppressing mechanism will depend on the specific model.

V. CONCLUSIONS

At long last, signals for $x_D$ and $y_D$ have been observed. These experimental findings, although greatly welcome, whet our appetite for ever more precise determinations. Hopefully these will be forthcoming, so we can put aside any lingering concerns that all the excitement has been the result of statistical fluctuations.

The SM analysis, as is so often the case, is not without its difficulties. At the quark level, theoretical analysis in the dimension six sector through NLO gives $x_D \sim y_D \sim 10^{-6}$. These values are tiny compared to the reported experimental signals. It is evident that the triple sum over the operator dimension $d_n$, the QCD coupling $\alpha_s$ and the mass expansion parameter $z$ of Eq. (4) is slowly convergent. This approach remains inconclusive at best.

A more promising avenue is to study $y_D$ with the hadronic degree of freedom. This yields a plausible, and quite possibly correct, explanation for reaching the $y_D \sim 0.01$ level. Again, however, the effect of strong interaction uncertainties mars predictive power.

Finally, the work of Refs. [1, 2] has explored which NP models can yield sizable values for $x_D$, $y_D$ and...
which cannot. Charm mixing data has been found to infer useful constraints on NP parameters spaces, and as should be clear to all, provides a most welcome addition to the High Energy Physics community.

Acknowledgments

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[22] These two papers contain extensive references of contributions not possible to cite here.