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Johan Carlstrom

Julien Garaud

Egor Babaev
University of Massachusetts - Amherst, babaev1@physics.umass.edu

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Non-pairwise intervortex interaction forces

Johan Carlstrom¹, Julien Garaud²,³, Egor Babaev²,¹

¹Department of Theoretical Physics The Royal Institute of Technology Stockholm SE-10691 Sweden
² Department of Physics University of Massachusetts Amherst MA 01003 USA

We demonstrate the existence of a new kind of non-pairwise multivortex interaction forces, which are present between superconducting vortices along with pairwise vortex interactions. We show that the multibody forces are especially important in compact vortex clusters in two-component type-1.5 superconductors and result in extremely rich physics of multivortex bound states.

The crucial importance of topological excitations in the physics of superfluidity and superconductivity made quantum vortex solutions in the Ginzburg-Landau and Gross-Pitaevskii models perhaps the most studied examples of topological solitons (defined as a localized lumps of energy characterized by a topological invariant). These well studied vortex solutions are frequently used as generic testing objects for High Energy Physics and Cosmological models. In that broader context, especially spectacular theoretical works attempted on numerous occasions to identify topological solitons with particles. There the particle-solitons are lumps of energy which enjoy a topological protection against radiating their energy. Moreover, in these constructions the interaction forces emerge without an addition of new fields, directly from the underlying field theory in a fashion similar to how e.g. effective Yukawa forces arise between vortices in a London superconductor. The most successful example is the model proposed by Skyrme where topologically conserved charge was associated with conserved barion number [1] and nuclei appear as bound states of topological solitons. In these constructions Skyrmions are quantized. The objects of unit topological charge are then associated with nucleons which are a spin-½ fermions [2].

Superconducting vortices, in spite being know to form a variety of “aggregate” vortex states: vortex liquids, glasses etc still do not show nearly as complex multibody bound states as real matter or ground states of topological defects in e.g. the Skyrme or Faddeev models [1]. The roots of the limited diversity of ground states of vortex matter in superconductors can be attributed to the smaller diversity of known intervortex interaction potentials. The recent developments in multicomponent superconductors aimed at realizing more complex stable strongly bound multi-solitonic states in the so-called “type-1.5 regime”. In that regime, due to existence of two components, vortex solutions were found that exhibit strongly non-monotonic interaction potentials between two vortices, with short-range repulsive and long-range attractive parts [3][5] and thus form vortex clusters in low magnetic fields. This physics recently received increased attention after the proposal by Moshchalkov et. al. [6] [7] that MgB₂ exhibits type-1.5 superconductivity. Using nonmonotonic two-body vortex forces calculated from the Ginzburg-Landau (GL) model, the formation of vortex clusters was studied in molecular dynamics simulations [6][8].

In this work we investigate the structure of multi-vortex bound states in a two-component superconductor beyond the validity of the linearized theory and find a new kind of non-pairwise multibody interactions which are getting especially pronounced at short intervortex separations. This results in extremely rich structures of the multi-soliton states, potentially as complex or in some respects more complex than those in other presently known models supporting multi-soliton bound states (in e.g. high energy physics). Furthermore it suggests that one can test nonpairwise interaction of topological solitons experimentally in the condensed matter systems.

We consider a Ginzburg-Landau model of a two-component superconductor with different values of intercomponent Josephson couplings.

\[
\mathcal{F} = \frac{1}{2} \sum_{i=1,2} |(\nabla + ieA)\psi_i|^2 + \frac{1}{2} (\nabla \times A)^2 + \alpha_i |\psi_i|^2 + \frac{1}{2} \beta_i |\psi_i|^4 - \eta |\psi_1||\psi_2|\cos(\theta_2 - \theta_1) \quad (1)
\]

Here \(\psi_{1,2} = |\psi_{1,2}|e^{i\theta_{1,2}}\) represent the superconducting components coupled by the gauge field \(A\) and the Josephson coupling \(\eta\). The model exhibits type-1.5 superconductivity when the penetration length scale is smaller than one of the characteristic length scales of the density variation and also the conditions for short range repulsion and thermodynamical stability are satisfied [3][5]. Then, in a range of parameters, vortices with similar circulation have interaction that is attractive at long range (driven by density-density interaction) but repulsive at short range (due to current-current and magnetic interaction) [3][5].

The two component superconductivity arises in a number of physical contexts. In two-band superconductors \(\psi_{1,2}\) corresponds to superfluid densities in two different bands (for a derivation of such theories from microscopic two-band models see e.g. the recent review [9]). Note that in two-band materials the intercomponent coupling dictates that the symmetry is \(U(1)\). Therefore the standard mean-field description of the phase transition could be expected to be given by a single-component GL theory. However we are interested here in the magnetic response...
of the system, which by its nature is a finite-length scale property. As demonstrated in [4,5], since the interaction between vortices in multi-band GL theory is a competition of several exponentially localized modes, the "leading order" meanfield approximation of two-band theory by single-component models in a number of cases is qualitatively wrong. That is, even a tiny coupling to the second component (e.g. arising in some cases only in the next order in \((1 - T/T_c)\)) can produce a small but long-ranged attractive force between vortices. There are also different kinds of interband couplings allowed by symmetry (mixed gradient, density-density couplings etc) whose influence on the vortex physics was studied in [6]. It was demonstrated in [6] that for a general two-component effective potential the model in general has three length scales: magnetic field penetration length and two characteristic length scales associated with the density variations [5]. Although the model [1] does not include all the terms allowed by symmetry in a generic case, it is clear from the analysis of a model with generic potentials [7] that the "minimalistic" model [1] captures qualitatively the essential "three length scales" physics of more general multiband models as well.

We also consider below the theory where Josephson coupling, as well as other inter-band specific couplings are forbidden on symmetry grounds but the condensates are coupled by the vector potential. Such theories possess two independent critical temperatures and independently diverging coherence lengths. They are highly relevant in the context of the theories of liquid metallic hydrogen, deuterium and their mixtures [10], which are the subjects of renewed experimental pursuit. Similar models appear in the physics of neutron star interiors, where the two fields represent protonic and \(\Sigma^-\) hyperon Cooper pair condensates and are phenomenologically relevant for observed rotational dynamics of pulsars [11].

Before we proceed to discuss the multivortex interactions, let us remind some general properties of multicomponent Ginzburg-Landau theory. In case of \(U(1) \times U(1)\) the key feature of the model is that one-flux quantum? vortices, which are induced by magnetic field are composite: i.e. in the ground state have a core around which both phases wind by \(2\pi\), i.e. \(\Delta \theta_1 = 2\pi; \Delta \theta_2 = 2\pi\) and can be viewed as a bound state of two fractional vortices [10] [12]. In the London limit (i.e. neglecting spatial variations of \(|\psi_{1,2}|\)) they are logarithmically bound states of two fractional flux vortices with a number of interesting consequences for phase transitions [10] [12] [13]. If the core in one component is larger than the magnetic field localization length scale while the other core is smaller than the magnetic field localization the \(U(1) \times U(1)\) system supports in some parameter range type-1.5 superconductivity [8]. When Josephson coupling \(\eta|\psi_1||\psi_2| \cos(\theta_2 - \theta_1)\) is added, it energetically prefers to lock phases and the \(U(1) \times U(1)\) the symmetry breaks down to to \(U(1)\) and one flux quantum vortices become a tightly bound composite object made of fractional vortices which, in the London limit attract each other with a much stronger (asymptotically linear) potential [12].

The number of discovered multiband superconductors is rapidly growing, some of which may be of type-1.5. However, one cannot vary the intercomponent Josephson coupling in these materials in a controllable way. The experimental realization of type-1.5 physics where analogous parameter can actually be tuned in a wide range (in order to access all the different regimes outlined below) is a system of interlaced Josephson-coupled type-I/type-II multilayers shown on Fig. 1.

The current paradigm within which vortex matter in superconductors or superfluids is understood relies on the assumption that interactions in a system of vortices is a superposition of pairwise forces. Indeed the most usual analysis of interaction between well separated topological solitons involves linearization. By nature of this approximation, the interaction in a system of multiple vortices is a superposition of two-body forces. Similarly, modern theories of phase transitions in quantum fluids in terms of vortex loops proliferation are formulated in the context of the London model. This is an approximation where fluctuations of densities are neglected and intervortexinteraction is reduced to a superposition of two-body forces. Here we demonstrate the existence of much more complicated nonpairwise forces between superconducting vortices arising along with the pairwise interactions. As a result the vortex clusters exhibit an immensely rich hierarchy of distinct high topological charge solitonic states.

First, let us present a highly accurate numerical study of a three body problem (for three composite one-quantum vortices, each characterized by the phase wini-
dictions $\Delta \theta_1 = 2\pi; \Delta \theta_2 = 2\pi$ at short separations where the linearized theory does not work. Our main focus is the type-1.5 regime, however we also investigate a type-II multicomponent superconductor.

The nonpairwise interaction between vortices was investigated as follows: First a vortex pair is fixed in the center of the system. A third vortex is then inserted, and the energy is minimized with respect to all the degrees of freedom, except the positions of the centers of the vortex cores in the dominant component. The procedure is then repeated for a different positions of the third vortex. This gives the dependence of the system energy on the position of the third vortex in a XY plane, plotted in Figs. 2-4. The first panel gives the total interaction energy, the second panel gives a sum of pairwise interaction. The third panel gives the difference between these energies which shows the presence of nontrivial non-pairwise (three body) interaction. To minimize the effects of discretization errors the calculation was performed on a high resolution grid of up to $8 \times 10^6$ points, with lattice spacing $h \sim \xi_1/100$. We used 4-16 hours on a 8-core cluster node to relax each data point in the interaction potential. We only report the numerically most accurate data which in a lattice calculation requires a certain parameter dependent minimal vortex separation. Consequently in the interaction energy plots Figs. 2-4 no data is given for too closely placed vortices.

In all the regimes Figs. 2-4 we found diverse and pronounced nonpairwise interaction forces.

To investigate how the presence of nonpairwise interactions along with non-monotonic two-body forces affects the structure formation, we investigated solutions for $N$-vortex bound states in several type-1.5 regimes in highly accurate numerical simulations. In these simulations the variational problem was defined using a finite element formulation provided by the Freefem++ framework using Nonlinear Conjugate Gradient method. A steepest descent minimization scheme was also tested, leading to qualitatively similar results. We used two kind of initial conditions: (i) a giant $N$-quanta vortex or (ii) various configurations of well separated single vortices. Animations showing the evolution of the system from the initial configuration to the vortex clusters in the energy minimization process are available at [13].

The figures (Fig. 5-9) show the ground state of multiple flux quanta in $U(1) \times U(1)$ as well as in Josephson-coupled $U(1)$ models. Consider first the case of two active bands ($\alpha_{1,2} < 0$). This case shows very interesting geometrical properties of the ground state configurations (shown on Fig. 5-7). One can clearly see that with growing number of vortices, the local vortex structure strongly depends on the number of vortices in a cluster. The striking feature which is manifest on all the figures (Fig. 5-7) is the coexistence and competition between type-II-like behavior of the first condensates which attempts to form a regular vortex lattice and type-I-like behavior of the second condensate. Namely the second condensate mimics the formation of a single large normal domain. Also like in a genuine type-I superconductor, this component has current concentrated only on the domain boundary and prefers a circular boundary. However in this type-1.5 system the competition between type-I and type-II physics at the boundary results in neither hexagonal nor circular boundary. The next visually striking effect is that the vortex solutions are very different inside the vortex cluster and on the cluster boundary.
which shows up especially clear in the current density.

This numerical analysis shows also different qualitatively new physics which arises in a cluster, which is not captured by a linear analysis. This is the appearance of gradients of the phase difference $\nabla(\theta_1 - \theta_2)$ as is clearly seen on the panels f from the plotted quantity $\text{Im}(\psi_1^* \psi_2)$. One of the mechanisms of the generation of the phase difference which we observed was associated with splitting of the vortex cores of the components $\psi_{1,2}$ driven by a magnetic repulsion. It leads to a dipole-like configuration of the phase difference of the two components. This splitting exists in cases of zero as well as finite Josephson coupling, though it is smaller in the later case. However it was not the only mechanism for nontrivial phase difference generation. More complicated configurations like phase difference “quadrupoles” also were observed. The presence of gradients in the phase difference, along with the gradients of the relative density is known to
Figure 5: The ground state of a $N_v = 9$ flux quanta configuration in type-1.5 $U(1) \times U(1)$ superconductor (i.e. $\eta = 0$). The parameters of the potential are $(\alpha_1, \beta_1) = (-1.00, 1.00)$ and $(\alpha_2, \beta_2) = (-0.60, 1.00)$, while the electric charge is $e = 1.48$. The physical quantities being represented here are $a$ the magnetic flux density; $b$ (resp. $c$) is the density of the first (resp. second condensate) $|\psi_1|^2$. $d$ (resp. $e$) shows the first (resp. second) norm of supercurrent, while $f$ is $\text{Im}(\psi_1^* \psi_2)$ which is nonzero when there appears the phase difference between both components. The same quantities are displayed in this order in the next plots (Fig. 5–Fig. 9). Note the induced nonzero phase difference pattern at the cluster boundary caused by competing interactions.

lead in multicomponent superconductors to contributions from self-generated Faddeev-Skyrme like terms to magnetic energy density [15]. This makes the energetics of the vortex cluster boundary a very complicated nonlinear problem.

Note that in single-component type-I superconductors the domains of normal phase have a circular boundary only when the effects of stray fields are neglected. It is well known from the theory of single-component type-I superconductors that the stray fields typically lead to formation of stripes of the normal phase rather than circular domains [16], though other geometries are also possible [17]. Similarly, this “type-I” aspect of this physics, in realistic experimental setups especially for thin films/bilayers like those shown on Fig. 1 should result in vortex stripes rather than vortex clusters formation.

Next we report the regime which, at first glance, would be expected to be most unfavorable for nontrivial vortex cluster physics. In this example, shown on Fig. 9 one passive band (i.e. $\alpha_2 > 0$) is coupled to the first band with extremely strong Josephson coupling $\eta = 7.0$. Such strong coupling introduces a strong energy penalty for disparities of the condensate variations. Identifying the ground states in this regime is numerically more complicated than in the previous cases. We get a flat and complicated energy landscape and the outcome of a straightforward energy minimization strongly depends on initial guess. Since, in a realistic physical situation this kind of energy landscape should reflect itself primarily in nonuniversal, history-dependent structure formation, we report in the Fig. 9 the typical non-universal outcome of the energy minimization. Starting from a compact initial guess (in this case a giant vortex initial condition) the energy minimization yields a pattern with stripe-like elements instead of vortex clusters. Note that even in this regime, the system exhibits self-induced gradients of the phase difference, in spite of the strong Josephson coupling.

In conclusion, vortices in condensed matter systems represent one of the most studied examples of topological solitons. The usual approach to describe interactions between topological solitons (superconducting vortex being one particular example) is based on pairwise interaction [1]. In this work we demonstrated the existence of multibody forces which are present along with pairwise forces between superconducting vortices. The non-pairwise interaction forces are especially important in type-1.5 regimes where the interplay between type-I-like and type-II-like physics results in non-monotonic two-body interactions and formation of vortex clusters and stripes. We showed that there is a very rich physics associated with the vortex clusters, namely N-quanta clusters can be quite different from simple superpositions of N single vortex solutions but rather represent a sequence of unique charge-N topological solitons. Besides vortex clusters in the type-1.5 regime, non-pairwise forces should be important for understanding forces in compact configurations of pinned vortices in type-II regimes.

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Figure 6: The ground state of a $N_v = 12$ flux quanta configuration using the same parameter set as in Fig. 5. Here the global 8-folded discrete symmetry of the cluster has been broken toward a less symmetric configuration (a 3-folded discrete symmetry) in favor of a regular vortex lattice in the cluster. There is again a competition between type-I-like (normal circular cluster with a boundary current) and type-II-like tendencies (vortex lattice).

Figure 7: Magnetic ground state of an $N_v = 9$ vortex configuration with a stronger electric charge coupling $e = 1.55$ (parameters of the potential are the same as in Fig. 5). This shows the behavior of the system with respect to a charge increase. Increasing the electric charge decreases penetration length and thus pushes the system towards type-I regime i.e. with more circular boundary.

Figure 8: Magnetic ground state of an $N_v = 9$ vortex configuration with the parameter set given by Fig. 5 but with $\epsilon = 1.59$ and added Josephson coupling $\eta = 0.1$. Although the Josephson term introduced an energy penalty for phase difference, it has little effect on the vortex cluster boundary where the high magnetic pressure still generates strong phase difference gradients.

Figure 9: A bound state of an $N_v = 25$ vortex configuration in case when superconductivity in the second band is due to interband proximity effect and the Josephson coupling is strong $\eta = 7.0$. Other parameters are $(\alpha_1, \beta_1) = (-1.00, 1.00)$, $(\alpha_2, \beta_2) = (3.00, 0.50)$, $\epsilon = 1.30$. 