The development and evaluation of curriculum materials to teach specified objectives of carrying and borrowing in mathematics to selected elementary school pupils.

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THE DEVELOPMENT AND EVALUATION OF CURRICULUM MATERIALS TO TEACH SPECIFIED OBJECTIVES OF CARRYING AND BORROWING IN MATHEMATICS TO SELECTED ELEMENTARY SCHOOL PUPILS

A Dissertation Presented

By

CURTIS JUNIUS MORRIS

Submitted to the Graduate School of the University of Massachusetts in partial fulfillment of the requirements for the degree of

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May 1979

Education
The development and evaluation of curriculum materials to teach specified objectives of carrying and borrowing in mathematics to selected elementary school pupils

A dissertation presented

By

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ABSTRACT

THE DEVELOPMENT AND EVALUATION OF CURRICULUM MATERIALS TO TEACH SPECIFIED OBJECTIVES OF CARRYING AND BORROWING IN MATHEMATICS TO SELECTED ELEMENTARY SCHOOL PUPILS

(May 1979)

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The purpose of this study was to develop and evaluate curriculum materials designed to teach specified objectives of carrying and borrowing in mathematics to selected elementary school pupils. Specifically, the purpose of this study was: (1) to determine appropriate objectives of carrying and borrowing in mathematics for selected elementary students; (2) to develop curriculum materials for accomplishing the identified objectives; and (3) to evaluate the effectiveness of the materials for helping selected students master the fundamental operations of carrying and borrowing as defined by the objectives.

In order to insure that appropriate objectives were chosen, the investigator conducted a review of the literature which includes: (1) the relationship between curriculum and instruction and the developmental stages children encounter as they progress from one conceptual level of mathematical understanding to the next; (2) identification of the quantitative skills associated with carrying and borrowing as found in the literature; and (3) the identification of qualitative skills actually...
used to solve problems requiring an operational understanding of carrying and borrowing through the application of the Program Evaluation Review Technique (PERT).

On the basis of the literature review, the investigator identified two objectives judged most appropriate for the scope of this study. The objectives were to master the operations of carrying and borrowing in addition and subtraction through teaching process. The initial target population was second and third grade students identified as having problems with carrying and borrowing by the school system.

The second portion of the study was concerned with the development and field testing of the curriculum materials designed to teach the process of carrying and borrowing. The field testing procedure included two field tests. The first field test was conducted with ten below-average ability elementary grade mathematics students in grades one through five. The results of the field test indicated that the materials were ready for further testing and evaluation.

The final evaluation of the curriculum materials was conducted at the Martin Luther King Elementary School in Providence, Rhode Island. A below-average ability second grade mathematics class and a below-average third grade mathematics class were selected for the evaluation.

The two major research questions and the results for each question were as follows:

1. Did a majority of the students in the below-average ability second grade mathematics class which served as the experimental group for this study master the terminal objectives of the cur-
riculum materials?

The answer to question one is "yes." Six students, or seventy percent of the experimental group, scored twenty-two out of thirty-three examples correct.

2. Did a majority of the students in the below-average ability third grade mathematics class which served as the experimental group for this study master the terminal objectives of the curriculum materials?

The answer to question two is "yes." Eight out of nine students, or eighty-nine percent, scored twenty-seven examples right out of thirty-three.

The results of this study indicate that MATH-EZE, the curriculum materials, are promising as a supplementary tool for teaching carrying and borrowing. Recommendations were advanced that the present study should be replicated with a larger and stratified random sample drawn from different ability level students in order to determine the level of confidence of the curriculum materials for promoting learning of carrying and borrowing in mathematics.
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CHAPTER I
INTRODUCTION

Today, it is difficult to evaluate the quality of public education available to American school children. There are as many accepted truths espoused on this subject as are viewpoints, all of which serve to highlight the lack of agreement regarding the importance of quality. Nevertheless, no one calls into question the fact that student failure to read, write, and compute is a key element which shapes and modifies public opinion concerning the relative value of education.

Increasing public concern over the worth of formal education is manifested through parental examination of the school's impact to develop the innate capacities of their children to a point which enables them to solve functional problems associated with everyday living in a complex society. The present level of public concern for the education of the nation's children is probably the best indicator of the overall inadequate quality of public education available in America today.

Many critical writers on the deteriorated condition of education in America such as Goodman (1962), Holt (1964, 1967), Kohl (1969), and Schwab (1972), report descriptions of ineffective educational practices to which children are subjected in the name of education. Inappropriate curriculum materials and instructional techniques are specifically cited as examples of these ineffective educational practices which impact upon the level of student achievement. These factors are considered to be partly responsible for causing a lack of motivation which is the
foundation of student failure and, in turn, strengthens negative attitudes toward learning and school.

Recent public information shows that current methods of teaching mathematics are by no means immune to the problem of student failure and schools are under constant parental pressure to return to teaching basic arithmetic computation skills. Awake (1975) reports findings of a study which clearly shows that only thirty percent (30%) of the seventeen-year olds sampled in the nation's schools could solve a simple multiplication problem involving decimals. In theory, these findings are connected with the inability of students to skillfully carry when adding and multiplying whole numbers in our base ten numeration system, a skill which is expected to be taught and mastered in the lower elementary grades.

A Health, Education and Welfare (HEW, 1975) report concludes that more than 23 million Americans cannot read, write, or compute at a minimum level. Based upon 1969 census figures, 23 million people would include all of the inhabitants of New York City, Chicago, Los Angeles, Detroit, Houston, Baltimore, Dallas, Washington, D.C., Cleveland, Indianapolis, Milwaukee, and San Francisco.

Educational Testing Services (ETS, 1976) is fully equipped to attest to a trend toward declining student achievement on the basis of lower achievement test scores obtained on standardized achievement tests. This revelation has sparked the development of minimum proficiency level programs by many state departments of education throughout the nation.

In 1976, the Florida legislature was one of the first to pass an
Educational Accountability Act which provides for the testing of not only high school juniors, but also, all third, fifth and eighth graders in the state in order to measure students' basic academic achievement.

The New York Times (1977), in an article entitled Tests in Florida Indicate Johnny Can't do Math, reports that "forty-two percent of the 19,000 juniors in Dade County (Miami) who took the standardized test in October failed the math portion. In some Miami schools, more than seventy percent (70%) of the junior class failed the mathematical portion of the statewide proficiency test."

Since this time, there has been a rash of suits filed in both state and federal courts by irate parents who are charging that public schools are failing to appropriately educate their children. In response, public school systems, in conjunction with state departments of education, are instituting statewide proficiency tests which will determine if a graduating student receives a certificate of attendance instead of a diploma.

The investigator recognized that the success point in education is the power to attend to things that are in themselves indifferent, by arousing a feeling of interest in the task to be accomplished. Improving mathematical achievement associated with the operations of carrying and borrowing is the focus of the present study. In order to successfully accomplish this purpose the study will develop and test selected curriculum materials for their effectiveness toward this end. In short, this study will systematically develop an experimental curriculum and instructional package to facilitate learning of the fundamental operations essential to carrying and borrowing correctly through the teach-
The twentieth century, frequently referred to as the "age of instance," has incredibly increased man's reliance upon instant cures which require little, if any, human toil. These cures are responsible for making most things easy while, at the same time, failing to teach process; i.e., the ability to accomplish from scratch. Consequently, this product or answer orientation has contributed to uncontrollable failure throughout the curriculum in schools across the nation.

Public dissatisfaction with the overall quality of education has generated the present controversy over the relative importance of teaching the process of arriving at solutions in American education. The teaching of process is an analytical method which increases the chances that each step is fully recognized and understood by the learner in terms of conceptual framework. More importantly, it is generally understood that traditionally, it has been the understanding of process that has permitted learners to move sequentially from simple to more complex learning tasks.

In response, a number of educators are emphasizing the need to highlight the teaching of process or "know how" to improve the level of student achievement. Rubin and Parker (1966) suggest that "the substance of our proposition is that process...is, in fact, the highest form of content and more appropriate base for curriculum change. It is through the teaching of process that we can best portray learning as a perpetual endeavor, and not something which terminates with the end of school."
Bruner (1971), in *The Process of Education Revisited*, confirms the vital need to teach process in American schools and suggests that any subject might be taught in some form. Through experience, he observed that "it was discovered again and again how difficult it was to get to the limit of children's competence when the teaching was good... No wonder then that we concluded that any subject could be taught in some honest form to any child at any stage of development. This did not necessarily mean that it could be taught in its final form, but it did mean that basically there was a courteous translation that could reduce ideas to a form that young students could grasp."

Useful beginnings have been initiated in the development of effective curriculum materials by many researchers, but more improvements and expansions are needed. Further accomplishment of this task requires that the schools accept a leadership role in the development of curriculum materials and instructional methods which are so easy and so well organized that all learners can succeed. At the same time, the materials must be sufficiently mature and realistic in order to gain the acceptance of students.

In mathematics, which is the subject of the present study, there is an outstanding need to test curriculum materials and instructional procedures that can teach carrying and borrowing, since both are essential skills needed to successfully master the process involved in developing an operational understanding of multiplication and division.

**Purpose of the Study**

The purpose of this study is to develop and evaluate curriculum
materials designed to teach specified objectives of carrying and borrowing in mathematics to selected elementary school pupils. The investigator: (1) determines selective objectives of carrying and borrowing in mathematics; (2) develops curriculum materials for accomplishing the identified objectives; and (3) evaluates the effectiveness of the materials through the use of pretest and posttest measures for helping selected elementary school students carry and borrow.

Definition of Terms

This section provides contextual definitions of important terminology that is essential to the investigation.

Objectives. Objectives, as used in this study, refers to instructional outcomes stated in terms of performance (Mager, 1971). Specifically, the type of instructional outcome this study is concerned with is the ability to carry and borrow if given a problem presented in a specified written form requiring the specified application of mathematical reasoning.

Curriculum Materials. Curriculum materials refers to a product including the ensuing four aspects: (1) outlined instructional objectives; (2) diagnostic tests to determine the extent to which students have reached the defined objectives; (3) instructional materials to achieve the objectives and a clear, but specified procedure for utilizing the materials; and (4) posttest to measure whether the students have achieved mastery of the stated objectives.
Carrying. Carrying in the decimal notation system is the transfer of sequential place value amounts to the next higher place so that the addends will produce the correct sum in addition and multiplication.

Borrowing. Borrowing in the decimal notation system is the transfer of one higher place value amount to a lower place value amount so that the subtrahend can be extracted from the minuend to arrive at the difference.

Significance of the Study

While Sputnik I (1957) sparked citizen concern and national interest regarding the methods applied in teaching mathematics in public and private schools, each concern was relatively shortlived. These concerns were convincingly assuaged with the thrust for the need for more knowledge which saw its way into our school systems through the application of teaching students "modern math." Modern math caught students and educators ill-prepared, forcing teachers to enroll in special courses involving methods of teaching this newly-developed subject matter. Several problematic results have been the outcome of the infusion of modern math into our school systems: (1) confusion as far as students are concerned; (2) embarrassment for parents based upon a clear lack of understanding of modern math; and (3) a demonstration of reasonable doubt in the minds of many educators as to the instructional effectiveness of modern math in the absence of an adequate knowledge of basic math.

This study is significant because it endeavors to systematically develop alternative curriculum materials designed to teach basic computation skills associated with carrying and borrowing. These fundamen-
tal skills are crucial for the computation of all mathematical problems whose sums are ten and greater. Moreover, these skills are absolutely necessary for solving multiplication and division problems successfully.

Secondly, this study is important because it attempts to provide teachers with valuable information for determining the appropriateness of the objectives for teaching carrying and borrowing to elementary school pupils.

Thirdly, with the preeminent adoption and use of the metric system in the United States, this study is significant as an instructional approach because the basic skills for applying these operational concepts are also required for solving similar problems in metric computations. The materials complement the instruction of that aspect of math which deals with carrying and borrowing in the decimal system of notations as well as the metric system of measures.

Finally, the study is significant because it may suggest ways in which teacher education programs may want to prepare future teachers in methods for teaching carrying and borrowing to below-average ability students.

Design of the Study

Below is the outline of procedures which will be followed to accomplish the purpose of the study. The outline is focused directly on: (1) a review of literature to determine the objectives for carrying and borrowing in addition and subtraction and the application of the Program Evaluation Review Technique (PERT) to determine the actual skills used; (2) what steps will be involved in developing the curriculum materials
for accomplishment of the objectives identified; and (3) how the effectiveness of the materials will be evaluated.

The process for isolating and identifying the essential steps involved in reaching the terminal objectives of carrying and borrowing is to be determined through a critical review of the literature which is directly concerned with teaching these vital skills. To understand the steps involved in the solution process, objectives yielded by the literature review will be compared to the steps uncovered by application of PERT. Both methods are important in communicating the critical activities needed to achieve the objectives of carrying and borrowing with elementary school students.

Part two of the study involves the actual development of curriculum materials which are designed to teach the identified objectives of carrying and borrowing. The procedure followed by the investigator is a modified version of that used by London (1975) to develop curriculum materials and it is diagrammed in Illustration 1. (See Illustration 1). The identification of activities necessary to reach the terminal objectives of carrying and borrowing is the first and most important step. Step two is the development of a draft of the curriculum materials which are engineered to achieve the identified objectives.

The format used to develop the curriculum materials is a variation of that used by Lowerre, Scandura (1972) and London (1975). That procedure is separated into four aspects: (1) the materials include a pretest for the specified objectives involved; (2) initial instruction is based directly on the student's performance on the pretest; (3) each student studies the process of carrying and borrowing in solving more
Illustration I

Procedure for Developing the Curriculum Materials

Start

Identify Terminal Objectives and Activities

Application of Program Evaluation Review Technique to the Process Involved in Carrying and Borrowing

Development of Curriculum Materials Designed to Teach Carrying and Borrowing

Field Test Materials

Are Materials Adequately Effective?

Yes

Curriculum Materials Revised for Final Evaluation

Finish
difficult problems only after mastery of simple examples; and (4) the materials include a posttest to measure whether the specified objectives have been adequately met. Step three of the design procedure is to field test the materials. The final activity of the testing includes questioning by the investigator of the students after completion of the materials to ascertain difficulties encountered by the students. Step four of the procedure determines whether the materials were effective. The materials are considered ready for final revision and evaluation if a majority of ten students achieve the stated objectives. If the materials do not meet that criteria, they are returned to the second step for re-evaluation and refinement.

Part three of the study evaluates the effectiveness of the curriculum materials in achieving the stated objectives. Selected students with undemonstrated skills are instructed with the curriculum materials and then are tested to ascertain whether the objectives have been mastered. The materials are to be judged effective if a majority of the selected students achieved the objectives.

The other aspects of the design in need of further clarification are addressed below: (1) sampling and arrangements—how the sample of students are selected and under what conditions and arrangements are the curriculum materials to be administered; (2) instrumentation—what test is used to evaluate the effectiveness of the materials; (3) limitations of the study—listing of the specific limitations of the study; and (4) rationale—reasoning for the use of decision-oriented design. These four remaining aspects of the design of the present study are addressed below.
Sampling and Arrangements. The selected sample to be studied will be relatively small and taken from one school because of the economic constraints placed on the investigator. To counterbalance this factor, the investigator will apply the criteria used by London (1975) to maximize the possible generalizability of the study to other samples. That is to (1) investigate the willingness of public schools within Providence, Rhode Island, to participate in the study, and (2) select the school (from the set of schools which indicate a willingness to participate in the study) which would result in a sample which would seem to maximize the possibility of generalizability of the study.

The school will be selected not only on the basis of demographic considerations, but also on the basis of instructional arrangements followed in the school. A minimum of thirty students are selected on the basis of the pretest which will determine those who did not demonstrate skills associated with carrying and borrowing in solving addition and subtraction examples.

The curriculum materials are introduced into the student's curriculum under the following conditions: the students are required to receive self-instruction, the treatment, for a period of four weeks (two for carrying and two for borrowing), in a separate room with the investigator overseeing the process and answering operational questions. More specifically, the instructional program will occur daily for twenty regular school days for fifty minutes per session.

Instrumentation. The investigator will use the computation sub-test of the Stanford Diagnostic Mathematics Test as both pre and post measures to ascertain whether the specified objectives have been mastered by the
students involved in the study. This particular instrument is considered appropriate since it meets statistical measures to insure validity and reliability.

In the pretest situation, this instrument will be used in determining which students are selected for treatment as well as ascertaining the percentage of the student population that has mastered the skills associated with carrying and borrowing. The scores for the experimental group will then serve as the pretest to measure for comparison against the posttest scores after treatment. In this regard, the instrument is the most important measure for evaluating the effectiveness of the materials associated with the objective of carrying and borrowing.

Limitations of the Study. This study had certain limitations:

1. The curriculum materials were tested with a small sample. Consequently, the generalizability of this study to other populations is limited.

2. The students' retention of the objectives over a substantial period of time, and their ability to transfer their learning to other situations was not tested.

3. The effectiveness of the curriculum materials was not compared to other approaches for achieving the objectives of carrying and borrowing.

4. There were no controls to separate out the influence of the teacher.

5. Time spent conducting the study was limited.

Rationale. This particular design is appropriate because the present study is decision-oriented rather than conclusion-oriented. It is im-
important to understand that a decision-oriented study is a research effort conducted to accomplish a specific and feasible goal. In this study, the specific and feasible goal is the development of curriculum materials designed to teach specified objectives of carrying and borrowing in mathematics to selected elementary school pupils. Cronbach and Suppes (1969) state that "The excellence of the product being developed...should be the ruling concern of the decision-oriented inquiry...Rigor in (decision-oriented) research is likely to express itself differently than the rigor of conclusion-oriented research (p.170)."

Had the evaluation portion of this study been conclusion-oriented, the present study would be endeavoring to show a significant difference on some measure(s) in favor of the treatment group over the other students (control group) using a traditional approach. Had that been the emphasis, a rigorous statistical design would most definitely be required. Notwithstanding, the present study is decision-oriented and the prime focus of the evaluation is to manifest that the developed curriculum materials (when administered under the exact same conditions explained above) are effective in accomplishing the identified objectives.

To recapitulate, the present study is primarily occupied with the development of effective curriculum materials and the largest portion of time will be used to select appropriate objectives, develop the materials and field test the curriculum developed.

The subsequent chapters are the result of conducting the study as it is outlined in Chapter I. Chapter II deals with determining the appropriateness of selected objectives of carrying and borrowing for second grade pupils; Chapter III discusses the development of the cur-
riculum materials and the instrumentation used; and Chapter IV is a description of the evaluation of the curriculum materials. Ultimately, Chapter V reports the findings and the implication for additional research on developing and evaluating curriculum materials to teach carrying and borrowing to elementary school pupils.
The purpose of this chapter is to provide the theoretical base upon which the study rests and discuss the importance of the relationship between curriculum and instruction and the developmental stages children encounter as they progress from one conceptual level of mathematics understanding to the next. It will discuss the contemporary views of mathematics teachers' and researchers' on the present status of the effectiveness of mathematics curriculum and their perceptions of what needs to be done to improve student achievement in mathematics. In addition, the chapter will review the literature to identify those skills necessary to reach the terminal objectives of carrying and borrowing in addition and subtraction, as a first step toward the development of curriculum materials to improve students' performance in basic mathematics.

In particular, it is the intent to let the literature identify those quantitative skills associated with carrying and borrowing. While a qualitative measure of those skills actually used to solve problems requiring an operational understanding of carrying and borrowing are assessed through the application of the Program Evaluation Review Technique (PERT). As a planning tool, PERT is used in conjunction with the literature review to assist in the development of curriculum materials which are appropriate as treatment for helping students to successfully develop an understanding of how to apply the skills required to carry
and borrow.

The present study is designed to systematically develop and test alternative curriculum materials for improving students' mathematical skills in the performance of the essential operations of carrying and borrowing in addition and subtraction. The findings generated through the literature search and the application of the Program Evaluation Review Technique (PERT) are used as the basis upon which to select the specific objectives to be accomplished by the students involved in the study. The objectives suggest that the curriculum materials should allow for direct participation and practice by students at both the individual and group level. In order to place emphasis on direct and active student participation, the curriculum materials acquired a semi-concrete abstract design. Chapter III provides a full and more detailed description of the curriculum materials and the process involved in the final development.

Theoretical Frame of Reference

The purpose of this section is to provide a discussion of the theoretical framework espoused by Piaget and Bruner, in addition to other supporting researchers. Together, these theoretical and practical frameworks are used to form the basic rationale which undergirds the thinking and the development of the present curriculum materials. The rationale behind the study is that any subject can be taught in some honest form, and that by moving from the concrete to the abstract level of thought in mathematics students can improve their general mathematical abilities, and specifically, those skills required for success in carrying and borrowing in addition and subtraction. Discussed herein
is an area of theoretical research which the investigator applied to the
development of specific curriculum materials to measure the importance
of carrying and borrowing in understanding and accurately solving simple
addition and subtraction algorithms. The focus is one of assessing the
role that a genuine understanding of carrying and borrowing plays in
successfully solving addition and subtraction problems.

It is generally accepted that the skills of carrying and borrowing
are subsumed and addressed under that phase of mathematics instruction
that deals with teaching the operations of addition and subtraction.
While mathematical literature has devoted little attention to examining
the role of carrying and borrowing as independent operations, there is
sufficient achievement test data that indicates these skills are not
sufficiently developed under the broad concepts of addition and subtrac-
tion either.

The existence of learning deficiencies in basic mathematics is evi-
denced through the general inability of students to make a smooth transi-
tion in moving from counting to successful computation in addition and
subtraction. This fact is supported nationally by the poor performance
students manifest on standardized mathematics achievement tests. It is
highly suspected that the inability of many students to correctly apply
the concepts of carrying and borrowing has had the rippling effect of
preventing students from mastering the operations of not only addition
and subtraction, but also the more difficult operations of multiplication
and division. In theory, a conceptual understanding of carrying
and borrowing, then, is an essential prerequisite to mastering the fun-
damental concepts of basic mathematics which allow a student to progress
on the mathematics continuum based on genuine ability to apply the operations of math rather than memorization of mathematical facts or tables.

Attention is further drawn to that information needed to clearly understand the connection between curriculum and instruction as it relates to the developmental stages children experience during the learning cycle, when thought patterns are provoked, stimulated and developed.

Since 1960, the contributions of Bruner and Piaget in the area of cognitive development research have continued to play a major role toward improving curriculum planning and development. These contributions are largely responsible for changing the way educators view the child in relationship to the learning process which the child is expected to master. Theoretically, both Bruner and Piaget consistently encourage innovative thinking in the construction of curriculum materials as a practical way to meet students at their levels of cognitive development and move them forward. Research data shows that where a child begins in the learning process is probably the most decisive element in determining how much a child will learn in relationship to mastery of required skills.

Today, Piaget is commonly referred to in most discussions associated with cognitive development and Piaget's research is well respected and utilized by a wide range of professionals who are pursuing an active interest in cognitive development and its relationship to both concrete and abstract learning processes. In fact, Piaget's research findings dominate much of the present thinking about how children learn and the
type of behaviors that are consistent with various stages of cognitive development, as exhibited by children under close observation.

Piaget's research is the foundation for recent psychiatric theories for the treatment of cognitive defects in the development of mental illness. Serban (1977) points out that a major factor in the new theories has been an application of the pioneering research of Piaget on the development of thinking patterns in children. "Freud got it the wrong way around, some psychiatrists are now asserting. Emotional disorders do not cause people to think in the bizarre, illogical patterns typical of mental illness. Rather, it is illogical or otherwise defective thinking patterns that cause emotional or behavioral problems."

These findings provide an entirely new aspect to the momentous subject of learning by suggesting that illogical or otherwise defective thinking patterns are the prime cause of individual failure, and, thereby, lead to a range of emotional disorders which affect one's ability to learn.

It would seem, therefore, to be perfectly evident that the more individualized and direct the efforts of education become, the closer education will approach its true goal of developing individual skills and knowledge. Thus, a program to correct illogical thinking patterns is certainly a task which would, by contributing to literacy, benefit both the student and society.

The nation's school systems should be called upon to upgrade the basic quality of education available in America today. Ideally, the schools can work to correct illogical thinking patterns by helping
students, via more direct instruction, bridge the existing learning gaps encountered in moving from concrete to abstract thinking, by developing curriculum materials geared to developing mastery of specific objectives.

At the elementary level, special attention must be devoted to the development of curriculum materials that can accommodate private individual needs as well as interactive group needs. It is important to note that there is a need to develop models to bridge the gap between concrete and abstract understanding. Children at the lower elementary school age cannot handle problems presented on a purely verbal or abstract level. At this age, it is argued, children's thinking depends rather on concrete perceptible data and generally involves an internal manipulation of the data. Consistent with this reasoning, Piaget (1951) termed the thinking of children at this stage "concrete operational thought." The stage is called "concrete operational" since the necessary logical thought is based in part on the physical manipulation of objects.

Gorman (1972), in support of Piaget's research, points out "that through inductive thinking, we start with concrete objectives, or specific instances, and derive a generalization from them. The elementary school child can grasp principles and relationships if his reasoning is mainly inductive."

Concurrently, Kline (1945) points out that "psychologically the teaching of abstractions first is all wrong. Indeed, a thorough understanding of the concrete must precede the abstract. Abstract concepts are meaningless unless one has many and diverse concrete interpretations well in mind. Premature abstractions fall on deaf ears."

Copeland (1974) indicates that children may be introduced to addi-
tion at the concrete level by joining sets of objects and noting the resulting number. This "joining" of sets is still another mathematical operation called union of sets. Copeland believes that teachers should be familiar with the type of laboratory materials children need at the concrete operational level in order to learn mathematical concepts.

Engler (1961) indicates that discovery is a key element in extending mathematical understanding and application; noting that fundamental conceptual and process development must move from concrete experience to the use of semi-concrete or representative materials to thinking through to written computation. Engler believes these steps should be systematically applied to every idea and fact that must be learned in arithmetic. May (1974) also makes the point that discovery is a key element in extending mathematical understanding and application; further suggesting that this understanding must be taught by means of gradual development of concepts, beginning at the most concrete level and eventually moving to the abstract. Repeatedly, Piaget outlines, through qualitative research, any number of concepts about time, space, measurement, mathematics and so forth, which measure the level of thought and identify the cognitive thinking patterns children apply to the solution of specific problems. Throughout, Piaget's work is associated with an examination of the internal thought processes developed and applied by children at different stages of their cognitive development.

In Toward a Theory of Instruction, Bruner (1966) states, "Unquestionably, the most impressive figure in the field of cognitive development today is Jean Piaget." Here, Bruner is alluding to Piaget's brilliant formal description of the nature of knowledge which children ex-
hibit at each stage of cognitive development. This research is, by and large, one of the most important contributions to our understanding of the relationship between curriculum development and intellectual growth.

The distinction to be made between Piaget and Bruner in terms of the knowledge base which supports their respective positions is epistemological versus psychological. In short, Piaget concentrates on the nature of knowledge per se; that is, knowledge as it exists at different points in the development of the child. Bruner, on the other hand, is considerably more interested in the processes that make cognitive growth possible.

Bruner's most provocative espousal is that there is an appropriate version of any skill or knowledge that may be imparted at whatever age one wishes to begin teaching—however preparatory the version may be. This posture, if taken seriously, is of far reaching importance in developing curriculum materials that are effective in reducing both concrete and abstract ideas to a level of comprehension attainable by any learner.

In response to Bruner's position, Jennings (1967) indicates that "its probably possible to accelerate learning but maximum acceleration is not desirable. There seems to be an optimum time. What this optimum time is will surely depend on each individual and on the subject matter."

In the foreward to Young Children's Thinking, Piaget states: "In the area of logico-mathematical structures, children have real understanding only of that which they invent themselves, and each time we try to teach them something too quickly, we keep them from reinventing
it themselves. Thus, there is no good reason to try to accelerate this development too much; the time which seems to be wasted in personal investigation is really gained in the construction of methods."

Bruner believes that research of the last decade makes it clear that the idea of readiness is a mischievous half-truth. This position is supported on the basis of research that shows one teaches readiness or provides opportunity for its nature, one does not simply wait for it to occur. It remains clear, however, that improvement in education can come about only through a general willingness on the part of educators to alter the familiar patterns of instruction and through attitudinal change on the part of teachers toward the development and use of innovative teaching models.

Bruner (1966) concludes "that new models are formed in increasingly powerful representational systems. It is this that leads me to think that the heart of the educational process consists of providing aids and dialogues for translating experiences into more powerful systems of notation and ordering. And it is for this reason that I think a theory of development must be linked both to a theory of knowledge and to a theory of instruction or be deemed to triviality."

In outlining future direction, Bruner acknowledges that the task of educators is to recognize that discovering how to make something comprehensible to the young is only a continuation of making something comprehensible to ourselves in the first place. This simply means that understanding and aiding others to understand are the opposite sides of the same coin.

Crescimbeni (1965) recommends games and activities initially for
making arithmetic a successful, pleasurable experience. Stressing further, that regardless of the types or kinds of materials that are developed for basic curriculum instruction, enrichment aids (games, etc.) are necessary as supplemental and motivational learning devices for children. In essence, Crescimbeni urges teachers to recognize that utilization of sensory aids assists the child to visualize the relationships involved in a particular problem situation, and that acts as an incentive to learning by appealing to as many of his senses as possible.

Copeland (1974) indicates that the teacher should provide a learning situation that will provoke the desired learning by the child when he is ready. This will involve concrete materials and a proper questioning technique in order that the child may disengage for himself the mathematical structure involved as he handles or manipulates objects.

Rosenbloom (1967) concludes that "the implications of Piaget's theories for mathematics education have not yet been realized. Studies by competent researchers involving American children are badly needed. New curricular materials, based on sound psychological evidence should be written. And, in teacher education, more work involving Piaget's theories and their implications would serve as landmarks in improving instruction in the elementary school."

Contemporary Views of Educators. The next logical question is what are contemporary mathematics teachers' and researchers' views on the present status of the effectiveness of mathematics curriculums and their percep-
tions of what needs to be done to improve student achievement in mathematics? In a new report from the National Council of Teachers of Mathematics (NCTM, 1977) and the Mathematical Association of America, which is based on a year-long study of high school math departments, school districts are urged to make special provisions to assist students when math deficiencies are first noticed. It also recommends regular homework assignments using problems that reinforce manipulative skills and using calculators to make computations less tedious with active parental support to oversee that the assignments are completed and turned in.

This particular study further concludes that mathematics instruction for students should be improved because of the detrimental effect on the individual student and on the entire class when students are advanced without appropriate achievement. In effect, mathematics teachers are calling for an end to social promotions and a return to assignment of grades which reflect student achievement in math.

A new report from Research for Better Schools (1977) says an analysis of the textbooks used in most math classrooms would not reflect the content that is being taught because teachers often find themselves devising their own supplementary materials, usually due to budget restraints. And "evaluation for placement and assessment of pupil progress also seems to be dominated by teacher-made measures and teachers' informal perceptions," reports Education USA, December 5, 1977.

Lazarus (1977) concludes that "the vast majority of the American population is still quantitatively illiterate, coping badly with the public issues that are quantitative at root: the energy problems, inflation, unemployment, the arms race, and many more. Most adults express
their dislike for mathematics openly and freely; most avoid mathematics whenever they can. Countless college students give up promising careers simply to avoid mathematical requirements. And each generation of school children treads in their parent's footsteps, deciding by the time they reach high school that mathematics is just for the brainy, and not worth the trouble for anyone else."

One of the most recent curriculum innovations in math education is Chisanbop math. Chisanbop math is a Korean method of counting by fingers introduced in this country for the first time in 1978. The developer reasoned that fingers could be developed as a natural calculator by giving each one a numerical property in line with the system of tens. Thus, in Chisanbop, the fingers of the right hand each have a property of one (with the thumb a cumulative five), while the left hand carries the tens (with the thumb a cumulative 50). When worked together, the two hands can handle all basic mathematical calculations—adding, subtracting, multiplying, dividing—up to 99.

The charge has been made that the Chisanbop program could inhibit the development of calculating skills. However, Lachterman (1978) indicates that there is a point at which the mind definitely takes over and draws a parallel with touch typing or piano playing. And, in the most recent set of controlled tests conducted by Mount Vernon schools, Lachterman says that "the accuracy of the Chisanbop group was outstanding."

The ninth Annual Gallup Poll (1977) of the public attitude toward public schools, conducted by the Gallup Poll and The Charles F. Kettering Foundation, has examined the Back-to-Basics movement. In response
to the question, do you favor or oppose this Back-To-Basics movement, all groups in the population expressed overwhelming approval of the movement. Table I gives the results based on the national totals of forty-one percent (41%) who were familiar with the term. (See Table I). It shows that at the national level, eighty-three percent (83%) of those familiar were in favor of a return to the basics.

Wootan (1965) points out that the problem of finding the correct level of abstraction appropriate to the cognitive readiness of the student is a very real one, and the definitive answer, if such exists, has not yet been found. This study endeavors to test an alternative curriculum model which is designed to ensure that students come to understand the meaning behind the process required for carrying and borrowing in addition and subtraction.

**Determining the Specified Objectives of Carrying and Borrowing in Math**

The purpose of this section is to provide a brief historical perspective on carrying and borrowing and to provide a description of the mathematical rules and the steps identified through the literature review for carrying and borrowing in computing simple addition and subtraction of whole numbers. Subsequently, the Program Evaluation Review Technique (PERT), commonly referred to as backchaining, is also applied as a qualitative measure to determine the actual process used in the solution of specified problems in basic addition and subtraction requiring an operational knowledge of carrying and borrowing. The Program Evaluation Review Technique (PERT) is expected to be instrumental in the de-
### Table 1

**Back-to-Basics Poll**

<table>
<thead>
<tr>
<th></th>
<th>Percentage In Favor</th>
<th>Percentage In Opposition</th>
<th>Percentage Did Not Know-No Answer</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>NATIONAL TOTALS</strong></td>
<td>83</td>
<td>11</td>
<td>6</td>
</tr>
<tr>
<td><strong>SEX</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Men</td>
<td>83</td>
<td>10</td>
<td>7</td>
</tr>
<tr>
<td>Women</td>
<td>83</td>
<td>11</td>
<td>6</td>
</tr>
<tr>
<td><strong>RACE</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>White</td>
<td>84</td>
<td>10</td>
<td>6</td>
</tr>
<tr>
<td>Non-White</td>
<td>75</td>
<td>20</td>
<td>5</td>
</tr>
<tr>
<td><strong>AGE</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>18-29 years</td>
<td>79</td>
<td>14</td>
<td>7</td>
</tr>
<tr>
<td>30-49 years</td>
<td>82</td>
<td>12</td>
<td>6</td>
</tr>
<tr>
<td>50 years &amp; over</td>
<td>87</td>
<td>7</td>
<td>6</td>
</tr>
<tr>
<td><strong>COMMUNITY SIZE</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1 million &amp; over</td>
<td>78</td>
<td>12</td>
<td>10</td>
</tr>
<tr>
<td>500,000 - 999,999</td>
<td>77</td>
<td>13</td>
<td>10</td>
</tr>
<tr>
<td>50,000 - 499,999</td>
<td>85</td>
<td>12</td>
<td>3</td>
</tr>
<tr>
<td>2,500 - 49,999</td>
<td>88</td>
<td>5</td>
<td>7</td>
</tr>
<tr>
<td>Under 2,500</td>
<td>85</td>
<td>9</td>
<td>6</td>
</tr>
<tr>
<td><strong>EDUCATION</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Grade School</td>
<td>93</td>
<td>6</td>
<td>1</td>
</tr>
<tr>
<td>High School</td>
<td>84</td>
<td>9</td>
<td>7</td>
</tr>
<tr>
<td>College</td>
<td>81</td>
<td>13</td>
<td>6</td>
</tr>
<tr>
<td><strong>REGION</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>East</td>
<td>77</td>
<td>15</td>
<td>8</td>
</tr>
<tr>
<td>Mid-west</td>
<td>89</td>
<td>6</td>
<td>5</td>
</tr>
<tr>
<td>South</td>
<td>85</td>
<td>11</td>
<td>4</td>
</tr>
<tr>
<td>West</td>
<td>81</td>
<td>11</td>
<td>8</td>
</tr>
</tbody>
</table>
velopment of the curriculum materials which are, in turn, expected to be effective in teaching the concepts of carrying and borrowing to selected elementary school pupils. The common characteristic shared by the selected students is their inability to perform these tasks consistently and accurately prior to application of the developed treatment. The historical derivation of "carry" is taken from the sixteenth century phrases "keepe in minde" and "keeping reposed in memorie." Sanford (1930) determined that "the origin of math was probably utilitarian, but from a very early period men seem to also have been interested in the abstract relationships between numbers." In early Greece, the practical computation of the merchant was called logistic, while the theoretical work of the scholar was called arithmetic. In the countries of the ancient world and in many parts of the modern world as well, computation with numerals was paralleled by work with the abacus. Another device widely used was the counting board. As far as can be determined, both of these devices have served to aid in the teaching of addition and subtraction by providing visual presentation and physical manipulation of the factors involved in the process of carrying and borrowing.

Carrying and Borrowing. When you went to school, you were probably taught to "carry" and to "borrow." Accordingly, in basic arithmetic, most people were simply taught to carry whatever it was, or to borrow whatever was needed. Many educators believe this approach is somewhat valid, but they contend that the terms carry and borrow do not really tell what happened. In other words, they suggest that what happens when one carries and borrows can more appropriately be described as group-
ing, regrouping or renaming. For example, if you put eight pennies on the table and they are counted and the amount confirmed, you can call them eight pennies. However, if you add two more pennies, you now have ten pennies and you can exchange them for a dime. In other words, you have grouped your ten pennies into one dime (or one ten). If you were to write this, you also rename ten ones as one ten.

Likewise, what is referred to as borrowing is often referred to as renaming. While renaming is said to be more a precise term which provides insights into what is really happening, as long as the child understands what he is really doing, in such computations, the terminology does not really matter. It is the child's insight into our system of numeration, however, that is the key to his understanding of this process.

While the movement to modern math encompassed a change in terminology associated with basic math, it also effectively served to remove many parents as partners from the process of teaching basic mathematics to their children and providing assistance in the preparation of homework assignments. This study accepts the merits of the more precise terminology which describes the operations of carrying and borrowing, yet, the investigator believes that returning to using the actual terms carrying and borrowing has the potential to again make parents partners in the teaching of mathematics.

Developing a conceptual understanding of borrowing is an essential skill not only for subtraction, but also for the mastery of division. This study views borrowing as the most important skill in subtraction, while all other related skills form part of its subset.
While some educators are using the process of regrouping as a technique to teach the skill of borrowing, most Americans continue to rely on the traditional method described below:

Example: Subtract 15 from 53

Step I  
53
-15
Write the problem. Starting on the right, note that 5 cannot be subtracted from 3, because 5 is larger than 3.

Step II  
53
-15
So borrow 1 ten from 5, leaving 4 tens. By placing the 1 in front of the 3, you are adding the 10 to the 3. The 3 is now 13.

Step III  
53
-15
Now subtract 5 from 13 = 8 and 1 from 4 is 3.

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Collectively, there are a number of mathematical skills associated with performing the operation of carrying in addition. These skills range from simple number identification and place value to the more complex skill of carrying equivalent amounts to the next higher place. This study identifies carrying as the most important skill in addition and relegates number identification, place value and other related skills as subsets to the actual skill of carrying. Further, mastery of the process of carrying is essential for an understanding of successive addition, commonly referred to as multiplication.

When adding, it is frequently necessary to "carry" amounts from the lower place value to a higher place in order to arrive at the correct sum. The following example reviews the current method of carrying in addition:

Example: Add 16 and 37

Step I  
16
37
Write the problem, starting on the right, add the ones, 6 plus 7, and get 13.
Step II

16
37
\[ \frac{53}{16} \]

Step III

Add \(1 + 1 + 3 = 5\)

Graham (1958) provides about the clearest explanation of the conceptual understanding required for carrying and borrowing in addition and subtraction through the use of three simple rules for each operation. Below appears Graham's definition of each operation and the rules of application from which the necessary skills are induced.

**Rule A**

Write the numbers to be added so that like orders of units stand in the same column.

**Rule B**

Commencing with the lowest order, or at the right hand, add each column separately, and if the sum can be expressed by one figure, write it under the column added.

**Rule C**

If the sum of any column contains more than one figure, write the unit figure under the column added, and add the remaining figure to the next column.

From Graham's description of the general rules for addition, the following three skills are indicated:

**Skill**

The ability to write and identify numbers in the order of their place value.

**Skill**

The ability to add different combinations of numbers correctly.

**Skill**

The ability to carry amounts to the next highest column in order of place value.

Graham (1958) defines subtraction as the process of taking one number called the subtrahend from another number called the minuend. The results thus obtained, or "difference" between the two numbers, is called the remainder; thus:

\[
\begin{array}{c}
10 \text{ minuend} \\
-7 \text{ subtrahend} \\
\hline
3 \text{ remainder}
\end{array}
\]
Rule A  Write down the sums so that the units stand under the units, the tens under the tens, etc.

Rule B  Begin with the units, and take under from the upper figure and put the remainder beneath the line.

Rule C  But, if the lower figure is the larger, add ten to the upper figure, and then subtract and put the remainder down. This borrowed ten must be deducted from the next column of figures where it is represented by 1.

From Graham's description of the general rules for subtraction, the following skills are indicated:

Skill - The ability to write and identify numbers in the order of their place value and to differentiate between the minuend and the subtrahend.

Skill - The ability to subtract different combinations of numbers correctly.

Skill - The ability to borrow amounts from the next highest column or place value position.

Today, mathematics is referred to as the universal language of technology and, consequently, mathematics is increasingly a prerequisite for an indepth understanding of such scientific issues. At a more functional level though, mathematics is, indeed, also necessary for the average citizen to participate in an enlightened technological society at an informed and practical day to day level.

Application of the Program Evaluation Review Technique. To further, and more specifically, review the skills required to successfully carry and borrow when adding and subtracting, the Program Evaluation Review Technique (PERT) is engaged to assist in identifying these skills applied and in the development of the curriculum materials for this experiment. Through the process of backchaining, PERT will specifically aid this
investigation in the following ways:

- defining the skills necessary for carrying and borrowing in specific terms;
- sequencing these skills within the general areas of addition and subtraction;
- communicating in a precise, visual manner the necessary skills to achieve the goals of carrying and borrowing in addition and subtraction.

Backchaining, the key element in the PERT technique, is a process used to specifically identify each step in the process in its proper order of occurrence toward the completion of a specific objective or goal. Remembering backwards is difficult, and our minds generally tend to think slowly and carefully. When thinking backwards, one does not have the tendency to forget that there are in-between steps, usually remembering every step in its proper reverse order.

In PERT, each of these steps are referred to as activity and every project consists of a chain of activities. Projects may be very complicated, with many activities required, or relatively simple, requiring only a few activities. Described below is the chain of activities associated with carrying and borrowing as uncovered by the application of PERT or the backchaining technique to addition and subtraction problems requiring carrying and borrowing to arrive at the correct answer. (See Appendix 1).

### Activities Associated with Carrying in Addition

<table>
<thead>
<tr>
<th>Project Completed</th>
</tr>
</thead>
<tbody>
<tr>
<td>(9-10) answer completed</td>
</tr>
<tr>
<td>(8-9) add numbers in 100's column</td>
</tr>
<tr>
<td>(7-8) carry 100's from ten's to 100's</td>
</tr>
<tr>
<td>(6-7) record the answer in ten's column</td>
</tr>
<tr>
<td>(5-6) add numbers in ten's column</td>
</tr>
<tr>
<td>(4-5) carry ten's from units to ten's column</td>
</tr>
</tbody>
</table>
(3-4) record units under units column
(2-3) add the number of units
(1-2) problem written

Project Started

Activities Associated with Borrowing in Subtraction

Project Completed

(9-10) answer completed
(8-9) subtract numbers in 100's column
(7-8) borrow if necessary
(6-7) record in answer in ten's column
(5-6) subtract numbers in ten's column
(4-5) borrow if necessary
(3-4) record units under units column
(2-3) subtract the number of units
(1-2) problem written

Project Started

Objectives of the Study. With the current trend toward behavioral objectives and accountability, there has been increased emphasis upon task analysis in the educational setting. The two operations serve to complement each other in that task analysis identifies the components of performance which may then be specified in observable terms. The teacher's responsibilities increase from merely providing information to also arranging the educational environment in support of measurable performance (Mager, 1962).

The Program Evaluation Review Technique (PERT) was selected based on its potential capacity to make learning of specific skills more direct by identifying the essential steps. As a planning tool, PERT proved to be an effective way to uncover the steps students actually use in the solution of specific problems involving carrying and borrowing. It is an important tool for assisting educators with that aspect of
curriculum development which involves task selection.

Task selection is a well known process of deciding which objectives or tasks are important for students to study and master. Selection of educational tasks determines the scope of what is learned by participating students. It is for this reason that the tasks or activities should be not only sequential, but that the tasks also be measurable in order to provide for academic accountability.

Essentially, this process allows one to adhere to Bruner's position by putting evaluation into use as we identify frequently used steps in the solution process. This procedure overcomes the drawbacks of the traditional teaching-learning curriculum process and its inability to clearly identify the sequential skills necessary to reach specified educational objectives.

A number of specified objectives required for carrying and borrowing will be the primary focus of those curriculum materials designed to accomplish the overall objectives of the study. It is important to note, however, that the skills uncovered by the PERT process are supportive of the set of skills induced from Graham's three rules for carrying and borrowing. The following list contains the specified performance objectives required for individual success at each step along the learning sequence in addition and subtraction to be accomplished by students involved in the study.

Specified Objectives

1. Given an addition problem requiring carrying, the child will:
   a. Display and/or recognize the problem in its correct order of place value.
b. Count the total, carrying when needed, and place the correct sum below the equation line.

2. Given a subtraction problem requiring borrowing, the child will:
   a. Display and/or recognize the problem in its correct order of place value.
   b. Subtract the minuend from the subtrahend, borrowing if needed, and place the answer below the equation line.

In summary, the reviewed research tends to support the following conclusions:

1. That in order for students to improve their general mathematical abilities curriculum materials should move them from the concrete to the abstract mathematical level of thought.

2. That abstract concepts are meaningless unless one has many and diverse concrete interpretations well in mind.

3. That anything can be taught in some form to anybody as this directly relates to the development of curriculum materials to teach specified objectives.

4. That educators are recommending regular homework assignments as a way to reinforce manipulative skills.

5. That "a theory of development must be linked to a theory of knowledge and to a theory of instruction or be deemed to triviality."

6. That the Program Evaluation Review Technique (PERT) is important as a curriculum planning tool for uncovering specific skills.
CHAPTER III
THE DEVELOPMENT AND FIELD TESTING OF
THE CURRICULUM MATERIALS

One important purpose of this study was to clearly identify those skills necessary for performing the operations of carrying and borrowing in addition and subtraction. A second purpose was to select those objectives to be accomplished by the study. Both the skills and the specific objectives were identified and reported in Chapter II.

Chapter III is now concerned with the development and field testing of the curriculum materials designed to achieve the selected objectives. The following aspects of that process are discussed: (1) selection of the objectives; (2) identification of an instructional strategy for achieving the objectives; and (3) development and field testing of the curriculum materials.

Selection of the Objectives

The review of the literature suggested the need for the development of instructional materials to achieve a significant portion of the objectives specific to this investigation. The mastery of the concepts of carrying and borrowing in simple form by elementary grade students was considered to be the most appropriate objective for this investigation.

The review of the literature suggested the following reasons for that choice: (1) generally, students at the elementary grade level are experienced with concrete learning and ready to assimilate more
abstract thought; (2) a growing number of students are unable to successfully compute simple math problems in addition and subtraction which necessarily involve carrying and borrowing; (3) there exist no proven effective curriculum materials to achieve the objective and provide that crucial bridge between concrete and abstract understanding; and (4) mastery of the operations of carrying and borrowing in simple addition and subtraction is a prerequisite for understanding and solving multiplication and division problems.

In sum, the investigator inferred from the literature that the development of effective curriculum materials to achieve mastery of carrying and borrowing in addition and subtraction represents an important step toward the development of curriculum materials which address a wider range of objectives in basic mathematics. For example, while the curriculum materials are specifically designed to assist students in learning how to carry and borrow, the mathematical operations of addition, subtraction, multiplication, and division can be taught through the use of these materials as well.

To facilitate the mastery of carrying and borrowing in addition and subtraction, two terminal objectives were identified through the review of the literature and application of the Program Evaluation Review Technique:

**Objective One:**

A majority of the students are expected to answer correctly eight out of twelve of the addition test items on the computation subtest of the Stanford Diagnostic Mathematics Test.
Task One: Each student is expected to be able to display and/or recognize a given addition problem in its correct order of place value.

Task Two: Each student is expected to count the addends, carry when needed, and then place the correct sum below the equation line.

For example, given instructions to find the sum of eighteen and four, each student should be able to display and/or recognize the problem in acceptable mathematical form:

\[
\begin{align*}
18 \\
+4 \\
22
\end{align*}
\]

The correct answer is twenty-two.

Objective Two:

A majority of the students are expected to answer correctly eight out of twelve of the subtraction test items on the computation subtest of the Stanford Diagnostic Mathematics Test.

Task One: Each student is expected to be able to display and/or recognize a given subtraction problem in its correct order of place value.

Task Two: Each student is expected to subtract the minuend from the subtrahend, borrow if needed, and then place the correct answer below the equation line.

For example, given instructions to find the difference between fifteen and seven, each student should be able to display and/or recognize the problem in acceptable mathematical form:
The student should then subtract the subtrahend from the minuend, borrow when needed, and then place the correct remainder below the equation line.

\[
\begin{array}{c}
15 \\
-7 \\
\hline
8
\end{array}
\]

The correct answer is eight.

**Instructional Strategy**

A review of other instructional approaches in basic mathematics indicates that a sound instructional strategy is necessary for the development of effective curriculum materials. The investigator feels that the following principles are consistent with literature sources and with the investigator’s approach to curriculum development:

1. It is essential that the instructional approach place emphasis on active discovery by the student of the operations of carrying and borrowing through actual problem solving or practice.

2. It is necessary that instructions in the prerequisite skills required for the mastery of the terminal objectives precede instruction in the terminal objectives.

3. It is also important that the curriculum materials minimize the amount of and the emphasis on new terminology.

4. It is required that students are instructed in less difficult forms of the operation before proceeding to more difficult forms of the operation.
5. It is absolutely required that students demonstrate mastery of each objective before proceeding to instruction which assumes mastery of that objective.

6. It is important to the investigator to develop effective curriculum materials which can be replicated and used in classrooms with minimal effort and difficulty on the part of the classroom teacher(s) involved.

The curriculum materials developed are consistent with the implications of the foregoing principles. These materials are designed to be non-graded and have as their operational objectives and educational goal, as previously stated, the improvement of basic instruction in carrying and borrowing in the processes of addition and subtraction.

The key to the achievement of effective instructional materials that are consistent with students' performance levels is accomplished through providing diversity in individual instruction and practice. The fact that all children, regardless of how they are grouped, are not homogenous with respect to any given ability, is reflected in the materials. Consequently, these materials are designed to challenge underachievers as well as more advanced students in basic mathematics.

It is hypothesized that if students are provided with curriculum materials that have a wide range of diversity and which are commensurate with individual abilities, students are capable of continuous development of their abilities. The instructional strategy undergirding these materials allows each student to remain at a particular level of instruction until mastery of that level moves the student to the next. When the student has proven to his individual satisfaction that he can
master the math problems presented, the student is required to demonstrate this mastery in order to move to the next level of instructional difficulty. This instructional strategy is intended to strengthen those prerequisite skills and abilities, the lack of which would hinder the child from making his normal progress in carrying and borrowing in addition and subtraction.

**Development and Field Testing of Curriculum Materials**

One of the main criticisms the investigator formulated upon a review of existing instructional materials which are intended to improve a student's ability to carry and borrow was the inability of such materials to extend the notion of one to one correspondence. One to one correspondence employs semi-concrete representation to aid in teaching basic arithmetic operation, and is used in every numeration system in the world today. While the goal of one to one correspondence is to aid the learner through perceptible manipulation and it has been proven generally effective, there is one major limitation to the method as it is currently designed; namely, the concept of one to one correspondence becomes cumbersome and confusing when attempting to represent numbers with values larger than twenty.

The second criticism of the existing materials to improve students' ability to carry and borrow was the inability of these materials to extend the idea of place value through a practical, yet manipulative format. An understanding of the system of place value in mathematics is fundamental to an understanding of the relationship between the number and the numeral. Both the notion of one to one correspondence and the
idea of place value are essential to understanding carrying and borrowing in addition and subtraction. In effect, what the investigator discovered is the need for a representational system which can present both the abstract and the semi-concrete and/or the relationship between the number and the numeral simultaneously; that is, an improved system for displaying one to one correspondence in a simple but effective format. The development of materials which meet these requirements and which serve to teach carrying and borrowing is available through curriculum materials which are referred to as MATH-EZE. A sample of the MATH-EZE materials is included in the Appendices. (See Appendix 2).

MATH-EZE was designed to help students bridge the gap between concrete and abstract understanding as it relates to carrying and borrowing in addition and subtraction through the use of a semi-concrete-abstract system of linkage. The MATH-EZE materials capitalize on the use of visual representations denoting sets of objects that are used to a considerable extent in working with children in basic mathematics. These visual representations, or correspondences, help children to communicate both the idea of a group of objects and to formulate a description of the number/numeral relationship. This built-in aspect overcomes the tendency for telling rather than teaching basic mathematics facts.

From the theoretical base, which is, in part, derived largely from Bruner, there is a strong belief that telling is not teaching, and that as children use good, open-ended materials their intelligence grows and basic concepts are developed. What MATH-EZE offers is a set of curriculum materials which will provoke a student's curiosity and at the same maintain his interest and his motivation while also developing his in-
telligence.

In commenting on the fact that teaching cannot be done by telling, Almy (1967) clearly states that "it is a great mistake to suppose that a child acquires the notion of numbers and other mathematical concepts just from teaching. On the contrary to a remarkable degree, he develops them himself, independently and spontaneously."

Experts agree that students should be given an opportunity to discover many of the principles of arithmetic which they are expected to learn and use. Learning styles have been identified as important in the discovery process and are perceived as useful in facilitating an instructional strategy for encouraging the discovery process. It is generally accepted that ideas which pupils discover for themselves, individually or through class participation or discussion, make sense to them while rules which are committed to memory by rote methods are soon forgotten. MATH-EZE is designed to encourage the discovery approach in learning basic arithmetic operations through its capacity to represent thought-provoking problems. In addition, it attempts to facilitate opportunities for self-discovery of the principles and structures of arithmetic through manipulation of its semi-concrete-abstract materials.

The development of MATH-EZE parallels the direction taken by an inventor long ago who became exasperated by the untidy clutter of papers and documents on his desk and decided to solve a problem which had existed for centuries. Instead of using his engineering knowledge to create a double-spring, loaded, water-cooled semi-automatic paper organizing device, this inventor overcame the natural tendency to over-complicate matters, and with a piece of bendable wire invented the utterly simple,
completely efficient paper clip. As with the development of the paper clip, simplicity is the underlying building block of MATH-EZE.

In addition to an emphasis on simplicity, MATH-EZE materials are designed to be challenging, self-teaching, self-correcting, and to motivate students to spend long periods of time working with them. The materials provide for a three-stage sequence of learning that students can use to develop specific mathematical skills. The beginning stage is sensorimotor where contact with concrete manipulative materials is provided; the second is a perceptible stage where contrasting stimuli are presented; and the third is the ideational-representational phase, where the student is allowed to deal with objects and ideas with a minimum of concrete and perceptual support data. The emphasis is on providing an open-ended process that gives students freedom to use the process in a variety of creative ways.

The investigator believes the most distinguishing feature of MATH-EZE is that it is an innovative system for "pencil-less mathematics" that places emphasis on cognitive development. The concept of pencil-less mathematics was built into the design in the development of MATH-EZE to overcome two of the obvious excuses students use for not completing math assignments. With MATH-EZE materials students can solve problems in basic mathematics without the need for either pencil or paper. The investigator notes that MATH-EZE is a practical calculating system requiring active student participation to arrive at solutions with each step clearly recorded so that mistakes can be corrected.

With the current interest of educators in promoting and encouraging increased parental participation as a way to improve literacy in America,
it is the hope of the investigator that this method of teaching basic mathematics could become a unifying factor in reviving the parent-child-teacher partnership for the learning of basic mathematics skills.

The design also addresses the fact that children teach one another very effectively and that a program which restricts this activity deprives students of the opportunity to learn from each other. As Piaget (1964) notes, "...nobody knows better than a professor that the best way to learn something is to teach it." MATH-EZE allows for student collaboration on the solution of problems in basic math.

In developing MATH-EZE, the investigator used two field testing procedures to insure the effectiveness of the final product. The first field test was informal and was conducted simply to determine the adequacy of the first draft of the curriculum materials. Each student was presented with a set of MATH-EZE materials. Students were then informed of the nature and purpose of the field test and encouraged to identify confusing aspects of the materials, as well as point out unclear instructions. The investigator provided initial instruction concerning the use of MATH-EZE, noted difficulties encountered, and talked with each student about their experiences prior to completion of the field-testing of the materials.

Upon completion of the first field test, which involved five students, the investigator decided the curriculum materials were adequate to field test with a minimum of ten students. The second field test was then arranged to approximate as closely as possible an actual classroom situation. The curriculum materials were judged ready for further evaluation if a majority of the participating students are able to solve
16 of the 24 test items presented.

In this investigation, two field tests were necessary before the curriculum materials were judged adequate for final evaluation. The first field test was with five students, and the second test was with ten students. In the remaining part of this chapter, the results of the two field tests will be summarized.

**Initial Construction of the Curriculum Materials**

**And the First and Second Field Tests**

A discussion of the initial instructional strategy is included in an earlier section of this chapter. The following additional characteristics of the actual materials are, however, important to note:

1. The curriculum materials consist of:
   a. 56 flash cards to accomplish the terminal objectives of carrying and borrowing in addition and subtraction.

2. For each terminal objective, a tentative list of necessary prerequisite skills are arranged into a hierarchy of difficulty:
   a. practice in number and numeral recognition.
   b. practice in counting using written numerals in ordering sequences of numerals, corresponding to order of number(s).
   c. practice in the so-called "arithmetic fact(s)"; and
   d. number sentence construction, and problem solving.

3. The initial materials were reviewed for technical correctness and appropriateness of the instructional strategy. Relevant suggestions were incorporated in the version to be used for the first field test.
First Field Test. The first field test was conducted in March, 1978, with a group of neighborhood children in Providence, Rhode Island. The group of students ranged from second to fifth grade levels. The students received instruction about how to use MATH-EZE and were given time to explore and become familiar with the materials. This group of students was chosen because: (1) this investigator had previously established rapport with the students based upon residence and (2) the residential proximity allowed the students to spend a long period of time examining and using the materials.

The test was conducted without regard to the students' demonstrated ability or lack thereof. Student records were not available nor were they solicited or necessary for the first phase of the field testing of the materials. The main purpose of the initial field testing was to determine if the materials were suitable for accomplishing the task of teaching carrying and borrowing in addition and subtraction. The investigator wanted to learn whether or not students at different grade levels could easily identify with the materials and the instructions for their use.

Each student received individual instruction for approximately thirty minutes. The time was split between addition and subtraction. The students were briefed about the nature of the field test and encouraged to ask questions and identify areas of confusion. The investigator took notes during the field test and talked with each of the students about their experience prior to completion of the field-testing of the materials.

As expected, the results of the field test suggested numerous struc-
tural and language changes in the curriculum materials. However, the results did not suggest the need for any basic change in the instructional strategy previously identified. Some of the more important changes suggested were: (1) consistency in the semi-concrete representational system; (2) consistency in the color schemes of the materials to avoid distracting student attention; and (3) reduction in the size and dimension of the flash cards to avoid their being too cumbersome to handle in an effective and efficient manner.

Second Field Test. The second field test was conducted in conjunction with the South Providence Tutorial, Inc., in Providence, Rhode Island. The student population of the tutorial program includes approximately 150 kindergarten through sixth grade students. The investigator worked with ten students in grades one through four with below-average ability levels in addition and subtraction. These students were selected from the four grade levels because: (1) it offered a diverse population from which to select a sample; and (2) the heterogeneity of the group of students allowed the investigator to include students with different levels of understanding of the process of carrying and borrowing in addition and subtraction. The students were selected for the field test on the basis of their willingness to participate and the judgement of the cooperating teacher regarding the students' math abilities. Consequently, the ten students were given a pretest related to the terminal objectives to identify their current levels of achievement.

Over a period of two weeks, the students met daily after school for fifty minutes of work with the MATH-EZE materials. Initially, the in-
vestigator gave the students one day of general instructions for each objective, which included a request that students identify areas of confusion. During the remainder of the field test, the investigator assigned specific problems to be solved and answered general process questions only. No direct instructional assistance was provided by the investigator. Each student received verification when the assigned problem was correct from the investigator and was then assigned a more difficult problem upon successful completion. For those students who experienced difficulty in completing assigned problems, they were asked to explain to the investigator the process they undertook to arrive at their answer. The investigator indicated the point at which the error occurred and asked the student to retrace his/her steps. The basic data for the students and the results of the field test are summarized in Table 2.

In summary, indicated in Table 2, at least a majority of the students passed the posttest and correctly answered at least 16 out of 24 examples; therefore, the curriculum materials were judged ready for the final evaluation of the effectiveness of the curriculum materials. (See Table 2). Chapter IV reports the results of the final evaluation of the curriculum materials.
<table>
<thead>
<tr>
<th>Measure</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
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<td>Pretest Raw Scores</td>
<td>8</td>
<td>11</td>
<td>12</td>
<td>5</td>
<td>13</td>
<td>9</td>
<td>6</td>
<td>12</td>
<td>7</td>
<td>10</td>
</tr>
<tr>
<td>Posttest Raw Scores</td>
<td>18</td>
<td>21</td>
<td>19</td>
<td>15</td>
<td>22</td>
<td>20</td>
<td>14</td>
<td>21</td>
<td>16</td>
<td>17</td>
</tr>
<tr>
<td>Was Posttest Passed?</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>No</td>
<td>Yes</td>
<td>Yes</td>
<td>No</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
</tbody>
</table>
This chapter reports the results of the final evaluation of the curriculum materials. First, the investigator will provide an overview of the final evaluation. Secondly, the investigator will report the results of the evaluation of the overall effectiveness of the curriculum materials. Thirdly, additional findings generated as a result of the statistical test for analysis of variance and t distribution will be discussed.

**Overview of the Final Evaluation**

**Preparation.** In preparation for the final evaluation, there were no changes made in the curriculum materials, MATH-EZE. The materials were judged appropriate to teach the specified objectives of carrying and borrowing to selected elementary students. (See Appendix 2).

The accompanying MATH-EZE instructional booklet was, however, edited, and some parts were rewritten in order to provide maximum clarity to those using MATH-EZE for the first time. The booklet provides instructions for the application of MATH-EZE to the four basic operations of addition, subtraction, multiplication, and division. (See Appendix 3).

Finally, Test Form A and Test Form B of the Stanford Diagnostic Mathematics Test, Computation Sub-test were selected to be used as the pretest and posttest to measure any change in student achievement in
addition and subtraction. Both the pretest and posttest consist of thirty-three items which relate to the two terminal objectives of the study. The results of these tests were subject to a decision-making statistical model. In addition, the data was subjected to tests for analysis of variance and t distribution.

The Sample and the Setting. The investigator selected the Martin Luther King Elementary School in Providence, Rhode Island for the final evaluation of the curriculum materials. The Martin Luther King Elementary School is one of twenty-seven elementary schools which serve metropolitan Providence, the capital city of Rhode Island. The school has an enrollment of approximately five hundred and ninety-six kindergarten through third grade students. The school was chosen from a field of ten based upon the following reasons: (1) students were familiar with an individualized approach to learning mathematics; (2) many students were in need of a special after-school tutorial program to help them improve their achievement in addition and subtraction; and (3) the staff displayed a willingness to cooperate with the investigator in the conduct of the study. The preceding conditions convinced the investigator that the students were likely to view the curriculum materials developed to accomplish the objectives of this study as individualized and non-threatening.

On the basis of the results of the field test and the resources of the investigator, it was decided to evaluate MATH-EZE at two grade levels; one class of second grade students and one class of third grade students. The materials were tested with these two groups because of
the small sample of second graders available to participate in the after-school program. It is important to note that all participants in the study scored under the fifty-two percentile on the California Test of Basic Skills which was administered by the Providence School Department in March, 1978. The results were made available in May, 1978, and were obtained by the investigator from the School Department.

The sample for the final evaluation of MATH-EZE consisted of one treatment group of nine second graders and one treatment group of nine third grade students. In addition, two classes similar in number served as control groups. The basic data for each of the classes identifies pretest and posttest achievement and is summarized in Tables 3 and 4. (See Tables 3 and 4). These tables provide information concerning sex, I.Q., pretest scores, posttest scores, and attendance information. The I.Q. scores were measured by the Peabody Picture Vocabulary Test, Form A, and was administered in June of 1978. All this data was subject to evaluation on the basis of a decision-oriented research model.

Design. In order to understand the appropriateness of the design of the final evaluation, it is important to realize that the present study is decision-oriented rather than conclusion-oriented. A decision-oriented study is an investigation carried out to accomplish a specific practical goal. In this study, the specific practical goal is the development of curriculum materials designed to teach the specified objectives of carrying and borrowing to selected elementary school students. Cronbach and Suppes (1969) note that "the excellence of the product being developed...should be the ruling concern of the decision-oriented inquiry...Rigor in (decision-oriented) research is likely to express it-
<table>
<thead>
<tr>
<th>Student</th>
<th>Sex</th>
<th>I.Q.</th>
<th>Raw Score</th>
<th>Raw Score</th>
<th>Attendance</th>
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<tr>
<td></td>
<td></td>
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<td>Pretest</td>
<td>Posttest</td>
<td>Absent</td>
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<tr>
<td>1. A</td>
<td>F</td>
<td>108</td>
<td>13</td>
<td>15</td>
<td>1</td>
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<td>2. B</td>
<td>F</td>
<td>111</td>
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<td>3. C</td>
<td>M</td>
<td>121</td>
<td>9</td>
<td>26</td>
<td>0</td>
</tr>
<tr>
<td>4. D</td>
<td>F</td>
<td>88</td>
<td>22</td>
<td>22</td>
<td>3</td>
</tr>
<tr>
<td>5. E</td>
<td>M</td>
<td>81</td>
<td>10</td>
<td>20</td>
<td>1</td>
</tr>
<tr>
<td>6. F</td>
<td>M</td>
<td>88</td>
<td>18</td>
<td>23</td>
<td>9</td>
</tr>
<tr>
<td>7. G</td>
<td>M</td>
<td>118</td>
<td>19</td>
<td>21</td>
<td>4</td>
</tr>
<tr>
<td>8. H</td>
<td>M</td>
<td>71</td>
<td>19</td>
<td>22</td>
<td>10</td>
</tr>
<tr>
<td>9. I</td>
<td>M</td>
<td>93</td>
<td>24</td>
<td>30</td>
<td>8</td>
</tr>
</tbody>
</table>

**Treatment Group**

| 1. J    | M   | 100 | 28        | 25        |
| 2. K    | F   | 85  | 10        | 4         |
| 3. L    | M   | 85  | 14        | 16        |
| 4. M    | F   | 106 | 24        | 24        |
| 5. N    | F   | 90  | 13        | 14        |
| 6. O    | M   | 93  | 20        | 20        |
| 7. P    | F   | 97  | 18        | 14        |
| 8. Q    | F   | 78  | 20        | 23        |

**Control Group**

Table 3

Basic Data on Second Grade Students
In Control and Experimental Classes
(Final Evaluation)
Table 4

**Basic Data on Third Grade Students**
**In Control and Experimental Classes**
**(Final Evaluation)**

<table>
<thead>
<tr>
<th>Student</th>
<th>Sex</th>
<th>I.Q.</th>
<th>Raw Score Pretest</th>
<th>Raw Score Posttest</th>
<th>Attendance</th>
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<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Absent</td>
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<tr>
<td><strong>Treatment Group</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1. AA</td>
<td>F</td>
<td>91</td>
<td>20</td>
<td>25</td>
<td>1</td>
</tr>
<tr>
<td>2. BB</td>
<td>F</td>
<td>96</td>
<td>27</td>
<td>31</td>
<td>1</td>
</tr>
<tr>
<td>3. CC</td>
<td>M</td>
<td>96</td>
<td>23</td>
<td>30</td>
<td>0</td>
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<tr>
<td>4. DD</td>
<td>F</td>
<td>71</td>
<td>19</td>
<td>28</td>
<td>2</td>
</tr>
<tr>
<td>5. EE</td>
<td>M</td>
<td>83</td>
<td>19</td>
<td>28</td>
<td>3</td>
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<tr>
<td>6. FF</td>
<td>F</td>
<td>81</td>
<td>30</td>
<td>32</td>
<td>0</td>
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<td>7. GG</td>
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<td>30</td>
<td>1</td>
</tr>
<tr>
<td>8. HH</td>
<td>F</td>
<td>106</td>
<td>30</td>
<td>29</td>
<td>6</td>
</tr>
<tr>
<td>9. II</td>
<td>M</td>
<td>96</td>
<td>25</td>
<td>27</td>
<td>10</td>
</tr>
<tr>
<td><strong>Control Group</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1. JJ</td>
<td>M</td>
<td>91</td>
<td>31</td>
<td>32</td>
<td>32</td>
</tr>
<tr>
<td>2. KK</td>
<td>F</td>
<td>73</td>
<td>30</td>
<td>26</td>
<td>26</td>
</tr>
<tr>
<td>3. LL</td>
<td>M</td>
<td>99</td>
<td>25</td>
<td>31</td>
<td>31</td>
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<tr>
<td>4. MM</td>
<td>F</td>
<td>116</td>
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<td>31</td>
<td>25</td>
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<tr>
<td>5. NN</td>
<td>F</td>
<td>98</td>
<td>28</td>
<td>28</td>
<td>28</td>
</tr>
<tr>
<td>6. OO</td>
<td>F</td>
<td>116</td>
<td>27</td>
<td>29</td>
<td>29</td>
</tr>
<tr>
<td>7. PP</td>
<td>F</td>
<td>100</td>
<td>22</td>
<td>23</td>
<td>23</td>
</tr>
</tbody>
</table>
self differently than the rigor of conclusion-oriented research."

An appropriate emphasis of the evaluation portion of this study is to determine whether the curriculum materials, MATH-EZE, (when given to the sample identified earlier) is effective in achieving the stated terminal objectives. Therefore, the two major research questions to be answered in this study are:

1. Did a majority of the second grade students (treatment group) master the terminal objectives of the curriculum materials?
2. Did a majority of the third grade students (treatment group) master the terminal objectives of the curriculum materials?

The major aspects of the two research questions are clarified with the following definition of terms:

1. Mastery of the terminal objectives of the curriculum materials means that the student correctly answered twenty-two out of thirty-three examples at the second grade level and twenty-seven out of thirty-three examples at the third grade level on the posttests.
2. A majority in numerical terms will be reached at the seventy percent level.
3. Because of the small size of both samples, the seventy percent criteria was considered to be met if \( X \) students mastered the terminal objectives, where

\[
N \times X = (X)
\]

\( N \) = the number of students in the sample
\( X = \) seventy percent of \( N \)
\( (X) = \) the largest whole number at which \( X \) can be rounded.
For example, there were nine students in each of the two sample groups; therefore, \( N = 9 \); \( X = 0.70 \times 9 = 6.3 \); and \( (X) = 6 \)

\[
N \times X = (X) \\
9 \times 0.70 = (X) \\
6.3 = (X)
\]

4. For the purpose of this study, the use of the curriculum materials did not include any provisions for instructional assistance by the investigator beyond initial introduction of how to use **MATH-EZE** and to answer process questions.

5. In order to establish that the students mastered the terminal objectives due to exposure to the curriculum materials rather than due to other contributing factors, the control groups were given the pretest with no treatment. They were administered the posttest nineteen days later.

In order to obtain additional information concerning the effectiveness of the curriculum materials, additional statistical findings were generated by applying the tests for analysis of variance and \( t \) distribution. The specific results are reported in a later section.

**Operation of the Final Evaluation.** The process for selecting the classes to be involved in the study consisted of the following steps: (1) the School Department was contacted by the investigator; (2) the principal at Martin Luther King School and the investigator explained the nature of the study to parents and determined which parents were willing to cooperate; (3) the principal identified all those second and third grade students who were in need of additional help in carrying and bor-
rowing; and (4) the classes to participate in the study were selected randomly from those not eliminated in steps two and three.

The investigator held two meetings with the principal to insure her cooperation and to insure that the conduct of the experiment and the testing procedures were educationally sound. At the meetings, the timetable for the experiment was determined and the investigator's role was defined, the principal was given a copy of the curriculum materials, and any outstanding questions were addressed.

The role of the investigator during the experiment was defined as follows: The investigator was expected to give out materials; to administer and score tests; to answer students' questions which did not necessitate instruction; to record any unusual questions or behavior by students; and to encourage the students to work at their normal pace. The investigator also assumed responsibility for assigning math problems for students to solve on the basis of their pretest performance. Since this was an after-school project and would not disrupt the regular mathematics program, it was decided to allow a maximum of twenty school days for the experiment, including two days for testing and eighteen days for self-instruction.

On Monday, June 5, 1978, the pretest was administered. The instructional period covered the next eighteen days ending on June 29. On Friday, June 30, 1978, the posttest was administered.

It is important to note that the curriculum materials were tested under the least desirable conditions, during the last month of the school year. The extremely hot weather and the excitement of the pending summer vacation are two examples of the adverse conditions with which stu-
dents had to contend during the course of the experiment.

**Evaluation of the Overall Effectiveness of the Curriculum Materials**

As previously stated, the two major research questions to be answered in this study are:

1. Did a majority of the students in the second grade mathematics class which served as the treatment group for this study master the terminal objectives of the curriculum materials?
2. Did a majority of the students in the third grade mathematics class which served as the treatment group for this study master the terminal objectives of the curriculum materials?

The answer to both questions is "yes." In the second grade treatment group, nine students were included in the final evaluation. Six students, or seventy percent of the second grade treatment group, scored twenty-two or higher, two scored twenty or higher, and one scored fifteen out of thirty-three examples right on the posttest. An examination of the basic data for both groups going into the experiment reveals the following comparisons. The treatment group had an average I.Q. score of 97.6, and the control group averaged 91.7.

On the pretest measure, the treatment group scored an average raw score of 15.7 out of 33 examples correct for an average percentile rank of 42.2 percent. The control group scored an average raw score of 18.3 out of 33 examples correct for an average percentile score of 53 percent for the group. Thus, the control group scored an average of 2.6 more examples right than the experimental group for a percentile difference of 10.6 percent. (See Table 5).
Table 5

<table>
<thead>
<tr>
<th></th>
<th>Treatment</th>
<th>Control</th>
<th>Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Raw Score</td>
<td>Raw Score</td>
<td>Raw Score</td>
</tr>
<tr>
<td>Group Average (Pre)</td>
<td>15.7</td>
<td>18.3</td>
<td>2.6</td>
</tr>
<tr>
<td></td>
<td>42.2</td>
<td>53</td>
<td>10.6</td>
</tr>
<tr>
<td>Group Average (Post)</td>
<td>22.4</td>
<td>17.5</td>
<td>4.9</td>
</tr>
<tr>
<td></td>
<td>72.6</td>
<td>52</td>
<td>20.6</td>
</tr>
<tr>
<td>Increase</td>
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<td>-0.8</td>
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<tr>
<td></td>
<td>30.4</td>
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</tbody>
</table>
On the posttest measure, the treatment group scored an average raw score of 22.4 out of 33 examples correct and a group percentile of 72.6 percent. The control group scored an average raw score of 17.5 out of 33 examples correct and a group percentile score of 52 percent. Thus, the treatment group scored an average of 2.3 more examples right than the control group and a group percentile difference of 10.00 percent.

A comparison between the results of the raw scores and group percentile on the pretest and posttest shows a 6.7 increase in the raw score and a 30.4 group percentile increase within the experimental group. The control group showed .8 of a percent decrease in raw score and a 1 percent loss in group percentile. It appears that there was an overall increase in achievement of 2.3 more correct examples in favor of the experimental group.

In sum, the treatment group overcame the 2.6 difference in achievement in favor of the control group on the pretest and went on to answer 2.3 more correct examples than the control group on the posttest.

In the third grade treatment sample, nine students were included in the final evaluation. Eight students scored twenty-seven or higher, and one scored twenty-five out of thirty-three examples right on the posttest. An examination of the basic data for both groups going into the experiment revealed the following comparisons. The treatment group had an average I.Q. score of 90.6 and the control group averaged 99.0.

On the pretest measure, the experimental group scored an average raw score of 24.4 out of 33 examples correct for a group percentile of 34.2 percent. The control group scored an average raw score of 27.2 out of 33 examples correct and a group percentile of 46.8 percent for
the group. Thus, the control group scored an average of 2.8 more examples correct than the treatment group for a percentile difference of 12.6 percent. (See Table 6).

On the posttest measure, the treatment group scored an average raw score of 28.7 out of 33 examples correct for a group percentile rank score of 55.5 percent. The control group scored an average raw score of 28.1 out of 33 examples correct for a group percentile of 52 percent. Thus, the treatment group scored an average of 0.6 more examples correct than the control group for a percentile difference of 1.3 percent.

A comparison between the results of the raw scores and percentile ranks on the pre and posttests show a 4.3 increase in the raw score and a 21.4 percent increase in achievement within the treatment group. The control group showed .9 of a percent increase in raw score and a 7.4 percent increase within the control group. There was an overall increase in achievement of 11.3 percent manifested on the posttest in favor of the treatment group.

In sum, the treatment group overcame the 2.8 difference in achievement in favor of the control group on the pretest and went on to answer 1.3 more correct examples than the control group on the posttest.

The treatment group at the second grade level was in attendance on the average of 13.3 days and the third graders were in attendance on the average of 15.3 days out of the 18 instructional days.

Additional Findings

The investigator was intrigued with the fact that both the second
### Table 6

#### Comparison of Pre and Posttest Results

<table>
<thead>
<tr>
<th></th>
<th>Treatment</th>
<th></th>
<th>Control</th>
<th></th>
<th>Difference</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Raw Score</td>
<td>%</td>
<td>Raw Score</td>
<td>%</td>
<td>Raw Score</td>
<td>%</td>
</tr>
<tr>
<td>Group Average (Pre)</td>
<td>24.4</td>
<td>34.2</td>
<td>27.2</td>
<td>46.8</td>
<td>2.8</td>
<td>12.6</td>
</tr>
<tr>
<td>Group Average (Post)</td>
<td>28.7</td>
<td>55.5</td>
<td>28.1</td>
<td>54.2</td>
<td>0.6</td>
<td>1.3</td>
</tr>
<tr>
<td>Increase</td>
<td>4.3</td>
<td>21.3</td>
<td>0.9</td>
<td>7.4</td>
<td>2.2</td>
<td>11.3</td>
</tr>
</tbody>
</table>
and third grade treatment groups answered more examples correct on the posttest than the control group. Originally, the control groups in both cases scored higher on the pretest measure. Consequently, the investigator decided to generate a set of null hypotheses and to apply the statistical tests for analysis of variance and t distribution.

In short, the effectiveness of the curriculum materials was analyzed from a statistical point of view for significance in accepting or rejecting the generated null hypotheses. The null hypotheses were identified and the results for each question for each group of students is reported and discussed. In addition, any other relevant data will be provided.

The results are reported in the following order: (1) the results of the test for analysis of variance to estimate the treatment effect for the second and third graders, and (2) the results of the t distribution test for statistical significance between group means for selected characteristics. The selected characteristics were: (A) I.Q., (B) sex, (C) attendance, and (D) rate of improvement.

**Application of Test For Analysis of Variance.**

**Null Hypothesis A:**

There is no positive correlation between changes in student achievement and use of the curriculum materials for the second grade experimental sample. This null hypothesis is rejected since $F = 6.90$ and the critical value of $F$ is $4.168$. The null hypothesis is rejected at the .01 level of significance.

**Null Hypothesis B:**
There is no positive correlation between changes in student achievement and use of the curriculum materials for the third grade experimental sample. This null hypothesis is not rejected since \( F = 1.962 \) and the critical value of \( F \) is 2.48. In this case, while there was no significant statistical difference in achievement for the total sample, there was a measurable increase in achievement.

**Application of Test For t Distribution.**

**Null Hypothesis C:**

There is no positive correlation between I.Q. scores and use of the curriculum materials for the second grade experimental sample. This null hypothesis is not rejected at the 95 percent level of confidence. The standard deviation was 27.86 and the t distribution was 1.63.

**Null Hypothesis D:**

There is no positive correlation between I.Q. scores and use of the curriculum materials for the third grade experimental sample. This null hypothesis is not rejected at the 95 percent level of confidence. The standard deviation was 10.01 and the t distribution was 3.15.

**Null Hypothesis E:**

There is no positive correlation between sex and use of the curriculum materials for the second grade experimental sample. This null hypothesis is not rejected at the 95 percent level of confidence. The standard deviation was 27.30 and the t distribution was 1.64.

**Null Hypothesis F:**

There is no positive correlation between sex and use of the cur-
riculum materials for the third grade experimental sample. This null hypothesis is not rejected at the 95 percent level of confidence. The standard deviation was 11.13 and the t distribution was 1.71.

Null Hypothesis G:

There is no positive correlation between attendance and use of the curriculum materials for the second grade experimental sample. This null hypothesis is not rejected at the 95 percent level of confidence. The standard deviation was 23.58 and the t distribution was 3.35.

Null Hypothesis H:

There is no positive correlation between attendance and use of the curriculum materials for the third grade experimental sample. This null hypothesis is not rejected at the 95 percent level of confidence. The standard deviation was 8.22 and the t distribution was 3.83.

Null Hypothesis I:

There is no positive correlation between rate of improvement and use of the curriculum materials for the second grade experimental sample is not rejected at the 95 percent level of confidence. The standard deviation was 28.90 and the t distribution was 2.83.

Null Hypothesis J:

There is no positive correlation between rate of improvement and use of the curriculum materials for the third grade experimental sample. This null hypothesis is rejected at the 95 percent level of confidence. The standard deviation was 3.40 and the t distribution was 4.26.

In summary, the results of the statistical tests for significance
indicate that the curriculum materials assisted in improving the level of student achievement in addition and subtraction. It must be remembered, however, that the sample was small, and that any connection between increased achievement and the curriculum materials in no way implies a cause and effect relationship. Most importantly, the findings of the study provide baseline data for use in comparisons with the results of future studies. Comparing results over time can help to determine whether or not the use of these curriculum materials will produce increased achievement in selected operations of mathematics.
The purposes of this chapter are to report the findings of this research, to discuss their implications, and to identify significant additional areas of research suggested by this study.

Summary

Briefly stated, this is a study of the development and evaluation of curriculum materials designed to teach carrying and borrowing in addition and subtraction to selected elementary school pupils. Specifically, the study's aim was: (1) to determine appropriate objectives of carrying and borrowing for selected elementary school pupils; (2) to develop curriculum materials for accomplishing the identified objectives; and (3) to evaluate the effectiveness of the materials for helping these pupils carry and borrow as determined by the objectives.

The two major research questions to be answered in this study were accepted on the basis of the evidence from the test scores and on the other findings presented.

The findings of the investigation showed that six out of the nine students at the second grade level reached the terminal objective of the first research question. Also, eight out of the nine students at the third grade level reached the terminal objective of the second research question. In each case, mastery of the terminal objectives study means that the student correctly answered twenty-two out of thirty-three ex-
amples at the second grade level and twenty-seven out of thirty-three examples at the third grade level on the posttest. Because of the small size of both samples, the seventy percent criteria was considered met if six students demonstrated mastery of the terminal objective. These data suggest an affirmative answer to the two major research questions stated.

Additional Findings. Since the original two research questions discussed above were affirmative, the investigator generated ten null hypotheses to be subjected to the test for analysis of variance and the t distribution test for statistical significance between group means for selected characteristics. The selected characteristics were: (A) I.Q.; (B) sex; (C) attendance; and (D) rate of improvement. The results of applying these tests reveals that two of the ten null hypotheses were rejected and the other eight were accepted.

In summary, the results of the statistical tests for significance indicate that the curriculum materials assisted in improving the level of student achievement in addition and subtraction. It must be remembered, however, that the sample was small, and that any connection between increased achievement and the curriculum materials in no way implies a cause and effect relationship. Most importantly, the findings of the study provide baseline data for use in comparisons with the results of future studies. Comparing results over time for example can help to determine whether or not use of the curriculum materials will produce increased achievement in the promotion of learning carrying and borrowing in mathematics.
Implications of the Findings

One purpose of this study as stated in the initial chapter was to determine appropriate objectives of carrying and borrowing for selected elementary school pupils.

In order to insure that appropriate objectives were chosen, the investigator conducted a critical review of the literature including: (1) identification of those activities necessary to reach the terminal objectives of carrying and borrowing in addition and subtraction; (2) theoretical assumptions concerning the relationship between curriculum and instruction and the developmental stages children encounter as they progress from one conceptual level of mathematics understanding to the next; and (3) application of the Program Evaluation Review Technique (PERT) as a qualitative measure of the skills associated with carrying and borrowing in addition and subtraction.

On the basis of the review of the literature, the investigator identified two objectives judged most appropriate for the scope of the present study. The terminal objectives were:

1. Given an addition problem, the student will:
   (A) Display and/or recognize the problem in its correct order of place value.
   (B) Count the total, carrying when needed, and place the correct sum below the equation line.

2. Given a subtraction problem, the child will:
   (A) Display and/or recognize the problem in its correct order of place value.
   (B) Subtract the minuend from the subtrahend, borrowing if needed, and place the correct answer below the equation line.
This purpose was most important in the fulfillment of the second purpose of the study. It is the view of the investigator that the use of the Program Evaluation Review Technique holds much promise as a planning tool for identifying qualitative steps involved in the process of reaching specified objectives.

The second purpose of the study was to develop curriculum materials for accomplishing the identified objectives. The second portion of the study then, was concerned with the development and field testing of these curriculum materials.

One of the main criticisms the investigator formulated upon a review of existing instructional materials which are intended to improve a student's ability to carry and borrow was the inability of such materials to extend the notion of one to one correspondence. One to one correspondence employs semi-concrete representation to teach basic arithmetic operations and is used in every number system in the world today. While the goal of one to one correspondence is to aid the learner through perceptible manipulation and it has been proven generally effective, there is one major limitation to the method as it is currently designed; namely, the concept of one to one correspondence becomes cumbersome and confusing when attempting to represent numbers with values larger than twenty.

The second criticism of the existing materials to improve students' ability to carry and borrow which the investigator made was the inability of these materials to extend the idea of place value through a practical, yet manipulative format. An understanding of the system of place value in mathematics is fundamental to an understanding of the relation-
ship of the number to the numeral. Furthermore, both the notion of one to one correspondence and the idea of place value are essential to understanding carrying and borrowing in addition and subtraction. In effect, what the investigator discovered is the need for a representational system which can present both the abstract and the semi-concrete and/or the relationship between the number and the numeral simultaneously; in other words, an improved system for displaying one to one correspondence in a simple but effective format. The development of materials which meet these requirements and which serve to teach carrying and borrowing in this study is available through curriculum materials which are referred to as MATH-EZE.

MATH-EZE was designed to help students bridge the gap between semi-concrete and abstract understanding as it relates to carrying and borrowing in addition and subtraction. The MATH-EZE materials capitalize on the use of visual representations denoting sets of objects that have traditionally been used to a considerable extent in working with children in basic mathematics. These visual representations, or correspondences, help children to communicate both the idea of a group of objects and to formulate a description of the number/numeral relationship. This built-in aspect, then, overcomes the tendency for telling rather than teaching basic mathematics facts.

The field testing procedure for the curriculum materials included two field tests. For the first field test students were given general instructions about how to use the curriculum materials. The students were then asked to set up and solve simple addition and subtraction problems involving the operations of carrying and borrowing. If the
solution was incorrect, the investigator provided minimal procedural
guidance, but encouraged the student to retrace the steps used in find-
ing the solution.

The second field test was with ten students. The students were
given MATH-EZE materials and an instructional booklet, but no addition-
al instruction was provided by the investigator beyond providing an-
swers to process questions. The second field test established that
the curriculum materials may be effective with students with below-
average abilities in basic math. The second field test was conducted
with pairs of below-average ability students in grades one, two, three,
and four. The results of the second field test indicated that the ma-
terials were ready for final evaluation.

During the study, the following observations were noted with re-
spect to the effectiveness of the curriculum materials designed to teach
the operations of carrying and borrowing in addition and subtraction.
The investigator noted that students seemed to enjoy working with
the materials and expressed favorable opinions during the exit interview.
Students tended to use the semi-concrete aspect of the materials to move
from counting to computing and to practice number facts. Students al-
so seemed to react favorably to manipulation of the materials and tend-
ed to be able to find and correct mistakes quicker as practice increased.

Implications for Further Research

Conduction of the present study has revealed the need for further
investigation into the development and testing of curriculum materials
to teach specified objectives. Additional studies that would extend the
meaning of this research to educators are discussed in the remainder of
this chapter. There should be studies: (1) to extend the investigation of the Program Evaluation Review Technique (PERT) as a tool in the development of curriculum materials; (2) to extend the investigation of the curriculum materials, MATH-EZE, for teaching carrying and borrowing in multiplication and division and (3) to extend the investigation of the curriculum materials to use in teaching handicapped and slow learners of all ages.

A viable follow-up study might well be a replication of the present investigation. Based upon a more rigorous statistical research design, an investigation would be designed to document more fully the relationship between the use of the curriculum materials and increased achievement. It is suggested that a much larger and more representative sample of elementary school students be used to form the reference group. The students included in the sample should represent a complete cross-section of elementary school students. Also, it is recommended that sufficient numbers of each type of student be included in the sample so that it would be possible to speak about the patterns of achievement with more confidence. The total sample, then, would most likely include over 1000 students. In addition, a better instrument could be developed to more precisely measure achievement associated with the terminal objectives.

Another related investigation is the study of the Program Evaluation Review Technique (PERT) as a tool in the development of curriculum materials. Such studies should provide further understanding about the dynamics of curriculum development at different stages of a student's schooling.
Chapter I suggests that there is a need for more precise and specific curriculum materials to improve achievement. The PERT process used in this study seems generally adequate for an initial investigation. However, continued systematic application of PERT to the development of curriculum materials would be desirable. It is hoped that additional research will result in information that will complement, contradict or expand the objectives used in the present investigation. Gathering more information about PERT as a curriculum planning tool will result in a greater understanding of the dynamics of curriculum development. Another area of research related to this study is concerned with extending the investigation of the curriculum material to carrying and borrowing in multiplication and division. A longitudinal study of a sample of elementary school pupils would enable educators to determine whether achievement changes or remains constant over a number of years in applying the operations of carrying and borrowing in addition and subtraction as well as multiplication and division. Further, it would be possible to identify changes resulting from strategies planned to bring about change through periodic assessments.

It will be recalled that in Chapter II there was emphasis placed on the need to move from concrete to abstract understanding in the development of the curriculum materials. This study developed materials that are flexible enough to allow students to use the materials for learning any of the basic math skills. The materials are simple enough to move learners from basic counting to computing. The investigator believes that the materials are appropriate and should be tested in teaching basic skills to handicapped and slow learners of all ages. It could
then be determined if the materials are effective in meeting learners at different levels of cognitive development and moving them forward.

Further research questions arise regarding the use of the curriculum materials for improving achievement in basic mathematics. For example, what type of children benefit most from the curriculum materials? Will a change in presentation result in corresponding changes in student achievement? These questions are related to differences in instructional strategy and students. Further research must be done to determine the relevance and validity of such questions for understanding the educational impact the curriculum materials have on the learner.

The present study demonstrates that the development of curriculum materials can be helpful in meeting specified objectives of learning. If the existence of these findings was supported by other research, educators would have valuable information for identifying objectives and developing curriculum materials. It is likely that many changes in educational programs would be deemed desirable.

It is hoped that the present study will be useful in stimulating research on curriculum development. It is here that research should enable educators not only to understand the process of curriculum development, but also to apply such understanding to the development of simple but effective curriculum materials.

Curriculum materials at the elementary school level are as different as the students who use them. Only when educators understand the influence of curriculum materials on different students will it be possible to improve achievement and provide the necessary level of remediation to improve learning in general.
BIBLIOGRAPHY
BIBLIOGRAPHY


Byrne, M.A., "The Development of a Measure of the Familiarity of Mathematical Terms and Symbols." Doctoral dissertation, Purdue University, 1970.


APPENDICES
APPENDIX 1

PERT Charts
PERT CHART: Activities Associated with Borrowing in Subtraction

<table>
<thead>
<tr>
<th>ACTIVITY NUMBER</th>
<th>ACTIVITY</th>
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</thead>
<tbody>
<tr>
<td>(9-10)</td>
<td>answer completed</td>
</tr>
<tr>
<td>(8-9)</td>
<td>subtract numbers in 100's column</td>
</tr>
<tr>
<td>(7-8)</td>
<td>borrow if necessary</td>
</tr>
<tr>
<td>(6-7)</td>
<td>record the answer in ten's column</td>
</tr>
<tr>
<td>(5-6)</td>
<td>subtract numbers in ten's column</td>
</tr>
<tr>
<td>(4-5)</td>
<td>borrow if necessary</td>
</tr>
<tr>
<td>(3-4)</td>
<td>record units under units column</td>
</tr>
<tr>
<td>(2-3)</td>
<td>subtract the number of units</td>
</tr>
<tr>
<td>(1-2)</td>
<td>problem written</td>
</tr>
</tbody>
</table>

Project Completed

Project Started
PERT CHART: Activities Associated with Carrying in Addition

<table>
<thead>
<tr>
<th>ACTIVITY NUMBER</th>
<th>ACTIVITY</th>
</tr>
</thead>
<tbody>
<tr>
<td>(9-10)</td>
<td>Project Completed</td>
</tr>
<tr>
<td>(8-9)</td>
<td>answer completed</td>
</tr>
<tr>
<td>(7-8)</td>
<td>add numbers in 100's column</td>
</tr>
<tr>
<td>(6-7)</td>
<td>carry 100's from ten's to 100's</td>
</tr>
<tr>
<td>(5-6)</td>
<td>record the answer in ten's column</td>
</tr>
<tr>
<td>(4-5)</td>
<td>add numbers in ten's column</td>
</tr>
<tr>
<td>(3-4)</td>
<td>carry ten's from units to ten's column</td>
</tr>
<tr>
<td>(2-3)</td>
<td>record units under units column</td>
</tr>
<tr>
<td>(1-2)</td>
<td>add the number of units</td>
</tr>
<tr>
<td></td>
<td>problem written</td>
</tr>
</tbody>
</table>

Project Started
APPENDIX 2

Sample of the Curriculum Materials

The curriculum materials, MATH-EZE, carton of 56 flash cards with an equal sign, number facts (five cards for numbers 0-9), and operational symbols (one each for addition, subtraction, multiplication, and division).
Equal

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MATH-EZE

MATH-EZE is a set of flash cards designed for simplicity and for learning basic number facts. It is a new practical approach for learning basic math facts, and can be used by beginners, students or adults. It is also a convenient way to master addition, subtraction, multiplication and division. This method builds confidence as it develops skills in basic mathematics.

There are 56 cards with number facts and math symbols. The numerals run from 0 through 9 and there are four basic operation symbols.

The operation symbols are as follows:

<table>
<thead>
<tr>
<th>Add</th>
<th>Subtract</th>
<th>Multiply</th>
<th>Divide</th>
</tr>
</thead>
<tbody>
<tr>
<td>+</td>
<td>-</td>
<td>×</td>
<td>÷</td>
</tr>
</tbody>
</table>

Plus Minus Times Divide

This method will serve as a starting point for beginners, reinforcement for students and adults.

How To Use MATH-EZE

The best way to begin is to let the player (especially beginners) familiarize himself with the cards. This can be done by having him or her stack the cards which show the same number in their respective piles. This process will also help the player deal with problems more efficiently, simply because he will have quick access to the cards as needed. Once the player has accomplished the pile stacking, give him a number to present. For example 682. The player should indicate the number by finding the appropriate cards and placing them in their proper order of place value.

EXAMPLE

6 8 2
At this point tell the player that the position of each card indicates its place value. That is, the card on the right represents two \textit{units} (ones), the middle card represents eight \textit{Tens}, and the remaining card indicates six \textit{hundreds}.

\textbf{EXAMPLE}

\[
\begin{array}{ccc}
6 & 8 & 2 \\
\text{Hundreds} & \text{Tens} & \text{Units}
\end{array}
\]

The instructor announces "Six hundred eighty-two", — the player has to associate:
Six (6) with the hundreds' column or place
Eighty (8) with the 'Tens' place and
Two (2) with the units' place.

Continue this process until the player shows that he has understood this exercise.

\textbf{ADDITION.} Start by giving the player a simple addition problem:

\[
\begin{array}{c}
1 \\
\hline
+ 2 \\
\hline
\_ 3
\end{array}
\]

*Note: Always use the appropriate operation sign to indicate the proper operation:

\[
\begin{array}{c|c|c|c}
\text{Add} & \text{Subtract} & \text{Multiply} & \text{Divide} \\
+ & - & \times & \div
\end{array}
\]

For beginners use the following number sentence method in the elementary stages:
EXAMPLE

\[
\begin{array}{c}
\text{Add} \\
\text{1 + 2 = 3}
\end{array}
\]

After the player masters the simple problems, challenge his/her intellect by giving him/her progressively harder problems.

SUBTRACTION: Once the player has begun to understand simple addition, he/she should be introduced to the subtraction process. Begin by giving the player a simple subtraction problem and continue by giving harder problems as he/she shows mastery.

EXAMPLE

\[
\begin{array}{c}
\text{Subtract} \\
\text{5 - 2 = 3}
\end{array}
\]

When mastery of addition and subtraction is accomplished, the player should be introduced to multiplication or successive addition.

MULTIPLICATION: Phase three of this project is to teach multiplication. However, one should not attempt this phase until the player achieves simple mastery of addition and subtraction. The rules are similar to those used in the other two activities. Just remember to start on simple problems and continue to harder ones as the player develops skills relating to the solving of simple problems.

EXAMPLE

\[
\begin{array}{c}
\text{Multiply} \\
\text{2 \times 3 = 6}
\end{array}
\]

The instructor should explain that two times three is the same as adding two sets of three's or three sets of two's.
DIVISION: Division cannot be successfully accomplished without simple mastery of the other operations. Remember to start slowly and continue to harder problems as the player shows complete understanding.

**EXAMPLE**

\[
\begin{array}{c}
9 \\
\div \\
3 \\
\hline
\end{array}
= \begin{array}{c}
3 \\
\hline
\end{array}
\]

or

\[
\begin{array}{c}
3 \\
\hline
9 \\
\hline
0 \\
\end{array}
\]

The instructor should explain to player that nine divided by three is the same as finding how many threes are contained in nine.

HOW TO CHECK

**ADDITION:** Add the numeral indicated on each card, labeled A and B, or count the boxes on each of these cards. The total should equal the same amount as on the answer card labeled C.

\[
\begin{array}{c}
A \\
\hline
1 \\
\hline
B \\
\hline
2 \\
\hline
C \\
\hline
3 \\
\end{array}
\]

**SUBTRACTION:** Count the boxes on the cards labeled B and C. They should equal the number of boxes on card labeled A.

\[
\begin{array}{c}
A \\
\hline
4 \\
\hline
B \\
\hline
2 \\
\hline
C \\
\hline
2 \\
\end{array}
\]

Page 7
DIVISION: Check your answer by multiplying the quotient by the divisor. That product equals the original number (dividend).

\[ \text{Dividend} \div \text{Divisor} = \text{Quotient} \]

\[
\begin{array}{c}
\text{Dividend} \\
9 \\
\end{array}
\]

\[
\begin{array}{c}
\text{Divisor} \\
3 \\
\end{array}
\]

\[
\begin{array}{c}
\text{Quotient} \\
3 \\
\end{array}
\]

MULTIPLICATION: Check this answer by either taking two number. Three cards from the pile and adding the blocks on the two cards or taking three number. Two cards from the pile and adding the blocks on them. This number should equal the number on the answer card.

\[ \begin{array}{c}
\text{Add} \\
3 \\
\text{Equal} \\
3 \\
\text{Add} \\
+ \\
3 \\
\text{Equal} \\
6 \\
\end{array} \]

\[ \begin{array}{c}
\text{Add} \\
2 \\
\text{Equal} \\
2 \\
\text{Add} \\
+ \\
2 \\
\text{Equal} \\
6 \\
\end{array} \]