

1927

Making algebra educative

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Making Algebra Educative

Walter G. Buchanan

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MAKING ALGEBRA EDUCATIVE

Walter G. Buchanan, B. Sc.

Submitted as a Thesis for the Degree of
Master of Science

MASSACHUSETTS AGRICULTURAL COLLEGE

AMHERST, MASSACHUSETTS

1927

INTRODUCTION

Question In September 1924 I began teaching mathe-
 Rises matics in the Methuen High School, Methuen,
 Massachusetts. During that year I taught
 two classes in beginners' Algebra. The text-
 book used was Hawkes, Luby, and Touton - First Course in Al-
 gebra. (see Outline # 1 page 10). At the end of the school
 year I found myself very much dissatisfied with the work ac-
 complished. I felt that too much time had been devoted to drill
 exercises and not enough emphasis placed on the understanding
 of the why and wherefore. In fact so much time had been devoted
 to the acquiring of skills in algebraic computations that
 scarcely any time was left for the solution of ordinary quan-
 titative problems which require insight and mental power
 rather than skill in symbolic manipulation.

 In an endeavor to remedy the situation all
 Taking Steps the teachers of mathematics in the Junior and
 Senior High Schools met in conference with
 the Superintendent of Schools several times during the school
 year 1925 - 1926 and formulated an outline of material to be
 covered. (see Outline # 2 pages 12 and 19). The outline was
 based on the textbook already in use - Hawkes, Luby, and Tou-
 ton - and was an attempt to eliminate such material as we felt
 was of questionable value for the future needs of the pupil,

and also to present the material in a more understandable manner. We did not lay very much stress on the order of teaching each subject in the final draft of our outline; but we did emphasize certain very definite objectives in the teaching of the various topics.

Again I was dissatisfied, for the same
 Educative big question still remained:- how can we make
 Effort Algebra truly educative? All real progress
 in Algebra teaching must center around the
 answer to this question. It is a matter of presenting material
 which is adapted to the child mind, and of presenting it in a
 way that the pupil knows what it is for and knows why he pro-
 ceeds as he does, so that he never has to hide behind rules
 for operation. Algebra must be for him a subject which teach-
 es him the "what for" and the "why", at least as soon as the
 "how". Algebra must appear as a simple and direct language,
 very useful in stating facts and rules, and in asking and an-
 swering questions, according to the laws of common sense.

If we are to accomplish this real im-
 Judgement provement in Algebra teaching we must
 Pronounced thoroughly revise the old order of present-
 ing the material which is illustrated by
 Outline # 1 page 10. This outline is the Table of Contents of
 our textbook - Hawkes, Luby, and Touton - First Course in
 Algebra. Although Outline # 1 is an outline of a specific

text it is typical of the old order of presenting material to beginning classes in Algebra. Furthermore, if we are to accomplish this real improvement in Algebra teaching we must thoroughly revise the old method of presenting the material to beginning classes in Algebra.

An attempt to accomplish both of these ends

Remedy	is discussed fully on pages 22 to 56. The
Attempted	outline of this discussion is Outline # 3 on
	page 20. Throughout the entire development

of this Outline # 3 the emphasis has been placed on the understanding. Each idea or process, if it is to mean anything to the student, must be developed at the moment when it is needed, and put to work at once. New processes must be discovered and introduced as the occasion demands, thus linking the "what for", the "why", and the "how" into an understanding whole. In fact, the order should be this; first, a new mathematical difficulty; second, a study of the situation so that we may find the way out; and third, the discovery of the new process which overcomes the difficulty.

Psychologically it is the understanding

Value of	fact which must be repeated if we want to
Understanding	make Algebra truly educative. The Algebra
	classroom must be a workshop where the in-

structor directs the activities of the pupils and where the textbook provides the materials and tools with which these pupils work. The organization of the text and the methods of

presenting the working materials play a large part in facilitating instruction and in the resultant achievement of individuals of the class.

REVIEW OF LITERATURE

The National Committee in their Report on the Reorganization of Mathematics in Secondary Education state on page 10:- "The primary purpose of the teaching of mathematics should be to develop those powers of understanding and of analyzing relations of quantity and of space which are necessary to an insight into and control over our environment and to an appreciation of the progress of civilization in its various aspects, and to develop those habits of thought and of action which will make those powers effective in the life of the individual".

Continuing on page 11 the Report states:- "Work should be limited to those processes and to the degree of complexity required for a thorough understanding of principles and for probable applications either in common life or in subsequent courses which a substantial proportion of the pupils will take".

Texts published since the appearance of the statements of the National Committee, which was organized in 1916, show an attempt on the part of the authors to find some material which can be used in the work of the 7th and 8th Grades. The Junior High School movement has made this necessary. Typical among these Junior High School authors are Scherling and Clark. They have a three volume series of mathematics which is widely used in the 7th, 8th, and 9th Grades - Junior High School.

They have placed their emphasis on the second paragraph above, quoted from the Report of the National Committee. The work has been "limited to those processes and to the degree of complexity" for those pupils who take the work. They have succeeded in taking the same old material and cutting out the harder portions, thus making the work easier. This material, however, they have presented in the same order as before.

Readers' Guide of periodical Literature contains too large a number of articles written within the last 15 years to incorporate in this thesis. There are, however, a few well known authors who have published one or more textbooks which are used in school. Prominent among these are:-

Barber - Everyday Algebra.

Barber - Teaching Junior High School Mathematics.

Clark - Mathematics in the Junior High School.

Report of the National Committee on Reorganization of
Mathematics in Secondary Education.

Schorling - The Teaching of Junior High School Mathematics.

Schorling & Clark - Modern Mathematics, Algebra. (3 vol).

Stone - Junior High School Mathematics.

Vosburgh & Gentleman - Junior High School Mathematics.

A few of the authors of recent Algebra texts, as named above, have changed the old order of presenting the material somewhat; but none of them have followed the order as presented in this thesis on page 10.

THE PROBLEM

Out of the dissatisfaction with the results of the Algebra teaching we had done, and our first efforts at improving that teaching, a question took shape. Will better results come if the order of presentation of topics is changed and the method of development remodeled? We had groped about in a state of half-seeing for two years. It was an opportunity too good to be passed and the whole situation was summed up in this problem: What effect will a change in the order of topics presented have upon the pupils' accomplishment in First Year Algebra based upon the idea of understanding at every step? Such a change necessitates an accompanying change in the method of procedure. This point is so evident that the expression "order of presenting" will be understood to stand for both throughout this thesis.

In order to determine the effectiveness of the procedure which involved changing the order of presenting the material covered in first year Algebra the work was arranged as follows. There were five beginning classes in first year Algebra, which for convenience have been designated as Numbers I, II, III, IV, and V. These five classes were under the direction of three teachers. Class # I was taught by myself, Class # II by a second teacher, and Classes # III, #IV, and # V by a third teacher.

All five classes used the same textbook - Hawkes, Luby, and Touton - First Course in Algebra. Classes # III, # IV, and # V followed Outline # 1 (see page 10), which presents the material of our text in the order according to the Table of Contents. In presenting each topic listed in Outline # 1 the objectives and abilities emphasized in Outline # 2 (see pages 12 to 19) were stressed. Five achievement tests were given at varying times during the months of February and March to each of the classes. Classes # I and # II followed Outline # 3 (see page 20) which is the order of presenting the material for beginning Algebra with the aim to make it more truly educative. The same textbook was used - Hawkes, Luby, and Touton - First Course in Algebra. The same objectives and abilities emphasized in Outline # 2 were stressed, and the same achievement tests given to these classes as ^{were} given to the other three classes. Two teachers, each using this outline, eliminate the personal equation. The evidence of achievement for all classes is given in Table # 1, page 59. Outline # 1 is the Table of Contents for our textbook. Outline # 3 is the newly arranged order of presenting the material for the first year of Algebra. Outline # 2 is the common ground between the two outlines. Without regard to the order of subject matter as given in Outline # 1, Outline # 2 lists the several abilities and objectives which should be emphasized in teaching the various topics of first year Algebra. Outline # 3 embodies all of these principles and objectives as formulated in Outline # 2. It differs from the latter only in that the order

of presentation of subject matter is so organized that each algebraic fact is linked to, and its treatment developed from the algebra fact immediately preceding. This gives a logical sequence and a natural gradation, which makes the work simple and easy for the beginner. The order of presentation of topics then is the varying factor in this discussion, as the material for all classes is the same.

Early in the course Terman Group Test of Preliminary Mental Ability - Form A - was given to each individual in the five classes. A copy of this Checks test will be found at the end of this thesis.

At varying intervals throughout the year Final Check each class has been given the same set of Achievement Tests and at the same time. These tests were prepared to test the several abilities and objectives listed in Outline # 2 pages 12 to 19. A copy of these tests will be found on pages 74 to 78. The results of these tests are summarized in Tables # 1, # 3, and # 4, pages 57 to 64.

Considering all the conditions and factors Statement of in the above experiment the problem for Problem which a solution is being attempted may be stated briefly as follows:- Is it possible to improve the understanding and achievement of pupils in beginning Algebra by a change of order in the presentation of material?

OUTLINE # 1

Table of Contents - First Course in Algebra-

Hawkes, Luby, and Touton.

Outline # 1 is a copy of the Table of Contents - First Course in Algebra - Hawkes, Luby, and Touton. It gives the order in which the subject matter is treated in this particular text. Although this is an outline of a specific text it presents the material in the typical order of a large number of first year Algebras. The Introductory Chapter of 17 pages is devoted to the definition of a large number of terms which are to be used in later chapters. The second chapter of 15 pages gives more definitions and also rules for the operation of signed numbers, these rules to be used in later chapters. The student thus becomes accustomed to do things by a set of rules or directions, and does not use his reasoning powers to any great extent. The complete Table of Contents follows:-

Chapter	Page
I Introduction - - - - -	1
II Positive and Negative Numbers - - - - -	18
III Addition - - - - -	33
IV Simple Equations - - - - -	39
V Subtraction - - - - -	49
VI Identities and Equations of Condition -	54

VII	Parentheses - - - - -	64
VIII	Multiplication - - - - -	70
IX	Parentheses in Equations - - - - -	79
X	Division - - - - -	87
XI	Equations and Problems - - - - -	95
XII	Important Special Products - - - - -	105
XIII	Factoring - - - - -	113
XIV	Solution of Equations by Factoring - - -	137
XV	Fractions - - - - -	148
XVI	Equations Containing Fractions - - - - -	175
XVII	Graphical Representation - - - - -	200
XVIII	Linear Systems - - - - -	217
XIX	Square Root - - - - -	240
XX	Radicals - - - - -	250
XXI	Quadratic Equations - - - - -	270
XXII	Ratio and Proportion - - - - -	282

OUTLINE # 2

Objectives in Teaching First Year Algebra.

In an endeavor to make our Algebra teaching more effective in Methuen all the teachers of the Junior and Senior High Schools met in conference with the Superintendent of Schools several times during the school year 1925 - 1926. Each teacher was assigned one or more topics from Outline # 1, the Table of Contents of our textbook - Hawkes, Luby, and Touton - and asked to formulate certain definite objectives and abilities that should be stressed in the teaching of that particular topic. Our final draft resulted in Outline # 2 which is a rather comprehensive outline for the work to be covered in First Year Algebra. We did not pay any particular attention to the order of topics in our outline. We were, however, aiming to formulate certain very definite objectives in the teaching of the various topics.

A copy of Outline # 2 follows. The pages refer to Hawkes, Luby, and Touton - First Course in Algebra; but it is recommended that many additional reference books be used.

I Objectives in teaching the Formula.

To develop the following abilities:-

1. To develop certain rules of mathematics and to translate them into formulas. Pages 4, 9.

2. To translate certain formulas into rules of mathematics. Page 3.
3. To evaluate certain formulas; that is, to find the values of certain letters when the values of the others are known. Page 6.
4. To derive one formula from another.
5. To represent by a graph certain formulas of a type no more difficult than $F = 1.8C + 32^{\circ}$.
6. To understand the idea of the dependence of one quantity upon another.

II Objectives in teaching equations of the First Degree in one unknown.

To develop the following abilities:-

1. To translate into equations form certain conditions stated in words. This involves two abilities:
 - a. To give the answer to such questions as this: "What is the cost of 8 pencils at k cents each?" ($C = 8k$).
 - b. To write the correct equation for problems similar to this: "Twice a number increased by five is equal to twelve. Find the number. ($2n + 5 = 12$)." Pages 9, 10, 11, 15, 60, 61, 63, 64, 65.
2. To translate into words certain simple types of equations.
3. To solve, algebraically, equations of the type $y = ax + b$. Pages 13, 14, Chapter V.
4. To understand the graph of the equation $y = ax + b$ as the graph of a linear equation and as a method of solving the family of equations when y is given any particular value. Chapter XVII.
5. To use equations in solving such applied problems of Algebra as are of real use in business or in science.

6. To distinguish clearly between an identity and an equation of condition. Chapter VI
 Note: This means the ability to appreciate the equation of condition as "the interrogative sentence of Algebra" and the identity as "the declarative sentence of Algebra".
7. To solve simple equations containing common or decimal fractions.

III Objectives in teaching graphs.

To develop the following abilities:-

1. To interpret pictorial graphs of various kinds. Chapter II.
2. To interpret and to draw: Bar graphs; Broken line graphs; Curve line graphs; and, Simple circular graphs. Chapter II.
3. To make a comparison of two or more graphs on the same piece of paper using the same coordinate axes.
4. To locate points using the conventional x and y coordinate axes. This involves the ability to understand the relation between a point and the number pair (x, y). Chapter XVII.
5. To understand, appreciate, and use directed numbers in graphic work.
6. To interpret the intersections of certain graphs as points whose coordinates represent real roots of the corresponding equations, and their nonintersection as indication of unreal or imaginary roots.
7. To construct the graph of the equation $y = ax + b$ and to explain its meaning.
8. To construct the graph of the function $y = ax^2 + bx + c$.
9. To construct the graphs of two simultaneous equations of the type $a'x + b'y = c'$, $a''x + b''y = c''$ and to explain how the graphs show whether the equations are solvable.

10. To use graphs as a rough check on algebraic operations.
11. To read values from a graph quickly and accurately.
12. To construct a graph for the purpose of solving practical problems.
13. To use the graph for the purpose in such allied fields as general science, social science, and the like.

IV Objectives in teaching Directed Numbers.

To develop the following abilities:-

1. To understand and interpret the meaning of directed numbers and to represent them on an algebraic scale, thus extending the notion of numbers. Pages 32, 33, 34, 35.
2. To use directed numbers in a practical way.
3. To add two directed numbers in a column or horizontally. Pages 34, 35, 36, 37, 44, 45, Chapter III.
4. To subtract one directed number from another in a column or horizontally. Pages 38, 39, 40, 44, 45, Chapter III.
5. To combine directed numbers by addition and subtraction in a column or horizontally.
6. To multiply one directed number by another in a column or horizontally. Pages 41, 42, 44, 45.
7. To divide one directed number by another, the operation being taught as the inverse of multiplication. Pages 42, 43, 44, 45.
8. To understand the use of directed numbers in a few simple cases taken from physics or mechanics. Chapter II.
9. To understand what is meant by the "absolute value" of a number. Page 22.

10. To appreciate the use of directed numbers in connection with formulas, thus extending their use.
11. To distinguish between plus and minus signs as signs of quality and signs of operation. Chapters VII, IX.
12. To remove not more than two sets of parentheses where directed numbers are involved, the case being treated as a convenient way of indicating addition or subtraction.

V Objectives in teaching the Fundamental Operations on Algebraic Polynomials.

To develop the following abilities:-

1. To multiply and divide in the manner indicated for directed numbers. Chapters VIII, X.
2. To understand the ordinary symbols of aggregation, not over two "nests of parentheses" being involved.
3. To divide one simple polynomial by another simple one. Since the case has no important applications in elementary algebra, it should be briefly treated.
4. To understand that factoring is the inverse of multiplication. For example $10x^2 + 19x + 6 = (2x + 3)(5x + 2)$ because $(2x + 3)(5x + 2) = 10x^2 + 19x + 6$. In other words the identity is reversed. Chapters XII, XIII.
5. To understand that factoring is important in transferring one formula into another which is easily evaluated.
6. To factor by taking out a common monomial factor.
7. To factor the general quadratic trinomial $ax^2 + bx + c$.
8. To factor the difference of two squares.

The so-called "type products" and "class" of factoring should not be overemphasized. We really need only two or three types of factoring and these can be handled simply by reversing the process of multiplication.

9. To understand that the fundamental operations are only a means of reaching some kind of result, mere skill in manipulation not being the purpose of the work. Only the types needed later, either in life problems, or in other parts of Algebra, should be taught. Their relation to the work on the formula should be explained.
10. To check solutions and thus to realize when a topic has been fairly mastered.

VI Objectives in the teaching of Fractions.

To develop the following abilities:-

1. To understand a fraction as an indicated division.
2. To understand the underlying principle in connection with the change of signs in the terms of a fraction. For example

$$\frac{+a}{+b} = \frac{+a}{-b} = \frac{-a}{+b} = -\frac{+a}{-b}$$

3. To reduce a fraction to lower terms, and, in few cases to higher terms.
4. To reduce an improper fraction to a mixed expression or to an integral expression.
5. To reduce a mixed expression to an improper fraction.
6. To reduce fractions to equivalent fractions having the L.C.D. when the L.C.D. can be found by inspection.
7. To add and subtract fractions having the same denominator and having different denominators when the L.C.D. can be found by inspection and where the denominators are either monomials, or else polynomials of the three ordinary factoring types, i. e. those which are the product of two binomials.
8. To multiply and divide fractions of simple type.

9. To simplify complex fractions which are not more complicated than those which would occur in the pupils' work with formulas, or in checking fractional equations where the root is a fraction.
10. To check all results.

VII Objectives in teaching Fractional Equations.

To develop the following abilities:-

1. To clear an equation of fractions the denominators being arithmetic or literal numbers, the work being limited to cases where the L.C.D. can be found by inspection. Chapter XVI.
2. To solve numerical equations containing common or decimal fractions of a simple type.
3. To solve equations containing fractions with binomial denominators.
4. To solve applied problems leading to fractional equations.
5. To derive one formula from another, that is, to solve literal equations.
6. To evaluate formulas involving fractions.
7. To check all results.

VIII Objectives in teaching Simultaneous Linear Equations.

To develop the following abilities:-

1. To understand by means of graphic representation simultaneous, inconsistent, and equivalent equations in two unknowns. Chapter XVIII.
2. To solve by addition and subtraction simultaneous equations in two unknowns, having integral or fractional (common or decimal) coefficients.
3. To obtain the equation of a straight line through two points whose coordinates are known.

4. To solve applied problems involving simultaneous equations in two unknowns.
5. To check all results.

OUTLINE # 3

An Outline for 9th Grade Algebra.

This outline has been made from the extended Outline # 2 given on pages 12 to 19. The objectives in teaching each of the topics listed have already been stated in Outline # 2. The topics of Outline # 2 have been rearranged so that each algebraic fact is linked to, and its treatment developed from, the algebra fact immediately preceding. The student learns through Formulas the very first day what Algebra really is. The work in Simple Equations, Fractional Equations, Simultaneous Equations, and so on, follow directly, thus making a logical sequence and a natural gradation in the subject matter.

Outline # 3, as followed by Classes # I and # II, follows:-

- I Formulas.
- II Simple Equations.
- III Fractional Equations.
- IV Simultaneous Equations.
 - a. Addition of directed numbers.
 - b. Subtraction of directed numbers.
 - c. Graphs.
 - 1. Formula graph (linear equation).
 - 2. Bar graph.
 - 3. Broken and curved line graphs.
 - 4. Circle graphs.

V Quadratic Equations.

a. Multiplication.

b. Division.

c. Factoring.

* VI Radicals.

* VII Practical Computations.

*VIII Numerical Trigonometry.

* Optional topics.

WHAT ALGEBRA IS

The preceding pages state the conditions giving rise to the experiment; the conditions of the experiment; and offer suggestions for the rearrangement of the material to be offered. The following pages are devoted to the details of the method of procedure in carrying Outline # 3 into actual practice, and point out how the student in following this topical order works understandingly rather than as an automaton.

Classes # III, # IV, and # V found in following Outline # 1 (the old plan) that the first 17 pages of the textbook were devoted to the definition of terms to be used in later chapters. The next 15 pages gave rules for the addition, subtraction, multiplication, and division of positive and negative numbers. The following chapters took up these four operations separately and involve examples with long polynomial expressions. The students all the while were only automatons, doing all their work by rules and having no conception of the why or the wherefore of anything done.

Classes # I and # II, however, in following Outline # 3 (the revised plan) were told the very first day what Algebra is, and they were told in language which they could understand and appreciate. And, furthermore, when they started to do their assignment they did not have to follow a set of rules. They had a very definite objective and knew why they proceeded in each

case as they did. The following illustrates how we put Outline #3 into practice in Methuen High School this year. It is our first class exercise in beginning Algebra.

Algebra has been defined as a written language or shorthand in which the sentences are called formulas or equations. It has also been defined as a symbolic method of solving problems. These two definitions mean practically the same thing. If we turn the first definition around it will read something like this, "Formulas state facts briefly", and if we explain the second definition it says, "we use symbols (signs which stand for numbers, words, or thoughts) because they make our work easier and save space". If I should ask you to write one hundred thirty-seven dollars and sixty-five cents, you would write \$137.65. Again, if I should ask you to multiply seven hundred sixty-two by three hundred fifty-one, you would write it this way:

$$\begin{array}{r} 762 \\ \times 351 \\ \hline \end{array}$$

The figures 762 and 351 stand for the words seven hundred sixty-two and three hundred fifty-one respectively, and the sign \times stands for the words multiply by. We took our written directions and translated them into symbols. Why? Because it is easy to multiply with figures, while it would be very difficult to multiply if we had to use words only. The result of the multiplication is also obtained in figure-symbols which can be translated back into words whenever we choose. Problem-solving in mathematics is performed in this way. We take word-problems, we assign symbols to the quantities and relations described. Then

we perform the operations indicated so as to obtain the result we desire.

The work with formulas as discussed in the next two pages was then taken up as a part of this first day's assignment.

FORMULAS

If we consider the familiar fact usually stated as follows: The area of a rectangle is equal to its base times its height, can we write this fact more briefly by using the shorthand of Algebra? Instead of the area of the rectangle we will write R ; instead of the base we will write b , and so forth. We can now state our fact briefly, $R = b \times h$, or still better, $R = bh$. To save time in Algebra, letters may be written side by side to indicate multiplication. We must be careful to give attention to the units in any formula. If b means the number of feet in the length of a floor, and if h means the number of feet in the width, R will mean what? We must be sure that our answer is expressed in the correct units.

Find the area of a floor 18.2' by 12.5'.

This is a very simple everyday problem, but in studying it we shall find the best way to set down our work and to keep a systematic record so that any mathematician can tell exactly what we have done.

$R = bh$	*Estimate	Computation	Check
$R = 18.2 \times 12.5$	$20 \times 12 = 240$	18.2	12.5
		<u>12.5</u>	<u>18.2</u>
$R = 227.50 \text{ sq.ft.}$		910	250
		364	1000
		<u>182</u>	<u>125</u>
		227.50	227.50

First: Write the formula.

Second: Substitute the numbers for the letters.

Third: Make an estimate of your answer.

Fourth: Multiply as indicated. Perform your multiplication neatly and carefully.

Fifth: Check your computation, and leave it as a part of the record.

*Note:- The estimate of the answer should be insisted upon. Its value is great in the beginning of the term for the following reasons:-

1. It obliges the pupil to read the problem attentively.
2. It livens the classwork. The various estimates all tabulated on the board arouse a great deal of interest.
3. It is excellent practice in getting the child to appreciate the size of numbers.
4. It develops the ability to grasp a situation as a whole.
5. It enables the pupil to put his decimal point in the correct place without having to remember a rule.

The assignment given for the second day was:-

Bring in to class at least three formulas with a problem adapted to each.

The formulas submitted the second day contained not only those found in mathematics and science texts, but those used by everyday workmen. Two boys in one division nearly came to blows because each insisted that the other's formula for making cement blocks was incorrect. Among the formulas which were brought in and which were used are the following:-

$S = a^2$. The area of a square.

$T = \frac{1}{2}bh$. The area of a triangle.

$C = 2\pi r$. The circumference of a circle.

$A = \pi r^2$. The area of a circle.

$S = 2\pi rh$. The area of the curved surface of a cylinder.

$V = Bh$. The volume of a rectangular solid.

$V = e^3$. The volume of a cube.

$I = prt$. Simple interest.

$F = 1.8C + 32^{\circ}$. Converting Centigrade degrees to Fahrenheit.

EQUATIONS

The next logical step in the development of the course in Algebra should be the equation. In preceding lessons with the formula the pupil will have translated given formulas into complete English sentences, and he will also have reduced to formulas many common rules and laws of business, mathematics, and science. He has been unconsciously making equations all the while, and he is quite ready to understand that an equation suggests a question.

The subject of equations may well be developed from the following problem:- A rectangular garden is to be laid out to contain 918 square feet. It is 34' wide. How long must it be made?

The formula required is $R = bh$.

The problem tells us that $R = 918$ and that $b = 34$, and says that $h = ?$.

Substituting these numbers in the formula, we get $918 = 34h$. We no longer think of this equation as a formula, stating a fact. We think of it as suggesting a question.

What question does it suggest? If $34h$'s are 918, how much is one h ?

If we find the answer to our question we have solved an equation.

(1)	$R = bh$	Estimate	Computation	Check
(2)	$918 = 34h$	$900 = 30 \times 30$	$34)918(27$	27
(3)	$27 = h$		$\begin{array}{r} 68 \\ 238 \\ \hline 238 \end{array}$	$\begin{array}{r} 34 \\ 108 \\ \hline 81 \end{array}$
(4)	$h = 27 \text{ feet}$			$\begin{array}{r} 81 \\ 918 \end{array}$

Take another problem: The bottom of a coal bin measures 96 sq. ft. How high must it be made to hold 432 cu. ft. when even full?

The formula required is $V = Bh$.

The problem tells us that $V = 432$ and that $B = 96$, and says that $h = ?$. The question suggested is: If 96h's are 432, how much is one h? If we solve and check this equation we find that h is 4.5 feet.

After a few problems like these have been considered write the equation, $5x = 20$, and ask the pupil to state the question it asks. His answer will be: If five x's are 20, how much is one x? He can tell you immediately what the answer is and also that it checks. He is now ready to practice on a few simple equations.

A problem to illustrate the next step in the development of our Algebra is: Frank and William earned \$90 working together. Frank earned four times as much of the money as William did. How much should each receive?

First, list the quantities mentioned.

Number of dollars William gets.

" " " Frank "

" " " both together get.

Second, represent each quantity by an abbreviation or symbol.

D = no. \$ William gets.

Then $4D$ = " " Frank "

$D + 4D$ = " " both together get, that is, 90.

Third, make an equation.

Fourth, solve the equation.

$$(1) \quad D + 4D = 90$$

$$(2) \quad 5D = 90$$

To get this equation think,
one D and $4D$'s make $5D$'s.

$$(3) \quad D = 18 \quad (2) \div 5$$

Write the directions for getting equation three before you write the equation. You will save yourself much trouble in Algebra if you always decide first whether to add, subtract, multiply, or divide, and then write the directions, and finally carry them out.

$$(4) \quad 4D = 72 \quad (3) \times 4$$

Why did we get this equation? What are the answers to the problem? Check them.

The fact that problem solving in Algebra is not the same as in Arithmetic may well be pointed out to the pupil at this time. In solving a problem in Arithmetic you decide whether you are to multiply or divide or what process you are to perform, and then, step by step, reason your way to your answer.

In Algebra you do not start this way at all; instead you write an equation and then solve the equation.

For many kinds of problems the algebraic method is much better. It leaves less to carry in your head. It keeps a very brief record of the steps taken in the solution. It gives the best way to explain the solution. It enables you to solve harder problems.

It will help you with more difficult problems if you form the habit of taking these three steps:-

1. List the quantities.
2. Express them algebraically.
3. Form an equation.

These facts all appeal to the child and they create in him a confidence that is very much worth while. His attitude toward his work is decidedly different from that of the boy or girl who does his daily assignment simply because the book or the teacher told him to do it that way.

The pupil is now ready to practice on equations like these: $x + 4x = 35$.

This equation suggests the question, if one x and four x 's make 35, how much is x ? In solving this problem the pupil thinks, one x and four x 's make five x 's, so he writes

$$(1) \quad x + 4x = 35$$

$$(2) \quad 5x = 35$$

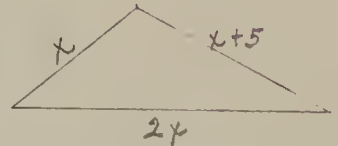
$$(3) \quad x = 7 \qquad (2) \div 5$$

He gets the 7 by thinking, if five x's equal thirty-five how much is x ? Practice with this type of equation and problem should follow.

The next type of equation may be illustrated by the problem:-
The sides of a triangle are x , $2x$, and $x + 5$.
The perimeter is 69". Find the length of each side.

The pupil writes the equation

$$(1) \quad x + 2x + x + 5 = 69.$$



He knows he can unite the x , the $2x$, and the x and write $4x$, but he cannot unite the $4x$ and the 5 because he knows we can unite only terms which are alike. His second equation then becomes

$$(2) \quad 4x + 5 = 69.$$

This equation suggests the question, if $4x + 5$ make 69, how much is x ? In order to find out how much $4x$ is the pupil reasons that he has 5 too much, so that we must subtract 5. He reasons too that if he subtracts 5 from one side of his equation he must also subtract 5 from the other side in order to keep the values the same. His third equation is then

$$(3) \quad 4x = 64 \quad (2) \quad - 5$$

$$(4) \quad x = 16 \quad (3) \quad \div 4$$

$$(5) \quad 2x = 32 \quad (4) \times 2$$

Check

$$(6) \quad x + 5 = 21 \quad (4) + 5$$

One side 16"

second side 32"

third side 21"

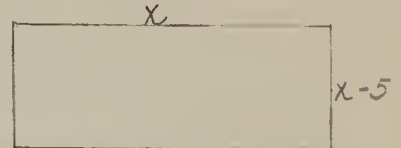
perimeter 69"

A problem of the next type is: The width of a rectangle is 5' less than the length. The perimeter is 170'.

Find the dimensions of the rectangle.

$$(1) \quad x + x - 5 + x + x - 5 = 170$$

$$(2) \quad 4x - 10 = 170$$



This last equation suggests the question, if $4x$ less 10 makes 170, how much is x ? In finding out how much $4x$ is we are "short" 10, and we must therefore add 10. Equation three then reads

$$(3) \quad 4x = 180 \quad (2) + 10$$

Check

$$(4) \quad x = 45 \quad (3) \div 4$$

lengths $45' \times 2 = 90'$

$$(5) \quad x - 5 = 40 \quad (4) - 5$$

widths $40' \times 2 = \underline{80}'$

perimeter 170'

Drill in these types of equations should follow.

PARENTHESES IN EQUATIONS

The introduction of the parentheses may be illustrated by the problem: To pay \$3.25 with halves and quarters, I used 2 more halves than quarters. How many coins of each kind did I use?

Let x = no. of quarters

Then $x + 2$ = " " halves

$25x$ = " " cents in quarters

To express the number of cents in $x + 2$ half dollars we must multiply $x + 2$ by 50. A convenient way to indicate this multiplication is this

$50(x + 2)$ = no. of cents in halves

Then $25x + 50(x + 2)$ = " " " " all = 325.

$$(1) \quad 25x + 50(x + 2) = 325$$

$$(2) \quad 25x + 50x + 100 = 325 \quad \text{The expression } 50(x + 2) \text{ means } 50 \text{ times } x \text{ and } 50 \text{ times } 2.$$

$$(3) \quad 75x = 225 \quad (2) - 100 \quad \text{Check}$$

$$(4) \quad x = 3 \quad (3) \div 75 \quad 3 \text{ quarters} = 75 \text{ cents}$$

$$(5) \quad x + 2 = 5 \quad (4) + 2 \quad 5 \text{ halves} = \underline{250} \quad "$$

total = 325 "

The parentheses are used to bring two or more quantities together into one quantity. This fact comes as a logical conclusion in the child mind rather than as a given rule which he must follow in his problem solving.

PROBLEM SOLVING

In order to be able to solve problems readily, you must know how to solve equations. In solving an equation you add, subtract, multiply, and divide, taking care to "be fair to each side" of the equation. If you change the value of either side, you must make a corresponding change in the value of the other side.

As minus signs occur in the equation from time to time, the pupil becomes more and more familiar with them, and finds that his common sense invariably tells him what they mean, and how to deal with them. In the equation $4(x - 2) = 15$, he performs the indicated multiplication without difficulty. Four times x is $4x$, and 4 times "short 2" leaves him short 8. In the form $15 - 3(x + 2)$, he observes that the terms within the parentheses are to be first multiplied by 3, and then the result subtracted from 15. The first step gives $15 - (3x + 6)$. The subtraction gives $15 - 3x - 6$. In the form $17 - 4(x - 2)$, the first step gives $17 - (4x - 8)$. Starting the subtraction, he gets $17 - 4x$. He has now subtracted too much for he was told to subtract 8 less than $4x$. He has subtracted the whole of $4x$. He has subtracted 8 too much. He must correct this by adding 8. This gives him $17 - 4x + 8$. If we unite similar terms our expression becomes $25 - 4x$. The 17 was increased by subtracting the shortage of 8. We see then that "subtracting a shortage is the same as adding".

The pupil does not think of parentheses as something to be "removed", but rather as a convenient means of giving directions as to what operations are to be performed. He does not remove the parentheses by rule, but carries out the instructions which they help to give.

This manner of thinking about the minus sign lays a real foundation for understanding both subtraction and multiplication later, and meanwhile it enables him to do such multiplication and subtraction as is necessary in the solution of his equations. And, furthermore, it convinces him that Algebra is a subject which one can think out for himself.

As a result of this procedure (problem solving), the young people come through the opening chapters of their Algebra with the feeling that the subject is not juggling, but common sense.

FRACTIONAL EQUATIONS

We are now ready for equations containing fractions. This problem makes a good introduction to the subject: I am thinking of a number. Half of it is 7. What is the number?

This is a very simple problem which you can answer mentally. But if you will study the written solution you will learn an important lesson in Algebra.

The statement may be translated into this equation

$$(1) \quad n/2 = 7 \quad n/2 \text{ is read "n over 2". It means n divided by 2, or one half of n.}$$

In equation one we have $\frac{1}{2}$ of n . We want the whole of n .

What must we do?

$$(2) \quad n = 14 \quad (1) \times 2 \quad \text{By multiplying both sides of our equation by 2 we keep the same value and get rid of the fraction.}$$

Another problem: One half of a certain number added to one fifth of it makes 28. Find the number.

$$(1) \quad n/2 + n/5 = 28$$

If we multiply (1) by 2, we shall be rid of one denominator.
 If we multiply (1) by 5 we shall be rid of the other denominator.
 If we multiply by 10 we shall be rid of both denominators. Performing this multiplication we get

$$(2) \quad 5n + 2n = 280 \quad (1) \times 10$$

Check

$$(3) \quad 7n = 280$$

$$40/2 + 40/5 =$$

$$(4) \quad n = 40 \quad (3) \div 7$$

$$20 + 8 = 28$$

\therefore 40 is the number.

It soon becomes quite evident to the pupil that if the two sides of an equation are equal, and that if we multiply both sides by the same number, the two products will still be equal. This fact the pupil uses in the solution of his fractional equations. It is a thing he discovered, however, and not a rule that he was told to follow. He learns readily to handle fractional equations with binomial numerators and with both positive and negative signs. He finds no difficulty in handling an equation of this type:

$$(1) \quad 2x/3 - (2x + 1)/7 = x - 2$$

$$(2) \quad 14x - (6x + 3) = 21x - 42 \quad (1) \times 21 \quad \text{He reasons that}$$

it was necessary to multiply $2x + 1$ by 3, and then to subtract the result. As yet he does not try to multiply and subtract at the same time.

EQUATIONS CONTAINING TWO UNKNOWNNS

The work outlined thus far took us, this year, to the Thanksgiving Recess to cover. More supplementary material from other texts might be used to advantage and thus extend the work in Simple and Fractional Equations to the Christmas Vacation.

However, the next problem brought before the pupils was this:- Five Algebras and two French books cost \$9. At the same prices, three Algebras and two French books cost \$7. Find the cost of each book.

If a = no. \$ 1 Algebra costs

and if f = " " 1 French book costs

$5a$ = " " 5 Algebras cost

$2f$ = " " 2 French books cost

also $3a$ = " " 3 Algebras cost

From this information we get this pair of equations:

$$(1) \quad 5a + 2f = 9$$

$$(2) \quad 3a + 2f = 7$$

From anything we have learned thus far we can not solve this pair of equations unless we can get rid of one of the letters so that we have only one unknown left. Can you see any process, addition, subtraction, multiplication, or division, which will enable us to do this? Yes, almost before the question is asked

someone suggests that we subtract the second equation from the first. Following this suggestion we get:

$$(3) \quad 2a = 2 \quad (1) - (2)$$

$$(4) \quad a = 1 \quad (3) \div 2$$

This gives us the value of a . To find the corresponding value of f we substitute 1 for a in (1).

$$(5) \quad 5 + 2f = 9 \quad (4) \text{ substituted in } (1)$$

$$(6) \quad 2f = 4 \quad (5) - 5$$

$$(7) \quad f = 2 \quad (6) \div 2$$

These answers check in both (1) and (2).

Problems of this type should follow for practice. These problems at first, should have either the coefficients of the first terms the same, or the coefficients of the second terms the same. After a little this should not be true, and then the pupil will discover that he must make the two coefficients of one letter the same before he adds or subtracts the equations.

ADDITION AND SUBTRACTION IN ALGEBRA

The examples in the preceding exercises show us that addition and subtraction are sometimes necessary in solving equations. You should become skillful in adding and subtracting positive and negative numbers. This fact is very apparent to the pupil and he has a real incentive for learning to add and subtract algebraic numbers.

Add:

$$\begin{array}{r}
 4 \quad -4 \quad 2a \quad -2a \quad -5y \quad 3x \quad -x \\
 \underline{3} \quad \underline{-3} \quad \underline{3a} \quad \underline{-5a} \quad \underline{-6y} \quad \underline{x} \quad \underline{-7x}
 \end{array}$$

When the signs of the terms to be added are alike, the addition is the same as in Arithmetic except that we prefix what sign? A pupil in answering this question formulates his own rule for the addition of signed numbers. He does not learn a rule first and then attempt to apply it. His rules are logical conclusions which come as the result of experience and thought, and are not mere words which he finds in bold type or italics at the beginning of each new chapter, and which must be memorized and rigidly and mechanically followed.

The two following examples illustrate another situation in the addition of signed numbers and call for the formation of another rule.

Add:

$$\begin{array}{r} -3 \\ \underline{5} \end{array} \quad -3 + 5 \text{ gives a "balance" of } 2.$$

$$\begin{array}{r} -10 \\ \underline{7} \end{array} \quad 7 \text{ added to a shortage of } 10 \text{ leaves a shortage of } 3.$$

When the signs of the terms are different, the sign of the smaller ? or larger ? number determines the sign of the answer.

Subtract:

$$8x + 5 - (3x + 2) \quad \begin{array}{r} 8x + 5 \\ \underline{3x + 2} \end{array}$$

$$9x + 6 - (8x + 9) \quad \begin{array}{r} 9x + 6 \\ \underline{8x + 9} \end{array}$$

If we take 9 from 6 it leaves a shortage of ? . Notice that Algebra enables us to do what we considered impossible in Arithmetic.

$$9x + 7 - (4x - 5) \quad \begin{array}{r} 9x + 7 \\ \underline{4x - 5} \end{array}$$

Remember that subtracting a shortage is the same as adding.

Subtract:

$$\begin{array}{r} 8 \\ \underline{4} \end{array} \quad \begin{array}{r} 4 \\ \underline{8} \end{array} \quad \begin{array}{r} -5 \\ \underline{-3} \end{array} \quad \begin{array}{r} -5 \\ \underline{3} \end{array}$$

Add:

$$\begin{array}{r} 8 \\ \underline{-4} \end{array} \quad \begin{array}{r} 4 \\ \underline{-8} \end{array} \quad \begin{array}{r} -5 \\ \underline{3} \end{array} \quad \begin{array}{r} -5 \\ \underline{-3} \end{array}$$

Compare the upper row with the lower row and explain the rule: In subtracting, think of the sign of the subtrahend as changed and add.

LEARNING TO MULTIPLY AND SUBTRACT AT ONE STEP

In solving the following pair of equations, a pupil found the answers 3 and -5. Was he right?

$$(1) \quad 4x - 3y = 27$$

$$(2) \quad 9x + 4y = 7$$

In checking the first equation he wrote

$$4(3) - 3(-5) = 27$$

12 He then said if I do the next step it gives $-(-15)$, and when I do the following step it gives $+15$. But he also said that he thought he could multiply the -5 by -3 , and subtract the result, at one step. Can you?

Do these multiplications and subtractions at one step:-

$$-3(-6); -1(-1); -8(-12\frac{1}{2}); -8(-2); -15(-6).$$

You will observe that when we find the value of $-3(-6)$ at one step, we sometimes say that we multiply -3 by -6 . Notice that it is difficult to give a practical meaning to multiplication by a negative number. You understand what it means to multiply 3 by 6, or -3 by 6; in the first case you take 3 six times; in the second, you take -3 six times. You can give no such meaning to -3 times -6 , but you can perform the operation at one step, and you can speak of it as multiplication by -6 . Be sure that you understand this process as a multiplication and a subtraction.

The law for signs for multiplication can be readily understood and may be stated by the pupils as follows: The product of two factors with like signs is positive; the product of two factors with unlike signs is negative.

GRAPHS

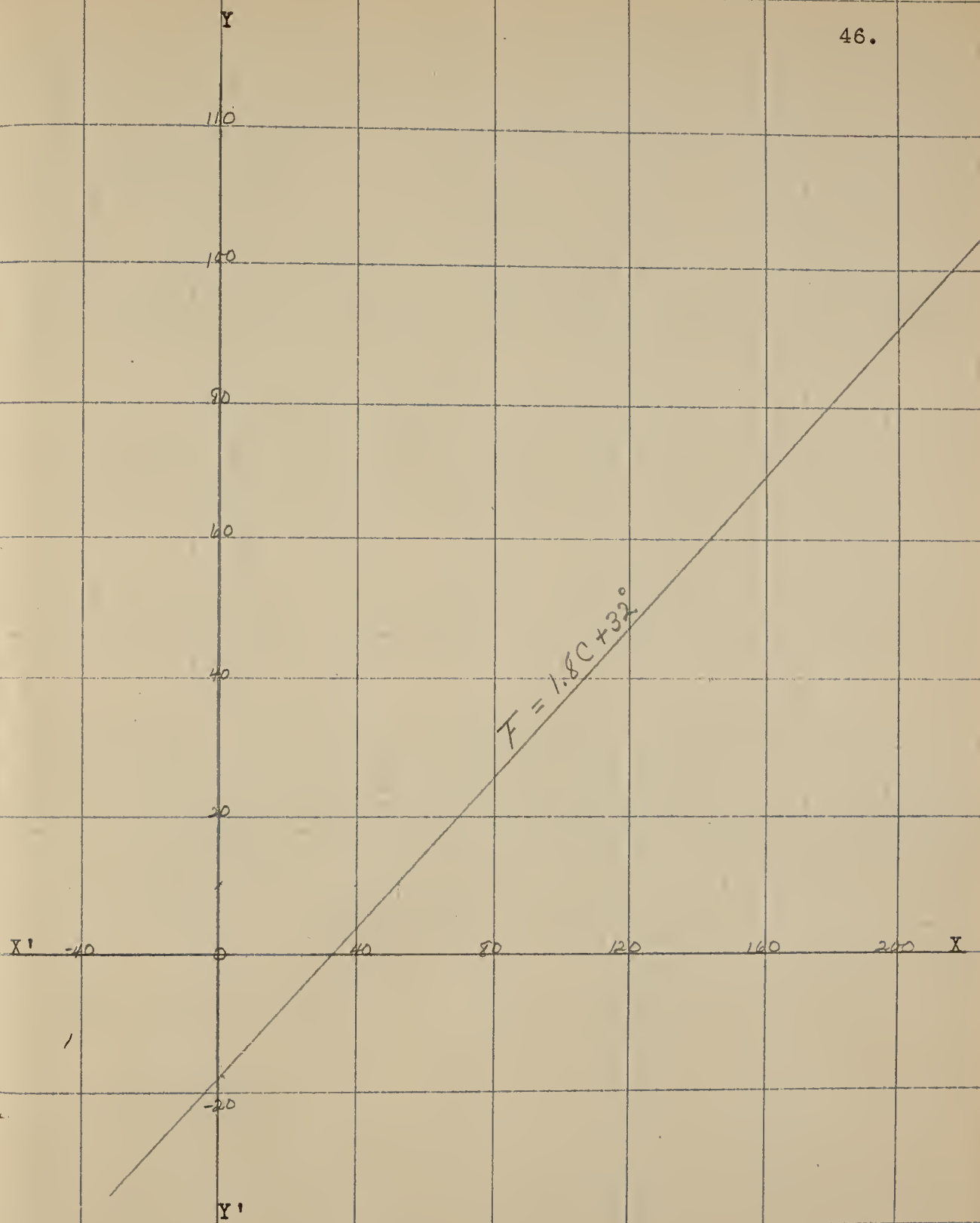
The relations between numbers may often be made more vivid by pictures because it teaches through the eye. Such a picture is called a graph. We are already familiar with the formula $F = 1.8C + 32^{\circ}$, expressing the relation between the Fahrenheit and Centigrade thermometers.

We can substitute values for C and then find the corresponding values for F as in the table below:

C	F
-5	23
0	32
5	41
10	50
15	?
20	?
40	?
100	?

On the horizontal line XX' mark off to the right of the point O the values above zero of the Fahrenheit thermometer and to the left the values below zero of the Fahrenheit thermometer. On the vertical line YY' , which passes through the point O, mark off above O the values above zero of the Centigrade thermometer and below O the values below zero of the Centigrade thermometer.

If we locate the points just found in our table on a piece of squared paper and join the points, we have the graph of the above formula. The graph pictures the relation between



The temperatures as recorded by the Centigrade and Fahrenheit thermometers.

Answer the following questions by reading the graph of the Centigrade and Fahrenheit thermometers.

- a. When the Fahrenheit reading is 40° , about what is the Centigrade reading?
- b. When the Centigrade reading is 20° , about what is the Fahrenheit reading?
- c. When $C = -25^{\circ}$, what is F ?
- d. Why is it necessary to go below the horizontal axis?
- e. Why is it necessary to go to the left of the vertical axis?
- f. The normal temperature of the body is 98° Fahrenheit.

What is the corresponding Centigrade reading?

- g. The boiling point of water is 212° F.

What is the boiling point on the Centigrade thermometer?

- h. Water freezes at 0° on the Centigrade scale. At what temperature does it freeze on the Fahrenheit scale?

The question now arises, can a problem have a negative answer?

Suppose we are speaking about profit and loss. How would you interpret an answer of -3 dollars? Suppose it was a problem about gain and loss by a football team. How would you interpret an answer of -6 yards? Give other illustrations. A number which is preceded by a minus sign is called a negative number. When the temperature on the Fahrenheit scale is 14° ,

what is the reading on the Centigrade scale? Your answer will be a negative number. Explain what it (-10) means. How does the graph represent negative numbers?

Another problem: Draw two axes, on one of which x extends to 20 and on the other y extends to 20. Find a point for which $x = 2$ and $y = 5$. Locate the point $(4,6)$. (This means the point where $x = 4$ and $y = 6$). Locate the points $(3,4)$; $(0,2)$; $(15,15)$; $(20,0)$. Practice until you can locate any point quickly. Select any point on the chart and mention the two numbers which describe it or locate it. Be sure to mention the value of x before the value of y .

Now extend the x scale to -20 at the left of the origin, and the y scale to -20 below the origin. Locate these points: $(-5,-4)$; $(5,-4)$; $(7,-3)$; $(-15,2)$. Continue this exercise until you can quickly locate a point in any of the four quarters or quadrants.

SOLVING PROBLEMS AND EQUATIONS BY GRAPHS

We found that the graph of the formula $F = 1.8C + 32^{\circ}$ was a straight line. In a similar way we can draw graphs of such equations as:

$$1. \quad 2x + y = 4$$

$$2. \quad -7x + 4y = 3$$

$$3. \quad ax + by = c$$

These equations which contain the first power and no higher power of an unknown number are called linear equations of the first degree, and we shall find that their graphs are always straight lines.

It is interesting to see how easily graphs solve pairs of equations containing two unknown quantities. Consider this problem: Find the cost of a barrel of apples and a barrel of potatoes when one barrel of apples and four of potatoes cost \$12. and three barrels of apples and two of potatoes cost \$16.

There are five quantities to list. Since the cost of a barrel of apples and of a barrel of potatoes have no known relation in our problem, we shall use two unknowns, a and p . Explain each:

a = no. \$ 1 barrel of apples cost.

$3a$ = " " 3 " " " "

p = " " 1 " " potatoes "

$4p$ = no. \$ 4 barrels of potatoes cost.

$2p$ = " " 2 " " " "

Explain equations (1) and (2). Show that each is a translation of one statement of the problem.

$$(1) \quad a + 4p = 12$$

$$(2) \quad 3a + 2p = 16$$

To solve these equations we will draw their graphs. Fill in the blanks in the table.

$$(1) \quad a + 4p = 12$$

$$\text{or} \quad a = 12 - 4p$$

a	p
?	0
0	?
?	1

$$(2) \quad 3a + 2p = 16$$

$$\text{or} \quad 3a = 16 - 2p$$

a	p
?	0
0	?
?	5

Draw the two graphs on the same pair of axes. Find from the graphs, the values of a and p at their intersection. Test them to see if they will check in both (1) and (2). Test them in the problem. Do they meet both the conditions of the problem?

The pairs of equations in the following examples might have come from problems somewhat similar to the problems above. Have each pupil state a problem for each example, and solve by graphs.

$$1. \quad \begin{aligned} x + 3y &= 7 \\ 2x + 5y &= 12 \end{aligned}$$

$$2. \quad \begin{aligned} 2x + 4y &= 16 \\ 3x + 5y &= 21 \end{aligned}$$

$$3. \quad \begin{aligned} 8x + 9y &= 69 \\ 3x - y &= 4 \end{aligned}$$

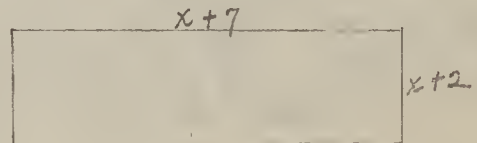
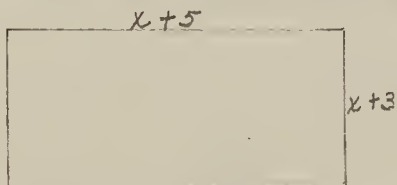
$$\begin{array}{l} 4. \quad x - y = 5 \\ \quad 4x + 3y = 41 \end{array}$$

Having introduced the subject of graphs by means of the formula and having shown the usefulness of graphs in the solution of pairs of linear equations, the subject of commercial graphs may well be considered. The pupil gets a great deal of pleasure and much valuable training in looking up statistical data and in the making of bar, line, and circle graphs. He also enjoys finding graphs already made and interpreting them.

EQUATIONS CONTAINING x^2

You have already learned what equations are for and much about how they do their work. You have learned to solve linear equations with one unknown and with two unknowns. We are now to learn about a third kind of equation, namely, equations containing x^2 . As you know, x^2 comes from multiplying x by x . This new kind of equation often results from multiplication.

The two rectangles represented below are known to have the same area. Let us see if we can find their dimensions.



$(x + 3)(x + 5) =$ no. sq. ft. in area of first.

$(x + 2)(x + 7) =$ " " " " " " second.

$$(1) \quad (x + 3)(x + 5) = (x + 2)(x + 7) \quad \text{Why?}$$

$$(2) \quad x(x + 5) + 3(x + 5) = x(x + 7) + 2(x + 7)$$

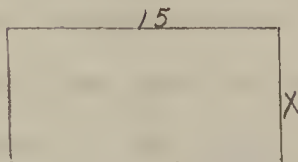
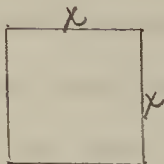
$$(3) \quad x^2 + 5x + 3x + 15 = x^2 + 7x + 2x + 14$$

$$(4) \quad 8x + 15 = 9x + 14 \qquad (4) - x^2$$

Complete and check. (In solving linear equations with one unknown, you may now be able to take two or more steps at once).

Another problem: A rectangle is 15' long and of unknown width. A square of the same width contains 36 square feet

less than the rectangle. What is the width of the rectangle?



Your list should contain the area of the square and of the rectangle. Can you state a fact which can be translated into an equation? (In stating it, use the word equals.)

(1) $15x - x^2 = 36$ State (1) in words. This equation may be written in this form:

(2) $0 = x^2 - 15x + 36$ (1) + $x^2 - 15x$

(3) $0 = (x - 12)(x - 3)$ Multiply these two factors in order to make sure that the right-hand member has the same value as in (2). Equation (3) will be true if $x - 12 = 0$, or if $x - 3 = 0$. Show it.

(4) $x - 12 = 0$

(5) $x = 12$ (4) + 12

(6) $x - 3 = 0$

(7) $x = 3$ (6) + 3

Check both answers. There are two answers to the equation in our problem. The rectangle may be either 12' wide or 3' wide.

This problem gave us a new kind of equation, called a quadratic equation. Quadratic means "having to do with a square". A complete quadratic expression or equation contains an x^2 term, an x term, a known term and no other terms. The

solution of a quadratic equation gives two answers or roots. We shall see whether they both will always check the equation, and whether they both will always meet the conditions of the problem. In order to get (3) from (2) we factor the right-hand number. Factoring is the reverse of the process of multiplication. We shall now proceed to learn to factor so that we may be able to solve quadratic equations in this way.

In multiplying $x + 5$ by $x + 3$ we get the identity $(x + 5)(x + 3) = x^2 + 8x + 15$. Where did the 8 come from? Where did the 15 come from?

In the identity $(x + a)(x + b) = x^2 + (a + b)x + ab$, a stands for any number and b stands for any number. In the right-hand member explain the coefficient of x . Explain the last term.

To the student of Algebra this identity says that when we multiply $x +$ any number by $x +$ any number, the coefficient of the x (in the product) will be the sum of the two numbers, and the last term will be the product of these two numbers.

Factoring is the reverse of multiplication. To factor $x^2 + 8x + 15$ we write $(x + \quad)(x + \quad)$. Then we find the pairs of factors of 15 and select two whose sum is 8.

The pupil should now practice factoring problems of this type.

Study the two identities below and explain:

$$(x - 4)(x - 3) = x^2 - 7x + 12.$$

$$(x - a)(x - b) = x^2 - (a + b)x + ab.$$

As soon as the pupil has a thorough understanding of the steps taken in these two identities he should practice factoring problems of this type.

Study the three identities given below and explain:

$$(x + 5)(x - 3) = x^2 + 2x - 15$$

$$(x - 5)(x + 3) = x^2 - 2x - 15$$

$$(x + a)(x - b) = x^2 + (a - b)x - ab$$

Why does the last term in each identity have a minus sign?

Why does the first $2x$ have a plus sign and the second $2x$ have a minus sign?

To factor $x^2 + 2x - 15$ we first write $(x + \quad)(x - \quad)$.

How do we know that one sign will be plus and the other minus? Second, we find the pairs of factors of 15 to be ? and ? Third, we select a pair whose difference is 2.

The pupil is now ready to practice factoring problems of this type. As soon as some skill has been acquired in these three types of factoring further practice should be given in solving quadratics by factoring. The pupil will discover that this is a very practical application of his factoring skills.

Study this multiplication until you can tell where each term in the product came from:

$$\begin{array}{r}
 2x + 5 \\
 \underline{3x - 7} \\
 6x^2 + 15x \\
 \quad - 14x - 35 \\
 \hline
 6x^2 + \quad x - 35
 \end{array}$$

$$(2x + 5)(3x - 7) = 6x^2 + x - 35$$

Practice this kind of multiplication until you can give the products readily. We are now ready to factor this kind of quadratic trinomial.

RESULTS

In accordance with the plan given on pages 7 and 9 five Achievement Tests were given to each of the classes in beginning Algebra. These tests are some which I have prepared after studying various Standard Achievement Tests. The results of these tests may be found in the accompanying tables. These tests cover all the objectives and abilities as formulated in Outline # 2 pages 12 to 19, with the exception of the subjects of Graphs and Simultaneous Equations.

The first column of Table # 1 below lists the tests given as Numbers 1, 2, 3, 4, and 5. Each class is given two columns. The left hand column for each class shows the average number of examples tried in each of the five tests. The right hand column shows the average percentage of correct answers received by each class in each of the five tests.

In Test # 1 (see page 74), which contains 10 questions, Class # I tried an average number of 8 examples, Class # II tried an average number of 7 examples, Class # III tried an average number of 9 examples, Class # IV tried an average number of 8 examples, and Class # V tried an average number of 8 examples. The highest average in the number of examples attempted was 9, and this by Class # III. The lowest average in the number of examples attempted was 7, and this by Class # II. Classes # I, # IV, and # V, each attempted an average of 8 examples.

In Test # 2 (see page 75), which contains 10 questions, Class # I tried an average number of 7 examples, Class # II an average of 6 examples, Class # III an average of 8 examples, Class # IV an average of 7 examples, and Class # V an average of 8 examples. Here the highest average in the number of examples attempted was 8, and this number by Class # III as well as by Class # V. Class # II attempted the smallest average number of examples which was 6. Classes # I and # IV each attempted an average of 7 examples.

In Test # 3 (see page 76), which contains 10 questions, Class # I tried an average of 8 examples, Class # II an average of 7 examples, Class # III an average of 7 examples, Class # IV an average of 7 examples, and Class # V an average of 7 examples. In this test the highest average in the number of examples attempted was 8, and this by Class # I. Each of the other classes attempted an average number of 7 examples.

In Test # 4 (see page 77), which contained 12 questions, Class # I tried an average number of 10 examples, Class # II an average of 9 examples, Class # III an average of 12 examples, Class # IV an average of 10 examples, and Class # V an average of 11 examples. In this test the time allowed for completion was evidently too long as the entire number of examples in the test was attempted by Class # III. Class # V attempted an average number of 11 examples, and Classes # I and # IV each attempted an average of 9 examples.

In Test # 5 (see page 78), which contained 8 questions, Class # I tried an average number of 6 examples, Class # II an average number of 5 examples, Class # III an average of 7 examples, Class # IV and average of 6 examples, and Class # V an average of 7 examples. In this test the highest average in the number of examples attempted was 7 and this by Classes # III and # V. The lowest average in the number of examples attempted was 5 and this by Class # II. Classes # I and # IV each attempted an average of 7 examples.

The table shows that there is not a very wide variation in the average number of example attempted. All five classes appear to be fairly equal in the rate at which they work. Class # III is apparently the fastest working group as they either lead or tied in the largest number of examples attempted. The slowest working group is apparently Class # II as they attempted the smallest number of examples. They were not, however, far behind in the average number of examples attempted as they never were less than one example behind in any one of the five tests.

Table # 1

	Class # 1		Class # II		Class # III		Class # IV		Class # V	
	Av No	Av %	Av No	Av%	Av No	Av %	Av No	Av %	Av No	Av %
Test	Tried	Right	Tried	Right	Tried	Right	Tried	Right	Tried	Right
#1	8	75	7	70	9	55	8	56	8	56
#2	7	73	6	75	8	54	7	57	8	60
#3	8	70	7	70	7	57	7	57	7	57
#4	10	68	9	70	12	66	10	70	11	68
#5	6	75	5	73	7	50	6	50	7	50

The right hand column for each class shows the average percentage of correct answers for each test. Here there is a wide variation in the percentages received except in Test # 4.

In Test # 1 the average percentage of correct answers for Class # I was 75, the average percentage of correct answers for Class # II was 70, the average percentage of correct answers for Class # III was 55, the average percentage of correct answers for Class # IV was 56, and the average percentage of correct answers for Class # V was 56.

In Test # 2 the average percentage of correct answers received by Class # I was 73, by Class # II 75, by Class # III 54, by Class # IV 57, and by Class # V 60.

In Test # 3 the average percentage of correct answers received by Class # I was 70, by Class # II 70, by Class # III 57, by Class # IV 57, and by Class # V 57.

In Test # 4 the average percentage of correct answers received by Class # I was 68, by Class # II 70, by Class # III 66, by Class # IV 70, and by Class # V 68.

In Test # 5 the average percentage of correct answers received by Class # I was 75, by Class # II 73, by Class # III 50, by Class # IV 50, and by Class # V 50.

The highest average percentage received by any one class is 75 and the lowest 50, a variation of 25%. It will be observed that there is a relatively close agreement between the

marks received in Classes # I and # II. Class # I received two ratings of 75%, one rating of 73%, one rating of 70%, and one rating of 68%. Class # II received one rating of 75%, one rating of 73%, and three rating of 70%.

There is also a relatively close agreement between the marks received in Classes # III, # IV, and # V. Excepting in Test # 4, in which all five classes rated between 70% and 66%, Class # III received one rating of 57%, one rating of 55%, one rating of 54% and one rating of 50%. Class # IV received two ratings of 57%, one rating of 56%, and one rating of 50%. Class # V received one rating of 60%, one rating of 57%, one rating of 56%, and one rating of 50%.

It is thus observed from this data that the average percentage of correct answers obtained in Classes # I and # II is much higher than the average percentage of correct answers received in Classes # III, # IV, and # V. Furthermore, it will be observed from the following Table # 2 that the Classes # I and # II, although ranking highest in the Achievement Tests as indicated above, have the lowest averages in I. Q.

Table # 2, below, shows that Class # I has an average I. Q. of 99, Class # II has an average I. Q. of 96, Class # III has an average I. Q. of 107, Class # IV has an average I. Q. of 101, and Class # V has an average I. Q. of 100.

Table # 2

Class #	I	Average I. Q. -	99
Class #	II	Average I. Q. -	96
Class #	III	Average I. Q. -	107
Class #	IV	Average I. Q. -	101
Class #	V	Average I. Q. -	100

This table shows that the highest average I. Q. for any one class was 107 and that the lowest average I. Q. for any one class was 96. There is then a very close agreement between the I. Q. for each class, In fact there is such a close agreement in the I. Q. for each class that any difference in the I. Q. cannot account for any difference in accomplishment.

By using the average percentage of correct answers in each of the five Achievement Tests as listed in Table # 1 page 59 Table # 3 has been made. Table # 3, below, shows that Class # I ranked highest in Test # 1, ranked second in Test # 2, was tied for first place with Class # II in Test # 3, was tied for third place with Class # V in Test # 4, and ranked highest in Test # 5. Class # II ranked second in Test # 1, ranked highest in Test # 2, was tied for first place with Class # I in Test # 3, was tied for first place with Class # IV in Test # 4, and ranked second in Test # 5. Class # III ranked last in Test # 1, also in Tests # 2 and # 4,

was tied for third place with Classes # IV and # V in both Test # 3 and in Test # 5. Class # IV was tied for third place with Class # V in Test # 1, ranked fourth in Test # 2, was tied for third place with Classes # III and # V in both Tests # 3 and # 5, and was tied for first place with Class # II in Test # 4. Class # V was tied for third place with Class # IV in Test # 1, ranked third in Test # 2, was tied for third with Classes # III and # IV for third place in both Tests # 3 and # 5, and was tied for third place with Class # I in Test # 4.

Table # 3

	Class ranking 1st	Class ranking 2nd	Class ranking 3rd	Class ranking 4th	Class ranking 5th
Test 1	I	II	IV & V	-	III
Test 2	II	I	V	IV	III
Test 3	I & II	-	III, IV, V	-	-
Test 4	II & IV	-	I & V	-	III
Test 5	I	II	III, IV, V	-	-

Using Table # 3, above, as a basis of comparison we may let 5 represent the highest possible score that any one class can attain, 4 represent the second highest, 3 represent third place, 2 represent fourth place, and 1 represent fifth place. Then, since Class # I was first in Test # 1, second in Test # 2, tied for first in Test # 3, tied for third in Test # 4, and first in Test # 5, the total score of Class # I is 18. This is shown in Table # 4 below. Since Class # II was second in

Test # 1, first in Test # 2, tied for first in both Test # 3 and Test # 4, and second in Test # 5, the total score of Class # II is 18. In the same way Class # III obtains a total score of 5, Class # IV obtains a total score of 8, and Class # V obtains a total score of 8. These scores are summarized as follows:-

Table # 4

Score for Class # I	5	+	4	+	$2\frac{1}{2}$	+	$1\frac{1}{2}$	+	5	=	18
Score for Class # II	4	+	5	+	$2\frac{1}{2}$	+	$2\frac{1}{2}$	+	2	=	18
Score for Class # III	1	+	1	+	1	+	1	+	1	=	5
Score for Class # IV	$1\frac{1}{2}$	+	2	+	1	+	$2\frac{1}{2}$	+	1	=	8
Score for Class # V	$1\frac{1}{2}$	+	3	+	1	+	$1\frac{1}{2}$	+	1	=	8

This shows very clearly the close agreement between Classes # I and # II, the two classes that are following Outline # 3 (the revised plan), and also the close agreement between Classes # III, # IV, and # V, the three classes that are following Outline # 1 (the old Plan). It also brings out the wide variation between the classes using Outline # 3 and the classes using Outline # 1.

DISCUSSION OF RESULTS

It seems necessary at this point to set up a proper defense of the facts shown and noted in the part of this thesis just preceding, namely, Results.

The effort to demonstrate the superior worth of the revised arrangement of topics and method of development necessitated the employment of the following factors one of which - the revision factor - is left free, the others are allowed to operate in the experiment only under control or check.

Early in the course Terman Group Test

Mental Capacity of Mental Ability - Form A - was given
of Classes to each individual in the five classes.

A copy of this test will be found at the end of this thesis. It is a test prepared by Lewis M. Terman of the Stanford University in California. It is a standard test designed to measure the mental ability of individuals in Grades 7 to 12. The results of the test as given follow:-

Student	I. Q.	Student	I. Q.	Student	I. Q.
#1	100	#12	100	#23	109
#2	89	#13	84	#24	106
#3	122	#15	104	#25	99
#4	101	#16	92	#26	91
#5	98	#17	106	#27	89
# 6	88	#18	103	#28	85
#7	107	#19	105	#29	89
#8	113	#20	86	#30	84
#9	107	#21	89	#31	87
#10	100	#14	103	#32	99
#11	99	#22	96	#33	90

Student	I. Q.	Student	I. Q.	Student	I. Q.
#34	103	#66	100		
#35	103	#67	118	#99	102
#36	86	#68	93	#100	122
#37	95	#69	107	#102	106
#38	91	#70	110	#103	85
#39	101	#71	96	#104	110
#40	96	#72	86	#105	90
#41	93	#73	100	#106	100
#42	85	#74	93	#107	91
#43	85	#75	108	#108	104
#44	85	#76	93	#109	104
#45	101	#77	106	#110	100
#46	90	#78	98	#111	94
#47	103	#79	117	#112	91
#48	82	#80	98	#113	113
#49	114	#81	99	#114	108
#50	95	#82	91	#115	100
#51	94	#83	87	#116	120
#52	98	#84	87	#117	100
#53	101	#85	93	#118	110
#54	117	#86	100	#119	115
#55	99	#87	102	#120	101
#56	124	#88	85	#121	84
#57	101	#89	101	#122	81
#58	89	#90	100	#123	82
#59	99	#91	109	#124	100
#60	112	#92	98	#125	89
#61	109	#93	106	#126	101
#62	98	#94	107	#127	99
#63	115	#95	94	#128	83
#64	102	#96	114		
#65	115	#97	108		
		#98	97		

The list of students making up the five Making of Sections classes was arranged in the office of the principal and had no connection whatever with the mental classification of the individuals. In fact the Terman Group Test of Mental Ability was not given to the Algebra Classes until the third week of the school year.

The five classes in beginning Algebra were under the direction of three

Teachers Hand-
ling Work

teachers. Class # I was taught by myself and Class # II was taught by a second teacher. These two classes fol-

lowed Outline # 3 (the revised topical outline). Inasmuch as two teachers followed the same outline throughout, the personal equation has been eliminated. This fact is further emphasized by the close agreement of the results of Classes # I and # II as given in the Tables on pages 59 and 62.

Classes # III, # IV, and # V were taught by a third teacher. These three classes followed Outline # 1 (the topical order as given in the Table of Contents of our text).

Text and
Material
Covered

The same text - First Course in Algebra - Hawkes, Luby, and Tauton - was used in each of the five classes. The same objectives and abilities as formulated in Outline # 2 were stressed in the teaching of the various topics, Outline # 2 being a comprehensive outline covering the work for First Year of Algebra. The records of this experiment do not cover the whole year's work. The work with Graphs and Simultaneous Equations has been omitted as the work includes only a record of the first 24 weeks of school.

Achievement
Tests

At varying intervals throughout the year each class was given the same set of achievement tests. These tests were prepared to test the several abilities and objectives

listed in Outline # 2 with the exception of the subjects of Graphs and Simultaneous Equations. These tests are not standardized tests but are some which I have prepared after studying a large number of standard achievement tests. The following tests are among those that have proved helpful in this work:-

Courtis Standard Tests.

Dearborn Group Test of Intelligence.

Hotz Algebra Scales.

Illinois Standardized Algebra Tests.

Monroe Diagnostic Tests in Arithmetic.

Monroe Standardized Reasoning Tests in Arithmetic.

National Intelligence Tests.

Otis Group Intelligence Tests.

Rogers Test of Mathematical Ability.

Rugg and Clark Tests in First Year Algebra.

Stone Reasoning Tests.

Woody Arithmetic Scales.

Woody - McCall Mixed Fundamentals.

Classes # III, # IV, and # V began at the beginning of our textbook - Hawkes, Luby, and Method of Touton - and studied the various topics as Procedure they were presented by the authors. In other words they received early in the course a large number of definitions of terms which were to be used in the course. They also found rules to be followed in the various operations which they

were to perform. The operations of addition, subtraction, multiplication, and division with positive and negative numbers were early ones to be performed. The objectives as formulated in Outline # 2 (pages 12 and 19) were stressed throughout the whole course.

Classes # I and # II did not begin on the first page of the textbook. Through the medium of the Formula they were introduced to the subject of Algebra. The Formula is the first topic listed in Outline # 3 (the revised topical order) page 22. It is used to introduce the subject of Algebra and to show what Algebra really is. It leads directly into the solution of simple equations. These are solved by a series of logical steps without the suggestion of juggling or artificiality. Simple equations are followed by equations containing fractions and these by linear pairs. The subject of linear equations affords the objective for addition and subtraction, and also for graphs. Quadratic equations, which follow directly in order, furnish the incentive for multiplication, division, and factoring. The last two named topics are taught as the opposite of multiplication. And in this way the four fundamental operations of addition, subtraction, multiplication, and division are introduced when the student needs them as an aid in his work. Thus to an unusual degree each topic is linked to, and its treatment developed from, the topic immediately preceding. This gives a logical sequence in which each idea or process is developed at the moment when it is needed, and put to work at once.

Briefly stated Classes # III, # IV, and # V followed Outline # 1 (the old order) and Classes # I and # II followed Outline # 3 (the Revised order).

The entire group used the same textbook throughout the year, had the same objectives stressed in the study of the various topics, took the same Group Test at the same time, and were examined by the same Achievement Tests at the same time.

All the classes have been given the same material. The order of presenting it has been different for Classes # III, # IV, and # V from that for Classes # I and # II. The former have followed Outline # 1 (the old order) and the later have followed Outline # 3 (the revised order).

CONCLUSION

The only varying factor in this experiment is the order of presenting the material to the classes. With the mental capacities of the classes involved in close agreement, with the teachers handling the work arranged in check, with the text and materials covered the same for all classes, with the same Achievement Tests given to all classes at the same time, and with Classes # I and II showing higher rank on the basis of these tests than Classes # III, # IV, and # V, it seems fair to conclude that an improvement in the understanding and achievement of pupils in these beginning classes in Algebra has been produced by changing the order in the presentation of the material.

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Wentworth - Junior High School Mathematics.

Young - The Teaching of Mathematics.

TEST # 1

Place a circle around the answer you believe correct.

Use the margin of the paper for figuring if necessary.

1. The cost of n articles at c cents is, $n+c$; nc ; n/c
 c/n ; $n-c$.
2. A basketball team played n games and lost 15. The
number of games won was $n+15$; $15-n$; $n/15$; $15n$.
3. A man is x years of age. His age 8 years from now will
be $8x$; $x-8$; $8-x$; $x/8$; $x+8$.
4. The quotient of a divided by b is, ab ; $a-b$; a/b ; b/a .
5. The product of 3 times a number n, diminished by 6 is,
 $n/3 - 6$; $3n-6$; $3n+6$; $3/n - 6$.
6. A boy has 50 cents and then gives away c cents. He
has $50-c$; $50/c$; $c-50$; $c/50$.
7. A rectangle is l feet long and w feet wide. Its per-
imeter is $l+w$; lw ; $2l+w$; $2l+2w$; $l+2w$.
8. Six diminished by twice a number is equal to $2n-6$;
 $2n+6$; $6-2n$; $6 - 2/n$.
9. The number of trees in an orchard of r rows is n. The
number of trees in each row is n/r ; $n-r$; r/n ; $n+r$; rn .
10. The cost c of one article when the total cost T of n
articles is known is n/T ; T/n ; T/n ; Tn .

TEST # 2

Place a circle around the answer you believe correct.

Use the margin of the paper for figuring if necessary.

1. In the formula $V = lwh$, if $l = \frac{1}{2}$, $w = 6$, $h = 5$,
then V is 15, 60, 20, 12, 30.
2. In the formula $V = e^3$, if $e = 2$, then V is 6, 8, 12, 4, 10.
3. In the formula $A = \frac{1}{2}bh$, if $b = 0.5$, $h = 12$, then A is
3. 0.3, 30, 12, 6.
4. In the formula $r = d/t$, if $d = 100$, $t = 20$, then r is
 $1/5$, 5, 10, 20, 25.
5. In the formula $S = 6e^2$, if $e = 2$, then S is 12, 144,
24, 12, 36.
6. In the formula $V = Bh$, if $B = 60$, $h = 1/3$, then
 V is 180, 20, 30, 60, 120.
7. In the formula $S = 2\pi rh$, if $h = 10$, $r = 14$, then
 S is 880, 440, 220, 22, 15. (use $\pi = 22/7$).
8. In the formula $t = d/r$, if $d = 80$, $r = 5$, then t is
400, 40, 80, 16, 75.
9. In the formula $A = lw$, if $l = 0.5$, $w = 8$, then A is
40, 4, 0.4, 16, $\frac{1}{2}$.
10. In the formula $A = \frac{1}{2}h(b + b')$, if $b = 6$, $b' = 4$, $h = \frac{1}{5}$,
then A is 10, 5, $5/2$, 20, $2/5$.

TEST # 3

Place a circle around the answer you believe correct.

Use the margin for figuring if necessary.

1. If $2x+1=7$, then $x = 4, 3, 5, 7/2, 1/2$.
2. If $x+5\frac{1}{2}=6$, then $x = 11\frac{1}{2}, 1\frac{1}{2}, \frac{1}{2}, 10\frac{1}{2}, 15$.
3. If $z+2=6$, then $z = 8, 3, 4, 12, 2$.
4. If $2x-3=2$, then $x = 2/5, 1/2, 2, 5/2, 3$.
5. If $x-0.5=6$, then $x = 11, 1, 6.5, 5.5, 0.3$.
6. If $y-1\frac{1}{2}=3\frac{1}{2}$, then $y = 4, 2, 3, 5, 6$.
7. If $2y-8=1$, then $y = 4\frac{1}{2}, 7, 11, 18, 5$.
8. If $n-2=1.6$, then $n = 3.6, 0.8, 3.2, 1.8, 4$.
9. If $cx-6=4$, then $x = 2c, 10c, 10+c, 10/c, 10-c$.
10. If $x-2\frac{1}{3} = 3\frac{1}{3}$, then $x = 1, 3, 6, 5\frac{2}{3}, 5\frac{1}{3}$.

TEST # 4

Place a circle around the answer you believe correct.

Use the margin of the paper for figuring if necessary.

1. The algebraic sum of 4 and -9 is, -5, 5, -4, +14.
2. If $x+3=8$, then $x =$ 5, 7, 4, 3, -3.
3. The product of $a^3 \cdot a^4$ is $1/a$, a^{12} , a , $3a$, a^7 .
4. If $x^2-9=0$ then $x =$ 0, 1, 3, 9, -1.
5. a^5/a^3 is the same as, a^8 , a^2 , $5/3$, a^3 .
6. $(a-b)^2 = a^2+2ab+b^2$, a^2-ab+b^2 , a^2-b^2 , $a^2-2ab+b^2$.
7. The sum of $5a-8a+10a-12a$ is, $3a-2a$, $2a$, $-25a$, $-5a$.
8. $(-2a)^3 = 8a^3$, $-8a$, $-2a^3$, $-6a^3$, $-8a^3$.
9. $-8a^{10}/2a^2 = -4a^5$, $-4a^8$, $-6a^8$, $6a^5$, $-8a^3$.
10. Given $a=3$, $b=1$, $c=5$, then $2a-5b+c$ equals 16, 10, 6, 8, 9.
11. $(9x-18x^4)/-3x = 3-6x^3$, $-3-6x^3$, $-3+6x^3$, $3x-6x^3$.
12. When $xm=n$, then $x = mn$, m/n , n/m , $m-n$.

TEST # 5

Place a circle around the answer you believe correct.

Use the margin of the paper for figuring if necessary.

1. $(x+3)(x-5) = x^2-8x-15, x^2-2x+15, x^2+2x-15, x^2-2x-15.$
2. $(2x-1)(2x+1) = 4x^2-4x+1, 4x^2-1, 4x^2+4x+1.$
3. $(4-x^2) = (2x-1)^2, (2-x)(2+x), (2-x)^2, (2+x)^2.$
4. $(7-x)(x+7) = 49-x^2, x^2-49, x^2-14x+49, x^2+49.$
5. $(x^2+y^2) = (x+y)(x+y), (x+y)(x-y), (x^2+y^2), x(x+y)$
6. $c^2-cd-6d^2 = (c-2d)(c+2d), (c-3)(c+2), (c-3d)(c+2d).$
7. $(x^3-4x) = x(x^2-4x), x^2(x-4), x(x-2)(x+2), 2x(x-2).$
8. $(x^2-8x+16) = (x+8)(x-2), (x+4)^2, (x-8)(x-2), (x-4)^2.$

TERMAN GROUP TEST OF MENTAL ABILITY

For Grades 7 to 12

Prepared by Lewis M. Terman, Stanford University, California

EXAMINATION: FORM A

1. Name
First name Last name
2. Boy or girl Grade High or Low
3. Age last birthday Date of birthday
Month Day Year
4. Name of city (or county)
5. Name of school
6. Name of teacher
7. Date of this examination 19.....
Month Day Year

Do not turn the page until you are told to.

TEST	SCORE	REMARKS OR FURTHER DATA
1. Information		
2. Best Answer		
3. Word Meaning		
4. Logical Selection		
5. Arithmetic		
6. Sentence Meaning		
7. Analogies		
8. Mixed Sentences		
9. Classification		
10. Number Series		
Total		

TEST 1. INFORMATION

Draw a line under the ONE word that makes
the sentence true, as shown in the sample.

SAMPLE. Our first President was

Adams Jefferson Lincoln Washington

- | | | | |
|----|---------------------------------------|---|----|
| 1 | Coffee is a kind of | bark berry leaf root | 1 |
| 2 | Sirloin is a cut of | beef mutton pork veal | 2 |
| 3 | Gasoline comes from | grains petroleum turpentine seeds | 3 |
| 4 | Most exports go from | Boston San Francisco New Orleans New York. | 4 |
| 5 | The number of pounds in a ton is | 1000 2000 3000 4000 | 5 |
| 6 | Napoleon was defeated at | Leipzig Paris Verdun Waterloo | 6 |
| 7 | Emeralds are usually | blue green red yellow | 7 |
| 8 | The optic nerve is for | seeing hearing tasting feeling | 8 |
| 9 | Larceny is a term used in | medicine theology law pedagogy | 9 |
| 10 | Sponges come from | animals farms forests mines | 10 |
| 11 | Confucius founded the religion of the | Persians Italians Chinese Indians | 11 |
| 12 | The larynx is in the | abdomen head throat shoulder | 12 |
| 13 | The piccolo is used in | farming music photography typewriting | 13 |
| 14 | The kilowatt measures | rainfall wind-power electricity water-power.... | 14 |
| 15 | The guillotine causes | death disease fever sickness | 15 |
| 16 | A character in "David Copperfield" is | Sindbad Uriah Heep Rebecca Hamlet | 16 |
| 17 | A windlass is used for | boring cutting lifting squeezing | 17 |
| 18 | A great law-giver of the Hebrews was | Abraham David Moses Saul | 18 |
| 19 | A six-sided figure is called a | scholium parallelogram hexagon trapezium.... | 19 |
| 20 | A meter is nearest in length to the | inch foot yard rod | 20 |

Right

TEST 2. BEST ANSWER

FORM A

Read each question or statement and make a cross
before the BEST answer, as shown in the sample.

-
- SAMPLE { Why do we buy clocks? Because
 1 We like to hear them strike.
 2 They have hands.
 X 3 They tell us the time.

- 1 Spokes of a wheel are often made of hickory because
 - 1 Hickory is tough.
 - 2 It cuts easily.
 - 3 It takes paint nicely.
- 2 The saying, "A watched pot never boils," means
 - 1 We should never watch a pot on the fire.
 - 2 Boiling takes a long time.
 - 3 Time passes slowly when we are waiting for something.
- 3 A train is harder to stop than an automobile because
 - 1 It has more wheels.
 - 2 It is heavier.
 - 3 Its brakes are not so good.
- 4 The saying, "Make hay while the sun shines," means
 - 1 Hay is made in summer.
 - 2 We should make the most of our opportunities.
 - 3 Hay should not be cut at night.
- 5 If the earth were nearer the sun
 - 1 The stars would disappear.
 - 2 Our months would be longer.
 - 3 The earth would be warmer.
- 6 The saying, "If wishes were horses, beggars would ride," means
 - 1 Wishing doesn't get us very far.
 - 2 Beggars often wish for horses to ride.
 - 3 Beggars are always asking for something.
- 7 The saying, "Little strokes fell great oaks," means
 - 1 Oak trees are weak.
 - 2 Little strokes are best.
 - 3 Continued effort brings results.
- 8 A steel battleship floats because
 - 1 The engines hold it up.
 - 2 It has much air space inside.
 - 3 It contains some wood.
- 9 The feathers on a bird's wings help him to fly because
 - 1 They make a wide, light surface.
 - 2 They keep the air off his body.
 - 3 They decrease the bird's weight.
- 10 The saying, "A carpenter should stick to his bench," means
 - 1 Carpenters should not work without benches.
 - 2 Carpenters should not be idle.
 - 3 One should work at the thing he can do best.
- 11 The saying, "One swallow does not make a summer," means
 - 1 Swallows come back for the summer.
 - 2 A single sign is not sufficient proof.
 - 3 Many birds add to the pleasures of summer.

Right × 2 = *Score*

TEST 3. WORD MEANING

When two words mean the SAME, draw a line under "SAME."
 When they mean the OPPOSITE, draw a line under "OPPOSITE."

SAMPLES	fall — drop	<u>same</u> — opposite	
	north — south	same — <u>opposite</u>	
1	expel — retain	same — opposite	1
2	comfort — console	same — opposite	2
3	waste — conserve	same — opposite	3
4	monotony — variety	same — opposite	4
5	quell — subdue	same — opposite	5
6	major — minor	same — opposite	6
7	boldness — audacity	same — opposite	7
8	exult — rejoice	same — opposite	8
9	prohibit — allow	same — opposite	9
10	debase — degrade	same — opposite	10
11	recline — stand	same — opposite	11
12	approve — veto	same — opposite	12
13	amateur — expert	same — opposite	13
14	evade — shun	same — opposite	14
15	tart — acid	same — opposite	15
16	concede — deny	same — opposite	16
17	tonic — stimulant	same — opposite	17
18	incite — quell	same — opposite	18
19	economy — frugality	same — opposite	19
20	rash — prudent	same — opposite	20
21	obtuse — acute	same — opposite	21
22	transient — permanent	same — opposite	22
23	expel — eject	same — opposite	23
24	hoax — deception	same — opposite	24
25	docile — submissive	same — opposite	25
26	wax — wane	same — opposite	26
27	incite — instigate	same — opposite	27
28	reverence — veneration	same — opposite	28
29	asset — liability	same — opposite	29
30	appease — placate	same — opposite	30

Right Wrong Score

TEST 4. LOGICAL SELECTION

FORM A

In each sentence draw a line under the TWO words that tell what the thing ALWAYS has. Underline TWO, and ONLY TWO, in each line.

- SAMPLE. A man always has
body cap gloves mouth money
- 1 ~~A~~ horse always has
 harness hoofs shoes stable tail 1
 - 2 A circle always has
 altitude circumference latitude longitude radius 2
 - 3 A bird always has
 bones eggs beak nest song..... 3
 - 4 Music always has
 listener piano rhythm sound violin 4
 - 5 An object always has
 smell size taste value weight 5
 - 6 Conversation always has
 agreement persons questions wit speech 6
 - 7 A banquet always has
 food music persons speeches toastmaster 7
 - 8 A pistol always has
 barrel bullet cartridge sights trigger..... 8
 - 9 A ship always has
 engine guns keel rudder sails 9
 - 10 ~~A~~ debt always involves
 creditor debtor interest mortgage payment 10
 - 11 A game always has
 cards contestants forfeits penalties rules..... 11
 - 12 A magazine always has
 advertisements paper pictures print stories..... 12
 - 13 A museum always has
 animals arrangement collections minerals visitors..... 13
 - 14 A forest always has
 animals flowers shade underbrush trees 14
 - 15 A citizen always has
 country occupation privileges property vote..... 15
 - 16 Controversy always involves
 claims disagreement dislike enmity hatred..... 16
 - 17 War always has
 airplanes cannons combat rifles soldiers 17
 - 18 Obstacles always bring
 difficulty discouragement failure hindrance stimulation .. 18
 - 19 Abhorrence always involves
 aversion dislike fear rage timidity 19
 - 20 Compromise always involves
 adjustment agreement friendship respect satisfaction ... 20

Right.....

TEST 5. ARITHMETIC

Find the answers as quickly as you can.
Write the answers on the dotted lines.
Use the bottom of the page to figure on.

- 1 How many hours will it take a person to go 66 miles at the rate of 6 miles an hour? *Answer*
- 2 At the rate of 2 for 5 cents, how many pencils can you buy for 50 cents? *Answer*
- 3 If a man earns \$20 a week and spends \$14, how long will it take him to save \$300? *Answer*
- 4 $2 \times 3 \times 4 \times 6$ is how many times as much as 3×4 ? *Answer*
- 5 If two pies cost 66 cents, what does a sixth of a pie cost? *Answer*
- 6 What is $16\frac{2}{3}$ per cent of \$120? *Answer*
- 7 4 per cent of \$1000 is the same as 8 per cent of what amount? *Answer*
- 8 A has \$180, B has $\frac{2}{3}$ as much as A, and C has $\frac{1}{2}$ as much as B. How much have all together? *Answer*
- 9 The capacity of a rectangular bin is 48 cubic feet. If the bin is 6 feet long and 4 feet wide, how deep is it? *Answer*
- 10 If it takes 7 men 2 days to dig a 140-foot ditch, how many men are needed to dig it in half a day? *Answer*
- 11 A man spends $\frac{1}{4}$ of his salary for board and room, and $\frac{3}{8}$ for all other expenses. What per cent of his salary does he save? *Answer*
- 12 If a man runs 100 yards in 10 seconds, how many feet does he run in $\frac{1}{5}$ of a second? *Answer*

Right $\times 2 =$ *Score*

TEST 6. SENTENCE MEANING

FORM A

Draw a line under the right answer, as shown in the samples.

SAMPLES	{	Is coal obtained from mines?	<u>Yes</u>	No	
		Are all men six feet tall?	Yes	<u>No</u>	
1		Does a conscientious person ever make mistakes?	Yes	No	1
2		Is an alloy a kind of musical instrument?	Yes	No	2
3		Is scurvy a kind of medicine?	Yes	No	3
4		Are mysterious things often uncanny?	Yes	No	4
5		Are destitute persons often subjects of charity?	Yes	No	5
6		Are anonymous letters ever properly signed?	Yes	No	6
7		Is the mimeograph sometimes used by stenographers? ..	Yes	No	7
8		Is a curriculum intended for horses?	Yes	No	8
9		Are proteids essential to health?	Yes	No	9
10		Does "perfunctory" mean the same as "careful"? ..	Yes	No	10
11		Are premeditated deeds always wicked?	Yes	No	11
12		Do alleged facts often require verification?	Yes	No	12
13		Are sheep carnivorous?	Yes	No	13
14		Are aristocrats subservient to their inferiors? ...	Yes	No	14
15		Are venerable people usually respected?	Yes	No	15
16		Is clematis sometimes cultivated?	Yes	No	16
17		Are ultimate results the last to appear?	Yes	No	17
18		Are cerebral hemorrhages helpful to thinking?	Yes	No	18
19		Are all people religious who have hallucinations?	Yes	No	19
20		Are intermittent sounds discontinuous?	Yes	No	20
21		Are sable colors preferred for nations' flags?	Yes	No	21
22		Does social contact tend to reduce eccentricities?	Yes	No	22
23		Are tentative decisions usually final?	Yes	No	23
24		Is rancor usually characterized by persistence?	Yes	No	24

Right.....Wrong.....Score.....

TEST 7. ANALOGIES

SAMPLES	{	Ear is to hear as eye is to
		table <u>see</u> hand play
	{	Hat is to head as shoe is to
		arm coat <u>foot</u> leg

Do them all like samples.

-
- | | | | |
|----|----------------------------|------------------------------------|----|
| 1 | Coat is to wear as | bread is to | |
| | | eat starve water cook..... | 1 |
| 2 | Week is to month as | month is to | |
| | | year hour minute century | 2 |
| 3 | Monday is to Tuesday as | Friday is to | |
| | | week Thursday day Saturday..... | 3 |
| 4 | Tell is to told as | speak is to | |
| | | sing spoke speaking sang | 4 |
| 5 | Lion is to animal as | rose is to | |
| | | smell leaf plant thorn..... | 5 |
| 6 | Cat is to tiger as | dog is to | |
| | | wolf bark bite snap..... | 6 |
| 7 | Success is to joy as | failure is to | |
| | | sadness luck fail work | 7 |
| 8 | Liberty is to freedom as | bondage is to | |
| | | negro slavery free suffer | 8 |
| 9 | Cry is to laugh as | sadness is to | |
| | | death joy coffin doctor | 9 |
| 10 | Tiger is to hair as | trout is to | |
| | | water fish scales swims..... | 10 |
| 11 | 1 is to 3 as | 9 is to | |
| | | 18 27 36 45..... | 11 |
| 12 | Lead is to heavy as | cork is to | |
| | | bottle weight light float | 12 |
| 13 | Poison is to death as | food is to | |
| | | eat bird life bad | 13 |
| 14 | 4 is to 16 as | 5 is to | |
| | | 7 45 35 25..... | 14 |
| 15 | Food is to hunger as | water is to | |
| | | drink clear thirst pure | 15 |
| 16 | b is to d as | second is to | |
| | | third later fourth last..... | 16 |
| 17 | City is to mayor as | army is to | |
| | | navy soldier general private | 17 |
| 18 | Here is to there as | this is to | |
| | | these those that then..... | 18 |
| 19 | Subject is to predicate as | noun is to | |
| | | pronoun adverb verb adjective..... | 19 |
| 20 | Corrupt is to depraved as | sacred is to | |
| | | Bible hallowed prayer Sunday | 20 |

Right.....

TEST 8. MIXED SENTENCES

FORM A

The words in each sentence below are mixed up. If what a sentence means is TRUE, draw a line under "TRUE." If what it means is FALSE, draw a line under "FALSE."

SAMPLES	{	hear are with to ears	<u>true</u>	false	
		eat gunpowder to good is	true	<u>false</u>	
1		true bought cannot friendship be	true	false	1
2		good sea drink to is water	true	false	2
3		of is the peace war opposite.	true	false	3
4		get grow they as children taller older.	true	false	4
5		horses automobile an are than slower.	true	false	5
6		never deeds rewarded be should good	true	false	6
7		four hundred all pages contain books.	true	false	7
8		to advice sometimes is good follow hard	true	false	8
9		envy bad greed traits are and	true	false	9
10		grow an than strawberries oak tree higher	true	false	10
11		external deceive never appearances us	true	false	11
12		never is man what show a deeds.	true	false	12
13		hatred bad unfriendliness traits are and	true	false	13
14		often judge can we actions man his by a	true	false	14
15		in are always American cities born presidents	true	false	15
16		certain always death of cause kinds sickness	true	false	16
17		are sheet blankets as as a never warm.	true	false	17
18		never who heedless those stumble are	true	false	18

Right *Wrong* *Score*

TEST 9. CLASSIFICATION

SAMPLES { 1 bullet cannon gun sword pen~~cil~~
 2 Canada Chi~~cago~~ China India France

In each line cross out the word that does not belong there.
 Cross out JUST ONE WORD in each line.

1	Frank James John Sarah William	1
2	Baptist Catholic Methodist Presbyterian Republican ..	2
3	automobile bicycle buggy telegraph train	3
4	Collie Holstein Shepherd Spitz Terrier	4
5	hop run skip stand walk	5
6	death grief picnic poverty sadness	6
7	bed chair dish sofa table	7
8	hard rough smooth soft sweet	8
9	mechanic doctor lawyer preacher teacher	9
10	Christ Confucius Mohammed Moses Cæsar	10
11	butterfly hawk ostrich robin swallow	11
12	cloth cotton flax hemp wool	12
13	digestion hearing sight smell touch	13
14	down hither recent up yonder	14
15	anger hatred joy pity reasoning	15
16	Australia Cuba Iceland Ireland Spain	16
17	Dewey Farragut Grant Paul Jones Schley	17
18	give lend lose keep waste	18

Right.....

TEST 10. NUMBER SERIES

$$\text{SAMPLES } \left\{ \begin{array}{ccccccc} 5 & 10 & 15 & 20 & 25 & .30. & .35. \\ 20 & 18 & 16 & 14 & 12 & .10. & .8. \end{array} \right.$$

In each row try to find out how the numbers are made up, then on the two dotted lines write the TWO numbers that should come next.

1st Row				8	7	6	5	4	3
2d Row				3	8	13	18	23	28
3d Row				$11\frac{3}{4}$	12	$12\frac{1}{4}$	$12\frac{1}{2}$	$12\frac{3}{4}$	
4th Row				8	8	6	6	4	4
5th Row				1	2	4	8	16	32
6th Row				4	3	5	4	6	5	7
7th Row				16	8	4	2	1	$\frac{1}{2}$
8th Row				8	9	12	13	16	17
9th Row	7	11	15	16	20	24	25	29	
10th Row	31.3	40.3	49.3	58.3	67.3	76.3			
11th Row				$\frac{1}{25}$	$\frac{1}{5}$	1	5		
12th Row				3	4	6	9	13	18

Right $\times 2 = \text{Score}$

