Modeling and Modifying Day-to-Day Travel Behaviors: Empirical Results and Methodological Advances

MODELING AND MODIFYING DAY-TO-DAY TRAVEL BEHAVIORS: EMPIRICAL RESULTS AND METHODOLOGICAL ADVANCES

A Dissertation Presented

by

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Dedicated to my family.

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ABSTRACT

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The increasing availability of individual-level longitudinal data provides the opportunity to better understand travelers' day-to-day learning process of their choice alternatives, which enables potentially more accurate predictions of choice patterns in a network with uncertainties. In this thesis, an instance-based learning (IBL) model for travel choice is developed within route-choice context, where on each day a traveler's decision depends on her entire choice history in the past. Learning in this model is based on the power law of forgetting and practice, which is shown to be capable of capturing various psychological effects embedded in travelers' day-to-day learning process, including the recency effect, hot stove effect and payoff variability effect. Estimation results based on empirical data show that the IBL model reveals

higher sensitivity to perceived travel time and achieves better model fit compared to a baseline learning model. Cross-validation experiments suggest that the forecasting ability of the IBL model is consistently better than the baseline learning model.

Despite the above-mentioned advantages of the IBL model, the common problem of missing initial observations in longitudinal data collection can lead to inconsistent estimates of perceived value of attributes in question, and thus inconsistent parameter estimates. In this thesis, the stated problem is addressed by treating the missing observations as latent variables. The proposed method is implemented in practice as maximum simulated likelihood (MSL) correction with two sampling methods in an instance-based learning model for travel choice, and the finite sample bias and efficiency of the estimators are investigated. Monte Carlo experimentation based on synthetic data shows that both the MSL with random sampling (MSLrs) and MSL with importance sampling (MSLis) are effective in correcting for the endogeneity problem in that the percent error and empirical coverage of the estimators are greatly improved after correction. The methods are applied to an experimental route-choice dataset to demonstrate their empirical application. Hausman-McFadden tests show that the estimators after correction are statistically equal to the estimators of the full dataset without missing observations, confirming that the proposed methods are practical and effective for addressing the stated problem.

Apart from modeling travelers' day-to-day learning process for travel choice, day-to-day driving behavior intervention is also studied in this thesis. A study of Mitigation Techniques to Modify Driver Performance to Improve Fuel Economy, Reduce Emissions and Improve Safety was undertaken as part of the Massachusetts Department of Transportation (MassDOT) Research Program. Major conclusions include: 1) Real-time feedback has a significant effect in reducing speeding and aggressive acceleration. 2) Training has a significant effect in reducing idling rate in the first month after training. 3) Combining training and feedback is expected to significantly improve fuel economy, reduce emissions and improve safety.

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CHAPTER 1

INTRODUCTION

1.1 Background and Motivation

Travelers make choice decisions based on their knowledge about the environment that is mainly learned through experience and constrained by their cognitive capability. The decision-making process is believed to be dynamic and involves consistent information acquisition and learning. For example, a newcomer to a city makes route-choice following the GPS device's recommendation, while after becoming a seasoned resident, she can recall past experience when making repeated choice and connect existing route segments to form a new route even if the destination is new. Thus, the formation of the decision-making process is indispensable in understanding travelers' choice behavior and predicting the overall choice patterns. In the meantime, the ever-increasing availability of smartphones and other wearable sensors provides abundant individual-level longitudinal data to help improve and validate travel choice models.

The mainstream travel choice models mainly focus on cross-sectional analysis of choice behaviors, where the impact of random attribute variability (e.g., travel time) on repeated choice is either completely ignored or inadequately captured. A number of studies have been conducted since the route-choice learning model was first introduced to the transportation community. Such studies either focus on the theoretical analysis of the convergence properties of the models or are inconsistent with the psychological findings on human memory decay. Thus, learning models that are able to sufficiently utilize the individual-level longitudinal data to precisely capture travelers' day-to-day learning process following mainstream psychological findings are in great demand for more accurate choice pattern predictions.

In such learning models, a traveler's perception of an alternative's attribute (e.g., travel time) evolves over time based on all her past experience with the alternative. As such, estimation of such a model requires data of travelers' complete choice histories. Longitudinal data collection in real life, however, inevitably starts midstream. Specialized data collection targeted at newcomers (e.g., new employees or students) to a region might provide the needed data, but such efforts are difficult to implement. The missing initial observations can cause endogeneity problem, which leads to inconsistent estimate of the perceived value of the attribute in question, and thus inconsistent parameter estimates. The serial correlation of error terms over time further complicates the problem. Therefore, correction methods that address the methodological challenge imposed by the complete history dependency of choices need to be developed for consistent estimation of learning model parameters with missing initial observations.

At a higher level, transportation has a major impact on our society and environment, contributing 70% of U.S. petroleum use, 28% of U.S. greenhouse gas (GHG) emissions of Transportation Statistics. (2013), and over 34,000 fatalities and 2.2 million injuries in 2011 (n.d.). In addition to the use of more fuel-efficient vehicles and alternative fuels, fuel consumption and CO_2 emissions can be lowered through promoting eco-driving, which typically involves of operating a vehicle in a more efficient, safe and environmentally friendly manner. Therefore, aside from modeling travelers' day-to-day route-choice behavior, this thesis also aims to gain insights on driving behavior intervention to promote eco-driving. Specifically, A project undertaken as part of the Massachusetts Department of Transportation Research Program is complete. The objective of the project is to adopt static and dynamic mitigation techniques to modify driver performance to improve fuel economy, reduce emissions and improve safety. The effectiveness of the two general types of techniques is evaluated based on their performance cross test and control groups. Recommendations are made on the widespread deployment of real-time feedback devices and eco-driving training programs. Furthermore, the individual-level longitudinal GPS data obtained from this project can be utilized to validate learning models in future studies.

1.2 Thesis Contributions

The contributions of the thesis to the knowledge base of learning-based models for travel choice are summarized as follows:

1. The original IBL model for simplified binary lottery choices is extended to an econometric model for travel choice in a general route-choice network, where spatial learning is explicitly considered and rigorous statistical tests can be performed.

2. The proposed IBL model is able to capture the learning attributes residing in travelers' choice behaviors that are either overlooked or misinterpreted in existing route-choice models. The adaptation of a psychologically sound learning theory enables a better understanding of the impact of travel time variability on repeated route-choice.

3. Empirical data is used to compare the proposed model to an existing learning model and demonstrate the model's applicability.

The contributions of this thesis to the knowledge base of the intial condition problem in learning models with complete history dependency are summarized as follows:

1. A practical and theoretically sound correction method is developed and assessed using the proposed IBL model. To the best of our knowledge, the stated problem is tackled for the first time.

2. Two sampling methods are proposed for the correction method, with the aim of avoiding the problem of the curse of dimensionality that arises as the number of missing days grows.

3. The suitability of the proposed method is confirmed using Monte Carlo experimentation on synthetic data, and its applicability is demonstrated using a laboratory experimental dataset.

Besides the modeling aspect of travelers' choice behaviors, this thesis also gains insights on intervening day-to-day driving behavior to promote safe and eco-driving. A large scale field project with a three-phase experiment was conducted. Both static and dynamic mitigation techniques were adopted and evaluated across test and control groups. Insightful recommendations are made for driver intervention to promote safe and eco-driving.

1.3 Thesis Structure

This thesis is structured as follows. The IBL model is proposed first, followed by the correction method for the initial condition problem in learning models with complete history dependency. Then, the MassDOT project that aims to intervene day-to-day driving behavior to promote safe and eco-driving is introduced. The thesis is closed with conclusions and future research directions.

Chapter 2 develops an IBL model for day-to-day travel choice. A literature review on travel time variability and route-choice models and the instance-based learning theory (IBLT) is provided first to show the gap between existing travel choice models and mainstream psychological findings. The model is then specified within a binary route-choice context with perceived travel time being the only attribute that evolves over time and other attributes assumed fixed from day to day. The model features, including its capability of capturing recency effect, hot stove effect, and payoff variability effect are illustrated. Then, computational experiments based on synthetic data are conducted to show that the model parameters can be consistently retrieved and ignoring learning can result in different predictions of overall traffic patterns. The IBL model is then compared with a baseline learning model using an experimental dataset of repeated route-choice to show that the IBL model achieves better model fit and has better forecasting ability.

Chapter 3 develops and assesses a correction method for the initial condition problem in learning models with complete history dependency. A literature review on endogeneity and importance sampling is provided first. The cause of the intial condition problem is then illustrated using the IBL model developed in Chapter 2. The estimation biases of the parameters are demonstrated using synthetic data. The correction method with two sampling approaches, i.e., the MSLrs and MSLis, are proposed within the IBL framework and their effectiveness and computational efficiency are assessed using Monte Carlo experimentation. Sensitivity analysis are conducted to investigate the impact of sampling size in random sampling and number of high probability choice sequences in importance sampling. In the end of this chapter, the proposed correction methods are applied to empirical data to prove their applicability and effectiveness.

CHAPTER 2

AN INSTANCE-BASED LEARNING (IBL) MODEL FOR TRAVEL CHOICE

Availability of individual-level longitudinal data provides the opportunity to better understand travelers' day-to-day learning behavior, enabling more accurate predictions of traffic patterns in a network with random travel times. Most econometric route-choice models focus on cross-sectional analysis of route-choice behaviors, where travel time variability is either ignored or assumed static over time. A number of studies have been conducted on learning models for route-choice in recent years. However, the weighting scheme of past experience is often inconsistent with the mainstream psychological theory. Therefore, learning models that are able to sufficiently utilize the individual-level longitudinal data to precisely capture travelers' day-to-day learning process following psychological findings are in great demand for more accurate traffic pattern predictions.

In this chapter, an IBL model for day-to-day travel choice is developed. A literature review on travel time variability and route-choice models and the instance-based learning theory (IBLT) is provided first to show the gap between existing route-choice models and mainstream psychological findings. The model is then specified within a binary route-choice context with perceived travel time as the only attribute that evolves over time and other attributes assumed fixed from day to day. The model features, including its capability of capturing recency effect, hot stove effect, and payoff variability effect are illustrated. Then, computational experiments based on synthetic data are conducted to show that the model parameters can be consistently retrieved and ignoring learning can result in different predictions of overall traffic patterns. The

IBL model is then compared with a baseline learning model using an experimental dataset of repeated route-choice to show that the IBL model achieves better model fit and has better forecasting ability.

2.1 Literature Review

2.1.1 Travel time variability and route-choice models

Travel times are inherently uncertain, due to random disruptions such as incidents and bad weather, and random behavior of travelers. The psychological literature has distinguished two types of decision under uncertainty/risk. The first is decision from description, where the probabilistic distribution of the payoff for each option is explicitly described to the decision maker, e.g., a 50% chance of winning $100 and a 50% chance of losing $20. Bounded-Rational theories, such as Prospect Theory (Kahneman & Tversky, 1979; Tversky & Kahneman, 1992), have been applied to study decision under description and travel choice modeling, e.g., Ben-Elia & Shiftan (2010) and Gao et al. (2010). The second type is decision from experience, where the uncertain outcomes of chosen actions are experienced by instead of described to decision makers. Past studies have shown that decision from experience and decision from description can result in very different, sometimes even opposite risk attitudes (Barron & Erev, 2003; Erev & Barron, 2005; Rakow & Newell, 2010).

Route-choice decision making is a typical example of decision from experience. Travelers make route-choice decisions based on their knowledge about the environment that is mainly learned through experience and constrained by their cognitive capabilities. The decision-making process is dynamic and involves information acquisition and assimilation. For example, a newcomer to a city follows a GPS device's recommendation. However, once becoming a seasoned resident, she can recall past experience and connect existing route segments to form a new route even if the destination is new. The process of learning about the decision environment is indispensable in understanding travelers' route-choice behavior and predicting the overall resulting traffic patterns. Meanwhile, the ever-increasing availability of smartphones and other portable sensors provides individual-level longitudinal data to help improve and validate route learning and choice models.

Most econometric route-choice models focus on cross-sectional analysis of route-choice behaviors, e.g., Path-Size Logit (Ben-Akiva & Ramming, 1998; Ben-Akiva & Bierlaire, 1999), C-Logit (Cascetta et al., 1996), Cross-Nested Logit (Vovsha & Bekhor, 1998), and Logit Mixture Ramming (2001); Bekhor et al. (2002); Frejinger & Bierlaire (2007). Travel time variability, if considered, is usually static and travelers are assumed to have the same knowledge of travel time distribution, such that the temporal relation between the current choice and past experience are ignored (Abdel-Aty et al., 1995; Bates et al., 2001; Lam & Small, 2001; Liu et al., 2004; Gan & Bai, 2014; Tilahun & Levinson, 2001; Carrion & Levinson, 2012; Fosgerau, 2015).

A number of studies have been conducted since the route-choice learning model was first introduced to the transportation research community. Some studies focus on theoretical analysis of the convergence properties of the models and thus impose relatively strong assumptions on learning and choice behavior without considering travelers' actual cognitive capacity (Horowitz, 1984; Cascetta & Cantarella, 1991; Yang & Zhang, 2009). Most empirical studies are conducted using experimental data on single-origin-destination (OD) networks with two or three routes and minimum overlapping (Avineri & Prashker, 2005; Bogers & van Zuylen, 2005; Ben-Elia & Shiftan, 2010; Lu et al., 2011, 2014), with the exception of a series of studies by Mahmassani and collaborators where there are successive switching options between three parallel routes (Mahmassani & Liu, 1999). Some simulation studies deal with more general networks, but the critical problem of spatial knowledge acquisition and its impact on route-choice in a realistic network setting is not properly addressed (Ben-Akiva et al., 1991; Emmerink et al., 1995; Jha et al., 1998; Ben-Elia & Avineri, 2015).

The weighting scheme of past experience has evolved along with learning models over time. Horowitz (1984) proposed using a weighting scheme to quantify the relative importance of the recent and distant travel experience, yet no specific psychological theories was referred to validate the weighting scheme. Later on, both Chang & Mahmassani (1988) and Iida et al. (1992) found that the more recent travel experience is more important than distant travel experience. However, they did not explicitly analyze how travelers develop perceptions of travel time variability. More recent learning models often embed perception updating mechanisms to quantify the weighting scheme of past experience. The dominant descriptive models in the literature (in contrast to a normative model such as Bayesian updating) speculate that the perceived travel time at time t is a convex combination of the perceived travel time and experienced travel time at time $t-1$ (Ben-Akiva et al., 1991; Emmerink et al., 1995; Nakayama et al., 2001; Avineri & Prashker, 2005; Bogers & van Zuylen, 2005; Lu et al., 2014). The convex combination updating is equivalent to an assumption of exponential decay of memory, which is inconsistent with the psychological theory that human memory decay follows a power function rather than an exponential function (Wickelgren, 1976; Newell & Rosenbloom, 1981; Anderson & Schooler, 1991; Rubin & Wenzel, 1996).

2.1.2 Background of instance-based learning theory (IBLT)

The instance-based learning theory (IBLT) was developed to explain decision making in complex and dynamic situations, where individuals make repeated choices attempting to maximize gains over the long run (Gonzales & Lebiere, 2005; Gonzalez et al., 2003). An instance is broadly defined by the context, decision, and outcome of a previous choice that is encoded in the declarative memory (i.e., memories that can be consciously recalled such as facts and verbal knowledge). Learning resides in the activation mechanism that relies on the frequency and recency of past choices, i.e., more recent and frequent instances are more active in memory. According to the IBLT, a decision-making process contains the stages of matching, evaluation, selection and execution. In the matching stage, based on their levels of activation, instances that are relevant to the current decision context are retrieved and blended to produce perceptions of options. Memory decay is captured by the power law of forgetting.

IBLT is often implemented within the Adaptive Control of Thought-Rational (ACT-R) cognitive architecture (Anderson & Lebiere, 1998), which incorporates a set of mechanisms that can be used to develop models of learning and performance. The different mechanisms used to retrieve instances, evaluate alternatives, and apply feedback are central to IBLT. A number of models have been implemented within the ACT-R architecture and demonstrated close approximations to human decision making in multiple tasks (Gonzales & Lebiere, 2005; Lebiere et al., 2007; Martin et al., 2004). More recent models have been implemented to account for repeated choices (Lebiere et al., 2007; Stewart et al., 2009). An IBL model implemented in the ACT-R architecture was the winner of a competition of predicting repeated binary lottery choice decisions (Erev et al., 2010). Since the aforementioned models are limited by the complexity of the ACT-R architecture, later on, Lejarraga et al. (2012) proposed a simplified version of the winning IBL model, where the decision context is not utilized in instance retrieval.

As preferable as the aforementioned IBL models are in accounting for decision making in dynamic environments, they are all developed for experimental psychology and tailored for binary lottery choices. Route-choice in a network context is much more complex. First, multiple factors (e.g., travel cost and ease of driving) besides travel time can affect travelers' decisions and need to be accounted for. Secondly, spatial knowledge is learned over time and can be carried over from one origin-destination (OD) pair to another. Thirdly, model parameter estimation for the existing IBL models is usually conducted by fitting data at an aggregate level and thus rigorous statistical tests can not be performed.

2.2 Model Specification and Features

2.2.1 Model specification

An econometric IBL model is developed for route-choice in a general network based on the simplified version of the IBL model proposed by Lejarraga et al. (2012). For illustrative purpose, the perceived travel time is the only attribute that evolves over time and other attributes are assumed fixed from day to day. Other attributes that vary over days can be incorporated easily.

A path is composed of multiple segments and psychological studies show that people can integrate segment knowledge to obtain path knowledge (Golledge, 1999). Due to the idiosyncrasy nature of learning, system-wide traffic prediction based on learning models requires storage of a copy of network attributes for each simulated traveler. On one extreme, if experience is coded at the link level for each traveler, the model will become intractable fairly quickly. On the other extreme, if experience from all travelers is blended in a single collective memory, the important issue of spatial knowledge heterogeneity is ignored which potentially lead to misunderstanding of route-choice behaviors.

A model that trades off model realism with tractability is proposed. In a general road network, a particular day's experience is the vehicle trajectory. A major road segment generally contains a number of links (e.g., a stretch of highway between two major interchanges, an arterial road between two neighborhood centers). Experience on a major road segment is stored in a traveler's memory and is individual-specific. Experience on minor road links from all travelers is stored in a collective memory and is not individual-specific. The trajectory does not need to cover the complete major road segment that defines the instance, and prorated travel time will be used if only part of the segment is traversed. The underlying assumption is that human beings tend to use categorization to simplify knowledge representation. Spatial knowledge from one OD is naturally carried over to another OD through experience on common major road segments.

A traveler n is faced with the problem of choosing one path from a choice set D for a given OD on each day t starting from day 1. Each road segment has an underlying random travel time whose realizations are independent from day to day, and independent across segments. The traveler experiences the realized travel times on the segments of the chosen path on a given day, and has no knowledge of the realized travel time on non-chosen paths.

An instance is defined as a past experience of segment s on a chosen path on day t' and its associated outcome (realized segment travel time), $x_s(t')$. The experience is scaled up for the whole segment if only part of the segment is experienced. An instance is stored in the declarative memory of the traveler, and its activation decays over time following a power law. Specifically, on day t, its activation is $(t - t')^{-d}$, where the decay parameter d captures the rate of forgetting in that a smaller d value translates into higher activation in memory and $t - t'$ measures the recency of the experienced travel times (smaller $t - t'$ values represent higher recency)[1].

Eq. (2.1) shows the weight function of an experienced travel time from a past day t' for traveler n, where the denominator is a summation of activations over all past experiences on segment s. It shows that recency and frequency jointly define the weight of a specific travel time value, i.e. more recent and frequent experienced travel times are more active in memory. On day t, the perceived travel time of segment s is the weighted average of realized travel times of all past days when segment s

[1]The definition of activation is slightly different from its original version in Anderson et al. (2004), due to an adaption of the theory to a presentation format that the transportation research community is more familiar with. However, the final equation that determines the perceived travel time is the same as using the original definition of activation.

is experienced as shown in Eq. (2.2). For implementation simplicity, the perceived travel time of path i for traveler n on day t is assumed as the sum of the prorated perceived travel times on all segments of the path as shown in Eq (2.3). For notation simplicity, perceived travel times on all segments are indexed by individual n, however it should be noted that only major road segment perceptions are individual-specific.

$$w_{ns}(t,t') = \frac{a_{ns}(t')(t-t')^{-d_n}}{\sum_{\tau=0}^{t-1} a_{ns}(\tau)(t-\tau)^{-d}} \tag{2.1}$$

where:

t**:** index of the current day, $t = 1, ..., K$

t'**:** index of a previous day, $t' = 0, ..., t-1$

$w_{ns}(t', t)$**:** weight of the experienced travel time on day t' for the perceived travel time on day t for traveler n and segment s

d_n**:** decay parameter for traveler n that captures the rate of forgetting, $d_n > 0$

$a_{ns}(t')$**:** a binary indicator. It is 1 if traveler n chose segment s on day t' and 0 otherwise

$$b_{ns}(t) = \sum_{t'=0}^{t-1} w_{ns}(t', t) x_s(t') \tag{2.2}$$

$$b_{ni}(t) = \sum_{s} b_{ns}(t)\delta_{si} \tag{2.3}$$

where:

$b_{ns}(t)$**:** perceived travel time of segment s on day t for traveler n

$x_s(t')$**:** experienced travel time of segment s on day t

$b_{ni}(t)$**:** perceived travel time of path i on day t for traveler n

δ_{si}**:** the fraction of segment s on path i. It is equal to zero if segment s does not overlap with path i

In the IBL model proposed in Lejarraga et al. (2012), the activation is calculated for each outcome of an alternative and perturbed by a noise term (see Appendix A). The utility of the alternative is the sum of all the observed outcomes weighted by their probability of retrieval. Although the perturbed activation makes the choice probabilistic, no closed form expression exists for the choice probability. Parameter estimation is based on fitting aggregate choice shares, and thus the properties (consistency and efficiency) of the estimator cannot be established, and no rigorous statistical tests can be carried out. A mixed Logit model of IBL for route-choice is developed, where the noise is an additive term to the systematic utility instead of a multiplicative term to the activation of the perceived travel time. Maximum likelihood estimation can be then performed based on disaggregate choice data.

Eq. (2.4) shows the utility function with the parameter vector $\boldsymbol{\phi} = \{d, \beta, \boldsymbol{\alpha}'\}$, where the noise term ϵ is i.i.d. extreme over time, individual and alternatives. The systematic utility is linear in the perceived travel time b that varies from day to day and other attributes $\boldsymbol{z}$ that are constant over time (e.g., toll price and number of traffic lights). Panel effect is accounted for by the random parameters that vary over travelers but are fixed across the observations from the same traveler. Path overlap is taken care of by path size S_i. Eq. (2.5) shows the probability of individual n choosing the sequence of alternatives $I = \{i_1, i_2, \dots\}$. The coefficients vary over travelers with density function $f(\boldsymbol{\phi})$ and the unconditional choice probability is the integral of the product of conditional probabilities over all possible values of $\boldsymbol{\phi}$. The difference between the IBL model and a learning-free model is that the IBL model treats the perceived travel time as a variable that varies from day to day, while in a learning-free model perceived travel time is assumed fixed over time. It is straightforward to extend the utility function to include other attributes that vary from day to day, such as perceived fuel consumption.

$$U_{ni}(t) = V_{ni}(t) + lnS_i + \epsilon_{ni}(t) = \beta_n b_{ni}(t) + \boldsymbol{\alpha}_n' \boldsymbol{z}_{ni} + lnS_i + \epsilon_{ni}(t) \tag{2.4}$$

where:

$U_{ni}(t)$**:** random utility of path i for traveler n on day t

$V_{ni}(t)$**:** systematic utility function of path i for traveler n on day t

β_n**:** coefficient of travel time for traveler n, a random coefficient over travelers and fixed over time for a given individual

$\boldsymbol{z}_{ni}$**:** observed variables relating to path i and traveler n that do not vary from day to day

$\boldsymbol{\alpha}_n$**:** a vector of coefficients for variables $\boldsymbol{z}_{ni}$ for individual n, random coefficients over travelers

S_i**:** path size for path i

$\epsilon_{ni}(t)$**:** noise terms being i.i.d. extreme over time, individuals and alternatives

$$P_{nI} = \int_{\boldsymbol{\phi}} \left(\prod_t \frac{e^{V_{ni}(t)+lnS_i}}{\sum_j e^{V_{nj}(t)+lnS_j}} \right) f(\boldsymbol{\phi}) d\boldsymbol{\phi} \tag{2.5}$$

2.2.2 Model features

In this section the model features are demonstrated in a binary route-choice situation. To focus on illustrating the learning mechanism of the model, several assumptions are made: 1) the perceived travel time is the only variable in the systematic utility, 2) each path has only one segment, 3) parameters are fixed over travelers, and 4) there is no overlap between paths. The travel time of Path 2 is assumed deterministic to represent a highly reliable path, while Path 1 is risky with random travel time. The decay parameter d is set to 0.5, which is the conventional value used in most ACT-R models (Gonzalez & Dutt, 2011; Lejarraga et al., 2012; Erev et al., 2010). The choices on the first two days are set to Path 1 and 2 respectively so that initial perceptions of the path travel times are obtained and the learning process is triggered.

An Iillustrative example

Table 2.1 illustrates an application of the model over 5 days. The objective travel time of Path 1 is normally distributed with a mean of 20 and standard deviation of 3, and that of Path 2 is fixed at 22. The coefficient of travel time β is set to -0.4, which is in the same magnitude as empirical values in the literature Frejinger & Bierlaire (2007); Ben-Akiva et al. (2015). The initial perceptions of the two paths are gained by enforcing the selection of them on the first two days, i.e., 20.7 and 22.0 respectively. On day 3, either path has one past instance experienced, and its weight is simply 1.00. The perceived travel time of either path is therefore the realized travel time, 20.7 and 22.0 respectively. The choice probabilities of the two paths are calculated following Eq. (2.5) and through random sampling Path 2 is chosen with a realized travel time of 22.0 (Path 2 has deterministic travel time). On day 4, there is still only one instance for Path 1 from day 1 and thus the perceived travel time remains at 20.7. There are indeed two instances for Path 2, but since Path 2 is deterministic, the perceived travel time remains at 22.0. The choice probabilities remain the same as on day 4 and through random sampling Path 1 is chosen with a realized travel time of 32.3. On day 5, there are two instances for Path 1 from days 1 and 4 respectively and their weights are calculated following Eq. (2.1) as 0.449 and 0.551 respectively. The perceived travel time on Path 1 is then calculated as the weighted sum of the

Table 2.1 Application of the IBL Model in A Binary Route-Choice Network over 5 Days

Day t	Choice	Experienced travel time $x_i(t')$		Weight of experienced travel time on current day $w_{ni}(t', t)$ using Eq. (2.1)		Perceived Travel time $b_{ni}(t)$ using Eq. (2.2)		Choice probability P_{ni} using Eq. (2.5)	
		Path 1	Path 2	Path 1	Path 2	Path 1	Path 2	Path 1	Path 2
1	1	20.7						1.00	0.000
2	2		22.0					0.000	1.00
3	2		22.0	$w_{n1}(1,3) = 1.00$	$w_{n2}(2,3) = 1.00$	20.7	22.0	0.627	0.373
4	1	32.3		$w_{n1}(1,4) = 1.00$	$w_{n2}(2,4) = 0.414$ $w_{n2}(3,4) = 0.586$	20.7	22.0	0.627	0.373
5	2		22.0	$w_{n1}(1,5) = 0.333$ $w_{n1}(4,5) = 0.667$	$w_{n2}(2,5) = 0.449$ $w_{n2}(3,5) = 0.551$	28.4	22.0	0.0718	0.928

two instances as 28.4. Perceived travel time on Path 2 remains constant and choice probabilities are calculated as 0.0718 and 0.928 respectively.

Hot stove effect

The hot stove effect was first described by Mark Twain. "A cat who sits on a hot stove will never sit on a hot stove again. But he won't sit on a cold stove, either." Erev & Barron (2005) explained the hot stove effect as in the absence of information about forgone payoffs, bad outcomes have a lasting effect because they inhibit future updating of the tendency to select this alternative. In other words, bad outcomes remain in memory, thus prevent people from exploring the alternative. Multiple studies have shown the existence of hot stove effect (Barron & Erev, 2003; Erev & Barron, 2005; Fujikawa, 2009; Denrell & March, 2001).

Figure 2.1 demonstrates the hot stove effect captured by the IBL model. The travel time of Path 1 follows $Normal(20, 9)$ and that of Path 2 fixed at 22. The travel time coefficient is set at -100 so that the choice is almost deterministic. The blue solid line and red dashed line represent the perceived travel times of Path 1 and 2 respectively. The black dots represent the experienced travel times of Path 1 when it is chosen. Over the first few days, the traveler switches between the two paths depending on the perceived travel times. On day 5, however, the very bad experienced travel time of Path 1 (close to 30) makes her never choose the path again despite its shorter mean travel time. Thus, the presence of the hot stove effect leads the traveler to deviate from minimizing the expected travel time.

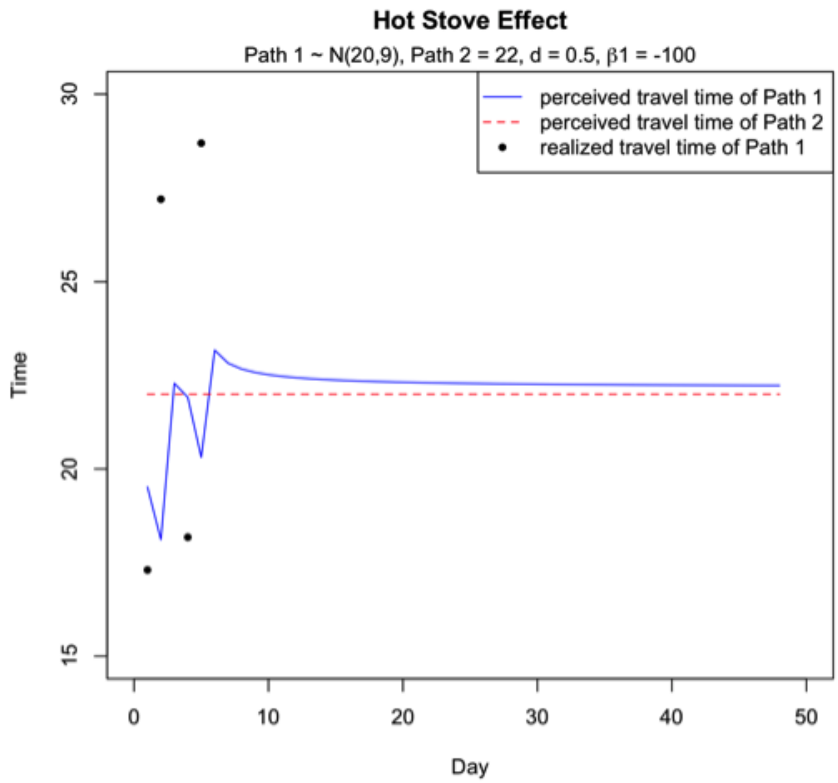


Figure 2.1 Hot Stove Effect.

Payoff variability effect

Psychological studies have found that when payoff variability is large, choice behavior moves toward random choice, and this effect is particularly strong when the variability is associated with the high payoff alternative (Erev & Barron, 2005; Haruvy & Erev, 2001). Several studies have verified the robustness of the payoff variability effect in travelers' route-choice behavior (Katsikopoulos et al., 2002; Avineri & Prashker, 2005; Ben-Elia et al., 2008). To show that the IBL model can effectively capture the payoff variability effect, the binary route-choice problems are simulated

Table 2.2 Problem setting of payoff variability effect

Problem 1	**Problem 2**
Path 1: 20 minutes with variation (low payoff)	Path 1: 20 minutes with variation (high payoff)
Path 2: 18 minutes with certainty (high payoff)	Path 2: 22 minutes with certainty (low payoff)

with various standard deviations of Path 1 travel time. For each problem, 1000 sets of 20-day choices are generated using the IBL model, and the choice probability of Path 1 on each day is calculated as the fraction of Path 1 choices out of the 1000 sets on that day.

Table 2.2 presents the problem setting of the payoff variability effect, where Path 1 is the low payoff alternative in Problem 1 and the high payoff alternative in Problem 2. The left graph in Figure 2.2 presents the result of Problem 1. When the objective travel time of Path 1 is reliable (e.g., standard deviation is 1), its choice probability stays close to 0 over time. When Path 1 has higher variability, its choice probability starts off higher and converges to 0 gradually. Therefore, Path 1 becomes more attractive when it is riskier. The right graph shows the simulation result of Problem 2. When Path 1 is highly reliable (e.g., standard deviation is 1), its choice probability is close to 1 at all time. However, as its objective travel time gets unreliable, the choice probability goes to a lower value. Therefore, Path 1 becomes less attractive when it is riskier. An interesting phenomenon is that when the objective travel time of Path 1 becomes highly unreliable (e.g., standard deviation larger than 5 shown as the blue and green lines), the choice probability decreases from over 0.5 to below 0.5 over time, which suggests that increasing the variability of the high payoff alternative could reverse the choice preference. Compared to Problem 1, the payoff variability effect in Problem 2 is much stronger. The two facets of the payoff variability effect are both well captured by the IBL model.

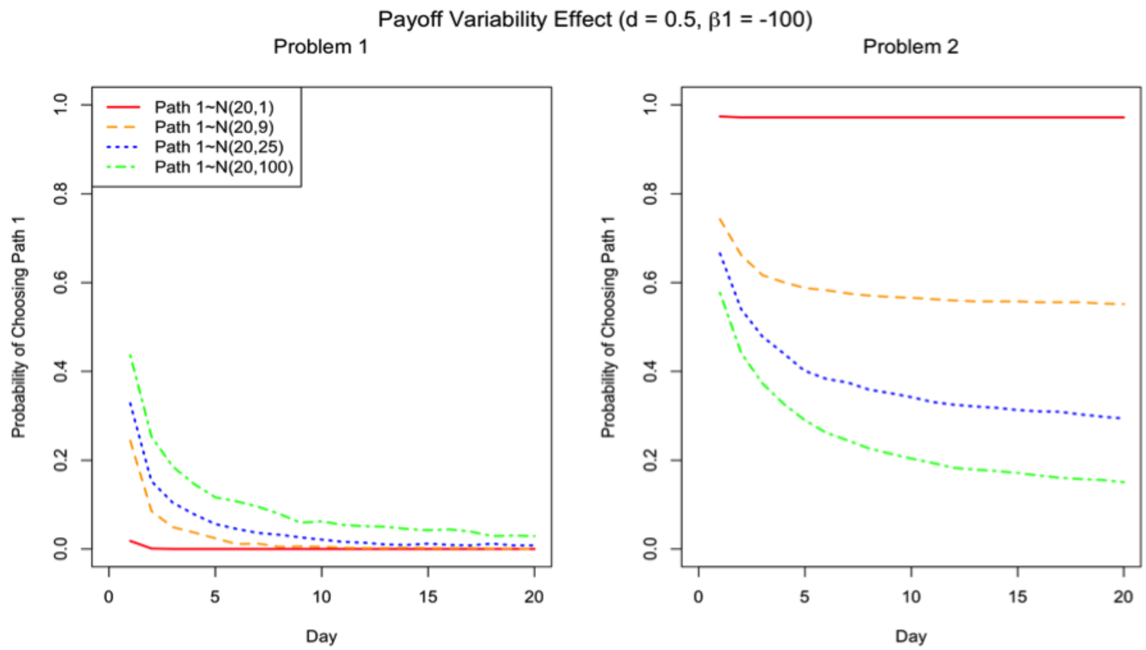


Figure 2.2 Payoff Variability Effect.

2.3 Computational Experiments Based on Synthetic Data

Synthetic data from a two-OD network in the Boston area is generated to demonstrate the identifiability of the IBL model parameters and differences in predicting traffic patterns compared to learning-free models. In Figure 2.3, the OD marked with red paddles (OD1) is a work trip from home in Watertown to Massachusetts General Hospital in Boston, with two path alternatives Path 1 and Path 2. Path 1 is an 8.4-mile local path that is composed of two major road segments Soldiers Field Road and Storrow Drive, landmarked by the Beacon Street Bridge, Boston University Bridge, and Longfellow Bridge. Path 2 is an 8.8-mile path with a considerable portion of toll road, the Massachusetts Turnpike. OD1 represents a daily work trip, and the traveler's perceptions of the travel time distributions of the two paths evolve over time. The OD marked with green paddles (OD2) is an occasional recreational trip from a friend's house in Brookline to the New England Aquarium in Boston with two path alternatives Path 3 and Path 4. The major road segments of Path 3 are defined by Commonwealth Avenue and Storrow Drive, and Path 4 contains two major road segments of Route 9 and Downtown Boston. In this experiment, spatial knowledge carryover is explicitly considered in that although the recreational trip is an entirely new OD to the traveler, its overlap with the regular work trip Path 1 (Storrow Drive passing the Hatch Shell) alters her perception of travel time distribution of Path 3.

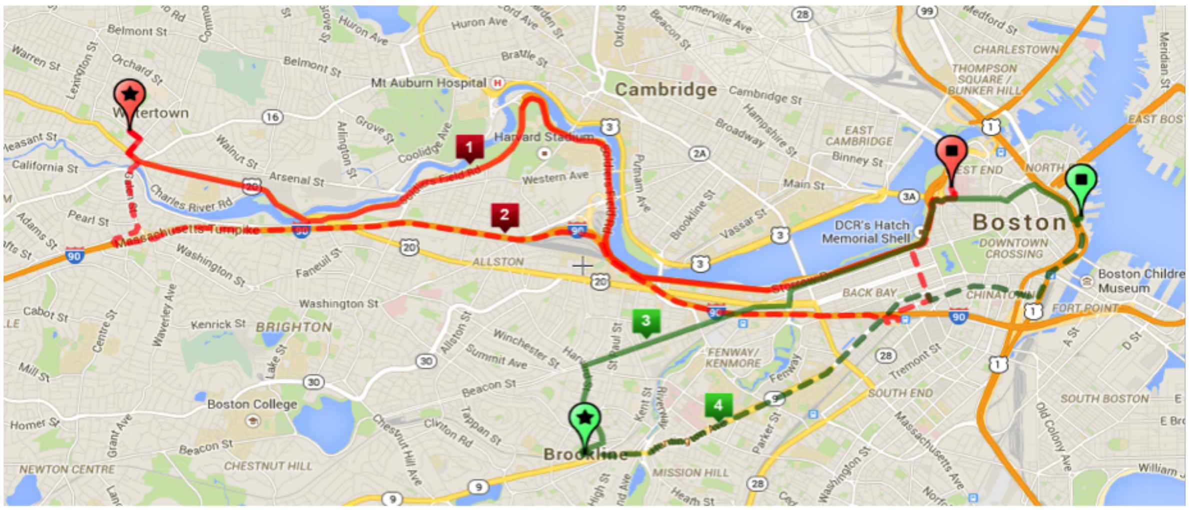


Figure 2.3 A Two-OD Network with Overlapping.

The perceived travel time, b, and toll price, c are included as explanatory variables. An error component, η, normally distributed over individuals with a mean of zero and a standard deviation of σ is added to the utility functions of Path 1 and Path 3 to account for the panel effect. The estimation parameters are $\boldsymbol{\phi} = \{\sigma, d, \beta_{time}, \beta_{cost}\}$. The original path size is used to account for path overlapping, as in Eq. (2.7) (Ben-Akiva & Bierlaire, 1999).

$$U_{ni}(t) = \beta_{time} b_{ni}(t) + \beta_{cost} c_i + lnS_i + \epsilon_{ni}(t) + \eta_{ni} \tag{2.6}$$

Where:

β_{time}: coefficient to travel time

β_{cost}: coefficient to toll price

$\epsilon_{ni}(t)$**:** error terms being i.i.d. extreme over individuals, alternatives and time

η_{ni}**:** zero-mean error components on Path 1 or 3, independent over individuals and alternatives .

$$S_i = \sum_{a \in \Gamma_i} \frac{l_a}{L_i} \frac{1}{\sum_{j \in C} \delta_{aj}} \tag{2.7}$$

Where:

S_i**:** path size for path i

Γ_i**:** the set of links in path i

l_a**:** the length of link a

L_i**:** length of path i

δ_{aj} link-path incidence variable. It is 1 if link a is on path j, 0 otherwise

2.3.1 Observation generation

The postulated true value of the decay parameter d follows its conventional value of 0.5 (Gonzalez & Dutt, 2011; Lejarraga et al., 2012; Erev et al., 2010), and those of the travel time coefficient β_{time} and toll coefficient β_{cost} are set at -0.4 (following empirical studies such as Frejinger & Bierlaire (2007); Ben-Akiva et al. (2015)) and -1.2 respectively , such that the value of time (VOT) is 0.333 $\$/min$, which is of similar magnitude as those reported in empirical studies (see, e.g., Gomez-Ibanez et al. (1999)). η_{ni} is assumed to follow a standard normal distribution, i.e., $\sigma = 1$. The toll price for each path is uniformly sampled between 0 and 20. For OD1, 50 sets of 98-day observations are generated with the postulated IBL model. For each set of observations, the travel time of Path 1 follows normal distribution with the mean uniformly sampled between 10 and 300 and the standard deviation uniformly sampled between 0.1 and 0.5 times the mean. The mean travel time of Path 2 is uniformly sampled between 0.5 and 2 times the corresponding mean travel times of Path 1, and the standard deviation uniformly sampled between 0.1 and 0.5 times the mean. For OD2, 980 sets of 5-day observations are generated after a 100-day experience with OD1. Path 3 contains an overlap segment with Path 1 and a non-overlap segment. For each set of observations, the mean and variance of the overlap segment are uniformly sampled between 0.5 and 1 times those of Path 1. The travel time on the non-overlap segment is normally distributed with the mean uniformly sampled from 5 to 100 and standard deviation uniformly sampled from 0.1 to 0.5 times the corresponding mean. The mean travel time of Path 4 is uniformly sampled between 0.5 and 2 times the corresponding mean travel times of Path 3, and the standard deviation is uniformly sampled between 0.1 and 0.5 times the mean. The dataset contains path travel times with adequate variabilities and the difference between the two mean travel times vary from negative to positive such that both the risk averse and risk seeking facets of the payoff variability effect can be captured.

2.3.2 Model estimation

Two baseline models are also estimated and later used in prediction for comparison. A mixed Logit model that assumes travelers' full information of the underlying travel time distributions is estimated for OD1. The utility function is a linear combination of the objective travel time mean, standard deviation (with a parameter β_{sd}), and toll price. Notice that the IBL model does not include explicitly a measure of travel time variability such as travel time standard deviation, as the impact of travel time variability is embedded in the learning process, such as the hot stove and payoff variability effects demonstrated in an earlier section. To show the impact of ignoring spatial knowledge carry-over from one OD to another, a no-carryover learning model that does not consider travelers' familiarity with Path 1 when traveling on OD2 is also estimated. Similar to the IBL model, a zero-mean normally distributed error component over individuals is added to the utility function of Path 1 or Path 3 for the two baseline models to account for panel effect, while all other parameters are fixed over travelers. Note that the estimation was done separately on two different ODs to isolate the impacts of two different types of simplifications. In real-life applications, data from all ODs are pooled.

Biogeme Python 2.2 Bierlaire (2003) is used for model estimation. Table 2.3 presents the estimation results. The t-tests for the IBL model are against the true values (shown in parenthesis next to the parameter) for both ODs, while the t-tests for the full-knowledge and no-carryover models are against zero. For both ODs, the IBL model can consistently retrieve the true parameter values within two standard errors. Compared to the IBL model, the adjusted ρ^2 of the two baseline models are both lower. The standard deviation of the error component is much higher in either baseline model than in the IBL model, suggesting that the baseline model that ignores part or all learning is trying to capture the heterogeneity over individuals resulting from idiosyncratic spatial knowledge through a more variable random error component. Travelers' sensitivity to travel time and toll price are underestimated. The VOTs are 0.189 $\$/min$ and 0.275 $\$/min$ respectively, which are lower than the true value of 0.333 $\$/min$. For the full-knowledge model, the numerical value of the coefficient to travel time standard deviation is very small, since it assumes a fixed risk attitude and cannot capture the payoff variability effect that manifests as both risk seeking and risk averse depending on the choice context. For the no-carryover model, the estimate of the decay parameter d is smaller than the true value. The conjecture here is that, to some extent, prior experience with Path 1 from the work trip stabilizes the perception of Path 3 that overlaps with Path 1. Since the no-carryover model ignores prior experience with Path 1, the forgetting rate has to be lower to achieve similar stability of perception.

Table 2.3 IBL Model Estimation Results based on Synthetic Data

Parameter	**OD1**		**OD2**	
(true value)	**IBL**	**Full-knowledge**	**IBL**	**No-carryover**
σ (1.0)	0.956	4.00	1.34	10.6
Robust std err	0.163	0.375	0.353	2.68
t-test	0.270	10.7	0.960	3.95
β_{time} (-0.4)	-0.376	-0.148	-0.483	-0.228
Robust std err	0.0229	0.0158	0.0586	0.0481
t-test	1.04	-9.37	-1.42	-4.74
β_{cost} (-1.2)	-1.14	-0.506	-1.45	-0.828
Robust std err	0.0710	0.0503	0.192	0.207
t-test	0.852	-10.1	-1.30	-3.99
d (0.5)	0.550		0.519	0.164
Robust std err	0.0384		0.0160	0.143
t-test	1.30		1.19	1.14
β_{sd}		-0.0428		
Robust std err		0.0170		
t-test		-2.51		
Initial log-likelihood	-3396	-3396	-3396	-3396
Final log-likelihood	-358	-1074	-122	-507
Adjusted ρ^2	0.893	0.683	0.963	0.850
No. of parameters	4	4	4	4
Sample size	4900	4900	4900	4900

* t-tests for the IBL model are against the true values;
t-tests for the full-knowledge and no-carryover models are against 0;
BIOGEME Bierlaire (2003) is used for model estimation.

2.3.3 Prediction

The path share and average travel time predicted by the four estimated models in a specific network setting are compared to show that the IBL model can lead to different prediction results that potentially better interpret travelers' risk attitude in route-choice behaviors. To avoid confusion, in this section travel time stands for the experienced travel time and objective travel time stands for the underlying travel time of a path.

All predictions are based on a period of 50 days to gain a representative picture of the traffic patterns. In OD1, the travel time of Path 1 is normally distributed with a mean of 25, and that of Path 2 is deterministic at 20 to represent a highly reliable path with a toll of \$3. In OD2, Path 1 is assumed as a segment of Path 3. The travel time of the non-overlap segment is fixed at 10 such that the travel time of Path 3 is normally distributed with a mean of 35. Path 4 has a deterministic travel time of 30 and a toll price of \$4. The stand deviation of Path 1 and Path 3 varies from 1 to 10 to represent a wide range of travel time uncertainties. At $VOT = 0.333\$/min$, Path 1 and Path 3 are the risky path with superior systematic utility in their respective OD despite their longer objective travel times, while Path 2 and Path 4 are the safe path. For each OD, 100 sets of 50-day travel times from the underlying distributions are sampled. For each set of the 50-day travel times, 200 travelers' route choices and perceived travel times are simulated following the specific models. Path share is calculated based on the 200 travelers' choices on each day and then averaged over 50 days. Travel time is averaged over both travelers and days. The expected path share, path share standard deviation, mean and standard deviation of average travel time are calculated based on the 100 sets of 50-day travel time realizations.

Figure 2.4 presents the impacts of travel time variability on the expected share for the four models. In Figure 2.4 (a), the solid line represents the share on Path 1 predicted by the IBL model. It follows that the path share of the risky path decreases as its objective travel time standard deviation increases. This suggests that travelers are risk aversion and more travelers switch to the safe path when the risky path becomes highly unreliable. This downward trend is more obvious when the travel time standard deviation is very large (i.e., greater than 8), because under such conditions very bad outcomes are likely to happen and the hot stove effect captured by the IBL model makes the travelers never choose the risky path again once having experienced a very bad travel time. The dashed line represents the share on Path 1 predicted by the full-knowledge model. Compared to the IBL model, the full-knowledge model underestimates the share on the risky path as well as travelers' sensitivity to the travel time variability. The small negative estimates of the travel time and toll coefficients tend to even up the utilities of the two paths such that the choice probability is close to random. The insensitivity to travel time variability is due to the small estimate of the travel time standard deviation coefficient. Figure 2.4 (b) shows the expected share on Path 3 predicted by the IBL model and no-carryover model. Compared to Path 1, the expected share on Path 3 predicted by the IBL model yields a more rapid decreasing trend. This is because travelers recall their past experiences from the work trip when making choices for the recreational trip, so that the choice pattern is more extreme with respect to travel time variability. The path share predicted by the no-carryover model is more random and steady with respect to the travel time variability because the small numerical value of the travel time coefficient makes path share insensitive to the perceived travel times and thus is much less affected by the underlying standard deviation.

Figure 2.4 (c) presents the change in path share standard deviation with respect to the travel time variability. Since the path share standard deviation of the two paths in the same OD are always equal in a binary network, paths from the same OD are presented in one plot. For both ODs, the IBL model predicts an upward rend in path share standard deviation with respect to the travel time variability. This is because as the travel time becomes uncertain, travelers' experiences become more divergent and thus their choices are also more divergent. It is expected that the full-knowledge model predicts zero path share standard deviation, since travelers perceive the true mean and standard deviation which are not affected by any particular realizations. The dashed line in Figure 2.4 (d) shows that the no-carryover model predicts very small and steady path share standard deviations with respect to travel time variability. This is because the small numerical value of the travel time coefficient adds massive noises to the choice rule, which makes the path shares fairly stable within each set of the realizations. It is concluded that the path share standard deviations predicted by the two simplifying models are insensitive to the objective travel time variability of the risky route.

Figure 2.5 presents the prediction results of the expected average travel time and its standard deviation with respect to the objective travel time variability by the four models. Figure 2.5 (a) and (b) yield very similar patterns as their corresponding plot in Figure 2.4, which is intuitive because the expected average travel time is directly related to the expected path share. For example, as more travelers switch to the safe path whose objective travel time is shorter than the risky path, the expected average travel time decreases accordingly. In Figure 2.5 (c) and (d), it is expected that the standard deviation of average travel time increases with respect to the travel time variability. In Figure 2.5 (c), the upward trend is more rapid for the IBL model than the full-knowledge model. This is because the path share of the risky path predicted by the IBL model is constantly higher than the full-knowledge model, such that with the increase of the travel time standard deviation the average travel times predicted by the IBL model is also higher. In Figure 2.5 (d), when the travel time variability gets large, the standard deviation of average travel time predicted by the IBL model is larger than the no-carryover model despite its higher predicted share on Path 3. This is because the hot stove effect captured by the IBL model makes the travelers

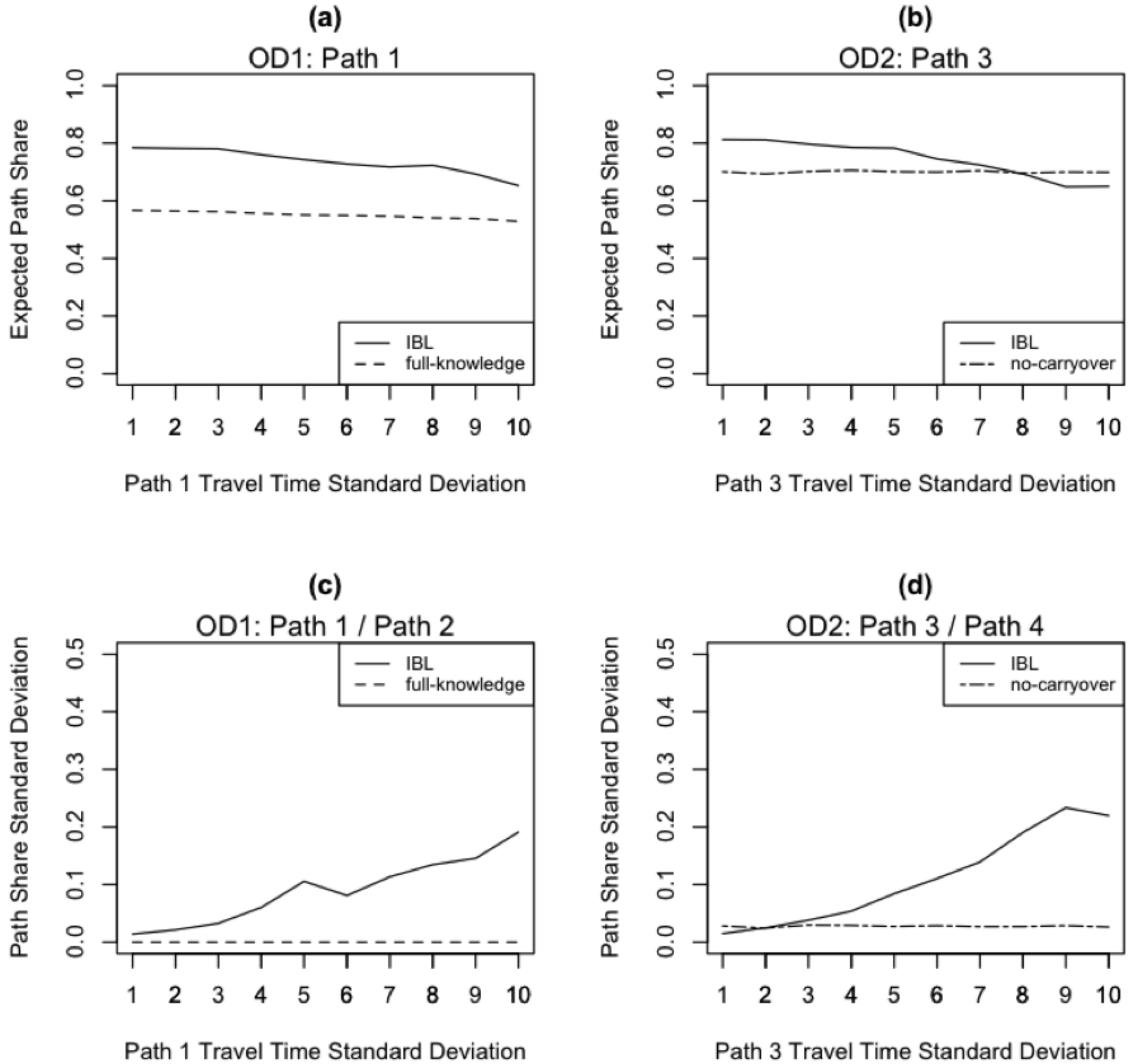


Figure 2.4 Impacts of Objective Travel Time Variability on Path Share.

choose the risky path only if very favorable outcomes are experienced, thus the more divergent travel times have larger standard deviations.

2.4 Model Estimation Based on an Experimental Dataset

To demonstrate the applicability of the IBL model and its potential in more precisely capturing travelers' learning process, the model is estimated using an experimental dataset for repeated route-choice and the estimation results is compared with a learning model in the literature.

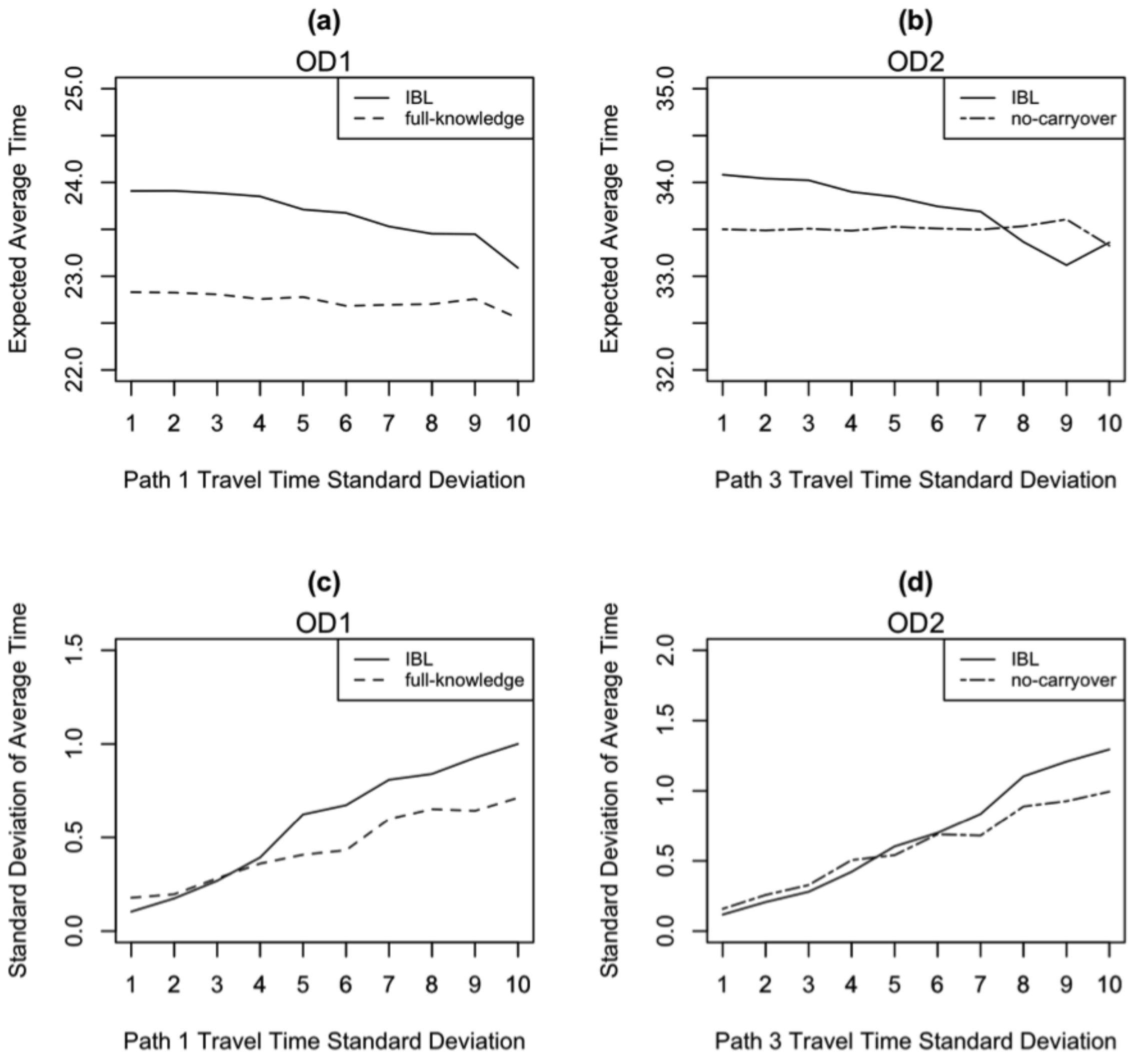


Figure 2.5 Impacts of Objective Travel Time Variability on Average Travel Times.

2.4.1 The experimental dataset

The data used in this section is an experimental dataset described in (Ben-Elia & Shiftan, 2010). In the experiment, forty-nine participants were faced with three scenarios of binary route-choice as presented in Table 3.3. A small degree of variation was programmed (± 5 or ± 15 min around the mean) to simulate a simple variable message sign (VMS). Each scenario included 100 trials so in total each participant completed 300 trials. The participants were randomly assigned to the informed and non-informed group to run through 1 out of the 6 (=3!) possible orders of the scenar-

Table 2.4 Hypothetical travel time scenarios of the experimental data set

Scenario	Travel time ranges (minutes)	
	Route F - 25 min.	Route S - 30 min.
Fast & Safe	± 5	± 15
Fast & Risky	± 15	± 5
Low-Risk	± 5	± 5

ios. For each choice situation the informed group (24 participants) received real-time information about the travel time range (the minimum and maximum travel times) for each of the two routes, while the non-informed group did not. Following the choice, a feedback was received regarding the "actual" travel time on the chosen route but not of the alternative one. This travel time was randomly drawn from the distribution of the travel time range. The non-informed group (25 participants) received the same feedback.

2.4.2 Model specification

The baseline model

The baseline model closely follows that in Ben-Elia & Shiftan (2010), except that only the coefficient to the mean travel time is treated as random for estimation efficiency. Eq. (2.8) shows the utility functions. On a given day t, mean travel times (MEANS, MEANF) are specified as the average of the travel times obtained in each choice trial and for each route from the simulated VMS according to the scenario design. Feedback travel times (TIMEF, TIMES) are the travel times displayed following each participant's choice. The stickiness (STICK) represents inertia, i.e., repetition of previous behavior. Learning in the long run is defined as a function of all previous outcomes which reflects the effect of memorization, and a cumulative weighted average (CWA) of the preceding choices is specified as a harmonic average. See Ben-Elia & Shiftan (2010) for the specification of CWA. Two different levels of experience are also specified with dummy variables to represent distinct behavioral tendencies in the short (first 10 trials) and long (last 50 trials) runs. Low experience (EXL) reflects choices within the first 10 trials and high experience (EXH) reflect choices in the last 50-100 trials. Sensitivity to variability of the travel times is represented using dummy variables indicating the travel time ranges (LRISK and FRISKY).

$$\begin{cases} U_{SLOW}(t) = \beta_{MEAN}(\sigma_{MEAN})MEANS(t) + \beta_{TIMES}TIMES(t) \\ U_{FAST}(t) = \beta_{MEAN}(\sigma_{MEAN})MEANF(t) + \beta_{TIMEF}TIMEF(t) \\ \qquad +\beta_{LRISK}LRISK(t) + \beta_{FRISKY}FRISKY(t) + \beta_{EXL}EXL(t) \\ \qquad +\beta_{EXH}EXH(t) + \beta_{STICK}STICK(t) + \beta_{CWA}CWA(t) \end{cases} \tag{2.8}$$

The IBL model

In Eq. (2.9), the IBL model has the same specification as the baseline model for fair comparison, except that the feedback travel times (TIMEF, TIMES) are replaced with the perceived travel time T_{FAST} and T_{SLOW} following Eqs. (2.1) through (2.3). The feedback travel time can be viewed as a simplified version of the perceived travel time calculated using the IBL model, where the forgetting rate is very high and thus only the latest experienced travel time is activated.

$$\begin{cases} V_{SLOW}(t) = \beta_{MEAN}(\sigma_{MEAN})MEANS(t) + \beta_{TIMES}T_{SLOW}(t) \\ V_{FAST}(t) = \beta_{MEAN}(\sigma_{MEAN})MEANF(t) + \beta_{TIMEF}T_{FAST}(t) \\ \qquad +\beta_{LRISK}LRISK(t) + \beta_{FRISKY}FRISKY(t) + \beta_{EXL}EXL(t) \\ \qquad +\beta_{EXH}EXH(t) + \beta_{STICK}STICK(t) + \beta_{CWA}CWA(t) \end{cases} \tag{2.9}$$

2.4.3 Estimation results

The IBL and baseline learning models are estimated for the informed and non-informed groups respectively and the results are shown in Table 2.5. The memory decay parameter d are estimated at 1.11 and 1.64 respectively, both are within the typical range Lejarraga et al. (2012). For both scenarios, the IBL model yields better model fit than the baseline model represented by the adjusted ρ^2.

The ratio of the feedback (or perceived) travel time coefficient over the stickiness coefficient shows the relative importance of learning over inertia. The IBL model reveals a dramatically larger (more than 10 times) ratio than the baseline model in both scenarios. Consider the travel time coefficients on the slow route (β_{TIMES}). The ratios are -0.288 and -0.0135 for the IBL and baseline models respectively for the informed group, and -0.150 and -0.0124 for the IBL and baseline models respectively for the non-informed group. The drastic difference between models are similar when travel time coefficients on the fast route (β_{TIMEF}) are used. The IBL model thus seems to suggest a much larger role of learning compared to inertia than the baseline model.

Comparing the ratios across the informed and non-informed groups could suggest how information impacts with learning. Both the IBL and baseline models reveal a higher ratio for the informed group than the non-informed group, suggesting that information facilitates learning. The IBL model suggests a larger benefit of the information than the baseline model does, given that the ratio from the IBL model almost doubles with information (-0.150 vs. -0.288), while that from the baseline model only increases slightly with information (-0.0125 vs. -0.0135).

2.4.4 Cross validation

Cross validation is performed for both models for the informed and non-informed groups respectively. For each group, 10 sets of data are generated. In each set, 2/3 of

Table 2.5 Estimation Results for the Experimental Dataset

Parameter	Informed		Non-informed	
	IBL	**Baseline Model**	**IBL**	**Baseline Model**
d	1.11		1.64	
Robust std err	0.230		0.884	
t-test	4.81		1.85	
β_{MEAN}	-0.353	0.528	-0.0894	0.577
Robust std err	0.220	0.129	0.204	0.136
t-test	-1.60	4.09	-0.430	4.24
σ_{MEAN}	0.269	0.241	0.142	0.152
Robust std err	0.0372	0.0324	0.0200	0.0213
t-test	7.23	7.46	7.04	7.07
β_{TIMES}	-0.123	-0.0601	-0.134	-0.077
Robust std err	0.0414	0.0192	0.0541	0.0212
t-test	-2.98	-3.25	-2.71	-3.95
β_{TIMEF}	-0.228	-0.0731	-0.181	-0.104
Robust std err	0.0372	0.0224	0.0681	0.0184
t-test	-6.12	-3.67	-2.65	-5.66
β_{STICK}	0.427	4.45	0.893	6.22
Robust std err	0.537	1.03	0.356	0.458
t-test	0.800	4.33	2.51	13.6
β_{FRISKY}	0.806	0.554	0.128	-0.0261
Robust std err	0.294	0.376	0.281	0.243
t-test	2.75	1.47	0.450	-0.112
β_{LRISK}	2.11	1.73	0.398	0.297
Robust std err	0.444	0.437	0.238	-0.237
t-test	4.75	3.97	1.67	1.25
β_{EXL}	-0.495	-0.613	-0.854	-0.813
Robust std err	0.221	0.193	0.113	0.117
t-test	-2.24	-3.18	-7.52	-6.98
β_{EXH}	0.0912	0.135	0.849	0.783
Robust std err	0.165	0.152	0.135	0.132
t-test	0.554	0.893	6.28	5.93
β_{CWA}	2.86	3.29	1.47	1.89
Robust std err	0.784	0.782	0.707	0.468
t-test	3.64	4.21	2.08	4.04
Initial log-likelihood	-4941	-4941	-5147	-5147
Final log-likelihood	-1398	-1435	-2274	-2328
Adjusted ρ^2	0.715	0.708	0.556	0.546
No. of parameters	11	10	11	10
Sample size	7128	7128	7425	7425

Table 2.6 Average Cross Validation Result of the IBL and Baseline Models

		Informed		Non-Informed	
		IBL	Baseline	IBL	Baseline
Average over 10 estimation datasets	FLL	-875	-905	-1385	-1421
	Adjusted ρ^2	0.731	0.722	0.576	0.565
Average over 10 prediction datasets	FLL	-735	-769	-932	-965
	Adjusted ρ^2	0.557	0.545	0.427	0.408
No. of parameters		11	10	11	10

the participants' data are randomly chosen as the training set for model estimation, while the remaining 1/3 of the participants' data are used as the validation set for prediction. The random coefficient to the mean travel time is drawn 1000 times and simulated likelihood is calculated. Log-likelihood and adjusted ρ^2 are computed to compare estimation and prediction quality.

Table 2.6 shows the estimation and prediction results averaged over the 10 sets. Compared to the estimation results from the full dataset, the adjusted ρ^2 of the estimation datasets is about the same for both models and both groups, while those of the prediction datasets are noticeably lower. This is expected as the prediction test is in general a stricter test than the estimation test. For both informed and non-informed groups, the IBL model has higher adjusted ρ^2 for prediction. Table 2.7 shows the estimation and predictions results of each set of training and validation data in detail. For each set, the IBL model consistently performs better than the baseline learning model in terms of prediction adjusted ρ^2.

2.5 Summary

An instance-based learning (IBL) model for route-choice is developed based on the power law of forgetting and practice. Experiments based on synthetic datasets show that the true parameter values of the IBL model can be consistently retrieved and the model can potentially predict different traffic patterns compared to non-

Table 2.7 Cross Validation Results of the Baseline Learning Model and IBL Model

			Informed		Non-Informed	
			IBL	Baseline	IBL	Baseline
Set 1	Estimation	FLL	-916	-949	-1607	-1646
		Adjusted ρ^2	0.719	0.709	0.509	0.497
	Prediction	FLL	-639	-704	-1014	-1127
		Adjusted ρ^2	0.614	0.585	0.378	0.310
Set 2	Estimation	FLL	-867	-882	-1355	-1659
		Adjusted ρ^2	0.736	0.729	0.585	0.493
	Prediction	FLL	-825	-832	-985	-1041
		Adjusted ρ^2	0.503	0.488	0.395	0.362
Set 3	Estimation	FLL	-848	-899	-1241	-1349
		Adjusted ρ^2	0.739	0.724	0.620	0.588
	Prediction	FLL	-6499	-744	-857	-870
		Adjusted ρ^2	0.608	0.598	0.473	0.466
Set 4	Estimation	FLL	-986	-879	-1367	-1270
		Adjusted ρ^2	0.697	0.730	0.582	0.611
	Prediction	FLL	-680	-710	-905	-924
		Adjusted ρ^2	0.589	0.580	0.444	0.433
Set 5	Estimation	FLL	-857	-1012	-1497	-1399
		Adjusted ρ^2	0.737	0.690	0.542	0.572
	Prediction	FLL	-666	-682	-936	-944
		Adjusted ρ^2	0.598	0.588	0.425	0.421
Set 6	Estimation	FLL	-952	-899	-1409	-1540
		Adjusted ρ^2	0.708	0.724	0.569	0.529
	Prediction	FLL	-706	-708	-898	-908
		Adjusted ρ^2	0.574	0.562	0.448	0.443
Set 7	Estimation	FLL	-834	-980	-1322	-1436
		Adjusted ρ^2	0.743	0.699	0.595	0.561
	Prediction	FLL	-591	-623	-959	-977
		Adjusted ρ^2	0.642	0.630	0.411	0.401
Set 8	Estimation	FLL	-829	-863	-1294	-1441
		rho	0.745	0.735	0.604	0.559
	Prediction	FLL	-883	-921	-841	-896
		Adjusted ρ^2	0.469	0.457	0.483	0.450
Set 9	Estimation	FLL	-798	-858	-1129	-1317
		Adjusted ρ^2	0.754	0.736	0.654	0.597
	Prediction	FLL	-834	-848	-979	-985
		Adjusted ρ^2	0.498	0.479	0.399	0.396
Set 10	Estimation	FLL	-866	-832	-1627	-1155
		Adjusted ρ^2	0.734	0.745	0.503	0.646
	Prediction	FLL	-875	-913	-9487	-980
		Adjusted ρ^2	0.473	0.479	0.418	0.399

learning models. The IBL model is compared with a baseline learning model using an experimental dataset of repeated route-choice. Estimation results show that the IBL model suggests a larger role of learning compared to inertia and achieves better model fit. Cross validation experiments suggest that the forecasting ability of the IBL model is better than the baseline learning model.

CHAPTER 3

THE INITIAL CONDITION PROBLEM WITH COMPLETE HISTORY DEPENDENCY IN LEARNING MODELS FOR TRAVEL CHOICE

In a learning model such as the IBL model introduced in Chapter 2, a traveler's perception of an alternative's attribute (e.g., travel time) evolves over time based on all her past experience with the alternative. When forming the perception, each past experience with the alternative takes a weight in memory and the perception is a weighted average of all past experience. The weighting scheme of past experience is specific to the learning model in use. Compared to non-learning models where the perception of an alternative is static over time, estimation of a learning model requires data of travelers' complete past experience with the alternatives. Longitudinal data collection in real life, however, inevitably starts midstream, and rarely includes subjects' complete choice histories. Specialized data collection targeted at newcomers (e.g., new employees or students) to a region might provide the needed data, but such efforts are difficult to implement. In the case of incomplete data, the missing initial observations can lead to biased estimate of the perceived value of the attribute in question, and thus inconsistent parameter estimates. Note that the majority of empirical studies on learning models for travel choice are based on experimental data in a laboratory, where subjects make choices from "day" and thus the stated problem does not exist. In this chapter, the initial condition problem in learning models illustrated and correction methods are proposed and assessed using both synthetic and empirical data.

In this chapter, a literature review on endogeneity and importance sampling is provided first. The cause of the endogeneity problem due to missing initial observations is then illustrated using the IBL model developed in Chapter 2. The estimation biases of the parameters are demonstrated using synthetic data. Two correction methods, i.e., the MSLrs and MSLis, are proposed within the IBL framework and their effectiveness and computational efficiency are assessed using Monte Carlo experimentation. Sensitivity analysis are conducted to investigate the impact of sampling size in random sampling and number of high probability choice sequences in importance sampling. In the end of this chapter, the proposed correction methods are applied to empirical data to prove their applicability and effectiveness.

3.1 Literature Review

3.1.1 Endogeneity problem with learning models

An econometric model is said to suffer from endogeneity when the systematic part of the utility is correlated with the error term. The variables that cause the correlation are called the endogenous variables. Endogeneity can lead to inconsistent estimation of model parameters, since changes in the error term are misinterpreted as changes of the endogenous variable. Endogeneity is common in discrete choice models (e.g., probit, logit, nested logit) as the assumption that the explanatory variables are independent from the error term is often violated. Guevara (2010) classifies endogeneity into three types based on their causes: (1) Omission of the variables that are correlated with some observed variables; (2) Simultaneous determination of multiple variables; and (3) The propagation of measurement errors in explanatory variables to the error term. Several correction methods have been developed to solve endogeneity problems (e.g., Guevara & Polanco, 2016; Heckman, 1978; Berry et al., 1995; Schenker & Welsh, 1988; Brownstone, 1991; Guevara, 2010; Fernández-Antolín et al., 2016) . The endogeneity problem this thesis tackles can be classified within the third group, a special case in which endogeneity arises because the researcher has an incorrect measure of the attributes of the alternatives perceived by the decision makers.

Solving the initial observation problem for dynamic panel data discrete choice models is known to be a difficult task. Most existing studies deal with first-order Markov process where the dependent variable is only lagged once. The major focus of these studies is that the initial condition is not exogenous due to correlation of error terms over time. Therefore, if there is no serial correlation, first-order Markov process model would not suffer from the problem. For example, Heckman (1981a) and Lee (1997) examined the problem of initial conditions in a time-discrete data stochastic process when serially correlated unobservable variables generate the process. Correction methods were proposed and tested with Monte Carlo experiments. More of such studies can be found in the reference list (e.g. Blundell & Bond, 1998; Wooldridge, 2005; Honore & Kyriazidou, 2000; Carro, 2007). In the learning models for travel choice, a current decision depends on the entire history of past experience, defined as a Polya process in Heckman (1981b). The complete history dependence makes the initial condition problem more challenging than those in the existing studies. The model will suffer from the initial observation problem even without serial correlation. To the best of our knowledge, no solution has been developed to date.

In this chapter, the proposed method is based on noting that the likelihood function of this problem can be written as a sequence of integrals over the conditional distribution of the possible choices on the missing days. This multifold integral is then maximized using a variation of the maximum simulated likelihood (MSL), which is described in detail by Train (2009). The MSL numerical estimation method has reached great popularity in the past 15 years, thanks to the significant improvement in computational power. This method has been mainly used for the estimation of Logit Mixture models aimed to account for random coefficients or different error component. The application of the method in this thesis is different from the usual ones, although all the conditions for consistency described in Train (2009) are extendable, e.g., the need for having the number of draws growing faster than the square root of the sample size. Despite its popularity, the MSL is not exempt from drawbacks. For example, MSL estimators have a downward bias for a finite number of draws, and they may suffer from empirical identification problems, both in the form of false empirical identification and lack of empirical identification. More importantly for this application, MSL may suffer from the problem known as the curse of dimensionality, which in this case implies that the number of draws required for estimation grows exponentially with the number of missing days, quickly making estimation impractical. This problem is shared by all estimation methods based on simulation. This issue will be illustrated and investigated with Monte Carlo experimentation.

Two sampling methods are proposed for the correction. The MSL random sampling (MSLrs) method randomly draws a set of missing choice sequences following the learning model and a simple average of the simulated choice probabilities is used in the simulated likelihood. This sampling method is expected to suffer from the curse of dimensionality as the number of missing days grows. To overcome this limitation, the MSL importance sampling (MSLis) method is proposed. It can be seen as a variation of the kernel conditional density nonparametric estimator proposed by Rosenblatt (1969) and enhanced by Hyndman et al. (1996). In this case, instead of randomly simulating a large enough number of missing choice sequences to evaluate the Logit Kernel function, a small number of sequences with high probability of occurrence are sampled and the kernel, conditioning on the said probability are evaluated.

3.2 Endogeneity Due to Missing Initial Observations

For illustrative purpose, the IBL model developed in Chapter 2 is defined within a binary route-choice context without path size and random parameters in this chapter (see Eq. (3.1)).

$$U_{ni}(t) = V_{ni}(t) + \varepsilon_{ni}(t) = \beta_{time} b_{ni}(t) + \alpha' \boldsymbol{z}_i + \varepsilon_{ni}(t) \tag{3.1}$$

Suppose the data are collected from day C. It is likely that the travelers have already accumulated some experience with the alternatives prior to day C. In such cases, the dataset only contains observations from day C to day K, while those from day 1 to day $C-1$ are missing. In this chapter, the dataset without missing observations is referred as the full dataset, and that with missing observations is referred as the cutoff dataset. Variables in the cutoff dataset are all denoted with asterisks (*), while those of the full dataset are denoted without asterisks. In this section, the cause of endogeneity due to missing initial observations is derived and the estimation biases are demonstrated.

3.2.1 Cause of endogeneity

The true likelihood of the cutoff dataset is the one shown in Eq. (3.2). However, this likelihood is impractical to compute because the true perceived travel time $b_{ni}(t)$ cannot be calculated. Recall that for the full dataset, the perceived travel time of alternative i on day t is the weighted average of all past instances (Eq. (2.2)). Instead of $b_{ni}(t)$, a curtailed version $b^*_{ni}(t)$ could be used, resulting in the modified likelihood shown in Eq. (3.3), where maximization will not retrieve consistent estimators of the model parameters.

$$\ell_{NC} = \sum_{n=1}^{N} \sum_{t=C}^{K} log \left[P_n(1|t, \{1,2\})^{a_{n1}(t)} \big(1 - P_n(1|t, \{1,2\})\big)^{1-a_{n1}(t)} \right] \tag{3.2}$$

$$\ell^*_{NC} = \sum_{n=1}^{N}\sum_{t=C}^{K} log\left[P^*_n(1|t,\{1,2\})^{a_{n1}(t)}\big(1-P^*_n(1|t,\{1,2\})\big)^{1-a_{n1}(t)}\right] \tag{3.3}$$

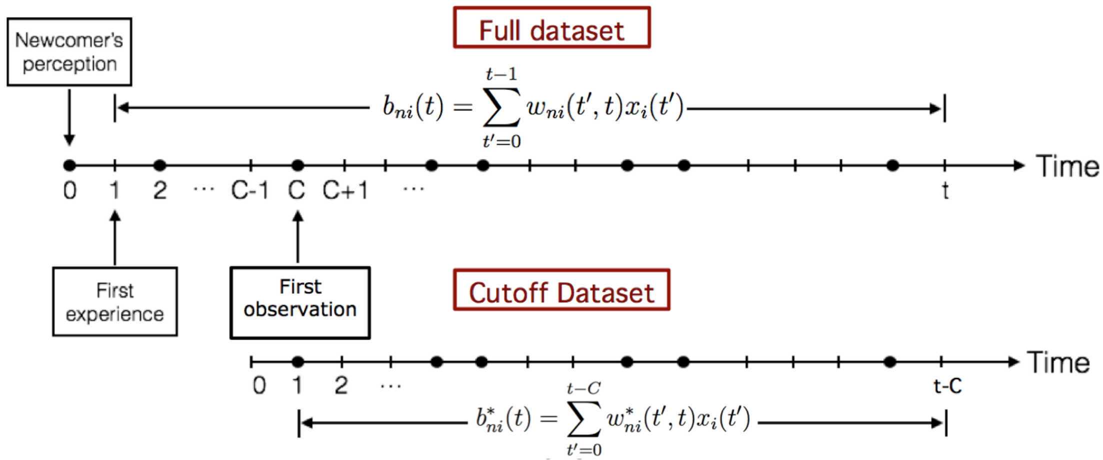
To illustrate the problem, consider that the perceived travel time can be written as the sum of the weighted average of the perceived travel time derived from the instances from day 0 to day $C-1$ and the perceived travel time derived from the instances from day C to day $t-1$ as in Eq. (3.4).

$$b_{ni}(t) = \sum_{t'=0}^{C-1} w_{ni}(t',t)x_i(t') + \sum_{t'=C}^{t-1} w_{ni}(t',t)x_i(t') \tag{3.4}$$

In the cutoff dataset, an initial perception b_i^{IP} is assumed to happen on day $C-1$ to approximate the perceived travel time prior to day C (it is effectively assumed zero if experiences prior to day C are simply ignored), and the perceived travel time at day t is the weighted average of the initial perception and instances happened from day C to day $t-1$ (Eq. (3.5)). The absolute value of activation of an observed instance (that occurs on or after day C) stays the same as in the full dataset, however it is normalized over a smaller set of instances including the assumed initial perception on day $C-1$, as shown in Eq. (3.6). Therefore, the weights of the observed instances are scaled up compared to their true weights in the full dataset. Figure 3.1 illustrates the measurement difference in perceived travel time between the full dataset and the cutoff dataset.

$$b^*_{ni}(t) = w^*_{ni}(C-1,t)b_i^{IP} + \sum_{t'=C}^{t-1} w^*_{ni}(t',t)x_i(t') \tag{3.5}$$

Figure 3.1 Measurement difference in perceived travel time between the full dataset and cutoff dataset



$$w^*_{ni}(t', t) = \frac{a_{ni}(t')(t - t')^{-d}}{(t - C + 1)^{-d} + \sum_{\tau=C}^{t-1} a_{ni}(\tau)(t - \tau)^{-d}} \tag{3.6}$$

where:

$w^*_{ni}(t', t)$**:** weight of the experienced travel time on day t' for the perceived travel time on day t for alternative i traveler n in the cutoff dataset

$w^*_{ni}(C - 1, t)$**:** weight of initial perception on day $C - 1$ for the perceived travel time on day t for alternative i for traveler n in the cutoff dataset

$b^*_{ni}(t)$**:** perceived travel time of alternative i on day t for traveler n in the cutoff dataset

b_i^{IP}**:** initial perception of alternative i

The discrepancy between the perceived travel time in the cutoff dataset $b^*_{ni}(t)$ and that of the full dataset $b_{ni}(t)$ is propagated to the error term, such that the error term in the utility function of the cutoff dataset $e_{ni}(t) = \varepsilon_{ni}(t) + \beta_{time}(b_{ni}(t) - b^*_{ni}(t))$ is correlated to the systematic part of the utility function. Thus, the perceived travel time is the endogenous variable, and the model that omits the missing initial observations can be seen as a model that suffers from a special case of endogeneity due to measurement error.

3.2.2 Experiments based on synthetic data

The impact of the endogeneity problem on parameter estimates is illustrated using synthetic datasets. Since VOT has important policy indication, toll price is included in the utility function as an attribute that is constant over time to exemplify travel cost. VOT is calculated based on the perceived travel time coefficient β_{time} and toll coefficient β_{cost}. The estimator of VOT is used to investigate the effectiveness of the correction method. The true value of the decay parameter d follows its conventional value of 0.5, and the true values of the perceived travel time coefficient β_{time} and toll coefficient β_{cost} are postulated at -0.4 and -1.2 respectively. The underlying travel time distributions are generated following truncated normal distribution.

100 datasets are generated following the true model. For each dataset, 200 sets of 50-day observations are generated. For each set of observations, the travel time of Path 1 follows a normal distribution with the mean uniformly sampled between 10 and 50 and the standard deviation uniformly sampled between 0.1 and 0.3 times the mean. The mean travel time of Path 2 is uniformly sampled between 0.8 and 1.2 times the corresponding mean travel times of Path 1, and the standard deviation uniformly sampled between 0.1 and 0.3 times the mean. The travel time distributions of both paths are truncated at half of its mean travel time to mimic a distribution with a lower bound set by the free flow travel time. The toll price of both paths is uniformly sampled between \$0 to \$10. Without any other information, the mean travel time of an alternative is the best approximation one can find to use as the initial perception. For simplicity, the decay parameter d is fixed at its true value and only the travel time coefficient β_{time} and toll coefficient β_{cost} are estimated. Unreported Monte Carlo experiments show that the decay parameter d can be retrieved in the full dataset,

and in Section 3.4.3 d is estimated in the empirical dataset. The software R-3.2 is used for both data generation and estimation throught the research, and Broyden-Fletcher-Goldfarb-Shanno (BFGS) algorithm is used for likelihood maximization.

Table 3.1 shows the estimation results of the full dataset and cutoff datasets with a variety of number of missing initial observations. The average, percent error from the true value, p-value against the true value, and empirical coverage of each estimate are reported. The empirical coverage is calculated as the percent of the tests among the 100 repetitions where the null hypothesis that the estimator is equal to its true value is accepted with 95% confidence. For the full dataset, the percent errors of the model parameters and VOT are all very small. Both the empirical coverages and p-values suggest the retrieval of the true values with 95% confidence. For the curtailed model, however, all the metrics suggest that the null hypothesis of the retrieval of the true value is rejected even when only 1 observation is missing. Thus, it is concluded that the missing initial observations can cause the endogeneity problem in a learning model and this problem gets more severe as the number of missing observations increases.

3.3 Maximum Simulated Likelihood (MSL) Method

Realized travel times are assumed observable, since traffic monitoring devices are generally available to obtain travel time measurements. Therefore, the choice histories prior to day C are the only latent variables. The MSL method uses simulation to integrate out the latent variables. The likelihood function of the IBL model with missing observations can be written as a sequence of integrals over the conditional distribution of the possible missing choices. The multivariate integration is carried out numerically through simulation, and an iterative algorithm is utilized to find the maximum simulated likelihood. At each iteration, the log-likelihood function needs to be evaluated for a given trial values of the parameters. A set of choice sequences prior to day C is obtained based on a specific sampling method for the log-likelihood

Table 3.1 Endogeneity due to missing initial observations

Parameter	# of missing observations	Average	Percent error	p-value	Empirical coverage (%)
β_{time} **(-0.4)**	0	-0.399	0.0433	0.873	96
	1	-0.339	15.2	<1e-05	59
	5	-0.273	31.9	<1e-05	1
	10	-0.241	39.8	<1e-05	0
	15	-0.216	46.0	<1e-05	0
	20	-0.200	49.9	<1e-05	0
	25	-0.183	54.3	<1e-05	0
	30	-0.167	58.2	<1e-05	0
	35	-0.151	62.2	<1e-05	0
	40	-0.131	67.3	<1e-05	0
β_{cost} **(-1.2)**	0	-1.199	0.05	0.780	94
	1	-1.168	2.61	<1e-05	91
	5	-1.172	2.31	<1e-05	92
	10	-1.183	1.39	<1e-05	97
	15	-1.181	1.58	<1e-05	96
	20	-1.183	1.41	<1e-05	96
	25	-1.177	1.92	<1e-05	93
	30	-1.169	2.59	<1e-05	94
	35	-1.161	3.25	<1e-05	91
	40	-1.147	4.39	<1e-05	92
VOT (0.333)	0	0.333	0.00168	0.944	99
	1	0.390	12.9	<1e-05	66
	5	0.233	30.2	<1e-05	3
	10	0.204	38.9	<1e-05	0
	15	0.183	45.1	<1e-05	0
	20	0.170	49.1	<1e-05	0
	25	0.156	53.3	<1e-05	0
	30	0.143	57.1	<1e-05	0
	35	0.130	60.9	<1e-05	0
	40	0.114	65.7	<1e-05	0

*Estimation results are based on 100 repetitions.
The nominal value of empirical coverage is 95%.
P-values are calculated against true values.

function. The total probability theorem is used to obtain an estimator of the log-likelihood corresponding to the trial values of the paramenters. The consistency of this method can be demonstrated using an approach equivalent to the one described in Train (2009). The algorithm is described in detail below. As it occurs with other methods to correct for endogeneity in discrete choice models, the proposed MSL method will consistently recover the linear utility coefficients, in general, only up to a scale (Guevara & Ben-Akiva, 2012). For example, if the utility considers travel time and travel cost of each route, then only the ratio of their coefficients, i.e., the VOT, will be consistently recovered with the proposed method, but not the individual coefficients. Conversely, the decay parameter should be fully recovered because of the nonlinear way in which it defines the normalized weights in Eq.2.1.

The random sampling and importance sampling approaches are proposed to implement the MSL method. The random sampling method follows the simulation approach described by Train (2009) for the Logit Mixture model. It sequentially simulates the missing choice sequences prior to day C following the IBL model with given trial values of ϕ. Sample R times to form the choice sequence set H_n. For each simulated choice sequence h_n, the likelihood of observing choices starting from day C is calculated as $P_n(i|t,\{1,2\},h_n)$. Due to the nature of random sampling, the simulated log-likelihood is thus

$$\hat{P}_n(i|t,\{1,2\}) = \frac{1}{R}\sum_{h_n \in H_n} P_n(i|t,\{1,2\},h_n) \tag{3.7}$$

The importance sampling approach can be better described if the complete enumeration method, a special case of importance sampling that quickly becomes impractical as the number of missing days grows, is reviewed first. The complete enumeration method finds the set H_n by enumerating each possible choice sequence that could have been chosen prior to day C by each traveler n. The probability of occurrence of each possible choice sequence, π_{h_n}, is the product of the sequence of conditional choice probabilities, shown in Eq. (3.8). Based on the total probability theorem, the choice probability to be considered in the likelihood function can be calculated as a weighted average of the conditional choice probabilites and their respective probability of occurence π_{h_n} as in Eq. (3.9).

$$\pi_{h_n} = \prod_{t=1}^{C-1} P_n(i|t, \{1,2\}, h_n^1, h_n^2, ..., h_n^{t-1}) \tag{3.8}$$

$$\hat{P}_n(i|t, \{1,2\}) = \sum_{h_n \in H_n} P_n(i|t, \{1,2\}, h_n)\pi_{h_n} \tag{3.9}$$

The complete enumeration method becomes quickly impractical as the set of unique choice sequences grows exponentially in the number of missing observations. To avoid this limitation, the importance sampling method defines the choice sequence set H_n by keeping a subset of the full choice sequence set with high probability of occurrence. Based on the total probability theorem, the choice probability to be considered in the likelihood function is calculated as in Eq. (3.10). The choice sequences in H_n are sampled by simulating the missing choices prior to day $C-1$ sequentially following the IBL model with given trial values of ϕ. If a choice sequence for a given traveler n is drawn twice, the second draw is discarded to keep the sequence set unique. The sequence set H_n is fixed over MSL iterations, and can be re-sampled after a certain number of MSL iterations. In practice, the choice of sampling size shall depend on the number of missing observations and number of high probablity choice sequences. Since the sampling process is independent of the estimation procedure and once a choice sequence is sampled, it can be reused for any given number of high probability sequences, the rule of thumb is to sample a large number of times to cover the sampling distribution of the choice sequences as much as possible.

$$\hat{P}_n(i|t, \{1,2\}) = \frac{\sum_{h_n \in H_n} P_n(i|t, \{1,2\}, h_n)\pi_{h_n}}{\sum_{h_n \in H_n} \pi_{h_n}} \tag{3.10}$$

Maximum Simulated Likelihood Algorithm with Random Sampling or Importance Sampling

Given the initial trial values ϕ_0, which could be gathered, e.g., from the estimators of the curtailed model.

Iteration $k = 1$

1. Obtain a choice sequence set H_n for the missing days $t = 1$ to $t = C - 1$ for each traveler n following the IBL model.

For random sampling, H_n is sampled at each iteration. For importance sampling, H_n is fixed over iterations.

2. For each choice sequence $h_n \in H_n$

i. For each day $t \geq C$, calculate the perceived travel time $b_{ni}(t)$ using the weights $w(t', t)$ and the sampled choices from h_n,

that is, $a_{ni}(\tau) = a_{ni}^{h_n}(\tau)$ for $\tau < C$.

ii. Calculate the choice probabilities for the current choice sequence h_n for each day $t \geq C$ as $P_n(i|t, \{1, 2\}, h_n)$

3. Based on the chosen sampling method, the choice probability $\hat{P}_n(i|t, \{1, 2\})$ to be considered in the likelihood function is calucated

based on $P_n(i|t, \{1, 2\}, h_n), \forall h_n \in H_n$.

4. Find new trial values ϕ_k to maximize the following simulated likelihood to retrieve the estimators $\hat{\phi}$:

$$\tilde{\ell}_{NC}^{MSL} = \sum_{n=1}^{N} \sum_{t=C}^{K} log \left[\left(\hat{P}_n(1|t, \{1, 2\}) \right)^{a_{n1}(t)} \left(\hat{P}_n(2|t, \{1, 2\}) \right)^{1-a_{n1}(t)} \right]$$

$k = k + 1$ to repeat steps 1-4 till convergence. For importance sampling, the set of choice sequences H_n can be re-sampled

after enough number of iterations.

3.4 Computational Experiments

The MSLis method can be seen as a variation of the kernel conditional density nonparametric estimator proposed by Rosenblatt (1969) and enhanced by Hyndman et al. (1996). In this case, instead of drawing a large number of choice sequences with potentially very low probability of occurrence, the effort is concentrated on drawing a small number of choice seuqences with large probability of occurrence and evaluating the kernel, conditioning on the said probability. Monte Carlo evidence provided in the following section shows that this modification is critical to avoid the problem of the curse of dimensionality as the number of missing observations grows, achieving a full recovery of the model parameters up to a scale with feasible estimation time.

3.4.1 Monte Carlo experimentation based on synthetic data

The effectiveness of the MSL using the two sampling methods, i.e. MSLrs and MSLis, is investigated using the same cutoff datasets as in Section 3.2.2. The experimentation was conducted using the Massachusetts Green High Performance Computing Center (MGHPCC)[1] clusters. For the reported results in Table 3.2, 1,400 jobs (100 datasets $\times$ 7 different numbers of missing observations $\times$ 2 methods) were submitted to the center specifying 4GB of memory per job. The estimates before correction given the specific number of missing observations are used as the starting values for all experiments.

[1] http://www.mghpcc.org

Table 3.2 reports the estimation results before and after applying correction. For the MSLrs method, the choice sequence is sampled 2000 times. It should be noted that this does not necessarily mean that 2000 draws will be enough for a general case, neither even for the synthetic problem at hand. Because of the curse of dimensionality, the number of draws is a dimension of the problem that needs to be investigated in a case by case basis. After correction, the percent error of VOT is generally more than 5 times better than before correction. The empirical coverage is greatly improved although it is still below the nominal value of 95%. The null hypothesis that the estimator is statistically equal to the true parameter value is rejected at all numbers of missing observations. This result is interpreted as evidence that, although consistency is achieved with the proposed correction method, the curse of dimensionality precludes formal recovery of the population parameters for the finite sample size. The MSLis method is proposed as a potential cure for the curse of dimensionality issue for this particular problem. The empirical results suggests that, for the problem at hand, the issue is satisfactorily resolved.

For the MSLis method, the complete enumeration sampling method is used for up to 5 missing days (32 unique choice sequences), and the importance sampling method is used when the number of missing days is 10 (1024 unique choice sequences) and above. The choice sequence set is generated by simulating the missing choices using random numbers following the IBL model. If a choice sequence for a given traveler is drawn twice, the second draw is discarded, since the set must contain unique choice sequences. For 10 and 15 missing days, the choice sequence is sampled 1000 and 2000 times respectively, and 20 and 100 high probability sequences are used in the estimation procedure. It should be noted that the setting does not necessarily mean that it will be enough for a general case, and the choice of sampling size and number of high probability choice sequences shall depend on the specific setting of the problem. The impact of number of high probability choice sequences on the effectiveness of the correction methods is preliminarily investigated in Section 3.4.2. After correction, the percent error of VOT is consistently below 1% and the empirical coverage of VOT is almost always above the nomial value of 95%. The p-values (¿0.05) suggest that the null hypothesis that the estimator is statistically equal to the true value is not rejected. Thus, for the problem at hand, the curse of dimensioanlity issue is satisfactorily resolved.

In the experimentation, the runtime of the MSLis is significantly shorter than that of the MSLrs. For the MSLis, since the full choice sequence set grows exponentially in the number of missing observations, the sampling size and number of high probability sequences required to statistically retrieve the true value of VOT is also expected to grow rapidly. Therefore, the runtime of larger numbers of missing observaitons is significantly longer but still 10 times smaller than that of the MSLrs, which also does not fully recover the population parameters.

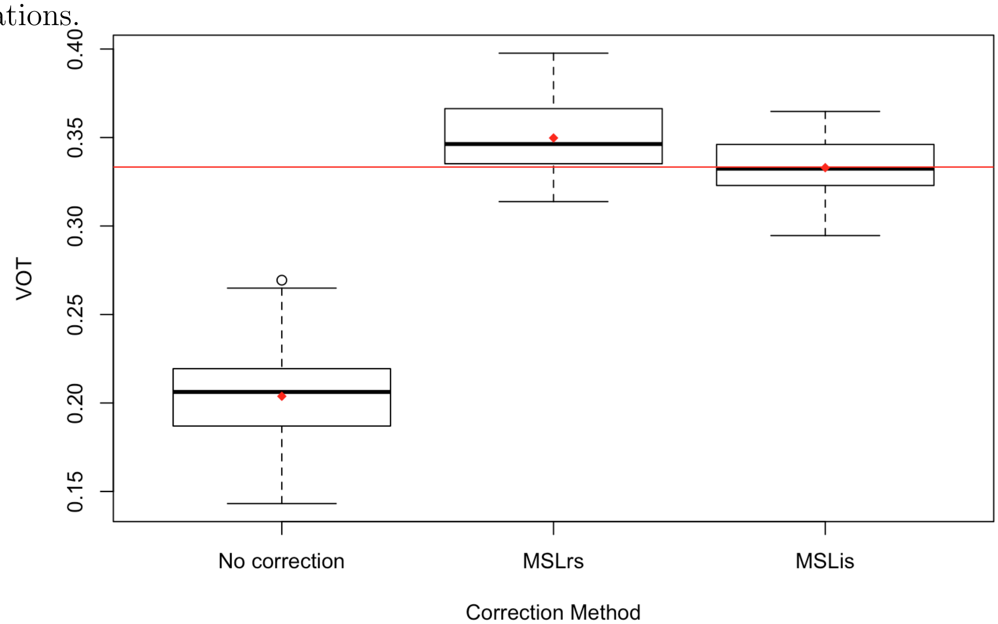
The box-plots of VOT in Fig.3.2 show the sampling distributions of VOT before and after the corrections with 10 missing initial observations. The red diamonds are the means of the estimators. It is shown that the population value of the VOT (0.333) is not covered by any point of the whole empirical distribution of the model without correction, not even by its outliers. The result is substantially improved after the MSLrs correction, in that not only the mean and median are much closer to the true population value but also the population value falls within the upper and lower (25%) quartiles. After the MSLis correction, the mean and median of the estimators are almost equal to the true population value and the population value falls within the upper and lower (25%) quartiles, confirming again that the proposed MSLis can retrieve the population parameters.

Table 3.2 Monte Carlo experimentation results

Correction Method	**# of missing obs**	**Parameter**	**Average**	**Percent error**	**p-value**	**Empirical coverage**	**Runtime**	**Sampling method**
No correction	1	β_{time} β_{cost} VOT	-0.339 -1.17 0.290	15.2 2.16 12.9	<1e-05 <1e-05 <1e-05	59 91 66	12.9 sec	
	2	β_{time} β_{cost} VOT	-0.313 -1.17 0.269	21.7 2.87 19.3	<1e-05 <1e-05 <1e-05	23 93 31	12.4 sec	
	3	β_{time} β_{cost} VOT	-0.296 -1.17 0.254	29.5 2.84 27.4	<1e-05 <1e-05 <1e-05	7 90 13	12.2 sec	
	4	β_{time} β_{cost} VOT	-0.281 -1.17 0.242	29.5 2.84 27.4	<1e-05 <1e-05 <1e-05	2 91 2	11.8 sec	
	5	β_{time} β_{cost} VOT	-0.273 -1.17 0.233	31.9 2.31 30.2	<1e-05 <1e-05 <1e-05	1 92 3	11.8 sec	
	10	β_{time} β_{cost} VOT	-0.241 -1.18 0.204	39.8 1.39 38.9	<1e-05 0.00303 <1e-05	0 97 0	9.18 sec	
	15	β_{time} β_{cost} VOT	-0.216 -1.18 0.183	46.0 1.58 45.1	<1e-05 <1e-05 <1e-05	0 96 0	7.56 sec	
MSLrs	1	β_{time} β_{cost} VOT	-0.407 -1.21 0.337	1.89 0.899 0.991	<1e-05 0.00005 0.00123	93 94 93	278 min	2000 draws
	2	β_{time} β_{cost} VOT	-0.411 -1.22 0.338	2.76 1.27 1.48	<1e-05 <1e-05 0.00003	90 90 90	299 min	
	3	β_{time} β_{cost} VOT	-0.416 -1.22 0.341	3.90 1.68 2.21	<1e-05 <1e-05 <1e-05	82 86 90	320 min	
	4	β_{time} β_{cost} VOT	-0.420 -1.23 0.343	5.01 2.20 2.77	<1e-05 <1e-05 <1e-05	75 88 88	332 min	
	5	β_{time} β_{cost} VOT	-0.424 -1.23 0.345	6.10 2.48 3.56	<1e-05 <1e-05 <1e-05	76 85 88	345 min	
	10	β_{time} β_{cost} VOT	-0.436 -1.25 0.350	8.89 3.79 4.92	<1e-05 <1e-05 <1e-05	71 82 85	744 min	
	15	β_{time} β_{cost} VOT	-0.444 -1.27 0.354	11.1 6.12 6.27	<1e-05 <1e-05 <1e-05	66 79 83	1209 min	
MSLis	1	β_{time} β_{cost} VOT	-0.403 -1.21 0.332	0.102 1.67 0.232	<1e-05 <1e-05 0.676	96 89 97	0.541 min	Complete enumeration: 2 choice sequences
	2	β_{time} β_{cost} VOT	-0.406 -1.22 0.332	1.72 2.05 0.296	<1e-05 <1e-05 0.447	94 84 95	0.952 min	Complete enumeration: 4 choice sequences
	3	β_{time} β_{cost} VOT	-0.410 -1.233 0.332	2.44 2.76 0.271	<1e-05 <1e-05 0.549	93 84 96	1.87 min	Complete enumeration: 8 choice sequences
	4	β_{time} β_{cost} VOT	-0.413 -1.24 0.332	3.15 3.58 0.363	<1e-05 <1e-05 0.476	90 74 97	2.85 min	Complete enumeration: 16 choice sequences
	5	β_{time} β_{cost} VOT	-0.414 -1.25 0.331	3.42 4.12 0.675	<1e-05 <1e-05 0.220	92 69 94	6.70 min	Complete enumeration: 32 choice sequences
	10	β_{time} β_{cost} VOT	-0.402 -1.21 0.333	0.607 0.743 0.111	0.216 0.00613 0.809	94 91 95	23.3 min	Importance sampling sampling size:1000 choice sequence:20
	15	β_{time} β_{cost} VOT	-0.396 -1.20 0.331	1.05 0.338 0.689	0.0503 0.253 0.168	93 96 95	149 min	importance sampling sampling size:1000 choice sequences:100

* Estimation results are based on 100 repetitions. The nominal value of empirical coverage is 95%.
P-values are calculated against true values. A p-value greater than 0.05 indicates no statistical difference.
Runtime is the average of one repetition.

Figure 3.2 Box-plots of VOT before and after corrections with 10 missing intial observations.



3.4.2 Sensitivy analysis to other simulation assumptions

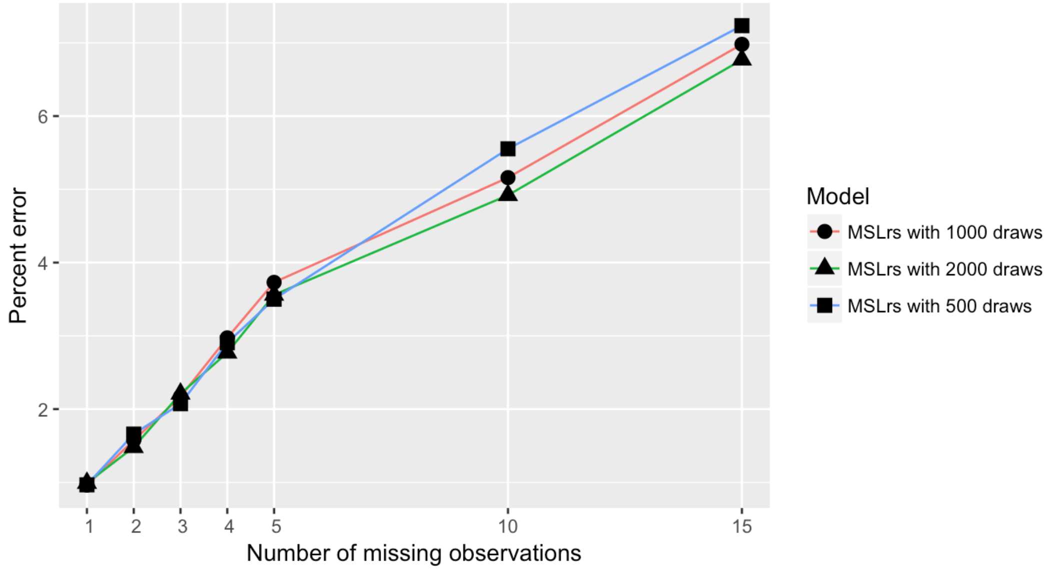
Sampling size in random sampling

In Section 3.4.1, the choice sequence is sampled 2000 times for the MSLrs method. Fig.3.3 investigates the impact of sampling size on the percent error of VOT using 500, 1000, and 2000 draws. The estimators based on 2000 draws are generally better than those based on 500 and 1000 draws, but the improvement is not significant. Theoretically, as the sampling size increases, the quality of the estimators should also increase, however, this can be very computationally expensive.

Number of high probability choice sequences in importance sampling

For the MSLis method, the impacts of the size of high probability choice sequence set on the percent error of VOT and runtime are investigated for 10 missing observations. In this experiment, the choice sequence is sampled 1000 times to represent an adequate sampling size. In Fig. 3.4, as the number of high probability sequences increases from 2 to 50, the percent error decreases from close to 5% to below 1%.

Figure 3.3 Impact of Number of Draws on Percent Error of VOT.



When the number of high probability sequences is greater than 20, the percent error increases slightly. The hypothesis is that the inclusion of the choice sequences with very low probability of occurrence may cause numerial issues in the estimation procedure. It should be noted that not all numbers of high probability choice sequences can statistically retrieve (i.e., p-value¿0.05) the true VOT value. The runtime increases with the number of high probability sequences. When computational efficiency is a major concern, it is recommended to reduce the number of high probability sequences for large number of missing observations.

3.4.3 Computational experiments based on empirical data

To confirm the applicability of the proposed methods, the IBL model is estimated using the experimental dataset described in Ben-Elia & Shiftan (2010). For illustrative purpose, only the data of the informed group is used for the estimations. In the experiment, twenty-four participants were faced with three scenarios of binary route-choice as presented in Table 3.3. A small degree of variation was programmed (± 5 or ± 15 min around the mean) to simulate a simple variable message sign (VMS). Each scenario included 100 choices so in total each participant completed 300 trials. For

Figure 3.4 Impact of Number of High Probability Choice Sequences on Percent Error of VOT and Runtime with 10 Missing Observations.

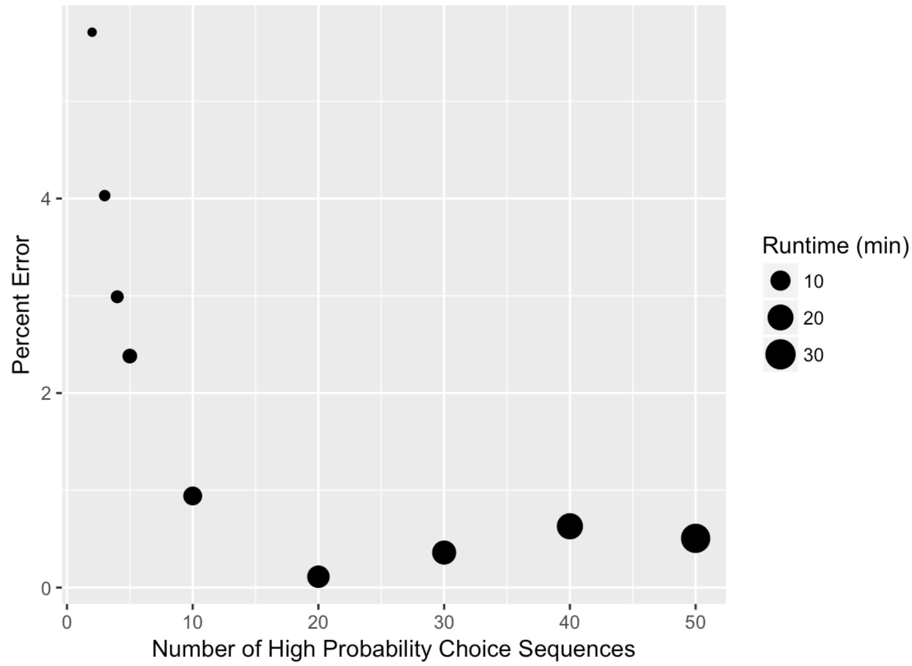


Table 3.3 Hypothetical travel time scenarios of the experimental dataset

Scenario	Travel time ranges (minutes)	
	Route F - 25 min.	Route S - 30 min.
Fast & Safe	±5	±15
Fast & Risky	±15	±5
Low-Risk	±5	±5

each choice situation the participants received real-time information about the travel time range (the minimum and maximum travel times) for each of the two routes. Following the choice, a feedback was received regarding the "actual" travel time on the chosen route but not of the un-chosen one. This travel time was randomly drawn from the distribution of the travel time range.

A simplified version of the IBL model developed in Chapter 2.4.2 is specified. Eq.(3.11) shows the utility functions for the two paths. On a given day t, T_{SLOW}, T_{FAST} are the perceived travel times for the two paths respectively and are non-linear functions of the decay parameter d. Note that d is estimated in the experimental dataset, as opposed to fixed in the Monte Carlo tests. The cumulative weighted average (CWA) of the preceding choices is used to reflect travelers' trends to repeat past choices. See Ben-Elia & Shiftan (2010) for specification. Sensitivity to variability of the travel times is represented using dummy variables for the scenarios, LRISK for Low-Risk and FRISKY for Fast & Risky. Although the model is a simplification of that developed in Chapter 2.4.2, the perceived travel time and CWA of the preceding choices that have complete history dependency are kept in the model to assess the proposed correction methods.

$$\begin{cases} U_{SLOW}(t) = \beta_{TIMES} T_{SLOW}(t) \\ U_{FAST}(t) = \beta_{TIMEF} T_{FAST}(t) + \beta_{LRISK} LRISK(t) \\ \qquad + \beta_{FRISKY} FRISKY(t) + \beta_{CWA} CWA(t) \end{cases} \tag{3.11}$$

The cutoff dataset is generated by removing the first 10 observations for each participant. The IBL model is estimated using the full dataset and the estimates are assumed to be the "true" parameter values. The cutoff dataset is used to estimate the IBL model before correction and after the MSLrs and MSLis correction methods. The sampling size for the MSLrs method is 1000, and the sampling size and high probability sequences are set to 1000 and 20 respectively for the MSLis method. With the correction methods applied, the estimates obtained from the cutoff dataset are used as the priors to mimic real-life practice. The Hausman-McFadden test (Hausman & McFadden, 1984) is used to exam whether the estimators of the cutoff dataset are statistically equal to the estimators of the full dataset. Table 3.4 presents the estimation results. For the full dataset, all estimates are statistically significant according to the t-test against 0. For the cutoff dataset, before correction the null hypothesis of Hausman-McFadden test that the estimators are statistically equal to the estimators of the full dataset model is rejected (95% confidence, degree of freedom of 6, and critical value of 12.59). After applying the correction methods, the null hypothesis of the Hausman-McFadden test is accepted, meaning the estimators are statistically equal

Table 3.4 Estimation results based on empirical data

Experiment	**Metric**	d	β_{TIMEF}	β_{TIMES}	β_{LRISK}	β_{FRISKY}	β_{CWA}
Full dataset	Estimate	1.28	-0.198	-0.086	0.864	0.405	5.71
	Std. error	0.137	0.015	0.013	0.144	0.120	0.192
	t-test (against 0)	9.31	-13.2	-6.62	6.00	3.38	29.7
10 missing observations, no correction	Estimate	1.19	-0.205	-0.0697	0.805	0.276	6.74
	Std. error	0.140	0.0176	0.0156	0.168	0.132	0.245
	Hausman-McFadden test	22.7					
10 missing observations, MSLrs correction	Estimate	1.180	-0.212	-0.0843	0.782	0.300	6.35
	Std. error	0.129	0.016	0.014	0.152	0.126	0.227
	Hausman-Mcfadden test	9.96					
10 missing observations, MSLis correction	Estimate	1.20	-0.196	-0.0852	0.842	0.312	6.24
	Std. error	0.149	0.0144	0.0164	0.213	0.161	0.213
	Hausman-McFadden test	8.56					

to the estimators of the full dataset model. The difference between the estimators of the curtailed model and corrected models is expected to be larger as the number of missing initial observations increases. Finally, note that the finite sample bias of the MSLis correction is notably smaller than that of the curtailed and the MSLrs models.

3.5 Summary

Learning-based models that capture travelers' day-to-day learning process in repeated travel choice can suffer from the common problem of missing initial observations in longitudinal data collection that leads to inconsistent estimate of the perceived value of the attribute in question, and thus inconsistent parameter estimates. In this chapter, the MSL with two sampling methods is developed and assessed to address the endogeneity problem due to missing initial observations in learning models with complete history dependency. An IBL model in recent literature is used for its capability of precisely capturing travelers' learning process in repeated choice and model complexity.

Monte Carlo experimentation based on synthetic data shows that the proposed method drastically reduces the finite sample bias of the estimators compared to the curtailed model. For the MSLrs method, a size distortion that reflects in p-values against the true VOT value is detected, which suggests the inefficiency of the sampling method makes the method suffer from the curse-of-dimensionality problem. In contrast, the MSLis method can retrieve the true VOT value. Moreover, the computational efficiency of the MSLis is significantly better than the MSLrs method. The impacts of the sampling size in the MSLrs method and number of high probability choice sequences in MSLis are investigated. Empirical results suggest that when the number of missing observations is large, the number of high probability sequences in MSLis should be limited for computational efficiency. The two methods are also applied to empirical data to demonstrate their applicabilities. Estimation results show that the estimators after correction are statistically equal to the estimators of the full dataset model.

CHAPTER 4

DAY-TO-DAY DRIVING BEHAVIOR INTERVENTION

This chapter is based on collaborative work with Tao Jiang

4.1 Project Overview

Apart from modeling travelers' day-to-day travel behavior, this dissertation also investigates travelers' day-to-day driving behavior intervention. A study of Mitigation Techniques to Modify Driver Performance to Improve Fuel Economy, Reduce Emissions and Improve Safety, was undertaken as part of the Massachusetts Department of Transportation Research Program. This program is funded with Federal Highway Administration (FHWA) Statewide Planning and Research (SPR) funds.

Transportation has a major impact on our society and environment, contributing 70% of U.S. petroleum use, 28% of U.S. greenhouse gas (GHG) emissions (Bureau of Transportation Statistics., 2013), and over 34,000 fatalities and 2.2 million injuries in 2011 (Environmental Protection Agency (n.d.).). MassDOT is a major contributor to energy use and greenhouse gas emission, the state-owned vehicle fleet consumes a fair amount of fuel each year. Thus, investigating techniques which could improve fuel economy, reduce emission and improve safety is in urgent need. This in furtherance is of MassDOT's mission and goals of the GreenDOT implementation plan.

This project investigates the effectiveness a combination of static and dynamic eco-driving techniques to modify driver performance to improve fuel economy, reduce emission and improve safety. The static eco-driving technique refers to eco-driving training and follow-up email tips, the dynamic technique is real-time in-vehicle feedback device, which could display drivers' driving performance in several detailed categories, i.e., acceleration, braking, cornering, lane handling and speeding, so as to help modifying their driving behavior instantaneously. Evaluation of effectiveness of the two general types of techniques would be made based on difference in performance cross test and control groups between each experiment phase.

The project involves 133 MassDOT vehicles installed with in-vehicle tracking devices provided by GreenRoad Technology, Inc. Two types of behavior interventions were tested as mentioned in the previous paragraph: in-vehicle real-time feedback and classroom eco-driving training with follow-up email tips. Then a two-factor, two-level design results in four groups with vehicles assigned randomly from the 133 vehicles with five major and four minor factors which would affect fuel economy equally distributed across groups. All four groups went through three chronological phases: 1) Phase I (baseline): 6/1-7/27/2015, no real-time feedback, no eco-driving training, 2) Phase II (intervention period): 7/28-10/09/2015, real-time feedback was provided to two groups and training was conducted for two groups, followed by bi-weekly eco-driving tip emails, and 3) Phase III (off period): 10/10/2015-02/01/2016, real-time feedback was turned off and eco-driving tips discontinued.

4.2 Literature Review

4.2.1 Factors affecting fuel economy, greenhouse gas and air pollutant emission and safety

Fuel consumption (FC) factor is defined as the volume of fuel consumed for a vehicle to travel a unit distance (gallon per mile or liter per kilometer). Similarly, the CO2 emission factor is defined as the mass of CO2 emission for a unit distance traveled (gram per mile or gram per km).

Speed, instantaneous speed especially, is a major factor affecting fuel economy and greenhouse gas emission. Tong et al. (2000) studied four different instrumented vehicles under four standard driving modes. The FC factor was monotonically decreasing until the maximum speed and the optimum fuel efficiency range approached at least 60-70 km/h for the petrol passenger car and diesel van. During acceleration process, FC was more than 80% higher than that during cruising for passenger car, slightly higher than that of deceleration for the petrol passenger car and van, and almost the same as that of deceleration for the diesel van. In addition, the NOx, HC, and CO emission factors decreased as the instantaneous speed increased, and the decrease rate became more gradual as the speed increased, similar to the trends for FC.

Ericsson (2001) conducted a comprehensive study in an average-sized Swedish city about factors that affect fuel consumption and emissions, which is based on real-traffic data of 2,550 journeys and 18,945 km of driving of five passenger cars. By using factorial analysis, only the factor for speed 50-70 km/h was found to have a significant negative effect on fuel-use and CO2 emissions. This indicated that the most fuel efficient cruise speed is in the range of 50-70 km/h. The stop factor was highly significant, suggesting that idling was a very important contributor to FC and CO2 emission. Ericsson (2001) also demonstrated that HC emissions were primarily affected by factors for acceleration with high power demand and extreme acceleration, none of the speed factors were significant for either NOx or HC emissions. This suggests that acceleration increased pollutant emissions more than it increased FC and CO2 emissions.

Idling a vehicle for any amount of time significantly reduced efficient fuel economy for a trip, as Saboohi & Farzaneh (2009) implied. In an experiment that lasted 276 seconds, an additional fuel consumption of 0.33 liters (0.08 gallons) was detected. Thus, for every hour of idle running for an average passenger car, 4.3 liters (1.14 gallons) of petrol was burnt. Another experiment mentioned in Sivak & Schoettle (2011) monitored vehicles on a 16 km course. By turning off the engine during each of the ten idle periods, lasting two minutes each, there was a 19% fuel economy improvement.

In addition, speed is an important factor in road safety. At high speeds the time to react to changes in the environment is shorter, the stopping distance is larger, and maneuverability is reduced. Aarts & Van Schagen (2006) provided a literature review of studies on the speed-crash risk relationship. The authors noted an Australian study by Fildes et al. (1991) that applied a self-report method. Drivers with different driving speeds were stopped and asked about their history of road crashes during the last 5 years. The relationship had the shape of an exponential function, and the similar trend was also reported in other studies (see, e.g., Kloeden et al. (2002)). Later Taylor et al. (2002) suggested that accident frequency increased with driving speed to the power of approximately 2.5.

Aggressive acceleration and deceleration are also leading factors in contributing fuel wasting and extra pollutant emissions. Wang et al. (2011) found that the FC factors were the highest at acceleration, modest at cruise speeds and the lowest at deceleration for non-idling buses. Kim & Choi (2013) estimated critical values of aggressive acceleration about FC factors and came to the same conclusion as in Wang et al. (2011). Aside from that, Wang et al. (2011) also found similar relationships between pollutant emissions and acceleration as those between FC and acceleration.

Quick acceleration and deceleration also lead to higher crash risk. Since they increase the potential for loss of vehicle control and reduce the time available to the driver to respond to the actions of other drivers and to take evasive actions to avoid a crash should a conflict materialize. Conclusions taken from researches done by Elvik (2006) and Bagdadai & Varhelyi (2011).

4.3 Methodology

4.3.1 Experiment design

All vehicles in the field test were owned by MassDOT with a designated driver so that potential behavioral changes could be properly attributed to interventions. Vehicle types were restricted to sedan, SUV, van, and pick-up truck, where heavy trucks and state police vehicles were explicitly excluded. Two types of behavioral interventions were tested: in-vehicle real-time feedback and classroom training with follow-up email tips. A two-factor, two-level factorial design results in four groups: 1) Receive in-vehicle feedback and eco-driving training, 2) Receive in-vehicle feedback but no eco-driving training, 3) Receive eco-driving training but no in-vehicle feedback, and 4) No eco-driving training and no in-vehicle feedback. Vehicles were randomly assigned to groups with four major and four minor factors that could potentially affect fuel economy and safety performance were counterbalanced. The major factors were: 1) Vehicle type (sedan, SUV/van, and pick-up truck), 2) Manufacture year (2000-2004, 2005-2009, and 2010-2015), 3) Fuel type (gasoline and hybrid), and 4) Driving distance in Phase I (baseline period). The four minor factors were: 1) Driver gender (male and female), 2) Age (21-30, 31-40, 41-50, 51-60, 61+), 3) Vehicle carrying weight typically (<100 lb, 100-200 lb, 200-300 lb, and >300 lb), and 4) Previous eco-driving feedback device experience (yes or no). During the whole study period, all groups went through three chronological phases: 1) Phase I (baseline phase): 6/1-7/27/2015 (8 weeks), No eco-driving interventions provided, 2) Phase II (intervention phase): 7/28-10/9/2015 (10 weeks), Real-time feedback was provided to two groups throughout Phase II, and classroom training was conducted for two groups at the beginning of Phase II, followed by bi-weekly eco-driving tip emails from the eco-driving trainer, and 3) Phase III (off phase): 10/10/2015-02/01/2016 (16 weeks), Real-time feedback was turned off and eco-driving tip emails discontinued.

4.3.2 Data Collection

Each driver contributed one data point in each phase for fuel economy, idling rate, overall safety score and each safety score by category. The daily raw data were obtained from GreenRoad Central (an online user interface where driver performance could be retrieved), and will be averaged over all days for a given phase. A variety of reports were available on a daily basis regarding fuel economy, idling, and safety performance in GreenRoad Central. In addition, a customized Amazon EC2 database was created by GreenRoad for this particular study, which provided the following information every 30 seconds: vehicle location coordinates with timestamps, cumulative fuel consumption, fuel economy, and cumulative traveling distance. This allows for analysis based on geographic location.

4.3.3 Regression analysis

After all the data were cleaned up. The researcher carried out multiple linear regression analysis to test whether the two interventions, eco-driving training and real-time feedback are effective in improving fuel economy, reducing idling rate and improving drivers' safety performance. The response variables are fuel economy percentage change, vehicle idling rate percentage change and safety score percentage change, which are defined as follows:

$$\text{response variable} = \frac{\text{value of variable in phase II or III} - \text{value of variable in phase I}}{\text{value of variable in phase I}} \tag{4.1}$$

Three dummy variables, corresponding to training, feedback, and interaction effect of training and feedback, were used as explanatory variables. Traditionally, a dummy variable equals 1 if a driver receives the corresponding intervention, and 0 otherwise. The interaction variable equals 1 if a driver receives both interventions, and 0 otherwise. In this study, the level of significance was chosen as 0.10, with the interpretation that p-value equals or less than 0.10 indicating significant effect, otherwise not.

Intuitively, the multiple linear regression for the changes between Phases I and II test the short-term effect of interventions, while for the changes between Phases I and III test the long-term effect. In addition, Phase II was further divided into two periods: first month (July 28 - Sep 9, 2015) and second month (Sep 10 - Oct 09, 2015). Analysis was also done for hybrid / non-hybrid vehicles separately, and based on vehicle types, namely, SUV, pick-up truck and sedan. Lastly, data collected from "Express way" and "Local way" were treated separately, since traffic condition and vehicle performance are quite different between the two types of road.

4.4 Analysis Results

4.4.1 Short-term effect

Safety

Safety analysis results (Table 4.1) show that overall safety score has been reduced in Phase II due to feedback at a 1% level of significance. Specifically, the positive effect of feedback in reducing speeding score was significant at a 0.01% level during phase II. Note that a lower safety score means safer behavior.

Further analysis by vehicle type (sedan, SUV, pickup truck) shows that pickup trucks benefit the most from real-time feedback.

1. Feedback reduced overall safety score for pickup trucks at a 5% significance level, during Phase II, while the effect was not significant for sedans or SUVs.

2. Feedback reduced acceleration score for pickup trucks at a 10% significance level during Phase II, and the effect sustained in Phase III.

Table 4.1 Regression analysis result of safety performance measures

-	**Overall safety score for all vehicle (Phase I/II)**		**Overall safety score for pickup truck (Phase I/II)**		**Acceleration score for all vehicle (Phase I/II)**		**Acceleration score for pickup truck (Phase I/II)**	
	Estimate	p-value	Estimate	p-value	Estimate	p-value	Estimate	p-value
Intercept	0.127	0.033	0.136	0.15	0.07	0.6	0.3297	0.15
Training	0.025	0.787	-0.087	0.517	-0.058	0.782	-0.51	0.124
Feedback	-0.284	0.002*	-0.32	0.023*	-0.35	0.088*	-0.64	0.055*
Training & Feedback	0.071	0.589	0.275	0.158	0.0002	0.999	0.414	0.381
-	**Acceleration score for sedan (Phase I/III)**		**Acceleration score for pickup truck (Phase I/III)**		**Braking score for all vehicle (Phase I/III)**		**Braking score for SUV (Phase I/III)**	
	Estimate	p-value	Estimate	p-value	Estimate	p-value	Estimate	p-value
Intercept	-0.08	0.77	0.8	0.001	0.37	0.017	-0.76	0.025*
Training	0.58	0.2	-0.9	0.009*	-0.24	0.31	0.056	0.87
Feedback	0.77	0.1	-0.68	0.039*	-0.53	0.019*	-0.76	0.025*
Training & Feedback	-1.28	0.046*	0.72	0.13	0.52	0.11	0.36	0.46
-	**Speeding score for all vehicle (Phase I/II)**		**Speeding score for SUV (Phase I/II)**		**Speeding score for pickup truck (Phase I/II)**		**Speeding score for pickup truck (Phase I/III)**	
	Estimate	p-value	Estimate	p-value	Estimate	p-value	Estimate	p-value
Intercept	0.11	0.07	0.02	0.83	0.18	0.079	0.11	0.49
Training	-0.014	0.88	0.22	0.16	-0.16	0.25	-0.26	0.28
Feedback	-0.36	1.04e-4*	-0.32	0.04*	-0.46	2.17e-03	-0.4	0.08*
Training & Feedback	0.12	0.38	-0.034	0.88	0.4	0.055*	0.53	0.12

3. Feedback reduced speeding score for pickup trucks at a 0.01% significance level during Phase II, and the effect sustained in Phase III (at a 10% significance level).

Idling

Result of idling rate change (Table 4.2) shows that training have a positive effect in reducing idling rate in the first month of Phase II at a 10% level of significance. Idling rate is a major contributor to fuel inefficiency, so reducing idling rate could potentially lead to improvement of fuel economy. The in-vehicle feedback device did

Table 4.2 Regression analysis results of idling rate and fuel economy percentage change

-	Idling rate of all vehicle (PhaseI/First month of Phase II)		Fuel economy of sedan (Phase I/First month of Phase II)		Fuel economy of SUV (Phase I/First month of Phase II)		Fuel economy of hybrid vehicles (Phase I/II)	
	Estimate	p-value	Estimate	p-value	Estimate	p-value	Estimate	p-value
Intercept	0.549	0.019	-0.015	0.765	-0.04	0.052	0.132	0.118
Training	-0.724	0.033*	-0.095	0.252	0.008	0.778	-0.167	0.161
Feedback	-0.472	0.15	-0.023	0.777	0.046	0.0705*	-0.17	0.138
Training & Feedback	0.734	0.109	0.196	0.091*	0.011	0.781	0.28	0.077*

not provide feedback on idling and only monitors it, thus it is not surprising that feedback does not have any effect in reducing idling rate. The classroom training session discussed idling as a major factor and the first two follow-up tip emails were mostly about idling with clear guidelines. This suggests that targeted education on an implementable behavioral change could be effective.

Fuel Economy

The effect of real-time feedback has a positive impact on fuel economy for SUVs in the first month of Phase II at a 10% level of significance (Table 4.3). The combination of feedback and training, however, has a positive effect in improving fuel economy for sedans in the first month of Phase II at a 10% significance level, and for hybrid vehicles throughout Phase II at a 10% significance level. Based on geographic location, the combination of training and feedback has a positive effect in improving fuel economy for pickup truck in the first month of Phase II at a 10% level of significance on "Local Way". And it improves the fuel economy for hybrid vehicle in the first month of Phase II at a 10% level of significance on "Express Way" (Table 4.3).

Classroom training provided drivers with a systematic treatment of eco-driving theories and practices, while real-time feedback provided immediate indication of driving performance. On one hand, it takes conscious effort and practice to translate what is learned in a classroom training session to real-world behaviors. On the other

Table 4.3 Regression analysis results of fuel economy percentage change based on geographic location

-	**Fuel economy of pickups on "Local Way" (Phase I/First month of Phase II)**		**Fuel economy of hybrid vehicles on "Express Way" (Phase I/First month of Phase II)**	
	Estimate	p-value	Estimate	p-value
Intercept	0.037	0.333	0.004	0.77
Training	-0.049	0.33	-0.009	0.65
Feedback	-0.087	0.091	-0.008	0.71
Training & Feedback	0.118	0.094	0.052	0.09

hand, real-time feedback might be difficult to understand if drivers are not familiar with energy-efficient driving styles. It is thus hypothesized that a combination of the two interventions could overcome the shortcomings of each intervention, that is, drivers do not need to make conscious effort but are rather reminded to change behaviors and can understand what to change based on the real-time feedback. The exception for SUVs could be the reason that the major driving areas of SUVs are construction or other working sites. Generally, the terrain characteristics or the traffic conditions of working zones are typically less-than-ideal. Thus drivers tend to receive more alerting indicators, and also have a larger chance of modifying behaviors while driving. The above result provid es some preliminary support to these hypotheses.

Remarks

As suggested by results for idling rate and safety scores, training has a positive effect on reducing idling, while feedback has a positive effect in reducing speeding and aggressive acceleration. Idling, speeding, and aggressive acceleration are major contributors to fuel inefficiency, GHG emissions and unsafe driving, according to the literature synthesis. It is plausible that the goal of improving fuel efficiency, safety and reducing emissions is more likely to be achieved when all three factors are accounted for, and thus combined training and feedback is needed.

4.4.2 Long-term effect

Drivers no longer received any feedback or eco-driving tip emails in Phase III and eco-driving training has passed for 10 weeks. The regression analysis of the change from Phase I provides evidence as to whether the intervention have long-term effects. From the regression analysis results, there was no significant positive improvement in fuel economy or idling rate. While some safety improvements sustained (Table 4.2). In general, the effects diminished in Phase III. This suggests that drivers tend to slip back to old driving habits after feedback devices were turned off, and effect of training diminishes in a couple of months after in-classroom training.

4.4.3 Cost saving estimation

Aggressive acceleration, speeding and idling are major factors that would affect fuel economy according to studies by Tong (3), Ericsson (4) and Saboohi (5). As there are no direct measurements of estimating fuel savings on reduction in first two factors, we focus on the idling factor.

From the regression results, we could see that the idling rate has been reduced by 17.5% compared with control group due to training factor (Table 4.2). Assuming all conditions in (5) apply here, where 1.14 gallons of petrol was burnt for every hour of idling running of average passenger car (5). Using average idling rate in control group during Phase I as baseline (which is 0.0713, not shown in table). We could estimate the fuel savings per hour driving as follows. This indicates that 0.039 dollar would be saved per hour driving for an average passenger car, with an average gas price of 2.75 $/gallon in Western Massachusetts currently.

$$\text{Unit Saving} = \frac{1.14\text{Gallon}}{1\text{Hour Idling}} * 0.0713 \frac{\text{Unit Idling Time}}{\text{Unit Driving Time}} * 0.175 = 0.0142 \frac{\text{Gallon}}{\text{Hour Driving}} \tag{4.2}$$

4.5 Conclusions and Recommendations

4.5.1 Conclusions

Based on the analysis in the previous section, several major conclusions could be drawn as below:

1. Real-time feedback had a highly significant effect in reducing speeding. The effect however disappeared after the feedback was discontinued. According to the conclusion from the literature synthesis, abiding by speed limits on highways not only can significantly reduce crash risk, but also improves fuel economy and reduces emissions (50-90 km/h has emerged as optimum fuel consumption and emission speed ranges from the literature).

2. Real-time feedback had a moderately significant effect in reducing aggressive acceleration and lane handling. The effect however disappeared after the feedback was discontinued. According to the literature synthesis, aggressive acceleration significantly increases fuel consumption, CO_2, NO_x, HC, and CO emissions, and is a contributor to crash risk.

3. Training had a moderately significant effect in reducing idling rate in the first month after training. The effect disappeared after the first month. According to the conclusion from the literature synthesis, idling (stops) or driving at a very low speed significantly worsens fuel consumption and emissions.

4. Combined classroom training and real-time feedback had a moderately significant effect in improving fuel economy for hybrid vehicles. The effect disappeared after the feedback was discontinued.

5. In the long run, eco-driving not only helps reduce fuel consumption and emissions, but also contributes to reduced accidents because of smoother and less

aggressive driving behavior. Savings due to reduced accident costs and insurance premiums should also add to the long-term benefits of implementing eco-driving.

4.5.2 Recommendations

Widespread deployment options for real-time feedback devices

Based on major conclusions #1 and #2 in conclusion section, we make the following recommendations regarding the widespread deployment of real-time feedback devices. Three options are available with regard to the deployment of real-time feedback device with the increment of system complexity, namely feedback device only, feedback device with periodic self-evaluation and feedback device, periodic self-evaluation with fleet manager monitoring.

For real-time feedback device-only option, the vehicles will be installed with the feedback device only. While no inspection or evaluation of driver performance would be provided during driving or after that, or drivers will not be able to get access to their driving records. This greatly protects drivers' privacy and reduce the cost of system. But the effectiveness might be the least significant overall. Adding the other two options with feedback device would possibly serve as stimuli to achieve a better driving performance so as to save more fuel and reducing emissions. But drivers' privacy will not be guaranteed and the system cost would be larger.

Widespread deployment options for eco-driving training

Majorly two options are readily available for the deployment of eco-driving training, online training and classroom training. The cost for online training course is lower than classroom training as no travel costs (and energy consumption) for the trainer or trainees are needed. Drivers have the time flexibility and can take the course at their own pace. Also, online training can be as effective as classroom training if properly designed. For classroom training, the cost might be higher than online training, but it allows for face-to-face interactions that usually promotes better learning effects. Additionally, the classroom training could be delivered by fleet members who received training from training program vendors (refers to trained by trainer option in literature synthesis section), which would reduce costs and the training would be easy to implement logistically.

CHAPTER 5

CONCLUSIONS AND FUTURE DIRECTIONS

5.1 Research Summary

In this thesis, an instance-based learning (IBL) model for travel choice is developed in a route-choice context based on the power law of forgetting and practice. This model is shown to be capable of capturing the recency, hot stove and payoff variability effects embedded in travelers' day-to-day learning process. Experiments based on synthetic datasets show that the true parameter values of the IBL model can be consistently retrieved and the model predicts different traffic patterns compared to a model that completely ignores learning and a learning model that ignores spatial knowledge carryover. The IBL model is also compared to a baseline learning model using an experimental dataset of repeated route-choice. Estimation results show that the IBL model suggests a larger role of learning and achieves better model fit. Cross validation experiments suggest that the forecasting ability of the IBL model is better than the baseline learning model.

Learning-based models with complete history dependency can suffer from the common problem of missing initial observations in longitudinal data collection that leads to inconsistent estimate of the perceived value of the attribute in question, and thus inconsistent parameter estimates. In this thesis the MSL with two sampling methods is developed and assessed to address the stated problem. The IBL model is used for its capability of precisely capturing travelers' learning process in repeated choice and model complexity. Monte Carlo experimentation based on synthetic data shows that the proposed method drastically reduces the finite sample bias of the estimators compared to the curtailed model. For the MSLrs method, a size distortion that reflects in p-values against the true VOT value is detected, which suggests the inefficiency of the sampling method makes the method suffer from the curse-of-dimensionality problem. In contrast, the MSLis method can retrieve the true VOT value. Moreover, the computational efficiency of the MSLis is significantly better than the MSLrs method. The impacts of the sampling size in the MSLrs method and number of high probability choice sequences in MSLis are investigated. Empirical results suggest that when the number of missing observations is large, the number of high probability sequences in MSLis should be limited for computational efficiency. The two methods are also applied to empirical data to demonstrate their applicabilities. Estimation results show that the estimators after correction are statistically equal to the estimators of the full dataset model.

Another aspect of this thesis is to gain insights on day-to-day driving behavior intervention. A study on mitigation techniques to improve ruel economy, reduce emissions and improve safety was undertaken as part of the Massachusetts Department of Transportation Research Program. Major conclusions include: 1) Real-time feedback has a significant effect in reducing speeding and aggressive acceleration. 2) Training has a significant effect in reducing idling rate in the first month after training. 3) Combining training and feedback is expected to significantly improve fuel economy, reduce emissions and improve safety.

5.2 Future Research Directions

In this thesis, an exploratory effort in understanding, specifying and applying the IBL model is presented in route-choice context. For the model to be operational, practical considerations need to be accounted for, as discussed below.

First, choice set generation in a real network needs to be considered in contrast to the well defined choice set in a binary choice context. A number of choice set generation methods have been developed to serve this end (Ramming, 2002; Bekhor et al., 2002). More recent studies have focused on the dynamic formation of choice set, where the addition or deletion of alternatives during the learning and repeated choice process is explicitly considered (Han et al., 2008, 2011). The IBL model also has potentials to be expanded and applied to the dynamic formation of choice set.

Second, past experience gained from work trips is retrieved for the occasional leisure trip to illustrate that spatial knowledge can be carried over from one trip to another. In real practice, the context of the experience (e.g., AM/PM/mid-day/other, weekday/weekend) will be considered and a matching score between the context in an instance and the current context is calculated such that only instances over a certain matching threshold can be retrieved.

The research on the correction method for the intitial condition problem in learning models with complete history dependency can be extended in the following directions.

First, since the runtime of the MSLis increases as the number of missing observations grows, the possibility of limiting the number of missing initial observations to be simulated needs to be investigated. Due to the nature of the model that more recent and frequent outcomes take larger weights in memory, only the omission of recent instances will cause estimation biases for practical purposes. Thus, the hypothesis is that only a certain number of unobserved instances prior to the first observation needs to be simulated to improve the estimators up to a desired threshold.

Second, we would like to explore alternative correction methods. For example, the Multiple Imputation (MI) principle proposed by (Little & Rubin, 1987) can be used to develop a correction method using importance sampling. In this method, instead of simulating the likelihood as with MSL, each simulated choice sequence is used to estimate the model parameters via maximum likelihood estimation. The vectors of estimators obtained from all choice sequences are used to build the sampling distribution of the estimators with standard complete-data methods. Other alternative methods similar to the method proposed by (Guevara & Ben-Akiva, 2013a,b) may also be explored.

APPENDIX
WEIGHT FUNCTION OF THE ORIGINAL IBL MODEL

In the IBL model proposed by Lejarraga et al. (2012), the activation of a past instance is regulated by a noise term $\mu_n(t')$, a random variable distributed between 0 and positive infinity. The instance is defined at the path level.

$$\widetilde{w_{ni}}(t', t) = \frac{\mu_n(t')(t - t')^{-d}}{\sum_{\tau=0}^{t-1} \mu_n(\tau)(t - \tau)^{-d}} \tag{A.1}$$

Where:

$\widetilde{w_{ni}}(t', t)$: perturbed weight of the experienced travel time of day t' for the perceived travel time on day t for traveler n and path i

$\mu_n(t')$: a noise term added to the weight function, $\mu_n(t') = \left(\frac{1-\gamma_n(t')}{\gamma_n(t')}\right)^{\sigma}$

$\gamma_n(t')$: a uniformly distributed random variable between 0 and 1

σ: a free noise parameter

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