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## Examining Small-group Discourse Through the Lens of Students,Äô Beliefs about Mathematics and The Instructional Triangle

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Examining Small-group Discourse Through the Lens of Students' Beliefs  
about Mathematics and The Instructional Triangle

A Dissertation Presented

by

JENNIFER ERICSON

Submitted to the Graduate School of the  
University of Massachusetts Amherst in partial fulfillment  
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## DEDICATION

To my patient and loving partner, Donald

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## ABSTRACT

### EXAMINING SMALL-GROUP DISCOURSE THROUGH THE LENS OF STUDENTS' BELIEFS ABOUT MATHEMATICS AND THE INSTRUCTIONAL TRIANGLE

FEBRUARY 2022

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Standards-based mathematics instruction [SBMI], with opportunities to engage in high-quality mathematical discourse about meaningful tasks, leads to increased student achievement. However, not all students participate in mathematical discourse with high-quality. Esmonde & Langer-Osuna (2013) found that students participate in discourse with various degrees of engagement based upon individual beliefs. Boaler (2006b) found that SBMI in a detracked setting improves students' beliefs about mathematics.

The purpose of this descriptive case study was to illustrate the phenomenon of incongruent written curricula commitments across grade bands in a school district by examining small-group discourse and students' mathematical related beliefs. This study compared students' mathematical related beliefs across two schools in the same district and small-group discourse across four classrooms within those two schools. Analyzing small-group discourse quality through the lens of students' beliefs and the degree to which lessons were representative of SBMI illuminated the interconnectivity of discourse, beliefs, and the instructional model.

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# CHAPTER 1

## INTRODUCTION

*“[The universe] cannot be read until we have learnt the language and become familiar with the characters in which it is written. It is written in mathematical language, and the letters are triangles, circles and other geometrical figures, without which means it is humanly impossible to comprehend a single word.”* -- Galileo Galilei-Opere II Saggiatore (Mathematical Association of America, n.d.)

Over 30 years ago, the National Council of Teachers of Mathematics [NCTM] put forth a vision for mathematics instruction to improve mathematics education (NCTM, 1989). These principles of high-quality mathematics education were later defined in NCTM’s, *Principles and Standards for School Mathematics* (NCTM, 2000). Included with these principles was a vision of how students should engage in meaningful mathematics learning, by being “actively involved in making sense of mathematics tasks by using varied strategies and representations, justifying solutions, making connections to prior knowledge or familiar contexts and experiences, and considering the reasoning of others” (NCTM, 2014, p. 11). This vision requires students to be active participants in their mathematical discourse community as they make, refine, and explore conjectures based on evidence (NCTM, 2000; NCTM, 2014). Furthermore, students are expected to communicate, orally and in writing, their ideas and results effectively (NCTM, 2000; NCTM, 2014). In order for the vision set forth by NCTM to be incorporated in classroom practice, mathematics educators and researchers identified a need for school mathematics to change from a traditional learning paradigm, noted for a teacher-centered lesson with direct instruction, to a

standards-based learning paradigm, rich in inquiry and meaningful tasks and with a shift towards student-centered learning, to engage students and promote learning (e.g., Boaler, 2015; Artigue & Blomhøj, 2013). This recommended shift in instruction is called standards-based mathematics instruction [SBMI] (NCTM, 1989; 2000; 2014).

Access to such high-quality learning opportunities is unequal and is correlated with demographics such as ethnicity, race, socioeconomic status, geographic location, culture, and language (Cai, Wang, Moyer, Wang, & Nie, 2011). Students' opportunities to learn [OTL] is cited as the best predictor of student learning, yet the construct of OTL is underdefined (Cai et al., 2011). Proxies for students' OTL have included time spent on a topic (Carroll, 1963), the characteristics of the tasks (Stein, Grover, & Henningsen, 1996), and quality of learning opportunities (Tate, 2005; Wang, 1998). Cai et al. (2011) state that any definition of classroom OTL would need to focus on the instructional triangle -- the interaction among mathematical tasks, teaching, and students (Cohen, Raudenbush, & Ball, 2003). This interaction includes teachers' mathematical knowledge for teaching, time used to reach the mathematical goal, a student-focused implementation of the task that includes connections between multiple representations, and opportunities for students to explain their mathematical thinking to move towards a deeper understanding of the mathematical goal (Walkowiak, Pinter, & Berry, 2017). Additionally, this interaction includes the prior knowledge and dispositions of the students which may affect engagement and ability to access the learning opportunity (Cai et al., 2011). Therefore, OTL is "the relationship between an individual with both a mind and a body and the environment in which the individual thinks, feels, acts, and interacts" (Gee, 2008, p. 81). This sociocultural perspective of learning considers how new information integrates with prior knowledge, the mental and public representations of the information, opportunities to practice with the representational means of the information, and the

affective filter of the learner--the psychological filter that either facilitates or hinders learning (Gee, 2008).

## **1.1 Rationale for the Study**

Mathematics educators, mathematicians and other stakeholders have debated for more than a century over what should be taught in the K-12 mathematics curriculum, to whom it should be taught, how it should be taught, and how it should be learned (Munter, Stein, & Smith, 2015; NCTM, 1970). The debate continues today with some educational experts supporting standards-based instructional practices while others supporting a more traditional instructional approach (Munter et al., 2015).

Despite a call for a standards-based instructional model by the NCTM (1989, 1991, 2000, 2014), as well as research reporting positive academic outcomes related to standards-based mathematics instruction (e.g., Huntley, Rasmussen, Villarubi, Sangtong, & Fey, 2000; Krupa & Confrey, 2017; Schoen, Ziebarth, Hirsch, & BrckaLorenz, 2010; Wood, Williams, & McNeal, 2006), there continue to be proponents of a traditional direct-instruction model (Munter et al., 2015). While mathematics educators may agree on NCTM's vision of engaging all students in meaningful mathematics learning, traditional teaching practices and curriculum utilization do not necessarily support such opportunities to learn (Munter et al., 2015; NCTM, 2014).

Even with high-quality learning opportunities, students' attitudes and dispositions towards mathematics may allow one student and not another to benefit from an OTL (Cai et al., 2011). Feeling fear, boredom, nervousness, or resistance to change – having a high affective filter- can diminish students' opportunity to learn by blocking the intake of new information, while feeling empowerment, safety, willingness to change, and risk-taking –having a low affective filter – can

increase students' opportunity to learn (Gee, 2008). Additionally, many students still do not recognize themselves as being good in mathematics (Boaler, 2015; Darragh, 2014; Mendick, 2005). Students' negative beliefs about what it means to be good at mathematics are founded in many experiences, including the masculine engendering of mathematics (Mendick, 2005), classroom practices (Darragh, 2014, Maltese & Tai, 2010), the classroom environment, and an instructional focus on procedural fluency over conceptual understanding (Boaler, 2015; Darragh, 2014). Perhaps the misunderstanding of what it means to be "good at math", and thereby not identifying as being a person who is good at mathematics, influences how students engage with and conceptualize mathematics (Schunk & Richardson, 2011). These negative beliefs can have powerful, and oftentimes negative, consequences (Schoenfeld, 1989). Student engagement with mathematics within an SBMI environment leads to more positive beliefs about the practice of mathematics as well as about themselves as doers of mathematics (Boaler, 2006a, 2006b, 2008; Boaler & Greeno, 2000; Hand & Gresalfi, 2015). Students with more positive beliefs about mathematics and their competency in mathematics are more likely to remain in the STEM pipeline (Maltese & Tai, 2010). Research is needed to identify what individual beliefs about and attitudes towards mathematics affect their learning opportunities in general and more specifically in SBMI environments (Cai et al., 2011).

Student participation in mathematical discourse is considered integral to student engagement with mathematics tasks and is one of the hallmarks of SBMI (NCTM, 2014). Mathematics discourse includes "the purposeful exchange of ideas through classroom discussions, as well as through other forms of verbal, visual, and written communication" (NCTM, 2014, p. 29). In this study, the term discourse will mean verbal communication: the talking, listening, and agreeing/disagreeing portion of discourse. This constraint on the discourse activities considered is

consistent with prior research by Gresalfi, Martin, Hand, & Greeno (2009), Webb, Nemer & Ing (2006), Kazemi & Stipek (2001), and Jansen (2008). For example, Webb et al. (2014) compared teachers' verbal discourse in whole class discussions with students' verbal discourse in small-group settings. Jansen (2008) compared students' verbal participation in whole class discussions with their beliefs about discourse. Kazemi & Stipek (2010) analyzed teachers' verbal discourse and press for higher order responses from students, such as justifications and generalizations. Gresalfi et al. (2009) analyzed verbal discourse to document the ways that agency and accountability were distributed in mathematics classrooms.

The degree to which high-quality mathematics discourse -- that is discourse which include justification of reasoning, predicting, relating to prior knowledge, adding to or challenging the thinking/reasoning of others, and generalizing -- occur in a classroom is a positive predictor of elementary through high-school students' achievement on standardized tests (Webb et al., 2014; Weaver, Dick, & Rigelman, 2005).

Additionally, small-group discourse can increase students' opportunities to learn if it provides equitable access that small-group participation (Esmonde & Langer-Osuna, 2013).

However, students' beliefs about mathematical discourse and the locus of knowledge can impact their participation in whole-class discourse (Jansen, 2008). Students' prior experiences with the norms of whole-class discourse can facilitate or impede their participation in small-group discourse (Michaels, O'Connor, & Resnick, 2008). Thus, despite implementation of pedagogical practices and classroom environments that support high-quality mathematical discourse, such as SBMI classrooms, students may participate in the discourse with varying degrees of engagement based upon individual beliefs and experiences, leading to unequal opportunities to learn mathematics (Esmonde & Langer-Osuna, 2013; Michaels et al., 2008).

## 1.2 Problem Statement

Although there is consensus among researchers, educators, and policymakers that all students need equitable access to high-quality mathematics instruction, what that instruction looks like can differ across classrooms, schools, districts, and states. Perceptions of high-quality mathematics instruction vary with each of the stakeholders (Munter, 2014).

Research has shown that participation in high-quality mathematics discourse promotes equity in opportunities to learn mathematics, and that students' affective filters, beliefs, and identities impact their classroom participation and opportunities to learn. There have been few studies investigating the relationship between standards-based mathematics instruction, students' beliefs and their views about themselves as mathematics learners, and small-group discourse. One recent study by Esmonde and Langer-Osuna (2013) concluded that students had preferences in instructional models, SBMI or traditional, and engaged with the mathematics when those methods and tasks matched their understanding of what it meant to do mathematics. They also concluded that students' positioning of authority allowed them to engage in small-group discourse at a higher-quality than students who did not hold positions of authority. Their study, with a focus on positioning, did not investigate students' mathematics related beliefs and how those beliefs influence small-group discourse.

Long-term research (e.g., Boaler, 1998, 2006a, 2006b, 2008) has suggested that SBMI provides positive outcomes concerning students' beliefs about identity and what it means to do mathematics. The students in Boaler's research at Railside High School (2006a, 2006b, 2008) experienced a department-created reform-oriented curriculum with a focus on open-ended tasks to promote conceptual understanding and group work. While Boaler did not use the terminology SBMI, her description of lessons included words that are indicative of SBMI; complex instruction,

group-worthy tasks, math tools, multiple representations, shared responsibility, mathematical communication, conjecturing questioning, and revising. Students in the SBMI pathway reported a positive change in their mathematics related beliefs [MRB], with students stating the inquiry with open-ended tasks in a group setting afforded them equitable access to the mathematics (Boaler 2006a, 2006b, 2008). Eighty-four percent of the students at Railside agreed that “Anyone could be good at math if they try,” compared to 52% of students in the traditional environment (Boaler, 2006a). Given that Boaler’s broad scale study did not look specifically at small-group discourse and did not look at a wide range of MRB, further research is warranted regarding SBMI, beliefs, and small-group discourse.

Webb et al. (2014) explored the relationship between student participation in elementary classroom discourse and teachers’ instructional practices. The extent to which students provided detailed explanations for their thinking and the extent to which students engaged with the thinking of their peers was analyzed. Additionally, the interaction between teacher and student was analyzed to determine whether the teacher employed questioning techniques to encourage students’ engagement with others’ ideas. Webb’s study did not look at student-student discourse in the absence of the teacher, nor did it contain any student belief component. Given Cai et al.’s (2011) findings that some students within the same learning environment take up the opportunity to learn while others do not, further research is warranted regarding practices that promote high-quality discourse and students’ beliefs.

The current study augments Boaler’s study and Emonde and Langer-Osuna’s study by analyzing a wide range of MRB and by focusing on student small-group discourse within an SBMI environment. It also addresses missing elements from Webb et al.’s study, by including careful

attention to student-student interactions and their beliefs. The current study explored students' participation in small-group discourse, with a lens of students' MRB and the instructional triangle.

### **1.3 Purpose of Study and Research Questions**

This study was a case study of within district incongruent written curriculum commitments across the grade bands. This phenomenon was studied by examining middle and high school students' mathematics-related beliefs, the quality of small-group discourse, and the enactment of lessons in a mixed methods approach. As such, there are two specific questions the study addressed.

1. What mathematics-related beliefs do students hold in a school district with incongruent written curriculum commitments in middle school and high school?

The first main research question sought to explore the mathematics-related beliefs that result from a change in written curricula commitments and instructional models. The following sub-questions assisted with the exploration of this first main question:

- a. What are students' beliefs about the nature of learning mathematics?
- b. What are students' beliefs about the speed/ability of knowledge acquisition?
- c. What are students' beliefs about their mathematical identity, mathematics self-efficacy, and mathematics interest?

For research question 1, it was hypothesized that middle school students would hold mathematical beliefs consistent with more traditional/hybrid instruction and high school students would hold mathematical beliefs consistent with SBMI based on their experienced with the enacted curriculum students. This study was also guided by a second main research question:

2. In what ways do students participate in small-group discourse as viewed through the lens of their mathematics related beliefs and *The Instructional Triangle*?

The second main research question sought to explore the outcomes of students' small-group discourse during collaborative inquiry within various classrooms.

#### **1.4 Overview of Methodology**

A descriptive case study design was used to better understand, illustrate, and analyze the mathematical beliefs and identities and small-group discourse of middle and high school students in one school district with incongruent written curricula commitments across grade bands. Two hundred eighty-eight middle and high school students participated in a survey to investigate similarities and differences in beliefs of these students. Four focus classrooms were observed to determine the extent to which enacted lessons represented high quality SMBI. Within the four focus classrooms, eight groups of students (n=21) were observed during small-group inquiry within the lessons to compare the quality of discourse across the four settings.

#### **1.5 Significance of Study**

Research on standards-based mathematics instruction has shown that it can lead to positive academic achievement outcomes for students (Huntley et al., 2000; Krupa & Confrey, 2017; Schoen et al., 2010; Wood et al., 2006). However, the focus of most research was on overall student achievement and not changes in students' mathematics-related beliefs. Schoen and colleagues' research (2010) did examine the mathematical beliefs of students who experienced SBMI with an NSF-funded curriculum in both middle school and high school and found that participants held beliefs consistent with SBMI. However, those beliefs were only measured at the end of their 3<sup>rd</sup> and 4<sup>th</sup> year of high school, with no initial measurements to analyze if there were changes in beliefs over time. High school students who experienced standards-based mathematics instruction in middle

school continue to hold beliefs about the importance of conceptual understanding after experiencing four years of a more traditional direct-instruction model in high school (Moyer, Robison, & Cai, 2018). However, there is little research about students' beliefs when the opposite occurs, when students experience a more traditional instructional model in middle school and standards-based mathematics instruction in high school (Boaler, 1998; Huntley et al., 2000). Studies are needed to further understand small-group discourse as situated within standards-based instructional models and as mediated by students' MRB.

Small-group discourse is one component of providing equitable access to mathematics participation by increasing student opportunity to engage in mathematical discourse (Esmonde & Langer-Osuna, 2013). Research on student mathematical discourse shows that it can increase opportunities for student engagement with mathematics tasks (Boaler, 2006a; Chi & Menekse, 2015; Chiu, 2008; Cobb, Boufi, McClain, & Whitenack, 1997; Esmonde & Langer-Osuna, 2013). However, middle school students who do not value the importance of high-quality discourse in mathematics or who believe that the source of knowledge resides in the teacher tend to participate less in whole-class discourse (Jansen, 2008). The current study adds to the existing literature that examines mathematical discourse in small-group inquiry settings at both the middle and high school level and adds to the literature by examining the discourse through the lens of *The Instructional Triangle* as well as students' MRB.

## **1.6 Organization of Dissertation**

This first chapter identified the problem of scarce information about small-group discourse and students' beliefs within the enacted curriculum. Chapter 2 contains a review of literature relevant to this study. The literature reviewed includes research about the mathematics curriculum,

students' mathematics-related beliefs, small-group discourse, and standards-based instructional practices. Chapter 3 describes the methodology, instrumentation, selection of participants, data collection and analysis used to conduct this research. Chapter 4 presents lesson observations analysis of the four focus classrooms as well as the results of the student survey and student interviews. Observation results comprise descriptive statistics of SBMI features measured by the lesson observation tool and themes of teacher-student and teacher-content interaction. Survey results encompass descriptive and inferential statistics as an aggregate of the survey and then disaggregated by school, gender, and tracking. Cross-class comparisons of students' beliefs are made. Student interviews are summarized by major emerging themes. Chapter 5 examines small-group discourse of the eight small-groups during collaborative inquiry. It includes selected artifacts of student discourse to support the analysis. Finally, Chapter 6 contains a discourse of key findings of this analysis, considerations for possible future studies that would explore longer-term or greater scope of settings, and limitations and constraints of the study.

## **CHAPTER 2**

### **REVIEW OF THE LITERATURE**

This chapter provides a review of the literature that identifies some of the contributing factors that impact students' opportunities to learn mathematics as they relate to the enactment of curriculum and students' beliefs. The first sections focus on the theoretical and conceptual frameworks. This is followed by a discussion of written curriculum materials, referred to throughout as curricula. Section four expounds upon characteristics of effective standards-based mathematics instruction that are integral to this research: high cognitive demand tasks, collaborative inquiry, and high-quality student discourse. Sections five and six review the literature on students' mathematics-related beliefs, specifically epistemological beliefs and mathematics identity. Additionally, the literature presented in this chapter includes discussion of the relationship between beliefs and students' participation in classroom practices. The argument is made that studies are needed to further understand how a change from a traditional instructional model to a SBMI model impacts students' beliefs and discourse participation.

#### **2.1 Theoretical Framework**

This case study of within district incongruent written curriculum commitments across the grade bands was guided by a sociocultural perspective on language and development. It was also guided by *The Instructional Triangle* (Cohen et al., 2003), the interaction between students, teacher, and content as situated within the enacted curriculum.

### **2.1.1 A Sociocultural Perspective**

A sociocultural perspective on knowledge and learning regards the development of knowledge as a relationship between an individual and its environment, filtered through the individual's prior experiences and beliefs (Gee, 2008). Learning is a process that structures and shapes cognitive activity leading to development (Vygotsky, 1978). Two major components of sociocultural learning that are central to the study include: (1) learning originates in social, historical, and cultural interactions, and (2) psychological tools, especially language, mediate the development of higher mental functions (Vygotsky, 1978). Learners actively construct understanding and knowledge based on their own questioning, exploring, and evaluating in relation to their experiences with mathematical activity and other learners (Foote, Vermette, & Battaglia, 2013; Fosnot & Perry, 2005).

#### **2.1.1.1 Opportunity to Learn**

Traditionally, opportunity to learn has been operationalized as the amount of time allocated to and the quality of classroom instruction (Carroll, 1963), the extent to which the content of instruction overlapped with the content of assessments (Husen, 1967), and/or the content emphasis (Stevens, 1996). Gee (2008) theorizes knowledge and opportunity to learn from a sociocultural perspective. Knowledge has traditionally been viewed in terms of mental representations stored in one's mind. Using this perspective, students have the same opportunity to learn if they have been exposed to the same content. Gee challenges this traditional view with a sociocultural view. Gee states that new information needs to be integrated with prior knowledge to make sense and that if new information cannot be tied to prior knowledge, then that new information is not learned well or at all. Gee also challenges the traditional view about representations. Gee states that some ways of

representing information are more efficient or effective than others and that students have different capacities to form efficient representations based upon prior experiences.

A third point that Gee (2008) makes about OTL is that there is a difference between input and intake. Input is the content to which a learner is exposed. If this content is not processed by the learner, then it is not learned. Input that had been processed leads to learning and is called intake. There are various reasons why content remains input and not intake. One reason is prior knowledge where learners cannot tie the new knowledge to prior knowledge. A second reason is that a learner may resist using the input due to some perceived threat to their sense of self, whether it be individual, social, or cultural self. When that threat, the affective filter, is low, then input is allowed to become intake. When the threat is high, input is not properly processed. The learning situation itself, the classroom environment, may cause the affective filter to rise for some students. Additionally, students' views of themselves and their identities in relationship to the content may cause the affective filter to rise. Students with higher affective filters, due to the learning environment or their own beliefs, do not have the same OTL as students with lower affective filters because the content is only input and not intake. Therefore, examining learning necessitates exploring the relationship between the individual, their thoughts, beliefs, and actions, and the environment in which they are learning.

### **2.1.1.2 Language**

Psychological tools are one of the cornerstones of Vygotsky's sociocultural theory of learning. Psychological tools are symbolic artifacts, including gestures, language, sign systems, mnemonic techniques, and decision-making systems, which serve as mediators for higher mental functions (Vygotsky, 1986). A symbol is a social object which represents something else and is

intentionally used to communicate with others or oneself (Charon, 2004). Language is “a set of words used for communication and representation. Words –symbols that are spoken or written— are the basis for all other symbols” (Charon, 2004, p. 52). All other forms of interpersonal communication to express ideas and emotions, such as gestures, expressions, and writing, relate back to words (Hertzler, 1965). “Fundamentally, the spoken word, or its equivalent or functioning counterpart in the languages that do not have “words”, is the only all-inclusive and basic medium of communication” (Hertzler, 1965, p. 30).

The use of tools, including psychological tools, to interact with the environment relies upon mediation and internalization. Mediation is a process in which humans use tools to indirectly interact with their environment and other human beings (Vygotsky, 1978). Vygotsky identified three types of mediators, cultural tools, psychological tools, and other human beings (Kozulin, 2003). Psychological tools are internally oriented and are used to organize the individual mental processes (Vygotsky, 1978). When speech is incorporated into an action, that action becomes transformed and is organized along new lines (Vygotsky, 1978, p. 24).

The use of language facilitates the co-construction of knowledge in social settings, both to communicate with others and to construct meaning (John-Steiner & Mahn, 1996). The co-construction of knowledge requires interdependence between individual and social processes, occurring in a cultural context, mediated by tools and symbols such as language (John-Steiner & Mahn, 1996). Communicative language transforms into inner speech and then into verbal thinking by appropriating and integrating this newly acquired information into pre-existing schema (Vygotsky, 1986).

Language-based mediation is first interpsychological, between people on a social level, and then intrapsychological, on an individual basis (Vygotsky, 1978). This transformation from

interpersonal to intrapersonal is the result of prolonged development and is referred to as internalization (Vygotsky, 1978). When psychological tools become internalized, they retain their initial social and communicative nature (Vygotsky, 1978). Student-student interaction and dialogue serves as a mediator (Vygotsky, 1978). By working with others, the learner adopts the socially shared experiences (interpsychological) and then internalizes the knowledge (intrapsychological) (Vygotsky, 1978).

### **2.1.1.3 Social Interactions**

All learning is negotiated in a social, cultural, and historical context (Vygotsky, 1978). Students use their background knowledge as a lens through which to interpret mathematical activity, and therefore many interpret the activities differently (Voigt, 1994). Mathematical understanding is a social product which grows out of how one student interacts with another student over an object or concept and is established through discourse (Voigt, 1994). While individual participants may not have the same initial understanding as other group members, an opportunity to negotiate meanings, to reflect on representations, and to create new insights develops through the use of language (Voigt, 1996; Vygotsky, 1978). Discourse between students allows for clarification of meaning and for a negotiated construction of meaning (Voigt, 1996). Once intersubjectivity is established, the negotiated meanings move from explicit to implicit (Yackel & Cobb, 1996). Vygotsky referred to this as an act of enculturation (John-Steiner & Mahn, 1996).

### **2.1.2 The Instructional Triangle**

Central to this study's research was the assumption that instruction and learning consists of "interactions among teachers and students around content, in environments" (Cohen, Raudenbush,

& Ball, 2003, p. 122). *The Instructional Triangle*, depicted in Figure 1, illustrates the interaction between teachers, students, other students, and content. Instruction and learning require all three elements and is the interaction between them (Cohen & Ball, 1999). The relationships within *The Instructional Triangle* include (a) teacher to the content, (b) teacher to students, (c) student to teacher, (d) student to other students, and (e) student to content (Cohen et al., 2003). The scope of the current study examined student-student interaction, in the form of discourse during small-group inquiry, around the content in varying quality enacted curricula.

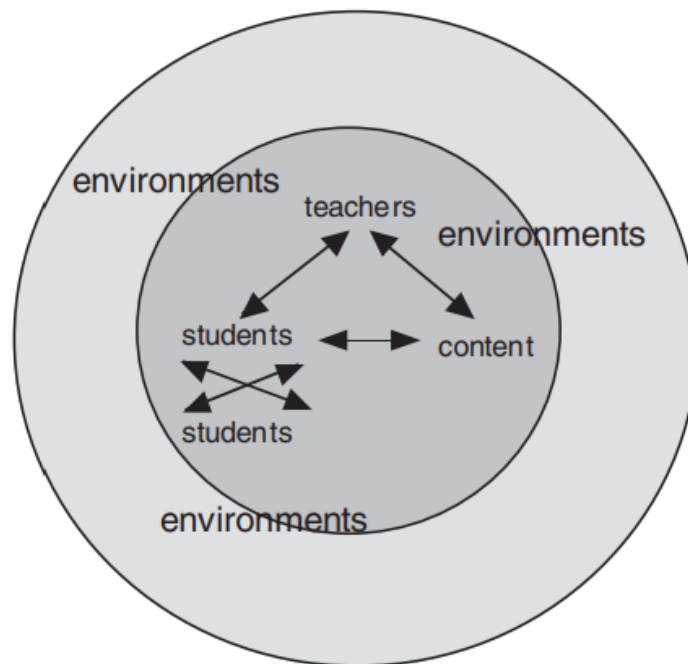


Figure 1 The Instructional Triangle (Cohen et al., 2003)

In this study, the focus is on student-student mathematical discourse during small-group inquiry. Those discourse cannot be viewed in isolation, but rather, must be viewed within the context of the teacher, the content, and the environment (SBMI), as displayed in Figure 1. Differences in student outcomes can be attributed to differences in the use of resources (Cohen et al., 2003).

### 2.1.2.1 Teachers and Teaching

Teachers hold a myriad of intellectual and personal resources which influence their interactions with students and their implementation of lessons (Cohen & Ball, 1999). Included in these resources are their conceptions of knowledge, content knowledge, familiarity with students' knowledge, pedagogical content knowledge (Shulman, 1986), and capacity to establish classroom environments that support learning (Cohen & Ball, 1999).

Ball, Thames, & Phelps (2008) expanded upon Shulman's (1986) model of pedagogical content knowledge to theorize about mathematical knowledge for teaching and its structure. Ball et al. theorized that mathematical knowledge for teaching is divided into two domains. Subject matter knowledge includes *common content knowledge*, the mathematical knowledge and skill used in settings other than teaching, *specialized content knowledge*, the mathematical knowledge and skill unique to teaching, and *horizon content knowledge*, the awareness of how mathematical topics are related over the span of mathematics included in the curriculum (Ball et al., 2008, p. 391).

Pedagogical content knowledge includes *knowledge of content and students*, the knowledge about what students are likely to think and what they will find confusing, *knowledge of content and teaching*, the knowledge of the mathematical content and the design of instruction that will maximized student learning, and *knowledge of content and curriculum*, the knowledge of a variety of instructional materials designed for the teaching of particular subjects and topics at a given level as well as the knowledge of the curriculum being taught in other subject areas (Ball et al., 2008 p. 391).

Plans for instruction are based on teachers' prior understandings and experiences, beliefs, instructional designs, and implementation (Cohen et al., 2003). Researchers have studied different aspects of teaching, including but not limited to pedagogical content knowledge, content

knowledge, beliefs, and identities, to understand the teaching that occurs within classrooms. Some research has been focused on understanding standards-based mathematics teaching (e.g., Polly et al., 2013) and some on investigating teaching practices that increase students' OTL (e.g., Webb et al., 2014).

Wilkins (2008) found that teachers' mathematical content knowledge had a negative relationship with their use of inquiry-based instructional methods while their beliefs in the effectiveness of inquiry-based instruction had a positive relationship with their use of inquiry-based methods, which mediated the negative effect of content knowledge. Polly, et al. (2013) examined teachers' beliefs about mathematics instruction and learning and their corresponding enacted practices and found alignment of beliefs and practices. Charalambous (2015) found that limitations in either teacher content knowledge or pedagogical content knowledge can mediate the positive effect the other component can have on teaching practices and that beliefs, alone, could not compensate for low content knowledge or pedagogical content knowledge.

What teachers know, believe, and can-do shapes the OTLs they provide to their students (Cohen & Ball, 1999).

### **2.1.2.2 Students**

Students' experiences, understandings, and engagement are critical to learning (Cohen & Ball, 1999). Students' resources, which include their experiences, prior knowledge, beliefs, and habits of mind, influence how they respond to materials and teachers (Cohen & Ball, 1999). Students who are explicitly taught how to reflect on ideas, express themselves clearly, and listen carefully are capable of using materials, teachers, and other students' work more advantageously (Cohen & Ball, 1999; Cohen et al., 2003).

Student learning includes not only the content knowledge and skills that are frequently assessed on tests, but also the attitudes, beliefs, and identities that have developed as the result of instructional experiences (Remillard & Heck, 2014). Identities are subsumed under the heading of affective factors and include persistence and interest in mathematics as well as motivation to learn mathematics (Cobb, Gresalfi, & Hodge, 2009) and are formed when interacting with the mathematics content (Tarr, et al., 2008). Teachers' beliefs about what it means to "do" mathematics, to teach mathematics, to be a student of mathematics, and to learn mathematics impact how teachers implement instruction in the classroom (Ernest, 1989). While the focus of this research is not on teachers' epistemological beliefs of mathematics, it is those beliefs that are reflected in decisions on how to implement the curriculum and ultimately impacts the experiences of the students (Ernest, 1989; Schoenfeld, 1989). This study focuses on the beliefs and identities formed by students and the relationship of these beliefs and identities to the quality of discourse within small groups. Instructional practices and the enacted curriculum are by no means the only factors influencing learning and beliefs, but they do have a profound impact on learning (Remillard & Heck, 2014; Stein, Remillard, & Smith, 2007; Tarr et al. 2008).

Beliefs and identities affect how students interact with the curriculum and therefore impact learning (Gee, 2008; Hand & Gresalfi, 2015). Therefore, a study of students' beliefs in relationship to their small-group discourse necessitates an examination of the enacted curriculum.

### **2.1.2.3 Content**

Content refers to what students are engaged in. This can include texts, media, tasks, and questions posed to students (Cohen & Ball, 1999, p. 4). The nature of the task/problem, the development of the concepts, and the use of multiple representations shape what students can do

and learn (Cohen & Ball, 1999; Stein et al., 2007). What the teacher intends to occur in the classroom does not always happen. The teacher and students interact with each other and the content, bringing with them their beliefs about learning, teaching, and knowledge acquisition, to create something that might be different than what is in the written curriculum or what the teacher intended (Cohen et al., 2003; Stein et al., 2007). This is called the *enacted curriculum*. The written curriculum and the enacted curriculum are discussed in the sections following the conceptual framework.

## **2.2 Conceptual Framework**

The conceptual framework that guides this study is influenced by a sociocultural perspective of learning, especially the role of language in the social context (Vygotsky, 1978), where learning is filtered through prior experiences and beliefs (Gee, 2008). It is also influenced by the work of Cohen et al. (2003) and the interactions that occur in *The Instructional Triangle*.

This conceptual framework, as shown in Figure 2, situates learning within the enacted curriculum, being influenced by both students' beliefs and their discourse. It is a modified version of *The Instructional Triangle* with enacted curriculum replacing environment. The environment is comprised on potential influences on and resources for instruction (Cohen & Ball, 1999). It includes the school, with its departments, material resources, grade-level groupings, and other teachers. It also includes parents, district policies, state requirements, professional norms, and school leadership (Cohen & Ball, 1999; Cohen et al., 2003). While these are important influences on the learning environment, this study examined the enacted curriculum as it shapes and is shaped by the student, teacher, and content.

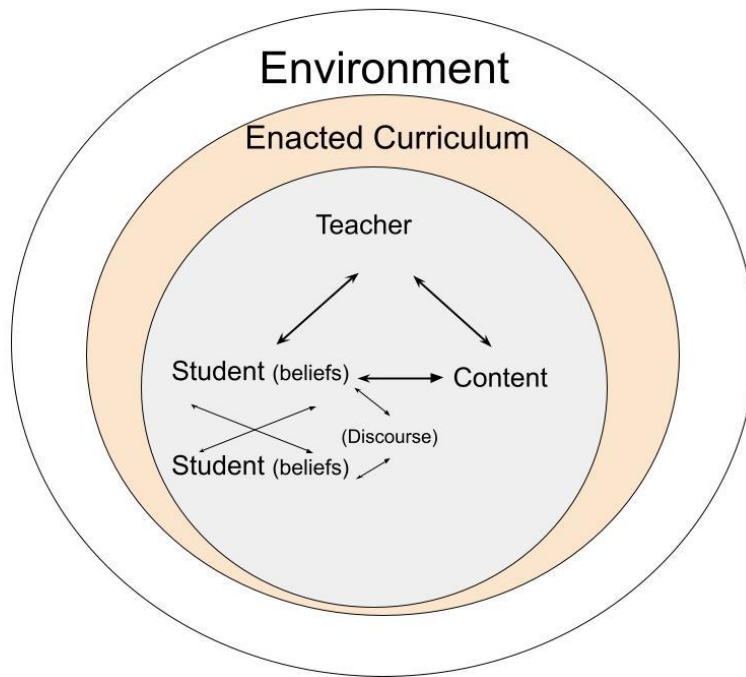


Figure 2 Conceptual Framework

The focus of this study was student-student interaction, namely small-group discourse, and beliefs to examine the effect of incongruent written curriculum commitments within a school district. The decision to focus on small-group discourse was influenced by Vygotsky's (1978) theory that all communication stems from verbal language. Student-student interaction and dialogue serves as a mediator for learning. The learner adopts the socially shared experiences and then internalizes the knowledge (Vygotsky, 1978). There is a bi-directional relationship between students' beliefs and their discourse. What students believe about the nature of mathematics and themselves as mathematicians can impact their discourse participation (Hoffman, 2004; Jansen, 2008). The types of discourses in which students engage can impact their beliefs about themselves as mathematics learners and the nature of mathematics (Boaler 2006a, 2006b, 2008). While the

focus of the study was on small-group discourse and beliefs, based upon *The Instructional Triangle*, one needs to also consider the roles of the teacher and the content.

## **2.3 The Written Curriculum**

The written curriculum refers to the material resources used by teachers in the classroom (Stein et al., 2007). This includes textbooks as well as curriculum materials and instructional guides, which focus on the pedagogy and mathematics (Stein et al., 2007). Textbooks are generally developed and marketed by commercial publishing companies, while curriculum materials are often designed by mathematics experts and researchers (Reys & Reys, 2007, Stein et al., 2007). The written curriculum is important to consider when researching the enacted curriculum, as commercially published textbooks and programs are the starting point for instruction for over 90% of teachers (Banilower et al., 2018). The written curriculum, which is generally classified as standards-based or traditional (Moyer, Robison, & Cai, 2018), is explored in the next section.

### **2.3.1 Types of Written Mathematics Curricula**

Written curricula, most often in the form of textbooks, serve two important functions: a curricular aspect, a logical mathematical progression, and a conceptual aspect, the development of cognitive structures in the learner (Van Dormolen, 1986). Curricular aspect refers to the selection of content and sequencing of topics in the textbook and conceptual aspect refers to the presentation of the content, the tasks for students, and the guidance for teachers in the teachers guides (Chavez, Tarr, Grouws, & Soria, 2013). There are three major types of written mathematics curricula: traditional, NSF- funded (standards-based), and hybrid curricula (Chavez et al., 2013). Each of these types have a different philosophical approach to teaching mathematics and often a different organization of content.

Traditional curricula tend to focus on understanding a concept as a computational algorithm through direct instruction (Cai et al., 2011). Direct instruction includes definitions, algorithms, and worked examples (Moyer et al., 2018). In traditional written curricula, lessons generally begin with definitions and worked examples. (Grouws, et al., 2013). Students then practice the procedures with many decontextualized problems with the opportunity to work with others sometimes offered but not mandatory (Grouws, et al., 2013; Moyer et al., 2018). Once students have mastered lower-level skills, then, and only then, do students have the possibility of engaging in higher-order thinking, reasoning, and problem solving (Stein et al., 2007). In a traditional curriculum, typically, each of the content strands such as algebra and geometry are separated into separate courses. However, there are integrated curriculum materials that reorganize content but maintain more traditional practices.

In the late 1980s, NCTM published its first Standards documents (NCTM, 1989), providing recommendations for reforming K-12 school mathematics. With support from the National Science Foundation [NSF], 13 mathematics curricula (the written curriculum) were developed to align with these recommendations (Reys & Reys, 2007). Unlike traditional curricula, the NSF-funded curricula were field tested and underwent substantial revisions prior to commercial distribution (Reys & Reys, 2007). These NSF-funded curricula are often referred to as standards-based or reform curricula due to their alignment with NCTM's standards (Nie, Cai, & Moyer, 2009). NSF-funded curricula are used in about 25% of K-5, 11% of grade 6-8, and less than 1% of grade 9-12 classrooms that utilize a commercially published textbook (Banilower et al., 2018).

In an NSF-funded curriculum, the focus of instruction shifts from rote memorization to conceptual understanding, reasoning, and problem solving (Cai et al., 2011; Stein et al., 2007; Remillard, 2005). NSF-funded curricula support standards-based mathematics instruction by including frequent group work that provides students opportunities engage in mathematical

discourse, the use of manipulatives and technology, student-invented algorithms, and generating multiple approaches to solve the task (Chavez et al., 2015; Moyer et al., 2018; Stein et al., 2007; Remillard, 2005). Lessons in NSF-funded curricula generally follow a launch-explore-summarize format (Grouws, et al., 2013). Lessons may span several days and assume that students will work collaboratively in small groups (Grouws, et al. 2013). The teacher's role is to facilitate learning as students explore, conjecture, and reason about non-routine, often real-life problems (those without obvious solution pathways) (Moyer et al., 2018). Rather than being segregated into separate courses, content strands are generally integrated with each other to highlight connections (Moyer et al., 2018). To be successful in a standards-based curriculum environment, students need to change their vision of mathematics from a focus on operations and algorithms to a focus on patterns and real-world applications (Moyer et al., 2018).

Publishers of "traditional curricula" also claim that their curricula are standards-based, with references in the teachers' guides as to how the materials correspond with content and process standards with minimal changes in the presentation of the content (Nie et al., 2009). Content standards are the mathematics students are expected to know and process standards are how students should engage with the mathematics to understand the content (NCTM, 2000). While both standards-based curricula, NSF-funded curricula, and traditional curricula claim to correspond to the NCTM content and process standards, there are fundamental differences in features of standards-based and traditional curricula (Nie et al., 2009). Differences include order and manner of presentation, balance between concepts and procedures, and organizational style.

Hybrid curricula include some reform-oriented elements and some traditional elements. Their organization may be subject-specific like traditional curricula, or they may be integrated (Chavez et al., 2013). Lessons may be designed around longer explorations followed by direct

instruction, definitions, and computational practice (Chavez et al., 2013). Many of the newer textbooks that claim to be standards-based would be classified as hybrid (Nie et al., 2009).

The different types of written curriculum have different pedagogical philosophies (Cai et al., 2011; Chavez et al., 2015; Moyer et al., 2018). NSF-funded curricula, with a focus on a balance of conceptual and procedural understanding, small group collaborative work, and traditional curricula, with a focus on students receiving definitions and worked examples before independent practice, would be considered incongruent curricula due to the difference in the instructional philosophies. With incongruent curricula commitments across grade bands students can experience discontinuities during boundary crossing (Jansen, Herbel-Eisenmnan, & Smith III, 2012). A boundary is a “socio-cultural difference leading to discontinuity in action or interaction (Akkerman & Bakker, 2011, p. 133). In this example, the boundary would be the two settings with different types of written curriculum. This discontinuity could lead to students’ experiencing a change in their roles in each setting (Jansen et al., 2012). Students adapt in different ways to changes in pedagogical practices in different ways. Students may or may not take up affordances in the new setting, affecting engagement and identification.

### **2.3.2 Student achievement**

All NSF-funded curriculum projects were required to conduct evaluations of the effectiveness of their materials (Reys & Reys, 2007). Students in NSF-funded curricula environments generally perform as well or better than students in traditional curricula environments on computational skills and outperformed them on non-routine problems requiring problem solving skills (e. g., Cai et al., 2011; Chavez et al., 2013; Krupa & Confrey, 2017). Studies comparing

written curricula often lack the requisite classroom observations to ensure fidelity in the enactment of the curriculum (Stein et al., 2007).

Boaler and Staples (2008) followed 300 high school students in a traditional curriculum environment and 300 high school students in a reform-based curriculum environment. The terminology reform-based curriculum is used here instead of NSF-funded curriculum as the reform-based curriculum was a combination of an NSF-funded and a reform-oriented curriculum. Students in both pathways were given tests of middle school mathematics at the beginning of high school. The tests were designed to have an equal proportion of question types from each of the teaching approaches. Students entering the reform-based curriculum pathway scored significantly lower on their assessments than students entering the traditional pathway. At the end of the first year, students in the two pathways were given a test on algebra. There was no difference between the two groups indicating that students in the reform-based curriculum pathway had made greater gains than students in the traditional pathway. At the end of the second year, students were given a test on algebra and geometry. Students in the reform-based curriculum pathway outperformed the students in the traditional pathway.

Krupa and Confrey (2017) looked at Algebra I and II end-of-course assessments of North Carolina students ( $n = 10,399$  and  $n = 9,127$  respectively). Over half the teachers in this sample participated in the North Carolina Integrated Mathematics Project, and used an integrated NSF-funded curriculum. At the end of Algebra I or Course 2, for the integrated pathway, students in the integrated pathway scored higher on the end-of-course assessment than students in the traditional pathway. At the end of Algebra II or Course 3, there was no difference in achievement between the two pathways. Of note, the race achievement gap had dissipated for students in the integrated

pathway but not for students in the traditional pathway by the end of Course 3/Algebra II. This study did not include classroom observations.

Chavez et al. (2013), as part of the COSMIC study, looked at the effect of curriculum upon student achievement when they had a choice in following an integrated, NSF-funded curriculum pathway or a traditional pathway (n = 2242). Two assessments were administered to students at the end of Course 3 or Algebra II. One assessment was researcher developed and focused on functions and multiple representations. The second assessment was a nationally standardized test, which focused on quantitative problem solving. Students in the integrated pathway scored significantly higher on the researcher developed assessment as students in the traditional pathway. On the nationally standardized test, there was no significant difference between the two groups. There were no classroom observations, with the researchers relying upon self-reported teachers' practices. Students whose teachers' reported aligning their practices put forth in the NCTM Standards (2000) scored higher on both assessments.

Cai et al. (2011), as part of the LieCal project, analyzed the impact of curriculum on students' learning in middle school (n = 1,284). Teachers either utilized an NSF-funded curriculum, Connected Mathematics Program (Lappan, Fey, Fitzgerald, Friel, & Phillips, 2003), or a traditional curriculum. Students were administered researcher designed open-ended and multiple choice assessments at the beginning of 6<sup>th</sup> grade and at the end of the 6<sup>th</sup>, 7<sup>th</sup>, and 8<sup>th</sup> grade. Researchers conducted 620 lesson observations over the three years. The students in the NSF-funded pathway had greater gains on open-ended tasks and the same gains on computational tasks as students in the traditional pathway.

Huntley and colleagues (2000), as part of the initial curriculum development evaluated the effects of the algebra experience for high school students (n = 593) in either an NSF-funded or

traditional curriculum pathway. At one site, students were randomly placed into the NSF-funded pathway or the traditional pathway. At other sites, students were given the option of entering the NSF-funded pathway at the beginning of 9<sup>th</sup> grade. At some sites, students with weaker aptitude and interest were tracked into the NSF-funded pathway. The effect of the algebra experience was measured by giving students a three-part assessment. On contextualized problems and the collaborative complex modeling task, students in the NSF-funded pathway outperformed students in the traditional pathway. On symbolic manipulation problems, students in the traditional pathway outperformed students in the NSF-funded pathway. Researchers did not conduct classroom observations but relied on teachers' interviews to determine the enactment of the curriculum.

Utilizing an NSF-funded curriculum does not ensure that standards-based pedagogies are employed (Tarr et al., 2008) and many studies about written curriculum effectiveness lack the requisite classroom observations to ensure fidelity in the enactment of the curriculum (Stein et al., 2007). In Boaler and Staples's (2008) study, that included over 600 hours of classroom observations, there were significant variations in the enactment of the curriculum between teachers utilizing the same curriculum. Cai et al. (2011) analyzed the portion of the lesson dedicated to conceptual understanding or procedural focus. Little was mentioned about the enactment of the tasks and fidelity to the philosophy of the written curriculum. Teachers may implement the lesson in such a way that they alter the tasks, purposefully or inadvertently, to decrease the cognitive demand (Henningesen & Stein, 2002). Furthermore, teachers believe they are implementing a standards-based curriculum with fidelity of pedagogy, but revert to traditional teaching pedagogies (Moyer et al., 2018; Tarr et al., 2008).

Generalizations about an NSF-funded curriculum cannot therefore be made without the requisite classroom observations to ensure that implementation of the written curriculum is

consistent with the philosophical tenets of learning of that curriculum (Stein et al., 2007). For these reasons, one cannot assume that because a school/district has adopted an NSF-funded curriculum or a traditional curriculum, that students experience that curriculum in the same manner and have the same opportunity to learn mathematics as students in other schools/districts or even other classrooms within the same school. It is necessary to also examine the enacted curriculum as this typically has the greatest impact on student achievement (Remillard, 2005; Remillard & Heck, 2014; Stein et al., 2007; Tarr et al. 2008). The next section addresses the enacted curriculum with standards-based mathematics instruction.

## **2.4 The Enacted Curriculum**

While curriculum materials (the written curriculum) impact students' OTL and achievement, how teachers implement the content may be a more important factor (Hunsader & Thompson, 2014; Remillard & Heck, 2014). The enacted curriculum is jointly constructed by the interaction between the teacher, the students, and the content, *The Instructional Triangle* (Cohen et al., 2003).

The enacted curriculum is the result of the interactions within *The Instructional Triangle* (Cohen et al., 2003). It is multi-faceted with dimensions including: “(a) the mathematics; (b) instructional interactions and the norms that govern them; (c) the teacher’s pedagogical moves; and (d) the use of resources and tools” (Remillard & Heck, 2014, p. 136). The mathematics refers to the content and how it is represented, for example: demonstration, practice, connections to prior concepts, conceptual connections, and proof (Hiebert et al., 2003). Instructional interactions refer to the interaction between the teacher, the students, and the task (Cohen, Raudenbush, & Ball, 2003; Remillard & Heck, 2014). These interactions include the nature of discourse (Wood et al., 2006),

social interactions (Esmonde & Langer-Osuna, 2013), and classroom norms (Cobb & Yackel, 1996; Franke, Kazemi, & Battey, 2007).

Teacher pedagogical moves refer to how the teacher structures the task, which in turn shapes the representation and investigation of the mathematics (Remillard & Heck, 2014).

Pedagogical moves influence the teacher-student interaction in *The Instructional Triangle* (Cohen et al., 2003). Tools and resources refer to the various physical (e.g., texts, manipulatives), technological (e.g., dynamic software), and cognitive tools (e.g., mnemonic devices, algebraic symbols, algorithms) that are used by teachers and students during instruction (Remillard & Heck 2014). A standards-based mathematics instructional model, aligned with NCTM's (1989, 1991, 2000, 2014) vision of mathematics teaching and learning improves students' opportunity to learn mathematics (e.g., Boaler, 1989; Borko, Stecher, Alonzo, Moncure, & McClam, 2005; Tarr et al., 2008). What follows is a review of the literature about standards-based mathematics instruction.

#### **2.4.1 Standards-Based and Traditional Mathematics Instruction**

Mathematics learning is a socially constructed active process, where knowledge is built upon experiences within the classroom (NCTM, 2014). In an SBMI model, students' opportunities to learn are supported by collaboration with peers on high-cognitive demand tasks with discourse that includes conjectures, explanations, challenges, generalizations, and abstractions (Munter et al., 2015, Weaver et al., 2005). Students' productive struggle with these tasks without premature teacher intervention increases student understanding (Munter et al., 2015; NCTM, 2014; Warshauer, 2015). Through small-group and whole classroom discourse, shared deep understandings of the mathematical concepts develop (Cobb et al., 1997).

SBMI differs from traditional, direct instruction in many ways, as summarized in Table 1 (Munter et al., 2015). In a traditional instruction model, classroom instruction is teacher-centered with the teacher as the locus of authority who transmits the mathematical knowledge to the students (Moyer et al., 2018; Munter et al., 2015). Teachers provide explicit definitions and algorithms, and students practice these procedures (Moyer et al., 2018; Munter et al., 2015). The pace of lessons is brisk with group unison responses to teacher questions (Munter et al., 2015). This is routinely referred to in the literature as direct instruction (Munter et al., 2015). Classroom discussions in the traditional classroom often take the form of initiation-response-evaluation (IRE), wherein the teacher poses a question, a student answers it, and the teacher evaluates the response (Sinclair & Coulthard, 1975).

Conversely, with SBMI, the teacher is the facilitator of discourse intended to bring the important mathematical concepts to the foreground (Munter et al., 2015; NCTM, 2014). Students often work in small groups on tasks and then share their solution trajectories (Munter et al., 2015; NCTM, 2014). They are expected to reason and justify their thinking as well as challenge the reasoning of others, including the teacher, when there is disagreement (Munter et al., 2015; NCTM, 2014).

In the next sections, four characteristics of effective standards-based mathematics instruction –collaborative problem solving, cognitive demand, complex instruction, and high-quality student discourse -- are presented. Small-group discourse during collaborative problem solving is the subject of this study. Cognitive demand of the task drives the type of discourse that is possible during collaborative problem solving (Henningesen & Stein, 2002). Complex instruction is imperative to provide equitable access to collaborative problem solving and high-quality discourse (Boaler, 2006a; 2006b, Cohen & Lotan, 1995). Equitable access to high-quality discourse provides

for equitable access to opportunities to learn (Esmonde & Langer-Osuna, 2013) and is predictive of higher achievement (Weaver et al., 2005).

Table 1 Characteristics of Standards-Based and Traditional Mathematics Instruction (Munter et al., 2015)

Area of distinction	Standards-based Instruction	Traditional Instruction
Role of discourse	Fundamental to knowing and learning mathematics. Students need time to talk about their thinking, questions, and arguments.	Teacher-led discourse. Students explain to the teacher how problems are solved. Student discussion is not necessary.
Role of group work	Group work, with complex problems, provides opportunities for students to talk and listen to each other.	Small-group work is optional and follows guided instruction as an opportunity to practice procedures. Students are grouped based upon similar abilities.
Sequencing of mathematical topics	Determined by both the structure of mathematics and students' developmental levels.	Determined by the structure of mathematics.
Role of representations	Representations are used for illustrating mathematical ideas and for "thinking with". Often created in the moment.	Representations are used to illustrate mathematical ideas, not to think with or to anchor problem-solving conversations.
Order of mathematical instructional tasks	Students are given time to wrestle with big ideas, without teacher interference.	Students practice problems similar to what the teacher has demonstrated.
Creativity	Learning pathways emerge from student questions and discoveries.	Student learning pathways are determined by the teacher. Student questions that drive instruction are not emphasized.
Purpose of diagnosing student thinking	Students' thinking and activity are sources of ideas to drive instruction.	Teacher diagnoses the cause of errors and intervenes on the likely cause.
Role of the teacher	Provide tasks that introduce new ideas and deepen conceptual understanding. Orchestrate discussions to make mathematical ideas available to all students. Sequence activities to position students as autonomous learners.	Articulate clear objectives and connections to previous topics, present requisite concepts, demonstrate how to solve the problem type, provide opportunities for guided practice, provide corrective feedback.

## 2.4.2 Mathematical Tasks and Cognitive Demand

Mathematical tasks are activities designed to focus students' attention on a specific mathematical concept (Stein et al., 2007). Not all tasks provide students with equal opportunities to think and learn (Stein, et al., 1996). Low-level cognitive demand tasks include rote memorization and/or procedures without connections whereas high-level cognitive demand tasks include procedures with connections and/or doing mathematics (Stein et al., 1996). Doing mathematics refers to complex tasks that do not have a predictable solution trajectory or worked-out example to replicate procedures (NCTM, 2014). In SBMI, these mathematical tasks tend to be higher-level cognitive tasks that are non-routine in nature and promote mathematical reasoning, multiple representations, and problem solving by providing multiple entry points and solution pathways (NCTM, 2014).

Implementing high-level cognitive demand tasks provides for greater student learning and student-student interaction than the utilization of tasks that are rote in nature (Boaler & Staples, 2008; Boston & Wilhelm, 2015; Henningsen & Stein, 1997; Munter et al., 2015). Teachers have the potential of reducing the cognitive demand of a task during its implementation, even in standards-based classrooms (Stein et al., 1996; Warshauer, 2015). The amount and type of scaffolding and guidance afforded to students can either maintain or diminish the cognitive demand of the task during enactment (Stein et al., 1996; Warshauer, 2015). High cognitive demand tasks often take longer to implement than routine problem solving (Henningsen & Stein, 2002). Insufficient time to make sense of and grapple with a task can change the focus of the task to procedural thinking (Henningsen & Stein, 2002). As the focus of complex problem-solving shifts towards quickness of problem solving, students' perceptions of what it means to do mathematics is impacted (Henningsen & Stein, 2002; Schoenfeld, 1989). Students may believe that ability in

mathematics is equated with speed and that all mathematics problems can be solved within minutes (Schoenfeld, 1989).

### **2.4.3 Collaborative Problem Solving**

Collaborative learning is “a reciprocal process of exploring each other’s reasoning and viewpoints in order to construct a shared understanding of the task” (Goos, 2000, p. 39). The use of collaborative problem-solving, also referred to as collaborative inquiry, results in the construction of new understandings and higher learning outcomes as compared to individual work (Gillies, 2004; NCTM, 2014). Collaborative learning differs from cooperative learning in that cooperative learning allows for students to divide a task into separate parts to be worked on individually whereas collaborative learning occurs when students share their ideas through continuous communication and the co-construction of understanding with the goal of jointly solving a challenging problem (Damon & Phelps, 1989).

Collaborative problem-solving provides a greater opportunity for students to talk about and listen to various solution methods than teacher-directed instruction (Munter et al., 2015). Successful collaborative learning is characterized by students clarifying, elaborating, and justifying their ideas for the benefit of the group; students asking their peers for help in finding errors and seeking feedback on their ideas; and students attempting to understand their peers’ thinking (Goos, 2000, p. 41).

The selection of the group task for collaboration is important (Lotan, 2003). The task should foster interdependence among the group members, with no one student being able to complete the task individually as well as the whole group can, by requiring group consensus and supporting dialogue among the members (Damon & Phelps, 1989; Tomlinson, 2018). There is both

group and individual accountability (Cohen, Lotan, Scarloss, & Arellano, 1999). Groups are expected to present to the class and teach others what they have learned. Group members are expected to report about their work in the group.

A collaborative mathematics environment with students and teachers comfortable with struggle is a deviation from typical U.S. mathematics instruction (NCTM, 2014). It requires students to have a different understanding of what it means to be a successful learner and teachers to have a different understanding of what it means to be an effective teacher of mathematics (NCTM, 2014).

Boaler's research at Railside High School (Boaler, 2006a; 2006b) focused on the interaction between SBMI and mixed-ability group work. Overall student achievement was greater with mixed-ability grouping, with more gains made by lower-ability students. Not only did achievement grow, but student understanding of what it meant to do mathematics changed. Through detailed interviews with students, Boaler chronicled repeated stories about how students' focus changed from individual achievement to group achievement, and how the contributions of all students in the classroom were valued (Boaler & Staples, 2008). Similar findings were reported in her research in the United Kingdom (Boaler, 2008), where she used the term 'relational equity' to refer to students treating each other with respect and responsibility. Additionally, Boaler, Wiliam, & Brown (2000) found less polarization within the classroom when there was mixed-ability grouping. For these types of changes in student beliefs, certain classroom structures must be implemented for successful collaborative problem-solving to occur.

### 2.4.3.1 Classroom social norms

Classroom social norms are the co-constructed rules for the participation within the classroom (Cobb & Yackel, 1996). Examples of social norms include publicly sharing interpretations and solution trajectories, explaining, justifying solutions, and making sense of other students' arguments (Cobb & Yackel, 1996). While the teacher may initiate such social norms, students shape the norms through their participation; thus, these norms are established through a joint social construction (Cobb & Yackel, 1996). By participating in these social norms, students reorganize their beliefs about their own roles, others' roles, and the general nature of mathematics (Cobb, Yackel, & Wood, 1989). As these beliefs change, how students participate in the social norms can change, thereby changing the nature of the social norms themselves (Cobb & Yackel, 1996).

In productive collaborative inquiry, special social norms may emerge:

- a. Collaboration norm: To learn more, mathematics requires collaboration (Fukawa-Connelly, 2012). Collaborative work involves individual accountability, and participants are expected to reach mutual consensus through argumentation (Kazemi & Stipeck, 2008; Tatsis & Koleza, 2008);
- b. Avoidance of threat norm: one is expected not to impose a threat towards one's partners during collaborative work (Tatsis & Koleza, 2008);
- c. Presenter responsibilities norm: the presenter is prepared to explain and defend the groups' work and to respond to questions (Fukawa-Connelly, 2012); and
- d. Audience responsibilities norm: members of the audience are expected to attend to the presentation, ensuring their understanding by asking clarifying questions (Fukawa-Connelly, 2012).

Students' beliefs about their roles, the roles of others, and the general nature of mathematical activity in school are influenced by the social norms of the classroom (Cobb & Yackel, 1996; Yackel & Cobb, 1996).

### 2.4.3.2 Sociomathematical norms

Sociomathematical norms are those classroom norms that are specific to mathematical activities and are connected to the quality of mathematical contributions (Cobb & Yackel, 1996). They include what counts as a different mathematical solution, a sophisticated mathematical solution, an efficient mathematical solution, and an acceptable mathematical explanation (Cobb & Yackel, 1996; Yackel & Cobb, 1996). As students respond to a call for a different mathematical solution, for example, both the teacher and the class decide if the solution is different, implicitly negotiating the sociomathematical norm of mathematical difference (Cobb & Yackel, 1996; Yackel & Cobb, 1996). This negotiation fosters intellectual autonomy, “students’ awareness of and willingness to draw on their own intellectual capabilities when making mathematical decisions and judgements” (Cobb & Bowers, 1999, p. 9), and increases students’ opportunities to learn by promoting mathematical discourse (Yackel & Cobb, 1996). Sociomathematical norms can impact students’ mathematical beliefs and values by changing the locus of authority from the teacher to the class (Cobb & Yackel, 1996; Yackel & Cobb, 1996).

Sociomathematical norms necessary for investigative tasks include the following:

- a. An explanation consists of a mathematical argument and centers on students’ mathematical activity with entities, rather than a procedural description or summary (Kazemi & Stipeck, 2008; Lopez & Allal, 2007);
- b. Mathematical thinking involves understanding connections between multiple strategies (Kazemi & Stipeck, 2008, p. 16);
- c. Errors provide opportunities to reconceptualize a problem, explore contradictions in solutions, or pursue alternative strategies (Kazemi & Stipeck, 2008, p. 16);
- d. Investigating mathematics requires a creative approach (Partanen & Kaasila, 2015);
- e. Collaborative inquiry requires students to explain to their partner what they understood and did not understand about the problem and to ask their partners questions about their problem-solving procedures (Lopez & Allal, 2007); and

- f. Investigating mathematics may require approaches in addition to symbolic methods (Partanen & Kaasila, 2015).

These sociomathematical norms, which can be influenced by students' and teachers' mathematical beliefs and values (Cobb & Yackel, 1996; Ernest, 1989; Fives & Buehl, 2016), provide support for collaborative problem-solving.

#### **2.4.4 Complex Instruction**

Small-group discourse can increase the opportunity for equitable access to participation for all students (Cohen & Lotan, 1995; Esmonde, 2009; Esmonde & Langer-Osuna, 2013). The presence of collaborative inquiry does not ensure access to mathematics for all students, particularly for students who do not have high status in the classroom (Cohen & Lotan, 1995; Esmonde, 2009). Without attention to status, high-status students can dominate group interactions over low-status students (Cohen & Lotan, 1995). Status characteristics may be based on race, gender, or social class or they may be based on perceived academic ability (Cohen & Lotan, 1995; Esmonde, 2009; Langer-Osuna, 2016). Students with perceived low-status are routinely ignored or not given a turn to participate in the activity which in turn lowers participation and decreases opportunities to learn for these students (Cohen et al., 1999; Langer-Osuna, 2016). Students with directive (social) or intellectual authority, as evaluated by group members and/or the teacher, position themselves as credible sources of mathematical knowledge and can control which ideas are pursued during collaborative mathematics problem-solving (Langer-Osuna, 2016).

Complex instruction [CI] is a pedagogical model of instruction designed to produce equal status in heterogeneous groupings (Cohen & Lotan, 1995). One facet of CI is 'multiple ability treatment' (Cohen & Lotan, 1995). Public statements, by the teacher, enumerate the many different

skills required to be successful on the task and that no one student possesses all the necessary skills. All students, however, have at least one of the necessary skills for the group to be successful. A second characteristic of CI involves ‘assigning competence’ (Cohen & Lotan, 1995; Esmonde, 2009; Langer-Osuna, 2016; Lotan, 2003). When teachers observe low-status students performing well, intellectually, on aspects of the task, or having unique solution trajectories, their public positive evaluation of the low-status student can help to change the expectations of competence for both the high and low-status students (Cohen & Lotan, 1995).

In collaborative learning classrooms with mixed-ability groups, where attention to status, position, and complex-instruction is provided, there is less polarization in students’ feelings towards school and there is greater equity in access to opportunities to learn (Boaler, 1998; Boaler et al., 2000; Langer-Osuna, 2016). In classrooms where status is not addressed, patterns of early group discourse can become normative with high status members dominating the interactions and lower status group members withholding their ideas, thereby reducing the group’s generation of new ideas (Chiu, 2008) and students’ OTL (Boston & Wilhelm, 2015; Esmonde, 2009; Langer-Osuna, 2016).

#### **2.4.5 Student Discourse in Collaborative Problem-Solving**

Student participation in high-quality mathematics discourse is a predictor of mathematics achievement (Weaver & Dick, 2009). Analogous to categorizing the cognitive demand of a mathematical task (Stein et al., 1996), discourse can be categorized according to the level of the cognitive demand of the thinking required to participate in the discourse (Bishop, Hardison, & Przybyla-Kuchek, 2016; Weaver et al., 2005, Webb, 2002). Low-quality discourse relates to contributions of recalled facts, short answers to a direct question, statements of results, or mathematical procedures (Bishop et al., 2016; Henningsen & Stein, 2002; Weaver et al., 2005; Webb, 2002). High-quality discourse pertains to deeper levels of mathematical connections and

includes justifications, conjectures, argumentations, or generalizations (Bishop et al., 2016; Henningsen & Stein, 2002; Weaver et al., 2005, Webb, 2002). A taxonomy of low- and high-quality discourse types can be found in Table 2.

When group members engage in discourse that includes argumentation, challenge and justification, the focus of learning shifts from a quest for the correct answer -- lower level of cognitive demand -- to understanding and meaning-making -- higher level of cognitive demand (Boaler, 2006a, 2006b; Cobb, Stephan, McClain, & Gravemeijer., 2001; Henningsen & Stein, 2002; Yackel, 2004). When the social norms and mathematical practices of whole class discourse include higher quality discourse, student discourse in collaborative learning groups reflects those social norms (Cobb et al., 2001; Kazemi & Stipek, 2008; Webb et al., 2006). The degree to which high-quality mathematics discourse -- generalizing and justifying -- occurs in a classroom is a positive predictor of middle-school and high-school student achievement (Weaver & Dick, 2009).

Table 2 Quality of Discourse  
(Weaver et al., 2005; Webb, 2002)

Low-Quality Discourse	
Type	Explanation
Answering	Short answer to a direct question
Statement	Simple statement or sharing of work without an explanation of how or why
Explaining	States how problem was solved with no justification of validity of procedure
Questioning	Asks a question to clarify understanding
High-Quality Discourse	
Type	Explanation
Challenging	Challenges the validity of a mathematical idea or procedure
Relating	Connects to prior knowledge
Predicting	Makes prediction based on understanding of the mathematics behind the problem
Justifying	Justifies validity of mathematical idea by providing explanation of thinking
Generalizing	Makes a statement to shift from a specific example to the general case

Participation in group discourse has varying impacts on opportunities to learn (Esmonde & Langer-Osuna, 2013). Jansen (2008) researched student participation in whole-class and small-group discourse in classrooms using an NSF-funded curriculum. Jansen found that students who believe that student-student or whole class discourse that focuses on argumentation or justification is not part of mathematics or who view participating in the discourse as threatening may not participate in the discourse during collaborative problem-solving.

Even if students hold the belief that discourse that focus on argumentation or justification is an important part of mathematics, they may not participate in the small-group discourse for various reasons. Group members must feel confident enough, in their knowledge and status, to challenge the thinking of others (Chiu, 2008). Small-group discourse can be affected by participants' knowledge and commitment to understanding (Nussbaum, 2008). Ways in which teachers may support students' discursive involvement in group dialogue by ensuring positive identity building needs further research (Langer-Osuna, 2016). Some dialogue patterns among group members may promote learning, while other patterns may inhibit learning (Chi & Menekse, 2015). For example, passive discourse -- simply agreeing with what has been stated -- has less of an effect on learning than active discourse -- repeating what has been stated --, or constructive discourse-- building upon what has been stated (Chi & Menekse, 2015). Certain set roles within groups - such as explainer or reporter -can increase a student's participation in the discourse, whereas other roles - such as recorder or "menial worker" - can inhibit a student's participation in the discourse (Wood, 2013).

The establishment of standards-based learning environments is not sufficient to explain student participation in small-group discourse. Students' mathematics-related beliefs, as alluded to in the theoretical framework and discussed in detail below, can impact the quality and quantity of

their participation in the discourse, and ultimately can impact their opportunities to learn mathematics.

## **2.5 Mathematics-Related Beliefs**

A study of student learning necessitates an examination of their beliefs as beliefs and self-perception act as filters of input and OTL (Gee, 2008). Beliefs are part of the affective domain but have stronger cognitive components than do emotions and attitudes (De Corte, Op't Eynde, & Verschaffel, 2002). Student mathematics-related beliefs (MRB) are the beliefs students construct about the subjective, affective, and epistemic aspects of learning mathematics which include (a) beliefs about mathematics education, (b) beliefs about the self in relation to mathematics, (c) beliefs about the social context of mathematical learning, and (d) epistemological beliefs (De Corte et al., 2002). What one believes about mathematics education is influenced by one's beliefs about what mathematics learning and teaching are like (Cobb et al., 1989; Hofer & Pintrich, 1997). Beliefs about the self in relation to mathematics include expectancy, value, self-efficacy, and identity beliefs (De Corte et al., 2002; Edroğan & Sengul, 2014; Oyserman, Elmore, & Smith, 2012). Beliefs about the social context of mathematical learning include views about classroom and sociomathematical norms (Cobb & Yackel, 1996), the role of the teacher, and the role of the student (Cobb et al., 1989). Epistemological beliefs include beliefs about the nature of knowledge and the nature of knowing (Hofer & Pintrich, 1997). Epistemological beliefs will be discussed first, followed by mathematical identity.

## **2.6 Mathematical Epistemological Beliefs**

Mathematical epistemological beliefs [EB] are beliefs about the nature of mathematical knowledge and the nature of knowing (Hofer & Pintrich, 1997). They include beliefs about the

ability to acquire knowledge, the speed of knowledge acquisition, the certainty of knowledge, the structure of knowledge, and the source of knowledge (Buehl & Alexander, 2005; Schommer, 1990). Beliefs have been referred to in the literature as dualistic vs. relativistic (Perry, 1968), reproductive vs. constructivist (Cano, 2005), naïve vs. sophisticated, less constructivist vs. more constructivist, inappropriate vs. appropriate, or non-availing vs. availing, most of which may have negative connotations (Muis, 2004). In the rest of this paper, the terms non-availing and availing beliefs will be used because these terms are non-judgmental and reflect the relationship of the belief to learning outcomes. Non-availing beliefs have no influence or a negative influence on learning; while availing beliefs have a positive influence on learning and are assumed to support higher-order thinking (Muis, 2004). A learner's beliefs about the nature of knowledge and knowing can be availing, non-availing, or a mixture of both availing and non-availing beliefs (Muis, 2004; Schommer, 1990).

There are varying hypotheses regarding the structure of epistemological beliefs. Researchers who adhere to developmental stage theories of epistemological beliefs, based on the work of Perry (1968), suggest that beliefs about knowledge and knowing move through a specified sequence as one's ability to make meaning evolves (Hofer, 2001). Beginning in a *dualistic* position where all questions have right-or-wrong answers and authority figures hold the truth, beliefs may develop to a *multiplistic* position where there may be multiple or no solutions to a problem; then to a *relativistic* position where answers are right or wrong based on the context, and that knowledge is highly complex, tentative, and derived through reason; and finally to *commitment within relativism* where there is an integration of knowledge from other sources with personal experiences (Limon, 2006; Zhu, 2017).

Rather than viewing epistemology as a solitary unit that develops in sophistication over time, some researchers believe that epistemological beliefs consist of various dimensions, or a systems of beliefs theory (Hofer & Pintrich, 1997). In this view, epistemological beliefs are a multidimensional system of semi-independent beliefs about the nature of knowledge and the process of knowing that fall along a continuum (Hofer & Pintrich, 1997). The four dimensions of this multi-dimension system are: (1) certainty of knowledge; (2) simplicity of knowledge; (3) source of knowledge; and (4) justification for knowing (Hofer & Pintrich, 1997). The dimensions of students' beliefs are semi-independent as students may hold availing beliefs about one dimension while simultaneously holding non-availing beliefs about another dimension (Hofer & Pintrich, 1997; Schommer, 1990).

Researchers that maintain a systems of beliefs theory view MRB as relatively stable and enduring (Hofer & Pintrich, 1997), suggesting that MRB are resistant to change. The beliefs themselves are the cognitive unit that can be analyzed and measured by interviewing students or conducting surveys (Hofer & Pintrich, 1997). Developing more availing EB requires a destabilization of non-availing beliefs by challenging the existing beliefs with ideas about learning and knowledge that foster cognitive conflict and disequilibrium (Hofer, 2006). This study adheres to the more contemporary, multidimensional system of semi-independent beliefs following the work of Schommer (1990).

### **2.6.1 Epistemological Beliefs and Learning**

High school and college students generally hold non-availing beliefs about mathematics and the learning of mathematics (Muis, Trevors, Duffy, & Ranellucci, 2016). This is important because beliefs are influential in how an individual perceives and approaches tasks and problems

and are a strong predictor of behavior (Nespor, 1987) and achievement (Schoenfeld, 1989; Schommer, 1990).

One of the first studies of students' MRB occurred over 30 years ago, at the same time NCTM published its first recommendations to improve mathematics education. Schoenfeld (1989) surveyed 230 high school students and interviewed 21 of those participants about the attributions of success or failure in mathematics and their perceptions of mathematics. Analysis of the data found that students held many non-availing beliefs about mathematics, including mathematics is learned by memorization, problems should be solved quickly, and mathematics ability was innate. This innate ability belief was later referred to by Dweck (2008) as a fixed mindset. Cano (2005) found that the average male high school student held a fixed mindset, and the average female student held a growth mindset.

Grouws, Howald, and Colangelo (1996) created a survey to measure students' conceptions of the nature of mathematical knowledge, the character of mathematical activity, and the essence of learning mathematics. The instrument is discussed in greater detail later in this section. Grouws et al. surveyed 55 mathematically talented high school students and 112 high school students in grade 9 through 11 who were enrolled in either an integrated mathematics course or an algebra course (algebra students). From this sample, he interviewed 19 students to expand on their survey responses. Differences between the talented students and the algebra students existed in their response patterns to the composition, structure, and doing mathematics scales. Talented students placed importance on the underlying concepts and ideas of mathematics while algebra students equated mathematics with a collection of rules. Talented students viewed mathematics as a coherent system with connections between concepts while algebra students viewed it as discrete pieces. Most talented students viewed doing mathematics as a sense-making process whereas algebra students

viewed mathematics as a rule following and memorization process. The beliefs of the algebra students, those in a traditional curriculum setting, resembled those of the students in Schoenfeld's (1989) study. The students in Grouws et al. (1996) study believed that if they could not solve a mathematics problem quickly, that spending more time on it would not help. Grouws et al. did not disaggregate the data of the algebra students by those following a traditional pathway and those following an integrated pathway although it was presumed by the researchers that all students experienced a traditional instructional model.

Star and Hoffman (2005) administered the same survey as Grouws et al. (1996) to 134 students who had completed three years of NSF-funded curriculum in middle school. No classroom observations were made as the teachers and their pedagogy were familiar to the researchers. They compared their results to Grouws's results and found that students from an NSF-funded background had beliefs that were more aligned with reform-oriented ideas on the survey. This study was significant because it was one of the first to consider curriculum and beliefs. It was also significant because it considered the beliefs of middle school students.

Schoen et al. (2010) analyzed high school students' beliefs after completing year 3 and year 4 of an NSF-funded curriculum. The students in this study were unique in that the feeder middle schools implemented an NSF-funded curriculum. After seven years of an NSF-funded curriculum, students believed that most mathematical ideas are related. They tended to believe that understanding concepts and ideas is crucial to solving mathematics problems. Male students held a stronger belief than female students that mathematical concepts are related. Female students held a stronger belief than male students that understanding concepts is key to solving mathematics problems, students learn mathematics by asking questions and analyzing mistakes, and mathematics is more about problem solving than memorizing formulas.

Nearly 30 years after Schoenfeld's (1989) report, in a small-scale study, Muis et al. (2016) surveyed secondary and tertiary students (n = 34) regarding their general and mathematical beliefs. Students responded to prompts about the certainty/simplicity of knowledge, source of knowledge, justification of knowledge, and attainability of knowledge. Students were then interviewed to explain their responses to the survey. Students generally believed there was only one correct answer, though three (3) believed there may be multiple approaches to solve a mathematics problem. They also believed mathematics knowledge is certain, unchanging, and handed down by authority figures with rules and algorithms to follow. Students believed the only justification they needed was to follow the formulas and practice. Muis's study did not consider the written or enacted curriculum in conjunction with beliefs.

Moyer et al. (2018), as part of the LieCal project, interviewed 44 students at the end of 12<sup>th</sup> grade and compared responses to those given when students were entering 9<sup>th</sup> grade. Of the 44 students, 18 had experienced a middle school NSF-funded curriculum pathway and 26 had not. All students experienced a non-NSF-funded curriculum pathway in high school. Students were asked about the nature of mathematics, among other things. Students' vision of mathematics was aligned with the curriculum and teaching they experienced in middle school.

These recent findings of Muis et al. (2016) and Moyer et al. (2018) are consistent with previous and current research with anecdotal evidence, interviews, and survey results (e.g., Grouws et al., 1996; Schoenfeld, 1989) for students experiencing a traditional instruction model, suggesting that little has changed in students' mathematical beliefs over 30 years. These non-availing beliefs conflict with the notions about mathematics and its learning put forth by the NCTM in *Principles and Standards* (2000) and *Principles to Action* (2014) and may adversely influence students' learning and identity formation.

Students' epistemological beliefs can have far-reaching effects. For example, if students believe that they are not "math" people, then they are less likely to persist when mathematics becomes difficult (Schoenfeld, 1989), and they are less likely to participate in classroom discourse (Hoffman, 2004; Jansen, 2008). Students who believe that learning is the acquisition of knowledge imparted by the teacher view public sharing of their thinking as risky for fear of being wrong (Hoffman, 2004). However, students who hold availing "speed of learning" and "ability to learn" mathematical beliefs are more likely to believe that problem solving requires effort and understanding and is useful (Boaler, 2006b; Schommer-Aikins, Duell, & Hutter, 2005). Not only do students with availing "speed of learning" and "ability to learn" mathematical beliefs have more confidence in their ability and less math anxiety (Chinn, 2012; Schommer-Aikins & Duell, 2013), but they are better at problem solving due to perseverance and multiple solution trajectories (Cano, 2005; Phillips, 2001; Schommer-Aikins et al., 2005).

### **2.6.2 Beliefs and the Classroom Experience**

Students' beliefs about mathematics are shaped, in part, by their experiences in the classroom (Boaler, 2006a; Cano, 2005; De Corte, Verschaffel, & Depaepe, 2008; Muis, Franco, & Gieus, 2011). As students participate in the social and sociomathematical norms of the classroom, their personal beliefs and mathematical identities begin to develop (Yackel & Cobb, 1996). Students who experience traditional teaching, with a focus on procedural knowledge and a toolbox perspective, generally have more non-availing beliefs than students who experience standards-based or problem-solving curricula, with a focus on construction of knowledge and conceptual understanding (Greene, Muis, & Pieschl, 2010; Muis & Foy, 2010; Star & Hoffman, 2005).

Direct instruction in problem-solving heuristics and small-group discourse with complex problem-solving opportunities can positively impact elementary and middle school students' beliefs that mathematics is more than memorization of facts and procedures, and that challenging problems can be solved through perseverance (De Corte et al., 2008; Higgins, 1997; Mason & Scrivani, 2004). Mathematics experiences with ill-structured tasks serves to challenge high school students' epistemological beliefs about the nature of mathematics (McGregor, 2014). Post-secondary students' beliefs about the certainty of knowledge, the attainment of knowledge, and the justification of knowledge are influenced by the procedural and conceptual emphasis in the learning environment (Muis et al., 2011). Changing the learning environment to an inquiry-based problem-solving environment can have positive impacts on students' beliefs about mathematics (De Corte et al., 2008; McGregor, 2014; Muis et al., 2011). A change in mathematical beliefs is speculated to result in a change in how students engage in mathematical practices, and therefore might result in a change in students' level of mathematical achievement (Cano, 2005; Phillips, 2001; Schommer-Aikins et al., 2005).

Tracking in mathematics can lead to negative beliefs. Students in lower tracked mathematics courses often hold non-availing beliefs about the nature of mathematics and their mathematics identities as compared to students in accelerated or honors courses (Boaler, 2006b; Grouws et al., 1996). A change in classroom learning environment from a traditional classroom to a standards-based classroom can positively affect students' mathematics epistemological and identity beliefs, even if they are in the lower tracked course (Boaler, 2006b). Additionally, by de-tracking classes and incorporating an SBMI learning environment, students begin to accept and appreciate the perspectives and contributions of different students, promoting equity in the classroom (Boaler, 2006b).

### **2.6.3 Measures of Mathematical Epistemological Beliefs**

Direct measurements of epistemological beliefs do not exist and must be interpreted from the individuals' statements, responses to questions/surveys, and/or from observed behavior (Pajares, 1992). There can be differences between individuals' self-reported beliefs and observed beliefs (Leach, Millar, & Ryder, 2000). Students may be unaware of their beliefs or may not be able to articulate their beliefs (Buzeika, 2004). Observed epistemological beliefs measure the application of the beliefs in a particular context, customarily when the individual is supported through established classroom practices and sociomathematical norms (Limon, 2006). For example, classroom social and sociomathematical norms which govern how one participates in mathematical activities and discourse may override a student's epistemological beliefs about the source of knowledge authority, resulting in the student seeking confirmation of the correctness of an answer from group members rather than from the teacher.

Given that research has shown that there are differences in students' epistemological beliefs based upon the domain (Caspi, 2002; Hofer, 2006; Trautwein & Ludtke, 2007), domain-specific surveys have been developed. Many of these surveys, as addressed below, measure mathematics-related beliefs and not epistemological beliefs.

#### **2.6.3.1 Fennema-Sherman Mathematics Attitudes Scales**

Prior to 1976, there were instruments available to measure students' global attitudes, but there were no well-defined instruments developed to specifically measure students' attitudes towards mathematics (Fennema & Sherman, 1976). Fennema and Sherman (1976) developed a domain-specific nine-scale survey to measure middle school and high school students' attitudes towards mathematics learning, with specific interest in gaining information about female students'

beliefs towards learning mathematics and variables affecting advanced mathematics course selection by all students. The nine scales of this survey included measures of (a) students' perceptions of the teacher's support; (b) perceptions of mother's support; (c) perceptions of father's support; (d) mathematics as a male domain; (e) consequences of mathematical success; (f) confidence in their own ability to learn mathematics; (g) their mathematics anxiety; (h) their effectance motivation, the desire for effective interaction in the classroom environment; and (i) the usefulness of mathematics (Fennema & Sherman, 1976, p. 325-326). While cited in over 1400 journal articles (Google Scholar, 2017) as the basis of other belief surveys, this survey is more a measurement of attitudinal factors rather than students' EBM (Fennema & Sherman, 1976).

### **2.6.3.2 Indiana Mathematics Belief Scale**

The Indiana Mathematics Belief Scale (IMBS) is considered the first instrument to measure students' beliefs about the discipline of mathematics or about how mathematics is learned (Kloosterman & Stage, 1992). The IMBS was developed to measure post-secondary level students' epistemological beliefs related to motivation and achievement on mathematical problem solving based on the assumption that certain beliefs increase motivation to learn to solve mathematical problems and other beliefs decrease this motivation (Kloosterman & Stage, 1992). Five scales comprise the IMBS measuring students' beliefs: (a) that they can solve time-consuming mathematics problems; (b) there are word problems that cannot be solved with simple, step-by-step procedures; (c) understanding concepts is important; (d) word problems are important in mathematics; and (e) effort can increase ability in mathematics (Kloosterman & Stage, 1992, p. 109-111). These scales are based on the work of Schoenfeld (1985, 1989) and his findings that students believe mathematics problems can be solved in less than five minutes and that they should

accept procedures without trying to understand how they work. Internal consistency for each scale ranged from 0.54 to 0.84 (Kloosterman & Stage, 1992), with two of the five scales - “word problems are important” and “there are word problems that cannot be solve with simple, step-by-step procedures” - having an unacceptable reliability levels of less than 0.70 (UCLA: Statistical Consulting Group, n.d.).

### **2.6.3.3 Conceptions of Mathematics Inventory**

The Conceptions of Mathematics Inventory (CMI) was developed to quantify high school students' EBM and conceptions of learning based on research by Schoenfeld (1989) (Grouws et al., 1996). The CMI, which includes several NAEP items, a subset of the Fennema-Sherman Usefulness of Mathematics Scale (Fennema & Sherman, 1976), and items from the IMBS (Kloosterman & Stage, 1992), was intended to measure seven dimensions of learning and knowing in mathematics: (1) the composition of mathematical knowledge [Is the focus conceptual understanding or procedural fluency with facts and algorithms?]; (2) the structure of mathematical knowledge [Is mathematics a set of isolated pieces or a coherent system?]; (3) the status of mathematical knowledge [Is mathematics dynamic or static?]; (4) doing mathematics [is mathematics a process of sensemaking or following rules and algorithms?]; (5) validating ideas in mathematics [Can mathematics be validated through individual reasoning or does it require an external authority?]; (6) student perceptions of learning mathematics [Is mathematics learned through constructing and understanding or through memorization?]; and (7) the usefulness of mathematics. No reliability studies of the CMI were reported for this process.

An attempt to validate the CMI instrument was conducted to establish baseline values for students' beliefs and conceptions from both standards-based learning environments and traditional,

direct-instruction learning environments (Star & Hoffman, 2002). This validation study contained no reliability analysis, factor analysis or Cronbach's alphas of the CMI (Star & Hoffman, 2002). The effect of a standards-based curriculum on middle school students' epistemological beliefs about mathematics was then measured using the CMI to determine that students with SBMI experiences align with the standards-based ideas on the CMI and that students with traditional instructional experiences do not (Star & Hoffman, 2005). Star and Hoffman's (2005) study had several limitations, including low Cronbach's alphas for the seven scales in the CMI, ranging from 0.28, poor, to 0.87, acceptable, and lack of classroom observations to determine the fidelity of curriculum implementation.

An updated CMI, the CMI-R, was used to assess university students' EBM and conceptions of learning (Briley & Thompson, 2009). Using factor analysis, three of the original scales from the CMI were combined to form the Doing, Validating, and Learning Mathematics (DVLM) scale and three were combined to form the Nature of Mathematics (NOM) scale. Only the Usefulness of Mathematics (UOM) scale, which originates from Fennema & Sherman (1976), remained as in the original CMI. Acceptable Cronbach's alphas for each of the three new scales ranged from .71 to .85 (Briley & Thompson, 2009). To date, the CMI-R has only been validated to measure university students' EBM and conceptions of learning.

#### **2.6.3.4 Mathematics Problem-Solving Beliefs Scale**

The IMBS (Kloosterman & Stage, 1992) and Fennema-Sherman Usefulness of Mathematics Scale (Fennema & Sherman, 1976) were combined by Schommer-Aikins et al. (2005) to form a single instrument, the Mathematics Problem-Solving Beliefs Scale (MPSB). Twenty-four of the items loaded onto six factors with a minimum Cronbach's alpha of 0.55: (a)

quick/fixed learning; (b) studying aimlessly; (c) effortful math; (d) useful math; (e) math confidence; and (f) understanding math concepts. This instrument, which is specific to problem-solving, was used in conjunction with Schommer's (1990) general EQ survey to predict problem-solving performance. Beliefs of middle school students, unlike those of older students, emerged into a single factor - quick/fixed learning - which was a strong predictor of problem-solving beliefs (Schommer-Aikins et al., 2005). This instrument measures both students' general epistemological beliefs and the beliefs they hold about problem-solving that affect their motivation to persist with difficult problems.

Beliefs about the nature of mathematics influence and are influenced by beliefs about how mathematics is learned and how mathematics is taught (Pehkonen, 2001). Evidence suggests that students' conceptions about what it means to do mathematics and their epistemological stances about the nature of mathematics knowledge are connected to their attitude toward STEM careers and to their achievement in mathematics (Schommer-Aikins et al., 2005). Students' epistemological beliefs are shaped and situated in classroom practices (De Corte et al., 2008; Greene et al., 2010; McGregor, 2014; Muis et al., 2011). Epistemological beliefs are not the only mathematics-related beliefs that affect students' achievement. The following section reviews the literature about students' beliefs about their identity and self-concept.

## **2.7 Students' Mathematics Identities**

Mathematics-related beliefs include beliefs about the nature of mathematics, about how mathematics is learned, and about oneself as a learner and user of mathematics (Pehkonen, 2001). Students' beliefs about mathematics can have far-reaching effects. Beliefs about oneself as a learner and user of mathematics and about one's competence in mathematical ability can impact future

course selections and even career decisions (Tyson, Lee, Borman, & Hanson, 2007). In the previous section, epistemological beliefs were explored. In this section, student beliefs about their mathematical identities will be addressed. The following sections provide a working definition of mathematics identity and outline factors that affect the formation of mathematical identity.

Research around identity has been heavily influenced by the foundational psychological developmental work of Erik Erickson (1968). Research has also been influenced by the sociocultural perspective where identity develops within a community of practice, such as the classroom (Cobb & Hodge, 2011). The curriculum and classroom routines can position students and shape their roles/identity in the classroom (Cobb & Hodge, 2011; Grootenboer, Lowrie, & Smith, 2006). Identity can be defined as being recognized by oneself and others as a certain “kind of person” in a particular context (Gee, 2000, p. 99). Gee further explained that there are four ways to view identity:

- a. Nature-identity: a state developed from forces in nature (born state)
- b. Institution-identity: a position authorized by authorities within institutions (authorized by a position)
- c. Discourse-identity: an individual trait recognized in the dialogue of /with individuals, and
- d. Affinity-identity: experiences shared in the practice of a group (Gee, 2000, p. 100).

These four identities are interrelated and can simultaneously exist (Gee, 2000). Since discourse and affinity identities form from an individual’s interaction with norms, practices, relationships, and contexts (Hand & Gresalfi, 2015), it is reasonable to believe that identities are fluid and continue to change in response to the environment. In education, students’ identities have been theorized to link to learning and persistence (Boaler & Greeno, 2000; Cribbs, Hazari, Sonnert, & Sadler, 2015).

Mathematics identity is concerned with how students come to understand what it means to do mathematics, to be a doer of mathematics (Cobb et al., 2009). Research suggests that students' mathematics identity has a positive correlation with their persistence in mathematics and STEM-related fields (Cass, Hazari, Cribbs, Sadler, & Sonnert, 2011). Literature on students' mathematics identity includes relationships with classroom community and pedagogy (Gresalfi, Barnes, & Cross, 2012; Grootenboer, 2013; Schoenfeld, 2014), multiple identities (Cobb & Hodge, 2011; Martin, 2000), and career choices (Cass et al., 2011). When students believe that they can do mathematics and that they belong, they develop a positive mathematics identity (Boaler, 2015; Boaler & Staples, 2008). Students with positive mathematics identities have higher levels of performance and participation (Cass et al., 2011; McGee & Martin, 2011).

Mathematics identity may be unique from other educational identities due to the commonly held belief that mathematics intelligence is innate and not learned (Dweck, 2008). Dweck (2008) refers to the belief that mathematics ability is innate as a fixed mindset and the belief that mathematics ability increases with increased effort as a growth mindset. Students, and much of society, think it is acceptable to believe that one is not good at mathematics or that mathematics ability is genetic (Goldin, Epstein, Schorr, & Warner, 2011; Rattan, Good, & Dweck, 2012).

Students' mathematics identities can come from a sociohistorical level: the ways in which parents and community members respond to mathematics send positive and negative messages, implicitly or explicitly, about the importance of mathematics learning (Martin, 2000). These identities can come from the classroom level: differences in norms and sociomathematical norms (Carlone, Haun-Frank, & Webb, 2011); teacher questioning practices (Gresalfi et al., 2012); classroom structures such as ability grouping (Boaler & Staples, 2008); established participation structures (Engle & Conant, 2002); and the microcultures established in classrooms (Boaler &

Greeno, 2000; Martin, 2000). Traditional pedagogies, such as teacher-directed questioning, require students to forfeit their agency to follow classroom routines to succeed (Boaler & Greeno, 2000). Finally, these understandings can develop with respect to different activities (Hand & Gresalfi, 2015). Different mathematical activities afford different opportunities for identity expression (Barton & Tan, 2010; Nasir & Hand, 2008; Polman & Diane, 2010). Classrooms that support students with SBMI promote positive mathematics identities (e. g., Barton & Tan, 2010; Boaler & Staples, 2008; Carlone et al., 2011; Gresalfi et al., 2012; Hand & Gresalfi, 2015).

An integral facet of mathematics identity is the individual's mathematics self-concept. In mathematics, self-concept refers to "the belief systems regarding mathematics and one's sense of self as a thinker in general and a doer of mathematics" (Schoenfeld, 2014, p. 405). Mathematics achievement, mathematics attitude, motivation, education level of parents, socio-economic level of the family, quantity and quality of instruction, and classroom environment are all factors affecting mathematics specific self-concept and identity (Erdogan & Sengul, 2014). An emergent model of self-concept suggests that achievement and academic self-concept are co-constructed and mutually reinforcing (Marsh & Martin, 2011). Skills and past mastery experiences influence self-concept, which in turn influences the ways students learn and acquire skill. Recent longitudinal studies appear to support this reciprocal model between mathematics self-concept and achievement (Arens et al., 2016).

Work that connects the classroom environment and students' experiences with SBMI, with their evolving beliefs about mathematics is still emerging in mathematics education research. The previous sections included a discourse of standards-based learning environments which, from research (e.g., Boaler & Greeno, 2000; De Corte et al., 2008; Greene et al., 2010; Muis & Foy, 2010; Mason & Scrivani, 2004), can promote availing beliefs and positive mathematics identities.

## **2.8 Summary and Implications of Literature Review**

Research in students' beliefs about mathematics in conjunction with the enacted curriculum provides important data for districts contemplating changes in the written curriculum or instructional models. Students' experiences in the classroom form the basis of their beliefs, which then impact their achievement as well as their mathematical identity. These beliefs and identities can ultimately impact students' decisions to persist in mathematics-based STEM fields.

This literature review situated the need for additional research on the interactions among MRB, SBMI, and small-group discourse. The literature review on students' MRB indicates that there are few studies that considered MRB in conjunction with the enacted curriculum. Instead, many studies measured students' beliefs quantitatively with a survey and compared the results to some measure of achievement. Classroom factors, such as the quantity and quality of SBMI, were not explored. There were few studies, e.g., Hoffman (2004) and Jansen (2008), that specifically examined beliefs and discourse. The literature review on identity revealed the importance of the classroom environment on students' development of mathematics identity. While prior studies indicate the importance of SBMI on the development of a positive mathematics identity and whole-class participation, little research has been conducted regarding students' behavior in collaborative work in conjunction with their mathematics identity. Finally, the literature review indicated the need for attention to group structures to ensure that all students have access to opportunities to learn in a standards-based learning environment.

Research shows that middle school students who experience a traditional mathematics learning environment hold less-availing beliefs about mathematics (Star & Hoffman, 2005). On the other hand, research indicates that students experiencing a standards-based middle school curriculum hold availing beliefs about the importance of conceptual understanding; and that these

beliefs remain with them after experiencing a traditional high school curriculum (Moyer et al., 2018). There is little research about changes in mathematical beliefs and identities for students experience incongruent written curricula commitments across grade levels, specifically the adaptation of a hybrid written curriculum in middle school and an NSF-funded written curriculum in high school.

The research by Schoen and colleagues' (2010) about students' MRB was limited to NSF-funded curriculum and similar SBMI models at both the feeder middle school and the research site high school. Research by Boaler and colleagues (2006a, 2006b, 2008) did not look at the learning environment at the middle school level. Based on the timeline of when Boaler and colleagues conducted their research and anecdotal descriptions by student of their experiences in middle school, one may infer a traditional middle school instructional model and an SBMI high school instructional model was present for the students at Railside High School.

This case study of within district incongruent written curricula commitments across the grade bands sought to fill an essential gap that has not been addressed, examining both the ways in which students participate in small-group discourse during collaborate inquiry and their beliefs systems within the learning environment. Esmonde and Langer-Osuna (2013) found that participation in small-group discourse were influenced by mathematics identity and beliefs about how mathematics is learned. Previous research has shown that structures for whole-class discourse are replicated in small-group discourse (Cobb et al., 2001; Kazemi & Stipek, 2008; Webb et al., 2006). What is missing, and what this study sought to fill, was an examination of small-group discourse together with an examination of the students' MRB and the learning environment.

## **CHAPTER 3**

### **METHODOLOGY**

The previous chapter described the current literature on mathematical beliefs, collaborative inquiry, and curriculum as well as the theoretical and conceptual frameworks. These frameworks, and gaps in the literature, lead to the rationale for a descriptive case study of within district incongruent written curricula commitments across the grade bands by examining students' mathematical beliefs and small group discourse.

This chapter begins with the presentation of the rationale for the descriptive case study design and the research questions guiding the study. Then, the school setting where the research was conducted, the research sample, and the participants are detailed. Next, the data collection processes for the survey, interviews, classroom observations, and small-group discourse is expounded upon, including the justification for using surveys, interviews, classroom observations, and the instrumentation. The chapter ends with a description of data analysis methods.

#### **3.1 Research Questions**

This mixed methods case study sought to describe the effects of within district incongruent written curricula commitments across the grade bands by examining students' mathematical beliefs and small-group discourse. Case studies (a) investigate a contemporary phenomenon, (b) cope with a technically distinctive situation in which there will be many more variables of interest than data points, (c) rely on multiple sources of evidence for triangulation of data, (d) benefit from prior theoretical propositions, and (e) are used when the boundaries between the subject of study and the context are not clear (Yin, 2014, pp. 16-17). In this study, conditions a, c, and e are met. This study examined how students engaged in small group-discourse and their mathematical beliefs. The

contextual conditions, that of incongruent written curricula commitments across schools within one district, was especially relevant to the study of the students' beliefs and discourse patterns. The boundaries of the enacted curriculum and students' beliefs and discourse patterns overlap.

Case study research can be used in exploratory, descriptive, explanatory, or evaluative modes (Yin, 2014). A descriptive case study protocol was chosen for this research to illustrate the phenomenon of incongruent written curriculum commitments, NSF-funded curriculum at the high school level and non-NSF-funded curriculum at the middle school level. This case study collected multiple data points (surveys, interviews, classroom observations, discourse analysis) with multiple units of analysis (tracking level, grade level, classroom level, group level, student level). Multiple methods (both quantitative and qualitative) were used to analyze the data.

The research questions that aided in the examination of within district incongruent written curricula commitments were:

1. What mathematics-related beliefs do students hold in a school district with incongruent written curricula commitments in middle school and high school?
  - a. What are students' beliefs about the nature of learning mathematics?
  - b. What are students' beliefs about the speed/ability of knowledge acquisition?
  - c. What are students' beliefs about their mathematical identity, mathematics self-efficacy, and mathematics interest?
2. How do students participate in small-group discourse within various enacted curricular models, and how is this participation related to their mathematical beliefs?
  - a. In what ways are students' mathematical beliefs related to the nature and quality of discourse during collaborative inquiry?
  - b. In what ways is the enacted curriculum related to the nature and quality of students' discourse during collaborative inquiry?

### 3.2 Setting

A public-school district located in western Massachusetts was selected as the site for this study. Autumn School District students have the choice of attending the public high school, a vocational high school, a charter school, or one of several area private schools. The demographics of the high school and the middle school are similar. Autumn High School has an enrollment of approximately 950 students in grades 9 to 12 (Massachusetts Department of Elementary and Secondary Education [MDESE], 2018). The student population is predominantly white, 59%, with approximately 37% classified as high needs and 17% not having English as their first language (MDESE, 2018). Autumn Middle School has an enrollment of approximately 420 students in grades 7 and 8 (MDESE, 2018). The student population is predominantly white, 56%, with approximately 46% classified as high needs and 19% not having English as their first language (MDESE, 2018). About 25% of students in both the middle and high schools are economically disadvantaged. Autumn High School requires two full-year mathematics courses for graduation.

Autumn School District was chosen because the apparent schism in written curriculum philosophies between elementary, middle, and high school as determined by the district-wide adopted written curricula. The high school implemented an NSF-funded curriculum, Interactive Mathematics Program [IMP] (Fendel, Resk, Alper & Fraser, 2015) with varying degrees of fidelity. While use of a standards-based curriculum is not the only means to implement SBMI, classrooms utilizing standards-based curricula with fidelity are more likely than non-NSF funded curricula to utilize SBMI (Moyer et al., 2018). In contrast, the official textbook for the middle school was Big Ideas Math (Larson & Boswell, 2010), a non-NSF funded curriculum. Big Ideas Math is a hybrid curriculum as described by Chavez et al. (2013). Each lesson occurs over two days. The first day is an extended student exploration. The second day provides direct instruction with algorithms,

definitions, and computation practice. Collaborative work during exploration is not routinely suggested in the teacher's manual. Teachers at AMS did not routinely implement lessons from the Big Ideas Math curriculum but rather drew from a myriad of resources. Prior to the adoption of Big Ideas Math in 2015, AMS implemented Connected Mathematics Project (Lappan, Fey, Fitzgerald, Friel, & Phillips, 2009), an NSF-funded middle school curriculum. The five elementary schools in the district implemented an NSF-funded mathematics curriculum. Implementation of the elementary school curriculum was not a focus of this research and therefore is not addressed. In this school district, students could experience discontinuities due to incongruent written curricula when transitioning from elementary school (NSF-funded) to middle school (non-NSF-funded) and then again from middle school to high school (NSF-funded). Entry into APS followed Institutional Review Board (see Appendix A for IRB approval), APS superintendent, AMS and AHS principal approval.

The setting for the classroom observations was four (4) classrooms, two seventh-grade classrooms and two ninth-grade classrooms. Each of these classrooms are described in detail in the participant section.

### **3.3 Participants**

All 7<sup>th</sup> through 10<sup>th</sup> grade students were invited to complete a beliefs survey and participate in a follow-up interview, see Table 3. Those students with continuous APS enrollment from seventh to ninth grade were included in the survey analysis. Eighteen (18) survey eligible students were selected for interviews. Four (4) teachers and their students were selected to participate in the classroom observation/discourse analysis portion of the study.

Table 3 Study Participants

	Ms. Luurtsema	Mr. McGinnis	Additional AMS students surveyed	Ms. Kraft	Ms. Chenette	Additional AHS students surveyed
School	Middle school	Middle school	Middle school	High school	High school	High School
Total Students Invited	20	19	374	16	17	441
Students Who Took Survey	13	19	248	16	14	60
Eligible Participants**	12	16	215	14	11	20
Interview Participants	2	2	8	2	2	2
Students who participated in small- group observations	4	6	N/A	4	5	N/A

\*\* Eligibility based on continuous enrollment

### 3.4 Data Collection

Determining students’ beliefs and attitudes about mathematics is complex and requires several different forms of data. In addition, examining student discourse during small-group inquiry requires a different form of data. Therefore, a mixed-methods case study approach with a concurrent strategy to collect and analyze gathered data was employed. Utilizing a mixed-method design permitted equal emphasis on both the quantitative and the qualitative data, allowing for: (a) the triangulation of the information collected during this research study; (b) the examination of overlapping and different facets of beliefs and identities; and (c) the discovery of paradoxes or contradictions in the data (Greene, Caracelli, & Graham, 1989). Making inferences about students’ beliefs and attitudes based solely on survey data does not provide sufficient information as to the experiences impacting the formation of those beliefs (Muis et al., 2016). There are often differences in stated beliefs and observed behaviors (Leach et al., 2000), as well as lack of awareness of or ability to articulate the belief on the part of the student; however, these beliefs can be apparent in

observed behaviors (Buzeika, 2004). As a result, both quantitative and qualitative data were drawn together to generalize the results of the study. The relationship between research questions, data collection, and data analysis is shown in Table 4.

Table 4 Research Questions, Data Collection, and Data Analysis

Research Question	Data Collections	Data Analysis Method
RQ1: What mathematics-related beliefs do students hold in a school district with incongruent written curricula commitments in middle school and high school?	Survey	Inferential statistics -Kruskal-Wallis H Test/Mann-Whitney U Test (non-parametric testing) -school -gender -tracking -gender and school -gender and tracking -tracking and school -tracking, school, and gender
	Student Interviews	Thematic analysis of experiences and beliefs
RQ2: How do students participate in small-group discourse within various enacted curricular models, and how is this participation related to their mathematical beliefs?	Classroom observations	Coding of discourse according to quality of discourse taxonomy
		Comparison of discourse with results from survey  SBMI lesson analysis using lesson observation instrument

### 3.4.1 Survey

All students from the seventh grade to tenth grade mathematics classes were invited to participate in the on-line survey. Teachers of 10<sup>th</sup>-grade students not in a mixed 9<sup>th</sup>/10<sup>th</sup> grade class did not provide class time for students to participate in the survey. As a result, this limited the

number of 10<sup>th</sup> grade survey participants who had attended Autumn public schools since 7<sup>th</sup> grade to 12. As this number represented less than 10% of the 10<sup>th</sup> grade class, their responses were not used in the analysis of students' mathematics-related beliefs.

The student survey consisted of two segments, a mathematical beliefs section and a mathematics identity section. The mathematical beliefs section of the student survey was taken from two established mathematical beliefs surveys, the Conceptions of Mathematics Inventory (Grouws et al., 1996) and the Indiana Mathematics Belief Scale (Kloosterman & Stage, 1992). The mathematics identity portion of the survey was taken from the NSF-funded Factors Influencing College Success in Mathematics Project (Cass et al., 2011). The survey asked participants to rate their agreement with 23 statements about mathematics epistemological beliefs on a 6-point Likert scale from "Strongly disagree" to "Strongly agree" (see Appendix B for complete questionnaire). Fifteen items were worded to represent non-availing beliefs. Participants were asked to rate their affinity to 13 mathematics identity statements on a 6-point Likert scale from "Not at all me" to "Exactly me." The survey was administered electronically, and statements were randomized to minimize the effect of survey response fatigue (Egleston, Miller, & Meropol, 2011). Statements presented in non-availing terms, such as "In mathematics, the teacher has the answer and it is the student's job to figure it out," were reverse scored prior to the statistical analysis.

Before using the survey information, the survey data was visually inspected for patterns representing possible non-accurate responses. If a student merely responded, "strongly disagree", "disagree", "slightly disagree", "slightly agree", "agree", or "strongly agree" to all questions, the average response before reverse coding was 3.39, 3.61, 3.08, 3.92, 3.36, and 3.64 respectively. Those students whose average response was one of these six were visually inspected to look for such patterns of responses and removed from the data set prior to analysis. This resulted in the

removal of five middle school respondents – four male students, one female student, three seventh graders, and two eighth graders – and one male ninth-grade respondent. Fifty (50) of 9321 survey response items, 0.005%, of middle school responses and 11 of 1911, 0.005%, of high school responses were missing. Given the completeness of the data set, missing data was imputed with the median for that survey item. All students had at least an 80% response rate and were retained in the data set.

The scales used in both the CMI and the IMBS suffer from low Cronbach's alpha, indicating low reliability (Schommer et al., 2005; Star & Hoffman, 2005). Given an adequate sample size, a confirmatory factor analysis was conducted to verify the existence of two latent variables with good model fit – fixed ability and the nature of learning mathematics-- as suggested by the work of Schommer and associates (2005). Question 1 and question 8 were removed from the model due to extremely low loading onto any latent variable. The decision was made to create two latent variables, “fixed-ability/quick-learning” and “nature of mathematics/conceptual understanding”, each being the mean of the indicators for that scale. Fixed-ability/quick-learning variable [FA] represents students' beliefs about the speed of learning mathematics and whether the ability to learn mathematics is a fixed trait. This variable had a Cronbach alpha of .699. The nature of mathematics/conceptual understanding variable [NOLM] represents students' beliefs about the locus of authority and knowledge in the classroom as well as the importance of conceptual understanding and justifying one's thinking. This variable had a Cronbach alpha of .708. Thus, the two factors, FA and NOLM, that emerged from the new survey had acceptable inter-item correlation.

The mathematics identity portion of the student survey was taken directly from the NSF-funded research project Factors Influencing College Success in Mathematics [FICSM] (Cass et al.,

2011). The three subconstructs of mathematics identity, mathematics self-efficacy, and mathematics interest from FICSM were equal weight averaged to create a composite mathematics identity. This decision was based on the literature by Cass et al. (2011) indicating that all three subconstructs equally contributed to a student's overall mathematics identity. Confirmatory factor analysis was not conducted for these constructs due to prior research confirming model validity (Cass et al., 2011). Student's composite mathematics identity [CID] was coded as either negative, neutral, or positive as seen in Table 5.

Table 5 Student Composite Mathematics Identity Ratings

CID	Negative	Neutral	Positive
Survey Score	1.0-2.8	2.8-4.2	4.2-6

### 3.4.2 Interviews

To effectively research the mathematical beliefs and identity formation of secondary students experiencing various instructional models, in-depth interviewing to encourage students to recreate their experiences and to elaborate on their survey responses through focused, yet open-ended questions was employed. All survey participants were invited to take part in the mathematics identity interviews. Mathematics identity was determined by students' survey responses. Students with a Composite Mathematics Identity [CID] below 2.8 were considered to have a negative mathematics identity. Students with a CID between 2.8 and 4.2 were considered to have a neutral mathematics identity. Students with a CID above 4.2 were considered to have a positive mathematics identity. Students were stratified based on gender, grade, and mathematics identity, and randomly selected from their stratum. For the interview stratification, the decision to not create strata for non-binary or other was due to having only one respondent with that gender classification at each school volunteering for an interview. The use of a stratified purposeful sample is intended to

capture major variations among the population as well as to identify similarities and differences in beliefs and identities (Merriam & Tisdell, 2016). The ultimate selection of participants was based upon their (1) composite mathematics identity score, (2) availability during the timeframe required for this study, and (3) willingness to be interviewed for this study.

Seventy-six (76) middle school students and 37 high school students volunteered to participate in an interview. There were no volunteers in the negative mathematics identity stratum for seventh-grade male. The researcher enlisted the assistance of a classroom teacher to encourage a student who fit the negative mathematics identity stratum for seventh-grade male to participate in the interview. Interviews were conducted before, during, and after school during the months of May and June. The interview protocol may be found in Appendix C. Interviews, each lasting about 20 minutes, were audio recorded and transcribed. All names have been replaced with pseudonyms. Analyses from the transcribed interviews are presented in Chapter 4.

A semi-structured interview guide was prepared and utilized, based on the literature review and the sociocultural theoretical framework for the study. This was to ensure that similar questions were asked of each participant (Merriam & Tisdell, 2016). Open-ended questions permitted students an opportunity to share their experiences in mathematics and their thoughts about the nature of mathematics. The goal of the interviews was to develop a better understanding of factors influencing students' beliefs and identities. The protocol for student interviews involved first questioning students regarding their general beliefs about mathematics knowledge and learning. Then students were asked about their responses to specific questions from the student survey, and their reason for selecting such a response. Students were asked to provide specific examples of experiences that impacted the response. In this manner, the students' responses were contextualized within the question and their experiences (Muis et al., 2016). Additionally, students were asked to

compare their experiences in mathematics classes over the course of their formal schooling. All interviews were recorded using a digital recorder and transcribed verbatim. All names have been replaced with pseudonyms.

The unit of analysis of interview data was students' responses to each interview question. A constant comparison method of analysis was employed to categorize, code, and delineate and connect categories (Tesch, 1990). For each interview question, student responses were initially categorized. Then the different categories were sorted into larger groups with corresponding themes. Finally, the themes were sorted into positive, negative, and neutral experiences, or availing versus non-availing beliefs, within each interview question. For example, initial coding for responses about why someone is good in mathematics resulted in categories such as math is fun, math is enjoyable, and math is interesting. These categories were condensed into one group of "student enjoys math." This category was labeled as availing. The other category for this question came from initial coding such as math comes easily, math makes sense, natural ability, and solves problems easily. These categories were condensed into one group of "student is good at math." This category was labeled as non-availing. Meaningful commonalities and differences in responses and experiences are reported. When beneficial, direct quotations from students are presented in the results.

### **3.4.3 Classroom Observations**

Lessons in four focal classrooms (see Table 6), two seventh-grade classrooms and two high school IMP2 classrooms – one regular level and one honors level for each grade – were observed over the course of six weeks from April to June. The IMP2 honors class was comprised exclusively of ninth-grade students. The IMP2 college prep class contained mostly ninth-grade students, with

four 10<sup>th</sup>-grade students. Both classes will herein be referred to as ninth-grade classrooms. Each classroom was observed twice a week for two weeks, and upon reaching saturation (Fusch & Ness, 2015) was then observed once a week for four more weeks, for a total of eight observations per classroom. As this observations period fell during and after the state standardized testing period for mathematics and science at both schools, the length of lessons varied from 40 minutes to 55 minutes. The decision was made by the researcher to ensure lessons were not observed on a day that students were testing nor on field trip days, as disrupted schedules might have impacted how students participated during mathematics class.

Table 6 Focal Classrooms

Teacher	Grade	Tracking
Ms. Luurtsema	7 <sup>th</sup>	Non-honors
Mr. McGinnis	7 <sup>th</sup>	Honors
Ms. Kraft	9 <sup>th</sup>	Non-honors
Ms. Chenette	9 <sup>th</sup>	Honors

The teachers were selected based on the recommendations of the mathematics department chair in their respective schools. Each teacher had more than 10 years of teaching experience and at least three years teaching the current curriculum. Prior to enrollment in this study, all teachers were observed on three different occasions to ascertain that their enacted curriculum was aligned to their written curriculum. The mathematics section selected to be observed was based upon teacher recommendation and scheduling.

The Science and Mathematics Program Improvement [SAMPI] (Jenness & Barley, 2003) instrument was used to document characteristics of lessons across each of the eight observations per teacher (see Appendix D for complete instrument). The lesson observation instrument, which has undergone validity and reliability studies, was designed to measure classroom culture as well as the

extent to which mathematics lessons attend to SBMI (Jenness & Barley, 2003) . Prior to each observation, a brief interview with the teacher was conducted to situate the lesson within the larger unit. Field notes were taken during the observations. After each observed lesson, a brief interview with the teacher was conducted to ascertain changes to the intended lesson, the extent to which the lesson was completed, and the extent to which students understood the concept of the lesson. A lesson observation form was completed after each lesson. This information was used to create the classroom vignettes for each of the four teachers. The average holistic lesson rating was used to determine the extent to which lessons were representative of SBMI. The rating scale was from 1, not at all reflective of a standards-based inquiry-oriented lesson., to 7, an excellent example of a high-quality standards-based inquiry-oriented lesson. Lessons with an overall rating between 3 and 5 were of average-quality SBMI.

#### **3.4.4 Small-Group Discourse**

For the data collection of small-group discourse observation, criterion sampling was utilized to select participants who matched the criteria for this study. For cases that require specific important criteria, criterion sampling is the best method for population sampling (Creswell & Clark, 2006). In this study, each of the participants needed to possess the following characteristics: (a) have participated in the student survey, (b) be in the seventh to tenth grade, (c) be currently taking a mathematics class, and (d) have had continuous enrollment in the APS system since seventh grade.

As one focus of this case study on incongruent written curriculum commitments across grade bands was through examining small-group discourse, recruiting seventh-to tenth-grade students who had continuous enrollment in the APS system since seventh grade fulfilled the requirements of this study. However, due to the lack of volunteers in one classroom to participate in

this portion of the study, one student (ninth-grade female) who volunteered but was new to the district was included in the discourse observation. This participant, who transferred to Autumn High School at the beginning of ninth grade, described a classroom environment in her previous school in California as a traditional classroom, with 40 students seated in rows working on worksheets after copying teachers' notes on specific algorithms. There was little to no student-student interaction nor work with manipulatives in her middle school experience. Her survey responses indicated a negative mathematics identity. The decision to include this participant was made based on the student's lack of experience with SBMI in middle school. A total of 21 students participated in the small-group discourse observations from four different classrooms. The 21 observation participants will be introduced in the analysis of the small-group discourse section of Chapter 5.

Discourse data were collected during classroom observations using video cameras. In one classroom, discourse data were collected using audio recording due to lack of student assent for video recording. Two different groups in each class were recorded during collaborative inquiry. The use of recording allowed for the capturing of collaborative inquiry of multiple groups per class. Video recording of the small-group inquiry enabled the researcher to document when groups huddled together over a task, as well as when students directed the group's attention to a particular aspect of the task.

Classroom observations of discourse were conducted using the lesson observation protocol developed by Weaver and colleagues (2005) and found in Appendix E. As multiple groups were studied, the groups were observed both in real-time and from the recordings. Two groups from each classroom volunteered to be part of the study. The discourse among the members of the selected groups was observed for the entire lesson, although the analysis of the discourse was limited to the small-group discourse. Only the verbal discourse of the group was analyzed. The time during which

students worked in small groups to solve problems was broken into five (5) minute episodes, or shorter if the whole episode lasted less than five minutes. Each episode was analyzed and coded as high-quality, low-quality, or mixed-quality discourse. A further discussion of the quality of discourse occurs in the Analysis of Data section below.

### **3.5 The Written Curriculum**

Autumn High School had utilized the Interactive Mathematics Program [IMP] (Fendel et al., 2015) for over 15 years. Prior to 2015, students had a choice of following a traditional mathematics pathway using a traditional curriculum or the IMP pathway. In 2015, AHS decided to discontinue the dual pathways, and as such, for the past four years, all students have followed the IMP pathway. IMP is an NSF-funded four-year, problem-based mathematics textbook that was published after more than a decade of research, pilot tests, field tests, revisions, and evaluations (Fraser, 2007). IMP focuses on supporting abstract understanding by embedding the mathematics into concrete experiences with a unit design focused on a main problem (Fraser, 2007). The IMP curriculum is designed such that students have opportunities to communicate their conjectures and explain their reasoning both orally and in writing (Fraser, 2007). Students are afforded the opportunity to actively engage with complex and realistic situations in a collaborative manner to reach deeper levels of understanding (Fraser, 2007).

Autumn Middle School adopted the use of Big Ideas Math (Larson & Boswell, 2010) in 2015. As a hybrid curriculum, Big Ideas Math is a more traditional curriculum than IMP. Each 2-day lesson begins with a guided discovery exploration encouraging conceptual understanding and is followed by a scaffolded lesson to develop procedural fluency (Larson & Boswell, 2010). Teachers at AMS did not routinely utilize the textbook or activities from the text, rather they gathered

resources from the internet or other curriculum. Prior to the adoption of Big Ideas Math, AMS used Connected Mathematics Project (Lappan et al., 2009), an NSF-funded curriculum. During this observation period, one seventh-grade classroom teacher (Mr. M) implemented lessons that reflected a traditional format of notes and worked examples followed Mathematics program (Dietiker et al., 2013), which was the first exposure these students had had to inquiry-based collaborative learning. The other seventh-grade classroom teacher (Ms. L) implemented lessons derived from Connected Mathematics Project (Lappan et al., 2009) as well as other activities gathered from the internet.

There was an incongruity between adopted written curriculum philosophies at the two schools with the high school implementing an NSF-funded curriculum and the middle school not. Additionally, there was an incongruity within the seventh-grade classrooms with teachers not adhering to a consistent and uniform curriculum.

### **3.6 Data Analysis**

To answer the research questions, coordination of multiple data sources and analyses was required. First, quantitative analysis of the survey responses was required. Secondly, qualitative analysis of the discourse transcriptions, interview responses, and field notes occurred. Making the connection between students' beliefs and identities and their discourse required both types of data collection and analyses.

#### **3.6.1 Survey Responses**

Thirty-six survey items were clustered to represent two constructs of epistemological beliefs and three constructs of mathematics identity, which were averaged to represent a composite

mathematics identity. SPSS, a quantitative software data analysis program, was used to conduct descriptive and inferential statistical analyses of the quantitative data.

The dependent variables for this study were the scaled factors created from the factor analysis of the survey results: composite mathematics identity [CID], fixed ability/fast learning [FA], and the nature of mathematics/conceptual understanding [NOLM]. Scaled factors were treated as interval variables (UCLA: Statistical Consulting Group, n.d). The dependent variables were inspected to determine if the assumptions were met for two-sample t-test analysis: random sampling, independence of observations, and sample size greater than 40 with no outliers. These conditions were met and thus, differences in beliefs by gender, grade, or tracking were analyzed used a two-sample t-test. The variable of gender and tracking was inspected to determine if the assumptions were met for ANOVA analysis: normality, homogeneity of variance, and independence of observations. Two latent variables – FA and NOLM -- as well as the three subconstructs of CID – mathematics identity [MID], mathematics self-efficacy [MSE], and mathematics interest [MINT] – did not meet the assumptions for factorial ANOVA. Levene’s test determined the data to be heteroscedastic. Additionally, the individual survey statement results were heteroscedastic. While ANOVA can be used with this violation to analyze the data when sample sizes are equal, the sample sizes for gender and tracking were not equal. Thus, the gender-tracking data was analyzed utilizing nonparametric testing, specifically a Kruskal-Wallis H test [KWH] to compare mean ranks with Dunn-Bonferroni post hoc tests for between group differences. The assumptions that need to be met to use a Kruskal-Wallis H test are (1) the dependent variable is measured at the ordinal or continuous level, including Likert scales; (2) the independent variables consist of two or more categorical, independent groups; and (3) independence of observations. The survey data met these assumptions. The distribution of the dependent variable by each group within

the independent variable was checked to determine whether the medians or the mean ranks of the data were to be compared (Kruskal-Wallis H Test using SPSS Statistics, 2018). Given that the distribution of data of each group had different shapes and variabilities, the mean ranks of the groups were compared. Significance was set with a  $p$  value less than 0.05 to reject the null hypothesis.

Each scaled factor and the individual survey questions were analyzed to determine if there were differences in students' beliefs either by gender, high school versus middle school, and/or honors versus non-honors. The null hypothesis in each case was that the mean ranks of the responses were the same across the categories of the independent variable. The alternative hypothesis was that the mean ranks of the responses were higher for high school students than for middle school students, for honors students than non-honors students, and for male students than female students. Twenty (20) students were not included in the gender analyses due to missing data for gender. Eight (8) students, who identified as neither male nor female, from across all grades were not included in the gender analyses due to the small number for that sample size but were included in analyses not including gender.

This research study included several categorical independent variables collected from the student survey. These categorical variables include the following:

- a. Gender: Recorded students' gender as male, female, or other. Initially students could choose from transgender, non-binary, or other but was condensed to other due to the small grouping size (N=8);
- b. Grade level in school;
- c. Enrollment: Measures if the student has been enrolled continuously in APS since seventh grade (binary); and

d. Honors course: Recorded student's enrollment in an honors or non-honors mathematics course (binary).

### **3.6.2 Interview Responses**

When student interviews were completed, verbatim transcripts of each student's responses were created. This formed the qualitative data set for research question 1. After transcribing the groups' discourse, the daily level of discourse was rated as either low, mixed, or high. The transcripts and the coding were analyzed for emerging themes and patterns pertaining to student beliefs and identity formation (Creswell & Clark, 2006).

For the first coding round, key words/phrases were highlighted, and open coding was used by placing notations next to portions of the transcript that might be useful in answering the research questions to see what categories or themes emerged. Once initial categories were formed, the categories were named and organized in a flow chart. Codes included portions of the theoretical framework, the research questions, and notable quotations.

Generative themes were developed from within each code, with the theoretical framework used to guide the data analysis. Drawing from Merriam and Tisdell (2016), analysis was guided by the following questions:

- a. What themes arise within each code?
- b. Are there outliers?
- c. How does this address the research questions?
- d. How will this finding be triangulated with survey data?
- e. What do I do with this information?

### 3.6.3 Lesson Observations

As a part of the data collection, direct observations of mathematics classes for the four focal cases and the participating students were conducted twice a week for two weeks and then once a week for four additional weeks. The notes from each of the observations, in conjunction with the transcripts, were used to measure the extent to which the lesson was representative of SBMI. The SAMPI lesson observation instrument was completed immediately after each lesson, with each lesson scored to provide an overall lesson rating and to create a classroom portrait. Additionally, each lesson was rated for implementation, content, and classroom culture. Means for each of the four major categories was calculated to provide an overall SBMI rating for each classroom.

### 3.6.4 Discourse Analysis

Students' small-group discourse was audio or video recorded. When the classroom observations were completed, verbatim transcripts of each group's small-group verbal discourse were coded according to the discourse taxonomy developed by Weaver and associates (2005). Each 5-minute interval of small-group discourse was coded on a scale of 0-3, as seen in Table 7. Then each group was assigned a total daily score for discourse level, a holistic score as opposed to the absolute mean of each interval. The discourse data was analyzed both on the group level, to create a group portrait, as well as on the individual level, to make connections to the individual beliefs and identities.

Table 7 Discourse Participation Score Taxonomy

Daily Score	0	1	2	3
	Little to no participation in group's discourse	Discourse is mostly of low-quality	Discourse is a mix of low-quality and high-quality	Discourse is mostly of high-quality

### **3.7 Summary**

The purpose of this case study of within district incongruent written curriculum commitments across grade bands was to explore students' mathematics-related beliefs and discourse participation during collaborative inquiry. The mixed-methods case study methodology suited this investigation by framing which data were necessary for providing closed-ended responses as well as open-ended stories and meanings, as well as by facilitating an understanding of the perspectives and beliefs of the participants. Surveys, interviews, and direct observations were used to collect data. Students' mathematics-related beliefs were investigated using surveys, interviews, and observations. Through direct observation of student participation in collaborative inquiry, the students' discourse was coded and analyzed. With these pieces of information, a description of students' MRB and small-group discourse was created.

Chapters 4 and 5 present the details and summary of the data analysis with a focus on the research questions. Survey responses are analyzed using both descriptive and inferential statistics. Portions of the interview transcripts and classroom discourse transcripts are used to demonstrate the participants' reflections and the level of their discourse participation in collaborative inquiry. Classroom observations are used to create portraits of the enacted curriculum and to frame the extent to which the enacted curriculum is representative of SBMI. Finally, Chapter 6 synthesizes the results to answer the research questions and discusses the findings in relation to the literature, along with implications of this study, limitations, and suggestions for further research.

## CHAPTER 4

### THE ENACTED CURRICULUM AND MATHEMATICS RELATED BELIEFS

This chapter provides an analysis of the enacted curriculum in the focus classrooms and of students' mathematical beliefs data as described in Chapter 3. The purpose of the study was to explore the relationship between students' mathematics-related beliefs, discourse participation during collaborative inquiry, and the enacted curriculum. Field notes and the lesson observation SAMPI instrument were used to determine the extent to which lessons were representative of SBMI. Two data sources were used to determine the mathematics-related beliefs held by middle and high school students in the Autumn School District, student surveys and interviews. The chapter begins with an introduction to the four focus classrooms, the observed enacted curriculum in each of those classrooms and then survey and interview results. The survey section is organized by each of the six latent variables: FA, NOLM, CID, MID, MSE, and MINT.

#### 4.1 The Enacted Curriculum

The enacted curriculum, the interaction of teacher and students with each other and the content, was the portion of the *Instructional Triangle* environment analyzed in this study. This section begins with an introduction to the four focus classrooms. It is followed by a comparison of the classrooms along overall and individual lesson observation instrument indicators. Excerpts from field notes are used to support indicator ratings. Additionally, themes that emerged from field notes are presented. Teacher-student interactions were analyzed using lenses of teacher-group interactions, scaffolding, and high expectations for all students. Teacher-content interactions were analyzed using lenses of inclusion of investigative mathematics tasks, instructional model, and

planning for substantive student-student interactions. Student-content interactions were analyzed from the perspective of student-student interactions enhanced learning/understanding of the topic and students working collaboratively.

It was anticipated that the enacted curriculum would mirror the adopted written curriculum as seen in Table 8. Analysis of the lessons in the focus classrooms indicated that there was not a clear correspondence between the enacted a and the written curriculum, which is discussed in following sections.

Table 8 Anticipated Comparison of Instructional Models and Written Curriculum

	Hybrid Written Curriculum	NSF-Funded Written Curriculum
Blended Direct Instruction & SBMI	XX	
SBMI		XX

#### 4.1.1 Introduction to the Focus Classrooms

Four classrooms, two 7<sup>th</sup>-grade classrooms and two 9<sup>th</sup>-grade classrooms, served as the setting for the classroom observation and small-group discourse analysis portion of this study. At each grade level, there was one honors classroom and one non-honors classroom. Table 9 provides a visual representation of this distribution of the four classrooms.

Table 9 Focus Classrooms

	Ms. Luurtsema	Mr. McGinnis	Ms. Kraft	Ms. Chenette
Non-honors	X		X	
Honors		X		X
7 <sup>th</sup> Grade	X	X		
9 <sup>th</sup> Grade			X	X

Each of the four focus classrooms were observed to determine the extent to which enacted lessons aligned to standards-based instructional practices. In each classroom two groups were observed eight times from late April to early June of the school year. Some of the observations (1-2 in each classroom) occurred prior to state testing, with the rest of the observations occurring after state testing.

#### **4.1.1.1 Introduction to Ms. Luurtsema's Classroom**

Ms. Luurtsema (Ms. L) has been teaching for a total of 12 years in the APS system, with the last six years at Autumn Middle School. She has had experience teaching with the prior NSF-funded curriculum as well as the current written curriculum. Resources for her lessons came from both types of curricula, as she was not satisfied with the resources of the current hybrid curriculum. Lessons in the geometry and probability units were observed for this study.

The 20 students in Ms. L's 7<sup>th</sup>-grade non-honors classroom chose to sit in groups of two to four per table. A visitor in Ms. L's classroom during this observation period would typically see students seated together working on an inquiry-based task followed by a teacher-led reflection. Students generally worked independently, checking with their groups as needed rather than working collaboratively on the task. The atmosphere in the classroom was light-hearted with students comfortable to socialize with each other and with Ms. L. Prior to the study, a visitor to the classroom would have seen the same students seated together practicing procedures that had been explicitly taught during direct instruction. Students generally worked independently, checking with their groups to verify answers. Then Ms. L would lead a whole class discussion to review answers.

Doing mathematics in Ms. L's classroom, prior to state testing, meant that students were provided with direct instruction and then provided opportunities to apply this knowledge working

on procedurally focused worksheets. While students were pushed for higher order thinking during whole class discussions, only three students routinely engaged in these higher order whole-class discussions with many of the other students off-task. Students frequently talked with each other while peers shared their results. Therefore, established classroom norms included waiting for further assistance from either Ms. L or through the whole class discussions when faced with non-routine problems. Also, they included not participating in whole-class discussions unless called upon by Ms. L. Ms. L was the locus of authority in the classroom. She would evaluate the veracity of a student's response and then restate it for the whole class to hear.

#### **4.1.1.2 Introduction to Mr. McGinnis's Classroom**

Mr. McGinnis (Mr. M) has taught at Autumn Middle School for 12 years after a mid-life career change. Students were arranged in groups of three decided by the teacher. Lessons in this 7<sup>th</sup>-grade honors classroom of 19 students generally began with a teacher-led period of direct instruction followed by a set of practice problems and a review of the answers. The pace of the work was quick as this was the "advanced" class. In multiple observations, Mr. McGinnis was heard to say, "I will give you 30 seconds to solve this problem and then Seymour will share his answer." Resources from his lessons come from various traditional textbooks. Students routinely worked independently, checking with table mates only to verify answers.

During the observation period, the transformations unit was drawn from the College Preparatory Mathematics program [CPM] (Dietiker et al., 2013). CPM is not an NSF-funded curriculum but is more investigatory in nature than traditional curricula. This was the first time that year that lessons from the CPM curriculum were used with the class. There was an established

classroom practice of independent work/inquiry without collaboration that carried over into the CPM lessons.

Established norms in this classroom included working quickly and independently. Speed and accuracy were rewarded with praise from Mr. M. Doing mathematics looked like a race with students trying to be the first to solve the problem. Doing mathematics did not include productive struggle and inventive algorithms. Discussions flowed through the teacher with the teacher being the locus of authority. Students' sharing of ideas was in response to a direct question to them from Mr. M. Mr. M told students exactly what formulas and rules they needed and demonstrated how to use this information to solve mathematical problems. Students took notes and then replicated the procedures with low cognitive demand tasks, mostly procedures without connections. Sociomathematical norms for students sharing work included explaining each of the steps but did not include providing justification or reasoning for the steps.

#### **4.1.1.3 Introduction to Ms. Kraft's Classroom**

Ms. Kraft (Ms. K) has been teaching for 15 years, the last three years at Autumn High School. The IMP curriculum was new to her, and this was the third year she has taught it. She taught IMP1 and IMP2 to ninth- and 10<sup>th</sup>-grade students, as well as AP Statistics to 12<sup>th</sup>-grade students. Ms. K's IMP2 classroom consisted of 16 ninth- and tenth-grade students arranged in groups of two to four of the students' choosing. Resources for the IMP2 lessons came mostly from the IMP program. There was little outside supplementation to the curriculum. Students generally worked collaboratively on open-ended tasks with their small group and then presented their findings during the closure of the lesson.

The observed lessons were from the Fireworks unit in IMP2 (Fendel et al., 2015), focusing on quadratic expressions, equations, and functions. In this unit, students worked on inquiry-based tasks, sometimes working cooperatively and sometimes working collaboratively. Other lessons were focused on procedural fluency, such as translating between different forms of the quadratic equation. For those lessons, students would work independently, checking in with their group for answers and assistance.

Although the unit and curriculum provided lessons that focused on investigative or hands-on activities, many of the observed lessons occurred on days where the written curriculum was more procedural in nature than typical. Additionally, some of the pedagogical choices made by Ms. K resulted in a lowering of the cognitive demand of the task and an implementation of the lesson that was below-average SBMI. Most lessons involved students moving between the various representations of quadratic equations, tabular to graphic to symbolic, as well as various symbolic forms. Pacing occasionally did not allow for adequate closure. Students were pushed for higher order thinking during whole class discussions. Many students volunteered to share their thinking.

During activities, students were on task and attended to the mathematics. Groups engaged in mostly cooperative work, with two groups observed to be working collaboratively. Students chose their grouping, ranging in size from two to four students. While this freedom of choice did not promote heterogeneous groupings, it did provide the opportunity for students to work with peers with whom they felt most comfortable. As there were no assigned roles within the groups, certain students consistently acted as the leader in their group and controlled the direction of the inquiry. Doing mathematics in Ms. K's class meant trying multiple pathways until the task was completed, making connections between multiple representations and prior knowledge, engaging in discussion with group members, and presenting findings to the class. Classroom norms included active

participation by all group members and the expectation that any one of the group members may be called to present the group's findings.

Observed lessons routinely began with a review of the previous night's homework or the previous day's classwork if there had been no closure. Groups then began the next investigation in the IMP2 unit, preceded by a launch, a brief introduction to the task by Ms. Kraft. Whole class discussions were limited to the introduction of the investigation and the synthesis of the lesson. Students' questions during whole class discussions were routinely turned back to the class to foster further discussion. Ms. K ensured that each question was answered to the student's satisfaction before moving on.

#### **4.1.1.4 Introduction to Ms. Chenette's Classroom**

Ms. Chenette (Ms. C) taught IMP1 and IMP2, ninth- and 10<sup>th</sup>-grade students. She has been teaching for 16 years in the APS system, with the last six years at the high school. The 17 students in this IMP2 honors classroom were arranged in groups of three that were randomly selected. All students in this class were 9<sup>th</sup> graders. Resources for lessons came from the IMP curriculum, with little outside supplementation to the curriculum. Students worked collaboratively on open-ended tasks and presented their findings during the closure of the lesson.

The observed lessons took place within the Geometry by Design unit from IMP2 (Fendel et al., 2015), focusing on Euclid's postulates and theorems. In this unit, students routinely worked collaboratively on inquiry-based tasks for a significant portion of the class, and then shared out their thinking. There were, however, a minority of lessons that focused on construction of triangles where students worked independently on their constructions and then discussed their thinking. Students were encouraged to share partial thinking, within their groups as well as during whole

class discussion, to stimulate solution trajectories. Questions by students were routinely turned back to the class or the group to solicit explanations.

Classroom norms included ensuring that everyone in the group understood the mathematics and could articulate the reasoning for the solution. All group members were expected to participate in the presentation of their group's work. Learning mathematics required persistence and trying different strategies.

Every lesson began with a "Get to Know You Question," so that students and the teacher could interact in non-mathematical ways and to build community. Observed lessons began with a "Get to Know You Question (GTKYQ)," where students discussed with their peers' questions such as "Is society better off now with all the available technology?" and "At what age do you think you would want to live alone?" The GTKYQ was followed by a short whole class discussion enabling peers and teachers to interact outside of the mathematics. The previous night's homework, or previous day's classwork if there had been no closure, was reviewed, with each group presenting a teacher-chosen section of the work. Groups then began the next investigation in the IMP2 unit. Whole class discussions were limited to the introduction of the investigation, the launch, and the synthesis of the lesson. Student questions were routinely turned back to the class to foster further discussion. Ms. Chenette ensured that the student's question was sufficiently answered to their satisfaction before moving on. Students presented their findings during the synthesis of the lesson.

#### **4.1.2 Overall Lesson Observation Results**

Overall, the observed lessons in the focus classrooms exemplified from below average to above average representation of SBM. The mean rating for each category from the lesson observation tool for each classroom is displayed in Table 10. The implementation, culture, and

overall lesson ratings were lower in the middle school classrooms than the high school classrooms. There was little difference in the ratings of the two middle school classrooms for these indicators. There was a larger difference in the ratings of the two high school classrooms across these same indicators.

Table 10 Mean Summary Findings of Focus Classrooms

Teacher	Grade	Honors/ Non- honors	Implementation	Content	Culture	Overall Rating
Ms. Luurtsema	7	NH	3.0	5.4	3.8	3.0
Mr. McGinnis	7	H	3.4	4.3	3.6	2.9
Ms. Kraft	9	NH	3.6	4.9	5.1	4.5
Ms. Chenette	9	H	5.3	5.5	6.1	5.6

#### 4.1.3 Planning and Organization of the Lesson

The planning and organization of the lesson pertains to the organization and structure of the lesson (Jenness & Barley, 2003). These pedagogical decisions can determine how students will engage with each other, the teacher, and the content.

##### 4.1.3.1 Substantive Student-Student Interaction

One hallmark of SBMI is student-student interaction. The *substantive student-student interaction* indicator measured whether a lesson encouraged student-student interaction rather than focusing only on individual or whole group work (Jenness & Barley, 2003). In lessons that support substantive student-student interactions, one would see students actively working together in small

groups or pairs to complete tasks, conversing about what they are learning, and the teacher raising questions and posing problems that encourage student-student interaction (Jenness & Barley, 2003).

The lessons at AHS encouraged substantive student-student interactions (see Table 11). All lessons in Ms. K's classroom, except for lesson 7, began with a launch followed by extensive time for students to work together on the assigned task. In lesson 7, Ms. K guided students through a rocket problem with which students struggled in completing the previous day. In Ms. C's classroom, all lessons were designed such that students spent most of the class time working in small groups, discussing the task.

At AMS, half of the observed lessons in Ms. L's and Mr. M's classrooms were designed to encourage student-student interaction. In Ms. L's classroom, four of the eight lessons encouraged substantive student-student interactions. In lesson 1, students worked independently on a worksheet. After each problem, Ms. L provided her algorithm and solution. Lesson 3 was structured as a whole class guided discussion about an area problem. Ms. L lead the discussion and ensured class understanding and completion of computations before moving to the next step. In lesson 6, students worked independently on creating a non-rectangular prism container to fit certain specifications. In lesson 7, Ms. L lead the class in games of "PIG" and "SKUNK" rather than having each group play the games.

In Mr. M's classroom, four of the eight lessons also encouraged substantive student-student interactions. Lesson 1 was structured as direct instruction and note taking for an introduction to transformations. In lesson 2, students completed a worksheet on inequalities independently. Lesson 3 was an inquiry task to develop generalizations about transformations that students completed independently. Lesson 8 found students working independently with a computer simulation and reading an article on probability.

Table 11 Planning for Substantive Student-Student Interactions

	Lesson 1	Lesson 2	Lesson 3	Lesson 4	Lesson 5	Lesson 6	Lesson 7	Lesson 8
Ms. L	No	Yes	No	Yes	Yes	No	No	Yes
Mr. M	No	No	No	Yes	Yes	Yes	Yes	No
Ms. K	Yes	Yes	Yes	Yes	Yes	Yes	No	Yes
Ms. C	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes

#### 4.1.3.2 Investigative Mathematics Tasks

Investigative mathematics tasks are another hallmark of SBMI. The *investigative mathematics task as an essential element of the lesson plan* indicator measured whether students needed to gather information and use it to understand a concept (Jeness & Barley, 2003). Students might need to pose questions and gather, record, analyze, and/or share information that helps to answer the task, identify characteristics of something, compare the task to related issues or situations, and/or apply what they learn to real-world situations (Jeness & Barley, 2003).

Investigative tasks were an essential component in 25 of the 32 observed lessons (see Table 12). Essential elements of the lessons in Ms. C’s classroom included investigative tasks in all observed lessons except for lesson 2. Students discussed with their small groups whether geometric statements were true or false. In Ms. K’s classroom, three lessons were not investigative in nature, but rather focused on procedural fluency in moving between various forms of quadratic equations. All four of these lessons at AHS were implemented as small-group activities as suggested by the written curriculum.

At AMS, the investigative tasks were not part of the adopted written curriculum. In Ms. L’s classroom, all lessons were investigative except lesson 1. In that lesson, students completed worksheets individually. In Mr. M’s classroom, lessons 1 and 2 were non-investigative, where students completed worksheets individually. Three-quarters of lesson 3 included Mr. M discussing

definitions of transformations and reviewing procedures on determining equations for translations and reflections. None of these four non-investigatory lessons were from Big Ideas. The worksheets were department developed worksheets. All the investigatory lessons in Mr. M’s classroom came from CPM, but for lesson 8. The investigatory lessons in Ms. L’s classroom were based on CMP activities or other activities found on the internet.

Table 12 Were Investigative Tasks an Essential Component of the Lesson

	Lesson 1	Lesson 2	Lesson 3	Lesson 4	Lesson 5	Lesson 6	Lesson 7	Lesson 8
Ms. L	No	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Mr. M	No	No	No	Yes	Yes	Yes	Yes	Yes
Ms. K	Yes	Yes	Yes	No	Yes	No	Yes	Yes
Ms. C	Yes	No	Yes	Yes	Yes	Yes	Yes	Yes

#### 4.1.4 Implementation of the Lesson

Implementation is concerned with how the teacher carries out the lesson (Jenness & Barley, 2003). Well implemented lessons are likely to enhance student learning through effective and engaging interactions throughout the lesson. Questioning and dialog that occurs during student-teacher interactions emphasize higher-order thinking and connections to prior knowledge. Classroom expectations are clear, with student-student interactions focusing on lesson content. Pacing of lessons allow for adequate time for reflection on and closure of the lesson.

Implementation was a relative weakness for observed lessons for Ms. L, Mr. M, and Ms. K (See Table 10), Their implementation of lessons was below-average SBMI while Ms. C’s implementation of lessons was above-average SBMI. This difference between Ms. C’s lessons and the other lessons was significantly different (Ms. L:  $p = 0.001$ ; Mr. M:  $p = 0.008$ ; Ms. K:  $p = 0.028$ ;  $\eta^2 = 0.431$ ). Two indicators within implementation were of particular interest: “*Periods of student-student interaction were focused on pertinent lesson content and enhanced individual*

*understanding of it,” and “The teacher communicates high expectations for all students, challenging all students to engage in problem solving, questioning, and the generation of knowledge.”*

#### **4.1.4.1 Student-Student Interactions**

In a classroom that is SBMI aligned, small group work is organized so there is appropriate and substantive student interactions that lead to improved understanding of concepts (Jenness & Barley, 2003). Students would be working together to solve a problem, sharing their learning, and supporting each other, with no one student dominating the group. The difference between this indicator and the one in the earlier section is that this indicator measures the degree to which students are engaged in collaborative problem solving and the previous indicator measure whether the lesson was planned for collaborative problem solving. The rating of this indicator for each lesson is provided in Table 13.

In both 7<sup>th</sup>-grade classrooms, the average rating on this indicator was below average (Ms. L:  $\bar{x} = 2.8$ ; Mr. M:  $\bar{x} = 3.3$ ) indicating that the student-student interactions were not very productive. In both 9<sup>th</sup>-grade classrooms, the average rating on this indicator was above average (Ms. K:  $\bar{x} = 4.4$ ; Ms. C:  $\bar{x} = 5.8$ ) indicating that the student-student interactions were productive. The difference between both 7<sup>th</sup>-grade classrooms and Ms. C was statistically significant (Ms. L:  $p < 0.001$ ; Mr. M:  $p = 0.002$ ;  $\eta^2 = 0.501$ ). At AMS, even though students were seated in groups and were expected to work cooperatively, in general, their interactions did not lead to improved understanding of concepts. Collaborative work was a new expectation in both 7<sup>th</sup>-grade classrooms. Prior to state testing, students were expected to work independently. In contrast, students in both 9<sup>th</sup>-grade classrooms had worked collaboratively since the beginning of the school year. Classroom norms

had been established for how groups interact and their responsibility for the understanding of each group member.

Table 13 Productive Student-Student Interactions

	Lesson 1	Lesson 2	Lesson 3	Lesson 4	Lesson 5	Lesson 6	Lesson 7	Lesson 8	Mean
Ms. L	2	3	2	3	6	2	1	3	2.8
Mr. M	2	2	3	4	3	4	6	2	3.3
Ms. K	4	3	5	4	3	5	6	5	4.4
Ms. C	6	4	5	6	6	7	6	6	5.8

#### 4.1.4.2 High Expectation for All Students

The role of the teacher in SBMI is to orchestrate discussions to make mathematical ideas available to all students (Munter et al., 2015). In doing so, a variety of students are asked to participate with a range of avenues for participation. Students are asked to rethink ideas, challenge the teacher and each other, and to lead the class and/or groups. All students are challenged to engage in problem solving and the generation of knowledge. This indicator rates the extent to which the teacher asks students to participate in the lesson in these ways. The rating for each lesson is provided in Table 14.

Table 14 High Expectations Ratings

	Lesson 1	Lesson 2	Lesson 3	Lesson 4	Lesson 5	Lesson 6	Lesson 7	Lesson 8	Mean
Ms. L	2	3	3	3	3	3	3	3	2.9
Mr. M	2	3	3	5	4	5	4	3	3.6
Ms. K	4	4	5	4	4	5	4	4	4.3
Ms. C	6	5	5	6	7	6	7	6	6.0

Observed lessons in Ms. L's classroom did not indicate that there were high expectations for all students ( $\bar{x} = 2.9$ ). During whole class discussions, only the students who volunteered were called on to respond. Groups were not asked to lead the class, explaining their findings. Instead, groups reported to Ms. L, and she "translated" their thinking for the whole class. Ms. L accepts answers only from many students but required explanations with answers for just a few students. During small-group work time, three groups consistently did not attend to their work and waited for whole class discussions.

Lessons in Mr. M's classroom were rated just below average ( $\bar{x} = 3.6$ ) for the teacher communicating high expectations for all students. In the four lessons that were designed to provide substantive student-student interactions, the ratings for this indicator were average (4's) and above-average (5's). These were the lessons that were from CPM and contained investigative tasks. In these four lessons, Mr. M asked for students to share which transformations they used to move the pre-image to the image. At times Mr. M would limit student discussion and funnel their thinking towards his transformation. In the four lessons that were not designed to provide substantive student-student interactions, the ratings for this indicator were below average (2's and 3's). In these lessons, Mr. M determined which student would answer the question before asking all students to work on the question. Tasks were of low cognitive demand, providing definitions or procedures without connections. Students were not asked to justify their reasoning, and alternate solution pathways generated by students was not encouraged.

Observed lessons in Ms. K's classroom were rated slightly above average ( $\bar{x} = 4.3$ ). All small groups, except for one, were routinely on task. Groups were expected to present their findings. Their findings were not filter through Ms. K, but rather presented directly to their peers. Ms. K would position herself towards the back of the classroom. Ms. K pushed students for higher

order thinking during whole class discussions, asking them to provide reasoning for their responses. Ms. K often responded to student provided solutions with, “Did any group approach it differently?” During teacher lead discussions, many students volunteered to share their thinking, although not all. Lessons were not rated higher than five because students were not encouraged to push beyond their initial answers to dig deeper into questions and possibilities.

In Ms. C’s classroom, observed lessons were rated as above average ( $\bar{x} = 6.0$ ). Students were pushed for higher order thinking during whole class discussions. Many students volunteered to share their thinking during these discussions. During the activities, students were on task and attended to the mathematics. Questions directed to Ms. C were turned back to their peers for answering. During group presentations, all group members were expected to participate in some fashion. Students were encouraged to create alternate pathways for proofs or to work together to challenge conflicting proofs. On challenging proofs, Ms. C often asked the group to explain what they had done so far and then would ask one student to try to convince the others on the validity of their reasoning.

#### **4.1.5 Culture of the Classroom**

The culture of the classroom is concerned with the classroom climate, the level of engagement of students in activities, and the nature of the working relationships among students and between students and the teacher (Jenness & Barley, 2003). Classrooms cultures that respect and value students’ ideas, questions, and contributions and that encourage students to generate ideas and conjectures allow students to take risks in their thinking. When teachers and students have strong collaborative relationships, teachers facilitate and not dominate the discourse and activities. The two high school classrooms rated higher for average overall classroom culture than the two

middle school classrooms (see Table 10). The classroom culture in the two observed 9<sup>th</sup> grade classrooms supported students in risk-taking, encouraging students to generate ideas and make conjectures even if they were not confident that their statements were correct. There were two indicators of interest where both 7<sup>th</sup> grade classrooms rated below average and both 9<sup>th</sup> grade classrooms rated above average. The difference in each indicator is discussed in the following sections.

#### **4.1.5.1 Students Showed Respect for and Valued Each Other's Ideas**

To ensure that all students have equitable access to mathematics, all students need to feel safe from ridicule when discussing their ideas or asking questions. In classrooms with high level of student-student respect, students accept each other's ideas without ridicule or trying to impose their own ideas (Jenness & Barley, 2003). Students feel comfortable asking questions. They discuss alternate ideas and challenge ideas with counter arguments. For three of the classrooms, the average rating for this indicator was above average. This suggests that the teacher has taken the time to set classroom norms that are inclusive of all mathematical participation. No ideas are dismissed without examining the reasoning behind the response.

However, the average rating in Ms. L's classroom was below average ( $\bar{x} = 3.0$ ). During whole class discussions, for example, when a student gave an obviously wrong answer, some of their peers twittered. Groups that did not understand the task sat quietly until Ms. L circulated around to them. They did not actively seek out her help. Students did not self-advocate when the whole class discussion moved quickly, and they were confused. This was evident when Ms. L would ask if there were any questions and there would be no student questions. Then Ms. L would

check for understanding by asking a specific student to restate or explain the problem and they could not.

#### 4.1.5.2 Collaborative Student-Student Relationships

When working on an investigative task, students working collaboratively coordinate their efforts, work as a team, solicit ideas from each other, and share responsibilities (Jenness & Barley, 2003). All students have a role in the group and no one student dominates the group. Students in the two 7<sup>th</sup> grade classrooms showed limited collaborative relationships and below-average collaborative relationships, and students in the two 9<sup>th</sup>-grade classrooms showed above-average collaborative relationships (see Table 15). The difference in collaborative student-student relationships rating between both 7<sup>th</sup> grade classrooms and both 9<sup>th</sup> grade classrooms was significant with a large effect size ( $p < 0.001$ ,  $\eta^2 = 0.770$ ).

Table 15 Extent of Collaborative Student-Student Relationships

	Lesson 1	Lesson 2	Lesson 3	Lesson 4	Lesson 5	Lesson 6	Lesson 7	Lesson 8	Mean
Ms. L	1	2	1	2	3	2	1	2	1.8
Mr. M	1	1	2	3	3	3	4	1	2.3
Ms. K	4	5	5	5	4	5	4	4	4.5
Ms. C	6	4	5	6	6	6	7	4	5.5

In the observed lessons in Ms. L’s classroom, students did not work collaboratively. Students were often seen working individually on the task despite urging from Ms. L to work together. There was no observed discussion about small-group work norms. Group responsibility was not evident as some group members completed their task while other group members did not. There were observed instances where one student explained to another student what the task was. Additionally, there were instances where one group member knew how to solve the task and

showed the other group members the procedure. In this case, that one group member dominated the group.

In Mr. M's classroom, students struggled to work collaboratively in the lessons that were designed for student-student interaction. On the first day of the new transformation CPM unit, students were told, "Take 30 to 45 seconds to write down your understanding of the words on this page." Mr. M then provided the class with the preferred definition of each rigid transformation. Then students were told, "I will give you 2 minutes to silently work on problems 1 and 2. Don't talk to your neighbors yet." Rather than being given time to talk to their small group, Mr. M made the decision to begin a whole class "we do" of transformations 1 and 2. With 10 minutes remaining in class, students were told, "I want you to work on 3 and 4 with your group. You can work independently but I want you to check and confirm with your group." This was the first encounter students had with CPM and collaborative learning. They were given explicit permission to work independently rather than collaboratively.

During the next observation, where students were moving from preimage to image on GeoGebra, see Figure 3, students were not directed to work with their groups. As part of the directions, students were told, "If your team, partner, individual gets two solutions raise your hand and I will come check." This statement gave students permission to continue to work independently. Some students worked independently, and other groups worked collaboratively on this task. Only in observation 7 were students directed to work collaboratively and were provided only one set of manipulatives for the group. Small-group work norms and responsibilities were not addressed. Not setting norms and responsibilities, along with mixed messages about working independently, may have impeded the formation of a cohesive collaborative working group.

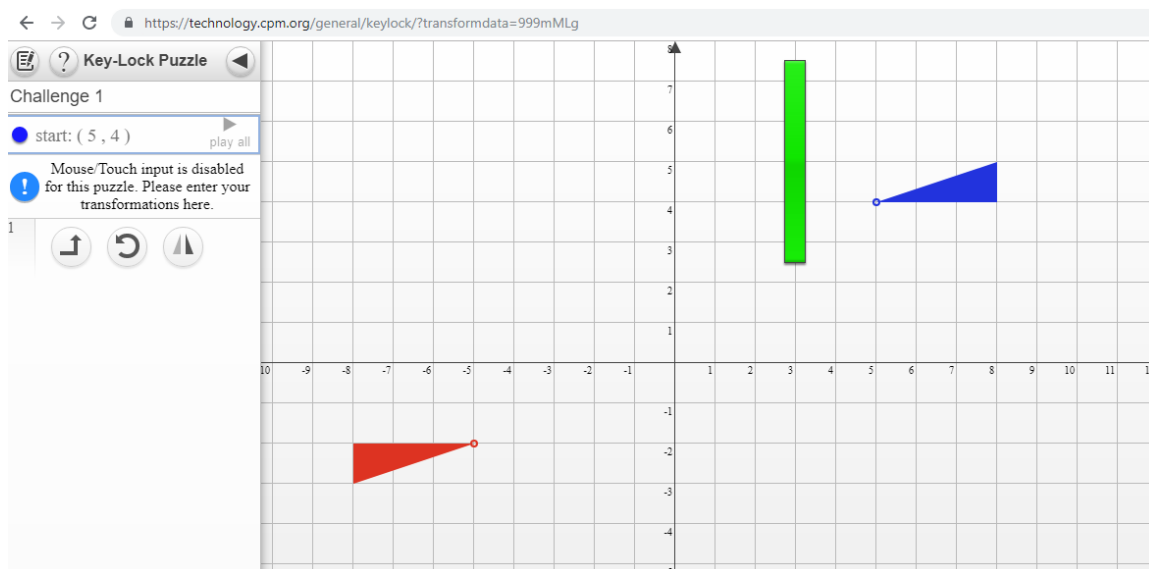


Figure 3 Example of CPM Transformation Task

In Ms. K’s classroom, groups were observed to be working in a collaborative relationship in all observations. The difference in the fours and fives rating was the amount of collaboration within a group. There were explicit instructions each day for students to work with their group. At times, Ms. K would limit resources, such as one graphing white board or one piece of graph paper, for the group. This pedagogical decision supported to collaborative nature of the group.

An example of a lesson rating four was observation 4-changing from vertex to standard form. One pair began working independently on their tasks. After a few minutes, the two students checked results, and one noticed that the other has solved the first task differently. For task 2, their answers did not agree. The pair worked through, justifying why they performed each step, huddling over one students’ work. What began as the pair working independently morphed into the pair working collaboratively.

Observed lessons that rated a five began collaboratively and remained collaborative. For example, in observation 6, Ms. K released the groups with the directive, “One person from your

group needs to get one piece of graph paper and a ruler.” One observed group of four, two female and two male students, began the task with each solving their assigned problem. Then the group discussed the answer, helping the one student who arrived at a wrong answer. They group members did not tell him the answer but explained to him how he arrived at the wrong answer. The group leader then asked for a volunteer to graph the quadratic equation. The person graphing asked for guidance from the group. No one person dominated the group.

Groups in Ms. C’s classroom worked together collaboratively on a routine basis. The exceptions were observation 2—determining true statements – and observation 8 – review of homework and portfolio piece. The lesson in observation 2 was planned for substantive student-student interaction. Groups were directed to talk about each statement and decide whether it was true or false. One group was observed to work independently and then compare answers at the end. They only talked about statements where group members differed on their answers. The other four groups discussed each statement together to decide on a group response. They justified their reasoning and continued to explain until dissenting group members were swayed.

The remaining observed lessons in Ms. C’s classroom ranked above average or exemplary for collaborative group work. The established classroom norm included Ms. C asking any group member to explain their task to the class. In this way, groups were responsible for each group member’s understanding. Groups took the time to ensure that each group member felt comfortable with the group’s decision. For example, in observation 4 students were asked to construct various triangles. When Ms. C released the groups to work on the task, a triad began talking over each other in harmony. One member began with the steps, then another chimed in, and then the third. When they had all the steps one member asked the group why the technique worked. The group had to

explain three different ways until the first member understood it. They created diagrams so he could see what they were explaining to him.

#### 4.1.6 Themes from Field Notes

Field notes were a source of comparison of the enacted curriculum in the focus classrooms. Analysis of the enacted curriculum revealed four major themes. These themes included instructional model, teacher-group interactions, and scaffolding. Field notes and quotes from transcriptions to illustrate the differences within themes are provided in the next sections.

##### 4.1.6.1 Instructional Model

There were three major instructional models observed in the focus classrooms. Direct instruction as an instructional model was evident in three lessons observed in middle school classrooms and one in a high school classroom. A hybrid model was more common as an instructional model in Ms. L’s lessons. It was also evident in two of the high school lessons. The rest of the high school lessons, one of Ms. L’s lessons, and four of Mr. M’s lessons followed an SBMI model where the tasks were open ended. Table 16 shows the distribution of instructional models for each focus classroom.

Table 16 Instructional Model of Observed Lessons

Teacher	Direct Instruction	Hybrid	SBMI
Ms. L	1	6	1
Mr. M	2	3	3
Ms. K	1	2	5
Ms. C	0	0	8

#### 4.1.6.1.1 Direct Instruction

Direct instruction, with the teacher providing guided notes and worked examples prior to student practice, was seen in both middle school classrooms and in Ms. K's classroom. Direct instruction accounted for 16% of the instructional models observed in the focus classrooms. These lessons began with a worked example from the teacher followed by student practice with similar problems.

For example, the introduction to the transformations unit in Mr. M's classroom was direct instruction on the three types of rigid transformations. Mr. M began:

[W]e are going over 3 types of geometric transformations which are when you take an image without changing its size or shape, but you adjust its position. One type is called a translation. We will call it a slide. We slide it over. I translated the image of myself as I slide to the right and forward. To get from the first image to the second we went 9 to the right and 4 up. Now turn to translation page. I will do one with you and then you will do the rest real [sic] quick in your packet. (Mr. M, Observation 1)

During the "I do" phase, some students worked ahead, others took notes, and several expressed confusions that were not addressed by Mr. M. This pattern of "I do" and then "You do quickly" repeated with the other rigid transformations in this lesson.

An analogous pattern was observed in Ms. L's classroom. In an observed lesson where students worked on a review worksheet, there was a procedural focused and direct instruction. Ms. L began the worksheet by guiding students through problem #1, "The average temperature on Jupiter is -162 degrees Fahrenheit. The average surface temperature on Saturn is 46 degrees less than on Jupiter. What is the average surface temperature of the 2 planets?" Ms. L asked the class

how to find average and then asked for the rule for when there is a positive and a negative to evaluate the expression  $-162 - 46$ . Without giving students wait time, Ms. L answered her own question, “The big one wins the sign battle. Add the 2 numbers together.” A student interrupted to ask why she would add the numbers when it is a subtraction problem. Ms. L responded with, “You are just adding them. Trust me.” The student persisted to clarify her confusion and Ms. L continued her minimizing of the student’s quest for understanding with, “Trust me. I am going to tell you.” The student left class with an algorithm but not an understanding as to why the algorithm worked.

Comparably, the lesson in observation 5 in Ms. K’s classroom began with a worked exemplar for completing the square of  $x^2 - 6x$ . She showed them how to set up the area model, in Figure 4, to find the missing constant. After explicit instruction, students were allowed to work on the remaining four squares, looking for a pattern to find  $c$ . During the summarization, students volunteered that they had found  $c = (b/2)^2$ .

	x	-3
x	$x^2$	$-3x$
-3	$-3x$	?

Figure 4 Completing the Square Exemplar

In each example, the teacher was present at the front of the room, writing the necessary steps on the whiteboard. Student participation was encouraged during the “We do” portion of the lesson. When students provided alternate strategies, the strategy was labeled as inferior with a response by the teacher or by no response by the teacher. For example, when a student suggested to only translate one vertex of a figure and then redrawing the figure rather than translating all vertices,

the teacher's response was, "Some people find that more disorienting than drawing each separately. The advantage to my method is that if you make a mistake translating your first dot, you will correct it when you do the other dots."

#### **4.1.6.1.2 Hybrid Model**

At the middle school level, eight observed lessons followed a hybrid format. At the high school level, two lessons followed a hybrid format. Ten total observed lessons (31%) followed a hybrid format. In a hybrid format, the lesson began with an inquiry task. This period of inquiry was followed by direct instruction. In the middle school cases, the inquiry task was prefaced by a long launch phase where algorithms that the teacher felt were necessary to solve the task were reviewed. When students showed signs of struggling in both Ms. L's and Ms. K's classroom, the inquiry phase stopped and the teacher led the class through the task, step by step.

The lesson in observation 5 in Mr. M's classroom was an introduction to dilations. Rather than beginning the lesson with definitions and rules, as had the introductory lesson on rigid transformations, the lesson began with students thinking about how a shape would change if the coordinates were multiplied. Students made predictions about the size and location on the image in comparison to the pre-image. After graphing the dilation and sharing findings, the class took notes on dilations. Then students completed an inquiry task using DESMOS. They observed the effect on the pre-image when both coordinates were multiplied by a whole number, a fraction, a negative number, and a negative fraction. Finally, students completed guided notes with rules to memorize for each type of coefficient.

In Ms. L's classroom, the goal of the lesson in observation 4 was for students to determine the best deal on three different size pizzas. Ms. L takes 15 minutes to steer students to finding the

area and the circumference of each pizza. After students work for 30 minutes, Ms. L has students share what they computed for the area and circumference of each pizza. Three groups that had not completed any calculations copied the information onto their worksheet. Students were then directed to divide the area by price and the circumference by price to determine the best deal for the pizzas. This inquiry task was highly structured with Ms. L providing the class with each step needed to be performed. This format, where students are walked through the inquiry task, was found in the other hybrid lessons in Ms. L's classroom.

Analogous to Ms. L's lesson, the inquiry task for the lesson in observation 3 in Ms. K's classroom was stopped to guide the class through the first scenario. Once the class had a worked example for setting up the diagram and equation for the square growing 3 meters north/south and 4 meters east/west, groups returned to collaboratively working on the remaining scenarios. Ms. K was explicit in her exemplar, telling students that growing 3 and 4 meters meant  $(x + 3)(x + 4)$  and drawing the diagram in Figure 5. She then directed students to start each scenario as  $x^2$  and then add or subtract based on the description.

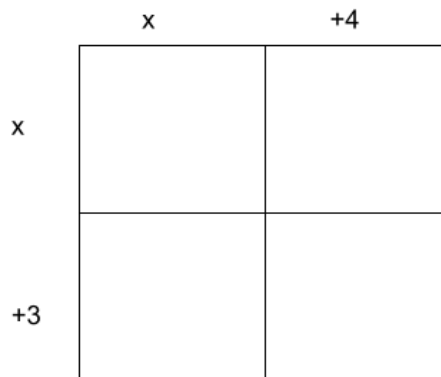


Figure 5 A Lot of Changing Sides

#### 4.1.6.1.3 SBMI Model

An SBMI model was observed in 10 of the 12 high school lessons, in three of Mr. M's lessons, and in one of Ms. L's lessons. In an SBMI model, students collaborate with peers on high-cognitive demand tasks with discourse that includes conjectures, explanations, challenges, generalizations, and abstractions (Munter et al., 2015, Weaver et al., 2005). Students' productively struggle with these tasks without premature teacher intervention (Munter et al., 2015; NCTM, 2014; Warshauer, 2015). The teacher positions students as autonomous learners (Munter et al., 2015). Representations are used to illustrate thinking and are often spontaneous (Munter et al., 2015). An exemplar from each of the focus classrooms follows.

One observed lesson in Ms. L's classroom was categorized an SBMI model. In observation 6, students were charged to construct a gumball container holding 90 cc, with only one portion a rectangular prism. Students were given three class periods to design, propose, and construct the drafts, nets, and container. They needed to justify that their container met the constraints of the project. Students revised drafts as needed, seeking help from Ms. L and their peers as needed. This is the only observed lesson where students were given complete autonomy in how they completed the task.

In an example of an SBMI lesson in Mr. M's classroom, observation 6, students made predictions about the size and location on the image in comparison to the pre-image when the  $x$ -coordinate was multiplied by a constant, when the  $y$ -coordinate was multiplied by a constant, and when both the  $x$ - and  $y$ -coordinates were multiplied by different coordinates. After a debrief on this activity, groups worked with various quadrilaterals to determine which were similar (same constant for both  $x$ - and  $y$ - coordinates in dilation) and which were non similar (different constant for  $x$ - and  $y$ -coordinates in dilation). This lesson was categorized an SBMI model because groups were

working collaboratively on investigatory tasks. Students made predictions and tested out their predictions. Groups determined similarity in different ways. One group found the ratio of side lengths. Another group matched angles.

In Ms. K's classroom, an example of an SBMI model, lesson 1, students explored the effects of the value of  $h$  and  $k$  in  $y = (x-h)^2 + k$ . Groups worked collaboratively to graph four variations of this equation. Students had the autonomy to determine what  $x$  values they would use to create the graphs. Then students were asked to generalize the effects of  $h$  and  $k$  on the equation. Groups were paired up to discuss their generalizations before they were asked to present their generalizations at the board in front of the whole class. This lesson was categorized an SBMI model because groups were working collaboratively on an investigatory task. Students had autonomy as to how to proceed. They dealt with abstractions, multiple representations of a quadratic equation, generalized, and justified their generalizations to their peers. When groups questioned their graph, Ms. K referred them to other groups to discuss findings rather than making the statement as to whether their graph was correct.

In an example of an SBMI model lesson in Ms. C's classroom, observation 7, students were tasked to prove  $\triangle LAC \approx \triangle NEC$  given  $LC = NC$  and  $LE = AN$ , see Figure 6. Groups struggled with this proof for ten minutes. Ms. C stopped the class and asked groups to share out what they had found so far. Volunteers restated the givens. They also decided that they could use *SAS* or *SSS* to prove triangle congruence. As students added to the discussion, the whole class created the proof using *ITT* without Ms. C's assistance. When the class proof was completed, two students shared a different proof which was celebrated by Ms. C. This was categorized an exemplary SBMI model because Ms. C did not step in and guide students to the proof. The proof

was organically generated by the class discussion with students building off the thinking of their peers. Alternate proofs were accepted and celebrated.

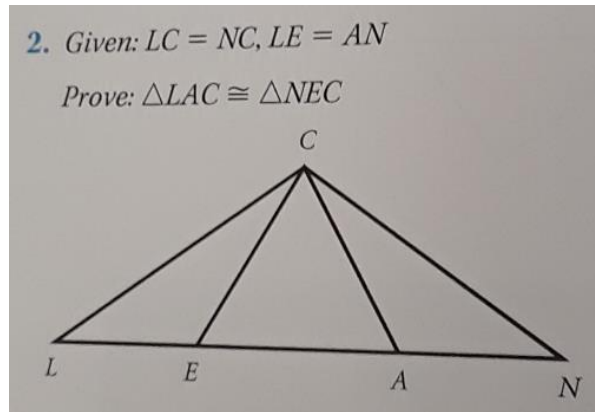


Figure 6 Triangle Proof

#### 4.1.6.2 Teacher-Group Interactions

Two themes emerged from the data analysis regarding teacher-group interactions. Teachers either addressed the group as one cohesive unit or addressed members individually. In the former, the teacher would ask students what the group had discussed or ask group members to restate what other group members had stated. In the latter, the teacher would address concerns with one student then address concerns with the next student, never connecting the two concerns.

Ms. C routinely addressed the group as one cohesive unit. When she was called over to a group for a question or when she was visiting a group, she began her interaction with, “What have you discussed/found so far?” In the lesson for observation 6, proving the *Isosceles Triangle Theorem*, a group was struggling with starting with a midpoint for the proof, see Figure 7. When Ms. C approached the table Student 1 remarked that, “This is so confusing!” Ms. C’s response was to ask the student to share how she was starting the proof. Student 2 explained that she had drawn a diagram on the table and marked all the lines. Ms. C asked her to tell Student 1 what she saw going

on in the diagram. She then tells Student 2, “If you are getting stuck, ask Student 1 to show you her diagram again. What is she is seeing, what she is looking at?” In this example and in other observed instances, Ms. C turned the question back to the group to help the student that is confused. When another group was struggling to find the postulate that was used to solve  $x + 8 = 15$ , Ms. C turned the question to the whole class. After the class discussion, one group member still questioned the appropriate postulate. Rather than answer the question herself, Ms. C waited a few seconds. At this point the other group members had begun explaining the rational for using the *Addition/Subtraction Postulate*.

## Another Way to Prove the Isosceles Triangle Theorem

There is often more than one way to prove a theorem.

1. Another way to prove the Isosceles Triangle Theorem starts by locating the midpoint of  $\overline{AB}$  and labeling the midpoint  $M$ . Which postulate justifies this?

Figure 7 Proving the Isosceles Triangle Theorem

In contrast when group members in Ms. L’s classroom were confused, Ms. L addressed the students individually. For example, in observation 2, students were directed to work with their group to determine the area of a circle using a 1-cm cube. One group member remarked to Ms. L that she was finding the diameter and squaring it. Ms. L explained that she didn’t want her to use the formula but rather the cube to estimate the area. Immediately, the other group member asked if circumference times radius equaled the area of the circle. Ms. L explained to him that she didn’t want him to use the formula. This pattern of addressing each group member was seen in other observations, such as observation 4 – the pizza lesson. When multiple groups asked for the same help from Ms. L, she stopped the lesson and provided students with a step-by-step solution. This

attention to the individual and not the whole group by Ms. L was seen in every observation. This individual attention was in contradiction to Ms. L's request that groups work collaboratively.

Mr. M exhibited a comparable pattern in addressing groups as Ms. L. Group members were often, but not always, addressed individually. For example, in observation 5 – dilations, a student stopped Mr. M as he passed by stating, "I have no idea." Mr. M addressed the student individually, telling her to write her prediction if the coordinate moved from (2, 2) to (4, 4). Then a different student from the same group asked Mr. M a question which he answered. Mr. M never turned the questions back to the group as Ms. C had done. In observation 3, students were encouraged but not required to work collaboratively. The same student stopped Mr. M to announce that she had finished the translation and to ask for the next step. Mr. M told her to proceed to the next problem and then helped another group member with the translation. Had Mr. M attended to the group as a cohesive unit, the response might have looked like, "Good, how is the whole group doing on this task?" And when the other group member asked for help, the response might have been, "What has your group talked about so far?" This attention to the individual and not the whole group by Mr. M was seen in every observation.

Ms. K was the most variable in addressing groups. Some groups she would address the student and others she would address the group. For example, in observation 5 – completing the square – Ms. K addressed one group member who was having difficulty in completing the square for  $x^2 - 5x$ . After explaining the procedure to that student, the other group member asked the same question. Ms. K repeated the algorithm to the second group member. In this instance, Ms. K addressed each group member individually. However, in observation 1, Ms. K addressed groups as a single unit. Towards the end of the explore she asked a group to explain how  $y = x^2$  changes when it becomes  $y = x^2 - 5$ . One group member explained that it was the same, but it moved down 5.

Immediately the other group member stated, “It shifted all the points by 5 on the y-axis.” During the exchange, Ms. K never directed a question to one student but rather to the whole group. In response, the group worked seamlessly to answer her questions, building on each other’s statements.

#### **4.1.6.3 Scaffolding**

Scaffolding is an instructional strategy for supporting learners that is based upon Vygotsky’s sociocultural theory of learning (Vygotsky, 1978). When appropriate supports are provided to students, the student can learn more that they would be able to on their own (Vygotsky, 1978). However, as teachers assist struggling students/groups, they may change the cognitive demand of the task (Boston & Wilhem, 2015; Stein et al., 2006). Providing appropriate, just-in-time scaffolding can preserved the cognitive demand of the task; whereas over-scaffolding can diminish the cognitive demand (Boston & Wilhem, 2015; Stein et al., 2006). This over-scaffolding can occur by students pressing the teacher for explicit steps to perform or by the teacher taking over the challenging aspects of the task (Stein et al., 2006). Ms. C was the only teacher who consistently maintained the cognitive demand of the task with appropriate scaffolding. The three other teachers either consistently over-scaffolded or occasionally over-scaffolded.

In Ms. C’s classroom, the cognitive demand of the tasks remained high during the implementation of the lesson. Launches of the lessons were brief, taking no more than five minutes. Students spent approximately 30 minutes on the task/proof before presenting out to their peers. When groups had questions, it was phrased as “we don’t understand/know” as opposed to “I don’t understand/know.” In responding to groups’ questions, Ms. C often asked the group to explain what they have tried or what their thinking was before providing any scaffolding. This strategy of just-in-time scaffolding served to maintain the cognitive demand of the task. For example, students arrived in class confused about a task they had been assigned for homework. Very few had been able to

determine the length of  $AC$  in Figure 8. Ms. C opened the discussion by asking students what they had noticed, what they were supposed to figure out, and what was tricky? After students shared, Ms. C sent them back to their groups to work on the task using the information from the whole class discussion. Her provision of appropriate scaffolding served to maintain the cognitive demand of the task.

Rectangle  $ABCD$  is inscribed in a quarter-circle as shown.  $BC = CE = 5$  units.

Can you accurately determine the length of diagonal  $\overline{AC}$  without measuring? Explain your answer.

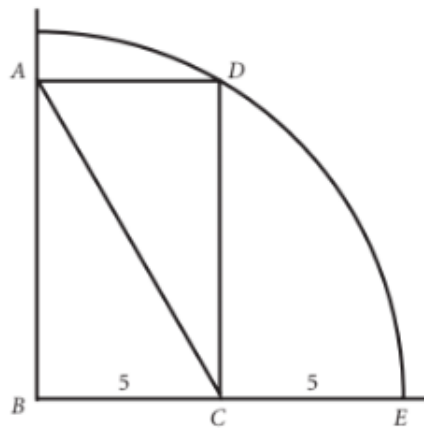


Figure 8 Homework assignment

Ms. K occasionally over-scaffolded tasks. When more than one group was confused, such as in observation 3 described in section 4.1.6.1.2, Ms. K stopped the group work and provided a specific procedure for setting up and solving the tasks. She took over the thinking on the task and routinized the task for students. This over-scaffolding lowered the cognitive demand of the task. Had she had students share out what had and hadn't worked and then resumed the group work, as Ms. C had done, the cognitive demand of the task would have remained high. Groups could have built upon the thinking of other groups.

Lessons in both Mr. M and Ms. L's classrooms were consistently over-scaffolded during the launch phase as well as during the implementation. Introductions to the tasks took 10 to 20 minutes, ensuring that students knew the procedures that they would need to complete the tasks. Additionally, when students had questions, they asked as individuals, "I don't understand" and not as a group. Both teachers then guided the student through the process to the next step. These teacher actions of extended launch to routinize the problem and providing guided steps during the problem-solving process served as over-scaffolding and diminished the cognitive demand of the task. An example of an extended launch in observation 4 in Ms. L's classroom, described in section 4.1.6.1.2, took 15 minutes. Rather than allowing groups to decide on how to determine the best deal for the pizza, Ms. L fore-fronted the investigation with a whole class discussion about area and perimeter. She then directed groups to find the area and perimeter [sic] of each pizza. Later in the same lesson, Ms. L directed groups to divide the cost by area and the cost by circumference to determine the best deal.

In Mr. M's classroom, the lesson in observation 8 was the *Monty Hall Problem*, where students needed to decide whether it was better to keep their "door" or switch their "door" when a second door was revealed as a goat. The lesson began with a quick explanation of Let's Make a Deal and the goat problem. Mr. M asked students whether they should switch or stay with their original choice. Rather than moving directly to the computer simulation to explore that question, Mr. M read to students an article about why it was better to switch. This over-scaffolding diminished the cognitive demand of the task by revealing the correct choice and the reason behind the choice. Students used the computer simulation to prove the correct answer. Then students read another article about why the greater probability of winning remained with switching. Students were not asked to try to explain this phenomenon prior to reading the article. Twice within the same

lesson, the opportunity to engage students in high cognitive demand thinking was removed by pedagogical decisions.

#### **4.1.7 Transitioning to Student Beliefs**

This section contained the analysis of the enacted curriculum. Data from the lesson observation tool provided a comparative of the degree to which observed lessons in the focus classrooms resembled SBMI. Lessons in both 7<sup>th</sup>-grade classrooms were less aligned to SBMI as compared to the 9<sup>th</sup>-grade classrooms. Overall implementation was a relative weakness for Ms. L, Mr. M, and Ms. K. Lessons in both 9<sup>th</sup>-grade classrooms held high expectations for all students, providing for an equitable opportunity to engage with the mathematics. Lessons in both 7<sup>th</sup>-grade classrooms held lower expectations for students, providing more opportunity to engage with the mathematics for students who volunteered in whole class discussions. Students who were quiet or reticent were not invited into the discussions. Additionally, in both 9<sup>th</sup>-grade classrooms, student-student interactions were focused on the content and were collaborative in nature. Students valued the thinking of others and routinely expressed their opinions and ideas. One group member did not dominate over the group. In the 7<sup>th</sup>-grade classrooms, student-student interactions were not collaborative in nature. Students worked individually and may or may not have checked answers with group members. Students in Ms. L's classroom appeared to not value the thinking of others, as evidenced by individuals talking over the speaking students. They did not routinely express their opinions and ideas as only three students participated in whole class discussions with frequency.

In the next section of this chapter, the analysis of students' mathematics related beliefs is presented in the next section. Each of the latent variables is presented, with overall descriptive statistics and then inferential statistics comparing school, gender, and tracking.

## 4.2 Students' Beliefs - Survey

A student survey was administered to seventh-, eighth-, and ninth-grade students at APS over a period of one week in May. Two hundred eighty-eight (288) eligible students completed the survey. The demographical breakdown for each grade level is found in Table 17.

Table 17 Survey Demographics

Gender	Tracking	7 <sup>th</sup>	8 <sup>th</sup>	9 <sup>th</sup>	Totals
Female		58	47	27	132
	Honors	37	28	12	77
	Non-honors	21	19	15	55
Male		66	45	22	133
	Honors	27	22	11	60
	Non-honors	37	25	11	73
Other		18	5	0	23
	Honors	9	0	0	9
	Non-honors	9	5	0	14
Total		142	97	49	288
% of Grade (Not of eligible students)		65%	51%	21%	

The student survey consisted of 23 mathematics-related beliefs statements and 13 mathematics-related identity statements. Students were asked the degree to which they agreed with a series of statements about the nature of learning mathematics, the importance of conceptual understanding, the locus of knowledge and authority in the classroom, the nature of mathematics ability, and speed of knowledge acquisition. Confirmatory factor analysis revealed two latent variables existing in student responses: 1) nature of mathematics/conceptual understanding [NOLM], and 2) and fixed ability/quick learning [FA]. Students were also asked to indicate the degree to which they identified with statements about their mathematics identity, ability, and interest. These three subconstructs – mathematics identity [MID], mathematics self-efficacy [MSE], and mathematics interest [MINT] – were combined to form a composite mathematics identity construct [CID].

For completeness, the results from the student survey are reported for each of the six latent variables. Both descriptive statistics and inferential statistics of the latent variables, disaggregated by the independent variables, are reported. A comparison of the summary beliefs of the four focus classrooms is included with the discussion about the general student population. Aggregate and individual scores range from 1 to 6. Scores less than 3.5 are considered non-availing -- hindering or neutral impact on learning. Scores greater than 3.5 are considered availing -- helping or promoting learning. When analysis of the data was between two groups, such as gender, school, or tracking a t-test was performed to determine significance of difference of means. Significance was set at  $p < 0.05$  and Cohen's  $d_s$  was calculated to determine effect size. Given the data did not meet the assumptions for parametric testing, Kruskal Wallis-H analysis was used for inferential analysis of the data with more than two groups, such as gender and tracking, with significance set at  $p < 0.05$  after Dunn-Bonferroni corrections. The distribution of the dependent variable by each group within the independent variable was checked to determine whether the medians or the mean ranks of the data were to be compared (Kruskal-Wallis H Test using SPSS Statistics, 2018). Given that the distribution of data of each group had different shapes and variabilities, the mean ranks of the groups were compared.

#### **4.2.1 Fixed Ability**

Fixed Ability beliefs [FA] refer to students' beliefs about ability in mathematics. Students with non-availing beliefs,  $FA < 3.5$ , agree with statements such as, "If you are ever going to be able to understand something in math, it will make sense to you the first time you hear it" or "It doesn't matter how hard a student tries, you are either born good at math or not." Students with availing beliefs,  $FA > 3.5$ , agree with statements such as, "Hard work can increase one's ability in math" or

“It doesn’t matter how long it takes to solve a problem as long as you figure it out.” Dweck (2008) refers to these non-availing beliefs as a fixed mindset and these availing beliefs as a growth mindset.

Students at Autumn Middle School and Autumn High School held availing beliefs about mathematics ability, see Table 18. The average belief for the composite variable FA was availing, greater than 3.5, for each examined population. Additionally, the average belief for each of the indicators that comprise FA was availing for each examined population.

Table 18 Average Fixed Ability Beliefs by Focus Classroom, School, Tracking, and Gender

	Ms. L	Mr. M	MS	Ms .K	Ms. C	HS	Honors	Non- honors	Male	Female	Agg
Fixed Ability	4.2 <sup>b</sup>	4.4	4.7 <sup>b</sup>	4.6	4.7	4.7	4.8	4.5	4.8	4.7	4.8
FA1	4.8	4.9	4.9	4.6	4.9	4.6	4.9	4.4	4.5	4.8	4.6
FA2	4.2 <sup>b</sup>	4.4 <sup>b</sup>	5.0 <sup>b</sup>	4.5	5.4	5.1	5.2	4.9	5.1	5.0	5.1
FA3	4.6	4.6	5.0	4.9	4.9	5.0	5.1	4.8	5.0	5.0	4.7
FA4	4.0 <sup>b</sup>	4.9	4.9 <sup>b</sup>	5.0	5.2	5.0	5.1	4.7	5.1	4.9	5.0
FA5	4.4	4.3	4.5	4.4	4.3	4.5	4.4	4.6	4.6	4.3	4.5
FA6	4.3	4.8	4.6	4.6	4.5	4.8	4.9	4.6	4.7	4.8	4.7
FA7	4.0 <sup>b</sup>	4.6	4.7 <sup>b</sup>	4.7	4.9	4.7	5.0	4.4	4.8	4.7	4.3
FA8	4.0	4.1	3.9	3.5	4.1	4.0	4.0	3.8	3.8	4.1	4.0
FA9	4.6	5.0	4.7	4.9	4.5	4.9	5.0	4.6	5.0	4.7	4.8

<sup>b</sup> = statistical difference between class and school      Agg = aggregate

#### 4.2.1.1 FA Beliefs in The Focus Classrooms

There were no significant differences of the mean scores between the four classrooms on all the fixed ability indicators. However, when comparing classrooms to their school, there were statistically significant differences between some beliefs of students in the 7<sup>th</sup> grade focus classrooms and their school. In Mr. M’s and Ms. L’s classroom, there was one indicator, FA 2, “It

doesn't matter how hard a student tries, they are either born good at math or not," that was significantly lower than the average beliefs of students at AMS. In both focus classrooms, students' beliefs were still availing for this indicator, but were in the slightly availing category rather than the availing category for the average of AMS students. Additionally, there were three more statistically significant differences between the beliefs of students in Ms. L's classroom as compared to AMS students. Those differences were in overall FA, FA4, "It doesn't matter how long it takes to solve a problem as long as you figure it out.," and FA 7 "Ability in math increases when one studies hard.," ( $p = 0.016, 0.002, \text{ and } 0.41$  respectively). The reason for these significant differences may lie within previous experiences. They may also lie within the over-scaffolding in Ms. L's classroom where students were not provided sufficient time to productively struggle with the mathematics before Ms. L routinized the problem.

#### **4.2.1.2 FA Beliefs by Gender and/or Tracking**

At AMS, there were numerous differences in students' FA beliefs along gender and along tracking. There were two indicators, FA1, "If you are ever going to be able to understand something in math, it will make sense to you the first time you hear it," and FA8, "It is important to convince yourself of the truth of a math statement rather than to rely on the word of others," where female students held statistically more availing beliefs than male students ( $p = 0.03$  and  $0.012$  respectively). The effect sizes of these differences were medium ( $d = 0.28$  and  $0.32$ ).

There were four indicators with significant differences in student beliefs along tracking. For each of these indicators, AMS honors students held more availing beliefs than AMS non-honors students. The four indicators were FA4, "It doesn't matter how long it takes to solve a problem as

long as you figure it out,” FA6, “Working on difficult problems only pays off for the really smart students,” FA7, “Ability in math increases when one studies hard,” and FA9, “By trying hard, one can become smarter in math.” ( $p < 0.001$ ,  $= 0.07$ ,  $< 0/001$ , and  $< 0.001$  respectively). Whether these were beliefs that developed over time at AMS or were established beliefs from elementary school is not known.

Disaggregating by gender and tracking, see Table 19, reveals seven significant differences between groups at AMS. For the composite variable FA, and for six of the nine indicators, female or male non-honors students held FA beliefs that were on average less availing than female or male honors students. Five of those indicators were the same ones with significant differences in just gender or just tracking analysis. The differences in the indicators were evenly split between female non-honors students and male non-honors students. The average non-honors female student, which held availing beliefs, was less likely to agree that effort increases ability in mathematics than the average honors student. The average non-honors male student, which held availing beliefs, was more likely than the average honors student to believe that learning or understanding quickly and solving problems fast is a sign that a student is good at math.

At AHS, there were no significant differences in students’ beliefs when disaggregated by gender, tracking, or gender and tracking. While observations of all classrooms were not part of the scope of this study, it is possible that the written curriculum implemented by AHS fosters positive beliefs for students who might feel marginalized or less competent in a hybrid or more traditional instructional setting.

Table 19 Middle School Average Fixed Ability Beliefs by Gender and Tracking

	Interaction (mean score)		<i>p</i> -value	$\eta^2$
	Group 1	Group 2		
Fixed Ability	Female NH (4.4)	Male H (5.0)	0.001	0.075
		Female H (4.9)	0.042	
FA1	Male NH (4.0)	Male H (4.7)	0.002	0.049
		Female H (4.9)	0.046	
FA3	Female NH (4.6)	Male H (5.3)	0.016	0.042
FA6	Female NH (4.3)	Male H (5.1)	0.026	0.062
		Female H (5.0)	0.007	
FA7	Male NH (4.1)	Female H (4.4)	0.008	0.048
FA8	Male NH (3.5)	Female H (4.2)	0.083	0.034
FA9	Female NH (4.7)	Male H (5.3)	0.010	0.056

#### 4.2.2 Nature of Learning Mathematics

Nature of Learning Mathematics beliefs [NOLM] refer to students' beliefs about how one learns mathematics and what mathematics looks like in a classroom. It includes students' beliefs about the locus of authority and knowledge in the classroom as well as the importance of conceptual understanding and justifying one's thinking. Students with non-availing beliefs, NOLM < 3.5, agree with statements such as, "You can only learn math when someone shows you how to solve a problem" or "Learning to do math problems is mostly a matter of memorizing the steps to follow." Students with availing beliefs, NOLM > 3.5, agree with statements such as, "Time used to investigate why a solution to a math problem works is time well spent" or "Justifying the statements a person makes is an important part of math."

Table 20 Average NOLM Beliefs by Focus Classroom, School, Tracking, and Gender

	Ms. L	Mr. M	MS	Ms. K	Ms. C	HS	Honors	Non-honors	Male	Female	Agg
NOLM	4.0	4.4	4.5	4.3	4.5	4.4	4.6 <sup>e</sup>	4.3 <sup>e</sup>	4.5	4.5	4.5
NOLM1	4.7	4.5	4.6	5.0	4.7	4.7	4.7 <sup>e</sup>	4.4 <sup>e</sup>	4.8	4.6	4.7
NOLM2	3.6 <sup>b</sup>	4.7	4.5 <sup>b</sup>	3.8	4.8	4.5	4.8 <sup>e</sup>	4.2 <sup>e</sup>	4.5	4.6	4.6
NOLM3	4.3	5.1	4.8 <sup>d</sup>	4.5	4.6	4.5 <sup>d</sup>	5.0 <sup>e</sup>	4.5 <sup>e</sup>	4.9	4.8	4.8
NOLM4	3.3 <sup>b</sup>	4.2	4.3 <sup>b</sup>	4.2	4.1	4.3	4.5 <sup>e</sup>	4.1 <sup>e</sup>	4.4	4.4	4.4
NOLM5	4.2	3.7	3.9	4.9	4.9	3.7	3.9	3.7	4.1 <sup>c</sup>	3.6 <sup>c</sup>	3.9
NOLM6	4.6	5.3	5.1	3.4	3.7	5.3	5.4 <sup>e</sup>	4.9 <sup>e</sup>	5.1 <sup>c</sup>	5.4 <sup>c</sup>	5.2
NOLM7	3.5	3.7	3.7	4.7	5.3	3.5	3.6	3.7	3.6	3.7	3.6
NOLM8	5.1	4.8	5.1 <sup>d</sup>	3.3	3.5	4.8 <sup>d</sup>	5.1 <sup>e</sup>	4.9 <sup>e</sup>	5.1	5.0	5.1
NOLM9*	4.3* <sup>b</sup>	5.1*	5.0 <sup>b</sup>	4.9	5.4	5.2	5.3 <sup>e</sup>	4.9 <sup>e</sup>	5.0	5.3	5.2
NOLM10	3.7	4.1	4.1	3.8	4.0	4.2	4.3 <sup>e</sup>	4.0 <sup>e</sup>	4.2	4.1	4.2
NOLM11	4.3	4.4	4.5	4.1	3.7	4.2	4.7 <sup>e</sup>	4.1 <sup>e</sup>	4.6 <sup>c</sup>	4.3 <sup>c</sup>	4.5
NOLM 12	4.0	4.1	4.0	3.6	4.1	4.0	4.1	3.8	3.8	4.1	4.0

*Italic = non-availing* \* = statistical difference between two classrooms <sup>b</sup> = statistical difference between class and school <sup>c</sup> = statistical difference between gender <sup>d</sup> = statistical difference between schools <sup>e</sup> = statistical difference between tracking Agg = aggregate

Students at Autumn Middle School and Autumn High School held availing beliefs about the nature of learning mathematics, see Table 20. The average belief for the composite variable NOLM was availing, greater than 3.5, for each examined population.

#### 4.2.2.1 NOLM Beliefs in The Focus Classrooms

The only statistically significant difference in average beliefs between classrooms occurred for indicator 9 in Ms. L’s and Mr. M’s. Students in Mr. M’s classroom were more likely to agree ( $\bar{x}$  = 5.1) with the statement, “When a student’s method of solving a math problem is different from the teacher’s method, both methods can be correct,” whereas the students in Ms. L’s classroom were more likely to slightly agree ( $\bar{x}$  = 4.3) with the statement. Ms. L’s classroom had the lowest average

rating of the four classrooms for this indicator. From the classroom observations of the enacted curriculum, Ms. L would stop lessons to provide the class with specific algorithms to complete tasks. While Mr. M would point out the superiority of his procedures/definitions/algorithms, he never stopped investigations to provide such procedures. Ms. K and Ms. C were observed to praise students for using methods with which they were most comfortable. These differences in scaffolding and classroom cultures supporting alternate solution trajectories may be a factor contributing to students' lower rating on this indicator.

There were three indicators where the difference between Ms. L's classroom and the whole 7<sup>th</sup> grade was statistically significant. For each indicator, Ms. L's classroom average was less than AMS's average. For the first indicator, NOLM3 "It doesn't really matter if you understand a math problem if you can get the right answer," Ms. L's classroom average was just above the neutral mark of 3.5. This would indicate that students in this classroom valued correct answers (a grade orientation) over understanding. For the second indicator, NOLM4 "Justifying the statements a person makes is an important part of math," Ms. L's classroom average was non-availing. Students in this classroom did not believe that justification is an important part of math. This belief ties with NOLM3 in that the focus was on correct answer getting and not understanding or justifying. For the last indicator, NOLM9 "When a student's method of solving a math problem is different from the teacher's method, both methods can be correct," Ms. L's classroom average was availing but still less than AMS's average. These differences in Ms. L's students' beliefs and AMS's students' beliefs may be due, in part, to the enacted curriculum. As described in the enacted curriculum section, Ms. L front-loaded lessons with the procedures she wanted students to use to solve their task. Additionally, when groups struggled on the task, she would stop the inquiry and guide the whole class through the problem. Since Ms. L was the person justifying the mathematics and

providing the algorithms, it is conceivable that students felt that it was not their role to justify and create unique solutions.

#### **4.2.2.2 High School NOLM Beliefs**

There were few statistically significant differences in NOLM beliefs across gender or tracking at the high school level. There was one gender difference and one tracking difference in NOLM beliefs at AHS.

Male high school students held beliefs that were more availing than female students on Q5, justifying the statements a person makes is an important part of math, ( $\bar{x} = 4.0$  and  $3.0$ ,  $p < 0.001$ ,  $d = 0.85$ ). Average female high school students held non-availing beliefs and male students held availing beliefs.

High school honors students held beliefs that were more availing than high school non-honor students on Q10, when a student's method of solving a math problem is different from the teacher's method, both methods can be correct ( $\bar{x} = 4.4$  and  $3.7$ ,  $p = 0.0495$ ,  $d = 0.50$ ). The effect size of this tracking differences was medium. Within the focus classrooms, there was not a significant difference in these beliefs. The enactment of the curriculum throughout AHS was not observed. The enactment of the curriculum in the two focus classrooms was similar in that both teachers welcomed praised students for unique methods for solving a math problem.

There were no significant differences in high school students' fixed ability beliefs based upon the interaction of gender and tracking.

#### 4.2.2.3 Middle School NOLM Beliefs

There were more statistically significant differences in NOLM beliefs across gender or tracking at the middle school level than were present at the high school level. There were four gender differences and 16 tracking differences in students' NOLM beliefs at AMS.

Comparable to the average female student at AHS, the average female student at AMS held beliefs that were more availing than the average male student on Q10, that “a person who doesn't understand why an answer to a math problem is correct hasn't really solved the problem,” ( $\bar{x} = 4.10$  and  $3.70$ ,  $p = 0.029$ ,  $\eta^2 = 0.02$ ). The average AMS female student also held beliefs that were more availing than the average male student on Q19, that “time used to investigate why a solution to a math problem works is time well spent,” ( $\bar{x} = 4.68$  and  $4.33$ ,  $p = 0.02$ ,  $\eta^2 = 0.03$ ). Conversely, the average female student held beliefs that were less availing on Q11, that “if students ask questions in math class, it means they didn't listen to the teacher well enough,” ( $\bar{x} = 5.00$  and  $5.40$ ,  $p = 0.002$ ,  $\eta^2 = 0.05$ ). Additionally, the average female AMS student held beliefs that were less availing than the average male student on Q16, that “when a student's method of solving a math problem is different from the teacher's method, both methods can be correct,” ( $\bar{x} = 5.02$  and  $5.29$ ,  $p = 0.033$ ,  $\eta^2 = 0.02$ ). The effect sizes of these differences were small, except for Q11 which was medium.

Like the average AHS female student, the average AMS female student valued the importance of why over strict memorization more than the average AMS male student. However, AMS female students were less likely to value alternate solutions and asking questions to clarify thinking than the average AMS male student. This difference was not seen at AHS suggesting that the written and enacted curriculum may impact these beliefs.

There were significant differences between male and female students' NOLM beliefs when disaggregated by tracking and school. At the middle school level, where a hybrid curriculum was utilized, there were significant differences in NOLM beliefs between students based on tracking and gender. For each of these significant differences, as reported in Table 21, the average male or female non-honors student held less-availing beliefs than the average male or female honors student. The effect sizes of these significant differences were medium. Four interactions were between female non-honors students and male and/or female honors students. Three interactions were between male non-honors students and male and/or female honors students. For each of these significant differences, the average male or female non-honors student held less-availing beliefs than the average male or female honors student. This suggests that the differences are tracking based rather than gender based, with the possibility of the difference having roots in the enacted curriculum.

Observed lessons in the honors classroom were more aligned with SBMI than observed lessons in the non-honors classroom. Lessons in non-honors classroom were over-scaffolded to a greater extent than in the honors classrooms. If this was true across AMS, these may be factors influencing the beliefs of honors students being more availing than the beliefs of non-honors students.

Table 21 Middle School Average NOLM Beliefs by Gender and Tracking

	Interaction (mean score)		<i>p</i> -value	$\eta^2$
	Group 1	Group 2		
NOLM	Female NH (4.3)	Male H (4.7)	0.001	0.111
		Female H (4.7)	0.001	
	Male NH (4.3)	Male H (4.7)	0.002	
		Female H (4.7)	0.003	
NOLM2	Male NH (4.0)	Male H (4.9)	0.005	0.061
		Female H (4.7)	0.027	
NOLM3	Female NH (4.6)	Male H (5.2)	0.017	0.051
NOLM4	Female NH (4.0)	Male H (5.2)	0.011	0.068
		Female H (5.1)	0.015	
NOLM6	Male NH (4.7)	Male H (5.2)	0.028	0.080
		Female H (5.5)	< 0.001	
NOLM9	Male NH (4.7)	Male H (5.3)	0.011	0.065
		Female H (5.3)	0.002	
NOLM10	Female NH (3.7)	Female H (4.4)	0.030	0.036
NOLM11	Female NH (4.1)	Male H (5.0)	< 0.001	0.082

### 4.2.3 Mathematics Identity

Mathematics identity beliefs refer to beliefs that students hold regarding their perception of being a “math person”, their mathematics self-efficacy, and their mathematics interest. Students with non-availing identity beliefs (negative identity) agree with statements like, “Math makes me nervous” or “I wish I did not have to take math.” Students with availing identity beliefs (positive identity) agree with statements like, “I find math interesting” or “I see myself as a math person.” The composite mathematics identity is comprised of three subconstructs, mathematics identity, mathematics self-efficacy, and mathematics interest.

Students at Autumn Middle School and Autumn High School held positive mathematics identities, see Table 22. The average beliefs for the construct CID and subconstruct MSE were availing, greater than 3.5, in the aggregate and across gender. Students held a neutral MID,  $x = 3.5$ , with the average female student holding a negative MID,  $x = 3.4$ . For MINT, the average student held a positive MINT,  $x = 3.8$ , with the average female student holding a neutral MINT,  $x = 3.5$ .

Table 22 Average Identity Beliefs by Focus Classroom, School, Tracking, and Gender

Indicator	Ms. L	Mr. M	MS	Ms. K	Ms. C	HS	Honors	Non-honors	Male	Female	Agg
CID	3.0 <sup>b</sup>	4.0	3.8 <sup>b</sup>	3.2	3.4	3.6	4.0	3.4	4.0	3.6	3.8
MID	2.7	3.8	3.6	2.9	3.0	3.2	3.9	3.1	3.6	3.4	3.5
MID1	2.8 <sup>*b</sup>	4.3 <sup>*</sup>	3.8 <sup>bd</sup>	3.1	3.1	3.2 <sup>d</sup>	4.2	3.1	3.8	3.6	3.7
MID2	2.3	3.6	3.5	2.9	2.5	3.0	3.7	3.0	3.5	3.2	3.4
MID3	2.3 <sup>*</sup>	3.8 <sup>*</sup>	3.4 <sup>d</sup>	2.7	2.6	2.8 <sup>d</sup>	3.7	2.8	3.3	3.3	3.3
MID4	3.2	3.9	3.9	3.4	3.6	3.6	4.9	3.8	4.0	3.8	3.9
MID5	2.3 <sup>*b</sup>	3.4 <sup>*</sup>	3.4 <sup>b</sup>	2.5	3.2	3.2	3.8	2.9	3.6	3.2	3.4
MSE	3.3 <sup>*b</sup>	4.3 <sup>*</sup>	4.0 <sup>b</sup>	3.3	3.4	3.9	4.2	3.7	4.2	3.8	4.0
MSE1	3.1 <sup>*b</sup>	4.8 <sup>*</sup>	4.0 <sup>b</sup>	3.6	4.1	4.0	4.2	3.8	4.1	4.0	4.1
MSE2	3.0 <sup>*b</sup>	4.2 <sup>*</sup>	4.0 <sup>b</sup>	3.3	3.4	3.7	4.2	3.7	4.2	3.7	4.0
MSE3	3.6 <sup>b</sup>	4.6	4.3 <sup>b</sup>	3.7	3.8	4.2	4.5	4.0	4.5	4.2	4.3
MSE4	3.0	3.8	3.7	2.9	3.5	3.6	3.9	3.4	4.0	3.3	3.7
MINT	3.2	4.0	3.8	3.3	3.7	3.6	4.0	3.4	4.0	3.5	3.8
MINT1	3.5	4.1	3.8	3.6	4.0	3.8	4.2	3.4	4.1	3.7	3.9
MINT2	3.1	4.1	3.9	3.5	3.6	3.8	4.2	3.6	4.1	3.7	3.9
MINT3	2.4	3.8	3.5	2.8	3.2	3.2	3.7	3.2	3.8	3.2	3.5
MINT4	3.1	4.1	3.9	3.3	3.8	3.7	4.1	3.5	4.2	3.6	3.9

*Italic = non-availing* \* = statistical difference between two classrooms

<sup>b</sup> = statistical difference between class and school <sup>d</sup> = statistical difference between schools

Agg = aggregate

No statistically significant differences existed between male and female students, and between honors and non-honors classes for all constructs and indicators. There were two indicators where AMS students held identity beliefs, MID1 (parents viewing them as math people) and MID3 (friends viewing them as math people), that were statistically higher than AHS students.

Unlike the FA and NOLM constructs and their individual indicators, three indicators for MID had non-availing mean aggregate scores. The average AMS and AHS student did not see themselves as a math person, MID5, nor did they believe their classmates, MID2, and friends, MID3, see them as math people. Additionally, all aggregate scores were below 4.5, indicating only slightly availing beliefs.

#### **4.2.3.1 Identity Beliefs in The Focus Classrooms**

There were statistically significant differences in the beliefs of students in the two 7<sup>th</sup>-grade classrooms but not in the two 9<sup>th</sup>-grade classrooms. The difference in average beliefs for indicators MID1, MID3, and MID5 in Ms. L's and Mr. M's classrooms was statistically significant. The average student in Ms. L's classroom did not believe their family, relatives, or friends saw them as a math person, whereas the average student in Mr. M's classroom did. Additionally, the average student in Ms. L's classroom strongly disagreed with the statement, "I am a math person." The average student in Mr. M's classroom was almost neutral about that statement.

The difference in the average mathematics self-efficacy subconstruct in Ms. L's and Mr. M's classrooms was statistically significant. The average student in Ms. L's classroom had a negative MSE and the average student in Mr. M's classroom had a positive MSE. Additionally, the indicators MSE1, "Math makes me nervous," and MSE2, "I am someone who is good at math" had statistically significant differences between the two classrooms.

Unlike at AMS, there were no statistically significant differences in the beliefs of students in Ms. C's and Ms. K's classrooms. The positive classroom culture that encouraged students to make conjectures and ask questions and the emphasis on understanding at the high school level have the potential to lessen the anxiety and nervousness of the non-honors students. When wrong or incomplete answers are used to further discussions and are treated in a non-judgmental fashion, students in Ms. K's classroom, who previously may not have believed they were someone who was good at math, may have improved self-efficacy. While not significantly significant, for each identity indicator the average in Ms. K's classroom was either the same as or higher than the average in Ms. L's classroom.

There were statistically significant differences between average beliefs of students in Ms. L's classroom and students at AMS for CID, MID1, MID5, MSE1, MSE2, and MSE3. For each indicator, the average beliefs were lower in Ms. L's classroom than for the school. To determine if these significant differences were a classroom effect or a tracking effect, the data were compared between Ms. L's classroom and all non-honors students at AMS. There were no statistically significant differences between the beliefs of students in Ms. L's classroom and other non-honors students at AMS. This finding suggests that the difference lies within tracking status at AMS. Whether this phenomenon was due to a school culture that elevated the status of honors students or due to differences in school-wide enacted curricula was not a focus of this study and is not known.

#### **4.2.3.2 High School Identity Beliefs**

At Autumn High School, there were few differences in students' identity beliefs when disaggregated by gender or by tracking. Although not statistically significant, AHS female students held identity beliefs that were less availing than AHS male students for every identity indicator except MID2, classmates seeing them as a math person, and MID3, friends seeing them as a math

person. There were three (3) identity beliefs that are statistically different when students are disaggregated by gender, MSE2, “I am someone who is good at math,” MSE4, “Setbacks in math do not discourage me,” and MINT4, “I enjoy learning math,” see Table 23. The effect size for these differences is medium to large.

Table 23 Differences in High School Identity Beliefs by Gender

Question	Sig	d	N	Female (N=27)	SD	Male (N=22)	SD
MSE2	0.02	0.59	49	3.3	1.31	4.0	1.02
MSE4	0.04	0.50	49	3.9	1.42	4.4	0.75
MINT4	<0.001	0.82	49	3.0	1.55	4.2	1.21

Honors and non-honors students at AHS held similar identity beliefs, but for two. AHS honors students were more likely to want to take mathematics than AHS non-honors students ( $\bar{x}$  = 4.0 and 3.1,  $p = 0.03$ ,  $d = 0.60$ ) and were less likely to state that mathematics makes them nervous ( $\bar{x}$  = 4.3 and 3.4,  $p = 0.01$ ,  $d = 0.65$ ). When disaggregated by gender and tracking, there were no statistically significant differences in male and female honors and non-honors students’ beliefs.

#### 4.2.3.3 Middle School Identity Beliefs

Unlike the high school, there were many statistically significant differences in students’ identity beliefs at AMS based upon gender and/or tracking. Male students held identity beliefs that were more positive than female students’ identity beliefs for three of the four constructs and nine of the 13 indicators. There was no difference in AMS male and female students’ beliefs about their MID but there were differences about their CID, MSE, and MINT. Male AMS students held more positive CID, MSE and MINT beliefs ( $p < 0.001$  for all;  $d = 0.37$ ,  $0.34$  and  $0.40$  respectively). This difference in identity beliefs based upon gender was not as prevalent at AHS.

Analysis of identity beliefs by tracking indicated that there were statistically significant differences for all constructs and all but one indicator at AMS. For each of these significant differences, the average honors student held positive identity beliefs, while the average non-honors student held negative identity beliefs for all constructs and all indicators, except MSE1, “Math makes me nervous,” and MSE3, “I understand the concepts I have studied in math.” While being statistically different, both the average honors student and non-honors student held slightly positive beliefs. The only indicator, MID4, “My teacher sees me as a math person,” had not difference between the average honor and non-honors student. This pattern of differences between honors and non-honors students at AMS was not seen at AHS.

A further disaggregation of data by gender and tracking indicated statistically significant differences for all constructs and all but two indicators. For each of these significant differences, as reported in Table 24, the average male or female non-honors student held less-availing beliefs about their mathematics identity than the average male or female honors student. The effect sizes of these significant differences were medium to large. At the high school level, where an NSF-funded curriculum was utilized, there were no significant differences in mathematics identity beliefs between students based on tracking and gender.

Observations of the focus classrooms revealed differences in the implementation of lessons and the culture of the classrooms. Observed lessons at AHS were rated as above-average SBMI and those at AMS were rated as below-average SBMI on the lesson observation instrument. If non-observed classrooms follow similar ratings in their school, the over-scaffolding of lessons, focus on correct answers over conceptual understanding, non-investigatory tasks, and non-collaborative classroom culture that does not place high expectations for all students seen at AMS may give rise to the differences in beliefs based on gender and tracking. The NSF-funded curriculum at AHS, if

implemented with integrity, may support more positive mathematics identities in groups that traditionally hold less positive, or negative, identities, namely female and lower-tracked students.

Table 24 Middle School Average Identity Beliefs by Gender and Tracking

	Interaction (mean score)		<i>p</i> -value	$\eta^2$
	Group 1	Group 2		
CID	Female NH (3.3)	Male H (4.3)	<0.001	0.110
	Male NH (3.6)	Male H (4.3)	0.002	
MID	Female NH (3.1)	Male H (4.1)	<0.001	0.083
	Male NH (3.4)	Male H (4.1)	0.009	
MID1	Female NH (3.2)	Male H (4.4)	<0.001	0.170
	Female NH (3.2)	Female H (4.1)	0.020	
	Male NH (3.3)	Male H (4.4)	<0.001	
	Male NH (3.3)	Female H (4.1)	0.041	
MID2	Female NH (3.0)	Male H (4.0)	0.001	0.131
	Male NH (3.4)	Male H (4.0)	0.023	
MID3	Female NH (2.9)	Male H (3.9)	0.004	0.162
	Female NH (2.9)	Female H (3.7)	0.015	
	Male NH (3.0)	Male H (3.9)	0.004	
	Male NH (3.0)	Female H (3.7)	0.015	
MID5	Female NH (2.8)	Male H (4.2)	<0.001	0.180
	Male NH (3.1)	Male H (4.2)	<0.001	
MSE	Female NH (3.6)	Male H (4.4)	0.001	0.061
MSE2	Female NH (3.4)	Male H (4.4)	0.003	0.114
MSE3	Female NH (3.9)	Male H (4.8)	0.003	0.115
MSE4	Female NH (3.2)	Male H (4.2)	0.003	0.123
MINT	Female NH (3.1)	Male H (4.5)	<0.001	0.110
	Male NH (3.5)	Male H (4.5)	0.004	
MINT1	Female NH (3.4)	Male H (4.5)	0.004	0.070
	Male NH (3.6)	Male H (4.5)	0.007	
MINT2	Female NH (3.4)	Male H (4.5)	<0.001	0.133
	Male NH (3.7)	Male H (4.5)	0.034	
MINT3	Female NH (3.0)	Male H (4.3)	<0.001	0.155
	Male NH (3.4)	Male H (4.3)	0.004	
MINT4	Female NH (3.4)	Male H (4.6)	<0.001	0.150
	Male NH (3.7)	Male H (4.6)	0.011	

#### **4.2.4 Transitioning to Student Interviews**

There were no significant differences in students' beliefs when comparing all AMS students to all AHS students. These findings, when taken at face value, obscure salient differences among students' beliefs based upon tracking and/or gender. When disaggregating the data by gender and tracking, differences in students' beliefs appeared. At AMS, the implementation of the hybrid adopted written curriculum was not observed. Students in the various classrooms learned mathematics from various non-aligned sources. The observed lessons were less aligned to SBMI than the observed lessons at AHS. At AHS, all students, regardless of tracking in an honors or non-honors course, learned mathematics from the same NSF-funded curriculum.

At AHS, there were no significant differences in students' beliefs when analyzed for gender. At AMS, there were significant differences in students' beliefs when analyzed for gender. Female middle school students held beliefs that were less availing than male middle school students about their composite mathematics identity, their mathematics self-efficacy, and their mathematics interest.

Honors students held beliefs that were more availing across all the latent variables than non-honors students. When further disaggregated by school and honors course status, there were significant differences in the sub-groups with medium to large effect sizes. There were significant differences in the beliefs of middle school honors students and middle school non-honors students but no differences between the beliefs of high school honors students and high school non-honors students.

The interaction between gender and tracking was significant at the middle school level but not at the high school level. At the middle school, male honors students held identity beliefs that were more availing than other students. This was followed by female honors students, male non-

honors students, and then female non-honors students. The average female and male non-honors student consistently held beliefs that were less availing than their female and male honors counterparts. At AHS, male honors students did not consistently hold beliefs that were the most availing. For three of the identity beliefs, the average AHS male non-honors student held beliefs that are more availing than other students. The average AHS female honors student held beliefs that are more availing than other students for five of the identity beliefs. The average AHS male honors student held beliefs that are more availing than other students for the remaining five identity beliefs.

The difference between genders and honors/non-honors students at the middle school level but not at the high school in conjunction with the written curriculum is discussed in Chapter 6. In the next section of this chapter, the analysis of student interviews is presented.

#### **4.3 Students' Beliefs - Interview Results**

As described in Chapter 3, six students from each grade, seventh to ninth, at APS were interviewed to expand upon their survey responses and the mathematical experiences contributing to their mathematics identity. Mathematics identity was determined by students' survey responses. Students with a composite mathematics identity [CID] below 2.8 were considered to have a negative mathematics identity. Students with a CID between 2.8 and 4.2 were considered to have a neutral mathematics identity. Students with a CID above 4.2 were considered to have a positive mathematics identity. The average interview lasted 20 minutes and was conducted at times convenient for the student. This section is organized according to interview questions.

Table 25 Interview Participants

Name	Gender	Grade	NOLM	FA	CID	MID	MSE	MINT	CID Identity	Honors	Code [CID grade honors gender]
AJ*	M	7	4.8	4.5	5.5	5.4	5.3	5.8	+(high)	H	H7HM
Alyssa	F	7	6.0	5.4	6.0	6.0	6.0	6.0	+(high)	H	H7HF
Austin*	M	7	4.4	4.1	4.1	2.8	4.5	5.0	Neutral	NH	N7NHM
Cat	F	7	5.1	4.5	3.8	3.0	3.8	4.5	Neutral	NH	N7NHF
Chris	M	7	4.7	4.6	1.6	2.8	1.0	1.0	-(low)	NH	L7NHM
Casey	F	7	4.3	4.3	1.7	2.4	1.8	1.0	-(low)	H	L7HF
Trevor	M	8	4.8	5.9	5.0	5.0	5.0	5.0	+(high)	H	H8HM
D'andrea	F	8	4.8	4.8	4.5	3.6	5.0	4.8	+(high)	NH	H8NHF
Jack	M	8	4.8	4.5	4.0	4.4	4.8	2.8	Neutral	NH	N8NHM
Jessica	F	8	5.0	4.5	3.8	2.2	3.3	6.0	Neutral	H	N8HF
Theo	M	8	4.7	4.5	2.6	1.4	2.5	3.8	-(low)	H	L8HM
Abby	F	8	4.7	5.5	2.2	1.6	3.3	1.8	-(low)	NH	L8NHF
Raoul	M	9	3.8	3.5	5.2	4.4	5.3	6.0	+(high)	NH	H9NHM
Anjelique	F	9	4.4	4.5	4.4	4.8	4.3	4.3	+(high)	NH	H9NHF
Max*	M	9	5.0	4.4	4.1	3.8	5.0	3.5	Neutral	H	N9HM
Sally*	F	9	4.8	5.0	4.0	3.8	4.0	4.3	Neutral	H	N9HF
Kaydon	M	9	4.3	4.5	2.4	1.4	2.8	3.0	-(low)	NH	L9NHM
Patrice*	F	9	4.5	4.9	1.9	1.0	3.0	1.8	-(low)	NH	L9NHF

\* Participated in small-group discourse observations

The 18 participants are listed in Table 25, along with their gender, grade, survey latent variables, honors course status, and code, for reference. There was one female and one male student from each of the three grade levels representing each of the three mathematics identities – positive/high, neutral, and negative/low. All interviewees had attended APS since seventh grade,

and, except for Sally, had attended one of the elementary schools in the APS district. Students in seventh and eighth grade were presumed to have experienced mostly traditional direct instruction learning based upon classroom observations and students' descriptions of a typical day in their current mathematics class. Ninth-grade students, except for Sally, were presumed to have experienced mostly SBMI based upon classroom observations and students' descriptions of a typical day in their current mathematics class. Sally's description of a typical day in her AHS classroom resembled traditional direct instruction.

#### **4.3.1 Importance of Justifying Answers**

Students had varying degrees of agreement with the relationship between being able to justify their procedures and it being representative of them having mastered the content. On the survey, all the interviewees, except Chris (L7NHM), agreed that justifying statements was an important aspect of mathematics. When interviewed, one-third of students ( $n = 6$ ) felt that it was acceptable to be able to solve a problem but not justify their procedures. There was no difference in the need to justify their procedure based upon gender or grade-level. However, there was a difference in the need to justify their procedure based upon honors course status. Five (5) of the six students who stated non-availing beliefs about justifying were non-honors students, with two major themes emerging from their responses (see Table 26). Of those students that expressed this opinion, four felt that the inability to justify their reasoning was due to an inability to express their thinking and not representative of not mastering the content. They believed that some people are just naturally better at justifying their thinking than others. D'andrea (H8HF) stated, "If you can solve a math problem, you know how to do it. Explaining may come naturally but it might be hard for someone." The other two students, both seventh graders, thought that justifying why was not important, but that it was important to follow the given procedures. Chris (L7NHM) shared, "You

don't really need to know why when solving a math problem. You just need to keep practicing, so you follow the right steps." A similar sentiment was echoed by Kat (N7NHF).

Table 26 Not Necessary to Justify Answers

	7 <sup>th</sup>	8 <sup>th</sup>	9 <sup>th</sup>
Only need to follow the algorithm	L7NHM N7NHF		
Not justifying signaling difficulty with expression		L8NHF H8HF	L9NHM H9NHM

While the seventh-grade students believed it was enough to follow an algorithm and not need to justify their thinking, the eighth- and ninth-grade students, regardless of whether they held positive or negative mathematics identities [CID], valued the justification process. They recognized that either they or others might have difficulty with the justification process.

Twelve (12) students believed that you needed to be able to justify your solution trajectory as proof that you truly know and understand the material. Three themes for why you needed to justify your answers emerged from students' responses (see Table 27). The first theme that emerged was that the justification process was a means of proving your answer was correct as a life skill (n = 3). Jessica (N8HF) stated, "If you can't explain why you did something a certain way you haven't learned the math. You need to vocalize why you did it a certain way. That is important to be able to do that in life." This was dual coded as the third theme.

The second theme was being able to apply the reasoning to a new situation, transferability (n = 3). Alyssa (H7HF) reasoned:

You don't really know the math as much if you can't explain why you did it. That means you just found the answer and you don't know how you got there. You

don't really know what you are doing and probably couldn't do it again. Unless you knew the steps that you took. Understanding why is important.

The third theme that emerged was partial understanding (n = 7). Inability to justify a solution trajectory was an indication of partial understanding. Angelique (H9NHF) shared (bold added to show emphasis in verbal response):

If you can't explain why you are doing a certain step you are doing, you partially learned the math, but you don't fully understand the math. You know **how** to do it, which is **technically** learning but **not fully learning** because you don't know why.

Table 27 Reasons to Justify Answers

	7 <sup>th</sup>	8 <sup>th</sup>	9 <sup>th</sup>
Proving correct/life skill	L7HF	N8HF	N9HF
Can't transfer to new situation	H7HF	H8HM	L9NHF
Only partial understanding	H7HM N7NHM	L8HM N8NHM N8HF	N9HM H9NHF

Responses were equally split across grade levels and mathematics identities for the different reasons one needs to justify their answers. There were differences in responses based upon gender. Female students saw the justification as a vehicle to prove their answer was correct and as a life skill. Male students saw justification of answers as proof of complete understanding of the concept.

Honors students were more likely to believe that it was necessary to justify your answers (n = 8) than non-honors students (n = 3). Additionally, only one (1) honors student believed that it was not always necessary to justify one's answers while five (5) non-honors students held that belief.

### 4.3.2 Strategies for Moving Past a Difficult Problem

Students at APS utilized a variety of problem-solving heuristics, such as trying to regroup, reassessing the problem, utilizing resources such as notes, or simplifying the problem, when they encountered a roadblock (See Table 28). Two-thirds of students (n= 12) reported using more than one problem-solving heuristic, with a mean of 2.1 heuristics. AJ (H7HM) shared, "...I will skip over it and finish the other work. Then I would come back to it. I find drawing a picture of the problem itself is useful. Simplifying the problem is also helpful." Abby (L8NHF) added, "If I have a difficult problem, I ask my classmates about it or I ask the teacher. Sometimes I go back to my notes."

Students (n = 11) indicated that they would work with peers before asking the teacher for help. Responses about the nature of peer help from students who experienced standards-based learning environments was slightly different than responses of students who had not. Students in a standards-based learning environment indicated that peer help meant collaboration and brainstorming about ways to solve the problems. Raoul (H9NHM) expressed his opinion with:

When I have a difficult math problem, that is when the group work comes in. I try to talk it out with my group. I like working with Igor. If we can't figure it out, then we ask the teacher.

This sentiment was echoed by all the ninth-grade students except Kaydon (L9NHM). Students in seventh or eighth grade were more likely to turn to their peers to explain the problem or algorithm, than to work collaboratively on the problem. This is evidenced by Austin's (N7NHM) response, "If I get a difficult math problem, sometimes I ask the teacher. Sometimes I might ask someone else, someone around me. They will show me how to do the work." The middle school students who

would ask their peers for help were overwhelmingly non-honors students (5/7) with an equal split across gender.

Not all students responded that they would seek help from the teacher. Only 2/3 of seventh and eighth graders indicated that they would ask the teacher for help, whereas all ninth-grade students except Max (N9HM) would. Much like the request for help from peers, the type of help students expected from their teachers was different based upon the classroom learning environment. Students at AMS looked to the teacher for specific steps to take. For example, Jessica (N8HF) stated, "I ask the teacher to help me to go through the steps." Conversely, students in a standards-based learning environment were more likely to seek help from the teacher to clarify their thinking. A typical response high school student response was from Kaydon (L9NHM):

When I get a hard problem, I usually ask the teacher for help. She is really good at explaining it and setting you up for the right path. And then something clicks in my mind. She doesn't tell me all the answers or all the steps to take. That is what it should be, not to tell the answers but to help guide you.

More non-honors seventh-grade students (n=3) would ask their teacher for help than honors seventh-grade students (n=1). Conversely, more honors eighth-grade students (n=3) than non-honors students (n=1) would ask their teachers for help if they were stuck on a problem. In all grades, the gender split was about equal.

One middle school student, Trevor (H8HM), and one high school student, Kaydon (L9NHM), saw the teacher as the sole source of knowledge in the classroom and would go straight to the teacher with difficulties. They did not use their peers as a resource. Trevor (H8HM) stated:

I tend to ask Mr. N for help. I am trying to learn different strategies like asking my peers before Mr. N or looking back at my notes for step by step when it is a

different situation but a similar concept. We haven't been encouraged to do that before, so it is difficult for me.

Skipping the problem and returning with a new perspective or focus was a problem-solving heuristic used by half of the ninth-grade students, but only one-quarter of the middle school students. Max (N9HM) stated:

If I get stuck on a problem, I try to work through it. Like yesterday in class, I needed to take a step back and come back an hour or 2 later and try a different approach. Thirty is the rule. If I can't solve something in 30 minutes, I will sleep on it or go to dinner and then try it again. Usually, I then have a different approach.

Students who would skip the problem and return with a new problem-solving strategy held high or neutral CID. While the ratio of honors to non-honors students in ninth grade using this technique was representative of the ninth-grade sample, the ratio was not representative of seventh- and eighth-grade students. The two eighth-grade students who would use this technique were both non-honors students.

Focusing on the algorithm was a problem-solving heuristic employed by half of the seventh-grade students ( $n = 3$ ). Alyssa (H7HF) said she thinks about the steps to work it out with the numbers that are there, while AJ (H7HM) tries to remove the extraneous information. Additionally, one-quarter of middle school students ( $n=3$ ) would refer to their notes or textbook when stuck on a difficult problem as compared to only one ninth-grade student. Middle school students ( $n=12$ ) stated that doing mathematics was solving equations or problems like  $2 + 2$ . High school students ( $n=6$ ) stated that doing mathematics was solving problems, in class as well as in the world. This difference in opinion about what it means "to do" mathematics and experiences in how

mathematics is taught in the classroom may contribute to seventh graders' focus on following the algorithm and ninth graders' focus on approaching the problem from a new perspective.

High school students, regardless of their MID, employed more problem-solving heuristics than middle school students. High school students were also more likely to work collaboratively with peers or their teacher for a solution rather than asking for a procedure to employ. Middle school students were more likely to focus on the algorithm and their notes to solve a difficult problem.

Table 28 Problem-solving Heuristics

	7 <sup>th</sup>	8 <sup>th</sup>	9 <sup>th</sup>
Skip problem and return to it later with different approach	H7HM	H8NHF N8NHM	H9NHF N9HM L9NHF
Ask a peer	N7NHF N7NHM L7NHM	H8NHF N8NHM L8HM L8NHF	H9NHM H9NHF N9HF N9HM L9NHF
Focus on the algorithm/steps	H7HM H7HF L7HF		
Go back to notes or textbook	H7HF	N8HF L8NHF	N9HF
Ask the teacher	H7HF N7NHM L7NHF L7NHM	H8HM N8HF L8HM L8NHF	H9NHF H9NHM N9HF L9NHM L9NHF

There was no difference between the mean of heuristics used by honors and non-honors middle school students ( $\bar{x} = 2$ ). At the high school, honors students employed slightly more heuristics ( $\bar{x} = 2.5$ ) compared to non-honors students ( $\bar{x} = 2.25$ ). Male students reported using a mean of 1.9 problem-solving heuristics with no difference between the grades. Female students reported using a mean of 2.6 problem-solving heuristics. Disaggregating female students by grade,

ninth graders reported using a mean of 3 heuristics, eighth graders using 2.3 heuristics, and seventh graders using 2 heuristics.

#### **4.3.3 Times When Student Felt They Just Could Not Do Mathematics**

Students were asked to recall a time when they felt they just could not do mathematics. Their responses ranged from having no instances, to specific instances, to a general feeling of not being able to do mathematics (see Table 29). Students with positive mathematics identities could not recall incidents that made them feel defeated by mathematics. Many felt that mathematics had always come easily for them. A few of them stated they became annoyed or discouraged when they did not master the concepts as quickly as others. AJ (H7HM) recalled a time in elementary school, “I do remember long division in 5<sup>th</sup> grade and how hard it was to learn. I wouldn’t call it traumatic. I was annoyed that I couldn’t learn it as fast as other things. I eventually learned how though.”

Five (5) students with neutral and negative mathematics identities recounted elementary school experiences where their lack of understanding made them feel like they could not do mathematics. Max (N9HM) recalled a time in elementary school, “In second grade we started geometry and I felt so lost and so stupid. Oh, the area was 10 and I was like what? How did you do that? I felt so stupid and lost. I was so discouraged.” These students cited not understanding the topic or the pacing as moving too quickly as a reason why they felt discouraged or that they could not be successful in mathematics. Once the students mastered the topic, they no longer felt defeated. No high school student, even those with negative MID, felt they could not do mathematics.

Middle school students (n=3) were less able to articulate specific instances where they felt defeated and discouraged by mathematics. Theirs was more of an overall feeling of defeat by the subject. Seventh grader Chris (L7NHM) stated, “I always feel like I can’t do math about half the

time. Lots of problems make me feel that way. Probability is super hard. I am good at multiplication.” And seventh grader Kat (L7HF) echoed, “Lots of experiences, like hard problems, make me feel like I can’t do math. I just can’t solve it and figure it out. I don’t feel good then. Math is an enemy.” These three seventh-grade students had repeatedly experienced instances that built upon each other of not feeling successful with mathematics.

Patrice’s (L9NHF) experience was teacher related. She recalled:

There have been experiences, NOT where I didn’t feel like I couldn’t do math, but that I was TOLD that I was doing it wrong even though I got the right answer. One time in 4<sup>th</sup> grade, we had these tests. Multiplication, memorizing the times tables. I am Brazilian and we write our ones differently, like an American 7. To ME the distinction was clear. But the teacher, the extra teacher at the school that would come in from time to time to tutor us, was like, ‘That is wrong because you wrote  $9 * 9 = 87$ . You don’t know your times tables.’ And I was like, ‘No, that is a 1. It is 81.’ I would have to argue with her that it was a 1. And so, my regular teacher would have to step in, and she would just have me take it over again as a compromise. When you already think that you are bad at math, it is demoralizing that they tell you that you are doing it wrong when you have the right answer.

Patrice was the only student who recounted a negative teacher experience in elementary school. Despite Patrice’s experience, she held a slightly non-availing belief about her mathematics self-efficacy. While she believed that she could do most of the mathematics with effort and support, her interest and identity in mathematics beliefs were strongly non-availing.

Table 29 Reasons for Feeling Bad About Mathematics Ability

	7 <sup>th</sup>	8 <sup>th</sup>	9 <sup>th</sup>
Never felt bad	H7HM H7HF N7NHM	H8HM H8NHF N8NHM	H9NHM H9NHF
Confused or pace too fast but now feel at ease with mathematics		L8NHF N8HF	L9NHM N9HM N9HF
Always seems difficult	L7HF L7NHM N7NHF		
Bad experience with a teacher in elementary school			L9NHF
Used to feel successful, now does not feel as successful		L8HM	

As students transitioned to the standards-based instruction in high school, they were more likely to feel success and overcome previous negative experiences in mathematics. Even Patrice, who experienced traumatic interactions with a teacher after she moved here from Brazil, cited becoming frustrated with the current mathematics, but not a feeling of defeat. Four students in AMS classrooms said they continued to be overwhelmed by the mathematics and that it was too difficult for them to be successful, regardless of their honors course status.

#### 4.3.4 Fixed Ability versus Growth Mindset

All interview participants showed availing beliefs about fixed ability versus growth mindset on the survey. Students had a variety of explanations as to why some people are good at mathematics and others are not (see Table 30). Reasons cited included prior/current experiences and effort, reflecting a growth mindset, and logical versus creative brains and natural ability, reflecting a fixed ability mindset.

Half the students (n= 9), including all eighth-grade students, cited student effort and/or patience as one the reasons why some students are good at mathematics and others are not, indicating a growth mindset. As Alyssa (H7HF) stated:

It isn't that some are good in math, and some aren't. It is more like if you have the patience to be able to focus to put the steps together. Sometimes math takes time.

Some don't have that patience if they just want to be doing something else or want to get on to the next thing. Rather than stay and work on it.

For Alyssa, Sally (N9HF), and Abby (L8NHF) effort was combined with enjoying mathematics as a reason for being good at mathematics. For these three females, the learning environment making mathematics fun was an equal factor with effort.

Prior learning experiences, either in the classroom or in informal settings, were cited by high school students (n=3) more than middle school students (n=1) as a reason some students were better at mathematics than others. Developmental play as a toddler may have primed them to be better at mathematics (Sally N9HF). Only high school students were able to verbalize the types of experiences that may have led to an enhanced mathematical ability. Chris (L7NHM) cited better experiences but could not elaborate as to the nature of those experiences. Additionally, students good at mathematics may have had better teachers as explained by both Max (N9HM) and Anjelique (H9NHF). Good teaching was only referred to by high school students (n=3). One student, Casey (L7HF), articulated a developmental aspect of mathematics ability:

Some are good at math because it is easier for them to learn. Brains develop at different speeds. That is why we have Flex\*\* (honors) classes. If you don't get it now, then you might get it later. \*\* (7<sup>th</sup> grade flex mathematics is AMS's terminology for the advanced mathematics class.)

This interpretation was slightly different than effort or experience, as it viewed mathematics ability as a function of brain development. Although one is exposed to mathematics now, there may be a delay in the cognitive processing so that students might appear to not be good at mathematics. This belief is a combination of a growth mindset, that you might understand mathematics later, and a fixed mindset, that it is easier for them to learn.

Seven (7) students provided a fixed ability reason for why some students are better at mathematics than others despite reporting growth mindset beliefs on the survey. One theme that emerged was natural ability. Belief in the importance of natural ability was equally distributed across the grades and gender, but not by honors course status. Four students, Raoul (H9NHM), Jack (N8NHM), Kat (N7NHF), and Casey (L7HF), believed that mathematics just came easier for some people. Jack and Casey cited effort and brain development as contributing factors. Raoul and Kat, both non-honors students, held the belief that innate ability was the only reason why some students were good at mathematics and others were not. Additionally, Raoul was the only interviewee who agreed with the survey statement that “It doesn’t matter how much a student tries, they are either born good in math or not” and disagreed with “Working on difficult problems only pays off for the really smart students.”

Another fixed ability theme that emerged from student responses was logical/methodical brains versus creative/language brains. Three students, AJ (H8HM), Anjelique (H9NHF), and Patrice (L9NHF), stated that people are hard wired to think a certain way; this is slightly different from natural ability. This was a belief in a neurological difference in how brains process information. This left-side, right-side brain thinking indicates a fixed mindset. Both Anjelique and Patrice also cited good teaching at the elementary level as a contributing factor. AJ only cited

logical brains versus creative brains as the reason why some students are better at mathematics than others.

Table 30 Beliefs About Mathematics Ability

	7 <sup>th</sup>	8 <sup>th</sup>	9 <sup>th</sup>
<b>Growth Mindset</b>			
Prior/current learning experiences	L7NHM		H9NHF N9HM N9HF
Liking mathematics	H7HF	L8NHF	N9HF
Developmental	L7HF		
Good teaching			H9NHF L9NHF N9HM
Effort	H7HF N7NHM	H8HM H8NHF N8HF N8NHM L8HM L8NHF	N9HM L9NHM N9HF
<b>Fixed Ability Mindset</b>			
Natural ability	N7NHF L7HF	N8NHM	H9NHM
Logical brain	H7HM		H9NHF L9NHF

Of the seven students stating fixed ability explanations for mathematics ability, five (5) were non-honors students. Gender and personal mathematics identity did not factor into this fixed ability explanation. In the high school, all three students who provided fixed ability explanations were non-honors students. Two of those students, females, also provided growth mindset explanations. Honors course status did not factor into the growth mindset explanations for ability explanation. Students expressing a growth mindset explanation for mathematics ability were equally split between honors and non-honors courses. High school students provided a mean of 1.8 explanations for mathematics ability while middle school students provided a mean of 1.3 explanations.

#### 4.3.5 What is a Math Person?

Students had typical responses to what makes a person a math person (see Table 31). Responses were that the person was good in mathematics and/or enjoyed mathematics. The definition of a math person as someone who was good at math indicates less-availing beliefs, a fixed-mindset. There appeared to be patterns based on mathematics identities, as well as grade and gender. All students in ninth grade stated that a math person was someone who enjoyed mathematics. Two of the ninth-grade students, both females, stated a math person might also be someone who was good at mathematics. This additional definition of a math person as possibly someone who is just good at mathematics indicated that these two ninth-grade students held less-availing beliefs than the other four ninth-grade students who did not include this option. This indication of holding less-availing beliefs is supported by their neutral/negative CID.

In middle school, three (3) students believed that a math person was only someone to whom mathematics came easily. All three identified themselves as “math people.” Additionally, only middle school students (n=2), believed that math people both enjoyed and were good at mathematics. These two middle school students were female non-honors students who did not identify as “math people.” No high school students held the requirement that the person be both good in mathematics and like mathematics.

Only students with neutral or negative CID responded that a “math person” was one who either enjoyed mathematics or was good at mathematics. Opening the definition of a “math person” to one who is good at mathematics is indicative of a less-availing/fixed-mindset belief. Additionally, it validates these students’ survey results of negative CID.

Table 31 What is a "Math Person"?

	7 <sup>th</sup>	8 <sup>th</sup>	9 <sup>th</sup>	
Enjoys math	L7NHM H7HM	N8NHF L8HM	H9NHM N9HM	H9NHF L9NHM
Is good at math	H7HF	H8HM H8NHF		
Enjoys math OR is good at math	L7HF N7NHM	N8NHM	L9NHF	N9HF
Enjoys math AND is good at math	N7NHF	L8NHF		

#### 4.3.6 Direct Instruction versus Standards-Based Instruction

Students had well-formed opinions about how they best learned (see Table 31). All students had had prior experience with direct instruction, having attended AMS since seventh grade. Abby (L8HF) was the only interviewed student that did not attend one of the APS elementary schools but had attended AMS since the beginning of seventh grade. Sally (N9HF) described a typical day in her current mathematics class as analogous to her middle school experience. Class began with a warm-up and quick review of homework, followed by a mini lesson with notetaking and whole class practice of examples before individual practice on worksheets. Sally was the only high school interviewee who was not a student of one of the two observed high school classroom teachers. The other ninth-grade interview participants described a typical SBMI format in their current mathematics class.

Seven (7) students, three from middle school and four from high school, believed that learning is better attained when they are given a challenging problem to figure out in a collaborative setting followed by whole class discussion. All students who exclusively preferred SBMI held positive mathematics identities, except for Max (N9HM). Anjelique (H9NH) shared:

I like how we do math this year. Working on a problem and figuring it out and then discussing it as a class. I did not like it in middle school when we had notes and then just practiced the same thing over and over. It was boring.

Two (2) students, one seventh grader and one ninth grader, thought there should be a balance between an exploratory activity first and direct instruction. Kaydon (L9NHM) felt that:

I learn better when I have a balance between working on these confusing problems in IMP and when she gives us notes for the procedures. I learn better when I get to work in groups, and we can talk about it [confusing IMP problems].

Austin (N7NHM), who participated in the small-group discourse portion of the study, agreed with that sentiment, adding that “sometimes it helps to explore first.”

The two high school students who preferred direct instruction with notes and guided practice were Sally (N9HF) and Patrice (L9NHF). Sally had not had the SBMI experience of the other high school interviewees, so perhaps did not have a basis for comparison. Patrice had the traumatic experience in elementary school. Patrice, also a participant in the small-group discourse portion of the study, did like some of the challenging IMP2 problems but had not had the experience of quality collaborative work. She chose to work with the same student most of the year. They customarily worked in parallel, checking answers, as described in detail in the next chapter.

Middle school students, regardless of grade, gender, or honors course status, generally felt that mathematics is best learned with direct instruction. The exceptions to this were Austin, who preferred a mixed approach, and AJ (H7HM) and D’andrea (H8NHF) who preferred SBMI. When asked about the teacher’s role in the classroom, middle school students responded that it was the teacher’s job to teach them how to solve problems, to give them notes explaining each of the steps, and to provide a couple of examples. Casey (L7HF) stated:

I think it is better to get the notes first. You don't know the steps. The teacher's role is making sure the students understand what to do. And how to do it. Not just let them figure it out themselves.

Middle school students' comments indicate they may not have had enough much exposure to SBMI to make a true comparison. As Jack (N8NH) stated, "I think that it is best that teachers give notes first to explain how to solve problems. It is the way I have always had it. It works."

Interestingly though, Jack later talked about making mathematics class more interesting by providing students with real-life scenarios and letting them figure out how to solve it rather than being told, "Here is the thing to do. Here is my example. Follow it." While there was comfort in the familiarity of direct instruction, there was a yearning to make mathematics more interesting and applicable to life by including some SBMI.

Table 32 Direct Instruction Versus Standards-Based Instruction

	7 <sup>th</sup>		8 <sup>th</sup>		9 <sup>th</sup>	
Direct instruction	H7HF L7HF	N7NHF L7NHM	H8HM N8HF L8NHF	N8NHM L8HM	N9HF	L9NHF
Standards-based instruction	H7HM		H8NHF		H9NHM H9NHF N9HM	
Mixed instruction	N7NHM				L9NHM	

#### 4.3.7 Summary of Interviews

Student interviews occurred during May and June of the school year at a time convenient for the student. In all, 15 prompts were provided to 18 students and their responses analyzed. Each prompt provided data about students' past experiences with mathematics as well as their views towards mathematics and mathematics instruction. Interviews lasted between 10 and 20 minutes, with high school students providing more in-depth responses than middle school students.

Differences in some beliefs aligned with gender, mathematics identity, grade in school, and/or honors course status (see Table 33). First, differences in beliefs aligning with mathematics identity will be summarized, followed by differences aligning with gender, grade in school, and honors course status.

Overall, students with positive mathematics identities valued standards-based learning with collaborative inquiry. Students with positive mathematics identities named teachers and family members as sources of support in their mathematics journey and did not recount experiences where they felt defeated by mathematics. Additionally, students with positive mathematics identities were apt to employ problem-solving heuristics, such as stepping back from the problem, using a different approach, or referring to notes and/or textbook, before turning to someone else for help. They believed that it is important to justify their steps in problem-solving, not only as demonstration of understanding of the material but as a life-skill needed outside school.

Table 33 Summary of Differences in Interviewees' Beliefs

	Mathematics identity	Gender	Grade in school	Honors course status
Importance of justifying answers	X	X		X
Heuristics employed to move past difficult problems	X		X	X
Times when student felt they could not do math	X		X	
Fixed ability vs growth mindset		X	X	X
What is a math person?		X	X	X
Direct instruction versus standards- based instruction	X		X	

Students with neutral or negative mathematics identities felt more secure with a traditional instructional model, with direct instruction and guided practice. They cited specific classroom experiences and difficulties with a concept that provoked feelings of discouragement or defeat, some of which persisted through the current study. Students with neutral or negative mathematics identities relied on asking peers or their teachers for guidance when faced with a difficult problem rather than utilizing other problem-solving heuristics. Additionally, they believed that it was possible to know the mathematics and not be able to justify one's reasoning. Some did not feel it was important to justify their reasoning and others cited difficulties with expressing the reasoning despite believing that they knew the concept.

Gender differences in beliefs and responses to interview questions existed. Middle school female students believed that a math person was someone who was naturally good at mathematics, with some adding the requirement that they also enjoy mathematics. Male students believed that a math person was someone who enjoyed mathematics. In response to the question about why some students were good at mathematics, only females maintained that liking mathematics contributed to a person's ability in mathematics. Females who felt that it was important to justify procedures and answers pointed out that this is a means to prove the validity of their argument and that it is an important life skill. Males who felt that it was important to justify procedures and answers indicated that a lack of justification signals only partial understanding.

Students' beliefs about the best way to learn/teach mathematics were aligned with the teaching style in their current classroom. Middle school students, whose teachers customarily utilized direct instruction, believed the best way to learn mathematics was through direct instruction; high school students, whose teachers routinely employed SBMI believed mathematics was best learned with SBMI. High school students believed that a math person was someone who

exclusively enjoyed mathematics, whereas middle school students' beliefs were a mix of enjoyment of and/or ability in mathematics.

All eighth-grade students believed that effort was the reason some students were good at mathematics. Only two students added an additional factor, liking mathematics and natural ability, with both criteria being necessary. Seventh-grade students were less unified in their beliefs, with half of them believing differences were due to natural abilities and the other half thinking differences were due to brain hardwiring. Like eighth graders, seventh graders only provided one factor for differences in ability. Half of the middle school students who held fixed ability mindsets did not offer additional growth mindset reasons for differences in ability. Ninth graders were more sophisticated in their beliefs. They cited multiple reasons for differences in ability, with only one student citing natural ability alone. Ninth-grade students believed that prior and current learning experiences and teachers have an impact on a students' mathematical ability.

When looking back at times when they did not feel like they could do mathematics, high school students were able to recall specific instances where they felt discouraged or defeated by mathematics. Seventh-grade students, who did not feel like they could do mathematics, were not able to recall a specific instance. Rather, they reported a sustained overall feeling of discomfort with mathematics.

High school students reported using multiple problem-solving strategies when encountering difficult problems. When they worked with a peer, it was generally to collaborate and brainstorm. Middle school students would turn to a peer or a teacher to direct them to the correct algorithm or point out errors in their arithmetic. Seventh graders were the only students who stated they would focus on the known algorithm, looking back at notes if need be.

Student enrollment in an honors course led to differences in beliefs as compared to non-honors courses. Non-honors course students were more likely than honors course students to believe that it was not necessary to justify thinking. Middle school non-honors students were more likely than honors students to ask a peer or teacher for help rather than employ a more sophisticated problem-solving heuristic. Additionally, more non-honors students than honors students held fixed ability beliefs.

#### **4.4 Transitioning to Chapter 5**

In this chapter, the analysis of classroom observations and the enacted curriculum was presented. Results of the survey were reported with attention to differences between school, gender, and tracking for the latent variables of fixed ability, mathematics identity, and the nature of learning mathematics. Finally, the student interview analyses were reported with attention to mathematics identity, gender, grade level in school, and tracking.

Observed lessons at AMS differed in instructional style and classroom norms from observed lessons at AHS. Seventh-grade lessons followed direct instruction, hybrid instruction, and SBMI models, with nine of 16 observed lessons following a hybrid model. Ninth-grade lessons followed an SBMI model in 13 of the 16 observed lessons. Lessons at AHS were planned with substantive student-student interactions that supported student learning through collaborative inquiry tasks. Classroom norms supported questioning, challenging, conjecturing, and generalizing about the topic. Students were challenged with high expectations and were positioned as the locus of authority. As such, students valued the thinking of others and were willing to take academic risks. Students persisted on tasks with minimal teacher interference.

Lessons at AMS were planned with substantive student-student interactions in half the observed lessons. Collaborative work was not supported by teachers' interactions with students nor by teachers' over-scaffolding of lessons. Classroom norms did not place the student/group as the locus of authority. Some students were challenged with high expectations and others were not. Lessons focused on speed over conceptual understanding.

Differences in composite mathematics identity, as well as its subconstructs of identity, self-efficacy, and interest, existed between APS male and female students. Male students held more-availing beliefs than female students. When disaggregated further by school, there were differences in these beliefs between middle school female and male students, but not between high school female and male students. Additionally, there were differences in these beliefs between honors and non-honors students. When disaggregated further by school, there were differences in these beliefs between middle school honors and non-honors students, but not between high school honors and non-honors students. These differences, that were found between sub-populations in middle school but not in high school, may indicate a positive impact of SBMI on shaping the beliefs of female students and lower-tracked students.

Students in the observed seventh-grade non-honors class held beliefs about FA, NOLM, and mathematics identities that were the least availing. Non-availing beliefs were supported by the teacher-content and teacher-student interactions in the way lessons were implemented. Pedagogical decisions kept the locus of authority and knowledge in the hands of the teacher and not of the student. Decisions about the launch of tasks over-scaffolded the tasks and provided students with specific procedures to solve the task. Routinizing tasks when students struggled diminished the cognitive demand of the tasks. These teacher moves served to support students' non-availing beliefs about mathematics ability, the nature of learning mathematics, and their mathematics identities.

Conversely, students in the observed ninth-grade non-honors class held beliefs about FA, NOLM, and mathematics identities that were more availing than students in the 7<sup>th</sup>-grade non-honors class. Pedagogical decisions allowed the locus of authority and knowledge to shift from the teacher to the student. Tasks were not over-scaffolded as often as in the 7<sup>th</sup>-grade classroom. Only on two occasions were the tasks routinized and the cognitive demand diminished. Teacher-content and teacher-student interactions served to support students in developing beliefs that were more positive in nature.

Interview results indicated that there were differences in beliefs based upon gender, mathematics identities, and tracking. Students who held positive mathematics identities favored SBMI practices and employed more sophisticated problem-solving heuristics than students who held neutral or negative mathematics identities. Non-honors students were more likely than honors students to state that it was acceptable to not justify answers, held fixed ability mindsets, and sought out teacher direction for an algorithm when stuck on a problem. Female students were more likely to describe math people as people who are good at mathematics while male students were more likely to describe them as people who enjoy mathematics. Additionally, female students stated that mathematics ability was a combination of genetics and experience. Male students were more likely to associate ability in mathematics with genetics and effort.

In chapter five, the results of the classroom observations and analysis of the small-group discourse will be presented. In chapter six, the results will be discussed in connection with the literature and the theoretical framework.

## **CHAPTER 5**

### **SMALL-GROUP DISCOURSE**

The purpose of this case study of within district incongruent written curricula commitments was to explore student beliefs and quality of small-group discourse across settings. In the previous chapter the analysis of the enacted curriculum of four focus classrooms and APS students' MRB were presented. In this chapter the analysis of small-group discourse is presented, grounded in the enacted curriculum and group members' MRB. Data pieces collected for this part of the study includes classroom observations, field notes, and transcripts of student-student discourse. The methodology for small-group discourse analysis was presented in Chapter 3.

This chapter begins with discourse exemplars from the classrooms. The exemplars are followed by a close look at the discourse, the nature of the groups, and the enacted curriculum in each classroom. This analysis seeks to explain within class variations in discourse patterns. After the individual classroom analyses, comparisons are made across groups by mathematics identity beliefs. This analysis seeks to explain across-class variation in discourse patterns.

#### **5.1 Quality of Discourse Exemplars**

In each of the four focus classrooms, two groups of students were observed during small-group inquiry eight times from late April to early June of the school year. Some of the observations (1-2 in each classroom) occurred prior to state testing, with the rest of the observations occurring after state testing. There could be low-quality, high-quality, or a mixture of low and high-quality discourse during each of the five-minute units of analysis.

Low-quality discourse includes providing a short answer to a direct question, making a simple statement or assertion without sharing why, explaining a mathematical idea with procedures

without justification, and/or asking questions to clarify personal understanding of a mathematical idea.

An example of low-quality discourse from this study comes from Ms. C's classroom, observation 2, where groups are tasked with justifying why given statements are true or false. In this lesson, Don, Taz, and Max worked together with one group member reading the statement aloud, including their response of true or false. Ms. C approached the group and reminded them that one person is reading the statement aloud and then the group talks about it and writes down what is agreed upon. When Ms. C leaves, the group continued with stating the answer without any reasoning. The discourse in this increment was rated low quality because students did not provide a justification for their true or false responses.

High-quality discourse includes making statements or asking questions that challenge the validity of a mathematical idea or procedure, making connections or seeing relationships to prior knowledge, making predictions or conjectures based on the understanding of the mathematics behind the problem, justifying the validity of a mathematical idea or procedure by providing an explanation of the thinking that led to the idea, and/or making a statement that is a shift from a specific case to the general case.

An example of high-quality discourse from this study comes from Mr. M's classroom, observation 7, where groups were determining the series of transformations that moved pre-image A to its image B. After several false starts, Sam suggested to the group that they could reflect the pre-image over the y-axis, translate it 2 units up along the y-axis, and then multiply it by 4. AJ agreed and added further explanation to convince the third group member, Marc, by saying:

Look Marc... if we reflect it here (points to y-axis on Marc's paper), it is going to be here (outlines it on Marc's graph). Then move it 2 units up. It is in this box

(draws the lower  $\frac{1}{4}$  of image B). Then if we multiply it by 4, then we would have 4 [boxes] (draws four pre-image A's in image B).

The discourse in this interaction was rated high quality because AJ used drawings to explain his thinking and to justify his and Sam's suggested transformations.

An example of mixed-quality discourse from this study comes from Ms. K's classroom, observation 2, where groups were writing an equation given the vertex,  $(-3, 1)$ , and another point on the parabola,  $(-2, 3)$ . In this example, Yusef and Patrice were working together to determine the equation. Patrice noted that the parabola had shifted over  $-3$  on the  $x$ -axis so that it would be  $-(x-3)$ . This snippet of the group interaction is rated high quality because Patrice explained her thinking for arriving at  $(x-3)$ . Yusef set up the equation  $y = a(x-h)^2 + k$ . Patrice suggested  $(x-3)$  and Yusef corrected her with  $(x-3)+1$ . This one snippet of the group interaction was rated low quality because Yusef stated the answer with no justification. Patrice must ask Yusef why he included  $+1$ . Yusef did not include that the vertex had shifted up 1 unit on the  $y$ -axis from the origin, which is represented by  $k$  in the equation. During the five-minute increment, there were instances of low-quality discourse and high-quality discourse. The overall rating for this increment was 2, mixed-quality discourse.

## **5.2 Small-group Discourse Analysis**

The analysis of the quality of small-group discourse in each of the four focus classrooms was conducted using the discourse taxonomy found in Appendix E. Included in the analyses are connections to students' MRB and the enacted curriculum. Table 34 contains the summary findings from the classroom observation instrument for the focus classrooms.

Table 34 Summary Findings of Focus Classrooms

Teacher	Grade	Honors/ Non- honors	Implementation	Content	Culture	Overall Rating	Instructional Model
Ms. Luurtsema	7	NH	3.0	5.4	3.8	3.0	Mostly Hybrid
Mr. McGinnis	7	H	3.4	4.3	3.6	2.9	Even Mix of Direct, Hybrid, and SBMI
Ms. Kraft	9	NH	3.6	4.9	5.1	4.5	Mostly SBMI
Ms. Chenette	9	H	5.3	5.5	6.1	5.6	SBMI

### 5.2.1 Quality of Discourse in Ms. Luurtsema’s Classroom

Observed lessons in Ms. L’s classroom were rated below average for SBMI. A summary of the findings from Chapter 4 is provided to situate the small-group discourse in the enacted curriculum. All lessons, except for one, were planned for investigative tasks to be an essential component of the lesson. However, only half the observed lessons were planned for substantive student-student interactions. In none of the lessons that were planned for substantive student-student interactions did students interact collaboratively. Ms. L routinely decreased the cognitive demand of the investigative task by either over-scaffolding during teacher-group interactions, stopping the investigation to provide explicit steps to solve the task because multiple groups were struggling, or conducting an extended launch of the task which provided groups with specific algorithms and procedures to solve the task. The established classroom norms included waiting for the teacher to intervene with assistance rather than problem solving as a group and whole class discussions limited to a few students who routinely volunteered. Additionally, the groups were not accountable for settling on a mutually agreeable strategy, nor were they accountable for explaining the method chosen by the group.

Eleven students participated in the beliefs survey. Five students held negative identity beliefs, five students held neutral identity beliefs, and two students held positive identity beliefs.

Examining the dialogue among students during small-group inquiry indicated that the small-group discourse in this classroom was limited and of low quality. The daily discourse quality ratings for the two focus groups are found in Table 35. Discussions between students, when they existed, were procedural in nature. The sharing of ideas and the construction of knowledge between group members was not apparent. Group members conducted their work independently, despite being urged by Ms. L to work collaboratively. Additionally, students rarely checked their work with their group. Instead, students waited until the whole class summary to determine if their answer was correct.

Table 35 Observation Summary of Small-group Discourse in Ms. L’s Classroom

Observation	1	2	3	4	5	6	7	8	Average
Austin/ Trinity	0	0	1	0	1	1	1	1	0.63
Elasha/ Hillary	0	1	1	1	0	1	1	0	0.63
Instructional Model	D	H	H	H	H	S	H	H	
PSSI	N	Y	N	Y	Y	N	N	Y	

D = direct instruction H = Hybrid Model S = SBMI PSSI = Planned student-student interaction

Below is the in-depth analysis of the discourse practices of the discussions occurring among the two groups during small-group inquiry, with events chosen as representative of the quality and quantity of discourse. Consistently, both groups engaged in minimal discourse. When partners talked with each other, it was to ask a question or to state an answer. There was minimal high-quality discourse that included challenging the thinking of the partner, justifying thinking, explaining answers, generalizing, or relating concepts to prior knowledge.

### 5.2.1.1 Trinity and Austin

Trinity and Austin worked together as partners for most of the seventh-grade school year. Trinity held beliefs about the nature of mathematics, mathematics ability, and her mathematics identity that were more availing than Austin's beliefs (see Table 36). Austin held availing beliefs about his MSE and MINT but non-availing beliefs about his MID. Overall, Austin held a high-neutral mathematics identity [CID] and Trinity held a positive mathematics identity. Both students reported earning an A on the last class assessment.

Table 36 Mathematical Beliefs of Austin and Trinity

	Gender	Grade	NOLM	FA	CID	MID	MSE	MINT
Trinity	F	7	5.0	5.0	4.5	4.2	4.5	4.8
Austin*	M	7	4.3	4.2	4.1	2.8**	4.5	5.0

\* participated in interview \*\* non-availing beliefs

Over the course of this study, the discourse between Trinity and Austin was minimal, with little collaboration on or discourse about the mathematical tasks. When there was discourse during small-group inquiry time, the discourse generally remained at a low quality. Trinity and Austin would check to determine if they arrived at the same answer but there was minimal discussion as to how to proceed in the task. If there was a difference in their answers, there were no verbal exchanges to reach a consensus.

The following episode, from observation 4, presents a pattern seen throughout the observations of working individually despite being directed by Ms. L to work collaboratively. The inquiry task was to determine the better deal between a 10-inch pizza for \$9.50, a 14-inch pizza for \$13.50, and a 16-inch pizza for \$15.00. Ms. L began by introducing the task, ensuring that students understood that the 10, 14, and 16 referred to how wide the pizza was, not the number of slices, or area, or circumference. Students were given 20 minutes to work with their table to determine which

pizza was the better deal. During the introduction to the problem, Trinity and Austin worked independently, trying to solve the problem, and not attending to the whole class discussion. They were not redirected to the launch of the lesson and the generation of strategies to solve the task. This was a high cognitive demand, group-worthy task on which students were explicitly instructed to work collaboratively with their table mates. The small-group portion of the lesson is chronicled below, with the full transcript in Appendix F.

After 14 minutes of launching the task, groups were released to work collaboratively on the task. Trinity and Austin continued to work independently. After a few minutes, Trinity looked at Austin's paper to see if she was on the right track. There was no discussion between them despite different approaches to the task. Ms. L stopped the class to review the two formulae groups should be using to solve the task because three of the five groups were struggling to begin the task. Students were directed to divide the work, with each person finding the circumference and area of one pizza. Once that was accomplished, students were told to find the ratio of circumference to price and area to price to determine the best deal. This enactment of the lesson shifted the cognitive demand of the task from doing mathematics to procedures with connections to meaning (Stein, Smith, Henningsen, & Silver, 2009). Ms. L had reduced the cognitive demand and group-worthiness of the task by routinizing the problematic aspects of the task.

After the direct instruction, Ms. L circulated the room, stopping to assist groups as needed. When she approached Trinity and Austin, Ms. L asked the pair if they had a method for how they were going to compare all of them [the 3 pizzas]. Trinity shook her head no and Ms. L asked them to talk to each other about possible strategies. She said, "After you get the measurements of all the circumferences, before you start to really decide which one is a better deal than the other, talk about what you are going to use as your criteria. OK?" Austin agreed. After Ms. L left, Trinity and Austin

continued to work independently until they reached an impasse. They still did not speak to each other about what they had tried and did not work. Both students sat quietly until Ms. L gathered the students' attention for a whole class discussion.

This episode was chosen because it depicts Trinity and Austin's reticence to exchange ideas. Ignoring the directive from Ms. L to talk to each other about the problem, neither Austin nor Trinity initiated a conversation. They continued to work independently. Even when they had completed the task, there was no discussion between the two about what they found or how they were going to determine which one was the better deal. The daily quality of discourse rating for this lesson was zero because the group members did not interact with each other.

Throughout the lesson observations, it appeared that established norms for this dyad were to work independently, as quickly as possible, and to sit quietly until Ms. L was ready to summarize the lesson. During whole-class discussions, both students eagerly and confidently engaged in IRE patterns with the teacher. During small-group inquiry, both students attempted to have Ms. L affirm their work or direct them on the next step. Sometimes Ms. L redirected their questioning back to the other partner. Other times she interacted with Austin and Trinity individually. There were inconsistencies in the expectations of how this dyad should interact with the teacher during small-group inquiry.

Besides pedagogical moves by Ms. L that allowed the pair to not engage in small-group discourse, there were student beliefs that come into play. Given their positive mathematics identities, reported academic success, and request to work with each other all year long, one might expect more dialogue between Austin and Trinity than what was observed. Austin held some non-availing beliefs about the nature of mathematics and his identity in mathematics that may have impeded his participation in small-group discussions. He believed that learning mathematics

consisted of memorizing steps and that if he was to understand the mathematics, it would make sense to him the first time he heard it. Austin also believed that the teacher was the source of knowledge and that working on difficult problems paid off for only the “really smart” students. The uncertainty of open-ended tasks in conjunction with his beliefs that he would understand the first time and could memorize the steps needed to solve the task may have impeded Austin’s participation in collaborative inquiry. These beliefs were evident when Austin did not ask Trinity about problems, rather he waited for the teacher to assess his work. If Austin could complete the task quickly and without questions, then it indicated to him that he was good at mathematics.

Trinity held non-availing beliefs that may hampered her participation in small-group discussion. Trinity believed that it was more important to be able to solve a problem quickly than to spend time trying to figure out the problem. If Trinity stopped to discuss points of confusion with Austin, it may have signaled to her that she was not good at mathematics. Understanding the first time, solving problems quickly, and the teacher being the source of knowledge were all non-availing beliefs that may have influenced how this pair interacted during small-group work. Both students worked diligently to be the first group in class to complete the work, but rarely interacted with each other. Attempts by Ms. L to have the pair engage in student-student discussion was met with silence. Established classroom norms, implementation of the lesson, pedagogical moves by Ms. L, and students’ MRB were factors in the lack of discourse between these two students.

#### **5.2.1.2 Elasha and Hillary**

Elasha and Hillary were good friends and anytime there was group work, the two opted to work together. Elasha held availing beliefs about the nature of mathematics and held a growth mindset. Hillary held slightly availing beliefs about the nature of mathematics and held a fixed

mindset. Both students held non-availing beliefs about their MID, MSE, and MINT (see Table 37). Both students held negative composite mathematics identity beliefs, in the lowest 10% of survey respondents. Elasha reported earning a B and Hillary an F on the last class assessment.

Table 37 Mathematical Beliefs of Elasha and Hillary

	Gender	Grade	NOLM	FA	CID	MID	MSE	MINT
Elasha	F	7	4.2	5.4	2.1**	2.6**	2.0**	1.8**
Hillary	F	7	3.9	2.9**	1.0**	1.0**	1.0**	1.0**

\*\* non-availing beliefs

Unlike Trinity and Austin, Elasha and Hillary routinely engaged in conversation during small-group inquiry. The discourse, however, was often off topic. When Elasha and Hillary were on topic, the discourse between them was of a low quality, focusing on procedures with little justification of thinking. Hillary typically turned to Elasha for procedural assistance, with little collaboration on or discourse about the mathematical tasks.

During a lesson observation on developing the formula for the area of a circle, there was some mathematical discourse between Elasha and Hillary, although the discourse was mostly of a lower quality, focusing on how to complete the task. See Appendix F for the complete transcription. Elasha and Hillary both created circles on their papers. Hillary made a mark on the edge of the circle to mark where she was going to draw her diameter but then stopped and said, “I don’t know how to do this.” Elasha let Hillary know that she did and showed Hillary how to mark off the diameter using the cube. Hillary mimicked the procedure. Each girl stated their diameter before continuing the task using different strategies. Hillary used the cubes to go around the perimeter and then filled up the circle. Elasha created rows of cubes above and below the diameter. After a few minutes, Hillary stated, “This doesn’t work to fill it in.” Elasha concurred, “It doesn’t work filling it

in, you have gaps.” Both girls then stopped and waited for Ms. L to lead the class in how to fill the circle with the 1-cc cube.

During this activity, Hillary felt comfortable enough to express her confusion as to how to start filling in her circle. Elasha quickly helped Hillary to get going. This episode was rated as low-quality discourse with a focus on how to fill in the circle. If the pair had discussed the advantages and disadvantages of the two different strategies for estimating the area of the circle, the discourse would have been rated at a higher quality level. Instead, Hillary expressed frustration with her method and copied Elasha’s work. There is no discussion, reasoning about the problem, or justification for the procedures used. This discourse example was representative of the lesson observations and served to illustrate the quality of the discourse in this pair.

When unsure of how to start a task, Elasha and Hillary did not discuss possible strategies. Both students waited until Ms. L, or the class, provided a clear strategy. This appears to be an established classroom norm as other groups in the class were observed to wait until they were told a direct procedure to follow before beginning the task. Unless prodded by Ms. L, neither Elasha nor Hillary discussed their reasoning during small-group inquiry. Additionally, neither student volunteered during whole-class discussions. Hillary acquiesced to Elasha’s role as the more knowledgeable of the two and deferred to her problem-solving strategies. When Ms. L interacted with the pair, it was to provide specific directions. Ms. L did not ask the pair what they had tried or what they had discussed. The teacher-student interaction was uni-directional with Ms. L retaining the locus of authority and knowledge.

Given their negative mathematics identities, Hillary’s lack of previous success on assessments, and an established classroom norm that the teacher would routinize the problematic aspects of the task by providing a clear procedure to follow, the quality of discourse between the

two students is not unexpected. Hillary held some strongly non-availing beliefs about mathematics and her identity in mathematics. She believed that learning mathematics was not about understanding the problem but rather achieving the correct answer, and that time spent on understanding why a solution works was not time well spent. This belief was evident when she sought procedural guidance from Elasha and did not ask why Elasha was approaching a task a certain way. Hillary had a fixed ability mindset, thinking that students are either born good at mathematics or not. According to her survey responses, hard work did not improve mathematical ability and working on hard mathematics problems was only beneficial for the “really smart” students. Hillary responded on the survey that neither herself, her peers, her teacher, or her family saw her as a “math person.” Given those beliefs, it is not surprising that Hillary stopped her problem-solving attempts as soon as she felt any struggle. If she believed she was not a math person or good at math, then hard work would not benefit her. Additionally, the pedagogical choices of Ms. L to routinely stop the investigations rather than let groups continue to productively struggle served to reinforce Hillary’s negative beliefs. She did not receive messages from her teacher that gave her permission to struggle.

Elasha firmly believed that success and understanding in mathematics was a product of the effort invested in problem-solving, indicating a growth mindset. Non-availing beliefs held by Elasha included that the teacher was the source of knowledge and that it was acceptable to not understand why an answer was correct. Additionally, Elasha did not believe that students could think through a problem together until they could reason why the answer was correct. These non-availing beliefs were evident in how Elasha waited to be told by Ms. L what algorithm to use and did not engage in discussion with Hillary about solution strategies. While Elasha did not see herself or think her peers and friends saw her as a math person, she believed that her parents and teacher

did see her as a math person. Despite earning above-average grades, Hillary's negative mathematics identity beliefs may be founded in the tracking at AMS or by teacher decisions about the implementation of tasks.

The lack of discourse between the Elasha and Hillary when they did not know how to start a task is reflective of their negative beliefs about the teacher as the source of knowledge, hard work not increasing ability, working on difficult problems only being productive for the "smart" students, and students not being able to think through a problem to come to a consensus. Neither student held the belief that they could be the source of knowledge nor that they could try different trajectories to arrive at a solution. This non-availing belief may have been established after of nearly a year of traditional direct instruction, as described by Ms. L, and an established pattern of being allowed to wait for Ms. L's input before beginning or continuing a task. Ms. L's routine lowering of the cognitive demand of the task by routinizing the problematic aspects of the task reinforced these non-availing beliefs. It is also conceivable these may have been beliefs that were established in elementary school and were a carry-over from the elementary school.

### **5.2.1.3 Comparison of the Groups in Ms. Luurtsema's Classroom**

The lesson style in the units of observation in Ms. L's class was a departure from the norm, with the observations occurring after state testing. Ms. L described lessons throughout the year as a blend of direct instruction with some inquiry. The observed geometry and probability lessons were inquiry-based, with a mean overall lesson observation rating of just below average ( $\bar{x} = 3.0$ ). Even with the move towards inquiry lessons, the level of mathematical discourse in the two groups during small-group inquiry was minimal to non-existent. This suggests a residual pattern of behavior established during traditional instruction.

Trinity and Austin were compliant in completing classroom tasks but did so independently. They looked at each other's work, predominantly without speaking, to confirm they had the same answers. Hillary and Elasha frequently were passive about their learning, waiting to be told the next procedure by Ms. L. The few times they were actively participating in the inquiry, Hillary expressed her confusion with the mathematics and Elasha attempted to help Hillary. Elasha was more compliant than Hillary in completing work.

There were few instances where the groups interacted on a collaborative level, discussing their thinking, or justifying their answers. At best, the pairs checked in, for the most part by only looking at their partner's paper, to ensure that their responses were correct. Pairs mirrored each other in their interactions and discourse, even though their mathematics identities were markedly different. They did not engage in higher quality discourse, in which they reasoned through answers, justified their thinking, or generalized about the mathematics.

Each group member held some non-availing beliefs about the NOLM that are connected. Austin and Hillary believed that working on difficult problems was beneficial for only the "smart" students. Austin and Elasha believed the teacher was the locus of authority and knowledge. Elasha and Hillary believed that arriving at the correct answer was more important than understanding the mathematics behind the algorithms, and Austin believed that mathematics was the memorization of steps. These non-availing beliefs have the potential to undermine investigative mathematics if the classroom environment does not support productive struggle. On some level, each of these students held beliefs that "hard" math was for "smart" people, and that if one memorized the steps needed to solve a particular type of problem, then the conceptual understanding behind the mathematics was not as important. Additionally, the teacher was the source of mathematical knowledge, not students.

In this classroom, the enactment of the curriculum and established classroom norms supported these non-availing beliefs. Tasks were over-scaffolded, tasks routinized, and group productive struggle was minimized. Groups were not treated as single entities, rather Ms. L addressed individual group members. Students were aware that if they waited enough time, Ms. L would provide an algorithm to solve the task. These teacher-content and teacher-student interactions negatively affected the student-content and student-student interactions.

### **5.2.2 Quality of Discourse in Mr. McGinnis's Classroom**

Observed lessons in Mr. M's classroom were rated below average for SBMI. Two lessons followed a direct instructional model, three a hybrid instructional model, and three an SBMI model. Five lessons were planned for investigative tasks to be an essential component of the lesson. However, only half the observed lessons were planned for substantive student-student interactions. In those lessons, students collaborated on investigative tasks some of the time. Other times, students work independently on the tasks. After launching the task, Mr. M would say, "You may work independently but check your work with your group before moving on." Statements such as this gave groups permission to not work collaboratively on the tasks and to not engage in high-quality discourse. Observed lessons in Mr. M's classroom were over-scaffolded during the launch phase as well as during the implementation. Mr. M would guide students through the process to the next step rather than referring the student back to their group to generate ideas. When students shared strategies and ideas, Mr. M would accept the strategy or idea as correct and then say, "That works, but I would..." or "I think you should do it this way instead." The established classroom norms gave praise to students who finished quickly and accurately. Alternate strategies were not routinely sought out nor celebrated. Additionally, the groups were not accountable for settling on a mutually agreeable strategy, nor were they accountable for explaining the method chosen by the group.

Sixteen students participated in the beliefs survey. Eight students held neutral identity beliefs, and seven students held positive identity beliefs. No students held negative identity beliefs.

Examining the dialogue among students during small-group inquiry indicated that the small-group discourse in this classroom varied according to the group. The daily discourse quality ratings for two focus groups are found in Table 38. One group consistently engaged in mixed to high-quality discourse during small-group inquiry, despite the absence of planning for substantive student-student interactions. The other group did not participate in small-group discourse in three of the four lessons that were not planned for student-student interactions. They participated in low to mixed-quality discourse in the other lesson not planned for student-student interactions and the four lessons planned for such interactions.

Table 38 Observation Summary of Small-group Discourse in Mr. M’s Classroom

Observation	1	2	3	4	5	6	7	8	Average
AJ, Sam, Marc	2	2	3	2	3	3	3	1	2.4
Nola, Luis, Kaitlin	0	0	2	1	1	2	2	0	1.0
Instructional Model	D	D	H	S	H	S	S	H	
PSSI	N	N	N	Y	Y	Y	Y	N	

D = direct instruction H = Hybrid Model S = SBMI PSSI = Planned student-student interaction

Below is the in-depth analysis of the discourse practices of the discussions occurring among the two groups during small-group inquiry, with events chosen as representative of the quality and quantity of discourse. The first two observed lessons followed a direct instruction model. The other observed lessons followed either a hybrid model, where students completed an investigatory task before receiving direct instruction, or an SBMI model. Lessons one through three and lesson 8 were not planned for substantive student-student interaction.

### 5.2.2.1 Nola, Luis, and Kaitlin

Nola, Luis, and Kaitlin worked together as a triad for this study. Mr. M routinely reassigned groups throughout the year, so it was not the first time some of them had worked together, but it was the first time all three had worked together as one group. All three group members held availing or strongly availing beliefs about mathematics ability and the nature of learning mathematics (see Table 39). Furthermore, all group members held high-neutral to positive mathematics identity beliefs. Of the three group members, Kaitlin ranked the lowest for all composite identity beliefs. Luis held low-neutral beliefs about his MID while Nola held high-positive identity beliefs. Group members reported earning an A or B on the most recent assessment.

Table 39 Mathematical Beliefs of Nola, Luis, and Kaitlin

	Gender	Grade	NOLM	FA	CID	MID	MSE	MINT
Nola	F	7	4.9	5.0	5.8	5.6	5.8	6.0
Luis	M	7	4.7	4.8	4.2	3.2**	4.3	5.3
Kaitlin	F	7	4.9	6.0	3.8	3.0**	3.8	4.5

\*\* non-availing beliefs

Over the course of this study, the quality of discourse between Nola, Luis, and Kaitlin was varied, with little collaboration on some days and more collaboration on others. The chosen

episodes highlight how the pedagogical choices in setting up the tasks impacted the discussions within this group, supporting group members to participate in a manner to which they were accustomed or in a manner consistent with SBMI.

The first example, from observation 4, is representative of the struggle the triad had in moving towards collaborative work. In this lesson, triads were directed to work with their group to complete a series of rigid transformations to move from a pre-image to an image using Desmos. The nature of this first activity was procedural, albeit set up as a puzzle. Students were provided the set of transformations to follow. Luis worked by himself, while Nola and Kaitlin worked with each other to quickly complete the activity. The second activity in this lesson (see Figure 9) was performing a given series of transformations on graph paper. Groups were again instructed to work together to perform the transformations and to determine if a different set of transformations could result in the same image. The complete transcription of the activity is found in Appendix F.

The episode begins with Luis asking Mr. M if they are supposed to be working together. Despite an affirmation, the group worked independently on this second task. Nola looked to see what Luis was doing and erased her work because it was different than Luis's. There was no conversation between Nola and Luis regarding the difference in their results. When finished, Luis put down his head and waited for the rest of the group and Mr. M to verify his work. When Mr. M came by to check on the group, Luis asked him to verify if his rotation was correct. Mr. M pointed out to Luis that he rotated around the origin and not the given point in the problem. After Luis finished corrected his rotation, Nola looked at it and realized that hers looks different again. This time she stated that Luis's rotation was wrong without any justification for her challenge. Luis maintained that his rotation was correct without any justification. This was the limit of the discourse between the group members. All other discourse occurred between student and teacher.

The only group interaction was when Nola asked to see Luis's work at, and then attempted to correct him. Each member of the group worked independently with little discussion among the members, except to check if their work was correct. Since there was limited student-student interaction which consists only of verifying answers without justifications, this segment was rated as a 0 for limited discourse.

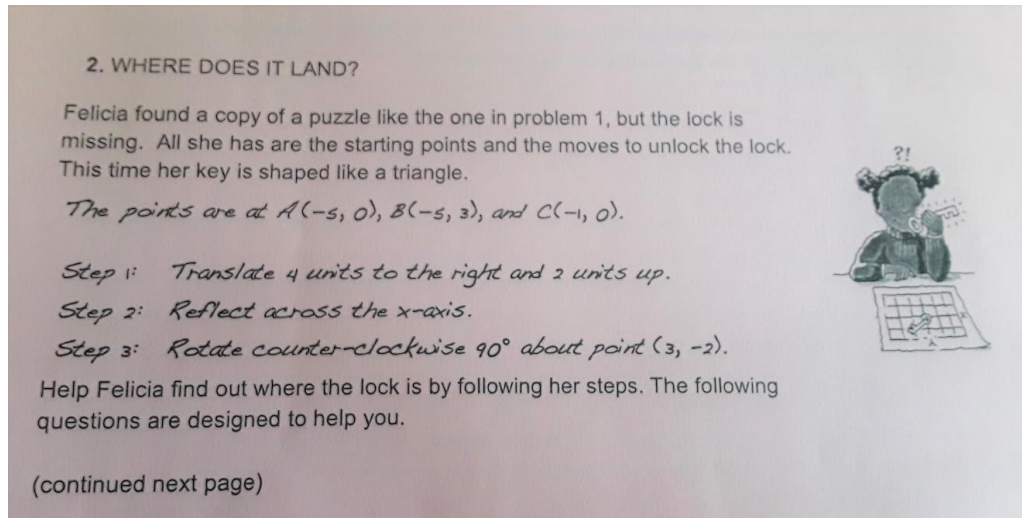


Figure 9 Where Does it Land?

This example sheds light onto several important aspects of this groups' discourse patterns. First, throughout the observation period, members of this group sought to verify their answers were correct. Rarely did group members explain the thinking and reasoning behind their statements. Secondly, Mr. M did not redirect the group members to collaborate when it was apparent that they were working independently. Initially, Mr. M stated, and confirmed to Luis, that groups were to work collaboratively on activity 2. The lack of redirection allowed this group to continue to do mathematics in isolation, limiting students' access to high-quality discourse. Finally, the dialogue about the mathematics was between student and teacher rather than student and group, or teacher and group, and continued in this manner for the entire lesson. When a group member had a question, the question was directed to the teacher. The teacher responded to the student individually

and not the group as an entity. Students were allowed to each use their own laptop for the Desmos activity and were provided their own graph paper to complete the task, thus interdependence within the group was not established. As the task was procedural in nature, the need for discourse within the small group was minimal. Groups were not required to come to a consensus, rather individuals shared out their solutions during the wrap-up.

Identify Similar Shapes Name \_\_\_\_\_

1.. CHANGING SHAPE

Your task: Work with your team to make predictions about what you could do to the coordinates of the shape at right to make it look stretched or squished, and what actions will keep the shape the same. Make predictions for the situations presented below.

- What do you think will happen if only the x-coordinates are multiplied by 3?
- What do you think will happen if just the y-coordinates are multiplied by 2?
- What do you think will happen if the x- and y-coordinates are multiplied by different numbers, like 2 for x and 3 for y?

Test the predictions your team made. On a sheet of graph paper, graph each of the shapes described below.

Figure 10 Predicting Transformations

The second example, see Table 40, is representative of how the triad transitioned to a mixture of lower- and higher-level quality discourse and a more collaborative working environment. In this observation, groups worked on predicting how a shape moves when dilated, and then graphing to verify the prediction (see Figure 10). Rather than working independently as depicted in the previous example, the triad worked collaboratively to make meaning and to generalize movements when the x- coordinates were multiplied by a coefficient, when the y- coordinates were multiplied by a coefficient, and when the x- and y-coordinates were multiplied by different coefficients.

Table 40 Nola, Luis, and Kaitlin Predicting Transformations (Observation 6)

Line	Time	Speaker	What is said ( <i>What is done</i> )	Small- group Discourse rating
52	20:00	Mr. M	Begin on the first page working with your table group. Do part 1. You will need the graph paper. You have 4 minutes and then we will review as a class. <i>Nola gets graph paper for the group.</i>	3
56		Mr. M	Make predictions about what you could do to the coordinates of the shape at the right to make it look stretched or squished and what actions will keep the shape the same. <i>Group reads the first page.</i>	
59		Nola	What if we only multiply the x-coordinates?	
60		Luis	It would move to the right [prediction]	
61		Nola	Okay.	
62		Kaitlin	Hold on. Let me graph this.	
63		Nola	If we multiply them all by 3, it is going to go up and to the right because....	
64		Luis	It just goes to the right [challenge]	
65		Kaitlin	What do you mean? [question]	
66		Luis	It says only the x [is multiplied by 3] [justification]	
67		Nola	Yes. It will be to the right and...	
68			If y-coordinates are multiplied by 2, it will move up [statement]	
69		Luis	What if you multiply x by 2 and y by 3?	
70		Nola	Then I think since y is multiplied by a larger number, 3, it will move farther up and only a little bit to the right. [justification]	
71		Kaitlin	OK. Also, I think that if the x...	
72		Luis	Taller and wider are my thoughts. [prediction]	
73		Nola	I think it is going to go a little to the right. [prediction]	
74		Kaitlin	Yes, it is going to go right but it is also going to stretch a little, umm. <i>Shows with hands growing diagonally.</i> The y is multiplied but the x is not so it will get taller and narrower. [justification] <i>Gestures with hands.</i>	

In this example, all three group members participated in the high-quality discussion. This task was a group-worthy task that promoted discourse within the group as students needed to make predictions about the results of the transformations prior to conducting them. While there was a “correct” answer, the task was phrased as “What do you think will happen. . . .” with the opportunity to graph the transformation after the predictions had been made. The discourse was rich, with members justifying their responses and challenging each other’s answers when the given answers did not make sense. There was a fundamental shift from the teacher as the locus of knowledge to the group negotiating and constructing their own knowledge. The triad negotiated disagreements without asking the teacher for the “correct” answer.

In this example, Mr. M had removed himself from the group discussion. This pedagogical move forced the group members to rely upon each other and provided them with access to higher-quality discourse. Mr. M also prefaced the task with a command for students to work with their groups. The cognitive demand of this task was higher, 4-doing mathematic, than the task in the previous example, 2- procedures without connections. All these teacher-content and teacher-student interactions helped this group work collaboratively and engage in higher-quality discourse.

Subsequent observations found the group working as individuals, reverting to the first example for quality of discourse. Reasons for the reversion were two-fold. First, Mr. M was inconsistent with his expectations. In the very first lesson from this CPM unit, observation 3, Mr. M let groups make the decision to work collaboratively or independently. This set the precedent that collaboration was not essential. Given that lessons prior to the study followed a direct instruction model, per Mr. M, the first experience with SBMI did not impress upon students the expectation for collaboration. In subsequent CPM lessons, observations four through seven, Mr. M directed students to work with their group. Some lessons, he enforced this expectation, such as in

observation 6. Other lessons, such as in observation 4, he did not. A second possible, though less likely, reason for the reversion was a slight shift in students' beliefs. Beliefs are slow to change (Pajares, 1992). The group may have begun the shift to valuing collaborative inquiry but then reverted to independent work during subsequent observations.

Despite holding high neutral or positive mathematics identity beliefs and reporting receiving an A or a B on the most recent class assessment, Nola, Kaitlin, and Luis struggled to consistently engage in high-quality discourse when directed to do so by Mr. M. All three group members held non-availing beliefs about the teacher or textbook as the locus of authority and knowledge. This belief was evident in many instances, both early and late in the observation cycle, when Mr. M was called to verify that the group was on track or if an individual's answer was correct. Both Luis and Nola held non-availing beliefs that justifying answers was not as important as obtaining the correct answer in mathematics. Kaitlin, on the other hand, focused on the procedure and justifying why the answer made sense despite slightly believing that one could successfully solve a problem without understanding the algorithm. Luis and Nola's non-availing belief about justification was seen when the two would check their answers but then not discuss why there might be a difference in the answers. Luis was concerned about his answers being correct, but when pushed by Kaitlin would justify his reasoning. Luis and Nola also held non-availing beliefs that speed in problem solving was equated to being good at mathematics. Nola, more so than Luis, focused on solving the problems quickly and frequently gave directives to speed up the process. Whether these beliefs emerged from or were reinforced by the norms of the classroom is not known. Kaitlin, holding availing beliefs for all NOLM and FA indicators except the aforementioned belief, was alone in her key NOLM and FA availing beliefs. Although she tried to engage the group in higher-quality discourse, she was not always successful.

This mixed gender group operated under the established norms of the classroom. They worked, for the most part, independently and silently. Speed combined with accuracy was of the utmost importance. This group did not adapt well to the change in the pedagogical focus of the classroom from teacher directed to student/group inquiry. The established norms were so ingrained in this group that even when a task was group-worthy, such as in observations six and seven, and even when directed by the teacher to work collaboratively, the group struggled to engage in the high-quality discourse associated with SBMI. There did not appear to be an issue of status of one student dominating the group. Instead, it was understood and accepted that mathematics was an individual endeavor and the purpose for discourse was to verify that one was correct or to ask what the next step in the process was. Discourse was not seen as a tool for developing deeper understanding.

#### **5.2.2.2 AJ, Marc, and Sam**

AJ, Marc, and Sam worked together as a triad for this study. Like the other triad, it was the first time all three had worked together as a group. All members of this mixed gender triad held availing or strongly availing beliefs about mathematic ability, the nature of mathematics, and mathematics identities (see Table 41). Their CID beliefs for 5.0 to 5.5 places all members in this group in the top 15% of survey respondents.

Over the course of the observations, the discourse between AJ, Marc, and Sam was less varied than the discourse between Nola, Luis, and Kaitlin, with there being more collaboration among this triad. The episode to follow is indicative of a higher quality discourse present during small-group inquiry as compared to the same segment between Nola, Luis, and Kaitlin.

Table 41 Mathematical Beliefs of AJ, Marc, and Sam

	Gender	Grade	NOLM	FA	CID	MID	MSE	MINT
AJ*	M	7	4.8	4.5	5.5	5.4	5.3	5.8
Marc	M	7	4.4	5.0	5.0	5.0	5.5	4.5
Sam	F	7	5.1	5.5	5.5	5.0	5.5	6.0

\* Participated in interview

The exemplar for AJ, Marc, and Sam’s group comes from observation 2, where students were directed to work independently on graphing inequalities. This group engaged in mixed-quality discourse, see Appendix F, despite being directed to work independently. As a comparison, Nola’s group engaged in no discourse, see Table 38. The lesson began with Mr. M providing guided notes on graphing inequalities. Mr. M gave the class two minutes to graph the two practice inequalities. When all three students were done, they checked in with each other even though Mr. M did not direct them to do so. Sam asked for clarification from the group, “If it is filled in, it is included? And if it is open, it is not?” This was a question for her own clarification that could have been answered with a simple yes or no from her group. Instead, AJ explained to Sam, “If it is open, it is not included. You don’t fill in the circle because that number doesn’t work. It doesn’t make it equal. It is like the center of a donut, there isn’t anything there to eat.” Sam continued with her questioning of the group about the inequality,  $-3x < 39$ . AJ began to explain his procedure when Marc adds that he solved it differently. The group reasoned that they would arrive at the same solution regardless of which strategy was used. In this case, the group was making connections between the two strategies to further their understanding of solving inequalities. One method was not privileged over the other.

This example illustrates the functionality of this group. Even when not explicitly directed to work together and support each other, Marc, AJ, and Sam engaged in small-group discourse to make meaning of the mathematics. No one person in the group had a higher status. Group members felt comfortable asking each other questions to gain greater understanding. An example of the

consistently higher quality discourse among the group members is found in this next snippet, based on Figure 11 and complete transcript found in Appendix F.

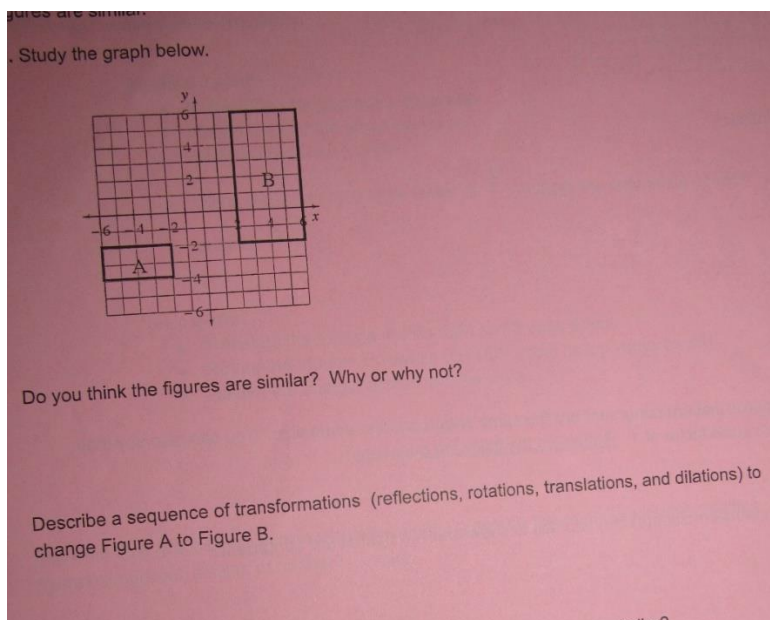


Figure 11 Dilations Observation 7

In observation 7, groups are tasked to work collaboratively to determine the sequence of transformations, including a dilation, to change Figure A to Figure B. Marc expressed confusion as to how to move from Figure A to Figure B. Sam suggested, “We have to reflect it over the y-axis. And then move it 2 units up along the y-axis. Then multiply by 4.” This statement was immediately added to by AJ:

Look Marc... if we reflect it here. *(Points to y-axis on Marc’s paper.)* It is going to be here. *(Outlines it on graph.)* Then move it 2 units up. It is in this box. *(AJ draws the lower 1/4 of rectangle B.)* Then if we multiply it by 4, then we would have 4 [boxes]. *(Draws in four boxes).*

With AJ’s justification of drawing the intermediary transformations, Marc was able to then join in the conversation. Sam asked what the dilation factor would be, and AJ responded, “2y, 1x because

x stays the same. If you just say dilate, it assumes all of the vertices are multiplied.” Marc was able to challenge AJ that it needed to be 4y because “2 would only get it to here (*pointing on AJ’s paper*). It needs to be 4 to get it all the way.”

In this episode, the group struggled productively to determine the necessary transformations. While they did not find an exact series of transformations, every member of the group contributed and offered suggestions for a solution. No solutions were arbitrarily dismissed, but rather were justified with diagrams or further explanations. The group determined that there was an issue with vertices with negative coordinates prior to the dilation, causing it to grow “downwards” rather than remain stationary along the y-coordinates. Mr. M’s premature stopping of the task prohibited the successful completion of the task. During the whole class debriefing of the problem, no group had completed the task and Mr. M led students through his set of transformations to move from Figure A to Figure B. The small-group discourse was rich with conjectures, challenges, justifications, and reasoning for the full ten minutes that students are allotted to work on the task.

This mixed gender group moved past the established classroom norms and engaged in SBMI in a meaningful manner. In their group, doing mathematics meant that one needed to justify one’s reasoning. Additionally, there could be various solution trajectories for a given task. The teacher was not the locus of authority. Instead, each group member had the agency to validate or challenge the thinking of others. The group rarely called Mr. M over to verify answers or to decide which group member’s answer was correct. This groups’ norms allowed for the triad to engage in higher-quality discourse during small-group inquiry. This suggests that beliefs may have a role in how this group interacted with each other.

AJ, Sam, and Marc all held positive mathematics identity beliefs, including all the sub-constructs and indicators. This was different than Nola's group where Kaitlin did not identify as a math person and did not think her parents saw her as a math person. Marc was the only group member to hold the non-availing belief that the teacher was the locus of knowledge in the classroom and that someone who was good at mathematics solved problems quickly. AJ held the non-availing belief that if he was to understand something in mathematics that it would make sense to him the first time. There were no overlapping non-availing beliefs among the group members. Perhaps this phenomenon allowed the availing beliefs of the other two group members to override the non-availing beliefs of the one group member. While Marc might have looked to Mr. M for assistance, his group filled the gap as soon as they realize that Marc is confused. This action preempted Marc from seeking assistance from Mr. M.

### **5.2.2.3 Comparison of the Groups in Mr. McGinnis's Classroom**

The level of mathematical discourse in the two triads during small-group inquiry was markedly different despite all six students holding similar availing beliefs about the nature of mathematics and mathematics ability (see Tables 38, 39, and 41). They held slightly different mathematics identity beliefs. Luis and Kaitlin held high-neutral beliefs about their composite mathematics identities, and Nola, AJ, Sam, and Marc held positive composite mathematics identity beliefs. Additionally, Luis and Kaitlin held non-availing beliefs about being a math person. All six students reported either an A or B on the last class assessment.

The discourse between Nola, Kaitlin, and Luis was generally of a lower quality over the course of the observations. During procedural work, the discourse was either non-existent or limited to verification of answers with no justification, reasoning, or generalizations. During the activities

intended to promote collaborative work, the triad struggled to engage in high-quality discourse and work collaboratively. The established classroom norms, prior to SBMI, was that mathematics was an individual endeavor to practice what had been explicitly taught by the teacher. This group's norms mirrored those of the established classroom norms, with a focus on speed and accuracy over understanding. The discourse shifted to higher quality midway through the unit when the nature of the task was group-worthy but was not sustained in subsequent inquiry using group-worthy tasks. The pre-existing patterns of discourse were less malleable in this group in the presence of SBMI.

The regression to lower-quality discourse after attaining higher-quality discourse may have been a residual effect of established classroom norms as previously explored and/or student beliefs. Both Nola and Luis held non-availing beliefs about needing to justify their work and equating speed in problem solving to mathematics ability. There was less cohesion in the group work where understanding by all group members was not a priority. This group struggled to maintain the higher-quality discourse. Towards the end of the unit, when the work could be completed collaboratively, the triad worked independently. When resources were limited, such as in observation seven when there was only one set of manipulatives for the group to share, the group was forced to work collaboratively. One positive shift in behavior that seemed more durable was the change in the locus of authority. There was a shift from asking the teacher to verify all answers to checking within the group.

In contrast, the discourse in the mixed-gender triad of AJ, Marc, and Sam began with mixed-quality discourse and transitioned to and remained at a higher quality throughout the unit. During mostly procedural work (observations 1, 2, and 8), there was a lower quality of discourse with a focus on checking answers but with rare instances of checking in with Mr. M to verify answers. There was an emphasis on the entire group understanding the procedures and why they

worked. The triad was able to negotiate strategies and come to a consensus about answers. Once the instructional model shifted to the collaborative work, the triad transitioned smoothly into an equitable work and discourse stance. The triad was observed multiple times huddled over one member's paper, debating possible strategies, and pointing out justifications for their answers. This collaborative environment and higher quality of discourse was sustained for the duration of the unit and was a marked deviation from established classroom norms. The triad made connections to prior knowledge and made many predictions as to the outcome of transformations. Even when their predictions and strategies resulted in needing to backtrack, the triad did not assign blame but worked together to try a new route.

Given the same established classroom norms and sociomathematical norms, and given the same shift in instruction towards SBMI, the resultant discourse patterns of the two triads were of markedly different qualities. The discourse patterns of the mixed gender triad of Sam, Marc, and AJ were of a higher quality during the initial traditional instruction and progressed to a sustained higher level during SBMI. On the other hand, the discourse patterns of the mixed gender triad of Kaitlin, Nola, and Luis were of a lower quality during the initial traditional instruction and did not sustain at a higher level during SBMI. Their group norms for discourse and engaging with the mathematics mimicked the established classroom norms during traditional instruction. Individual non-availing beliefs about not needing to justify answers and speed in problem solving being equated with ability held by both Nola and Luis surfaced during small-group inquiry. These non-availing beliefs, supported by established norms, led to discourse that was focused on correct answers and finishing quickly rather than on sense-making, conjecturing, and generalizing.

Teacher-student and teacher-content interactions hampered collaborative work with high-quality discourse in Nola's group. At times Mr. M would address the group as individuals and at

other times as a single entity. Collaborative tasks were prefaced with permission to work independently with group verification of answers. Tasks were prematurely stopped, with Mr. M providing his preferred solutions. If students did not work quickly, they would not finish the task prior to whole class discussions. These pedagogical moves served to support non-availing beliefs that the teacher was the locus of authority and knowledge, that people who are good at mathematics problem-solve quickly, and that justifying answers are not important. Nola and Luis held some of these reinforced non-availing beliefs whereas AJ and Sam did not. Teacher moves in conjunction with pre-existing beliefs may explain the differences in the quality of discourse between the two groups.

### **5.2.3 Quality of Discourse in Ms. Kraft's Classroom**

Observed lessons in Ms. K's classroom were rated above average for SBMI ( $\bar{x} = 4.5$ ,  $SD = 0.53$ ). One lesson followed a direct instructional model, two a hybrid instructional model, and five an SBMI model. Six lessons were planned for investigative tasks to be an essential component of the lesson, seven lessons planned for substantive student-student interactions. In those seven lessons, students were on task and collaborated on investigative tasks. One lesson, when students were confused about the previous night's homework, was a whole class discussion and solving of the task, thus the one direct instruction lesson. After launching a task, Ms. K would direct students to either begin the task individually for five minutes and then work with their group or to complete the whole task with their group. During this group work, about half of the groups worked collaboratively and about half worked cooperatively. Lessons ended with groups presenting their findings to the whole class and explaining their reasoning.

There were two instances where Ms. K over-scaffolded the tasks for students. The first instance was in observation 3, as discussed in Chapter 4 - hybrid model section. The other instance

was the observation when students were confused about the previous night’s homework. Rather than allowing groups time to discuss their work and to share strategies, Ms. K immediately led a whole class discussion where she guided students through the problem. At times Ms. K would guide students through the process to the next step rather than referring the student back to their group to generate ideas. For different groups, Ms. K would refer the student back to their group, asking the group to explain again their reasoning. Student thinking and creativity were celebrated in Ms. K’s classroom. Ms. K was heard to remark, “I see many different strategies for expanding your quadratics. I love that you are using the method that makes sense to you.” When groups presented their work, Ms. K would ask if anyone had questions for the group, giving the group the authority in the discussions. She would also seek out alternate strategies and made connections between the strategies that groups used.

Fourteen students participated in the beliefs survey. Five students held negative identity beliefs, seven students held neutral identity beliefs, and two students held positive identity beliefs. This distribution of CID beliefs resembled the distribution in Ms. L’s classroom.

Table 42 Observation Summary of Small-Group Discourse: Ms. K's Classroom

Observation	1	2	3	4	5	6	7	8	Ave
Allen and Riley	2	3	2	1	1	2	3	1	1.9
Patrice and Yusef	1	1	2	1	1	2	2	1	1.4
Instructional Model	S	S	H	S	D	S	H	S	
PSSI	Y	Y	Y	Y	Y	Y	N	Y	

D = Direct Instruction H = Hybrid Model S = SBMI PSSI = Planned student-student interaction

Examining the dialogue among students during small-group inquiry indicated that the small-group discourse in this classroom was mostly of low to mixed-quality. The daily discourse quality ratings for two focus groups are found in Table 42. One group engaged in mixed to high-

quality discourse during small-group inquiry during five observations. The other group engaged in mix-quality discourse in three observations. In the remaining observations, both groups engaged in low-quality discourse. The discussion begins with Patrice and Yusef, the group with the lower quality discourse, and is followed by Riley and Allen.

### 5.2.3.1 Patrice and Yusef

Patrice and Yusef had worked together all year long. The two were good friends, enjoyed each other’s company, and shared similar interests in music, art, and drama. Patrice was born in Brazil, moving to the area when she was in early elementary school. She was bi-lingual in English and Portuguese. Patrice and Yusef held availing beliefs about the nature of learning mathematics and have growth mindsets. Patrice held negative CID beliefs and Yusef held low-neutral neutral CID beliefs, ranking the pair in the lowest 15% of the data set. Additionally, Patrice and Yusef held non-availing beliefs about their MID, MSE, and MINT (see Table 43). Yusef’s and Patrice’s MID beliefs, as low as Hillary’s and Elasha’s in Ms. L’s class, ranked in the lowest 5% of the data set.

Table 43 Mathematical Beliefs of Patrice and Yusef

	Gender	Grade	NOLM	FA	CID	MID	MSE	MINT
Patrice*	F	9	4.3	4.9	1.9**	1.0**	3.0**	1.8**
Yusef	M	9	4.0	3.9	2.6**	1.2**	3.3**	3.3**

\* participated in interview \*\* non-availing beliefs

Over the six weeks of lesson observations, Patrice and Yusef worked well with each other, as a rule working cooperatively rather than collaboratively. Patrice volunteered more than Yusef did during whole class discussions. During small-group inquiry, they engaged in a mixture of high- and low-quality discourse, although most of the discourse was low-level, or procedural, in nature,

regardless of the nature of the task. Even on tasks that were “doing mathematics”, the pair limited their discussions to the procedures necessary to complete the task. Early in the observation cycle, questions about their work were routinely referred to Ms. K, rather than to each other, as both appeared fragile with their understanding of quadratic functions. Over the course of the unit, there were less questions directed towards Ms. K, but she was still seen as the locus of authority.

Early in the observation cycle, the focus of observation 3 was writing a quadratic equation to model a lot in a subdivision based upon written descriptions of how the original square lot was changed. This episode is representative of the lower quality discourse between Patrice and Yusef and their interactions with Ms. K. After a brief introduction to the task, students were directed to work with their group to write the factored form representing the change in the lots’ dimensions and then convert from factored form to expanded form. Yusef and Patrice began the task working independently.

After a few minutes of group work, Ms. K stopped the class to provide whole class guided instruction for problem 1. After Ms. K modeled the procedure, Patrice and Yusef resumed their independent work. Patrice asked Yusef what to do if the problem says smaller. Yusef responded with, “You subtract.” This response kept the interaction at lower quality discourse with just a question on procedure and a simple answer. There was no justification associated with Yusef’s answer. Ms. K, overhearing the conversation, interjected with praise and then explained, “So, this is your length times your width.  $x + 3$  here and  $x + 4$  there (*pointing to diagram in Patrice’s notes*).” Yusef asked Ms. K, rather than Patrice, if he should multiply them. Ms. K’s initial insertion into the group’s discussion shifted the discourse from student-student to student-teacher. All subsequent questions during the small-group time are then referred to Ms. K.

Ms. K decision to pause the small-group inquiry after five minutes of struggle to detail each step in solving the first problem served to lower the cognitive demand of the task by providing students with a specific procedure. Had groups shared out their strategies, thus far, what worked, what did not work, and incomplete ideas, groups could have generated new ideas. In that manner, students would have retained the locus of authority and could have built upon each other's thinking. This diversion from the intended curriculum changed the nature of the task from one of inquiry, necessitating collaborative thinking of the group, to a task that could be solved individually or cooperatively.

Ms. K's intrusion in the discourse between Patrice and Yusef also reinforced her role as the locus of authority. The pair had not requested her assistance. Rather Ms. K was passing by, checking on groups. She began the interaction with praise for their work up to that point. Then she told them their next step, without them asking for the next step. Had she questioned the pair about their next move, Patrice and Yusef would be positioned as mathematically competent. It is possible that Patrice and Yusef interpreted this unsolicited direction as a signal that they were not mathematically competent to complete the task without assistance. It also reinforced some non-availing beliefs that the group held about learning mathematics and about their mathematics identities. Recall that Patrice shared traumatic mathematics experiences in middle school during student interviews.

In this second example of the discourse between Patrice and Yusef, collaborative work and higher quality discourse were present. The episode, from observation 4, began with Patrice and Yusef working on the warm-up of  $(x + 3)(x - 1)$ . Patrice struggled to begin the problem. Yusef explained that the problem can be solved the way they have done other problems, pointing to an area model template in his notes. In this way he related the current task to previous work. Patrice

challenged him that the problem did not involve multiplication. Yusef justified his initial response by explaining that the parenthesis meant multiplication. Yusef and Patrice did not maintain this higher quality of discourse throughout the lesson, when simplifying expressions such as  $(x + 3)^2$  or writing equations like  $y = (x + 5)^2 + 2$  in standard form. The group reverted to procedural statements, in accordance with the procedural nature of the task, to verify that their answers were correct. Overall, the observation was rated 1, mostly low-quality discourse, despite the first unit of analysis being rate 3, mostly high-quality.

There was a slight shift in the questioning pattern in this observation. After a public sharing of algorithms for moving from vertex form to standard form, Patrice and Yusef used their notes from the warm-up to complete the tasks in the lesson. While the choice of this warm-up helped to ensure the success of this and other groups, it served to routinize the task by providing an algorithm to solve the tasks. The decision by Ms. K to enact the curriculum in such a manner served to change the task from a potentially collaborative one, where students might have wrestled with how to move from vertex/factored form to standard form, to a task that was individual or cooperative in nature. This pedagogical decision likely served to keep this group's discourse at a superficial level. Additionally, Ms. K assigned only sections 1 and 2 from the task, the procedural portion of the task. Section 3 asked students to find a quadratic function that made a displayed graph. This portion of the task, seen in Figure 12, was more open-ended and lent itself to higher quality discourse. Students might have needed to multiple attempts to find the correct quadratic function for the graph. They would have needed to make conjectures as to how the shifts in the graph would affect the function. This pedagogical decision limited the activity to the more procedural portions of the task, thereby limiting the potential quality of discourse.

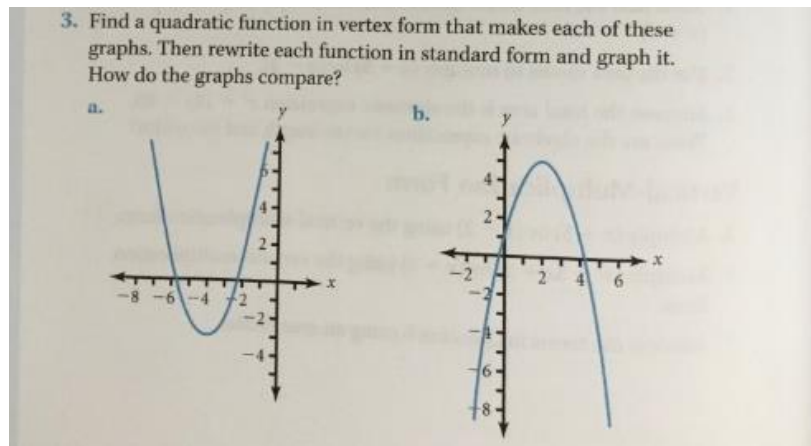


Figure 12 Writing a Quadratic Equation from a Graph

Survey responses revealed that Patrice and Yusef held slightly different NOLM beliefs. Both students held non-availing beliefs about mathematics being a process of memorizing algorithms. This belief is seen in the group's notetaking and referring to notes to assist in solving tasks. Patrice also held slightly non-availing beliefs about it being acceptable to not understand why an answer was correct, about the textbook or teacher being the locus of authority, and about only being able to learn mathematics when someone shows her how to solve a problem. These beliefs may have been rooted in negative elementary school experiences, as discussed by Patrice during her interviews. These beliefs were reinforced by Ms. K's over-scaffolding and routinizing of tasks. Additionally, when Ms. K worked with this group, she would tell them the next step they should perform, rather than eliciting their ideas for next possible steps. Patrice's beliefs about the locus of authority and needing to be shown how to solve a problem were evident in her constant pursuit of reassurance from Ms. K.

Patrice's apprehension about non-routine problems and the journey down various pathways to solve such tasks may have been a manifestation of her non-availing belief that speed in problem solving is equated to ability in mathematics. Yusef, who did not share this non-availing belief,

displayed similar apprehension. It is possible that both student’s ill-ease with non-routine problems stemmed from their belief that mathematics is a process of memorizing steps. Both students reported that mathematics makes them nervous, and that setback discouraged them.

### 5.2.3.2 Riley and Allen

Riley and Allen worked with each other most of the year. The two students got along well. Both students held availing beliefs about NOLM and FA. They held different mathematics identity beliefs. Riley identified as a math person with a positive mathematics identity. Allen held a neutral mathematics identity with lower scores on each of the subconstructs of identity than Riley (see Table 44). Both students held availing beliefs about their MSE.

Table 44 Mathematical Beliefs of Riley and Allen

	Gender	Grade	NOLM	FA	CID	MID	MSE	MINT
Allen	M	9	3.7	5.0	3.3**	3.0**	3.8	3.0**
Riley	M	9	4.5	4.7	4.4	4.4	4.5	4.3

\*\* non-availing beliefs

The discourse between Allen and Riley during small-group inquiry was of a higher quality than that of Patrice and Yusef (see Table 45). In the first observation, Riley and Allen explored shifts in a parabola when a constant was added to  $x$  before squaring and after squaring,  $x^2 + 5$ ,  $x^2 - 5$ ,  $(x + 5)^2$ , and  $(x - 5)^2$  (see Table 45). This task was a combination of Parabolas and Equations II and Parabolas and Equations III in CMP2. The implementation of the tasks aligned with the written curriculum, with little modification by Ms. K. The focus of Allen and Riley’s conversation was related to past experiences with drawing  $x^2$  [in Parabolas and Equations I] and creating tables. It also focused on patterns that they saw in the movement of the parabola based on changes to the equation.

In this example, the student-student discourse was of high quality, rich in connections and suppositions. Riley and Allen shared the work equally, with each contributing to the conversation. Both students generalized about the movement of the parabolas. When Allen and Riley were not certain about their results, they checked in with another group, rather than with Ms. K. This pattern of checking in with peers rather than Ms. K may be an indication that the pair believed that the locus of authority could reside in students as well as the teacher. Both students believed that they did not need someone to show them the exact procedures to learn mathematics, giving them confidence in investigative tasks. This type of higher quality discourse was consistent throughout the observations when the task from IMP2 is of a collaborative nature.

When the task from IMP2 was procedural in focus, Riley and Allen continued to discuss the task. That discussion, however, was of mixed quality with fewer generalizations. During the observation that focused on moving between vertex form and standard form of a quadratic equation, Riley and Allen worked independently but checked in with each other after each problem. The nature of the tasks was procedural, leading to an assumption that the discourse would be of a lower quality, focusing on procedures. That assumption was false, as the dyad engaged in mixed-quality discourse that included challenging, relating, and justifying. See Appendix F for a transcription of the discourse.

In this observation, Riley and Allen began by working independently. After completing several problems, they check in with each other to ensure they are solving the problems correctly. When their answer for problem 2A differs, Allen challenged Riley method as missing the  $bx$  portion of  $ax^2 + bx + c$ . Allen justified his challenge by referring to the warm-up problem and Ms. K's reminder to the class to not forget the middle term. Allen further justified his challenge by creating an area model for the multiplication. Allen was not able to persuade Riley and looked to

Ms. K as the arbitrator rather than seeking out the assistance of another group. The dyad related back to prior experiences and justified their reasoning in their attempt to make sense of the movement from vertex to standard form. The result was a mixed-quality discourse for this lesson rather than the low-quality discourse that was seen with Patrice and Yusef.

Table 45 Riley and Allen: Shifting Parabolas (Observation 1)

Line	Time	Speaker	What is said ( <i>What is done</i> )	Small- group Discourse rating
	15:10		<i>Riley takes the white board and creates a coordinate grid</i>	3
65		Allen	Let's use 1, 2, and 3 for our table.	
66		Riley	Shouldn't we also use negative numbers too, so we get the swoosh? [Relating]	
67		Allen	I guess. It needs to be accurate in creating the graph. <i>Riley draws the parabola based on Allen's table.</i>	
69		Allen	Why did you draw arrows [at the end]? [Question]	
70		Riley	Because it continues past -2 to 2. [Justification]	
72		Ms. K	<i>To group.</i> Explain how it changed. Be specific.	
73		Riley	It is the same, but it moved down 5. [Statement]	
74		Allen	It shifted all the points by 5 on the y-axis. Y is up and down. Let's check with the other fam [other group assigned to problem b] to be certain. <i>Group checks in with another group in the classroom</i>	
77	20:20	Allen	If you add, it goes up [Generalization]	3
78		Riley	And if you subtract it goes down. Do we have to do generalizations for the others? [Generalization] <i>They start the next two functions.</i> <i>Allen completes the table and Riley draws the parabola.</i>	
84		Allen	If you add inside the parentheses before squaring, it moves to the left. [generalization]	
85		Riley	And if you subtract inside the parentheses, it moves right. [generalization]	

Throughout the observation period, Allen and Riley engaged in discourse of mixed quality. There were three instances when their discourse is of a lower quality, focusing on answers. Two of these instances occurred when the IMP2 task was procedural in nature. The third instance was in

observation 8. The task in observation 8 was an application task mirroring the task in observation 7 when the discourse was of a higher quality. Groups struggled with the task in observation 7. In observation 8, the pair worked independently, following their notes from the previous lesson. The other five lesson observations revealed small-group discourse rich with reasoning, justifications, and connections to prior work.

Although Allen held mathematics identity and NOLM beliefs that were less availing than Riley's beliefs, he customarily took the role of leadership in this pair. He challenged Riley to justify his answers and to engage with the mathematics at a conceptual level. Both boys held non-availing beliefs about the teacher as the locus of authority but held availing beliefs that students that don't agree can reason through to find a solution. This contradiction in beliefs emerged as a pattern in that when they were confident in their work and they could agree, they checked in with other groups. When they could not come to a consensus, they checked in with the teacher to verify who was correct.

Contrary to their non-availing beliefs about mathematics being a process of memorizing the steps to follow, Riley and Allen actively engaged with the curriculum to generalize and to relate back to prior problems. Allen held very strong non-availing beliefs about the importance of understanding why a solution works or of conceptual understanding, as evidenced by his survey responses.

However, he routinely challenged Riley to justify why he used an algorithm. This was a contradiction in stated and observed beliefs but may be the influence of both their beliefs that it is important to justify statements in mathematics. The joint belief had the potential to override a single member's belief. It is also possible that the established classroom norms that students justify their responses helped support the evidenced behavior of justification.

### 5.2.3.3 Comparison of the Groups in Ms. Kraft's Classroom

There were more differences in identity beliefs of the four students than there were in their NOLM and ability beliefs. Patrice and Yusef held negative mathematics identities. Riley and Allen held high-neutral or positive mathematics identities. Looking within the NOLM variable at the individual indicators, there were similarities among the students' beliefs that may help to explain similarities discourse patterns. All four students held the non-availing belief that learning mathematics is mostly a matter of memorizing the steps to follow. This belief has the potential hinder students in an SBMI setting where the daily focus in on non-routine or investigative tasks. In the IMP2 curriculum, tasks that do not have obvious solution trajectories are often the focus of the lesson. Implementation of tasks is crucial in maintaining the cognitive demand. There were only two observations of Ms. K over-scaffolding and routinizing the non-routine tasks. She allowed time for groups to productively struggle with the tasks. She also expected groups to share out what they found and allowed for the class to build understanding together. One student, alone, was not asked to explain the solution, but rather multiple students were asked to participate in the generation of understanding. These pedagogical decisions helped to support students in their investigative tasks by providing the message that mathematics takes time and that several attempts may be needed to find a correct solution. This also supported students' belief that it does not matter how long it takes to solve a problem as long as they figure it out.

The way the dyads engaged with the mathematics followed similar patterns, but Riley and Allen engaged in a slightly higher quality of discourse than Patrice and Yusef. In all observations, both members of the dyads actively engaged in the mathematics and the discourse. Five of the lessons during the observation period included tasks that were of a procedural nature, shifting the interactions from a collaborative perspective to a cooperative one. However, this did not relegate the

discourse to lower quality. While some of the discourse was focused on statements, answers, questions, and explanations, there were instances with both dyads when the discourse included generalizing, justifying, challenging, predicting, and relating to previous work.

Patrice and Yusef were actively engaged with the mathematical tasks in every observation. Ms. K specifically instructed students to work with their groups on the day's task. Despite those directives, most observations witnessed Patrice and Yusef beginning the task separately. After a few minutes of individual work, one of the pair would either check in with the other regarding the answer/appropriate procedure or would begin a think aloud protocol. This transitioned into a dialogue focused on the steps needed to complete the task, with the discourse between them of a procedural nature. Only in observations three, four, and seven was the discourse of a mixed quality. The nature of the task in observation seven was an application task. The resulting discussion included some instances of challenging, relating, and justifying, as Yusef and Patrice struggled to apply their knowledge of quadratics to a real-life situation. The nature of the task in observation one was conceptual in nature and lent itself to higher quality discourse. However, this did not translate into higher quality discourse between Patrice and Yusef. The dyad needed to complete each of the squares to see the effect of adding a constant within the square or after squaring the variable. The focus for this pair was procedural, attending to graphing the quadratics properly from a table, rather than looking for patterns and making predictions.

Throughout the observation period, Allen and Riley enthusiastically engaged with the assigned mathematical tasks. The quality of their discourse mirrored Yusef and Patrice's for observations three through six, and eight. These were the lessons that scored lower for implementation on the lesson observation instrument, suggesting that Allen and Riley's discourse patterns varied with the enacted lesson more than Yusef and Patrice's. For observations one, two,

and seven, the quality of their discourse was higher than that of Yusef and Patrice. The tasks in observations one, two, and seven were of higher cognitive demand, doing mathematics, than the tasks in observations two through six. In these observations, Allen and Riley engaged in mixed or higher quality discourse, which was of a higher quality than that of Patrice and Yusef. In observation one, Allen and Riley were adept at creating a table and then graphing. After creating one graph for adding to  $x^2$ , they were able to make predictions for subtracting from  $x^2$ . This same ease in prediction occurred when adding and subtracting before squaring. As their efforts were not concentrated on creating the table and graph, like Patrice's and Yusef's were, they were able to spend time engaging in the predictions and generalizations that were the focus of the task.

In observation two, the difference in the quality of the dyads' small-group discourse was the greatest. The task in this observation was more procedural than the task in observation one. Groups were to write the formula for the graphed quadratics using vertex form (see Figure 13). Yusef and Patrice struggled with determining the vertices of the parabolas and as such had multiple attempts at writing the equation for just one parabola. They also struggled with the arithmetic associated with solving for  $a$  in the vertex form. Riley and Allen, on the other hand, did not struggle with determining the vertices nor with the arithmetic. They quickly solved for  $a$  and wrote their vertex form equations. This afforded them the opportunity to engage in a higher quality discussion during which they justified their results and compared the different formulae.

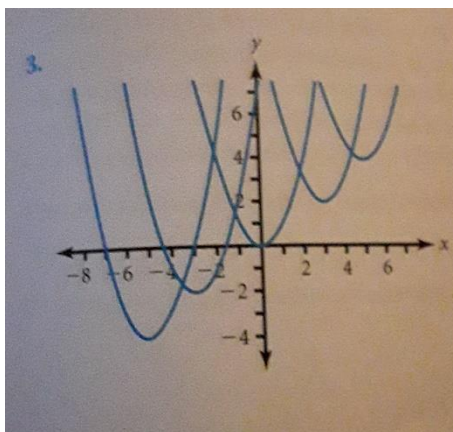


Figure 13 Observation 2 Task: Writing Vertex Form of Graphed Quadratics

There was little difference in the quantity of the dyads' small-group discourse. The difference lay in the quality of the discourse. Besides beliefs, the differences in the quality of the dyads' discourse could be attributed to their respective mathematical ability/confidence. During lessons when the task was non-routine, Yusef and Patrice needed to concentrate on the arithmetic, whereas Riley and Allen did not. Yusef's and Patrice's added layer of processing did not permit them to focus on higher quality discourse the way Riley and Allen could. When the nature of the task was procedural, with or without context, both groups engaged in similarly lower quality discourse.

#### 5.2.4 Quality of Discourse in Ms. Chenette's Classroom

Observed lessons in Mr. C's classroom were rated above average for SBMI ( $\bar{x} = 5.6$ ,  $SD = 0.53$ ). All lessons followed an SBI model consistent with the written curriculum's recommendation. Seven of the lessons were investigative in nature and all were planned for substantive student-student interactions. The student-student interactions were collaborative in nature and enhanced student understanding. Students were pushed for higher order thinking with Ms. C constantly

asking students to justify their reasoning. Groups were encouraged to interact with other groups when needing assistance.

Eleven of the students in the observed classroom completed the survey. Three held negative mathematics identity beliefs, seven held neutral mathematics identity beliefs, and one held positive mathematics identity beliefs. The distribution of CID beliefs was different from Mr. M's seventh-grade honors classroom. In this ninth-grade honors class, there were students that held negative mathematics identity beliefs, while no seventh-grade honors student in the observed classroom held negative identity beliefs.

Student discourse during small-group inquiry was generally of a varied quality (see Table 46). Each group engaged in high quality discourse for most of the small-group time during three observations. They also engaged in low quality discourse for most of the small-group time during three or four observations. Some of the lower quality discourse occurred during lessons of triangle construction, review of theorems, and the introduction to proofs, activities that were less investigatory in nature. Below is the in-depth analysis of the discourse practices of the discussions occurring among the students during small-group inquiry. The events were chosen as representative of the quality and quantity of discourse during those inquiries. Both groups presented similar patterns in the quality of their discussions.

Table 46 Observation Summary of Small-group Discourse: Ms. C's Classroom

Observation	1	2	3	4	5	6	7	8	Average
Sally and Kelly	1	1	3	2	3	3	2	1	2.0
Taz, Max, and Donald	1	1	3	1	3	3	2	1	1.9
Instructional Model	S	S	S	S	S	S	S	S	
PSSI	Y	Y	Y	Y	Y	Y	Y	Y	

S = SBMI PSSSI = Planned student-student interaction

### 5.2.4.1 Sally and Kelly

Sally and Kelly were good friends and were excited for any opportunity to work together. Both Sally and Kelly held growth mindsets and slightly availing beliefs about the nature of mathematics (see Table 47). Additionally, they both held non-availing beliefs about their mathematics identity, self-efficacy, and interest. Sally held a negative composite mathematics identity and Kelly held a neutral one, with Sally being on the upper end and Kelly on the lower end of the ranges. This placed the pair in the lowest 20% of survey respondents for CID. Both students reported B's on the last class assessment.

Table 47 Mathematical Beliefs of Sally and Kelly

	Gender	Grade	NOLM	FA	CID	MID	MSE	MINT
Sally*	F	9	4.0	4.6	2.6**	1.8**	2.8**	3.3**
Kelly	F	9	3.7	4.5	2.8**	3.0**	2.8**	2.8**

\* participated in student interview \*\* non-availing beliefs

Over the six weeks of lesson observations, Sally and Kelly work well with each other, engaging in mixed-quality discourse. The quality of discourse was evenly distributed between low, mixed, and high quality. Kelly appeared to have a better understanding of constructing proofs and took the time to ensure that Sally understood. Kelly also volunteered more than Sally during whole class discussions.

The first example of the discourse, from observation 3, found the pair working on finding the length of a diagonal of a rectangle with one vertex at the center of a circle and the opposite vertex on the circle (Figure 14).

Rectangle  $ABCD$  is inscribed in a quarter-circle as shown.  $BC = CE = 5$  units.

Can you accurately determine the length of diagonal  $\overline{AC}$  without measuring? Explain your answer.

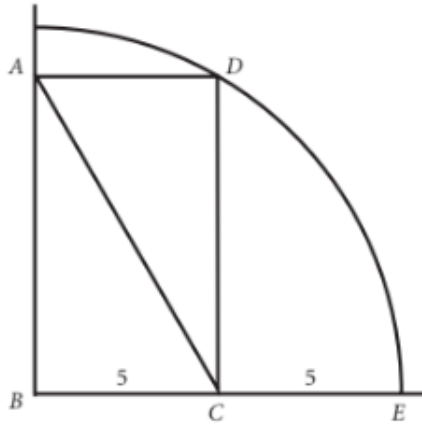


Figure 14 Rectangle Inscribed in a Circle

This problem was assigned as the homework problem. Like most other students in the class, Sally and Kelly were stumped when they worked on it individually, at home. During class, students were given five minutes to collaborate and reason through the problem with their group. The transcription of the entire segment is found in Appendix F. What follows is representative of high-quality discourse between Sally and Kelly as they struggled to find the length of  $AC$ .

The two students began by sharing what they tried at home. Sally suggested, “If these are 90 (*pointing to  $\angle CBA$  and  $\angle DCB$* ) then these have to be 90 too (*pointing to  $\angle ADC$  and  $\angle BAD$* ). They are all 90 because it is a rectangle” Sally justified her statement by relating it back to her knowledge of rectangles. This allowed Kelly to conjecture that they could use ASA to prove  $\triangle ABC$  was congruent to  $\triangle DBC$ . While this knowledge did not help the pair complete the proof, this exchange of ideas illustrates the culture of the classroom to take risks and share ideas, even if not fully formed.

This was not the intended lesson for the day. The intended lesson for the day was delayed when Ms. C determined that multiple students struggled with this proof individually. She made the decision to have groups discuss their false starts in solving the proof. As she walked around the room, she noticed that several groups were exploring if  $AC$  was an angle bisector of  $\angle BAD$  and  $\angle DCB$  or have decided that it was. Rather than correcting individual groups, she brought the question to the whole class and entertained their reasoning. The class determined that it would be an angle bisector if and only if the rectangle was a square. Additionally, not once did she assume the role of authority and tell them that  $AC$  and  $BD$  were both diagonals for the same rectangle and therefore equal in length. Rather, she allowed groups to productively struggle in remembering that relationship. While it may have been quicker and easier for her to point out the obvious, to some, relationships between  $BE$  and  $BD$  and between  $BD$  and  $AC$ , she allowed students the time to grapple with this non-routine task.

In the above episode, early in the unit on proofs, the discourse between Sally and Kelly was of a high quality. The discourse was filled with justification and reasoning to determine that  $AB = DC$  in the first part of the episode. They continued to work on the proof, although they were not successful in determining the length of  $AC$  because they did not recognize that  $BD$  was also a radius of circle  $B$ . Later in the same episode, the pair made connections to their understanding of isosceles triangles with one 90-degree angle to determine whether all intersecting angles of the diagonal of a rectangle were 45-degree angles. They made their thinking explicit by pointing to portions of their diagrams. One suggestion was not privileged over another suggestion in this dyad, allowing for full participation by both members. In this next example, the class was working on developing logical arguments to create proofs. Currently, students were using written proofs and column proofs, with whichever they felt more comfortable. Additionally, students had the option of

no scaffolding or some scaffolding where they were given statements for the proof, but not the postulate or theorem that supported the statement. The task for the day was to prove the Isosceles Triangle Theorem given only that AC was congruent to CB. The book gave a hint to start by finding the midpoint of AB and labeling it M (Figure 15).

## Proving the Isosceles Triangle Theorem

The Isosceles Triangle Theorem states:

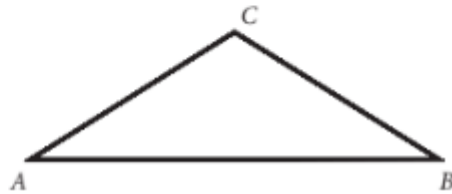
*If two sides in a triangle are equal, then the angles opposite those sides are equal.*

Does that seem reasonable?

Let's prove this theorem using our established postulates and theorems. We start by drawing a diagram and stating what is given and what is to be proven.

*Given:*  $\triangle ABC$  with  $\overline{AC} \cong \overline{BC}$

*Prove:*  $\angle A \cong \angle B$



### Another Way to Prove the Isosceles Triangle Theorem

There is often more than one way to prove a theorem.

1. Another way to prove the Isosceles Triangle Theorem starts by locating the midpoint of  $\overline{AB}$  and labeling the midpoint  $M$ . Which postulate justifies this?

Figure 15 Proving the Isosceles Triangle Theorem

Kelly and Sally struggled with the difference between creating a midpoint and assuming a point is the midpoint. Ms. C provided some “just in time scaffolding,” but did not over-scaffold their thinking process. She referred them back to each other to restate their reasoning and to listen to how their partner was reasoning. Sally and Kelly made several false starts in their attempt at the

proof but successfully completed it. Sally and Kelly generalized about isosceles triangles and the relationship between side lengths and angle measures. With statements about congruence, the pair challenged each other and justified their statements. This example exemplifies the expectation that students will persevere in their discussions as the creators of mathematical knowledge.

Despite not identifying as math people, this pair engaged in higher quality discourse on a routine basis, as evidenced by the above examples. Additionally, both students held the non-availing belief that mathematics is memorizing a series of steps. This belief is challenged by NSF-funded curricula by providing non-routine tasks. The pair also held the non-availing belief that two students cannot reason the correct answer when initially they disagree. This belief was not evidenced in their discourse. Both students engaged in justifications, conjectures, and generalizations as they work through their proofs.

Non-availing beliefs do not appear to have influenced the discourse between Sally and Kelly. Most segments of analysis of the discourse between Kelly and Sally were scored as mixed or high-quality. Established classroom norms and high quality SBMI appeared to promote discourse between Kelly and Sally even when they were unsure of the mathematics or disagreed. Rarely did they look to the teacher to confirm their reasoning despite their common belief that the teacher and textbook were the locus of authority. Kelly and Sally were expected to try multiple pathways until they completed their proofs. Ms. C asked for their thinking to help them clarify their processes and did not tell them the next step they should take. She did not step in and routinize the problematic aspects of the task. With her scaffolding of providing steps without justification, students needed to reason why that next step could be taken. The pair utilized the scaffolding occasionally. Every proof was expected to be solved publicly within the group, rather than individually and then checked with the group.

### 5.2.4.2 Taz, Donald, and Max

This group of three male students worked together at various times throughout the school year. Donald and Max played on the same spring sports team, so had a relationship that extended beyond the classroom. All three students held availing NOLM and FA beliefs. Max and Donald held neutral mathematics identity beliefs [CID = 4.1 and 4.2] while Taz held positive mathematics identity beliefs. Max and Donald were on the high end of the neutral range and Taz was on the low end of the positive range, indicating little difference between their mathematics identity beliefs. Their differences in subconstructs rested in their MSE and MINT beliefs, with Taz holding a lower MSE beliefs than his group and Maz holding a neutral mathematics interest. All reported A's on the last class assessment.

Table 48 Mathematical Beliefs of Taz, Donald, and Max

	Gender	Grade	NOLM	FA	CID	MID	MSE	MINT
Donald	M	9	5.1	5.4	4.2	3.2	5.5	4.0
Taz	M	9	4.7	4.8	4.5	3.8	4.5	5.3
Max*	M	9	5.0	4.4	4.1	3.8	5.0	3.5

\* participated in interview

Donald, Taz, and Max worked well together. Donald regularly led the discussions with Taz and Max providing their perspectives on the problems. There appeared to be an open exchange of ideas, and if one group member did not understand the other two members, they restated their thinking to provide support. During whole class discussions, Max and Donald volunteered more frequently than Taz. When presenting their work to the class for discussion, all three group members participated equally. The quality of discourse over the observations was either low-quality or high-quality, with only one observation rated as mixed quality.

In this example, near the end of the observation period, the Max, Taz, and Donald were observed working on the proof in Figure 16. This was one of the proofs assigned for homework the previous night. After a quick check at the beginning of class, Ms. C realized that most students have struggled with this proof. She gave the class five minutes to work on the proof with their group, analogous to the situation in observations 3 in the previous section.

**2. Given:**  $LC = NC$ ,  $LE = AN$

**Prove:**  $\triangle LAC \cong \triangle NEC$

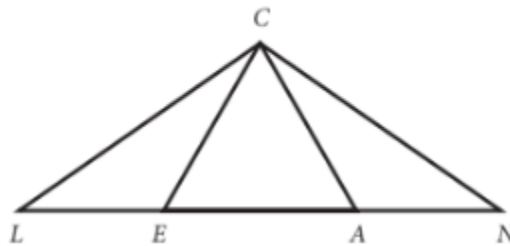


Figure 16 Proof Worked on by Donald, Taz, and Max

This segment of discourse was unique in that Donald did not come to lead the discussion with a solution in mind. In fact, he stated that he did not know where to go from the givens. Ms. C suggested that Donald restate the givens, and a possible proof trajectory emerged from the free exchange of ideas between the group members, see Table 49.

This segment of discourse is representative of the high-quality discourse of this group. The pattern of disagreement, challenging, agreement in this group was seen in all the observed lessons, except observation 2. The students were comfortable stating alternate views and they worked together to convince each other why their view was correct. This allowed for frequent episodes of mixed or high-quality discourse. Ms. C's role was to refocus the discussions back to the proof. This shifted the locus of authority from Ms. C to the group.

Table 49 Observation 7: Donald, Taz, and Max

Line	Time	Speaker	What is said ( <i>What is done</i> )	Small- group Discourse rating
29	9:25	Donald	On this one [#2], I just wrote down what was given and then I stopped. I wasn't sure where to go from there.	3
30		Ms. C	Can you give your group those statements? Maybe they can work with them [ <i>leaves</i> ]	
31		Don	LC = NC, LE = AN Given Wait, then can we say they are even? No. Wait, if we say 2 sides are equal, doesn't that mean that the angles are equal? [relating]	
34		Taz	What angles do you mean? [question]	
35		Max	These are opposites! [ <i>pointing at <math>\angle L</math> and <math>\angle N</math></i> ] [statement]	
36		Taz	The big triangle is isosceles. [statement]	
37		Donald	So, the base angles are equal [relating]	
38		Taz	Why?	
39		Donald	ITT [justification]	
40		Taz	Then we have SAS, right?	
41		Max	Only for triangles LEC and ANC...I am not sure where... [answer]	
42		Donald	We could say EC = AC. [statement]	
43		Taz	How does that help? [challenge]	
44		Max	Wait! Can we say EA = EA and so LE + EA = EA + AN? Like we had the other day with those algebra proofs? [relating]	
45		Donald	Yes. And then we can use SAS to prove it. [statement]	
46		Max	So, we have $\angle L \approx \angle N$ , LA = EN, and LC = CN. [justification]	

In this short proof, no one came to the table with a solution. Together, the triad related their understandings of isosceles triangles and congruent base angles to determine that  $\triangle LAC$  and  $\triangle NEC$  were congruent. At line 42, Donald built upon Max's answer that  $\triangle LEC$  and  $\triangle ANC$  were congruent to state "EC = AC." Max then built upon that new knowledge to establish LA = EN.

Max led the proof of SAS using  $\angle L$  and  $\angle N$ . Taz's questions to clarify his understanding enabled Donald and Max to explain their reasoning, thereby illuminating the proof for the triad.

The triad did not request assistance from Ms. C when they had no strategy to move forward. Ms. C overheard their statement that they did not know how to proceed and suggested that one of the group members read the givens aloud to the group. Then she walked away, allowing the triad to productively struggle over the task. In doing so, she shifted the authority to create knowledge from herself to the group. During the whole class discussion, groups shared different proofs that  $\triangle LAC$  and  $\triangle ANC$  were congruent. Differences were celebrated and analyzed to determine the efficiency and elegance of the proofs.

Max, Taz, and Donald held high neutral/positive mathematics identities. Donald and Max held lower CID beliefs than Taz. Max and Taz held the same MID beliefs, which were more availing than Donald's MID beliefs. Despite holding the most positive CID beliefs, Taz held the lowest MSE beliefs of the group. This lower MSE belief may be evident in the discourse patterns of the group. During small group discourse, Donald and Max did most of the justifying and relating while Taz did most of the questioning. Taz held non-availing beliefs about the speed of knowledge acquisition and the appropriateness of rote memorization of algorithms. These beliefs, along with his lower MSE, may have limited Taz's participation in the discourse to that of lower quality. His questions were mostly for his clarification rather than as a challenge to the other's thinking. If he believed that people who are good at mathematics learn it quickly and by rote memorization, he may have viewed the negotiation of thoughts as a sign that one is not good at mathematics. Max believed that the textbook or teacher was the locus of authority. He also disagreed with the statement that trying harder can make one smarter in mathematics and that it does not matter how long it took to solve a problem as long as you figured it out. Max's non-availing beliefs were not

evident in his small-group participation. He did not look to Ms. C for support when he was confused. Instead, he looked to his group for clarification. He also persisted when confused about a proof. In his student interview, Max stated that sometimes he needed to walk away and come back with fresh eyes to complete a proof.

#### **5.2.4.3 Comparison of the Groups in Ms. Chenette's Classroom**

The quality of discourse in the two groups was remarkably similar given the differences in group members' beliefs. Kelly and Sally held less-availing beliefs on all the latent variables as compared to Max, Donald, and Taz. Kelly and Sally held slightly availing beliefs about NOLM while Taz, Donald, and Max held availing beliefs. Both dyad members held low-neutral or negative mathematics identities while all three triad members held high-neutral or positive mathematics identities. Furthermore, the dyad held non-availing beliefs about their MSE while the triad held availing beliefs about their MSE. These differences would suggest a finding of different quality of discourse in the two groups, but that was not evident.

Kelly and Sally were inclined to check in with Ms. Chenette more often than the triad. This may have been the observed effect of their non-availing beliefs about mathematics being a set of procedures to follow or their non-availing beliefs about the teacher and/or textbook being the source of knowledge. This may also have been an observed effect of an all-female group associating with an all-female teacher. The dyad did not arrive in class with completely solved proofs as frequently as the triad. They attempted the proofs at home but were perplexed and had partial proofs. Despite the lack of success individually, the dyad engaged in the discourse of relating, justifying, and generalizing to complete the proofs together in class. In the triad, Taz occasionally did not come to

class with a completely solved proof, but through the discussions he was able to complete the proofs as well.

Both groups routinely engaged in mixed to high-quality discourse. The patterns of low, mixed, and high-quality discourse were similar across the two groups. Small-group discourse for both groups was of lower quality when the lesson was not investigatory in nature. Small-group discourse was of mixed or higher quality when the task was investigatory. The nature of the task, rather than Ms. C's implementation of the task or the classroom culture, appeared to be a limiting factor on the quality of discourse.

Ms. C's planning for and implementation of lessons aligned with the tenets of the IMP curriculum. She kept the launch of the task short and focused which enabled groups sufficient time to grapple with the complex tasks. Classroom norms ensured that no one was done with the proof until all group members were done and understood the proof. Groups were expected to present their proofs to the class during the summarization phase of the lesson. Groups were expected to work collaboratively and wrestle with the complex tasks before seeking assistance. They were encouraged to seek out assistance from other groups before seeking assistance from Ms. C. When Ms. C addressed the group, she addressed it as a single entity, emphasizing the collaborative nature of the work. These teacher-content and teacher-student interactions supported all students, even those with negative mathematics identities and non-availing FA and NOLM beliefs, in engaging in higher-quality discourse.

### **5.2.5 Transitioning to Cross-Classroom Comparisons**

In this section, a close look at each group within classrooms revealed similarities between groups in all classrooms, except for Mr. M's classroom. Groups in Ms. L's classroom routinely engaged in no discourse or low-quality discourse. Groups in Ms. K's classroom routinely engaged

in low to mixed-quality discourse. Groups in Ms. C's classroom engaged in high-quality discourse or low-quality discourse in accordance with the nature of the task. In Mr. M's classroom, one group engaged in low to mixed-quality discourse while the other group engaged in mixed to high-quality discourse.

Some of the discourse patterns were explainable by beliefs. Others were explainable by the *Instructional Triangle* and teachers' pedagogical decisions. When classroom learning environments included direct instruction, over-scaffolding, and a lack of emphasis on collaborative work, students' non-availing beliefs surfaced and were reflected in the quality of small-group discourse. Even students holding positive mathematics identities were limited in the quality of their discourse. When classroom learning environments were aligned with SBMI, students' non-availing beliefs were less influential in the quality of small-group discourse.

In the next section, cross-classroom comparisons are made to explore how tracking and the enacted curriculum impact the discourse quality in groups with similar mathematics identity beliefs.

### **5.3 Cross-Classroom Comparisons**

In the previous section small--group discourse in the four classrooms was examined. It was hypothesized that groups' discourse would be driven by the mathematical beliefs held by students as well as the *Instructional Triangle*. This section compares groups along mathematics identity beliefs to identity similarities and differences in the quality of discourse among groups with similar beliefs.

### 5.3.1 Comparison of Groups with Negative Mathematics Identity Beliefs

There were three groups of students, in different classrooms, that held negative mathematics identity beliefs: (a) Elasha and Hillary, (b) Patrice and Yusef, and (c) Sally and Kelly. All members in the groups placed in the lowest quintile of the identity beliefs held by APS students. The first group was in Ms. L's 7<sup>th</sup>-grade non-honors class. The second group was in Ms. K's 9<sup>th</sup>-grade non-honors class. The third group was in Ms. C's 9<sup>th</sup>-grade honors class. Group members held similar composite mathematics identity beliefs. There were similarities in key non-availing NOLM and FA beliefs, particularly beliefs about memorization, locus of knowledge, and speed indicating ability. For each of these indicators, as shown in Table 50, one or both group members held non-availing beliefs about these ideas. Analysis of small-group discourse revealed low-quality discourse for one group, mixed-quality discourse for the other two groups.

Table 50 Negative Identity Students' Beliefs for Key NOLM and FA Indicators

Student	Tracking	Quality of Discourse	Composite Mathematics Identity	Mathematics is a series of steps to memorize	Teacher is the locus of knowledge	Speed in problem solving equals ability
Elasha	NH	0.63	2.1	A	A	A
Hillary	NH	0.63	1.0	NA	NA	NA
Patrice	NH	1.4	1.9	NA	NA	A
Yusef	NH	1.4	2.6	NA	A	A
Kelly	H	2.0	2.8	NA	A	A
Sally	H	2.0	2.6	NA	NA	A

H = Honors NH = Non-honors A = Availing NA = Non-availing

All students, except for Elasha, held the non-availing belief that mathematics was a series of steps to memorize. All students, except for Hillary, held the availing belief that speed in problem solving did not equate to mathematics ability. In each group, one group member held the non-

availing belief that the teacher was the locus of knowledge while the other group member did not. Given these similarities, it is reasonable to hypothesize that the groups would engage in similar quality discourse during small-group inquiry.

Analysis of the small-group inquiry indicated that the quality of discourse of these three groups was markedly different, see Figure 17. Elasha and Hillary routinely engaged in low-quality discourse,  $\bar{x} = 0.63$ . The focus of their discussion was Elasha telling Hillary what procedure to use to complete the investigative task. Patrice and Yusef engaged in just below-average discourse,  $\bar{x} = 1.4$ . At times their discourse was of low-quality, with an emphasis on procedures. Unlike the exchanges between Elasha and Hillary, Patrice and Yusef's exchanges were bi-directional. At times, Yusef would explain the procedure needed to complete the task. Other times, Patrice would be the explainer. The pair often engaged in a think aloud protocol where they worked on the task individually while speaking the procedures. In three of the eight observations, Patrice and Yusef engaged in mixed-quality discourse that included justifications, generalizations, and connections.

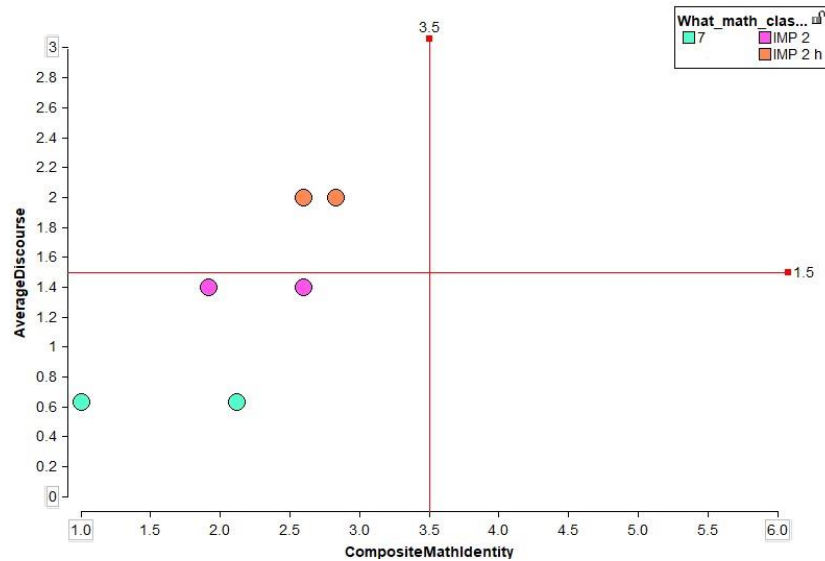


Figure 17 Discourse Quality of Students with Negative Mathematics Identity Beliefs

Both groups were tracked in the non-honors section of mathematics. Both groups had one group member that ranked the lowest in the data set for composite mathematics identity; Hillary and Patrice each had a mean of 1.0 for their CID. Yet the way the two groups engaged with investigative tasks was different.

In Ms. L's 7<sup>th</sup>-grade classroom, Elasha and Hillary were accustomed to a more traditional teaching style prior to the observation period. The observed implementation of investigative tasks and overall lesson rating for SBMI was below average, as rated by the lesson observation tool. Lessons were often over-scaffolded, either with a prolonged launch that included a strategy to solve the task or with an interruption of the lesson to provide direct instruction and the procedure needed to solve the task. Classroom norms included waiting for the teacher to provide specific guidance before beginning the task. Ms. L retained the locus of authority when interacting with this group, telling them the next steps they needed to perform. Not all students were expected to participate in whole class discussions about the task. These teacher-student and teacher-content interactions served to support the negative mathematics identity beliefs as well as the non-availing NOLM and FA beliefs held by both students. As such, these beliefs, in conjunction with an enactment of the curriculum that did not promote substantive student-student collaboration, relegated the discourse to a superficial level.

Conversely, in Ms. K's 9<sup>th</sup>-grade classroom, Patrice and Yusef were accustomed to an SBMI style since the beginning of the school year. Daily tasks, taken directly from the written curriculum, were investigative in nature, with the overall lesson rating above average for SBMI. Only two of the eight lessons were over-scaffolded. Lessons were planned with substantive student-student interactions. Some of the times Ms. K retained the locus of authority when interacting with this group by telling them the next steps they needed to perform. Other times, she would ask the

pair to explain what they had tried and what they proposed to try and help them solidify their thinking. Patrice and Yusef were expected to present to the class their solution for a task. These pedagogical moves shifted the onus of learning onto Patrice and Yusef. They were expected to work together on the tasks. While their negative mathematics identity beliefs and their non-availing beliefs about mathematics being memorization of a series of steps hindered the discussion at times, the classroom norms and Ms. K's pedagogical decisions served to overcome some of the belief impediments. It is conceivable that if the pair were in a classroom that did not promote student authority and student-student discourse that their quality of discourse would resemble that of Elasha and Hillary.

The last group of Sally and Kelly, in the 9<sup>th</sup>-grade honors class, held nearly neutral mathematics identity beliefs. Like Patrice and Yusef, both students in this pair held the non-availing belief that mathematics was memorization of a series of steps. Sally, like Patrice, held the non-availing belief that the teacher was the locus of knowledge in the classroom. The analysis of the discourse quality of this group found variability in quality of the discourse. At times, this pair engaged in high-quality discourse, mixed-quality discourse, and low-quality discourse. The mean rating of their discourse was 2.0-mixed quality. This pair was closer in beliefs to Patrice and Yusef than Elasha and Hillary. Yet the quality of their discourse was higher than that of Patrice and Yusef.

In Ms. C's classroom, observed lessons were the most aligned to SMBI. The above-average lessons were consistently planned for substantive student-student interactions. Ms. C was not observed to over-scaffold lessons, providing instead just in time scaffolding. Her interactions with this group positioned the students as the locus of authority, asking them to explain to their partner their thinking and rationale. Alternate proofs were celebrated. Questions were turned back to the group or to the whole class. Ms. C did not intervene and provide her opinion about how the

group should solve the proof. The classroom culture supported risk-taking and moving out of one's comfort zone. The nature of the tasks, geometric proofs, as well as Ms. C's pedagogical moves allowed this group to engaged in high-quality discourse despite their non-availing beliefs.

### **5.3.2 Comparison of Groups with High-Neutral or Positive Mathematics Identities**

Survey analysis indicated that three groups, each in a different classroom, were composed of students that all held high-neutral or positive mathematics identities: (a) Trinity and Austin, (b) AJ, Marc, and Sam, and (c) Taz, Max, and Donald. The first group was in Ms. L's 7<sup>th</sup>-grade non-honors classroom. The second group was in Mr. M's 7<sup>th</sup>-grade honors classroom. The third group was in Ms. C's 9<sup>th</sup>-grade honors classroom. There was a greater variation in key NOLM and FA beliefs, as seen in Table 51, as compared to the groups of students with negative mathematics identities. Although fewer than half of the students held the non-availing belief that mathematics is a series of steps to memorize, at least one member of each group held such non-availing belief. Over half of the students, unlike the students holding negative mathematics identity beliefs, held the non-availing belief that speed in problem solving is an indicator of ability. Both Trinity and Austin held this non-availing belief, and at least one group member of the other three groups held this belief. At least one group member and all the group of Nola, Luis, and Kaitlin held the non-availing belief that the teacher is the locus of knowledge.

Table 51 Positive Identity Students' Beliefs for Key NOLM and FA Indicators

Student	Tracking	Quality of Discourse	Composite Mathematics Identity	Mathematics is a series of steps to memorize	Teacher is the locus of knowledge	Speed in problem solving equals ability
Trinity	NH	0.63	4.1	A	A	NA
Austin	NH	0.63	4.5	NA	NA	NA
AJ	H	2.4	5.5	A	A	A
Marc	H	2.4	5.0	NA	NA	NA
Sam	H	2.4	5.5	A	A	A
Kaitlin	H	1.0	3.8	NA	NA	A
Luis	H	1.0	4.2	A	NA	NA
Nola	H	1.0	5.8	A	NA	NA
Max	H	1.9	4.1	A	NA	A
Taz	H	1.9	4.5	NA	A	NA
Donald	H	1.9	4.2	A	A	A

H = Honors NH = Non-honors A = Availing NA = Non-availing

Small-group discourse analysis revealed low-quality discourse for two groups, and mixed-quality and higher quality discourse for the other two groups, see Figure 18. The group of Nola, Kaitlin, and Luis, in Mr. M's classroom, routinely engaged in low quality discourse,  $\bar{x} = 1.0$ . The group of Trinity and Austin, in Ms. L's classroom, also routinely engaged in low quality discourse,  $\bar{x} = 0.63$ . Both these groups were in the 7<sup>th</sup>-grade classrooms where the overall quality of lessons was below-average SBMI. The distinguishing belief between these two groups and the other two groups is the members' beliefs about speed problem solving. Trinity, Austin, Nola, and Luis all held non-availing beliefs that speed in problem solving is equated to mathematics ability. Additionally, Austin, Nola, Luis, and Kaitlin held non-availing beliefs about the teacher as the locus of knowledge. The overriding group belief for these NOLM and FA beliefs were non-availing. Implementation of lessons in the two classrooms supported these non-availing beliefs. In Mr. M's classroom, students were given limited time to work on tasks. Groups often did not complete the

task before Mr. M would provide a possible solution. In Ms. L's classroom, when students struggled in establishing a possible solution trajectory, she would stop the class and provide that solution trajectory. In both cases, the message to students was that if they did not solve the task quickly, i.e., before the teachers' explanation, then they were not good at math. Taking the time to challenge the thinking of others or justifying one's own thinking is time away from problem solving. If the goal, explicitly or implicitly stated, in both these classrooms was to complete work quickly then conversations needed to be on a superficial level. Classroom norms did not support justification of answers or working collaboratively.

These students' positive mathematics identity beliefs were supported by a more traditional instructional setting that existed prior to and at the early part of the observation cycle. How they understood what it meant to be good at mathematics was challenged when the teachers changed to inquiry-based learning. Neither teacher fully developed what it meant to work collaboratively on investigative tasks. As such, the students attempted to negotiate discussions and attend to tasks under an old teaching/learning paradigm. Additionally, the students received mixed messages from their teachers about learning in an SBMI paradigm. Groups were treated as individual members. Speed in problem solving was still privileged. Collaborative work was not consistently encouraged. Over-scaffolding to routinize the tasks occurred on a regular basis. These teacher-content and teacher-student interactions served to hinder the development of higher quality discourse as students interacted with each other and the content.

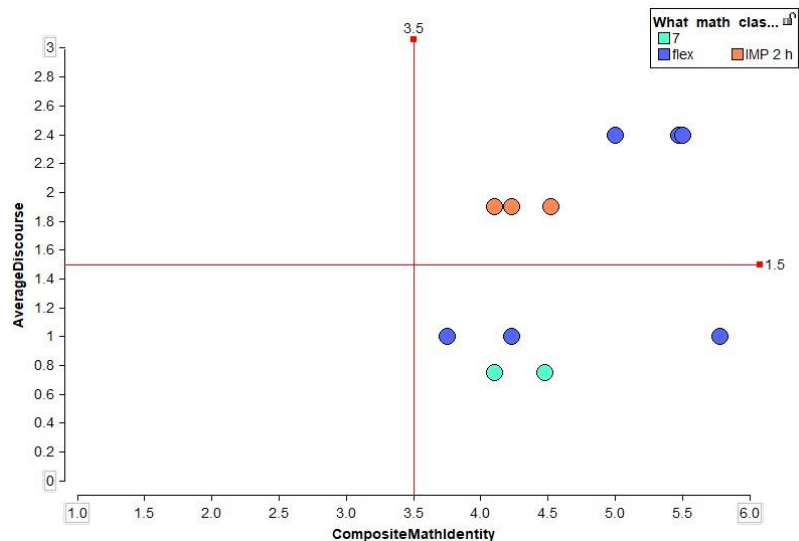


Figure 18 Discourse Quality of Students with Positive Mathematics Identity Beliefs

The group of Taz, Max, and Donald, in Ms. C's classroom, consistently engaged in mixed-quality discourse,  $\bar{x} = 1.9$ . In this group, only one member held a non-availing key NOLM or FA belief. The other two members held availing key NOLM or FA beliefs. As the overriding belief of the group was availing and Ms. C treated the group as a single entity, the learning environment and group beliefs were consistent in supporting higher quality discourse. Ms. C's implementation of lessons was of above-average SBMI. The nature of the tasks, direct from the written curriculum, were not always investigative in nature. In lessons that were more procedural in nature, the group engaged in low-quality discourse. When the tasks were investigative in nature, the group engaged in high-quality discourse.

The group of AJ, Marc, and Sam, also in Mr. M's classroom, routinely engaged in higher quality discourse,  $\bar{x} = 2.4$ . All group members held positive mathematics identity beliefs, unlike the group of Nola, Luis, and Kaitlin and the other groups. Like the group of Taz, Max, and Donald, there were no overlapping non-availing NOLM or FA beliefs among group members. The

overriding belief of the groups was availing. This allowed the group to transcend the below-average SBMI lessons and engage in higher quality discourse.

### 5.3.3 Comparison of Non-Honors Classrooms

Ms. K and Ms. L both taught non-honors sections of mathematics. In each classroom, one pair consisted of students who held negative mathematics identities, Elasha/Hillary and Patrice/Yusef, and one pair consisted of students who held positive/neutral mathematics identities, Austin/Trinity and Riley/Allen. The plot of discourse quality by average overall lesson observation rating in the two non-honors classrooms is seen in Figure 19.

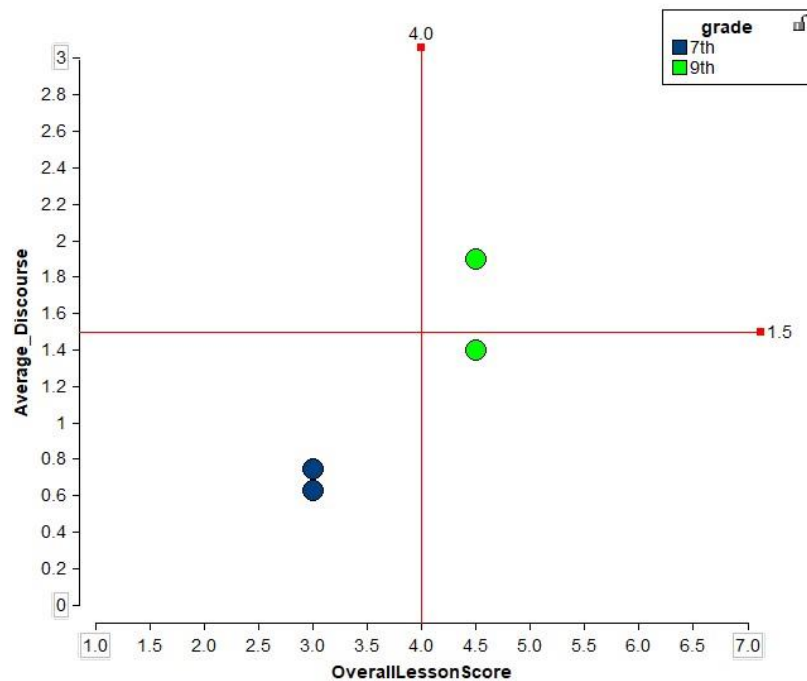


Figure 19 Groups' Average Discourse in Non-honors Classrooms

The implementation of lessons in the two non-honors classrooms differed. Lessons in Ms. K's classroom rated above average ( $\bar{x} = 4.5$ ,  $SD = 0.53$ ) overall on the lesson observation instrument. Students experienced above average standards-based lessons throughout the

observation cycle. Lessons began with a short introduction followed by group work. The expectation was that all students would engage in small-group inquiry and would present their findings to the class at the front of the room. Groups presented their findings to the class and not to Ms. K who then restated the findings for the class.

Lessons in Ms. L's classroom rated below average ( $\bar{x} = 3.0$ ,  $SD = 0.93$ ) overall on the lesson observation instrument. Students experienced below average standards-based lessons throughout the observation cycle. Additionally, Ms. L acknowledged that lessons before the end of year assessments, during the observation period, were more traditional in nature. The observed lessons began with a long introduction followed by group work. Groups did not present their findings. Sharing out of approaches and solutions went from individual to Ms. L to the class. The pedagogical moves of the two teachers differed. Ms. K's lessons were more closely aligned with SBMI, with a short launch, long exploration, and student-presented summary. Ms. L's lessons were more closely aligned with traditional instruction, with a long launch, short exploration, and teacher-presented summary.

Patrice and Yusef, in Ms. K's class, were similar in their beliefs to Elasha and Hillary in Ms. L's class. All four students held negative mathematics identities. None of them identified as math people. The ways in which Patrice and Yusef engaged with the mathematics and participated in small-group discourse was in stark contrast with the ways in which Elasha and Hillary engaged. This difference was discussed in section 5.3.1.

Riley, Allen, Trinity, and Austin held comparable beliefs. Trinity and Riley held positive mathematics identity beliefs. Austin and Allen held neutral mathematics identity beliefs. Allen, Austin, and Riley held non-availing beliefs about the teacher being the source of knowledge and

non-availing beliefs about mathematics being a process of memorizing steps. All four students held availing beliefs about mathematics ability.

Given similarities in beliefs and achievement, one might expect the interactions between Riley and Allen to parallel those of Trinity and Austin. When comparing these two groups, there was a stark difference in the amount and quality of the small-group discourse. Therefore, other factors besides ability/achievement and beliefs need to be considered. Trinity and Austin completed their work in relative silence and rarely engaged in mathematical discourse about tasks or answers. They did not utilize the small-group time to make meaning of the mathematics together. They did not challenge each other's thinking. The only justification came when Ms. L pushed the individual to explain. Questions and statements from Ms. L were not directed to the pair but to an individual. Riley and Allen routinely worked collaboratively or checked in with each other during independent work and engaged in higher quality discourse. Given the nature of the tasks, one would have expected Trinity and Austin to engage in more discourse and at a higher quality than Riley and Allen. The tasks that Trinity and Austin worked on lent themselves well to collaboration; they were inquiries. The tasks that Allen and Riley worked on were more procedural in nature. However, the opposite occurred. There was more discourse between Allen and Riley than between Austin and Trinity.

Considering the matched pairs in Ms. L's and Ms. K's classrooms, the differences in how students engaged with mathematics transcended beliefs and abilities. The culture and established norms in Ms. K's classroom supported students with lower mathematics identities and achievement to engage in mathematics with higher-quality discourse. The expectation was such that students justified their thoughts and productively struggled with non-routine tasks. Ms. K's routinization of problematic aspects of some tasks and decisions to omit certain tasks did serve to constrain some of

the discourse to that of a lower quality. Nonetheless, groups worked cooperatively to ensure that each member understood the mathematics and that any one of them could present the solution strategy. Ms. L's pedagogical decisions about implementation inhibited groups' abilities to work collaboratively on inquiry tasks. Additionally, established classroom norms of working independently and seeking assistance from Ms. L hampered students' collaborative work. These, in turn, stifled the quantity and quality of discourse during small-group work.

#### **5.4 Summary**

The hypothesis that students' composite mathematics identity [CID] would be related to their small-group discourse was not necessarily so (see Figure 20). Students who held availing composite mathematics identities engaged in both high-quality and low-quality discourse. Students who held non-availing composite mathematics identities engaged in both high-quality and low-quality discourse. Seventh-grade students, apart from the group of AJ, Marc, and Sam, tended to have lower quality discourse than 9<sup>th</sup> graders. Factors other than beliefs, such as teacher-student and teacher-content interactions, were examined.

Students in the classes that least resembled standards-based learning environments, middle school classrooms, engaged in levels of discourse that were different than those in the classes that more closely resembled high-quality standards-based learning environments, as measured by the lesson observation instrument.

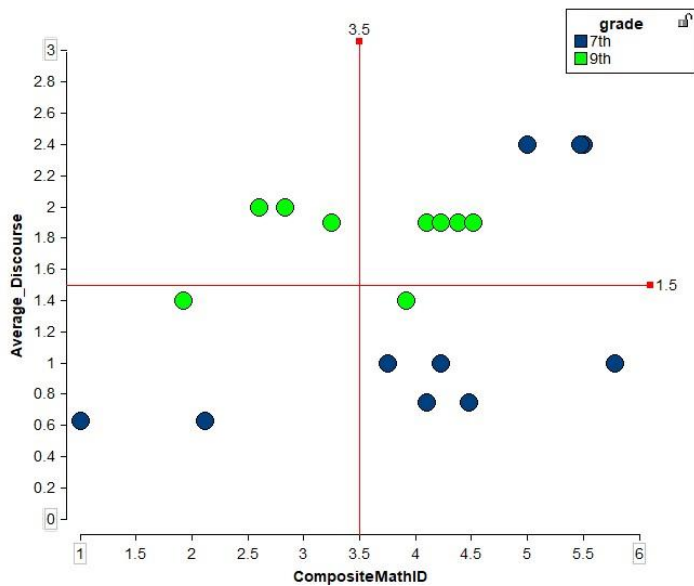


Figure 20 Average Student Discourse Compared to Composite Mathematics Identity

All high school groups, except for one, engaged in above-average, higher quality discourse, see Figure 21. That one group engaged in slightly below average quality discourse. In each case, the overall lesson score was above average for SBMI. All middle school groups, except for one, engaged in lower quality discourse. In each case, the overall lesson score was below average for SBMI. When lessons were of below-average SBMI quality, groups with overriding non-availing key NOLM or FA beliefs engaged in low-quality discourse. When lessons were of above-average SBMI quality, groups, regardless of their key NOLM or FA beliefs, engaged in higher quality discourse.

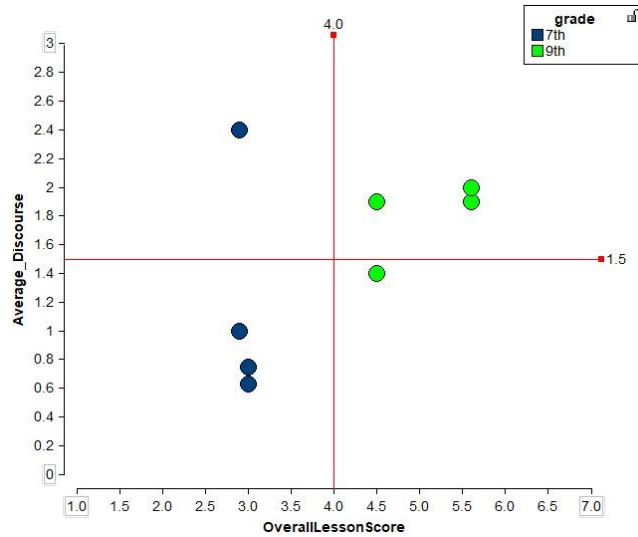


Figure 21 Overall Lesson Observation Score Compared to Average Group Discourse

There appeared to be a strong relationship between the classroom culture and the quality of discourse in small groups. In Figure 24, as the rating for classroom culture increases, the quality of small-group discourse increases. The culture in the high school classrooms scored highly on the lesson observation instrument. These students experienced a classroom culture that supports SBMI and learning. The classroom culture at the high school included established classroom norms promoting justification, argumentation, connections, and generalizations. The established classroom norms at the middle school supported a culture where students looked to the teacher to provide explicit examples and verification of the correctness of their work. Observation of middle school lessons revealed instances where students did not begin work until the teacher told them the specific procedure to follow.

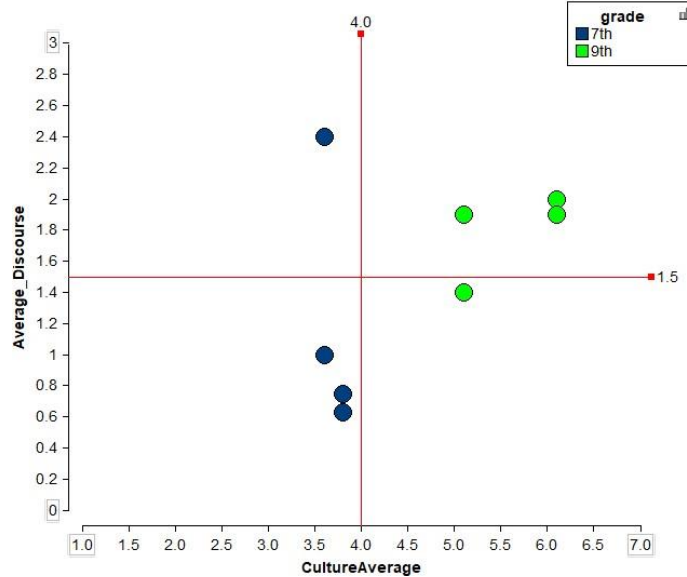


Figure 22 Overall Culture of Lessons Compared to Average Group Discourse

Mr. McGinnis’s lessons rated the lowest, for summary, implementation, and culture ratings, on the lesson observation instrument. Some students who held positive mathematics identities, such as the triad of AJ, Marc, and Sam, engaged in higher quality small-group discourse. Other students who held positive beliefs about their mathematics identities, the triad of Nola, Luis, and Kaitlin, engaged in lower quality small-group discourse, even when the lessons aligned with SBMI. Ms. Luurtsema’s lessons rated the second lowest, for summary, implementation, and culture ratings, on the lesson observation instrument. Both groups in that classroom engaged in no to low-level discourse during small-group inquiry. In both these cases, the level of discourse was lower than or commensurate with students’ beliefs about mathematics and their mathematics identities. The enacted curriculum and established classroom norms, both middle school classrooms, may have inhibited students’ participation in collaborative small-group inquiry and failed to support students’ learning and/or the expectation to engage in high quality discourse.

In the two high school classrooms, where lessons were rated above average on the lesson observation instrument, students engaged in a different type of discourse during small-group inquiry. Ms. Kraft's lessons were rated just above average for the summary rating. The classroom culture rated above average on the lesson observation instrument. Her implementation of the IMP lessons was of slightly below-average quality. Her pedagogical decisions tended to change the nature of the tasks from doing mathematics to practicing procedures. The observed discourse was a mixture of low- and high-level quality; this, despite group members who held negative mathematics identities, beliefs that the locus of authority resided in the teacher or textbook, and beliefs that mathematics was a process of following rote procedures. Established classroom norms and average SBMI supported students to engage with collaborative inquiry despite their beliefs, as evidenced by her higher overall classroom culture rating.

Ms. Chenette's lessons were rated the highest on the lesson observation instrument. Lessons were enacted with high fidelity to the IMP curriculum. Her pedagogical decisions supported the nature of the tasks as intended. Classroom norms included active group participation and focused on student generation of knowledge. Students in her class were observed to routinely engage in mixed to higher quality small-group discourse. Like Ms. Kraft's class, one of the two groups in Ms. Chenette's class held negative beliefs about their mathematics identity. The two groups in Ms. Chenette's class participated in small-group inquiry with approximately the same quantity and quality of discourse. In both of these classrooms, the quality and quantity of small-group discourse was higher than or commensurate with students' beliefs about mathematics and their mathematics identities. The small-group discourse average of both groups was also of a higher quality and quantity than in either of the seventh-grade classrooms.

There were differences in the quality and quantity of the small-group discourse between seventh-grade honors and non-honors groups and ninth-grade honors and non-honors groups (see Figure 14). A comparison of seventh-grade discourse, ninth-grade discourse, honors discourse, and non-honors discourse is explored in depth in this section.

## CHAPTER 6

### DISCUSSION AND IMPLICATIONS

The purpose of this mixed-methods case study was to describe the effects of within district congruent written curricula commitments across the grade bands by examining students' mathematical beliefs and small-group discourse. This chapter begins with a summarization of the major findings from Chapters 4 and 5. Those findings will then be related to findings of other studies. Following the discussion, implications of this this research and suggestions for further topics of study will be examined. The findings were in response to the two research questions:

1. What mathematics-related beliefs do students hold in a school district with incongruent written curricula commitments in middle school and high school?
  - a. What are students' beliefs about the nature of learning mathematics?
  - b. What are students' beliefs about the speed/ability of knowledge acquisition?
  - c. What are students' beliefs about their mathematical identity, mathematics self-efficacy, and mathematics interest?
2. How do students participate in small-group discourse within various enacted curricular models, and how is this participation related to their mathematical beliefs?
  - a. In what ways are students' mathematical beliefs related to the nature and quality of discourse during collaborative inquiry?
  - b. In what ways is the enacted curriculum related to the nature and quality of students' discourse during collaborative inquiry?

The conceptual framework guiding this study speculated that mathematics learning is dependent upon each of the following: (a) *The Instructional Triangle*, (b) small-group discourse, and (c) students' mathematics related beliefs, all of which are situated within the enacted curriculum. Individually, each of these has the potential for impacting students' learning (Cohen et al., 2003; Esmonde & Langer-Osuna, 2013; Gee, 2008; Remillard & Heck, 2014; Tarr et al., 2008;

Weaver et al., 2005). The literature review made the case for conducting a research study examining the connections between *The Instructional Triangle*, small-group discourse, and MRB. Findings, analyzed through a sociocultural lens, point to students' participation in small-group discourse being rooted in their own mathematics-related beliefs as well as shaped by teacher-student-content interactions. These claims are evidenced within the data collected through survey responses, individual interviews, lesson observation ratings, and small-group discourse quality analysis. Data revealed ways in which students engaged in discourse within their small group as they worked on mathematical tasks within the enacted curriculum to negotiate sense-making varied within and between classrooms.

## **6.1 Summary of Findings**

### **6.1.1 Answering Research Question One**

The first research question for this study addressed the mathematics-related beliefs held by secondary students at Autumn Public Schools: *What mathematics-related beliefs do students hold in a school district with incongruent written curricula commitments in middle school and high school?* Answering this question required analyses of the enacted curriculum and students' beliefs.

#### **6.1.1.1 Standards-Based Mathematics Instruction**

Findings indicate that the enactment of lessons and quality of SBMI can vary by teacher. While some teachers, such as Ms. Chenette, enacted consistent levels of high quality SBMI lessons, others, such as Mr. McGinnis and Ms. Luurtsema, enacted variable levels from low quality to average quality SBMI lessons. One-half of the lessons in the 7<sup>th</sup>-grade classrooms and approximately 94% of the lessons in the 9<sup>th</sup>-grade classrooms were designed to encourage

substantive student-student interactions. Investigative tasks were a focus in all classrooms, with approximately 84% of observed lessons incorporating such tasks.

Implementation of lessons were a relative weakness for the 7<sup>th</sup>-grade teachers. Students did not work collaboratively on a consistent basis in these two classrooms. Collaborative work was a new expectation for these students with which they struggled. In addition, the 7<sup>th</sup>-grade teachers inconsistently reinforced the expectation of collaborative work. At the high school level, collaborative work was a year-long expectation and was an established classroom norm. High expectations of all students varied by teacher, providing different, inequitable opportunities to learn for students. The 7<sup>th</sup>-grade classrooms provided lower OTL, with some students held to higher expectations of participation and engagement than others.

#### **6.1.1.2 Themes in Observations**

Themes from observation notes indicate that the level of scaffolding in classrooms vary by teacher. High school teachers provided appropriated scaffolding that did not diminish the cognitive demand of the task by routinizing the problem. Middle school teachers over-scaffolded tasks, which diminished the cognitive demand of the task and reinforced the belief that mathematics is a series of steps to be memorized.

A similar theme appears in the interaction of teacher and groups. Middle school teachers interacted with individual students in the group and not with the group. This interaction, coupled with a lack of accountability for the group to come to a consensus and present their findings, promotes individual work rather than collaborative work. High school teachers routinely interacted with the group as a single entity. When students had questions, the teacher asked what the group had discussed or asked group members to explain their thinking to the student with the questions.

This type of interaction, in conjunction with classroom norms that groups negotiate their findings and then present them to their peers, supports collaborative work with inter-dependency. The degree to which high school teachers maintained this type of interaction were varied, with Ms. Chenette consistently interacting in this manner and Ms. Kraft occasionally interacting with individual students as opposed to the group.

### **6.1.1.3 Students' Mathematics Related Beliefs**

Autumn Public Schools was chosen as the site for this study because of its incongruent written curricula commitments. The middle school had adopted a hybrid written curriculum and the high school had adopted an NSF-funded written curriculum. It was hypothesized that students at Autumn Middle School would hold different, less availing, beliefs about mathematics than students at Autumn High School. However, because the middle school had previously implemented an NSF-funded curriculum, there was some carryover or residue as seen in activities and pedagogical actions that impacted the enactment of the hybrid curriculum. That is, the enacted curriculum at each school was not as distinctly different as was hypothesized. This is a non-trivial finding that makes interpreting results of this study especially complicated.

Overall, the average student at Autumn Middle School and Autumn High School held availing beliefs about the nature of mathematics learning and fixed ability/growth mindset. Additionally, they held slightly positive mathematics identity, mathematics self-efficacy, and mathematics interest beliefs. Differences existed between the average male and female students' beliefs as well as between the average honors and non-honors students' beliefs at the middle school level but not at the high school.

At AHS, there were no significant differences in students' beliefs when analyzed for gender. At AMS, there were significant differences in male and female students' beliefs. The average AMS female students held beliefs that were less availing about their composite mathematics identity, self-efficacy, and interest as compared to the average AMS male student.

There were no statistically significant differences between the beliefs of the average honors student at AHS and AMS or the average non-honors student at AHS and AMS. At AHS, there were no significant difference in honors and non-honors students' beliefs. However, at AMS, the average honors student held MRB that were more availing than the average non-honors student.

There were statistically significant differences between the beliefs of the average student in the two 7<sup>th</sup>-grade classrooms but not between the average student in the two 9<sup>th</sup>-grade classrooms. The average student in the non-honors 7<sup>th</sup>-grade classroom was less likely than the average student in the honors 7<sup>th</sup>-grade classroom to agree that when their method of solving a math problem is different from the teacher's method that both methods can be correct. The average student in the non-honors 7<sup>th</sup>-grade classroom was also less likely than the average student in the honors 7<sup>th</sup>-grade classroom to believe their family, relatives, and friends see them as a math person. They are also more likely than the honors student to state that mathematics makes them nervous.

### **6.1.2 Answering Research Question Two**

The second research question focused on the analysis of the quality with which students participate in small-group discourse: *How do students participate in small-group discourse within various curricular models, and how is this participation related to their mathematical beliefs?* Students' mathematics-related beliefs and the implementation of lessons were attended to as explanatory factors for the ways in which students participate in small-group inquiry.

The quality of small-group discourse varied within some classrooms and across classrooms. Within a given classroom, and therefore within an enacted curriculum, there were some observable similarities in the ways in which students participated in the small-group discourse.

Both groups of students in Ms. Luurtsema's classroom were observed to engage in low-quality or no discourse, despite vast differences in the groups' overriding mathematical identity beliefs. One group's members held positive mathematical identity beliefs and the other negative mathematical identity beliefs. Key non-availing NOLM and FA beliefs were supported by the below-average SBMI implementation of lessons. Classroom norms did not support productive struggle or high-quality discourse.

Both groups of students in Ms. Chenette's classroom were observed to routinely engage in similar-quality discourse despite differences in the groups' overriding mathematical identity beliefs. One group's members held positive mathematical identity beliefs. The other group's members held negative mathematical identity beliefs. For students in this classroom, the nature of the discourse was related to the investigative nature of the task. High quality SBMI lessons served to impede key non-availing NOLM and FA beliefs from lowering the quality of the discourse.

Both groups of students in Ms. Kraft's classroom engaged in low to mixed-quality discourse during small-group inquiry. One group's members held negative mathematical identity beliefs. The other group's members held low-neutral and positive mathematical identity beliefs. The group with the negative mathematical identity beliefs participated in slightly below-average discourse. Their struggle with the arithmetic appeared to limit the quality of their discourse. The group with the mixed identity beliefs participated in higher quality discourse. They did not struggle with the arithmetic, and, as such, could devote more of their small-group time engaged in making connections, justifying solutions, and challenging the thinking of each other.

The quality of discourse in Mr. McGinnis's classroom was different for the two groups. One group's members all held positive mathematical identity beliefs. The other group's members held a mixture of neutral and positive mathematical identity beliefs. In this classroom, students' key NOLM and FA non-availing beliefs, in concert with below-average SBMI implementation of lessons, hindered the discourse quality of the group with mixed beliefs.

The small-group discourse in the 7<sup>th</sup>-grade classrooms was lower than that in the 9<sup>th</sup>-grade classrooms, comparing either the two non-honors classrooms or the two honor classrooms. Comparison across the two 7<sup>th</sup>-grade classrooms found similarly low-quality discourse, but for one group. Comparison across the two 9<sup>th</sup>-grade classrooms found similarly higher quality discourse.

## **6.2 Discussion**

Findings from this mixed-methods case study suggest that students' mathematical beliefs and participation in high quality small-group discourse is related to *The Instructional Triangle* and the quality of SBMI as well as to each other. Beliefs, classroom norms, and teacher moves either support or diminish the quality of small-group discourse.

### **6.2.1 Mathematical Related Beliefs**

Previous research about high school students' mathematical beliefs have often been conducted with the absence of classroom observations (e.g., Cano, 2005; Chinn, 2012; Schommer-Aikins & Duell, 2013). AMS students' availing mathematical beliefs were in contradiction to several studies (Darragh, 2014; Greene et al., 2010; Muis et al., 2011; Muis & Foy, 2010; Schoenfeld, 1989; Star & Hoffman, 2005), which found that students in traditional mathematics learning environments tended to hold non-availing beliefs about mathematics. AHS students'

availing beliefs about mathematics were consistent with what is known about high school students experiencing SBMI (e.g., Boaler, 2006a; 2006b; McGregor, 2014; Muis et al., 2011).

### **6.2.1.1 Gender differences**

Gender differences existed in students' beliefs about their composite mathematics identity, with female students having less positive mathematics identities, mathematics self-efficacy, and mathematics interest than their male counterparts. Female students at AMS held neutral mathematics identity and interest beliefs and slightly positive mathematics self-efficacy beliefs, while male students held slightly positive identity and positive self-efficacy and interest beliefs. This aligned with what is known about mathematics identities and gender (Arens, et al., 2016): female students generally hold less-positive perceptions of their mathematics identities and self-efficacy than male students. However, while there were differences between female and male students' identity beliefs in the middle school traditional learning environment, these differences were not found at the high school level. The absence of differences in female and male high school students' beliefs, in contradiction to what was previously known, suggests that SBMI may be a way to mitigate the erosion of female students' mathematics identities and to provide equitable opportunities to learn across gender.

There were no gender differences in students' beliefs about mathematics ability being fixed, regardless of school. Both female and male students in both schools held availing beliefs about mathematics ability. This contrasts with earlier findings by Cano (2005), which indicated that male high school students hold a non-availing stance about innate mathematics ability while female high school students hold an availing stance about it. Finding a growth mindset across all students is an encouraging finding, as students with availing beliefs about mathematics ability and speed of

knowledge acquisition have less mathematics anxiety and more confidence in their ability (Chinn, 2012; Schommer-Aikins & Duell, 2013), as well as perseverance in the face of difficult tasks (Cano, 2005; Phillips, 2001; Schommer-Aikins & Duell, 2013).

### **6.2.1.2 Honors differences**

In the 7<sup>th</sup>-grade classrooms, honors students held beliefs that were more-availing than non-honors students for the six latent variables. There was no significant difference between the beliefs of honors and non-honors students for any of the six latent variables in the 9<sup>th</sup>-grade classrooms. Grouws (1996) found that mathematically talented high school students, those in honors classes, held more-availing beliefs than students in non-honors courses about the nature of learning mathematics. The type of learning environment was not described, nor was it part of the investigation, in Grouws' study. It could be inferred that it was a traditional learning environment by Grouws' use of the term "high school algebra students" and "mathematically talented students." In this study, 7<sup>th</sup>-grade honors students held beliefs that were more-availing than 7<sup>th</sup>-grade non-honors students, supporting Grouws' (1996) findings. However, the similarity of non-honors and honors students' beliefs in the 9<sup>th</sup>-grade classrooms contradicts Grouws' findings. This similarity between the 9<sup>th</sup>-grade honors and non-honors students' beliefs is consistent with the findings by Boaler (2006a) and Boaler and Staples (2008): high quality SBMI in a detracked setting served to elevate students' MRB.

The average APS student holds the non-availing beliefs that mathematics is memorizing a series of steps and that the teacher is the locus of knowledge. These findings are commensurate with previous research (Muis et al., 2016; Schoenfeld, 1989). Ideally, the 9<sup>th</sup> grade AHS students' beliefs

about memorization and the locus of knowledge will become more availing as they experience additional years of high quality SBMI.

Encouraging findings include students at APS not holding the unavailing belief that being good at mathematics means arriving at the answer quickly. This contrasts with early findings (Darragh, 2014; Schoenfeld, 1989) suggesting a subtle positive shift at APS that mathematics takes time. This shift is important as students who believe mathematics problem solving takes time will persist when faced with challenging tasks (De Corte et al., 2008; Mason & Scrivani, 2004).

### **6.2.2 Discourse Quality**

Groups in both 9<sup>th</sup>-grade classrooms were observed to engage in high-quality discourse. The length of the high-quality discoursed varied from sporadic instances to sustained discourse throughout the small-group inquiry. The participation in discourse that includes challenges, justifications, and connections shifts the focus of learning from obtaining the correct answer to meaning making (Boaler, 2006a, Cobb et al., 2001). The higher quality discourse allows the 9<sup>th</sup>-grade students a greater opportunity to learn than the lower quality discourse in the 7<sup>th</sup>-grade classrooms (Esmonde & Langer-Osuna, 2013).

The enacted curriculum, including the nature of the tasks and pedagogical moves that impact the cognitive demand of the tasks (Stein et al., 1996; Warshauer, 2015), as well as established classroom culture and norms about what it means to do mathematics (Cobb et al., 2001; Kazemi & Stipek, 2008; Webb et al., 2006) appears to be a predictive factor for consistent high-quality discourse. In the two 9<sup>th</sup>-grade classrooms, learning environments supported productive struggle and small-group discourse during inquiry-based learning. Classroom norms were such that students were expected to work collaboratively and to engage in high-quality discourse where they

justified their reasoning, generalized, and connected new learning to prior experiences (Damon & Phelps, 1989; Tomlinson, 2018). Tasks were implemented so that students worked collaboratively (Damon & Phelps, 1989; Tomlinson, 2018). Groups, as well as individual students, were held accountable for the learning and presentation of findings (Fukawa-Connelly, 2012; Kazemi & Stipeck, 2008; Tatsis & Koleza, 2008). The teacher moved from being the source of knowledge, providing direct guidance, to that of a facilitator of knowledge.

The 7<sup>th</sup>-grade learning environments established a culture of independent work methods but dependent learning. Classroom norms were such that quick completion of tasks was rewarded with praise. Students expected the teacher to provide direct instruction on algorithms for them to replicate on similar problems. When challenged with new expectations, inquiry-based tasks, students had difficulty adjusting. The established norms constrained students' ability to work collaboratively or there was not a clear expectation that students had to adjust. Tasks were implemented in such a way that interdependence and collaboration was not essential. For example, Ms. Luurtsema, in lesson 2, made the pedagogical decision that each student would draw their own circle and have a centimeter cube to find the area of the circle. If this task were implemented such that the group drew one circle and had access to one centimeter cube, then students would be interdependent and would need to work collaboratively. The teacher remained the source of knowledge, often routinizing the task. If students appeared to be struggling with a mathematical task, Mr. M would provide them with a procedure to follow as opposed to having groups share out strategies that they were trying.

Identity, ability, and nature of learning mathematical beliefs, in conjunction with the enacted curriculum, are also important factors supporting high-quality discourse. This finding supports the conclusions made by Esmonde & Langer-Osuna (2013) and Jansen (2008) that found

that students' beliefs impact their participation in mathematics discourse. In the 7<sup>th</sup>-grade classrooms, the lack of high-quality discourse may lead to limited opportunities to learn for students with negative mathematics identities, although this study did not measure learning. The established norms allowed them to passively wait for direct instruction from the teacher. Students with positive mathematics identities varied in the quality of their discourse. When the implementation of the task promoted collaborative inquiry, the quality of discourse was at a higher level than when the implementation promoted individual work. In the 9<sup>th</sup>-grade classrooms, there was little difference in the quality of discourse of students. There was no evidence that beliefs impacted the quality of discourse during small-group inquiry. In all classrooms, the quality of small-group discourse was reflective of whole-class discussions and norms. This mirrors the findings of Kazemi & Stipek (2008).

### **6.3 Implications for Practice**

The purpose of this mixed-methods case study was to examine a school district with incongruent written curricula commitments by investigating students' mathematics-related beliefs and their patterns of discourse during small-group inquiry. Incongruencies in written curriculum can happen within a grade level, within a school, or across schools. It is commonly seen when students transition between elementary to middle school, middle to high school, and high school to college. It can also be seen when students transfer to different schools within a district or to a different school district.

Student engagement with high quality discourse is an essential component of mathematics achievement and persistence (Cano, 2005; Phillips, 2001; Schommer-Aikins et al., 2005). Access to high quality discourse is an issue of equity, allowing for greater opportunities to learn mathematics

(Esmonde & Langer-Osuna, 2013; Gee, 2008). The following are important outcomes of this study that translate directly into implications for practice.

Students in the 7<sup>th</sup>-grade classroom with non-availing beliefs and negative mathematics identities, as well as those with availing beliefs and positive mathematics identities, exhibited lower-level discourse than students in a standards-based classroom. These differences were apparent in both honors (accelerated) courses and non-honors courses. Classroom norms for participating in small-group inquiry need to include high quality discourse. Students need to be held accountable for maintaining discourse that includes features such as justifying answers, relating to prior knowledge, and challenging the thinking of others. As the teacher cannot be present for all discourse within all groups, those norms need to be developed so that students' experiences within the small group enable them to achieve a higher level of mathematical understanding. Additionally, students need consistent support when transitioning to collaborative work until those norms are established.

Given the differences between the implementation of lessons at the two schools, school districts that utilize a more traditional or hybrid curriculum need to incorporate more facets of standards-based instruction into their practices. When teachers modify tasks to increase the cognitive demand and to shift the task from "procedures without connections" to "doing mathematics," students can engage in that higher-quality discourse. This access to higher-quality discourse can increase students' opportunity to learn mathematics (Esmonde & Langer-Osuna, 2013) and is a predictor of increased mathematical achievement (Wood et al., 2006). Additionally, limiting resources, groups are forced to collaborate rather than work independently or cooperatively, supporting more discourse. This was seen in environments that were representative of just above average standards-based lessons, indicating that even modest shifts in the enacted curriculum may affect student participation.

## **6.4 Limitations and Delimitations of Study**

Limitations of this study arise from the data, the method of analysis, and from the methods and how the findings were interpreted. One major limitation of this study is the change in instructional methods partway through the classroom observations and the lack of fidelity implementing the written curriculum at the middle school. Timing of observations at the high school where lessons were anticipated to follow SBMI characteristics led to observations of a unit that followed a more procedural focus. As such, the lesson characteristics at the two locations did not fall into completely distinct categories. A second major limitation of this study is the small sample size for the small-group discourse analysis. Students in these classrooms were reluctant to participate in the observation portion of the study. All students who volunteered to be observed were included in the study. Ideally, there would be enough volunteers to allow a selection of observation participants. A third major limitation of this study is the small survey sample size for the high school. Over twice as many eighth-grade students and nearly three times as many seventh-grade students participated in the survey as ninth-grade students. This discrepancy was partly because fewer ninth-grade students were willing to take the survey. Additionally, of the students who did take the survey, more students at the high school level were ineligible to be included in the analysis of the data because they had not attended APS continuously since seventh grade. The findings in this study about collaborative inquiry and small-group discourse, enacted curricula, and students' mathematical beliefs are representative of students and classrooms in one school district. They may not be generalizable to other school districts.

The findings in this study may not be generalizable to other classrooms within the district, as there are variations in how teachers choose to implement the written curriculum. As with most classroom research designs, the classroom teachers may be a confounding variable. In this research

context, the classroom teachers at the middle school were well-liked and were adept at making connections with students. They both had previous experience with standards-based curricula and attempted to integrate some inquiry activities into the traditional curriculum. An additional confounding variable, parental pressure to obtain honor roll or status of being in an honors (accelerated) class, was not considered. Being an honors student, with pressure to achieve high grades or class status, may have impacted how students participated and their compliance during the small-group inquiry. This increase in participation may have impacted achievement, which in turn impacted beliefs and identities, rather than the beliefs impacting the participation (Cano, 2005; Phillips, 2001; Schommer-Aikins et al., 2005).

Additionally, students volunteered to participate in all aspects of the study rather than being randomly selected. Students were recruited to participate in interviews due to lack of volunteers for certain strata. This may limit the generalizability of the findings for those with non-availing mathematics-related beliefs or negative mathematics identities. All observed students were not interviewed due to conflicting schedules. This limited the triangulation of survey, interview, and observation of beliefs. Finally, findings may not be generalizable, due to the much smaller sample of high school students versus middle school students.

It would have been ideal to conduct this study early in the school year. In the middle school, access was constrained due to end-of-year state testing. In the high school, access was also constrained due to EOY state testing and district finals. At both schools, field trips, assemblies, half-days, and end of the year events disrupted lessons and opportunities to observe lessons. Additionally, three of the four teachers sat on a committee to select a new mathematics curriculum. This committee met during the school day. As such, the days the committee met were days that were not available for observation.

Delimitations of this study are the boundaries set on the purpose and scope of the study. The delimitations of this study include focusing solely on student beliefs. Parental and teacher beliefs were not considered. Lesson observations occurred over the second half of the school year, after classroom norms for group participation had been well established. It is not known whether student beliefs and identities had been influenced by the enacted curriculum over the school year, nor was this a purpose of the study. The NSF-funded curriculum at the elementary schools was not examined for fidelity of enactment and its possible role in shaping students' beliefs and identities.

### **6.5 Further Research**

To support and build on this study, further research should include repeating the present study design in different research contexts with larger sample sizes. This research provided a small window into students' beliefs, and into how those beliefs and the enacted curriculum impact small-group discourse. It also created tension between expected results and actual results. This tension, as previously discussed, is theorized to be based upon the middle school teachers' prior experience with an NSF-funded curriculum and their incorporation of SBMI into the hybrid curriculum. Selecting a site where there is less crossover of instructional models might lead to more clearly defined differences in beliefs and discourse patterns.

It is necessary to explore whether the availing beliefs and positive mathematics identities held by the middle school students in APS are a function of this school district or if this indicates a general shift in students' beliefs. Are Autumn Middle School students' beliefs shifting due to small changes in teaching pedagogy towards a standards-based learning environment? Alternatively, are these availing beliefs a carryover from a standards-based curriculum employed at the elementary schools? A third possibility is a positive impact of other factors, such as parents, teachers, and

society, upon the students at AMS. A longitudinal study measuring students' beliefs and identities entering and leaving Autumn Middle School could help to clarify some of these uncertainties. Further research across different school districts is warranted to determine if the positive shift in beliefs exists outside of APS.

This is just one study exploring high school students' beliefs in a standards-based learning environment after experiencing a more traditional learning environment in middle school. The results show there was no or minimal change in beliefs and identities for high school students. Repeating the study earlier in the year, prior to standardized testing and review for end of year assessments, might increase the participation rate of ninth-grade students. It would also allow for a more accurate observation of lessons across units. Additional research should include students beyond the ninth grade, as beliefs and identities are slow to change.

As access to high quality discourse is one avenue for creating equitable access to mathematics, additional research is needed to determine variables that affect the level of student participation. What other factors besides students' beliefs and identities affect participation? Will similar results emerge where students with non-availing beliefs and negative identities participate at a higher level in a standards-based instructional model compared to those in a hybrid instructional model? Studies on a larger scale and of a different scope are necessary to determine if this phenomenon of the enacted curriculum mitigating students' beliefs truly exists or is an anomaly of APS. The relationship between the nature of the tasks during small-group work and the quality of discourse could be further studied.

High school students who experienced SBMI in middle school continue to hold beliefs about the importance of conceptual understanding after experiencing four years of a more traditional direct-instruction curriculum in high school (Moyer et al., 2018). One goal of this study

was to discover if students' non-availing beliefs from middle school persisted in high school despite the change to SBMI. However, this goal could not be addressed because students at AMS held availing beliefs. If this study could be replicated, the researcher would fully analyze the survey data prior to conducting interviews rather than at the conclusion of the data collection. Additionally, research is needed, with the same type of setting, but one in which students in middle school hold non-availing mathematics-related beliefs, to determine if results contrary to those in Moyer's (2018) study results exist. This would add to the limited literature about students' beliefs when the opposite occurs, when students experience traditional instruction in middle school and SBMI in high school (Boaler, 1998; Huntley et al., 2000).

The impact of high-quality discourse upon student learning was not measured in this study. Future work, with pre- and post-assessments to measure learning, could include a longer study coordinating beliefs, discourse quality, and learning.

## **6.6 Conclusions**

In this study the quality of small-group discourse among classrooms varied with the implementation of the curriculum, including pedagogical decisions/skill of the teacher, and the established classroom norms. The researcher acknowledges that studying the relationship between the enacted curriculum and classroom discourse is complex. On the other hand, evidence showcased by this work corroborates results of Boaler (2006a, b), Cobb et al., (2001), Kazemi & Stipek (2008), Langer-Osuna (2016), Lopez & Allal (2007), and Webb et al. (2006) and points to the fact that lesson implementation influences students' nature and quality of discourse. In the case of this study, differences in small-group discourse in the middle school and the high school were in alignment with differences in the nature of the lessons.

The study suggests that students' beliefs and identity may also influence the nature of the discourse, as found by others (Chiu, 2008; Esmonde & Langer-Osuna 2013; Jansen, 2008; Nussbaum, 2008). There were differences in the quality of small-group discourse within the same classroom. Even when students are experiencing the same lesson, they are experiencing differently based on their beliefs and other societal and cultural factors (Gee, 2008). Although within the same classroom, students have different experiences with the same lesson, the same pedagogical decisions/skill, and the same established classroom norms. These differences in small-group discourse may be attributed to differences in identity, ability, and nature of learning mathematical beliefs.

Teasing out the separate contributions of lesson implementation versus beliefs on discourse is neither possible nor the intent of this research because both are integral parts of the enacted curriculum. Working on building both students' beliefs and SBMI high quality practices may have positive effects on students' mathematical discourse and needs to be studied in more detail by others.

## APPENDIX A IRB APPROVAL



University of Massachusetts Amherst  
108 Research Administration Bldg.  
70 Butterfield Terrace  
Amherst, MA 01003-9242

Research Compliance  
Human Research Protection Office (HRPO)  
Telephone: (413) 545-3428  
FAX: (413) 577-1728

### Certification of Human Subjects Approval

**Date:** April 10, 2019  
**To:** Jennifer Ericson, Teacher Educ & Curriculum Stud  
**Other Investigator:** Sandra Madden, Teacher Educ & Curriculum Stud  
**From:** Lynnette Leidy Sievert, Chair, UMASS IRB

Protocol Title: Examining the Effect of the Enacted Curriculum and Students' Beliefs about Mathematics on Small Group Participation and Discourse  
Protocol ID: 2019-5441  
Review Type: EXPEDITED - NEW  
Category: 7  
Approval Date: 04/10/2019  
No Continuing Review Required  
UM Proposal #:

This study has been reviewed and approved by the University of Massachusetts Amherst IRB, Federal Wide Assurance # 00003909. Approval is granted with the understanding that investigator(s) are responsible for:

Consent forms - A copy of the approved consent form (with the IRB stamp) must be used for each participant (Please note: Online consent forms will not be stamped). Investigators must retain copies of signed consent forms for six (6) years after close of the grant, or three (3) years if unfunded.

Use only IRB-approved study materials (e.g., questionnaires, letters, advertisements, flyers, scripts, etc.) in your research.

Revisions - All changes to the study (e.g. protocol, recruitment materials, consent form, additional key personnel), must be submitted for approval in e-protocol before implementing the changes. New personnel must have completed CITI training.

Final Reports - Notify the IRB when your study is complete by submitting a Final Report Form in the electronic protocol system.

Serious Adverse Events and Unanticipated problems involving risks to participants or others - All such events must be reported in the electronic protocol system as soon as possible, but no later than five (5) working days.

Annual Check In - HRPO will conduct an annual check in to determine the study status.

Please contact the Human Research Protection Office if you have any further questions. Best wishes for a successful project.

## APPENDIX B STUDENT SURVEY

Student Assent

Name

What is the best time for you to participate in an interview (about ½ hour)?

Before school

After school

During lunch

During guided study/study hall

Math beliefs: Read each statement carefully and pick the response which best describes your beliefs about each item. There are no right or wrong answers.

-Strongly disagree -Disagree -Slightly disagree -Slightly agree -Agree -Strongly agree

In math, the teacher has the answer, and it is the student's job to figure it out.

Learning to do math problems is mostly a matter of memorizing the steps to follow.

NOLM

When 2 classmates don't agree on an answer, they can usually think through the problem together until they have a reason for what is correct.

It doesn't really matter if you understand a math problem if you can get the right answer.

Understanding the concepts in math is just as important as knowing the procedures.

Justifying the statements a person makes is an important part of math.

A person who doesn't understand why an answer to a math problem is correct hasn't really solved the problem.

If students ask questions in math class, it means they didn't listen to the teacher well enough.

You know something is true in math when it is in a book or an instructor tells you.

When learning math, it is helpful to analyze your mistakes.

When a student's method of solving a math problem is different from the teacher's method, both methods can be correct.

You can only learn math when someone shows you how to solve a problem.

Time used to investigate why a solution to a math problem works is time well spent.

It is important to convince yourself of the truth of a math statement rather than to rely on the word of others.

FA

If you are ever going to be able to understand something in math, it will make sense to you the first time you hear it.

It doesn't matter how hard a student tries, they are either born good at math or not.

Hard work can increase one's ability to do math.

It doesn't matter how long it takes to solve a problem as long as you figure it out.

If a student cannot solve a math problem quickly, then spending more time on it won't help.

Working on difficult problems only pays off for the really smart students.

Ability in math increases when one studies hard.  
You know someone is good at math if they can solve a problem quickly.  
By trying hard, one can become smarter in math.

Math Identity: Read each statement carefully and pick the response which best describes your beliefs about each item. There are no right or wrong answers.

-Not at all like me – Not like me – Not really me – Kind of like me – Like me – Exactly like me

MINT

I wish I did not have to take math.  
I find math interesting.  
I look forward to taking math.  
I enjoy learning math.

MID

My parents and relatives see me as a math person.  
My classmates see me as a math person.  
My friends see me as a math person.  
My math teacher sees me as a math person.  
I see myself as a math person.

MSE

Math makes me nervous.  
I am someone who is good at math.  
I understand the concepts I have studied in math.  
Setbacks in math do not discourage me.

What is your gender?

Female

Male

Transgender

Non-binary

Other: \_\_\_\_\_

What grade are you in? 7<sup>th</sup> 8<sup>th</sup> 9<sup>th</sup> 10<sup>th</sup>

Have you attended (Autumn) Public Schools continuously since 7<sup>th</sup> grade? Yes or No

What math class are you currently in?

What grade did you get on your last math unit test or quiz? A B C D F

## APPENDIX C INTERVIEW QUESTIONS

**Prior to starting the interview, researcher asks for participant's assent:**

**Introduction:**

“Thank you for agreeing to participate in this interview. The goal of this interview is to talk to you about your experiences in mathematics. The interview is expected to take between 20 and 25 minutes. You do not have to answer any questions that make you feel uncomfortable.

Is it OK if I record our discussion? [If yes, turn on microphone and repeat the question so it is recorded]

**Statement of Assent:**

Do you provide your assent and consent to participate in this interview? Please say yes or no.

When I transcribe this interview, meaning type up the audio recording with your response, I will replace your name with a pseudonym. Both the audio file and the transcriptions will be saved on a password-protected cloud storage, only accessible to my faculty sponsor and me.

Do you have any questions before we begin?”

What name would you like for your pseudonym?

What is your most favorable subject? Why?

If you didn't have to take math, would you? Why or why not?

What is working/not working for you in learning math this year?

Describe a typical day in math class.

What did you like and dislike about the math lessons?

How do you interact with your peers and the teacher?

Compare your current experiences in math with experiences in previous years.

When faced with a difficult math problem, what has helped you work through the problem?

What does it mean to be a math person?

How might you use math in your career?

Why are some people good at math and some people aren't good at math?

What does it mean to “do math”?

What is the teacher's role in the math class?

Is there someone who has encouraged you with your struggles/journey in math?

Have there been experiences that make you feel that you can't do math?

Have you learned math if you can't explain why you are solving a problem a certain way?

How could you make math class more interesting?

How do you think math should be taught? Do you think that you should be taught/shown how to solve problems first and then have to solve similar problems (practice) or is it better to work with others to figure it out?

## APPENDIX D LESSON OBSERVATION INSTRUMENT

Code Number: \_\_\_\_\_

**SAMPI—Western Michigan University** (MODIFIED FOR NOYCE/UMASS FEB 2015)

K-12 LESSON OBSERVATION DEBRIEFING FORM--Version B (Snapshot)

Complete this form using the observer's notes and information from the pre- and post-observation interviews. Use the "Definitions of Indicators" tool as a reference. Complete as soon after the observation session as possible.

DATE OF OBSERVATION \_\_\_\_\_ OBSERVER \_\_\_\_\_

TIME OF OBSERVATION: Start \_\_\_\_\_ End \_\_\_\_\_ GRADE LEVEL \_\_\_\_\_ No. Students in Class \_\_\_\_\_

### INFORMATION ABOUT THE LESSON AND CLASSROOM

1. Core subject matter of the lesson:
2. In a few sentences, describe the lesson observed—objectives, primary activities, where this lesson fits in the overall unit of study.
3. Indicate MAJOR ways that student activities were conducted over the entire course of the lesson.  
 Whole group activity     Small group activity     Pairs of students     As individuals
4. Rate the arrangement of the room relative to how well it facilitates student interactions.

1	2	3	4	5	6	7
Inhibits interactions				Facilitates interactions		

5. Indicate the primary intended purpose(s)—not specific objectives—of this lesson based on the pre- and post-observation interviews and what's learned during the observation.

- Identify prior student knowledge
- Introduce new concepts
- Develop understanding of concepts
- Review concepts
- Learn processes/skills related to the subject matter
- Learn vocabulary/specific facts
- Show how a concept applies in the real-world
- Develop appreciation for the core ideas of the subject matter of the lesson
- Develop awareness of contributions of experts in the subject matter from diverse backgrounds
- Other. Describe: \_\_\_\_\_

6. Briefly describe the instructional materials used in the lesson (e.g., textbooks, modules, kits, software, web-based materials, equipment/supplies, audio-visuals). Give specific names/publishers of materials being used.

**KEY ELEMENTS OF THE LESSON**

In this section, rate each of the indicators or answer the questions in four areas: planning/organization, implementation, content, and classroom culture. Note that any single lesson may not provide enough evidence for every indicator or question. In that case, check the DON'T KNOW box (but only as a last resort). Note any other indicators you consider important to the lesson. Use the "Definitions of the Indicators" tool for clarification.

**PLANNING/ORGANIZATION OF THE LESSON**

1. Does the lesson come directly from a pre-packaged program (i.e., kit, text series, modules, web-based program) with very few teacher modifications?

Yes	No	Don't Know
-----	----	---------------

If yes, name of program and specific lesson.

2. Rate the adequacy of classroom resources (supplies, equipment) to support the lesson.

1	2	3	4	5	6	7
Few resources						Many

resources

Supporting evidence for rating:

3. Was the lesson organized to provide substantive teacher-student interactions?

Yes	No	Don't Know
-----	----	---------------

If yes, what is the evidence?

4. Was the lesson organized to provide substantive student-student interactions?

Yes	No	Don't Know
-----	----	---------------

If yes, what is the evidence?

5. Were investigative tasks essential elements of the lesson plan (e.g., manipulation of information to help make sense of content, elements of problem-solving situations, connections to real-world experiences)?

Yes	No	Don't Know
-----	----	---------------

If yes, what is the evidence?

6. Was the lesson organized so that it appropriately addressed students' experiences, developmental levels, preparedness, and/or learning styles?

Yes	No	Don't Know
-----	----	---------------

If yes, what is the evidence?

7. Was the lesson organized so that it appropriately addressed issues of access, equity, and/or diversity?

Yes	No	Don't Know
-----	----	---------------

If yes, what is the evidence?

8. Were adequate precautions taken and procedures incorporated to assure student safety in conducting the lesson?

Yes	No	Don't Know
-----	----	---------------

If yes, what is the evidence?

9. Did the lesson incorporate student and/or teacher use of technology (i.e., computers, video/digital cameras, monitoring equipment, calculators)?

Yes	No	Don't Know
-----	----	------------

Note: If incorporation of technology was a major part of the lesson, complete the TECHNOLOGY TO SUPPORT THE LESSON section on PAGE 12 of this form.

10. Other comments about lesson planning/organization or other indicators of importance.

**IMPLEMENTATION OF THE LESSON**

1. The teacher appeared confident in facilitating this lesson.

1	2	3	4	5	6	7
Limited confidence			Great confidence			

Supporting evidence for rating:

  
Don't Know

2. Periods of teacher-student interaction were probing and substantive (questioning and dialog emphasized higher-order thinking and deep understanding and exposed students' prior knowledge).

1	2	3	4	5	6	7
Weak student-teacher interaction			Strong Student-Teacher interaction			

Supporting evidence for rating:

  
Don't Know

3. Classroom management was effective in engaging students in the lesson.

1	2	3	4	5	6	7
Limited effectiveness			Very effective			

Supporting evidence for rating:

  
Don't Know

4. The pace of the lesson was appropriate for the developmental levels of the students.

1	2	3	4	5	6	7
Poorly paced			Well paced			

Supporting evidence for rating:

  
Don't Know

5. Periods of student-student interaction were focused on pertinent lesson content and enhanced individual understanding of it.

1	2	3	4	5	6	7
Interaction not productive			Interaction very productive			

Supporting evidence for rating:

  
Don't Know

6. The lesson was organized so there was adequate time for students and/or the teacher to reflect on the lesson and its content.

1	2	3	4	5	6	7
Little or no time devoted to reflection			Considerable time devoted to reflection			

Supporting evidence for rating:

  
Don't Know

7. The lesson was organized so there was adequate time for wrap-up and closure of the lesson.

1	2	3	4	5	6	7
Little or no time devoted to closure				Considerable time devoted to closure		

Supporting evidence for rating:

Don't Know

8. Teacher makes connections between the content and the students' culture, community and families.

1	2	3	4	5	6	7
Little or no connections to student culture				Strong connections to students' culture		

Supporting evidence for rating:

Don't Know

9. The teacher communicates high expectations for all students, challenging all students to engage in problem solving, questioning and the generation of knowledge.

1	2	3	4	5	6	7
Expectations not sufficiently high				High expectations for all students		

Supporting evidence for rating:

Don't Know

10. Other comments about lesson implementation or other indicators of importance.

**OVERALL RATING FOR IMPLEMENTATION OF THE LESSON**

The overall rating represents the observer's best summary judgment of the appropriateness and quality of the lesson IMPLEMENTATION. Overall ratings are not necessarily intended to be the numerical average of the ratings of the indicators for Implementation of the Lesson. There may be other factors that influence an overall rating.

1	2	3	4	5	6	7
Implementation of the lesson not at all consistent with best practice in standards-based inquiry-oriented teaching and learning				Implementation of the lesson very consistent with best practice in standards-based inquiry-oriented teaching and learning		

## CONTENT OF THE LESSON

1. The content of the lesson was important and worthwhile.

1	2	3	4	5	6	7
Trivial content			Important content			

Supporting evidence for rating:

  
Don't Know

2. Students were intellectually engaged with important ideas related to the focus of the lesson.

1	2	3	4	5	6	7
Limited engagement			Significant engagement			

Supporting evidence for rating:

  
Don't Know

3. The subject matter was portrayed as a dynamic body of knowledge enriched by conjecture, investigation, analysis, and/or proof/justification.

1	2	3	4	5	6	7
Limited portrayal			Strong portrayal			

Supporting evidence for rating:

  
Don't Know

4. The teacher showed an understanding of the concepts and content that were the focus of the lesson and the topical/conceptual area being addressed by the lesson.

1	2	3	4	5	6	7
Limited understanding			Strong understanding			

Supporting evidence for rating:

  
Don't Know

5. The lesson included connections between concepts/content in this lesson and/or previous or future lessons in the overall unit or topic being addressed.

1	2	3	4	5	6	7
Weak connections			Strong connections			

Supporting evidence for rating:

  
Don't Know

6. The lesson included connections between this lesson and/or other areas of the same subject and/or other subjects.

1	2	3	4	5	6	7
Limited connections			Strong connections			

Supporting evidence for rating:

  
Don't Know

7. The lesson incorporated applications of the content/concepts of the lesson to real-world situations.

1	2	3	4	5	6	7
Limited applications			Strong applications			

Supporting evidence for rating:

  
Don't Know

8. The lesson included abstractions (theories and models) as appropriate.

1	2	3	4	5	6	7
Few abstractions			Many abstractions			

Supporting evidence for rating:

  
Don't Know

9. Other comments about lesson content or other indicators of importance.

### OVERALL RATING FOR CONTENT OF THE LESSON

The overall rating represents the observer's best summary judgment of the appropriateness and quality of the lesson CONTENT. Overall ratings are not necessarily intended to be the numerical average of the ratings of the indicators for Content of the Lesson. There may be other factors that influence an overall rating.

1	2	3	4	5	6	7
Insignificant or trivial content addressed in the lesson			Significant content consistent with standards and benchmarks addressed in this lesson			

**CLASSROOM CULTURE IN WHICH THE LESSON WAS CONDUCTED**

1. Active participation of all students was encouraged and valued.

1	2	3	4	5	6	7
Participation not encouraged/ not valued			Participation strongly encouraged/valued			

Supporting evidence for rating:   
Don't Know

2. The teacher showed respect for and valued students' ideas, questions, and/or contributions to the lesson.

1	2	3	4	5	6	7
Limited respect/value			Great respect/value			

Supporting evidence for rating:   
Don't Know

3. Students showed respect for and valued each other's ideas, questions, and/or contributions to the lesson.

1	2	3	4	5	6	7
Limited respect/value			Great respect/value			

Supporting evidence for rating:   
Don't Know

4. The classroom climate for the lesson encouraged students to generate ideas, questions, conjectures, and/or propositions.

1	2	3	4	5	6	7
Climate discouraged students			Climate encouraged students			

Supporting evidence for rating:   
Don't Know

5. Student-student interactions reflected collaborative working relationships.

1	2	3	4	5	6	7
Limited collaborative relationships			Strong collaborative relationships			

Supporting evidence for rating:   
Don't Know

6. Teacher-student interactions reflected collaborative working relationships.

1	2	3	4	5	6	7
Limited collaborative relationships			Strong collaborative relationships			

Supporting evidence for rating:   
Don't Know

7. The teacher's language and behavior showed sensitivity to issues of gender, race/ethnicity, special needs, and/or socio-economic status.

1	2	3	4	5	6	7
Little sensitivity			Strong sensitivity			

Supporting evidence for rating:

Don't Know

8. Teacher-student interactions reflect teacher knowledge of and appreciation for students' lives outside of the classroom including knowledge of family, culture and the life of the community.

1	2	3	4	5	6	7
Little knowledge and appreciation			Strong knowledge and appreciation			

Supporting evidence for rating:

Don't Know

9. Other comments about classroom culture or other indicators of importance.

**OVERALL RATING FOR CLASSROOM CULTURE**

The overall rating represents the observer's best summary judgment of the appropriateness and quality of the CLASSROOM CULTURE. Overall ratings are not necessarily intended to be the numerical average of the ratings of the indicators for Classroom Culture. There may be other factors that influence an overall rating.

1	2	3	4	5	6	7
Classroom culture not supportive of student learning			Classroom culture very supportive of student learning			

**USE OF TECHNOLOGY TO SUPPORT THE LESSON**

Complete this section when information and other electronic technology are used in a major way to support the lesson being observed.

1. List the major types(s) of technology hardware being used by the teacher and students to support the lesson.

Teacher:

Students:

2. List the major type(s) of software or programs being used to support this lesson (such as word processing, spreadsheets, mapping software, desktop publishing, PowerPoint, video production, dynamic geometry, dynamic statistics). Be as specific as possible about the software version being used.

3. Student technology use arrangement:

Computers

- Whole group activity (i.e., all students in lab setting). Students per computer? \_\_\_\_\_
- In groups of 2-4 at classroom computers.
- Individual activity (single student working at computer or students taking turns)

Other (video camera, video editor, Palms, etc.)

- Whole group activity.
- Small groups working together with equipment.
- Individual activity (single student using equipment or students taking turns)

4. Indicate the primary intended purpose(s) for which technology was used.

- Production: Students create a product (publication, web page, presentation, video, model, maps, etc.)
- Presentation: Teacher and/or students present (PowerPoint, video, music, publication)
- Communication: Students use Internet/email to communicate with peers, experts, and other audiences.
- Internet Research: Use the Internet to gather information.
- Original Research: Use monitoring or recording devices to gather data.
- Data Compilation/Analysis: Use technology to organize and analyze data.
- Visualization: Use graphing calculators or visualization software to see or manipulate relationships or objects.
- Other. Describe: \_\_\_\_\_

5. If this lesson is part of a curriculum unit or series of lessons, is technology used to support other lessons in the unit or series?  Yes  No

6. In using the technology and/or accessing information through technology, were students limited to specific procedures or sources devised by the teacher or dictated by the instructional materials? (Note: This may vary by grade or student skill level).

Yes	No	Don't Know
-----	----	------------

7. Technology resources were adequate to support the lesson.

1	2	3	4	5	6	7	
Inadequate resources					Many resources		

Supporting evidence for rating:

Don't Know

8. Technology use was effectively integrated into this lesson (not an "add-on" or novelty).

1	2	3	4	5	6	7	
Poor integration					Very effective integration		

Supporting evidence for rating:

Don't Know

9. The use of technology enhanced student learning of the lesson's core concepts/content.

1	2	3	4	5	6	7
Did little to enhance learning			Strongly enhanced learning			

Supporting evidence for rating:

--

Don't Know

10. The use of technology supported real-world application of the lesson concepts/content.

1	2	3	4	5	6	7
Did little to support			Strongly supported			

Supporting evidence for rating:

--

Don't Know

11. Technology use enhanced the ability of students to collaborate with each other.

1	2	3	4	5	6	7
Did little to enhance			Strongly enhanced			

Supporting evidence for rating:

--

Don't Know

12. Classroom management was effective in engaging students in the use of the technology.

1	2	3	4	5	6	7
Limited effectiveness			Very effective			

Supporting evidence for rating:

--

Don't Know

13. The teacher shows skills and ability in using technology to support student learning (consider both technical skills and lesson design).

1	2	3	4	5	6	7
Limited skills/ability			Strong skills/ability			

Supporting evidence for rating:

--

Don't Know

14. Other comments about use of technology or other indicators of importance.

**OVERALL RATING FOR USE OF TECHNOLOGY TO SUPPORT THE LESSON**

The overall rating represents the observer’s best summary judgment of the appropriateness and quality of the USE OF TECHNOLOGY TO SUPPORT THE LESSON. Overall ratings are not necessarily intended to be the numerical average of the ratings of the indicators for Use of Technology to Support the Lesson. There may be other factors that influence an overall rating.

1	2	3	4	5	6	7
Use of technology has little effect on teaching and learning in this lesson						Use of technology greatly enhances teaching and learning in this lesson

**OPTIONAL SUMMARY RATING OF THE LESSON**

Depending on how the data from the observation of lessons are going to be used, the observer may want to do a summary rating of the entire lesson, based on the ratings of the four major elements (five elements, if the technology support material is used). If the purpose of the set of observations is to get an overview of the nature and quality of lessons being conducted, the summary rating can be useful. However, unless the number of the set of lessons is fairly large (an adequate proportion of the classrooms being sampled and selected randomly) generalizing from the summary ratings of the sample to the entire set of classrooms is problematic. The summary rating is useful for looking at change over time among all the classrooms, as long as the sampling is credible.

The summary rating represents the observer’s best judgment of the quality of the lesson. This rating is not necessarily intended to be the numerical average of the ratings of the indicators for the four elements: planning/organization, implementation, content, and classroom culture. There may be other factors that influence the summary rating.

**SUMMARY RATING OF THE LESSON**

1	2	3	4	5	6	7
Overall, the lesson was not at all reflective of a standards-based inquiry-oriented lesson.						Overall, the lesson was an excellent example of a high quality standards-based inquiry-oriented lesson.

Note: Modifications to this instrument by UMass/Noyce Team include:

- 1) Removed references to Social Studies and Language Arts;
- 2) Added two indicators to Lesson Implementation related to culturally responsive teaching (#8 & 9);
- 3) Added one indicator to Classroom Culture related to culturally responsive teaching (#8), and
- 4) Added dynamic geometry and dynamic statistics software to the list of technologies in the Technology section (#2); and
- 5) Minor formatting changes.

**APPENDIX E**  
**DISCOURSE TAXONOMY CODES**

**Mode of Discourse**—Who the student is addressing during the mathematical discourse.

<b>Code</b>	<b>Definition</b>	<b>Explanation</b>
T	Student to Teacher	Student primarily addresses the teacher even though the entire class or group hears the student’s comments.
SS	Student to Student	Student addresses another student
SG/C	Student to Group	Student addresses a small group of students or the entire class.
IR	Individual Reflection	Student documents his or her reflections about mathematics in writing.

**Types of Mathematical Discourse**

<b>Code</b>	<b>Level</b>	<b>Definition</b>	<b>Explanation</b>
A	Low	Answering	Student gives a short answer to a direct question from the teacher or another student.
S	Low	Making a statement	Student makes a simple statement/assertion or shares his or her work with others and the statement or sharing does not involve an explanation of how or why.
E	Low	Explaining	Student explains a mathematical idea or procedure by stating a description of what he or she did, or how he or she solved a problem, but the explanation does not provide any justification of the validity of the idea or procedure.
Q	Low	Questioning	Student asks a question to clarify his or her understanding of a mathematical idea or procedure.
C	High	Challenging	Student makes a statement or asks a question in a way that challenges the validity of a mathematical idea or procedure. The statement may include a counter example. A challenge requires someone else to reevaluate his or her thinking.
R	High	Relating	Student makes a statement indicating that he or she has made a connection or sees a relationship to some prior knowledge or experience.
P	High	Predicting or Conjecturing	Student makes a predication or conjecture based on their understanding of the mathematics behind the problem.
J	High	Justifying	Student provides a justification for the validity of a mathematical idea or procedure by providing an explanation of the thinking that led him or her to the idea or procedure.
G	High	Generalizing	Student makes a statement that is evidence of a shift from a specific example to the general case.

**APPENDIX F**  
**DISCOURSE ANALYSIS OF CHAPTER 5 EXEMPLARS**

Ms. Luurtsema Observation 4: Trinity and Austin

Line	Time	Speaker	What is said ( <i>What is done</i> )	Small- group Discourse rating
74	13:40	Ms. L	And then, make a judgment, is there a pizza that is the better deal regardless of circumference or area? <i>(During this whole time, Austin and Trinity have been working independently to solve the problem.)</i>	
76		Ms. L	That is 3 things I have given you to do.	
77		Ms. L	That sounds like a lot, but you have a team of people at your table.	
78		Ms. L	Are you ready to work with them?	
<hr/>				
	14:10		<i>Trinity and Austin continue to work independently. After a few minutes, Trinity looks at Austin's paper to see if she is on the right track. There is no discussion between them. At 19:20 Ms. L calls the class's attention to review the two formulas they should be using. Austin continues to work with Trinity watching him.</i>	0
<hr/>				
109	21:36	Ms. L	Do you guys (addressing both) have a method for how you are going to compare all of them?	0
110		Trinity	Not really. <i>Shaking head no.</i>	
111		Ms. L	So, I want you guys to talk to each other about that	
112		Ms. L	So, after you get the measurements of all the	
113			circumferences, before you start to really decide which one is a better deal than the other, talk about what you are going to use as your criteria. OK?	
114		Austin	Okay <i>After Ms. L leaves, Trinity and Austin continue to work independently</i>	
<hr/>				
	27:00		<i>Both appear at an impasse. Stop working and sit quietly waiting for the whole class discussion without discussing with each other the different methods they tried.</i>	0
<hr/>				
	35:00		<i>Ms. L calls class to begin whole class discussion</i>	

Ms. Luurtsema Observation 2: Elasha and Hillary

Line	Time	Speaker	What is said ( <i>What is done</i> )	Small- group Discourse rating
32	8:50	Ms. L	Let's use this information to find the area of a circle.	1
33			<i>Hillary uses a water bottle to make her circle. Elasha uses a plate to create her circle. Hillary marks where to start on the circle to find the diameter</i>	
34		Hillary	I don't know how to do this. [statement]	
35		Elasha	I do. [statement] <i>Shows Hillary how to mark off the diameter using the cube. Stephanie mimics the procedure</i>	
36	10:10	Hillary	Mine is 8 ½ cubes long. [statement]	1
37		Elasha	Mine is 10. [statement] <i>Hillary uses the cubes to go around the perimeter and then fills up the circle. Elasha creates rows of cubes above and below the diameter.</i>	
38		Hillary	This doesn't work to fill it in. [statement] <i>Erases all her work and copies Elasha's work</i>	
39		Elasha	It doesn't work filling it in, you have gaps.	
40		Elasha	I feel like you have to multiply by something to get the area. [statement]	

Mr. McGinnis Observation 4: Nola, Luis, and Kaitlin

Line	Time	Speaker	What is said ( <i>What is done</i> )	Small- group Discourse rating
	45:00		<i>Problem 1 done independently</i>	
170	46:10	Nola	[to Mr. M] Are we doing this with our team?	0
171		Mr. M	Yes	No student-
172		Luis	[to Nola] Team is your table group. <i>Luis continues to work independently. So do Nola and Kaitlin</i>	student discourse.
175		Nola	[to Mr. M] A block is $1/4^{\text{th}}$ . What is that supposed to mean? <i>Nola looks to see what Luis is doing and erases her work. Luis puts down his head and waits for the rest of the group and Mr. M</i>	Only teacher-student discourse.
178		Luis	[to Mr. M] If I rotate it around this point, wouldn't it be here [points to paper]	
179		Mr. M	You are rotating around the point [points to directions]	
180		Luis	Oh! (3, -2)	
		Mr. M	That is just here [points to (3, -2) on Luis's paper] <i>Luis erases the last rotation and rotates around (3, -2) incorrectly.</i>	
184		Nola	[to Luis] What does yours look like? [question] <i>Luis shows Nola his work.</i>	
186		Nola	[to Mr. M] I finished problem 2! <i>Mr. M checks her work</i>	
189		Nola	Should I work on problem 3?	
190		Mr. M	Yes. Wait! You did a clockwise rotation. It was supposed to be counterclockwise. <i>Luis and Nola correct their rotations.</i>	
192		Mr. M	[to Kaitlin] This is the x-axis here. <i>Points to graph.</i>	
193		Kaitlin	I thought if you reflected across here [pointing to graph] that was across the y-axis	
194		Mr. M	<i>Pointing to graph</i> This is the x-axis, and this is the y-axis...	
199		Nola	[to Luis] That reflection should be lower on the graph. [statement]	
200	49:30	Luis	No, this is correct. [statement] <i>Nola erases her work and copies Luis's reflection</i>	

*even though it is incorrect. The group works independently without conversation until the wrap-up.*

Mr. McGinnis Observation 7: AJ, Sam, and Marc

Line	Time	Speaker	What is said ( <i>What is done</i> )	Small- group Discourse rating
55	25:00	Mr. M	Start with part 1 of the classwork. Describe a sequence of transformations to get from Figure A to Figure B. Work in your table groups for this part.	3
57		Marc	It rotated 90 degrees. NO! Wait! Yeah 90 degrees.	
58		Sam	Clockwise	
59		AJ	I have an idea.	
60		Marc	Around?	
61		AJ	We could move it 8 units to the right and 2 units up. And multiply the y-axis by 4.	
62		Sam	What?	
63		AJ	We could move it 8 units to the right. Is it going from A to B?	
64		Sam	We have to reflect it over the y-axis. And then move it 2 units up along the y-axis. Then multiply by 4.	
66		AJ	That is what I was thinking. Look Marc... if we reflect it here. <i>Points to y-axis on Marc's paper.</i> It is going to be here. <i>Outlines it on graph.</i> Then move it 2 units up. It is in this box. <i>AJ draws the lower 1/4 of rectangle B.</i>	
68		Marc	Oh.	
69		AJ	Then if we multiply it by 4, then we would have 4 [boxes]. <i>AJ draws in 4 boxes.</i>	
70		Marc	That is easier.	
71		Sam	And dilate it what?	
72	30:00	AJ	2y, 1x because x stays the same. If you just say dilate, it assumes all of the vertices are multiplied. [justification]	3
75		Marc	But only y because you are not dilating x.	
76		Sam	1x	
77		AJ	You would write 1x	
79		AJ	4y?	
80		Marc	<i>Pointing on AJ's paper.</i> 2 would only get it to here. It needs to be 4 to get it all the way.	

81 AJ I just realized that this is on the negative side. SO, it  
would actually go down and we would have to translate  
it up.

82 Sam Why don't we translate it...?

83 Marc And multiply it by....

84 AJ Yes, but if we translate it over then there is that little bit  
that is underneath [the x-axis].

86 Marc What if you multiply it by negatives?

87 AJ If we multiply it by -2 that should go....

88 Sam No, if you multiply it by -4, it would go up.

89 AJ But that would put the shape.... You would have to  
translate the figure 2 units down.

90 Marc UGH

91 Sam Then we just translate it 2 units down.

92 AJ 4 times -4....  
This is interesting because in step 2 we actually  
translated it 2 units up. So maybe those cancel out each  
other? Let's see. Nope. Nope. We have to go up and  
down.

93 35:00 *Mr. M calls class to debrief the problem.*

Ms. Kraft Observation 4: Riley and Allen

Line	Time	Speaker	What is said ( <i>What is done</i> )	Small- group Discourse rating
63	12:00	Ms. K	In your groups do #1-3. Then we will reconvene in about twenty minutes. <i>Riley and Allen begin to work independently.</i>	2
64		Allen	When we are both done with 2, we should compare before we go onto 3. What did you get for 2A? $(x+5)^2 + 2$	
66		Riley	$y = x^2 + 27$ [answer]	
67		Allen	What about the middle figure? This x. [challenge] <i>Allen points to his work and the 10x.</i>	
68		Riley	Uh, its...	
69		Allen	Don't you do this? [challenge] <i>Shows how he solved <math>(x + 5)(x + 5)</math></i> And then add the 2?	
70		Riley	I did the $5^2$ part too. [statement]	
71		Allen	We have 4 more minutes! This is the same as this. [Relating back to a previous problem from the warmup $(x+3)(x-1)$ ] Wouldn't you just do this <i>Allen shows his area model</i> and then condense it? [challenge] Ms. K said don't forget the middle term. [Justification]	
74		Riley	I am not sure. <i>Allen and Riley huddle over Riley's work</i>	
75		Allen	I think we are supposed to do it like the warm-up and add the 2. I might be wrong though. NO, wait. I don't know. Let's check with Ms. K. <i>Allen calls Ms. K over.</i>	
76		Allen	Can you check to see if what we did is correct because we got a different answer for 2A?	

Ms. Chenette Observation 3: Kelly and Sally

Line	Time	Speaker	What is said ( <i>What is done</i> )	Small- group Discourse rating
18	3:00	Kelly	I couldn't figure it out. I thought it was 10. Then I realized that I didn't know what BD was, so I wrote $ac = ?$ [statement]	3
		Sally	I tried $a^2 + b^2 = c^2$ . I thought that $BC = CE = 5$ but I didn't know what AB was to use the Pythagorean theorem. [relating]	
		Kelly	I didn't have a protractor to measure the angles. It looks like $\angle B$ is a right angle so we could use the Pythagorean theorem. And $\angle C$ is a right angle too. [relating]	
		Sally	If these are 90 [ $\angle B$ and $\angle C$ ] then these have to be 90 too [ $\angle D$ and $\angle A$ ]. [justification]	
		Kelly	Wait, this is 180...so that means...nothing.	
		Sally	They are all 90. It is a rectangle. [Relating/justification]	
		Kelly	Maybe we can use ASA to prove these [ $\triangle ABC$ and to $\triangle DBC$ ] are the same [Conjecture]	
		Sally	I took notes on that. [Gets out notebook]	
		Kelly	What do we know?	
		Sally	That $\angle B$ and $\angle C$ are both 90 degrees.	
		Kelly	And that $AB = DC$ . But now I am stuck.	
55	15:30	Ms. C	Do we know if these [pointing to the angles created by the diagonals of the rectangles $\angle ABD$ and $\angle DBC$ ] are both 45 degrees? Discuss this with your group.	
		Sally	No, they aren't. IF it were a square, it would be, but a rectangle, no. These two sides are equal, but these aren't the same [points to sides of rectangle] [Justification]	
		Kelly	They can't be 45 degrees. IF it were a square then you could be sure! Because all 4 sides are the same, and we would have isosceles triangles making them both 45 degrees. [Generalization]	

Ms. Chenette Observation 7: Taz, Max, and Donald

Line	Time	Speaker	What is said ( <i>What is done</i> )	Small- group Discourse rating
29	9:25	Donald	On this one [#2], I just wrote down what was given and then I stopped. I wasn't sure where to go from there.	3
30		Ms. C	Can you give your group those statements? Maybe they can work with them [ <i>leaves</i> ]	
31		Don	LC = NC, LE = AN Given Wait, then can we say they are even? No. Wait, if we say 2 sides are equal, doesn't that mean that the angles are equal? [relating]	
34		Taz	What angles do you mean? [question]	
35		Max	These are opposites! [ <i>pointing at <math>\angle L</math> and <math>\angle N</math></i> ] [statement]	
36		Taz	The big triangle is isosceles. [statement]	
37		Donald	So, the base angles are equal [relating]	
38		Taz	Why?	
39		Donald	ITT [justification]	
40		Taz	Then we have SAS, right?	
41		Max	Only for triangles LEC and ANC...I am not sure where... [answer]	
42		Donald	We could say EC = AC. [statement]	
43		Taz	How does that help? [challenge]	
44		Max	Wait! Can we say EA = EA and so LE + EA = EA + AN? Like we had the other day with those algebra proofs? [relating]	
45		Donald	Yes. And then we can use SAS to prove it. [statement]	
46		Max	So, we have $\angle L \approx \angle N$ , LA = EN, and LC = CN. [justification]	

## REFERENCES

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