Engaging Students in Mathematics Conversations: Discourse Practices and the Development of Social and Socialmathematical Norms in Three Novice Teachers' Classrooms

ENGAGING STUDENTS IN MATHEMATICS CONVERSATIONS: DISCOURSE PRACTICES AND THE DEVELOPMENT OF SOCIAL AND SOCIALMATHEMATICAL NORMS IN THREE NOVICE TEACHERS' CLASSROOMS

Dissertation Presented

by

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DEDICATION

This dissertation is dedicated to my mother, Frances O'Connor Quinn, and my father, James B. Quinn.
Thank you.

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ABSTRACT

ENGAGING STUDENTS IN MATHEMATICS CONVERSATIONS: DISCOURSE PRACTICES AND THE DEVELOPMENT OF SOCIAL AND SOCIALMATHEMATICAL NORMS IN THREE NOVICE TEACHERS' CLASSROOMS

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Research on learning to teach mathematics reveals that mathematics teaching is a complex process (Lerman, 2000) and classroom teaching and learning is a "multifaceted, extraordinarily complex phenomenon" (O'Connor, 1998, p. 43). Moreover, research reveals that the mathematics reform agenda has had an impact on what happens in the mathematics classroom, however, the impact has been superficial (Kazemi & Stipek, 2001) with teachers often retaining their pre-reform habits and attitudes in regards to mathematics teaching and learning (O'Connor, 1998). This study examined the reform discourse practices that three novice teachers, who had been enrolled in a reform based methods course during their preservice teacher education program, adopted, adapted, or ignored as they attempted to engage students in mathematical conversations. Data sources included interviews, field notes, artifacts, and transcripts of videotaped classroom lessons. The primary research questions guiding this study included: 1) What reform-oriented discourses practices do novice teachers, who participated in a reform-based mathematics methods course adopt? What practices do they adapt? What practices do they ignore as they engage students in mathematics conversations? and 2) What issues and challenges surface as novice teachers begin to enact reform-oriented discourse practices? Results indicated that despite holding beliefs that reflect the basic tenets of mathematics reform, theses novice teachers represent a continuum of practices ranging from traditional to reform. Evidence suggests that adopting the reform-oriented practice of eliciting different solutions was critical in the development of social norms that reflect mathematics reform. Eliciting different solutions served to focus classroom conversations on meaningful student generated explanations and justifications. Moreover, evidence suggests that enacting the practice of eliciting different solutions was instrumental in enacting other reform-orientated practices associated with the development of reform-oriented socialmathematical norms. Lastly, results indicate that the pressures of teaching in an underperforming school, as defined by state standardized high stakes tests, can impact a novice teacher’s ability and willingness to adopt mathematics reform practices.

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INTRODUCTION

A Vision for School Mathematics

> Imagine a classroom where… teachers help students make, refine, and explore conjectures on the basis of evidence and use a variety of reasoning and proof techniques to confirm or disprove those conjectures. …Students are flexible and resourceful problem solvers. Alone or in groups and with access to technology, they work productively and reflectively, with the skilled guidance of their teachers. Orally and in writing students communicate their ideas and results effectively (National Council of Teachers of Mathematics [NCTM], 2000, p. 3).

The vision for school mathematics, proposed and supported by the National Council of Teachers of Mathematics, is an ambitious one but, more importantly, it is one that requires knowledgeable teachers who can orchestrate and cultivate such a promising vision (NCTM, 2000). The council acknowledges the challenge that such a vision imposes, describing it as “enormous” (p. 4) while at the same time recognizes the valuable impact realizing such a vision will have on how U.S. students are prepared to live in an ever-changing world. The technological boom of the 1980s and 1990s has made quantitative information, available at first to only a limited number of people, readily available to the general public (NCTM, 2000). The need to understand mathematics more deeply is critical, if the United States and its citizens are to keep pace with an ever-changing technologically driven global environment.

To meet the challenge, advocates of mathematics reform have proposed a vision of classroom teaching and learning that differs significantly from past visions and will require teachers to reassess their role in the classroom and their role in bringing the vision of reform to fruition. The role of the mathematics teacher in a reform-oriented classroom is in sharp contrast to the traditional role most teachers are familiar with (Clarke, 1997). Rather than being a transmitter of facts and procedures, the teacher in a reform-oriented classroom is a facilitator who actively engages students in constructing mathematical knowledge (NCTM, 1989, 1991, 2000). To shift from a transmitter role to a facilitator role will call for teachers to develop a sense of what McClain and Cobb (2001) call "*knowing in action*" (p. 236). Knowing in action is the ability to "capitalize on opportunities for mathematical learning that emerge from students' activity and explanations" (p. 236). Meeting the challenges of reform will entail engaging teachers throughout their professional career in the types of rich mathematical thinking and learning that we expect students to be actively engaged in. It is only when teachers are able to think deeply about mathematics can we expect students in the classrooms to do the same for "teachers are key players in changing how mathematics is taught and learned in schools" (NCTM, 1991, p. 1).

Teacher education is a critical point in a teacher's career trajectory and a place where change can be made (Luft, 2007; Ma, 1999). How do novice teachers, who participated in a reform-based mathematics methods course, develop a mathematical teaching practice that reflects the vision set forth by the NCTM? This is a critical question and one of high priority, if we are to expect students to be engaged in mathematics in meaningful ways.

Statement of the Problem

Mathematics teaching is a complex process (Lerman, 2000) and classroom teaching and learning is a "multifaceted, extraordinarily complex phenomenon"

(O'Connor, 1998, p. 43). Furthermore, mathematics reform recommendations proposed and supported by the National Council of Teachers of Mathematics (1989, 2000) add another layer of intricacy to the already complex nature of mathematics teaching. Reform recommendations require teachers to think about teaching and learning in ways that are new and unfamiliar to many U.S. teachers. Moreover, reform recommendations call for teachers to interact with students in ways that foster a shared negotiation of the mathematical content – a concept many teachers have not experienced in their own mathematical education.

Reform recommendations are problematic in that teachers have experienced a traditional approach to learning mathematics in most, if not all, of their mathematics classes (Kirschner, 2002). Teachers' past mathematical experiences are deeply rooted within a transmission model of teaching and learning and are in direct contrast to the model of teaching and learning proposed by advocates of reform (Scanlon, 2003). These formative experiences have had a significant impact on the way that teachers teach, with teachers often teaching the way they were taught (Ball, 1988; Ball, Lubienski, Mewborn, 2001).

Although the reform agenda initiated by National Council of Teachers of Mathematics has had an impact on what happens in the mathematics classroom, the impact has been superficial (Kazemi & Stipek, 2001). O'Connor (1998) articulates the superficiality of the reform efforts in her statement, "the accouterments of practice may change, and the terminology changes, but often teaching practices retain their prereform character, in spite of attestations by practitioners that they have changed" (p. 43). Ball (1988) states that teacher education itself is a "weak intervention" (p. 2) with teachers often teaching mathematics the way they were taught despite reform efforts in mathematics teacher education.

The problem for novice teachers becomes how to develop and sustain a teaching practice based on mathematics reform given their wealth of traditional experiences in school mathematics. Much research has been done to study how children develop a deep conceptual understanding of mathematics; however, little research has focused on how novice teachers develop classroom cultures that foster a conceptual understanding of mathematics and in which both students and teachers engage in rich contextual discourse (Putman & Borko, 2000) – cultures advocated by reform initiatives.

Examining how a mathematical teaching practice develops during the formative first years of teaching will aid in understanding how teacher preparation programs based on reform practices impact the culture of the mathematics classroom. Additionally, understanding the issues and challenges that novice teachers face as they begin developing a mathematical teaching practice based on reform recommendations will uncover the reform practices that novice teachers adopt, adapt, or completely ignore while developing a mathematical teaching practice. Ball et al. (2001) recommend shifting the research lens from teachers to the practice of teaching in order improve and inform policy, practice, and teacher education. As such, this study focused its lens on the practice of teaching thus addressing the recommendation by Ball et al. (2001).

Overview of this Study

This descriptive case study examined the practices used by three novice teachers to engage their students in mathematics conversations. Using video observations the study captured the discourse practices the participants used and examined the reform orientation of such practices. The thesis of this study was that novice teachers who had participated in a reform-oriented mathematics methods course would enact discourse practices that engage students in productive mathematics conversations. This study fills a void in the literature on alternative patterns of discourse in mathematics classrooms (Cazden & Beck, 2003) as it sought to code, quantify, and qualify the discourses practices of teachers attempting to implement mathematics reform practices.

Purpose

The purpose of this descriptive case study was to examine and describe the discourse practices used by three novice teachers as they attempted to implement mathematics reform initiatives that engaged students in productive mathematics conversations. Additionally, this study was interested in examining the issues and challenges that surfaced for the participants as they attempted to enact mathematics reform practices.

This study provides an analysis of the parallel mathematical activity of three novice teachers as they engage in the work of developing a mathematical teaching practice under the auspice of mathematics reform. Understanding the practices that novices' adopt, adapt, and/or ignore is essential in understanding the complexity of such an endeavor. Moreover, examining the issues and challenges novice teachers face as they transition from students of mathematics to teachers of mathematics is essential to developing teacher education programs and professional development experiences that support novice teachers’ ability and willingness to engage and enact mathematics reform practices.

Research Questions

This study is a microanalysis of videotaped lessons that were captured in order to describe the discourse practices of three novice teachers as they began to develop a teaching practice under the auspice of mathematics reform. The research questions guiding this study were:

1. What reform-oriented discourse practices do novice teachers who participated in a reform-based mathematics methods course adopt? What practices do they adapt? What practices do they ignore as they engage their students in mathematics conversations?
2. What issues and challenges surface as novice teachers attempt to enact reform-oriented discourse practices?

Scope and Significance of the Study

This inquiry is a follow-up to a pilot study that took place in the spring of 2006 at a large research institution in northeast U.S. During that time the participants were preservice teachers enrolled in a one-year intensive teacher preparation program. This study followed the participants from their preservice years into their second year of teaching in two different urban school districts. The second year was chosen as an entry point because it is a critical year in the development of a teaching professional, as the annual turnover rate of novice teachers in the first three years of teaching is as high as 50% for those teaching in urban school districts (Haberman & Richards, 1990).

The study is significant in that it provides an analysis of how a mathematical teaching practice develops, from the theoretical preservice years to the practical beginning years of teaching. Bauersfeld (1993) suggests that if university teacher preparation programs want teachers to develop classroom cultures that facilitate students' mathematical conceptual development, then preservice teacher training programs must develop these types of mathematical communities in their programs as well. The participants in this study were encouraged to be agents of change during their preservice teacher preparation program. They were immersed in a subculture that fostered the development of social and socialmathematical normative behaviors consistent with reform initiatives. This study examined and documented the practices that the participants used and more over attempted to sort out the social and socialmathematical norms that the participants' practices fostered within their own classrooms. The results will inform policymakers and teacher educators as to the issues and challenges novice teachers face as they begin to move from the theoretical preservice teaching years into the practical and often daunting first years of teaching.

The study was particularly interested in understanding the significance of the interactions between the novice teacher and her students as the novice begins to navigate the mathematics curriculum with students. Ball and Forzani (2007) refer to this type of research as *"research in education"* (p. 520) and distinguish it from research that is "*related* to education" (p. 530). The authors advocate for this type of research and conclude that it is “necessary for the production of the sort of disciplined knowledge that might contribute directly to solutions to pressing problems in education” (p. 530).

In addition, this study gives significant value and voice to the novice teacher’s experience as she attempts to develop into an effective teacher of mathematics, thus contributing to a body of research that attends to the process of “*becoming* a teacher” (McLean, 1999, p. 55).

Assumptions of the Study

An assumption underlying this study is that mathematics teaching and learning is a socially constructed endeavor, negotiated by the members of the classroom community. I am in agreement with Shane (2002) who holds “a holistic view of the distinctive culture of the mathematics classroom as being initiated, maintained, and developed by the teacher” (p. 119). Classroom culture in this study refers to the shared normative behaviors that are continually being constituted and reconstituted among members of the community. These norms account for the common knowledge that exists within the classroom community and, as such, are not specifically explained and delineated up front as a set of rules to follow but are constituted as members of the community work together.

Additionally, this qualitative case study was embedded in the assumption that reality is continually changing, thus it cannot be observed as a static, fixed entity (Merriam, 1998). Reality is not an “objective phenomenon waiting to be discovered, observed, and measured” (p. 202) by the qualitative researcher. Reality is ever changing and inextricably tied to the each individual’s construction of reality. Thus, it is imperative in qualitative research to address the quality and validity of the research methods and design as well as the quality and validity of the researcher, as she is inextricably tied to the reality of the research project.

Lastly, it was assumed that preservice teachers *do* learn about reform initiatives within the context of a mathematics methods course and that this learning will influence the discourse practices that they use while teaching mathematics.

Establishing Trustworthiness

As a means of establishing credibility, the following forms of trustworthiness were utilized throughout the data analysis process:

1. Participant Validation: Emergent findings were shared with the participants allowing them the opportunity to elaborate, question, or correct information gathered (Rossman & Rallis, 2003).
2. Participant Collaboration: Participants were involved in all aspects of the research from conceptual design through writing up the findings to increase internal validity and “match reality” (Merriam, 1998, p. 201) of the situations being observed.
3. Triangulation: Multiple data sources, methods, and points in time were used to aid in triangulation. As the data was analyzed cross-checking occurred across all data sources looking for corresponding information.
4. Peer Review: As data was analyzed and interpreted a “critical friend” (Rossman & Rallis, 2003 p. 69) served as an intellectual peer reviewer helping me to stay on track and keep me honest in my analysis and interpretation.

5. Clarifying Researcher Biases: Throughout the research process I continually make clear and addressed my own biases in relationship to the data being collected. Biases cannot be eliminated from any qualitative research (Rossman & Rallis, 2003) nor should they be, as they are an important part of the researcher's reality. However, it was imperative that as the researcher I examined and make clear my biases and addressed each as it pertained to the data collected and the interpretations made.
6. Audit Trail: Reliability in qualitative research lies in the ability of "outsiders to concur that, given the data collected, the results make sense" (Merriam, 1998, p. 206). Explaining and describing in detail how data was collected, how categories develop, and how important decisions were made increased the reliability of this study (Merriam, 1998).

Definitions of Terms

1. Practice: In this study the term practice refers to the discourse practices that the participants used to engage students in mathematics conversations.
2. Preservice teacher**:** An individual who is studying to be a teacher and enrolled in a teacher preparation program at a college or university.
3. Novice teacher: An individual who is in the first 3 years of classroom teaching. By definition a novice is considered to be in the beginning stages of learning an activity.

4. Social norms**:** The ways in which members of a group interact with each other. As such, social norms regulate the participation structure within classrooms (Yackel &Cobb, 1996).
5. Socialmathematical norms**:** Socialmathematical norms regulate students' mathematical activity within classrooms (Yackel & Cobb, 1996).
6. Taken-as-shared expectation**s:** When normative behaviors are continually being constituted and reconstituted among members of a group the ways of doing things within the group come to be taken-as-shared among all members (Cobb, Yackel, & Wood, 1993).

Organization of the Dissertation

The remainder of this dissertation is divided into seven chapters. The second chapter presents a literature review examining the following areas: mathematics reform, classroom discourse, knowledge needed to teach mathematics, and teacher learning and the process of change. The third chapter presents the theoretical framework grounding this study as well as the research methodology and the methods of data collection and analysis used to conduct this study. Chapters four, five, and six present the results of the individual case studies. Chapter seven presents a cross-case analysis and lastly chapter eight provides a discussion of key findings and concluding remarks regarding implications of the study and recommendations for further research.

CHAPTER 2

LITERATURE REVIEW

The purpose of this chapter is to situate the study, via the literature, by discussing the theoretical and practical constructs that were drawn upon in making sense of the teaching practices that three novice teachers used to engage students in mathematics conversations. This chapter will present a review of the literature in the following four areas: mathematics reform, classroom discourse, knowledge needed for teaching, and teacher learning and the process of change. Drawing from a wide literature base in mathematics education, the first section will examine the tenets of mathematics reform and discuss the complexity of adopting mathematics reform practices. Section two will present a review of the literature regarding the knowledge base needed to teach mathematics and make the argument that research is needed to understand how such knowledge develops during the novice teaching years. In section three, the literature addressing classroom discourse will be examined as a means to illuminate the discourse practices that teachers use to stimulate productive mathematical conversations in the classroom. Here the argument will be made that studies are needed to discern, quantify, and qualify reform discourse practices from traditional ones. Lastly, in section four, a review of the literature on teacher learning and the process of change will be explored. More specifically, the review will address the resiliency of beliefs and argue that understanding how beliefs impact a novice teachers’ enactment of reform practices is warranted. The chapter will culminate with a return to the questions guiding this study.

Mathematics Education Reform

Traditional classroom teaching practices have been based on what Freire (1997) refers to as the "banking concept of education" (p. 53) whereby teachers are the depositors and the students take on the role of depositories. The interaction in this type of classroom is linear in that the teacher fills students with information and the students passively wait to be filled. Freire (1997) delineates the following classroom practices that are embedded within the banking concept of education:

a) The teacher teaches and the students are taught;
b) The teacher knows everything and the student know nothing;
c) The teacher thinks and the students are thought about;
d) The teacher talks and the students listen. (p. 54)

The practices delineated by Paulo Freire have dominated the educational landscape in the United States and have had significant influence over how teachers themselves were taught mathematics as students.

In 1989 the National Council of Teachers of Mathematics issued a call for reform in mathematics education, thereby calling for a reform in classroom practices as well. The vision of mathematics education advocated by the reform movement is one of inquiry where both teachers and students engage in rich mathematical discourse negotiated by all members of the classroom community. The goal of the reform movement is to develop autonomous thinkers and learners (Warfield, Wood, & Lehman, 2005). The NCTM (1991) developed a set of professional standards as a means to guide mathematics teachers toward implementing reform recommendations. The NCTM (1991, p. 1) Professional Standards document mapped out the following discourses practices to guide teachers as they engaged students in productive mathematics conversations:

The teacher of mathematics should orchestrate discourse by:

- posing questions and tasks that elicit, engage, and challenge students' thinking;
- listening carefully to students' ideas;
- asking students to clarify and justify their ideas orally and in writing;
- deciding what to pursue in dept

There is a stark contrast between traditional and reform classrooms, and the contrast runs deeper than differences in classroom practices. In traditional classrooms the banking concept of education serves the purpose of eliminating or at least minimizing student autonomy so that students are conformed by education rather than transformed by the process (Freire, 1997). However, the classroom practices envisioned by the reform movement seek to transform students into autonomous learners capable of thinking deeply about mathematical ideas and solving problems effectively and efficiently without being taught particular methods or procedures (Warfield et al., 2005). The problem with such a vision is that the teachers expected to enact the vision are unfamiliar with autonomy on a personal level. Teachers are the products of the banking concept of education, and the practices they bring to the classroom are the byproducts of such a system. Additionally, teachers enter into reform movements with different types of knowledge and beliefs that often conflict with reform (K. Davis, 2003) thereby influencing their understanding and enactment of reform initiatives.

Although the reform agenda advocates shifting from rote memorization of basic mathematical facts to a conceptual understanding of mathematical topics and concepts, the historical modernist past has been the dominant force shaping teachers' mathematical experience. Many novice teachers in classroom today were educated during the height of the reform movement; however, the reform initiatives have had little impact on how mathematics is taught (Hardy, 2004). The changes that have been made in school mathematics have been superficial and "students persistently experience a fundamental curriculum of fact learning and routinized computations where they are expected to be consumers of established mathematical truths" (p. 104).

Novice teachers are being asked to teach mathematics in a significantly different way from how they learned mathematics. Their formal mathematics learning was based on the model that knowledge acquisition is authoritarian in nature, is transmitted from teacher to student, and is in sharp contrast to the model of cultural participation supported and accepted by the mathematics education community (Yackel & Cobb, 1996). How novice teachers embark on the journey toward reforming their own habits and assumptions about how mathematics should be taught and learned is of central importance, if we expect students to engage in the types of mathematical discourse advocated by reform initiatives.

Mathematics classrooms, by nature of the subject, may be problematic to reform. Burton (1995) asserts that the dominant view of mathematics is one of objectivity and such a view fosters the belief that "mathematical 'truths' exist and the purpose of education is to convey them into the heads of the learners" (p. 276). The discipline of mathematics with its philosophical roots in modernist objectivity seems to favor the banking concept of education that Freire (1997) and reform practices push against. Moreover, the absolutist foundation of mathematics and the assertion that mathematics is an objective body of knowledge makes it an untenable place to implement reform practices, albeit worthy, without a significant shift in the discipline itself. The absolutist perspective implies that truth in mathematics is absolute and unchanging. Such a view does not allow for critical questioning by the common individual (Ernest, 1998) and thus affords the dominant discourse to continue to dictate the mathematical conversations inside and outside of the classroom.

Only recently has the absolute nature of mathematics been under question. Ernest (1998) emphatically argues against the absolutist view in favor of what he terms a “fallibilist” (p. 10) view of mathematics. Viewing mathematics as fallible means to accept the notion that what is considered knowledge today may very well “lose its modal status as true or necessary” (Ernest, 1998, p. 10). Accepting the fallibility of mathematics frees one to explore and invent ways of coming to understand mathematics, rather than being constrained by the assumption that mathematics is a fixed set of rules that one must discover.

Learning to teach in ways that reflect reform recommendations is a difficult task for the novice teacher, as many factors influence a developing mathematical teaching practice. According to Ball, Hill, and Bass (2005) the quality of a mathematical teaching practice depends largely on the mathematical content knowledge of the teacher. The following section will explore the knowledge base needed to teach mathematics in concert with reform recommendations.

Teaching Practices that Foster Reform

Most classroom teachers have had little, if any, experience constructing mathematical knowledge in ways recommended by the reform movement (Scanlon, 2003). However, mathematics teacher educators, cognizant of this fact, are engaging preservice teachers in rich mathematical learning opportunities so that they can begin to develop a teaching practice based on reform recommendations. As an elementary mathematics teacher educator, I have had the opportunity to work with preservice teachers as they begin to grapple with their own limited mathematical understandings, in relationship to the standards set forth by NCTM (1989, 1991, 2000). These future educators are being asked to teach mathematics in a significantly different way from how they learned mathematics. Their formal mathematics learning was based on the model that knowledge is transmitted from teacher to student and is in sharp contrast to the model of cultural participation supported and accepted by the mathematics education community (Yackel & Cobbs, 1996). Bauersfeld (1993) describes this new model as,

> a model of participating in a culture rather than a model of transmitting knowledge. Participating in the process of a mathematics classroom is participating in a culture of using mathematics, or better: a culture of mathematizing as a practice. (p. 4)

To develop a community such as the one described by Bauersfeld (1993), the social interactions that occur between students and students and students and teacher must be different from the ones that have dominated the American educational landscape. According to interactionist theory, social interactions are a critical part of any mathematical activity (Voight, 1995). Thus understanding the social interactions that happen within the mathematics classroom is an important line of inquiry.

Social and Socialmathematical Norms

Paul Cobb, Erna Yackel, and Terry Wood (1993) have extensively researched the “social reality of mathematics classrooms” (p. 22) and have focused their analysis on student learning as it is negotiated within a group. As a result of their work in elementary mathematics classrooms, the researchers have articulated the ways in which teachers and students work together to develop a taken-as-shared reality within the context of mathematics. Moreover, the extensive work done by Cobb and his colleagues over the last 17 years provides a means to analyze classroom interactions in terms of two constructs – social and socialmathematical norms (see Cobb, Boufi, McClain, & Whitnack, 1997; Cobb, Stephan, McClain, & Gravemeijer, 2001; Cobb & Whitenack, 1996; Cobb, Wood, Yackel, & McNeal, 1992; McClain & Cobb, 2001; Yackel & Cobb, 1996). .

In analyzing mathematics interactions in classrooms where reform mathematics initiatives are present, Cobb et al. (1993) found that "mathematics does not consist of timeless ahistorical facts, rules, or structures but is continually negotiated and institutionalized by a community of knowers" (p. 28). Learning mathematics is seen as a constructive process that combines the work of individuals as they negotiate the mathematical content within a group bound by normative behaviors. Furthermore, Yackel (2000) explains that the taken-as-shared understandings that developed in the classrooms under examination were "normative understandings regarding expectations and obligations for social interactions and for specifically mathematical interactions" (p. 4).

Cobb et al. (1993) make an important distinction between the constructs of social norms and socialmathematical norms. Social norms can be observed across all subject areas and are not specific to mathematics. Social norms constitute the ways in which members of a group interact and converse with each other through everyday experiences and situations. The interactions and conversations are between student and student and between teacher and student. Social norms are continually reconstructed within the context of concrete situations and become expectations of the way in which classroom members interact with one another (Yackel, 2000). In such classrooms students' thinking is of high priority and is valued and used to foster learning on the part of students as well as on the part of the teacher. According to Clement (1991) legitimizing students' non-formal ways of thinking about mathematics offers the teacher an opportunity to "gain a more realistic appreciation for the nature of their reasoning and understanding" (p. 423).

When a teacher asks a student to explain her thinking to the class, the teacher constitutes a social norm – sharing of ideas is expected and valued. Such norms include the following ways of interacting in the classroom:

1. Explain and justify ones' reasoning;
2. Listen and attempt to understand others' explanations;
3. Indicate non-understanding of an explanation and ask for clarification; and
4. Indicate when a solution is considered to be invalid and explain the reasons for the assertion. (Cobb, Stephan, McClain, & Gravejeijer, 2001)

In essence, these ways of interacting with others become the "shared reality" within the classroom and the expectations become taken-as-shared (Yackel, 2000).

On the other hand, socialmathematical norms constitute that which is specific to students' mathematical activity (Yackel & Cobb, 1996). For example, a socialmathematical norm would require that a student understand "what counts as an acceptable mathematical explanation" or argument **(**Yackel, 2000, p. 14). By eliciting student explanations (social norm) teachers can help students to understand what an acceptable mathematical answer or argument sounds like (social mathematical norm). Socialmathematical norms include such normative understandings as what counts as a mathematically different, sophisticated, efficient, or elegant solution (Yackel & Cobb, 1996).

For a teacher to develop socialmathematical norms in the classroom she must personally possess a deep conceptual understanding of the mathematics involved. She must know how to create and orchestrate mathematical experiences so that students' responses are elicited for the purpose of understanding individual student thinking and for fostering the concepts of mathematical justification and argumentation as a way of communicating mathematically to members of the classroom community.

Students must be able to defend their mathematical responses with viable arguments, and equally important is the teacher's ability to interpret students thinking and pose questions that encourages students to refine and make use of their mathematical proposals. In essence, students are given the opportunity to act like mathematicians proposing, arguing, and defending their own mathematical ideas. When students engage in this kind of mathematical discourse they learn what counts as a mathematical argument and in doing so learn how to justify their own mathematical ideas through the process of explanation and justification (Yackel, 2001).

The constructs of social and socialmathematical norms advanced by the work of Paul Cobb, Erna Yackel, and Terry Wood (1993) in the early 1990s and more recently by Cobb et al. (2001) provides a framework for exploring the ways in which novice teachers orchestrate a taken-as-shared environment compatible with mathematics reform initiatives. The work by Cobb and his colleagues has focused primarily on analyzing the collective mathematical leaning of students within mathematical communities of practice. However, the researchers acknowledge that it is a skillful teacher who can orchestrate communal mathematical conversations that foster diverse thinking and reasoning. According to Cobb et al. (2001) a skillful teacher is the "primary motor of the collective mathematical learning of the classroom community" (p. 117). Moreover, the classroom teacher is responsible for facilitating and guiding discussions as well as developing her students' abilities to "engage in productive mathematical discourse" (McClain & Cobb, 2001, p. 236). Research is needed on the discourse practices teachers use to engage students in such productive discourse.

Classroom Discourse

Traditional Classroom Discourse

Traditional classroom lessons follow a predictable pattern of discourse that has been described by Mehan (1979) as the IRE question-answer sequence or initiate-reply-evaluate. In his seminal work on discourse in the classroom Hugh Mehan detailed an interactional pattern between teachers and students that begins with a Question with a Known Answer (QWKA) that in turn prompts a student response and ends with an evaluation by the teacher. He labeled this pattern the Initiation, Respond, Evaluate or IRE sequence. In the classrooms he studied, this pattern emerged as a dominating force, and subsequent research has indicated that the IRE pattern is alive and well in many classroom not only here n the United States but other countries as well (see Abd-Kadir & Hardman, 2007; Baynham, 2006; Inagaki, Morita, & Hatano, 1999; Nassaji & Wells, 2000).

In traditional classrooms, the pattern begins with the teacher initiating the sequence with a question; the student responds with an answer; and the teacher ends the sequences with an evaluation of the students' response (see Figure 2.1).

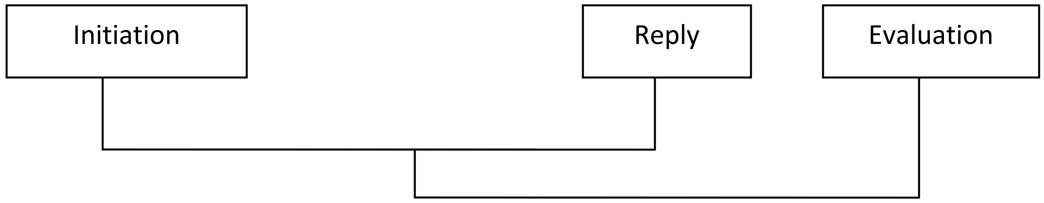


Figure 2.1 The sequential organization of a typical three-part structure (Mehan, 1979, p. 286)

Moreover, the IRE discourse pattern is often attributed to a questioning sequence where the initial question is in the form of a Question with a Known Answer (QWKA) and it is the students' responsibility to "match the questioner's predetermined knowledge, or at least fall within previously established parameters" (Mehan, 1979, p. 286). Analysis of the IRE questioning pattern can shed light on the social norms that have been constituted in classrooms where the pattern is present. When a student is asked a QWKA he or she is put into the position of trying to guess the teacher's predetermined answer (Mehan, 1979). The answer that the student gives is "conditionally relevant" (p. 286) to the question the teacher asked and the evaluation that the teacher offers is conditionally relevant to the students reply. The following description by Mehan sheds light on the IRE structure:

> [O]ne item in a pair is conditionally relevant upon the other if, given one item in the pair, the presence of the second is expected. For example a summons calls forth a response – the ringing of the telephone virtually demands that it be answered; the offering of a greeting seems to compel its return; the asking of a question demands a response. (p. 287)

Others have researched the IRE sequence and found that the third move in the sequence does not have to be used solely as a means to evaluate student responses but can serve a variety of functions depending on the goal of a particular activity (Nassaji &

Wells, 2000). Wells and Arauz (2006) found the third move in the traditional IRE sequences looks different in classrooms where the goal was to "create communities of inquiry" (p. 388). In such classrooms, Wells and Arauz found that the third move in the sequence was used as a means to follow up on a student's response rather than evaluate it. In order for classroom discussions to run smoothly and orderly, a particular discussion genre is needed and as such the initiate, respond, follow-up (IRF) sequence serves the purpose of managing classroom conversations, as does the IRE, however, using the third part of the sequence as a place to asks follow up questions lends itself to a more inquiry based practice (Wells & Arauz, 2006).

Reform-oriented Classroom Discourse

If the teacher is the primary motor generating the collective learning in the mathematics classroom, then examining what the teacher says while working with students is an important line of inquiry into how teachers generate mathematical learning and understanding. Pirie and Schwarzenegger (1988) define mathematical discussion as "purposeful talk on a mathematical subject in which there are genuine pupil contributions and interaction" (p. 461). However, the researchers excluded teacher-led talk from their analysis of mathematical discussions because of the contention that "teacher-led talk … is not genuine discussion because typically it consists not of pupils formulating their own opinions but of pupils guessing the correct answer required to satisfy questions posed by the teacher" (p. 460). I would argue that teacher-led talk in a reform-oriented classroom is critical to examine, as it is the means by which classroom discussion is generated.

The NCTM (1991) has called for teachers to orchestrate discourse in mathematics classrooms so that "reasoning and arguing about mathematical meanings" (p. 2) becomes the norm. In reform-oriented classrooms students are expected to be actively engaged in the role of *talker* rather than in the traditional passive role of *listener*. Teachers whose classrooms are oriented toward reform would, in theory, expect students to make conjectures, explain, and justify their divergent methods of solution; argue for the appropriateness of their methods; and attempt to understand the methods posed by their classmates and teacher. In such classrooms, emphasis on correct answers moves to the background while mathematical reasoning moves to the foreground. In their extensive review of the literature on nontraditional classroom lessons Cazden and Beck (2003) found no examples of research studies that "coded and quantified alternative patterns" (p. 174) of discourse in nontraditional classrooms classroom. Moreover,

> classroom discourse now counts as much more than just background context for individual students' learning. It has become an essential social process by which students accomplish complex conceptual and communicative goals. (p. 166)

In their work examining discussions in the mathematics classroom Martin, Towers, and Pirie (2006) characterized collective mathematical understanding as a "creative and emergent improvisational process" (p. 149), and they assert that this collective mathematical understanding can be observed as it happens in classroom discussion. Mathematics reform calls for teachers to be able to create communities whereby students engage in productive mathematical discourse with their teachers as well as with their peers. Cobb and his colleagues have also hypothesized that mathematical understanding in classrooms emerges as teachers and students communicate and interact together with mathematical content.

In an attempt to reconcile the function of "telling" in constructivist classrooms Lobato, Clarke, and Ellis (2005) have reformulated the process of telling in terms of its function in a particular conversation, its conceptual rather than procedural content, and its relationship to students' mathematical understanding at a particular point in time. As such, the researchers propose that when teachers initiate and elicit students' mathematical understandings and reasoning telling can be useful to the development of the students' mathematical understanding. Telling, in constructivist classrooms, serves to develop the students' mathematics rather than communicate the teacher's mathematics.

Discourse in a reform-oriented classroom functions to bring together teachers and students as they engage in conversation about mathematics. As such, what happens collectively in the classrooms impacts why and how individual students come to learn mathematics. In their research linking mathematics education to complexity science, B. Davis and Simmet (2003) suggest:

> in terms of the range of complex forms, the teachers main attentions should perhaps be focused on the establishment of a classroom collective – that is, on ensuring that conditions are met for the possibility of a mathematical community. Such an emphasis is not meant to displace concerns for individual understanding. The suggestion, rather, is that the individual learner's mathematical understandings might be better supported – not compromised – if the teacher pays more attention to the grander learning system. (p. 146)

Examining the discourse practices that are used by novice teachers is critical to providing a snapshot as to how mathematics reform practices are being implemented in classrooms by the newest members of the teaching profession. Developing a model as to how novice teachers, fresh out of reform-based teacher preparation programs, are enacting mathematics reform is an important line of inquiry. Feiman-Nemser (2001), suggests that if teacher education programs have been successful, than "beginning teachers will have a compelling vision of good teaching and a beginning repertoire of approaches to curriculum, instruction, and assessment consistent with that vision" (p. 1029).

Knowledge Needed to Teach Mathematics

The knowledge base necessary to teach reform-based mathematics has gained increasing attention from scholars in the last two decades. According to the National Research Council (2001) teachers need an "elaborated, integrated knowledge of mathematics, a knowledge of how students' mathematical understanding develops, and a repertoire of pedagogical practices that take into account the mathematics being taught and how students learn it" (p. 381). The following section will describe the intricate and interrelated forms of knowledge needed to teach mathematics for conceptual understanding.

Knowledge of Mathematics

What is it that prospective teachers know and understand about mathematics and what is it that prospective teachers *need* to know and understand in order to instruct students in elementary mathematics? Understanding the mathematical content knowledge that prospective elementary teachers bring to the mathematics curriculum is essential in developing and designing preservice mathematics education courses. In order to facilitate prospective teachers' learning and understanding of mathematics, reviewing research with respect to prospective teacher's mathematical content knowledge is critical (Simon, 1993).

In the following two seminal studies Ball (1990) and Ma (1999) investigated the mathematical knowledge of elementary preservice and inservice teachers in order to understand what these two groups of teachers comprehend about basic mathematical concepts and procedures and the underlying relationship inherent between the two. Ball (1990) delved into the thinking and understanding that preservice teachers possess as they enter teacher education programs. Her work is important in helping teacher educators understand the knowledge that preservice teachers bring to their prospective teacher education programs. Drawing on data from the Teacher Education and Learning to Teach study (TELT), Deborah Ball (1990) studied what prospective teachers bring to their teacher education program in terms of mathematical content knowledge. Results of interviews and questionnaires given to 252 prospective elementary and secondary teachers revealed that the mathematical understanding of teachers in the United States is "rule-bound and thin" (p. 451). As a result of her research, Ball challenged the following three common misconceptions regarding the teaching of school mathematics:

1. Traditional school mathematics is not difficult.
2. Pre-college education provides teachers with much of what they need to know about mathematics.
3. Majoring in mathematics ensures subject matter knowledge. (p. 450)

Ball (1990) concluded that the mathematical content knowledge that prospective teachers possess is "inadequate for teaching mathematics for understanding" (p. 464). Moreover, Ball concluded that majoring in mathematics did not ensure a solid understanding of the elementary mathematics curriculum. As a result of this research, a question that comes to the forefront is: How and when do prospective teachers learn to teach mathematics for understanding? If prospective teachers' past mathematical content knowledge is bound by rules and weak, how can we expect teachers to develop a teaching practice that is rich, deep, and conceptually based?

In her book, *Knowing and Teaching Elementary Mathematics*, Ma (1999) compared the “knowledge packages” that teachers from China and the United States bring to their teaching practice. Using the TELT interview questions, Liping Ma compared the procedural and conceptual knowledge of Chinese and U.S. elementary mathematics teachers. Her findings revealed that the Chinese teachers possessed a deeper conceptual and procedural understanding of the elementary mathematics curriculum than did their American counterparts. The U.S teachers tended to be procedurally focused and most showed competence in the area of algorithmic manipulation in whole number subtraction and multidigit multiplication. However, when it came to more advanced topics, such as division with fractions and area and perimeter, the U.S. teachers lacked the procedural skills as well as the conceptual understanding needed to solve such problems.

In contrast, the Chinese teachers demonstrated not only a strong procedural understanding of all four topics but a conceptual understanding as well. In addition, the Chinese teachers were adept at delineating the relationship between the conceptual and procedural aspects of the elementary mathematics curriculum. U. S. teachers’ mathematical understanding tended to be procedurally based, lacking in any conceptual sense-making, whereas the Chinese teachers’ mathematical understanding was steeped in conceptual flexibility as well as procedural proficiency.

Both Ball (1990) and Ma (1999) concluded that teacher preparation programs and professional development programs must address the insufficient “knowledge packages”

(Ma, 1999, p. 17) that U.S. teachers possess. In order for U.S. students to become competent in the area of mathematics, teachers must begin to grapple with the conceptual underpinnings of the discipline of mathematics. If we expect students to reach the standards set forth by NCTM, it is critical that elementary teachers, who, in a sense, lay the mathematical foundation, engage deeply in mathematics so as to gain a deep procedural as well as conceptual understanding of the subject.

Research reveals that subject matter expertise does not ensure that teachers will be effective in the classroom. According to Nathan and Petrosino (2003) "expertise in a subject area may make educators blind to the learning processes and instructional needs of novice students and that educators with such expertise are often totally unaware of having such a blind spot" (p. 906). The expert blind spot hypothesis states that those with expertise in a subject area tend to utilize formal methods of analysis that serve the needs of the discipline but do not necessarily serve the developmental learning needs of the students. Nathan and Petrosino explored the expert blind spot construct in a study of 48 preservice teachers enrolled in nationally ranked teacher education program. Results revealed that teachers with more subject matter knowledge expertise tended to view symbolic reasoning and mastery of formal procedures as a prerequisite for word problems which is in contrast to actual performance by students. The research by Nathan and Petrosino (2003) calls into questions the relatively popular process of licensing teachers on the basis of subject-matter expertise alone. A teacher must possess a deep understanding of the subject; however, it must not be at the expense of understanding the developmental learning needs of the students.

Knowledge of Students

Knowledge of students and how students learn is an intricate piece of the mathematics teaching and learning puzzle (Kagan, 1992; NRC, 2001). Teachers need to understand the mathematics that they teach, how students learn mathematic, and the places students travel on their journey toward mathematical proficiency (NCR, 2001). Fosnot and Dolk (2001) call this journey the “landscape of learning.” According to Fosnot and Dolk throughout this journey, teachers must think about the mathematical concepts they want students to know and understand; for instance, multiplication and division. Along the way the teacher must be aware of and understand “landmark strategies” that students may use in order to reach an understanding of the concept (repeated addition, skip counting). Once a teacher knows where students are in their thinking she must acknowledge these strategies and offer students opportunities to explore the connections between their strategy (repeated addition, skip counting) and multiplication and addition. It is a skillful teacher who understands not only the mathematics but also the ways in which students in general navigate this rich landscape. Teachers’ knowledge of mathematics must be coupled with knowledge of how students learn and make sense of mathematics.

Knowledge of Pedagogical Practices

Knowledge of pedagogical practices that foster reform initiatives is another intricate part of the teaching and learning process (Shulman, 1987). Teachers must have a working knowledge of what needs to be taught at a particular grade level and how to plan, conduct, and assess learning in the mathematics classroom (National Research

Council, 2001). The National Research Council also advises that teachers must be proficient in understanding and implementing state curriculum standards as well as be knowledgeable of the resources available to help foster learning. Additionally, teachers must be able to orchestrate and manage classroom discourse and develop a community whereby students are free to explore and make conjectures about the mathematics in which they engage in the classroom and in the world. Managing this type of classroom discourse is a skill that teachers must develop in order to teach mathematics for understanding.

Shulman (1986) refers to the intersection of content and pedagogy as pedagogical content knowledge (PCK). The concept of PCK has widened the educational lens, capturing the process of teaching and learning in a more focused format. Veal and MaKinster have provided the following operational definition of PCK:

> Pedagogical content knowledge is the ability to translate subject matter to a diverse group of students using multiple strategies and methods of instructional assessment while understanding the contextual, cultural, and social limitations within the learning environment. (p. 10)

Veal and MaKinster (1999) have developed two taxonomies to help understand the role of PCK in science education, and these taxonomies can be useful in discerning the mathematical knowledge needed for teaching as well. The General Taxonomy of PCK is organized in a hierarchical manner with pedagogy as the foundation - "the teaching skills and pedagogy that should be developed by all teachers" (p. 6). These pedagogical strategies are not aligned with any content area and can move from one content area to another, thus they are not considered PCK just yet. The next level moves to general PCK, which is now related to a content area, making it more specific than pedagogy. Next is a level that encompasses domain-specific PCK, which includes, for example, the domain of chemistry within the content of science. Finally, the taxonomy reveals a topic-specific PCK and, for example, includes the topic of oxidation within the domain of chemistry, which is within the content of science. Utilizing this general taxonomy in mathematics may be a useful and valuable measure when trying to discern and articulate the mathematical knowledge needed for teaching.

The second taxonomy developed by Veal and MaKinster (1999) is the Taxonomy of PCK Attributes, and it too is hierarchical in design and details the PCK characteristics of the expert. Content knowledge is the foundation of this taxonomy, indicating that content knowledge is a necessary precursor to pedagogical content knowledge. Next in the taxonomy and embedded within content knowledge is knowledge of students. Knowledge of students refers to the level of knowledge needed to analyze, understand, and interpret students' misconceptions and or errors. Veal and MaKinster make a strong case for embedding content knowledge and knowledge of students by proposing that "student errors and misconceptions are more easily recognized when a teacher knows the content topics and concepts" (p. 8).

Lastly, embedded within content knowledge and knowledge of students are the attributes of assessment, context, environment, nature of science, classroom management, curriculum, pedagogy, and socioculturalism. These attributes are not arranged in a hierarchical manner but are thought to be fluid in nature and able to be developed at various times during a teacher's career. Researchers in mathematics education can utilize the two taxonomies developed by Veal and MaKinster (1999) to help guide their understanding of how PCK develops in mathematics teachers.

Research indicates that PCK evolves and develops through the act of teaching (Fennema & Franke, 1992; Lee, Brown, Luft, & Roehrig, 2007). However, according to Lee et al. how such practices develop is not clear. Lee et al. advocate for studies that examine beginning teachers to document the development of pedagogical teaching practices in relation to the content being taught.

Mathematical Knowledge for Teaching

Effective mathematics teachers possess more than mathematical content knowledge; they also possess "mathematical knowledge for teaching" (Hill, Rowan, & Ball, 2005, p. 373). This mathematical knowledge for teaching involves understanding and interpreting a wide array of student explanations, judging the content of textbooks, and utilizing models and representations accurately to explain mathematical concepts and procedures (Hill et al., 2005). In their study of mathematical knowledge for teaching Hill et al. found that a teacher's mathematical knowledge for teaching was significantly correlated to student achievement. The study examined the results of teacher surveys and student achievement data from 115 elementary schools during the 2001–2004 academic years. The survey included questions regarding "the knowledge that teachers *use* in the classroom rather than general mathematical knowledge" (p. 387).

Results indicated that teachers' knowledge for teaching mathematics was a significant predictor of student achievement in grades 1 and 3. Additionally, the study also found that teachers' mathematical knowledge for teaching outweighed "proxy measures such as courses taken" by teachers (p. 400) when looking at student achievement. Hill et al. (2005) recommend that "a new generation of process-product studies designed to answer questions about how teachers' mathematical behavior – in particular, their classroom explanations, representations, and interactions with students' mathematical thinking – might affect student outcome" (p. 400). Other scholars are calling for this same type of evidenced based research, connecting teachers' knowledge with student learning (Cochran-Smith & Fries, 2005; Ma, 1999).

Clearly, there is a need to understand the knowledge base necessary to teach mathematics so that students gain a deep understanding of the mathematical content, rather that the superficial understanding that most U.S teachers and adults possess (Ball et al., 2005). However, it will be equally important to understand how and when teachers develop the specialized mathematical knowledge needed for teaching.

This section explored the various types of knowledge that teachers need in order to teach mathematics for conceptual understanding rather than rote memorization. In summary, teachers need to have a deep understanding of the mathematics they teach, an understanding of how students learn mathematics, a repertoire of pedagogical practices, and a specialized mathematical knowledge for teaching. How novice teachers engage in developing the knowledge needed for teaching is a critical question to consider, if mathematics reform is to have a significant impact on what happens in the classroom. In the following section, the teaching practices that foster reform recommendations will be examined.

Teacher Learning and the Process of Change

The roles and expectations for teachers today differ significantly from the roles and expectations that teachers were held to in the past. Today, teachers are expected to teach students for conceptual understanding rather than rote memorization of procedures and this practice differs from the roles and expectations that were held in the past (Bransford, Brown, & Cocking, 2000). Most teachers in the United States have not developed a deep conceptual understanding of mathematics or what Ma (1999) refers to as a profound understanding of fundamental mathematics (PUFM). According to Ma (1999), PUFM is a precursor to teaching mathematics for conceptual understanding. Teachers must engage in experiences that challenge their preexisting beliefs, attitudes, and understandings of mathematics in order to develop a profound understanding of fundamental mathematics. The trajectory that teachers embark on must be measured in years rather than in months “for the bulk of what teachers must learn will necessarily come only in their own classroom with their own students (Fosnot & Shifter, 1993, p. 193).

Influence of Beliefs on a Developing Teaching Practice

The practice of teaching is a complex and daunting prospect; however, the practice of teaching mathematics can be an anxiety driven prospect as well. Research focusing on how teachers learn to teach mathematics indicates that a mathematical teaching practice is profoundly influenced by the beliefs and attitudes an individual harbors toward the subject of mathematics (Vacc & Bright, 1999). As with most belief systems, preservice teachers’ beliefs about mathematics stem from past experiences often portraying vivid emotional situations (Nesbit & Bright, 1999). The following excerpts reveal the emotional connections one preservice teacher had regarding learning mathematics and teaching mathematics (Grassetti, 2007).

Emma reflecting on self as a mathematics learner:

> By saying I never felt comfortable with math and it made me uncomfortable, I mean it literally made me nervous. I was convinced I was a 'bad' math student and I didn't like taking part in something I knew I wouldn't be successful in. I tried to avoid math at all cost, hence why I opted not to take it senior year in high school.

Emma reflecting on her mathematics placement:

> That you were not a strong math student, this is the room where kids with IEPs would be and support people would come in from the resource room, and, now, I never received any of the services, but I was in that class, which was fine, but it was just one of those labels. I think there were probably seven College Level I classes and there were maybe two College Level II courses. So everybody knew.

Emma reflecting on self as a mathematics teacher:

> I was terrified of having to teach mathematics, afraid that I would be giving children wrong information and that I was not confident enough to deliver effective instruction.

It is no wonder that Emma was fearful of teaching mathematics. Past experiences in mathematics instilled in her the belief that she was “bad” at doing math, thus making her “terrified” about the prospect of teaching math to her students. According to Ernest (1988), the mental models that teachers possess regarding mathematics teaching and learning significantly impact their ability to implement reform initiatives set forth by the National Council of Teachers of Mathematics. Ernest articulates the relationship between what a teacher believes about the nature of mathematics and the teacher’s developing mathematical teaching practice as being recursive in nature. What a teacher believes about the nature of mathematics is “transformed into classroom practices” (p. 3).

Emma’s response typifies the sentiments of the preservice teacher. In her study of the understandings that preservice teachers bring to teacher education, Ball’s (1990) participants expressed similar feeling. For example, Cathy, an elementary teacher candidate in Ball's study reported, "I am really worried about teaching something to kids I may now know. Like long division – I can *do* – it but I don't know if I could *teach* it because I don't know if I really *know* it or know how to word it" (p. 449). Addressing the beliefs and attitudes of preservice teachers is the first step in guiding them toward understanding how their beliefs critically influence their instructional teaching practice.

There are many factors at work shaping and reshaping a teacher's beliefs about the practice of teaching mathematics. Anderson (1998) developed a model, based on research findings, regarding factors influencing teachers' beliefs and practices. She argues that a teachers' reported beliefs are influenced by their actual beliefs and experiences, the advice they have received from others, and the curriculum that they are teaching. Additionally, Anderson argues that teachers' classroom teaching practices are influenced by their reported beliefs and by the opportunities and constraints available to them in their teaching context. The factors involved act as filters that ultimately impact a teachers' day-to-day decision making, or teaching practice, and are not easily separated out (Anderson, 1998). Anderson's model offers a window into understanding the interconnected web that makes up a teacher's belief system and the complexity in which a teaching practice is shaped and reshaped as it confronts long held beliefs and practices and adjust to reform recommendations.

To explore the "range of beliefs about problem solving that elementary school teachers hold as well as to describe their classroom practices" (Anderson, 1998, p. 1) Anderson gathered questionnaire data from 174 teachers, conducted interviews with 9 of the teachers, and finally selected 2 teachers from the 174 to participate in classroom observations and additional interviews. The questionnaire data revealed that teachers' beliefs about mathematics teaching and learning were more in line with traditionally held beliefs and values than with those proposed by NCTM. For example, allowing students to invent their own strategies when solving mathematical problems is advocated by NCTM; however, the majority of participants in Andersons study reject this as a viable strategy that they would foster in their own classroom. This result indicates a tension between what teachers believe and what mathematics education faculty recommends (Anderson, 1998). Addressing this tension is a necessary step in understanding how a mathematical teaching practice develops into one that fosters the reform initiatives proposed by NCTM.

What is missing from Anderson's model is the influence of the social context in which teachers have been immersed in as students of mathematics or as members of society. Although Anderson may have inexplicitly included the social within the context of "experiences" by encompassing Andersons' entire model within the social makes the social circumstances of the teacher explicit and therefore available for scrutiny. Teachers have had years of past mathematical classroom experiences that have had a profound influence their beliefs and consequently, their actions in the classroom. As Cochran-Smith and Fries (2005) point out, researchers need "to make links between what teachers know and believe and how they develop professional practice in the context of different school and classrooms" (p. 90). Additionally, Cochran-Smith and Fries say that research is needed to understand, "the intricate ways that knowledge, beliefs, and professional practices are related to pupil learning and other outcomes" (p. 90).

Resiliency of Beliefs

Beliefs can be resistant and difficult to change (Foss & Kleinsasser, 1996; Scott, 2005); however, it is imperative that teachers engage in the reflection necessary to begin to understand how their own beliefs and values about teaching and learning affect the students that they teach (Foss & Kleinsasser, 1996). As Elliot (1992) so eloquently stated, teacher educators must “raise core beliefs to a conscious level, examine them as we feel compelled, and then act in accordance with them” (p. 6). Teaching is “both a moral and political activity” (Feldman, 2000, p. 606), thus teacher educators should feel morally compelled to guide preservice teachers in the process of self-reflection, so that underlying beliefs and values are given a chance to surface. “Change is hard” (K. Davis, 2003, p. 3), and change is less likely to happen if one’s beliefs are not challenged and addressed.

These new roles and expectations will require teachers to undergo a type of conceptual change when it comes to learning and teaching elementary mathematics. How do teachers, who have experienced mathematics in terms of rules and procedures, come to an understanding of the conceptual underpinnings of the elementary mathematics curriculum? How does conceptual change happen? The research on teacher learning is a new area of study; however, the research that is available provides a window into how teachers begin to change their practices (Bransford et al., 2000). In her study of science teachers’ ability to implement reform recommendations, K. Davis (2003) found that “it is critical when implementing reform to consider the length of time teachers may need to reconsider their long held beliefs and approaches” (p. 23). Change is not something that comes about overnight; rather, it takes systematic refection and consideration on the part of the teacher in order to change one's long held beliefs about mathematics teaching and learning.

According to Bransford et al. (2000), teachers learn in the context of teaching. The framework grounding this study is a social constructivist one. Using the theoretical framework of social constructivism offers an opportunity to explore and give significance to the *practice of teaching* as teachers engage with students and mathematical content. Although social constructivism is a theory of learning, it is inextricably tied to teaching. How mathematical knowledge is enacted in the classroom can and must be informed by theories on how people learn. Using a social constructivist lens allows for the assumption that learning to teach mathematics is a socially constructed endeavor "embedded in the social world of human interaction" (Ernest, 2004, p. 25). Meeting the challenges of mathematics reform will require that we engage prospective teachers in the types of mathematical discourse and experiences that we expect them to foster and provide in their own classrooms. Drawing upon social constructivist theory provides an opportunity to view mathematics teaching and learning with the supposition that human reality is fundamentally conversational (Shotter, 1991) and culturally situated in everyday experience and practice as teachers and students interact with mathematical content.

Teachers make adjustments to their teaching practice by monitoring, analyzing, and reflecting on their everyday teaching. This practical experience offers teachers opportunities to gain new knowledge and understanding of students while at the same time gaining a deeper understanding of their own teaching practice (Bransford et al., 2000). Taking into consideration that teachers learn while teaching, research in mathematics education is needed to study how teachers grapple and ultimately come to terms with a reform mathematics curriculum. Understanding the developmental process that teachers go through, while teaching a reform mathematics curriculum, might shed light on what, if any, conceptual changes occur.

In a case study analysis of 10 preservice teachers completing their student teaching semester, Hartman (2004) found that portfolios were instrumental in enriching the methods course experience for prospective teachers. Moreover, the portfolios revealed how the participant teachers' beliefs impacted their teaching practice, thus enabling Hartman, the researcher/methods instructor, to address the participant teachers' beliefs in ways that provided opportunities to engage the participants in the process of reflecting on their practice. One of the participants highlighted in the analysis was using a standards based curriculum called *Connected Math Program*. Hartman documented the participant's desire to revise curriculum materials to make them more accessible to students. However, in the process, the participant took away much of the "cognitive complexity" (p. 2) of the mathematical task, thus interacting with the reform-based curriculum in ways that impeded the mathematical learning process for students. Hartman's study reveals the need to engage preservice teachers with reform based curriculum in ways that challenge their beliefs about teaching and learning mathematics, so that when they utilize curriculum materials based on reform recommendations they do not revise the curriculum to reflect their own traditional model of teaching and learning.

When considering how to encourage new teachers to adopt reform recommendations, it would seem imperative for preservice teachers to be actively engaged in working with and exploring reform based mathematics on a personal level (Foss & Kleinsasser, 1996) so that they begin to develop a teaching practice based on "new interactive ways of learning" (Klein, 2004, p. 35) thereby challenging the "linear transmission of knowledge" (Elliott, 1992, p. 5).

Some research has indicated that preservice teachers do not change their beliefs about how mathematics is learned, thus their teaching practice begins to develop under the guise of a traditional model of teaching and learning while other studies indicate that change is possible. In their study of 22 preservice teachers enrolled in mathematics methods course, centered on the NCTM (1989) curriculum and evaluation standards, Foss and Kleinsasser (1996) found little evidence to support a change in the preservice teachers' knowledge, beliefs, and practice in the context of a mathematics methods course. Data gleaned from interviews, observations, and video-taped teaching episodes revealed that: 1) preservice teachers' conceptions of mathematics remained constant; 2) pedagogical content knowledge did not change; 3) traditional notions of mathematics remained intact; and 4) preservice teachers agree with the importance of new approaches to teaching and learning garnered in a methods course but do not implement such approaches in their own teaching practice. Foss and Kleinsasser argue that the results from their study point to a "symbiotic" (p. 441) relationship between preservice teachers' beliefs and their actual teaching practice. The researchers caution that the relationship is not one that will ultimately foster the types of teaching practices espoused by teacher educators who support the agenda put forth by the National Council of Teachers of Mathematics.

Timmerman (2004) researched the influence of three interventions within a reform based mathematics methods on 24 preservice elementary school teachers' beliefs regarding the knowledge needed to teach mathematics. Using pre- and post-course survey data, Timmerman found that the interventions, which included problem solving journals, structured interviews, and peer teaching episodes, “facilitated change in the prospective teachers beliefs, with a shift toward reform-orientated mathematics education perspectives” (p. 1). Timmerman concludes with recommending that teacher preparation courses challenge preservice teachers’ beliefs about how mathematics is taught and learned by engaging preservice teachers in the aforementioned interventions as a means to shift their beliefs toward reform teaching practices.

Inquiry-based teaching practices, which focus on active student engagement, are prevalent in many mathematics methods courses but the impact of such practices on teachers’ beliefs and their teaching practice is questionable (Klein, 2004). According to Klein (2004), the relationship between teacher education programs and actual teaching practice is much more complex than is revealed in the literature. The promise of change in the direction of reform may be a promise never realized, in part because teacher education may inadvertently “reproduce old epistemological and ontological assumptions” (p. 37).

Reflecting on Radcliff-Brown’s (1952) theory of structural-functionalism may shed some light on Klein’s (2004) position. Radcliff-Brown utilized an organic analogy to divide society into parts thus highlighting the important functionality of the individual parts of a society to the survival of the society as a whole. Radcliffe-Brown argued that every activity in a society has a function, therefore, to understand the reason for an activity, one must uncover the activity’s function in relationship to the larger society.

Why would teacher education programs reproduce old epistemological and ontological assumptions about the process of teaching and learning? Using

Radcliffe-Brown's (1952) argument, one must look at the function of reproducing old epistemological and ontological assumptions in order to uncover the function of such an activity. Reflecting back on Shotter's (1991) postulation that our reality is continually being constituted for us by the dominant discourse and our obligation to that discourse becomes a moral one that we must uphold, may help uncover the function of reproducing old epistemological and ontological assumptions about the process of teaching and learning mathematics? It seems plausible that the function of such inadvertent practices in teacher education is to uphold and keep in place the dominant mathematical discourse. Shotter contends that we must uphold the dominant discourse or risk being "treated as in some way socially incompetent and be sanctioned accordingly" (p. 508).

According to Hart (1998) in order for a system to function institutions must maintain "boundaries and standards of acceptable behaviour, and this maintenance can be overt but is mostly covert" (p. 117). Education is one such institutional system that aids in maintaining the boundaries and standards of acceptable behavior of our society thus it is understandable, albeit unacceptable, that teacher education would help to covertly keep these standards in place. Klein (1994) suggests that to bring new "hope and purpose to mathematics", teacher preparation programs must "open up new discursive space where old authorities and stereotypical thinking are challenged and where the possibility of creating something new arises" (p. 45).

Another obstacle in the process of change occurs when a teacher's beliefs are overshadowed by the particular goals of a school community (Karaagac & Threlfall, 2004). Karaagac and Threlfall argue that “goals can drive actions more than beliefs do” (p. 149). In their study of Turkish teachers, these researchers found that the mandated curriculum often “overwhelmed” the teacher’s beliefs about good teaching practices. In one episode, even though the teacher believed the mandated curriculum was “not a healthy way to teach mathematics” (p. 145), the curriculum still prevailed. The teacher was clearly aware of the tension between his beliefs about how mathematics should be taught and learned and his teaching practice but, according to the study results, he was not bothered by this apparent tension. The authors explained the teacher’s seemingly contradictory practice by focusing attention on the sociocultural aspects involved. The mandated curriculum was being used by the teacher as a cultural tool to help student pass state exams. The teacher’s job in teaching at the private school was to help student pass a test; therefore, the work place “goal” overshadowed his beliefs about good teaching (Karaagac& Threlfall, 2004).

If novice teachers are placed in schools where the goal is to “pass the test” then it may be difficult for them to demonstrate the understandings gleaned from a methods course. The novice teachers’ beliefs may have changed or be in the process of changing, but the change may be in belief only and not in practice because of the confines of the school context.

In a previous study Grassetti (2007) examined the extent to which preservice teachers (N=28) believed various reform-oriented social and socialmathematical norms had been developed in their mathematics methods course. Survey results revealed that the participants had a high degree of belief that reform-oriented norms had developed in their methods course. These results led the researcher to question whether or not preservice teachers would begin to constitute such norms within their own teaching practice. To establish a connection between the survey results and the participants' actual teaching practice, two participants volunteered to participate in in-depth interviews and classroom observations while student teaching. Grassetti (2007) found that the two participants were able to implement some reform teaching practices associated with reform-oriented social and socialmathematical norms. Consider the following exchange that took place in Ms. Arielle's mathematics class while she was student teaching in a second grade classroom. In this exchange Ms. Arielle is facilitating a conversation focused on finding different solution to the same problem (p. 19).

1.	Ms. Arielle:	How much is 50 cents worth?
2.	Students:	Fifty pennies
3.	Ms. Arielle:	Can we count to 50 cents by pennies?
4.	Students:	[Students count from 1 to 50]
5.	Ms. Arielle:	Is there a *different* way we can get to 50 by using *different* coins?
6.	Students:	Yes, by nickels – [Students count by fives].
7.	Ms. Arielle:	Yes, counting by nickels is a *different* way to get to 50. Are there other *different* ways that could make 50?
8.	Student:	Yes, we could count by dimes. 10, 20, 30, 40, 50.
9.	Ms. Arielle:	Okay, that's a *different* way too. Is there another different way to 50?
10.	Student:	You could count by count by quarters and say 25 50.
11.	Ms. Arielle:	Yes, that's *different.* Is there another *different* way?
12.	Student:	You could count by half dollars because one half is 50.
13.	Ms. Arielle:	To Class (With excitement in her voice): So then, you are telling me that there are *different* ways we can make 50?
14.	Students (Very excited): YES	

In this exchange Ms. Arielle helping her students to understand that problems can have many more than one solution. As seen in Turns 5, 7, 9, and 11, she requested that student share a different way to make 50 cents using different coins. Moreover, as students were sharing their different ways they offered explanations as to how the coins added up to fifty cents "You could count by dimes: 10…." And in another teaching episode Ms. Arielle said to the whole class:

> Kids I want you to get your pencils and come back to the rug. Working with a partner you are going to show *different* ways to make 45 using quarters, dimes, nickels and pennies (p. 19).

The observational notes recorded for this observation also noted that students were developing an idea as to what it meant to have a different solution. The field notes for the above observation stated:

> Observation Ms. Arielle 3.23.06 - At first, students seemed perplexed by the thought of working with a partner and what it meant to have a different solution. After a bit more explaining students began working when suddenly a voice erupted over the voices of the other students saying "I did it look! I get it, it's *different*! I made forty-five cents in a *different* way than Mia did!

In the exchanges above, Ms. Arielle was helping her students to see the value of finding different mathematical ways of solving the same problem. Ms. Arielle shared in an informal interview that the students she was assigned to during this student teaching practicum had little experience working with a partner and/or finding different ways to come up with the same answer. Even though these students had little experience with this way of doing mathematics, Ms. Arielle was determined to establish taken-as-shared expectations – even if they only lasted the two weeks while she was master teaching.

How is it that Ms. Arielle came to the understanding that finding different ways to solve a problem is beneficial to student learning? When asked to reflect on the learning she did in her mathematics methods course she said,

> Before this course if I saw a student solve a problem differently from what I was taught, I would not have known what to do or say. I would have probably said, "Okay but this is how it has to be done." I always thought that there was only one

> way to get to a correct answer. Now I understand that students think you know differently and come to understand mathematics in very different ways (Gressetti, 2007).

In her methods course, Ms. Arielle began to realize that different ways of finding the same answer was valued and, as evident in the dialogue above, begins to establish this same norm within her classroom.

Ms. Arielle was on her way to establishing a teaching practice based on taken-as-shared expectations and socialmathematical norms. I observed Ms. Arielle at the very beginning of her two-week Master Teaching experience and already she was setting up an environment that was conducive to higher order mathematical learning. In summary, the assumption that mathematics teaching and learning is socially constructed by members of the classroom community as they engage with mathematical content offers a window into examining the social interactions orchestrated by the novice teacher as she works at implementing reform initiatives into her developing mathematical teaching practice. These interactions can be examined in terms of social norms and socialmathematical norms and how such norms are constituted by the novice teacher within the mathematics classroom.

Emma too experienced the value of solving problems in different ways in her mathematics methods course and shared the following during her interview:

> Growing up, I was taught that there was only one right way to get an answer. This frustrated me, as I often found correct answers a different way than other people. After taking this course, I now know that learning mathematics is not going to be the same for everyone and I need to make room for many ways for solving problems, so that students can find a way that fits them best (Grassetti, 2007, p. 21).

It was evident that Emma was able to take this learning experience and implement it in her classroom teaching practice. During an observation I noted that Emma was beginning the let her students know that different answers would be accepted and valued. Consider the following exchange revealing Emma's emphasis on finding different solutions (Grassetti, 2007, p. 21).

Emma:	Today you will be working on writing your own division problems. You get to be the expert because this is YOUR problem. (Emphasis by Emma)
Emma:	Brian has such a neat way of solving division problems. Brian can you show us how you solve a division problem.
Brian:	If my problem is 8/2 = then first I would draw 8 circles (draws 8 circles for the class) then I would group the circles in two's because that is what I am dividing by. He then circles groups of 2 on his paper. Then I count how many groups I have and that is my answer. Counts 4 groups of 2 for his answer of 8/2 = 4.
Emma:	Thank you Brian. Asks class: Is this the only way to solve a division problem?
Students:	NO!

As I circulated the room I noticed a student who was using Brian's method on her own problem. The following is an excerpt from my observational notes:

> Emma Observation 3/20/06. Another student tried out Brian's method with 72/6 = 12. She drew 72 circles and then circled groups of 6 and came up with 12 groups. This student did not have any problem with drawing or counting her circles. She said that she never thought to do division in this way before but after listening to Brian explain his answer she decided to try this *different* way [Student emphasized the word different with her tone of voice].

It is evident that the taken-as-shared expectations that Emma was developing within her classroom was starting to shape the ways in which students were thinking about and learning mathematics. Asking Brian to explain his method (social norm) to the class offered another student insight into solving division problems in a different way.

Research in the area how a mathematics teaching practice develops over time and how participating in a preservice mathematics methods course influences novice teachers as they begin to develop a mathematics teaching practice is underdeveloped (Clift & Brady, 2005). In their critique of research on methods courses and field experiences, Clift and Brady advocate for qualitative studies that follow teachers into their first years of teaching as a "means to move beyond a focus on belief to research on action and to longer term connections between the teacher education program content and recommended practice and actual teaching practice after graduation" (p. 336). Moreover, even less is understood about how a mathematical teaching practice ultimately impacts student learning. The researchers go on to say that we cannot make conjectures about how a teaching practice impacts student learning if we have little information as to how it develops from the theoretical preservice years to the practical first years of teaching and the challenges that that arise as novice teachers "adopt and adapt recommended practices" (Clift & Brady, 2005, p. 336).

How does a classroom teacher begin to grapple with and possibly change her often long standing experiential beliefs and practices? In his model of practical conceptual change, Feldman (2000) argues that for change to occur a teacher must first become dissatisfied with her current practice. This dissatisfaction opens the door for new practical theories to be introduced. If the new theory is found to be beneficial as well as "illuminating and enlightening" (p. 606) to the teacher's practice, then it has a chance of being accepted and acted upon.

Summary

Mathematics teaching is a complex process (Lerman, 2000) and classroom teaching and learning is a “multifaceted, extraordinarily complex phenomenon” (O’Connor, 1998, p. 43). The decisions and actions of elementary mathematics teachers have a powerful impact on what students learn in the mathematics classroom (NCTM, 2000). Naturally, the decisions teachers make in the classroom are influenced by their habits, attitudes, and beliefs regarding how mathematics should be taught and learned. Research indicates that most classrooms teachers have experienced a traditional approach to learning mathematics in most, if not all, of their elementary, middle, high school, and college classes (Kirschner, 2002).

Novice teachers are being asked to teach mathematics in a significantly different way from how they learned mathematics. Their formal mathematics learning was based on the model that knowledge acquisition is authoritarian in nature, is transmitted from teacher to student, and is in sharp contrast to the model of cultural participation supported and accepted by the mathematics education community (Yackel & Cobb, 1996). How novice teachers embark on the journey toward reforming their own habits, attitudes, and beliefs about how mathematics should be taught and learned is of central importance, if we expect students to engage in the types of mathematical discourse advocated by reform initiatives.

After a careful and critical review of the literature on learning to teach mathematics, I have found ambiguity in how novice teachers learn to develop a mathematics teaching practice that reflects the vision set forth by NCTM (1989, 2000). Although the literature on learning to teach is vast and provides pertinent documentation, it does not attend to how novice teachers adopt, adapt, or ignore reform recommendations as they begin to develop their mathematics teaching practice. Examining such development will aid in understanding how to support novice teachers in the beginning stages of teaching. Systematic documentation of how a teaching practice develops is needed to understand the complexity involved in crafting a teaching practice under the auspice of mathematics educational reform.

The literature reveals that there is a need to examine what teachers say and do while engaged in the act of teaching (Cohen, Raudenbush, & Ball, 2003). Most learning-to-teach studies focus on beliefs, knowledge, and dispositions with little attention given to how teachers develop communities that foster reform (Putman & Borko, 2000). Research in this area will lead to an understanding of the issues and challenges that novice teachers encounter as they begin to enact teaching practices compatible with reform recommendations. As such, it is important to study the actions of novice teachers as they learn to orchestrate a community based of reform recommendations.

In summary, the assumption that mathematics teaching and learning is socially constructed by members of the classroom community as they engage with mathematical content offers a window into examining the social and socialmathematical interactions orchestrated by the novice teacher as she works at implementing reform initiatives into her developing mathematical teaching practice. These interactions can be examined in terms of social norms and socialmathematical norms and how such norms are constituted by the novice teacher within the mathematics classroom.

The results of the pilot study by Grassetti (2007) indicated that preservice teachers did learn about reform teaching practices while participating in teacher education courses and what was learned impacted their developing teaching practice during student teaching. This research study followed the teachers from the pilot study into their second year of teaching to examine if they were able to take what was learned in teacher education and infuse it into their teaching practice. The following chapters will present the results of individual case studies of Ms. Duncan, Ms. Arielle, and Ms. Quinn. All three novice teachers participated in the survey portion of the pilot; however, Ms. Arielle also participated in the observation portion of the pilot study. As such, this was her second time being observed by the researcher and as such she may have been more accustomed and comfortable with having an observer in the classroom.

CHAPTER 3

METHODS AND PROCEDURES

The purpose of this chapter is to describe the overall research methodology, beginning with the theoretical framework grounding this study followed by a description of the methods of data collection and analysis. The methods described in this chapter offered an opportunity to examine the teaching practices of three novice teachers as they taught elementary mathematics. More specifically, this study examined the discourse patterns that emerged as the participants engaged in mathematical conversations with their students. My interest in this study stems from my own past experiences as an elementary mathematics teacher, who found the reform movement enlightening as well as challenging, and from my current experiences as a teacher educator who has encountered many preservice teachers who have little confidence in their capability to teach mathematics successfully. In the following section a description of the theoretical perspectives grounding this study will be presented.

Theoretical Framework

The focus of this study was to describe the discourse practices that novice teachers used to encourage students to make conjectures and explain, justify, and argue different mathematical solutions. As such, it was appropriate to couple case study methodology with discourse analysis as a means to tease out the patterns of practice that emerged as teachers engaged with students in mathematical conversation. The focus on discourses analysis is grounded in the theoretical framework of social constructivism.

From a social constructivist perspective mathematics is viewed as a social activity. Moreover, social constructivism acknowledges “that human language, rules, and agreement play a key role in establishing and justifying the truths of mathematics” (Ernest, 1991, p. 43). Examining the ways in which novice teachers engaged students in mathematical conversations and the social and socialmathematical norms that such conversations established was the focus of this research study. The following section will discuss the theoretical framework of social constructivism as well as appropriateness of coupling case study methodology with discourse analysis.

Social Constructivism

The focus of this study was to describe the discourse practices that three novice teachers used to guide their students in constructing an understanding of mathematical concepts. More specifically, this study was interested in describing the discourse practices that three novice teachers drew upon to engage their students in mathematics conversations. The theoretical framework guiding this study was that of social constructivism, which views mathematics as a social construction and conversations as being essential to mathematical knowledge construction: “Social constructivism has adopted conversation as an underlying metaphor for epistemological reasons, to enable the social aspects of mathematical knowledge to be adequately treated within the philosophy of mathematics” (Ernest, 1998, p. 274). Looking through the lens of social constructivism allowed for the conversation within the mathematics classroom to take center stage. The fundamental assumption that mathematics is inherently conversational is grounded in the following three principles proposed by Ernest (1991):

1. The basis of mathematical knowledge is linguistic knowledge, conventions, and rules, and language is a social construction,
2. Interpersonal social processes are required to turn an individual's subjective mathematical knowledge, after publication, into accepted objective mathematical knowledge,
3. Objectivity itself will be understood to be social. (p. 43)

Social constructivism provided a theoretical framework to study the teacher-student discourse patterns that emerged during whole group discussions. If, as Shotter (1991) postulates, human reality is fundamentally conversational, then examining the conversations taking place in the mathematics classroom would shed light on the reality constituted by novice teachers as they attempted to develop a teaching practice under the auspice of mathematics reform. Looking through the lens of social constructivism focused the researcher's eye on the social and social mathematical norms that were being developed by the teachers and students as they engaged in mathematics conversations. Thus this research study viewed mathematics teaching and learning as constructive process combining the work of individuals as they negotiated mathematical content within a group bounded by normative behaviors (Cobb et al., 1993).

Examining the discourse practices that novice teachers used during whole group conversation and inferring the normative behaviors, both social and socialmathematical, that such practices fostered was of central importance to this study. The conversations that teachers orchestrate are rich spaces to discern the types of normative behaviors that are being constituted within classrooms. Classroom conversations provide teachers with rich instructive opportunities, albeit, how such conversations develop in reform mathematics classroom is relatively unknown (Cazden & Beck, 2003).

Rationale for Case Study

The overall research approach followed the descriptive case study tradition of qualitative research. Descriptive case study provided the researcher with the means to gather rich, descriptive, and holistic data on the everyday practice (Rossman & Rallis, 2003) of novice teachers in their second year of teaching. Moreover, collective case study methods provided the framework for examining the occurrences of reform-based teaching practices enacted by three novice teachers as they engaged with students in mathematical conversation. According to Merriam (1998), case studies are “an especially good design for practical problems – for questions, situations, or puzzling occurrences arising from everyday practice” (Merriam, 1998, p.11). Becker (1968) states that case studies can be used to “arrive at a comprehensive understanding of the group under study” and “to develop general theoretical statements about regularities in social structure and process” (p. 233). Lijphart (1971) refers to descriptive case studies as “moving in a theoretical vacuum; neither guided by established or hypothesized generalizations nor motivated by a desire to formulate general hypotheses” (p. 691), albeit acknowledging the usefulness of descriptive case studies in indirectly contributing to theory building.

Given that the purpose of this study was to gather a comprehensive understanding of the teaching practices of novice teachers, case study provided an appropriate methodological approach, as it offered the means to study such problems of practice. Case study approach was selected rather than ethnography or phenomenology because the study focused on the teaching practices that novice teachers adopted and/or adapted rather than an in-depth exploration of a single individual or cultural phenomenon. Moreover, the boundedness of this study, novice teachers, as defined in the literature as the first three years of teaching, lent itself to the use of case study design. Additionally, case study offered the flexibility to explore the reform-based teaching practices that were constituted in three novice teachers' classrooms.

In summary case study methods offered the means to gather rich descriptive data on the everyday practice of teaching, while discourse analysis served as a vehicle to analyze the "language patterns carried out" (Cirillo, 2008, p. 57) as novice teachers orchestrated mathematics conversations within their classrooms. Moreover, these methodologies provided a framework to make inferences as to the social and socialmathematical norms that developed in each classroom.

Research Questions

The questions guiding this study are as follows:

1. What reform-oriented discourse practices do novice teachers who participated in a reform-based mathematics methods course adopt? What practices do they adapt? What practices do they ignore as they engage their students in mathematical conversations?
2. What issues and challenges surface as novice teachers begin to enact reform-oriented discourse practices?

The purpose of asking such questions was to develop an understanding how novice teachers begin to craft a teaching practice based on mathematics reform principles and standards. The thesis of this study is that teachers who graduated from a reform-oriented teacher preparation program will adopt practices that can be described as reform-oriented mathematics teaching practices. Moreover, it is hypothesized that some practice will be adapted to fit the needs of the novice teacher while other practices will be ignored because of issues and challenges that novices face in the beginning years of developing their teaching practice.

Methods

Participants and Setting

A purposeful sampling method was used to select the participants for this study. In a purposeful sample the assumption is that the researcher "wants to discover, understand, and gain insight" (Merriam, 1998, p. 60) into a particular case, thus purposefully selecting participants that fit a set criteria is critical. Because this study was interested in examining the reform practices that novice teachers use, it was essential to choose participants in the beginning of their teaching careers. Thus, number of years teaching was a criterion used to select participants. Participants were all in their second year of teaching when this study began putting them at the midpoint of the novice experience. Moreover, because this study was interested in understanding how novice teachers implement reform teaching practices, it was critical to recruit participants who had a solid understanding of mathematics education reform principles. Thus, the second criterion set was that participants would need to have experience with mathematics reform principles and practices. To meet this criterion, the participants in this study were selected from the Collaborative Teacher Education Program (CTEP) – an early childhood/elementary teacher preparation program. Graduates of the CTEP were chosen because of the program's commitment to preparing "beginning teachers who are reflective practitioners, committed to meeting the learning needs of diverse students, and motivated to be agents of change in the school communities in which they work" (CTEP, 2007, ¶1). Moreover, the participants were chosen because all three had been enrolled in a graduate level reform based mathematics methods course of which I was one of the instructors. Thus, I had firsthand knowledge and experience with the core principles of the course and its orientation toward mathematics reform.

Four novice teachers were originally recruited for this study from a pool of CTEP preservice teachers (N=28) who had participated in the researcher's pilot study. Shortly after the study began, one of the participants left teaching to become a university admissions counselor. The impetus for the career change was her desire to pursue a doctorate in social justice education. Taking the position at the university provided her with the financial resources she needed, in the form of tuition and fee waivers, to fund her doctoral ambitions. As a result, the study included three participants rather than the intended four.

The three sites in which the participants were teaching were similar in that they were urban schools serving a minority, low-income populations of students. Moreover, each school was struggling to make annual yearly progress (AYP) as measured by scores on the Massachusetts Comprehensive Assessment System (MCAS). The following provides a description of the participants and setting. Two of the school sites were public elementary school, and one was a publically funded charter school.

<u>Ms. Duncan</u>

At the time of this study, Ms. Duncan was a third grade teacher at Sunrise Elementary School, located in an urban school district in Massachusetts. Sunrise

Elementary had been struggling to make annual yearly progress (AYP) for the past three years and according to Ms. Duncan, if the school did not make AYP by the following year, the principal would be replaced. At the time of this study the school was under review and subjected to much scrutiny at the district and state levels as well as by an outside consulting firm that had been hired to help the school make its AYP. Table 3.1 provides a general description of Sunrise Elementary School (Massachusetts Department of Education Directory Profiles).

Table 3.1 Description of Sunrise Elementary School (2008-2009)

Enrollment by Gender		**Enrollment by Race**	
Males:	201	African America:	13.2%
Females:	217	Asian:	1.2
Total Enrollment:	418	Hispanic:	58.1
		White:	22.2
		Multi Race Non Hispanic:	5.3

Ms. Duncan described herself as Native American with her father being reservation-born. Ms. Duncan shared that her fathers' childhood inspired her to work with underprivileged students, thus prompting her to seek out a teaching position in a large, urban school district. She grew up in a white, affluent community and attended public schools there. Ms. Duncan received a bachelor's degree in sociology with a minor in education from a large northeastern university. After graduating, she continued on at the university and earned a M.Ed. in elementary education. Her future plans include eventually return to school for a Ph.D.

Ms. Quinn

When this research study began, Ms. Quinn was in her second year of teaching first grade at Maple Elementary School located in an urban school district in Massachusetts. However, shortly after the study began, Ms. Quinn learned that because her school district was reorganizing, she would be reassigned to teach fourth grade math at Morningstar Elementary School the following school year. As a result of her transfer, observations were not scheduled until Ms. Quinn settled in at Morningstar. Ms. Quinn described herself as a white teacher of middle-class background. Ms. Quinn had never considered teaching in an urban environment and believed she would apply to schools similar to the one in which she attended. However, after having the opportunity to student-teach in an inner city school, Ms. Quinn decided that the urban environment was where she could have the most influence and impact on students' lives. Table 3.2 provides a general description of the student body at Morningstar Elementary (Massachusetts Department of Education Directory Profiles).

Table 3.2 Description of Morningstar Elementary School (2008-2009)

Enrollment by Gender		**Enrollment by Race**	
Males:	306	African America:	3.3%
Females:	307	Asian:	0.3
Total Enrollment:	613	Hispanic:	91.2
		White:	4.6
		Multi Race Non Hispanic:	0.7

Ms. Arielle

Ms. Arielle was born in Puerto Rico and lived in the United States for four years during elementary school. She returned to the U.S. years later with her husband when they both entered graduate programs. Ms. Arielle was a mother of a young son and described her family as being part of the working, middle class. Ms. Arielle would someday like to pursue a doctorate in education.

When this study began Ms. Arielle was teaching first grade at Achievement for All Charter School, located in an urban school district in Massachusetts. Achievement for All was housed in the upstairs rooms of a Baptist Church until a more suitable location could be purchased. The school was located in a predominately African American neighborhood and served, for the most part, students who lived in the surrounding area. Table 3.3 provides a general description of Achievement for All Charter School in terms of student demographics.

Table 3.3 Description of Achievement for All Charter School (2008-2009)

Enrollment by Gender		**Enrollment by Race**	
Males:	201	African America:	51%
Females:	217	Asian:	0
Total Enrollment:	418	Hispanic:	25
		White:	18
		Multi Race Non Hispanic:	5

It is important to note here that all three of the participants were enrolled in an integrated methods seminar in which I was the instructor. The two-semester seminar was part of the participants' graduate program and was intended to provide a course focusing on integrating curriculum, such as language arts, math, science, and social studies in the elementary classroom. The students in the course were in a cohort and as such developed a strong bond with each other as well as with me over the course of their graduate program in teacher education. This bond proved to be both an asset and a challenge. It was an asset in that the participants in this study were quite comfortable with me as an instructor, thus observing them in their classrooms during their critical second year of teaching did not cause the participants undue stress. It was challenging in that because I had developed such a strong bond with each of the participants, I needed to continually check my assumptions and biases as I was analyzing and interpreting the data. In an effort to address researcher bias, I shared written summaries of my work with the participants and asked for their feedback. Moreover, during data analysis and interpretation, I continually reflected with a critical friend who helped me to reflect deeply on the assumptions and biases I brought with me to this study. It should also be noted that Ms. Arielle was also a student in my section of a reform-based methods course, while Ms. Duncan and Ms. Quinn were enrolled in another section of the same course. All sections of the methods course were anchored by the following objectives:

As a result of actively participating in Education 691R students will:

- Construct a more extensive and integrated understanding of the "big ideas" of elementary mathematics;
- Become more knowledgeable and skillful in assessing their students' understanding, skill, and learning of mathematics and use this assessment to guide instruction;
- Increase their ability to use constructivist pedagogy and a variety of teaching techniques to teach mathematics;
- Learn skills, vocabulary, procedures, and concepts included in the elementary mathematics curriculum;
- Increase their comfort, confidence, and enjoyment in learning and teaching mathematics;
- Become more able to address equity issues and to enable all students to be successful learners through their teaching of mathematics; and
- Continue to develop as on-going, reflective learners and practitioners.

Although both sections of the methods course were based on the aforementioned objectives, the instructors focused class readings, discussions, and activities using different texts. In my section, in which Ms. Arielle was a student, we used a case study approach wherein participants read case studies of teachers and students as they engaged with mathematics using the text *Developing Mathematical Ideas: Building A System of*

Tens (Shifter, Bastable, & Russell, 1999). In the section that Ms. Duncan and Ms. Quinn were enrolled students read and responded to various readings focused on mathematics reform, such as *Knowing and Teaching Elementary Mathematics* (Ma, 1999).

Data Sources and Collection

A multiple case study design was used to provide a thick, rich description of the types of practices novice teachers use to engage students in mathematics conversations. Moreover, the multiple case study design allowed for a cross-case analysis of the data as a means to reveal generalizations and discern commonalities and differences among the three novice teachers participating in this study (Miles & Huberman, 1994). According to Merriam (1998), data collection in case study research usually includes the following three strategies: observing, interviewing, and analyzing artifacts. All three methods of data collection were utilized in this research study as a means to gather rich descriptive data about the case under study. Data were collected from the following sources: videotaped classroom observations, audiotaped individual and focus group interviews, participant lesson plans, participant reflections, and the Mathematical Knowledge for Teaching assessment (MKT).

The above sources were mindfully chosen for their ability to offer pertinent information about the culture of the participants' mathematics classroom and to aid in the process of triangulation. Literature on qualitative case study methodology offered insights into the ways of gathering and analyzing the data collected for this study (Creswell, 1998; Lijphart, 1971; Merriam, 1998; Rossman & Rallis, 2003).

Data were collected during the second and third years of the participants' teaching career. This time period was purposefully chosen as it constituted a mid-point or transition period in terms of experience. The second-year teacher is no longer bogged down by the newness and challenges of being a first-year teacher, and she is able to begin to focus on and perfect her craft toward effectiveness. However, the second-year teacher is not yet considered to be an experienced, effective teacher. Research on the qualities of effective teachers indicates that it is the years between three and eight that are attributed to teacher effectiveness (Stronge, 2004) thus making the first three years a formative time in a novice teacher's career. Table 3.4 shows the types of data collected and the time in which it was gathered.

Table 3.4 Data Sources and Dates Collected

Data Sources / **Participant**	**Individual Interview**	**Focus Group Interview**	**Videotaped Observations and Field Notes**	**Lesson Plans and Video Reflections**	**MKT/Minutes to Take Test**
Ms. Arielle	June 25, 2008	December 5, 2008	April 14, 16, 2008 June 4, 5, 12, 2008	Plans for all lessons. Written reflection on one videotaped lesson	January 2, 2009 Minutes 16:12:54
Ms. Duncan	June 25, 2008	December 5, 2008	April 28, 2008 May 1, 2008 June 10, 11,12, 2008	Plans for all lessons Written reflection on one videotaped lesson	December 28, 2008 Minutes 17:08:37
Ms Quinn	June 26, 2008	December 5, 2008	November 3, 5, 2008 March 20, 23 2009	Plans for all lessons Written reflection on one videotaped lesson	December 6, 2008 Minutes 17:22:31

Interview Data

Individual Interviews

At the beginning of this study, each participant was interviewed for approximately 90 minutes, using an in-depth formal interview format (See Appendix A). All interviews were audio-recorded and transcribed by a professional transcriber for later analysis. Interviewing, as described by Kahn and Cannell (1957, p. 149), can be viewed as "a conversation with a purpose." This description is fitting to use as the metaphor of conversation was central to this research study. The purpose of the in-depth formal interview was to engage the participants in conversation as a means to develop a sense of the participants in terms of their self as a learner of mathematics and their reflections of practice in light of mathematics reform. Additionally, the interview attempted to tease out issues and challenges the participants believed impacted their ability to teach mathematics in a manner that was orientated toward reform. The interview followed recommendations set for by Marshal and Rossman (1995) in that a small number of topics were explored during the interview to gain the participants "meaning perspective" (p. 80) on the issues discussed.

Focus Group Interview

The three participants in this study also participated in a two-hour semi-structured focus group interview. The purpose of the focus group was to gather information as to the issues and challenges that novice teachers face as they attempt to develop a teaching practice under the auspice of mathematics reform. Focus groups work under the assumption that attitudes, beliefs, and values are not formed within a vacuum (Rossman

& Rallis, 2003), thus sharing one's understanding with others in the same situation provides rich and valuable discourse and data. Focus groups are most productive when the participants are comparable to each other, are supportive of one another, and when one-on-one interviews may not yield as much information (Creswell, 1998). The teachers in this study were comparable in that they were second-year, novice teachers; shared a past history in their graduate teacher education program; and taught in urban school districts. See Appendix B for the Focus Group Interview Protocol.

Classroom Observation Data

Videotaped Observations

All classroom observations were videotaped by the researcher and transcribed by a professional transcriber. These videotaped lessons represent an essential data source for this study. They were recorded during the second and third year of the participants' teaching career with a ZR 800 digital video recorder positioned strategically during each observation to best capture what the teacher said to students as she orchestrated whole group conversation.

The videotaped data set for Ms. Arielle and Ms. Duncan consisted of five lessons while the videotaped data set for Ms. Quinn consisted of three lessons. The data set for Ms. Quinn was limited because of a district reorganization that included transferring Ms. Quinn to another school shortly after this study began. Consequently, a mutual decision was made by the researcher and Ms. Quinn that videotaping would not be scheduled until Ms. Quinn was settled into her new teaching assignment the following fall. Moreover, although four observations were completed for Ms. Quinn only three were appropriate for video analysis. When the researcher arrived on March 20, 2009 Ms. Quinn’s document camera had broken, consequently altering her planned lesson in favor of a last minute individual worksheet/test prep lesson. As a result, this lesson did not generate whole class conversation among teacher and students, thus it was not used in analysis. Although the researcher and Ms. Quinn attempted to schedule additional observations, MCAS test preparation and administration was getting underway and a mutual time could not be found. As a result, the researcher was unable to observe again after March 23, 2009.

Field Notes

In this study field notes were utilized as a means to “capture as much detail as possible” (Rossman & Rallis, 2003, p. 196) about the physical setting in which the observations took place. The field notes were used to write contact summary forms as suggested by Miles and Huberman (1994), assisted in summarizing the details of the observation, and helped to generate questions to guide the next observation. See Appendix C for a copy of a contact summary form.

Documents and Artifacts

Lesson Plans

Lesson plans were collected for each of the observed lessons as a way of gathering data on the “material culture” (Rossman & Rallis, 2003, 9) of the classroom and as a means to support the triangulation of data. The material culture, according to Rossman and Rallis includes such artifacts that serve as the “written record” (p. 198) of a person life. Lesson plans provided a written record of what participants expected to happen during each lesson and as such constituted valuable source of information and meaning. These data were used to develop an understanding of what the participants deemed important in each lesson before the act of teaching took place. Moreover, the lesson plans provided information as to what the participants expected students to learn during each lesson.

Participant Videotaped Lesson Reflections

The researcher provided each participant with one videotaped lesson and asked them to review the tape and complete a reflection based on a set of guiding questions (see Appendix D). The purpose of the videotaped reflection was to discern the participant's reaction to their teaching and to assess their perception of reform practices.

Mathematical Knowledge for Teaching Assessment (MKT)

Although the questions guiding this study did not specifically focus on the participants' mathematical knowledge for teaching per se, it seemed important to discern whether or not the teachers participating in this study possessed the types of knowledge needed to teach elementary mathematics. Research over the last 20 years indicates that elementary teachers possess a weak knowledge of mathematics and that the knowledge needed to teach mathematics is different from the knowledge needed by mathematicians (Ball, Hill, & Bass, 2004). The MKT instrument aims to assess the mathematical knowledge needed for teaching. More specifically, the MKT instrument was designed to measure teachers' ability to assess student work, to represent mathematical ideas and operations, and to explain mathematical rules and/or procedures.

Participants were administered Mathematical Knowledge for Teaching instrument developed by researchers at the University of Michigan (Hill, Schilling, & Ball, 2004). Although the intent was to give a pretest in May 2008 and a posttest March 2009 to assess how mathematical knowledge for teaching develops over time, this did not happen, and only one testing session was administered. The test developers required prospective users to attend a two-day training session at the University of Michigan. Training sessions were only held two times per year and the next available one was not offered until November 2008. As a result, the researcher could not administer the MKT instrument until late 2008 and early 2009. This meant the posttest would have been administered with only a two month lapse in time between pre- and posttest sessions, thus making any type of analysis of growth of mathematical knowledge over time invalid. However, the test was administered once and used to gain a baseline understanding of the participants' mathematical knowledge for teaching while in the novice years (1-3) and as a means to assess issues and challenges the participants had with assessing student work, representing mathematical ideas, and understanding mathematical rules and procedures – three domains addressed by the MKT instrument.

It is important to note that the MKT instrument was designed to measure quantitatively a teacher's mathematical knowledge as it relates to teaching mathematics to school age children. The measure was not developed to publicly demonstrate teachers' ability or lack thereof and researchers must agree not to use the measure in such a way. See Appendix E for a sample of released items from the MKT assessment.

The MKT instrument was administered using the online Teacher Knowledge Assessment System (TKAS), specifically designed for administration of the MKT instrument. After attending the training session at the University of Michigan, the researcher was given an access code to log onto the TKAS and set up the site for test administration. When using the TKAS online administration site, a test administer can not specify which form a participant will receive (A or B), as the system randomly assigns a form to each participant.

To gain an understanding of the participants' knowledge of essential elementary concepts, the 2004 Elementary Number Concepts and Operations – Content Knowledge test was chosen. Once the researcher set up the test administration site, an email was sent to participants with an URL link for them to follow and log into the TKAS site. Participants were then instructed, via the site, how to create their own user accounts, allowing them to gain access to the assessment. Once participants had set up an account, they could access the assessment at a time most convenient for them. Participants had between December 6, 2008 through January 9, 2009 to log on and take the assessment.

Once logged onto the site, participants were randomly assigned equated forms A or B. The assessment consisted of 14 multiple choice questions; however, some questions contained more than one correct response thus in total participants answered 25-26 questions per assessment. The participants were instructed to imagine that they were responding to real time classroom situations; thus, they were asked to spend no more than one to two minutes per question. Moreover, the participants were instructed to choose the answer that best reflected what they would say or do at the moment. The TKAS site revealed that participants spent between 16 and 17 minutes taking the MKT assessment.

Data Analysis Procedures

Video Observation Analysis

Quantitative methods were used to quantify the distribution of practices three novice teachers used to engage their students in mathematics conversations. Although useful in proving a general summation of the teaching practices used, quantitative results provided little information as to the reform orientation of the practices or the social and socialmathematical norms such practices constituted. Discourse analysis provided the means to analyze the video transcripts of the mathematics lessons. According to Silverman (1993), discourse analysis is the analysis of “recorded talk” (p. 120). Of interest in this research study was what novice teachers say to students to engage them in mathematics conversations. In reform-oriented classrooms intellectual autonomy is fostered by teachers who initiate and guide the “development of a community of validators” (Cobb et al., 2001, p. 124) who are adept at using argumentation to establish mathematical claims rather than relying on the teacher or text book to validate claims. The purpose of the analysis was to examine and document teacher led talk in relationship to mathematics reform recommendation

Data were analyzed using an inductive analysis approach. More specifically, the researcher brought “sensitizing concepts” (Patton, 1990, p. 391) to utilize as a point of reference while analyzing the data. These sensitizing concepts were generated from the literature on reform teaching practices and learning to teach mathematics. As described by Patton to apply inductive analysis using sensitizing concepts, the researcher examines how the concepts are manifested “in a particular setting or among a particular group of people” (p. 391). As the researcher read through the transcripts, the data were colored by the sensitizing concepts that were applied, similar to placing a colored transparency placed over the transcripts before reading. Since this case study was specifically searching for the types of reform teaching practices that novice teachers used as they engaged students in mathematics conversations, it was important to analyze the data with following sensitizing concepts:

- Elicited different mathematical solutions
- Elicited student explanations and justifications
- Developed idea of what counted as a mathematically different solution
- Developed idea of what counted as an acceptable mathematical explanation and justification.
- Developed idea of what counted as mathematically efficient or sophisticated solution.

In an effort to focus on the discourses practices that the participants used to engage their students in mathematics conversations a “data reduction” (Miles & Huberman, 1994, p. 10) strategy was used to focus the analysis. Data from the videotaped lessons were reduced to include only those instances when participants were engaged in whole class mathematics conversations with students. For the purpose of this study, whole class conversation was defined as being led by the teacher and encompassing whole class interaction rather than small group or individual interaction. Whole class conversation, as defined above, was selected for analysis for its ability to shed light on the practices that the participants used to engage students in whole class conversations focused on mathematics. As a result of this decision, video footage of students working in small groups was not used in analysis.

The decision to include only whole group conversations in the analysis reduced the video observation data to approximately 369 minutes of observation data. Ms. Duncan engaged students exclusively in whole group instruction/conversation, thus there were 144 minutes of data available for analysis. Ms. Arielle engaged students in whole group and pair/small group work during each observation, thus 116 minutes of whole class instruction/conversation were available for analysis. And lastly, Ms. Quinn also engaged students in whole group and pair/small group work during each observation, thus there were 109 of whole class conversation available for analysis.

The digital videos were downloaded onto a Dell Computer and edited by the researcher using Windows Movie Maker. The researcher made one full length clip of each video and one that was cut into 5-minute video clips. Each clip was separated by a screen marked Clip 1 and so on. Next, the edited video containing the 5-minute clips was transferred to a compact disc and sent to a professional transcriber who was instructed to transcribe the videos verbatim into the 5-minute clips noted on the compact disc. In the interim, while the video tapes were being professionally transcribed, the researcher watched each video tape in its entirety to get a general sense of how the lessons unfolded.

When the transcripts were returned to the researcher, the videos were watched again, this time in 5-minute clips with the addition of the audio transcripts. During this viewing the researcher used the aforementioned sensitizing concepts to conduct an “exploratory analysis” (Cobb & Whitenack, 1996, p. 217) of each 5-minute clip. At this point in the analysis, the researched identified instances where the participants attempted to utilize a reform-oriented discourse practice. The following is an example of the researcher’s first attempt at identifying instances of the practice if eliciting different solutions. While watching one of Ms. Arielle’s video clips, the researcher highlighted in the transcript the following statement she made immediately after a student had shared how he solved a math problem:

T: Who did something different?

This statement was coded as EDS – elicits different solution. From here, the researcher looked for other instances that could be coded as EDS. Such statements as, "Anybody do it differently? Who has another way? or There are a couple of ways we could do this. What's one?" were all assigned the code EDS. At this point there was no analysis as to whether or not the elicitation generated a different solution only that the elicitation had been made.

Next the researcher went back to the first instance of EDS in a transcript and conducted a deeper analysis as to what transpired before and after the coded EDS to discern if the elicitation generated a different solution. The following is what came before and after a coded EDS segment.

<u>Before EDS Segment</u>

1. S: I put 5 dots, and then when you put it up again, I took away 2 dots and it equals 3.
2. T: So let's see. Did you remember that this is a 10 frame?
3. S: [Student shakes head yes]
4. T: That we have 1, 2, 3, 4, 5 squares and 1, 2, 3, 4, 5 squares to have 10. So you did 5 dots first [draws 5 dots on overhead] and then you took away 2? [Erases 2 dots]
5. S: Yes.
6. T: Okay. Nice! Who did something different? (EDS)

<u>After EDS Segment</u>

7. S: Seven in my head, and then I took 7 – then I said, "1, 2, 3."

By examining what came before and after the coded EDS segment, the researcher was able to discern the efficacy of the EDS to generate a different solution method. This exchange was coded as one that did generate a different solution method. The researcher then went through the rest of transcript and conducted the same type of before and after analysis on each coded EDS to see if the pattern of generating a different solution held for each of the remaining EDS coded segments.

A similar analysis was used for instances of eliciting an explanation and justification (EX/J). Here the researcher coded segments of the transcript where the teacher asked students questions pertaining to why or how they solved a particular problem. For example, the statement "How do you know 7 plus 5 equals 12?" was coded as EX/J. Again, after coding all instances of EX/J the researcher went back to the first instance and conducted deeper analysis to discern if the elicitation generated a student explanation and/or justification.

The analysis of what counted as a mathematically different solution referred the researcher back to the coded segments of elicited a different solution (EDS). In this stage of analysis the researcher examined what the teacher said when a solution offered by a student was not mathematically different from one previously shared. For example, if a teacher made a commented, such as, "Well that's the same as what Brian did," it was coded as developed idea of a mathematically different (MD). In this example the teacher was letting the student know that his or her contribution was not acceptable because it was mathematically the same as one previously shared.

After coding the instances of MD, the researcher went back to the beginning to examine the patterns that emerged within the coded segments of MD. For example, at times a teacher might loosely mention that a contribution was not different while other times she might attempt to explain how the solutions were the same or different from ones previously shared. Moreover, the researcher looked again at all instances of EDS to examine in more depth the MD between solutions. For example, a transcript may not have generated many codes with MD; however, when going back over the transcripts it was clear that the majority of coded EDS segments generated mathematically different solutions, thus the teacher did not have make MD comments.

To analyze what counted as an acceptable mathematical explanation and justification, the researcher reexamined the coded segments of elicited an explanation and justification (EX/J) to discern how such elicitations obligated students to respond. When Ms. Arielle elicited explanations and justifications, students were obligated to act on numbers either conceptually or physically as though in a physical reality. For example, when students explained their solutions, they often used words, such as grabbed, bunched, squeezed, took, and put to explain how they mathematically solved a problem. Moreover, Ms. Arielle continually asked students to “show what you did” as they explained their solutions.

Finally, analysis of what counted as an efficient or sophisticated solution referred the researcher back to the coded segments of explanations and justifications to discern if the teacher made note of solutions that were more efficient or sophisticated than ones previously shared. For example, the comment “Did that take you a long time to do it that way?” was coded as efficient/sophisticated (E/S) and was considered direct in that the teacher made a direct reference to the efficiency of a solution method. However, oftentimes this analysis was quite subtle often occurring in comments, such as, “Oh goodness this is what Reba did.” Again, after coding for these direct and subtle instances of E/S, the researcher went back and examined each instance in more depth and compared subsequent instances refining conjecture with each new instance examined.

Although analyzing the data using the aforementioned sensitizing concepts provided a means to see reform practices within the discourse patterns of classroom conversations, it should be noted that whenever a researcher imposes concepts on data, there is a chance that the data analysis report portrays a picture of how the researcher experiences the participant's world rather than portraying the way in which the participant experiences her world (Patton, 1990). To address this concern, video transcripts were also analyzed using "indigenous concepts" (Patton, 1990, p. 390). These concepts are native to the data set and are generated by the participants and emerge as analysis is taking place. According to Patton, every program has certain patterns that when paid attention to by the researcher, provide meaningful glimpses into the participant's world. As the researcher mined the date set for instances of the aforementioned reform-oriented practices, a set of practices indigenous to the participants came to light. The following indigenous patterns emerge through data analysis:

- Asked a Question with a Known Answer
- One word correct answers
- Explained student thinking
- Evaluated/accepted students' response
- Indicated that math is rule bound
- Made mathematical thinking public

These indigenous practices were examined in much the same way as the reform practices were analyzed. Practices were noted and then reexamined to discern the impact of such practices on the mathematics conversations that ensued in each of the participants' classrooms. For example, in Ms. Duncan's classroom, asking questions with a known answer (QWKA) dominated her classroom teaching practice. Each coded segment of QWKA was examined to discern the type of response it generated from students. This analysis revealed that asking QWKA obligated students to supply a one word correct answer.

Interview Analysis

As was the case with the video recordings, all individual and focus group interviews were transcribed verbatim by a professional transcriber. Once transcribed, the researcher listened to each tape-recorded interview. Then the researcher listened a second time to each tape, aided by the verbatim transcripts. Lastly, the researcher manually coded each transcript so as to remain close to the data set while engaged in data analysis (Douglas, 2008). The interview data were analyzed line by line in an effort to categorize meaning with respect to the participants' thoughts on mathematics reform and the issues and challenges they experienced as novice teachers (Rossman & Rallis, 2003). The researcher attempted to capture reoccurring patterns that emerged by searching for meaningful pieces of data that addressed the research questions (Merriam, 1998). Once the interviews were analyzed a narrative interpretation was written and given to each participant in an effort to validate the analysis made (Rossman & Rallis, 2003).

Documents and Artifacts Analysis

Lesson Plans

Participant lesson plans were used to aid in triangulation of the data. Using a scoring guide that was developed by the researcher and based on NCTM Curriculum and

Professional Teaching Standards (1991, 2000), each lesson plan was analyzed in terms of participation structure, whether whole group small group or a combination, lesson objectives, and lesson assessment and given a rating from 1 (traditional) to 3 (reform) (see Appendix F). A summary table was constructed as a means to view and compare the participants' lesson plans in terms of their reform orientation (see Appendix G).

Mathematical Knowledge for Teaching

The MKT instrument was not designed to make highly accurate statements about individual teachers; however, it can be used to compare groups of teachers' mathematical knowledge with an average teachers' ability and to make statements about how a group of teachers performs at a given point in time. This study was concerned with a particular point in time – novice teaching years – thus, gathering information regarding the participants' mathematical knowledge for teaching provided a snapshot of this time period and offered an opportunity to make assess issues and challenges the participants experienced in relationship to the MKT assessment questions.

When developing the MKT instrument, the designers deliberately made half of the items on the assessment more difficult than the average teachers' ability and a small number of items more difficult than most teachers' ability (MKT Training Manual). According to the test developers, the overall reliabilities of the Number Concepts and Operations - Content Knowledge Instrument were adequate at .70 or higher. The assessment had a mean of 0.000 and a standard deviation of 1.000. Scores on the MKT assessment will be presented as IRT scores rather than raw frequencies or number correct. Before being allowed to administer the assessment, the researcher signed a contract, stipulating that raw frequencies or number correct would not be used in reporting results. Moreover, the researcher agreed not to use the assessment to “publicly demonstrate teachers’ ability or lack of ability in mathematics” (MKT Terms of Use Contract).

Once IRT scores were calculated for each participant, test questions were analyzed in an effort to examine areas of strength and challenge for each participant in relationship to assessing student work, representing mathematical ideas and operations, and explaining mathematical rules and/or procedures. The intent was to determine if the participants were particularly strong or challenged in one of the aforementioned three areas the assessment measured. For example, if a participant was strong in the area of representing mathematical ideas, did this strength show though during the classroom observations? Again, raw scores or number correct were not revealed in any particular area but provided a general statement as to the participants’ areas of strength and possible areas of concern in comparison to their actual classroom teaching practices.

Gaining Entry and Informed Consent

Gaining Entry

Before making contact with the building principals, the participants first requested permission to participate in the proposed research study. Once permission was granted, the researcher provided each principal with a brief description of the study and was available to answer any questions and address any concerns that the principals had. The principals granted permission via email and did not express any type of concern with allowing the teachers to participate. The research met each principal briefly on the first day observations and was not contacted afterwards regarding concerns or questions.

Informed Consent

Through the informed consent process, the researcher specified how and when the research would be conducted. Participants were given a copy of the consent form before the study began and asked to sign, date, and return to the researcher (see Appendix H). The informed consent helped to establish guidelines so that the research followed a clear and detailed timeline. Two schools had on file blanket permission slips for classroom videotaping, and all caregivers in the two schools had given permission for their child to be videotaped for educational purposes. One school did not have such a policy, thus a letter was sent home to all caregivers in Spanish and in English (see Appendices I and J). Once all permission slips were returned, videotaped observations began.

Confidentiality

Qualitative research is research in action and takes place in the field with real individuals living and working in the settings explored (Rossman & Rallis, 2003). The promise of confidentiality is two-fold: to ensure confidentiality in the written material and to ensure confidentiality in the spoken word. In an effort to ensure confidentiality, pseudonyms were used for the participants, schools, and students mentioned and used in all field notes, transcriptions, analysis, and written texts as well as when the researcher discussed this study with colleagues and advisors.

Researcher Profile

My theoretical orientation is in line with social constructivism as defined by Cobb et al. (1993). The process of teaching and learning inherently encompasses social interactions between student and student and teacher and student. These inherent social interactions influence knowledge construction and therefore must be taken into consideration when trying to understand the process of teaching and learning. Although learning is an individual endeavor, with the student constructing his/her own individual understanding, knowledge construction is influenced by the social interactions inherent in the teaching and learning process.

In mathematics the assumption tends to be that some students “get math” and other students “don’t get math.” This assumption has limited the opportunities for women and minority students in the field of mathematics. I work with preservice elementary teachers in different mathematics methods courses. Most of these future teachers reveal that they never understood math and are terrified of having to teach this subject to their future students. The majority of the students that I teach are female, as are the majority of elementary school teachers. Elementary school teachers lay the foundations for future mathematical exploration, making this study of novice teachers as they begin to develop a teaching practice critical to the research literature.

Summary

In summary, this chapter outlined the proposed research design, methodology, data collection, and analysis methods that will be used to answer the following questions:

1. What reform-oriented discourse practices do novice teachers who participated in a reform-based mathematics methods course adopt? What practices do they adapt? What practices do they ignore as they engage their students in mathematics conversations?
2. What issues and challenges surface as novice teachers attempt to enact reform-oriented discourse practices?

CHAPTER 4

MS. DUNCAN

When this study began, Ms. Duncan was in her second year of teaching third grade at Sunrise Elementary School, located in a large urban school district in Western Massachusetts. Ms. Duncan's had 21 students in her class, 14 boys and 7 girls. Seventeen of her 21 students were in her math class while the other 4 students were placed in classes for students in need of special services in mathematics. The following sections will present the analysis of the teaching practice used by Ms. Duncan to engage students in mathematical conversations. First will be an analysis of the teaching practices that fostered the development of social norms followed by an analysis of the practices that fostered the development of socialmathematical norms in Ms. Duncan's classroom. The focus of this section will be to address the following research question:

- What reform-oriented discourse practices do novice teachers who participated in a reform-based mathematics methods course adopt? What practices do they adapt? What practices do they ignore as they engage their students in mathematics conversations?

Practices Fostering Social Norms

Social norms constitute the "participation structure" (McClain & Cobb, 2001, p. 244) within classrooms and are not specific to the discipline of mathematics. By analyzing the practices Ms. Duncan used to engage her students in mathematical conversation the participation structure of the classroom emerged. Table 4.1 shows the distribution of practices constituting the ways in which Ms. Duncan and her students were obligated to participate in classroom conversations as well as the number of times each practice was used per hour.

Table 4.1 Observed Practices Fostering Social Norms: Ms. Duncan's

	Observation						
Practice	OB 1	OB 2	OB 3	OB 4	OB 5	Total	Per Hour
Elicited Student Explanation/Justification	2	4	1	11	4	22	9.2
Elicited a Different Solution	0	0	0	9	0	9	3.7
Asked a QWKA	82	39	42	29	37	229	95.4
Evaluated Student Response	52	29	26	26	17	150	62.5
Accepted Student Response	18	14	14	21	20	87	36.3
Explained Student Thinking	15	7	16	18	10	66	27.5

QWKA - Question with a Known Answer

As seen in Table 4.1 Ms. Duncan rarely utilized the reform-oriented practice of asking student to solve problems in different ways with only 9 attempts observed indicating this was not a developed aspect of her teaching practice. Moreover, because Ms. Duncan did not elicited different solution methods from students, she afforded herself little opportunity to elicit student explanations and/or justifications with only 22 attempts observed. Rather than elicit explanations and/or justifications from students, Ms. Duncan adapted this practice and instead, as seen in Table 4.1, explained student thinking by providing 66 detailed explanations and/or justifications. Consequently, this practice made Ms. Duncan the mathematical authority in the classroom.

As seen in Table 4.1 the dominant practice used by Ms. Duncan was asking a question with a known answer (QWKA) with 229 such questions asked over the course of five observations. Accordingly, Ms. Duncan was obligated to evaluate the responses students gave to her QWKA with 150 evaluations provided by Ms. Duncan. Lastly, Ms. Duncan accepted student responses 87 times over the course of the five observations, and although this practice could be considered a reform-oriented one, in Ms. Duncan's classroom it had the flavor of an evaluation, thus it was categorized as a traditional practice. Because the dominant practices used by Ms. Duncan were traditional, a fast paced question and answer quiz-like participation structure emerged.

The following section will examine each of the practices in more detail as a means to shed light on the social norms that were constitute as a result of the practices Ms. Duncan used. First under review are the practices associated with mathematics reform: Elicited Different Solutions and Elicited Explanations and Justification, followed by an examination of the practices associated with traditional classrooms: Asked a QWKA and Evaluated/Accepted Student Response. Examining such practices through the filter of social norms provided a framework to describe the participation structure that emerged in Ms. Duncan's classroom.

Elicited Different Solutions

Obligating students to solve problems differently is a reform-oriented social practice and as seen in Table 4.1 Ms. Duncan only used this practice 9 times over the course of the five observations. Moreover, what stands out about this result is that the practice of asking students to solve problems differently only happened during Observation 4, indicating that Ms. Duncan has not adopted the practice of eliciting different solutions. Although clearly not an adopted practice, it is important to examine the ways in which she attempted to elicit different solutions as a means to discern the social norms fostered as a result. The following will examine the way in which Ms. Duncan attempted to elicit different solutions to the same problem.

The focus of this particular lesson was on the different ways to solve multiplication problems. The objective and assessment of this lesson was written as follows in Ms. Duncan's lesson plan (June 11):

> Objective
> As way to move toward multiplication of greater numbers, students will create, explain, and review various ways to multiply using single-digit numbers.
>
> Assessment of Student Learning
> During the group lesson, students will volunteer their strategies for multiplying numbers. While this happens, I will be recording on chart paper and using student models to reinforce the concept. Student modeling not only assesses the expertise of the student volunteers and their thought process, but also creates a platform to re-teach or review other learners. Individual marker boards are used to assess the student's ability to choose appropriate strategies and use them appropriately to solve multiplication problems they do not have memorized. Small individual mistakes or flaws in strategy selection are often caught during this portion of the lesson and immediate intervention is often possible. A homework sheet that reinforces skills and provides additional practice is an assessment of whether or not the student has taken what they need away from the lesson**.**

Ms. Duncan's objective, as stated in her lesson plan, was to encourage students to solve 3 x 32 in "various ways," and she planned to assess their understanding by recording the strategies that students volunteered. This statement indicates that from the outset Ms. Duncan was eliciting different strategies that students *might* use rather than eliciting the different ways students *actually* solved the problem. Moreover, analysis revealed that Ms. Duncan's focus was on eliciting the different types of manipulatives students could use to solve 3 x 32. Consider the following exchange that reveals the focus on manipulatives rather than on students' divergent ways of actually solving the problem. In this exchange, Ms. Duncan asked her students to share the different ways they could solve 3 x 32 without using a pencil or a paper.

1. T: There are a couple of ways we could do this. Jacob what's one way?
2. S: Calculator
3. T: Calculator. Absolutely. You could pull out your calculator and you saw me do that in a lesson yesterday. I needed to use it to do a quick math problem. You can always use a calculator if you have one, but if you are taking a big test like the MCAS math test, you can't use a calculator then. What's another way?
4. S: Mental math.
5. T: Mental math! Great job! Another way?
6. S: You could use your fingers.
7. T: You could use your fingers. Another way?
8. S: A multiplication chart.
9. T: A multiplication chart. We have one of those hanging on the wall. You could absolutely do that, but for a number this big our multiplication chart would have to go on for a long way, wouldn't it? It would have to be three times as big if we wanted to use a multiplication chart to solve this one. So other ways. You guys have named like 10 different ways to multiply. That's a lot. I haven't heard from Helen. What's another way?
10. S: You could use rows of.
11. T: You could use rows of. For a point question what is the name of the object that we draw – the name of the object we draw when we are using rows of in place of the multiplication symbol? Rows of?
12. S: An array
13. T: An array is correct! Great job! Okay, great minds seem to think alike because I heard a lot of people who had the same idea. There are a couple more ways. What's another way?
14. S: You could act it out.
15. T: Acting it out. But do we have enough students to act this problem out?
16. Ss: No!
17. T: No, but do we have enough fingers and toes in the room to act it out? So you can use your body in so many different ways. We used our bodies the most in this room when we were learning our division and multiplication properties, and wasn't it so much better to do it that way when we were learning about dividing by zero and dividing by one?

18. Ss: Yes!
19. T: What’s another way?
20. S: Objects.
21. T: That’s the last one I was thinking of.

Although in the exchange above Ms. Duncan attempted in Turns 1, 3, 5, 7, 9, 13, and 19 to elicit different ways to solve 3 x 32, it was clear that her focus was on the strategies or manipulatives students could use rather than on the different ways students actually solved the problem. Moreover, students understood her elicitation as a request for a strategy or manipulative rather than a request for them to share their solution methods. For example, in Turn 1 Ms. Duncan stated that there are several ways to solve 3 x 32 and asked Jacob “What’s one way.” Jacob responded in Turn 2 with the one word answer “calculator,” which prompted Ms. Duncan to evaluate his response positively in Turn 3 by saying, “Calculator Absolutely.” The positive response by Ms. Duncan indicated to Jacob and the group that this was an acceptable response. From this point forward the discussion centered on the strategies and manipulatives that could be used to solve the problem, such as mental math, multiplication chart, fingers, an array, and objects. However, what is lacking here is any discussion about how students would use the manipulatives or strategies to solve 3 x 32. Lastly, Ms. Duncan posed a calculational problem for students to solve rather than a contextual problem, indicating that in this classroom mathematics meant calculating answers rather than solving problems.

Ms. Duncan did not elicit from students different solutions to the problem 3 x 32 but instead elicited different strategies and/or manipulatives that students *could* use. As a result, Ms. Duncan was not able to bring to the fore of the conversation what students understood about the problem 3 x 32. After recording on chart paper the different strategies that students had shared, Ms. Duncan directed students to pick one of the strategies and solve the problem. As seen from the transcript except below, she tightly controlled how students would go about solving 3 x 32 (Observation 4, p. 7).

1. T: I'm going to let everybody solve different parts of this problem or a different way of solving the problem. Okay? And we're going to see if we all come out to the same answer. So raise your hand if you want to try doing it this way [pointing to one of the ways written on the chart paper]. [Students raise hands] Okay. Do you need a piece of paper for that?
2. Ss: No.
3. T: Don't you need to show me how you did it?
4. Ss. Yes.
5. T: Okay, go get a piece of paper and any piece – yellow or whatever, anyone who wants to do this, you need paper and pencil – go ahead and do it. The next one would be this [pointing to another way listed on chart paper] Paper and pencil go get it. I think I should take the yellow paper. The yellow paper.
6. Ss: [Students go to back of room and get the yellow paper]
7. T: So, you're adding 32 three times, go ahead. If you're doing mental math, go for it.
8. Ss: [Students doing mental math get up from circle and go to their desks].
9. T: If you're counting these [cubes] raise your hands. [Students raise hands] Okay. That's a good one. It's math! [referring to using cubes to solve the problem]
10. Ss: [Students who decide to count the cubes go to the back of the room to get yellow paper]
11. T: So it looks like I have two or three girls – wait-wait-wait – people who are counting these you don't need paper. You just need to count them. So we have four quiet minutes right now for you to do whatever calculations that you're doing, but everyone needs an answer when time is up.

Ms. Duncan managed this lesson in such a way that she offered students little opportunity to engage in the problem solving process. First, she presented students with a calculational problem rather than a contextual problem to solve. Second, she generated a list of strategies students could use and then instructed them to pick one and calculate the answer. Moreover, she tightly controlled the materials that students needed to use to solve the problem. What is interesting to note in this exchange is that students seemed to be dependent upon Ms. Duncan to tell them what materials were needed, indicating students had little autonomy in the process. Consequently, Ms. Duncan provided herself with little opportunity to assess how individual students would have solved 3 x 32 if presented with the problem without being shown different solution strategies first. As a result, she was not able to discern students' individual reasoning methods as she asked them to pick one of the listed strategies. As a result, all students were successful in finding the answer, and as we will see in subsequent analysis, solving problems with success was a key aspect of Ms. Duncan's teaching practice.

The reform practice of eliciting different solutions to the same problem fosters a participation structure whereby students are obligated to solve mathematical problems in different ways and then expected to explain and justify their methods to the class. Ms. Duncan's attempts at eliciting different solutions fostered an obligation to state a different strategy or manipulative one could use rather than an obligation share how one solves the problem. Moreover, because the elicitation did not obligate students to solve the problem using their own reasoning, the attempt to elicit different solutions was not tied to an expectation that students would have to explain and justify how they were thinking about the problem, thus productive mathematical conversations never materialized. Rather, the discussion was more of a question and answer session than a productive mathematical conversation.

The following section will examine Ms. Duncan's attempts at implementing the reform-oriented practice of eliciting explanations and justifications as a means to bring student thinking and understanding into the public discourse space to be considered by all members of the class.

Elicited Explanations and Justifications

Ms. Duncan attempted to create situations where students were encouraged to explain and/or justify their solutions; however, her attempts created situations for instruction instead and served as a mechanism for Ms. Duncan to steer lessons in a particular direction. As seen in Table 4.1 over the course of five observations Ms. Duncan made 22 attempts to elicit from students an explanation or justification with 50% of the elicitations occurring during Observation 4, indicating this was not a regular practice for Ms. Duncan.

Although scant in number, it is important to examine the instances of explanation and justification to uncover the social norms such practices fostered. The following exchange took place during a lesson on representing fractional parts of a set. The objective of this lesson, as stated in Ms. Duncan's lesson plan, was for students to "identify fractional parts of sets or groups and divide sets to show fractional parts" (Observation 1, p. 1). This was a whole group lesson with students working independently answering questions posed by Ms. Duncan's. Each student had an individual white board where they would write down their answers to Ms. Duncan's questions and hold it up for her to evaluate. Ms. Duncan began the following exchange with a request for an explanation as to how students would go about drawing a set to represent 5/8 of the set as suns.

1. T: Another one! And, this will be a springtime picture. I would like you to draw any springtime objects you would like but for your picture and your set I will require you to make let's see – okay – I would like five eighths of your set to be suns. And you can have any other springtime object other than that. So, you could have trees, flowers, birds, bees, butterflies. Five eighths must be suns.

I am interested in how people's brains are working. Does someone want to raise their hand and explain what they did first? If you knew that you were going to be drawing five suns, did you draw all five suns first? Or, did you draw the other objects first or did you make a sun and then another object. Tell me if people did a certain order with their work.

2. S: I did a sun, then a flower, and then a butterfly.
3. T: So you did a sun, then a flower, and then a butterfly. So you're going to have to keep counting as you go along to add suns. So you might need to do some subtraction here. (Observation 1, p. 14)

In Turn 1 Ms. Duncan began this exchange with a very detailed description of what she would "require" from her students. In the second paragraph of Turn 1, Ms. Duncan indicates she is requesting an explanation when she says, "Does someone want to raise a hand and explain what they did first?" However, before nominating a student to respond, she explains several ways that students could have approached the problem when she said, "Did you draw all five suns first or did you draw the other objects first or did you make a sun and then another object?" She, in effect, gave several explanations before offering students an opportunity to explain what they had done. Moreover, her initiation for an explanation placed the emphasis on steps when she said, "Does someone want to explain what they did first?" rather than on eliciting an explanation of student thinking or understanding.

When a student responded in Turn 2 with a method for drawing the set, Ms. Duncan accepted what the student said by repeating it, then commented that the student would need to use subtraction to solve the problem: "So you're going to have to keep counting as you go along to add those suns. So you might need to do some subtraction here." In this exchange we do not know how the student approached the problem or how the pattern was ultimately used to represent a set with 5/8 suns. Moreover, from the student's response, we do not know if she used addition or subtraction to figure out how to draw the set.

This was a missed opportunity for Ms. Duncan. In this exchange she could have encouraged the student to explain how drawing a sun, a flower, and butterfly helped her to represent 5/8 of the set as suns. Moreover, by not asking the student to explain further, she may have unknowingly de-legitimized the student's method by suggesting that the student needed to do subtraction because, from the student's explanation, it is not evident that subtraction was used to draw the set. Consequently, Ms. Duncan may have inducted the whole class into the idea that this type of problem required subtraction, thereby de-legitimizing any other method that could have been used. I conjecture that this interaction was a move by Ms. Duncan to ensure that her students followed a certain procedure when completing this type of problem.

The following exchange took place immediately after the aforementioned one (Observation1, p. 15). In this exchange Ms. Duncan further inducted the students into the idea that subtraction is the method of choice for this type of problem. Here she attempted to elicit a justification for subtraction as a means to steer the direction of the lesson.

1 T: Why would you need to subtract – can you think why you would you need to subtract here?
2 S: Because if you have five suns…
3 T: I need more people to listen to this – because this guy is going to explain something really important – go ahead.
4 S: You gotta have five suns but you gotta have three different objects.
5 T: Exactly. So what you did to say there is three different objects that aren't suns is you had to do eight take away those five suns. So he did his whole set and he took away the amount he knew that was going to be suns and how many were left over is what he wants to draw for extra objects.

In this exchange Ms. Duncan attempted to elicit a student justification when she asked in Turn 1, "Why would you need to subtract?" but analysis reveals that the intent was not to bring the student's justification into public view, but the intent was to steer students toward the operation of subtraction. When the student responded in Turn 4 to Ms. Duncan's initiation with, "You gotta have five suns, but you gotta have three different objects," Ms. Duncan followed up with an evaluation in Turn 5, "Exactly," and proceeded to explain to the class what the student had done. Ms. Duncan took the student's white board and erased his work and continued with, "So what you did to say there is three different objects that aren't suns is that you had to do eight take away those five suns," and wrote 8 – 5 = 3 on the board. Ms. Duncan used this opportunity as an instructional one rather than an opportunity to elicit a student explanation/justification and in doing so missed an opportunity to bring the student's thinking into the public discourse space to be considered by peers. Moreover, Ms. Duncan made it clear to students that subtraction was the method to choose in this situation when she said in Turn 5, "You had to do eight take away those five suns." In this exchange, although Ms. Duncan attempted to elicit a justification, she effectively moved the situation from one of justification toward one of instruction.

In the next exchange Ms. Duncan was introducing the lesson on the meaning of multiplication. Here, in the introduction, we can see how Ms. Duncan created situations for instruction rather than situations for explanation or justification. Although Ms. Duncan began the lesson saying she was interested in what students were thinking and wanted them to explain how they would solve 3 x 32, analysis revealed the intent of her initiations were clearly instructional. This exchange highlights the way in which Ms. Duncan set up the lesson so that instruction rather than student explanations would be the focal point (Observation 4, p. 1).

1. T: So, we're going to start by refreshing our memory. Okay. The first question I have and it's a point question is: What is the name of the answer in multiplication? What is the name of the answer in multiplication?
2. S: The product.
3. T: The product – so anytime we're doing this it's to find the product. So as soon as we add the equals sign it tells us we're looking for a product or an answer. And those words are synonyms meaning the exact same thing. So your product, Amy, is just your answer. Now, what I want to talk about today is ways to multiply. What multiplying really means. And, I want to see how well you have this idea in your brain.

   So think back to multiplication when we made this sign and I'm going to let this sign perhaps refresh your memory just a little bit. This is kind of our synonym sign for multiplication. And that synonym sign shows us all the different meanings that this one symbol can carry. So go ahead and just take a minute to refresh your memory and read that sign again before you raise your hand to answer my next question.

   So, just look at it. Look at those different meanings because I am going to take it down in a minute. And give you kind of a quiz on it. Okay. Now, after looking at that multiplication poster, and kind of bringing all of those ideas back to the front of your brain, I want you to look back at today's poster. Look at these two multiplication products - multiplication problems. How would you solve these problems? And when I ask that I don't want to tell me how I would set it up to solve it, I want you to explain to me what it means to do this? You're looking at 3 times 32 – so what does that mean to you? What would you do to get that answer?

In the aforementioned exchange Ms. Duncan began the lesson in Turn 1with a "point question," signaling to her students that they would be rewarded for a correct answer to her question. Ms. Duncan displayed a multiplication poster that showed several different equivalent meanings for multiplication (groups of, repeated addition) so that students could refresh their memories as to "what multiplying really means." By using the poster Ms. Duncan had predetermined for her students the different ways in which they should be thinking about multiplication. Moreover, Ms. Duncan went on to imply in Turn 3 that the purpose of multiplication is to find a product and the purpose of the day's lesson was to see how well they had memorized the different meanings that the multiplication symbol carries. Again, in the third paragraph of Turn 3, Ms. Duncan reiterates the importance of getting the information into their brains and ups the ante by stating that she would give students a quiz on it later. And, her last statement, “What would you do to get the answer?” alerted student to the fact that her overarching objective focused on obtaining a correct answer rather than developing proficiency in explaining and justifying their mathematical ideas. It is also important to note that in this one exchange Ms. Duncan uttered three hundred and thirty one words to two words uttered by a student as it reveals the dominating voice in this classroom was Ms. Duncan’s.

The following exchange took place shortly after the previous one and highlights the tight control Ms. Duncan had over her lessons and how such control served to hinder her ability to effectively elicit explanations form her students.

1. T: So that’s one suggestion or idea. What else? What else do we do with multiplication?
2. S: 32 times 3 gives you 32 three times.
3. T: And by that do you mean a different operation than multiplication?
4. S: Yes.
5. T: What operation?
6. S: Addition.
7. T: Addition, he says. So when he says you can do 32 three times he means I can take 32 plus 32 plus 32. And that would give me the same answer**.**

Ms. Duncan began in Turn 1 with an initiation suggesting that there are specific things “we do with multiplication.” This signaled to her students that she was looking for something particular rather than signaling that she wanted to understand how they were thinking. Moreover, her question suggested a correct answer was being requested. In Turn 2 a student replied to her initiation with, “Thirty two times 3 gives you 32 three times.” Ms. Duncan’s follow up move in Turn 3, “And by that do you mean a different operation than multiplication?” effectively took the focus off of the student’s response and directed the conversation toward the concept of addition as repeated addition.

Recall that at beginning of this lesson Ms. Duncan had displayed a multiplication poster showing several different equivalent meanings for multiplication (groups of, repeated addition) so that students could refresh their memories as to “what multiplying really means.” Repeated addition was prominently displayed on the poster, thus the poster serves as a means to instruct students as to how to respond to her questions. Ms. Duncan wants the method of repeated addition to be brought into the conversation; however, the student’s explanation of “32 times 3 gives you 3 three times” did not explicitly indicate repeated addition. Rather than asking the student to explain further, Ms. Duncan asked a question with a known answer (QWKA) in Turn 5 to which the student responded with a correct answer in Turn 6. The focus is on the idea of multiplication as repeated addition rather than on an explanation of what the student was thinking when he said, “32 times 3 gives you 32 three times.”

In the above example Ms. Duncan missed a valuable opportunity to elicit a meaningful student explanation and instead placed within the discourse space the methods she wanted students to be attuned to. If she had probed further and asked the student to explain what he meant when he said “32 times 3 gives you 32 three times,” it would have given prominence to his voice and quite possibly could have brought out the concept of multiplication as being equivalent to repeated addition. Instead, Ms. Duncan orchestrated the conversation in such a way that she became the explainer of the student’s idea and put the student in the role of passive listener of his own idea.

In the following exchange Ms. Duncan attempts to elicit an explanation regarding the relationship between multiplication and addition. Although she does elicit an explanation, she does not follow up on the student's response but uses the opportunity as an instructional one rather than one that would develop her students' ability to offer reform-oriented acceptable mathematical explanations (Observation 4, p. 2). This exchange took place immediately after the aforementioned one regarding 32 x 3 being the same as 32 three times.

1. T: Okay, so what does that mean about multiplication?
2. S: It means multiplication and addition are best friends.
3. T: Yes, and I am going to be adding up these points for people giving great answers. I have one point for Jami; I've got one for Raymond; and one for Cameron. Thank you guys. So. Because multiplication and addition are best friends because multiplication and addition are best friends, we could switch one for the other.

In this exchange Ms. Duncan began by attempting to elicit an explanation of the relationship between multiplication and addition; however, she accepted an answer in which the relationship between the two operations was never addressed. She let the group know that she considered the student's answer an acceptable mathematical one, rewarding the student with a point for his answer. The students' response in Turn 2 alludes to a memorized answer rather than an explanation of the relationship between multiplication and addition, thus we do know exactly what the student means or understands about the relationship between multiplication and addition. Ms. Duncan could have asked a follow up question as a means to develop the student's ability to offer an explanation that addressed the mathematics of the problem. Instead, she follows up by giving more weight to points for good answers than instead of addressing the student's response. Moreover, Ms. Duncan ended the sequence by stating that multiplication and addition are “best friends” and could be switched for one another. Her statement does not mathematically address the concept of multiplication as repeated addition, thereby leaving the students to infer what she meant by her statement, “We could switch one for the other.” Moreover, “although multiplication *is equivalent* to repeated addition multiplication is not repeated addition” (Jacobson, 2009, p. 69), thus stating that one switching for the other could be problematic and could possibly lead students to develop a mathematical misconception.

Although Ms. Duncan developed her lesson around eliciting student explanations and conceptual understanding, analysis reveals that her implementation was controlled and traditional in nature and rarely resulted in a productive student explanation. Consider the rationale that Ms. Duncan wrote in her lesson plan for Observation 4:

> This investigation is taught to reinforce the conceptual idea of multiplication before having to teach a technical algorithm to multiply with greater numbers. This helps students grasp the underlying idea of what multiplying is, while sharpening their strategic reasoning skills and their ability to find a solution without using an algorithm. If students understand what it means to multiply, they can never get the wrong answer! (Lesson Plan, June 11, Observation 4)

Ms. Duncan’s intentions, in writing, were reform-oriented; seeking to help students understand the conceptual underpinnings of multiplication; however, her actual implementation of the lesson was tightly controlled and focused on instruction. Correct answers, rather than developing students’ conceptual understanding of multiplication, became the focus. Moreover, her attempts to elicit explanations and justification did not focus on how students solved 32 x 3 or on how they understood the operation of multiplication. Later on in the lesson, Ms. Duncan asked her students to choose one of the methods listed on the poster and solve 32 x 3. Again, this was a very teacher directed lesson in that students were instructed first how to solve the problem and then given the opportunity to choose a set method of solution. We will return to this particular lesson again when analyzing the socialmathematical norms constituted by such teacher directed practices.

As seen in Table 4.1 Ms. Duncan made 22 attempts to elicit student explanations and/or justifications; however, analysis reveals that the attempts were controlled and traditional in nature. The exchanges above indicate that, at times, Ms. Duncan did elicit students' explanations and justifications; however, she did not use the elicitations as opportunities to bring students' thinking into the public discourse space as recommended and advocated by mathematics reform. Rather, her actions are instructional and do not effectively help to create situations where student's explanations and justifications were made public. When Ms. Duncan reviewed one of her videotaped lessons she reflected on her ability to elicit student thinking she said,

> The idea exchange that transpired during the review was mainly me reminding the class what brought us up to the day's lesson, and when I called on students, it seemed like I was looking for an almost 'memorized' set of rules, tips, or reminders. During this lesson, I almost NEVER encouraged the students to share their ideas with one another (Lesson Reflection, p. 1)

Ms. Duncan was surprised when she realized that she rarely asked students to explain their thinking because she believed that sharing was an integral part of her teaching practice. In an interview Ms. Duncan shared that she felt it was important to develop her student's abilities to share and articulate their thinking to each other indicating that her beliefs in theory do not match her beliefs in practice. Consider the following quote in which Ms. Duncan was asked to describe a social norm that she develops in her classroom:

> I think it's important for them to be able to articulate themselves and verbalize their thoughts or questions on their "aha" moments. So I think a norm is definitely acceptance from every student in the room of whatever anybody else has to say. So they need to have that in order for us to have conversations, and things like that, in the classroom. So I think that's important. And I also think explaining and questioning are huge in that. So I try to do less direct instruction and less telling them and more asking questions and letting the questions kind of create the pathway towards what I'm trying to get them to understand. So I use questioning, reflection, and a lot of student models and student examples, so I try to let my students teach each other when possible and learn from each other and review from each other (Interview, p. 4).

As stated previously, Ms. Duncan only asked for an explanation or justification in the follow up spot four times. This happened when Ms. Duncan requested an explanation from a student who had already provided her with a correct answer to her initial question. Upon analysis of these exchanges it became apparent that before Ms. Duncan asks for an explanation or justification as a follow up move, she first needed to provide the student with a bridge in the form of a positive evaluation so that the student could step into the unfamiliar territory of explanation and justification. The following three exchanges reveal how Ms. Duncan bridges her students before eliciting an explanation in the follow up spot.

The following excerpt was taken from the second observation on May 1, 2008 (p. 2) as Ms. Duncan and her students were continuing their work on fractions and sets. What is interesting about this exchange is that although Ms. Duncan elicited a justification, she first informed the student that he was correct in his answer .

1. T: So there are two left. Okay. So this is my whole set of students right here. My whole set. It shows me that there are a couple left. So what does that tell you if we were to break these into a couple of pieces and give people more? Will they have more than one bar or less than one bar?
2. S: More than one bar.
3. T: Manuel is right. And, Manuel, how do you know it is definitely more than one?

4. S:If you have one bar and you break it and give them each a piece they will each have more than one bar.
5. T: And now, Manuel has introduced the lesson perfectly.

This exchange was very different from most of the other exchanges that ensued in Ms. Duncan's classroom. In this instance, Ms. Duncan first asked a QWKA in Turn 1 to which a student gave a correct answer in Turn 2. Next, in Turn 3, Ms. Duncan probed further and asked the student how he knew that the answer was more than one bar, thereby effectively eliciting a justification rather than just a correct answer; however, before she asked him to justify his answer, she prefaced the request with, "Manuel is right." This is significant in that correct answers dominated the landscape of this classroom, and the students and Ms. Duncan were all well aware of this fact. Without first indicating that the answer was correct, the student may have inferred that the answer was incorrect, thus inhibiting him from continuing with the conversation.

Because Ms. Duncan rarely asked for explanations or justification, I conjecture that she was compelled to preface her request with a positive evaluation before she could ask Manuel to justify his answer. Had Ms. Duncan just asked Manuel, "How do you know it is more than one?" he may have interpreted this request as questioning the legitimacy of his answer rather than as a request to explain or justify his answer. Supporting this conjecture, I present the following exchange that took place on May 1, 2008 during Observation 2 (p. 6) in which Ms. Duncan again elicited a justification prefaced with an evaluation of the student's response.

1. T: How many pieces of hearts are there? Amy?
2. S: One half.
3. T: One half yes! – and how did you know that this was one half, honey**.** How did you know it was going to be a "2" denominator?
4. S: Inaudible

5. T: Good girl. Well you were thinking of looking at the shape and just by looking at it you can tell if the other piece were there you'd have how many equal parts.

Although this affirmation was not as direct as the previous one, the tone of Ms. Duncan's voice was positive and her affirming, "Yes," and smile indicated to Amy that her answer of one half was correct. I conjecture that these affirmations serve two purposes. First, they serve to constitute the norm that responding with a correct answer is important in this classroom. These particular discussions illuminate for all members of the community that the goal of mathematics is to have correct answers and correct answers please the teacher. Second, they serve as a bridge allowing students to venture into new territory – that of explaining or justifying their answer. Analysis of the five classroom observation indicates that the practice of eliciting correct answers is consistent and paramount across all of the lessons. Moreover, as stated previously, Ms. Duncan's goal was to help her student answer questions with success, and she expressed concern that allowing students to share their thinking might cause confusion. Thus, when Ms. Duncan attempted to elicit an explanation or justification she had to offer a bridge if she wanted her students to successfully respond to her elicitation.

The practice of affirming the correctness of an answer before eliciting an explanation or justification fits with the goal of developing students' abilities to answer questions with success. Ms. Duncan needed to let her students know that they had answered a question successfully before asking them to venture into the unfamiliar territory of explanation/justification. Analysis revealed that there were a few instances when Ms. Duncan did not, at first, offer a bridge in the form of a positive evaluation before attempting to elicit an explanation or justification, resulting in student hesitation until a bridge was offered. Consider the following exchange that took place on June 10, Observation 3 (p. 10) as students were involved in a lesson on probability. In this exchange Ms. Duncan is referencing a homework problem the students had completed the previous evening.

1. T: Right – it was six pennies in a cup and they asked what was least likely. Was it the least likely to have 5 tails and 1 head, 1 tail and 5 heads, 3 tails and 3 heads or 6 tails? Which one was the least likely?
2. T: Which one was the least likely?
3. S: Six tails.
4. T: Why?
5. S: Because it is least likely?
6. T: You're got the right answer. Can you compare it to this James? It is least likely because it is 6 tails. That would mean it all came out to be one thing. That would kind of be like Jacob's here. I didn't say that Jacob was wrong; did I tell you were wrong today? No. I was watching you and you really were just getting heads over and over and over again. But that's what I said. Is it likely that that will happen every time you flip coins? Not likely. So this is very unlikely but it is still possible, but it yeah, it's possible that maybe you can get all those 6 coins if all 6 of them will be tails?

Because Ms. Duncan did not, in Turn 3, offer a positive evaluation of the student's answer, he was hesitant to give an explanation and his response in Turn 4 was in the form of a question rather than as an explanation. Picking up on his hesitation, Ms. Duncan realizes that she needs to bring him back into the discourse by offering him a bridge and provides him with an evaluation of his original response in Turn 6 when she said, "You've got the right answer." However, in this case Ms. Duncan does not ask James to continue explaining or justifying his answer of six tails in favor of taking over the explanation for him. A social norm that has taken root in Ms. Duncan's class is that before attempting to explain or justify a solution, students need to be assured that their answer is correct.

Although eliciting explanations and justifications was not a practice Ms. Duncan had adopted, she believed that getting students to explain their thinking was very important; however, for her this happened more in writing than in open classroom discussions. Moreover, Ms. Duncan believed that her students never became comfortable with taking risks in her classroom, thus asking them to verbally explain their thinking was a difficult task. She said,

> I think it would be great if there was a little bit of an increased comfort level for risk-taking, because I know that in my classroom this past year, I had some students with amazing higher level mathematical reasoning skills. And to use that and to let them teach was really great and I just wish that I could have kind of instilled a little bit more risk taking, because what I was getting back on paper was really, really great and I would continue to share that with the kids. But the students, I don't think, were as good at defending their answers verbally I guess I'd say that I think that our program's a little bit more based toward explaining in writing. Does that make sense? (Interview, 6/25)

Here Ms. Duncan revealed that the math program her school has adopted placed more emphasis on written communication and less on verbal communication. Moreover, Ms. Duncan believes she would need to do much more up front work in order for her students to feel more confident with oral expression of ides. When asked what she would need to do to foster more communication in the classroom she said,

> And if I did a lot more modeling of getting the kids to verbally explain and justify their answers, and then maybe at the same time be writing as well so that we can kind of do those two things in correspondence, I think that might kind of strengthen the skill. But as of right now, in reflection, I think I am really focused more as a teacher on getting them to write to explain as opposed to getting them to verbally explain it. I think I just saw it as more valuable for testing and things like that. (Interview, 6/25)

The pressure of standardized testing seems to be inhibiting Ms. Duncan from implementing the practice of eliciting explanations and justification from her students because, as she sees it, written explanations are more valuable for testing purposes than are verbal explanations. There are competing forces influencing Ms. Duncan when it comes to fostering the reform-oriented social norm of eliciting student explanations and justifications. On the one hand she believes that it would be productive for her students to have opportunities to share their mathematical ways of reasoning, yet on the other, she feels compelled to control the classroom discourse so that students do not learn the wrong information; consequently, productive mathematical conversations never take shape.

Ms. Duncan also realizes that her students do not take risks, and she wishes she had spent more time instilling in them a more risk-taking attitude so that they would have developed the skill of explaining and justifying their thinking. There is evidence to support Ms. Duncan's conjecture that her students are not risk-takers. Consider the following exchange that reveals one student's struggle to take a risk when called upon to answer a question.

1. T: Nathan – tough question. What is 1 x 10?
2. S: [Nathan does not respond to the question]
3. T: One group of 10. I'm not checking you – don't worry. If I had one group with 10 in it, how many do I have? It's not a trick, just say the number. You can't mess it up – just count by 10 one time. What's the first number you say? Start him off guys. How do you skip count by 10s? Ready? Go ahead.
4. S: 10-20-30-40-50
5. T:He's doing it. Good job Nicholas. Nicholas what is the first number you said – 10x1 – one group of 10. Let's go ahead and just use a symbol. I have one group of 10. Show me 10 fingers. Do you have any more groups of them?
6. S: No
7. T: Okay, so how many are there?
8. S: One
9. T: Okay – well not how many groups – how many fingers?
10. S: Ten?
11. T: There you go. Easy enough. Good job Nathan. Nathan did what one of our students did yesterday where he made it harder than it really was

> because I was asking the question. Now let’s keep going skip counting by 10s – okay.

In this exchange Nathan struggled to take a risk because he has learned that his job in conversation with Ms. Duncan is to provide a correct answer and not provide an explanation as to what he is confused about or thinking about. It seems that Nathan does not understand what Ms. Duncan is looking for and rather than taking a risk and possibly providing an incorrect answer, he remains silent while she tries unsuccessfully in Turn 3 to elicit a response. Several times she mentions that he cannott mess up and that she is not trying to trick him, indicating that there are times when a student can mess up and that sometimes she does try to trick them. When Nathan finally responded in Turn 8 with an answer, Ms. Duncan in Turn 9 evaluates his response as being correct but not what she had wanted. Nathan finally in Turn 10 provided a correct answer to which she replied, “There you go - easy enough!” This was not an easy exchange for Nathan, and although he finally after 10 turns provided a correct answer, there is no evidence to suggest that he developed a deeper understanding of the concept of 10 x 1 than before the exchange began.

Ms. Duncan maintains such tight control over conversation that it is very difficult for students to take risks and offer explanations regarding how they were thinking about a particular questions or problem. Consider the comment Ms. Duncan made during the focus group interview when asked to describe some core practices that she utilizes while teaching mathematics.

> I really believe in the power of student leadership in learning and for me a practice is definitely having complete control and complete attention in order to teach. So that comes first for me and then questioning. I think questioning is really important to practice for me in math. And just that idea of like, you know, children being able to take risks to share, children leading their learning and

> answering questions for each other and, you know, helping to just scaffold, you know, their peers. (Focus Group Interview, p. 19)

According to Ms. Duncan, she believes “in the power of student leadership” but “having complete control and complete attention” is her most important practice. Here Ms. Duncan acknowledges a conflict, however unknowingly, between her beliefs and her practices. Ms. Duncan’s need for control is evident in the way in which she adapts the practice of eliciting student explanations in favor of taking over the role of explaining student thinking. The following section will examine how Ms. Duncan controls the classroom discourse by adapting the practice of eliciting student explanations by providing teacher generated explanations of students thinking.

Explained Student Thinking – An Adapted Practice

Ms. Duncan seems to understand the importance of explanations and justifications in fostering student understanding; however, she adapts the practice by providing detailed explanations of students’ thinking rather than letting students engage in such discourse. As seen in Table 4.1 there were 66 instances where Ms. Duncan supplied an explanation of a student’s response as opposed to asking students to explain their mathematical ideas and conjectures.

In this next exchange we go back to the lesson when students were working on solving 32 x 3. Cameron had chosen unifix cubes, from the list of strategies written on the chart paper, to use to solve 32 x 3. Ms. Duncan invited him to the front of the room to show to the class what he had done. As seen in the exchange below, rather than let Cameron explain to the class how and why he used cubes to solve the problem, Ms. Duncan takes over that role and explains for the class what Cameron had done.

1. T: Cameron can you bring your finished array forward? Cameron used cubes to solve the problem 32x3. And this strategy is exactly the same on paper as it is with cubes. Do you see how that works? So if you don't have cubes are you completely lost? All you need is a pencil and paper and you can draw your objects. Which was more fun Cameron drawing or building?
2. S: Building it.
3. T: So what we did was we took – we knew that the link cubes we play with everyday at recess are organized in strips of how many?
4. S: Ten
5. T: Thank you for raising your hand. Okay - So what we did is we took three 10s, right Cameron, and then we added two extras. So, over here we used sets of 5, but right here it was easier to count in sets of ten. So let's go ahead and count those. I think we might have one that is off by one. So, let's see. We have 10, 20, 30 …this one has one there we go. So now each has 30 tow right. So what we can do is we can count and if you notice the extras are what color?
6. Ss: Black
7. T: They are black. Okay. We are going to skip count and we are going to skip count by 10 to do the rest of them (Observation 4, p. 9).

What is concerning in this exchange is that although Ms. Duncan attempted to bring Cameron's thinking front and center by inviting him to show the whole class what he had done. Cameron only uttered 3 words to Ms. Duncan's 184 words; thus, she was the explainer of his solution, and Cameron became a passive listener. When watching the video clip, Cameron is seen standing next to Ms. Duncan swinging his arms and looking around the room as she explains how he used the cubes to solve 32 x 3 to the rest of the class. Moreover, Ms. Duncan uses the pronoun "we" thereby taking part ownership of Cameron's solution. This is significant in that, again, Ms. Duncan is positioned in a place of authority even when considering Cameron way of solving 32 x 3. In the above exchange the only part of the solution that Cameron is asked to comment on is to tell the class which was more fun building or drawing it. This was an opportunity for Ms. Duncan to develop Cameron's sense of autonomy; however, the opportunity was lost when Ms. Duncan took over for Cameron and explained his solution.

In the next exchange a student was sharing how he solved 32 x 3 by breaking apart 32 and multiplying 30 times 3 and 2 times 3. Again, rather than eliciting an explanation Ms. Duncan acknowledged the strategy as a good one and proceeded to explain what the student had done (June 11, p. 2).

1. T: What's one way George?
2. S: We can break down 32 into 30 plus 2
3. T: <u>Ahhhh.</u> He said we could break down 32 so we could make it into 3x30 and 3x2. That would be right. So what he said is that we could do an easier job. You could break apart the numbers – you could take the 32 and you know that it is separate because of those great addition properties that make the rules easy for us to follow. 30+2=32. So, it is okay to take that number and break it apart and multiply the part separately if there are easier numbers to work with – right?

   So let me show you what George means. He means to take the 32 and turn it into 30+2, then we could do 30 three times and 2 three times. That's a pretty easy job. Can you add 30 three times right in your head? What is 30x3? Right in your head. 30+30+30?

Ms. Duncan acknowledged in Turn 3 that she is pleased with George's strategy and then goes on to explain what George meant when he said that 32 could be broken down into 30 plus 2. George used 9 words in this exchange to Ms. Duncan's 184, thereby making her voice the dominant one in the explanation. Important to note in this exchange is when Ms. Duncan says in Turn 3, "So what he said was" and "So let me show you what George means," as these two utterances reveal that Ms. Duncan is in control of explaining how students solve problems.

In the following exchange Ms. Duncan attempts to elicit an explanation but again, a student explanation is ignored in favor of Ms. Duncan's explanation (Observation 5, p. 5).

1. T: Okay, Amy, what was the pattern you were naming, Honey?
2. S: All the numbers have three 0s except for that one.

3. T: Excellent. All of the numbers have three 0s, and she was great because she said, except the last one. Great job, Honey. So, here the reason that there's an extra 0 is why? Where do you see an extra 0? Where's the extra one? In this problem right at the end. Everybody else's answer has three 0s. She's right. But this one has one, two, three, four 0s. Why is that? Does anyone see an extra 0 on the other side of the equal sign that might balance it out? James, what do you see?
4. S: Because it has 10 times 100.
5. T: So are you looking at this 0, James?
6. S: Yes.
7. T: So, you're saying to me that if I took this 0, and I counted up all these together, I'd have one, two, three, four and on the other side of the equals sign to balance it out – I'd have one, two, three, four. So, Amy, that missing 0 came from the zero in the 10 – which is the only number here that is not a single digit number.

In this exchange Ms. Duncan attempts to elicit an explanation when she asks why there is an extra 0 in the last answer, however, before she calls on James, she steers the students toward the answer she is looking for when she says at the end of Turn 3, "Does anyone see an extra 0 on the other side of the equals sign that might balance it out?" In Turn 4 James offers a mathematical explanation as to why there was an extra 0 in the last answer, but rather than to ask James to explain further, Ms. Duncan only asks James to clarify if he is looking at the 10 in 10 x 100. When James responds affirmatively in Turn 6, Ms. Duncan proceeds to take over for James and tells the class what James had noticed. However, what James had noticed was deeper than just noticing there are extra 0s, as his comment refers to the multiplication of 10 x 100. Again, she misses an opportunity to probe James' comment further and bring what he is thinking about into the public discourse space for all to consider.

As evident in the aforementioned three exchanges, Ms. Duncan often takes over student explanations rather than allowing students to explain or justify their thinking and reasoning. Instead of eliciting explanations or justification from Cameron, George, and

James, Ms. Duncan opts to adapt the practice by taking on the role of explainer, and by doing so, puts Cameron, George, and James in the passive role of listeners. This practice effectively places Ms. Duncan at the center of students' ideas and as the mathematical authority n the classroom discourse.

During an interview Ms. Duncan shared that she often struggled with allowing students to explain how they solved a problem because she was not sure all students were ready to hear and process divergent ways of thinking.

> And my struggle with that is sometimes student's think at a slightly higher level but that level is out of reach for the other students, and I don't want to use that as a model, because if it works for that one brain, that's great. But it can make it more difficult for the other kids and more confusing, and so I do struggle with it. (Interview, p. 12)

As a means to alleviate such confusion, Ms. Duncan adapts the practice of eliciting student explanations by providing the explanation and/or justification for her students rather than risk having her students becoming confused by a student explanation. Analysis reveals that Ms. Duncan uses this practice as a means to control what is shared in the classroom. In her lesson reflection, she was surprised by the lack of student-to-student idea sharing. She said,

> Throughout the lesson, I encourage students to be active participants, but I viewed them mainly idea sharing with me, and not each other. During this lesson, I almost NEVER encouraged the students to share their ideas with one another. (Lesson Reflection, Observation 1)

And when a student solved problems in a way that might be difficult for her to explain she would ask the student to redo their solution so that it would make more sense to her. Consider the following lengthy comment Ms. Duncan made when she could not understand how to explain a student's solution.

> Let me see if there are seven shapes here. Okay. You're going to have to erase it. And you're going to have to draw something that makes a little bit more sense. Because I know what you're trying to do Bryan, and your brain does think outside of the box.
>
> Remember when Bryan did this and I said – what would the fraction be or how many 11s would you need to make one and the answer was 11 11s. But Bryan wrote on his whiteboard two – and we figured out that the reason he did that was because it does take two 11s to make one; there is one – two 11s in this fraction: 11/11.
>
> So you were right, I just needed to take my time to figure out what your brain was thinking, right? And that was what I was doing this time, and I think it could have been right, Bryan, but it wasn't carefully enough drawn. So, those extra lines made too many spots, but you got the idea, right? (Observation 1, p. 14)

In this instance Ms. Duncan could not understand what Bryan had done, and rather than ask him to explain, she asked him to erase his work and solve the problem in a way that made more sense to her. She acknowledged that Bryan sometimes thought outside of the box, but at the same time she let Bryan and the rest of the class know that thinking outside of the box is only acceptable when she can make sense of the solution. During an interview Ms. Duncan shared that she sometimes has difficulty understanding students who do things differently. She said,

> I have a student who does that for almost everything, and I have to say, it's kind of my instinct to be annoyed by it, but I think that's because I don't necessarily understand it myself, and it's taken a lot of really careful analysis of this child's work to figure out what he's doing that's making him get the right answer but solving the problem differently (Interview, p. 12).

Where does the instinct to become annoyed generate from in Ms. Duncan? When asked to reflect on this question, she revealed that her instinct to become annoyed stems from her own insecurities in mathematics. She said,

> I think being annoyed is only coming from my own insecurities or my own confusion, so if I'm teaching something one way and I've come to understand it that one way and someone does something different, it throws me off. So annoyed at the child, absolutely not, but annoyed at the fact that I have to take the

> time to learn an entire different way of solving this, yea. You know, just because I think it makes a little bit more work for you to be like, “Wait, what did you do?” And then you have to stop and think. (Interview, p. 12)

For Ms. Duncan understanding students’ divergent ways of thinking is difficult because she is insecure about her own mathematical ability; thus, when trying to figure out what her students are doing, she becomes confused. As a means to alleviate confusion for herself and for her students Ms. Duncan tightly controls the classroom discourse by taking on the role of explainer and putting her students in the role of passive listeners of their own ideas. When Ms. Duncan cannot explain how a student solved a problem she does not try to elicit explanation; rather, she obligates the student to ignore his own idea and do something that would make more sense to her and possibly to the rest of the class.

Ms. Duncan has harbored a fear of mathematics since her early elementary school years and considers herself to be weak in mathematics and strong in language arts. When asked during the individual interview to describe her experiences in mathematics she remembered math as being a “source of anxiety” and causing her much trepidation not only in school but at home as well. She said,

> I have to say academically math has never been my strongest suit. Learning math even as a child I remember math being a source of anxiety, and my father was a genius and wonderful at it, so he would always be the homework helper and was a little forceful, and I remember kind of not looking forward to math homework with him. (Interview, p. 1)

The experience with her father was a significant one for Ms. Duncan as she commented on it during the focus group interview as well again, mentioning that her father was a “genius.” She said,

> My father is a genius and when he would teach math to me when I was a child, he’d look at me like, “I just told you. Why don’t you understand?” You know, he couldn’t, he couldn’t conceptually understand why I didn’t get it on the first

> try. And I think that's probably how you would innately feel if you didn't, you know, have that more objective standpoint. (p. 7)

When reflecting on her early classroom learning experiences, she remembered specifically having trouble in third grade (which is the grade level she presently teaches) and feeling like a failure. She shared the following poignant memory:

> And when I got to the grade that I teach now, third grade, I had a really hard time learning my multiplication tables. And I think my perceived failure learning my multiplication tables actually kind of set up that whole fear of math as I grew older because I never really did poorly in it. I was never failing or getting low grades, but it was a struggle to even keep a "B" and that sense of fear continued on for me. (Interview p. 1)

Interestingly, because of her earlier experiences with math, Ms. Duncan believes she is a better math teacher in spite of her struggles with math. She believes that because math is not as easily accessible to her as language arts, she works harder at planning the lesson and breaking things down so that she has a concrete understanding of what she will be teaching. When reflecting on the notion of being a better math teacher because of her prior difficulty with math she said,

> As I told you before, as a student I am very language arts based. I am a writer and a reader, and those things come natural to me. But I am a better math teacher than I am a language arts teacher. And I have thought about this a lot and asked myself why? I think the reason is because math is not as accessible for me, and when I have to break it down into teachable terms for myself, I think it's easier to teach it in a more concrete way. When something does come more naturally to you, it is more difficult to articulate it or to access where that knowledge comes from – you just know. So for math actually I take every lesson, and I make sure I understand it and its concrete for me and I break it down into these really simplistic terms for myself and I think that helps the kids. (Interview p. 3)

Ms. Duncan believes that by breaking her lessons down into "simplistic terms" helps her students; however, analysis reveals that this practice inhibits Ms. Duncan from engaging her students in productive mathematical conversations. As a result of Ms. Duncan's insecurities in mathematics she overcompensates by planning every last detail of her lessons, making sure she understands the math concretely for herself before presently it to her students, thus leaving little room for productive mathematical conversations to emerge.

Ms. Duncan's sense of self as a learner of mathematics mirrors that of many elementary teachers (Ball, 1990). Her fear of mathematics is something she experienced early on in her educational career (third grade), yet its impact can be felt well into her college years when she changed her major from business to education because of a math requirement. Ms. Duncan's belief that she is not a strong math student is impacting her ability to adopt a reform-oriented teaching practice.

A question that arises here is that possibly Ms. Duncan's beliefs are warranted, and she does not possess a strong understanding of elementary math concepts. However, when analyzing results of the Mathematical Knowledge for Teaching instrument, it is evident that Ms. Duncan possesses a relatively strong understanding of the knowledge needed to teach elementary mathematics; however, an area of challenge was noted.

When analyzing the specific questions on the MKT assessment, Ms. Duncan is strongest in assessing student work, answering most of these questions with success. Ms. Duncan is also quite strong in representing mathematical ideas as she again answered the majority of this type of question successfully. An area of challenge for Ms. Duncan is found in explaining mathematical rules and procedures, as she was less successful in answering this type of question on the MKT (see Appendix E for samples of MKT released items).

Ms. Duncan was challenged when she was presented with rules and procedures in need of explaining, possibly causing her to maintain such tight control over the explanations that were shared within the mathematics classroom. If Ms. Duncan had difficulty with explaining mathematical rules and procedures, then she may have been challenged to understand the unconventional rules and procedures that students might use if she gave them the opportunity to explain and justify their solutions. As a result, Ms. Duncan chooses to provide the explanations and justifications possibly in an effort to address her own confusion with explaining rules and procedures. Ms. Duncan works hard at breaking down all of her lessons in ways that make sense to her. She said,

> Even though it's third grade math, I have to break it down into teachable terms for myself, and I look at every lesson, I make sure that I understand it, and it's concrete for me, and I break it into these really simplistic terms for myself, and when I do that for myself, I think it just comes out more audibly for the kids. (Interview, p. 4)

Possibly, because understanding conventional rules and procedures is not an area of strength for Ms. Duncan, she is challenged to open up the classroom discourse to student explanations and justifications. When teachers elicit explanations and justifications, they are obligated to try to make sense of how students are thinking and reasoning about mathematics. By maintaining tight control over the classroom discourse Ms. Duncan ensures that she does not become confused when working with students in the mathematics classroom.

Overall, Ms. Duncan did quite well on the MKT instrument, obtaining a score almost one standard deviation above the mean. This indicates that compared to the average teacher, her mathematical knowledge for teaching Elementary Number Concepts and Operations is strong; however, Ms. Duncan's perception of self as a mathematics learner is weak. This result suggests that a teacher's early school experiences in mathematics and the beliefs that such experiences instilled about ones' mathematical ability has far reaching effects and can impact the way in which a teacher is able to implement reform practices, regardless of a teachers' mathematical knowledge for teaching. Beliefs, in Ms. Duncan's case, overshadow her mathematical ability.

Ms. Duncan's early experiences in mathematics have had a formative impact on her confidence level in mathematics and, more troubling, in her ability to engage students in productive mathematical conversations. Ms. Duncan tightly controls what is shared in the classroom as a means to alleviate confusion for herself and for her students. To control the classroom discussion, Ms. Duncan relies very heavily on the traditional practice of asking QWKAs to stimulate conversations. The following section will examine this practice and its effect on the participation structure in the classroom.

Asked QWKAs and Evaluated/Accepted Student Responses

The traditional initiate, respond, evaluate (IRE) questioning pattern dominated Ms. Duncan's classroom discourse, and this pattern is predicated on the practice of asking QWKAs. Analysis reveals that the majority of conversations that Ms. Duncan initiates begin with a QWKA, and each required a correct response that was subsequently evaluated or accepted by Ms. Duncan. As seen in Table 4.1, Ms. Duncan asked 229 QWKAs. Moreover, Table 4.1 reveals that Ms. Duncan offerered150 evaluations of student responses and accepted 87 student responses. The accept move in this class has the flavor of an evaluation, with students understanding that the accept move indicates their answer is correct. These results indicate that Ms. Duncan's teaching practice follows the traditional IRE questioning sequence as described by Mehan (1979).

The following excerpt was taken from Observation 1 and highlights the IRE sequence dominating Ms. Duncan's teaching practice and how Ms. Duncan used QWKA to engage students in mathematics (see Figure 4.1). In this lesson the learning objective, as stated in Ms. Duncan's lesson plan, was for students to identify the fractional part of a set. Students were seated in various areas of the classroom with some students seated on the floor in the front of the room and others seated at their individual desks facing the front. In this exchange Ms. Duncan was setting the stage for the days' lesson, and she began with what would become a familiar IRE question and answer sequence (Observation 1, p. 1).

1. T: Alright, we're looking today at fractions and sets. I've showed you two different ways already to show fractions. One way was like this. We would draw a shape and the shape would be called a whole right? But then we would take that shape, just like multiplication and division does and we would make that shape divided into what kind of parts?
2. S: Equal.
3. T: They would be equal parts.

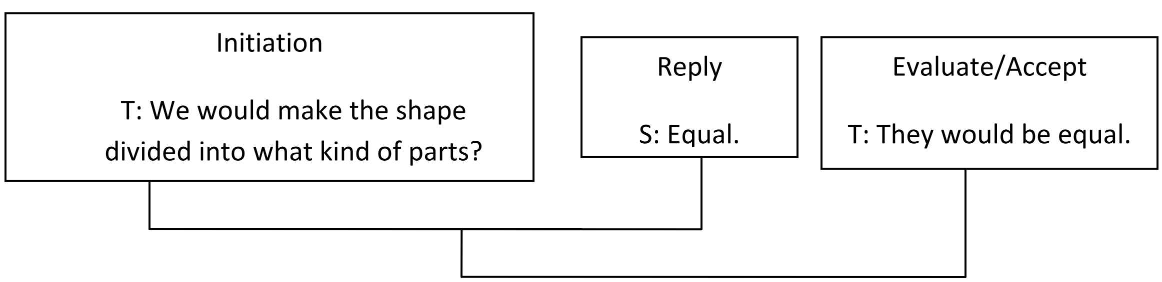


Figure 4.1 Example of an initiate, respond, evaluate (IRE) sequence: Ms. Duncan

Ms. Duncan began the sequence in Turn 1 with a QWKA, "We would make the shape divided into what kind of parts?" wherein a student supplied the correct answer in Turn 2, "Equal," prompting Ms. Duncan to follow up with the accept move in Turn 3, "They would be equal parts." Although Ms. Duncan does not, in this sequence, offer praise, such as "Good job" or "Correct," her matter of fact acceptance and her tone of voice indicated to the group that indeed the answer was correct.

The following exchange happened shortly after the aforementioned one and again reveals the IRE pattern that is dependent on asking a QWKA. Moreover, in this exchange Ms. Duncan offers evaluative feedback in the form of an "overt verbal evaluation" (Mehan, 1978, p. 286) affirming that the student's answer was correct. (Observation 1, p. 9)

1. T: So if I drew this rectangle what would my denominator be? Avery?
2. S: Ten.
3. T: Correct. So whatever it was I know my denominator is 10. Denominator is the easy part.

In Turn 1 Ms. Duncan begins the sequence by asking a QWKA and nominates Avery to answer. When Avery supplies the correct answer in Turn 2, Ms. Duncan follows up with an overt positive evaluation of Avery's response by uttering the word "Correct" in Turn 3, which effectively ends the sequence with Avery. In this sequence Ms. Duncan misses an opportunity to elicit an explanation from Avery as to why the denominator in this case is 10 or how she knew the denominator is 10. It is evident here that Ms. Duncan's main goal is to elicit a correct response to her question rather than to elicit a student explanation or justification of the response.

The following sequence is an example of an "extended sequence of interaction (Mehan (1979, p. 52). In this particular extended sequence, a student responded with an incorrect answer to Ms. Duncan's QWKA initiation. As a result, Ms. Duncan continues the exchange by asking several additional QWKA as a means to eventually elicit a correct response. (Observation 1, p. 2)

1. T: Because they're fourths does that number become the down denominator or the numerator?
2. S: Numerator.
3. T: The numerator? Think of what numerator says. What's the "N" in numerator remind us of?
4. S: North
5. T: North.
6. T: And north is …. Up right? (Student shakes head yes)
7. T: And what's D?
8. S: Down
9. T: Down. Down for denominator.
10. T: Now which number – the numerator – north or the denominator – down tells us the whole number - the whole number of equal parts in the shape?
11. S: Denominator
12. T: The denominator, you said? That's correct.

In this exchange Ms. Duncan initiates with a QWKA in Turn 1 for which an incorrect answer is given in Turn 2. As a means to encourage the student to respond with the correct answer Ms. Duncan follows up with a prompt in Turn 3, "Think of what numerator says," and extends the sequence by asking another QWKA, "What's the N in numerator remind us of?" The purpose of this question is to guide the student toward the correct answer of "denominator." The student answered correctly in Turn 4 with "North," to which Ms. Duncan offered an affirmative evaluation in Turn 5 by repeating what the student had said. In Turn 6 Ms. Duncan offers a second follow up QWKA, "And North is up right?" to which the student, being cued by Ms. Duncan, responds with an affirmative nod of yes. In Turn 7 Ms. Duncan asks another QWKA, "And what's D?" again with the intent to guide the student toward the correct response to the original initiation. The student responded correctly in Turn 8 with "Down" to which Ms. Duncan offers an affirmative evaluation in Turn 9 by repeating the word down. Finally, in Turn 10, Ms. Duncan goes back to her original initiation, however, clarified and expanded, "Now which number – the numerator – north or the denominator – down tells us the whole number - the whole number of equal parts in the shape?" and the student responded correctly in Turn 11 with "Denominator." Ms. Duncan ends the sequence with a final affirmative evaluation, thereby acknowledging that denominator was the correct answer.

The above is an example of how Ms. Duncan and her students interact in their day-to-day mathematics lessons. The practice of asking QWKAs develops the social norm that the teacher asks the questions, and the students supply the answers. Moreover, when QWKA dominate mathematical conversations, the participation structure becomes one of quick question and answer exchanges centered on correct answers, albeit at the expense of examining, developing, and bringing student thinking into focus. When reflecting on this particular lesson, Ms. Duncan was quite concerned with the ways in which she engaged students in mathematics, and she was especially concerned that her practice seemed to ignore student understanding. She wrote,

> Reflectively, I can see myself *stating* math lessons, as if they are what they are, black and white, set in stone. I felt guilty watching myself act like a lawyer "leading the witness" to say exactly what she wishes. I felt guilty and concerned because it didn't seem like I always gave my students the wait-time they needed to think on their own. (Lesson Reflection, p. 1)

Although in the above exchange Ms. Duncan follows up with embedded questions, they were all QWKAs, and the purpose of each was to lead the student toward the correct answer to her original QWKA. Although the student in the end was successful in supplying the correct answer, there is no evidence to suggest he understood the difference between the numerator and the denominator. In reflection Ms. Duncan was most surprised and concerned with her questioning. She said:

> My questioning was by far my largest surprise and concern. I'd like to say that this is not always my questioning method, and I do believe that I am growing to ask deeper questions; I have used Bloom's taxonomy to help deepen student

> thinking through questions. Still, I find that of all of the subjects I teach, it is math where I have the most difficulty asking questions that elicit deeper student thinking. (Lesson Reflection, p. 2)

When the participation structure in the classroom is dominated by QWKA, the types of follow up moves that teachers rely on are often evaluative in nature and serve to affirm or disconfirm a student's response. As seen in Table 4.1, Ms. Duncan's second most utilized practice was that of evaluating student responses followed by accepting student responses. This use of evaluations and acceptances made for a fast paced question and answer participation structure and left little room for exploration and examination of student thinking. Because Ms. Duncan relied on asking QWKAs in conversation with students she was, in a sense, obligated to evaluate their responses.

Summary

This section described the practices that Ms. Duncan used to "initiate and guide the development of social norms" (Yackel & Cobb, 1996, p. 460) within the classroom as she attempts to engage students in mathematical conversations. The most prominent practice is that of asking a QWKA, followed by evaluating and accepting student responses. Ms. Duncan rarely elicits from students an explanation or justification and when she does, analysis reveals that Ms. Duncan controls what students share. Ms. Duncan never attempts to make her student's thinking public because she is concerned that if she allows students to publicly share their thinking, it could cause confusion for her and her students. Moreover, there is no evidence in the observations to suggest that students are obligated to listen and attempt to understand a peer's solution, indicating this important reform-oriented practice is ignored.

The practices that Ms. Duncan uses constitute social norms that are traditional and are interactively established by Ms. Duncan and her students as they engage in mathematical conversations. As Ms. Duncan and her students engage in mathematical conversations, their interactions guide and constrain each other's activity and such interactions constitute the social norms within the classroom setting (Cobb, Wood, Yackel, & McNeal, 1992). Ms. Duncan's practices foster a participation structure whereby Ms. Duncan is obligated to transmit mathematical information to students and consequently students are obligated to passively receive the information she transmits. The dominant practices that Ms. Duncan uses and the social norms that regulate the participation structure in her classroom are summarized as follows:

- Teacher ask the questions and students' are obligated to provide correct answers.
- Teacher evaluate all student responses, making her the sole mathematical authority.
- Teacher provides students correct explanations and justifications and procedural steps to solve problems.
- Students practice what they learn from the teacher.
- Students are expected to listen for management reasons.

The practices that Ms. Duncan relies upon creates a participation structure that is very traditional in nature and serves to turn conversations about mathematics into quick questions and answer sessions where correct answers rather than productive mathematical conversations become the norm. When Ms. Duncan reflected on one of her videotaped lessons she too noticed the pervasive question and answer pattern noted here. When reflecting on her questioning practices she wrote,

> I was shocked to watch myself encourage ALL correct responses throughout my review and group lesson. Again, it was as if I saw the lesson as a prewritten script where I needed to evoke the exact words from children to make my modeling and example pieces fall into their prescribed places. I was hinting to students, motioning to answers, and just about telling them what to say. I would then positively reinforce. I left the directions, vocabulary, and

> samples on the board to assist students even during independent practice. I appear to be looking for memorization of what I had previously taught, as well as the ability to follow the necessary steps for the day's lesson and problems. (Lesson Reflection, p. 2)

In the quote above Ms. Duncan describes her teaching very accurately, and it is clear that when presented with the opportunity to reflect on her teaching practice she notices the same pattern of discourse that is revealed in analysis. When contemplating what might be prompting such a discourse practice she wrote,

> It's as if I have the fear that if students hear the wrong thing or get confused, they will learn the wrong way, then have to un/relearn, or become more confused and mixed up by what another child said. (Lesson Reflection, p. 2)

Ms. Duncan's concern that her students might hear the wrong information and/or become confused stems from the pressure she feels working in an underperforming school and trying to get her students to pass the high-stakes test that the state requires of all third graders. When reflecting on why her teaching practice is focused on eliciting correct answers from students she wrote,

> It is also important to point out the influence of high-stakes testing on my teaching. I would like to spend more time on investigation, but our schools are currently focused on students' testing performance above all. I have to prepare my students for the MCAS with a very real deadline, and while I am in fundamental disagreement of this practice, my job literally depends on it.
> I guess what I am saying is that schools that are underperforming seem to place so much value on correct answers to tests that teachers have no choice but to place the same burden on their students. (Lesson Reflection, p. 2)

It is clear from Ms. Duncan's lesson reflection that she feels enormous professional pressure to help her students to pass the Massachusetts Comprehensive Assessment System (MCAS) and appreciates the pressure her students are under to successfully pass state mandated exams. As a novice teacher, who has not yet earned tenure, the pressure she feels is real and quite possibly enough to cause her to ignore many reform practices in favor of more traditional ones that are geared toward transmitting information to students if an effort to help students pass exams.

The social norms that develop in a classroom have a formative effect on the types of socialmathematical norms that emerge. Classrooms where students are not encouraged to explain and justify their thinking or listen to and attempt to make sense of other's solutions are difficult places for reform-oriented socialmathematical norms to flourish, albeit traditional socialmathematical norms often take root. The following section will examine the practices that foster socialmathematical norms in Ms. Duncan's classroom.

Practices Fostering Socialmathematical Norms

As stated previously, socialmathematical norms are specific to the mathematics of the lesson and contribute to students' understanding of what counts as a different, efficient, and sophisticated mathematical solution as well as what counts as an acceptable mathematical explanation and justification (McClain & Cobb, 2001; Yackel & Cobb, 1996). Developing a taken-as-shared sense of what counts as a different, sophisticated, or efficient solution involves understanding when it is appropriate to contribute to a particular classroom conversation. In this sense students are obligated to compare their solution to others previously shared and determine if in fact their method was different before making a contribution to the conversation at hand. On the other hand, understanding what counts as an acceptable mathematical explanation and/or justification involves the act or process of contributing to the mathematical conversation (McClain &

Cobb, 2001). Here, students develop an understanding of what an acceptable mathematical explanation must entail. For example, an acceptable mathematical explanation in a reform-oriented classroom would require physical or conceptual actions on objects representing number. Table 4.2 shows the distribution of practices Ms. Duncan used that are associated with the constitution of classroom socialmathematical norms as well as the number of times each practice was used per hour.

Table 4.2 Observed Practices Fostering Socialmathematical Norms: Ms. Duncan

	Observation						
Practice	OB1	OB2	OB3	OB4	OB5	Total	Per Hour
Developed Idea of Mathematically Different	0	0	0	1	0	1	0.4
Developed Idea of an Efficient/Sophisticated Solution	0	0	0	4	0	4	1.7
Made Mathematical Thinking Public	0	0	0	0	0	0	0
Accepted One Word Answers	57	38	44	49	45	233	97.1
Indicated Math was Rule Bound	34	42	25	26	20	147	61.3

Number indicates number of times code occurred

As seen in Table 4.2, Ms. Duncan does not engage in the reform-oriented practice of helping student to understand what it means to have a mathematically different solution. Because Ms. Duncan rarely elicits different solutions (see Table 4.1), it was difficult for her to find opportunities to highlight when a solution was and was not different from one previously shared. Table 4.2 reveals that Ms. Duncan utilized, on occasion, the reform practice of helping students to understand what constitutes a mathematically efficient solution, albeit all of her attempts occurred during Observation

4. And lastly, Ms. Duncan has not adopted the reform-oriented practice of making student thinking public.

Ms. Duncan’s most utilized practices are traditional and serve to constitute traditional socialmathematical norms within her classroom. As seen in Table 4.2, Ms. Duncan accepted a one word answer 233 times, indicating that an acceptable mathematical explanation was a one word correct answer. Next, the traditionally situated practice of indicating math is bound by rules and procedures fosters the socialmathematical norm that mathematics is made up of facts and rules that must be memorized in order to be successful. This following section will examine instances of reform-oriented practices followed by an analysis of the traditional practices that Ms. Duncan uses and the socialmathematical norms that such practices constituted.

Mathematically Different

Table 4.2 reveals that Ms. Duncan did not adopt the reform-oriented practice of helping students to understand what counts as a mathematically different solution. This important reform-oriented practice involves guiding students toward understanding when a solution is and is not different from one already shared. When the teacher explicitly addresses the concept of mathematical difference with students, they begin to understand that they must compare their solutions to ones already shared before making a contribution to the conversation. Moreover, because Ms. Duncan rarely engages in the practice of eliciting different solutions (see Table 4.1), she affords herself little opportunity to develop this socialmathematical norm.

As seen in Table 4.2, Ms. Duncan did make one attempt during Observation 4 to establish what it means to have a mathematically different solution by acknowledging how two shared methods were the same. However, her attempt focused on the manipulative used rather than on the mathematics being represented.

> Okay. Now Cameron, can you bring your finished array forward? Cameron used this same strategy, and if you notice, this strategy is exactly the same on paper as it is with objects. Do you see how that works? So if you don't have objects, are you completely lost and hopeless forever? All you need is a pencil and a piece of paper, and you can draw your objects right on there for you. So this is our array that we drew, and this is the one Cameron built. (Observation 4, p. 9)

In this comment Ms. Duncan made an attempt to establish that drawing cubes to represent 3 x 32 was the same as using cubes to represent 3 x 32; however, there was no discussion as to how these two methods were mathematically the same. Ms. Duncan did not develop the socialmathematical norm of Mathematical Difference; consequently, the norm that developed was one of acceptance of any solution without distinguishing or acknowledging the mathematical difference or sameness between solutions. When Ms. Duncan was asked during an interview to describe the social norms that have developed in her classroom, she talked about the importance of accepting whatever students have to say. Ms. Duncan said,

> So I think a norm is definitely that, just acceptance from every student in the room of whatever anybody else has to say. So they need to have that in order for us to have like conversations and things like that in the classroom, so I think that's important. (Interview, p. 4)

The practice of accepting "whatever anybody has to say" is in conflict with the reform-oriented practices of helping students to understand when a solution is and is not different from ones previously shared in conversation. Because Ms. Duncan's students are accustomed to having their contributions accepted, they do not have to listen to others and compare their solutions before contributing to the conversation. Thus, her students make contributions without regard to the mathematics of their contribution. Ms. Duncan is attempting to develop her students' confidence in sharing, thus she finds it necessary to accept all solutions regardless of whether or not they are the same or different from ones already shared.

Mathematically Sophisticated/Efficient

Next, Table 4.2 reveals that Ms. Duncan made four comments over the course of five observations that referred to the efficiency of a solution, albeit all of her attempts occurred during the fourth observation, indicating that this is not a practice that she regularly uses. When utilizing the practice of eliciting different solutions, as she does in Observation 4, Ms. Duncan has an opportunity to bring to the fore of the conversation the notion of a more efficient and more sophisticated mathematical way of counting by fives rather than by ones. The following exchange takes place after Ms. Duncan had asked student to pick a strategy from a generated list and solve 3 x 32. This particular student chose to use tally marks to solve the problem (Observation 4, p. 8).

1. T: Can you share with us how you solved this problem right here? I saw you doing it one way at first. Can you tell them a little bit about how you started to do it?
2. S: I was counting the lines.
3. T: You were counting the lines – how many at a time?
4. S: One.
5. T: She was counting them one at a time. And, was that taking you a long time or a little time?
6. S: A long time.
7. T: It was taking her a long time; then *we* decided on a short cut, right? What was the short cut *we* decided on?
8. S: *We* counted in groups of five.
9. T: Yes, *we* counted in groups of five…

In this exchange Ms. Duncan creates a situation in Turn 1 whereby a student is asked to communicate aspects of her thinking that are not readily apparent to the other students and follows this up with an attempt to develop the idea of a more efficient and sophisticated solution. The student moves from counting by ones to counting by fives and Ms. Duncan successfully brings this more efficient and sophisticated counting strategy into the public discourse space. Ms. Duncan develops the idea of efficiency in different counting strategies again later on in the lesson when she refers back to this exchange and says, "So here we counted by 5s, but here it was easier to count by groups of 10." And later when it was more efficient to count by 2s, she said, "But here instead of counting by 1s, we can count by 2s because they're in groups of 2." This is an attempt by Ms. Duncan to utilize the reform-oriented practice of helping students to develop the idea the efficiency of counting strategies. Although her lesson in general is very controlled in that students are guided as to the different ways multiplication could be calculated, the lesson opens up a window of opportunity, and Ms. Duncan notices it and decides to bring the idea of efficient counting strategies to the attention of the rest of the group.

What is problematic about this exchange is the use of the first person plural pronoun *we*. Rather than giving ownership of the counting strategy to the student and instilling a sense of autonomy, Ms. Duncan interjects herself into the situation when in Turn 7 she said, "Then *we* decided on a short cut, right" and effectively took partial credit for the student's solution. Ms. Duncan did guide the student toward seeing that counting by fives was a more efficient counting strategy, however, why was it necessary to use the pronoun we? Possibly, because Ms. Duncan is the sole mathematical authority in the classroom, using the pronoun *we* gives the strategy the prominence it needs for students to see it as viable strategy to use in the future.

Acceptable Mathematical Explanation/Justification

A reform-oriented socialmathematical norm develops when teachers guide students toward understanding that an acceptable mathematical explanation or justification requires describing the actual physical or conceptual actions taken on mathematical objects as opposed to describing procedures performed (Cobb et al. 1992). This norm focuses on the mathematical activity that students engage in while solving problems and when established, aids in fostering intellectual autonomy in students (Cobb et al., 2001). In Ms. Duncan's classroom, students are rarely asked to offer an explanation or justification for the answers they give, thus the norm that consequently developes is that correct answers do not require explanations or justification. As a result, it can be inferred that one word correct answers suffice as acceptable mathematical explanations and justifications. As seen in Table 4.2, Ms. Duncan accepted 233 one word correct answers over the course of five observations. This result makes sense considering that Ms. Duncan relied on the practice of asking QWKAs to engage her students in mathematical conversations.

In Ms. Duncan's classroom an acceptable mathematical explanation does not require physical or conceptual actions on objects. Moreover, an explanation from students most often takes the form of one word answers rather than detailed explanations of how students solved a particular problem. The following is an example of an exchange where a one word answer was accepted, thus constituting the norm that one word correct answers were acceptable mathematical explanations.

One Word Correct Answers

In this exchange Ms. Duncan and her students are working on representing and writing a mixed number. Ms. Duncan displayed a poster with drawings of several different pictures of fruits in fractional amounts such as one whole orange and one third of another orange (Observation 2, p. 7).

1. T: So let's do our little practice down here and look at the other mixed numbers. I used some fruit and some different things to show you. What you need to do in mixed numbers is that you count up the whole shape first, and then you count up the remaining pieces to find out what the fractional amount is. So, the first job is to find out how many whole shapes are there – okay? The second job would be to count the equal pieces so that you'll know what your denominator is going to be. So let's start doing that here with my oranges. First of all, how many whole oranges are there? How many whole entire oranges are there? Not pieces of oranges. Just whole. Nick?
2. S: One.
3. T: There's one whole, and that's this first one here, right, Nick? But how many pieces is it divided into?
4. S: Three.
5. T: Three – so what would we call those pieces in fraction names?
6. S: One third.
7. T: Thirds. Good. Okay. So, here we've got one. So, I could go and write down that whole number here? See how I did it? One. And see how I made it taller than the other two numbers, too? Now, let's go ahead and name the fractions. So, now we're going to say one "AND" and Nick was kind enough to tell us the denominator that we're working with is thirds. So how many thirds are here? In this picture – how many thirds are there?
8. S: Silent [Student is looking at the picture not seeming to understand what Ms. Duncan is looking for].
9. T: You've got three whole thirds to make this one whole, and how many are here? Count the pieces. It's hard to see that separation here. Does that help you?
10. S: Two.

11. T: Exactly. So, between my helpers, we worked together to figure out that this is one AND two thirds oranges. Okay – so – let's look at the delicious limes – I'm going to count these pieces for you because people like to call it out. Now we're looking at limes, and they look very delicious, and I'm going to count the pieces here. Did anyone else count with me? Let's see if they can name that fractional amount.

The exchange carried on for 13 more Turns with students offering one word answers and Ms. Duncan providing positive evaluations of their one word answers. The focus of the exchange was on the procedures or steps that students would need to follow when writing fractional amounts. For example, in Turn 1 Ms. Duncan refers to what needs to be done first when she says, "What you need to do in mixed numbers is that you count up the whole shape first, and then you count up the remaining pieces to find out what the fractional amount is." And then again in Turn 7 she says, "So, I could go and write down that whole number here? See how I did it? One. And see how I made it taller than the other two numbers, too? Now, let's go ahead and name the fractions. So, now we're going to say 'one and'…" Although in the exchange students did act on objects by counting the pieces of fruit, they were never asked to explain how their actions helped them to solve the problem posed.

The socialmathematical norm that has developed in Ms. Duncan's classroom was that a one word correct answers are an acceptable mathematical explanation, as correct answers did not require explanations. Moreover, the students have come to expect that explanations are Ms. Duncan's job and not something that they are responsible for in the conversation. This is consistent with Ms. Duncan's goal of helping her students to answer questions with success and is reminiscent of a traditional mathematical practice that focuses on the activity of calculations and procedures rather than on the physical and/or conceptual action on objects. The following section will examine the socialmathematical norm that math is bounded by rules and procedures.

Mathematical Activity Bound by Procedures

Lastly, as seen in Table 4.2, Ms. Duncan made 147 references to math being bounded by rules and procedures that students need to learn, follow, and memorize. Moreover, Ms. Duncan continually tells students that they need to “get things into their head,” indicating that the memorization of steps is needed to know and understand mathematics. This result again points to a teaching practice that would foster traditional socialmathematical norms rather than reform-oriented ones. In the following exchange Ms. Duncan is having a discussion with her students about when and when not to use mental math as a means to solve math problems. In this exchange Ms. Duncan cautions those students who might be inclined to use mental math as a method to solve multiplication problems (Observation 4, p. 11).

1. T: Does mental math always work?
2. Ss: No!
3. T: Mental math is tough because it is easy to make a mistake in mental math and because the mistake is not written down, it’s pretty much right inside your brain, you can’t keep track of the mistake. So, mental math is really good if all of your multiplication facts are memorized and all of your addition facts are memorized. If you don’t have them memorized, it’s actually much harder to use mental math. So it’s another reason why it is good to get these in your memory. So what else is fast if you don’t trust mental math and you want to use pencil and paper?

In the exchange above Ms. Duncan may have inducted her students into the belief that mental math was not an appropriate strategy and that students need to use caution when attempting to try to figure out problems, such as 32 x 3 in their heads. When she asks if mental math could be trusted, all of the students joined in with a resounding “No,” indicating that they have learned this lesson well. Consequently, this comment may have thwarted some students' attempt at trying to use mental math in favor of a more traditional paper and pencil method.

Another aspect of this practice was demonstrated by Ms. Duncan's attention to the steps students need to know in order to do certain math problems. In the following three exchanges Ms. Duncan was working with her students on fractions and sets. The rationale for this lesson, as stated in Ms. Duncan's lesson plan on April 28, Observation 1, was written as follows:

> The class has just learned about fractional parts of whole and will now apply that understanding to *sets* and *groups.* They will build and examine the fractional sets within their own classroom group!

Ms. Duncan called a group of students up to the front of the class to represent a group where one out of the five students was wearing a red shirt. After assembling the group, Ms. Duncan s asked the class to tell her what fraction of the set was wearing red (Observation 1, p. 4).

1. T: Now I'm asking you, friends, what fraction of my set is wearing a red shirt? Or jacket? So you need to think of denominator means I count all of the students. Numerator means I count just the shaded amount or just what they are asking for. I'll repeat it one more time. What is the fraction that Cameron represents in this set? Now, what do you think?
2. S: One fourth?
3. T: Close. One is correct. The numerator is correct. Now when you do the denominator, you need to count everything including the shaded boy. So what would that denominator be then? [student hesitates and then Ms. Duncan calls on another student to respond] Lilly?
4. S: Five.
5. T: Five is correct. So, what I'm saying is that one – two – three – four – five students make up my one set. So, they are one full group, right? One whole group equals one whole.

In this exchange Ms. Duncan alerts students to the steps that they should use when trying to figure out fractional amounts. In Turn 1 she tells the students how to find the denominator and then how to find the numerator, yet little meaning is attached to the explanations. Moreover, she ends with "What do you think?" However, in Turn 2 when a student responds with an incorrect answer, she does not elicit his thinking. Rather she indicated that part of his answer was correct in Turn 3, "Close - The one is correct" and then immediately began in Turn 4 to explain what the student needed to do to find the denominator, "Now when you do the denominator…" In this exchange Ms. Duncan bypasses an opportunity to delve deeper into Cameron's thinking in favor of getting her students to follow steps when finding fractional amounts in a set. By not exploring the student's answer of ¼ she misses an opportunity to assess what the student did and did not understand about the set she had put together. It may have been a simple miscalculation or it may have been a much deeper misunderstanding, yet it went ignored in this instance.

In the following exchange Ms. Duncan attempts to address the confusion students are experiencing when representing fractional amounts in a set. The exchange again indicates the procedural and rule bound nature of Ms. Duncan's classroom.

1. T: So what we need to think of is this right here. This is going to be the key that I will leave up (whispering as your cheat sheet when you are working). So how many of you still get a little confused when you are about to write numerators and denominators?
2. S: [Many students raise their hands].
3. T: Thank you for the honesty. It's confusing. Remember we gave ourselves– N stands for north, which faces up and D stands for down, which faces down so that helps us to remember, but that does not always make it easy.

   So, I have this right here for your eyes, and if you are using your white board, and you're confused, and you're saying, "Hmmm – I can't remember which one I'm supposed to count them all up and write the number down." Which of these two numbers – numerator or denominator do you know – Ms. Henry always does first, or I always do first – it's different than what you might think – Ronnie?

4. S: Denominator.
5. T: I do the denominator first ‘cause it is easier for me to count one – two – three – four – five and put my five there. That’s always a sure thing for me. So, if I stood up a set of – one – two – three – four – just stand right up – I could instantly write my denominator without even asking a question, right? So, the denominator would be, George?

In this exchange Ms. Duncan begins the lesson in Turn 1 by showing her students a “cheat sheet” that could be referred to if students become confused during the lesson. When Ms. Duncan asks students if they are confused when trying to find the numerator and denominator, most of the students’ hands shoot up. Moreover, it is evident throughout the lesson that students are confused, even with the use of the “cheat sheet.” In Turn 3, Ms. Duncan acknowledges their confusion by reminding them that N – North –numerator goes on top and D-Down denominator goes on the bottom. Then Ms. Duncan gives students another hint to help them remember which number goes where - they could do what she always does first – the denominator.

What is concerning about this exchange is that Ms. Duncan never elicits from students what they are confused about and instead gives them hints or pointers to help them remember what to do when finding fractional amounts. Moreover, she misses opportunities to assess what students do and do not understand when trying to represent fractions of a set. The use of the “cheat sheet” is also problematic because it indicates that students need to memorize what is on the sheet because most likely it would not be something they would have access to at all times. Consider the following exchange that again references to the “cheat sheet” and indicates that there are steps you need to follow when working with fractions (Observation 1, p. 8).

1. T: So this is the cheat sheet part that you really want to look at. Okay. I’ve got all my hints in one chart. Here it says numerator is the number circled

or maybe it's the number colored or maybe it's the number wearing pigtails or maybe it's the number wearing red – whatever I say.

So, that's the numerator. Denominator in blue here and green is just the number in the set. However many there are or if you are looking at a shape like a circle or a rectangle, it's how many pieces that rectangle is divided into. Right? So if I drew this rectangle, what would my denominator be?

2. S: Ten.
3. T: Correct! So whatever it was, I know my denominator is 10. Denominator is the easy part. And then it actually even gets easier because then you just have to count up either the number shaded or what I asked for – so whatever is was pop it on top, okay.

Within the last four examples there is evidence to suggest that students are learning math at a procedural level rather than at a conceptual level and given a substantial amount of information to remember pertaining to the steps they need to take to figure out the numerator and the denominator. Moreover, in the fraction exchanges Ms. Duncan does not attempt to elicit student thinking or understanding of fractions and sets. For third graders this lesson is one in which much information is handed down to them in a step-by-step manner with no attention given to conceptual understanding about the meaning of fractions.

When Ms. Duncan reflected on the above lesson she was surprised at how procedural her teaching was; however, she felt that this type of teaching was necessary first before trying to develop students' conceptual understanding of fractions. Consider the following quote taken from Ms. Duncan's reflection of her videotaped lesson on April 28, Observation 1:

> I observed this entire lesson to focus around following procedures, using tips, reminders, cheat sheets (which is what I referred to in my example chart and before independent practice!), examples, and rules. The chart *shows* students how to solve a problem; I *tell* them what I would do to solve it. Much of my teaching uses algorithms, tips and rules, and I feel that in many mathematical cases, this is

> necessary (like in the case of mathematical properties). I also think that sometimes students are not yet developmentally capable of understanding some things conceptually, but they can find success if they learn the steps for solving and LATER uncover the conceptual “hows” and “whys.”(Lesson Reflection, p. 1)

The above quote from Ms. Duncan’s lesson reflection is in contrast to statements she made during an interview with regard to her own mathematical experiences in elementary school and later in a teacher preparation program. The following quote references Ms. Duncan’s experiences in a mathematics methods course where she had opportunities to investigate elementary math conceptually. When reflecting on her methods course she said,

> I was thinking back to what I said about my childhood experiences in math, and I think if I had had that [Investigations in math] and I didn’t see multiplication as just rote memorization or, you know, these steps need to be followed, I think that conceptually I would have understood better what was going on. And, if I had those strategies and tools myself as a student of just being able to multiply or picture groups or draw pictures to help, or something of that sort it would have become more concrete to me. But when I was taught, it was pure memorization. (Interview, p. 3)

And when considering the practicum placement she was assigned to, she noticed that students were delving deeper into the concept of multiplication than she had as an elementary school student. She said,

> So I think that I thought about my own experiences and then I watched the students that I was student teaching kind of delve into their investigation and really understand what a multiple was and what they were doing and repeated addition and things like that. (Interview, p. 3)

Ms. Duncan beliefs in theory seem to be in conflict with her beliefs in practice. In theory, Ms. Duncan believes that students should be allowed to investigate mathematical concepts before being introduced to mathematical procedures while in practice she believes that students may not be “developmentally capable” of understanding concepts but are able to find success with steps and procedures. Again, because the goal of Ms. Duncan's teaching practice is to help her students to, as she said, "answer questions with success" so that students ultimately pass state mandated exams, her beliefs in practice seem more suitable to her goal. Time is of the essence for Ms. Duncan, thus taking time to develop conceptual understanding would detract from her goal of passing exams because as she said,

> There is so much to learn in the third grade curriculum, and so little time to spend on each concept, that to take the time for conceptual exploration of every topic would make the school year never-ending. (Lesson Reflection, p. 1).

Summary

Socialmathematical norms describe the ways in which classroom members come to understand what counts as mathematically different, efficient, and sophisticated as well as what counts as an acceptable mathematical explanation and justification (Yackel & Cobb, 1996). Moreover, in a reform-oriented classroom acceptable mathematical explanations require students to explain their solutions in terms of physical actions on objects as opposed to the more traditional norm of describing procedures or calculations made (Yackel & Cobb, 1996). In Ms. Duncan's classroom students rarely were asked to solve problems in different ways thus they had little opportunity to construct an understanding of the concept of mathematical difference. On a few occasions Ms. Duncan did attempt to establish for students what it meant to have an efficient solution; however, because she did not obligate her students to solve problems in different ways opportunities to explore more efficient or sophisticated solutions did not take root in her classroom.

In Ms. Duncan's classroom students were not obligated to describe how they physically or conceptually acted on objects to solve mathematical problems. Moreover, the norm that was developed seemed to be that correct answers were acceptable mathematical explanations thus one word answers became the normative way in which students responded. The socialmathematical norms that have taken root in Ms. Duncan's classroom mirror norms that represent traditional ways of engaging in mathematics. Although in theory Ms. Duncan subscribed to reform-oriented practices her beliefs seem overshadowed by the pressure she feels to help her students pass state mandated exams. The following socialmathematical norms were constituted as a result of Ms. Duncan's practices:

- Correct answers did not warrant explanation or justification.
- All solutions, regardless of their mathematical difference were accepted and valued.
- Acceptable mathematical explanations and justifications were the responsibility of the teacher.
- Mathematics consisted of rules and procedures that were to be memorized and followed.

In summary, the socialmathematical norms that developed in Ms. Duncan's classroom mirrored traditional norms whereby the teacher was seen as the mathematical authority in the classroom and students were seen as passive receptacles waiting to be filled with information that was to be memorized in order to be successful. Correct answers played a central role and were more important than developing students' understanding of mathematical concepts and procedures. Moreover, correct answers became the norm for an acceptable mathematical explanations or justifications. When reflecting on of her videotaped lesson Ms. Duncan commented on her practice of eliciting correct answers over understanding. She wrote,

> I also even noticed a case where it looked like the child said the right answer, having no clue how or why what they said was true! I wish I had stopped to check for understanding more often during the first half of this lesson! (Lesson Reflection, p. 2)

Moreover, mathematical efficiency and sophistication was viewed from the standpoint of rules and procedures rather than from the divergent ways in which students came to know and understand mathematics. Ms. Duncan' practices were steeped in tradition thus the socialmathematical norms that were constituted were traditional as well and were not representative of mathematics reform.

The following section will address the issues and challenges that Ms. Duncan faced as a novice teacher attempting to develop a teaching practice under the auspice of mathematics reform.

Issues and Challenges

After analyzing Ms. Duncan's teaching practices as well as her interview discourse it became apparent that there were several issues and challenges that Ms. Duncan was faced with when attempting to develop her teaching practice under the auspice of mathematics reform. The following section will address the second research question guiding this study.

- What issues and challenges surface as novice teachers begin to enact reform-oriented discourse practices?

Top Down Pressure

At the time of this study Ms. Duncan was in her second year of teaching third grade in a predominantly Hispanic inner city school where 82% of the students were from low-income households. Her school had been struggling to make annual yearly progress (AYP) for the past three years, and according to Ms. Duncan, if the school did not make AYP by the following year, the principal would be replaced. Moreover, because the school had not made AYP for three consecutive years, it was labeled “underperforming” and in jeopardy of being taken over by the state if test scores did not improve. The pressure Ms. Duncan experienced working in an underperforming school seemed to have influenced her ability and possibly her desire to implement reform-oriented teaching practices. When reflecting on one of her teaching videos, she noticed the lack of student-to-student idea sharing and attributed it to the pressure of working in an underperforming school. She said,

> This opportunity to reflect made me question my reasoning behind the lesson’s lack of student-to-student idea sharing. I think that during my review, I don’t want students to hear or learn the “wrong” information, and that much of my teaching is clearly toward a goal of answering questions with success. This probably has to do with the stress put on me and my students to perform on standardized tests. (Lesson Reflection, p. 1)

As a novice teacher, the pressure of teaching in an underperforming school may have mitigated Ms. Duncan’s ability and desire to implement reform-oriented practices in favor of implementing more traditional practices where the teacher asks the questions and students provide the answers, which are subsequently evaluated as right or wrong by the teacher. Ms. Duncan was concerned that offering students opportunities to explain and justify their thinking might cause undue confusion for her students and possibly impact their performance on standardized test. As she said, her teaching is clearly oriented toward the goal of “answering questions with success,” thus the traditional practices she has adopted effectively match her goal. Adopting reform-oriented practices that obligate students to publicly explain and justify their mathematical reasoning is problematic for Ms. Duncan because of her genuine concern that students might “learn the wrong information.” In a reform-oriented classroom, students are encouraged to express their mathematical ideas and conjectures, thus teachers must be prepared to make room for “wrong” or naïve reasoning to enter into mathematics conversations. For Ms. Duncan “wrong” or naïve reasoning does not fit with her current instructional agenda, which at present, is focused on “clearly working toward correct answers” (Lesson Reflection, p. 1).

Because of its underperforming label, the state was already involved in overseeing some of the day-to-day operations of Sunrise Elementary School. When discussing the day-to-day pressure she experiences teaching in an underperforming school Ms. Duncan said,

> The State came in at the first level and mandated our schedules. So they took our schedule and they designed it themselves. So two months into the school year, they completely flipped our schedules upside down, changed specials, changed everything. So it was a big transition for the kids to go through. It was kind of like starting the year over again. And that created uninterrupted blocks so then the English language arts block could continue for two and a half uninterrupted hours and math for ninety minutes uninterrupted. So that way you got all the subjects back to back and in their entirety. So there’s supposed to be a sixty-minute lesson itself and then the half hour is an intervention piece and the intervention piece is mainly based around standardized testing. (Interview, p. 10)

Having the daily teaching schedule turned “upside down” by outside forces is a daunting and discombobulating experience for any teacher but especially for a novice who is in the early stages of developing her craft. In the beginning years of teaching, novices are learning to adapt to the daily routines of teaching and sudden unexplained changes places added pressure onto an already pressure filled experience. The change in schedule for Ms. Duncan and her students meant adding an extra half hour of time onto the math block. This extra half hour was, according to Ms. Duncan, an intervention piece focused on standardized testing.

As stated previously Ms. Duncan's goal in teaching is to help her students answer questions with success in an effort to ensure her students pass the Massachusetts Comprehensive Assessment System (MCAS). In Massachusetts students in grades 3 through 10 must take one or more subject matter tests each school year and must pass the English and math tests in order to graduate from high school. When asked what role the MCAS test has in her teaching Ms. Duncan said,

> They [MCAS] play a very large role because we're in the city. We're in an under-performing school. It's a very, very high Title One population of poor students. So because of that make up, we're really pushed hard when it comes time for standardized tests. It's kind of the "If you don't make the cut this time, then the State's coming in," and there's all these consequences kind of just waiting around the corner. So I'd have to say that that's definitely a source of anxiety. (Interview, p. 9)

Ms. Duncan is experiencing the anxiety that comes with being a novice teacher along with the anxiety of ensuring her students are successful in passing the MCAS test. Although Ms. Duncan said that she was not a proponent of letting high-stakes tests drive her teaching practice, she felt helpless to challenge the mandate because, as she said,

> It is also important to point out the influence of high-stakes testing on my teaching. I would like to spend more time on investigation, but our schools are currently focused on students testing performance above all. I have to prepare my students for the MCAS with a very real deadline, and while I am in fundamental disagreement of this practice, my job literally depends on it. I guess what I am

> saying is that schools that are underperforming seem to place so much value on correct answers to tests, that teachers have no choice but to place the same burden on their students. (Lesson Reflection, p. 4)

Although fundamentally her beliefs were in conflict with the practice of prepping students for standardized tests, Ms. Duncan realized the importance of the MCAS not just for her students but for her career as a teacher in the state of Massachusetts. Ms. Duncan was also under pressure to administer and grade various practice tests to get students ready to take the MCAS. She believed that much of this work was not a good use of her or her students' time. When reflecting on how test preparation impacted her teaching she said,

> Actually an exam was in my mailbox yesterday morning, and it said, "Give this exam during your math block today and return by this afternoon corrected." So that happens fairly often, and a lot of the times it would be the week before a test where they'd give us three prep activities and ask for us to correct all of them and complete item analysis. So I don't think that that was a good use of my time, because I was really giving that to someone else and they don't return it to the teachers for us to use for ourselves. So it was really just for somebody else's data collection. And times like that you would give up an entire math lesson. (Interview, p. 9)

And when reflecting on how the added burden of test preparation impacts her students learning she said,

> All of the test prep that students had, that was required of them, took away from my ability to, to really delve into things as deeply as I'd like to, to teach every lesson to mastery. You just couldn't do it in the time that was allotted because they were really forcing us to do certain test preps and I'd have to complete an item analysis and correct it myself and turn it back in. So that would take over certain lessons. (Interview, p. 9)

The pressure of teaching in an underperforming school was real for Ms. Duncan. She was required to administer, grade, and complete an item analysis on test preparation materials that were never returned to her to use as a means to assess her students' learning. Moreover, this was not something that took place a few times in a school year. For Ms. Duncan and her student test preparation was an ongoing intrusion into their mathematics class. When reflecting on the number of times she must administer these types of tests to students she said,

> There's three cycles of testing in terms of the district, and then there's two in terms of the State level with MCAS and things like that. So for every single chapter in our math program there's an MCAS prep test for each chapter. I only had to complete that, so that was 12 of those, and then I'd say about 4 or 5 other actual tests. The chapter preps would be about five questions and you could do that as a 15-minute warm-up at the beginning of a lesson. That wouldn't take up your whole day, but I'd say probably five or six times there were actual full tests that would take up a whole lesson and there's a program we were doing and we were supposed to give a half hour of our hour and a half math block per day to MCAS prep. And we had like prep books that we were to use and things like that. (Interview p. 9)

The MCAS test served as a source of TDP that impacted Ms. Duncan's ability to adopt reform-oriented teaching practices, and moreover, it served as a constant reminder that she was not a "good teacher." Reflecting on this notion of what makes a good teacher she said,

> And I think good is measured on a scale of perspective and relativity so like if you were to look at our MCAS scores from last year, the Principal would let you know that we didn't do a really effective job. None of our students were able to pass the third grade math MCAS but, but at the end of my year last year, I had one student who we were rotating for math, and he wrote me a beautiful card, and he said, because he was coming into my room only for math, and he, one line was like, "You make math my favorite subject." So in his perspective and in that perspective of the way that I taught, I made a good math teacher for the class that I had and I made learning fun for them. And so on that level, I do think that I was a good math teacher. (Focus Group Interview, p. 10)

Confronted with the reality that no third grade student in her school was able to pass the MCAS test and the insinuation that teachers had not done an “effective job,” Ms. Duncan needed to find away to gain some perspective on the situation. She chose to equate good teaching with making learning fun and interesting for her students. Making a meaningful connections with students meant more to her than students’ obtaining proficient scores on standardized tests.

Another top down pressure that has been imposed on Ms. Duncan came from a grant that her school was awarded to introduce the game of chess to elementary school students. Students in third grade were learning how to play chess, and although Ms. Duncan felt the experience was a valuable one, it infringed on her ability to keep on top of her math lessons. When reflecting on issues of concern for her math practice she said,

> Currently, other factors such as my chess grant (replacing 1 out of 5 weekly math lessons... the math program has exactly 180 lessons) are "squeezing me.” It put us behind and when the district tests us, they expect us to be up to the exact point in our pacing guides on that date--- being just 3 or 4 lessons behind can cause students to lose proficiency on these high stakes tests (Email communication, January 2009).

There are 180 school days in a school year, and Ms. Duncan’s math curriculum was comprised of 180 math lessons that she was required to teach; however, because of the chess grant, she was put behind one lesson per week, yet the district expectation was that she was to keep her class up to speed with the testing cycle. For a novice teacher this pressure was noteworthy.

The curriculum Ms. Duncan uses was also a form of Top Down pressure in that she was mandated to be on a particular page in the textbook on a given day and lesson plans were provided to her by the district. She was pressured to ensure that students were introduced to a significant amount of information in what she referred to as a “spiral teaching” manner. When commenting on the challenges, she faced implementing the mandated curriculum she said,

> There’s 180 math lessons and they’re expected to master every single one of those 180 lessons in time for MCAS in May, which doesn’t come before the 180 days. So you have to squeeze all of this information, and you’re really seeking mastery in such a short time, so for me, it’s that idea of reviewing and spiral teaching and making sure that everything that they did learn that they did have solid, that they’re not forgetting it and that it’s not falling to the wayside because there’s so many different things and concepts and lessons that we have to be pushed forward into. And we don’t really get a choice as to whether, “Okay, they didn’t understand today so I’ll take an extra day for that lesson”. It’s more an option of, “Okay, which two lessons can I fit into one block this week?” So for me, it’s time and mastery. (Focus Group Interview, p. 12)

Ms. Duncan believed that as a teacher, she was left with little choice as to how much time she could spend on a particular concept and that the curriculum drove her teaching. Consider the following conversation between Ms. Duncan and Ms. Arielle that took place during the focus group interview (p. 13). The participants were discussing the various curriculums that they use to teach math.

> Ms. Duncan: Do you get the idea that you’re told the curriculum is the Bible?
> Ms. Arielle: See, we’re told that the curriculum is **not** the Bible.
> Ms. Duncan: Well your school has a prescribed curriculum with daily lessons though, that you do have to use, right?
> Ms. Arielle: Well, we use it kind of like as a tool. But no, we don’t have to follow it.
> Ms. Duncan: That’s the way it should be.

Ms. Duncan was in a school where state testing cycles and mandated curriculum drove her mathematics teaching practices. Moreover, Ms. Duncan did have the autonomy to use other teaching materials or curriculum as a means to enhance her mathematics lessons. The researcher was aware that Ms. Duncan had access to the reform-based curriculum, *Investigations*, as it had been used previously as a supplement in the school. When the researcher asked to observe Ms. Duncan teaching a lesson from *Investigations* Ms. Duncan responded,

> So sorry (for my students, especially), but no more *Investigations* at Sunrise Elementary /per school improvement scheduling. At the beginning of the year, the teacher in the room next-door to me tried to use solely *Investigations* to teach math, and I believe she was really enjoying it until the school cut her short because it wasn't aligning exactly with the pacing guide. Whatever! (Email Correspondence, January 17, 2009)

For Ms. Duncan there was top down pressure in terms of standardized testing, which then filtered down to the day-to-day teaching of mathematics. Moreover, there was pressure to implement enriching programs such as chess; however, she was expected to cover the same amount of material so students were prepared for the MCAS test in May. Moreover, Ms. Duncan was pressured to teach using only the district adopted curriculum, thus utilizing a reform-oriented curriculum, even as a supplement, was not an option. Teaching in an underperforming school has placed much pressure on Ms. Duncan and as a result she experienced a conflict putting her beliefs into practice. The following section will examine Ms. Duncan's conflicting beliefs.

Conflicting Beliefs

Ms. Duncan was challenged to come to terms with two conflicting beliefs - one theoretical and the other practical. In theory, she espoused several reform-oriented beliefs; however, in practice and in reflection, she contradicted her espoused beliefs in favor of more traditionally focused ones. In conversation Ms. Duncan said that she believed that it is important to let students solve problems differently because,

> Some brains just don't compute the same way that other brains do, or don't think the same way or don't necessarily go about solving problems with the same method that other kids would. What I realized after kind of jumping on it quickly and thinking it was wrong because it wasn't what everybody else was doing, is that there's a lot of kids who actually aren't wrong but just use a kind of roundabout, different way of thinking. (Interview, p. 5)

She went on to say that this belief has prompted her, at times, to delve deeper into a student's response rather than just assume that a student is wrong when her or his answer is not what she had in mind. When reflecting on one particular student, she said,

> So I've learned to look more closely and to question myself and to just keep an open mind and know that there's not only one way to do things and that they actually always teach me different things. If you take your time to question where it came from [a student's answer] you can figure out that the student was thinking correctly but just explaining themselves differently. (Interview, p. 5)

However, in practice Ms. Duncan did not foster in students the idea that solving problems differently was an important aspect of mathematics. Moreover, when a student did solve a problem in a way that was different from what she had expected, her reaction was not one of acceptance, but one of conformity. Consider the following comment Ms. Duncan made to a student who had solved a problem differently from the way she had expected.

> Okay. You're going to have to erase it. And you're going to have to draw something that makes a little bit more sense. Because I know what you're trying to do, Bryan, and your brain does think outside of the box. (Observation 1, p. 14)

Rather than asking Bryan what he had done to solve the problem, Ms. Duncan looked at his white board and then erased what he had drawn and instructed him to draw something that made more sense to her. This is in direct conflict with her interview statement indicating she has learned to look more closely rather than jumping to the conclusion that a student's work is wrong. Ms. Duncan's beliefs about encouraging students to solve problems differently conflicts with her actual teaching practice.

Moreover, the beliefs that Ms. Duncan ascribed to during an interview after her second year of teaching have changed and reflect a more traditional view of teaching and learning. She shared in an interview that she believed as a teacher she needed to teach math conceptually first before introducing students to rules and procedures. When asked to reflect on something she took away from the mathematics methods course and implements into her teaching practice she said,

> I think the thing I take the most away from the methods course is to teach math not as an algorithm. To teach math as a concept and to allow students to really investigate the properties on their own and get a sense for it before really teaching them strategies and rules and steps to follow. I think that is one of the most important things I do, and I do believe it comes from that course probably. (Interview, p. 4)

At the end of her third year of teaching in an underperforming, low-socioeconomic school with a predominantly Hispanic student population Ms. Duncan's beliefs have changed. When asked to reflect on issues and/or challenges that have come up for her in her first three years of teaching she said,

> For me, the issue has been whether or not my students are capable of learning new information that requires higher level thinking skills. I have honestly found the opposite of what I had thought to be true during my master's degree. I have, surprisingly, found that it works better to teach an algorithm FIRST and THEN do the investigating and discussing of WHY. The third grade brain seems to succeed best using that method - weird, huh? (Email Correspondence, July 2009)

It seems that Ms. Duncan has addressed her conflicting beliefs by now subscribing to the belief that third graders are not capable of utilizing higher order thinking skills, thus she must resort to teaching algorithms first before addressing the whys of mathematics. What Ms. Duncan learned in her methods class has since been overshadowed by the pressure of working in an underperforming school that is heavily mandated by the district and state. Ms. Duncan has adjusted her beliefs to meet demands of teaching in an underperforming school and has adopted a traditional practice of teaching mathematics that fits more with the goal of preparing students to pass standardized tests.

Perception of Self as a Math Learner

Another issue that presented a challenge for Ms. Duncan was her perception of her own mathematical ability. Ms. Duncan has harbored a fear of mathematics since her early elementary school years and considers herself to be weak in mathematics and strong in language arts. She recalled memories of her father, who she believes is a genius when it comes to math, being forceful as a "homework helper" as well as school memories of failing to learn her multiplication tables in third grade. Both experiences, she believes, helped to foster a "fear" of math that followed her well into her college years. According to Ms. Duncan, when she entered college, she let a math requirement pave the way for her future career. She said,

> And in college actually the reason I went into education was because of the math requirement in my intended major. I was originally a business major, but the statistics course scared me so that very first semester, when I found out about the requirement, I switched my major to education. (Interview, p. 1)

Reflecting further on how her fear of mathematics paved the way for a career in teaching she seemed to acquiesce to her fear and reconciled her defeat by choosing a career path that she has come to enjoy. She said,

> I took a lower level math class my first semester and struggled though it, and it was kind of just enough to get me through my requirement, and after that I swore to myself I wouldn't do a higher level math than that course. That first course I took, I think it was pre-calculus, was really difficult for me and statistics was just another fear of mine. But I don't know how I would have done because I didn't do it – I was too afraid to take it. And I was really happy with that decision because, of course, now I am doing what I am doing. (Interview, p. 3)

Ms. Duncan's reflections suggest that early learning experiences in mathematics can have a negative effect on an individual's decision to engage in higher level mathematics courses and quite possibly the career one chooses. Although Ms. Duncan indicated that she was happy with her choice, and by all outward appearances she was, one must wonder if she truly wanted to pursue teaching as a career or if her fear of mathematics pushed her into her career choice.

Additionally, Ms. Duncan's belief that she is not strong in math was not supported by the Mathematical Knowledge for Teaching (MKT) instrument. Ms. Duncan obtained a score that was almost one standard deviation above the mean suggesting her mathematical knowledge for teaching is relatively strong compared when compared with the average teacher taking the MKT assessment. Ms. Duncan's perception of self as a math learner has overshadowed her actual mathematical knowledge for teaching.

Summary

In summary, as a novice teacher in an underperforming school, Ms. Duncan experienced a significant amount of top down pressure (TDP) to ensure that her students made gains on the state mandated MCAS test and moreover, believed her job depended on increasing test scores. Her daily schedule had been readjusted by the state two months into the year in an effort to increase daily MCAS preparation. Moreover, the added pressure of teaching chess to students while ensuring all 180 district designed and mandated math lessons were taught increased the pressure that she experienced. Ms. Duncan also resented but complied with the TDP to administer, grade, and complete item analysis for several mandated assessments handed over to her by the school math coordinator. She resented the intrusion on her time, as administering the assessments took away from her math teaching, and grading and item analysis took away from her preparation time. Moreover, she did not see the assessments as having real value for her teaching, as the assessments were never given back to her to use as a means to inform her teaching.

Ms. Duncan experienced a conflict between her beliefs in theory and her beliefs in practice. As such she developed a new belief, one that said her students were not developmentally ready to learn mathematics in a reform-oriented manner, thus students were in need of remediation before attempting reform practices.

Lastly, Ms. Duncan’s lack of confidence in her abilities in math, however unfounded, caused her to tightly control her math lessons leaving little room for student inquiry.

CHAPTER 5

MS. ARIELLE

When this study began, Ms. Arielle, as she was called by her students, was in her second year of teaching first grade at Achievement for All Elementary School. Achievement for All is a community-based, publicly funded, charter school located in an urban school district in Western Massachusetts. The school was temporarily housed in the upstairs rooms of a local Baptist Church until a more permanent facility could be purchased through the school's Capital Campaign. The student body is predominately African America (51%) with Hispanic (25%), White (18%), and Multi Race Non-Hispanic (5%) students making up the rest of the student body. Ms. Arielle had 22 first-graders in her classroom – 12 boys and 10 girls. The majority of students attending the school are considered low-income and live, for the most part, in the surrounding neighborhood.

Ms. Arielle was a member of the pilot study preceding this research project and was eager for the opportunity to continue to critically reflect on her mathematics teaching practice. She believed being part of this project would offer her an opportunity to think deeply about her practice and the ways in which she engaged students in mathematics.

The following sections will present the analysis of the teaching practices used by Ms. Arielle to engage her students in mathematical conversations. First will be an analysis of the teaching practices that fostered the development of social norms followed by an analysis of the practices that fostered the development of socialmathematical norms in Ms. Duncan's classroom. The focus of this section of the case study will be to address the following research question:

- What reform-oriented discourse practices do novice teachers who participated in a reform-based mathematics methods course adopt? What practices do they adapt? What practices do they ignore as they engage their students in mathematics conversations?

Practices Fostering Social Norms

As stated previously, social norms constitute the “participation structure” (McClain & Cobb, 2001, p. 244) within classrooms and are not specific to the discipline of mathematics. This study analyzed the teaching practices Ms. Arielle used to engage her students in mathematical conversations and made inferences as to the social norms that such practices constituted. Table 5.1 reveals the distribution of practices Ms. Arielle used while in whole group discussions as well as the number of times each practice was used per hour.

Table 5.1 Observed Practices Fostering Social Norms: Ms. Arielle

	Observation						
Practice	OB 1	OB 2	OB 3	OB 4	OB 5	Total	Per Hour
Elicited Explanation/Justification	23	35	4	13	3	78	41.1
Elicited Different Solution	12	8	16	4	2	42	22.1
Asked a QWKA	7	2	5	9	18	41	21.6
Accepted Student Response	1	0	14	2	14	31	16.3
Evaluated Student Response	1	2	1	0	1	5	2.6
Explained Students' Thinking	0	0	0	0	0	0	0

QWKA - Question with a Known Answer

As seen in Table 5.1, Ms. Arielle's most utilized practice was that of eliciting explanations/justification from students with 78 elicitations counted over the course of five classroom observations. Ms. Arielle's next most utilized practice was that of eliciting different solutions to the same problem with 42 requests made, followed by accepting student responses without evaluation 30 times. Although accepting student responses can be a traditional practice, as seen in the case of Ms. Duncan, analysis reveals that Ms. Arielle utilized this practice as a means to foster student autonomy. Ms. Arielle utilized the aforementioned practices in a reform-oriented manner, and as seen in Table 5.1, theses three practices accounted for the majority of practices she used while engaging her students in mathematical conversations.

As seen in Table 5.1, Ms. Arielle did use the traditional practice of asking QWKAs, however, analysis reveals that QWKAs were asked as a means to clarify student thinking rather than to test student knowledge. Lastly, as seen in Table 5.1, Ms. Arielle rarely evaluated student responses, with only four observations noted, and there were no occurrences where Ms. Arielle provided an explanation of students' thinking to the class. Although Ms. Arielle utilized the practice of revoicing students explanations and justifications, the practice was used as a means to bring student thinking into the public discourse space rather than as a means to provide a teacher directed explanation and/or justification.

The following section will examine each of these practices in more detail as a means to shed light on the social norms that were constituted as a result of the practices Ms. Arielle used. First under review are the practices associated with mathematics reform: Elicited Explanations and/or Justifications and Elicited Different Solutions followed by an examination of the practices associated with traditional classrooms: Asked a QWKA and Evaluated/Accepted Student Response. Examining such practices through the filter of social norms provides a framework to describe the participation structure that emerged in Ms. Arielle's classroom.

Elicited Explanations and Justifications

Ms. Arielle expected her students to explain and/or justify the way in which they solved math problems. As seen in Table 5.1, Ms. Arielle elicited explanations and/or justifications from students 78 times over the course of five observations, making this her dominant teaching practice. Moreover, because Ms. Arielle also obligated students to solve problems in multiple ways, she had ample opportunities to elicit explanations and/or justifications from her students. The following examples illustrate the way in which Ms. Arielle developed the social norm of obligating students to explain and/or justify their solution methods.

Exchange 1 comes from one of the earlier observations in the database in which Ms. Arielle and her students were negotiating what it meant to offer an explanation and a justification. The term negotiated is used here in that Ms. Arielle did not direct students how to offer explanations and justifications; rather, she negotiated the process by engaging students in conversation about the way in which they solved math problems. In this exchange Ms. Arielle drew upon a student's initial response to the following problem as a means to elicit explanations and justifications of the student's method of solution.

> Fourteen children were playing in the park. Then five children went home. How many children were still in the park? (Observation 2, p. 1)

1. T: Okay, so what were you thinking, Reba?
2. S: I figured it out by tally marks…I did…tally marks and took those away. I did 14 tally marks and took 5 away.
3. T: Okay, so we are going to make a chart, and it is going to be called Subtraction Strategies, because I want you guys to share what you did to figure this problem out. Reba said she did tally marks. How many tally marks did you start with Reba?
4. S: 14.
5. T: 14 – [looking at Reba's paper] so this is how Reba did it [draws tally marks and says] 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14. Okay, then what did you do?
6. S: I crossed out 5.
7. T: You crossed out 5 – let me see how you did it [looks over at Reba's paper then crosses out tally marks as she says] 1, 2, 3, 4, 5. Now why did you cross out 5?
8. S: Because that's how – because 5 children went home.
9. T: Then 5 children went home. And what does that mean when you cross them out?
10. S: It means those people don't count anymore.

In the above exchange several aspects of Ms. Arielle's practice are noteworthy. Significantly, this exchange does not focus on the answer to the problem, but instead focuses on the process that Reba used as she worked through the problem. From the very first question asked in Turn 1, "So what were you thinking Reba?" it is evident that Reba's thinking is the object of the conversation rather than the correct answer.

If the social norm in this classroom were to respond with a correct answer, then the sequence would have ended after Reba's initial response in Turn 2 with Ms. Arielle offering an evaluation such as "good job." Instead of using the third turn in this exchange as a place to evaluate Reba's response, as in a traditional IRE sequence, Ms. Arielle uses it as a place to focus on Reba's solution. In Turn 3 Ms. Arielle begins to chart Reba's ideas so that everyone in the class can examine how Reba solved the problem. Not only are students expected to publicly share their ideas, but Ms. Arielle is responsible for publicly recording their ideas so that students can consider and compare their solution methods to others previously shared. It was noticeable during this observation that Ms. Arielle was mindful of ensuring that what she wrote on the chart paper was what the student had drawn or recorded on their paper (see brackets in Turns 5 and 7).

Ms. Arielle continues to engage Reba in conversation by asking her to explain how she solved the problem in Turn 5 and then justifies her method in Turn 7. Moreover, each time that Ms. Arielle asks Reba to explain or justify, Reba is able to respond appropriately, indicating that she is prepared to explain and justify her method of solution. When Ms. Arielle asks in Turn 5, "Then what did you do?" Reba provides an explanation by saying in Turn 6, "I crossed out 5." Then in Turn 7, Ms. Arielle elicits a justification when she asks, "Now why did you cross out 5?" and Reba responds in Turn 8 with a justification, "Because that's how – because 5 children went home." And finally, in Turn 9, when Ms. Arielle asks "What does it mean when you cross them out?" Reba again responds appropriately with an explanation in Turn 10, when she replies, "It means those people don't count anymore." The exchange continued for several more turns, revealing the detailed manner in which Ms. Arielle engages students in the process of explanation and justification.

11. T: They don't count anymore because we are looking for what?
12. S: The answer.
13. T: What was the question? – what were we trying to find out?
14. S: How many children were left in the park?
15. T: How many children were left in the park? - So these went home [pointing to the crossed out tally marks on the chart paper]. So then what did you do?
16. S: And then I counted the rest of them, and then I …came up with 9.
17. T: 9, let's see [counts the tally marks not crossed out]. And I also see you have an equation on your paper. What is your equation?
18. S: 14 take away 5 equals 9 children.

19. T: 14 children minus 5 children equal 9 children. Raise your hand if you did like this with tally marks. Did anyone do it a different way? Something different?

In Turn 11 Ms. Arielle continued the conversation by asking Reba to explain how crossing out was related to the problem at hand when she commented, “They don’t count anymore because we are looking for what?” When Reba responded in turn 12 with “the answer,” Ms. Arielle effectively took the focus off finding the answer and placed it back on solving the problem when she asked in Turn 13, “What was the question? What were we trying to find out?” Reba responded in Turn 14 with, “How many children were left in the park,” which offered Ms. Arielle another opportunity to probe further. In Turn 15 Ms. Arielle confirmed that the purpose was to find out how many children were left in the park and then pointing to the chart paper that documented Reba’s work asked, “So then what did you do?” This question provides Reba an opportunity to explain that she counts the remaining tally marks and comes up with 9. Ms. Arielle notices that Reba had written an equation on her paper and asks Reba in Turn 17 to share her equation. Lastly, after Reba shares her equation in Turn 18, Ms. Arielle revoices Reba’s equation and inserts “minus” in place of “take away,” effectively providing students with more accurate terminology.

Adopting the practice of eliciting student explanations and/or justifications serves to focus the conversation on the process that students go through to solve math problems rather than on the correct answer to a particular problem. What is especially noteworthy here is that Ms. Arielle does not indicate that Reba had found the correct answer. Instead in Turn 19, Ms. Arielle asks, “Did anyone do it differently,” again taking the focus off a correct answer and putting it on the process students engage in to solve the problem.

From exchanges such as this, it was evident that students in Ms. Arielle's class were obliged to share their thinking and explain and justify their solution methods and students understood this obligation and responded accordingly. Moreover, students are obligated to listen to their peers because Ms. Arielle would always ask if anyone solved the problem differently. In order to effectively contribute to the conversation, students need to listen and compare their solution methods to ones already shared before volunteering to engage in the conversation at hand.

Ms. Arielle orchestrates conversations by following up student responses with requests that effectively obligate them to explain and/or justify how they solve problems. In the following exchange Ms. Arielle is orchestrating a whole class conversation focusing on the following problem:

Sally washed 8 paint brushes, and Pete washed 11 paint brushes. How many brushes did they wash? (Observation 2, p. 4)

1. T: Okay, who has a different way? What did you do? Nicholas?
2. S: I counted on.
3. T: Come up here and show the class how you counted.
4. S: I put 11 in my head [touched head and counted on 8 using his fingers].
5. T: Why did you put 11 in your head?
6. S: Because that's the big number, and Pete washed 11 brushes.
7. T: Okay and why did you count on 8?
8. S: Because Sally washed 8 brushes and I needed to … (Inaudible).
9. T: Okay, so that's another way you could use. You can put the big number in your head, and count on just like Nicholas did.

In Turn 1 Ms. Arielle "Okay. What did you do, Nicholas?" and Nicholas responded by naming the strategy he used. Ms. Arielle requested in Turn 3 that Nicholas "show" the class what he had done, and Nicholas responded by physically acting out his solution to the problem. Nicholas touched his head as though physically putting 11 inside and then used his fingers and counted on the number of brushes that Sally washed. Ms. Arielle focuses the conversation on Nicholas's thought process by asking him to justify his method when she asks in Turn 5, "Why did you put 11 in your head?" and again in Turn 7 "Why did you count on 8?" Nicholas responded appropriately to Ms. Arielle's requests, indicating that he understood his obligation in the conversation was to provide explanations and justifications of his method of solution.

To be successful in this conversation, Nicholas needs to reflect on his own thinking in relationship to the questions that Ms. Arielle asks and in relationship to the original problem. Moreover, to successfully engage in this conversation, Nicholas needs to understand his obligation at the onset, thus it can be inferred that the taken-as-shared reality in this classroom is that students are obligated to explain and justify their solutions. Again, as in the previous exchange, the focus is in the process students use rather than on the answer.

The following exchange again reveals the taken-as-shared reality that obligates students to explain and justify their solution methods. In this exchange Sally is sharing a different way to solve the following problem (Observation 2, p. 5).

Kim had 11 stamps in her collection. Then her Grandmother gave her 4 more stamps. How many stamps does Kim have altogether?

1. T: How about a different way?
2. S: Instead of 11 I did 10?
3. T: Why did you do 10? I don't see a 10 here? Where did you get the 10?
4. S: I took one from the 11 and then I added the plus 4…(inaudible)
5. T: Okay, and then what did you do?
6. S: I did 10 plus 5 equal 15.
7. T: How did you know 10 plus 5 equals 15?
8. S: I counted 10, 15.
9. T: How did you know to count 10, 15?
10. S: Because my mom taught me to count by 5s.

11. T: Okay, thank your mom. Another way we could solve this problem?

In this exchange Ms. Arielle asks the student multiple questions regarding how and why she solves the problem in the way that she does. What is important to note in this exchange is that when Ms. Arielle challenges the student in Turn 3 and asks, “Why did you do 10? Where did you get the 10 from?” she met the challenge by providing a justification in Turn 4, stating, “I took 1 from the 11 and added the plus 4….” In this exchange the student is cognizant of her thought process and is able to successfully provide a justification for using 10 in her solution method. Students in this classroom are well aware of their obligation to explain and justify their methods of solution and enter into conversations very well prepared even when challenged by the teacher.

Eliciting explanations and justifications in Ms. Arielle’s classroom serves to help students reflect on their own process and also provides the means by which students can consider a peer’s thought process. Moreover, eliciting explanations and justification helps Ms. Arielle to understand the divergent ways in which her students approach math problems. When asked to reflect on why she asks students to explain their thinking she said,

> Well, it’s actually doing two things, like sometimes I’m really trying to tell them to explain to me because I don’t know what they did. And I really want to understand because sometimes at first I’ll say, “Wait a minute….” Like I remember a couple of times I did say, “That’s too confusing. Let’s try to think of a simpler way,” but then once I stopped doing that and said “Okay we’ll try it,” and I kept digging more to see if they could really explain it, then I would really understand what it was they were doing. And it was simple, and I mean, they understood it, and then I got to understand it. So that helped me later on not say, “Well, you know, that’s too confusing,” and I would really try to get them to explain. So it was to help them understand what they did and maybe to get other kids to understand, but it was actually more for me, for me to really understand their thinking. (Interview, p. 11)

Eliciting explanations and justifications helps Ms. Arielle to develop an appreciation for the divergent ways in which her students go about solving math problems. Moreover, eliciting explanations and justifications prompts her to assess the "logic in their method" (Interview p. 11). Ms. Arielle also uses student explanations and justifications as a means to help other students who might be struggling with a particular problem or concept. During an interview she discussed how she uses her students' explanations and justification as a means to help other students in the class. She explained,

> I've learned how to use what they did, and if it works for them, it might work for someone else who's struggling who doesn't necessarily know what they could do and maybe the way I'm teaching them is not working. So I take what the other kid did and use it with that other student thinking maybe that would help them. (Interview, p. 12)

Moreover, Ms. Arielle utilizes student explanations and/or justifications as a means to assess if students understand a particular math concept. When reflecting on what she would consider a successful teaching moment, her response circled back to students sharing their explanations and she said,

> Just if I heard the students click, like when they were sharing something or when they were playing and actually figured out how what they were doing connected to the concept. Just if I heard the students say, "Oh now I get it or I got it" or when they explained it, they were excited about it. (Interview, p. 16)

The exchanges described in this section are indicative of the ways in which Ms. Ms. Arielle consistently elicits explanations and justifications from her first grade students. Significantly, many such exchanges begin with Ms. Arielle requesting that students share a different way to solve the same problem, thus the practice of eliciting explanations and justifications emerges from the practice of eliciting different solutions to the same problem. The following section will examine Ms. Arielle's practice of eliciting different solutions to the same problem.

Elicited Different Solutions

As a means to engage her students in mathematical conversations Ms. Arielle consistently utilizes the reform practice of eliciting from students different solutions to the same problem. As seen in Table 5.1, Ms. Arielle elicited different solutions 42 times over the course of five observations. Moreover, Table 5.1 reveals that Ms. Arielle elicited different solutions during each lesson observed; however, during Observation 4 and 5 this practice was utilized less often. Analysis revealed during these two lessons that students were in the process of learning to play a new math game, thus much of the conversation focused on directing the students how to play the game.

To engage students in sharing different methods of solution, Ms. Arielle would ask, "Did anyone do it differently?" (Observation 1, p. 2) or "Okay, how about another way to solve it – who has a different way?" (Observation 2, p. 5). The following is an example of the way in which Ms. Duncan elicited different solutions to the same math problem and an example of the way in which students responded to her request. Moreover, it reveals the way in which Ms. Duncan utilizes the practice of eliciting different solutions as a means to elicit explanations and justifications. In this exchange students had been given the following word problem and asked to share how they solved it. After eliciting one way to solve the problem, Ms. Arielle then asks students if anyone solved it differently.

> There were 14 children in the park. Then 5 children went home. How many children were left in the park? (Observation 2, p. 1)

1. T: Anyone have a different way? A different way?
2. S: I used the number line.
3. T: Can you show us how you used the number line?
4. S: [Students walks to the front of the room to use the large number line taped to the wall] I started on 14 and landed on 9.
5. T: How did you know to stop on 9?
6. S: Because I went back 5 for the kids that left the park.

As seen in Turn 1, Ms. Arielle elicits a different way to solve 14 – 5. When a student responded inTurn 2 with, "I used the number line" Ms. Arielle follows up with a request for the student to physically explain how he had solved the problem when she said, "Show us how you used the number line." When the student responded with an explanation in Turn 4, "I started on 14 and landed on 9," Ms. Arielle followed up again with a request for a justification in Turn 5, "How did you know to stop on 9?" to which the student supplied a justification in Turn 6 when he said, "Because I went back 5 for the kids that left the park." By engaging in the practice of eliciting different solutions Ms. Arielle is able to elicit explanations and justification regarding how her students go about solving problems.

Ms. Arielle's teaching practice is grounded in eliciting different ways to solve the same problem. According to Ms. Arielle, she used small individual white boards as a means for students to record and explain how they solved a problem. According to Ms. Arielle, the white boards "kinesthetically" engaged her students and also provided a

> visual right in front of them and then that serves also like whatever they did they can share because they have it right there and they can just go back to it. (Interview, p. 12)

Ms. Arielle acknowledges that ultimately an answer would end up on the white board, but what she is eliciting from students is how they arrived at an answer, as she is interested in their thought process or as she said, “Whatever they were trying to get at, whatever they were trying to do in their head goes onto the white board” (Interview, p. 12).

Another indication that Ms. Arielle places value on fostering the social norm of solving problems differently is revealed when she talks about the importance of finding different ways to solve the same problem. When asked if she encourages her students to find different methods of solution she responded with linking the question back to individual ways of knowing and thinking. The following quote is indicative of the value she places on individual thinking:

> You know, kids think differently and you know, the traditional algorithm is not the only way of getting to solution. I know students have different ways of thinking, and I know they do things differently, and also because if that way that we shared doesn’t work, it doesn’t really click with the student, maybe someone else’s way will click with another student who didn’t think of that way. It also let’s them know that there’s just not one right way to do it. There’s different ways, you can get to the answer in different ways, and they’re all fine or correct. (Interview, p. 19)

Moreover, Ms. Arielle indicates that the process of sharing different solutions can serve several different and important goals within the mathematics classroom, such as unpacking thinking, building confidence, and helping students to make sense of the mathematics being discussed. She said,

> I notice when kids are sharing, they realize how they got it [their answer] and that is important to them knowing that, you know, they were good in math and they understood. Then I also noticed how other kids also, you know, if they didn’t get it when I introduced the concept or while they were doing it they would sometimes get it when their peers were talking about it. (Interview, p. 8)

Because Ms. Arielle engages in the practice of asking students to solve problems in different ways, she naturally utilizes the practice of asking students to explain and/or justify their methods of solution. Thus, eliciting explanations and/or justifications emerged from the practice of eliciting different solutions.

Asked QWKAs and Evaluated/Accepted Student Responses

As seen in Table 5.1, Ms. Arielle did utilize the practice of asking QWKAs; however, QWKAs were not used as a means to evaluate student knowledge and to end a sequence, as in the traditional IRE sequence. For example, Ms. Arielle often asked a QWKA in conjunction with revoicing a student's solution as a means to clarify a student's method. The following example illustrates how she would continue a sequence to uncover what a student was thinking using a QWKA (Observation 1, p. 10).

1. T: So, wait – you add the three and the four and you would have how much in total? Three plus four equals what?
2. S: Seven
3. T: And then what would you do?
4. S: I would count on to 13.
5. T: You would count on to 13. Why would you count on to 13?
6. S: Because she had 13 stickers.

In this exchange Ms. Arielle asked a QWKA in Turn 1; however, rather than evaluating the student's response and ending the sequence, she follows up in Turn 3 with a question that allows the student to continue explaining what she had done. In Turn 5 Ms. Arielle continues the sequence by asking another follow up question that effectively requests a justification of counting on to 13 wherein the student replied in Turn 6 with a justification. Clearly, the focus was on how the student solved the problem and not on the students answer to the QWKA.

Ms. Arielle also asked QWKAs as a means to elicit different solutions to a particular problem. The following is an example of a QWKA that Ms. Arielle asked as a means to elicit several different solution methods (Observation 2, p. 4).

1. T: Okay, close your eyes. Kim had 11 stamps in her collection. Then her grandmother gave her 4 more stamps. How many stamps does Kim have altogether? Close your eyes and make an image. Let's think about it in our heads. Kim had 11 stamps and her Grandmother gave her 4 more stamps. How many stamps does Kim have altogether? Who can retell the story?
2. S: Kim had 11 stamps, and her Grandmother gave her 4 more. How many stamps does she have altogether?
3. T: Good. Who else thinks they can retell the story?
4. S: Kim has 11 stamps, and her Grandma gave her 4 more. How many does she have now?
5. T: Very good. Now I have a question: Is Kim going to have more than 11 or less than 11 stamps?
6. S: More.
7. T: Why do you think she will have more than 11?
8. S: Because she had 11, and she got more from her Grandmother, so it will be more than 11.
9. T. Okay, so it will be more. Now can someone come up and solve the problem?

Ms. Arielle began by asking a question that she clearly knew the answer to; however, she does not use the question as a means to elicit a correct answer. She uses it as a means to guide students toward understanding first what the question meant and second to elicit different solution to the problem posed. Moreover, Ms. Arielle does not ask students to solve the problem until Turn 9. After asking the QWKA in Turn 1, Ms. Arielle effectively took the focus off the answer by asking students to retell the story. After two appropriate retellings in Turns 2 and 4, Ms. Arielle overtly evaluated the retellings by saying "Good" in Turn 3 and "Very good" in Turn 5. Even at this point Ms. Arielle does not focus the students on solving the problem. Instead, she asks another QWKA in Turn 7 "Now I am going to ask a question: Is Kim going to have more than 11 or less than 11?"

When a student responds with a correct answer in Turn 8, Ms. Arielle does not evaluate the response by saying correct or good job. Instead, she effectively deepened the conversation by asking in Turn 8, “Why do you think she will have more?” When the student provides a valid explanation in Turn 8, Ms. Arielle accepts it with her comment “Okay,” and then asks the students to solve the problem.

In the exchange above Ms. Arielle used a QWKA as a means to elicit how students were thinking about the problem she posed and as a means to elicit different solutions. Moreover, Ms. Arielle generates 4 different solutions to the addition problem 11 + 4 = 15. It is evident from the exchange above that the focus of such conversations is on student thinking and understanding and not on eliciting a correct answer.

Ms. Arielle often engages her students in playing mathematical games and as a means to help them to understand how to play the game, she utilizes the practice of asking QWKA. In the following exchange Ms. Arielle asked several QWKAs as she is teaching students how to play a math game called Ten go Fish (Observation 3, p. 3).

1. T: Now you are going to look at your cards. Look to see if you have any cards that will make 10. Do I have any cards that will make 10? Nicholas?
2. S: 0 plus 10.
3. T: 0 plus 10. So I am going to take these 2 and going to put them on the side, and I am going to record again, and I am going to keep looking. Do I have any more cards that make 10?
4. S: No
5. T: No. So now it is my turn to ask my partner to see if he has a card I could use. And you know what. I remember before Susan said 3 plus 7 is 10. So then I know the opposite also 7 plus 3 so I am going to ask Danny – do you have a 3?

In the above exchange, as in the previous ones, Ms. Duncan doed not ask a QWKA as a means to assess her students’ mathematical knowledge; rather, she uses the questions to develop students' understanding of the combinations that make 10 and their understanding of how to play the game Ten Go Fish.

Lastly, the following exchange reveals how Ms. Arielle uses QWKA to give ownership of mathematical knowledge back to students. In this exchange Ms. Arielle asked a QWKA; however, rather than evaluate the response, she accepts what the student says in a matter-of-fact manner, thereby giving the impression that the student was a valid knower of mathematics (Observation 3, p. 3).

1. T: Anna, what number should I get to make 10 to go with this 4?
2. S: 6.
3. T: I am going to look up and yup I have a 6 – so like Anna said, "4 plus 6 equals 10".

The above exchange was part of a larger conversation between Ms. Arielle and Anna that incorporated five follow up moves by Ms. Arielle. At this point in the exchange Ms. Arielle asked a QWKA in Turn 1 to which Anna supplied the correct answer. Rather than evaluating the answer and placing herself in the position of mathematical authority, Ms. Arielle simply stated in a mater-of-fact manner that 6 was a choice and then gave ownership of the knowledge back to the Anna by ending her commentary with, "so like Anna said, '4 plus 6 equals 10'".

As seen in Table 5.1, Ms. Arielle did not often evaluate the correctness of student responses with only five overt evaluations given over the course of five observations. When she did offer an evaluation, it was stated positively, such as "good job" or "wow" and was used to express her excitement regarding a different solution that a student had shared. When a student offered an incorrect answer Ms. Arielle did not immediately evaluate, rather she asked the student to show her how she or he arrived at their answer, thus effectively putting the responsibility of assessing the correctness of an answer into the hands of the student. When discussing how she handles incorrect answers she said.

> I guess for a very long time I think it's more about the process of them practicing their methods, but if they do get the wrong answer, I just have them go back and recheck and do it all over again. You know, I don't just say, "Well that was wrong." I mean I have them go back and look at what they did and see if they can figure out where they went wrong. So I don't necessarily say, "Well, this is where you went wrong." I just have to say, "Well, double check your work." We're big on double checking your work. And if they find they've made a mistake, great, but if the student comes back and he still has the wrong answer, then I might have to, you know, point it out and sit down and figure out where he went wrong. (Interview, p. 17)

Ms. Arielle is interested in developing her students' ability to solve problems successfully as well as developing their ability to determine when an answer was incorrect. To do this she focuses on the process rather than on the answer and engages her students in assessing their own work. However, as evident in the quote above, she understands that there are times when, as the teacher, she needs to intervene and help a student figure things out.

In an effort to help students assess the correctness of their answers, Ms. Arielle utilizes the practice of accepting student responses rather than evaluating them. As seen in Table 5.1, Ms. Arielle used the practice of accepting student responses 31 times while in conversation with students. Ms. Arielle's acceptances are neutral in tone and do not take on the tone of an evaluation. The word she uses most often to accept a student response is "Okay." Her practice of accepting student responses constitutes the norm that students are "knowers of mathematical information" and can assess the correctness of their own responses. The following exchange is an example of how Ms. Arielle responds to a student's incorrect answer and how she helps the student to assess the correctness of his response (Observation 3, p. 8).

1. S: 20 minus 11 equals 10.
2. T: Okay, show me that on the number line where you will end up [as the classroom conversation continues with other students sharing different combinations that make 10 the student goes to the number line and works out 20 – 11].
3. S: Ms. Arielle, it's not 20 minus 11 equals 10. It is 20 minus 10 equals 10.
4. T: Okay.

Ms. Arielle's neutral comment in Turn 2 "Okay" served to accept Jacob's response rather than evaluate it. When Ms. Arielle said, "Okay," it was said in a very matter-of-fact manner without judgment or questioning attached to it. Next, Ms. Arielle handed the responsibility of figuring out the correctness of the response over to Jacob by directing him to the number line. When Jacob returned after using the number line, he reported on his findings, thereby taking ownership of not only his miscalculation but the correct calculation as well. By accepting student responses rather than evaluating them in terms of correctness Ms. Arielle is developing a sense of autonomy within her students. Her students are able to contribute to conversations as knowers of mathematics rather than only as receivers of information.

Summary

This section described the practices that Ms. Arielle uses to initiate and guide the participation structure in her classroom. The most prominent practice that Ms. Arielle uses is that of eliciting explanations and/or justifications from her students followed by eliciting different solutions to the same problem. Because Ms. Arielle consistently asks her students to solve problems in different ways she affords herself ample opportunities to ask students to explain how they solve a particular problem and justify why they use a particular method. Moreover, because Ms. Arielle asks students to share different methods of solution the social norm that is constituted implicitly obligates students to listen to how others solved a problem and compare their solutions before making a contribution to the conversation at hand.

Ms. Arielle at times utilizes practices that are considered traditional, such as, asking QWKAs and evaluating and accepting student responses, albeit she used these practices in reform-oriented ways. For example, she asks QWKAs as a means to clarify and highlight student thinking and not as a means to test student content knowledge as in traditional classrooms. Moreover, when Ms. Arielle uses the practice of evaluation, it is used to evaluate and highlight a student's method of solution, for example, "Wow this is what Reba did" rather than as a means to evaluate the correctness of a student's answer. Moreover, Ms. Arielle uses the practice of accepting student responses in ways that indicate that students are knowers of mathematics, and as such, this practice helps to foster student autonomy.

The practices that Ms. Arielle uses constituted social norms that reflect those advocated by mathematics reform. As Ms. Arielle and her students engage in mathematical conversations, they interactively establishe the following as taken-as-shared participation structure:

- Teacher elicited students' different solution methods
- Teacher elicited student explanations and justifications of their solutions
- Students were obligated to provide explanations and justifications of their solutions.

- Students possessed mathematical authority in that they were expected to assess the correctness of their solutions.
- Students were expected to listen to determine if their solution was different.
- Teacher was the authority in that she facilitated the conversations

The social norms listed above were not set out in advance as a set of rules that teacher and students were obligated to follow; rather, they are developed as Ms. Arielle and her students interact together in conversation. For example, because Ms. Arielle makes a practice of eliciting explanations and/or justification, students understand that when she asks them to share their solutions, the request also obligates them to explain and/or justify their solution methods. Consequently, when students engage in solving math problems they need to be cognizant of their process in order to successfully engage in subsequent mathematical conversations. Moreover, because Ms. Arielle utilizes the practice of asking students to share different ways to solve the same problem, students are obligated to listen to their peers solutions so that they can make comparisons between what they do and determine if their method is different from one previously shared. Accordingly, students are obligated to think about their process in comparison to their peers' process before making a contribution to the conversation.

Ms. Arielle's teaching practices constitute a participation structure that mirrors reform recommendations. Ms. Arielle consistently asks students to explain and justify their thinking orally and in writing. Moreover, there is an expectation that students listen to other's contributions before making one of their own. In this classroom, students understand that Ms. Arielle is interested in their thinking and were adept at offering up how they go about solving mathematical problems.

Lastly, the participation structure that developed in Ms. Arielle's classroom includes students participating in the process of assessing and evaluating the correctness of their solutions. As such, this fostered a sense of autonomy in students in that they do not rely solely on Ms. Arielle as the mathematical authority but appeal to their own sense of mathematical knowledge in judging the correctness of their solutions. The social norms that Ms. Arielle's practices foster are linked to the socialmathematical norms that ultimately emerge. The following section will examine Ms. Arielle's practices in relationship to the socialmathematical norms that emerged.

Practices Fostering Socialmathematical Norms

Socialmathematical norms constitute the understandings that students develop as to what counts as a mathematically different, efficient, or sophisticated solution. Moreover, what counts as an acceptable mathematical explanation and justification is considered a socialmathematical norm. Table 5.2 reveals the distribution of practices that Ms. Arielle used while engaging students in mathematical conversations as well as the number of times each practice was used per hour.

Table 5.2 Observed Practice Fostering Socialmathematical Norms: Ms. Arielle

	Observation						
	OB1	OB2	OB3	OB4	OB5	Total	Per Hour
Practice							
Developed Idea of Mathematically Different	4	1	1	3	1	10	5.3
Developed Idea of Mathematically Efficient/Sophisticated	3	0	1	4	1	9	4.7
Made Student Thinking Public	25	15	17	15	18	90	47.4
Accepted One Word Answers	1	0	15	1	12	29	15.3
Indicated Math was Rule Bound	0	0	0	0	0	0	0

Number indicates number of times code occurred

As seen in Table 5.2, Ms. Arielle utilized the following reform-oriented practices: Developed Idea of Mathematically Different; Developed Idea of Mathematically Efficient/Sophisticated; and Made Student Thinking Public. Ms. Arielle also utilized the traditional practice of: Accepted One Word Answers; however, there were no instances where she indicated that Math was Bounded by Rules. The following section will examine her practices and the socialmathematical norms that were constituted as a result of the practices she utilized as a means to engage her students in mathematical conversations.

Mathematically Different

Because Ms Arielle regularly asks her students if anyone has solved a particular problem in a different way, she affords herself several opportunities to comment on solutions that are not different from ones previously shared, thus constituting the norm of mathematical difference. Thus, the socialmathematical norm of mathematical difference emerges from the social norm of eliciting different solutions. As seen in Table 5.2, there were 10 instances where Ms. Arielle negotiated with her students the socialmathematical norm of mathematical difference. With this practice, Ms. Arielle is interactively establishing with her students what it means to have a different solution to the same problem. When she asks her students to share a different way to solve a problem she wants students to develop an understanding of what it means to have a different mathematical solution. During the interview Ms. Arielle commented on the importance of getting her students to understand what it means to share a mathematically different solution. She said,

> I have found that kids will just share the same type of way of getting the answer, and they'll keep just raising their hand to share the same one so it's more of, you know, not only them understanding, 'Well that's the same way.' I also want them to understand that I don't want them just to share because they want to share and hear their voice, I want them to know that I want a different way. (Interview, p. 19)

One way for her to establish what it means to share a mathematically different solution is by pointing out for students when a solution was not different. Although Ms. Arielle values students' thinking and encourages her students to share, she also wants them to develop an understanding of what an acceptable mathematical contribution sounds like when a request is made for a different solution. Although Ms. Arielle made 10 attempts to comment on the mathematical difference between solutions, there were missed opportunities when Ms. Arielle does not interject when a solution was mathematically the same as one previously shared.

The following is an example of Ms. Arielle establishing the socialmathematical norm of what constitutes a mathematically different solution after a student had shared a method that was not different (Observation 1, p. 2).

> 1.T: Anybody do it differently? Differently?
> 2.S: I did – I still did all of them and I took – I filled in all 10 and then I took away - I took away 7. And then I came up with 3.
> 3.T: That sounds like the way Andrew did it.

In this case, Ms. Arielle makes a point in Turn 3 of letting Mia and the rest of the class know that what Mia had shared is the same as what Adam had shared previously. This exchange suggests that Ms. Arielle is negotiating within the public discourse space that an acceptable contribution in this case means more than restating a previously shared solution. Although in this case she brings to the fore of the conversation that Mia's solution sounds similar to Andrew's, Ms. Arielle misses an opportunity to highlight how the two solutions are similar as well as misses an opportunity to help her students to make such distinctions in future exchanges.

Further along in the lesson, another opportunity surfaced when a student shared a method that was the same as one previously shared and again Ms. Arielle used this as opportunity to highlight that an acceptable different solution means that the solutions need to be mathematically different. Moreover, she made more of an attempt to highlight for students the similarity between a previously shared solution (Observation 1, p. 3).

1. S: I colored them all in, and then I counted them, and then I remembered there was one more so I erased 4 of them…
2. T: So you drew 10 dots, and then did you erased 4 of them first or all 5?
3. S: I erased 4 of them.
4. T: You erased 4 – then that's the way that Michael also did it, right. Did anyone do something different?

In the exchange above Ms. Arielle again publicly lets her students know that the solution shared was the same as Michael's; however, she expands a bit more in this case by asking in Turn 2, "Did you erase 4 of them first or all 5?" Here she was more direct with letting students know that by erasing four of the dots, the student had figured out the dot arrangement the same way that Michael had, thus his solution was mathematically the same.

In the next exchange a student responds to Ms. Arielle's request for a different way to solve the problem with a method that had been previously shared, and rather than ask the student to explain further, she let him know at the onset that his method was the same. Students were working to solve the following problem (Observation 2, p. 5):

1. T: Okay, so he put 11 in his head and counted on using his fingers 4 more stamps. Okay, How about another way to solve it. Who has a different way?
2. S: You could solve it counting on.
3. T: Yes, and that is the same as what Nicholas just did. He counted on. How about a different way?

Ms. Arielle's implicit expectation in these exchanges is that students are to share a solution that is mathematically different and that an acceptable different solution requires more than a restatement of a previously shared solution (Yackel & Cobb, 1996). Furthermore, theses exchanges serves to "guide the development of a taken-as-shared understanding of what was mathematically significant in such situations" (Yackel & Cobb, 1996, p. 463). Moreover, by negotiating such exchanges, Ms. Arielle was helping her students to understand that what was significant in such situations was not what each students had done but how their solution compared to ones previously shared. In order for students to share a mathematically different solution, they first need to attentively listen to others, compare their solution, and make a judgment as to whether or not it is appropriate to contribute to the conversation.

The following exchange supports the conjecture that students in Ms. Arielle's class are developing a taken-as-shared understanding of what it means to share a mathematically different solution. In this particular exchange a student challenged Ms. Arielle when she assumed that he had solved a problem in a particular way (Observation 1, p. 1).

1. T: So how many dots did you see Nicholas?
2. S: I saw 3 dots.
3. T: How did you know to draw 3 dots down? [colors in 3 dots down]
4. S: Because… I did it different. I didn't do like 3 dots.
5. T: What did you do?
6. S: I put 5 dots down, and then when you put it up again, I took away 2 dots, and it equals 3.

Here Nicholas' comment in Turn 4 indicates that he had listen to what Ms. Arielle said and concluded that what he actually did was mathematically different from what she assumed. Moreover, this exchange indicates that Nicholas is confident enough to suggest to Ms. Arielle that he did not solve the problem the way in which she suggested. Ms. Arielle's comment suggested that he added 3 dots to his white board, whereas Nicholas's explanation in Turn 6 showed that he drew 5 dots and then subtracted 2 dots to come up with his answer of 3. In order to respond to Ms. Arielle, Nicholas needed to reflect on his on his own thought process and compare it to what Ms. Arielle had suggested he did. Thus, in this exchange it can be inferred that Nicholas engaged in a higher level thought process – that of thinking about your own thinking.

As stated previously there are missed opportunities when Ms. Arielle does not comment on how solutions are mathematically the same. In these instances students are offering solutions that are mathematically the same, albeit they solved the problem using different strategies or manipulatives. The following exchange highlights a missed opportunity for Ms. Arielle. Students were working on solving the following problem:

> Fourteen children were playing in the park. Then five went home. How many children were still in the park? (Observation 2, p. 1)

Previously, a student had shared that he used 14 cubes and took 5 cubes away to solve the problem. The following is the next student who responded to Ms. Arielle's request for a different strategy (Observation 2, p. 2).

1. T: Okay. Another strategy you used?
2. S: I drew 14 kids, and then I took 5 away, and then it made 9 left, and I wrote the number sentence 14 minus 9 equals 5.
3. T: Anyone have a different way? A different way?

In this exchange Ms. Arielle asked in Turn 1 for a different strategy and in Turn 2 a student responded appropriately with a different strategy; however, the solution method was mathematically the same as the one previous shared. In this exchange Ms. Arielle may have realized that the solution was not mathematically different, but she held back comment because she had asked in Turn 1 for a different strategy and the student responded appropriately to her request. Ms. Arielle in Turn 3 ended by asking for a "different way" rather than a different strategy, possibly as a means to elicit a mathematically different solution. At this point, she is negotiating this norm for her students and for herself as well. Moreover, Ms. Arielle may be conflicted as to what it means to share a different strategy and what it means to share a mathematically different solution.

The socialmathematical norm of mathematical difference is a work in progress for Ms. Arielle and her first grade students. In the following exchange Ms. Arielle realizes that students are sharing the same mathematical solution to the way in which they counted a group of dots, and she attempts to steer their thinking in another direction in an effort to elicit a mathematically different solution. Here Ms. Arielle showed a group of dots on the overhead projector for five seconds and asked student to observe and draw on their white boards how they remembered the dots. The dots were arranged in a row of seven, a row of three, and a row of two (see Figure 5.1). When Ms. Arielle asked students to share how they remembered and counted the dots, three out of four students shared the same mathematical solution in that they all counted the dots one at a time and wrote 7 + 3 + 2 = 12 as their equation.

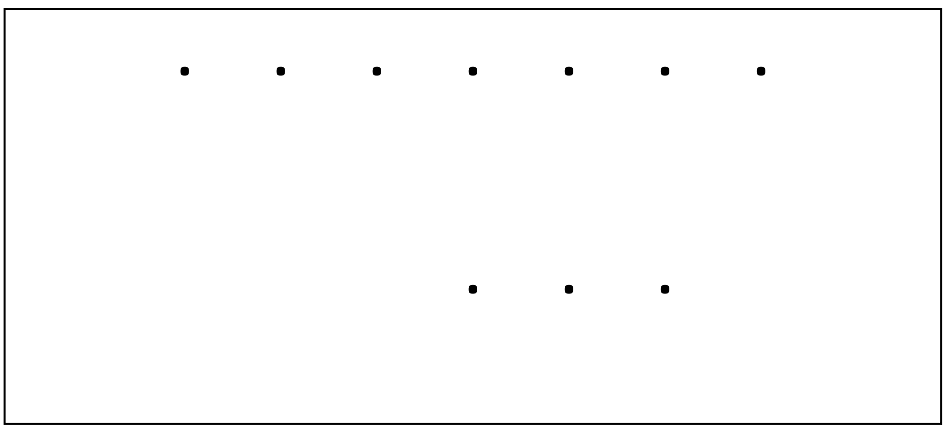


Figure 5.1 Dot formation shown on the overhead projector

Although Ms. Arielle commented each time saying, “That’s the same. You counted the dots one at a time,” students continued to offer the same solution. As a means to help her students develop an understanding of a mathematically different solution she shared the way that she had counted the dots, which may have prompted a student to make a mathematically different contribution to the conversation (Observation 1, p. 8).

1.T: Okay, that’s the same. You counted one at a time. Did anyone think that 7 plus 3 equals what? How much is 7 plus 3? [writes on overhead 7 + 3 =
2.Ss: (in unison) 10!
3.T: 10 and then 10 plus 2 equals what? [7 + 3 =10 and 10+ 2 =]

4.Ss: 12!
5.T: That's another way that I thought of it too. That's a different way. Hold on. Let's share one more way.
6.S: I did 8 plus 2 plus 2 equals 12 [pointing at overhead as though moving the dots around].
7.T: So, did you take one of these away? Over here [pointing to the group of 3 dots]?
8.S: Yes, 1 from the 3 there on to the 7, so that's 8.
9.T: So he did! Eight plus 2 plus 2 equals 12! That's another way!

In this exchange Ms. Arielle focuses her students' attention on a different way to see and count the dots by sharing that she saw the dots as 7 + 3 = 10 and 10 + 2 = 12. In Turn 6 a student offered a mathematically different way as he saw the dots as 8 + 2 = 10 and 10 + 2 =12. In can be inferred that the student listened to what Ms. Arielle shared and compared his solution to hers and made the determination that he had done something different ,thus prompting him to contribute to the conversation at that point.

In the next exchange Ms. Arielle asked Nick to share his solution but before contributing to the conversation he acknowledged that his solution might be the same as one just shared, indicating that he understood that his obligation was to share a mathematically different solution (Observation 1, p. 7).

1.T: A different way. What did you do, Nick?
2.S: I did - I almost did it the same. I put - I knew that 7 plus 3 plus 2 equaled 12.
3.T: How do you now that 7 plus 3 plus 2 equals 12?
4.S: Because it is the same… the 6. It is just one number up – it's just 2 plus 3 plus 7 – like 6 plus 6. It's 1 up and 1 down.
5.T: Okay, that's kind of like what Reba did. Did anybody do it a different way?

In this exchange when Ms. Arielle called on Nick and asked him how he solved the problem, he first let her know that his solution might be the same as the one just shared, and in fact it was. This is evidence that Nick was listening to the previous student and compared his solution with hers and determined that his solution was the same. It also supports the conjecture that Nick has developed an understanding of what a mathematically different solution means in this classroom.

What is interesting to note in these exchanges is that Ms. Arielle does not explain rules or procedures that students must follow in order to share a mathematically different solution; rather, she builds on what students say “in the moment” as a means to develop the socialmathematical norm of difference. The above exchanges are indicative of the way in which the norm of mathematical difference was being negotiated by Ms. Arielle and her students as they engaged in mathematical conversation.

Mathematically Efficient/Sophisticated

As seen in Table 5.2, Ms. Arielle made nine attempts to develop the socialmathematical norm of what counted as an efficient and/or sophisticated solution This norm was established implicitly by Ms. Arielle as she listened to students share their different mathematical solutions and commented on their solution methods. During Observation 4 Ms. Arielle made four comments that were coded as developing the idea of a mathematically efficient and sophisticated solution. During this lesson she and her students were examining how one student counted by 10s rather than by 1s when counting a set of objects. Guiding first grade students toward developing an understanding of counting 10 objects as one group of 10 is a critical, albeit challenging, aspect of the first grade curriculum. When engaging students in conversations about counting by 1s and counting by 10s, teachers are developing the socialmathematical norm of what counts as mathematically efficient and sophisticated way of counting. The following exchange takes place toward the end of the school year as students are in the process of developing an understanding of the efficiency of counting by 10. Ms. Arielle wrote the following objective and assessment in her lesson plan dated June 5, 2008:

> Objective:
> Students will: Add single digit numbers; organize objects to count them more efficiently; and counting by tens.
>
> Assessment:
> Observing how students find the total rolled: How do students figure out how many more cubes they need to complete a row; and how do students figure out how many cubes they have?

From the onset Ms. Arielle was interested in helping her students to develop this critical mathematical concept; however, the manner in which she engages her students in the process is noteworthy. The following exchanges reveal the way in which Ms. Arielle let the idea of counting by 10 develop naturally from students actions on objects. Moreover, the exchanges reveal the challenging nature of helping students to understand the concept of counting by groups of 10.

In the first exchange a student started to count by 10s, and Ms. Arielle considered this to be an efficient and more sophisticated way of counting and decides to make the counting strategy the focus of the conversation. It seems that Ms. Arielle senses that some of her students are beginning to notice the efficiency of counting by 10s, and she wants to make sure that everyone in the class has an opportunity to consider this new way of grouping and counting by 10s instead of 1s. The student was counting 25 unifix cubes that were grouped into 2 towers of 10 and 1 tower of 5 (Observation 4, p. 4).

1. T: How many do I have so far Levi?
2. S: [Counting to himself touching each tower]
3. T: [Smiling] Do it out loud so we can hear you.
4. S: [Says silently to himself] 10, 20, 25.
5. T: [Smiling] How did you know? How did you figure out? How did you count?

6. S: [Started counting again but this time by 1s]
7. T: [Stopping Levi] But did you count by 1? How did you count? I didn't see you count by 1s.
8. Ss: He counted by 10!
9. T: Yes, you counted by 10s! The very first time you counted this one by 10s. How many do I have in each tower? Ana?
10. S: 10. You have 10.
11. T: You have 10 in each tower. So can you guys try to count by 10s?
12. Ss: 10, 20.
13. T: Now if you remember there is 5 here and you can count by 5s. But if no, you can count by 1s. So let's try first by 10 then by 1s.
14. Ss: 10, 20, 21, 22, 23, 24, 25.
15. S: That's what I do!

In this exchange we see that Ms. Arielle set up a situation whereby students could count by 10 if they were ready to. When she asked Levi to count the towers, Levi touched each tower and said silently "10, 20, 25." Ms. Arielle then asked Levi to count out loud so that the entire class could hear how he counted. By taking up his method and bringing it to the center of the conversation, Ms. Arielle implicitly let the group know that what Levi had shared was important for them to consider. Ms. Arielle used Levi's attempt at counting by 10s as opportunity to pursue her pedagogical agenda (see Lesson plan objective) and as a way to focus her students' attention on a mathematical practice (counting by 10s) that is accepted by the wider community (Cobb et al., 1992). As Cobb et al point out, it is critical that the teacher intervene in such situations as she is the one who can "take the norms of the wider society as a reference and judge whether the children's constructions would be productive with regard to their further learning and mathematical enculturation" (p. 594).

In the exchange above Ms. Arielle is negotiating the socialmathematical norm that counting by 10s is a more efficient and sophisticated strategy than counting by 1s. I use the term negotiate here because as seen in the above exchange the efficiency of counting by 10s is not a concept that Ms. Arielle handed down to her students. Rather, she negotiated with Levi and the group what it means to count by 10s and as seen in Levi's response in Turn 6 the efficiency of counting by 10 is a difficult concept to negotiate. In Turn 5 Ms. Arielle asked Levi how he knew to count by 10s, prompting Levi to start counting the towers again, only this time by 1s. This was a new concept for Levi, thus he was moving back and forth between his new understanding (counting by 10s) and what he was sure of (counting by 1s).

In the ensuing exchange, which took place toward the end of Observation 4, Ms. Arielle attempted to bring up the concept of counting by 10s with Jeffrey; however, she realized that he was not ready and allowed him to count by 1s rather than by 10s (Observation 4, p. 6).

1.T: Okay, Jeffrey– how many do I need to count a group of 10? How many 1s do I need in a group of 10? In a group of 10 – how many do I need to count by 10? A group of 10?
2.S: 12?
3.T: In a group of 10. Just like what we do every morning. How many do I need to count by 10? Count by 10 – how many do I need to count by 10? In a group. How many does the group have to have?
4.S: [Looking at the tower of 10 Ms. Arielle is holding up]
5.T: A group of 10? 1, 2, 3, 4, 5, 6, 7, 8, 9, 10. For me to be able to count by 10s, I need to have 10 1s in a group. Ten 1s make a group of 10. So I have one group of 10, and I have another group of 10. So how many do I have so far, Jeffrey?
6.S: Two.
7.T: I have one group of 10 and another group of 10? Come over here and count.
8.S: [Touches each cube and says] 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20.
9.T: 20. So, he counted by 1 – very good. We have 20. So, Jeffrey, you know what we could also do? We could count by groups of 10 because when we make a tower of 10, we know all our groups look the same – they have 10 each – so instead of counting by 1 – it would be faster if we count by groups of 10. So, you can count 10, 20. How much is 10 plus 10?
10. S: 10 plus 10. [taking a bit of time before answering] Twenty!
11. T: 20 – you know how we count by 10? 10, 20!

In the above exchange Ms. Arielle tries to get Jeffrey to count by 10s but was unsuccessful. Watching her on the videotape, you can sense her frustration as she tries patiently to help Jeffrey count the towers by groups of 10s. Ms. Arielle finally calls Jeffrey over and asks him to physically count the towers of 10. However, Jeffrey counts the towers by 1s instead of 10s. In Turn 9 Ms. Arielle acknowledges Jeffrey's counting method as a good one, "He counted by 1, very good" and goes on to try to help Jeffrey develop the concept of counting by groups of 10. This is a challenging exchange for Ms. Arielle; however, rather than insisting that Jeffrey count by 10s, she allows him to continue to count in the way that made the most sense to him at that particular point in time.

Lastly, the next exchange again shows Ms. Arielle attempting to establish the socialmathematical norm of efficient counting strategies. Here she is confronted with students who are developing the understanding and others who are not yet ready.

1. T: So, how much do I have now? How many do I have – altogether.
2. S: 10, 20, 25.
3. T: See how he counted. Notice how he counted. By 10s and then he counted by 1s. Adam, can you come over here to help me with the next one. How many do we have now? Count it loud so everybody can hear you.
4. S: 10, 20, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39.
5. T: 39– Good job – Thank you, Adam. See how he counted, Jeffrey? He counted by 10s and when you can't count by 10s anymore, because you don't have any groups of 10, you can count by 1s.
6. T: [Adding another tower of 10] Now how many do I have now? I added a tower – I added a tower of 10? How many do I have now? How many do I have now, Amy?
7. S: [Looking at the towers of 10 but not responding]
8. T: Look, he said we had 39. I'm adding a tower of 10. How many do I have now?
9. S: [Touches cubes and counts]
10. T: Say it out loud so we can hear you.
11. S: 2, 4, 6…

12. T: Wait. You’re counting by 2s! But I thought we’re counting by 10s .Can we try counting by 10s?
13. S: I forgot how to count by 10s.
14. T: Okay, so show me how we do it by 2s

In the above exchange Ms. Arielle continues to work with her students in developing their understanding of counting by groups of 10. There were some students who were ready to count by a group of 10 and others who were clearly not ready for this new efficient more sophisticated strategy. Ms. Arielle senses this from her students and allows individuals the flexibility to count the towers in the way that made the most sense to them. When she comes upon a student who counted by 10s she made a point of acknowledging this new way of counting when she says, “Notice how he counted” or “See how he counted.” In this exchange Ms. Arielle and her students are negotiating the socialmathematical norm of mathematically efficient and sophisticated revealing a concept in the making.

At other times during a lesson Ms. Arielle implicitly maks a point of acknowledging that a solution is more sophisticated than ones previously shared. Consider the following comment she made after Reba shared a different way to count a series of dots arranged in groups of 7, 3, and 2 (see Figure 5.1). (Observation 1, p. 6)

1. S: Since I knew that 6 plus 6 equals 12, I just put it [inaudible]
I just put it from the 7 I took away 1 of the dots [inaudible]
And I put that 1 with the 2s, and I took away all those 3s –
and then I put all the 3s with the other ones [inaudible].
2. T: Oh, goodness. This is what Reba did. Reba says, wait a minute
1, 2, 3, 4, 5, 6 – I have 6 there, and if I take this 1 away, and I put it
with these 2, I have 3 here and 3 here, but now I’m going to put
this 3 with this 3 – 1, 2 3 - so in the end she has 6 plus 6 equals 12
[writes 6 + 6 = 12 on overhead]. Reba I would have never figure
out to do it that way.

In this exchange, the comment, “Oh goodness,” signaled to students that Ms. Arielle was pleased with Reba’s solution (Yackel & Cobb, 1996). Moreover, Ms. Arielle’s comment, “Reba I would have never figured out to do it that way” indicates that Reba’s solution a more sophisticated one. What is important to note here is that Ms. Arielle is not evaluating the correctness of Reba’s solution, rather she is negotiating the norm of mathematical sophistication for herself and her students by using what Reba had shared and making a point of bringing it into the center of the conversation. According to Voight (1995) exchanges such as this serve important functions in the mathematics classroom. Such exchanges help students to become aware of more sophisticated mathematical activity while concurrently leaving it up to the individual to determine if they are ready to engage with it. As will be seen in the next exchange, one student took up the method and used it to solve a different problem. In this exchange students were sharing different solution to the following problem (Observation 1, p. 9).

Alexis went to the store with her dad. She had 13 pennies to spend. She bought one sticker that cost 3 pennies and another sticker that cost 4 pennies. How many pennies did she have left?

1. T: A different way – Nicholas?
2. S: I know that 6 plus 6 equals 12, and then you add 1 more it equals 13. It’s the same. So if you add 1 more it would be 7 plus 6 equals 13.
3. T: Do you guys understand that?
4. S: Yes. No.
5. T: He says, “Wait a minute. I know that 6 plus 6 equals 12, and 12 plus 1 equals 13.” So 13 is 1 more than 12. So 13take away 7 – so 6 plus 6 is 13, so 6 plus 7 equals 13. So he knows that 13 take away 7 has to be 6. Very good.

Possibly Nicholas inferred from the exchange with Reba that Reba’s solution pleased Ms. Arielle, prompting him to take it up and use it to solve the word problem. Again, Ms. Arielle implicitly considers Nicholas’s solution sophisticated, prompting her to ask students if they understand what he had done. When Ms. Arielle said in Turn 5, "He said, 'Wait a minute. I know that 6 plus 6 equals 12," she signaled to the students that Nicholas had noticed something interesting and possibly sophisticated about the numbers that helped him to solve the problem. When Ms. Arielle revoices Nicholas's solution, she effectively makes it public so that others who are ready to take up the challenge have an opportunity to consider the solution method in more detail. Making student thinking public for others to consider is an important aspect of Ms. Arielle's practice, and it serves several functions. The following section will examine this practice in more detail.

Made Mathematical Thinking Public

As seen in Table 5.2, the practice of making Mathematical Thinking Public (MTP) was Ms. Arielle's most utilized practice with 90 of her comments being coded as such. As seen in the previous exchange with Reba, the practice of making MTP brought students' thinking into the public discourse space, and as such, this practice served several purposes. One such purpose was to bring a student's idea to the fore of the conversation so that the idea could be considered by others and comparisons between solution methods could be made. One way that Ms. Arielle accomplishes the goal of bringing students' ideas to the fore of the conversation to be considered and compared by all is by revoicing student explanations while at the same time recording on the overhead or large chart paper the student's solution method. The practice of revoicing and recording student methods serves to make student thinking public, prominent, and available for other students to consider and make comparisons.

The following exchange is an example of how Ms. Arielle utilizes the practice of making MTP as a means to develop students' abilities to consider another solution and make comparisons with their own method. In this exchange students were shown a series of dots on the overhead projector for five seconds and then asked to draw on their white board what they saw (see Figure5.2).

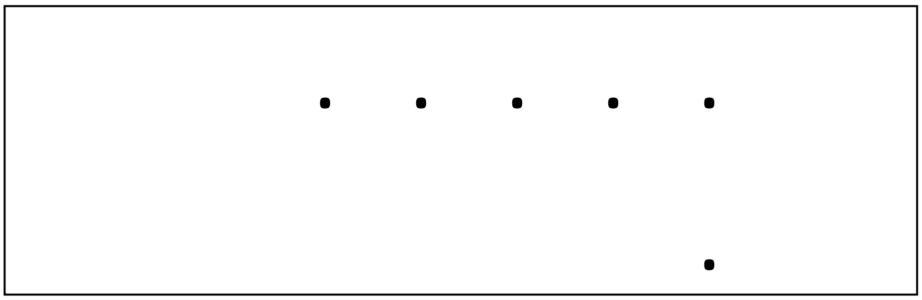


Figure 5.2 Dot formation shown on overhead projector

Ms. Arielle showed the dots again for five more seconds and instructed students to make any necessary changes and prepare to share how they remembered the dots (Observation 1, p. 3).

1. T: Double check. What do you remember? I see some people drew the 10 frame– some people just remember the dots. Let's see – I'm going to show you again. Now. Let's see – have a look around. Let me look around. Michelle – how did you remember to draw your dots?
2. S: I remembered there were 10 and I counted the empty spaces and there were 4 and then I took away the empty spaces from the 10 so that gave me 6 dots.
3. T: So she said… "Wait. Let me get a blank frame." So Michelle says I remembered the 10, and then I took away the empty spaces, and it was 4. Now 10 take away 4 is what Michelle?
4. S: 6.
5. T: 6. Anybody do something different?
6. S: (INAUDIBLE)
7. T: Hold on I hear a lot of talking.
8. S: When you covered it I remembered, and then I did the boxes and I did 5 dots, and then I said, "I think there's more" – I put 1 more. That's 6.
9. T: So, she said I saw 5 dots first, 1, 2, 3, 4, 5, and I remembered there was 1 more that equals 6 dots. That's how she remembered to make 6 dots. Ana?
10. S: I colored in all the dots, and then I counted them, and then I remembered there was 1 more so I erased the 4 of them and that's how I got it.

11. T: So, you drew 10 dots and then – did you erase 4 of them first or all 5?
12. S: 4.
13. T: 4 – then that's like the way Michelle also did it, right? Did anybody do something different? Allia?
14. S: I had (inaudible) I checked and there was 2 more.
15. T: Can you show me up here which 4 you had? And the 2 more. Tell us again – what did you do?
16. S: I had 4 minus 2 and (inaudible)
17. T: Wow. So, Allia saw these 4 first and then these 2. So she said, "I have 4, and then I saw a group of 2, so 4 plus 2 equals 6. Okay.

It was apparent from the conversation that ensued that students were comparing their solutions to ones previously share and making determinations as to whether or not their solution was different. In this exchange Ms. Arielle recorded how students saw the dots by coloring in the dots on the overhead projector and recording the equation that represented the arrangement (see Figure 5.3).

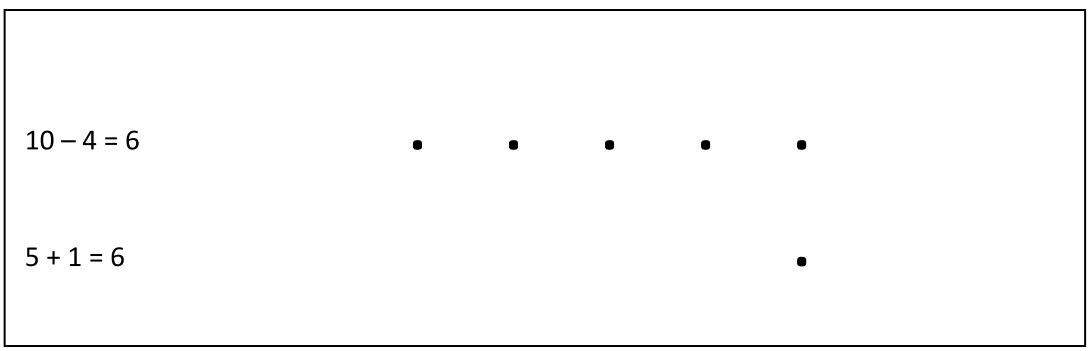


Figure 5.3 Ms. Arielle's recording of student work on the overhead projector

This recording strategy helped students to compare their work with the work of others, and as such, students were able to determine if their solution was the same or different. By publicly revoicing student solutions and recording the mathematical equations she effectively makes students' mathematical thinking available to others so that comparisons can be made. Moreover, by engaging in the practice of making MTP Ms. Arielle supports the development of the socialmathematical norm of mathematically different as she is able in Turn 13 to let a student know that what he had shared was the same as a solution previously shared. Making MTP also supported the socialmathematical norm of mathematically sophisticated. In Turn 17 Ms. Arielle's comment, "Wow," signaled to students that Allia's way of seeing the dots as a group of 4 plus a group of 2 was possibly more sophisticated, as Allia needed to conceptually rearrange the dots in order to come up with her equation.

Another aspect of making MTP served the purpose of obligating student to focus their explanations on the process they engaged in to solve a particular problem. Ms. Arielle requests that students "show" what they do while solving problems, thereby obligating students to physically act on objects representing numbers rather than only state their solution method. For example, when a student said, "I used cubes," Ms. Arielle responded with, "Can you get the cubes and *show* us?" This request to show was made quite often, and it effectively obligated students to make their MTP by physically acting on objects in their explanation. Ms. Arielle utilizes the practice of making MTP as she elicits explanations and/or justification from her students thus she helps students to understand that an acceptable explanation and/or justification required physical actions on objects representing numbers. The following section will examine how relying on the practice of MTP rather than on evaluating student responses helps to foster the socialmathematical norm of what counts as an acceptable mathematical explanation and/or justification in Ms. Arielle's classroom.

Acceptable Mathematical Explanation/Justification

In a reform-oriented classroom, an explanation and/or justification is considered acceptable when students explain and/or justify the process they used to solve a problem, thus one word/correct answers are not the focus in such classrooms. As indicated in Table 5.2, Ms. Arielle did accept one word/correct answers; however, as stated previously, these one word/correct answers are often elicited as a means to clarify the process that a student had used. Additionally, when Ms. Arielle introduces a new game, she accepts one word/correct answers as a means to teach students how a particular game is played. For example, when teaching students how to play the Less Than More Than game, she asked, “So, whatever number goes here will it be bigger or smaller?” and the students responded in unison “Smaller” (Observation 5, p. 3).

Although, Ms. Arielle does accept one word correct answers, for the most part, students are expected to provide explanations and/or justifications of the process they engage in to arrive at an answer because for Ms. Arielle the process is of the utmost importance. She indicated that before focusing on correct answers she devotes her time to assessing the process students use when solving a math problem. She said,

> I think explaining is important because explaining what they’re doing helps me in terms of me figuring out what their thinking is. I don’t know, just because I guess I like to focus more on the process than on the actual answer, just because maybe that process, like what they’re doing right now, will work for them right now, but it won’t for them each and every time in the future. So I think I want to explore more their way of thinking and explaining what they’re doing instead of why the answer’s right or wrong. (Interview, p. 12)

However, upon reflection Ms. Arielle acknowledged the importance of correct answers when determining if students had met a particular benchmark. She said,

> I mean, if the answer is wrong, it is important because I need to assess, when we’re doing the assessment, the student does not meet the benchmark if getting the right answer is part of meeting the benchmark and I mean, that’s why you’re teaching them because that’s what they need to get to - the right answer. (Interview, p. 16)

For Ms. Arielle an acceptable mathematical explanation and/or justification focuses on the process students engage in rather than on correct answers. Because Ms. Arielle is interested in the process students engage, she requires students to provide explanations as descriptions of their actions on objects. Thus, explanations are in the form of actions taken by students, either physically or conceptually, on objects that represented number. The following section will examine the way in which Ms. Arielle's students provided explanations and/or justifications as the physical or conceptual manipulation of objects representing number.

Mathematical Activity Bound by Actions

An acceptable explanation and/or justification in Ms. Arielle's classroom requires students to describe the actions that they take in order to solve a problem. As seen in Table 5.1, Ms Arielle elicited 78 explanations and/or justification over the course of five observations and most required some type of physical action on objects. Ms. Arielle requires her students to "show" what they had done by physically acting on objects that represented number.

In the following exchange Ms. Arielle was introducing a new card game where students were to use cubes to add numbers totaling 10, thus building a Tower of Ten. The objective of the game was for students to practice combinations of 10, but more importantly, to build towers of 10 and start to notice that they could count the towers as groups of 10. During this time the expectation was that students would listen to and watch as Adrianne physically explained her thinking. For Adrianne, the expectation was that an acceptable mathematical explanation meant physically manipulating numbers while explaining what she was doing and make adjustments when necessary. All of this work came from Adrianne with little direction from Ms. Arielle (Observation 3, p. 1).

1. T: I think I'm going to try 4, and you know what I'm going to do – I'm giving you cubes to help me out. We're going to use cubes to help me out. How could you use the cubes? I have 4, and I want to figure out how many more I need to make 10? [Gives cubes to Adrianne.]
2. T: Adrianne, tell us what you are doing.
3. S: I'm putting 4, and then I'm going to put like 8 on the top of 4.
4. T: Come on – we've got to be patient [some students were fidgeting and she brings their attention back to Adrianne.]
5. S: I put – I had – I put 4 cubes, and then I grabbed 8 cubes and counted them to see if it was 8, and it really was 8, and then I put them together like this, and then I counted. I counted 1, 2, 3, 4 and broke this apart. And then I counted this row and then this row and then I counted them all together and it equaled 10.
6. T: So, let's double check to make sure that there is 10. Put the two towers together; now let's double check to make sure that the total equals 10.
7. S: [Physically touches each cube as she counts and says] 1, 2, 3, 4, 5, 6, 7, 8, 9, 10 [stops counting and seems puzzled and then says] 11, 12.
8. T: Anna, we need to be patient. Children, can we scoot back to our places so Anna can see?
9. S: Because last time I had 8, so we doubled checked, and they equaled 11. So, I figured out if you take 1 cube away, then maybe it will equal 10 [takes off one cube and puts aside].
10. T: So, do you have 10 now. Let's see.
11. S: 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, [stops counting and seems puzzled then says] 11. [Takes 1 cube off].
12. T: She's double checking – that's why it is so important to double check.
13. S: I'm counting down – backwards— so I can see which one equals 10 [starts at the bottom of a tower and counts]. 1, 2, 3, 4, 5 6, 7, 8, 9, 10.
14. T: Okay, so we have 10 cubes. So, Adrianne, how many …so after you make your Tower of Ten break off [she breaks the tower] with the number you started with. So, Adrianne, what number should I get to make 10 that will go with the 4?

In this exchange we see Adrianne working through the problem as she physically manipulated cubes. Moreover, Adrianne acted autonomously as she worked through several attempts to make a Tower of Ten. At no time during the 3.5 minute exchange did

Adrianne appeal to Ms. Arielle for help when she was puzzled, as in Turns 7 and 11. Rather, Adrianne appeals to her own sense of authority, making new conjectures and adjustments as she works through the problem. Ms. Arielle's role in this exchange is to direct the class to the work of Adrianne (Turn 4 and 8), letting students know that patience is needed while at the same time guiding Adrianne as she works through the problem (Turns 6, 10, 12).

The way in which Adrianne explains her solution indicates that she understands that explanations should required physical manipulation of objects that represent numbers along with clear detailed explanations of what you are doing. In Turn 2 Ms. Arielle requests an explanation and Adrianne's response in Turn 3 indicates that she understands that she must explain how she physically manipulated the objects as she uses several words that reveal the physical nature of her work, "I' m putting 4, and then I am going to like put 8 on top of 4" (see also Turns 3, 8, 9, and 13). In Turn 5 Adrianne uses several words that describe the physical nature of the work that she is engaged in (grabbed, counted, put them together, broke apart) as she solves the problem, indicating that she developed an understanding that an acceptable mathematical explanation requires giving a detailed explanation of how one acts physically on objects representing numbers.

Moreover, Ms. Arielle often uses words in her instructions that signify to students that the physical manipulation of numbers is necessary. Consider the following way in which she describes for students how they should play the game Make Ten.

> Okay so we have 10 cubes – we have 10 cubes so how many –we have 4 and after she got the 10 cubes – after you make sure you have a tower of 10 cubes, break off the number that you have. So we are going to break off by 4. So what number should I get to make 10? (Observation 3, p. 2)

Here Ms. Arielle instructed student to “break off the number you have” and “so we are going to break off by 4,” indicating that students need to physically act when playing the game Make Ten. And when asking students to compare 10 + 6 and 9 + 7 she said, “Can you build 9 + 7?” as she hands cubes over to a student (Observation 5, p. 12).

When students offer explanations they are obligated to explain as though physically acting on the numbers they are working with. This next exchange reveals the way in which Ms. Arielle orchestrates explanations whereby students are obligated to provide explanations as descriptions of actions. In this exchange a student is explaining her solution to the following word problem (Observation 2, p. 6).

Kim had 11 stamps in her collection. Then her Grandmother gave her 4 more. How many stamps does Kim have altogether?

1.T: Okay, another way you could solve this problem. Anna?
2.S: I could do it with cubes.
3.T: Okay, can you come up and show us how you would use cubes to solve the problem.

4.S: [Counts out 11 cubes] 1, 2, 3, 4, 5, 6, 7, 8, 9, 10,, 11, and then I am adding 4 more. [Begins adding cubes one at a time]
5.T: [Says to the class] So, what do you think Anna is going to do with the cubes?
6.Ss: She is going to count them all to see what they equal.
7.T: Okay, let’s see what she does. Now what are you doing, Anna?
8.S: I counted them all, and I got 15.

As seen in the above exchange Ms. Arielle requires that Anna physically act when explaining how she solved a problem. When she asked in Turn 1 for another way to solve the problem, Anna responded with, “I could do it with cubes.” Rather than just accepting Anna’s response, Ms. Arielle requests that Anna “show” the class how cubes can be used to solve the problem. As a means to keep students engaged and focused on Anna’s explanation, Ms. Arielle asks in Turn 5, “So what do you think Anna is going to do with the cubes?" effectively obligating students to watch and listen to Anna's explanation. In this exchange Anna acts out for the group how she solved the problem with cubes, and as such, her explanation takes time and patience on the part of students and Ms. Arielle. Such exchanges are instrumental in developing the socialmathematical norm of what counts as an acceptable mathematical explanation or justification. The student providing the explanation is obligated to offer a description of her actions while the rest of the group is obligated to listen to the description and moreover make comparisons to her method. Students in this classroom know that they will be asked to share a different way, thus to make an acceptable mathematical contribution to the conversation, they needed to be aware of how their solution compares with their peers' solutions.

When students share that they solved a problem in their head, Ms. Arielle requires that they too offer a physical explanation. Consider the following exchange that took place immediately after the one above, where Ms. Arielle obligates a student to explain how he solved the stamp problem (Observation 2, p. 6).

1.T: Okay how about another way?
2.S: I did it in my head?
3.T: Can you tell us what you did in your head?
4.S: I took one from the 11 and that made 10. Then I added the 1 and the 4 and that was 5.
5.T: Okay, and then what did you do?
6.S: Then I did 10 plus 5 equals 15.

In this exchange when Ms. Arielle asks the student to explain what he had done in his head, he responds using the physical metaphor when he said, "I took 1 from the 11." Here the student explains his conceptual solution as though he physically took 1 off of the 11 to make it 10.

In the two aforementioned exchanges, Ms. Arielle elicits two mathematically different solutions to the same problem. Moreover, she expects that the two students explain the actions that they took as they went about solving the stamp problem. The first student explained the problem by physically taking the cubes, thereby acting out her actions, and the second student used the physical metaphor to explain how he had conceptually solved the same problem.

Significantly, later on in the school year, Ms. Arielle made an observation that is important to note here, as it constitutes the negotiation of a new socialmathematical norm. Previously, students had been obligated to explain and/or justify how they knew that two numbers, such as 6 and 4 equaled 10. Moreover, the expectation was that an acceptable mathematical explanation involved the physical manipulation of objects as a means to "show" how two numbers equaled 10. In the following exchange, students were coming up with different two addend combinations that equaled 10, and Ms. Arielle negotiated this new socialmathematical norm. Here, Ms. Arielle notices that some students do not need to physically manipulate objects anymore when finding different combinations of 10. (Observation 3, p. 1)

1. T: So, Susan, what would you do? So, it is your turn, Susan.
2. S: [Chooses the cards 9 and 1]
3. T: Now how did you know? How did you choose these two cards?
4. S: There is 9, and you have 1 more. You have 10.
5. T: She knows 9 plus 1 more equals 10.

In this exchange Ms. Arielle is negotiating the socialmathematical norm that will now obligate students to "know" several two addend combinations of 10. When Ms. Arielle said, "She knows 9 plus 1 more equals 10," she is negotiating this new norm with her students. What is interesting to note here is that Ms. Arielle uses her students' responses to her questions to negotiate this new norm rather than explicitly stating that students needed to know or memorize two addend combinations of 10.

Later on in the lesson Ms. Arielle commented again that knowing the combinations of 10 was acceptable and possibly expected from that point forward. Consider the following way in which Ms. Arielle negotiated this new socialmathematical norm. In this exchange students were sharing different two addend combinations of 10 while Ms. Arielle recorded their responses on chart paper (Observation 3, p. 5).

1. T: You know what? You guys are getting really good. You guys aren't even counting anymore. You don't need cubes and fingers. Now I am going to start with 5. Five plus what makes 10?
2. S: Five
3. T: 5 plus 5 equals 10. So let's keep doing a list. Let's keep doing a list. Who can give me another equation that equals 10? Denise.
4. S: 4 plus 6 equals 10.
5. T: 4 plus 6 [writes on chart paper]. Who can give me another one?
6. S: 11 take away 1.
7. T: 11 take away 1. I am going to put that one on the bottom [writes 11 – 1 = 10 on the bottom of the chart paper]. Did we have an 11 in our number cards?
8. Ss: No.
9. T: But you know what? She is totally right. She knows that 11 minus 1 equals 10. But I want you to think addition. We are going to do addition okay – addition equations. Let's see another combination?
10. S: 8 plus 2.
11. T: 8 plus 2 equals 10. Who has another addition equation?
12. S: 10 plus 0.
13. T: 10 plus 0. Another one?
14. S: 9 plus 1.
15. T: 9 plus 1. We have more. Anna?
16. S: 1 plus 7 plus 2.
17. T: What! [Acting surprised and excited] Let's see. 1 plus 7 plus 2. How much is 7 plus 1?
18. Ss: 8.
19. T: And look [points to chart]. We said that 8 plus 2 equals 10. So she did it! We are coming up with equations that have two addends. That means two numbers when you put them together they make 10. But she came up with an equation that has 3 addends. She has 1, 2, 3 numbers that when you add them together equal 10. Good job. Who has more? Can we try again with only two addends combinations? Only equations that have

two numbers that when you add them up equal 10. Look at what we have so far.

20. S: 7 plus 3.

21. T: 7 plus 3.

This exchange is significant in that previously, students had been expected to provide explanations and/or justifications of the two addend combinations of 10s. It can be inferred that as a result of students' past work explaining and justifying, they had developed a taken-as-shared understanding of such combinations of 10. During this lesson Ms. Arielle implicitly acknowledged that "knowing" two addend combinations that make 10 was now acceptable. And when a student offered in Turn 8 "Eleven minus one" Ms. Arielle's comment in Turn 9, "She knows 11 minus 1 equals 10," again acknowledges that knowing is acceptable. As evident in the above exchange, it was now acceptable to offer, without explanation and/or justification, two addend combinations of 10, whereas previously students were expected to explain and/or justify how they got to their solutions. In her lesson plan Ms. Arielle explicitly stated that developing fluency with two addend combinations was her main objective; however, the way in which she went about meeting this goal was student-centered rather than teacher directed. In her lesson plan she wrote:

> Student will develop fluency with 2-addend combinations of 10 and solve problems in which the total and one part are known and use addition notation to record their equations. (Lesson Plan June 4)

When reflecting on the videotaped lesson on June 4, Ms. Arielle noticed that she had met her stated objective and that many but not all students knew the two addend combinations of ten. She said,

> The cubes and pictures helped some students to manipulate the numbers to make two addend combinations of ten while those with better number sense relied only on the numbers on the cards. (Lesson Reflection, p. 1)

Socialmathematical norms are ever evolving and as seen here in the above exchanges, Ms. Arielle actively negotiated a new socialmathematical norm for what counted as an acceptable mathematical explanation and justification for two addend combinations of 10. Previously, students were obligated to provide explanations as actions on objects, whereas now, it is acceptable to “know” the two addend combinations and as such, knowing suffices as an acceptable explanation and justification.

Summary

The practices that Ms. Arielle uses are reform-oriented, thus the socialmathematical norms that were constituted are also oriented toward mathematics reform. Ms. Arielle adopts the practice of helping students understand what counts as a mathematically different solution, albeit this is a work in progress. In an effort to establish the taken-as-shared sense of what constitutes a mathematically different solution, Ms. Arielle regularly points out to students when a solution method is not mathematically different. However, analysis reveals that there are times when Ms. Arielle accepts solutions from students that are not mathematically different. Ms. Arielle, it seemed, is in the process of negotiating this norm for her students as well as for herself. It is evident that Ms. Arielle wants her students to develop a repertoire of strategies that they can use when solving particular problems, thus at times her pedagogical agenda is more focused on eliciting different strategies rather than on mathematically different solutions. And, at other times, when her intent seems more focused on eliciting mathematically different solutions she directly intervenes as a means to develop her students' understanding of this socialmathematical norm. Ms. Arielle's practice is positioned between these two worthy objectives, and as such, she is in the process of determining when to focus on strategies and when to focus on a mathematically different solution.

Ms. Arielle did make an attempt to develop the norm of what constitutes a mathematically efficient or sophisticated solution, although this was only observed happening in Observation 3. What is noteworthy here is that although Ms. Arielle's pedagogical agenda in Observation 3 is focused on introducing her students to the efficiency of grouping and counting by 10s, she does not let her agenda overshadow her students' readiness to understand this critical first grade concept. Moreover, the concept of counting by 10s is allowed to emerge from student work rather than being handed down by Ms. Arielle.

Ms. Arielle is dedicated to engaging in the practice of making students' thinking public, thereby making it prominent as well. This practice serves the purpose of bringing students' thinking to the fore of classroom conversations so that student solutions can be considered and compared by all. Moreover, this practice serves to require students to offer explanations and justification of the process they use to solve a math problem, concomitantly taking the focus off correct answers. As a result of this practice, the socialmathematical norm guiding what counts as an acceptable mathematical solution focuses on the actions students take on objects while solving mathematical problems. Ms. Arielle's students understand that their explanations must provide descriptions of their actions as though acting in a physical mathematical reality. When students explain or justify their solutions, they regularly use words and actions that indicate how they physically or conceptually manipulate objects representing numbers.

As a result of the practices Ms. Arielle use, the following socialmathematical norms are developed:

- Ms. Arielle does not consider all solutions as valid.
- Students are obligated to contribute solutions that are mathematically different.
- Students are obligated to focus their explanations of the mathematical processes they use to solve problems rather than the correct answer to the problem.
- Students are obligated to rely on their own mathematical knowledge to assess the correctness of a solution and are assisted, when necessary, by the teacher.
- Students are obligated to explain and justify their solutions as though acting in a physical mathematical reality.
- Ms. Arielle considers some solutions more efficient or sophisticated.

In summary, the socialmathematical norms that are constituted in Ms. Arielle's classroom mirror reform-oriented norms whereby the teacher orchestrates conversations focused on student thinking and understanding. Students in Ms. Arielle's classroom are adept at offering valid explanations and justifications of their mathematical processes indicating that students understand their socialmathematical obligations and act accordingly. Moreover, students understand that an acceptable mathematical explanation or justification requires a description of the actions they took on objects, as their explanations often sound like they are acting in a physical mathematical reality. For example, students use phrases, such as, I took, I grabbed, I clumped, and I put as they explain their methods of solution.

As a novice teacher ,Ms. Arielle has begun to develop a teaching practice that is firmly grounded and orientated toward mathematics reform. However, there are issues and challenges that surfaced for Ms. Arielle and as such played a part in the extent to which she is able to implement reform recommendations. The following section will address the issues and challenges Ms. Arielle faced as a novice teacher attempting to implement mathematics reform recommendations.

Issues and Challenges

After analyzing Ms. Arielle's teaching practices as well as her interview discourse, it became apparent that there were issues and challenges that she faced as she attempted to implement teaching practices that were aligned with mathematics reform. The following section will address the second research question guiding this study.

- What issues and challenges surface as novice teachers begin to enact reform orientated discourse practices?

Ms. Arielle's Background in Mathematics

According to Ms. Arielle, understanding the divergent ways that individual students comprehend mathematics has been a challenge and learning process for her. Because mathematics has always come easy to her, and she experienced a good deal of success in traditionally situated classrooms, she is cognizant that her beliefs could impact the way in which she approaches teaching. Unlike many elementary school teachers, math has been a source of enjoyment for Ms. Arielle, and she harbors fond memories of learning mathematics from as far back as she can remember. When reflecting on her past experiences with mathematics she said,

> Well, my mom’s a math freak. I mean she loves math, and ever since we were very little, the way she would explain math to us so that we would like it. She would always say that math was like a game, it was like, you know, just like a game trying to figure out what the answer was - you know, the fun part. So I never thought of math as something scary or that was boring that I didn’t like. It was always a game. (Interview, p. 1)

Moreover, Ms. Arielle’s school experiences in math were positive as well. Thinking back on her elementary school math classes she said,

> Math is something like, like logical to me - like it just makes reason. In school, I didn’t have to think too much about it. I can’t remember, like now, looking at the way we teach, I can’t remember if they told me that 2 plus 2 is 4, and I just learned it and memorized it, and I knew the answer or if I could really see that 2 plus 2 was 4, but I would just be good at addition, subtraction, multiplication, anything. But it just really came easy to me. (Interview, p. 2)

Ms. Arielle’s love of mathematics continued throughout high school where she was enrolled in a program geared for students who would pursue math or science in college. During high school she had a bit of trouble in mathematics, but she never doubted her ability and instead attributed the difficulty she experienced to her teacher. When reflecting on her high school experience she said,

> I took two maths in the same semester. I took geometry, and I don’t remember if it was calculus or trigonometry. Geometry I just loved. I loved the teacher. It was a male teacher. Geometry was like, once you knew the theorems and the postulates, it was reason. It was just so logical to me that I just totally got it. Now the other class, I did not like the teacher at all or the way she set up exams with like maybe one or two problems. And it was either a right or wrong answer, so even if like the whole problem wasn’t wrong but the answer was, she marked it wrong. It was just really long math problems and the process was really long, and it was just right or wrong. You would get the whole question wrong, no matter. She wanted it a certain way. (Interview, p. 4)

Ms. Arielle ended up dropping the math class, choosing to take it in the summer where she earned an A. In college she entered an animal science program and was successful even though she “flunked” a math class. Again, she did not attribute her failure to her ability to do math but to her inability to focus during the beginning years of college. Once she began to focus, she took the course again and was successful, earning an A in the course. Ms. Arielle graduated from college with a Bachelor of Agricultural Science with a focus on the animal industry.

Being confident and well prepared in mathematics does not always translate into effective teaching and is not without its drawbacks. According to Nathan and Petrosino (2003), educators who excel in a particular discipline may experience something called the expert blind spot. The expert blind spot hypothesis states that:

> Educators with advanced subject matter knowledge of a scholarly discipline tend to use the powerful organizing principles, formalisms, and methods of analysis that serve as the foundation of that discipline as guiding principles for their students’ conceptual development and instruction, rather than being guided by knowledge of the learning needs and developmental profiles of novices. (p. 906)

According to Ms. Arielle, she was always a good student of mathematics, as math came easy to her. She said in an interview, “I’ve always been a good student in math, and I just got it” (Interview, p. 1); however, as a teacher, she found mathematics much more challenging. When analyzing the results of the MKT instrument, Ms. Arielle’s score indicated that, as a novice teacher, she possessed an average understanding of the mathematics needed to teach Elementary Number Concepts and Operations as measured by the MKT instrument. However, it is possible that Ms. Arielle is struggling with the expert blind spot, as described by Nathan and Petrosino, in that the results of the MKT assessment indicated that she had difficulty with understanding students’ non-conventional ways of solving mathematics problems. Ms. Arielle was most challenged in the area of assessing students' non-conventional ways of solving math problems, as she answered this type of question with the least success.

According to Ms. Arielle, she never had difficulty understanding the rules and procedures associated with mathematics; moreover, she was able to successfully follow these rules and procedures all though her mathematics school experiences and found much success. Ms. Arielle was stronger in explaining rules and procedures and representing mathematical ideas ,as she answered more than half of these types of questions successfully. See Appendix E for an example from the MKT released items representative of the type of problem Ms. Arielle had the most difficulty with.

Understanding Divergent Ways of Thinking

Understanding the different ways that young children might think about and understand mathematics posed a challenge for Ms. Arielle; however, it is a challenge that she had considered and reflected upon since she enrolled in a mathematics methods course. While participating in a reform-oriented math methods course, Ms. Arielle had the opportunity to read and reflect on case studies of young children solving math problems in very different ways other than the traditional algorithm. This experience proved valuable to her, and subsequently, pushed herself to try out some of the methods for herself. She said,

> I remember the very first case study that I read. I had to re-read it over and over again because I didn't get what the kids were doing. I didn't get what they were thinking at all because to me, you know, 1 plus 1 is 2, and that was it. And it made sense, so I would try to figure out what they were doing, and then learn it myself in their way, and it took me a while. It wasn't until like maybe the third case study was I able to understand how the students were figuring out the

> problems. I remember once I got to participate and write how I mentally got an answer to a problem on the board, and I totally did it like a way that the students would do it, and it came easy. But never would have I done it that way before, before this course. Never. (Interview, p. 7)

In her methods course, Ms. Arielle began to look at mathematics differently and from the perspective of a student rather than from her own experiences as a learner and this new lens helped her begin to develop an understanding of other's might learn mathematics. The methods course was somewhat of a turning point for Ms. Arielle. The following quote highlights the way in which her perspective was reshaped:

> I loved the course. I think what I enjoyed most was the different ways that we could do math. When I was growing up, we didn't have like the manipulatives, you know, now we have like the base 10 blocks, and we have the cubes and all that. It was fun, you know. It was like, it's like a visual way to see things. (Interview, p. 7)

Ms. Arielle's experience in the methods course was very different from her previous school experiences, and she attributed her methods course to having "opened up" her mind in terms of understanding the divergent ways in which individuals think about mathematics. Ms. Arielle shared that when she worked in small groups in her methods course she had, for the first time, an opportunity to see how differently "other adults also figured out math." This experience helped her to understand math concepts more deeply and, moreover, helped her to appreciate the different ways others come to understand mathematics. Reflecting back on one particular experience she said,

> I remember one particular instance in our last class where we were working with fractions. I was trying to understand the relationship between a written fraction and a picture representation. I knew that the answer I got using the algorithm was correct, but I couldn't explain why. One of the other students in the class helped me understand using a visual explanation. When I understood what she was explaining I literally said, "Wow." (Interview, p. 7)

Ms. Arielle found these different ways of thinking about and doing math fascinating but also challenging.

> I learned math the regular way, and I understood it. Like, then I tried to push myself to do it a different way, and I couldn’t, so I would go back and do it my way. It wasn’t until like, you know, third or fourth class that I was able to do it, like you know, if it was 28 plus 32, I would do 20 plus 30 and then, you know add the ones. (Interview, p. 12)

Moreover, she reflected on how she now approaches math differently and does not rely solely on the traditional algorithm when trying to figure out everyday math problems. She said,

> Now I add by 10s first and then 1s, or however I want instead of doing the traditional algorithm. Even when I am just doing math anywhere, not even necessarily in class, and I was not able to do that before the course. (Focus Group Interview, p. 1)

Ms. Arielle is a confident and self-assured student of mathematics; however, she understands that being a good math student does not necessarily translate into being a good math teacher. She has had firsthand experience with the expert blind spot hypothesis and has worked to develop an understanding of how others might come to know and understand mathematics. Before enrolling in the methods course, Ms. Arielle said that if a student in her class solved a problem differently from the standard algorithm, she would have showed the student the “right way” to solve the problem by explaining the algorithm. She said,

> When I took the methods course, it is really interesting just because, I realized if I had just gone into teaching without that course, I wouldn’t have really understood the kids, like their way of thinking, and it would have been hard for me to understand why they didn’t get what I was trying to tell them because for me it was so easy. Just do this and that. So now I see math that’s like instead of just one answer it’s got like many answers or maybe ways to get to that answer. And I didn’t think that way before so now I find it even more interesting. (Interview, p. 6)

Ms. Arielle also believes because of her experience in her methods course she has developed her ability to empathize with her students more in spite of her confidence in mathematics. During the focus group interview she commented on how the methods course influenced her teaching. She said,

> I still love math. The only thing that changed for me is that as a teacher I think I would have been very frustrated with student before taking that course. After the course, I can relate to the student more. (Focus Group Interview, p. 7)

Ms. Arielle now believes that listening to and understanding students' divergent ways of solving math problems is essential to the work of teaching mathematics, albeit she acknowledges the difficulties she has experienced in enacting such a practice. She said,

> Sometimes I feel like I know where they're coming from, but it's hard for me to get them to say it and or explain it to the class, and sometimes it's the other way around, like I have no idea, and I'm really trying to have them explain it just so I can understand what they're doing. So I'm still, I think, I'm still struggling in that area a little bit. (Interview, p. 14)

Moreover, Ms. Arielle has experienced difficulty with trying to figure out how to move students along in their thinking and as such feels somewhat incompetent in this area. When reflecting on an area of challenge for her she said,

> When a kid is just completely stuck, nothing that I've taught him or listening to other students has really helped him, sometimes it's, I get a little frustrated and wonder, "Okay, well he should have gotten it by now. What can I do?" Or I wonder, "Maybe I didn't do it right" or Uh oh, so now what am I supposed to do now?" And sometimes I feel like I don't know what I'm supposed to do. So I'm still trying to, when a student doesn't get it like trying to come up with other ways or better ways for me to explain. (Interview p. 14)

As a novice teacher, Ms. Arielle has begun to develop a teaching practice that mirrors reform recommendations in that her practices are focused on eliciting the divergent ways that students reason about mathematics. A challenge for Ms. Arielle lies in understanding how students reason about mathematics because she has found that their thinking and reasoning is often very different from her own. Ms. Arielle feels responsible when she cannot understand her students' thinking and, consequently, questions her own ability as a teacher. Although eliciting and understanding student thinking is challenging for Ms. Arielle, she is addressing this through her day-to-day interactions with students and with mathematics.

Orchestrating Productive Conversations

Orchestrating productive mathematics conversations with first graders has proven challenging to Ms. Arielle and is an aspect of her teaching practice that she was continually trying to develop while this study was taking place. According to Ms. Arielle, managing conversations is "an ongoing issue" (Focus Group Interview, p.12) and one she continually reflected upon. Because Ms. Arielle's teaching practice does not follow the traditional IRE sequence where the teacher asks a question, a student responds, and the teacher evaluates, orchestrating conversations among first graders proves problematic. The exchanges highlighted in Ms. Arielle's case study reveal that she devotes a significant amount of time probing and encouraging students to explain their thinking, thus requiring the rest of the class to listen and stay engaged. When reflecting on what challenges her as a teacher she said,

> I guess one of my biggest challenges is actually getting kids interested in sharing AND listening to one another during discussions. The kids usually always want to share, but the rest of the students aren't always paying attention. Usually, discussions happen at the end of the lesson after the exploring phase. At this time, kids are starting to zone out. I do always have kids who always want to be called on, and are ready to share. I try to explain to the students why listening to

> each other will or may be helpful to them, but they don't always get it. Sometimes, I'll have the discussion at the beginning of the lesson. Having kids do the writing on the board or chart paper gets them interested. And I remind them that I will only call on "good listeners.” (Email Correspondence, July 2009)

And in all of the classroom observations, it was evident that orchestrating productive mathematics conversations with a group of six year old students was a challenging endeavor. Ms. Arielle often needed to stop the lesson to remind students how to engage in classroom conversations, and at times, these reminders interrupted the flow of the lesson. When trying to orchestrate such conversations, she often highlighted for students the importance of her being able to hear their explanations so that she could understand their thinking. Reflecting the significance she places on student thinking during one lesson she said to student,

> Ahhh…. Shhh…. it’s really hard for me to hear you guys think and tell me what you’re doing in math when there is so much talking – so much yelling at each other – there’s a lot of fooling around. People not really doing what they are supposed to so I’m spending all my time – I’m spending all my time reminding you what to do instead of helping you learn. Lewis, I’ll take that. (Observation 4, p. 5)

Orchestrating productive mathematics conversations is an issue for Ms. Arielle in that keeping all of her students engaged as she elicited from students detailed explanations and justifications is difficult and often creates situations where learning is halted in order to address student behavior. Ms. Arielle sometimes needs to completely stop a lesson and send children to their seats thus conversations would be interrupted.

> Let’s stop. I don’t feel like you guys are being very respectful. I have children – excuse me – playing instead of sitting. I have children talking. I have children making noises. I want everybody to go to your seats and put your heads down. (Observation 5, p. 7)

Interrupting her teaching to manage student behavior happened during four of the five lessons observed and caused Ms. Arielle a considerable amount of frustration. However, Ms. Arielle is determined to engage students in worthwhile mathematics conversations where sharing and listening are central to the learning process. Ms. Arielle's teaching practice is such that she expects her student to learn not only from her, but from themselves and their peers as well. Moreover, because Ms. Arielle adopts reform teaching practices that obligate her students to listen to others' mathematical reasoning and make comparison to their own reasoning, it is critical that she develop her ability to manage such interactions.

As a novice teacher Ms. Arielle sometimes seems overwhelmed at the prospect of managing such conversation; however, her commitment to reform is strong in that she continues to engage her students in rich mathematical conversations, even when faced with issues around engaging students in productive mathematical conversations. Rather than falling back on more traditional practices that would have afforded her more control over the interactions in the classroom, she holds fast to her reform practices as she continues to work on developing her ability to manage rich mathematical conversations with her first grade students.

Summary

The issues and challenges that have emerged for Ms. Arielle are ones that are focused inwardly on her teaching practice rather than outwardly on students or any perceived pressure from the administration. Ms. Arielle had been a strong student of mathematics, and as she said, she "just got it." As a result, she is challenged to understand her student’s divergent ways of reasoning about mathematics; however, she is aware of this challenge and is actively working at trying to understand how students are thinking about the problems she poses. And because she is actively attempting to understand her students’ mathematical reasoning, she focuses her practice on eliciting different solutions as well as explanations and justification of the solutions student shared. Consequently, she is challenged to manage productive classroom conversations among a group of six and seven year olds. However, she views this challenge as a teaching challenge rather than as a student problem, in that she believes it is possible for first grade students to engage in such conversations. Consequently, she worked on her teaching practices. Thus, Ms. Arielle does not view her students as problematic and in need of changing; rather, she views her practice as in need of refinement and change so that such conversations can happen more effectively.

CHAPTER 6

MS. QUINN

When this research study began, Ms. Quinn was in her second year of teaching first grade at Maple Elementary School, located in an urban school district in Western Massachusetts. However, shortly after the study got underway, Ms. Quinn learned that she would be reassigned to teach fourth grade math at Morningstar Elementary School the following school year. Understandably, being transferred to a new school caused Ms. Quinn a bit of frustration and being assigned to teach solely mathematics to fourth graders caused her trepidation as well. When reflecting on the move she said, “I get first grade math” and “I don’t have the training in *Investigations*, the curriculum used in 4^{th} grade, so I am going in blinded.” This was a difficult situation for Ms. Quinn and one in which she had little control.

As a result of her transfer, the decision was made to schedule observation once Ms. Quinn was settled in at Morningstar Elementary School. In terms of student demographics, Morningstar’s student population is predominately Hispanic (91.2%) with White (4.6%), African American (3.3%), Multi Racial (0.7%), and Asian (0.3%) comprising the rest of the student body. Ms. Quinn taught three sections of math each day to the fourth grade students on her team.

The following section will present an analysis of the teaching practices Ms. Quinn used to engage her students in mathematical conversations. First will be an analysis of the practices attributed to the development of social norms followed by an analysis of the practice attributed to the development of socialmathematical norms. This section of the case study will address the following research question:

- What reform-oriented discourse practices do novice teachers who participated in a reform-based mathematics methods course adopt? What practices do they adapt? What practices do they ignore as they engage their students in mathematics conversations?

Practices Fostering Social Norms

Social norms constitute the "participation structure" (McClain & Cobb, 2001, p. 244) within classrooms and are not specific to the discipline of mathematics. By analyzing the practices Ms. Quinn used to engage her students in mathematical conversation, the participation structure of the classroom emerged. Table 6.1 shows the distribution of practices that Ms. Quinn used as she engaged students in whole class conversations. Analyzing such practices reveals the way in which Ms. Quinn's students were obligated to participate in classroom conversations as well as the number of times each practice was used per hour.

Table 6.1 Observed Practices Fostering Social Norms: Ms. Quinn

	Observation				
Practice	OB1	OB2	OB3	Total	Per Hour
Developed Idea of Mathematically Different	1	0	0	1	0.5
Developed Idea of Mathematically Efficient/Sophisticated	1	0	0	1	0.5
Made Student Thinking Public	5	1	6	12	6.7
Accepted One Word Answers	25	24	45	94	52.2
Indicated Math was Rule Bound	4	3	7	14	7.8

QWKA: Question with a Known Answer

As seen in Table 6.1, Ms. Quinn utilized the traditional practice of asking QWKA 126 times over the course of three observations, making this the dominant practice she used to engage students in mathematics conversations. Her next most utilized practice was that of accepting student responses to the QWKA that she asked. With this practice Ms. Quinn would repeat a student's response in a positive or negative tone, thereby indicating the correctness of the response. Ms. Quinn utilized the reform-oriented practice of eliciting explanations and/or justifications 27 times, and analysis reveals that this practice had the potential to bring students' ideas and conjectures to the fore of the conversation or focus the conversation on correct answers. Although Table 6.1 shows that Ms. Quinn rarely requested that students share different solutions to the same problem, analysis reveals the eight attempts she did make were at times reform-oriented and served to bring student thinking into the public discourse space for all to consider. Lastly, Ms. Quinn did not engage in the traditional practice of providing explanations of student's thinking with zero occurrences observed.

The following sections will examine Ms. Quinn's practices in more detail as a means to shed light on their reform orientation and the social norms that such practices fostered. First under review will be the practices associated with reform recommendations: Elicited Explanations/Justifications, Elicited Different Solutions followed by an analysis of the traditional practices of Asked QWKA and Evaluated and Accepted Student Responses. Examining such practices through the filter of social norms provided a framework to describe the participation structure that emerged in Ms. Quinn's classroom.

Elicited Explanations and Justifications

In a reform-oriented classroom teachers “pose questions that elicit, engage, and challenge each student’s thinking” (NCTM, 1991, p. 1) and offer students opportunities to explain and justify their thinking as they work on solving mathematical problems. Although not a well developed aspect of her teaching practice, Ms. Quinn did at times ask her students to provide an explanation or justification to how they solved a problem. As seen in Table 6.1, Ms. Quinn elicited an explanation or justification from students 27 times over the course of three observations. The following four exchanges uncover the way in which Ms. Quinn elicited and then responded to students’ explanations and justifications revealing an adapted practice.

In this first exchange Ms. Quinn and her students are engaged in an activity called Quick Images. In this activity, using the document camera, Ms. Quinn showed students a series of dots for five seconds, covered them up, and asked students to draw or write a multiplication equation that would represent the number of dots that they saw (see Figure 6.1). Next, she uncovered the dots for five more seconds, asked students to look again and make any changes to their representation. Lastly, Ms. Quinn asked students to explain how they remembered what was projected on the screen.

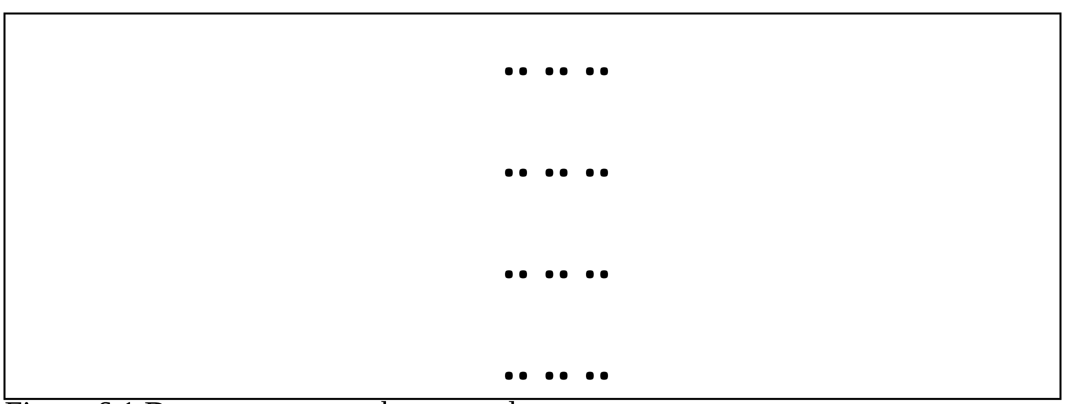


Figure 6.1 Dot arrangement shown on document camera

1. T: Who can explain what they saw when they looked at this and how you remembered what to do on your paper?
2. S: I saw like four squares in up and then … (inaudible) more to the side.
3. T: You saw four squares?
4. S: No like four dots … (inaudible)
5. T: In a group?
6. S: Yes
7. T: [Circles group on document camera] And then what?
8. S: Then I like looked at it again, and I like saw … (inaudible) looking down I look put four in circle then I put another row in the middle and then another row on the end and …(inaudible).
9. T: So, you saw it like this? [circles 3 groups of 12 on the document camera]
10. S: Yes.

In this exchange Ms. Quinn began in Turn 1 with a request for an explanation as to how students remembered seeing the dots on the document camera. Although not clear from the transcript, in Turns 2, 4, and 8, Mitchell provided Ms. Quinn and the class with a detailed explanation of how he remembered the dots. In Turns 3 and 5 Ms. Quinn asked follow up questions to help her and the class understand Mitchell's explanation. Moreover, Ms. Quinn notated Mitchell's explanation by circling on the screen 3 groups of 12 dots. Moreover, it was clear from the tape that many of the students were engaged in what Mitchell was describing because as he talked, students were looking at the screen and nodding their heads in agreement or pointing at the screen as if counting the dots and the groupings that Mitchell had described.

In the aforementioned exchange the practice of eliciting an explanation served to focus the conversation on Mitchell's thinking rather than on a correct answer. Moreover, the practice of recording student work aided in focusing the conversation on Mitchell's solution and served to help the rest of the students in the class make comparisons between their solution and Mitchell's solution. This exchange was productive for Mitchell, as he was afforded an opportunity to talk through his solution; for Ms. Quinn as she was given an opportunity to assess Mitchells' ability to connect the arrangement of dots to an equation; and for the group, as they had an opportunity to make comparisons between solution methods. An exchange such as this has the potential to become significant in the development of reform-oriented social norms within the classroom. However, to be significant, more such exchanges would be needed in order for students to have developed the taken-as-shared understanding that obligates students to explain and justify their thinking. As will be revealed in the following exchanges, Ms. Quinn's practice of eliciting explanations and justifications often turned into a traditional question and answer sequence focused on correct answers rather than student thinking.

In this next exchange, which came immediately after the aforementioned one, Ms. Quinn asked students to find an equation that would explain how Mitchell had seen the arrangement of dots. The significance of this exchange is that it illuminates an instance where Ms. Quinn asks her students to consider and explain what someone else is thinking; however, it also illuminates how Ms. Quinn often shifts the focus of a conversation from a student's explanation to a correct answer, thereby ignoring the meaning behind a student explanation (Observation 2, p. 3).

1. T: What could be your equation that would go along with the way Mitchell saw this image? What equation would go with this? Dina?
2. S: 3 times 4.
3. T: How many groups – how many circles do I have?
4. S: 3?
5. T: You've got that part right. But now the next number is what?
6. S: 3 times 3?
7. T: Is that… how many dots do you have in each circle?
8. S: 4?
9. T: In each circle?
10. S: 12? 3 times 12?
11. T: And what does it equal?
12. S: 36.

In Turn 1 Ms. Quinn asked students a QWKA in that she knew the equation representing Mitchell's explanation was 3 x 12. When Dina responded in Turn 2 with "3 times 4," Ms. Quinn ignored the response and in Turn 3 employed a traditional questioning sequence until Dina responded with a correct answer in Turn 10. It was evident in Turns 4, 6, 8, and 10 that Dina guessed at the answers to Ms. Quinn's questions because her responses were offered in a questioning tone. This is an example of a missed opportunity for Ms. Quinn as she could have engaged Dina in conversation as to how she saw the equation as 3 times 4, thereby focusing the conversation on Dina's thinking rather than on a correct answer. Moreover, the social norm that such an exchange constituted is that the purpose of engaging in conversation with Ms,. Quinn is to supply a correct answer rather than to share your thinking.

Examining the aforementioned exchange further illuminates an instance where Ms. Quinn seems obligated to offer a bridge in the form of a positive evaluation. In Turn 15, Ms. Quinn commented, "You got that part right, but now the next number is what?" With this comment Ms. Quinn offers Dina a "bridge" by indicating that at least part of her answer is correct, thus allowing Dina to continue with the exchange. Ms. Quinn offers Dina a bridge in an effort to keep her engaged and more importantly, in an effort to keep her focused on providing a correct answer. Moreover, offering Dina a bridge in the form of an evaluation as to the correctness of a response serves to establish the norm that correct answers are acceptable mathematical explanations.

The following exchange reveals another example where Ms. Quinn offers a student a bridge as a means to elicit an explanation but again the bridge serves to focus the conversation on a correct answer rather than on student thinking (Observation 2, p.

12) and further established the norm that correct answers take precedence over student thinking. Here students were engaged in figuring out what the dimensions of a 10 x 5 and a 10 x 6 array would be.

1. T: There's two 10s. We knew when we were doing our game, that for them to match they had to have the same factor – right? One of the factors had to be the same. So we know we're going to assign this side a 10. We now need to assign this side. An array dimension. What would that one be?
2. S: 11
3. T: Why is it 11? You're right, but why?
4. S: [No response]

In this exchange Ms. Quinn attempted to reassure the student that her answer was on the right track by providing her with a bridge in the form of a positive evaluation. Unfortunately, the bridge shifts the focus from what the student is thinking toward supplying a correct answer. When Ms. Quinn said, "You are right, but why?" she implicitly indicated that being right was important. It was unclear that the student does know why her answer of "11" was correct, thus she could not successfully continue with the conversation. Such an exchange indicates that students in Ms. Quinn's classroom are not accustomed to providing explanations and justifications of their responses thus when asked to, they are often not prepared. Moreover, Ms. Quinn rarely asks for explanations and justifications of correct answers, thus the norm constituted is that correct answers do not warrant an explanation or justification.

The next exchange again reveals the importance of answering Ms. Quinn's questions with a particular correct answer. In this exchange, although Ms. Quinn asks for an explanation, her comment indicates that she is eliciting a correct answer rather than a genuine student explanation. The exchange began with students examining a 6 by 4 inch grid displayed on the document camera with one fourth of the grid shaded in (see Figure

6.2). Ms. Quinn was attempting to engage her students in a conversation about how much of the grid was not shaded in (Observation 3, p.11).

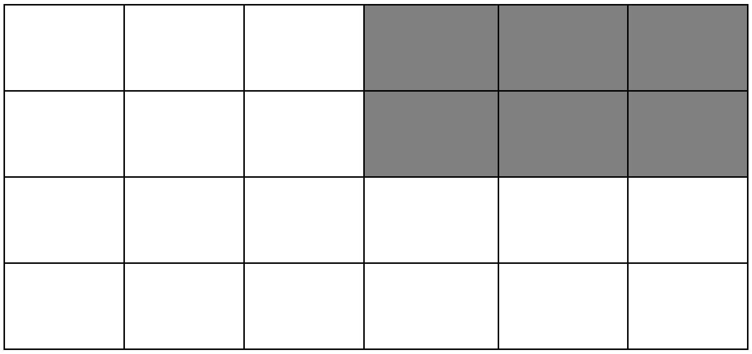


Figure 6.2 Grid representing ¼ shown on document camera

1. T: Okay. If we look at this and I know that we all agree that this is one fourth, right – how much of this is not shaded? What is the fraction of my rectangle is not shaded for this one?
2. S: 18 squares
3. T: No – Let's start with this – how many …
4. S: 3/4s
5. T: Well how do you know it's 3/4s, Jerry?
6. S: Because if you have it like that you just turn it the other way and you have them all the same right there … (inaudible)
7. T: Is that mathematical?
8. S: [Shrugs shoulders looking embarrassed] No.
9. T: No. How many parts are we breaking the whole up into?
10. S: 4
11. T: It tells us right there – right – so we have 4 parts. How many of it is not shaded?
12. Ss: 3
13. T: 3 – because of one of it is shaded and 3 is not. What would four 4ths equal? The whole thing, right! Because if I said to shade in four 4ths of the rectangle you would break it up into 4 parts and how many of those parts would you shade?
14. Ss: 4
15. T: 4 of them – so the whole thing would be shaded in. So this is 1 fourth that is shaded and 3 fourths is not shaded.

In this exchange Ms. Quinn asked a QWKA in Turn 1, and when a student responded with "18 squares" Ms. Quinn overtly evaluated the response with, "No." Seemingly the student transgressed a norm in this instance in that Ms. Quinn was looking for the specific response of “3 fourths.” Here, Ms. Quinn misses an opportunity to probe the student’s thinking further and ask him to explain what he means when he says “18 squares.” Moreover, she misses a valuable opportunity to highlight for her students the relationship between 18/24ths and 3/4ths.

In Turn 4, although Jerry responded with the correct answer, Ms. Quinn followed up utilizing the reform-oriented practice of eliciting a justification “Well, how do you know it’s 3/4th, Jerry?” Unfortunately, when Jerry offered a detailed explanation and justification in Turn 6, Ms. Quinn’s response “Is that mathematical?” served to shut down Jerry’s thinking, and as evident on the video, caused him some embarrassment. An exchange such as the one above serves to develop a participation structure that obligates students to offer correct answer to Ms. Quinn’s questions rather than explanations or justifications of their thinking. Although Ms. Quinn asked Jerry to explain how he knew the answer was 3/4ths, it was clear that her intend was not to elicit his mathematical thinking and reasoning, it was to elicit a particular procedural correct answer. When Jerry did not provide what she was looking for, she proceeded to walk the students through a step-by-step process for finding the non-shaded part of the grid.

In this last exchange Ms. Quinn is attempting to engage her students in examining the meaning of the terms numerator and denominator. Here she asks Jackson to consider and explain what the two terms mean; however, when Jackson responds with his interpretation, Ms. Quinn revises his response and then shifts the conversations toward the procedure used to find the numerator and denominator (Observation 3, p. 3).

1. T: So what does this tell us? So what do these numbers tell us? We know one is the numerator, and one is the denominator, but what does that mean? Jackson?
2. S: The denominator is the whole and the numerator is the…(inaudible)

3. T: Okay. So you said, ”The denominator is the whole”? So the denominator is not the whole but how many parts it’s broken up into right? So, in one half our denominator tell us how many parts we are breaking it up into. How many parts did we break this rectangle up into? For one half?
4. S: 2.
5. T: It tells us right here that we have to break it into two parts. The top number tell us how much we need to shade in. How many parts of our two were we shading in one half? Debbie?
6. S: 1.
7. T: 1, that’s the top. Now if we look at the 1/4th, and I’m going to write that number now that we know we have a numerator and a denominator, and it tells us something. The bottom number tells us how many parts we are breaking it into.

In this exchange Ms. Quinn began using the reform-oriented practice of eliciting an explanation as to what the denominator means to students. However, when a student offered a response in Turn 2 Ms. Quinn accepted the response with “Okay,” although it is evident that the response was not acceptable as she corrected it in Turn 2 and effectively turned the conversation into one of eliciting a procedural responses rather than focusing on student explanations. Here, Ms. Quinn misses an opportunity to bring the student’s thinking into the center of the discourse for all to consider in comparison to their own understanding as to the meaning of the terms numerator and denominator. Although Ms. Quinn’s lesson objective for Observation 3 was stated as follows: “Students will interpret the meaning of the numerator and the denominator of a fraction,” it is evident that Ms. Quinn directs students toward the meaning she wants them to learn rather than offering them opportunities to make their own interpretations based on their work with fractions.

Moreover, of significance here are the comments Ms. Quinn uses as she explains to students how to find the numerator and denominator. She said in Turn 5, “It tells us right here…” and “the top number tells us…” and in Turn 7, “The bottom number tells us,” which indicates to students that there are rules they must remember and follow in order to be successful in finding the numerator and the denominator. However, it is unknown from this exchange what students interprete or understand about the terms numerator and denominator.

The above four exchanges are representative of Ms. Quinn's attempts to implement the reform practice of eliciting explanations and justification. These exchanges reveal that Ms. Quinn does ask students to explain their thinking; however, her attempts serve to encourage students to give correct answers. Moreover, her practice serves to discouraged students from expressing ideas and conjectures that are not, as Ms. Quinn said, "mathematical." Ms. Quinn's practice of eliciting explanations and justifications is an adapted one in that her she often begins by asking for an explanation or justification but finishes the elicitation using a traditional question answer type format.

In an interview Ms. Quinn indicated that she personally believed eliciting explanations and justifications as to how students went about solving math problems helpes students to develop a deeper understanding of mathematics. She said,

> It all goes back to them understanding why they're doing it, not just this is my answer because I borrowed, and I carried, and I was told to cross out and make it a 9. Now it's building on the why. Now they're understanding the whole process behind it. (Interview, p. 7)

Moreover, Ms. Quinn also believed that the practice of eliciting explanations is a fascinating process for her as a teacher and that she learned a lot by listening to students explain their thinking about a particular problem.. She explained,

> Kids are amazing. Some of the things that they come up with, you know, I would have never have done it that way, and I would never have thought of it that way, but their little minds are going, and they come up with things that I would never think of. And actually seeing that, it's actually almost getting a glimpse of what is going on inside of their head (Interview, p. 7).

Ms. Quinn also believes that the practice of asking students to explain their thinking benefits other students in the class by giving them an opportunity to see the problem solved in multiple ways. She said,

> There are so many different ways of thinking that maybe the way I do it doesn't make any sense to them. But, this way might help another student too, because they might be lost in understanding what I'm talking about but they understand what this other student has found. (Interview, p. 9)

Although in theory Ms. Quinn values student generated explanations and justifications, in practice, her attempts are geared more toward correct answers and procedural explanations, thus her beliefs in theory are in conflict with her actual teaching practices. A question that arises here is: Why are Ms. Quinn's beliefs and teaching practices in conflict with each other? Examining the results of the Mathematical Knowledge for Teaching (MKT) assessment sheds light on this question.

Analyzing the specific questions on the MKT assessment, it is evident that Ms. Quinn is challenged in the area of assessing students' non-conventional ways of solving mathematical problems as she was unsuccessful in answering this type of question with success. Ms. Quinn is also challenged in the area of representing mathematical ideas, as she had difficulty with this type of question as well. Ms. Quinn's area of strength is in explaining rules and procedures, as she answered all questions of this type with success. As such, although Ms. Quinn values the practice of eliciting student generated explanations and justifications, she is challenged to understand students' non-conventional ways of explaining their mathematical ideas. It is possible that she has adapted the practice of eliciting student generated explanations and justifications in favor of eliciting explanations of conventional rules and procedures because understanding conventional rules and procedures is area where she is quite strong. See Appendix E for an example of released items from the MKT instrument representing these types of questions.

There were times during the three observations when Ms. Quinn was able to engage students in productive mathematical conversations by eliciting explanations and justifications as to how they solved a particular problem. This happened only when Ms. Quinn asked students to solve a problem is different ways. It seems that the practice of eliciting different solutions is instrumental in helping Ms. Quinn to orchestrate productive mathematics conversations focused on student thinking. The following section will examine the conversations that ensued when Ms. Quinn elicited different solutions.

Elicited Different Solutions

As seen in Table 6.1, Ms. Quinn only asked students to share different solutions to the same problem 8 times over the course of three observations, thus this was not a developed aspect of her teaching practice. However, analyzing Ms. Quinn's attempts at implementing the practice of eliciting different solutions sheds light on how such a practice helps Ms. Quinn to elicit and stay focused on student explanations and justifications. Moreover, examining the practice of eliciting different solutions reveals the significance of this practice in bringing student thinking to the fore of the conversation.

The following exchange returns to Observation 1 and the Quick Images activity. Recall that Ms. Quinn had shown an arrangement of 36 dots on the overhead screen and asked students to draw or write an equation to represented the number of dots they remembered seeing (see Figure 6.1). This activity provides Ms. Quinn with several opportunities to elicit different solutions to the same problem and an opportunity to elicit explanations and justifications as to how students solved the Quick Images problem. This exchange took place after a student explained how he saw the arrangement of dots as 3 x 12.

1. T: 3 times 12 is one way. Who saw it differently? There is more than one way to see these. Kerry?
2. S: In 1 group of 4 [circling the groups in the air as she speaks].
3. T: What do you mean 1 group of 4?
4. S: [Pointing at the screen and circling in the air] One little group of 4 dots.
5. T: Like that? [Circles a group of four dots and then another…]
6. S: [Nodding head and smiling] Yea like that.
7. T: So you saw it like this. [Circles 9 groups with 4 dots in each]
8. S: Yes.
9. T: What's the equation for this?
10. S: 4 times 9.
11. T: Why is it 4 times 9?
12. S: (Inaudible)
13. T: What does the 4 represent?
14. S: The 4 …(inaudible) there is a group of 4 and (inaudible) are 9 groups of 4.
15. T: Very good. Did anyone see it differently?

The dot activity is instrumental in aiding Ms. Quinn in eliciting from students different ways to represent 36 dots. Moreover, the activity encourages students to interpret how their classmates saw the dots, thereby offering them an opportunity interpret another's perspective. In this exchange, Ms. Quinn is successful at eliciting a different way to represent the dots as the previous student saw the dots as 3 x 12 and Kerri saw the dots as 4 x 9. Ms. Quinn's role as recorder of student ideas is very important because as she records Kerri's way of seeing the dots on the overhead screen, she provides the class with a visual representation that allows them to consider and make comparisons between their solution and Kerri's solution. Moreover, by recording the solution Ms. Quinn effectively makes Kerri's idea the focus of the conversation. Additionally, as a result of implementing the practice of eliciting a different solution, Ms.

Quinn is able to concurrently implement the practice of eliciting an explanation as evident in Turns 2 and 4 and a justification in Turn 11. And lastly, by eliciting different solutions she manages to keep the focus on student thinking rather than on a correct answer to the equation.

In this next exchange we return to Observation 3 and the lesson on finding 1/4th of a 6 by 4 inch grid. Recall that Ms. Quinn displayed a grid that had been cut in half vertically and horizontally with the upper right hand corner shaded in to represent one fourth (see Figure 6.2). After discussing the 1/4th shaded in, she asked students to see if they could find another way to represent 1/4th on the grid. Figure 6.3 represents the way in which the student in the next exchange found 1/4th in a different way.

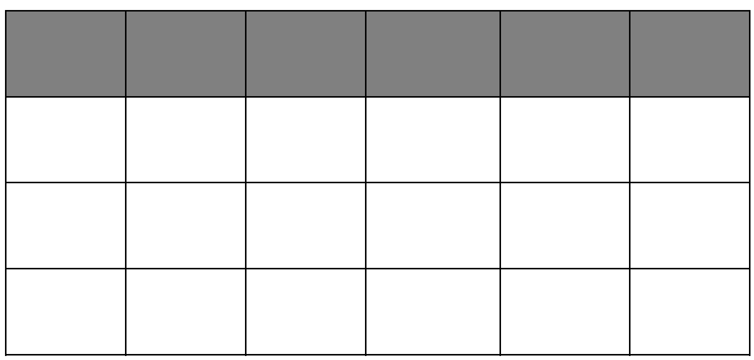


Figure 6.3 Student representation of ¼

1. T: My sandwich – I'm going to cut it into $1/4^{th}$. Hmmm…. You know I'm curious. Do you think we could find 1/4th a different way?
2. Ss: Yes.
3. T: Right now I want you to try and figure out if you can find 1/4th that looks different than this? Try to find 1/4th that looks different?
4. S: 4 x 6. There must be another way.
   [Students work individually at their tables while Ms. Quinn circulates reviewing how students are solving the problem].
5. T: Once you get it, see if you can find a different way.
6. T: Who would like to come up and show us what they did and explain how they figured it out? That means we are all paying attention. Vincent, come on up, buddy. You're going to explain this one.

7. S: Yeah. I solved this 4 by 6– so I looked right here and on this side there was six and right here there was 4 and I colored it in.

8. T: So how many parts do you have on this one?
9. S: 4.
10. T: Where are the parts? Can you show me?
11. S: [points and counts the rows down] 1, 2, 3, 4.
12. T: That's how you broke it up into parts?
13. S: [Points and counts how many in each row] 1, 2, 3, 4, 5, 6.
14. T: Where did your parts go – did you break …
15. S: Right here [points to the top row].
16. T: Oh, so you broke it apart this way. Do you remember what way this is called?
17. S: Vertical.
18. T: Not vertical…..
19. Ss: Horizontal.
20. T: Horizontal – making horizontal lines breaking up the rectangle horizontally and then you figured it out once you had your 4 parts that how many squares needed to be in each part?
21. S: 6.
22. T 6 – because you had 6 up there – you had 6 squares in this part and 6 squares in that part …good. Anybody find it a different way?
23. S: I found it the same way.

Ms. Quinn again successfully utilized the practice of eliciting a different solution, thereby creating opportunities to elicit explanations as justifications. What is noteworthy in this exchange is that Ms. Quinn stayed with Vincent for 24 turns, as he explained to the class a different way to show 1/4th. Moreover, as will be examined later in the section on practices fostering socialmathematical norms, Ms. Quinn made Vincent's thinking public when she asked in Turn 10, "Where are the parts? Can you show me?" And in Turn 14, "Where did your parts go? How did you break your rectangle the up?" Recording Vincent's work on the overhead screen provided the rest of the class a visual representation to go along with his explanation, thus students were offered an opportunity to make comparisons between Vincent's solution and their solution. As Vincent provided an explanation of his work, it was evident from the videotape that students were listening and comparing their solutions. And in Turn 23 when Ms. Quinn asked a student to share a different solution, he responded, “I found it the same way,” indicating he determined his method was the same. This was a successful exchange on four accounts for Ms. Quinn. First, she elicited a different solution; second, she elicited from Vincent an explanation and justification of his solution; third, she managed to keep the focus of the conversation on Vincent’s solution; and fourth, she made Vincent’s thinking public for the rest of the class to consider and compare.

This last exchange again reveals the critical role eliciting different solutions plays in orchestrating productive mathematics conversations. During Observation 3, after students shared different ways to show 1/4th on a 6 by 4 inch grid, Ms. Quinn asked if they could find different ways to show 1/8th of the same grid. After working on the problem individually for several minutes Ms. Quinn asked Cheryl to share her solution. Figure 6.4 represents how Cheryl solved the problem.

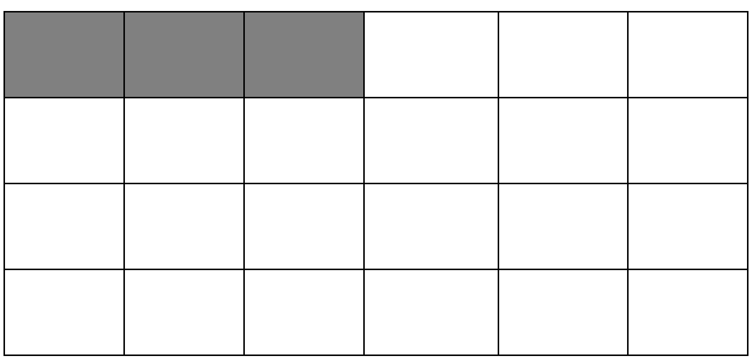


Figure 6.4 Student’s representation of 1/8

1. T: Okay, Cheryl, how did you do it?
2. S: I did 1-2-3-4-5-6-7-8 [counts 8 groups of 3] There’s 4 in a row [pointing to the 4 groups of 3 in each half of the grid].
3. T: So there’s 4 groups in each row. Okay. Did you just count that out and figure it out?
4. S: No. First I started with 4s, and I figured out that didn’t work, and then I did 1s, and I knew that wasn’t going to work.
5. T: Why wasn’t that going to work?

6. S: Because there are too many 1s in the whole box. So then I went to 2, and that didn't work. (inaudible – other students talking in background).

7. T: Okay, very good. So, if this is 1/8th, right? The shaded in part. So how much do you have here that is not shaded in? What's the fractional part that is not shaded in?
8. S: 7/8th?
9. T: How did you figure that out?
10. S: There's 7 right here, so I'm guessing 8 is the, is the umm denominator and then the numerator is the number that's left over.

Ms. Quinn begins with a request to share different solutions, thus she immediately focuses the conversation on student thinking rather than on correct answers. When Cheryl offers an explanation in Turn 2, Ms. Quinn follows up with a request for Cheryl to explain further, "Did you just count that out or figure it out?" This request helps to illuminate the process that Cheryl uses for other students and for Ms. Quinn as well. Next, when Cheryl states in Turn 4 that she knew 1s would not work, Ms. Quinn again follows up and asks, "Why wasn't that going to work?" thereby requesting that Cheryl justify her statement. Moreover, when Ms. Quinn asks Cheryl a QWKA in Turn 7 and Cheryl provides a correct answer in Turn 8, rather than evaluate the response and end the exchange, Ms. Quinn asks Cheryl to explain her thought process further by asking, "How did you figure that out?" Again, this provides Cheryl an opportunity to further explain her thinking to MS. Quinn and the class.

When Ms. Quinn utilizes the practice of eliciting different solution she is successful at keeping the focus of conversations on students' thinking. Moreover, when she elicits different solutions she successfully elicites student explanations and justifications as well. Furthermore, the practice of eliciting different solutions obligates students to compare their solution to ones previously shared, contributing to the conversation, thus students are engaged in a higher level of critical thinking. For Ms.

Quinn the practice of eliciting different solutions is critical to her ability to adopt other reform-oriented teaching practices. However, as seen in Table 6.1, Ms. Quinn only engaged in the practice of eliciting different solutions 8 times over the course of three observations; thus, it was not a developed aspect of her teaching practice. Ms. Quinn relies heavily on the traditional practice of asking questions with the known answer (QWKA). The following section will examine this practice in detail and how such a practice regulates conversations to quick question and answer exchanges.

Asked QWKAs and Evaluated/Accepted Student Responses

The prevailing participation structure found in Ms. Quinn's classroom is that of a traditional IRE question and answer pattern as described by Mehan (1979). In this pattern the teacher asks a question, nominates a student to respond, and follows up with an evaluation of the student response or another question or comment to help the student respond with a correct answer. Most often the initiation is begun with a QWKA in that the teacher asks a question in which she already knows the answer and works at eliciting this answer from her students. When the participation structure in the classroom is dominated by QWKA, the types of follow up moves that teachers rely on are often evaluative in nature and serve to affirm or disconfirm a student's response. As seen in Table 6.1, Ms. Quinn asked 126 QWKAs making this the prevailing practice that dominates the conversations she orchestrates with students. Moreover, Ms. Quinn evaluated and accepted student responses 71 times, making this her second most utilized practice. Figure 6.5 reveals a typical three part IRE sequence taken from Observation 3 (p.13).

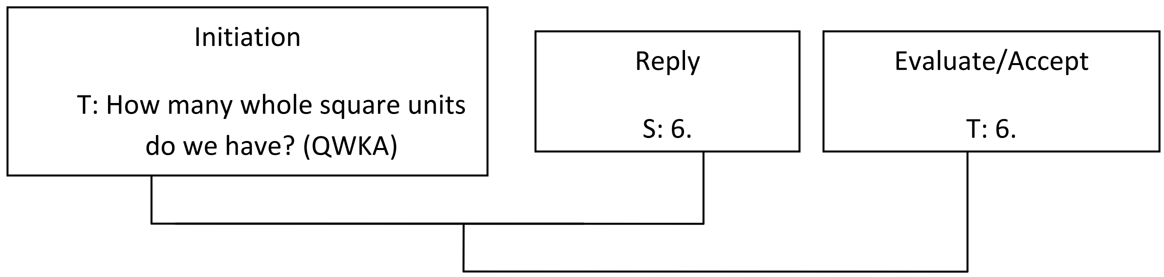


Figure 6.5 An initiate, respond, evaluate sequence: Ms. Quinn

The question that Ms. Quinn askes in the initiation phases is a QWKA, obligating students to respond with a correct answer. When the student nominated by Ms. Quinn said, “6” Ms. Quinn accepts the answer, thereby evaluating the correctness of the response. The following two exchanges revel how this pattern of discourse regulate the conversations in Ms. Quinn’s classroom.

In the following exchange Ms. Quinn began the lesson with a QWKA, thus Ms. Quinn had a particular answer in mind before the exchange began. As a result, the predetermined answer took precedence over student thinking and reasoning about fractions (March 23, p. 1).

1. T: What’s a fraction? John?
2. S: Something that’s 3/4ths… um it’s like half of something.
3. T: It’s what?
4. S: It’s half of something?
5. T: Would 3/4ths be half of something?[asked in a way that implied his response did not make sense]
6. S: No [Seems embarrassed by his response].
7. T: Who can help us explain what a fraction is a little bit more than that? I can’t believe I told you this at the end of Friday.
8. S: You did?
9. T: Do you remember what it was, James?
10. S: Nope.
11. T: A fraction is part of a whole or part of something.

In this exchange Ms. Quinn asked a QWKA in Turn1, however, in Turn 2 John offered a response that was a conjecture stating that he thought a fractions was “something that’s 3/4ths… um half of something.” Because Ms. Quinn asked a QWKA she was not looking for a conjecture but a correct answer to her question. As a result, Ms. Quinn misses a valuable opportunity to elicit John’s understanding of fractions and build on his conjecture. Her comment in Turn 7, however unintended, let the students know that she was not asking them to explain their thinking but to supply her with a correct answer. Moreover, when she says in Turn7, “I can’t believe I told you this at the end of Friday” she indicates to students a correct answer was being requested.

In an exchange such as this, Ms. Quinn and her student’s are interactively establishing social norms that regulate students’ participation in mathematics conversations to providing correct answers when asked a QWKA rather than explanations of one’s thinking. It is evident that John transgressed this norm when he provided an explanation of his current understanding of fractions, because Ms. Quinn responded by refuting his conjecture when she said “Would 3/4ths be half of something?” in a that-does-not-make-sense kind of tone. When she asked if someone else could “explain what a fraction is a little bit more than that” the taken-as-shared understanding of her request was that a correct answer and not an explanation was being requested. Evidently no one knew the answer Ms. Quinn was looking for, thus Ms. Quinn was obligated to provide it for students. Perhaps the most troubling aspect of this conversation is that Ms. Quinn does not make any attempt to understand John’s response and her questions in Turns 2 and 5 serve to shut down his thinking rather than to illuminate what it was he was trying to explain. Moreover, Ms. Quinn’s explanation of a fraction in Turn 11 seemed packaged and less meaningful than the one that John had attempted to explain to the class.

This next exchange again reveals the way in which asking QWKA obligates students in Ms. Quinn's class to respond with correct answers. In this exchange Ms. Quinn displayed on the document camera a 6 x 4 grid with the left hand side shaded in to represent one half (see Figure 6.6). The exchange began with Ms. Quinn asking her students to tell her how much of the grid the shaded in part represented (Observation 3, p.2).

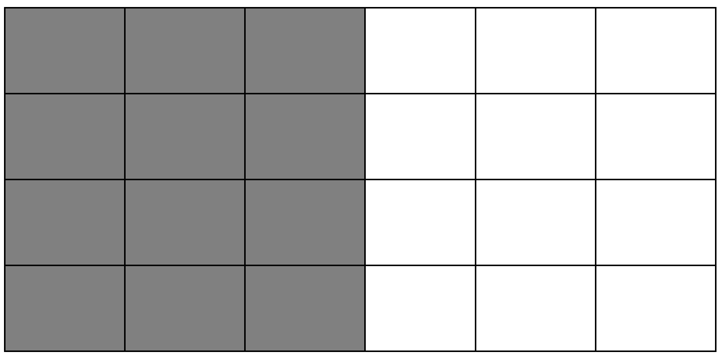


Figure 6.6 Grid shown on document camera representing ½

1. T: This part of my sandwich is gone [pointing to the shaded portion of the grid]. So this part of my sandwich I have eaten. What fraction of the rectangle is the shaded piece? What fraction of the rectangle is called the shaded part of this? What's the fraction that is shaded of this that I have eaten? How much of this have I eaten? Jonathan?
2. S: You ate 4/3rds?
3. T: 4/3rds? [asked in a way that implied 4/3rds did not make sense] Brian?
4. S: One half.
5. T: Why would it be one half, Brian?
6. S: Because it's half a sandwich.
7. T: Because it is half a sandwich. Right. So do you agree this is one half?
8. S: Yes.

In this exchange when Vincent responded to Ms. Quinn's question with "4/3rds" she repeats his response in a questioning manner, which indicates that 4/3rds is not the correct answer, and moreover that 4/3rds did not make sense. However, for Jonathan

4/3rds does make sense, yet he is left to wonder why his answer is not correct. It is possible that Jonathan, looking at the shaded in part, noticed that it was 4 down by 3 across, prompting his response of 4/3rds. By not probing his thinking further, Ms. Quinn left Jonathan and possibly others to wonder why the response was not accurate.

The interaction between Ms. Quinn and Jonathan serves to develop the taken-as-shared understanding that when asked a QWKA, the appropriate response is a correct answer and not a conjecture. Here, Jonathan transgressed this social norm, and Ms. Quinn’s responses serve to focus the conversation back to responding with correct answers. As evidence, when Ms. Quinn asks Brian to explain why the answer is one half, his response, “Because it’s half a sandwich” was vague and not a mathematical explanation; however, it was obviously what Ms. Quinn wanted as she indicated that Brian’s answer was correct. This is significant in that the socialmathematical norm of what counts as an acceptable mathematical explanation is developed here as well. In this exchange an acceptable mathematical explanation is a correct answer to Ms. Quinn’s question rather than an explanation of your mathematical thinking.

According to Ms. Quinn the language skills of her students are such that she feels it necessary to ask questions that require only a correct answer from students rather than an explanation or justification. Many of her students come from homes where Spanish is the language spoken, thus Ms. Quinn believes she can only engage them in a limited amount of detailed conversation. When reflecting on one of her teaching videos, Ms. Quinn noticed that she asked several questions that required her student to respond with a correct answer rather than an explanation. For Ms. Quinn this was a necessary practice to utilize because of her students’ English language skills. She said,

> My questioning strategies vary from both eliciting students' thinking to students giving me the correct answer. This is largely due to the fact that I have many students in my class who are English language learners, so it is often hard for them to understand detailed, wordy questions. I did see that with students whom I felt had a better grasp on the concepts, I was asking more questions to understand their thinking. (Video Reflection, p. 1)

The language abilities of Ms. Quinn's students play a role in the types of questions she feels she can ask and in the types of responses she believes students are capable of offering. Students with stronger English language skills are offered more detailed questions so that Ms. Quinn can understand their thinking; whereas students who are less skilled in English are often regulated to answering questions that have one particular correct answer.

Asking QWKA dominates Ms. Quinn's teaching practice, thus the overall participation structure mirrors a traditional classroom where the teacher asks questions and students respond with correct answers. Moreover, because Ms. Quinn asks QWKAs, she is obligated to evaluate and/or accept the responses students offer. As seen in Table 6.1, Ms. Quinn overtly evaluated responses offered by students 25 times and overtly accepted student responses 46 times over the course of three observations. Evaluated responses often took the form of an overt evaluation as in the following exchange. (Observation 1, p. 10)

1. T: What is a product?
2. S: It's the answer to two factors.
3. T: The answer to two factors when we multiply. Excellent.

Ms. Quinn overtly evaluated student responses by using phrases, such as "good job," "excellent," "right," or "very good" for a correct answer. For incorrect answers Ms. Quinn at times responded overtly by saying, "No" or "Nope" but at times she would accept a response; however, her acceptance often indicated that a response was incorrect or not what she was looking for. Consider the following exchange that took place after students had just finished counting around the room by 8s. Here Ms. Quinn asks a QWKA and although two students responded correctly, the responses were not what Ms. Quinn had in mind, thus prompting her to provide the correct answer (Observation 2, p. 8).

1. T: So if you go up by 2, what do you call all of these numbers?
2. S: Even numbers.
3. T: Well, they are even, but I want you to think about hmmm – when we're doing arrays in the beginning of the year. We were talking about arrays and prime numbers and positive numbers and square numbers and factors and multiples. What are all of these?
4. S: Numerical data?
5. T: It could be a kind of numerical data, but I am not talking about the data units we did. Good job remembering though. These are all multiples.

Although both students responded with answers that were correct, they were not the answers that Ms. Quinn was trying to elicit. Thus, Ms. Quinn accepted both; however, her acceptance took the form of an evaluation. When Ms. Quinn said, "Well they are even" and "It could be kind of numerical data," she effectively evaluated the answers as incorrect. Situations such as this reduce students' participation to that of guessers of answers rather than active participants in meaning making.

There are also times when Ms. Quinn moves on to the next QWKA and this practice serves to indicate that the previous response was correct. Students develope the taken-as-shared understanding that if their response is not questioned by Ms. Quinn, it is a correct response. As an example, consider the following exchange (Observation 1, p. 6).

1. T: What is 10 times 8?
2. S: 80.
3. T: What is 9 times 8?
4. S: 72.

5. T: 72.

In this exchange after a student provided a correct answer in Turn 2, Ms. Quinn immediately asked the next question, effectively evaluating the previous response. It seems likely that students in Ms. Quinn's classroom develop the taken-as-shared understanding of the multiples of 10, thus Ms. Quinn does not feel the need to overtly evaluate the response. This practice of moving on to the next QWKA serves the purpose of evaluation. However, Turn 5, Ms. Quinn overtly accepts the response of 72 by repeating it. This indicates that possibly the multiples of 9 are not taken-as-shared at this point in the school year, thus Ms. Quinn wants to let students know that the answer of 72 is a correct one. In the above exchange, asking the next QWKA serves as an evaluation of the previous response and repeating a student's response serves as an evaluation as well. The following exchange again reveals the way in which Ms. Quinn used the next QWKA as a means of evaluation.

1. T: What is 20 times 5?
2. S: 100.
3. T: Do we see an array?
4. Ss: Yes.

In Ms. Quinn's classroom students have developed the taken-as-shared understanding that when Ms. Quinn moves on to the next question their response had been evaluated as a correct one. Consequently, asking QWKA serves a dual purpose in MS. Quinn's classroom: 1) to elicit a correct answer and 2) to evaluate a previous student's response.

Summary

This section described the teaching practices that Ms. Quinn used to engage her students in mathematics conversations. Ms. Quinn's most prominent practice is that of asking QWKAs followed by accepting or evaluating student responses. Moreover, the practice of asking QWKAs serves to elicit correct answers and at times to evaluate the previous response given. As such, the conversations Ms. Quinn orchestrates with students often focuses on answering her questions rather than on student thinking. As a result, students in Ms. Quinn's classroom are, for the most part, obligated to produce correct answers rather than share how they went about solving problems. Thus the social norm constituted is one where students supply correct answers to the QWKA that Ms. Quinn asks.

Ms. Quinn at times utilizes the reform-oriented practice of eliciting explanations and justifications. Although this practice has the potential to focus conversations on student thinking, it also has the potential to focus student's thinking on correct answers. As a result of having a practice dominated by asking QWKA Ms. Quinn often misses opportunities to ask students to explain their thinking and instead asks questions that eventually leads student to respond with a correct answer. Possibly, because Ms. Quinn has difficulty assessing student's non-conventional ways of understanding mathematics, yet is strong in explaining rules and procedures, she steers away from probing student's thinking in favor of keeping conversations focused on rules and procedures that will produce a correct answer.

Ms. Quinn did elicit different solutions to the same problem, albeit only 8 times over the course of three observations. Analysis reveals this is her most successful attempt at also eliciting explanations and justifications of students thinking. Although clearly not a developed aspect of her teaching practice, when Ms. Quinn does elicit different solutions, she is more apt to keep the focus on student's thinking and not on correct answers.

Ms. Quinn relies heavily on the traditional practice of asking QWKA followed by accepting and evaluating student responses. As such, the social norms that develop are overwhelmingly traditional. Because of her over reliance of asking QWKAs, Ms. Quinn struggles to utilize reform-oriented practices and often her attempts focus on correct answers rather than on students' thinking. As Ms. Quinn and her students engage in mathematics conversations, they interactively establish the following taken-as-shared participation structure:

- When the teacher asks QWKAs, students are expected to provide correct answers and not explanations of their thinking.
- Teacher was the mathematical authority in that she evaluates student responses
- Listening is expected for management reasons.
- If teacher elicits a different solution, students are expected to provide an explanation and justification of their solution
- If teacher elicits a different solution, students are expected to listen to determine if their solution is different.

As stated previously, the social norms that are constituted are not set out as a list of rules that students must follow, rather such norms are interactively constituted as Ms. Quinn and her students engage in mathematics conversations. For example, because Ms. Quinn makes a practice of asking QWKAs, students develope the taken-as-shared understanding that an appropriate response requires a correct answer. When students transgresses this norm and provides an explanation of their thinking, Ms. Quinn responds negatively, thus indicating that this type of question is not eliciting their thinking. Such a norm often reduces students to guessing what Ms. Quinn wants for an answer. However, students develop the taken-as-shared understanding that when Ms. Quinn elicits different solutions, this type of elicitation obligates them to explain and justify their thinking. In these instances the conversations are focused on student thinking rather than on correct answers.

Ms. Quinn's teaching practices constitute a participation structure that is for-the-most-part traditional whereby the teacher asks the questions and students supply correct answers. However, reform-oriented conversations do emerge when Ms. Quinn elicits different solutions from students, indicating a practice that is not solely grounded in traditional practices. The social norms that Ms. Quinn's practices foster are linked to the socialmathematical norms that are allowed to flourish in Ms. Quinn's classroom. The following section will examine Ms. Quinn's practices in relationship to the socialmathematical norms that emerge.

Practices Fostering Socialmathematical Norms

As stated previously, socialmathematical norms are specific to the mathematics of a lesson and contribute to students' understanding of what counts as a different, efficient, or sophisticated mathematical solution as well as what counts as an acceptable mathematical explanation and justification (McClain & Cobb, 2001; Yackel & Cobb, 1996). Developing a taken-as-shared sense of what counts as a different, efficient, or sophisticated solution involves understanding when it is appropriate to contribute to the classroom conversation. In this sense students would be obligated to compare their solution to others previously shared and to determine if in fact their method was different or more efficient than ones previously shared. Understanding what counts as an acceptable mathematical explanation and/or justification involves the act of contributing to the mathematical conversation (McClain & Cobb, 2001). Here, students develop an understanding of what an acceptable mathematical explanation must entail. Table 6.2 shows the distribution of practices Ms. Quinn used that served to constitute the socialmathematical norms in her classroom as well as the number of times each practice was used per hour.

Table 6.2 Observed Practices Fostering Socialmathematical Norms: Ms. Quinn

	Observation				
Practice	OB1	OB2	OB3	Total	Per Hour
Developed Idea of Mathematically Different	1	0	0	1	0.5
Developed Idea of Mathematically Efficient/Sophisticated	1	0	0	1	0.5
Made Student Thinking Public	5	1	6	12	6.7
Accepted One Word Answers	25	24	45	94	52.2
Indicated Math was Rule Bound	4	3	7	14	7.8

As seen in Table 6.2, Ms. Quinn made one reference that could be viewed as an attempt to establish what it meant to share a different mathematical solution. Moreover, because Ms. Quinn only asked students to share different solutions (social norm) 8 times, she afforded herself little opportunity to establish the socialmathematical norm of what constitutes a mathematically different solution. Moreover, Ms. Quinn only made one reference that pointed out the efficiency of a solution, thus the students were not obligated to make comparisons between solutions to determine more efficient and sophisticated ones. Ms. Quinn did attempt to make students' thinking public; however, with only 12 attempts, this was not a developed aspect of her teaching practice.

Ms. Quinn's most utilized practices are traditional and serve to constitute traditional socialmathematical norms within her classroom. As seen in Table 6.2, Ms. Quinn accepted one word answers 94 times, indicating that such answers are considered acceptable mathematical explanations and justifications. Lastly, Table 6.2 reveals that Ms. Quinn made 14 references to math being governed by rules and procedures that needed to be remembered. The following sections will examine the instances of Ms. Quinn's reform-oriented practices followed by an analysis of the traditional practices that Ms. Quinn used and the socialmathematical norms that such practices constituted.

Mathematically Different

As seen in Table 6.2, there was one instance where Ms. Quinn commented that a student's solution was the same as a solution previously shared. Moreover, of the 8 requests Ms. Quinn made asking students to share a different solution, 7 were mathematically different. Possibly, this result indicates that students had developed the taken-as-shared understanding of what counts as a mathematically different solution because 7 of the 8 solutions shared were mathematically different. Moreover, the one case where a solution shared was not mathematically different, Ms. Quinn did not accept the solution as different. However, because Ms. Quinn only requests that students share different solutions 8 times, more data would be needed to make such an inference. However, examining the instance where she interactively established the socialmathematical norm of mathematically different will shed light the implicit lessons learned from such an exchange and the way in which Ms. Quinn negotiates what it means to share a mathematically different solution.

In the following exchange Ms. Quinn displayed on the document camera an arrangement of dots for five seconds and asked student to observe and then write down how they remembered the dots either using pictures, words, or an equation. Figure 6.7 shows the arrangement Ms. Quinn displayed on the screen in the front of the room (Observation 1, p. 6).

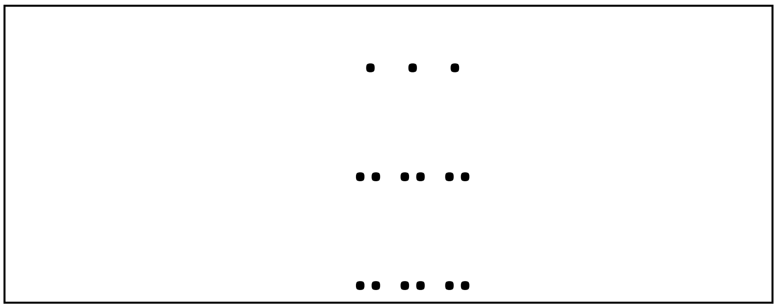


Figure 6.7 Dot arrangement shown on overhead

1. T: Alright, how did you see it, Jake?
2. S: Well, I saw it (inaudible) there is 3 groups like there is 3 groups with 3 in a group.
3. T: Like that? [Circles on the screen how Jake saw it] What would your equation be?
4. S: 3 times 7 equals 21.
5. T: Good. Did anybody see it a different way? Myra.
6. S: I saw it like a rocket ship.
7. T You remembered it like this? [Draws a line around the dots to look like a rocket ship.]
8. S: Yes.
9. T: So, would your equation be the same or different from Jake's?
10. S: It would be the same.
11. T: Yep. It would be the same.

The most striking feature of this exchange is the fact that by developing the social norm that problems have more than one solution, the socialmathematical norm of a mathematically different solution has an opportunity to emerge. For example, in Turn 5 Ms. Quinn asked, "Did anybody see it differently?" which obligates students to compare their solution with Jake's to determine if they had found a mathematically different one.

Myra responded in Turn 6 with a different way of seeing the dots “like a rocket ship,” but her resulting equation was the same as Jake’s. Ms. Quinn capitalized on the opportunity to negotiate with Myra and the group that an acceptable different solution should result in a mathematically different equation. Moreover, in Turn 9 Ms. Quinn gives responsibility of determining if the solution is mathematically different to Myra. Before Myra answered in Turn 10 she could be seen looking at the screen and comparing her rocket formation to the equation that Jake had shared before determining her equation was the same as Jake’s.

It is important to note that the role Ms. Quinn plays in this exchange is significant in that she helps the students to visually assess if their equation would be the same or different. Ms. Quinn circled Jake’s way of seeing the dots and drew lines in the shape of a rocket when Myra explained how she saw the dots. This seems to help Myra see that she in fact did have a mathematically different solution because the way she saw the dots could be mapped out differently on the overhead screen. Moreover, by recording the different ways student remembered the arrangement, Ms. Quinn effectively takes the focus off finding an answer and places it on bringing student ideas to the fore of the conversation to be considered and compared by all.

In the next exchange, which took place immediately after the aforementioned one, a student calls out that she has a different solution, thus indicating that the exchange between Ms. Quinn, Jake, and Myra was significant in helping to constitute the socialmathematical norm of what counts as a mathematically different solution (Observation 1, p. 6).

1. S: Miss I got a different one.
2. T: All right, Jayna.

3. S: I see 1s by themselves as one group [makes a motion in the air circling the top 3 dots].
4. T: Like that? Each one by itself? Like that? [Circles top row of dots].
5. S: Yes. And then the 2s the same things [circles in air as if circling the dots].
6. T: Like that [Circles the 2s in a column]
7. S: No, no like in a row – 3 (inaudible).
8. T: Okay, you come up and circle. [Jayna goes to overhead and circles 3 rows with 6 dots in each row].
9. T: What would the equation be for this one?

Immediately before this exchange began, Jayna can be seen on the videotape looking at her paper and then at the screen before she excitedly called out in Turn 1 "Miss, I got a different one." At this point in the conversation it was evident that criteria for communicating a different mathematical solution were developing (McClain & Cobb, 2001). For example, Jayna apparently realized that to contribute to the conversation, her solution needed to yield a different equation. It can be inferred that Jayna visually compared her solution to the ones already shared and made a determination that the way in which she saw the dots would yield a different equation, prompting her to offer to contribute to the conversation. Moreover, in Turn 6 when Ms. Quinn circled the dots in a column, Jayna interrupted and said, "No, No like in a row 3" indicating that she did not see the dots arranged in a column as did Jake and Myra but in a row. .

In sum, these were two productive exchanges that engaged the class in what it means to share a different mathematical solution. McClain and Cobb (2001) assert that exchanges such as these may have begun with the objective of eliciting different solutions (social norm); however, in the course of sharing different solutions, what counted as a different mathematical solution emerged. In these exchanges Ms. Quinn and her students interactively constituted the socialmathematical norm of what counts as a mathematically different solution. Both the explanations that students share and the recordings that Ms. Quinn makes on the overhead help to establish this socialmathematical norm.

Although the two exchanges reveal that Ms. Quinn did attempt to engage her students in conversations that fostered the socialmathematical norm of mathematically different, because she only elicited different solutions 8 times, the opportunities for her students to develop this understanding were limited. However, it should be noted that when asked to share a different solution, all but one student shared a mathematically different solution, indicating that students may have developed a taken-as-shared sense of this socialmathematical norm. Further observations would be needed to assert that this norm is well established in Ms. Quinn's classroom.

Mathematically Efficient/Sophisticated

As seen in Table 6.2, Ms. Quinn made only one attempt to point out when one solution was more efficient than another, indicating again that this was not a developed aspect of her teaching practice. Moreover, because she rarely asks students to share solutions that are different, she has limited opportunities to make such distinctions for students. However, when a student does offer an inefficient solution, Ms. Quinn uses it as an opportunity to negotiate the socialmathematical of what constitutes a mathematically efficient solution. Consider the following exchange that took place immediately after the exchange with Jayna (Observation 1, p. 7).

1. T: Joe, do you have another one?
2. S: Yes, you can do it by 1s.
3. T: We could do it by 1s, but would that be very efficient? Is that efficient going by 1s?
4. Ss: That would be hard.

5. T: The whole point in multiplication is that you don't have to keep adding 1 onto the next 1 to the next 1.
6. S: Yea, and you'd have to do 1 and 1 and 1…
7. T: Right. But that's going to take a while.

In this exchange Joe offered a mathematically different solution; however, it was considered a less efficient solution than the ones already shared. Ms. Quinn uses Joe's response as an opportunity to help students make a distinction between more efficient solutions, and as seen in the exchange, students were listening and attempting to make sense of this concept. When a student responded in Turn 4 with, "That would be hard" and in Turn 6 with, "Yea, and you'd have to do 1 and 1 and 1…," they seemed to be developing an understanding of the inefficiency of counting the group by 1s. Again, it should be noted that this is only one exchange, thus assumptions that this is a well developed norm in Ms. Quinn's classroom cannot be made. However, this exchange does indicate that Ms. Quinn is aware of the importance of developing student's abilities to assess the mathematical efficiency of solutions. Moreover, the above exchanges highlighting what counts as mathematically different and mathematically efficient solutions all emerge when Ms. Quinn implements the practice of eliciting different solutions. Significantly, when Ms. Quinn develops the social norm of sharing different solutions, she can capitalize on opportunities to develop the socialmathematical norm of what counts as a mathematically different and efficient solution. Thus, the practice of eliciting different solutions is fundamental to developing reform-oriented, socialmathematical norms within the classroom.

Made Mathematical Thinking Public

As seen in Table 6.2, Ms. Quinn attempted to implement the practice of making student's mathematical thinking public 11 times over the course of three observations, indicating that this again was not a well developed aspect of her teaching practice. Moreover, analysis reveals that Ms. Quinn's use of this practice has the potential to focus the conversation on student thinking as well as on correct answers.

In the following exchange Ms. Quinn asked a student to explain how she found a different way to represent 1/4th of a 6 x 4 grid. Here, not only did the student share her solution, Ms. Quinn revoiced her solution effectively, making the student's solution the focus of the conversation. Figure 6.8 shows the way in which the student represented 1/4th of the grid (Observation 3, p. 7).

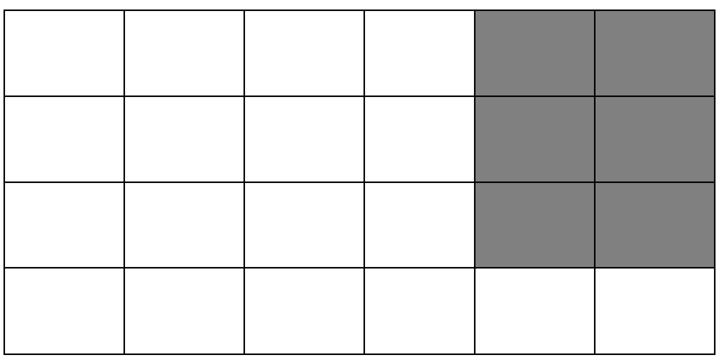


Figure 6.8 Student representation of 1/4

1. T: Cheryl, how did you solve it?
2. S: Inaudible (student provides an explanation but is too quiet to hear)
3. T: Okay, I want you to all follow along. Okay, so she took this how we had it horizontally, and she took it, and she flipped it so that it was vertical. So how many parts would this shape be broken into?
4. S: 4
5. T: 4 parts. So, I have a question for you? The 4 parts do not look the same. So does what she did work?
6. Ss: No! Yes!
7. T: It still works because they don't have to look exactly the same. They just have to be the same area. How many are here [Pointing to shaded area]?
8. Ss: 6.

9. T: How many here?
10. S: 6.
11. T: And how many here?
12. Ss: 6.
13. T: And this long line across the bottom?
14. Ss: 6.
15. T: So even though they don't look the same, do they have the same area?
16. Ss: Yes! No!
17. T: [Sounding frustrated] Yes, they are! This works - this is 1/4th.

In this exchange Ms. Quinn manages to make Cheryl's thinking public by asking her to display her grid on the document camera and explain how she found a different way to show 1/4th. Because Cheryl's explanation was too quiet for others to hear ,Ms. Quinn revoices it in Turn 3, thereby making Cheryl's thinking more acsessible to the other students. She then follows up by asking the students to consider if Cheryl's method did represent 1/4th. Up until that point in the conversation, Ms. Quinn's practices were oriented toward reform. However, when she receives a mixed response in Turn 6, she shifts to focus from eliciting student thinking toward getting students to believe that Cheryl's method was correct. At this point in the conversation Ms. Quinn misses an opportunity to illuminate what students did and did not understand about Cheryl solution. Rather than follow up and ask student to explain how Cheryl's representation did or did not show $1/4^{th}$, Ms. Quinn asks questions that she assumes will lead to the conjecture that Cheryl's method was correct. However, as seen in Turn 16, there were still students who were not convinced that the method represented 1/4th. Ms. Quinn ends the exchange, somewhat frustrated, by stating that Cheryl's method was correct, consequently negating any student who thought differently.

In the above exchange Ms. Quinn initiates the conversation by using the reform-oriented practice of eliciting student thinking; however, as the exchange continued she shifts the focus from how students view Cheryl's representation toward convincing students that Cheryl's' method was correct. In social situations such as this, students learn that providing correct answers to Ms. Quinn's elicitations takes precedence over understanding why an answer or solution method is deemed correct.

Acceptable Mathematical Explanations/Justifications

In a reform-oriented classroom an explanation or justification is considered acceptable when students describe the physical or conceptual actions taken on objects representing numbers as opposed to describing procedures performed (Cobb et al., 1992). This norm focuses on the mathematical activity that students engage in while solving problems and when established, aids in fostering intellectual autonomy in students (Cobb et al., 2001). For the most part, students in Ms. Quinn's classroom are expected to give one word correct answers to her elicitations, thus opportunity for them to act on numbers physically or conceptually is not offered. Consequently, the socialmathematical norm constituted is that one word correct answers are acceptable mathematical explanations.

One Word Correct Answers

As seen in Table 6.2, Ms. Quinn accepted 94 one word answers as she engaged with students in mathematics conversations. This result makes sense, considering that Ms. Quinn relies heavily on the practice of asking QWKAs and such questions require a correct answer. As a result of relying on QWKAs to initiate conversations, an acceptable mathematical explanation more often than not took the form of a one word correct answer rather than a detailed explanation of the actions students took to solve a particular problem. In the following exchange Ms. Quinn asked a student to figure out the dimensions of an array. As Ms. Quinn engaged the student in the conversation, she ignored incorrect answers and accepted correct ones, although it is unclear if the student understands why his answers are deemed correct by Ms. Quinn (Observation 1, p. 18).

T: So what would you do to complete this? How can you figure it out?
1. S: Times?
2. T: Okay, so what would the equation be or the expression be? Just think about the dimensions we talked about when using arrays. That's all these really are. Remember the across the top and the side. Can you figure out what the dimensions of the missing piece?
3. S: 25?
4. T: What would the dimensions be? How many across?
5. S: 12?
6. T: By how many down?
7. S: 12?
8. T: Nope – How many going down?
9. S: 2?
10. T: So what are the dimensions? What did you say going across?
11. S: 12 across times 2 going down.
12. T: Right, so that's the array you are looking for. The dimensions are 12 times 2 to complete this.

It can be inferred from this conversation that an acceptable mathematical explanation is one in which a correct answer is supplied regardless of whether or not the student understands why the answer is correct. As seen in Turns 3 and 7 the student responded with an incorrect answer and rather than ask for an explanation of the student's response Ms Quinn asked a QWKA in Turns 4 and 8 in an effort to elicit a correct answers. Although the student answered correctly in Turns 5 and 9, there was no indication that the student understood why such answers were correct. Cobb et al. (1992) referencing Krummheuer (1986) refer to such exchanges as working interims, whereby students learn to cooperate without understanding. The implicit, however unintentional, lessons learned in such an exchange are that answers do not require explanations or justifications. Moreover, correct answers take precedence over understanding and, as such, are considered acceptable mathematical explanations and justifications.

The following exchange examines how accepting one word answers turns conversations into question and answer sessions rather than productive mathematical conservations. In this exchange Ms. Quinn has asked students to tell her what they know about squares.

1. T: So, squares have sides that are equal. So, what else can you tell me about a square? What happens because it is a square? Cameron?
2. S: I don't know.
3. T: You don't know? John?
4. S: The sides are even.
5. T: Well, if the sides are equal they are even.
6. S: So, it is easier to make an X in a square … (inaudible)
7. S: What if I bring up lines of symmetry? If I cut my square this way, and I fold this, and if I use this as my line of symmetry, are both sides the same?
8. S: Yes.
9. T: And, if I go this way?
10. Ss: Yes.
11. T: Are they the same?
12. Ss: Yes.
13. T: So, what about the four parts that I have are they all the same?
14. Ss: Yes.
15. T: So, if we were doing this with a square, the "X" would work out well because all of our sides are the same in a square. There are how many lines of symmetry?
16. S: Four.
17. T: There are four lines of symmetry in a square. So, what are the other ways? Going vertical and horizontal, right? But with a rectangle, there are how many lines of symmetry? Think about it – look at it? How many ways can I fold that rectangle?
18. S: Six ways.
19. T: Oh my goodness!
20. S: Two ways.
21. T: Thank you Jacob. Two ways. Vertical and horizontal.

In this exchange Ms. Quinn shifts the focus of the conversation from eliciting student understanding to eliciting one word responses to her questions. As seen in Turn 7, John attempts to offer an explanation that might have proved productive; however, Ms.

Quinn does not follow up on John's response. Instead, Ms. Quinn introduces the term symmetry and then begins a question and answer sequence where students are obligated to provide correct one word answers to her questions without explanation or justification. Moreover, as seen in Turn 18, her demonstration of lines of symmetry do not help all students to understand the concept of symmetry, as when she asks how many lines of symmetry are in a rectangle, a student responded with "Six ways." What is interesting about this exchange is that it flows, in that students are able to supply her with correct answers and seem to know that this is their responsibility in this type of exchange. John's explanation is ignored in favor of eliciting correct answers to specific questions asked by Ms. Quinn.

Mathematical Activity Bound by Procedures

Lastly, as seen in Table 6.2, Ms. Quinn, at times, implies that math is bound by rules and procedures with 14 such implications. These were not overt attempts at spelling out rules and procedures but interactively constituted as Ms. Quinn engages with students in mathematics conversations. For example, when attempting to help students understand the concepts of numerator and denominator, she said, "Remember the denominator tells us how many parts and the numerator tells us how many to shade in" (Observation 3, p.4) and "One that's the top number" (Observation 3, p. 3). Although these comments are not stated overtly as rules and procedures to follow, the underlying message seems procedural. Such comments imply that remembering what each number in a fraction represented is all that is needed to understand the concepts of numerator and denominator. However, what is missing in these conversations is any discussion about the relationship between the numerator and the denominator. Developing an understanding of the numerator-denominator relationship helps students to develop an understanding of the concept of fraction size (Behr, Wachsmuth, & Post, 1985). If students are not offered opportunities to engage in conversations and activity about the relationship between the numerator and the denominator and only see them as being separate units, subsequent work with rational numbers may prove problematic.

Ms. Quinn also implies that once students learned something they should be able to remember it weeks later when called upon to do so. Consider the following exchange that took place when Ms. Quinn asked students to recall the term dimension (Observation 2, p. 10).

1. T: Dimensions – what are dimensions? What are dimensions?
2. S: Dimensions has to do with arrays?
3. T: Has to do with arrays. Can you be a little bit more specific? If I asked you the dimensions of my array what would you tell me?
4. S: I don't know.
5. T: What would that array be? We should know this. We learned this the first week of school, guys.

In this exchange Ms. Quinn suggests to students that they should be able to remember and give back when called upon definitions to mathematical terms that they learned during "the first week of school." Moreover, when a student offers that the term dimension is related in some way to an array, Ms. Quinn indicates that a more specific answer was needed. This is a missed opportunity for Ms. Quinn to develop her students' conceptual understanding of the term dimension in relationship to the term array. Because Ms. Quinn has focused much of her teaching on eliciting correct answers, she misses opportunities to explore her students' thinking and help them to make connections between concepts. Moreover, such an exchange constitutes the socialmathematical norm that remembering rules, procedures, and in this case definitions, is what you need to do to understand mathematics.

Moreover, Ms. Quinn explains rules and procedures to students, albeit without offering students opportunities to understand the purpose of such rules and procedures. For example, consider the following exchange centered on the use of parenthesis. (Observation 2, p. 11)

1. T: What are they called?
2. S: Parenthesis.
3. T: In my last group, I called them hugs because they remind me of a hug because it goes around the front and around the back like a big hug.
4. S: So it gives you a hug from around the back and around the front?
5. T: When you give a hug your arms go around like this right?
6. S: But why do you do that?
7. T: To show that you have to multiply these two numbers together and then multiply these two numbers together first, and then add them up.
8. S: But it says 1 times 12 and then you put the plus sign and then you do 5 times 12?
9. T: You are multiplying this together, and then you'll multiply this together, and then you take your two answers from that, and you add them together.

In this exchange Ms. Quinn told students what to do when confronted with parentheses in an equation in a procedural manner "multiply these two numbers together, and then multiply these two numbers together first, and then add them up" without addressing the function of parentheses in an equation. When a student commented in Turn 8 that the plus sign seemed to serve the same purpose as the parentheses, Ms. Quinn again stated the procedure to follow when encountering parentheses in an equation. When viewing the videotape, it is evident that the student in Turn 8 was no more convinced with Ms. Quinn's second attempt at an explanation than he was with the first. Focusing solely on the procedural aspects of how to remember what a parentheses looks like (a hug) and what to do when one is encountered (multiply these two first…) ignored the student's question of "why do you do that?" Ms. Quinn might have offered her students opportunities to examine the equation of 1 x 12 + 5 x 12 changing the placement of the parentheses and discussing the results. An exchange such as this one fosters the taken-as-shared understanding that following procedures is more important than understanding why a particular procedure is used.

During two of the three observations, Ms. Quinn engaged her students in learning how to play a multiplication game. The objective of the lesson was to help students develop "strategies for multiplying that involve breaking apart numbers" (Lesson Plan, 11/3/08). As she reflected on the lesson Ms. Quinn pointed out that students need to understand the "steps" of a process in order to move on in their understanding. She said,

> I felt that the students were uncovering concepts as they were realizing what arrays they needed in order to "complete" the center array that they were working on. This is the first step they needed to understand to get to the next point, which was breaking up larger multiplication problems into smaller more manageable ones.

For Ms. Quinn, math seems to be a series of steps that she feels responsible to teach and that she believes students need to learn before they can move on in the curriculum. Even when students are engaged in an activity whose objective is to help them develop their own personal strategies for solving multiplication problems, the focus is on the steps of the process rather than on the divergent strategies that students use.

Summary

This section described the teaching practices Ms. Quinn uses that are associated with the constitution of socialmathematical norms in the classroom. Socialmathematical norms are interactively negotiated as teachers and students engage in mathematical activity. Moreover, socialmathematical norms “regulate classroom discourse and influence the learning opportunities that arise for both the students and the teacher” (McClain & Cobb, 2001, p. 237). Socialmathematical norms include what counts as a mathematically different, efficient, and sophisticated solution and, as such, regulate when it is appropriate to contribute to a classroom conversation. The socialmathematical norms of what counts as an acceptable mathematical explanation and justification regulate the actual act of contributing to a conversation.

The teaching practices that Ms. Quinn uses help to constitute socialmathematical norms that were more traditional than reform and serve to regulate students’ mathematical activity toward contributing one word correct answers and providing procedural explanations and justifications. Moreover, the practices that Ms. Quinn used to regulate the participation structure constrain the types of socialmathematical norms that were able to flourish. For example, because Ms. Quinn does not consistently make a practice of eliciting different solutions (social norm), she has little opportunity to proactively foster the development of what counts as a mathematically different, efficient, and sophisticated solution (socialmathematical norms). Moreover, because Ms. Quinn’s relies heavily on the practice of asking QWKAs (social practice), students are regulated to providing correct answers in conversation rather than explanations and justifications of their mathematical thinking. Consequently, the socialmathematical norms that emerge regulate students to provide correct answers to Ms. Quinn’s questions; thus, it can be inferred that a correct answer is contribute to a conversation when you know the correct answer. Moreover, this over reliance on asking QWKAs regulates what counts as an acceptable mathematical explanation in Ms. Quinn’s classroom and again a correct answer becomes the normative explanation and justification. When students transgress this norm and instead offer an explanation of their thinking, Ms. Quinn responds with comments that serve to let them know an explanation of their thinking is not requested, but a correct answer was. As a result of the practices that Ms. Quinn uses, the following socialmathematical norms emerged:

- Students are obligated to explain their thinking when Ms. Quinn asks for different solutions; however, when students are asked a QWKA, they are obligated to respond with a correct answer and not with an explanation of their thinking.

- Math problems, for the most part, have one solution that students must learn and remember how to do.

- Mathematics is made up of rules and procedures that must be memorized.

Issues and Challenges

After analyzing Ms. Quinn's teaching practices as well as her interview discourse, it became apparent that there were issues and challenges that Ms. Quinn faces as she attempts to implement teaching practices that are aligned with mathematics reform. The following section will address the second research question guiding this study.

- What issues and challenges surface as novice teachers begin to enact reform orientated discourse practices?

Top Down Pressure

As a novice teacher Ms. Quinn experienced top down pressure from her district administration and such pressure impacts her ability to implement mathematics reform recommendations. In June of her second year of teaching, Ms. Quinn was informed that because of district reorganization, she was being transferred to a different school and would be teaching fourth grade math the following school year. The decision was made without any input from Ms. Quinn, and when asked how she felt about the move, her response was, “I am going in blinded” (Interview, p. 1) referring to the fact she did not have training in *Investigations*, the new reform-oriented curriculum that the district had recently adopted. When contemplating her transfer to a new school and reassignment as fourth grade math specialist, Ms. Quinn expressed concern when she said, “I am a little worried about next year because I don’t have the training” (Interview, p. 13). According to Ms. Quinn she lacked the foundation needed to teach using the *Investigations* curriculum because first grade teachers were not offered training in the first year of implementation. Expressing her frustration she said,

> We got the new *Investigations* – here you go. And then we find out there will be *Investigations* trainings one day a month for all the grade levels, grades two through five but no first grade training. (Interview, p. 13)

According to Ms. Quinn, first grade teachers were never informed why they were not included in the first year training cycle “No idea, no one ever said” (Interview, p. 13). Moreover, she recently was informed that as a result of being transferred to fourth grade she would miss out on receiving any training in the newly adopted curriculum. She said,

> I was speaking with the math coach in our building at the end of the year, and she said that next year first grade will get training, but that doesn’t help me any because I’ll be in fourth grade next year. (Interview, p. 13)

Ms. Quinn felt quite a bit of pressure when considering she was not trained in the new district adopted curriculum, and she would be expected to use it the following year as the fourth grade math specialist. The curriculum, according to Ms. Quinn, is extensive with much to know and understand. When reflecting how prepared she was to teach fourth grade math using the *Investigations* curriculum the following year she said,

> I don't have the building blocks that everybody else has. The beginning, I mean, there are pages in that book [*Investigations*] that I wouldn't even think to look at for, you know, different strategies. There was so much in that, and I don't know half of it because nobody ever sat down, I didn't get a chance to sit down a day each month and really go over and go into it so. (Interview, p. 13)

Moreover, Ms. Quinn expresses frustration with her new building principal in that when she asked when she could pick up the fourth grade math curriculum materials she was told "Come in in August, and you can get the stuff." (Interview, p. 18)

For Ms. Quinn the top down decision to move her to a new school and change her teaching assignment from generalist to math specialist was a frustrating one in that she felt ill-prepared in the use of the curriculum. Furthermore, the decision caused Ms. Quinn a good amount of anxiety because, according to Ms. Quinn, math was not her area of strength. When reflecting on her past experiences with school mathematics, she said,

> In elementary school I was, well we started tracking in fourth grade, and I was considered too high of a learner to be in the low group, but I was one of the lowest ones in the high group. So I was in that middle, I was hanging on the bottom rung of the ladder. My math teacher in fourth grade was not very nice, and she made me cry a lot because I couldn't do my math, and I struggled because I didn't have self-confidence, and she didn't help me out. (Interview, p. 1)

Besides being moved to a new school, there were other top down pressures that impacted Ms. Quinn's ability to enact reform-oriented teaching practices. The school district she was teaching in had been deemed underperforming according to the federal No Child Left Behind Act (NCLB) (n.d.) because the district has failed to make annual yearly progress (AYP) several years in a row. As a result, the district received a grade of "corrective action" in terms of accountability status on its 2008 NCLB report card and a "very low" performance rating on the Massachusetts Curriculum Assessment System

(MCAS) report card. As a result, the district was under enormous pressure to increase its NCLB accountability status and its MCAS performance rating. During the initial interview for this study, Ms. Quinn had just finished teaching first grade and, as such, indicated that she was inclined to go against the grain when it came to succumbing to the pressures of preparing students to pass standardized tests. She said,

> And in my district we, they had a curriculum map, and we were supposed to do all the stuff, and it was supposed to be done, you know, a whole unit was supposed to be done in five days. And I kind of took my own approach to that. So I would spend extra time making sure that they truly understood what they were doing because my mentality was I didn't want to just give them, like show them a quick picture of everything that they were supposed to learn in first grade, because I felt that it would benefit them more if they actually understood maybe half of what they were supposed to learn instead of not understanding any of it but have seen it all. (Interview, p. 4)

And as she reflected further as to why she made a conscious decision to ensure that her students understood something before moving on with the curriculum, she remembered back to her own school experiences and how she was pushed along without having a chance to understand. She explained,

> So, I maybe wasn't the best when it came to following what was scripted in my book, but I was there for the kids. And I wanted to make sure that they understood as much as they could and not leave them, and that's what I felt happened to me in a sense, was that in that fourth grade I was kind of shown everything, and nobody ever really made sure I understood it, so I struggled for the next few years trying to catch up, and I didn't want that to happen to those kids. (Interview, p. 5)

As a first grade teacher Ms. Quinn felt confident to go against the grain in an effort to ensure that her students developed an understanding of mathematics rather than pushing them along because a curriculum map said students should be at a certain point at a particular time in the school year. Moreover, she expressed that the support she received from her building principal was instrumental in helping her to follow her instincts as a teacher. She said,

> My principal that I had this year was very supportive of the teachers, and she would come in, and she would see what I was doing, and her main focus with the kids as well, which was nice because there was the higher ups that were telling her, you know, what should be going on and what was supposed to be going on. But she has the same mentality that I have, that we're not there to follow a map. We're not there to make sure that we hit this unit, that unit, that unit. It's to make sure that the kids understand something, so I didn't have the pressure in my building. My principal was kind of the buffer for that which was nice (Interview, p. 5)

The following year after her transfer to fourth grade there was a considerable change in how Ms. Quinn approached the subject of preparing students to pass standardized tests. In her new school Ms. Quinn was mandated to have her lesson plans for the following week on the principal's desk every Friday before the end of the school day. During an observation, the following announcement was broadcast over the school intercom: "May I have your attention please? Teachers, remember your lesson plans must be on the principal's desk by 3:30 today. Thank you" (Field Notes, 11/20/08). As an observer the message seemed condescending to teachers, and it was surprising to witness such an interruption during academic time, as the announcement was quite disruptive to students, as they were in the middle of a math lesson. When Ms. Quinn was asked about the the interruption, she shrugged it off as something that she "tries to ignore." However, it was evident that this new top down pressure from her building principal was impacting her teaching and thinking about how students should be taught math. Moreover, the pressure of being in a grade where standardized tests in mathematics were administered also seemed to have an impact of Ms. Quinn's beliefs about preparing student for upcoming tests.

Ms. Quinn's ability and desire to go against the grain, as she had the previous year while teaching first grade, was now challenged by the new constraints that were placed on her in her new school and her new grade and teaching assignment. As a fourth grade math teacher, she was now presented with the challenging responsibility of ensuring that her students made adequate progress on the Massachusetts Curriculum Assessment System (MCAS) test administered to all fourth graders in Massachusetts. Previously, when teaching first grade, Ms. Quinn indicated that standardized testing did not play a significant role in her teaching because as she said, "We don't start that in first grade. MCAS starts in third grade" (Interview, p. 15). However, when asked to reflect on the role of standardized tests as a fourth grade teacher, things had changed significantly for Ms. Quinn. She said,

> Standardized tests play a huge role in my teaching. As a new teacher unfamiliar with the test the first part of the year, I did not focus a lot on the test, but as the test got closer, I realized that my students were not prepared for the test, so I started to go over old tests, as well as every Friday gave the students an open response to work on. (Email Correspondence, July 2009)

After teaching fourth grade for six months, Ms. Quinn was well aware of the pressure she and students were under to perform well on the state mandated MCAS test. As such, Ms. Quinn adapted her teaching practice to ensure as best she could that students were prepared for the MCAS test. Moreover, Ms. Quinn indicated that for her students the pressure was not as strong to pass the test but to show improvement from the previous year. She said,

> There is somewhat of a pressure to have the kids pass the MCAS; however, because our school has been so low in the past, the greatest push is to have the students make some gains from the year before. Many of the students have not passed the year before so we are just striving to bring scores up. (Email Correspondence July 2009)

And the MCAS test was not the only test that Ms. Quinn and her students were responsible for ,as the Measurement of Academic Progress (MAP) was administered three times a year as well. When considering this testing cycle, Ms. Quinn found it to be a worthwhile endeavor for her and for her students. She explained,

> We also take the MAP test three times a year. This is a test that is all taken on the computer and because it is taken three times (fall, winter, spring), it is a great way to show the students' progress and help us see if the students are making gains or not. (Email Correspondence, July 2009)

Moreover, the reform-oriented curriculum that her school district adopted two years ago was also under scrutiny and being adapted to address the standards outlined in Massachusetts Curriculum Frameworks document. Ms. Quinn acknowledged that the curriculum was a good one but also indicated that it fell short in preparing student to pass the MCAS test. She explained,

> Like any program, there are gaps. There are a lot of holes that need to be supplemented, especially when you align *Investigations* with the Massachusetts Frameworks. Due to the gaps when aligned with the frameworks, it is necessary to pull from other sources so that the students are familiar with all of the concepts that they will see on the MCAS test (Email, July 2009)

Teaching in an underperforming school district comes with its own set of issues and challenges in that the district itself is under pressure to increase student performance on standardized test. When reflecting on how she managed the move and the reassignment she said, "Overall this year was a challenging one for me personally, starting in a new school, in a new grade level, and teaching a subject that I was not crazy over"(Email, July 2009).

Couple teaching in an underperforming school with being a novice teacher with little over two years' experience, a transfer to a new school, and a reassignment as math specialist, the pressure was real and posed a significant challenge to her ability to enact mathematics reform practices. Although Ms. Quinn believed in the tenets of mathematics reform the pressure of teaching in an underperforming posed a significant challenge to her as she attempted to enact reform teaching practices. The following section will examine Ms. Quinn's conflicting beliefs.

Conflicting Beliefs

Ms. Quinn never considered herself a very strong student of mathematics. Reflecting on her elementary school experiences in math, she said, "I struggled and I didn't have self confidence" (Interview, p. 1) and her lack of confidence did not change when she entered middle school. She said, "When I got into middle school, math just wasn't one of my favorite things" (Interview, p. 1). Because of her own difficulties with mathematics, Ms. Quinn seemed determined to offer her students a very different kind of school experience from the one she had. In retrospect, she indicated that she wanted her own students to have more opportunities to be able to understand the whys of mathematics – opportunities she never had in elementary school. She said,

> I want to show my kids why, what is done and why it is done. So that they have that understanding because I feel if I had been taught that when I was first introduced to mathematics I may have been able to gather more from it, been able to build on it more. So maybe I wouldn't struggle so much. (Interview, p. 4)

Ms. Quinn shared that it was not until she was enrolled in a graduate level reform-based mathematics methods course that she had opportunities to explore the conceptual nature of mathematics rather than just the procedural aspects of the discipline. According to Ms. Quinn it was in the methods course that she understood the underlying "why" of certain procedures she had performed routinely for years. Ms. Quinn reflected on the importance of developing a conceptual understanding of mathematics, and in her reflection, she attributed much of her present desire to teach for conceptual understanding to the methods course and the instructor. Consider the following poignant quote describing the learning she did and the way in which she wanted to take that learning and infuse it into her teaching practice.

> I actually learned why you borrow and carry in her class. I always knew what you had to do it, but she was the first person to actually explain why, you know, break it down into the 1s, and really, I was 23 years old when I finally figured out why I'm doing this and why I've done it for so many years. She explained it, and we had all of the manipulatives and the 1s and the 10s and the rods and everything and the mat, we had to carried things over, and I just found it amazing that I borrow and I carry whenever I need to just because I was taught you borrow and you carry, you borrow and you carry when you need to. Really, no one ever sat down and took that time. (Interview, p. 3)

Moreover, Ms. Quinn shared that in the methods course, she was offered opportunities to work with other students sharing how each went about solving a particular math problem. This way of learning and experiencing mathematics was significantly different from how she had experienced math in the past. When asked what aspects of the methods course were significant to her, she reflected,

> We worked a lot in small groups and the instructor didn't just tell us why, we explored it and talked about it amongst ourselves and figured out why and there was a lot of collaboration going on in there which was nice and which was different from the way I learned, you know with the teacher in front of the class showing me what to do and not why it was done (Interview, p. 4).

According to Ms. Quinn, the methods course experience was one in which she made a conscious decision to teach differently from how she was taught in elementary school. She said,

> It opened, it kind of like made a light go on in my head that you can't just teach kids 'this is how you do it.' You have to explain to them why because I don't

> want my kids 20 years later being like, ‘Oh my teacher never told me that.’ (Interview, p. 3)

And when asked to describe the most valuable piece of knowledge she took away from her methods course, she again mentioned the borrowing and carrying incident. Moreover, she linked this experience to her teaching practice, indicating that this experience was significant in helping her to shape her practice. She said,

> I think it goes back to that really understanding the borrowing and carrying and like I said, that made me realize that I want my kids to know why, I want them to understand all of it, the whole aspect of it. (Interview, p. 4)

Although Ms. Quinn espouses beliefs that are in concert with mathematics reform, she also holds beliefs in practice that are in conflict with reform. According to Ms. Quinn, most, but not all, students are capable of understanding concepts in mathematics, thus some students need to memorize information in order to be successful learners. When discussing the reform-oriented curriculum that her district has adopted, she shared that it is not appropriate for all students. She said,

> *Investigations* I think is good partly because it has, it goes into the whys of it. It’s not just, “Here’s the sheet, memorize it. We’re going to have a test on it on Friday.” But I also feel that there are some students that need the “Here’s what it is, memorize it.” Not all learners can have that way of thinking. A lot, some, not most of our kids but there are a few kids that are more of a concrete, they just need to see it, not, you know, sitting there exploring. (Interview, p. 12)

In practice, Ms. Quinn’s beliefs were in conflict with her beliefs in theory. Although Ms. Quinn experienced the value of understanding “why” in her methods course, when confronted with a diverse group of learners she seemed to fall back on the traditional practice of memorization as the key to learning mathematics. For students whom Ms. Quinn believes need a more memorization type of instruction, she is able to relinquish her

"I want my kids to know why" (p. 4) belief in favor of adopting the "some kids get confused by the more hands on exploring and are not able to tie it back to the big picture" (p. 12) belief. Ms. Quinn's belief that some students need to just memorize information is grounded in a desire to help all of her students learn and to alleviate confusion for students.

Ms. Quinn also believes that some students are more detail oriented than others, and, for this type of student, investigative experiences would be too much for them to deal with in the mathematics classroom. She explained,

> Well not all the students, I mean it's such a different variety of kids in the class. One of my brothers, he is into details, it has to be this way, and he's very organized, and the *Investigations*, hands on, things all over the place would throw him. I mean, he's 21 now, so he's, but that [*Investigations*] I would be too overwhelming. And especially with kids that have a lot of sensory motor, all the different math choices that *Investigations* has in one day would be too much. So I had a few kids like that, and I would change it so that we would only, we were all doing the same thing at once. Because if I had them going to three different stations in one day, trying to do three different things, it would be too much. So I think that there's a lot in there and for the average student in my class it was too much, so I would modify it in that way. (Interview, p. 13)

Here Ms. Quinn reflected on her belief that the reform-oriented curriculum adopted by the district was not appropriate for the average student in her class, thus she needed to modify it to meet their needs. However, one must wonder if lack of training in the district adopted curriculum *Investigations* has caused Ms. Quinn to believe that it is too much for the average student. Ms. Quinn was not afforded an opportunity to be trained in the curriculum, thus lack of training may be impacting her ability to implement it successfully. Moreover, by adopting the belief that the curriculum was ill suited to many of her students may have allowed her to justify enacting more traditional teaching practices.

Other obstacles that inhibited Ms. Quinn from enacting reform practices were the beliefs she held about the students she was responsible to teach. Ms. Quinn believed that many of her students were not willing to put in the effort necessary to learn mathematics for understanding. She said that engaging her students in productive mathematical conversations was problematic because,

> Many of my students do not want to give the effort to change themselves or question why or how something is happening. Many of them will just take the first answer they hear and go with that. Not questioning if it is right or not. This was rather difficult to get them to question more. (Email, July 5, 2009)

Ms. Quinn seemed to have found a way of dealing with her conflicting beliefs by adopting the new belief that students do not want to learn using reform practices. This new way of thinking helps her to stay true to her reform beliefs while at the same time allows her to enact traditional teaching practices.

Ms. Quinn has also experienced behavioral issues that impacted her ability to enact reform-oriented teaching practices. When asked to consider some of the challenges she has encountered during her first three years of teaching, she said,

> Listening to each student's ideas carefully is also sometimes difficult when there are behavior issues that are running the class. This was more so an issue this year, where I had new administration and many of my students who had repeat behavior issues were given warnings and sent back to class, which only disrupts the learning that is taking place. (Email, July, 5, 2009)

Issues with student behavior challenged Ms. Quinn and quite possibly impacted her ability to engage students in productive mathematical conversations. Enacting reform practices requires that teachers shift some of their control over to students in an effort to foster student autonomy. For Ms. Quinn this shift has been difficult thus traditional teaching practices have proved to be more conducive to managing student behavior.

Lastly, Ms. Quinn expressed sensitivity to the population of students who attend Morningstar Elementary School, and this sensitivity may be a factor that impacted her ability to implement reform practices. According to Ms. Quinn her students' lives are impoverished at home, thus they do not come to school with the tools needed to engage in higher level learning. She said,

> My students are so far behind, and they don't, you know, go home and watch Sesame Street. They go home and they watch whatever mom's watching. So they, whatever learning they're getting is just in school. There's nothing outside, so we have to work extra to try to get them there, and some of them, some of the kids got there, some of them didn't. (Interview, p. 7)

The belief that students' home life is not conducive to learning may have impacted Ms. Quinn's expectations of her students' mathematical abilities once they arrive at her classroom door. Ms. Quinn continually expressed concern for her student's home life and this concern, however genuine, may have a limiting effect on the reform practices she enacts in the classroom. Reflecting further on her students' home life she said,

> I try to get to know my students. Try to understand where they're coming from, especially in the setting that my school is in, there's a lot of hardship going on. So being able to talk to one of my students that lives, you know, they just moved to a shelter, and I will sit down and tell them, "I know things at home have changed. I know that whatever's going on at home is hard for you but you know, everything here is the same." And being able to relay that to him and for him to understand that you get where he's coming from, I think sets the tone for what's going to happen in the classroom. (Interview, p. 17)

Ms. Quinn was genuinely concerned for her students' well being; however, her concern may have impacted her ability to engage her students in higher level thinking and learning because, according to Ms. Quinn, her students were "so far behind" (Interview, p. 6). As a result, Ms. Quinn made decisions based on what she believes her students are capable of understanding and doing in mathematics. She said,

> The biggest thing was first figuring out what was useless for my kids. Having them copying down a hexagon and then drawing a picture of it and then vocabulary notebook too was pointless for the group of kids that I had. So I

> stopped vocabulary after a while. I mean we would talk about the vocabulary and my kids knew vocabulary. To me it was pointless to have them put it on a piece of paper because also the group of kids I had really had a hard time spelling. We had fine motor issues so too, so it was so taxing on them. (Interview, p. 14)

Here, Ms. Quinn lowers the expectations placed on students primarily because she believes it was too hard for her students due to issues with spelling and fine motor coordination, thus she eliminated the vocabulary aspect of the lesson. Moreover, because many of her students were labeled English Language Learners (ELL), she believes that much of the math curriculum is difficult for them to understand. As a result Ms. Quinn was challenged to implement curriculum assessments because of the language difficulties of her students. When reflecting on her students' ability to express their mathematical reasoning in writing, she said, "I had such a large ELL population when it came to the writing, many of them struggled" (Email, 7/5/09). Assessing student understanding was also problematic for Ms. Quinn because of her students' language skills. When reflecting on administering assessments that come with the *Investigations* curriculum she said,

> The assessments in *Investigations* are very wordy, so it was hard for my kids to understand what they were supposed to do, what they were asking. And they, we could do it all together, but that was defeating the purpose of actually having them do an assessment to see what they could do on their own, and because of all the needs in our class, having, asking them to sit down and do something on their own was the end of the world. (Interview, p. 15)

Summary

Ms. Quinn faced many issues and challenges as a new teacher. First, Ms. Quinn was teaching in an underperforming school district that placed a tremendous emphasis on ensuring that students pass the high-stakes test mandated by the state. Moreover, the district was failing in terms of the accountability standards set for by the No Child Left

Behind Act (2002) and, as such, was under pressure to increase its accountability rating by 2014. Second, Ms. Quinn's beliefs in theory conflict with her beliefs in practice. Although she believes in teaching mathematics differently from how she learned mathematics, in practice when confronted with a diverse group of students, she believes students did not want to change, thus she often fell back on traditional practices. Lastly, Ms. Quinn's genuine concern for her students' well being challenged her ability to enact reform practices. According to Ms. Quinn, her students entered her classroom already "too far behind" and their status as ELL students required her to cut out aspects of the curriculum that was too "wordy" and hard for them to understand. Ms. Quinn made adjustments to her teaching practice based on what she believes her students are capable of achieving.

In summary, Ms. Quinn is a dedicated novice teacher who expresses a desire to teach mathematics for conceptual understanding. She had personally experienced a very traditional form of mathematics teaching while a student and believes that her students deserve more than only a procedural learning experience. However, in practice, Ms. Quinn is challenged to implement reform practices and more often than not fell into a very traditional pattern of teaching. Ms. Quinn at times implemented reform practices and the results were promising, but her practice at this point in her teaching career is more situated within the traditional paradigm where the teacher asks QWKAs and students respond with correct answers, consequently leaving little room for productive mathematical conversations to emerge.

CHAPTER 7

CROSS-CASE ANALYSIS

This study was designed to examine and describe the teaching practices used by three novice teachers as they engaged their students in mathematics conversations. Looking through the lens of social and socialmathematical norms, the research questions guiding the study, addressed the reform orientation of the novice teachers' practices. Moreover, the research questions also examined the issues and challenges that emerged for the novice teachers as they attempted to implement reform practices into their teaching of mathematics. The purpose of cross-case analysis is to deepen one's understanding and increase one's ability to generalize across cases (Miles & Huberman, 1994). Although, with only three cases examined, to propose the study can be generalized is problematic, to say the least, looking across and between the three cases aids in painting a general picture of the novice experience with more detail and clarity than can be painted by examining a single case alone.

The participants were purposefully chosen for this study because all had graduated from the same M.Ed. program where they also earned an initial license to teach at the elementary level and had taken a reform-based mathematics methods course. It would seem plausible that they would resemble each other in terms of what they learned about teaching and learning mathematics and how what they learned has impacted their current teaching practice. On the other hand, because each of the participants entered the program with various experiences in mathematics and different beliefs and attitudes as to how mathematics should be taught and learned, it would also seem plausible to unearth differences among them as well.

Analysis reveals that the three novice teachers who participated in this study do implement reform teaching practices, albeit, the practices they use and the ways in which they implement reform practices varies. Consequently, the social and socialmathematical norms that are constituted in each participant's classroom reveal marked differences in the participation structure and student engagement with mathematics. Moreover, analysis reveals that one participant, Ms. Arielle, fully adopted reform practices into her teaching of mathematics while Ms. Duncan and Ms. Quinn at times adopted or adapted reform practices but more often than not, ignored practices associated with mathematics reform in favor of more traditional ones.

The following section will address the first research question guiding this study and examine the teaching practices that the participants used as they engaged students in mathematics conversations. Moreover, the social and socialmathematical norms that such practices constituted will be explored.

Research Question # 1

- What reform-oriented discourse practices do novice teachers who participated in a reform-based mathematics methods course adopt? What practices do they adapt? What practices do they ignore as they engage their students in mathematics conversations?

Reform Practices Constituting Social Norms

As stated previously, practices fostering social norms regulate the participation structure in classrooms. The three participants in this study all attempted to use reform practices; however, the practices they use and the ways in which they implement reform practices reveals marked variation. Table 7.1 compares the ways in which participants adopted, adapted, or ignored reform teaching practices associated with the development of social normative behaviors in classrooms.

As seen in Table 7.1, Ms. Arielle consistently adopted reform-oriented practice of eliciting different solutions as well as explanations and justifications of such solution methods. These practices serve to foster the understanding that to participate successfully in classroom conversations students need to be cognizant of their solutions as well as listen to and compare their peers' solutions to their own. Table 7.1 reveals that Ms. Duncan adapted and often ignored reform orientation teaching practices in favor of more traditionally situated ones that regulated student participation to providing correct answers to her questions. As a result, the focus of conversations in Ms. Duncan's classroom was on correct answers rather than on students' mathematical reasoning. Lastly, at times Ms. Quinn adopted reform-oriented teaching practices; however, for the most part she adapted or ignored such practices in favor of more traditionally situated ones. Significantly, when Ms. Quinn used the practice of eliciting different solutions, the practice is reform-oriented in that she elicits from students the different ways in which they solve problems. Moreover, during these instances, her elicitations for a different solution are coupled with an elicitation for an explanation or justification thus students' mathematical reasoning become the focus of the conversation. However, because eliciting different solutions is not a consistently utilized practice, Ms. Quinn does not have many opportunities to elicit student explanations and justification, thus the majority of conversations focus on eliciting correct answers rather than students' mathematical reasoning.

Table 7.1 Summary of Reform Practices Fostering Social Norms

	PRACTICES		
	Elicited Different Solutions	**Elicited Explanation and Justification**	**Social Norms Constituted**
Ms. Arielle	**Adopted** Consistently elicits different solutions to the same problem during each lesson. Practice is critical in helping to develop meaningful student explanations and justifications. Practice also helps to establish the socialmathematical norm of what counted as an acceptable different mathematical solution.	**Adopted** Consistently asks students to explain and justify how they solved math problems. Because she consistently asks students to share their different solution methods, she has ample opportunities to develop this practice. This practice is instrumental in helping to develop the socialmathematical norm of what counts as an acceptable mathematical explanation and justification.	Consistently elicits different solutions, thus students are obligated to respond with different and novel problem solving approaches. Concomitantly, students are expected to listen to and make sense of their peers' solutions. Students are expected to provide detailed explanations and justifications of their solutions. Teacher is the authority in that she facilitates conversations. Students are expected to assess the correctness of their solutions, giving them mathematical authority.
Ms. Duncan	**Adapted/Ignored** When used, the practice is adapted and focuses on asking students to name a different strategy rather than share different solution methods. Only elicited different solutions 9 times, thus practice was most often ignored. Dominant practice is asking questions with the known answer (QWKA).	**Adapted/Ignored** When used, the practice serves to steer lessons in a particular instructional direction rather than to elicit students' explanations and justifications. Adapted the practice by providing teacher generated explanations and justifications, thus ignoring student explanations and justifications.	Teacher asks QWKA and students' provide correct answers. Teacher evaluates responses, thus she is the sole mathematical authority. Teacher provides correct mathematical explanations and justifications. Students practice what they learned.
Ms. Quinn	**Used, but mostly Ignored** When used, the practice is instrumental in helping to develop meaningful student explanations and justifications. When used, the practice also helps to establish the socialmathematical norm of what counts as a different mathematical solution. However, only elicited different solutions 8 times, thus practice is most often ignored. Dominant practice is asking a QWKA.	**Used, but mostly Adapted** When the practice is used in conjunction with eliciting different solutions, meaningful student explanations and justifications are elicited. All other times, the practice is adapted and serves to elicit from students correct answers rather than explanations and justifications. Thus the norm becomes correct answers that do not warrant explanation.	When teacher asked QWKA students were expected to provide correct answer and not explanations of their thinking. Teacher evaluated student responses thus she was the mathematical authority. Only when teacher elicited different solutions were students expected to provide explanations and justifications of their thinking. Listening to peers' explanations of their solutions was expected to determine if one's solution was different.

The following section will examine the reform-oriented practices of eliciting different solutions and eliciting explanations and justification in more detail by examining the number of elicitations that each participant made per hour of whole group conversation. Moreover, this section will also examine the practices of asking questions with the known answer and evaluating/accepting student responses to discern the impact of such practices on students' participation in whole class conversation. As stated previously, Ms. Duncan was observed engaging students in whole group conversation for approximately 2.4 hours, Ms. Arielle for 1.9 hours, and Ms. Quinn for 1.8 hours. Moreover, this section will examine the social norms that such practice constituted.

Eliciting Different Solutions

Figure 7.1 reveals the number of times per hour of whole group conversation observed that each participant elicited a different solution. As seen in Figure7.1, Ms. Arielle consistently elicited different solutions with 22.1 elicitations made per hour of whole group conversation, and as such, this practice was the centerpiece of her teaching. The practice of eliciting different solutions offers Ms. Arielle numerous opportunities to elicit explanations and justifications of students' mathematical reasoning. Her elicitations were quite simple and took the form of, "Who did something different?" or "Anybody else do it differently?" These simple requests obligate students to provide a different solution from the ones previously shared. Thus, the practice of eliciting different solutions concomitantly obligates students to listen and make comparisons between a peers' solutions and their own before volunteering to contribute to a conversation.

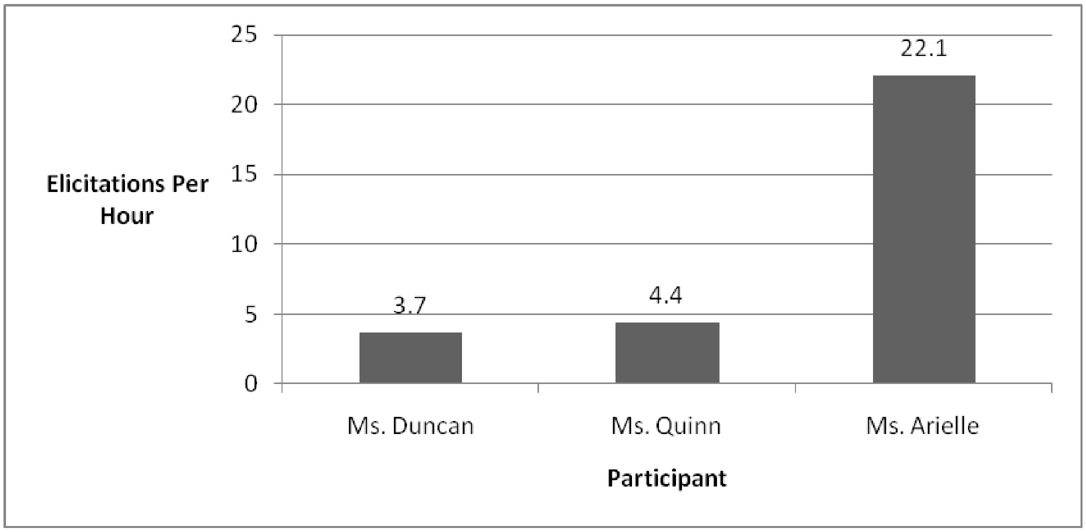


Figure 7.1 Number of elicitations for a different solution by each participant per hour of whole group conversations

To successfully engage in mathematics conversations, students in Ms. Arielle's classroom needed to be active listeners as well as active solution sharers. Moreover, the problems Ms. Arielle posed to students were contextual, and she gave students ample time to work through such problems before eliciting different solutions. The practice of eliciting different solutions is a critical one in that it provides Ms. Arielle with many opportunities to examine students' mathematical reasoning in a public way so that all students have ample opportunities to listen to others and to make comparisons between the solutions being shared.

Ms. Duncan only used the practice of eliciting different solutions during Observation 4 and averaged only 3.7 elicitations per hour of whole group conversation (see Figure7.1). During this observation, Ms. Duncan wrote 3 x 32 = on the board and before asking students to solve the problem, she asked, "There are many different ways we could do this. What's one?" The elicitation seemed to obligate students to name a strategy that could be used to calculate the answer. As evidence of this conjecture, students responded with one to two words answers, such as mental math, calculator, fingers, an array, acting it out, multiplication chart, objects, and tally marks. The timing of her elicitation also seems to constrain student responses, as her elicitation occurs before students have an opportunity to work through the problem. Consequently, the conversations that ensued do not focus on students' mathematical reasoning, rather they focus on strategies that could be used to calculate the answer. Moreover, Ms. Duncan attempts to elicit different solutions to calculational problems rather than to contextual ones. As a result, the discourse that develops in Ms. Duncan's classroom reflects what Cobb et al. (2001) refers to as "calculational discourse" (p. 134), wherein students are obligated to provide a "method or process for producing results" (p. 134). The conversations that ensue in her classroom focus on correct answers rather students mathematical reasoning. Ms. Duncan is clearly concerned with students producing correct answers, and as such, she emphasizes to students that different solutions are acceptable if the answer obtained is correct. She said,

> What we are going to do now is we're going to try multiplying bigger numbers. Okay. Once you get to your seats, we're going to do a mental math warm up, and then I am going to accept the answers however you get them. So you are going to put your answer right on your white board and no matter how you got there, I will accept it *if you got it right*, because, there are a lot of ways to do this *the right way*. (p. 11, emphasis added)

For Ms. Duncan, eliciting different solutions is an adapted and controlled endeavor as different solutions are acceptable only if the end result is a correct answer. If not, then the method is considered an inappropriate one to use. Because of the control that she exhibits, the practice of eliciting different solutions does not bring out students' mathematical reasoning, and as such, the practice is not oriented toward mathematics reform.

As seen in Figures 7.1, Ms. Quinn did not make a regular practice of eliciting different solutions with only 4.4 elicitations made per hour of whole group conversation. However, when she does utilize the practice of eliciting different solutions, she is successful in making students' mathematical reasoning and conjecturing the focus of conversations. Moreover, when Ms. Quinn requested a different solution 7 of her 8 requests generated mathematically different solutions, thus it seems that students have developed the taken-as-shared understanding that to contribute to this type of conversation, their solutions need to be mathematically different from ones previously shared. To successfully engage in conversations, students need to be active listeners as well as active solution sharers. For Ms. Quinn, the practice of eliciting different solutions is reform-oriented and produces conversations that focus on mathematically significant aspects of students' solutions rather than on correct answers. However, because Ms. Quinn does not make a regular practice of eliciting different solutions, her overall teaching practice is more traditional than reform-based.

For the participants in this study, the timing of elicitation is critical in producing not only a variety of mathematically different solutions but also in providing the teacher with opportunities to elicit explanations and justifications of students' mathematical reasoning. Because Ms. Arielle and Ms. Quinn ask students to solve problems first, before eliciting different solutions, their ensuing conversations focus on students' mathematical reasoning, rather than on correct answers. Ms. Duncan, on the other hand, elicits different solutions before giving her students an opportunity to engage in problem solving, thus her elicitation results in naming a strategy or manipulative that could be used to solve a problem rather than on students' actual problem solving approaches.

As stated previously, eliciting different solutions has the potential to provide participants with opportunities to examine students thinking further by eliciting explanations and justifications. The following section will examine the ways in which the participants attempted to implement this reform-oriented practice.

Eliciting Explanations and Justifications

Ms. Arielle used the practice of eliciting explanations and justifications more often per hour of whole group conversation than did Ms. Duncan and Ms. Quinn (see Figure 7.2). Mrs. Arielle used the practice on average 41 times per hour, indicating that this is a practice that she has adopted. As stated previously, eliciting different solutions is a hallmark of Ms. Arielle's practice, thus she has ample opportunity to elicit valuable explanations and justifications as to how students went about solving the math problems she posed. For Ms. Arielle, eliciting explanations and justifications is a well developed aspect of her teaching practice and more importantly, the practice actively fosters the development of students' autonomy within the mathematics classroom. Ms. Arielle's elicitations for an explanation or justification are connected to students' different solutions, thus conversations focus on students' mathematical reasoning rather than on correct answers. Moreover, Ms. Arielle's students have developed the taken-as-shared understanding that sharing a solution comes with an obligation to explain and justify their methods to Ms. Arielle and to their classmates. Students understand that to successfully engage in mathematics conversations, they need to be able to articulate the how and why of their solutions. Moreover, because students in Ms. Arielle's class expect to be challenged by Ms. Arielle with questions, such as "Why did you count on to 13?" or "How did you know 7 plus 3 plus 5 equals 12?" students need to be cognizant of their problem solving processes in order to successfully engage in mathematics conversations.

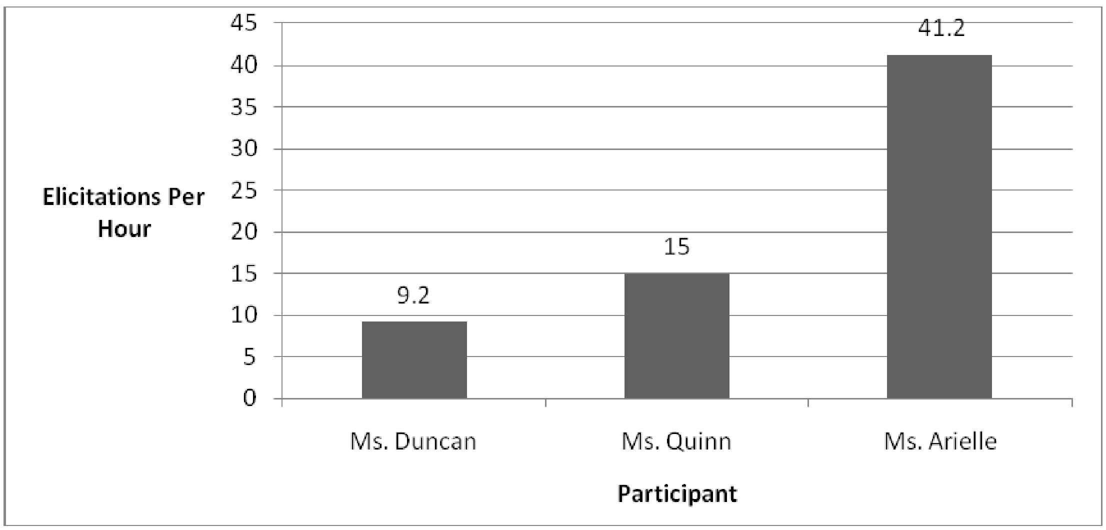


Figure 7.2 Number of elicitations for an explanation or justification by each participant per hour of whole group conversations

Ms. Arielle expects her students to verbally and physically explain and justify their solution methods using words, fingers, cubes, drawings, or other concrete objects in their explanations. This helps to bring students' mathematical reasoning out into the public discourse space to be considered and compared by other members of the classroom community. Obligating students to explain and justify in such a public and prominent manner helps the "listening students" to fulfill their obligation, which was to make comparisons between solutions shared and their own solution.

Students in Ms. Arielle's classroom do not depend on Ms. Arielle to evaluate their solution methods; rather, they depend on their own mathematical reasoning and use it to support their solution methods. As a result, students are autonomous in that they are responsible for arguing and defending their solutions to Ms. Arielle and their classmates. For example, when a student in Ms. Arielle's class said, "20 minus 11 equals 10," Ms. Arielle replied, "Okay, show me that on the number line." The student was challenged to provide support for his incorrect conjecture and after explaining how he used the number line, he revised his response and said, "Ms. Arielle, it's not 20 minus 11. It's 20 minus 10" (Observation 3, p. 8). Because Ms. Arielle consistently challenges her students to explain and justify their claims, students need to be cognizant of their mathematical reasoning to successfully engage in conversations. And as a result, students develop into autonomous problem solvers, capable of judging the correctness of an answer based on their own mathematical reasoning. Ms. Arielle has developed what Cobb et al. (2001) call a "community of validators" (p. 124) within her classroom. As such, claims made by students are not directly evaluated by Ms. Arielle; rather, students are expected to provide explanations and justifications of their mathematical reasoning in support of their claim rather than to "appeal directly to the authority of the teacher" (Cobb et al. 2001, p. 124).

Moreover, Ms. Arielle expects students to explain and justify claims regardless of their correctness. For example, when a student said "I know 7, plus 3, plus 2 equals 12," Ms. Arielle responded with "Well, how do you know that 7, plus 3, plus 2 equals 12?" Ms. Arielle expects that her students explain and justify regardless of whether their claim is correct. In Ms. Arielle's classroom, correct answers do not carry as much weight and require support and backing by students. Thus, the social norm that develops obligates students to explain and justify all claims, regardless of the correctness of the claim. As a result, correct answers are not a prominent part of the conversations that ensue in this classroom.

It is important here to draw attention again to Table 5.1. When looking at Observations 3 and 5, it is clear that Ms. Arielle did not elicit as many explanations and justifications as she did in the other observations. Findings suggest that Ms. Arielle and her students are in the process of refining the norm of explanation and justification. During these two lessons students were finding different two-digit combinations of 10. Previously, students were obligated to explain and justify how they knew two numbers equaled 10. However, during these two lessons Ms. Arielle let students know that it was now acceptable, and possibly expected, that they know combinations of 10 and such combinations no longer warranted explanation and justification. This is important to note in that it reveals that Ms. Arielle and her students were actively negotiating the social norms within the context of their work together.

Figure 7.2 reveals that Ms. Duncan used the practice of eliciting explanations and justifications on average 9.2 times per hour of whole group conversation; however, for Ms. Duncan, eliciting explanations and justifications often evolve into situations whereby she steers conversation in a particular instructional direction consequently, ignoring students' explanations and justifications. For example, after eliciting a student explanation, she said "So you might have to do some subtraction here," however, the student's explanation did not indicate that subtraction had been used in the solution. Although Ms. Duncan elicited an explanation, she used the explanation as an opportunity to introduce subtraction as a way to solve the problem and did not use the opportunity to examine her students' thinking.

Students in Ms. Duncan's class are not accustomed to being challenged by their teacher to explain or justify their mathematical reasoning, thus when challenged, they are often take this to mean that their answer is incorrect. In such situations, Ms. Duncan provides students with a "bridge" in the form of a positive evaluation as to the correctness of the student's initial response. Ms. Duncan seems compelled to evaluate the correctness of a response before asking for an explanation or justification. For example, before eliciting a justification of a students' response, Ms. Duncan said, "Manuel is right. And Manuel, how do you know it is more than one bar?" (Observation 2, p. 2)? Because Ms. Duncan's students are not accustomed to being challenged by their teacher before engaging in explanation or justification, students need a bridge in order to successfully proceed.

Ms. Duncan's elicitations for explanations and justifications are not grounded in a problem solving context, thus students are regulated to explain calculational problems rather than contextual ones. Moreover, Ms. Duncan rarely follows up students' responses with requests for further explanation or a justification. Instead, she evaluates responses as either being correct or not and then moves on to her next question. Consequently, the social norm that develops in Ms. Duncan's classroom is that correct answers to calculational problems are acceptable mathematical explanations and do not warrant further explanation or justification. Moreover, Ms. Duncan's is the sole mathematical authority in the classroom as she evaluates all student responses as being either correct or incorrect. It is evident in this classroom that students are not autonomous problem solvers, in that they depend upon Ms. Duncan to validate or invalidate their mathematical claims, rather than to appeal to their own mathematical reasoning.

For Ms. Duncan, the practice of eliciting student explanations and justifications is an adapted one in that she does make such requests, albeit, her attempts are controlling, often steering students toward what she wants them to know rather than trying to bring students' thinking into the discourse space of the classroom to be considered and compared by other members of the classroom community. When reflecting on one of her lesson videos, Ms. Duncan indicated that she does not asked students to explain and justify because, "I don't want students to hear or learn the "wrong information" (Lesson Reflection, p. 1). Because she is concerned with students hearing inaccurate information, it makes sense to Ms. Duncan to limit and control mathematical explanations. In an effort to limit student explanations and justifications, Ms. Duncan often takes over conversations by providing detailed explanations of students thinking. For example, when she asked a student, "How did you do it?" the student responded with "Counted by 5s." And rather than ask the student for further clarification she said,

> You counted by 5s in your head. That's exactly what I was doing. Great minds think alike. This is what I did inside of my brain. And I bet, Jami, you did the same thing. Did you go, 5, 10, 15, 20, 25, 30 and then stop because the next one would have been 35 and that's too high. So he is right! (Observation 4, p. 4)

The above example is representative of how Ms. Duncan manages to take over students' explanations by providing her own interpretation of what a student had done. As a result, students are regulated to the periphery of their own mathematical reasoning as they sit passively listening to Ms. Duncan explain and justify their thinking. Although Ms. Duncan does elicit explanations and justifications, the practice is controlled and does not serve to bring students' mathematical reasoning to the forefront of conversations.

As seen in Figure 7.2, Ms. Quinn used the practice of eliciting explanations and justification on average 15 times per hour of whole group conversation. Importantly, Ms.

Quinn is most successful using the practice of eliciting explanations and justification when such a request is coupled with a request for a different solution. When Ms. Quinn elicits different solutions, her focus is not on a correct answer but is on how students reasoned about a particular math problem. And, as stated previously, her elicitations for a different solution are based on contextual problems rather than on calculational ones, thus students are engaged in problem solving rather than performing steps or procedures to find an answer. However, when Ms. Quinn attempts to elicit an explanation or justification that is not connected to a request for a different solution, and wherein she has a particular answer in mind, she has difficulty understanding students' non-conventional explanations. For example, when she asked a student to explain how he knew the non-shaded part of a rectangle represented 3/4ths, she effectively elicited an explanation and justification of his thinking. However, when the student responded with, "Because if you have it like that, you just turn it the other way and you have them all the same right there…," (p. 11) she discounted the explanation and said "Is that mathematical?" In his explanation the student was literally taking the 1/4th that was shaded in and placing it 3 more times on the grid, showing there were 3/4ths left over. Rather than probe his thinking further, Ms. Quinn discounted his explanation as not being a mathematical one and then proceeded to walk students through a step by step procedure as to how to find the non-shaded part of the rectangle. Consequently, in such situations, Ms. Quinn's comments serve to let students know that explaining their thinking is not an acceptable response when she has not elicited a different solution.

Although Ms. Quinn believes that she valued students' explanations, "There are so many ways of thinking" (Interview, p. 9), when she attempts to elicit explanations and justifications, she rarely probes student reasoning further and instead focuses the conversation on correct answers or procedural explanations. For example, when a student attempted to explain what a fraction was by saying “Something that’s like 3/4ths …um half of something…” Ms. Quinn responded in an evaluative tone with, “Is 3/4ths half of something?” Rather than probe the student’s thinking further, Ms. Quinn discounted the student explanation and then provided a definition of the term fraction, “A fraction is a part of a whole…” (March 23, p. 1). Again, in such situations Ms. Quinn’s comment seems to indicate that if she has not elicited a different solution then providing an explanation of one’s reasoning is not acceptable.

Like Ms. Duncan, Ms. Quinn, at times, uses the practice of providing students with a bridge before eliciting an explanation or a justification. She was observed making comments, such as, “You’re right, but why?” Like Ms. Duncan’s students, Ms. Quinn’s students are not accustomed to being challenged by their teacher to support and justify correct answers, thus when asked to do so, they are not prepared. A bridge, in the form of a positive evaluation of their initial response, seems to provide them with the support they need to continue on in the conversation.

Unlike Ms. Arielle’s students, Ms. Quinn’s and Ms. Duncan’s students str not adept at explaining and justifying their mathematical thinking. Because both teachers rely heavily on the practice of asking questions with the known answer (QWKA) and such questions obligate students to provide only correct answers, students are not expected to provide support based on their mathematical reasoning. The following section will examine the practice of asking QWKAs and shed light on how such a practice regulates students to providing correct answers rather than explanations of their mathematical reasoning.

Asking Questions with a Known Answer

As seen in Figure 7.3, Ms. Duncan used the practice of asking questions with a known answer (QWKA) on average 95 times per hour of whole group conversation and Ms. Quinn used it on average 70 times per hour. Both Ms. Quinn and Ms. Duncan were bogged down with asking QWKAs, as this is the primary practice they use to engage students in mathematics conversations. QWKAs, which are pervasive in traditionally situated classrooms, require that students respond with a particular answer, and such questions often regulate students to guessing at answers rather than thinking through problems (Mehan, 1979). Moreover, both Ms. Quinn and Ms. Duncan tend to use the practice in such a way that their conversations with students turn into knowledge testing sessions rather that meaningful conversations focused on students' mathematical reasoning.

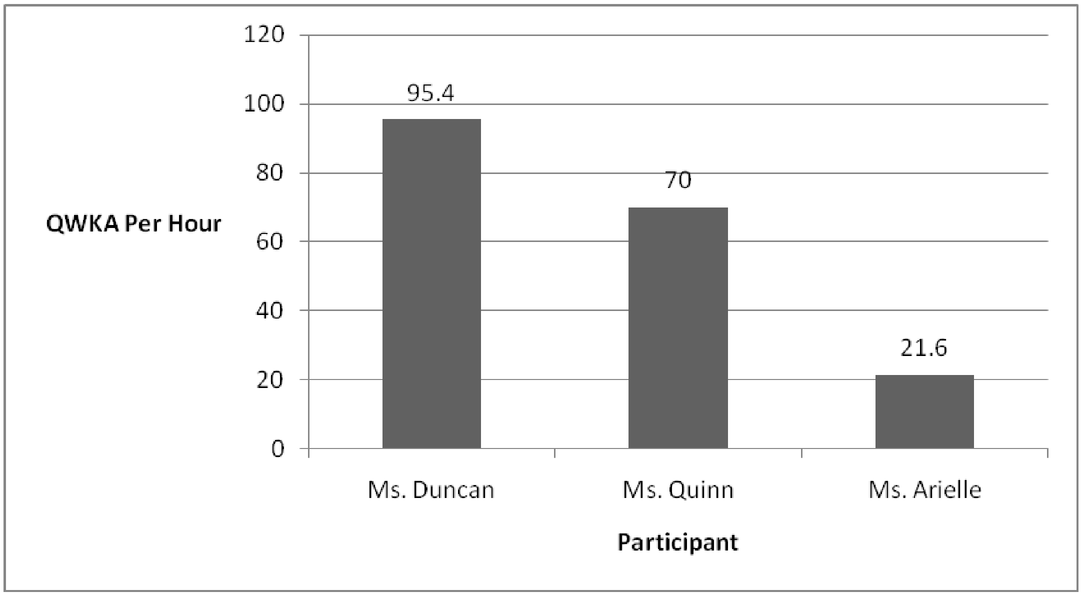


Figure 7.3 Number of QWKAs asked by each participant per hour of whole group conversations

As seen in Figure 7.3, Ms. Arielle used the practice of asking QWKAs on average 20 times per hour of whole group conversation. However, results indicate that, for Ms. Arielle, the practice of asking QWKAs was used within the context of student generated explanations and justifications. Thus, Ms. Arielle does not use the practice of asking QWKAs to test students' knowledge as did Ms. Duncan and Ms. Quinn; rather, she uses it as a means to clarify student explanations and justifications. Moreover, Ms. Arielle asks such questions in conjunction with eliciting different solutions, thus her questions are related to the explanations and justifications students were sharing. For example, during Observation 1 (p. 10) a student began an explanation with, "First, I would add the 3 and the 4," and Ms. Arielle followed up with, "So wait. You would add the 3 and the 4, and you would have how much in total, 3 plus 4?" When the student responded, "7," Ms. Arielle probed further and asked, "And then what would you do?" In this exchange Ms. Arielle does ask a QWKA when she asked how much 3 plus 4 totaled; however, the question is related to the student's solution and serves as a means to clarify the student's solution rather than as a means to test the student's knowledge. Moreover, after supplying the correct answer, Ms. Arielle does not overtly evaluate the student's response. Rather, she places the focus back on the student's problem solving method by saying, "And then what did you do?" In instances such as this, it is clear that QWKAs serve to clarify student's problem solving approaches as opposed to testing their knowledge.

Moreover, because Ms. Arielle's main objective is to elicit students' divergent ways of solving problems, her questioning practice is not focused on correct answers – her questioning practice is centered on students' mathematical reasoning about the problems she poses. For example, consider the following problem that Ms. Arielle presented to her students.

> Sally washed 8 paint brushes and Pete washed 11 paint brushes. How many brushes did they wash?" (Observation 2, p. 3)

After allowing students time to work individually to solve the problem, Ms. Arielle asked, "Okay, what would you do?" Although the question above is a QWKA, in that Ms. Arielle clearly knew that the answer was 19, she takes the emphasis off the answer and places it on students' mathematical reasoning when she asked, "Okay, what would you do?" With this comment, Ms. Arielle signals to students that she is eliciting their divergent ways of solving the problem rather than the correct answer. In the paint brush problem, students explained and justified several mathematically different solutions, and the conversations that were generated focused on students' mathematical reasoning rather than on the correct answer of 19. Moreover, these first grade students are adept at explaining and justifying their methods of solution to the rest of the class in consistently articulate ways.

When Ms. Arielle poses problems for students to consider, she often first asks the students questions about the context of the problem, such as “Will Kim have more or less than 11 stamps” or “What type of problem do you think this is” (Observation 1, p. 4)? Such questions serve to focus the students’ thinking on the contextual aspects of the problem rather than on the correct answer to the problem. Moreover, such questions engage students in a higher level of thinking in that students are obligated to reason about the problem before attempting to solve it (McClain & Cobb, 2001).

Ms. Arielle is the authority in the classroom in that she holds the sole responsibility of making decisions as to how to conduct and facilitate the mathematical conversations; however, she is not viewed as the sole mathematical authority in the classroom. Ms. Arielle’s students act in ways that indicate that they have developed a sense of autonomy in that they take responsibility for determining whether their answers are correct by explaining and justifying their mathematical reasoning. The process of explaining and justifying their reasoning helps students to find their own miscalculations and helps them to discern whether or not a method they used was efficient or not. Although Ms. Arielle does ask QWKAs, her practice focuses on her students’ mathematical reasoning rather than on correct answers.

The prevailing participation structure found in Ms. Quinn’s and Ms. Duncan’s looks quite different from Ms. Arielle’s and mirrors a traditional initiate, respond, and evaluate (IRE) question and answer sequence, as described by Mehan (1979). In Ms. Quinn’s and Ms. Duncan’s classrooms, such a pattern of discourse is often initiated using a QWKA, which in turn obligates students to respond with a correct answer. If students respond incorrectly, both Ms. Quinn and Ms. Duncan follow up by asking additional

QWKAs in an attempt to elicit the correct answer, regardless of whether students understand the meaning behind their answers.

Asking QWKAs turns Ms. Quinn's and Ms. Duncan's conversations into quick question and answer knowledge testing sessions rather than meaningful conversations focused on how students reason about mathematics. Moreover, as stated previously, when Ms. Quinn and Ms. Duncan ask QWKAs, they both are searching for a particular answer. Consequently, both participants often ignore or discount student responses that do not fit the answers they have in mind. Such a participation structure inhibits students' genuine participation in that students are regulated to guessing what the teacher is looking for rather than generating their own ideas and conjectures in relationship to problems presented to them to solve.

Ms. Duncan and Ms. Quinn developed teaching practices based on asking QWKAs. Such a practice regulates their students to be peripheral participants, in that QWKAs obligate them to provide specific answers rather than explanations or justifications of their mathematical reasoning. According to Mehan (1979)

> Because there is often only a single correct response to known information questions, and this answer is known in advanced of the questioning, teachers often find themselves "searching" for that answer, while students provide various "trial" responses which are in search of validation as the correct answer. (p. 291)

Griffin and Mehan (cited in Cazden & Beck, 2003)) describe classroom discourse as being "negotiated conventions – spontaneous improvisations on basic patterns of interactions" (p. 205). The negotiated conventions in both classrooms seems to regulate student participation to answering the teacher's questions correctly, rather than providing explanations or justifications of their mathematical reasoning. Moreover, the practice of asking QWKAs often regulates students' participation to guessers of answers rather than problem solvers. For example, Ms. Duncan often said to students, “Oh, you’re so close,” when a student gave an incorrect response to her QWKA. In theses exchanges she does not ask students to explain how they had arrived at their answer or provide any indication as to what it means to be “so close,” thus students are regulated to guess the correct answer rather than to use their mathematical reasoning.

QWKAs regulate Ms. Quinn’s students to be guesser of correct answers as well. For example, during Observation 2 (p. 8), attempting to elicit the answer of “multiples,” Ms. Quinn asked, “What do you call these numbers?” albeit, when one student responded “Even numbers,” and another responded, “Numerical data,” Ms. Quinn said, “Well they are even but…” and “It could be a kind of numerical data, but I’m not really talking about data - good job remembering the term though.” As such, asking QWKAs regulates her students’ participation to that of guessing the answer their teacher had in mind. Moreover, with such a practice, only the teacher could evaluate the correctness of a response because only she knew the specific correct answer. Thus, the practice of asking QWKAs provides a structure where the teacher is seen as the sole mathematical authority on whom student depend for evaluation. Such a structure leaves little room for fostering student autonomy.

Figure 7.4 reveals the number of evaluations of student responses that each participant offered per hour while in conversation with students. As seen in Figure 7.4, Ms. Duncan averaged 98.7 evaluations per hour, Ms. Quinn averaged 39.4, and Ms. Arielle averaged 18.9. Ms. Duncan’s and Ms. Quinn’s evaluations took two forms –overt, such as “excellent” or “good job” and subtle, such as “okay” or by repeating an answer back either positively, “6,” or negatively, “6?” Ms. Duncan offered 150 overt evaluations and 87 subtle evaluations, whereas Ms. Quinn offered 25 overt evaluations and 46 subtle evaluations. However, in Ms. Quinn's classroom, the practice of asking the next QWKA, without commenting on the previous response, serves as a positive evaluation. The negotiated convention in such situations seem to be that when Ms. Quinn moves on to the next question, without comment, the previous answer was correct. In the case of Ms. Quinn and Ms. Duncan, asking QWKAs limits students' participation to providing correct answers to such questions. Moreover, correct answers rarely warrant an explanation or justification from students in either classroom, thus student participation ends once a correct answer is provided.

Conversely, because Ms. Arielle's practice is not focused on asking QWKAs, her comments to students are not focused on evaluating the correctness of their responses; thus, conversations do not end once a correct answer is offered. Rather, students in Ms. Arielle's classroom are obligated to provide explanations and justifications of their divergent ways of reasoning about mathematics, regardless of whether or not their answers are correct. Figure 7.4 reveals that Ms. Arielle evaluated or accepted student responses 18.9 times per hour of whole group conversation. Moreover, Ms. Arielle rarely overtly evaluates the correctness of a student's responses, with only 5 such evaluations noted. However, during Observations 3 and 5, Ms. Arielle offered 28 acceptances of students' responses, most often in the form of "Okay." As stated previously, during these two observations Ms. Arielle and her students are negotiating a new norm that knowing combinations of 10 no longer requires an explanation or a justification; thus, during these two lesson she accepts students responses without requesting that they explain or justify their mathematical reasoning.

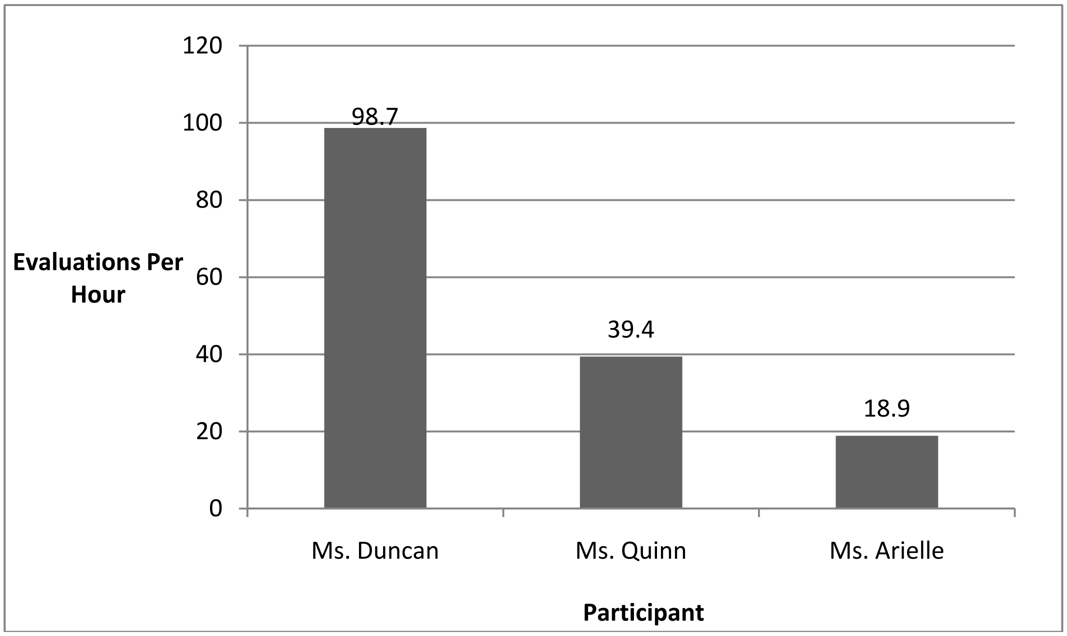


Figure 7.4 Number of evaluations/acceptances offered by each participant per hour of whole group conversations

Summary

The practices participants use to engage their students in mathematics conversations have the potential to enhance or limit their student's ability to actively engage in productive mathematics conversations. Moreover, the practice of eliciting different solutions significantly impacts the way in which students are obligated to participate in classroom conversations. Ms. Arielle's teaching practice is grounded in eliciting students' divergent ways of solving contextual problems. As a result, this practice affords Ms. Arielle ample opportunities to elicit from students explanations and justifications of their mathematical reasoning. Moreover, because Ms. Arielle consistently requests that students explain their mathematical reasoning, the negotiated convention becomes one where all responses, regardless of their correctness, warrant explanation and justification.

Ms. Duncan's practice of eliciting different solutions does not foster opportunities for her to engage students in productive mathematics conversations. For Ms. Duncan, the practice of eliciting different solutions obligates students to name a manipulative or strategy that they can use to solve a calculational problem rather than to explain and justify their actual solution methods. As a result, such exchanges do not provide Ms. Duncan with opportunities to elicit explanations and justifications of her students' mathematical reasoning. This practice is purposeful for Ms. Duncan, as she indicated she does not want her students to hear the wrong information, thus she steers away from asking students to explain and justify their thinking to the rest of the class. Ms. Duncan's practice is firmly grounded in the traditional practice of asking QWKAs, and eliciting correct answers is of the utmost importance for Ms. Duncan. Ms. Duncan's main objective is to help her students answer questions with success, thus asking QWKAs helps her to meet her instructional objective.

Ms. Quinn, at times, elicits different solutions, and it is in theses exchanges that she is most successful at engaging students in productive mathematics conversations. When Ms. Quinn utilizes the practice of eliciting different solutions, she capitalizes on opportunities to elicit explanations and justifications of students' mathematical reasoning and in such situations she is not focused on a correct answer, thus she allows her students' thinking to be the focus of conversations. However, Ms. Quinn's predominate practice is that of asking QWKAs, thus for the most part, her students are obligated to provide correct answers, rather than explanations and justifications of their mathematical reasoning.

The practices above constitute the social norms that regulate the participation structure in each of the participants' conversations with students. As such, the participation structure significantly impacted the socialmathematical norms that were allowed to emerge in each of their respective classrooms. The following section will examine the practices that help to constitute the socialmathematical norms that develop in each participant's classroom.

Reform Practices Constituting Socialmathematical Norms

As seen in summary Table 7.2, Ms. Arielle adopted practices that serve to make students' mathematical thinking the focus of classroom conversations. As will be discussed further, the practices she uses serve to foster the understanding of what count as a mathematically different solution, that explanations require conceptual or physical actions on objects representing numbers, and that some solutions are more efficient and sophisticated than others and such solutions are noteworthy. As such, the practices that Ms. Arielle uses foster socialmathematical norms that are reform-oriented. Summary Table 7.2 reveals that Ms. Duncan ignores reform-oriented practices, thus the socialmathematical norms constituted mirror traditional ones where the teacher asks QWKAs and students supply correct answers. Lastly, summary Table 7.2 shows that Ms. Quinn's adoption of reform practices was situational in that when she elicits different solutions, she makes students; mathematical thinking public for others to consider and compare. Moreover, in such situations students are adept at providing mathematically different solutions and explanations and justifications that reveal the physical and/or conceptual action they took on objects representing numbers. However, when Ms. Quinn asks a QWKA, reform-oriented socialmathematical norms do not regulate such exchanges; rather, traditional socialmathematical norms obligate students to provide a correct answer, and such answers do not warrant explanation or justification.

The following section will examine the practices of Made Mathematical Thinking Public, Indicated Solution was/was not Mathematically Different, and Indicated Solution was/was not Sophisticated, and the resulting socialmathematical norms that adopting, adapting or ignoring such practices constitute. As stated previously, Ms. Duncan engaged in whole class conversation for 2.4 hours, Ms. Arielle for 1.9 hours, and Ms. Quinn for 1.8 hours.

Table 7.2 Summary of Practice Fostering Socialmathematical Norms

	PRACTICES			
Participant	**Made Students' Mathematical Thinking Public (MTP)**	**Indicated Solution was/was not Mathematically Different**	**Indicated Solution was/was not Efficient or Sophisticated**	**Socialmathematical Norms Constituted**
Ms. Arielle	**Adopted** Consistently makes MTP. Accomplished by revoicing (O'Connor & Michaels, 1996) students' solutions and requesting students explain by evoking the physical metaphor in their explanation. The practice of MTP helps to develop a taken-as-shared understanding of what constitutes an acceptable mathematical explanation.	**Adopted** Consistently indicates when a solution is not mathematically different. When asked why she indicated when solutions were not different, she said, "I don't want them just to share because they want to share and hear their voice, I want them to think 'Well, I want a different way.'"	**Adopted** Explicitly and implicitly adopted the practice. Explicitly organizes lessons around efficient solutions; however, the implementation is tied to students' readiness to understand more efficient methods. Implicitly indicates when she considers a solution more sophisticated with, "Wow" or "Oh Goodness. This is how … solved it." Her comment signal to students that a solution is noteworthy.	An acceptable mathematical explanation or justification is one in which students conceptually or physically act on numbers to "show" how they solved math problems. Mathematically different solutions are the norm. Solutions that are not mathematically different are not considered acceptable. Efficient solutions are generated by students and include ones that made work "faster." Sophisticated solutions "wow" the teacher and are ones others should take note of.
Ms. Duncan	**Ignored** Does not make MTP. This is a purposeful choice in that she believes if she allows students to share their reasoning, other students might "hear the wrong information."	**Ignored** Students do not have opportunities to consider and compare different solutions, thus the concept of mathematically different does not emerge.	**Ignored** Made one attempt to acknowledge a more efficient solution, but her attempt was teacher directed rather than student centered.	Correct answers are considered acceptable and do not require explanation or justification. All contributions are accepted as valid and considered equal. Math is a pencil activity rather than something that students can act on and reason about.
Ms. Quinn	**Adopted/Ignored** When different solutions are elicited she is successful in MTP. In such situations, students explain and justify their mathematical reasoning in terms of physical actions on objects representing numbers. However, when she asks a QWKA, the focus turns to correct answers rather than MTP.	**Adopted/Ignored** Of the 8 different solutions shared, 7 were mathematically different. When a student shares a solution that is not mathematically different, she indicates to the student that she needs to compare her solution to one already shared. As a result, student realizes her solution is not mathematically different.	**Ignored** Does not make a practice of eliciting different solutions, thus she does not have opportunities to highlight when a solution is more efficient or sophisticated.	Socialmathematical norms change depending on the type of elicitation made. When different solutions are elicited, students are adept at providing mathematically different ones. In these situations an acceptable explanations and justifications is one in which students physically or conceptually describe their actions. However, when Ms. Quinn asks a QWKA, students are obligated to provide a correct answer and not an explanation of their mathematical reasoning. If students are asked to explain or justify in these situations the obligation is to explain or justify a particular procedure and rather than their mathematical reasoning. Thus eliciting different solutions is a critical practice in establishing reform-oriented socialmathematical norms. However, Ms. Quinn does not regularly elicit different solutions.

Making Mathematical Thinking Public

Ms. Arielle adopted the practice of making mathematical thinking public (MTP) with 47.4 instances noted per hour of whole group conversation (see Figure 7.5). In Ms. Arielle's classroom, the practice of making MTP helps students to focus their explanations and justifications on their problem solving process rather than on correct answers. Ms. Arielle consistently asks student to "show" what they had done while solving a problem, thereby making students' MTP for the rest of the class to consider and compare. As a result, others in the class are offered opportunities to listen to the public mathematical explanations and make a determination as to whether or not they had a different solution to share. Moreover, Ms. Arielle expects that students physically act on objects representing numbers rather than simply state their solution method. For example, when a student said that he had solved a problem by "counting on," Ms. Arielle responded with, "Come up here and show the class what you did" (Observation 2, p. 4), effectively obligating the student to physically act out his solution. In this case the student came up to the front of the classroom, touched his head and said, "I put 11 in my head and then counted 12, 13, 14, and 15" [using his fingers to count on to 15]. For students to successfully contribute to a conversation in Ms. Arielle's classroom, they are obligated to frame their explanations in terms of the physical actions they used to manipulate objects representing numbers. The following student's explanation from Observation 3 is illustrative of the way in which students understand that their obligation in conversation is to make their MTP to the rest of the class by explaining their solution in terms of the physical manipulation of objects (McClain & Cobb, 2001).

> I *put* – I *had* – I *put* 4 cubes, and then I *grabbed* 8 cubes and *counted* them to see if it really was 8 then *put* them together like this [snaps cubes together],

> and then I *counted*. I *counted* 1, 2, 3, 4 [touches each cube], and then *broke* this apart. And then I *counted* this row, and then this row, and then I *counted* them altogether, and it equaled 10. (Observation, 3, p. 1-2)

In this explanation, the student uses several words [in italics] that indicate that she understands her obligation is to explain her reasoning in terms of the physical manipulation of objects representing numbers. This contrasts with traditionally situated discourse in which students are obligated to explain in terms of “procedures for manipulating conventional mathematical symbols” (McClain & Cobb, 2001, p. 247). Because students are obligated to frame their explanations and justification in terms of the physical, an acceptable mathematical explanation is one in which the physical metaphor is evoked. By making students’ MTP, Ms. Arielle effectively helps students to develop this taken-as-shared understanding, in that they have ample opportunity to listen as their peers share their explanations and justifications and make inferences as to what Ms. Arielle considered acceptable. When a student said, “I did it in my head,” Ms. Arielle responded with, “Can you tell us what you did in your head.” When the student responded, “I *took* 1 from the 11,” it is evident that he understands Ms. Arielle’s request obligates him to evoke the use of the physical metaphor to explain what he had conceptually thought about in his head. In Ms. Arielle’s classroom, the practice of making MTP helps students to develop the taken-as-shared sense that acceptable explanations and justifications require that they explain as though acting in a “mathematical reality” (Cobb et al., 2001, p. 246) that they can manipulate and experience as problem solvers. The practice of making students’ MTP serves to develop the taken-as-shared understanding of what counts as an acceptable mathematical explanation or justification.

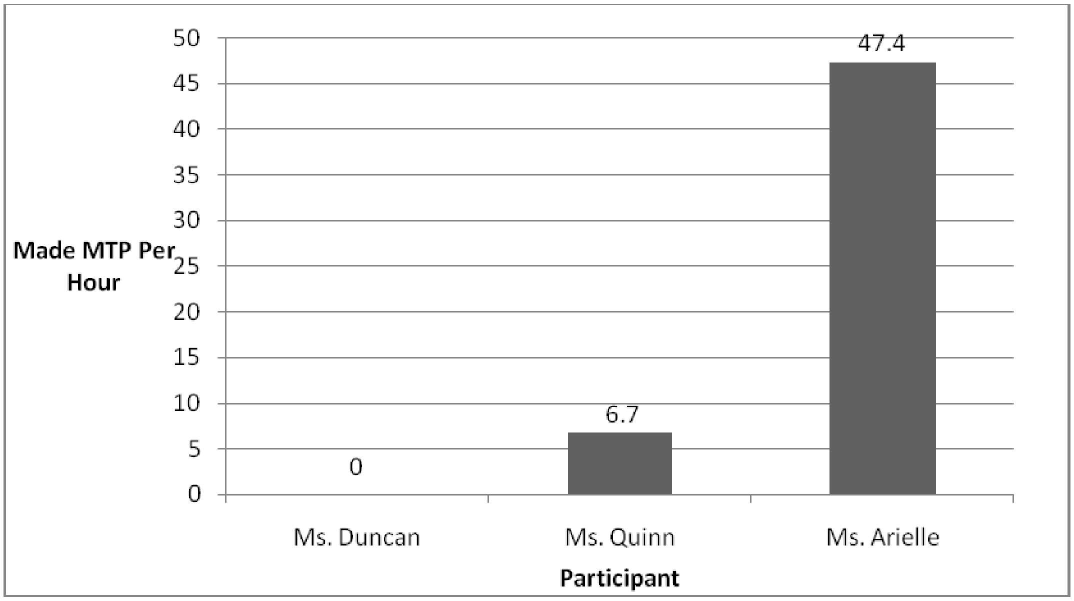


Figure 7.5 Number of times per hour of whole group conversations each participant made students' mathematical thinking public

On the contrary, as seen in Figure 7.5, Ms. Duncan never engages in the practice of making students' MTP. Moreover, she purposefully chose not to elicit student explanations and justifications in a public manner because she is concerned that if she did, students "might hear the wrong information" (Lesson Reflection). Consequently, students in Ms. Duncan's classroom do not have opportunities consider to their peers' thinking and reasoning about mathematics. Students in Ms. Duncan's classroom do not act on numbers as though they have a manipulative quality. Rather, students in this classroom are obligated to explain their work in terms of manipulating mathematical symbols and procedures. For example, during Observation 4 (p. 2), Ms. Duncan and her students were engaged in a conversation about fractions when she said, "And we remember this line has a meaning. What is that line's meaning? Just like a symbol we use in math, an operation?" And then later on in the same lesson she asked, "What is the fraction that Chris's represents in this set?" When a student responded with "1/4th," she said, "Close. One is correct. The numerator is correct. Now when you do the denominator, you need to count everything including the shaded part" (p. 4). Ms. Duncan engages her students in manipulating what Cobb et al. (2001) referred to as conventional mathematical procedures and symbols. As such, students are not offered opportunities to act on mathematics physically as problem solvers; rather, students are obligated to remember what to say and do mathematics.

As seen in Figure 7.5, Ms. Quinn did not regularly or consistently enact the practice of making MTP, with only 6.7 occurrences noted per hour of whole group conversation. Ms. Quinn is most successful at making MTP when she elicits different solutions. Moreover, it seems that in these situations it is acceptable for students to evoke the physical metaphor in their explanations. Consider the following comment Ms. Quinn made after a student had publicly explained her solution method (Observation 3, p. 7).

> Okay, so she *took* this [pointing to the shape on the document camera] how we had it horizontally, and she *flipped it* so that it was vertical. So how many parts would this shape be *broken* into?

With this comment, Ms. Quinn successfully revoices the student's solution, thereby making the student's MTP and available to be considered and compared by all members of the classroom community. Moreover, in her revoicing, she too evokes the physical metaphor using words, such as took, flipped, and broken. When Ms. Quinn asks her students to share different solutions, she effectively gives students opportunities to make their thinking public. Moreover, in these situations, students seem obligated to explain the physical actions they took on objects representing numbers. For example, when she elicits different ways of remembering a series of dots on the document camera, a student evokes the physical metaphor in his explanation when he says, "I *put* four in a circle then I *put* another row in the middle and then another row on the end" (Observation 1, p. 3). In this situation not only does the student explain his thinking in terms of the physical manipulation of objects representing numbers, he also uses his hands in the explanation as though actually moving the dots that are displayed on the document camera. When explaining their different solutions it is acceptable for the students to make their MTP and many did by evoking the physical metaphor. However, when Ms. Quinn asks students a QWKA, it is not acceptable for them to make their MTP. Instead, in these situations, students are obligated to provide Ms. Quinn with a correct answer. Ms. Quinn's students seem accustomed to this back and forth exchange where some elicitations obligate them to make their MTP and provide explanations evoking a physical metaphor while other elicitations obligate them to provide correct answers rather than explanations or justifications.

Indicating Mathematical Difference

Figure 7.6 reveals that Ms. Arielle utilized the practice of explicitly indicating when a solution was not mathematically different more often than did Ms. Quinn or Ms. Duncan with 5.3 occurrences observed per hour of whole group conversation. Ms. Arielle made 42 requests for students to share a different solution, and of the 42 solutions shared, Ms. Arielle pointed out that 10 were not mathematically different. As a result of this practice, Ms. Arielle proactively develops the reform-oriented socialmathematical norm that obligates students to share mathematically different solutions. Moreover, for the most part, Ms. Arielle's first grade students share solutions that are mathematically different, indicating that this socialmathematical norm is well established. Ms. Arielle's proactive role helps students to understand that a different solution requires more than a restatement of a previous solutions and moreover, helps students to be attuned to what was "mathematically significant" (Yackel & Cobb, 1996, p. 463) in a particular situation.

Because Ms. Arielle consistently asks students to share different solutions, she has many opportunities to listen to and make comments when students breach this critical socialmathematical norm.

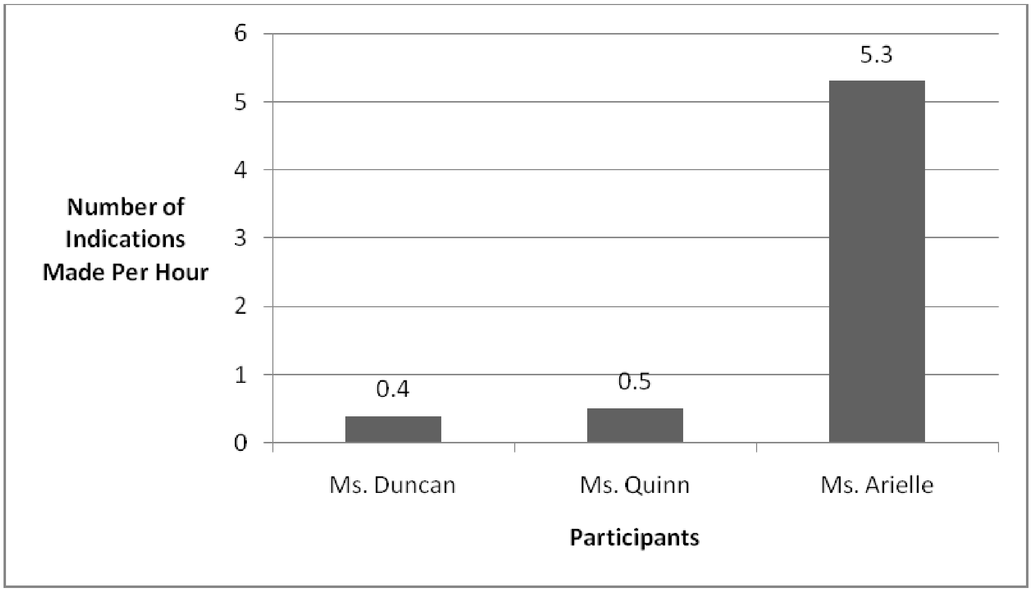


Figure 7.6 Number of times per hour of whole group conversations each participant indicated a solution was not mathematically different

In Ms. Arielle's classroom, the social practice of eliciting different solutions allowed the socialmathematical norm of mathematical difference an opportunity to emerge. For example, if a student shared a solution that was a restatement of a previous solution, Ms. Arielle would sometimes make a general comment such as, "Then that's the same way Michael also did it, right?" (Observation 1, p. 3) or "That sounds like the way Andrew did it" (Observation 1, p. 2). Such comments serve to inform students that a norm has been breached in that sharing different solutions requires more than repeating back what someone else had shared (Yackel & Cobb, 1996). However, these general comments do not specify how two solutions are the same. This result concurs with Cobb et al.'s (2001) reflection that developing the socialmathematical norm of mathematical difference can be problematic if teachers do not specify why some contributions are not accepted as different, as students are left to infer what counts as a different mathematical solution. Results indicate that Ms. Arielle seems aware of this issue and attempts to address more specifically why a solution was considered acceptable. A more explicit comment, such as "Yes, that's the same as the way that Nicholas did it. He counted on. Who did it differently?" (Observation 2, p. 5) helps students to understand that counting on had already been shared as a solution method.

With such comments, it was clear that Ms. Ms. Arielle does not consider all contributions equally valuable, and as she listens to students, she needs to "judge-in-action" (Cobb et al., 2001, p. 250) which contributions are mathematically different and which ones are not. In their research, Cobb et al. found that development of this socialmathematical norm was complex and problematic for the experienced teacher, thus observing it being used successfully by Ms. Arielle provides evidence that novice teachers are able to successfully implement complex mathematics reform practices. Although a social norm in Ms. Arielle's classroom obligates students to share different solutions, the socialmathematical norm on what counts as a mathematically different solution is continually negotiated and developed as Ms. Arielle proactively comments on the solutions that students share.

When asked why she made a point of indicating when a solution was not different, Ms. Arielle said that she wanted students to understand that sharing was more than hearing their own voice it was about listening to others and comparing solutions before volunteering to share. She said, "I also want them to understand that I don't want them just to share because they want to share and hear their voice. I want them to think, 'Well, I want a different way'" (Interview, p. 19). The practice of judging the mathematical difference between solutions is continually being developed as she asks students to share their different solutions.

Yackel and Cobb (1996) found that the development of the socialmathematical norm of what counts as a mathematically different solution is interactively constituted by both teacher and students. The ways in which a teacher responds to students' different solutions fosters the taken-as-shared understanding of what counts as a mathematically different solution. Yackel and Cobb go on to conclude that the different solutions that students share further develops the teacher's understanding of what counts as a mathematically different solution. The results of this study concur with that of Yackel and Cobb in that the socialmathematical norm of mathematical difference emerges in Ms. Arielle's classroom from the practice of eliciting different solutions and is interactively constituted by students and teacher. As such, eliciting different solutions plays a critical role in Ms. Arielle's practice in that it is the basis for establishing what counts as a mathematically different solution.

Ms. Duncan made only one attempt at indicating a solution was not mathematically different. Moreover, in this instance Ms. Duncan's indicated that it was acceptable to share solutions that were mathematically the same. She said, "If you notice this strategy is exactly the same on paper as it is with objects. So if you don't have objects… all you need is a pencil and paper" (Observation 4, p. 9). Comments such as this indicate to students that what was significant in such exchanges was the act of sharing rather than the mathematical difference or similarities between solutions. Ms. Duncan believes that it is important for her to accept "whatever anyone has to say" (Interview, p. 4), thus all contributions are considered equally valuable in her classroom.

Consequently, the socialmathematical norms guiding her students do not obligate them to listen to peers' solutions and make comparisons before contributing to a conversation.

Ms. Quinn too only utilized the practice of indicating when a solution was not different one time; however, it is important to note that of the 8 requests that Ms. Quinn made for a different solution, 7 were mathematically different. Moreover, when one student shared a solution that was not different, Ms. Quinn took a proactive role and helped the student to see how her solution was mathematically the same as one previously shared. When Ms. Quinn elicits different solutions, her students seem adept at proving ones that are mathematically different. However, because she only elicited different solutions 8 times over the course of three observations, more data would be needed to discern if the socialmathematical norm of mathematical difference was developed in her classroom.

Indicating a Solution is Efficient or Sophisticated

Ms. Arielle indicated that a solution was more efficient or sophisticated more often per hour of whole group conversation than did Ms. Duncan and Ms. Quinn (see Figure 7.7). Here again, because Ms. Arielle asks students to share different solutions, she has several opportunities to capitalize on solutions that she considers to be more efficient and/or sophisticated. Ms. Arielle implements this practice implicitly and explicitly depending on her lesson's instructional goals. For example, when a student shared a solution that was somewhat sophisticated, she commented, "Wow" (Observation 1, p. 3) or "Oh Goodness this is what Reba did" (Observation 1, p. 6). Comments such as these implicitly signal to students that they should take notice of what the student had done. Moreover, following her initial comment of excitement, she would re-voice what the student had said, thereby making the solution available once again for the whole class to consider.

At times Ms. Arielle was more explicit in her attempt to help students notice more efficient solutions. For example, when Ms. Arielle wanted students to begin to develop an understanding that counting by 10s was more efficient than counting by 1s, she explicitly picks up on a student's counting strategy and asks him to explain to the class how he had counted. Moreover, during this lesson, she deliberately set up situations that had the potential to bring the concept of counting by 10s into the conversation by students who were ready. What was most notable, however, was that Ms. Arielle allows her students to guide the counting by 10s conversation. Rather than impose this concept on her students she allows it to develop through her students' work with counting larger numbers. As a result, the development of this critical first grade concept is generated by students and, as importantly, attributes to students' work, thereby fostering her students' sense of autonomy in finding more efficient and sophisticated ways to engage with mathematics.

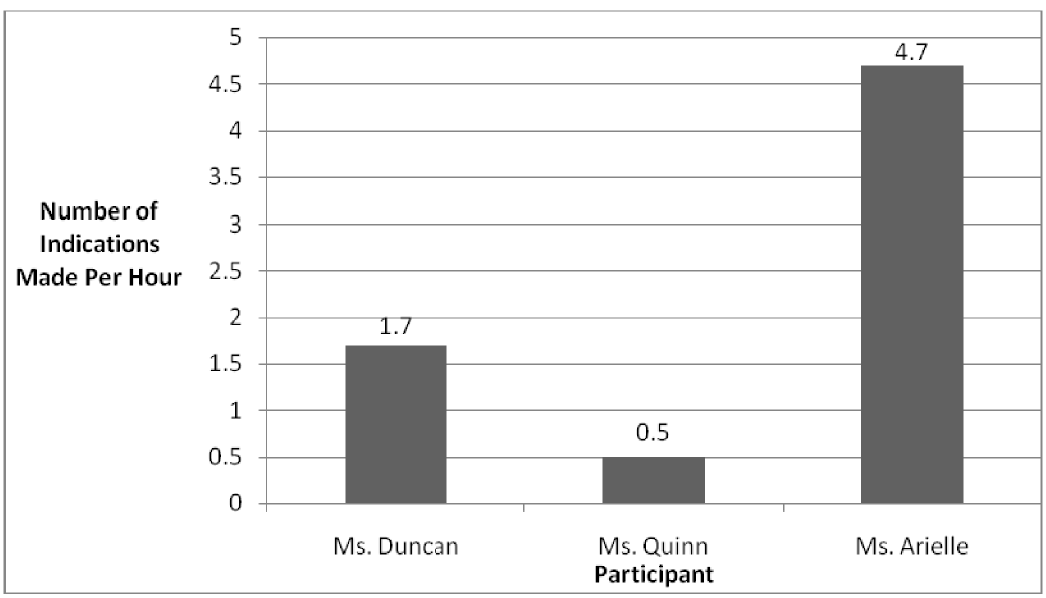


Figure 7.7 Number of times per hour of whole group conversations each participant indicated a solution was efficient or sophisticated

Ms. Duncan attempted to use the practice of indicating when solutions were more efficient only during Observation 4 (see Table 4.2). During this observation, although Ms. Duncan highlighted a more efficient counting strategy (counting by 5s rather than by 1s), she inserted herself into a student's solution by using the first person plural pronoun, "we," when she said, "Then *we* decided on a short cut right? What was the short cut *we* decided on?" By using the first person plural pronoun *we*, Ms. Duncan effectively took partial ownership the student's work. It is possible that because Ms. Duncan was seen as the sole mathematical authority in the classroom; she needed to take some credit for the solution so that students would consider it a viable one to utilize in the future.

Lastly, Ms. Quinn made one attempt to recognize when a solution was more efficient or sophisticated. Significantly, this opportunity presented itself while Ms. Quinn was eliciting different solutions, indicating that the practice of eliciting different solutions can yield valuable opportunities to develop students' understanding of mathematically efficient and more sophisticated solutions.

Summary

The practices that the participants adopt or ignore impact the socialmathematical norms that develop in each of their respective classrooms. Ms. Arielle consistently adopted the reform-oriented practices of MTP, indicating a solution was not different, and indicating a solution was more efficient and/or more sophisticated. As a result, the conversations that ensued in her classroom help students to understand that a different solution requires a mathematically different solution rather than a restatement of a previously shared solution. For students to offer mathematically different solutions, they need to be cognizant of their mathematical thinking as well as the mathematical thinking of their peers. Because Ms. Arielle consistently makes students' MTP, students have opportunities to hear what an acceptable explanation sounded like. It is clear that acceptable explanations and justifications require actions on objects representing numbers rather than the manipulation and of conventional signs and symbols. Moreover, Ms. Arielle resists evaluating students' responses as to being "right" or "wrong" and instead expects that students, through the act of explanation and justification, prove their solutions to the class. Students in this classroom are expected to listen as their peers explain and justify their mathematics reasoning. In Ms. Arielle's classroom, the adoption of reform-oriented discourse practices fosters the development of socialmathematical norms that are oriented toward reform.

Ms. Arielle's classroom reflects what Kazemi and Stipek (2001) refer to as a "high press for conceptual thinking" (p. 59) classroom. The researchers analyzed the mathematics conversations that took place in four low income elementary classrooms and found a high press for conceptual thinking when the following socialmathematical norms were developed:

> a) An explanation consists of a mathematical argument, not simply a procedural description;
> b) Mathematical thinking involves understanding relations among multiple strategies;
> c) Errors provide opportunities to reconceptualize a problem, explore contradictions, and pursue alternative strategies; and
> d) Collaborative work involves individual accountability and reaching consensus through mathematical argumentation. (p. 59)

The researchers found that in high press for conceptual thinking classrooms students' problem solving and conceptual understanding increased.

Ms. Duncan did not adopt reform practices of MTP, indicating a solution was not different, or indicating a solution was more efficient and/or sophisticated, thus the socialmathematical norms that flourish are traditionally situated. Such norms obligate students to remember and manipulate conventional procedures and symbols while engaging with mathematics. The social norms in Ms. Duncan's classroom are not oriented toward mathematics reform, thus reform-oriented socialmathematical norms do not have an opportunity to emerge.

Ms. Quinn's implementation of MTP, indicating a solution was not different, and indicating a solution was more efficient and/or more sophisticated was situational in that such practices only emerged when she elicited different solutions. In these instances, she is successful in orchestrating conversations that mirror a reform-oriented classroom; however, when using the practice of asking QWKAs, the socialmathematical norms that regulate the classroom conversation are traditionally situated. Because Ms. Quinn's practice is grounded in asking QWKAs rather than in eliciting students' different solutions, the classroom socialmathematical norms mirror more traditional norms rather than reform-oriented ones.

The importance of eliciting different solutions to the development of socialmathematical norms consistent with mathematics reform should be noted here. The practice of eliciting different solutions provides opportunities to elicit explanations and justifications of students' solutions and, as such, provide the space needed to develop the socialmathematical norms of what counts as a mathematically different solution and what counts as an acceptable explanation and justification. Figure 7.8 shows how Ms. Arielle uses the practice of eliciting different solutions as a space to elicit explanations and justifications and as a vehicle for the development of socialmathematical norms.

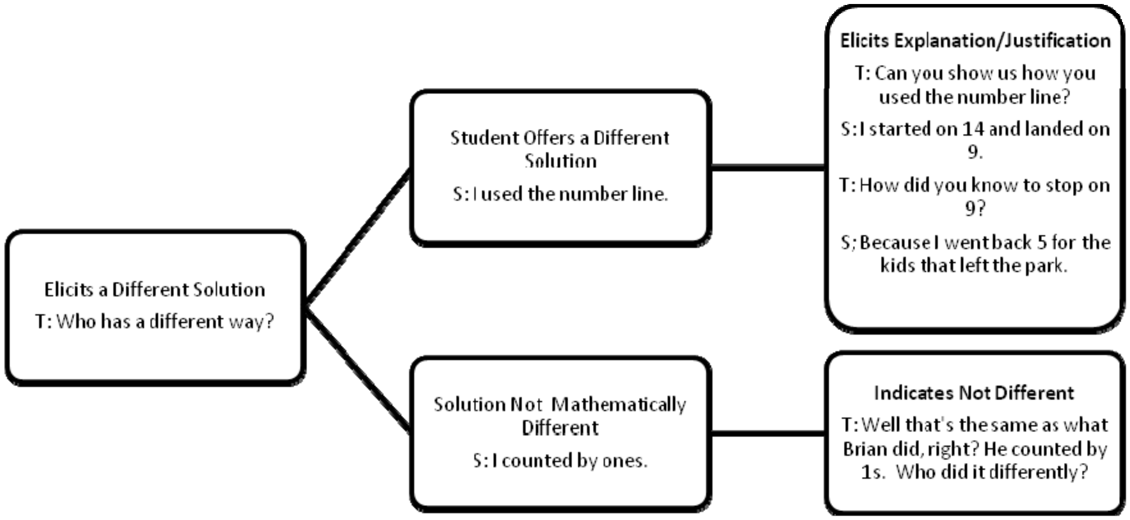


Figure 7.8 The trajectory of eliciting a different solution

In this example, Ms. Arielle requested that students share a different solution. She considered the top solution different and, as such, continued the conversation, asking for further explanation and justification. She develops the norm of what counts as an acceptable explanation by requesting the student physically act by using the number line to explain and justify his solution. In this exchange the student is obligated to provide an explanation by evoking the physical metaphor. However, Ms. Arielle considers the bottom response unacceptable as it is not mathematically different, thus she does not continue the conversation with the student. As such, students understand that Ms. Arielle does not consider all solutions equal, and moreover, their obligation is to provide a mathematically different solution rather than a repetition of a previously shared solution.

The following section will examine the issues and challenges that emerged for the participants as they attempted to adopt reform-oriented practices.

Research Question #2: Issues and Challenges

- What issues and challenges surface as novice teachers begin to enact reform-oriented discourses practice?

The practices that the participants used are related to the issues and challenges that they face as novice teachers attempting to implement reform initiatives. The beginning years of teaching, often referred to as the novice or induction years, are a challenging time in a teacher's career because of the dual nature of the position. At this point on the continuum, a teacher is teaching while simultaneously learning to teach (Feiman-Nemser, 2001). Moreover, besides being challenging, the novice years are also critical years in a teachers' development (Luft, 2007). The following section will examine the issues and challenges experienced by the participants as they began to develop a teaching practice based on mathematics reform recommendations.

Top Down Pressure Creates a Shift in Beliefs

Top down pressure (TDP), in this study, refers to the pressure that the participants experienced from school, district, and state administrative bodies. Ms. Duncan, Ms. Arielle, and Ms. Quinn were all teaching in low-income, minority schools that had yet to make annual yearly progress (AYP) in terms of students test scores on the Massachusetts Curriculum Assessment System (MCAS). The MCAS test is administered, beginning in third grade, to all Massachusetts students. Moreover, the test is considered "high-stakes" in that students who do not pass the test by graduation do not receive a high school diploma.

In the case of Ms. Duncan, TDP was a significant factor that seems to drive much of the day-to-day work at Sunrise Elementary School. An “underperforming” label, attached to Sunrise because of low MCAS scores, created a pressure-filled atmosphere for teachers and especially for a novice teacher like Ms. Duncan. She was well aware of her responsibility to ensure that students make gains on the MCAS test during the next round of standardized testing. During the focus group interview, she reveals some of the TDP she was under when she commented,

> I think that there’s a lot of pressure being put on us as teachers for mastery and the only way that they [the state] can assess mastery is by testing the children, so the children need to prove adequate on those tests, so they need to be capable, and we need to get them those tools and algorithms are a part of the tool kit. (Ms. Duncan, Focus Group Interview, p. 3)

Ms. Duncan feels compelled to ensure that her students were, as she said, “able to answer questions with success,” thus much of her teaching and interaction with students mirrors a question and answer type format. Although, fundamentally opposed to her school’s current practice of focusing on “students’ testing performance above all else” (Lesson Reflection, p. 4), Ms. Duncan believes that she does not have a choice in the matter, as her job “literally depends” on students increasing their scores on the MCAS test. Moreover, Ms. Duncan expresss genuine concern for her building principal, who, according to Ms. Duncan, is slated to lose her job the following year if test scores did not improve.

Ms. Duncan’s dominant question practice was asking QWKAs, and as such, this fit with her school’s emphasis on testing. After reflecting on one of her teaching videos, Ms. Duncan commented,

> I have to prepare my students for the MCAS with a very real deadline, and while I am in fundamental disagreement of this practice, my job literally depends on it. I guess what I am saying is that schools that are underperforming seem to place so

> much value on correct answers to tests, that teachers have no choice but to place the same burden on their students (Lesson Reflection, p. 2).

TDP has created a conflict between Ms. Duncan's beliefs in theory and her beliefs in practice. On one hand, she believes, "Brains don't compute the same way"…and "There's not only one way to do things," but in practice, she does not elicit different solutions. In theory, she believes it is important to "take your time to question" and "keep an open mind," but when confronted with a student's non-conventional solution she said, "You are going to have to erase it, and you are going to have to draw something that makes a little more sense." She said that she fundamentally, "believed in the "power of student leadership," but concurrently needs to maintain "complete control" in the classroom. In theory, she believes that it is important to "teach math as a concept and … let students really investigate before teaching rules and steps to follow," but in practice, she believes that her students are not "capable of learning new information that requires high level thinking skills." Once assigned to teaching in an underperforming school of low-income, minority students, Ms. Duncan's articulated beliefs in theory have difficulty coming to fruition in the classroom.

At the end of her third year of teaching, Ms. Duncan was able come to terms with her conflicting beliefs by readjusting her belief system. When asked to reflect on the issues and challenges that came up for her as she attempted to enact reform practices she said,

> I have honestly found the opposite of what I had thought to be true during my master's degree--- I have surprisingly found that it works better to teach an algorithm FIRST and THEN do the investigating and discussing of WHY. The third grade brain seems to succeed best using that method—weird, huh? (Email Correspondence, July 15, 2009)

Ms. Duncan's new belief that students need to be taught an algorithm first and then investigate is in direct conflict with her previously held position that students need to investigate concepts before learning rules to follow. After teaching for three years in a school that was deemed underperforming, Ms. Duncan had come to believe that third grade students are unable to engage in higher order thinking skills, thus mathematics reform, which encourages such higher level reasoning, and was not applicable to her group of students.

Ms. Quinn also experienced a significant amount of TDP to increase student test scores on the high-stakes MCAS test. The entire school district that Ms. Quinn was teaching in received an "underperforming" label by the state and a "correct action" grade on its 2008 No Child Left Behind (NCLB)(n.d.) report card. As a result of its low performance, the district entered a partnership with the state in an effort to improve student performance on standardized tests. However, Ms. Quinn's experience with TDP did not manifest itself until she was reassigned to a 4th grade math specialist position in another school within the district. Previously, while teaching first grade, Ms. Quinn said that there was pressure from the "higher ups" (Interview, p. 4) to follow district developed curriculum maps, but she often found herself going against the grain. She said,

> We had a curriculum map, and we were supposed to do all the stuff, and it was supposed to be done, you know, a whole unit was supposed to be done in five days. And I kind of took my own approach to that because I had kids that couldn't count to 10. So I would spend extra time making sure that they truly understood what they were doing because my mentality was I didn't want to just give them, like show them, a quick picture of everything that they were supposed to learn in first grade, because I felt that it would benefit them more if they actually understood maybe half of what they were supposed to learn instead of not understanding any of it but have seen it all. (Interview, p. 4)

While assigned to teach first grade, Ms. Quinn felt more able to resist the pressure to teach for coverage and instead tried to adopt a stance where she taught for understanding.

Ms. Quinn also indicated that when she was teaching first grade, her principal was very supportive of her actions and did not put pressure on the staff to teach to the MCAS test. She said,

> My principal that I had this year, was very supportive of the teachers, and she would come in, and she would see what I was doing, and her main focus with the kids as well, which was nice because there was the higher ups that were telling her, you know, what should be going on and what was supposed to be going on. But she has the same mentality that I had, that we're not there to follow a map. We're not there to make sure that we hit this unit, that unit, that unit. It's to make sure that the kids understand something, so I didn't have the pressure in my building. My principal was kind of the buffer for that, which was nice. (Interview, p. 5)

Although Ms. Quinn did not feel pressure as a first grade teacher, possibly because of the support she received from her building principal and because first graders are not required to take the MCAS, things changed once she was transferred to a different school as the fourth grade math specialist. After being reassigned to a new position and grade level, Ms. Quinn revealed that "standardized tests play a huge role in my teaching" (Ms. Quinn, Email Correspondence, August 17, 2009), and support from her new building principal was lacking. She recalled that when she requested, in June, to make an appointment to obtain the fourth grade curriculum materials for mathematics, she was told by her new principal to "come in August and get the stuff" (Interview, p. 18). His comment did not sit well with Ms. Quinn, as she was nervous about her new assignment and felt as though she was unprepared and unsupported. She said, "I don't have the training in the *Investigations*, so I'm going in blinded." Moreover, now that she was the math specialist, she had the sole responsibility and pressure of ensuring that all fourth grade students made gains on the mathematics portion of the MCAS test. This was a new reality for Ms. Quinn and quite different from what she had experienced in her previous first grade teaching assignment, thus adjustments in her beliefs were necessary. She said,

> At first, I did not focus a lot on the test, but as the test got closer I realized that my students were not prepared for the test, so I started to go over old tests, as well as every Friday gave the students an open response to work on. This is something that I am going to start earlier in the year this upcoming year in the hopes that if the students have been working on them for 8 months, it should just be second nature when they go to answer them during the test (Email Correspondence, August 17, 2009).

Testing was now a reality for Ms. Quinn, and as such she was obligated to adjust her teaching practice in an effort to ensure that her students made adequate progress on the state mandated testing. Previously, as a first grade teacher, her stance on following district mandated curriculum maps was somewhat critical; however, as a fourth grade teacher, her stance shifted. When asked as a first grade teacher how she handled the districts' focus on preparing students to make gains on the MCAS test, she said, "Maturity wise, I had some kids that were in the developmental age of three year olds, and I was supposed to be starting simple multiplication, and in my mind, I couldn't do that morally" (Interview, p. 6). Before experiencing the TDP pressure of the MCAS test, Ms. Quinn viewed her work in terms of her moral obligation to her students; however, once transferred to fourth grade, where testing was a reality of classroom life, her perspective on mandates and testing changed. When reflecting on the different tests that were mandated in fourth grade, Ms. Quinn seems to acquiesce to the pressure and rather than push back, she now seems pleased with the assessment measures that were being mandated. She said,

> We also take the MAP test three times a year. This is a test that is all taken on the computer and because it is taken three times (fall, winter, spring) it is a great way to show the students' progress and help us see if the students are making gains or not. (Email Correspondence, August 17, 2009)

As a fourth grade teacher, Ms. Quinn no longer has the luxury of pushing back on mandates and testing, as her new position requires that she ensure that all fourth grade students make progress on the many tests measuring their academic performance. Her stance seems to have shifted from one of moral conviction: “I couldn’t do that morally,” to one of compliance: “And it’s a great way to show the students’ progress.” Now, rather than pushing back on the system, Ms. Quinn has begun to push back on her students. When reflecting on the challenges she has experienced as a novice teacher she said, “Some of the biggest challenges I have faced in the past three years have been that many of my students do not want to give the effort to change themselves or question why or how something is happening” (Email Correspondence, July 5, 2009). In an effort to address the TDP pressure and responsibilities of her new fourth grade math specialist position, Ms. Quinn feels that she needs to shift her perspective on state mandates and testing.

Ms. Quinn’s experience teaching in an underperforming school has caused her to shift her perspective from one of resistance and questioning to one of compliance and blame, as she now believes the problem is with her students rather than with the system of mandates and testing. Moreover, she sees her dominant practice of asking QWKAs as a needed one for her students with limited English proficiency. When asked to reflect on one of her teaching videos, she justifies eliciting only correct answers for some students because, as she said,

> My questioning strategies vary from both eliciting students thinking to students giving me the correct answer. This is largely due to the fact that I have many students in my class who are English language learners, so it is often hard for them to understand detailed, wordy questions. (Lesson Reflection, p. 1)

Unlike Ms. Duncan and Ms. Quinn, Ms. Arielle did not experience TDP to ensure that her students would perform well on standardized tests. However, Ms. Arielle was a first grade teacher and testing was not a reality that she needed to deal with. When asked if she experienced pressure from her school administration to prepare students to perform on standardized tests, she said,

> I don't feel pressure because I am in first grade, and it's not something I have to directly deal with. I also hate the test so much that I try to pretend I don't care about it enough to feel pressured. (MCAS Reflection, p. 1)

Although personally Ms. Arielle does not experience pressure to ensure that her students perform well on mandated tests, she acknowledges that others in her school feel a significant amount of pressure. When reflecting on the impact of the MCAS test, she said,

> My colleagues and the administration do feel VERY [emphasis in original] pressured. All we talk about now is the MCAS! We even had someone who has success raising test scores come talk to us about curriculum mapping in order to get better scores. Basically, he teaches to the test. Our school will not abandon our constructivist philosophy, but we are looking for ideas to better our scores. Unfortunately, that's how our success will be measured. (MCAS Reflection, p. 1)

As a public charter school, Achievement for All has more flexibility in how they address the issue of increasing test scores, and as Ms. Arielle said, they are unwilling to give up their constructivist philosophy. However, even at Achievement for All, the pressure to increase tests scores was mounting. In this era of accountability, as measured by student performance on standardized testing, TDP is beginning to erode the confidence that the teachers and the administration have in using reform-oriented curriculum materials. According to Ms. Arielle, "Teachers feel students each year are better prepared in math but somehow fail the tests. Teachers are worried about the *Investigations* curriculum and how effective it is for test taking" (MCAS Reflection, p. 1). To meet the demands of standardized testing, Ms. Arielle shared that the administration recently hired two math specialists to work with faculty on their math teaching. Reflecting on the impact of such support, she said, “Between the both of them, I think we can improve our math teaching” (MCAS Reflection, p. 1).

Significantly, in Ms. Arielle’s school, the faculty and administration were concentrating their efforts on improving the teaching at Achievement for All, whereas Ms. Quinn’s and Ms. Duncan’s schools were concentrating their efforts on improving student scores. This difference is important to consider in that all three schools are serving predominantly Latino and African American students. When efforts are concentrated on improving student test scores, the underlying message is that students need fixing. However, when focusing efforts on the teaching practices used to engage students in mathematics, the work shifts from fixing the student toward considering the methods used to teach students. When discussing the achievement gap as it relates to African American students, Martin (2009) postulates,

> Closing the achievement gap is often translated as “raising African American children to the level of white children.” Under this directive, African American children are viewed as change worthy and in need of remediation in the direction of white children with respect to behaviors and beliefs. This idea insidiously frames mathematics education for African American children in terms of the standing and well being of white children rather than considering the needs of African American children as African American children. (p. 136)

If students are considered to be the problem, then education works at fixing the problem, thus “fixing” or “changing” the student. Ms. Duncan commented, “Many of my students are developmentally incapable of grasping some of the deeper concepts brought up in third grade” (Lesson Reflection, p. 2), indicating that she perceives her responsibility to be that of remediation, so that one day her students would be able to engage in higher level thinking and reasoning. Consequently, she is able to justify her lack of reform-oriented teaching methods while at the same time still holding to the belief that reform methods are valuable – just not for her students at this particular time. Ms.

Quinn believes that her students are just not willing to “change themselves or question why or how something is happening” (Email, July 5, 2009). Such a stance allows her to hold onto her reform-oriented beliefs, while concurrently justifying her traditionally centered practices.

Ms. Arielle, her colleagues, and the administration are cognizant of the pressures imposed by standardized testing and are working to ensure that accountability measures were met. However, as a group, they do not see their work as changing their students to meet state mandated testing demands; rather, they view their work as improving their practice so that students at Achievement for All will continue to engage in and with mathematics reform.

Mathematical Knowledge for Teaching

In an effort to obtain a snapshot of the participants’ mathematical knowledge for teaching elementary mathematics, the Mathematical Knowledge for Teaching (MKT) Elementary Number Concepts and Operations instrument was administered. The instrument, developed at the University of Michigan, was designed to provide users with the ability to make comments about “how groups of teachers perform at one or more points in time” (MKT Handbook). As such, the instrument was not designed to make comments regarding individual teacher’s mathematical knowledge for teaching. In this study, the instrument was used to discern if, as a group, the participants’ mathematical knowledge for teaching might have been an area of concern with respect to their ability to implement reform-oriented practices. The participants in this study obtained IRT scores, which were based on a mean of zero and a standard deviation of 1, consistent with those of the average teacher taking the assessment. The scores do not indicate that, as a group, the participants' mathematical knowledge for teaching is problematic, as their scores are above or just below the mean score achieved by all teachers taking this assessment.

However, when looking more closely at the individual test items, it is apparent that there are areas of challenge for the participants. For example, the MKT assessment was designed to measure teachers' knowledge related to representing mathematical ideas, explaining rules and procedures and assessing students' nonconventional work (See Appendix E for a sample of released test items). Ms. Duncan was proficient in all areas of the assessment; however, an area of challenge was related to explaining rules and procedures, thus this may have caused her to maintain such tight control over classroom explanations. If she had trouble explaining rules and procedures, then she may have difficulty allowing students to attempt to explain rules and procedures that she, herself, was unsure of. Consequently, Ms. Duncan provided all explanations of rules and procedures in a very detailed and direct manner to her students.

Most surprising was that Ms. Duncan was quite adept at assessing students' nonconventional work in mathematics; however, in practice she does not elicit or encourage student to solve problems in non-conventional ways. Ms. Duncan does not allow for students to share their different ways of reasoning because first, she thinks that some students think on a higher level, thus letting them explain their reasoning might confuse others, and second, she believes if she let students explain their nonconventional solutions, others might hear the wrong information, thus she keeps student explanations of their mathematical reasoning, for the most part, private. Possibly, the TDP Ms. Duncan experienced to prepare students to succeed on standardized test causes her to ignore students' divergent problem solving approaches, even though she dos adept at understanding them. Further exploration of this conjecture would be needed to understand why Ms. Duncan does not put into practice what she was equipped to do.

Additionally, Ms. Duncan's belief that she is weak in mathematics is not supported by the results of the MKT. Quite to the contrary, she scored almost one standard deviation above the average teacher taking the assessment. Consequently, Ms. Duncan's early mathematics experiences in which she perceived herself a failure seem to have long lasting, however unwarranted, effects on her perception of self as a math learner and quite possibly her perception of herself as a competent math teacher. Her inadequate feelings as a math learner may have caused her to have such tight control over classroom conversations and be more tentative to adopt reform-oriented practices that would require more student generated conversations about their mathematical reasoning. Again, further research is needed to examine this conjecture further.

Ms. Arielle's IRT score indicates that she was, at the time of this study, within the average range of MKT assessment. Her area of challenge is in assessing student's nonconventional work. Ms. Arielle was a successful student of mathematics throughout all of her schooling, and as such, she indicates that she has difficulty understanding the divergent ways that students represent their mathematical understanding. As she said, "I just got math," thus understanding how someone else does not is problematic for her. However, Ms. Arielle is aware of this challenge, as she indicated in an interview that this was a difficult area for her but one in which she was continually working to address. And, observations concur that she is actively working on understanding children's ways of reasoning about mathematics, as she consistently elicits students' mathematical reasoning, making it the focus of her classroom discourse.

Ms. Quinn is most challenged by the prospect of assessing student nonconventional work in mathematics, as she had great difficulty with answering this type of question successfully. As such, it seems plausible that Ms. Quinn would be tentative to fully adopt practices that encourage students to solve problems using non-conventional approaches and reasoning. Observations support this finding in that Ms. Quinn often discounts students' mathematical reasoning rather than attempt to explore it further. Ms. Quinn is most adept at explaining rules and procedures as she answered these questions quite successfully. Again, this result fits with Ms. Quinn's teaching practice, which was more focused on helping students to understand conventional rules and procedures.

Orchestrating Productive Mathematics Conversations

Orchestrating productive mathematics conversations proved problematic for all three participants; however, only Ms. Arielle considers it to be a challenge. Orchestrating such conversations is not perceived as problematic for Ms. Duncan or Ms. Quinn because, for the most part, they do not attempt to engage their students in such productive conversations. Because both Ms. Duncan and Ms. Quinn rely on the practice of asking questions with a known answer (QWKA), conversations are focused on eliciting correct answers rather than on engaging students in conversations focused on their mathematical reasoning. Moreover, the practice of relying on QWKA provides both Ms. Duncan and Ms. Quinn with the structure they need as novice teachers to manage and control student behavior. Students in both classrooms str obligated to respond to QWKAd with correct answers, which often are only one word utterances. Consequently, productive mathematics conversations have difficulty flourishing in such environments.

Conversely, Ms. Arielle does not use the practice of asking QWKAs as a basis for conversations with students; thus, she expects her students to engage in conversations by explaining and justifying their mathematical solutions. Ms. Arielle attempts to bring to light students' divergent ways of problem solving so that all members of the class can consider and compare multiple solution methods rather than one correct answer. As such, because she has adopted reform practices she needs to orchestrate several different aspects of the conversation in order for it to become productive. For example, Ms. Arielle needs to be mindful to ask questions that engage and challenge her students' reasoning, to listen to and decide what aspects of a student's reasoning needs further explanation or justification, to decide when to provide additional mathematical information or clarification, to decide when to revoice a student's solution either to clarify or highlight it as possibly more efficient or sophisticated than ones previously shared, and lastly, to monitor the rest of the group to ensure that all students are listening and engaged in the ensuing conversation. This proves to be a challenging and, at times, frustrating endeavor for Ms. Arielle. However, Ms. Arielle believes that her students are capable of engaging in such productive conversations, thus she seems to accept the challenge and the frustration as part of the work associated with developing a teaching practice based on reform.

Summary

The issues and challenges that the participants face reveal the complexity of attempting to implement mathematics reform practices while being a novice teacher. Although all three participants espoused reform rhetoric, only one, Ms. Arielle, was able to actively and consistently adopt mathematics reform practices into her teaching.

Moreover, Ms. Duncan and Ms. Quinn experienced a significant amount of TDP in terms of ensuring that their students make significant gains on the state mandated MCAS assessment. The pressure of working in an underperforming school, as novice teachers, seems to impact Ms. Duncan's and Ms. Quinn's ability and willingness to enact mathematics reform. Ms. Arielle, who was teaching in a publicly funded charter school, does not experience such pressure. Moreover, teaching first grade seems to lessen the pressure she feels because first grade students are not required to take the MCAS, yet. Ms. Quinn, who was teaching first grade when this study began, also indicates that as a first grade teacher she did not feel the MCAS pressure because her students were not yet tested. However, when she was transferred to fourth grade, she indicates that the MCAS tests suddenly played "a huge role" (Ms. Quinn, MCAS Reflection) in her teaching.

Managing productive mathematics conversations was a challenge for all three participants; however, only Ms. Arielle actively worked on addressing this issue. Ms. Duncan and Ms. Quinn actively resisted such conversations by adopting the traditional practice of asking QWKA.

CHAPTER 8

DISCUSSION

This concluding chapter will first return to the research questions that guided this cross-case study before summarizing the key findings. Second, this chapter will discuss the design and methods of this study as a means to discuss the limitations surrounding this body of work. Lastly, implications for practice and further research will be discussed.

The original research questions guiding this study were framed generally and sought to examine the teaching practices used by novice teachers to implement mathematics reform and discern the social and socialmathematical norms such practices constituted. However, as the study progressed, it became clear that a more focused approach would prove valuable in ascertaining the social and socialmathematical norms that developed in each participant's classroom. As such, rather than examine teaching practices in general, this study honed in on the discourse practices that the participants used to engage their students in mathematics conversations. Additionally, this study attempted to tease out the issues and challenges experienced by novice teachers as they attempted to develop a teaching practice under the auspice of mathematics reform. The following sections will provide a discussion of the key findings followed by a discussion of the study's limitations and its implications with regard to future research.

First and foremost, it is important to note that the participants in this study were dedicated teachers who had at the core of their practice the genuine desire to implement practices that were grounded in mathematics reform. The discussion here is not to find fault with the participants' practices, but to shed light on the issues and challenges that the participants experienced as they attempted to implement mathematics reform. As such, this discussion should not be read as a critique of individual teachers; rather, it should be read as a critique of the complexity involved in developing a teaching practice under the auspice of mathematics reform. With that said, as a result of careful and critical analysis of the discourse practices used by three novice teachers to engage their students in mathematics conversations the following general themes emerged:

1) The critical role of eliciting different solutions
2) The pressure of high-stakes testing obstructs reform

<u>The Important Role of Eliciting Different Solutions: A "Real Space"</u>

Findings reveal that the reform-oriented practice of eliciting different solutions, which was adopted by Ms. Arielle, used sparingly by Ms. Quinn, and adapted/ignored by Ms. Duncan appeared important to the adoption of other reform-oriented practices. The practice of eliciting different solutions served to provide a space (Schutz, 1997) wherein Ms. Arielle, and at times Ms. Quinn, had opportunities to appreciate and consider the different approaches students used while engaged in problem solving. Concomitantly, the practice of eliciting different solutions provided the necessary space to initiate conversations that focused on student generated explanations and justifications. Moreover, the practice provided the necessary space to address significant aspects of students' mathematical reasoning, thus opportunities to utilize practices that fostered the development of socialmathematical norms surfaced as well.

The metaphor of space is an important one to reflect upon when considering the critical role of eliciting the different solutions in reform-oriented classrooms. Schutz (1997), discussing Hannah Arendt's conceptualization of space, noted that "real space must allow individuals to appear in *unique* and unpredictable positions, not on some pre-determined cultural “field” no matter how dynamic” (p. 6). When a teacher adopts the practice of eliciting different solutions, as did Ms. Arielle, she opens the door to students’ unique, and at times unpredictable, mathematical reasoning and conjecturing. However, for the space to be “real,” a teacher must not expect students to offer solutions that follow a predetermined strategy or way of thinking or to focus solely on correct answers. Rather, she must be available to listen, address, and attempt to understand the unpredictability of students’ unique mathematical reasoning.

In this study, the practice of eliciting different solutions appears to be a necessary precursor to the implementation of other reform practices associated with the constitution of social norms, such as, eliciting explanations and justifications of students’ problem solving approaches. Moreover, the practice of eliciting different solutions made possible the enactment of other reform practices associated with socialmathematical norms, such as, indicating when a solution is not different, when a solution is more efficient or sophisticated, and indicating what counts as an acceptable mathematical explanation (see Figure 7.7). This finding is consistent with research by Kazemi and Stipek (2001) who found that practices fostering social norms, such as asking students to share and explain their different solution methods are necessary for mathematics reform to take root in classrooms. However, the researchers argue that such practices are superficial and alone will not “help students to build sophisticated understandings of mathematics” (p. 60).

I concur with Kazemi and Stipek in that sharing and explaining different solutions alone will not help students to develop sophisticated mathematical understandings. However, from the Kazemi and Stipek argument, it could be construed that the practice of eliciting different solutions is a superficial one to adopt. I argue that the practice of eliciting different solutions appears to be critical rather than superficial, as without it teachers would have difficulty calling upon other reform-oriented practices to engage students in mathematics conversations. Moreover, I argue that there is a distinction between the practice of asking students to *share* different solutions and the practice of *eliciting* different solutions and such a distinction is important to consider.

Asking students to share a different solution functions as a vehicle for expression, in that when students share a solution, they express to the class how they solved a problem. Conversely, eliciting different solutions serves “the function of drawing out students’ images, ideas, strategies, conjectures, conceptions, and ways of viewing mathematical situations” (Lobato et al., 2005, p. 111). With such a distinction, eliciting different solutions becomes more critical and important to consider when compared with sharing solutions, as the practice has the potential to draw out and build upon students’ unique and unpredictable mathematical reasoning. This critique should not be read as counter to the work of Kazemi and Stipek (2001); rather, it should be read as a means to fine tune a practice (sharing solutions) that teachers who have attempted to implement mathematics reform have adopted (Kazemi & Stipek, 2001; Pang, 2001) . Rather than consider the practice superficial, we must consider it critical, and as such, shift the focus from asking for students to share different solutions to eliciting from students their divergent methods of solution.

Asking students to share, places the teacher in a passive role; whereas, eliciting different solutions places the teacher in an active role, in that the practice focuses on actively drawing out students’ mathematical reasoning. As such, the practice does not end once a student shares his or her solution method. Research on classroom discourse during traditional “sharing time” provides insight into why in the mathematics classroom sharing different solutions has not produced the productive mathematics conversations envisioned by reform. Research has found that during “sharing time,” teachers often have difficulty understandings students’ “out-of-school experiences that are different from their own” (Cazden & Beck, 2003, p. 169) and, as such, are less appreciative of what their students share. Moreover, teachers reveal their lack of appreciation by offering general comments at the end of a student’s sharing. As such, because many U.S. teachers understanding of mathematics is “rule-bound and thin” (Ball, 1990, p. 451), they may have difficulty understanding solutions that are quite different from their own procedural methods, thus they may be less appreciative and end such exchanges with general comments, rather than actively developing a student’s line of reasoning. However, in a reform-oriented classroom enacting the practice of eliciting different solutions obligates students to share their solution methods, and as importantly, the practice obligates teachers to draw out and build upon students’ ideas and conjectures rather than simply to acknowledge the sharing.

Reflecting on the case of Ms. Arielle reveals that the practice of eliciting different solutions serves four functions that were critical and lay the foundation for productive mathematics conversations to develop. First, because Ms. Arielle’s practice is focused on eliciting different solutions, her students are obligated to develop their own novel approaches rather than depend on Ms. Arielle to provide step by step procedures. Thus, the practice of eliciting different solutions serves to communicate to students that problems could be solved in a multiplicity of ways. Second, inasmuch as the practice guides students toward understanding that problems could have many solutions, it also provides Ms. Arielle numerous opportunities to listen to and consider a diverse array of approaches and possibly helps her to develop an appreciation for and, as important, an understanding of the divergent ways her students reason about mathematics. This finding maps onto previous research by Borko, Jacobs, Eiteljorg, and Pittman (2008) who found that when teachers had opportunities, through video analysis, to reflect on the different approaches students use while problem solving, they began to “more deeply appreciate their own students’ unique approaches” (p. 432) in the mathematics classroom. Third, Ms. Arielle’s practice of eliciting different solution went deeper than the superficial practice Kazemi and Stipek (2001) refer to where teachers ask students to *share* different solution methods. Accordingly, the practice of eliciting different solutions serves to function as a “real space” (Schutz, 1997, p. 6) to draw out students’ unique and unpredictable mathematical reasoning rather than simply to obligate students to share their problem solving methods (Lobato et al., 2005). Lastly, for Ms. Arielle, adopting the practice of eliciting different solutions places the spotlight on students’ problem solving approaches rather than on correct answers during ensuing conversations. As such, conversations are tied to students mathematical reasoning and conjecturing rather than on correct answers to problems posed.

The importance of developing a practice that focuses on eliciting different solutions is supported in the literature on students learning in mathematics. In a study of elementary students’ proficiency with mental computation, Heirdsfield and Cooper (2004) found that flexibility and number sense were strongest in students who had “strong beliefs in their own self-developed and efficient strategies” (p. 461). The researchers recommend that teachers provide students with opportunities to develop their own solution methods rather than teaching procedures and rules to follow. Moreover, Henningsen and Stein (1997) support the significance of eliciting different solutions in that their research indicates that when classroom discourse is focused on students’ different solution methods, rather than on correct answers, students engage in a higher level of mathematical reasoning and thinking. Conversely, the researchers found that when classroom conversations focus on correct answers, as opposed to students' solution methods, there is a decline in higher order thinking and reasoning. Research, also suggests that to create a teaching practice that focuses on students understanding a key practice to develop is "eliciting from students representations of their thoughts so that the thought is 'on the table' for group consideration" (Cohen, 2004, p. 81). In this study, a single teacher, Ms. Arielle, consistently uses the practice of eliciting different solutions as a means to put student thinking and reasoning "on the table" (Cohen, 2004, p. 81). Moreover, the social and socialmathematical norms that are constituted in her classroom mirror norms advocated by researchers and advocates of mathematics reform.

It is important to consider, however, the complexity of enacting the practice of eliciting different solutions, as it requires much of the teacher who uses it to stimulate productive mathematics conversations. Fraivellig, Murphy, and Fuson (1999) concur that eliciting different solutions is a complex teacher move and one that requires sustained intentional effort to be productive. Acknowledging the importance and complexity of eliciting different solutions the researchers said,

> Clearly eliciting and then using students' descriptions of mathematical thinking is a complex and time-consuming task requiring patience, skill, and high levels of knowledge about individual children and about typical solution methods in major mathematical areas. Additionally, the elicitation of different solution procedures requires effective classroom management so that all students can become participants in problem solving discussions. A teacher must view the curriculum in terms of progression of teacher-supported and teacher-led solution methods by children rather than just in terms of covering the lessons in the textbook. Successful elicitation also requires a teacher who is willing and able to relax intellectual control sufficiently for children to respond with their own solution methods. (p. 167)

Ms. Duncan and Ms. Quinn predominantly use the practice of asking questions with a known answer (QWKAs) and as such, orchestrated conversations that were focused on correct answers and not on students' problem solving approaches or mathematical reasoning. However, it is important to note that when Ms. Quinn did utilize the practice of eliciting different solutions, the conversations that ensued focus on students' mathematical reasoning and thinking, rather than on correct answers. This finding highlights the critical nature of eliciting different solutions in that when used, even a teacher who is more apt at eliciting correct answers is able to shift her focus and bring students' mathematical reasoning and conjecturing into the conversation.

The results of this study indicate that developing a teaching practice upon the practice of eliciting different solutions appears to be critical in providing a "real space"(Schutz,1997, p. 6) whereby other reform-oriented practices can be called upon to stimulate productive mathematics conversations focused on students' unique and often unpredictable approaches to problem solving. Accordingly, in this study, the practice is considered critical rather than superficial. Moreover, the results of this study reveal that eliciting different solutions, although seemingly simple – "Who did it differently?" – is a complex practice that teachers must consider deeply and learn how to implement (Feiman-Nemser, 2001). Quite possibly because of the seeming simplicity of the practice, teachers have come to believe that by simply asking students to share solutions or name a strategy, as was the case of Ms. Duncan, they are effectively utilizing the practice. However, to genuinely adopt the practice of eliciting students' different solutions, a teacher must use it to draw out students' mathematical reasoning so as to put such reasoning up for consideration by all members of the community (Cohen, 2004; Lobato et al., 2005). Moreover, to effectively use the practice, one must listen to what students have to say and moreover, be available to respond appropriately to students' thinking and reasoning. By available, I mean able to make sense of, work with, and develop the student generated discourse that emerges from the practice of eliciting different solutions. Being available means much more than asking students to share their solution methods and may be the most difficult aspect of adopting the practice of eliciting different solutions.

There are particular factors that may mitigate one's ability and desire to adopt the practice of eliciting different solutions and as such, must be considered and examined in an effort to develop an understanding as to why the practice of eliciting different solutions might be adapted or ignored by novice teachers.

The Pressure of High-Stakes Testing Obstructs Reform

Adopting mathematics reform practices while concurrently being pressured to ensure that students make significant gains on state mandated high-stakes tests proved problematic for two of the participants in this study. Ms. Duncan and Ms. Quinn both experienced a significant amount of top down pressure from district and state administrative bodies to prepare students to make gains on the MCAS test. It seems that these novice teachers were unable and/or unwilling to adopt reform-oriented practices because of their schools' focus on test taking preparations. As such, both teachers adopted the traditional initiate, respond, and evaluate (IRE) questioning practice as their dominant mode of engaging students in mathematics conversations. This finding is consistent with earlier research on the effects of mandated testing on classroom discourse patterns. Poole (1994) found that mandated testing regulates classroom talk to the IRE sequence and to the reproduction of "school-valued knowledge" (p. 143). Moreover, Poole's research suggests that the IRE sequence "appears to inhibit holistic approaches (e.g., critical thinking or problem solving) to curricular topics” (p. 150). As such, the IRE sequence is a problematic practice for a novice teacher to adopt when considering that the mathematics reform movement advocates that teachers develop their students’ ability to think critically while engaged in problem solving.

As novice teachers, Ms. Duncan and Ms. Quinn were caught in a difficult space in that they had just stepped out of their preservice teacher preparation program equipped with reform-oriented ideas about mathematics teaching and learning, albeit, they stepped into schools that had been deemed underperforming because of lack of significant gains on the MCAS test. Even for an experienced teacher, such a dichotomy is difficult to navigate; to expect a novice teacher to surmount such a challenge without substantial support is problematic at best and futile to say the least.

Both Ms. Duncan and Ms. Quinn expressed a genuine desire to teach mathematics differently from how they were taught, and they attributed this change in thinking to their reform-based mathematics methods course. They want their students to develop a deeper more conceptual understanding of mathematics rather than only a shallow procedural understanding that reflected their own school experiences. In essence, they want more for their students than what they had experienced as students; however, the pressure of teaching in an underperforming school mitigated their ability and possibly willingness to adopt reform practices.

Feldman’s (2000) model of practical conceptual change warrants attention here, considering the dichotomy that presents itself when novices, armed with reform ideas and practices, begin teaching in underperforming schools fixated on test preparation. In his model of practical conceptual change, Feldman argues that being dissatisfied with one’s own educational experience and being engaged in reform-oriented methods during preservice teacher education may not be enough to bring about the "practical paradigm shifts" (p. 622) needed to enact reform-oriented practices. Feldman suggests that for a practical paradigm shift to occur, a teacher must first be dissatisfied with a practical theory that she holds. Once dissatisfied it is possible, according to Feldman, for a new theory to be taken up, if it proves to be "sensible… and …as beneficial" (p. 621), as other reasoned, practical theories may seem in a given situation. Lastly, for a paradigm shift to take root, a theory must also be "enlightening" (p. 621) when applied to a particular situation.

Findings suggest that Ms. Duncan and Ms. Quinn found reform practices sensible, beneficial, and enlightening while in a reform-based mathematics methods course, and as such, were tentatively able to take up the new practical theory of mathematics reform in that particular situation. However, once confronted with the realities of teaching minority students from low-income families in schools deemed underperforming and confronted with the responsibilities of ensuring that students make adequate gains on state mandated, high-stakes tests, both Ms. Duncan and Ms. Quinn resisted a practical paradigm shift possibly because in their current situation, this new practical theory of reform does not make sense nor is it beneficial or enlightening. Consequently, Ms. Duncan and Ms. Quinn are unable to undergo a practical conceptual change in their teaching situation because of the pressure they are under to raise the test scores of the low-income, minority students in their classrooms.

Delpit and White-Bradley (2003) argue that focusing solely on raising test scores in schools that educate urban children from low-income families has a "[d]ehumanizing effect on the ways teachers and students interact" (p. 283) and serves to regard students and teachers "as objects to be manipulated and 'managed'" (p. 284). Teachers and students are subjected to using scripted lessons from district adopted curriculum designed to raise test scores, and straying from the adopted curriculum is not allowed. When Ms. Duncan was asked during this study to teach a lesson from the reform-oriented curriculum, *Investigations*, of which the school was in possession, she said, “So sorry (for my students, especially), but no more *Investigations* at Sunrise Elementary/per school improvement scheduling” (Email Correspondence, January 17, 2009).

Because both Ms. Duncan and Ms. Quinn were externally pressured by administrators and outside consultants to prepare their students to pass the MCAS test, the IRE sequence may have been more relevant to their particular situation. Testing produces what Poole (1994) refers to as “knowledge objectification” (p. 143), wherein “testing encourages an objectifiable, value-free form of knowledge presentation” (p. 143). Consequently, where increasing scores on mandated tests is a reality of classroom life, reform practices, such as eliciting different solutions and eliciting explanations and justifications, are not practical practices to adopt because knowledge in such classrooms is not up for question or discussion.

Conversely, Ms. Arielle was able to undergo a practical conceptual change, and as such, effectively adopted mathematics reform practices into her teaching. Like Ms. Duncan and Ms. Quinn, while a preservice teacher, Ms. Arielle became discontented with her traditional school experiences in mathematics, thus she experienced her reform-based methods course to be sensible, beneficial, and enlightening. Albeit, when Ms. Arielle began teaching first grade in a public charter school that had yet to show significant gains on state mandated, high-stakes tests, she continued to find mathematics reform sensible, beneficial, and enlightening to her teaching practice and, moreover, to her students’ mathematical development. Why was Ms. Arielle able to anchor her teaching practice in mathematics reform while Ms. Duncan and Ms. Quinn were not? Possibly, because Ms. Arielle was a first grade teacher, she did not experience top down pressure directly because first grade students in Massachusetts are not required to take any portion of the MCAS test, yet. Findings from this study suggest that in states with high-stakes testing, grade level assigned may impact a teacher's ability to adopt reform practices. Further research is needed to support this claim. Cazden and Beck (2003) concur that research is needed to discern the impact of high-stakes tests on the patterns of discourse that emerges in such classrooms.

Mathematics Reform and the Question of Equity

One must now wonder how Ms. Duncan and Ms. Quinn were able to champion beliefs about the importance of mathematics reform while simultaneously developing teaching practices that were traditionally situated. The findings here are troubling, and we return to Feldman's (2000) work on practical conceptual change as a means to address this paradox. Discussing how practical paradigms guide a teacher's decision making, Feldman postulates that "Practical paradigms are analogous to the scientific paradigm described by Kuhn in that a community shares them. In some ways, they become the ethos of that community into which newcomers are indoctrinated" (p. 611). Newcomers, Ms. Duncan and Ms. Quinn, armed with their newly developed ideas about how mathematics should be taught and learned were no match for school communities that had been labeled "underperforming" previous to their arrival. The schools in which they were teaching were cloaked in an ethos of underperformance, and as such, practical paradigms about how to teach underperforming students were already well established and not easily challenged by the newcomers. According to Feldman (2000),

> Practical paradigms are quite tenacious; teachers do not easily modify them. They are shared by a community and are supported by other teachers, students, parents, and administrators. They may be applied in situations for which they are inappropriate, and their existence can cause teachers to be extremely selective in their observations on the situations and analysis for the problems. (p. 612)

Ms. Duncan and Ms. Quinn needed to reformulate their beliefs about mathematics reform in light of their current teaching situations. Both teachers needed to find a way to come to terms with their inability and/or unwillingness to enact reform practices while staying true to their beliefs that mathematics reform is a worthwhile goal. Findings suggest that both teachers reformulated their beliefs, albeit, in different ways, about mathematics reform by shifting their perspective to focus on their students rather than on their teaching practices.

Ms. Duncan came to believe that her students were developmentally incapable of understanding the conceptual underpinnings of elementary mathematics, thus they required step-by-step procedural instructions before they could be expected to engage with mathematics reform. Thus, Ms. Duncan's teaching practice was grounded in transmitting the formal conventions of mathematics to her students rather than developing their abilities to engage in higher order thinking. However, research does not support the postulation that some students cannot engage in higher order thinking and reasoning. Analyzing data gleaned from the QUASAR Project[1], Henningsen and Stein (1997) found that students from economically disadvantaged communities can engage in higher level mathematics thinking and reasoning when a given task is appropriate: when teachers provide support and scaffolding and when teachers consistently obligate students to provide explanations and to make meaningful connections all while ensuring that the

[1] QUASAR Project: Quantitative Understanding: Amplifying Student Achievement and Reasoning. University of Pittsburg under the direction of Edward A. Silver.

complexity of a task is not reduced. The type of discourse that Ms. Duncan believes her students need is what Henningsen and Stein refer to as fostering a “decline to procedural thinking without connection to meaning” (p. 541). Moreover, Henningsen a Stein suggest that this type of decline is related to a focus on correct answers, which Ms. Duncan admittedly focused on, and not to eliciting and validating students’ mathematical thinking and reasoning.

Ms. Quinn, on the other hand, now subscribes to the belief that her students just do not want to “change themselves” (Email, July 5, 2009), thus mathematics reform is difficult for her to enact, not because she does not believe it a worthy goal, rather, because she believed that her students would not take up the challenge. Both Ms. Duncan and Ms. Quinn reformulated their beliefs by adjusting their understanding of their students’ ability and desire to engage in reform practices and, as such, were able to hold onto the belief that mathematics reform is worthwhile, albeit just not for their students. This finding is supported by Feldman’s (2000) work with novice teachers, as he found that novices often “adjust their understanding of their teaching situation so that their practical theory continues to make sense” (p. 621). By adjusting their understanding of their students via their current teaching situations, Ms. Duncan and Ms. Quinn did not have to address the dissonance their traditional practices creates in relationship to their reform-oriented beliefs. Neither Ms. Duncan nor Ms. Quinn consider their practices as being problematic and, as such, were able to remain naïve as to the consequences their practices might have on their students. Such a disposition serves to “locate the problem within students themselves” rather than within the teaching practices teachers use with students (Boaler, 2002, p. 241).

Delpit (1988) questions how beneficial the reform movement is to disadvantaged students who stand outside of what she calls the “culture of power” (p. 280). According to Delpit, “[I]f you are not already a participant in the culture of power, being told explicitly the rules of that culture makes acquiring power easier” (p. 282). As such, Delpit argues that reform practices could function to deny those outside the “culture of power” access to the rules and codes necessary for achieving cultural power. However, Boaler (2002) contends that rather than discard teaching practices meant to foster a deeper conceptual understanding of mathematics, there needs to be a deeper examination of how reform teaching practices can be implemented more equitably. Although Delpit (2006) argues emphatically that schools need to ensure that low-income, minority students, who lack cultural power, obtain the basic skills needed to be successful in the United States, she decries the ways in which high-stakes tests have regulated learning in schools deemed as underperforming to a mechanized forms of teaching and learning.

Ms. Arielle’s teaching practices are reform-oriented in that she expects students to solve problems in different ways, explain and justify their thinking, and attend to significant aspects of their mathematical work, such as efficiency and sophistication of the solutions shared in conversation. What is most striking about Ms. Arielle’s practice is that it is open ended and focuses on students’ mathematical reasoning and thinking rather than on rules and procedures to follow. Lubienski’s (2000) research points to the possibility that white, middle-class students may be more adept at engaging in such open-ended discourse, thus they are more advantaged by reform practices than students with less power in society. However, I argue that because reform practices are based on the contention that ways of communicating are collectively negotiated as members of a community engage in conversation about significant aspects of their mathematical reasoning, reform practices may not be as difficult for students with less cultural power to engage with, as Lubienski suggests. Citing Bernstein, Lubienski (2000) reports, “Lower status families use *restricted codes,* language with implicit and context-dependent meanings that make sense in contexts in which emphasis is placed on community and common knowledge and values are assumed to be shared” (p. 456). As such, reform-oriented classrooms with their emphasis on emergent, taken-as-shared, social and socialmathematical norms may not prove problematic for students whose lives outside of school function in a similar manner.

As a Latina teacher who was positioned outside the culture of power, Ms. Arielle may have been more available to engage her students in mathematics reform. When reflecting on her practice, she commented, “I think I go slower, I try to use very clear words. I don’t know if it’s because English is my second language and I also have that in my mind, that I want to make sure that what I’m saying is understood” (Interview, p. 10). Possibly, Ms. Arielle is more available to listen, work with, and develop her students’ thinking because of her conscious effort to help her students to see the language of mathematics as clear and understandable, thereby making students’ engagement with it more productive. She often spends up-front time asking students to rephrase a problem before attempting to work on it, thus she does not leave students to interpret the meaning of problems on their own. Rather, students interpret meaning as a collective under the guidance of their teacher. The reform-oriented curriculum Ms. Arielle uses is representative of reform texts that have been said to be “extremely wordy and linguistically demanding” (Boaler, 2002, p. 249). In her research on mathematics achievement and social class, Boaler found that supporting students to make meaning of the contexts in which math problems are situated is a critical factor in low-income students' ability to be successful in reform-oriented classrooms.

Ms. Arielle's teaching practice could be viewed as what Paulo Freire (1997) referred to as "problem-posing education" (p. 64), wherein students are considered "critical co-investigators in dialogue with the teacher" (p. 62). Problem-posing education, according to Freire, is one where dialogue is "indispensable to the act of cognition" (p. 64). Ms. Arielle grounds her teaching in the reform-oriented social discourse practice of eliciting different solutions and follows up such solutions with requests for explanations and justifications, thus dialogue is a natural outgrowth. Students in this first grade classroom, who are relatively new to the formal educational process, appear to take up communicating their mathematical thinking and reasoning in ways that are advocated by mathematics reform researchers and educators.

Conversely, Ms. Duncan and Ms. Quinn's practices mirror what Freire referred to as "banking education" (p. 54). Banking education marginalizes students to the periphery in that they are considered receptacles waiting to be filled with knowledge by their teacher. The teacher asks questions, students provide answers, and the teacher, exercising her power, evaluates student answers as either correct or incorrect. Quoting Freire (1997), the banking concept of education regards students as

> individual cases, as marginal persons who deviate from the general configuration of a "good, organized, and just" society. The oppressed are regarded as the pathology of a healthy society, which therefore must adjust these "incompetent and lazy folk to its own patterns by changing their mentality. These marginals need to be "integrated," incorporated" into the healthy society they have forsaken. (p. 55)

In banking education, the power to learn is in the hands of the teacher rather than in the hands of the learner, as it is the teacher's responsibility to dispense what needs to be learned to students, and it is the student's responsibility to learn what is taught by the teacher. However, the power of the teacher has is not seen as problematic because she is using her power to enhance the knowledge of her students. Rather, it is viewed as necessary by the teacher and students. Foucault says of power,

> If power were never anything but repressive, if it never did anything but to say no, do you really think one would be brought to obey it? What makes power hold good, what makes it accepted, is simply the fact that it doesn't only weigh on us as a force that says no, but that it traverses and produces things, it induces pleasure, forms knowledge, produces discourse. It needs to be considered as a productive network which runs through the whole social body, much more than a negative instance whose function is repression. (Foucault as cited in Rainbow, 1984, p. 61)

Considering that the schools Ms. Duncan and Ms. Quinn were teaching in had been labeled underperforming, Foucault's analysis of power is fitting in that the banking practice is not viewed as repressive because its main purpose is to foster student achievement, which is arguably is a worthy goal. Moreover, a banking practice may be the one that best serves the goal of increasing student test scores on mandated, high-stakes tests, and as such, a banking practice may be a difficult one to resist for novice teachers teaching in underperforming schools.

Limitations

A limitation of this study is in its inability to generalize because the multiple case study design included only three participants. However, because of a gap in the literature documenting reform discourse practices (Cazden & Beck, 2003), a descriptive study with a small number of participants was appropriate to provide a foundation from which to generate future research. Moreover, the researcher's inexperience required a more manageable study, thus the number of participants was limited to address this concern.

A second limitation concerns the dual role that I played as the researcher and the participant's former instructor. As such, I needed to be cognizant at all times that, as former students, the participants may have been predisposed to pleasing the teacher, and as such, the results were interpreted with this in mind (Seidman, 1998). However, because I was the participant's former instructor, I had developed a positive relationship with each participant before video observations began; thus the participants may have been more willing to engage in this study during the often turbulent, novice years (Luft, 2007).

Lastly, this study was limited in that I was the sole researcher involved in this study. As such, even with my consistent attention to the biases and assumptions I brought to the study, it is possible that another researcher might not have come to the same conclusions. The incorporation of participant checks, a critical friend, and triangulated data sources all served to address this limitation.

Implications for Research on Practice

In her discussion of mathematics reform and issues of equity, Rochelle Gutierrez (2002) hailed teachers' practice as the key to equity in mathematics education. This is an imposing, yet promising statement. It is imposing in that Gutierrez's statement suggests that issues of equity can be addressed through examining teacher practice. This would require systematic and deep reflection on the part of researchers and teachers as to how practice does and does not promote mathematics equity in classrooms. Conversely, the statement is also promising in that it situates research and reflection squarely within practice, thus making teachers' practice the unit of analysis rather than individual teacher abilities (Gutierrez, 2002). According to Gutierrez,

> If the practice of teaching is not merely what teachers bring to the classroom (i.e. their beliefs, knowledge, lived histories, and personalities) but also is part teachers' membership in local communities how might we come to understand what it takes to enact particular practices, especially ones that relate to certain kinds of students or equity goals? I see effective teacher practice as the knowledge, skill, and commitment, to engage in the local context and community under a variety of conditions many unexpected. (p. 171)

Thus, research generated by such a stance will not focus on the individual teacher; rather, the focus will be on the practice of teaching within local communities. This is significant because such a research agenda would be obligated to examine how the practice of teaching is enacted differently across communities. An implication of this study relates to the enactment of reform discourse practices across school communities deemed as underperforming. The three novice teachers in this study were all situated in schools that had yet to make adequate yearly progress on state mandated, high-stakes, standardized tests. It seems that for Ms. Duncan and Ms. Quinn, the top down pressure they experienced from school, district, and state administrators, to ensure students made progress on standardized tests, possibly mitigated their ability and/or willingness to enact discourse practices associated with mathematics reform. Conversely, Ms. Arielle, who was teaching in a public charter school and felt supported by her school administrators and colleagues to continue with her constructivist practices, was able to withstand the pressure imposed by standardized tests and, as such, successfully adopted reform-oriented discourses practices. This implies a critical need to support novice teachers, especially those who teach in schools labeled "underperforming" beyond the teacher preparation years. Continuing to provide support to newly licensed teachers after teacher preparation would offer novices the scaffolding and guidance they need to develop and refine the reform practices they were exposed to during their preservice years. Novice teachers leave teacher education with ideas about teaching and learning that are often not supported in the environments in which they find themselves. Extending the relationship between novice teachers and Schools of Education beyond the preservice years would provide critical support for the reform ideas garnered in teacher preparation programs. Moreover, such a relationship would provide novice teachers the venue for addressing and critiquing their practice juxtaposed with the unexpected conditions that are embedded within the school community in which they work. The label “underperforming” is a challenging construct for any teacher to dismantle. Novice teachers need support and guidance dismantling the social underpinnings and implications of the construct of underperforming, so as not to see the label as synonymous with their students.

I agree with Peressini and Knuth (1998) that teachers need to be offered opportunities to view, analyze, discuss, and reflect upon videotaped episodes of their own teaching and the teaching of others so as to support their ability and willingness to enact reform practices. With the availability of technology and online course formats, teacher education programs could effectively offer a “first year in practice” capstone course that would provide novices the space they need to continue to engage in critical reflection on their developing mathematics teaching practice within their complex school communities. In an online course, novice teachers could videotape themselves as they teach mathematics and upload the videos onto a course site where, in conjunction with their instructors and peers, they could critically reflect on their emerging practice. This online format would address the issues raised by Luft (2007) regarding the difficulty of studying novice teachers. According to Luft, the prospect of participating in research on practice during the beginning years of teaching could be an overwhelming experience even for the most adept novice teacher. Providing novices with an opportunity to reflect deeply on their developing mathematics teaching practice via video analysis using an online venue may ease the trepidation of engaging in such an endeavor. There is mounting research pointing to the effectiveness of using video as “an artifact of practice” (Borko et al., 2008, p. 434) to examine how teachers can transform their teaching by reflecting on the issues and challenges inherent to adopting reform practices. Studies examining how novices can use video artifacts to develop their practice so that the “practical conceptual change” (Feldman, 2000, p. 616) they experienced as students in teacher education can continue to prove sensible, beneficial, and enlightening as they move into their roles as teachers.

A second implication of this work addresses the important role that the practice of eliciting different solutions can play in the implementation of other reform-oriented discourse practices. Those working with preservice and novice teachers must make a concerted effort to foster the disposition that mathematics problems can be solved in a multiplicity of ways, and to do that they must be genuinely dedicated to adopting the practice of eliciting such divergent methods of solution from the preservice teachers with whom they work. As such, the practice of eliciting different solutions is not superficial, as argued by Kazemi and Stipek (2001) when considering a novice teachers’ developing teaching practice. Rather, a teaching practice that is fundamentally dedicated to the practice of eliciting students’ divergent methods of solution lays the foundation for mathematics reform to take root. An appreciation of students’ different solution approaches can develop only if such solutions are afforded a venue to be heard and considered by the classroom community. The practice of eliciting different solutions is a critical one to adopt, develop, and refine if teachers are to build classroom communities that reflect mathematics reform. To say that a practice, such as eliciting different solutions, is both necessary and superficial is a contradiction of sorts, in that a practice cannot not be both necessary (essential, crucial) and superficial (trivial, unimportant). Consequently, if researchers consider the practice of eliciting different solutions superficial, then studies may cease to examine how this critical practice develops, and subsequently, teachers may cease to develop and refine their ability to elicit students' different ways of thinking and reasoning about mathematics.

As such, instructors in the methods and content courses required of preservice teachers must model the types of discourse practices that they expect preservice teachers to utilize in classrooms. Moreover, the constructs of social and socialmathematical norms and how such norms are developed through negotiated discourse in classrooms must be made public and be put up for discussion and critique in mathematics methods courses serving preservice teachers. Helping preservice teachers to develop a practice that first, values students' different mathematical ideas and conjectures and second, understands how to work with students' mathematical thinking and reasoning to stimulate productive discourse is critical if reform is to take root within the classroom. Moreover, guiding preservice teachers in the art of managing and orchestrating productive mathematics conversations that fosters critical dialogue between teachers and students is imperative to the development of reform recommendations. If, as Shotter (1991) postulates, human reality is fundamentally conversational, then guiding preservice teachers in the art of meaningful mathematics conversation would prove valuable.

This study reveals the importance of designing research studies that directly address the particular issues and concerns that are inherent to the novice teaching years. Much general research has been done on the effects of induction programs and mentoring in supporting novice teachers, but limited research has examined the complex issues and challenges that are particular to being a novice teacher (Luft, 2007) attempting to enact practices associated with mathematics reform. A question this study raises is why Ms. Duncan and Ms. Quinn do not make a practice of eliciting students' different solution approaches. Both Ms. Duncan and Ms. Quinn expressed a lack of confidence in their mathematical abilities, whereas Ms. Arielle indicated she was quite confident in her ability to engage with mathematics. In studies of math anxiety, researchers have found that confidence in one's mathematical ability is related to one's trust in their own mathematical instincts and judgments (Clute, 1984). As such, it is plausible that because Ms. Duncan and Ms. Quinn lacked confidence in their mathematical abilities, they did not trust their own judgments and instincts when accessing students' different ways of reasoning and thinking about mathematics. Consequently, lack of confidence may have caused them both to ignore the practice of eliciting different solutions in favor of initiating conversations focused on correct answers that they could assess as either right or wrong without having to address students' mathematical reasoning.

Moreover, in this study, lack of confidence trumped actual mathematical knowledge for teaching as assessed by the MKT instrument. Recall that Ms. Duncan scored almost one standard deviation above the mean on the Mathematical Knowledge for Teaching instrument, indicating that she possessed a relatively strong knowledge base for assessing students' novel ways of thinking and reasoning, thus eliciting and developing students different solutions should not have been problematic for her. It seems possible that Ms. Duncan's lack of confidence trumped her ability, as measured by the MKT instrument, to assess students' novel ways of solving problems. Conversely, Ms. Arielle, who obtained a score slightly below the mean, was quite confident in her mathematical abilities and did not shy away from eliciting and developing students' different solution methods.

Using this study as a reference point, I would recommend pursuing research that further examines the questions: “What prompts teachers to build their teaching practice on the foundation of eliciting students’ different solution methods? Is there a connection between a teacher’s level of confidence in mathematics and her decision to build her practice on eliciting different solutions?” and How does teaching in a school labeled “underperforming” impact a teacher’s ability and/or willingness to enact mathematics reform practices? Findings from this study indicate that there is a need for research that examines the impact of high-stakes testing on a novice teacher’s ability and willingness to implement mathematics reform practices. If the pressure of high-stakes testing obstructs reform, as was the case in this study, then the work done in teacher education has little chance to come to fruition.

Conclusions

The findings of this cross-case study underscore the complexity involved in developing a teaching practice under the auspice of mathematics reform. Moreover, the findings fill a gap in the literature about how novice teachers enact discourse practices associated with mathematics reform. Findings suggest that the top down pressure that novice teachers experience to ensure that students make significant gains on state mandated, high-stakes assessments can impact their ability and willingness to adopt mathematics reform practices.

Findings also reveal that adopting discourse practices associated with mathematics reform is possible for novice teachers, and when such practices are adopted, productive mathematics conversations focused on students’ mathematical thinking and reasoning emerge. This study found that situational factors, such as confidence in one’s mathematical ability, support from colleagues and administrators to adopt and use reform practices, and teaching at a level (first grade) wherein students are not yet tested may facilitate the adoption of discourse practices associated with reform.

Findings from this investigation illuminate the need for novice teachers to be supported while developing a teaching practice under the auspice of mathematics reform. Novices learn about reform practices during teacher preparation programs and begin to develop new beliefs about how mathematics should be taught and learned. However, once in the reality of classroom life, it is difficult for novices to put into practice what was learned without support. As such, it seems important for teacher preparation programs to become more connected to their graduates once out in the field teaching. With the reality of high states testing in schools across the United States (Cazden & Beck, 2003), novices may find themselves teaching in situations where the ethos of the school is cloaked in the top down label of underperformance. As such, attempting to enact mathematics reform as a novice, without support and guidance, may prove to be an insurmountable task. Schools of Education could be the link that enables novice teachers to continue to critically engage and reflect on practices associated with mathematics reform and how such practices can take hold in their particular classroom.

APPENDIX A

INDIVIDUAL INTERVIEW PROTOCOL

Time of Interview:
Date:
Place:
Interviewer:
Interviewee:

Before beginning the interview, outline the purpose of the interview and assure the participant of the confidentiality of the interview and their right to withdraw at any time.

1. Tell me a little about yourself in terms of being a math learner.

2. Describe your experiences in Education 691R: Promising Practices in School Mathematics.

3. Describe the most valuable piece of information, knowledge, and/or pedagogy that you took from the course? Why do you think that stands out as valuable?

4. Please share with me examples of how you have incorporated aspects from 691R into your teaching practice.

5. How would you describe your mathematical teaching practice?

6. What are the social behaviors that you expect students to develop in your math class?

7. How do you foster such behaviors?

8. Do you think it is important to have students explain their solutions to the class? What could you learn about their thinking and understanding?

9. Is it important for your students to be able to justify or defend their solutions to the class? What could you learn about students thinking by listening to their justification?

10. Can you describe for me the difference between explaining a solution and justifying a solution?

11. How do you encourage students to justify their solutions?

12. Suppose a student uses a method for solving a problem that you know would work every time but it is not the traditional algorithm. What would you say to that student?

13. Suppose a student came up with a method for solving a problem that you knew would not work every time? What would you do?

14. Describe your confidence level your first year of teaching math.

    a. What is your confidence level like now?

    b. What do you attribute the change to?

15. What have you learned about how students understand math from your experience teaching math?

16. What have you learned about yourself as a math teacher from your experience teaching math?

17. What issues or challenges have you come up against in to teach mathematics?

18. What resources have been available to you to help you develop your math teaching practice?

19. What role does standardized testing play in your teaching practice?

20. What role does the algorithm play in your teaching practice?

APPENDIX B

FOCUS GROUP INTERVIEW PROTOCOL

1. Describe for me how your methods course, EDUC 691R Promising Practices in School Mathematics, prepared you for your role as a mathematics teacher?

2. What practical skills did you develop in 691R? By practical I mean the everyday skills and methods for teaching math.

3. What theoretical understandings did you develop in 691R? By theoretical I mean the overall framework you developed in terms of how students learn mathematics and how that framework impacts your teaching.

4. Describe for me the core practices that you use when engaging your students in whole group mathematics conversations.

5. Reflect back on your beginning days of teaching. What would you say was your biggest concern regarding teaching mathematics? What was the biggest obstacle that you thought you might have to overcome?

6. Now as a two plus year teacher, what is the biggest concern you have teaching mathematics? What is your biggest obstacle?

7. Take a minute and think about the past two years teaching math and describe for me how your confidence has been impacted by the process of teaching?

8. What other issues or challenges besides the one you mentioned are of concern to you?

APPENDIX C

CONTACT SUMMARY FORM

Contact Summary Form

Ms. Duncan Observation 5/1/08

Write Up 5/2/09

1. Main issues of observation.
   When students don't have a correct answer, she says, "Looks like you were thinking of something really good, so I am going to give you some more time and if it comes back to you, pop up that hand." She has a format that she followed again today: Introduction, question answers session, using students to act out or model fractions, student practice.
2. Summary
   Some of her practices seem quite traditional. She gives them specific ways in which they must read the mixed number. "What you need to do in mixed numbers is you count up the whole shape first, then you count up the remaining pieces." So is this okay? Do students need this type of information to progress? Would this be called the shell or skeleton or foundation in that it is needed so that everything else can be attached or built upon in later lessons? Missed opportunity: If you were smart, you would already know the denominator. George? G: 4 T: Correct! She could have asked him why he got 4 or what did he do to get 4. This would have helped the kids that did not get why he said 4 and helped to develop his own thinking.
   She is giving them steps: First step is count the wholes: This is a traditional practice. Missed opportunity: J goes to count all of the squares and she says, No, just this one." Then later she tells him why she did that. Why not ask him why she did that?
3. Salient Points to Consider
   There are certain aspects of the lesson that she expects students to follow such as how to write a mixed number. But she engages the students and asks them why it is important to write it a certain way. Student responds that if you wrote it too close, it might look like 12/2 rather than 1 ½. I suppose you could call this a rule, but I am thinking it is a part of math that is very important – it is like she is teaching them how to write in the language of math as well as how to speak and hear it.
   When she gets a phone call and is occupied for a bit, the students are amazing!
4. Questions for next observation.
   Why does she feel that she needs to be so explicit with things such as how to read a mixed number? I am wondering if she notices that she is giving students steps to

follow. I might ask her to view this tape and give me her assessment. She talks a lot, and the students talk very little.

APPENDIX D

LESSON REFLECTION PROTOCOL

Here is a compact disc of one of the classes that I observed you teach. I would appreciate it if you could review the disc and then reflect on the following questions:

1. How much sharing of ideas do you notice between students and students and student and teacher?

2. Do you observe students collaboratively engaged in problem solving?

3. Or are students more engaged in learning steps or rules to a procedure like multiplication, subtraction, or writing fractions?

4. Is the majority of the lesson conducted in whole group? Is it a bit of whole group at the beginning and end with small group work in the middle, or is it whole group first and individual work second?

5. Are students using manipulatives to help them with understanding? What are the manipulatives helping them to understand?

6. Are students engaged in uncovering concepts? If yes, describe the conceptual understanding you think they are developing?

7. Take a look at your questioning strategies. Are you eliciting student thinking, or are your questions more geared toward students giving a right answer?

8. After reviewing the tape, what surprised you the most? What concerned you the most?

9. On a scale of 1 – 4 with 1 being traditional and 4 being reform, where do you rate this lesson?

10. Anything else that you think is important to reflect on please feel free.

APPENDIX E

MATHEMATICAL KNOWLEDGE FOR TEACHING RELEASED ITEMS[2]

Assessing Student Work

Imagine that you are working with your class on multiplying large numbers. Among your students' papers, you notice that some have displayed their work in the following ways:

Student A	Student B	Student C
$\begin{array}{r} 35 \\ \times 25 \\ \hline 125 \\ +75 \\ \hline 875 \end{array}$	$\begin{array}{r} 35 \\ \times 25 \\ \hline 175 \\ +700 \\ \hline 875 \end{array}$	$\begin{array}{r} 35 \\ \times 25 \\ \hline 25 \\ 150 \\ 100 \\ +600 \\ \hline 875 \end{array}$

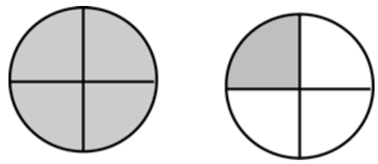
Which of these students would you judge to be using a method that could be used to multiply any two whole numbers?

	Method would work for all whole numbers	Method would NOT work for all whole numbers	I'm not sure
a) Method A	1	2	3
b) Method B	1	2	3
c) Method C	1	2	3

Represent Mathematical Ideas and Operations

[2] Measures copyright 2005, Study of Instructional Improvement (SII)/Learning Mathematics for Teaching/Consortium for Policy Research in Education (CPRE). Not for reproduction or use without written consent of LMT. Measures development supported by NSF grants REC-9979873, REC-0207649, HER-0233456 & HER 0335411, and by a subcontract to CPRE on Department of Education (DOE), Office of Educational Research and Improvement (OERI) award # R308A960003.

Mrs. Johnson thinks it is important to vary the whole when she teaches fractions. For example, she might use five dollars to be the whole, or ten students, or a single rectangle. On one particular day, she uses as the whole a picture of two pizzas. What fraction of the two pizzas is she illustrating below? (Mark ONE answer.)



a) 5/4
b) 5/3
c) 5/8
d) 1/4

Explain Mathematical Rules and Procedures

Ms. Harris was working with her class on divisibility rules. She told her class that a number is divisible by 4 if and only if the last two digits of the number are divisible by 4. One of her students asked her why the rule for 4 worked. She asked the other students if they could come up with a reason, and several possible reasons were proposed. Which of the following statements comes closest to explaining the reason for the divisibility rule for 4? (Mark ONE answer.)

a) Four is an even number, and odd numbers are not divisible by even numbers.

b) The number 100 is divisible by 4 (and also 1000, 10,000, etc.).

c) Every other even number is divisible by 4, for example, 24 and 28 but not 26.

d) It only works when the sum of the last two digits is an even number.

APPENDIX F

LESSON PLAN SCORING GUIDE

Level	Lesson Plan Level of Reform Practices
	Student Participation Structure: Teachers guide individual, whole group, and small group work (NCTM 1991). Students communicate their mathematical ideas to peers, teachers, and others (NCTM, 2000).
3	Students work in whole group sharing ideas and strategies, move to individual, pair, or small group working on worthwhile tasks, end with whole group sharing of ideas.
2	Students work in whole group listening to teacher, move to individual small group working on worthwhile tasks, ending with small group sharing or listening to teacher sum up lesson.
1	Students work in teacher directed whole group followed by individual practice of procedures and review sessions.
	Objectives: Lesson objectives address developing different strategies to solve problems for which the solution is not known in advance; reflecting on the process of problem solving (NCTM, 2000).
3	Lesson objectives focus on student generated approached to problem solving, developing fluency with number, and understanding of concepts.
2	Lesson objectives focus on developing students' ability to use particular strategies to solve problems.
1	Lesson objectives focus on students' abilities to perform certain procedures.
	Assessment Methods: Assessment is formative and enhances student learning. Assessment includes observing students as they engage in mathematics and is a part of classroom activity and not something that happens to students at the end of a lesson (NCTM, 2000).
3	Assessment is focused on *how* students are engaging with the mathematics throughout the lesson. Assessment happens while mathematics is happening.

2	Assessment is focused on whether students *can* perform particular strategies and is ongoing throughout the lesson.
1	Assessment is focused solely on students *practicing* what was learned and takes place at end of lesson.

APPENDIX G

SUMMARY TABLE OF LEVEL OF REFORM IN LESSON PLAN

Participant	Participation Structure	Lesson Objectives	Lesson Assessment	Level of Reform
Ms. Duncan (Grade 3)	All lessons were whole group for 40m ending with 20m individual practice/ homework review	1. Identify fractional parts. 2. Read a mixed number and use objects and picture to show mixed numbers 3. Observe patterns w/zero and practice finding products mentally. 4. Discern the likelihood of an event and use fractions to represent the probability. 5. Explain and review ways to multiply	1. Practice creating and translating images of mixed numbers. 2. Practice creating and translating images of mixed numbers. 3. Construct, observe, and practice patterns of multiples of 10, 100, 1000 4. Practice using terminology connected to probability using single digit numbers 5. Volunteer strategies to solve multiplication problems that are not memorized.	1
Ms. Quinn (Grade 4)	All lessons began with teacher instruction in whole group, moved to small group and/or individual work, and ended most times with whole group sharing	1.Develop strategies for multiplying using arrays to model multiplication 2. Develop strategies for multiplying using array to model multiplication 3. Find fractional parts of a rectangle and interpret meaning of numerator and denominator	1. Can student record correctly with an equation? 2. Can students identify the larger array? Can students break apart the larger array? Can student write an equation? Can student record their matches? 3. Can students find ¼ of a rectangle? Can students identify ¾ of a rectangle? Can student explain how they know its ¼?	2
Ms. Arielle (Grade 1)	All lessons began with whole group instruction, problem solving, sharing; moved to small group/pair work; ended with whole group sharing.	1. Develop strategies to add single digit numbers. Students will model the action in a subtraction (removal) situation 2. Students develop strategies for solving addition and subtraction problems. Students will develop strategies for recording solutions using standard notation (=, +, -). 3. Students will develop fluency with2-addend combinations of 10. Students will solve problems that total where one part is known and use addition notation (= +). 4. Develop strategies to organize objects to count more efficiently; Counting by 10s 5. Students think about numbers to 20 and how they relate to 10; students will determine equivalent expressions.	1 Observe whether students can count on to combine at least 2 single digit numbers; observe if students can determine the problem a given roll represents; observe the different tools and strategies students use. 2. Observe if students can count on or count back; observe if students use numerical reasoning like taking numbers apart into useful chunks; can students interpret when it is an addition or subtraction problem and show how they solved it? 3. Observe how students find combinations of 10; what addition strategies do they use? How do students determine that the game is over? Observe if they use addition notation. 4. Observe how students find the total rolled; how do students figure out how many more they need to complete a row; how do they? How many cubes they have? 5. Observe how students add – do they count all? Do they figure out where to record a given equation? Are students accurate in their use of notation?	3

APPENDIX H

PARTICIPANT INFORMED CONSENT

CONSENT FORM FOR VOLUNTARY PARTICIPATION

I volunteer to participate in this qualitative study and understand that:

1. I will participate in one focus group interview and one individual interview conducted by Mary Grassetti using a semi-structured interview format.

2. The questions I will be answering address my views on issues related to mathematics teaching and learning. I understand that the primary purpose of this research is to identify the issues and challenges that novice teachers face as they develop a mathematical teaching practice based on reform initiatives.

3. The interviews will be tape recorded to facilitate analysis of the data.

4. I will be observed and videotaped teaching mathematics lessons during the fall of 2008. Videotaping will facilitate analysis of the data.

5. I understand that Mary Grassetti will analyze the tapes.

6. I understand that I will provide Mary Grassetti with lesson plans for each lesson videotaped at least 24 hours before the video-taped observation.

7. I understand that I will reflect on one of my videotaped lessons and submit that reflection to Mary Grassetti.

8. I understand that I will take pre- and posttest assessing mathematical knowledge for teaching.

9. My name will not be used, nor will I be identified personally, in any way or at any time.

10. I understand that it will be necessary to identify participants in the study by general position and school district (e.g., a fifth grade teacher from an urban school district said …).

11. I may withdraw from part or all of this study at any time without consequence.

12. I have the right to review material prior to the oral exam or other publication.

13. I understand that the results from this study may be included in Mary Grassetti's doctoral dissertation and may also be included in manuscripts submitted to professional journals for publication.

14. I am free to participate or not to participate without prejudice.

______________________________ ______________________

Participant's Signature Date

______________________________ ______________________

Researcher's Signature Date

APPENDIX I

CAREGIVER INFORMED CONSENT ENGLISH VERSION

Dear Parents/Caregivers:

My name is Mary Grassetti, and I am a student at the University of Massachusetts Amherst. I am working on a research study investigating the different ways that teachers go about teaching mathematics to elementary school students. Your child's teacher has agreed to participate in my research study.

As a participant in the study, Ms. __________has agreed to allow me to videotape her while she is teaching mathematics. Although the focus of my research is on Ms. _______, your child may be in the video footage that I collect. The video will be used to analyze Ms. _________ teaching practice and some of the footage may be used in my dissertation presentation.

Please check off if you do or do not give permission, sign and date the form, and return to Ms._________ as soon as possible. If you have any questions, please do not hesitate to contact me.

Sincerely,

Mary T. Grassetti
(413) 531-5655
mgrassetti@educ.umass.edu

Please check one and return to your child's teacher.

_____ My child ,___________________, has permission to be videotape for this research project.
(Name)

_____ My child ,___________________, does not have permission to be videotaped.

Parent/Caregiver Signature: ____

Date: ____

APPENDIX J

CAREGIVER INFORMED CONSENT SPANISH VERSION

Estimado Padre/ Encargado:

Mi nombre es Mary Grassetti y soy una estudiante en la Universidad de Massachussets, Amherst. Estoy trabajando en un estudio de investigación sobre las maneras diferentes que maestros enseñan matemáticas a estudiantes de escuelas primaria. La maestra de su niño/a, la maestra ________, ha concordado en participar en mi estudio de investigación.

Como participante en el estudio, la maestra ________ ha concordado en permitirme grabarla mientras ella enseña matemáticas. Aunque el foco de mi investigación es en la maestra __________, su niño/a puede salir en la grabación. El video será utilizado para analizar la práctica educacional de la maestra ________y parte del video puede ser utilizado en mi presentación de la disertación.

Favor de marcar si usted le da permiso a su niño, firme su nombre y el día, y devuelve este permiso a la Sra. __________ lo mas pronto posible. Si usted tiene cualquier pregunta, por favor no vacile en contactarme.

Sinceramente,

Mary T. Grassetti
(413) 531-5655
mgrassetti@educ.umass.edu

Marque uno y se lo devuelve a la maestra de su niño.

_____ Mi niño/a, ___________________, tiene el permiso para salir en la grabación
Nombre de su hijo/a
para este proyecto de investigación.

_____ Mi niño/a, ___________________, no tiene el permiso para ser grabado en video.
Nombre de su hijo/a.

La firma del Padre/Encargado: ____________________

Fecha: _______________________

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