



University of  
Massachusetts  
Amherst

## What Happens When Demand Is Estimated with a Misspecified Model?

Item Type	article
Authors	Huang, DL;Rojas, C;Bass, F
Download date	2026-05-17 02:33:23
Link to Item	<a href="https://hdl.handle.net/20.500.14394/42658">https://hdl.handle.net/20.500.14394/42658</a>

# WHAT HAPPENS WHEN DEMAND IS ESTIMATED WITH A MISSPECIFIED MODEL?\*

DONGLING HUANG<sup>†</sup>

CHRISTIAN ROJAS<sup>‡</sup>

FRANK BASS<sup>§</sup>

We conduct Monte Carlo experiments to investigate the biases of assuming a misspecified demand model. We study continuous models (linear, log-linear and AIDS), and discrete choice models (logit) in the context of differentiated products and aggregate data. Estimating demand with the ‘wrong’ model yields varying degrees of bias in estimated elasticities, but the logit model can yield unbiased estimates for a certain size of the assumed market potential. Merger simulations confirm the key importance of market potential in logit estimation suggesting that a discrete choice model may be preferable even when the discreteness of the purchase decision is questionable.

## I. INTRODUCTION

EMPIRICAL ANALYSES OF MARKETS are increasingly becoming more tightly linked to theory. This structural approach to estimation often requires demand and supply estimates, and these estimates need to be precise if one wants inference to be reliable. While markets are comprised of demand and supply, demand estimation has become the main focus of empirical analyses. One reason for a greater interest in demand is that supply side data is less commonly available, especially at the product-level. In addition, advances have been made to make supply inference feasible with unobserved cost data (Bresnahan [1989]).

Demand estimates are the key ingredient in empirical analyses such as merger simulation and product introductions, yet the biases that may arise from employing the incorrect demand model have received little attention. Our analysis focuses on two strands of demand models that have become

\*We thank the Editor and an anonymous referee for helpful comments and suggestions. Huang would like to thank Yexiao Xu for many helpful discussions, and seminar participants at the University of Texas at Dallas. The usual disclaimer applies.

<sup>†</sup>Authors’ affiliations: The Lally School of Management & Technology, Rensselaer Polytechnic Institute, Troy, New York, 12180-3590, U.S.A.  
*email: huangd3@rpi.edu*

<sup>‡</sup>Department of Resource Economics, University of Massachusetts Amherst, 219A Stockbridge Hall, Amherst, Massachusetts 01003, U.S.A.  
*email: rojas@resecon.umass.edu*

<sup>§</sup>(Deceased). School of Management, University of Texas at Dallas, Texas, U.S.A.

popular in empirical studies of differentiated products: discrete choice models and representative consumer (also called continuous choice) models. Discrete choice models assume that consumers choose one unit of the brand that yields the highest utility. Continuous choice models, on the other hand, do not impose such restriction.<sup>1</sup>

While continuous choice models have a longer history in applied work, discrete choice models are now the more popular choice because of their parsimony in dealing with many own- and cross-price coefficients that typically arise with brand level data.<sup>2</sup> However, recent advances now allow researchers to reduce the number of parameters in continuous choice models to a manageable level (Pinkse, Slade and Brett [2002]; Rojas [2008]). We focus on *aggregate* data models (the econometrician only observes total quantity purchased in a market) because it is here that the purchase decision assumption is less clear to the researcher (i.e., the discreteness of the process may be verified with micro-level data).

In this paper, we use a simple controlled environment to study the biases that arise when demand is estimated with a misspecified model, especially when the assumed purchase decision is incorrect. We focus on the bias of the primary structural parameters of interest, price-elasticities, and study the implications of such biases in merger simulation. We consider four demand models that have been popular in empirical applications; three continuous choice models: linear, log-linear and the Almost Ideal Demand System (Deaton and Muellbauer [1980]) and one discrete choice model: logit. Table I presents a list of recent studies that have used these four types of functional forms (typically in more complex variations than the ones presented here) to model demand with aggregate data. We believe log-linear is an unpopular model partly because it implies a (restrictive) constant elasticity. In our simulations below, log-linear also produces results with particular patterns.

Our general approach is to generate equilibrium price and quantity data for four types of duopoly markets comprised of stochastic demand and supply (i.e., the error term and exogenous covariates come from a distribution). All four duopoly markets have the same cost function but a different demand specification, each corresponding to one of the four models we are interested in. The demand parameters of each market are calibrated so that own- and cross-price elasticities at the equilibrium prices and quantities are the same across the four duopolies. We consider several cases of own- and cross-price elasticities to assess the sensitivity of our results

<sup>1</sup> Discrete and continuous choice models differ in other dimensions but we shall loosely distinguish the difference between these two types of models according to their assumption about the consumer's purchase decision.

<sup>2</sup> Other advantages of discrete choice models are: a) consumer heterogeneity can be modeled in the most flexible form of discrete choice models (i.e., random coefficients), and 2) product introductions can be studied more easily.

TABLE I  
RECENT APPLIED DEMAND ANALYSES WITH DIFFERENT FUNCTIONAL FORMS

Author(s)	Year	Demand Functional Form
Porter	1983	Log-Linear
Gasmi, Laffont and Vuong	1992	Linear
Hausman, Leonard and Zona	1994	AIDS
Berry, Levinsohn and Pakes	1995	Logit
Hausman	1996	AIDS
Kadiyali, Vilcassim and Chintagunta	1996	Linear
Besanko, Gupta and Jain	1998	Logit
Kadiyali, Chintagunta and Vilcassim	2000	Linear
Nevo	2001	Logit
Sudhir	2001a	Logit
Sudhir	2001b	Logit
Hausman and Leonard	2002	AIDS
Chintagunta, Dube and Singh	2003	Logit
Besanko, Dube and Gupta	2003	Logit
Slade	2004	Linear
Dube and Manchanda	2005	Linear
Khan and Jain	2005	Logit
Chintagunta and Dube	2005	Logit
Rojas	2006	AIDS

to different parameterizations. We then use the generated data to consistently estimate demand with both the true and the other three misspecified models and analyze the biases defined by the difference between the estimated elasticities and the elasticities at the true parameters. We finally study the implications of demand misspecification on the accuracy of simulated post-merger equilibria.

We acknowledge the fact that the choice of demand functional form is typically driven by the application at hand and also by computational limitations, but the researcher often faces some flexibility in functional form choice. Our results should hence be useful to improve such choice for cases in which the researchers *can* select a model. Also, our results can illuminate the potential biases of prior applications in which the choice of demand functional form may be questionable: e.g., discrete choice models have been employed to study markets where it is very likely that consumers may purchase multiple units of a particular brand or multiple brands in the same shopping trip (e.g., breakfast cereals, beer, soft drinks).

In general, any type of misspecification will yield unreliable estimates and thus our results could be somewhat anticipated. However, in addition to the arguments given in the last paragraph, our approach to studying misspecification is important for at least two reasons. Quantifying the direction and magnitude of biases via simulation is important because an analytical method is not always capable of determining when biases are important. Our simulations indeed support this argument by showing not only when misspecification biases arise but also in which cases they can be reduced or eliminated. Second, our focus is on *structural* misspecification rather than statistical misspecification, which means that the parameters of

interest have an economic interpretation: this is important because applications such as merger simulation rely on having precise structural (instead of reduced-form) estimates. To account for the structural nature of markets, we adopt a stochastic environment where equilibrium price and quantity are determined by demand and supply and we estimate the economic parameters via instrumental variables.

An important reason why researchers have devoted important efforts to model demand for differentiated products has been the challenging issue of estimating numerous substitution parameters when there are many brands (Reiss and Wolak [2007]). Given these advances, the reader may thus deem our analysis of two products as a restrictive case, hence we must address this potential criticism. First, restricting the analysis to two products reduces the computational burden and eliminates other confounds in our experiment. For example, extending the experiment to three products implies calibrating a  $3 \times 3$  matrix of elasticities in each model which proved to be a non-trivial task. In addition, with more than two products, the IIA property of logit becomes a restriction that needs to be addressed with a model such as random coefficients. We attempted such simulations with some success but they raised a variety of issues that may be better addressed in a separate paper as they would distract from the central results of our simulations here.<sup>3</sup> Importantly, we have reason to believe that some of our main results here may extend to more complex models and to markets with more than two products. For example, our finding (below) that assuming a precise market potential for logit is crucial in the two product case appears to be just as important in some preliminary simulations with a 3-product random coefficients logit.

Our results indicate that biases in estimates of own- and cross-price elasticities are typically largest when ‘continuous choice’ models (linear, log-linear, AIDS) are used to estimate the discrete choice model (logit), and vice versa.<sup>4</sup> However, a pattern in the latter misspecification suggests that logit has an advantage over continuous choice models: when misspecified, logit’s

<sup>3</sup> Some of these issues are: 1) if data is generated with a random coefficients model and then estimated with the same model (i.e., no misspecification), frequent convergence problems arise if a different set of random draws (than the ones used to generate the data) are employed, 2) as the number of random draws increases (which would theoretically give more precise estimates) convergence becomes more problematic, 3) which technique for generating random draws is more desirable to solve issues 1) and 2).

<sup>4</sup> Misspecifications only produce ‘magnitude’ biases but no ‘sign’ biases (i.e., estimated own-price elasticities are positive and estimated cross-price elasticities are negative). The absence of sign biases is not general, however, as logit (and more complex variations of it – nested logit, random coefficients logit) always forces cross-price elasticities to be positive, even if the simulation assumed product complementarity. In addition, in section 4 we discuss a case when a ‘sign’ bias can arise when representative consumer models are used to estimate data generated by a logit model.

biases in own- and cross-price elasticities increase as the assumed market potential gets large or small but disappear as the assumed market potential approaches a specific value.<sup>5</sup> Intuitively, market potential gives the logit model a degree of freedom that allows it to approximate elasticities of models that do not assume a discrete purchase decision. However, this advantage can also play against logit when the econometrician only observes aggregate quantity data: if the assumed market potential is incorrect, logit may fail to recover the true own- and cross-price elasticities even when correctly specified.

Continuous choice models, when logit is the true model, produce own- and cross-price elasticities that are usually biased (20–30% larger or smaller than the true logit elasticity). However, there is a large variation in the estimated elasticities across simulations which renders all cross-price elasticities and some own-price elasticities statistically insignificant. As opposed to the logit model, continuous choice models estimates are not biased when correctly specified or even when the wrong continuous choice model (e.g., AIDS used to estimate linear) is employed.

We consider two types of merger simulations: a) post-merger prices, and b) the (reduced) marginal cost needed to keep post-merger prices at the pre-merger levels (also called ‘compensating marginal’ cost; Werden [1996]). In post-merger price simulations we find that in the presence of misspecification, logit appears to be a reasonably accurate model whereas continuous choice models tend to perform less accurately. Compensating marginal cost simulations are consistent with elasticity results: logit’s ability to predict correctly the true marginal cost reduction crucially depends on the assumed market potential, and continuous choice models tend to perform poorly when data is generated by logit.

The next sections present a brief review of related literature, the model and methods used, and a detailed description of the misspecification biases. We also discuss the economic intuition behind our results and the implications of the identified misspecification patterns for merger simulations and for applied demand research in general.

## II. PRIOR LITERATURE

Our work contributes to the broad literature on functional form misspecification which has shown that biases arise when the wrong functional form is assumed (e.g., White [1980]). Since our specific interest is in biases that arise in structural estimation of demand for differentiated products and the economic importance of these biases in merger simulations, we focus on the most relevant literature in these fields.

<sup>5</sup> This value is not the same for own- and cross-price elasticities, but usually close.

The usual practice when simulating post-merger equilibria is first to estimate demand with a particular functional form and recover (usually constant) marginal costs from firms' first order conditions in a Bertrand-Nash game. Post-merger prices are then calculated using the recovered marginal costs and the demand estimates. Applications of this type to particular industries include Hausman, Leonard and Zona [1994], Hausman and Leonard [1997], Nevo [2000] and Werden [2000]. These studies, however, typically do not explore the sensitivity of their results to other plausible demand specifications, which is one of the focuses of our study.

Some papers have begun to address the accuracy of different demand functional forms in predicting the *actual* post merger prices in industries for which pre and postmerger prices are available (Peters [2006]; Weinberg [2008]), and have, in general, found that merger simulations do not accurately predict the observed post merger prices.

The most related study to our paper is that of Crooke *et al.* [1999], as they also conduct Monte Carlo experiments to analyze how four different demand functional forms (the same ones we are considering here) give rise to different prices and elasticities after a merger occurs. Crooke *et al.* employ a setup that is similar to ours: oligopoly price competition with differentiated products and constant marginal cost. The authors set the pre-merger equilibrium prices, quantities and elasticities equal across models and then search for the post-merger prices and elasticities. Crooke *et al.* conclude that post-merger prices and elasticities depend heavily on the assumed functional form. As opposed to our work, however, Crooke *et al.* assume that equilibrium price and quantity are given deterministically (rather than stochastically) and that the researcher knows with certainty the parameters of the true demand model. Hence, our approach is different to that of Crooke *et al.* in two fundamental ways: a) we adopt an econometric methodology (i.e., we consistently estimate the structural demand parameters) and b) we analyze the consequences of structural misspecification (i.e., using the wrong demand functional form).

### III. THE MODEL

We adopt the general approach of recent empirical work of assuming price competition with differentiated products (e.g., Berry, Levinsohn and Pakes [1995]; Pinkse, Slade and Brett [2002]) and focus on a duopoly with single-product firms. Because our approach is econometric in nature, we define all equations with a stochastic error term.

#### III(i). Demand

*Logit Demand.* Consumer  $i$ 's utility of choosing brand  $j$  is given by:

$$u_{ij} = \beta_0 + \beta_x x_j - \alpha p_j + \zeta_j + \varepsilon_{ij}$$

where  $x$  is the observed product characteristic,  $p$  is price,  $\xi$  is the unobserved product characteristic<sup>6</sup>,  $\varepsilon$  is a stochastic term representing consumer  $i$ 's idiosyncratic utility component. We adopt Berry's [1994] approach of including the unobserved (to the econometrician) product characteristic in the utility function, which is correlated with the equilibrium price. The consumer has the option of not purchasing either good; the utility of the outside good is denoted  $u_{0i}$  and is normalized to a constant: 0. If  $\varepsilon_{ij}$  follows a type I extreme value distribution, the probability of consumer  $i$  choosing brand  $j$  (since we assume homogeneous consumers, we drop the  $i$  subscript) is:

$$\Pr_j = \frac{\exp(\beta_0 + \beta_x x_j - \alpha p_j + \xi_j)}{1 + \sum_{k=1}^2 \exp(\beta_0 + \beta_x x_k - \alpha p_k + \xi_k)} = s_j$$

where  $s_j$  denotes the quantity ( $q$ ) share of good  $j$  (i.e.,  $s_j = q_j / \sum_k q_k$ ); the second equality holds if each consumer buys one unit. Elasticities and price derivatives are defined as:

$$\mu_{jk} = \begin{cases} -\alpha p_k (1 - s_j) & \text{if } j = k \\ \alpha p_k s_k & \text{if } j \neq k \end{cases};$$

$$\frac{\partial s_j}{\partial p_k} = \begin{cases} -\alpha (1 - s_j) s_j & \text{if } j = k \\ \alpha s_k s_j & \text{if } j \neq k \end{cases}$$

For estimation purposes, we use Berry's transformed version of the market share equation:<sup>7</sup>

$$\ln s_j - \ln s_0 = \beta_0 + \beta_x x_j - \alpha p_j + \xi_j$$

Where  $s_0$  is the market share of the outside alternative of not buying any of the inside goods. It is important to note that the size of  $s_0$  indirectly determines own- and cross-price elasticities.

The term  $s_0$  is directly determined by the size of the normalized utility for the outside good,  $u_0$  (with a larger  $u_0$  implying a larger  $s_0$ ), and indirectly determined by the size of the constant term in the utility of the inside goods,  $\beta_0$  (with a larger  $\beta_0$  implying a smaller  $s_0$ ). While in the simulations below we calibrate  $s_0$  (and other parameters) so that elasticities are equal across demand specifications, the stochastic nature of demand implies that  $s_0$ ,  $s_1$  and  $s_2$  will vary in each simulation.

<sup>6</sup>We adopt the case of a coefficient equal to one for the unobserved product characteristic. Alternatively, one can specify  $\xi$  to have a coefficient. Our results are not sensitive to this assumption.

<sup>7</sup>This is the strategy adopted by most studies (e.g., Berry [1994]).

*Linear and Log-Linear Demand* The system of two demand equations is:

$$q_j = a_j + \sum_k b_{jk} p_k + \beta_x x_j + \xi_j, \quad j = 1, 2$$

where  $x$ , as in the logit case, can be thought of as a product characteristic or, more naturally, as a demand shifter and  $\xi$  is an unobserved demand shock which is analogous to the unobserved product characteristic of the logit model (i.e., it is correlated with price because of simultaneity). The log-linear demand system is identical to the linear except that quantity and price are replaced by their 'log' values. Elasticities and price derivatives are straightforward in these two cases.

*Almost Ideal Demand System (AIDS)* As with the logit model, there are two commodities (1 and 2) and an outside good (0). The AIDS model is defined in 'sales share' form ( $w_j$ ):

$$w_j = \alpha_j + \sum_k \gamma_{jk} \log(p_k) + \beta_j \log(X/P) + \xi_j$$

where :

$$\log P = \bar{\alpha} + \sum_l \alpha_l \log(p_l) + \frac{1}{2} \sum_l \sum_m \gamma_{lm} \log(p_l) \log(p_m)$$

$$w_j = \frac{p_j q_j}{X}; \quad \alpha_j = \tilde{\alpha}_j + \beta_x x_j; \quad X = \sum_k p_k q_k; \quad j, k, l, m \in (0, 1, 2)$$

We impose the theoretical restrictions of homogeneity and adding up:  $\sum_j \alpha_j = 1$ ;  $\sum_j \gamma_{jk} = \sum_k \gamma_{jk} = \sum_j \beta_j = 0$ ; and symmetry:  $\gamma_{jk} = \gamma_{kj}$ . The parameters of the outside good are defined according to the theoretical restrictions:  $\alpha_0 = 1 - \alpha_2 - \alpha_1$ ,  $\beta_0 = -\beta_2 - \beta_1$ ,  $\gamma_{10} = \gamma_{01} = -\gamma_{11} - \gamma_{12}$ ,  $\gamma_{20} = \gamma_{02} = -\gamma_{12} - \gamma_{22}$ ,  $\gamma_{00} = -\gamma_{01} - \gamma_{02} = -\gamma_{10} - \gamma_{20}$ .

Strategic interaction takes place between goods 1 and 2 and the price of the outside good is assumed to be fixed. To simplify computation, and without loss of generality, we set  $\bar{\alpha} = 1$ ,  $p_0 = 1$  and hold  $X$  constant. Our three demand equations are thus defined as:

$$q_j = \frac{(\alpha_j^* - \beta_j^*) + \sum_{k=1}^2 (\gamma_{jk}^* - \beta_j^* \alpha_k) \log(p_k) + \beta_j^* \log X - \frac{\beta_j^*}{2} \bar{P} + \xi_j^*}{p_j}$$

where  $\bar{P} = \gamma_{11}^* [\log(p_1)]^2 + \gamma_{12}^* \log(p_1) \log(p_2) + \gamma_{22}^* [\log(p_2)]^2$  and the superscript '\*\*' indicates that the original parameter has been rescaled by  $X$ . Elasticities and price derivatives are:

$$\mu_{jk} = \frac{1}{p_j q_j} \left[ \gamma_{jk}^* - \beta_j^* \left( \alpha_k + \sum_l \gamma_{kl} \log p_l \right) \right] - \delta_{jk}$$

$$\frac{\partial q_j}{\partial p_k} = \frac{1}{p_j p_k} \left[ \gamma_{jk}^* - \beta_j^* \left( \alpha_k + \sum_l \gamma_{kl} \log p_l \right) \right] - \delta_{jk} \frac{q_j}{p_j}$$

where,  $\delta_{jk} = \begin{cases} 1 & \text{if } j = k \\ 0 & \text{otherwise} \end{cases}$

The assumed ‘dollar’ market potential (i.e.  $X$ ) only enters elasticities as a rescaling factor for the estimated coefficients ( $\gamma_{jk}^*, \beta_j^*$ ). This is in contrast to the logit model where the non-linear specification does not allow different market potentials to be ‘absorbed’ via parameter rescaling.

### III(ii). *Supply*

Firm  $j$ ’s profit function is defined as:

$$\begin{aligned} \pi_j &= (p_j - mc_j)q_j - F_j \\ &= (p_j - mc_j)s_j \cdot M - F_j \end{aligned}$$

where  $\pi$  denotes profit,  $mc$  is marginal cost  $q$  is quantity,  $F$  denotes fixed cost, and  $M$  is the market potential for this duopoly market ( $M = \sum_{k=0}^2 q_k$ ). We assume that marginal cost is independent of quantity and employ Berry’s specification:

$$mc_j = e^{\gamma_0 + \gamma_x x_j + \gamma_\omega \omega_j + \sigma_c \xi_j + \sigma_\eta \eta_j}$$

where,  $x$  is the observed product characteristic,  $\omega$  is a cost shifter,  $\xi$  is the unobserved product characteristic,  $\sigma_c$  is the standard deviation of  $\xi$ ,  $\eta$  is the supply shock and  $\sigma_\eta$  is the standard deviation for  $\eta$ .

We also assume a static setting in which each firm maximizes its profit in each time period (we have omitted the time subscript for simplicity). Firm  $j$ ’s first order condition is:

$$(p_j - mc_j) \frac{\partial s_j}{\partial p_j} + s_j = 0$$

After replacing marginal cost gives the following supply equation in logarithmic form:

$$\log p_j = \log \left[ \left( \left| \frac{\partial s_j}{\partial p_j} \right| \right)^{-1} s_j \right] + \gamma_0 + \gamma_x x_j + \gamma_\omega \omega_j + \sigma_c \xi_j + \sigma_\eta \eta_j$$

## IV. MONTE CARLO EXPERIMENTS

### IV(i). *Data Generation*

For each of the four duopolies, we randomly draw 500 values for  $x$ ,  $\omega$ ,  $\xi$ , and  $\eta$  from a standard normal distribution. The marginal cost parameters are the same across the four duopolies:  $\gamma_0 = 1$ ,  $\gamma_x = 0.5$ , and  $\gamma_\omega = \sigma_c = \sigma_\eta = 0.25$  whereas the constants and the price coefficients of each demand model are calibrated so that the elasticities at the equilibrium prices and quantities are as close as possible across the four duopolies.<sup>8</sup>

We analyze symmetric duopolies and study three elasticity cases: medium own-price elasticity ( $\sim -1.70$ ) with medium cross-price elasticity ( $\sim -1.40$ ) [M/M]; medium own-price elasticity ( $\sim -2.00$ ) with low cross-price elasticity ( $\sim -0.60$ ) [M/L]; and high own-price elasticity ( $\sim -3.00$ ) with low cross-price elasticity ( $\sim -0.60$ ) [H/L].<sup>9</sup> The M/M case corresponds to one of the two cases in Berry's Monte Carlo experiment, which we consider our baseline.<sup>10</sup> The other two cases reflect values that are commonly found in concentrated oligopolies (see Crooke *et al.*).<sup>11</sup>

For each of the 500 draws (a 'data set'), we compute the equilibrium prices and quantities. The linear case has an analytical solution whereas the other three cases require numerical methods. We do this exercise 100 times for each elasticity case and each demand specification and take these data to estimation. There are, in total, 12 different cases (4 demand specifications  $\times$  3 elasticity cases), each with 50,000 data points (100 data sets of 500 observations each).

### IV(ii). *Estimation*

For each of the 100 data sets in each of the 12 combinations of demand specifications and elasticities, we compute four sets of structural parameters and elasticities, one with the true model and the other three with misspecified models. Elasticities are computed at the mean values of the 500 equilibrium prices and quantities of each data set.

<sup>8</sup> Because of their unit-free nature, we measure the misspecification bias in terms of elasticities.

<sup>9</sup> A recent empirical generalization of price elasticities (Bijmolt *et al.*, [2005]) found an average own-price elasticity of  $-2.62$  based on 1,851 elasticities documented in published research. Our definition of medium and low own-price elasticity is somewhat consistent with this finding. We are not aware of similar work on cross-price elasticities.

<sup>10</sup> Berry considers two cases in his simulation exercises, one with a coefficient of one for the  $\xi_j$  term in the indirect utility function and the other with a coefficient of three. Our logit model corresponds to Berry's first case. We were able to replicate Berry's results successfully for both cases.

<sup>11</sup> Our cross-price elasticity is larger than the mean cross-price elasticity that Crooke *et al.* obtain in their analysis of mergers in oligopolies composed of 4 (to 8) firms. The reason for our choice is that a duopoly is more likely to have a larger cross-price elasticity than an oligopoly with three or more firms.

Note that with aggregate data (the focus of this paper), only quantities of the inside goods ( $q_j$ ) are observed. Thus, market shares needed for the estimation of logit demand can only be defined after assuming a ‘market potential’  $M$  (i.e., the size of the market if all consumers chose an inside option). Put differently, when using the logit specification, the size of the outside alternative,  $s_0$ , is unknown; this is crucial as  $s_0$  indirectly determines elasticities in the logit model (see section 3.1). The usual approach to solving this problem is to assume a *single* value for  $M > q_1 + q_2$  and then to compute:  $s_j = \frac{q_j}{M}$  and  $s_0 = 1 - \sum_j s_j$ , but prior work usually provides little discussion as to the sensitivity of results to different market potential sizes. To investigate this issue, estimation with logit (both with and without misspecification) is carried out using numerous market potential sizes. In order to accommodate all 500 observations in each data set and given the lack of a universal procedure for computing the market potential, we define 100 market potentials in the estimation of each data set as:  $M_r = [\max(q_{1t} + q_{2t})]/r$ , where  $t = 1, \dots, 500$  and  $r = (0.01, 0.02, \dots, 0.99, 1)$ .<sup>12,13</sup>

It is important to point out that there is no theoretical counterpart in the data generation processes for the parameter ‘ $r$ ’. Rather we use it as an ad-hoc way to define various market potentials ( $M$ ). More importantly, there is no theoretical counterpart for  $M$  in continuous choice models as the data generating process only produces  $q_1$  and  $q_2$  (not an outside option), but an  $M$  needs to be assumed to proceed with logit estimation. Although in the case of logit generated data there is a ‘true’ market potential that comes in the form of the outside good share ( $s_0$ ), it varies with each observation because of the stochastic nature of the market (see section 3.1). Hence, even in a logit generated data set there is no *single* theoretical  $M$ .

We use two-stage least squares where the instruments include the own- and rival-cost shifter ( $\omega$ ) as well as the rival’s product characteristic ( $x$ ). When the data is generated by logit and estimated by continuous choice models, however, we exclude the rival’s product characteristic as an instrument. The reason for this exclusion is that logit’s particular functional form forces the rival’s product characteristic to be an omitted variable in the error term specified in the continuous choice models. This, in turn, violates the orthogonality condition required for a valid instrumental variable. If the

<sup>12</sup> When data is generated by logit, the formula employed is  $M_r = \left[ \max_t ((s_{1t} + s_{2t})k) \right] / r$ , where  $k$  is any scalar. We set  $k = 50$  because it is consistent with the market potential sizes observed in the other demand specifications.

<sup>13</sup> Simulated data have a broader range of values than real data because large random draws are inevitable. Hence, the use of this ad-hoc formula with real data will imply a smaller range of market sizes.

rival's product characteristic is included as an instrument, it can produce negative cross-price coefficients, a 'sign' bias.<sup>14</sup>

#### IV(iii). *Merger Analysis*

The merged firm has two first order conditions:

$$(1) \quad \sum_{k=1}^2 \left[ (p_k - mc_k) \frac{\partial s_k}{\partial p_j} \right] + s_j = 0, \quad j = 1, 2$$

The derivative term,  $\frac{\partial s_k}{\partial p_j}$ , and  $s_j$  are a function of the demand estimates and prices.<sup>15</sup> Hence, given a set of demand parameters, the unknowns in these two first order conditions are prices. Using the incorrect demand functional form to simulate post-merger prices can produce biased results even when pre-merger elasticities are estimated without bias. Intuitively, this arises because elasticities of different demand specifications react differently to a given price change, and price-elasticities are the main determinant of firms' profit-maximizing behavior (Crooke *et al.* [1999]). This additional 'extrapolation' bias can thus be confounded with the estimation bias that is the focus of our study. We consider an alternative merger simulation metric that is free of this problem (Werden [1996]): the marginal cost needed to keep post-merger prices at the pre-merger level, the 'compensating marginal cost;' this term is denoted  $mc_k^{comp}$  ( $k = 1, 2$ ) and is the solution to the 2-equation system:

$$(2) \quad \sum_{k=1}^2 \left[ (p_k^{pre} - mc_k^{comp}) \frac{\partial s_k}{\partial p_j} \Big|_{p_j=p_j^{pre}, p_k=p_k^{pre}} \right] + s_j(p_j^{pre}, p_k^{pre}) = 0, \quad j = 1, 2$$

Using the demand estimates of the each of the 4 models on each data set, we conduct a search for: a) the prices that would satisfy the two first order conditions in (1) and the marginal costs that would satisfy the two first order conditions in (2).

#### IV(iv). *Computational Details*

We encountered several difficulties in our Monte Carlo experiment. First, we experienced large computational times in the search for equilibrium prices and quantities for the post-merger AIDS case. Also, we occasionally found

<sup>14</sup> Negative cross-price elasticities are not uncommon in continuous choice models (see Hausman, Leonard and Zona [1994]; Hausman [1996[19]]).

<sup>15</sup> The only exception is the linear demand case, which has a derivative that does not depend on price.

unusual data points with very large prices or quantities in the log-linear case and non-convergence points in the AIDS case. As explained earlier, these difficulties are part of the reason why we restricted our analysis to three elasticity cases and a symmetric duopoly.

The M/M elasticity case and the log-linear demand are excluded from merger simulations. The M/M elasticity case often proved to be computationally intractable, which we attribute to the implausibly large cross-price elasticity ( $\sim 1.40$ ). Log-linear demand is also a restrictive specification as it forces elasticities to be identical at any point of the demand curve; this unique functional form creates large price and quantity outliers that are often difficult to deal with.

Also, to reduce computational time, and following Crooke *et al.*, we impose homothetic preferences ( $\beta_i = 0$ ) in the AIDS model.<sup>16</sup> Because of adding up, symmetry and homogeneity,  $q_0$  in the AIDS model is defined by an identity and hence does not need to be solved numerically. It is computed to check that the fixed total expenditures are greater than the combined sales of goods 1 and 2 ( $x > p_1q_1 + p_2q_2$ ).

## V. RESULTS

### V(i). *Elasticities*

Table II reports the means and standard errors of the elasticities across the 100 iterations of the Monte Carlo experiment. Because the duopoly we study is symmetric, we only report firm 1's own- and cross-price elasticities ( $\eta_{11}, \eta_{12}$ ). The data generating models are indicated in the first column of the table. The next columns report the elasticities calculated at the true parameters of each of the models. Note that the true elasticities are not exactly the same across the four demand models; this is because equilibrium prices and quantities are a function of stochastic terms (except for the deterministic log-linear case). The next four sets of columns in the table report the elasticity estimates obtained with each of the four demand models considered.

Recall that logit's estimates depend on the assumed market potential when aggregate quantity data ( $q_1$  and  $q_2$ ) are observed. As a consequence, the results of logit estimation when aggregate data is observed are not reported in column 1; instead figures 1–3 plot the mean logit elasticities against each of the 100 different market potentials considered, each figure corresponding to a demand specification and an elasticity case.<sup>17</sup> The only logit estimates reported in table II correspond to the case when the

<sup>16</sup> We also conducted a smaller number of iterations with  $\beta_i \neq 0$  and found similar results.

<sup>17</sup> In the case of logit demand, figures 1A, 2A and 3A report the elasticities computed with a logit model using aggregate quantity (not shares) data. Footnote 12 reports the procedure used to transform logit generated market shares to aggregate quantities.

TABLE II  
MONTE CARLO ELASTICITY ESTIMATES WITH CORRECT AND MISSPECIFIED MODELS

Data Generation Model	Elasticities at True Parameters						Elasticities of Estimated Model*															
	Cases**						Logit***				Linear				AIDS				Log-linear			
	Elasticity	M/M	M/L	H/L	M/L	H/L	M/M	M/L	H/L	M/L	M/L	H/L	M/M	M/L	H/L	M/M	M/L	H/L	M/M	M/L	H/L	
Logit	$\eta_{11}$ (own)	-1.69	-2.10	-2.99	-2.10	-3.01	-1.72	-2.10	-3.01	-1.22	-1.94	-2.49	-1.23	-2.55	-3.29	-1.29	-2.12	-3.24	-1.29	-2.12	-3.24	
		0.02	0.03	0.06	0.28	0.20	0.44	0.28	0.20	4.04	0.45	0.30	5.96	0.58	0.39	4.24	0.57	0.42	4.24	0.57	0.42	
	$\eta_{12}$ (cross)	1.41	0.64	0.60	0.64	0.60	1.44	0.64	0.60	0.96	0.68	0.46	0.95	0.87	0.53	1.12	0.84	0.59	1.12	0.84	0.59	
Linear		0.05	0.03	0.02	0.09	0.04	0.38	0.09	0.04	5.93	0.52	0.34	5.28	0.68	0.43	4.51	0.60	0.40	4.51	0.60	0.40	
	$\eta_{11}$ (own)	-1.69	-2.06	-2.99	-2.06	-2.99	-1.67	-2.06	-2.99	-1.67	-2.06	-2.99	-1.70	-2.10	-3.06	-1.69	-2.11	-3.14	-1.69	-2.11	-3.14	
		0.01	0.01	0.03	0.01	0.03	0.14	0.12	0.16	0.14	0.12	0.16	0.15	0.13	0.17	0.15	0.13	0.19	0.15	0.13	0.19	
AIDS	$\eta_{12}$ (cross)	1.43	0.62	0.61	0.62	0.61	1.43	0.62	0.61	1.43	0.62	0.61	1.45	0.63	0.62	1.44	0.63	0.64	1.44	0.63	0.64	
		0.00	0.00	0.00	0.00	0.00	0.08	0.04	0.07	0.08	0.04	0.07	0.08	0.04	0.07	0.08	0.04	0.08	0.08	0.04	0.08	
	$\eta_{11}$ (own)	-1.71	-2.05	-3.02	-2.05	-3.02	-1.71	-2.07	-3.06	-1.71	-2.07	-3.06	-1.71	-2.05	-3.01	-1.74	-2.09	-3.13	-1.74	-2.09	-3.13	
Log-linear		0.01	0.01	0.01	0.01	0.01	0.08	0.09	0.10	0.08	0.09	0.10	0.04	0.06	0.07	0.08	0.06	0.08	0.08	0.06	0.08	
	$\eta_{12}$ (cross)	1.44	0.62	0.62	0.62	0.62	1.51	0.63	0.64	1.51	0.63	0.64	1.44	0.62	0.62	1.56	0.64	0.66	1.56	0.64	0.66	
		0.02	0.00	0.00	0.00	0.00	0.07	0.03	0.04	0.07	0.03	0.04	0.03	0.03	0.03	0.07	0.03	0.03	0.07	0.03	0.03	
Log-linear	$\eta_{11}$ (own)	-1.70	-2.06	-3.00	-2.06	-3.00	-1.70	-2.14	-3.04	-1.70	-2.14	-3.04	-1.81	-2.22	-2.85	-1.75	-2.11	-3.00	-1.75	-2.11	-3.00	
		n/a	n/a	n/a	n/a	n/a	0.39	0.34	0.43	0.39	0.34	0.43	0.57	0.38	0.33	0.26	0.25	0.29	0.26	0.25	0.29	
	$\eta_{12}$ (cross)	1.43	0.62	0.60	0.62	0.60	1.42	0.63	0.60	1.42	0.63	0.60	1.67	0.68	0.56	1.44	0.62	0.60	1.44	0.62	0.60	
	n/a	n/a	n/a	n/a	n/a	0.18	0.17	0.19	0.18	0.17	0.19	0.29	0.20	0.17	0.13	0.13	0.14	0.13	0.13	0.13	0.14	

\*Computed with each model's estimated parameters. Grey cells denote estimated model is the same as data generation model.

\*\*M/M = Medium own-price and medium cross-price elasticities. M/L = Medium own-price and low cross-price elasticities. H/L = High own-price and low cross-price elasticities.

\*\*\*Results in this column assume the true (i.e., simulated) shares (and hence market potential) are observed by the econometrician when data is generated by logit. When the econometrician only observes aggregate quantity data, logit results depend on the assumed market potential; we consider 100 possible market potentials in figures 1 to 3 for this latter case.

Notes: Reported are the mean elasticities across 100 iterations (of 500 random draws each), standard errors are in italics. Standard errors are not computed for the true elasticities in the log-linear model because elasticities are fixed scalars.

econometrician observes market share data ( $s_0$ ,  $s_1$  and  $s_2$ ) and hence the correct market potential is used in every observation. To summarize, the grey cells in table II denote that the true data generating model is used to compute the elasticities (i.e., there is no misspecification).

Table II shows that, as expected, there are no biases when elasticities are estimated with the true model (and the correct market potential in the case of logit). When there is misspecification among the continuous choice models (linear, log-linear and AIDS), the biases are non-existent or small (when the true model is log-linear, AIDS appears to produce results that are slightly different from the true ones). When the true model is logit, the three continuous choice models estimate elasticities with some biases: linear tends to under-predict the magnitude of elasticities (except in the M/M cross-price elasticity case), whereas both AIDS and log-linear under-predict the magnitude of elasticities in the M/M case but tend to over-predict the magnitude of elasticities in the M/L and H/L cases. Importantly, in several of these cases elasticities are estimated with large standard errors, making them statistically insignificant.<sup>18</sup>

Turning to logit elasticity estimates in figures 1–3, we observe several patterns. First, there tends to be a monotonic relationship between market potential size and elasticity magnitudes: own- and cross-price elasticities are smaller in absolute value (i.e., biased toward zero) when market potential is large (small  $r$ ) and larger otherwise. Second, cross-price elasticity biases are more severe than the own-price elasticity bias, except in the M/L and H/L cases with logit-generated data. When there is convergence to the true parameters, both own- and cross-price elasticities are accurately estimated with similar market potential values, and in the case of logit with the same market potential value. Finally, when the difference in the absolute value of own- and cross-price elasticity increases a larger market potential is needed to accurately recover true elasticities.

Figures when data is generated by log-linear (1D, 2D, 3D) and in the M/M case (1A-1C) show somewhat different patterns than other figures. Logit estimates of log-linear data do not show important biases in own-price elasticity estimates but show large cross-price elasticity biases that do not decrease dramatically when market potential decreases (larger  $r$ ).<sup>19</sup> We attribute these patterns to the restrictive constant elasticity structure implied by the log-linear specification. Figures 1A–1C show that estimated own- and

<sup>18</sup> Continuous choice models' estimates are sensitive to outliers (e.g., large quantity data points), but excluding outliers from estimation did not improve their estimates of logits' elasticities.

<sup>19</sup> Because log-linear simulations sometimes produce large equilibrium quantity outliers, the market potentials tend to be larger than in other cases. To reduce this problem, market potentials in figures 1D, 2D, 3D exclude the largest 5% and the smallest 5% simulated quantity data points. We also modified these figures (not shown) by extending the range of  $r$  to 2, but convergence to the true parameters only occurs for the cross-price elasticity in the H/L case.

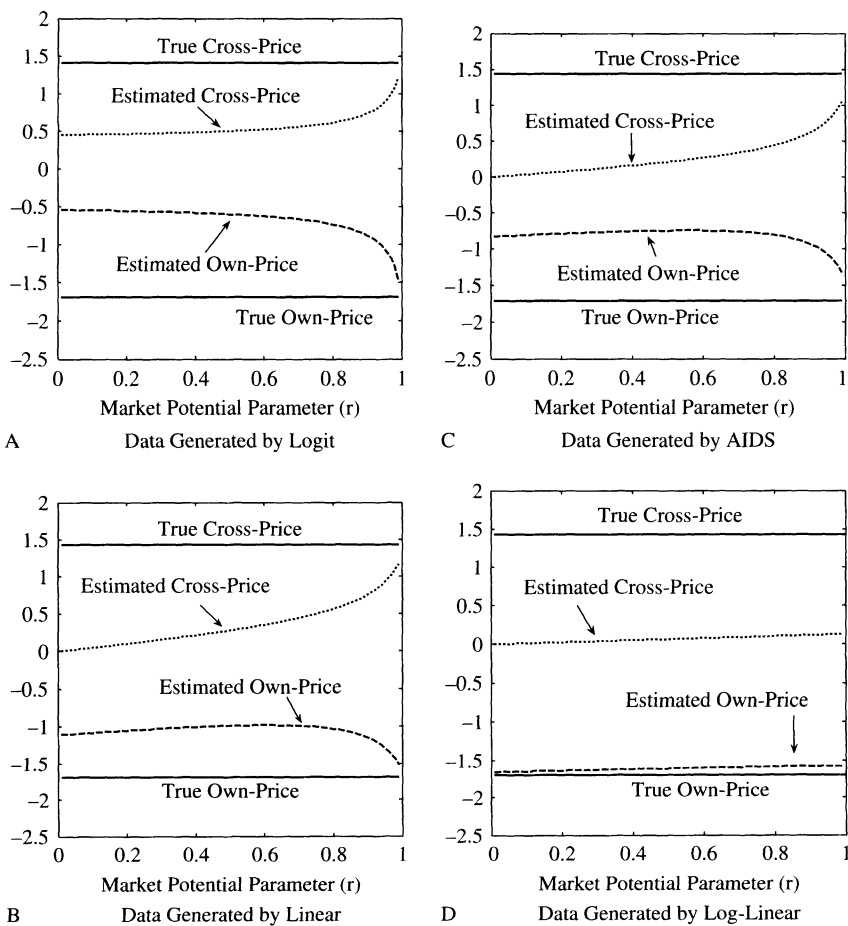


Figure 1

Logit Elasticities for Different Market Potentials, Medium-Medium Elasticity Case

Notes: Solid lines represent true elasticities. A point on the dashed and dotted lines represents the mean of the computed elasticities (own- and cross-, respectively) across the 100 simulated data sets under a given market potential. A larger market potential parameter ( $r$ ) indicates a smaller market potential.

cross-price elasticity estimates rapidly increase in absolute value as  $r$  approaches 1 (market potential becomes smaller) but do not fully converge the true parameters. The M/M case results need to be interpreted with care, however, as this case implies an implausibly large cross-price elasticity which may never be observed in actual markets.<sup>20</sup>

<sup>20</sup> A cross-price elasticity greater than 1 is rarely observed in empirical work; this value implies the unlikely effect that a 10% price increase will result in a larger than 10% increase in quantity sold by the competitor.

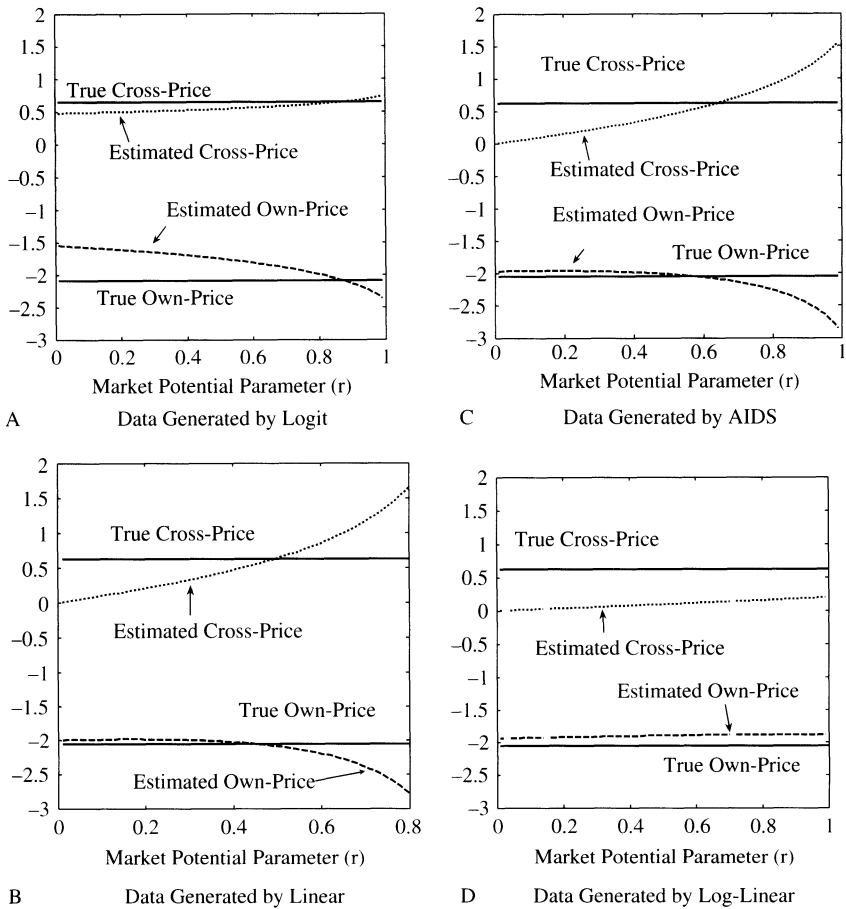


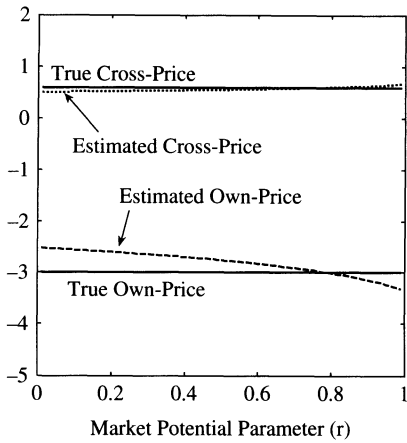
Figure 2

Logit Elasticities For Different Market Potentials, Medium-Low Elasticity Case

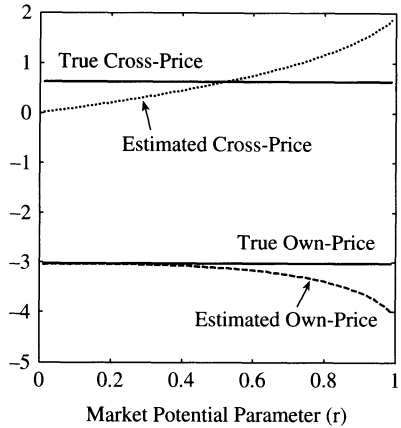
Notes: Solid lines represent true elasticities. A point on the dashed and dotted lines represents the mean of the computed elasticities (own- and cross-, respectively) across the 100 simulated data sets under a given market potential. A larger market potential parameter ( $r$ ) indicates a smaller market potential. The x-axis in the linear case is cut-off at 0.80 to allow for the same range on the y-axis (-3 to 2).

Results of table II and figures 1–3 indicate that, overall, biases are larger when the own- and cross-price elasticities are closer in absolute value (M/M case) and smaller when the own- and cross-price elasticities are further apart in absolute value (H/L case).

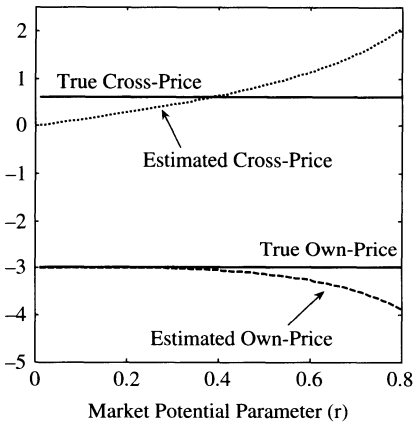
Overall, the results in this section suggest that the logit model *may* be able to recover the true elasticities of data generated by models that do not explicitly assume a discrete purchase decision. However, when there is no misspecification, logit *may* fail to recover the true own- and cross-price



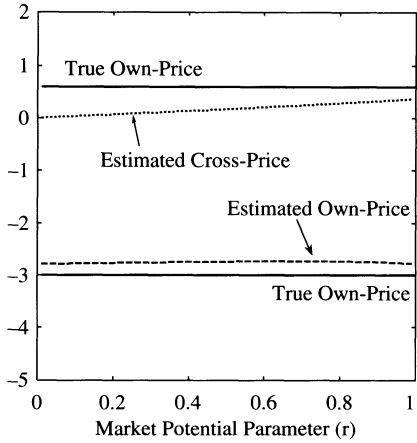
A Data Generated by Logit



C Data Generated by AIDS



B Data Generated by Linear



D Data Generated by Log-Linear

Figure 3

Logit Elasticities for Different Market Potentials, High-Low Elasticity Case

Notes: Solid lines represent true elasticities. A point on the dashed and dotted lines represents the mean of the computed elasticities (own- and cross-, respectively) across the 100 simulated data sets under a given market potential. A larger market potential parameter ( $r$ ) indicates a smaller market potential. The x-axis in the linear case is cut-off at 0.80 to allow for the same range on the y-axis ( $-5$  to  $2$ ).

elasticities while continuous choice models do not suffer from this potential pitfall. One reason for the observed patterns in these simulations is that continuous choice models estimate own- and cross-price elasticities with different parameters (4 in this case, 2 for own- and 2 for cross-price), whereas a single parameter is used in all logit elasticities. Thus, the different

continuous choice models tend to produce similar results among them, but they are different than those of logit.

#### V(ii). *Mergers*

We compare: a) the simulated post-merger equilibrium prices predicted by the true model with those predicted by the misspecified model, and b) the compensating marginal cost predicted by the true model with that predicted by the misspecified model. To focus on model misspecification only, in this section we assume that there is no market potential misspecification when logit is not misspecified (i.e., the econometrician observes the correct market potential when data is generated and estimated by logit).

We report three ‘precision’ statistics for the variable of interest ‘S’, where S is either the post-merger price or the compensating marginal cost. Because of symmetry we only report results for one of the two prices. The first statistic (Accuracy) measures the overall accuracy of the misspecified model in predicting the true ‘S’ and is equal to the median of squared deviations (MSD), where the a deviation is defined by the difference between the S predicted by the correct (*true*) model and the S predicted by the misspecified (*false*) model:  $deviation = (S^{true} - S^{false})$ .<sup>21</sup> The closer this statistic is to zero, the more accurate is the prediction of the misspecified model.

The second statistic looks at the extent to which the S predicted by the misspecified (*false*) model ‘under-predict’ the correct (*true*) S:

$$\% \text{Under} - \text{predicted} = 100 \times \left[ \frac{\sum_i I(S_i^{true} - S_i^{false})}{50,000} \right]$$

where  $I(R)$  is an indicator function that takes a value of 1 if argument  $R > 0$ , and  $i$  indexes each of the total 50,000 S’s (500 observations  $\times$  100 markets).<sup>22</sup> A third statistic measures whether the misspecified model correctly predicts the direction of change in S (positive or negative) predicted by the true model as a result of the merger<sup>23</sup>:

$$\% \text{Correct Change} = 100 \times \left[ \frac{\sum_i G(I(S_i^{true} - S_{pre,i}) - I(S_i^{false} - S_{pre,i}))}{50,000} \right]$$

<sup>21</sup> We use the median instead of the mean to eliminate the sensitivity of the statistic to extreme outliers.

<sup>22</sup> The ‘% over-predicted’ statistic is defined as  $(1 - \% \text{ under-predicted})$ .

<sup>23</sup> Analytically, given our demand and cost parameters, post-merger prices should always be higher than pre-merger prices when there is no misspecification. Since we are dealing with a stochastic environment, there are a few instances where the draws of the random terms are such that a price decrease is observed. This is why we adopt the word ‘change’ instead of ‘increase.’

where  $I(\cdot)$  is defined as before,  $G(R)$  is an indicator function that takes a value of 1 if argument  $R = 0$ .

Simulated pre-merger prices and marginal costs (not shown) across the four demand models are roughly equal and thus accuracy statistics across cases in a given elasticity case are readily comparable. In this section we exclude the log-linear model and the M/M for reasons explained in section 4. Also, to conserve space we only report the results of three market potentials ( $r = 0.25, 0.50, 0.75$ ).

*Post-Merger Prices.* Table III reports the results of the constructed statistics. Consistent with our elasticity findings, overall accuracy is greater in the H/L case, which has own- and cross-price elasticities further apart (in absolute value). When the true model is logit, the linear model is more accurate than AIDS in predicting post-merger prices in the M/L case (smaller MSD) but both continuous choice models predict approximately equally well in the H/L case. Somewhat surprisingly, when the true model is linear, logit's performance as measured by MSD is better than AIDS, except in the H/L case when  $r = 0.75$ , where logit's MSD is twice that of AIDS' but still very small. Also, when the true model is linear, logit and AIDS are equally accurate at predicting the direction of price changes but AIDS always over-predicts post-merger prices whereas logit's % of under-predicted post-merger prices varies with market potential. When the true model is AIDS, linear and logit models appear to do similarly well in predicting post-merger prices: MSD's are not too dissimilar, the direction of price change is almost always correctly predicted, and there tends to be under-prediction.

It is important to note that values of ' $r$ ' that yield elasticity estimates of logit that are close to the true elasticities of linear and AIDS (figures 3B and 3C) do not necessarily yield the highest accuracy in predicting the true post-merger prices of linear and AIDS. To see this, consider logit's accuracy in predicting post-merger prices when data is generated by AIDS in the H/L elasticity case. From figure 3C,  $r = 0.50$  appears as the market potential parameter (from the 3 being considered here) that would yield elasticities closest to the true values. However,  $r = 0.75$  yields a smaller MSD and a smaller fraction of under-predicted post-merger prices than  $r = 0.50$ . The reason for this result is the 'extrapolation' bias that was explained in section 4. Thus, we deliberately omit an analysis of the 'optimal' ' $r$ ' size that produces the highest accuracy in predicting post-merger prices.

Logit always predicts the correct direction of change in prices as a result of the merger. Interestingly, misspecification among continuous choice models (i.e., AIDS estimated with linear and vice versa) yield relatively large MSD in the M/L case, and in the H/L case such predictions are not much more precise than logit's. While the AIDS functional form has been claimed to be preferable to other continuous choice models because of its flexible

TABLE III  
PREDICTION ACCURACY OF POST-MERGER PRICES BY MISSPECIFIED MODELS

Data Generation Model	Accuracy Statistics***	Model Used for Post-Merger Price Prediction*															
		M/L			H/L			Logit**			Linear			AIDS			
		r = 0.25	r = 0.50	r = 0.75	r = 0.25	r = 0.50	r = 0.75	r = 0.25	r = 0.50	r = 0.75	M/L	H/L	M/L	H/L	M/L	H/L	
Logit	MSD	-	-	-	-	-	-	-	-	-	-	-	-	0.53	0.2	2.4	0.21
	% Under-predicted	-	-	-	-	-	-	-	-	-	-	-	-	56%	55%	32%	55%
Linear	% Correct Change	-	-	-	-	-	-	-	-	-	-	-	-	80%	62%	81%	64%
	MSD	0.013	0.014	0.026	0.0003	0.004	0.015	0.015	0.015	-	-	-	-	-	-	0.77	0.007
AIDS	% Under-predicted	100%	10%	0%	73%	0%	0%	0%	0%	100%	100%	100%	100%	-	-	0%	0%
	% Correct Change	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	-	-	100%	100%
	MSD	1.15	0.91	0.71	0.013	0.004	0.001	0.83	0.006	0.83	0.006	0.83	0.006	100%	100%	-	-
	% Under-predicted	100%	100%	100%	100%	96%	61%	100%	100%	100%	100%	100%	100%	100%	100%	-	-
	% Correct Change	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	98%	-	-

\*M/L = Medium own-price elasticity (~ -2) and low cross-price elasticity (~ 0.60) case. H/L = High own-price elasticity (~ -3) and low cross-price elasticity (~ 0.60) case.

\*\*r = market potential parameter. A lower r implies a larger market potential.

\*\*\*MSD = Median squared deviations, deviation = (post-merger price, true model) - (post-merger price, false model). % Under-predicted = % of times that the post-merger prices of the false model are lower than the post-merger prices of true model. % Correct change = % of times that the post-merger price of the false model correctly predicts the change in price from the pre-merger level to the post-merger level of the true model.

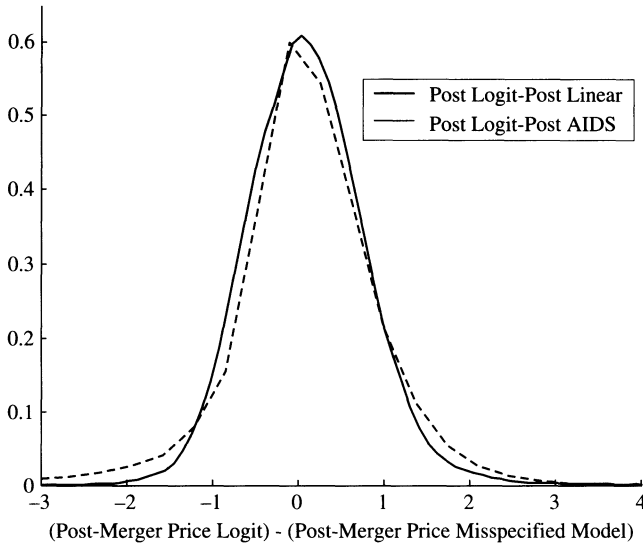


Figure 4

Distribution of Difference between True Post-Merger Price (Logit) and Misspecified Post-Merger Price (Linear or AIDS)

approximation to any demand system and the tight link of its parameters to theory, it is unclear that it performs better than linear demand when used to compute the post-merger prices of another model. Furthermore, when data is generated with linear demand, the logit model does an overall better job than AIDS at predicting post-merger prices.

We find that certain misspecifications produce consistent under-predictions or over-predictions of post-merger prices. When misspecified, the logit model tends to under-predict the post-merger prices for larger market potentials (smaller  $r$ ). A similar case arises when the linear model is used to compute the post-merger prices of data that is generated by AIDS. AIDS, on the other hand, *always* over-predicts post-merger prices when the true model is linear.

Figures 4 through 6 display the (kernel smoothed) distributions of the difference between the post-merger price predicted by the true model (logit: 4, linear: 5 and AIDS: 6) and the post-merger price as predicted by the incorrect models. These figures, which correspond to the H/L case<sup>24</sup>, confirm the patterns observed in the accuracy statistics. Linear and AIDS do approximately equally well in predicting logit post-merger prices and there are no clear under- or over-prediction patterns. AIDS over-predicts linear post-merger prices but under-prediction occurs in the opposite direction (i.e., when AIDS is the true model and linear is the incorrect model). Logit

<sup>24</sup> The patterns are similar in the M/L case figures.

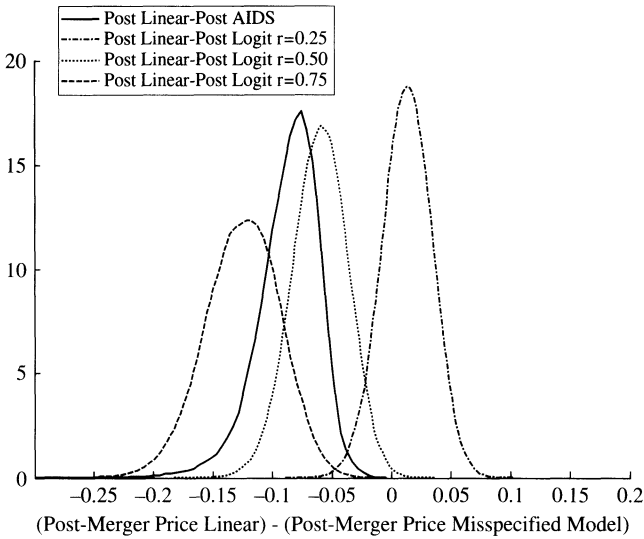


Figure 5  
Distribution of Difference between True Post-Merger Price (Linear) and Misspecified Post-Merger Price (AIDS or Logit)

predictions vary by market potential, yielding distributions more closely distributed around zero for  $r = 0.25$  and  $r = 0.75$ , for linear and AIDS data, respectively. The figures also provide a clearer picture of whole range of values of the difference in simulated post-merger prices: the x-axis range of figure 4 is one order of magnitude larger than in figures 5 and 6, confirming the relatively larger inaccuracy of continuous choice models when predicting logit's post-merger prices.

*Compensating Marginal Cost* Compensating marginal cost is not only a metric that is free of extrapolation bias but it is also more reliable because simple matrix inversion renders it easy and fast to compute (i.e., there are no convergence problems). Table IV shows the results of the computed statistics. Our findings here are very consistent with elasticity results. First, when data is generated by continuous choice models, logit can approximate well the compensating marginal cost, and in many cases produces MSD that are smaller than those of continuous choice models. As with elasticities, logit's approximation of continuous choice models' compensating marginal costs can improve with the choice of market potential: when data is generated by linear, a reasonable  $r$  for estimating the compensating marginal cost is 0.5 for the M/L case and (apparently) slightly lower than 0.5 for the H/L case. When data is generated by AIDS, a reasonable  $r$  for estimating the compensating marginal cost appears to be between 0.5 and 0.75 for the M/L case and about 0.5 for the H/L case. We simulated additional cases for

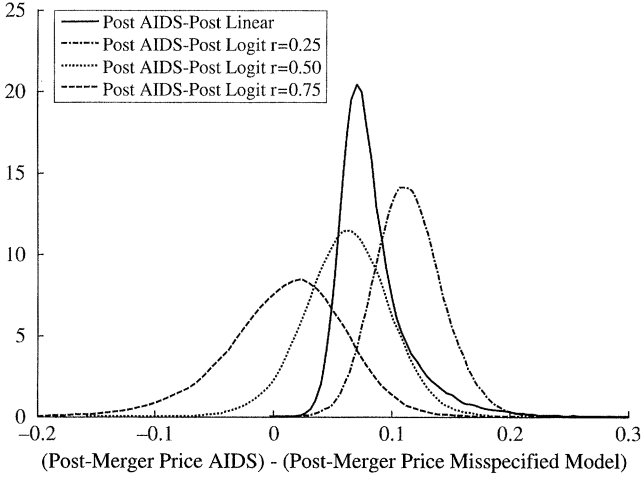


Figure 6

Distribution of Difference between True Post-Merger Price (AIDS) and Misspecified Post-Merger Price (Logit or Linear)

Notes:  $r$  = market potential parameter. A larger  $r$  implies a smaller market potential. Figures correspond to H/L case.

other values of  $r$  (not shown) and confirmed a u-shape relationship: there is an optimal value of  $r$  that yields minimal MSD (and no significant over-prediction or under-prediction) and values of  $r$  above and below this optimal level reduce predictive accuracy. Importantly, these optimal values of  $r$  correspond to the values of  $r$  that also yield elasticity estimates close to the true values.

Second, marginal cost reductions are well approximated when the wrong continuous choice model is used (e.g., AIDS used when linear is the true model). However, when data is generated by logit, continuous choice models approximate the true marginal cost reduction poorly (although with no under- or over-prediction patterns) as evidenced by the relatively large MSD statistic. Figures 7 through 9 display the (kernel smoothed) distributions of the difference between the compensating marginal cost predicted by the true model (logit: 7, linear: 8 and AIDS: 9) and the compensating marginal cost predicted by the incorrect models. These figures, which correspond to the H/L case<sup>25</sup>, confirm the patterns observed in table IV.

## VI. DISCUSSION

We investigate the implications of functional form misspecification in demand estimation when only aggregate data is observed. We focus on: a)

<sup>25</sup>The patterns are similar in the M/L case figures.

TABLE IV  
PREDICTION ACCURACY OF COMPENSATING MARGINAL COSTS BY MISSPECIFIED MODELS

Data Generation Model	Accuracy Statistics***	Model Used for Compensating Marginal Cost*											
		Logit**				Linear				AIDS			
		M/L		H/L		M/L		H/L		M/L		H/L	
		$r = 0.25$	$r = 0.50$	$r = 0.75$	$r = 0.25$	$r = 0.50$	$r = 0.75$	$r = 0.25$	$r = 0.50$	$r = 0.75$			
Logit	MSD	-	-	-	-	-	-	-	-	-	2.52	0.771	4.11
	% Under-predicted	-	-	-	-	-	-	-	-	-	48%	50%	26%
	% Correct Change	-	-	-	-	-	-	-	-	-	90%	99%	89%
Linear	MSD	0.092	0.005	0.142	0.009	0.002	0.045						
	% Under-predicted	0%	39%	100%	11%	72%	99%						
	% Correct Change	100%	100%	100%	100%	100%	100%						
AIDS	MSD	0.09	0.059	0.10	0.008	0.006	0.013	0.11	0.02				
	% Under-predicted	13%	34%	56%	14%	43%	64%	50%	49%				
	% Correct Change	100%	100%	100%	100%	100%	100%	100%	100%				

\*M/L = Medium own-price elasticity (~ 2) and low cross-price elasticity (~ 0.60) case. H/L = High own-price elasticity (~ - 3) and low cross-price elasticity (~ 0.60) case.

\*\*r = market potential parameter. A lower r implies a larger market potential.

\*\*\*MSD = Median squared deviations. deviation = (Compensating MC, true model) - (Compensating MC, false model). % Under-predicted = % of times that the Compensating MC of the false model are lower than the Compensating MC of true model. % Correct change = % of times that the Compensating MC of the false model correctly predicts the change in MC from the pre-merger level to the Compensating level of the true model.

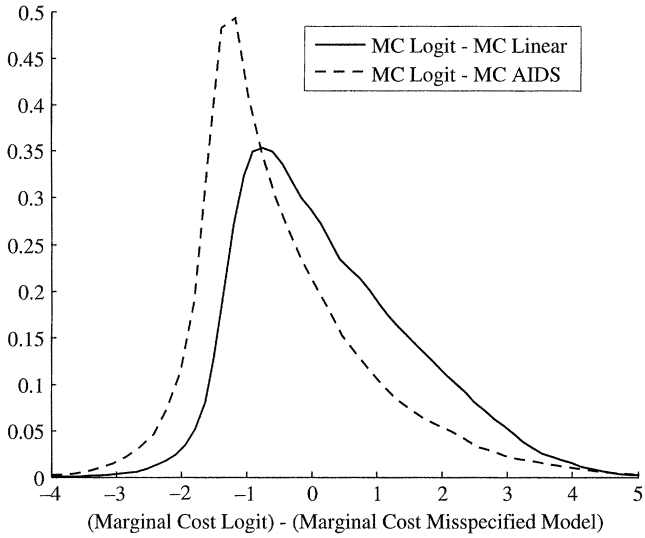


Figure 7

Distribution of Difference between True Compensating Marginal Cost (Logit) and Misspecified Compensating Marginal Cost (Linear or AIDS)

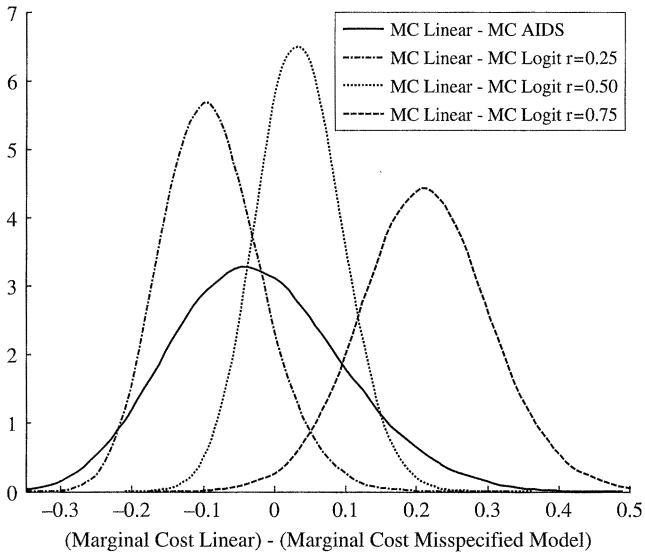


Figure 8

Distribution of Difference between True Compensating Marginal Cost (Linear) and Misspecified Compensating Marginal Cost (AIDS or Logit)

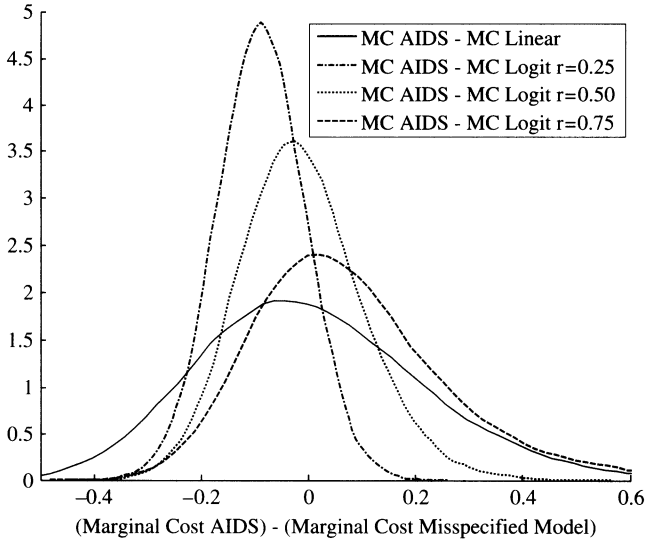


Figure 9

Distribution of Difference between True Compensating Marginal Cost (AIDS) and Misspecified Compensating Marginal Cost (Logit or Linear)

Notes:  $r$  = market potential parameter. A larger  $r$  implies a smaller market potential. Figures correspond to H/L case. Compensating marginal cost is defined as the (lower) post-merger marginal cost needed to keep post-merger prices at pre-merger levels.

biases in elasticity estimates, and b) accuracy in merger simulation of post-merger prices and compensating marginal costs. Consistent with the recent literature in empirical industrial organization, we adopt a structural approach: we center our attention in the consistent estimation of the economic parameters of the model. We study two strands of demand models that have been popular in the literature: discrete choice models (logit) and continuous choice models (AIDS, linear and log-linear).

A somewhat expected result is that biases can arise when continuous choice models (linear, AIDS and log-linear) are used to estimate demand with data that is generated with a discrete choice model (logit), and vice versa. A less obvious finding is that the assumed market potential in logit estimation is a key element in recovering the true elasticities. This result carries over to post-merger simulations using a compensating marginal cost metric. One conclusion of our study is that the market potential assumption needs to be more carefully addressed by researchers using the logit model with aggregate data.

In our simulations, the market potential that recovers the true own-price elasticity is not the same as the market potential that recovers the true cross-price elasticity (except when the true model is logit), but these market potentials are always close. In prior work, a usual practice in defining the

potential market is to use the whole population as potential consumers; for example, Berry, Levinsohn and Pakes use the driving population, Nevo [2001] assumes one serving of cereal times the population of the city, and Chintagunta *et al.* [2003] consider the weekly store traffic. Defining market potential in this way could produce an overestimated market potential as only a fraction of the population may consider purchasing a particular product at a given time (e.g., a family of two that already owns two cars or that has a low income level). This problem can be magnified in applications where researchers only have a sample of the data.<sup>26</sup> It is hence possible that the small magnitude of cross-price elasticities typically found in these studies may be due in part to a market potential that is too large.

A potential solution for obtaining a market potential in the logit model is to estimate it directly. This would amount to rewriting logit's estimable equation to include the market potential ' $M$ ', which enters  $s_j$  and  $s_0$ . Estimation of this parameter, however, needs to be carefully thought out as it is not clear what variation in the data provides the identification of this constant. We leave this task for future research, but conjecture that in more complex versions of logit (such as random coefficients) the estimation of this additional parameter is unlikely to add significant computational burden for the researcher (provided identification).<sup>27</sup>

A less optimal solution for market potential estimation is to carry out sensitivity analysis. Our results for the more plausible M/L and H/L cases (excluding the restrictive log-linear specification) show that when a reasonable market potential is assumed (a market potential between the optimal own-price elasticity market potential and the optimal cross-price elasticity market potential), a larger market potential (smaller  $r$ ) will not affect own-price elasticity but will bias cross-price elasticity towards zero whereas smaller market potentials (larger  $r$ ) will bias *both* elasticities away from zero. Researchers could then search for a market potential that behaves in this way. We provide this suggestion with caution as it may not hold for more complex cases.

Merger results suggest that employing a continuous choice model that is different from the one from which data is generated is no guarantee of obtaining results that are more accurate than those obtained by a logit model. Moreover, when the true model is logit, continuous choice models do a poor job at predicting the true post-merger prices. Again, with the logit model one needs to be very careful when defining the market potential as it can produce misleading results. For example, assuming too large a market potential (as it seems plausible in previous work) may lead authorities to

<sup>26</sup> For example, Nevo [2001] uses data from a sample of supermarkets which covers only a fraction of all grocery sales in a metropolitan area.

<sup>27</sup> Reiss and Wolak also suggest this approach but offer limited details on how it can be carried out.

allow a merger that must have been blocked otherwise. This is supported by our results: when  $r$  goes to zero post-merger prices are underpredicted and compensating marginal costs are overpredicted.

A motivation for this paper was to investigate the restrictiveness of assuming a discrete choice model when the purchase decision appears not to be discrete. Our results consistently suggest that such misspecification is not too important as long as market potential is carefully defined. Moreover, this result also appears to hold in our preliminary simulations with three goods and random coefficients logit (not shown here).

While our initial objective was not to choose a better model, we interpret our results as favorable to the logit model. The advantages of using a logit model when the true model is not logit are that it can recover true elasticities even when misspecified and that it tends to predict more accurate post-merger prices and compensating marginal costs. Also, when misspecified, continuous choice models can produce larger biases and even 'wrong-signed' elasticities if the instruments are not appropriately chosen (see section 4), while logit does not suffer from this latter pitfall. The fact that biases of using a continuous choice model could be larger than those from using a discrete choice model is a little counterintuitive since, in principle, the larger number of parameters in continuous choice models should provide them with the advantage of capturing substitution patterns more freely than a model that recovers them using only one parameter.

In sum, our results raise caution about inference in applications in which the assumed functional form is questionable. More importantly, our results should not only help researchers make better informed decisions about which model to choose but they should also help them draw more careful inference.

#### REFERENCES

- Berry, S., 1994, 'Estimating Discrete Choice Models of Product Differentiation,' *Rand Journal of Economics*, 25, pp. 242–262.
- Berry, S.; Levinsohn, S. and Pakes, A., 1995, 'Automobile Prices in Market Equilibrium,' *Econometrica*, 63, pp. 841–890.
- Besanko, D.; Gupta, S. and Jain, D., 1998, 'Logit Demand Estimation under Competitive Pricing Behavior: An Equilibrium Framework,' *Management Science*, 44, pp. 1533–1547.
- Besanko, D.; Dube, J. P. and Gupta, S., 2003, 'Competitive Price Discrimination Strategies in a Vertical Channel Using Aggregate Retail Data,' *Management Science*, 49, pp. 1121–1138.
- Bijmolt, T.; van Heerde, H. and Pieters, R., 2005, 'New Empirical Generalizations on the Determinants of Price Elasticity,' *Journal of Marketing Research*, 42, pp. 141–156.
- Bresnahan, T., 1989, 'Empirical Methods for Industries with Market Power,' in Schmalensee, R. and Willig, R. (eds.), *Handbook of Industrial Organization*, Vol. II (North-Holland, Amsterdam, The Netherlands).

- Chintagunta, P.; Dubé, J. P. and Singh, V., 2003, 'Balancing Profitability and Customer Welfare in a Supermarket Chain,' *Quantitative Marketing and Economics*, 1, pp. 111–147.
- Chintagunta, P. and Dubé, J. P., 2005, 'Estimating an SKU-level Brand Choice Model Combining Household Panel Data and Store Data,' *Journal of Marketing Research*, 42, pp. 368–380.
- Crooke, P.; Froeb, L.; Tschantz, S. and Werden, G., 1999, 'Effects of Assumed Demand Form on Simulated Postmerger Equilibria,' *Review of Industrial Organization*, 15, pp. 205–217.
- Deaton, A. and Muellbauer, J., 1980, 'An Almost Ideal Demand System,' *American Economic Review*, 70, pp. 312–326.
- Dube, J. and Manchanda, P., 2005, 'Differences in Dynamic Brand Competition Across Markets: An Empirical Analysis,' *Marketing Science*, 24, pp. 81–95.
- Gasmi, F.; Laffont, J. J. and Vuong, Q., 1992, 'Econometric Analysis of Collusive Behavior in a Soft-Drink Market,' *Journal of Economics and Management Strategy*, 1, pp. 277–311.
- Hausman, J. and Leonard, G., 1997, 'Competitive Analysis with Differentiated Products,' *George Mason Law Review*, 34, pp. 159–180.
- Hausman, J. and Leonard, G., 2002, 'The Competitive Effects of a New Product Introduction: A Case Study,' *Journal of Industrial Economics*, 50, pp. 237–263.
- Hausman, J.; Leonard, G. and Zona, D., 1994, 'Competitive Analysis with Differentiated Products,' *Annales d'Économie et de Statistique*, 34, pp. 159–180.
- Hausman, J., 1996, 'Valuation of New Goods under Perfect and Imperfect Competition,' in Bresnahan, T. and Gordon, R. (eds.), *The Economics of New Goods, Studies in Income and Wealth*, Vol. 58 (National Bureau of Economic Research, Chicago).
- Kadiyali, V.; Vilcassim, N. and Chintagunta, P., 1996, 'Empirical Analysis of Competitive Product line Pricing Decisions: Lead, Follow or Move Together?,' *Journal of Business*, 9, pp. 459–487.
- Kadiyali, V.; Chintagunta, P. and Vilcassim, N., 2000, 'Manufacturer-Retailer Channel Interactions and Implications of Channel Power: An Empirical Investigation of Pricing in a Local Market,' *Marketing Science*, 19, pp. 127–148.
- Khan, R. and Jain, D., 2005, 'An Empirical Analysis of Price Discrimination Mechanisms and Retailer Profitability,' *Journal of Marketing Research*, 42, pp. 516–524.
- Nevo, A., 2000, 'Mergers with Differentiated Products: The Case of the Ready-to-Eat Cereal Industry,' *Rand Journal of Economics*, 31, 395–421.
- Nevo, A., 2001, 'Measuring Market Power in the Ready-to-Eat Cereal Industry,' *Econometrica*, 69, pp. 307–342.
- Peters, C., 2006, 'Evaluating the Performance of Merger Simulations: Evidence from the U.S. Airline Industry,' *Journal of Law and Economics*, 49, pp. 627–649.
- Porter, R., 1983, 'A Study of Cartel Stability: The Joint Executive Committee, 1880–1886,' *Bell Journal of Economics*, 2, pp. 301–14.
- Pinkse, J.; Slade, M. and Brett, C., 2002, 'Spatial Price Competition: A Semiparametric Approach,' *Econometrica*, 70, pp. 1111–1155.
- Pinkse, J. and Slade, M., 2004, 'Mergers, Brand Competition, and the Price of a Pint,' *European Economic Review*, 48, pp. 617–643.
- Reiss, P. and Wolak, F., 2007, 'Structural Econometric Modeling: Rationales and Examples from Industrial Organization,' in Heckman, J. and Leamer, E., *Handbook of Econometrics*, Volume 6 (North-Holland, Amsterdam, The Netherlands).
- Rojas, C., 2008, 'Price Competition in U.S. Brewing,' *Journal of Industrial Economics*, 56, pp. 1–31.

- Slade, M., 2004, 'Market Power and Joint Dominance in U.K. Brewing,' *Journal of Industrial Economics*, 52, pp. 133–163.
- Sudhir, K., 2001a, 'Competitive Pricing in the Auto Market: A Structural Analysis,' *Marketing Science*, 20, pp. 42–60.
- Sudhir, K., 2001b, 'Structural Analysis of Manufacturer Pricing in the Presence of a Strategic Retailer,' *Marketing Science*, 20, pp. 244–264.
- Weinberg, M., 2008, 'An Evaluation of Merger Simulations,' working paper, University of Georgia.
- Werden, G., 1996, 'A Robust Test for Consumer Welfare Enhancing Mergers among Sellers of Differentiated Products,' *Journal of Industrial Economics*, 44, pp. 409–413.
- Werden, G., 2000, 'Expert Report in United States v. Interstate Bakeries and Continental Baking Co,' *International Journal of the Economics of Business*, 7, pp. 139–148.
- White, H., 1980, 'Using Least Squares to Approximate Unknown Regression Functions,' *International Economic Review*, 21, pp. 149–170.