Portfolio adjustment and panic behavior under true uncertainty

Portfolio adjustment and panic behavior under true uncertainty*

Se Ho Kwak[†]

August 29, 2022

Abstract

G.L.S. Shackle was one of the representative critics against probability-based economic theory, and influenced some Post-Keynesians and Austrians. During the 1980s and 1990s, his alternative framework was mathematically reconstructed by Katzner. In this paper, we will reformalize the Shackle-Katzner framework to explain the financial decision-making of the individual. For this, the portfolio diversification between two non-monetary assets will be explained by the reformalized model introduced here, and then moved to the analysis about a case of money and a non-monetary asset. Based on these findings, a few possible scenarios of panic behavior in the portfolio adjustment will be examined.

JEL Classification Code: B21, B50, D81, G11.

Keywords: Shackle; uncertainty; portfolio; panic; money

1 Introduction

Consider the following a pair of behaviors: In choosing an asset or a ratio for investment between two given assets, one having higher potential gain and loss than the other, the individual selects the former when she is optimistic about the market. Alternatively, when the dominant market expectation for the future is reversed from optimism to pessimism, the individual abruptly changes her mind so that she abandons the former and switches to the latter rather than just partially adjusting the ratio between two assets in the portfolio. When the change of sentiment regarding the future outcome from optimism to pessimism is not only abrupt but "even" gradual, such panic behavior can happen, as instanced by black Monday in the U.S. stock market in 1987. Such drastic adjustment of a portfolio from the high potential yield with high uncertainty to low yield in low uncertainty or even zero yield with certainty will be referred to hereafter as panic behavior. Panic behaviors have been observed frequently in

*I am specially thankful to Donald Katzner and Naoki Yoshihara

†Corresponding author. Department of Economics, University of Massachusetts Amherst, Crotty Hall, Amherst, MA, 01002, USA, E-mail: skwak@econs.umass.edu.

history of financial crises. Many prominent economists including Friedman and Schwarz (1963) and Kindleberger (1978) insisted that there must be an element of panic in financial crises.

There is influential theoretical research on financial crises[1]asserting that panic behavior is the result of self-fulfilling interactions of market participants and the resultant multiple equilibria. This is, for example, exhibited by the Diamond and Dybvid (1983) model of banking crises, as well as models of self-fulfilling currency collapse by Obstfeld (1996) and Krugman (1999). Rosser (1997) and Gallegati et al. (2011) constructs heterogenous agent models which, respectively, focus on the speculative bubble and herding behavior. In applied mathematics, local martingale models (Jarrow, Protter, and Shimbo, 2010; Jarrow, Kohia, and Protter, 2011; Protter, 2016), the disorder detection models (Shiryaev 2007) and the earthquake model (Gresnigt et al., 2015) have examined the mechanism of bubble and the ensuing crash. These models explain panic phenomena in the stock market in terms of the detection of a stock bubble and the optimal moment of exit before the realization of the crash. Recently, Gennaioli and Shleifer (2018) introduced the notion of diagnostic beliefs and neglected risk to explain financial fragility.

Strictly speaking, in the approaches mentioned so far, panic behaviors are not captured by the explanations of pure individual decision-making. The focus in the approach of self-fulfillment and multiple equilibria is mainly on the interaction of market participants rather than on the idiosyncrasies of individual decision-making. Basically, self-fulfilling behavior, mutual interaction of speculators and financial fragility indicate 'herding' not 'panic'. Herding is the result of group behavior and panic is the response by individuals. Although herding can reinforce panic, some individuals start panic behavior before herding. .

However, it is hard to describe the switching mechanism from one decision to another by the logic of pure individual decision theory without relying on the narratives of market participants corresponding to each situational and institutional context (Goetzman et al. 2016). For example, as a representative theoretical tool of microfoundation, in expected utility theory, the summation or integration of expected utilities shows only continuous changes rather than any discontinuity implying panic response unless an abrupt switch in the probability distribution or payoff values is supposed. But surely panic behavior does not happen only after a sudden, unexpected update in the information prescribing the probability distribution or payoff schedules but also after enduring long and gradual changes in anticipated future outcomes. At the right moment, panic happens suddenly, without notice in advance.

Expected utility theory is not the only theoretical tool employed in explaining individual decision-making. Recent developments in the applied mathematics of the theory of financial fragility also heavily rely on the notion of probability or a set of probabilities as the indicators of the likelihood of payoffs in valuing potential outcomes. But the availability of probability information in concrete

[1]For the survey of financial crisis literature including panic-based and fundamental-based approaches, see Goldstein (2013).

individual decision-makers has been persistently doubted (Keynes 1921; Knight, 1921; Mises, 1949; Lachmann, 1976; O'Driscoll and Rizzo, 1985; Davidson, 1983; 1991; Lawson, 1995; Katzner, 1998). In normative and descriptive contexts, the use of both objective and subjective probabilities is not credible for skeptics of probability calculus and expected utility theory in constructing 'realistic' explanation of individual behavior.

Specifically, G.L.S. Shackle is a major critic of probability and expected utility theory. If the probability is construed in terms of frequency, then the probability calculus cannot be utilized when the choice is a single, unique activity because the probability value has meaning only when an experiment is to be repeated a large number of times. But almost all economic decisions occur under unique and irreversible circumstances. Even when probability is thought of as subjective, Shackle has eloquently argued that most economic decisions are made under conditions of true uncertainty in which there is not enough information available to the decision-maker to be able to construct the stable exhaustive list of possible future outcomes. The reality in time requiring a decision must be unstable, and any change in the number of alternative outcomes must alter the probability distribution assigned to previously known outcomes. It is not possible, while preserving the axiom of probability, to represent $n > 2$ independent events whose realization and nonrealization are assessed as equally plausible with the identical probability value than $1/2$. Otherwise, the summation of probability would be bigger than unity.

Furthermore, representing the possibility of an event having no supporting evidence with zero probability is not a suitable way to reflect ignorance on the part of the decision-maker. The assignment of zero probability is relevant to the "knowledge" that the relative frequency or possibility of occurrence of a specific event is zero, and this is different from disbeliefs due to the lack of supporting information. Hence, instead of the problematic use of probability as a basis for expected utility theory, Shackle constructed his own alternative theoretical framework explaining human decisions on the premise of historical time, ignorance of future events, and a non-probabilistic measure of uncertainty (Shackle, 1954; 1969; 1972). Shackle proposed alternative notion of subjective response toward an uncertain future by what he called potential surprise. Potential surprise was defined by Shackle as the surprise the individual imagines today that he would feel in the future if a particular payoff were to come to pass.[2]

Shackle's criticism of probability was accepted by several Austrians and Post Keynesians (Lachmann 1976; Davidson 1983) and his theoretical framework was formalized with the language of modern theory of portfolio selection and of firm behavior by Ford (1983; 1994) and Vickers (1978; 1987; 1994). Katzner constructed a totally reformalized version of Shackean decision theory (Katzner 1986-7, 1987-88, 1989-90) and extended the range of its application to simultaneous behavior (Katzner 1995), the demand for money (Katzner 2001), firm behavior (Katzner 1990-91), and macroeconomic phenomena (Katzner 1998).[3]

[2] See Shackle (1972).

[3] This series of studies was combined and expanded in a single book (Katzner 1998).

In the Shacklean approach that we will construct here, panic behavior is explainable as the result of the existence of money in the choice option and the specific decision criteria of an individual focusing on only the stronger aspect out of coexisting optimism and pessimism on the potential payoffs from the asset portfolio. Optimism is observable in a bull market and pessimism is observable in bear market. As the terminating stage of a booming market arrives, the expectation of potential payoff is gradually eroded and finally goes through a turning point. At that point, the object of decision-making is totally reversed, away from high uncertainty and high yield towards maintaining the principal of one's investment. In order to elaborate this mechanism in the dimension of individual decision-making, the next section sets out Shacklean decision model along the line of reformalized model of Katzner' formalization (1986-7; 1987-88; 1989-90; 1998) by the author. In section 3, we will develop a model of portfolio selection for the case of two non-money assets. In section 4, we will illustrate the panic response in portfolio adjustment on the basis of the model established in section 3. In section 5, we will extend our analysis to the special case of decision-making between money and an arbitrary non-money portfolio.

2 A summary of Shacklean decision process

Potential surprise and payoff function [4]

In Shacklean framework, decisions are made in three steps. In **step 1**, encountering a problem of choice facing an uncertain future, the decision-maker recognizes a set $\boldsymbol{X}$ of all available actions and a set $\boldsymbol{\Omega}$ of all currently imaginable future states of the world. In Shacklean framework, the incompleteness of $\boldsymbol{\Omega}$ is premised.[5] To construct a measure of uncertainty over the hypotheses from that collection of imaginable future states, let $\boldsymbol{F^*}$ be a nonempty σ-field over $\boldsymbol{\Omega}$ and call an element of $\boldsymbol{F^*}$ a hypothesis. For any hypothesis $A \in \boldsymbol{F^*}$, a hypothesis $B \in \boldsymbol{F^*}$ is called rival to $A \cap B \neq \emptyset$. [6]

Definition 2.1: A collection of **rival hypotheses** is defined as a collection of hypotheses A_i such that

i) $A_1 = \emptyset$,
ii) A_i is nonempty hypothesis for each $i \neq 1$,
iii) For all $i \neq j$, A_i and A_j are rival hypotheses,
iv) $\bigcup_i A_i = \boldsymbol{\Omega}$.

[4] For the details on definitions introduced here, see Katzner (1998, pp.86-94).

[5] Savage (1954) also acknowledged that the axiomatic system of the subjective expected utility theory is related not to the 'grand world' which is the complete list of future states, but only to the 'small world' derived from the grand world. On the contrary, in Shacklean framework, because that 'small world' confines the decision-maker, Shackle negates the additivity and distributivity of probability calculus.

[6] A nonempty collection $\boldsymbol{F^*}$ of subsets of $\boldsymbol{\Omega}$ is called σ-field over $\boldsymbol{\Omega}$ if for any $A \in \boldsymbol{F^*}$ and countable collection $\{A_i | A_i \in \boldsymbol{F^*}\}$, it satisfies $A^c \in \boldsymbol{F^*}$.

In Katzner's reformalization of Shaklean decision theory, the null hypothesis $A_1 = \emptyset$ is not just an inevitable element of logics for the set-theoretic construction, but concretely reinterpreted as the residual hypothesis, *i.e.*, the currently unspecifiable but potentially possible hypothesis beyond the range of $\boldsymbol{\Omega}$.

Based on the recognition of available actions in $\boldsymbol{X}$ and hypotheses in $\boldsymbol{F}^*$ over $\boldsymbol{\Omega}$, the decision-maker imagines **(1)** the degree of surprise he/she would feel now upon the future realization of an element of a hypothesis in $\boldsymbol{F}^*$ and **(2)** the future payoff summoned by the realization of a state of the world together with his/her chosen action from $\boldsymbol{X}$.

Definition 2.2: The **potential surprise** is a function $s : \boldsymbol{F}^* \to [0, 1]$ satisfying these three conditions:
i) for all $A \in \boldsymbol{F}^*, 0 \leq A \leq 1$,
ii) for any collection A_i of nonempty subsets in $\boldsymbol{F}^*$, $s(\bigcup_i A_i) = \inf_i s(A_i)$ [7],
iii) if A_i is an exhaustive set of rival hypotheses, then $s(A_i) = 0$ for least one i.

The condition *ii*) of definition 2.2 is the counterpart to the additivity of probability measure. As a hypothesis includes the wider range of possible future states by combining multiple hypotheses, that hypothesis is regarded more plausible to be realized. But in Shacklean framework the implication of potential surprise is distinctive and even contrasting to probability. Thus, adding the potential surprise values of multiple hypotheses is not justifiable as the sense of probability.

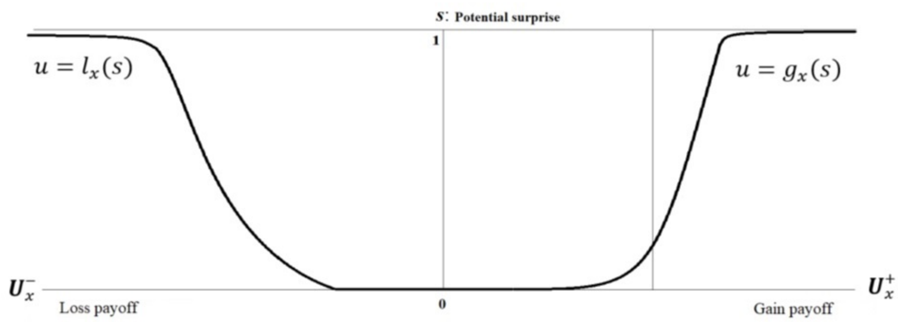
In condition *iii*), if a hypothesis A in $\boldsymbol{F}^*$ has zero potential surprise value, then the hypothesis A is said to be *perfectly possible*. If $s(A) = 1$, then the hypothesis A is said to be *perfectly impossible*. The condition *iii*) establishes the standardized cap to the degrees of belief in Shacklean framework. In Shacklean framework, the perfect possibility is represented by, unlike the unitary probability, zero. The zero potential surprise does not mean that the decision-maker has the strongest assurance about the realization of that event, but the decision-maker cannot confirm any evidence or scenarios, which block the progress of the situation toward that direction. In other words, in decision-maker's mind, future is definitely open to that direction, but the exact plausibility is unknown. Like this, the measure of uncertainty in Shacklean context, *i.e.*, the potential surprise is totally insulated from the premises of probability.

Another function defined on $\boldsymbol{\Omega}$ in conjunction with the space of acts $\boldsymbol{X}$ is the *payoff function* defined as $u : \boldsymbol{X} \times \boldsymbol{\Omega} \to \mathbb{R}$, where $\mathbb{R}$ is the space of real numbers. For a fixed act $x_o \in \boldsymbol{X}$, its possible payoff values for all $\omega \in \boldsymbol{\Omega}$ are specified as $u(x_o, \omega)$ or $u_{x_o}(\omega)$. Then, for an act $x \in \boldsymbol{x}$, we can obtain a locus describing the

[7] Although the domain of potential surprise is the family $\boldsymbol{F}^*$ of $\boldsymbol{\Omega}$ equipped with the completeness of set-operations, $\cap$, ^{c}, ultimately, it is no problem to suppose that the potential surprise values of arbitrary hypotheses are reduced to corresponding ones of singleton sets of $\boldsymbol{F}^*$, *i.e.*, $\boldsymbol{\Omega}$.Unlike continuous probability distribution, Shacklean framework can allow that the singleton set may have non-zero value of potential surprise. How to construct the functional representation of potential surprise 'order' is beyond the range of this study, and it will be explained in another paper by the author.

relation between possible gain/loss and potential surprise as in figure 2.1. For a given level of surprise value, we can pinpoint the maximum potential gain and loss, and they can be defined as a functional form as $u = g_x(s)$ and $u = l_x(s)$. We will call the graph of $u = g_x(s)$ and $u = l_x(s)$ the *potential surprise locus*.

As the size of gains or losses becomes larger, the degree of potential surprise increases because it can be regarded as more unrealistic. In other words, an extremely high gain or loss is treated highly implausible. Meanwhile, there is a range of 'perfectly possible' gains or losses around some "standard" level of payoff to be determined by the decision-maker.



[Figure 2.1]

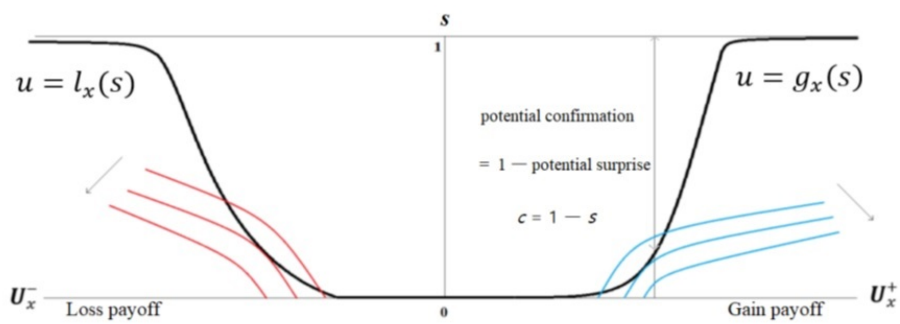
To formalize this idea, let u^o in $\mathbb{R}$ be the standard payoff relating to an act $\boldsymbol{x}$. For example, u^o may be the zero payoff in gambling or the real interest rate in the case of a financial investment. Without loss of generality, we can simply put $u^o = 0$ because we can restate any gains and losses as the distance from the standard payoff u^o. Then we can split the whole space $[0,1] \times \mathbb{R}$ of the potential surprise and payoff into the gain space, $[0,1] \times U_x^+$ and and the loss space, $[0,1] \times U_x^-$, where $U_x^+ = \{u \in \mathbb{R} : u \geq 0\}$ and $U_x^- = \{| \ u \ | \in \mathbb{R} : u \leq 0\}$. Simply, $U_x^+ = U_x^- = \mathbb{R}_+$ is the non-negative real space and $U_x^+ \cap U_x^- = 0$, which indicate the magnitude of all imaginable gains and losses respectively. We will denote the element of U_x^+ and U_x^- as $u_x^+, u_x^- \geq 0$ or simply, $u^+, u^- \geq 0$.

Attractiveness function, attractive index

Potential surprise indicates the degree of emotional disbelief without any premise of certain knowledge regarding frequency or possibility of such a state. Now, **step 2** functionalizes two variables - potential surprise and payoff - to specify in a well-defined way the importance of each outcome to the decision-maker.

For an arbitrary act x, each point $(s, u) = (s(\omega), u_x(\omega))$ in $[0,1] \times U$ may have different importance to the decision-maker. For example, at points in $[0,1] \times U_x^+$ it is sensible to think that that bigger payoffs with lower potential surprise have more importance to the decision-maker because that implies a more certain and successful outcome of the decision. In the geometric representation as figure 2.2, this is equivalent to the higher value of 1 minus potential surprise implying the higher belief. In $[0,1] \times U_x^-$, bigger losses with higher confirmation have more importance to the decision-maker because that imply a more threatening outcome in the future.

At this point, we need to introduce another measure of uncertainty defined on $\boldsymbol{F^*}$ called *potential confirmation*. The potential confirmation function was originally introduced by Katzner (1986-7, 1998) in his reformalization of Shacklean theory, and it is the analogue of probability in the Shackle-Katzner context. The potential confirmation of a hypothesis A in $\boldsymbol{F^*}$ is the degree of confidence or the absence of surprise that the individual imagines today that he would feel in the future if a payoff were to actually arise.[8] Although the potential confirmation function also has its own definition and independent meaning as potential surprise,[9] we will interpret it in the narrowest sense, *i.e.*, $c = 1 - s : A \in \boldsymbol{F^*} \to c(A) \in [0,1]$, because it is a geometrically hereditary result as we can see in figure 2.2. Thus the *potential confirmation locus* of an action $x \in \boldsymbol{X}$ can be defined as follows: for any $c_o \in [0,1], u^+ = g_x(c_o) = \sup\{u_x^+(\omega) : c(\{\omega\}) = c_o, \omega \in \boldsymbol{\Omega}\}$ and $u^- = l_x(c_o) = \sup\{|u_x^-(\omega)| : c(\{\omega\}) = c_o, \omega \in \boldsymbol{\Omega}\}$.



[Figure 2.2]

Hence at step 2 for each decision option x in $\boldsymbol{X}$, the decision-maker is thought to focus on the most important two pairs of each consisting of a potential confirmation and payoff values. The more important a (c, u) is, the greater ability it has to grab the decision-maker's attention. On the basis of this reasoning, we can introduce a pair of real valued functions $a = (a^-, a^+)$ assigning a non-negative numerical value to each $(c, u^+) \in [0,1] \times U_x^+$ or $(c, u^-) \in [0,1] \times U_x^-$. This combined function a is called the *attractiveness function.*[10] The determination of those pairs of gain and loss emerges from maximizing an attractiveness

[8] See Katzner (1986-7; 1987-88; 1989-90; 1998).

[9] The potential confirmation value $c(A)$ may have its own distinctive functional form than $1 - s(A)$ if it is necessary. For detailed explanation, see Katzner (1998) p.62.

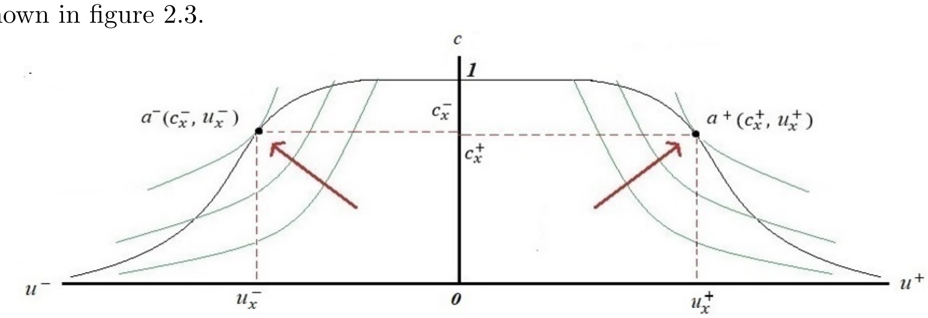
[10] In expected utility theory or other variant types of modern decision theories, the future value or importance of an act determined by both the payoff and its corresponding degree of uncertainty (probability), which is assumed to have the multiplicative form of those two

function subject to $u^+ = g_x(c)$ and $u^- = l_x(c)$. In each case, the higher the function value, the greater the attention-grabbing power. The properties of attractiveness a^+ and a^- can be assumed as $\frac{\partial a^+(c,u^+))}{\partial u^+}, \frac{\partial a^-(c,u^-))}{\partial u^-} > 0$ and $\frac{\partial a^+(c,u^+))}{\partial c}, \frac{\partial a^-(c,u^-))}{\partial c} > 0$, $u^+ \in U_x^+, u^- \in U_x^-$ and $c \in [0,1]$ where the partial derivative at a boundary point is defined as the right derivative.

For a given attractiveness function a^-, a^+ on $[0,1] \times U_x^-$ and $[0,1] \times U_x^+$ respectively, we can obtain a series of curves along which attractiveness values are constant. We will call those curves *iso-attractiveness contours* as described in figure 2.2 and 2.3. For any act x in $\boldsymbol{X}$ and the associated potential confirmation locus and attractiveness functions, the decision-maker can find the most attention-grabbing points $(c_x^+, u_x^+) \in [0,1] \times U_x^+$ and $(c_x^-, u_x^-) \in [0,1] \times U_x^-$ by solving the two constrained optimization problems below.

$$\begin{cases} \max\limits_{(c,u)} a^+(c,u) \text{ such that } u = g_x(c) \text{ where } (c,u) \in [0,1] \times U_x^+, \\ \max\limits_{(c,u)} a^-(c,u) \text{ such that } u = l_x(c) \text{ where } (c,u) \in [0,1] \times U_x^-. \end{cases} \tag{1}$$

The maximizing solutions occur at tangencies between the potential confirmation locus and iso-attractiveness contours of the attractiveness function as shown in figure 2.3.



[**Figure 2.3**: maximizing attractiveness *s.t* the potential confirmation locus]

Definition 2.3: The point (c_x^+, u_x^+) and (c_x^-, u_x^-) satisfying **(1)** are called the *focus-gain* of $\boldsymbol{x}$ and the *focus-loss* of $\boldsymbol{x}$, respectively. The set of pairs for focus-gain and focus-loss for all x in $\boldsymbol{X}$ is called the *attractiveness path.*

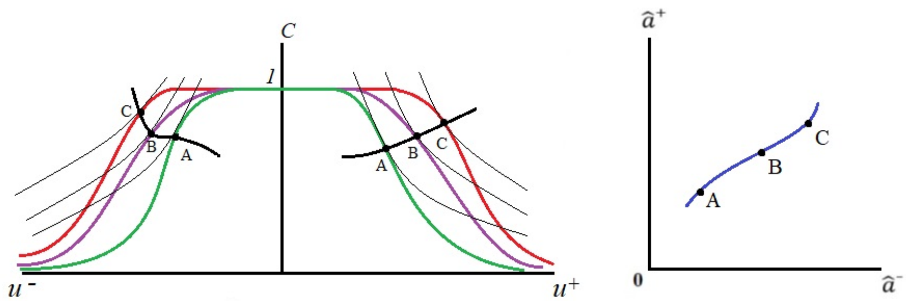
By maximizing the attractiveness function, the less important pairs of payoff and potential confirmation values is excluded, and the decision-maker's focus is restricted to only the most attractive pairs of payoff and potential surprise

numeric, *e.g.*, $expectation = probability \times payoff$. Rather than adopting the multiplicative function, Shackle defined attractiveness (in Shackle's original terminology, ascendancy) function in the abstract form. Meanwhile, when we apply Shacklean framework to the portfolio selection in next section, we will adopt the multiplicative attractiveness function.

values for each action, focus-gain and focus-loss. Then $[-a(c_x^-, u_x^-), a(c_x^+, u_x^+)]$ is the range of the importance (attractiveness) values for each x, and the focus-loss and gain are the arguments of two end points, *i.e.*, the minimum and maximum importance of each x.

Now we can redefine the domain of the attractiveness function to the entire set $\boldsymbol{X}$ of available actions by $\hat{a}^+, \hat{a}^- : \boldsymbol{X} \to \mathbb{R}$, $\hat{a}^+(x) = a^+(c_x^+, u_x^+)$ and $\hat{a}^-(x) = a^-(c_x^-, u_x^-)$ for all $x \in \boldsymbol{X}$. The collection of all pairs of focus-gain and focus-loss for all x in $\boldsymbol{X}$ generates the attractiveness path as the figure **2.4**.

Definition 2.4: Let $\mathbb{R}_+$ be the set of non-negative real numbers. The *attractiveness index* of an act $x \in \boldsymbol{X}$ is defined by $\hat{a} : \boldsymbol{X} \to \mathbb{R}_+ \to \mathbb{R}_+$ where $\hat{a}(x) = (\hat{a}^-(x), \hat{a}^+(x))$. The function values of $\hat{a}$ on $\boldsymbol{X}$ are assumed to form a connected path in $\mathbb{R}_+ \to \mathbb{R}_+$ called the *decision path.*



[**Figure 2.4**: the attractiveness path and the decision path]

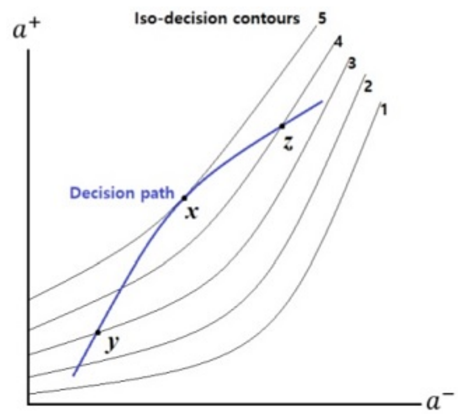
The attractiveness path and the decision path are illustrated in figure 2.4 for the cases in which the graphs of three potential confirmation locus are nested within each other. In figure 2.4, we can see the decision path along points A, B, and C derived from the attractiveness path points A, B, and C respectively. Given set of attractiveness foci for all selectable acts, the decision process reduces to evaluating and comparing foci pairs. For this process, we need to introduce another functional tool called the decision index.

Decision function, decision index

Now it is **step 3**. From the previous step involving the attractiveness function and the attractiveness index, we specified an attractiveness path and its corresponding decision path. The decision path in the plane of (a^-, a^+) [11] summarizes the information regarding the possible values of maximum and minimum attractiveness/importance from the given choice situation. For all possible pairs

[11] The plane on which the decision path is drawn in figure 2.4 and 2.5 can be represented by either $(\hat{a}^-, \hat{a}^+)$ or (a^-, a^+).

of $a^-(c_x^-, u_x^-), a^+(c_x^+, u_x^+)$ for each $x \in \boldsymbol{X}$, define a function $d : \mathbb{R}_+ \times \mathbb{R}_+ \to \mathbb{R}$ that assigns the rank of 'final value' for the decision-making. This function is said to be the *decision function.* Decision function reflects the preference of decision-maker regarding the optimistic and pessimistic aspect of each action. Then we can obtain a series of curves indicating each level of 'final' values. For an arbitrary $k \in \mathbb{R}$, the set$\{(a^-, a^+) \in \mathbb{R}_+ \times \mathbb{R}_+ : d(a^-, a^+) = k\}$ is called an *iso-decision contour.*[12] As we introduced the attractiveness index from the attractiveness function, we can extend the domain of the decision function to the entire set $\boldsymbol{X}$.



[**Figure 2.5**: Iso-decision contours and decision path]

Definition 2.5: The *decision index* on $\boldsymbol{X}$ is a function $\hat{d} : \boldsymbol{X} \to \mathbb{R}$ for which $\hat{d}(x) = d(\hat{a}^-(x), \hat{a}^+(x))) = d(a^-(c_x^-, u_x^-), a^+(c_x^+, u_x^+))$.

The final choice x^o in $\boldsymbol{X}$ is determined by solving the following maximization problem:

$$\max_{x \in \boldsymbol{X}} \hat{d}(x).$$

In figure 2.5, the act x has the highest value in decision index. Hence the decision-maker finally chooses x.

Summary of the decision process in Shacklean framework

The entire process of decision-making described in the Shacklean framework can be summarized as follows. For each act x, potential confirmation values and corresponding payoffs of possible future states are determined. Specifically, the decision-maker is imputed a standard value u_o determining each payoff as either gain or loss. These two series of information for each act x generate its potential confirmation locus. The importance of all imaginable scenarios for each

[12] Shackle called this *gambler-preference map.*

x are evaluated by attractiveness functions. By maximizing the attractiveness function subject to the mental constraint of the potential confirmation locus for each x, the decision-maker finds the pairs of potential confirmation values and corresponding gain and loss which give the highest degree of importance respectively. Defined on these pairs over all possible acts, the decision index assigns a final rank to each x in $\boldsymbol{X}$. The highest ranked act in $\boldsymbol{X}$ will then be chosen.

3 Portfolio adjustment under uncertainty

To illustrate the application of the decision model developed here, now we analyze the selection of a simple portfolio containing two non-monetary assets A, B whose nominal value is subject to change under uncertainty. In next section, we will analyze the selection of a portfolio containing money and a single non-monetary asset.[13]

Money has a stable nominal value while the nominal value of non-monetary assets is uncertain and subject to change. In this section, all the available money is invested to two non-monetary assets and the payoff (gain or loss) of the portfolio depends on the unknown future price of the non-monetary assets.

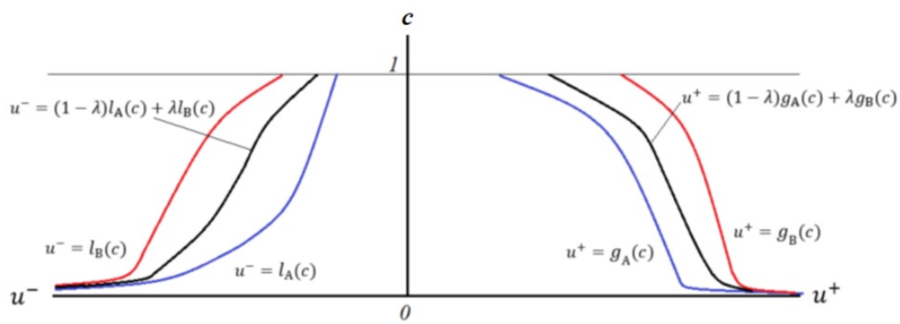
Suppose that fractions of available funds invested to non-monetary assets A and B are represented by the vector $(1-\lambda, \lambda)$ where $0 \leq \lambda \leq 1$. $\lambda = 0$ means that all available funds are invested in the asset A and $\lambda = 1$ means investing all into B. Then the set of all investment acts represented by the mixture ratio λ is $\boldsymbol{X} = \{\lambda : 0 \leq \lambda \leq 1\}$. As λ increases from 0 to 1, the potential confirmation locus continuously shifts from $g_A(c), l_A(c)$ to $g_B(c), l_B(c)$. The payoff of the portfolio depends on the future value of two assets and the mixture ratio λ.

Potential confirmation locus and the mixture asset

To simplify the situation, we assume the following property of the potential confirmation locus.

Assumption 1 The potential confirmation loci $u^+ = g_A(c), u^- = l_A(c)$ and $u^+ = g_B(c), u^- = l_B(c)$ are at least twice continuously differentiable.

[13]Discussions of portfolio selection in the Shacklean framework have appeared in Vickers (1978), Ford (1983; 1994) and Katzner (1998; 2001).



[**Figure 2.6**:potential confirmation locus of the mixture asset]

The potential confirmation locus for a mixture of two assets A, B can be written by

$$u^+ = (1-\lambda)g_A(c) + \lambda g_B(c), \quad u^- = (1-\lambda)l_A(c) + \lambda l_B(c). \tag{2}$$

As λ moves from 0 to 1, the potential confirmation locus continuously moves from $u^+ = g_A(c), u^- = l_A(c)$ to $u^+ = g_B(c), u^- = l_B(c)$ as figure 2.6.

It is worth noting that, being different to the random variable in probability theory, here a future event or a hypothesis are represented by each potential confirmation value $c \in [0,1]$ so that the portfolio return for an arbitrary mixture ratio λ is calculated by the weighted average of two payoffs of the asset A and B for each c. [14]

Portfolio attractiveness function

The next step is to describe the graphical pattern of the attractiveness path as the mixture ratio λ varies from 0 to 1. Recall that the attractiveness function evaluates pairs containing gains or losses with their associated potential confirmation value with respect to importance or attention-grabbing power. Following the custom of multiplicative functional form in the expected utility theory and other mainstream decision theories, we will specify a form of attractiveness function as following:

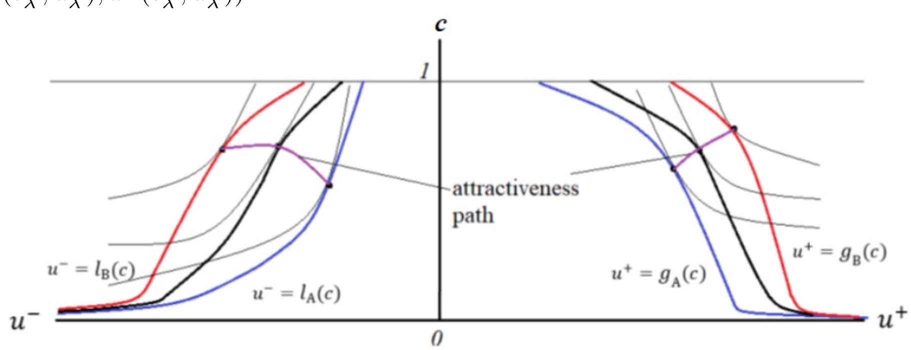
[14] In Katzner's formalization (Katzner 1986-87; 1998), he introduced a similar concept of random variable and defined the *potential surprise/confirmation density function*. But here we do not follow his methodology. Actually, a decision-maker does not care about the concrete name or contents of an event, but only its (speculated) degree of certainty and its corresponding payoffs. So each event of the future can be represented by either a payoff value as a random variable or a degree of certainty. Probability theory takes the former and here we choose the latter. Hence in this study, the event (or events) determining a payoff is represented by each $c \in [0,1]$. This stance determines the formula (2).

Definition 3.1: The *multiplicative attractiveness function*[15] a^+ on $[0,1] \times U_x^+$ and a^- on $[0,1] \times U_x^-$ is defined by $a^+(c,u^+) = c \cdot u^+, a^-(c,u^-) = c \cdot u^-$.

Note in definition 3.1 that u^+, u^- are positive because ${U_x}^+, {U_x}^-$ are the set of degree of gain and loss. Later when we reflect the attractiveness values in decision function and index, the sign of loss value will be included.

We can understand the multiplicative attractiveness function from a different viewpoint. Since the multiplicative attractiveness is the product, like the subjective expected utility, between the degree of subjective confidence of confirmation and the possible gain or loss, the attractiveness at the focus-gain and focus-loss can be interpreted in each act x as the maximum potential gain and loss out of all possible hypotheses about the future. [16]

The shape of attractiveness path for two non-monetary assets is hard to assess beforehand. The properties of two gain loci $g_A(c), g_B(c)$ and two loss loci $l_A(c), l_B(c)$ determine the pattern of the linear combination (2) and the trace of the attractiveness path as the figure 3.2. As the ratio λ combining two assets A and B changes from 0 to 1, the attractiveness path is drawn by tracing the points of tangency between the potential confirmation locus and the iso-attractiveness contour. We have called the tangency point for each λ the focus-loss and the focus-gain of λ and denoted them by $(c_\lambda^-, u_{\lambda^-}), (c_\lambda^+, u_\lambda^+)$. Then the attractiveness index for an act λ is represented as $\hat{a}(\lambda) = (\hat{a}^-(\lambda), \hat{a}^+(\lambda)) = (a^-(c_\lambda^-, u_\lambda^-), a^+(c_\lambda^+, u_\lambda^+))$.



[**Figure 3.2**: the attractiveness path of two non-money asset A and B]

The decision path is obtained by transferring the attractiveness path into

[15]The multiplicative functional form of uncertainty and payoff values, often called 'expectation' and derived from the notion of average, has been the universal way of evaluation since 17th century (See Hacking 1975). Expected utility (V.Neumann and Mogensern 1944; Savage 1954) is constructed as $p \cdot u$, where u is the payoff and p is its probability of realization. Many types of non-expected utility theories (Kahneman-Tversky 1979; Quiggin 1982; Yaari 1987, Hey 1984; Loomes-Sugden 1982) and generalized expected utility theory (Machina 1982) also use the multiplicative form with an adjusted probability function and an appropriate payoff or value function.

[16] Of course, if the attractiveness function does not take the multiplicative form, such identification between attractiveness and potential gain is not trivially justifiable.

$\mathbb{R}_+ \times \mathbb{R}_+$. The shape of the decision path will have a crucial role later in determining the optimal portfolio between two non-monetary assets, and it is determined by varying patterns of $\hat{a}^+(\lambda)$ relative to $\hat{a}^-(\lambda)$. For this task, we should analyze the attractiveness function through arbitrary mixture ratio λ. For a given confirmation loci $(g_A(c), l_A(c))$, $(g_B(c), l_B(c))$ of the assets A and B, the portfolio attractiveness functions of A and B is $A^+, A^- : [0,1] \times [0,1] \times \mathbb{R}$ where for$(\lambda, c) \in [0,1] \times [0,1]$, $A^+(\lambda, c) = (1-\lambda)[cg_A(c)] + \lambda[cg_B(c)]$ and $A^-(\lambda, c) = (1-\lambda)[cl_A(c)] + \lambda[cl_B(c)]$. Trivially, $A^+(0,c) = cg_A(c)$, $A^-(0,c) = cl_A(c)$, $A^+(1,c) = cg_B(c)$ and $A^-(1,c) = cl_B(c)$. Then we can obtain the following two properties regarding the attractiveness path.

Proposition 3.2

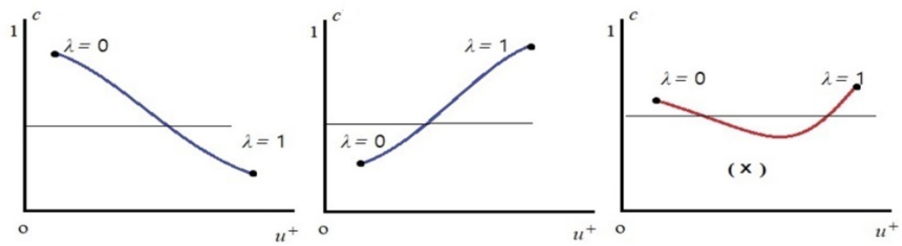
i) There exists a connected path $R^+(\lambda, c)$ in $[0,1] \times [0,1]$ such that for every $(\lambda^*, c^*) \in R^+(\lambda, c)$, if the mixture ratio $\lambda^* \in [0,1]$ is given, then $A^+(c) \mid_{\lambda=\lambda^*}$ is maximized at $c^* \in [0,1]$. For the loss side, $R^-(\lambda, c) \subset [0,1] \times [0,1]$ also exists in a similar manner.

ii) For any c^* there exists the unique pair λ_1^*, λ_2^* such that (λ_1^*, c^*) is contained in $R^+(\lambda, c)$ and (λ_2^*, c^*) is in $R^-(\lambda, c)$. In other words, for a potential confirmation c^*, the unique pair of mixture ratios λ_g^* and λ_l^* maximizes A^+ and A^-, respectively.

It is trivial to verify the above propositions. For *i*), since the differentiability of two confirmation loci implies that the portfolio attractiveness function is differentiable on $[0,1] \times [0,1]$. Then it implies that the partial derivatives with respect to λ, c and the directional derivative exist at every point in $[0,1] \times [0,1]$, so it is possible to take the direction having the maximum directional derivatives at the maximum point of $A^+(c) \mid_\lambda$ and $A^-(c) \mid_\lambda$ for each $0 < \lambda < 1$. Starting from $\lambda = 0$ and going toward $\lambda = 1$, we can obtain the paths $R^+(\lambda, c)$ and $R^-(\lambda, c)$.

For the proposition *ii*), suppose that for an arbitrary c_o there are $\lambda_{g1} \neq \lambda_{g2} \in [0,1]$ such that (λ_{g1}, c_o), $(\lambda_{g2}, c_o) \in R^+(\lambda, c)$. By the first order condition for the maximization of A^+ with respect to c, we have $\lambda_{g_1} \frac{d(c(g_B(c)-g_A(c))}{dc} + \frac{d(cg_A(C))}{dc} = \lambda_{g_2} \frac{d(c(g_B(c)-g_A(c))}{dc} + \frac{d(cg_A(C))}{dc}$ at $c = c_o$. To preserve the premise $\lambda_{g1} \neq \lambda_{g2}$, $\frac{d\{cg_A(c)\}}{dc} = \frac{d\{cg_B(c)\}}{dc}$ must hold. This is equivalent to $h(c) = ch'(c)$ where $h(c) = g_B(c) - g_A(c)$. By solving the simple differential equation, we can see that this holds only when $g_B(c) - g_A(c) = K/c$ for some constant K. But there is no economic reason to justify that the payoff of an asset is exactly higher or lower as K/c for each possible event than the other. In othwer words, unless the financial asset B is artificially and deliberately manufactured with respect to the payoff of the asset A in this way, *e.g.*, some purpose of financial engineering, such case is extremely exceptional and need not to be considered. Thus, by excluding this, we can assure that $\lambda_{g_1} = \lambda_{g_2}$, *i.e.*, λ_g is unique. The similar reasoning can be applied to A^-.

By proposition *3.2.i*), the attractiveness path previously defined by the payoff and the potential confirmation is now recharacterized in terms of the mixture ratio and the potential confirmation. Proposition *3.3.ii*) means that as the mixture ratio λ varies from 0 to 1, $R^+(\lambda, c)$ and $R^-(\lambda, c)$ gives strictly monotone attractiveness paths with respect to c, as in the first two graphs in figure 3.3 unless two confirmation loci intersect.



[**Figure 3.3**:the shape of the attractiveness path]

Slope of the decision path

On the basis of proposition *3.2.ii*), visualized in the first two diagrams of figure 3.3, we can introduce the portfolio confirmation function $m_+, m_- : [0,1] \to [0,1]$, $m_+(\lambda) = c^+$ and $m_-(\lambda) = c^-$ satisfying $(\lambda, c^+) \in R^+(\lambda, c)$ and $(\lambda, c^-) \in R^-(\lambda, c)$. The portfolio confirmation function m_+, m_- is 1-1 and assigns the value of confirmations for gains and losses, which maximizes the attractiveness, to each mixture ratio $0 \le \lambda \le 1$. Basically, the graph of the functions m_+, m_- from the mixture ratio to the confirmation is $R^+(\lambda, c), R^-(\lambda, c)$, which is the attractiveness path embedded in $[0,1] \times [0,1]$.

Previously, the potential confirmation function c expressed the relation between possible hypotheses for the future, which is defined over Ω, and their corresponding degree of subjective confirmation. Meanwhile, here the portfolio confirmation function $m_+(\lambda)$ and $m_-(\lambda)$ has been interpreted as the degree of subjective confidence of confirmation for the occurrence of the maximum potential gain (attractiveness at the focus-gain) and loss (attractiveness at the focus-loss) when a mixture ratio λ of the portfolio is given. These two functions m_+, m_- will be used when we describe the shape of the decision path in terms of the decision index $\hat{a^+}$ and $\hat{a^-}$.

For each λ, the portfolio payoffs on the attractiveness path can be specified with m_+, m_- as follows:

$$\begin{cases} G(\lambda) = (1-\lambda) g_A(m_+(\lambda)) + \lambda g_B(m_+(\lambda)), \\ L(\lambda) = (1-\lambda) l_A(m_-(\lambda)) + \lambda l_B(m_-(\lambda)). \end{cases} \tag{3}$$

Since we assumed the multiplicative attractiveness function, the portfolio attractiveness for each $0 \le \lambda \le 1$ is as follows:

$$\begin{cases} \hat{A}^+(\lambda) = A^+(m_+(\lambda), G(\lambda)) = m_+(\lambda)G(\lambda), \\ \hat{A}^-(\lambda) = A^-(m_-(\lambda), G(\lambda)) = m_-(\lambda)G(\lambda) \end{cases} \tag{4}$$

where $A^+(c^+, u^+) = c^+u^+$ and $A^-(c^-, u^-) = c^-u^-$ and (c^+, u^+) and (c^-, u^-) are on the attractiveness path.

Note that the values of l_A, l_B and L are non-negative because in calculating the attractiveness, we deal with only the magnitude of loss. Also, since we are dealing with attractiveness maximizing points on each λ, the confirmation values here have the superscript $+, -$.

The derivative of the attractive index in gain side with respect to the mixture ratio λ is as follows:[17]

$$\frac{\hat{A}^+(\lambda)}{d\lambda} = G(c^+)(1 + \eta_{c^+}^{u^+})m'_+(\lambda) \tag{5}$$

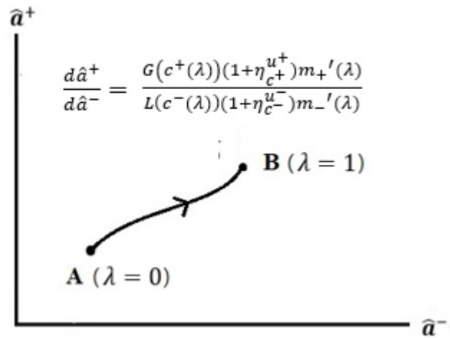
Here, $\eta_{c^+}^{u^+} = \frac{d(G(c^+)/G_c^+}{dc^+/c^+}$. Note that the one-to-oneness of m_+ implies that each level of mixture ratio can be identified with the corresponding level of confirmation. In other words, the selection of a certain portfolio - represented by λ - corresponds to the acceptance of that perceived degree of confidence. When an investor speculates about adjusting the portfolio slightly towards a greater degree of certainty, *e.g.*, substituting a portion of stock for MMF, this involves the sacrifice of potential gain and the alleviation of the potential loss in correspondence to that adjustment. In **(5)**, $\eta_{c^+}^{u^+}$ indicates the expected responsiveness of payoffs relative to changes of mixture ratio λ so its corresponding degree of confidence.

Similarly, we can obtain $L(c^-) = \frac{d\hat{a}^-}{d\lambda} = (1 + \eta_{c^-}^{u^-})$ for the loss side. Then the slope of the decision path is calculated as the following:

$$\frac{d\hat{a}^+}{d\hat{a}^-} = \frac{d\hat{a}^+/d\lambda}{d\hat{a}^-/d\lambda} = \frac{G(C^+(\lambda))(1 + \eta_{c^+}^{u^+})m'_+(\lambda)}{L(C^-(\lambda))(1 + \eta_{c^-}^{u^-})m'_-(\lambda)} \tag{6}$$

From now, we will narrow down the range of our analysis only to the upward sloping decision path. In the plan of $(\hat{a}^-, \hat{a}^+)$, the location of asset A is the lower left endpoint of the positive sloped decision path connecting A to B while the asset B lies at the upper right endpoint. This is a reasonable simplification in the consideration of the portfolio choice. If the decision path is negative-sloped or flat, it implies that the asset A has a lower maximum potential loss and a higher or equal maximum potential gain than the asset B. Since we assume the multiplicative attractiveness function, $\hat{a}^-, \hat{a}^+$ can be identified as maximum potential loss and gain. Thus, the negative or flat decision path implies that there is no need of trade-off.

[17] $\frac{d\hat{A}^+}{d\lambda} = m'_+(\lambda)G(\lambda) + m_+(\lambda)G'(\lambda) = G(\lambda)[1 + \frac{m_+(\lambda)}{m'_+(\lambda)}\frac{G'(\lambda)}{G(\lambda)}]m'_+(\lambda) = G(\lambda)[1 + \frac{g'(\lambda)/G(\lambda)}{m'_+(\lambda)/m_+(\lambda)}]m'_+(\lambda)$



[**Figure 3.4**: Iso-decision contours and decision path]

Assumption 2 $\quad \hat{a}(0) = (\hat{a}^-(0), \hat{a}^+(0)) < \hat{a}(1) = (\hat{a}^-(1), \hat{a}^+(1))$.

To illustrate factors determining the slope of the decision path, we now interpret terms of **(6)** one by one. First, $G(c^+(\lambda))/L(c^-(\lambda))$ is the ratio of gain and loss for each mixture ratio. It must be positive because L indicates only the magnitude of loss. Second, as we have seen in **(5)**, $m_+(\lambda)$ and $m_-(\lambda)$ specify the degrees of confirmation regarding the occurrence of the maximum potential gain and the maximum potential loss for an arbitrarily chosen mixture ratio λ. Since the sign of $m_+^{'}(\lambda)$ and $m_-^{'}(\lambda)$ do not change by proposition *3.2. ii)*, the sign of $\frac{m_+^{'}(\lambda)}{m_-^{'}(\lambda)}$ must be same for all $0 \leq \lambda \leq 1$. Furthermore, $m_+^{'}(\lambda)$ and $m_-^{'}(\lambda)$ should have the identical sign if the situation of trade-off is in effect. For instance, if $m_+^{'} > 0$ but $m_-^{'} < 0$, it implies that, for example, as the portion of asset B increases, the degree of confirmation for the maximum potential gain becomes stronger but the confirmation for the maximum potential loss becomes weaker. This means that the necessity for the portfolio diversification does not exist and switching all the fund to the asset B is the rational choice. So the choice situation no longer exists. Thus, we can assume the following.

Assumption 3 $\quad \frac{m_+^{'}(\lambda)}{m_-^{'}(\lambda)} > 0,\ 0 < \lambda < 1$

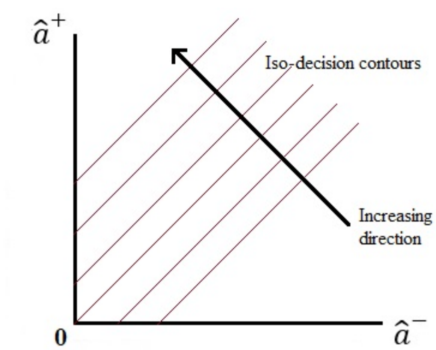
The purpose of assumptions 2 and 3 is to focus on the analysis of trade-off situation between A and B. Then the remaining factor influencing the slope of the decision path is $\frac{1+\eta_{c^+}^{u^+}}{1+\eta_{c^-}^{u^-}}$, which implies the relative volatility between the maximum gain and the maximum loss in the decision-maker's mind. The assumption 2 implies that $\frac{1+\eta_{c^+}^{u^+}}{1+\eta_{c^-}^{u^-}} < 0$ cannot be maintained through all $\lambda \in [0, 1]$, but the negative slope of it can be only partly possible. Under the assumption 2 and 3, the result (6) implies that the slope of decision path at each mixture ratio, the main factor determining the motivation of portfolio adjustment at each decision moment, is influenced by the volatility of gain and loss in response to the level of confidence as anticipated by the decision-maker's mind.

The feasibility of portfolio diversification

We have obtained the decision path on which each point is constituted by the maximum potential gain and the maximum potential loss for each λ. Now in order to obtain the final decision out of this locus of points, let us further specify the decision index.

Definition 3.3: The *additive decision index* $\hat{d} : \boldsymbol{X} \to \mathbb{R}$ is defined by $\hat{d} = \alpha\hat{a}^{+}(x) - \beta\hat{a}^{-}(x)$ for x in $\boldsymbol{X}$.

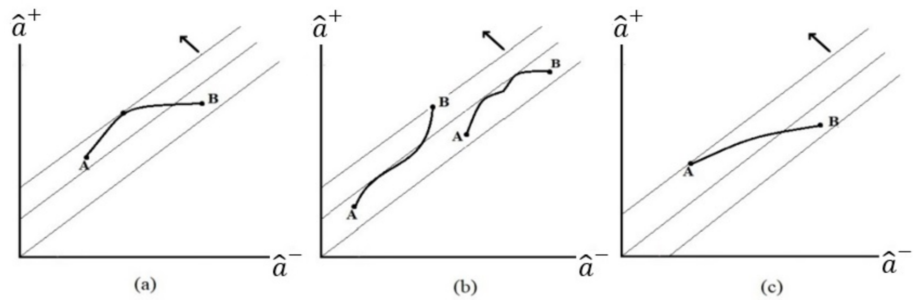
Here, $\alpha, \beta > 0$ indicate the relative weights to the maximum potential gain $\hat{a}^{+}$ and the maximum potential loss $\hat{a}^{-}$ respectively by the decision-maker. Although minus sign is included in the above definition, it means just simple summation of the maximum potential gain and the maximum potential loss, $-\hat{a}^{-}(x) < 0$. The additive decision index indicates how much the subjectively rejudged value of the maximum potential gain dominates the one of the loss-side in choosing a specific ratio between two assets.



[**Figure 3.5**: The iso-decision contours of the additive decision index]

Iso-decision contours of the additive decision index are represented by $z = \alpha\hat{a}^{+} - \beta\hat{a}^{-}$ for various constants z in the plane of $\hat{a}^{+}$ and a $\hat{a}^{-}+$ in figure 3.5. Each iso-decision contour is linear, and the slope is $\frac{\alpha}{\beta}$. If there is no subjective distortion in effect, then $\alpha = \beta$, and the iso-decision contours are 45^{o} sloped as figure 3.5. Movement up and to the left increases the value of z. The following proposition shows an example of portfolio composition between A and B under restricted condition as seen in the diagram (a) of figure 3.6.

Proposition 3.4 Let $\hat{d_A} = \alpha\hat{a}^+(\lambda) - \beta\hat{a}^-(\lambda)$ be the additive decision index on the space of the mixture ratio $\boldsymbol{X} = [0,1]$. Suppose a decision path from A to B satisfies $\frac{d\hat{a}^+}{d\hat{a}^-}(\lambda) > 0$ and $\frac{d^2\hat{a}^+}{d(\hat{a}^-)^2} \leq 0$. If $\frac{\alpha}{\beta} \in \{\frac{d\hat{a}^+}{d\hat{a}^-}(\lambda) : \lambda \in (0,1)\}$, then there exists $\lambda^* \in (0,1)$ such that $\hat{d}(\lambda^*) = \max\{\hat{d}(\lambda) : \lambda \in [0,1]\}$.



[**Figure 3.6**: the portfolio selection under the additive decision function]

It is easy to verify Proposition 3.4. Under the strict concavity of the decision path, the tangency between the decision path and an iso-decision contour of the additive decision index is unique. But, as in figure 3.6 (b) and (c), the tangency may exist at the corner of the decision path or the non-optimal positions. Thus, to guarantee 'actual' diversification, the mixture ratio of two assets at the tangency point ought to be between 0 and 1. Intuitively, the situation described in proposition 3.4 means that when the trade-off is in effect ($\frac{d\hat{a}^+}{d\hat{a}^-}(\lambda) > 0$), yet the merit of increasing portion of the asset B is gradually decreasing ($\frac{d^2\hat{a}^+}{d(\hat{a}^-)^2} \leq 0$), the buying asset B will stop at the moment which the potential maximum gain and loss offset each other in his subjective speculation (α, β). Beyond that portion λ^* of asset B, the maximum potential loss dominates the maximum potential gain.

As figure 3.7, even when the decision index has strictly quasi concave iso-decision contours, the optimal portfolio may not be diversified. [18] To guarantee

[18] The decision index generating strictly quasi-concave iso-decision contours can be defined as follows. Let $\hat{d_c}(\lambda)$ be a decision index from $X = [0,1]$ to $\mathbb{R}$. If for any distinct $\lambda_1, \lambda_2 \in \boldsymbol{X}$ with their corresponding $\hat{a}^+(\lambda_1)$, $\hat{a}^+(\lambda_2)$ and any μ where $0 < \mu < 1$, if $\hat{d_c}(\lambda_1) = \hat{d_c}(\lambda_2)$ implies $\hat{d_c}((1-\mu)a(\lambda_1) + \mu a(\lambda_2)) > \hat{d_c}(\lambda_1)$, then $\hat{d_c}$ is said to be the strictly quasi-concave decision index on $\boldsymbol{X}$. Then we can derive a similar proposition to proposition 4.11 for $\hat{d_c}$.

Let $\hat{d_c}(\lambda)$ be a smooth strictly quasi-concave decision index on the set of mixture ratio $\boldsymbol{X} = [0,1]$ to $\mathbb{R}$. Suppose a decision path from A to B satisfies $\frac{d\hat{a}^+}{d\hat{a}^-}(\lambda) > 0$ and $\frac{d^2\hat{a}^+}{d(\hat{a}^-)^2} \leq 0$. If the following condition holds, then there exists $\lambda^* \in (0,1)$ such that λ^* satisfies $\max\{\hat{d_c}(\lambda)\} = \hat{d_c}(\lambda^*)$, $\lambda \in [0,1]$.

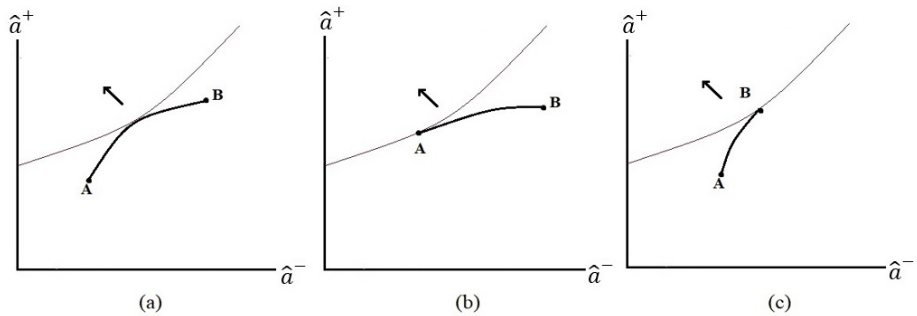
$$\{(\lambda, \frac{d\hat{a}^+}{d\hat{a}^-}(\lambda)) : \lambda \in (0,1)\} \cap \{(\lambda, \frac{\partial\{\hat{d_c}(\hat{a}^-,\hat{a}^+)\}}{\partial\hat{a}^+} / \frac{\partial\{\hat{d_c}(\hat{a}^-,\hat{a}^+)\}}{\partial\hat{a}^-}) \,|_{(\hat{a}^-(\lambda),\hat{a}^+(\lambda))} : \lambda \in (0,1)\} \neq \emptyset.$$

Here the former means the slope of the decision path and the latter is the slope of the iso-decision contour.

the portfolio diversification, similar premises as proposition 3.4 should be added.

The implication of the additive decision index is how much the maximal potential gain (the attractiveness at the focus-gain) dominates the maximal potential loss (the attractiveness at the focus-loss). Practically, the additive decision index suggests that the decision-maker has the ability to evaluate and compare the maximal potential gain and loss, and to calculate the exact difference between them. Not only that, the strictly quasi-concave decision index presumes even more advanced knowledge and ability of the decision-maker to assign varying degree of relative weights between gain and loss. On a specific iso-decision contour, as the potential maximum loss is increasing, it requires the premium of the higher maximum potential gain in order to compensate it, and this is represented by the steeper slope of iso-decision contour as $\hat{a}^-$ increases.

However, if the decision-maker can only naively guess the maximum potential gain and loss but cannot consider how the one dominates the other, then this situation may cause drastic panic-shifting between the two assets. Such characteristic of the naive decision-maker is reflected in a specific form of decision index. We will see this in the next section.



[**Figure 3.7**: the portfolio selection under the strictly quasi-concave iso-decision contours]

4 Portfolio adjustment and individual panic behavior

Maximal decision index and two non-monetary assets

To reflect the characteristic of naive investors, who are the main source of volatility in the stock market, we now introduce a new decision index as follows.

Definition 4.1: The *maximal decision index* $\hat{d_M} : \boldsymbol{X} \to \mathbb{R}$ is defined by

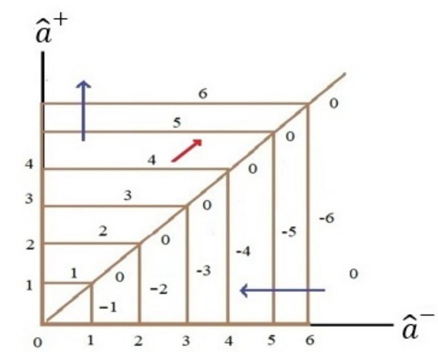
$$\hat{d_M} = \begin{cases} \max\{\hat{a}^-(\lambda), \hat{a}^+(\lambda)\} & \text{when } \hat{a}^-(\lambda) < \hat{a}^+(\lambda), \\ -\max\{\hat{a}^-(\lambda), \hat{a}^+(\lambda)\} & \text{when } \hat{a}^-(\lambda) > \hat{a}^+(\lambda), \\ 0 & \text{when } \hat{a}^-(\lambda) = \hat{a}^+(\lambda). \end{cases}$$

In terms of the multiplicative attractiveness index, the maximal decision index can be specified by the following.

$$\hat{d_M} = \begin{cases} \max\{m_-(\lambda) \cdot L(\lambda), m_+(\lambda) \cdot G(\lambda)\} & \text{when } m_-(\lambda) \cdot L(\lambda) < m_+ \cdot G(\lambda), \\ -\max\{m_-(\lambda) \cdot L(\lambda), m_+(\lambda) \cdot G(\lambda)\} & \text{when } m_-(\lambda) \cdot L(\lambda) > m_+ \cdot G(\lambda), \\ 0 & \text{when } m_-(\lambda) \cdot L(\lambda) = m_+ \cdot G(\lambda), \end{cases}$$

where m_+, m_- are portfolio confirmation and $G(\lambda)$,$L(\lambda)$ are the portfolio payoffs on the attractiveness path.

Here, the purpose of the minus sign is to make the attractiveness value back to the negative scale of loss in order to reflect in the functional value that the lower attractiveness (lower maximum potential loss) is the better outcome to the decision-maker.



[**Figure 4.1**: The iso-decision contours of the maximal decision index]

The intuition of definition 4.1 can be explained like this. When the naive decision-maker compares the maximum potential gain and loss, if the former outweighs the latter in the current market state, she will focus entirely on the maximum potential gain, excluding any consideration of loss whatsoever. This occurs due to their lack of any meaningful heuristic by which to compare the two magnitudes.

In other words, the decision-maker refers to only the dominant sentiment identified with the current optimistic market situation. The numbers arranged across 45^o line indicate the value of maximal decision index, *i.e.*, a sort of utility for the final decision, and the numbers through two axes are the attractiveness values (the magnitude of the maximum potential gain or loss). The horizontal iso-decision contours above the 45^o line means that the potential maximum gain dominates the potential maximal loss. In the area under the 45^o line, the loss exceeds the gain. Where the maximum potential gain and loss are equal, as on the 45^o, the situation may be regarded as neutral, *i.e.*, a zero value in the decision index. Let's divide the plane into three regions, call the set $\mathbb{O} = \{(\hat{a}^-, \hat{a}^+) : \hat{a}^- < \hat{a}^+\}$ *optimism*, $\mathbb{P} = \{(\hat{a}^-, \hat{a}^+) : \hat{a}^- > \hat{a}^+\}$ *pessimism*, and $\mathbb{I} = \{(\hat{a}^-, \hat{a}^+) : \hat{a}^- = \hat{a}^+\}$ *indeterminate*.

Now we are ready to explore the motivation of induvial panic behavior in portfolio adjustment in response to the gradual and continuous, but not sudden market change. It can be anticipated that the market change will update the potential confirmation and the corresponding payoff schedule. Then this involves the movement of the decision path.

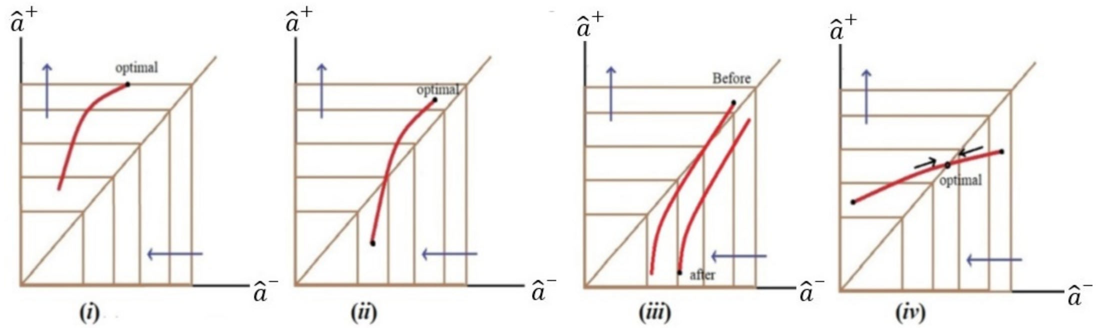
The following proposition explains the panic process of shifting between optimism and pessimism as changing information influences market sentiment and thus alters the decision path. The figure 4.2 exhibits each situation discussed in proposition 4.2

Proposition 4.2 Let $\hat{d_M}$ be a maximal decision index on $\boldsymbol{X} = [0, 1]$. Suppose a decision path from A to B satisfies $\frac{d\hat{a}^+}{d\hat{a}^-}(\lambda) > 0$. Then the followings hold.
i) **Being engrossed in optimism:**
If $\{(\hat{a}^-(\lambda), \hat{a}^+(\lambda)) : \lambda \in [0, 1]\} \subset \mathbb{O}$, then $\hat{d}_M(1) = \max\{\hat{d}_M(\lambda) : \lambda \in [0, 1]\}$.
ii) **Gradual but unrecognized change to pessimism:**
If $(\hat{a}^-(\lambda_o), \hat{a}^+(\lambda_o)) \in \mathbb{I}$, $\{(\hat{a}^-(\lambda), \hat{a}^+(\lambda)) : \lambda \in [0, \lambda_o)\} \cap \mathbb{P} \neq \emptyset$ and $\{(\hat{a}^-(\lambda), \hat{a}^+(\lambda)) : \lambda \in (\lambda_o, 1]\} \cap \mathbb{O} \neq \emptyset$, then $\hat{d}_M(1) = \max\{\hat{d}_M(\lambda) : \lambda \in [0, 1]\}$.
iii) **Panic adjustment**:
If $\{(\hat{a}^-(\lambda), \hat{a}^+(\lambda)) : \lambda \in [0, 1]\} \subset \mathbb{P}$, then $\hat{d}_M(0) = \max\{\hat{d}_M(\lambda) : \lambda \in [0, 1]\}$.
iv) **Gradual adjustment**:
If $\{(\hat{a}^-(\lambda), \hat{a}^+(\lambda)) : \lambda \in [0, \lambda_o)\} \subset \mathbb{O}$ and $\{(\hat{a}^-(\lambda), \hat{a}^+(\lambda)) : \lambda \in (\lambda_o, 1]\} \subset \mathbb{P}$, then $\hat{d}_M(\lambda_o) = \sup\{\hat{d}_M(\lambda) : \lambda \in [0, 1]/\lambda_o\}$. Furthermore, for any $\epsilon > 0$, $\lambda_o < \lambda < \lambda + \epsilon \implies \frac{d\hat{a}^+}{d\hat{a}^-} < 1$.

Proof. Verifying above propositions is geometrically trivial. For *i*), regardless of the value of $\hat{a}^-$, the value of decision index in $\mathbb{O}$ increases only when $\hat{a}^+$ increases. Since we know that $\frac{d\hat{a}^+}{d\hat{a}^-}(\lambda) > 0$, $\hat{d}_M(1) = \max\{\hat{d}_M : \lambda \in [0, 1]\}$. For *ii*), all points of decision path intersecting $\mathbb{P} \cup \mathbb{I}$ yield a lower value for the decision index than those of $\{(\hat{a}^-(\lambda), \hat{a}^+(\lambda)) : \lambda \in (\lambda_o, 1]\} \cap \mathbb{O}$. Since $\frac{d\hat{a}^+}{d\hat{a}^-}(\lambda) > 0$, $\hat{d}_M(1) = \max\{\hat{d}_M : \lambda \in [0, 1]\}$.

For *iii*), Since $\{(\hat{a}^-(\lambda), \hat{a}^+(\lambda)) : \lambda \in [0, 1]\} \subset \mathbb{P}$, and $\hat{a}^+ < \hat{a}^-$ for all $(a^-, a^+) \in \mathbb{P}$, the value of the decision index in the part of the decision path intersected with $\mathbb{P}$ is evaluated by only $\hat{a}^-$. Thus $d_M(0) = \max\{\hat{d}_M : \lambda \in [0, 1]\}$.

iv). For $\{(\hat{a}^-(\lambda), \hat{a}^+(\lambda)) : \lambda \in [0, \lambda_o)\} \in \mathbb{O}$, we have $\hat{a}^+(\lambda) > \hat{a}^-(\lambda)$. Due to $\frac{d\hat{a}^+}{d\hat{a}^-}(\lambda) > 0, \hat{d}_M(\lambda_o) = \sup\{\hat{d}_M(\lambda) : \lambda \in [0, \lambda_o)\}$. Verifying the other part for $(\lambda_o, 1]$ is similar. For the last statement, suppose that the conclusion does not hold, *i.e.*, $\frac{d\hat{a}^+}{d\hat{a}^-}(\lambda) \geq 1$. Then $\{(\hat{a}^-(\lambda), \hat{a}^+(\lambda)) : \lambda \in (\lambda_o, 1]\} \cap (\mathbb{O} \cup \mathbb{I}) \neq \emptyset$, which is contradiction. ■



[**Figure 4.2**]

As in proposition 4.2.i)(figure 4.2.(i)), as long as the decision-maker is optimistic, lacking any knowledge by which to compare the maximum potential gains and losses, their optimal decision is betting all-in on asset B. This behavior is frequently observed among naive investors following short-term bull market trends and ignoring potential reversals. Even with the onset of negative news, the decision-maker may stick to the current choice for a time. Once, however, the decision path begins shifting into the range of pessimism - the region $\mathbb{P}$ in in figure 4.8.(iii) - the investor entirely liquidates their holdings of B.

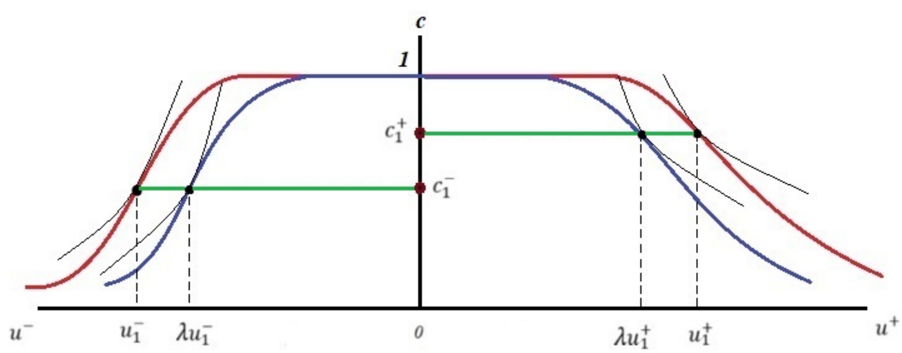
It is worth emphasizing that concentrating and switching to a specific asset happens here, even though the incentive to trade-off (positive sloping decision path) is still effective. Meanwhile, proposition 4.2.iv) describes a kind of interim status between two extreme choices but distinguished from the case of (ii). In the case of (iv), the momentum of potential gain is not strong enough to dominate the potential loss, so the decision path has relatively flatter slope. Thus, in spite of the higher potential gain of B than A, as the mixture ratio λ of B increases beyond λ_o, the anticipation of the investor is absorbed into the range $\mathbb{P}$ of negative sentiment. As mixture ratios approach to λ_o on $\mathbb{I}$ from both $\mathbb{P}$ and $\mathbb{O}$, the situation becomes gradually better off. If the decision path continuously moves through the diagonal line with intersecting $\mathbb{I}$ at a point, this case can be regarded as smooth adjustment of portfolio without any behavioral discontinuity.

In the portfolio selection between two non-monetary assets, we can explain the panic behavior as a myopic reaction of the naive investor. This is frequently observable in the reality so the behavioral implication of the maximal decision index is sensible *prima facie*. Furthermore, if money is included in the composition of the asset portfolio, the panic adjustment can happen even when the decision-maker is able to compare both aspects of potential gain and loss and to guess how the one dominates the other. Details will be discussed in the next section.

5 Money and panic

Now let's extend our analysis to the portfolio adjustment between money and a non-monetary asset. Here, the latter need not be restricted to only a single asset but it can be a diversified portfolio, say $\boldsymbol{P}$, made of arbitrary n assets. Properties of that asset portfolio $\boldsymbol{P}$ can be represented by its potential confirmation locus $g_P(c)$ and $l_P(c)$. In the appendix, the portfolio composed of n-assets is briefly explained. In this section, we focus on the case of money and a non-monetary assep portfolio $\boldsymbol{P}$.

By virtue of liquidity and stable nominal value, money is eligible as the universal alternative to other assets accompanying potential gain or loss. Suppose the quantity of money available to the decision-maker is 1 and any fraction λ of it may be invested in the non-monetary asset. The payoff (gain or loss) of the portfolio depends on the unknown future price of the latter. The multiplicative attractiveness function is intact here.



[**Figure 5.1**: the attractiveness path for money and a non-monetary asset]

Since the payoff of money ($\lambda = 0$) is fixed as 0 regardless of future states, the potential confirmation locus of money is the vertical line, $g_0(c) = l_0(c) = 0$. With respect to the non-monetary asset ($\lambda = 1$), the focus-gain (c_1^+, u_1^+) and the focus-loss (c_1^-, u_1^-) are the attractiveness maximizing pair of potential confirmation and payoff values obtained with the potential confirmation locus, $g_1(c)$,$l_1(c)$ as figure 5.1. Then the focus-gain $(c_\lambda^+, u_\lambda^+)$ and the focus-loss $(c_\lambda^-, u_\lambda^-)$ for any $\lambda \in (0, 1)$ are obtained by maximizing the multiplicative attractiveness function subject to the potential confirmation locus, $u^+ = g_\lambda(c) = \lambda g_1(c)$, $u^- = l_\lambda(c) = \lambda l_1(c)$. Thus the focus-gain and the focus-loss are given by $(c_\lambda^+, u_\lambda^+) = (c_1^+, \lambda u_1^+)$, $(c_\lambda^-, u_\lambda^-) = (c_1^-, \lambda u_1^-)$, [19] so $\hat{a^+}(\lambda) = a^+(c_\lambda^+, u_\lambda^+) = c_1^+ \cdot (\lambda u_1^+)$, $\hat{a}^-(\lambda) = a^-(c_\lambda^-, u_\lambda^-) = c_1^- \cdot (\lambda u_1^-)$. The attractiveness path on the gain

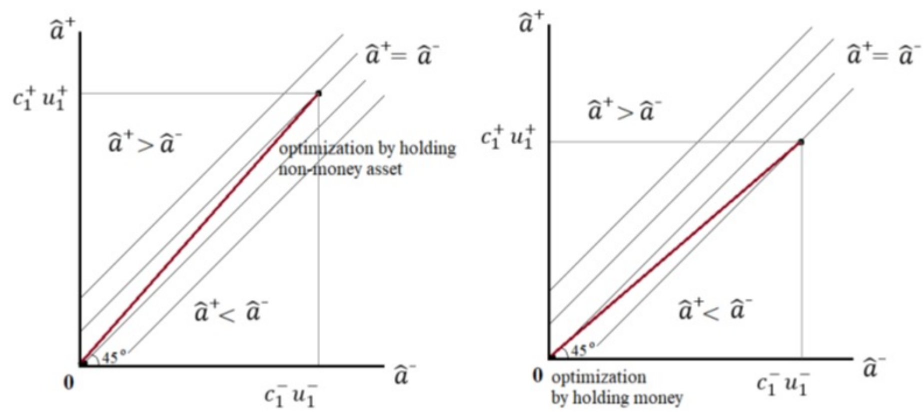
[19] Let $u^+ = g_1(c)$ be potential confirmation locus of the non-money asset and $a^+(c, u^+) = cu^+ = cg_1(c)$ be the attractiveness function on the gain side. Let c_1^+ be the attractiveness maximizing confirmation. For an arbitrary $\lambda \in (0, 1)$, $a^+(c, u^+) = a^+(c, g_\lambda(c)) = cg_\lambda(c) = c(\lambda g_1(c))$. Then, for $0 < \lambda < 1$, the maximization of the attractiveness function gives $c_\lambda^+ = c_1^+$ and its corresponding payoff is λu_1^+. A similar argument applies to the loss side.

side is the straight line connecting $(c_1^+, 0)$ and (c_1^+, u_1^+), while that on the loss side is the straight line between $(c_1^-, 0)$ and (c_1^-, u_1^-).

The decision path $\hat{a}(\lambda) = (\hat{a}^-(\lambda), \hat{a}^+(\lambda))$, $0 \leq \lambda \leq 1$ is obtained by transforming the attractiveness path in the plane of $[0,1] \times (U_x^+ \cup U_x^-)$ into $\mathbb{R}_+ \times \mathbb{R}_+$ where $\hat{a}^-(\lambda)$ and $\hat{a}^+(\lambda)$ are the attractiveness values of the focus-loss and focus-gain for each x. The slope of the decision path is $\frac{\hat{a}^+(x)}{\hat{a}^-(x)} = \frac{\lambda c_1^+ u_1^+}{\lambda c_1^- u_1^-} = \frac{c_1^+ u_1^+}{c_1^- u_1^-}$ for any $\lambda \in [0,1]$.

Since u_1^- is the magnitude of the loss, the decision path is positively sloped straight line segment connecting the origin to the point $(c_1^+ u_1^+, c_1^- u_1^-)$ as we can see in figure 5.2. Here, because money has only zero payoff, the origin $(0,0)$ is the pair of attractiveness values $\hat{a}(0) = (\hat{a}^-(0), \hat{a}^+(0))$.

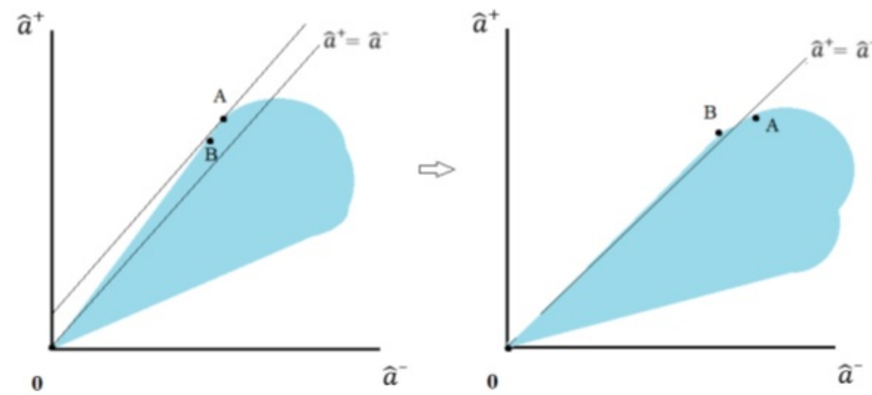
To analyze the final choice, let's adopt the additive decision index, $\hat{d}_A(\lambda) = \alpha\hat{a}^+(\lambda) - \beta\hat{a}^-(\lambda)$. The optimizing point as figure 5.2, is determined by the slope of the decision path in relation to those of the iso-decision contours. If the slope of the decision path is steeper than 45^o $(\alpha = \beta)$ as in the left graph in figure 5.2, in other words, the maximum potential gain of the non-monetary asset exceeds its maximum potential loss, so that keeping only the non-monetary asset is optimal. If the slope of decision path is less than 45^o as in the second graph in figure 5.1, then holding only money is the best choice.



[**Figure 5.2**: the optimal decision between money and a non-money asset]

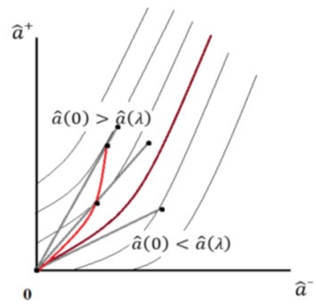
In the case of one non-monetary asset and money, panic behavior is now easily explained. Assume the investor has all of her money invested in the non-monetary asset as in the left diagram in figure 5.1. Interpret the effect on the investor from changing market conditions as decreasing the slope of the decision path. Then as soon as that slope drops below 45^o, the investor abruptly sells her entire stock of the non-monetary asset. It is the switch from the left diagram in figure 5.1 to the right diagram in figure that explains the investor's panic.

Note that the additive decision index presumes higher proficiency in calculating and comparing the potential gain and loss than the maximal decision index as supposed in the previous section. Evaluating and comparing the potential value of financial assets like stock and bond require the advanced knowledge and practical skill, and it is unrealistic to assume that a naive investor can conduct it. On the contrary, comparing directly the value of an asset with money is a simpler task, so that the additive decision index is acceptable simplification for considering the case of money and a non-monetary asset. Needless to say, under the maximal decision index, the panic can also occur at that moment the decision path enters the range of $\hat{a}^- > \hat{a}^+$. This means that when money is eligible as an element in composing the asset portfolio, the panic behavior can occur in both maximal and additive decision function. Money strengthens the volatility of the individual behavior in the portfolio selection.



[**Figure 5.3**]

In the case of n assets including money as explained in the appendix, as the market situation changes, the decision region can move and the optimized portfolio can emerge in the range of $\hat{a}^- > \hat{a}^+$. Unless direct movement from the previous optimizing point to the new one is institutionally allowed, the adjustment can go through the shift from A to 0 and then B. In this adjusting process, an abrupt market collapse can occur in the phase from A to 0, and then the market may recover soon via moving from 0 to B. As a matter of fact, this is frequently observable phenomenon in financial markets. Panic does not indicate just massive movement of fund from a market to another, *e.g.*, fund race from the commodity market to the corporate saving account in the commercial bank. Usually panic involves abrupt crash of an asset value due to the sudden selling of the asset and preparing the cash to purchase another asset in the next opportunity. Hence, here we can corroborate that the stability and liquidity of money functions as the hub connecting various types of asset markets, and may stimulate the panic adjustment during transferring process of fund through diverse asset markets.



[**Figure 5.4**: the decision path under strictly quasi concave decision function]

Then we can ask about this question: how about a more complicated form of decision index, for example, strict quasi-concave decision index (as the footnote 17)? If the attitude of the decision-maker regarding potential gain and loss varies smoothly, for example, giving more weight to gain than loss around the zero-interest level (as the shape of the iso-decision contours in figure 5.4), the optimal investment ratio continuously changes from some λ in $(0, 1]$ to 0. Then panic does not occur. We can obtain a smooth decision path as the red curve in figure 5.4.

6 Closing remarks

The Shacklean approach adopted here to explain the panic behavior in portfolio adjustment has a novel characteristic in comparison to expected utility theory. As it is mentioned in the introduction, the gradual and continuous change of payoff and probability distribution in expected utility theory is unable to explain the abrupt behavioral disconnection due to its fixed functional form of the valuation, the summation/integration of each probability×payoff. On the contrary, beyond the rigid summation/integration of expected payoffs, Shacklean portfolio theory can adopt various forms of decision index reflecting diverse behavioral tendencies of decision-maker. By virtue of this, the motivation of seemingly irrational herding behavior in panic can be explained by the individual decision-making, at least, under the context of given decision index, in Shackle's original terminology, gambler-preference map.

Then which decision index reflects the reality more exactly? If decision-makers can change the decision index as influenced by the market situation, then more various behavioral pattern can be explained. If we allow that the multiple groups of decision-makers can choose not just a specific portfolio determined by the mixture ratio λ but also a decision index itself, then Shacklean portfolio theory can be connected to evolutionary game theory, explaining panic behavior as evolutionary stable strategy related to a chosen decision index. Then it can explain how the individual panic behavior develops to the group herding behavior. This can be the next research agenda.

7 References

Davidson, P. (1983). Rational expectations, a fallacious foundation for studying crucial decision-making. *Journal of Post Keynesian Economics*, 5, 182–198.

Davidson, P. (1991). Is probability relevant for Uncertainty? A post Keynesian perspective. *Journal of Economic Perspectives*, 5, 129–143.

Diamond, D.W., and Dybvig, P.H. (1983). Bank run, deposit insurance, and liquidity. *Journal of Political Economy*, 91, 401–414.

Ford, J. L. (1983). *Choice, Expectation and Uncertainty*, Martin Robertson and company, Oxford.

Ford, J. L. (1994). *G.L.S.Shackle: The dissenting economist's economist*, Edward Elgar, Vermont.

Fishburn, P.C. (1981). *The Foundations of Expected Utility*, D.Reidel Publishing Company, Dordrecht.

Friedmann, M. and Schwartz, A.J. (1963). *A Monetary History of the United States, 1867-1960*, Princeton University Press, Princeton.

Gallegati, M., Palestrini, A., and Rosser, J. B. Jr. (2011). The period of financial distress in speculative markets: interacting heterogeneous agents and financial constraints.*Macroeconomic Dynamics*, 15, 60–79.

Gennaioli, N. and Schleifer, A. (2018). *A Crisis of Beliefs: Investor Psychology and Financial Fragility*, Princeton University Press, Princeton.

Goetzmann, W. N., Kim, D., and Shiller, R. J. (2022). Crash beliefs from investor surveys. *NBER Working Paper*, 22143.

Goldstein, I. (2013). Empirical Literature on Financial Crises: Fundamentals vs. Panic, in Caprio, G. J., Beck, T., Claessens, S., and Schmukler, S. (eds.), *The Evidence and Impact of Financial Globalization*, 523–534, Elsevier.

Gresnigt, F., Kole, E., and Franses, P. H. (2015). Interpreting financial market crashes as earthquakes: A new early warning system for medium term crashes. *Journal of Banking and Finance*, 56, 123–139.

Hacking, I. (1975). *The Emergence of Probability*, Cambridge University Press, Cambridge.

Hey, D. (1984). The Economics of optimism and pessimism: a definition and some application. *Kyklos*, 37, 181–205.

Jarrow, R., Protter, P., and Shimbo, K. (2010). Asset price bubbles in incomplete markets. *Mathematical Finance*, 20, 145–185.

Jarrow, R., Kohia, Y., Protter, P. (2011). How to detect an asset bubble. *SIAM Journal of Financial Math*, 2, 839–865.

Jarrow, R. 2016. Testing for asset price bubbles: three new approaches. *Quantitative Finance Letters*, 4, 4–9.

Kahneman, D., Tversky, A. (1979). Prospect theory: an analysis of decision under risk, *Econometrica*, 47, 263–292.

Katzner, D.W. (1986-87). Potential surprise, potential confirmation, and probability, *Journal of Post Keynesian Economics*, 9, 58–78.

Katzner, D.W. (1987-88). More on the Distinction between Potential Confirmation and Probability, *Journal of Post Keynesian Economics*, 10, 65–83.

Katzner, D.W. (1989-90). The Shackle–Vickers Approach to Decision-Making in Ignorance, *Journal of Post Keynesian Economics*, 12, 237–259.

Katzner, D.W. (1998). *Time, Ignorance, and Uncertainty in Economic Models*, The University of Michigan Press, Ann Arbor.

Katzner, D.W. (2001). The Demand for Money in an Uncertain World, in Earl, P.E. and Frowen, S.F. (eds.), *Economics as an Art of Thought: Essays in Memory of G.L.S. Shackle*, 269–298, Routledge, London and New York.

Keynes, J.M. (1921). *A Treatise on Probability*. Macmillan, London.

Kindleberger, G. L. (1978). *Manias, Panics, and Crashes: A History of Financial Crises* Basic Books, New York.

Knight, F.(1921). *Risk, Uncertainty and Profit*.Houghton Mifflin, New-York.

Krugman, P. (1999). Balance sheets, the transfer problem, and financial crises, *International Tax and Public Finance*, 6, 459–472.

Lachmann, L.M. (1976). From Mises to Shackle: an essay on Austrian economics and the kaleidic society, *Journal of Economic Literature*, 14, 54-62.

Lawson, T. (1995). Economics and expectations in Dow, S.C., and Hillard, J. (eds) *Keynes, knowledge and uncertainty*, 77–106, Edward Elgar, Aldershot.

Loomes, G., Sugden, R. (1982). Regret Theory: an alternative theory of rational choice under uncertainty. *Economic Journal*, 92, 805-824.

Machina, M. (1982). 'Expected Utility' analysis without the independence axiom. *Econometrica*, 50, 277-323.

Mises, L.V. (1949).*Human Action*, Yale University Press, New Heaven.

Obstfeld, M. (1996). Models of currency crises with self-fulfilling features. *European Economic Review*, 40, 1037–1047.

O'Driscoll, G.P., Rizzo, M. J. (1985). *The Economics of Time and Ignorance*, Basili Blackwell, Oxford.

Pratt, J. (1964). Risk aversion in the small and in the large, *Econometrica*, 32, 122–136.

Protter, P. (2016). Mathematical models of bubbles. *Quantitative Finance Letters*, 4, 10–13.

Rosser, J. B. Jr. (1997). Speculations on Nonlinear Speculative Bubbles. *Nonlinear Dynamics, Psychology, and Life Sciences*, 1, 275—300.

Quiggin, J. (1982). A theory of anticipated utility. *Journal of Economic Behavior and Organization*, 3, 323–343.

Savage, L.J. (1954). *The Foundations of Statistics*, J. Wiley and Sons, New York.

Shackle, G.L.S. (1949). *Expectation in Economics*, Cambridge University Press, Cambridge.

Shackle, G.L.S.(1954). *Uncertainty in Economics*, Cambridge University Press, Cambridge.

Shackle, G.L.S. (1969). *Decision, Order and Time in Human Affairs*, 2nd edition, Cambridge University Press, Cambridge.

Shiryaev, A. N. (2007). *Optimal stopping rules, Springer*, Berlin.

Vickers, D. (1978). *Financial markets in the capitalist process*, University of Pennsylvania Press.

Vickers, D. (1987). *Money capital in the theory of the firm*, Cambridge University Press, Cambridge.

Von Neumann, J., O. Morgenstern. (1944), *Theory of Games and Economic Behavior*, Princeton University Press, Princeton.

Yaari, M. (1987). The dual theory of choice under risk. *Econometrica*, 55, 95–115.

8 Appendix

Portfolio diversification among more than two assets under uncertainty

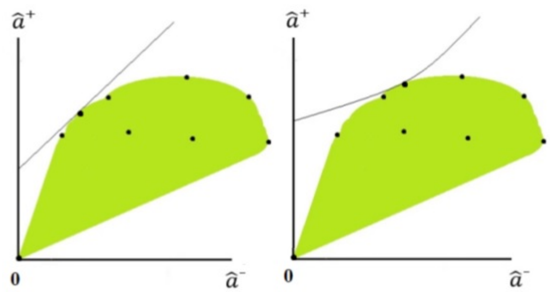
Suppose that there is an arbitrary set of n-assets and define the set of mixture ratios for portfolio diversification by $\boldsymbol{X} = \{\lambda = (\lambda_1, \lambda_2, \ldots, \lambda_{n-1}, 1 - \lambda_1 - \lambda_2 - ... - \lambda_{n-1}) : 0 \leq \lambda_1, \lambda_2, ..., \lambda_{n-1} \leq 1\}$.

The set of potential confirmation loci for gains and losses of each asset i is given by $g_i(c)$, $l_i(c)$ where $u_i^+ = g_i(c)$, $u_i^- = l_i(c)$ are all smooth on $(0, 1)$. Then a potential confirmation locus of a portfolio is defined by $u^+ = g_\lambda(c) = \sum_{i=1}^{n-1} \lambda_i g_i(c) + (1 - \lambda_1 - \lambda_2 ... - \lambda_{n-1}) g_n(c)$ and $u^- = l_\lambda(c) = \sum_{i=1}^{n-1} \lambda_i l_i(c) + (1 - \lambda_1 - \lambda_2 ... - \lambda_{n-1}) l_n(c)$ where $0 \leq \lambda_1, \lambda_2, ..., \lambda_{n-1} \leq 1$.

The multiplicative attractiveness function of the portfolio is defined by

$A^+(\lambda, c) = c\{\sum_{i=1}^{n-1} \lambda_i g_i(c) + (1 - \lambda_1 - \lambda_2 ... - \lambda_{n-1}) g_n(c)\}$ and $A^-(\lambda, c) = c\{\sum_{i=1}^{n-1} \lambda_i l_i(c) + (1 - \lambda_1 - \lambda_2 ... - \lambda_{n-1}) l_n(c)\}$ where $0 \leq \lambda_1, \lambda_2, ..., \lambda_{n-1} \leq 1$.

We can get the attractiveness range, which corresponds to attractiveness path in two asset case, including all attractiveness maximizing points, *i.e.*, the focus-gain $(c_\lambda^+, g_\lambda(c_\lambda^+))$ and the focus-loss $(c_\lambda^-, l_\lambda(c_\lambda^-)))$ for each λ in the n dimensional simplex $\boldsymbol{X}$. As the analogue to the decision path in the situation of two assets, we can obtain decision range by transferring attractiveness region to $\mathbb{R}_+ \times \mathbb{R}_+$.



[**Figure A.1**: the portfolio diversification on n assets]

Suppose every $g_i(c)$, $l_i(c)$ for all $i = 1, 2, \ldots .n$ are assumed to be at least twice continuously differentiable, we can say that $a^+ : [0,1] \times U_x^+ \to R_+$, $a^- : [0,1] \times U_x^- \to R_+$ are continuous and preserve the compactness and the connectedness of objects in $[0,1] \times R_+$. Then we can get a connected boundary of decision range, $\{(\hat{A}^+, \hat{A}^-) : \lambda \in \boldsymbol{X}\}$. Then for a given decision index, the optimal portfolio ratio is determined at the tangency between an iso-decision contour of a given decision index and the boundary of decision range. It is trivial that any point on the boundary of decision region can be represented by the convex combination of two mixture assets, where they are again convex combinations by n assets. Thus, the problem is ultimately reduced to the portfolio diversification between two assets and money.