Adverbs of Quantification as Generalized Quantifiers[1]

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In this paper I propose a way to incorporate adverbs of quantification into generalized quantifier theory. This will allow for the categorization of adverbs with respect to the generalized quantifier properties: conservativity, symmetry and persistence which in turn will lead to a natural expansion of the scope of certain semantic universals so as to include adverbs as well as determiners. Let me begin by introducing the two main ideas.

The first idea, popularized in the 80's by Barwise & Cooper, is that NP's denote generalized quantifiers, or sets of sets. For example, the NP, "every animal" will denote the set of supersets of the set of animals. "some animal" denotes the set of sets that have a non-empty intersection with the set of animals. "no animal" denotes the set of sets that have a null intersection with the set of animals. The denotations for these determiners can be found in (1):

(1) Determiner denotations:

Let E be the domain of discourse, some set of individuals. A is a subset of E.

$$\|every\| (A) = \{X \subseteq E : A \subseteq X\}$$

$$\|some\| (A) = \{X \subseteq E : A \cap X \neq \emptyset\}$$

$$\|no\| (A) = \{X \subseteq E : A \cap X = \emptyset\}$$

Assuming that VP's denote sets of individuals, a simple sentence is true if the set denoted by the VP is a member of the generalized quantifier denoted by the NP. For example "every animal barks" will be true, if the set of barkers is a member of the set of supersets of the set of animals. This will be the case if the set of barkers contains in it every animal.

The second idea attributed to David Lewis, is that adverbs of quantification, such as always and sometimes act as unselective binders for indefinites in their scope. To understand this compare the examples in (3) a and b.:

(3)
a. Always, if a $child_1$ feeds a dog_2, it_2 bites him_1.
b. Sometimes, if a $child_1$ feeds a dog_2, it_2 bites him_1.

(3a) makes a claim about all children-dog feeding pairs, while (3b) makes a claim about some children feeding some dogs. In each case, the children and the dogs get their quantificational force from the initial adverb. Similar results obtain with never, usually, often, seldom and rarely.

Let me note that I will try in this discussion to maintain the canonical form used in (3) in which an adverb is followed by what I will call an if/when clause and that is followed by a main clause. Also, I will be ignoring any temporal or causal relations between parts of these sentences.[2]

It has often been suggested that since these adverbs are quantifiers, one should be able to construe the combination of an adverb and a restrictive if/when clause as denoting a set of sets or a generalized quantifier. In order to do this, however, one first needs to say what these sets will contain.

In general, one can think of a sentence containing (indexed) indefinites as denoting a set of n-tuples of individuals. For example, "a $child_i$ feeds a dog_j" will denote the set of n-tuples whose i-th member is a child and whose j-th member is a dog that the child feeds. In Heim (1982) one finds an algorithm for generating such sets of n-tuples from English sentences, using the central idea that indefinites are simply predicates of individual variables.

We can now put our two ideas together to get so-called adverbial generalized quantifiers, as follows: An adverb combines with a restrictive if/when clause to denote a set of sets of n-tuples. If this generalized quantifier contains the set of n-tuples denoted by the main clause, the sentence is true. I return to the example in (3a) for illustration:

(3a) Always, if a $child_1$ feeds a dog_2, it_2 bites him_1.

Here we may use the denotation for "always" given in (4a) below, where R is just the denotation of "a $child_1$ feeds a dog_2". So "Always if a child feeds a dog" will denote the set of all supersets, Y, of the set of n-tuples whose first member is a child and whose second member is a dog that the child feeds. "it bites him" will simply denote a set of n-tuples each of whose second member bites its first member. Note, the subscripts, 1 and 2, allow us to talk about "the first" and "the second" members of the n-tuples. Notice also the parallel between the denotations for the generalized quantifiers for "always" and "every": in each case we have the superset relation. Simlarly, "sometimes" will form generalized quantifiers analogous to "some" and "never" works like "no", as follows:

(4) Adverb denotations:

Let N be the set of all n-tuples formed from E.
Let R be a subset of N:

a. $\|always\|(R) = \{Y \subseteq N : R \subseteq Y\}$

b. $\|sometimes\|(R) = \{Y \subseteq N : R \cap Y \neq \emptyset\}$

c. $\|never\|(R) = \{Y \subseteq N : R \cap Y = \emptyset\}$

d. $\|once\|(R) = \{Y \subseteq N : |R \cap Y| \geq 1\}$

e. $\|twice\|(R) = \{Y \subseteq N : |R \cap Y| \geq 2\}$

In the examples in (5) and (6), a when-clause is used as a restrictive clause:

(5) (At least) twice, when a $child_1$ beat a dog_2, it_2 bit him_1.

(6) When a $child_1$ beat a dog_2, it_2 bit him_1.

In both (5) and (6) the restrictive when-clause will denote the set of n-tuples whose first element is a child and whose second element is a dog that the child beats and the main clause will denote the set of n-tuples whose first element was bitten by its second element. Following the definition for "twice" given in (4e), sentence (5) will be true if the intersection of the sets denoted by the restrictive and main clauses contains 2 or more n-tuples. This is possible if there were at least 2 child-dog beating pairs that were also biting pairs. Notice, that if there was just one child-dog beating-biting pair, then intuitively sentence (6) should be true though (5) should be false. So it seems that a bare when-clause in the past tense as in (6) can be interpreted with a covert adverb meaning "once".

Something needs to be said at this point regarding the length of these n-tuples. N must be at least as large as the number of indefinites in the restrictive clause. Since there is no limit on the number of noun phrases in a clause it is usually assumed that we are dealing with infinitely long n-tuples. But here is the problem. It turns out that a clause with a finite number of NP's will denote an infinite set of such infinitely long n-tuples if it denotes any at all. Similarly the intersection of the denotations of two such clauses is also either an infinite set or the empty set. But notice for example, that sentences such as (5) and (6) differed in truth value depending on whether there was at least one or at least two n-tuples in the intersection. Allowing our n-tuples to be infinitely long makes the incorrect prediction that (5) entails and is entailed by (6). Similar problems arise with the adverbs "usually", "seldom" and "often" as was pointed out by Lewis himself.

In order to remedy this we need to eliminate the parts of our n-tuples that are doing no work. It turns out that all we need from our n-tuples are those elements that have the same index as the indefinites in the restrictive clause. Those indices are just Heim's selectional indices. To formalize this somewhat I introduce the notion of a "select-tuple". If you take an n-tuple and remove all the elements whose index is not a selectional index, you get the corresponding "select-tuple". The semantics for our adverbial generalized quantifiers is now revised replacing "n-tuples" with "select-tuples". Consider example (3a) again:

(3a) Always, if a child$_1$ feeds a dog$_2$, it$_2$ bites him$_1$.

Here there are only two selectional indices and so our select-tuples will just be pairs. "Always if a child feeds a dog" will denote the set of all supersets of the set of select-tuples, or pairs in this case whose first member is a child and whose second member is a dog that the child feeds. "it bites him" will denote a set of select-tuples whose second member bites its first member. This set is also a set of pairs.

Let me note in passing that the use of "select-tuples" in effect gives us what Heim calls "nuclear scope existential closure" as needed in an example like (7):

(7) Always, if a man$_1$ owns a dog$_2$, he$_1$ beats it$_2$ with a stick$_3$

Since the index "3" does not appear in the restrictive clause, there are only two selectional indices, 1 and 2. The main clause, "he$_1$ beats it$_2$ with a stick$_3$" will give us a set of n-tuples whose first member beats its second member with its third member, a stick. From these we derive a set of select-tuples, by removing all but the first and second members of the original n-tuples. This gives us a set of select-tuples or pairs in this case whose first members beat their second members with a stick. A pair will be in that set just in case some stick was used. The restrictive clause will simply give us a set of select-tuples, again pairs, each of whose first member is a man and whose second member is dog that the man owns. Given the definition of "always" the sentence is true if the man-dog pairs form a subset of the beating-with-a-stick pairs.

Reviewing then, we now have a way to view adverbs of quantification as generalized quantifiers, summarized below in (8). As indicated in (8c) the main and restrictive clauses will give us sets of select-tuples. The adverb combines with the restrictive clause to form an adverbial generalized quantifier or a set of sets of select-tuples, as in (8d). The whole sentence is true, if that set includes the main clause denotation, as is stated in (8e). A somewhat more compositional account of this can be found in the appendix.[3]

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(8) Scheme for evaluating "ADVERB, if/when P, Q"

a. selection indices ==> the set of indices on the indefinites in P.(cf. Heim(1982))

b. $\|P\|$, $\|Q\|$ ==> sets of infinite n-tuples.

c. $\|P\|^s$, $\|Q\|^s$ ==> sets of select-tuples gotten by removing from $\|P\|$ and $\|Q\|$ all elements whose index is not a selection index (more precisely, whose position does not correspond to a selection index).

d. $\|$ ADVERB + P $\|$ ==> a generalized quantifier arrived at by using the denotations in (4), with $\|P\|^s$ for R.

e. $\|$ADVERB, if/when P, Q$\|$ = 1, if $\|Q\|^s \in \|$ ADVERB +P$\|$.

Having chosen a way to view adverbs of quantification as generalized quantifiers, I can now categorize them with respect to some properties that NP generalized quantifiers are said to have. This will allow me then to consider the relevance of some NP universals to these adverbs. The properties I will be considering are: conservativity, symmetry, and persistence.

Given f, a function from sets to sets of sets and any sets X and Y

f is conservative iff $[X \in f(Y) \leftrightarrow (X \cap Y) \in f(Y)]$

Much of Keenan & Stavi's discussion of determiners, centers on the following universal:

(9) All determiners denote conservative functions.

This universal is supported by inferences of the type "D A's are B " iff "D A's are B A's" where D ranges over determiners. For example:<u>No astronauts are American</u> if and only if <u>No astronauts are American astronauts</u>. On the analysis I'm proposing, adverbs of quantification also denote functions from sets to generalized quantifiers. Do they also enjoy the property of conservativity?

To answer this question, we need to think about inferences that conform to the following pattern:

(A) ADVERB, when a $child_1$ feeds a dog_2, it_2 bites him_1

if and only if:

(B) ADVERB, when a $child_1$ feeds a dog_2, the $child_1$ feeds the dog_2 and it_2 bites him_1

The main clause of (B), "the $child_1$ feeds the dog_2 and it_2 bites him_1" will give us the set of child-dog pairs each of whose first element feeds and is bitten by its second element. This set is just the intersection of the sets denoted by the restrictive and main clauses of (A), as the conservativity condition would require.

This inference pattern in fact seems valid for English adverbs of quantification which demonstrates that they are conservative just like their counterparts in the NP domain.

Let me note here that the success of an analysis involving select-tuples in cases like the following:

(10) Always, when a man$_i$ owns a dog$_j$, he$_i$ is rich.

depends on the conservative nature of these adverbs to, in effect, fill in the missing reference to "a dog" in the main clause. Let me explain. Because there are two selectional indices in this case, namely i and j, the main clause will in fact denote the set of select-tuples, or pairs in this case, of rich men and any arbitrary object. However, by conservativity the truth of the sentence will depend only on the intersection of that set with the set of dog owning pairs, giving us pairs of rich men and the dogs they own, which is, intuitively, what matters here.

The next property I want to look at is symmetry. Given f, a function from sets to sets of sets and any sets X and Y, f is symmetric if and only if:

$$X \in f(Y) \longleftrightarrow Y \in f(X)$$

Since some is a symmetric determiner, Some astronauts are women implies and is implied by Some women are astronauts. Similarly, "sometimes" is a symmetric adverb:

Sometimes, when a child gets good grades, he is smart.
implies and is implied by:
Sometimes, when a child is smart, he gets good grades.

"Always" on the other hand is not a symmetric adverb, thus Always, when a child gets good grades, he is smart. does not imply Always, when a child is smart, he gets good grades.

Next, I introduce persistence. Given f, a function from sets to sets of sets and any sets A, B, X and Y

$$f \text{ is persistent iff } [A \subseteq B \ \ \& \ \ X \in f(A)] \longrightarrow X \in f(B)$$

"Some" is an example of a persistent determiner. Since New Yorkers form a subset of Americans, if some New Yorker owns a Corvette then some American owns a Corvette. Similarly, "sometimes" is persistent: sometimes, when a child force feeds a dog, it bites him. implies that sometimes, when a child feeds a dog, it bites him. since the set of force-feedings is a subset of the set of feedings.

I now want to introduce the following Determiner Universal and investigate its application to adverbs.

(11) Basic persistent determiners are symmetric.

What this universal does is eliminate the possibility of a basic determiner that means "not every" which would be persistent but not symmetric:

Persistent:
Not every mammal is a dog implies
Not every animal is a dog.

Non-symmetric:
Not every mammal is a dog does not imply
Not every dog is a mammal.

None of the determiners listed in (1) violate this universal. This universal can now be extended to adverbs:

(12) Basic persistent adverbs are symmetric

This holds straightforwardly for the non-persistent adverbs "always", "never" and "usually", and there is no one word adverb meaning "not always". As we have seen already "sometimes" is persistent and symmetric. Furthermore, the numerical adverb "twice" though restricted with respect to the types of clauses that it combines with, is shown in the example below to be persistent, on its "at least" reading:

(At least) twice, when a child force fed a dog, the dog bit him.

implies:

(At least) twice, when a child fed a dog, the dog bit him.

and it is symmetric, as predicted by the universal:

(At least) twice, when a child force fed a dog, the dog bit him.

implies and is implied by:

(At least) twice, when a dog bit a child,

<u>the child had force fed him.</u>

And similarly, for the adverb "once" and its covert counterpart.

Things are a bit complicated with the adverb "often". It turns out the adverb "often" is ambiguous between a proportional and a non-proportional or cardinal reading. The sentence <u>Often, if a farmer force feeds a dog, it bites him.</u> on its cardinal reading, means that for some context dependent x, at least x force-feedings end in bitings. On it's proportional reading, it means that for some context dependent p, at least p percent of force-feedings end in bitings. "Often" is persistent only on it's cardinal reading. Happily for our universal, "often" is also symmetric on it's cardinal reading. And so, when the meanings of "often" are taken individually, the universal is upheld.

I emphasize here that I am claiming "often" is ambiguous and not just vague and while I have not argued here for this ambiguity, I will mention that in a recent talk by Barbara Partee arguments were given for the same ambiguity in the determiners "few" and "many".

The universal in (12) raises another issue in the case of "sometimes" though not with its near relative "once". I have categorized "sometimes" as symmetric. However, it is unclear what status it should have in cases where the restrictive clause denotes the empty set. So, consider the symmetric pair (13) and (14):

(13). Sometimes, if a cat consumes Cat Chow,
it speaks Pig Latin.
(14). Sometimes, if a cat speaks Pig Latin,
it consumes Cat Chow.

Given that cats do not speak Pig Latin but do consume Cat Chow (13) is surely false. But what is the status of (14)? If "sometimes" was really an existential over n-tuples then sentence (14) should be false. It should have the same status as the sentences in (15):

(15)
a. Some cat that speaks Pig Latin consumes Cat Chow.
b. Some cat that speaks Pig Latin meows in meter.

I would be more inclined to say that (15) a and b are false while (14) is undefined or perhaps even true. But if (14) is true and (13) is false, then "sometimes" is not symmetric. This would mean that "sometimes" though persistent, is not symmetric hence the universal is falsified.

However, as in the case of "often" it turns out that just the assumption that makes "sometimes" non-symmetric makes it non-persistent as well.

To see this consider sentence (14) again. We're assuming that it is true because the restrictive clause denotes the empty set. Since the empty set is a subset of any set, by the definition of persistence given above, (14) will remain true no matter what we replace the restrictive clause with, IF "sometimes" is persistent. But clearly that is not the case, as the falsehood of (16) shows:

(16) Sometimes, if a cat is dead, it consumes Cat Chow

The result then is that if sentences like (14) are considered true then "sometimes" is neither persistent nor symmetric. If your intuition is that sentences like (14) are not true then "sometimes" is both symmetric and persistent. In either case the universal makes the correct prediction.

So far, I have discussed two universals that have been proposed for NP's and I have shown that they hold in the adverbial domain. This is somewhat surprising given that adverbs and determiners have so little in common from the viewpoint of syntax.

I would now like to consider a way in which the class of determiners differs from the class of adverbs. In doing this, I will make use of the term "number sensitive", introduced by Dag Westerstahl to capture a <u>pressuposition</u> of the definite determiners. These are his words:

> "The requirement for number-sensitivity is that the determiner ... is defined for the argument (set) which interprets the N, just in case this argument satisfies the condition corresponding to the syntactic number of the N" Westerstahl (1985), p. 65

To understand this, consider, if you will, the examples in (17) through (20).

(17) Some drug that was improperly tested was released by the FDA.

(18) Some drugs that were improperly tested were released by the FDA.

(19) The drug that was improperly tested was released by the FDA.

(20) The drugs that were improperly tested were released by the FDA.

The number of drugs that were improperly tested and were nonetheless released will no doubt affect the truth of all these sentences. Nevertheless, there are differences. In a situation where many drugs were improperly tested (17) and (18) might both be true, hence "some" is not sensitive to the syntactic number on the head noun. On the other hand, since "the" is number sensitive, in a situation where many drugs were improperly tested, (19) is undefined though (20) is not.

Next, we move to the adverbial domain and here I direct your attention to (21) and (22) to show that it is meaningful to talk of 'number' in the adverbial cases. (21) differs from (22) in the syntactic number of its indefinites and pronouns and indeed this difference has consequences. In a case where exactly one of the improperly tested drugs was released, (21) is true:

(21) Once, when a drug was improperly tested, the FDA released it anyway.

but, if only one drug was released, (22) is not true:

(22) Once, when drugs were improperly tested, the FDA released them anyway.

And now we ask: Are there any number-sensitive adverbs? That is, are there any situations in which an adverb + restrictor combination is undefined if in the singular but defined when the restrictive clause is pluralized?

I submit the following examples as partial evidence for the conclusion that: Adverbs are never number-sensitive.

(23)

a. Always, if a drug is tested, it gets approved.
Always, if drugs are tested, they get approved.

b. In the U.S, when a drug was improperly tested, it got approved.[4]
In the U.S, when drugs were improperly tested, they got approved.

c. The one time that a woman headed the F.D.A., when a drug was insufficiently tested, it did not get approved.
The one time that a woman headed the F.D.A., when drugs were insufficiently tested, they did not get approved.

d. Seldom/ Never, if a drug was improperly tested, did it get approved.

Seldom/ Never, if drugs were improperly tested, did they get approved.

In example (c) I have used an adverbial which by itself is definite. Yet, I still think there is no number sensitivity, so, if 20 drugs were insufficiently tested and none were approved, I would judge the first sentence true and not undefined.

This number-sensitivity universal clearly does not transfer into the NP domain, as (19) and (20) show, and one would want to know why this is so.

Appendix

For a more compositional account of adverbs of quantification:

1. Let restrictive and main clauses denote infinite n-tuples, as in Heim (1982)

2. If X is a set of n-tuples, then X^s is the set of select-tuples gotten from X by removing from each of its n-tuples those elements whose index is not in s, the set of selectional indices.

3. Use the following definitions instead of (4):

$\|Always_s\| (R) = \{Y \subseteq N : R^s \subseteq Y^s\}$

$\|Sometimes_s\| (R) = \{Y \subseteq N : (R \cap Y)^s \neq \emptyset\}$

$\|Never_s\| (R) = \{Y \subseteq N : (R \cap Y)^s = \emptyset\}$

$\|once_s\| (R) = \{Y \subseteq N: |(R \cap Y)^s| \geq 1\}$

$\|usually_s\| (R) = \{Y \subseteq N: |(R \cap Y)^s| > 1/2 \, |R^s|\}$

Consider again example (7) repeated below:

(7) Always, if a man_1 owns a dog_2, he_1 beats it_2 with a $stick_3$

Here s, the set of selectional indices, is {1,2}. R, the denotation of the restrictive clause will be the set of infinitely long n-tuples each of whose first member is a man and whose second member is a dog the man owns. Y, the denotation of the main clause, will be a set of infinitely long n-tuples each of whose first member beats its second member with its third member which is a stick. According to the denotation given above for "$always_s$", to see if (7) is true we need to form two sets of pairs, R^s and Y^s, by eliminating all but the 1st and 2nd members of all the n-tuples in R and Y. (7) is true iff R^s is a subset of Y^s. This will be the case only if the set of man-dog owning pairs is a subset of the set of beating-with-a-stick pairs.

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2. Let me offer a hint towards incorporating temporal relations into the approach I will take here. The basic pattern in these sentences is to have an indefinite in the restrictive clause coindexed with a pronoun in the main clause, both of which are bound by

the adverb to yield restrictive quantification. Tense marking is sometimes taken as a relation between a new, 'indefinite' variable over time intervals and a 'pronominal', reference variable over time intervals. So, every tensed clause has the ingredients for one of our restrictive or main clauses.

3. Due to space limitations, I will not cover cases like the following:

i. Usually, when a student$_1$ buys it$_4$, he$_1$ lends it$_4$ to a friend$_2$.

ii. Often, if a book is expensive, seldom, when a graduate student buys it$_4$, does he lend it$_4$ to a friend.

where the pronouns are free in the scope of the (closest) adverbs. To do this I would need to give satisfaction rather than truth conditions. See for example Heim (1982) pages 157-168 for an account of how to do this that would be compatible with the analysis presented here.

4. The adverbials in b and c are to be thought of as adverbs of quantification just as genitive phrases are treatable as determiners -- both form non-logical generalized quantifiers.

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