Not All Numbers Were Created Equal: Evidence the Number One is Unique

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Not All Numbers Were Created Equal: Evidence the Number One is Unique

A Thesis Presented
by
JENNA CROTEAU

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ABSTRACT

Not All Numbers Were Created Equal:
Evidence the Number One is Unique

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Universally across modern cultures children acquire the meaning of the words one, two, and three in order. While much research has focused on *how* children acquire this knowledge and *what* this knowledge represents, the question of *why* children learn numbers in order has been comparatively neglected. To address this question, a non-verbal anticipatory looking task was implemented. In this task, 35 14- to 23-month-old infants were assessed on their ability to form implicit category structures for the numbers one, two, and three. We hypothesized that children would be able to form the implicit category structure for the number one but not for two or three because sets of two and three objects would exceed the working memory capacities of infants. We found results consistent with this hypothesis; infants (regardless of age) were able form a category for sets with one object, as evidenced by their looking behavior while the looking behavior for the numbers two and three did not demonstrate a statistically significant pattern. We interpret our results as consistent with our hypothesis and discuss implications for parallel individuation, number acquisition theories, and the development of working memory resources.

Keywords: parallel individuation, number acquisition, subitizing, anticipatory looking, infants, object-based working memory

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CHAPTER I.

INTRODUCTION

Understanding the ontogenetic origins of number concepts has been a longstanding research project in cognitive science (Carey, 2009; Carey & Barner, 2019; Condry & Spelke, 2008; Dehaene, 2001; Feigenson et al., 2004; Leslie et al., 2008). For many, this interest stems from a unique set of qualities number concepts seem to possess. Numbers concepts are abstract, like concepts of causality and intentionality, in the sense that we cannot directly observe a number. For example, we can observe three bananas, but we cannot directly observe three in the absence of some instantiation--like a group of bananas. In more formal terms, numbers are symbols for 2nd order relations like exact equality, cardinality, and ordinality. That said, unlike other abstract ideas, numbers have precise and concrete definitions which are context independent in nature. As an illustration, the number two, at least when used to represent the quantity of items in a set, always means the same thing regardless of whether we are discussing two ideas, two cats, or two atoms. Put simply, what is two today will be two tomorrow. This well-defined yet abstract nature is what makes number concepts an excellent test case for understanding how abstract knowledge is acquired and transmitted across human development.

On the other hand, understanding the developmental origins of number concepts is not just a scientific curiosity. Early numeracy skills, like the ability to correctly identify and produce a number, are critical building blocks for later mathematical skills like arithmetic, fractions, and word problems even when taking other factors like fluid intelligence, socioeconomic status, and working memory into account (Duncan et al., 2007; Geary et al., 2013; Krajewski & Schneider, 2009; Watts et al., 2014). Importantly, the acquisition of early numeracy skills prior to the start of kindergarten has been shown to be predictive of mathematical achievement in the later

elementary school years (Aunio & Niemivirta, 2010; Aunola et al., 2004; Jordan et al., 2009; Nguyen et al., 2016; Slusser et al., 2019). Beyond the classroom, research has demonstrated that differences in early numeracy relate to high school math course selection and the probability of successful enrollment in college, both of which have longstanding implications for the well-being and happiness of an individual (Davis-Kean et al., 2022). Given these consequential downstream effects of early numeracy, understanding exactly how children learn about numbers is of practical concern.

In this paper we concern ourselves with the early number acquisition process. Specifically, we attempt to address why children learn the first few numbers in order. In contrast to other aspects of early numeracy, such as the ability to effectively count or use one-to-one correspondence, understanding why children learn small numbers in order has been relatively under-researched. We propose that domain-general cognitive limitations, like working memory capacity, as opposed to alternative explanations, can explain this acquisition pattern.

A. Learning Small Numbers in Order

Much research on the development of early number concepts focuses on a child's ability to produce appropriate quantities when asked for a certain number. These abilities are typically assessed using a language-based task called *Give-a-number* (*Give-N*), first popularized by Wynn (1990). In this paradigm, children are asked to give the experimenter *N* objects like, "Can you give Mr. Frog three buttons?" The number the experimenter asks for is titrated based on the child's response. For example, a child that responds incorrectly to a request for three buttons will be given a smaller number on the next trial while a child who responds correctly will be given a larger number. Children are assigned an N-knower level based on the largest number they can accurately produce. The proliferation of this paradigm has revealed that children universally

move through predictable number word knowledge stages (Almoammer et al., 2013; Barner, Chow, et al., 2009; Barner, Libenson, et al., 2009; Ceylan & Aslan, 2018; Condry & Spelke, 2008; Le Corre & Carey, 2007; Meyer et al., 2020; Negen & Sarnecka, 2009; Piantadosi et al., 2014; Sarnecka et al., 2007; Sarnecka & Carey, 2008; Sarnecka & Gelman, 2004; Sarnecka & Lee, 2009; Shusterman et al., 2022; Slusser & Sarnecka, 2011; Spaepen et al., 2018; Wynn, 1990, 1992b).

Initially, children are considered non-knowers, meaning that they do not know the meaning of any numbers. At this stage of number knowledge, children can often *route count*, or recite portions of the count sequence accurately (e.g., "One, two, three!") but without understanding that the words they utter refer to specific quantities. Rote counting relies on a child's procedural learning skills, specifically the ability to memorize a stable sequence of words. Sometime soon after the second birthday children begin transitioning from non-knowers to 1-knowers, meaning that they can reliably give exactly one object when asked for one object and do not give one object when asked for any other amount (Barner et al., 2007). Some months later, children become 2-knowers, reliably giving 2 objects only when asked for 2 objects. This protracted stepwise pattern of number acquisition continues until children become 4-knowers.

Instead of transitioning from 4-knowers to 5-knowers, children often become *cardinality principle (CP-) knowers*, meaning they have acquired the ability to give any number asked for as long as the number remains within in the bounds of the child's count sequence. Recently, the existence of 5-knowers, 6-knowers, and possibly even 7- and 8-knowers without cardinality principle knowledge has been documented (Gunderson et al., 2015; Krajcsi, 2021; Krajcsi & Fintor, 2023; Marchand et al., 2022; Mussolin et al., 2012; Rousselle et al., 2021; Wagner & Johnson, 2011); however, it is unclear what these findings represent. For example, it's possible

that some children are more conservative when it comes to generalizing a procedural rule like the cardinality principle. In other words, these children may require additional evidence (e.g., from the numbers 5, 6, etc.) to generalize the cardinality principle to their entire count sequence. This would be consistent with previous findings that individual differences in statistical learning are predictive of natural language acquisition as well as when a child generalizes a syntactic rule (Kidd, 2012; Kidd & Arciuli, 2015; Misyak & Christiansen, 2012; Siegelman, 2020).

Another limitation to these findings is that recently the Give-N task has come under scrutiny for inconsistencies in knower level assessment at the individual level (Krajcsi, 2021; Krajcsi & Fintor, 2023 Rousselle et al., 2021); however, it is possible that differences in task procedures such as prompting the child to recount the set of objects given, forgoing titration, or assessing a wider range of numbers could account for these differences. For example, other work has demonstrated that children appear to have partial knowledge of numbers just outside the child's knowledge range, indicating that the Give-N task may underestimate a child's number word knowledge (Wagner et al., 2019). If true, this would suggest that minor task differences could help or hinder a child with partial knowledge of a number. Consistent with this interpretation, the Give-N task has been found to be highly correlated with other primarily verbal number tasks (e.g., "What's on This Card Task"), indicating that the Give-N task is reliable assessment of children's number word knowledge (Marchand et al., 2022).

Another important caveat to these empirical findings is that children's abstract knowledge of number concepts is assessed using a language-based task. To succeed on a given trial, children must understand that the uttered number word corresponds to a precise number concept in addition to possessing the number concept asked for. That means that a child's general language development necessarily constrains that same child's performance on the Give-

N task. This is especially important given that children do not seem to possess absent verbal reference, or the ability to label mental representations such as 2nd order relations between sets, until around 18-22 months (Ganea et al., 2007; Luchkina & Waxman, 2021; Luchkina & Xu, 2021). In other words, even if children possess an accurate mental representation of a number, they would be unable to link this mental representation to a word or phrase until at least 18 months.[1]

Regardless of the above limitations, the step-wise progression of N-knower levels has remained a stable finding in number acquisition research. Children start out with no number knowledge, learn the number one, then two, then three and always in that order regardless of ethnicity, language, socioeconomic status, or nationality. This is in stark contrast to other related developmental patterns, such as the onset of cardinal principal knowledge, which seem to vary across individuals and cultures. Despite the relative stability of this pattern, very little work has theorized as to why children learn small numbers in order. In the next section we outline one popular existing acquisition framework and how that framework could account for this pattern.

B. The Exact Algorithms Hypothesis

The Exact Algorithms Hypothesis postulates that children construct the meaning of small number words by using existing cognitive representations and abilities, such as the ability to track objects, in qualitatively new ways. The most famous rendition of this theory is Susan Carey's bootstrapping hypothesis (Carey, 2004, 2009). By this theory, children are hypothesized to construct representations of small cardinalities (i.e., 1-3) using an innate cognitive mechanism

[1] Skeptics may argue that the reason infants do not demonstrate success with verbal labels for abstract relations before 18 months is because infants do not possess these concepts. These skeptics should note that work on preverbal infant's comprehension of abstract thematic roles such as agent, patient, and goal which suggests that representing abstract relations is likely possible, albeit constrained, without absent verbal reference (for a review see Rissman & Majid, 2019).

called *parallel individuation*. Parallel individuation refers to the ability to simultaneously attend to three distinct items with a high level of accuracy. Carey argues that this ability to attend to three items in parallel is not limited to objects (Carey, 2009, p. 232); however, most research has focused on object-based parallel individuation so we will constrain our discussion to this format. Representations of objects which have undergone parallel individuation are thought to have the format of an object-file, such that predicative information like the color, shape, and size of the object is "stored" in a file and bound to a particular location in the environment (for a review see Green & Quilty-Dunn, 2021). There is good evidence that the ability to keep track of 1-3 objects in the environment is present in some capacity very early in development, potentially even from birth (Chiang & Wynn, 2000; Feigenson, 2005; Feigenson, Carey, & Spelke, 2002; Feigenson & Carey, 2003, 2005; Van De Walle et al., 2000; Wynn, 1992a). For example, when 12- and 14-month-old infants observe an experimenter hide three objects in a box, they persist in their searching of that box after retrieving two of the objects—presumably in search of the third hidden object (Feigenson & Carey, 2003).

According to Carey, after children construct representations of small cardinalities using the parallel individuation system, these representations are subsequently stored in long-term memory. Once stored, children use these long-term memory representations in one-to-one correspondence with real life objects to identify numbers in the natural environment. Similarly, children use these representations to learn the meanings of number words. Specifically, children are hypothesized to notice that certain long-term memory representations (e.g., the representation of two items) tend to be activated when hearing a particular number word (e.g., the word "two"). As children slowly progress through different number knowledge stages, they supposedly reach an insight: moving one number up in the number line corresponds to adding one object to the

mental representation of the previous number. From this insight, Carey theorizes children bootstrap the cardinality principle, and with it, cardinal knowledge of all numbers within the child's count sequence, as well as *successor-principle* knowledge, or that each number is exactly one more than the number that precedes it.

While the original theory was highly influential and well-regarded for decades, it has recently fallen out of favor due to new evidence that contradicts the bootstrapping explanation. For example, it appears that children do not actually represent numbers as exactly 1 unit apart in the count sequence nor do they have a stable understanding of one-to-one correspondence until well after they have transitioned into being CP-knowers (Cheung et al., 2017; Jara-Ettinger et al., 2017; Spaepen et al., 2018). Latest renditions of this theory still retain core aspects of the original; the successor principle remains a key player as well as the importance of sociolinguistic inputs like productive numerical syntax and a stable count sequence (Carey & Barner, 2019). In this alternative formulation children supposedly acquire the successor-principle by applying their recursive representation of numerical syntax to an alternative format (i.e., to real world objects). Additionally, children are still assumed to use one-to-one correspondence via a stored mental representation to identify numbers in their environment.

One important assumption that Carey and others make about parallel individuation and the object-file system is that infants can represent sets of 1, 2, and 3 items well before they store these representations and use them productively in numerical contexts. Given that children are equally capable of identifying sets of 1, 2, and 3 items well before the first birthday, there is no a priori reason, at least by this framework, that children should learn small numbers in order; however, although the exact algorithms hypothesis doesn't make clear predictions about why

children learn small numbers in order, we can imagine a few possibilities compatible with the framework.

First, children may get more experience with some small number words as opposed to others. For example, children may hear the word "one" more than "two," and the word "two" more than "three." This pattern of environmental input would give young children more training on smaller numbers and therefore facilitate earlier acquisition of small numbers. Lending support to this idea, some analyses of natural language corpuses have revealed that this pattern of environmental input seems to be the case, at least in English (Anderson & Schooler, 1991; Dehaene & Mehler, 1992; Piantadosi, 2014); however, this explanation is not without critique. If the bottleneck for children's number acquisition is the number and quality of labeling events, one would expect that the rate that children hear number words is correlated with the speed with which those same children learn number words. One cross-linguistic study sought to directly test this hypothesis by comparing natural language corpuses of English, Japanese, and Russian (Sarnecka et al., 2007). Sarnecka and colleagues found that the frequency of different number words was not correlated with the age of transition into different number knowledge stages. For example, children do not become one-knowers faster in cultures where the word "one" is more frequent, casting doubt on this potential explanation.

Second, and perhaps relatedly, children may get more experience with small number instantiations. In other words, children encounter sets of one more often than sets of two and sets of two more often than sets of three. There is some evidence supporting this perspective, specifically that one can recover signature characteristics of number reasoning (e.g., Weber's law) if one assumes that the probability of encountering a given number reduces as one moves forward in the count sequence (Piantadosi, 2016). This type of explanation is consistent with

recent resource-rational models of number perception[2] which postulate that the statistical regularities of numbers in the environment shape the way adults and children perceive numbers (Cheyette & Piantadosi, 2020).

Finally, a third possibility is that the algorithms children must acquire to represent numbers precisely (e.g., one-to-one correspondence, counting, numerical syntax) somehow constrain this acquisition process. For example, it may be the case that children need to acquire a symbolic understanding of the number one before they can learn how to use one-to-one correspondence to identify other numbers. Alternatively, children may need to acquire either a symbolic iterative or recursive procedure to construct number representations larger than the number one (Buijsman, 2019). Why either would be the case is not immediately clear, but potentially representing "one" as a symbol reduces the cognitive load required when doing algorithmic procedures like one-to-one correspondence or iteration. That said, it's unclear why "two" should always come before "three" if the only cognitive bottleneck is learning the number one and algorithmic procedures.

C. Why do children learn small numbers in order?

The present study is designed to address why children learn small numbers in order. Current number acquisition frameworks spend little time addressing why children learn the first few numbers in order—even though this stable ordering is one of the most robust and reliable findings in the literature. We spent some time in the previous section speculating on what proponents of the Exact Algorithms Hypothesis might consider as reasons for this stable ordering. Importantly, none of the speculated reasons involved a neural or cognitive maturation

[2] These are also sometimes referred to as models of "unified number perception," in reference to the fact psychologists no longer need to postulate two systems of number representation under these theories.

explanation. In other words, we believe proponents of the Exact Algorithms Hypothesis would appeal to other potential bottlenecks such as the number and quality of opportunities to learn a number/number word or a constructivist process such as extracting an algorithm from the environment. We advance an alternative and potentially controversial hypothesis: children learn small numbers in order because they are not capable of representing and maintaining more than one object in working memory, at least in very early development.

Taken at face value, our hypothesis flies in the face of decades of research, which have collectively demonstrated very young infants' ability to accurately track up to three objects during occlusion events (Carey & Xu, 2001; Chiang & Wynn, 2000; Feigenson, 2005; Feigenson, Carey, & Hauser, 2002; Feigenson & Carey, 2003, 2005; Van De Walle et al., 2009; Wynn, 1992a; Xu & Carey, 1996; Zosh & Feigenson, 2015); however, recent work suggests that these object-file type representations can be fragile, especially when mapping object-files across events (for a review see Stavans et al., 2019). For example, 6-month-old infants can incorporate feature information such as color and shape into their representations of objects undergoing occlusion but when the occlusion event ends, like when the screen lifts to reveal the hidden objects, such feature-based information is not carried forward and cannot guide looking expectations (Kibbe & Leslie, 2019). By contrast, if the occlusion event continues to be ongoing, like when the screen lifts to reveal another transparent screen, such feature-based information can guide looking expectations (Wilcox & Chapa, 2002). Given that a hallmark of number concepts is their context independence, or that fact that they can apply to any kind of set, this type of event-mapping constraint around object-file representations could explain why children do not seem to learn the number one much earlier than 24 months.

Additionally, other research has suggested that object-based working memory capacity develops slowly across the toddler years (Cowan, 2016; Kibbe, 2015); however, before reviewing these findings it is important to distinguish between the two types of object-based working memory representations. In accordance with (Kibbe & Applin, 2022), we will use the terms "object-location bound" and "feature-location bound" to refer to these distinct representations; however, these representations are also sometimes called "object-based" and "feature-based or "where" and "what" representations (Green & Quilty-Dunn, 2021; Kibbe, 2015; Leslie et al., 1998; Scholl, 2001). Object-location bound representations can be thought of like pointers or indices in computer science, although it's likely that these representations contain some conceptual content too. Importantly, these representations primarily encode the location of an object with little featural content. For example, one might experience this type of representation if an object whizzes past one's head before disappearing into a bush. In this case the person in question is aware of the fact an object is hidden in the bush and likely has some high-level conceptual information about the object, such as the fact it was a ball; however, any precise high-dimensional information like the color, pattern, or texture of the object is not included. On the other hand, feature-location bound representations refer to precise high-dimensional representations of objects. This type of representation might be activated when one looks for a favorite book on a crowded library shelf.

Returning to our discussion on the development of object-based working memory representations, some research has found that 25-month-old children can maintain and track two feature-location based representations while 20-month-old children cannot (Cheng et al., 2019). Other work also suggests that the ability to simultaneously maintain two feature-location based representations is still developing between the ages of two and three (Kibbe & Applin, 2022).

This is not to say that infants aren't able to track up to three objects in parallel; to be clear, we believe that infants can and do use this ability (e.g., Wynn, 1992a). Instead, we would like to draw a distinction between "tracking" or "encoding" objects in parallel (i.e., object-location representations) from "representing" and "maintaining" those same objects (i.e., feature-location representations). In other words, tracking and encoding do not require the same cognitive resources as representation and maintenance. For example, when tracking objects infants and adults have the external world to help scaffold their internal processing whereas that scaffolding input is absent in the case of representation and maintenance because the objects and context are no longer present.

Another potential way to think about this cognitive bottlenecking hypothesis is that infants and toddlers do not possess the top-down control required to maintain multiple feature-location bindings in tandem. If we assume parallel individuation operates by assigning unique attentional indices to objects in the environment, then infants may be incapable of maintaining the content those attentional indices point to in the absence of environmental cueing. Under this hypothesis, the reason children would learn the number one first is because the ability to maintain one object in the absence of environmental cueing emerges first. Likewise, the reason the number two comes in before three would be because children develop the working memory capacity to maintain two representations before three. Of course, this explanation assumes that maintenance of working memory is a prerequisite to committing the contents of working memory to long term memory; however, we believe this is a plausible assumption to make.

D. The present study

To investigate why children learn small numbers in order, we designed a non-verbal anticipatory looking task to determine whether 14- to 23-month-old infants could form categories

for sets of one, two, and three objects. We specifically chose a non-verbal task to ensure that differences in language capacity did not interfere with our assessment of children's abstract number knowledge. We reasoned that if our hypothesis about working memory constraints was correct then infants would be able to form a category for the number one but not for the numbers two and three because of the working memory resources required to represent sets of two and three. As a brief preview, this is indeed what we found. Implications for the Exact Algorithms Hypothesis as well as parallel individuation are discussed.

CHAPTER II.

METHODS

A. Participants

Participants were recruited online via Lookit's internal database which automatically notifies parents when their child meets the eligibility criteria for a new study that is posted. Families were also able to find the study by searching through the public studies page on Lookit. We recruited 35 unique 14- to 23-month-old infants (19 female), although we only used data from 31 of the infants (see *Exclusion Criteria* for more information). Families primarily identified as White (44%), educated (98% reported at least a bachelor's degree level education), with primarily middle- to upper-level incomes (90% reported an annual income of $50,000 a year or more). Any participant that provided a valid consent video was compensated with a non-monetary Junior Scientist Certificate within 1 week of their final testing session.

We chose this age range with the intention of creating at least two age bins. Our age bins were generated with two considerations in mind. First, previous findings suggest that children begin distinguishing between singular and plural sets around 21 months (Barner et al., 2007; Li et al., 2009). Second, other work suggests that absent verbal reference, or the ability to link mental representations to language without perceptual anchoring, emerges somewhere between 16 and 18 months (see Luchkina & Waxman, 2021 and Luchkina & Xu, 2021 for reviews). Given that these two cognitive abilities may be necessary preconditions for the development of number concepts, we created three age bins: < 15 months (n=9, M=14.3 mo, range=14.0-14.8 mo), 15-18 months (n=14, M=16.4 mo, range=15.0-17.8 mo), and > 18 months (n=10, M=20.1 mo, range=18.2-22.9 mo).

Our target sample size was 20 participants per age bin. This number was calculated based on the effect size (d = .91) reported in Shukla & de Villiers (2021), a study which used an identical paradigm to the one we used. A preliminary power analysis conducted in G*Power based on this effect size and an alpha level of 0.05 indicated that at least 9 participants would need to be recruited for a within-subject *t*-test to achieve 80% statistical power (Faul et al., 2007). This same analysis also indicated at least 16 participants would need to be recruited for a between-subject *t*-test to achieve 80% statistical power.

1. Exclusion Criteria

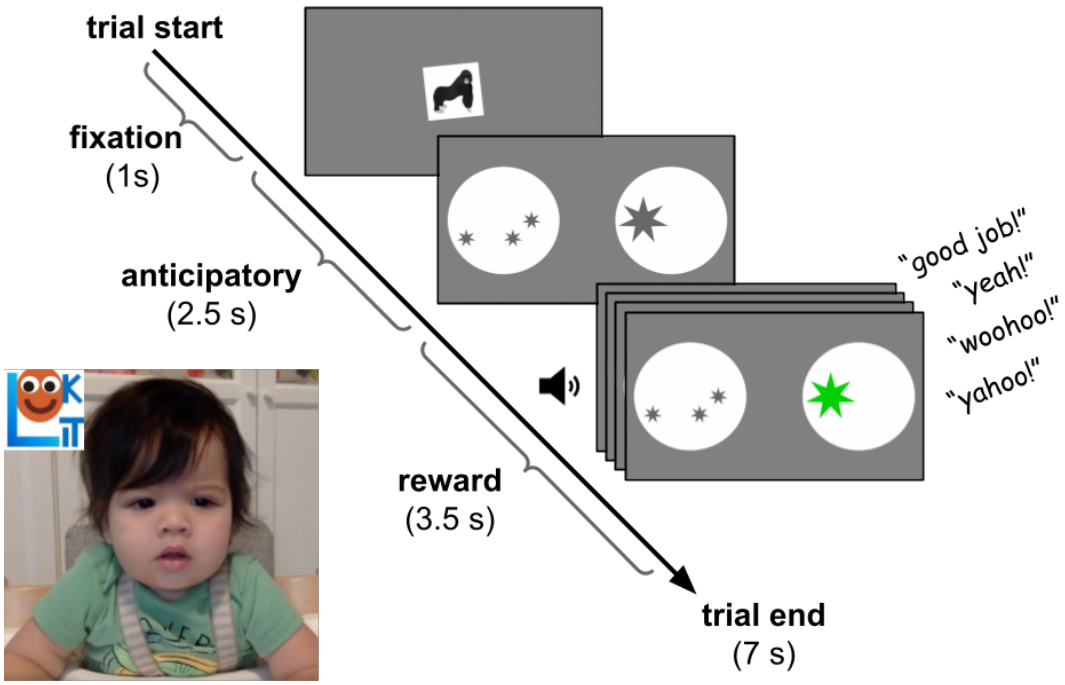
Exclusion criteria were based on Scott et al. (2017). Participants were excluded from analysis if they did not provide a valid consent video or equivalent email confirmation (n=0), were out of the age range (n=0), or their videos are unusable due to technical errors like slow frame-rate (n=2), missing videos (n=1), missing event timings (n=5) or poor video quality (n=3).

B. Implicit Concept Formation Paradigm

The task we designed was modeled after the implicit concept formation paradigm introduced by Shukla and de Villiers (2021). Importantly, this task has been successfully used in the past to assess other abstract knowledge such as agent/patient relationships as well as negation. In this paradigm, two categories of stimuli are used: target and non-target. What makes a stimulus a target is its category membership. For example, in the *One* condition of our experiment, image arrays with one shape were targets while image arrays with two and three shapes were non-targets (see Figure 1 for an example of stimuli used).

Figure 1.

Sample trial structure from implicit concept formation task (target = one)



A trial consisted of three distinct periods: fixation (1.5 s), anticipatory (2.5 s), and reward (3.5 s). During the fixation period, a small cartoon animal appeared at the center of the screen and tilted left and right. The anticipatory period followed next. During the anticipatory period, stimuli were randomly chosen from the target and non-target sets and projected on the left and right side of the computer monitor. Whether the target appeared on the left or the right side of the screen was counterbalanced and randomized across trials. Importantly, both target and non-target stimuli were gray-scaled during the anticipatory period. After the anticipatory period, the reward phase followed. During the reward phase, the target stimulus flashed bright colors while a child's voice provided positive feedback (e.g., "Woohoo! Yeah! Good job! Yahoo!") while the non-target stimulus stayed gray-scale. If infants can represent what all the target stimuli have in common (e.g., their set quantity), then the infant should begin to look towards the target stimulus before it lights up in anticipation of the reward.

The dependent measure for this paradigm is the proportional looking time [$time_{target}$ / $time_{non\text{-}target}$] for each trial during the anticipatory period. The dependent measure is proportional looking time rather than cumulative looking time to account for differences in looking preferences. For example, an infant may be biased to look to the right side of the screen but still demonstrate a different pattern of looking behavior depending on whether the target is present on the left or right side of the screen. Similarly, infants may not be very interested in the task and look away for much of the anticipatory period; however, they may still periodically check the target stimulus to see whether it has animated yet.

Generally, a statistically significant difference in proportional looking times between trials where the target appears on the left versus on the right indicates that infants can represent the "hidden" category that separates target stimuli from non-target stimuli (Shukla & de Villiers, 2021). It should be noted that the strength of this interpretation relies on the stimulus design; in other words, if the target and non-target stimulus vary on more than one-dimension (i.e., category membership), then it is unclear which dimension(s) infants are using to represent a difference between the target and non-target stimuli.

C. Procedure

We implemented our anticipatory looking task across three asynchronous online experiments hosted on Lookit, a virtual platform for developmental science experiments (Scott et al., 2017; Scott & Schulz, 2017). Families were encouraged to participate in all three experiments with email invitations for additional sessions sent out 2-5 business days after session completion. However, the dropout rate after the first experiment was 50%; therefore, we treated our data as between-subjects. Within each experiment families participated in two trial blocks. The first block was always the number block where the target category was either *One, Two, or*

Three. The second block was always the control block where the target category was either *Vertical* lines, *Horizontal* lines, or *45 Degree* angle lines. Three number conditions were administered across the experiments, where either the number *One, Two*, or *Three* was the target category with the other two numbers used as the non-target. Similarly, three control conditions were administered where three different line orientations served as target and non-target stimuli categories. Infants were randomly assigned to an initial condition and then moved through the other conditions in a fixed order (see Figure 2). The preregistration of this study is available at https://osf.io/rnksh/?view_only=9121e38a701c4799862146964e458792. All procedures were approved by the Institutional Review Board at the University of Massachusetts Amherst.

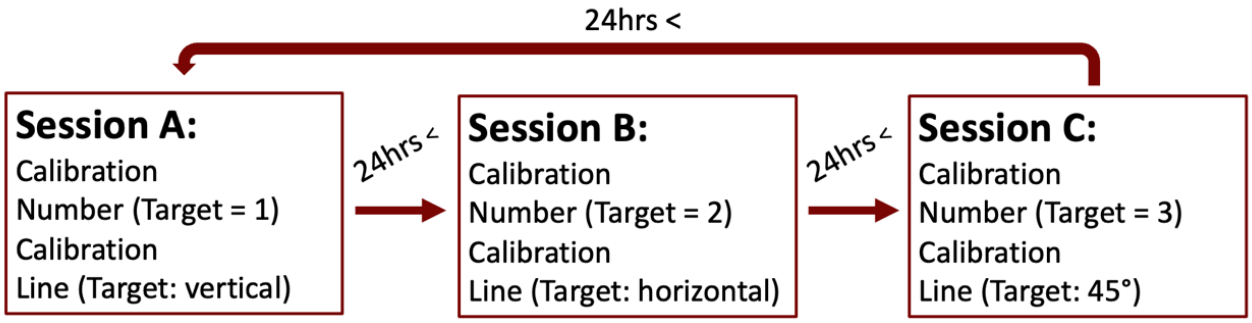
Each experiment had an initial consent and set-up phase. First, caregivers gave their consent by recording a verbal statement affirming they had read the consent document. Next, caregivers were given the opportunity to review the experiment before their child participated. We recorded this preview session to ensure that infants were not accidentally exposed to the experimental stimulus before the experiment began. Next, caregivers were asked to place their infant in a highchair or stroller or to sit with their infant on their lap. If caregivers chose to sit with their infant on their lap, they were instructed to hold their child over their shoulder and turn away from the computer screen to avoid influencing their child's looking behavior. Finally, caregivers completed one last to check to ensure that their child's face was in view of the webcam and then the experiment began. After the experiment was finished, caregivers were given an opportunity to retract their data.

Infants experienced two fixed-order blocks of the implicit concept formation paradigm (see Figure 2). The first block was always the number condition, while the second block was always the control condition (e.g., line orientation condition). Each block consisted of 32 unique

trials and took about 5 minutes to complete. This meant each experiment had a total of 64 trials (32 number condition trials and 32 line condition trials) unless the family exited the experiment early or paused and restarted a condition. Infants were randomly assigned a condition to start with. For example, an infant could receive Session A, Session B, or Session C first (see Figure 2). If a family returned for additional sessions, they moved through those sessions in a fixed order. So, if a baby started with Session A they next received Session B followed by Session C.

Figure 2.

Experimental Procedure



Families could pause the experiment and restart a block at any time. When a pause was initiated in a block, a new and random sequence of the exact same 32 trials was shown to the infant. Families could restart a block as many times as they wanted; however, we only included data from the first three attempts at a block. If a family exceeded this many pauses, we retained their first three attempts and excluded any additional. Of the 111 attempts made, only 9 attempts contained exactly one pause and 10 attempts contained two pauses. In other words, most of the time families did not pause the experiment (74%).

D. Stimuli

Number is necessarily confounded with non-numerical features like area and perimeter, making it difficult to determine whether number or some other feature is driving a behavioral response. In fact, there is debate about the extent to which number perception is separate from perception of these non-numerical features (for a recent review of the debate see Clarke & Beck, 2021). Because it is mathematically impossible to control for all dimensions of non-numerical features simultaneously, DeWind and colleagues (2015) devised a way of accounting for the individual contributions of these features in numerosity judgement tasks involving dot arrays. This statistical procedure relies on the fact that a dot array can be fully described as a linear combination of three independent dimensions: size (e.g., individual surface area, total surface area), spacing (e.g., sparsity, field area), and numerosity (e.g., how many dots are present). By logarithmically compressing these dimensions and applying a linear transformation to ensure the dimensions are orthogonal to one another, one can precisely quantify the contribution of each dimension (e.g., size, spacing, and number) to a participant's performance. The stimuli used in the number condition were constructed using a computer program based on this mathematical formulation to bolster the interpretation of any significant findings (Park, 2021). It should be noted that the perception of small numerosities, or *subitizing*, is not known to be influenced by size or spacing (although to some degree by spatial configuration). Nevertheless, we manipulated the three magnitude dimensions—number, size, and spacing—equally and systematically in this experiment to account for this.

For the number condition, stimuli consisted of two large white circles, each with a homogenous array of shapes inside. Six possible different shapes were used: circle, hexagon, triangle, square, cross, and an irregular star. The type of shape used on a given trial was consistent across the two sides of the screen; however, the size, spacing, and number of shapes

systematically varied. For example, one possible trial combination was one large hexagon on the left side of screen with three small hexagons on the right side of the screen. During the reward phase of a trial, the target stimuli changed to a random selection of four colors while a child exclaiming “Woohoo!”, “Yahoo!”, “Yippee!”, “Good job!” in a random order played.

For the line condition, stimuli also consisted of two large white circles, but this time each circle was filled with lines all going in the same orientation. The spatial frequency of the lines was the same across both sides of the screen; however, the line orientation varied. During the reward phase, the vertical lines changed to a random pattern of colored lines going in the same orientation four times while the same audio reward from the number condition played.

Stimuli for both the number condition and line condition were generated using PsychoPy’s Builder and Coder modules (Peirce et al., 2019). Because the experiment was posted on Lookit, videos of the entire number blocks and line blocks were created to maximize user experience. Stimuli videos were made by simultaneously using QuickTime Player screen recording and Loopback audio recording. Once created, videos were compressed and converted to the appropriate file types using the FFmpeg library. Altogether, three versions of each number and each line condition were constructed for each experiment (resulting in 18 different videos) to ensure that trial order did not influence children’s responses.

E. Qualitative Coding

Webcam recordings for each experimental session were downloaded and annotated using ELAN 6.3 (Sloetjes & Wittenburg, 2008). Video annotators were blind to study condition and experiment block. The webcam recordings were muted during coding to prevent video annotators from using audio cues to determine whether a trial was in the anticipatory phase or reward phase.

Each coder followed a set of shared guidelines for annotating infant and caregiver behaviors. A breakdown of the qualitative codes, criteria, and example behavior can be found in Tables 1 and 2 in the Appendix. In addition to these guidelines, coders were given three important instructions. First, coders were instructed to use the infant's pupil to determine where the infant was looking on the screen. Second, they were instructed to slow videos down to a 1/100 second frame increments to determine precisely where a given behavior began and ended. Finally, coders were instructed to use the AMBIG tier code sparingly, reserving it primarily for when the infant went out of frame, or the webcam resolution degraded such that the infant's eyes were no longer visible. Before using the AMBIG code, coders were required to review the calibration video to refresh their memory of where the edges of the computer monitor were for that infant. If coders could still not determine whether the infant was looking at the computer monitor or away, then they annotated the behavior using the AMBIG code.

CHAPTER III.

ANALYSIS

Our primary research question was why do children learn small numbers in order. We hypothesized that children learn the number one first because children lack the cognitive resources required to represent more than one object early in development. If our hypothesis is correct, then we would we predict that 14-23 month-old-infants will be able to represent the implicit category rule for the number one but not for the numbers two and three in our anticipatory looking paradigm. In other words, we predicted that infants would show statistically significant differences in their looking time ratios as a function of whether the target was on the left or right side only in the *One* condition. All analyses were pre-registered at https://osf.io/rnksh/?view_only=9121e38a701c4799862146964e458792 unless otherwise stated in text.

A. Number Block

To test this prediction, we first calculated the normalized left/right looking time ratio for each trial using the following equation: $looking\ ratio = \frac{LEFT}{LEFT+RIGHT}$ where LEFT refers to cumulative time spent looking at the left side of the screen during a given trial and RIGHT refers to cumulative time spent looking at the right side of screen during that same trial. If an infant spent 0ms looking left and 0ms looking right on a given trial, then the normalized left/right looking time ratio was set to .5 to ensure we did not divide by 0. We used this measure, as opposed to a raw $\frac{LEFT}{RIGHT}$ ratio or a raw $\frac{TARGET}{NON-TARGET}$ ratio, for a few reasons. First, this formula enhanced the interpretability of the results by ensuring that each observation of the normalized left/right looking ratio fell between 0 and 1, where a ratio equal to .5 indicated no bias, a ratio

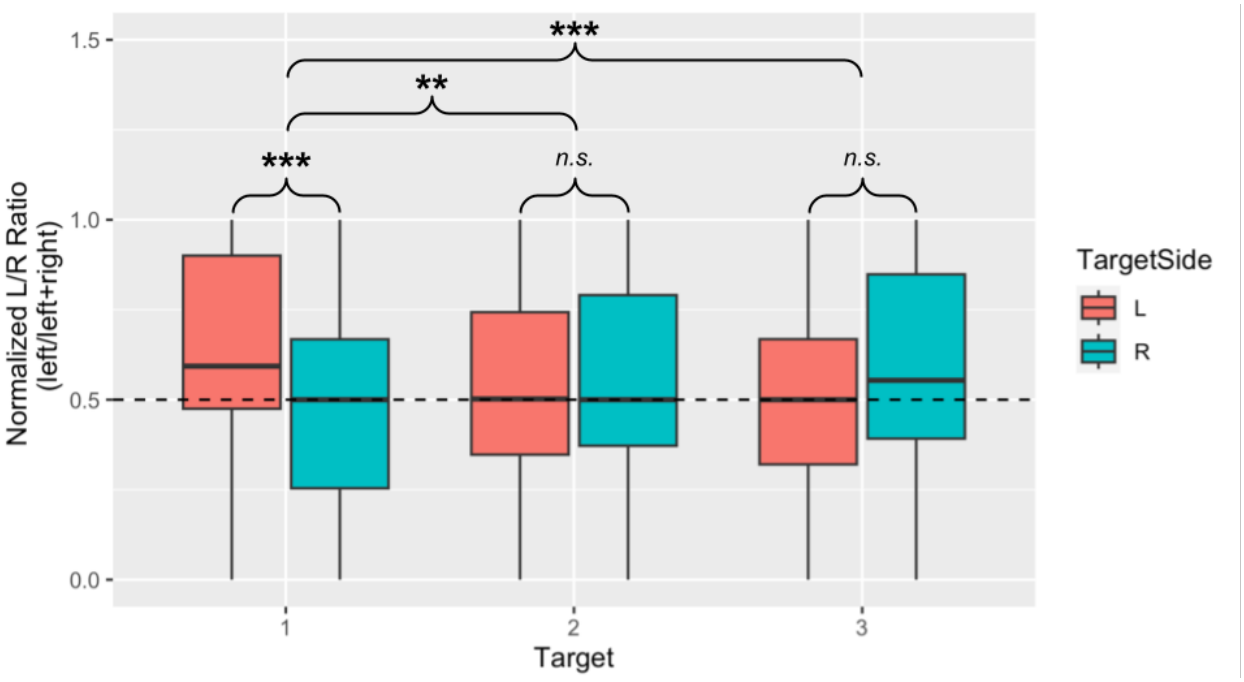
less than .5 indicated a right-ward looking bias and a ratio over .5 indicated a left-ward looking bias. Second, using the normalized formula ensured that the scaling was equivalent on both sides of the chance line. For example, a normalized ratio of .6 indicates a left-ward bias that is of equivalent size to a right-ward bias represented by a normalized ratio of .4. Using these normalized looking ratios, average left/right looking time ratios for each child were calculated. Because a target could appear on either the left or right side of the screen, two average ratios were calculated for each condition: a looking ratio for when the target appeared on the right and a looking ratio for when the target appeared on the left. Importantly, because infants may have a natural preference for the left or the right, what matters is not that the looking time ratio is significantly different from 0.5 but instead that the ratios are different depending on whether the target was present on the left versus the right.

We first looked at the number block data to see whether infants could represent the implicit category rule when the target category was the number one. A paired *t*-test was conducted on infant's average looking ratios in the *One* condition as function of whether the target appeared on the left or right (see Figure 3, target=1). Infants looked significantly more to the left when the target appeared on the left (M_{LEFT} = 0.62, SD_{LEFT} = 0.13) than when the target appeared on the right (M_{RIGHT} = 0.47, SD_{RIGHT} = 0.17), t(21) = 4.63, $p < .001$, $d = 1.1$. The Bayes factor for this effect was BF10 = 197, indicating extreme evidence for the alternative hypothesis. To determine whether there was an effect of age, a 2x3 (Target Side [left, right] x Age Bin [< 15 months, 15 – 18 months, > 18 months]) mixed-effects ANOVA was conducted. This ANOVA revealed only a significant main effect of target side, F(1, 19) = 23.01, $p = <.001$, $\eta^2_G = .26$. Importantly, the interaction of age bin and target side was not significant, F(2, 19) = 1.78, $p = .19$.

Next, we looked at the number block data for the number *Two* condition. A paired *t*-test was conducted on infant's average looking ratios as function of whether the target appeared on the left or right (see Figure BB, target=2); however, the results did not reach significance, t(15) = 0.41, p = .69, (M_{LEFT} = 0.57, SD_{LEFT} = 0.13; M_{RIGHT} = 0.55, SD_{RIGHT} = 0.12). The Bayes factor for this effect was BF10 = 0.27, indicating a moderate level of evidence for the null hypothesis. We also conducted a paired *t*-test on infant's average looking ratios as function of whether the target appeared on the left or right for the number *Three* condition (see Figure 3, target=3), but again, the results did not reach significance t(12) = -1.44, p = .17, (M_{LEFT} = 0.49, SD_{LEFT} = 0.16; M_{RIGHT} = 0.57, SD_{RIGHT} = 0.18). The Bayes factor for this effect was BF10 = 0.65, indicating an anecdotal level of evidence for the null hypothesis.

Figure 3.

Infants' (14- 23 months) average looking time ratios across number conditions



Note. The target side L refers to when the target is on the left, and R refers to when the target is on the right.

** $p < .01$. *** $p < .001$

Given the anecdotal evidence for the null hypothesis in both the two and three condition, we were interested in whether significant differences existed in average looking time ratios between the *One* condition and the *Two* and *Three* conditions. To investigate, we ran a 2x3x3 (Target Side [left, right] x Target [one, two, three] x Age Bin [< 15 months, 15 – 18 months, > 18 months]) mixed-effects ANOVA. This ANOVA revealed two significant effect on average looking behavior: an interaction between target and target side, $F(2, 42) = 9.17$, $p > .001$, $\eta_G^2 = .13$ and an interaction between target and age bin, $F(2, 42) = 94.70$, $p = .04$, $\eta_G^2 = .03$. Using a Bonferroni adjusted alpha level of .017 per test (.05/3), three planned contrasts were conducted on the target and target side interaction. First, there was a significant difference between the *One* condition and the other two conditions on average looking as a function of whether the target was present on the left or right, $t(42)=4.56$, $p < .001$. The pairwise comparison of the *One* condition with the *Two* condition was significant, $t(42)=3.71$, $p=.002$ and the pairwise comparison of the *One* condition with the *Three* condition was also significant $t(42)=4.75$, $p<.001$.

B. Line Block

Finally, we looked at the line block data. Unfortunately, it became clear early on in data collection that infants were bored of our task by the time the line condition started. Many parents and caregivers also directly reported this in the comments box during debriefing. Rather than have coders annotate all the line block data like we pre-registered, after they had annotated 16

participants we had them stop because it became clear that infants were not paying attention[3]. We also collapsed the line block data across the various conditions (i.e., horizontal, vertical, 45 degree) to increase the statistical power of the test. A paired *t*-test was conducted on infant's average looking ratios during the line blocks as function of whether the target appeared on the left or right; however, the results did not reach significance, t(15) = 0.77, p = .45, (M_{LEFT} = 0.58, SD_{LEFT} = 0.26; M_{RIGHT} = 0.53, SD_{RIGHT} = 0.08). The Bayes factor for this effect was BF10 = .33, indicating a moderate level of support for the null hypothesis.

[3] We stopped specifically at 16 participants because a priori power analyses indicated that 16 participants would be sufficient to conduct our planned *t*-tests at 80% statistical power.

CHAPTER IV.

DISCUSSION

The present study investigated whether 14- to 23-month-old infants could use small number representations to guide their looking behavior in an anticipatory looking paradigm. Specifically, we tested whether infants would be able to use representations of the numbers one, two, and three to anticipate which side of the screen was going to light up. Our hypothesis was that infants would be able to activate and maintain a representation of the number one (but not two or three) to successfully anticipate the target side. The results we found were consistent with this hypothesis.

First, infants showed a large significant difference in the amount of time they looked to the left versus the right as function of target side only in the *One* condition. By contrast, we found null results in the *Two* and *Three* conditions with a Bayes factor analysis indicating moderate and anecdotal support for the null hypothesis. Next, and most importantly, there was a significant interaction effect between target condition (i.e., target = *One, Two*, or *Three*) and target side, indicating that infants responded differently across the different target conditions. Planned contrasts revealed that this effect was driven by a difference between the *One* condition and the other two conditions. Taken together, these results suggest that there was a qualitative difference in infant performance in the *One* condition versus the *Two* and *Three* conditions. Specifically, infants were able to identify the target category when it was the number one but not when the target category was two or three.

A. Exact Algorithms Hypothesis

Our results have meaningful implications for the Exact Algorithms Hypothesis, especially with regards to parallel individuation. While previous work has consistently demonstrated that

infants are able to track up to ~3 items in parallel across occlusions of time and space, our results suggest that there are important limitations to this early developing ability. But, what could these limitations be? We suggest that the attentional indices, which facilitate the parallel individuation mechanism, are strictly bound to item identities. These bindings create what are known as "object-location" representations in the working memory literature. Importantly, because the attentional indices are bound to item identities, this prevents attentional indices from being represented and reasoned with abstractly—at least initially in development.

In previous work, infants were only required to re-identify already individuated objects. For example, in Wynn (1992a) infants were presented with dolls which disappeared behind a screen that later lifted to reveal either a consistent or inconsistent number of previously shown dolls. Our task differed from previous such tasks with regards to this matter. Rather than requiring infants to reidentify the expected objects using object-location bindings, infants in our study were required to maintain and use their representations relationally. This meant infants could not simply reidentify the objects they had seen previously. Instead, we suggest that there were two potential paths to successful anticipation in our task.

First, infants could have noticed that the side of the screen with *N* attentional indices activated, where *N* is the target number, was the side of the screen that would begin flashing bright colors. This proposal is most like what Carey and others have suggested with regards to the role parallel individuation plays in the development of number concepts. In fact, there is already some evidence that four-year-old children are better at comparing quantities of objects represented by parallel individuation than comparing quantities of objects represented by the analogue magnitude system (Cheung & Le Corre, 2018). However, we find this proposal unlikely for two reasons. First, it would require that infants represent and reason about their own

attentional resources, either subconsciously or consciously. To our knowledge, there is no evidence supporting that level of metacognition prior to age two. Additionally, if infants were deploying this strategy, then it is unclear why they would succeed in the *One* condition yet fail in the *Two* and *Three* conditions. One of the hallmarks of parallel individuation is that the attentional indices can operate concurrently and independently, making it such that identifying sets of three is just as easy as identifying sets of one. Given this defining characteristic, there is no a priori reason why it would be easier to notice when one attentional index is deployed versus three.

Alternatively, infants could have used their "object-feature" working memory resources to succeed. Practically, this would mean that infants encode the last trial's reward stimulus (e.g., two flashing triangles), maintain that stimulus across the fixation period, and then relate the previous reward stimuli to the two concurrently presented stimuli (e.g., one cross vs two crosses). At minimum, this requires infants to maintain two distinct working memory representations: the representation of the previously seen stimuli and the representation of the stimuli currently presented. We find this explanation more parsimonious for a few reasons. Previous work has found that infant's working memory resources are featurally limited and develop slowly across the first four years (Cheng & Kibbe, 2022; Cowan, 2016; Kibbe, 2015; Kibbe & Applin, 2022; Kibbe & Leslie, 2019). This suggests that the reason infants succeeded in the one condition but not in the two and three conditions was because infants only possess enough working memory resources to maintain and relate one feature-location binding. This interpretation is consistent with previous studies which have found that 24-month-old infants but not 20-month-old infants can represent two feature-location bindings (Cheng et al., 2019).

As it stands currently, the Exact Algorithms Hypothesis does not account for the results reported in this paper; however, that does not mean that the framework should be abandoned. Instead, we believe that the framework needs to be updated by revising the role parallel individuation plays. For example, our results are consistent with a large body of literature which has argued that subitizing requires online attentional resources—except perhaps for the number one (Chen et al., 2022; Egeth et al., 2008). It may be the case that infants are able to bind attentional indices to object-location representations while they lack the attentional resources required to maintain similar bindings to feature-location representations. In this case, parallel individuation is not an innate mechanism but instead the by-product of domain-general attentional maturation. If our interpretations are correct, we should see strong correlations with visual working memory capacity and whether a toddler can form the implicit category rule when the target is two or three. Future work might also investigate to what degree parallel individuation, and therefore subitizing, depends on constituent item identities.

B. Unified Number Sense Models

Our results also have meaningful implications for other acquisition models. As alluded to in the introduction, it has been recently proposed that humans possess a "unified number sense" as opposed to distinct mechanisms for parallel individuation and large magnitude estimation (Cheyette & Piantadosi, 2020). Our results present a challenge to this model in two distinct ways. First, the informational entropy of the sets used in our study was consistent across numbers and number conditions. Specifically, because the stimuli were homogenous in size and shape, the entropy of the sets should have been zero—regardless of whether the sets consisted of one, two, or three shapes. This means that the information processing demands required by infants to identify the numbers one, two, or three should have been equivalent. That said, it's possible that

the spatial frequency/location is an additional parameter which contributes the perception of entropy within a set.[4] However, we find it unlikely that spatial frequency alone was driving the effects reported in this study given that there were no statistically detectable differences between the *Two* and *Three* conditions.

Second, using a similar anticipatory procedure, previous work has demonstrated that infants are able to discriminate quantities at a 2:3 ratio by 9 months (Lipton & Spelke, 2003). Given that comparisons of 1 vs 3, 1 vs 2, and 2 vs 3 all fall at or under this ratio, infants should have been able to succeed in all conditions of our task if they were recruiting analog magnitude representations. Proponents of this theory might defend themselves by appealing to the differing need-probabilities of small numbers. In this case, we might expect the number one to be represented the most accurately because we encounter the number one more than two or three (Piantadosi, 2014). Future work should seek to replicate our paradigm with large magnitudes (e.g., 8, 16, and 24 shapes) to determine whether this defense is viable. If 14- to 23- month old infants can succeed in all three large magnitude conditions, it would be very difficult to appeal to need probabilities as a reason infants failed in the two and three conditions in the present study.

C. Language and Number Acquisition

Turning from specific acquisition theories to more general theory elements, our results support the notion that language has a role to play in the development of precise number concepts (Carey & Barner, 2019; Condry & Spelke, 2008; Guerrero & Park, 2023; Perszyk & Waxman, 2018; Watanabe, 2017). In our study infants failed to reason symbolically and

[4] It's unclear to us exactly what "informational entropy" means from a neuroscience perspective. In many of the studies which appeal to informational entropy, entropy is operationalized using categorical variables defined by the researcher's subjective experience (e.g., what kind of object an item is, whether a particular feature is present). Although several models that attempt to explain entropy perception exist (e.g., Finding Differences Model, Relational Generalized Context Model, etc.), we are still far from consensus on what informational entropy represents cognitively. For a relevant discussion of these issues, see Wasserman et al. (2021).

relationally with the numbers two and three. One potential explanation behind this failure that children construct these concepts via cultural inputs like language. Although one major limitation of our study is that we did not collect data on infants' language development, we also believe it to be highly unlikely that the youngest infants in our study (i.e., 14- to 18-month-olds) were producing any kind of number-related language. For example, previous work has shown that it isn't until infants are about 20- to 22-months that they begin understanding grammatical number, or in English the ability to distinguish between singular (e.g., "cat") and plural sets (e.g., "cats") (Almoammer et al., 2013; Barner et al., 2007; Barner, Libenson, et al., 2009; Li et al., 2009). Likewise, it is not until infants are about 2 years old that they begin appropriately using the word "one" to refer to sets of exactly one (Barner et al., 2007). If one fairly assumes that the infants in our study have not yet mastered grammatical number or number word knowledge, then it is possible that acquisition of these linguistic faculties is the driving force behind the acquisition of number concepts like two and three.

On the other hand, infants in our study did succeed at using the number one relationally—with no detectable differences in performance across age bins. This suggests that the number one may be unique in some respects. Other work unrelated to the present also supports this suggestion. For example, some work has reported that when subitizing, the number one seems especially immune to manipulations of attention when compared to the numbers two and three (e.g., Egeth et al., 2008). Additionally, it appears that other animals, like rhesus monkeys, also represent the number one exactly despite the fact they cannot represent other numbers with the same precision (Barner et al., 2008). These and other findings have spurred some scholars to theorize that children may have an innate representation of the number one, and that children later construct larger numbers using this innate representation of one and some kind

of recursive or iterative algorithm (Buijsman, 2019; Watanabe, 2017). While our results cannot directly prove or disprove these types of theories, our results are consistent with the predictions these theories make. Additional empirical work on the development of later number concepts as well as the ontological origins of the number one will need to be conducted before we can definitively rule in or out the proposals made by those like Buijsman (2019) and Wanatanbe (2017).

D. Shape Homogeneity

Our results also contribute some nuance to previous findings regarding infants' attention to number versus continuous extent when representing homogenous arrays. Previous work had reported that infants do not attend to the number of items in array when the array items of homogenous in nature (Feigenson, 2005; Feigenson, Carey, & Hauser, 2002). Our results suggest that infants pay attention to the number of homogenous items in an array when number is made salient by the task demands. In other words, infants may have failed to represent number in the original Feigenson, Carey, and Hauser (2002) experiments because graham crackers were used as stimuli. If infants wanted to maximize their graham cracker rewards, they might have picked the containers with the most graham cracker stuff as opposed to basing their decisions on quantity.

That said, additional work by Zosh & Feigenson (2014) suggests that the reason infants may fail to attend to number in the case of homogenous arrays is because infants experience "catastrophic forgetting." In other words, because the items are identical in homogenous arrays, there is more interference between items when it comes time to remember the array. This causes infants to misremember the number of items originally present. Given that we used homogenous arrays in our task, it's possible that the reason infants failed to successfully anticipate the target side of the screen in the two and three conditions was because infants had difficulty remembering

the previous target arrays. Future work should test this hypothesis by running the same task except with heterogenous arrays. If 14- to 23-month-old infants succeed in the two and three then, that would bolster the idea that working memory resources are a critical factor in the development of number concepts. It might also suggest that language is not necessarily required for the development of small number concepts.

E. Study Limitations

The present study was not without shortcomings. As previously noted, we did not collect language development measures nor did we collect working memory measures, making it hard to draw strong conclusions as to why children failed in the *Two* and *Three* conditions yet succeeded in the *One* condition. Another potential limitation was our stimulus design. We chose to use homogenous arrays of shapes placed within a white circle because we wanted the entropy between arrays to be consistent and we wanted the sets of shapes to be as distinct as possible. However, as we already noted in the previous section, the homogeneity of the shapes may have contributed to infants' failures in the *Two* and *Three* conditions. Furthermore, the use of a white circle may have been problematic as it has been suggested that common region, a Gestalt principle, may be able to explain the results we found.

Common region refers to the principle where items which fall under a common region become grouped together and represented distinctly from other regions in the visual field (Palmer, 2003). The criticism is that infants may not have individuated the shapes within the white circles and instead represented them as attributes of the white circle (i.e., the "common region"). Infants seem to possess an early developing principle of common region so it is not out of the question that it might operate this way (Bhatt et al., 2007); however, other work suggests that the principle of common region helps adults attend to the number of objects within the

common region (Pan et al., 2021). Given the adult findings, we find it highly unlikely that infants were representing the arrays as unindividuated masses. Future work could resolve this alternative interpretation by running a similar experiment but without explicitly grouping the arrays. If the pattern of results changed such that infants succeeded in all three conditions, that would indicate that the principle of common region was interfering with shape individuation. On the other hand, if infants failed to anticipate any of the targets across the conditions, that would indicate that common region was helping infants either distinguish the arrays or attend to the number of shapes within the arrays. If the same pattern of results were obtained, then this alternative explanation could be successfully ruled out.

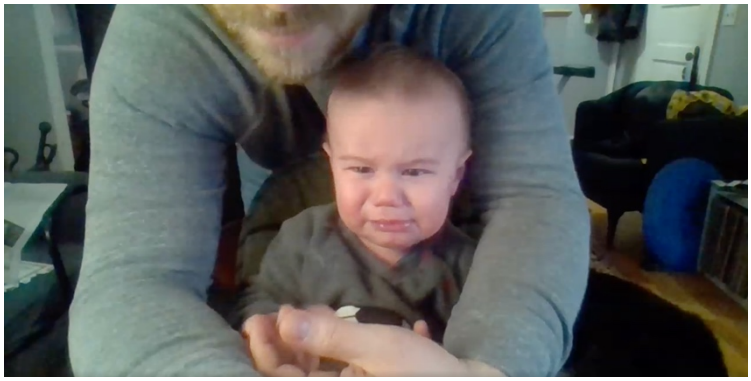
CHAPTER V.

CONCLUSION

In summary, this research contributes to existing number acquisition research by demonstrating that 14- to 23-month-old infants are capable of reasoning about the number one (but not two or three) relationally and abstractly. This finding suggests that the reason children learn small numbers in order is because children initially lack the working memory resources required to represent, maintain, and relate sets of more than one object.

APPENDIX
LOOKING BEHAVIOR CODES

Table 1.
Tier codes and criteria for annotating infant looking behavior

Tier Code	Criteria	Example
CRY	Infant crying or fussing	
MOVE	Infant actively trying to get out of highchair, stroller, or parent's lap; infant moving excessively and not looking at computer	
SLEEP	Infant fell asleep	

CENTER Infant looking at the center of the computer screen

RIGHT Infant looking at the right of the computer screen

LEFT Infant looking at the left of the computer screen

AWAY Infant not looking at screen; minor movements like slapping the highchair table or turning head are okay

EVENT Infant not looking at screen and caused by an external event (ex: loud noise, parent interference)

AMBIG Cannot determine where infant is looking due to poor video quality, infant moving out of frame, or any other reason

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