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# Growth cycles in mature and dual economies

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## Abstract

Mature economies may experience fluctuations, but the average medium and long run growth rate matches the natural rate. Like Kaldor's neo-Keynesian models, the Marx-Goodwin tradition explains this outcome by endogenizing the distribution of income and assuming that the accumulation of capital is increasing as a function of the profit share. The application of Goodwin cycles to developing economies may be hard to justify, however. The modified Goodwin models in this paper include relative-wage norms as a central element of wage formation. Norms change endogenously, leading to path dependence (hysteresis) in the stationary solution for the employment share of the modern sector. The effects of shocks – the sensitivity of the long-run outcome to initial conditions – may be amplified by non-linearities in the adjustment of wages to deviations of actual wages from the norm.

Key words: Goodwin cycles, wage norms, employment hysteresis  
JEL codes: E11, E32, O41

## 1 Introduction

Harrod's analysis of the dynamics of capitalist economies identified two distinct problems. The first problem concerned the absence of automatic adjustments of the 'warranted rate of growth' to the 'natural rate'. Assuming a constant saving rate out of income, a constant output capital ratio and a constant depreciation rate of capital, the equilibrium condition for the goods market determines a unique warranted rate of growth; only by a fluke will this warranted rate be equal to the growth rate of the labor force in efficiency units (the natural rate of growth). The second problem highlighted the likely instability of the warranted growth path when firms react to positive (negative) deviations of actual and desired utilization by increasing (decreasing) the rate of accumulation.

Many economies follow growth paths that seem to align the natural and warranted growth rates. The rich OECD countries may not always have full employment, but the employment rate fluctuates around a fairly high level, and these

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economies would come up against labor constraints if aggregate demand were to expand rapidly over periods lasting more than few years. Large-scale immigration could alleviate labor shortages but would almost certainly run into political constraints, and it is limited how fast and to what extent these economies would be able to draw new groups into the domestic labor market through changes in the retirement age, for instance, or increases in women’s participation rate. In these ‘mature economies’ with fluctuations around a near-full-employment trend the warranted growth rate appears to adjust to the natural rate, with perhaps some adjustment coming also from induced changes in the natural rate.

I have argued elsewhere that Harrod’s two problems open the way for a Keynesian theory of local instability and endogenous cycles in mature economies. This paper, however, focuses exclusively on Harrod’s first problem and the way it has been addressed by prominent post-Keynesian and neo-Marxian contributions, especially the literature inspired by Goodwin’s (1967) formalization of Marx’s general law of capitalist accumulation. Thus, I leave aside Harrodian instability issues and their role in cyclical fluctuations.

Robert Solow’s reconciliation of warranted and natural growth rates relied on adjustments in the output capital ratio as economies move along a smooth neoclassical production function. Rejecting this solution, post-Keynesian and neo-Marxian theories have based the reconciliation on differential saving rates out of wages and profits and the effects of endogenous changes in income distribution on the average saving rate. The relevance of these models to developing economies is questionable. One-sector models – whether of the Solow or Goodwin type – may become misleading in economies with small modern sectors and large reservoirs of underemployment in traditional and informal sectors. The very process of economic development is characterized by structural transformation, growth rates above the natural rate, and a gradual reduction of underemployment. These features also bring into question the feedback effects from the labor market to the accumulation rate that are central to the Goodwin cycle.

Section 2 discusses Solow’s solution to Harrod’s first problem. Section 3 outlines alternative Kaldorian and Marxian solutions, including the Goodwin model. Section 4 discusses the application of the Goodwin mechanism to developing economies. Section 5 concludes.

## 2 Reconciling warranted and natural growth rates

### 2.1 The Solow solution and neoclassical production functions

If  $s, \sigma, n, \delta$  denote the saving rate, the output capital ratio, the growth rate of the labor force in efficiency units and the depreciation rate, a reconciliation of warranted and natural growth rates requires that

$$s\sigma = n + \delta \tag{1}$$

The four terms in equation (1) cannot be set independently; at least one of them must be allowed to adjust to satisfy the equation.

Solow (1956, p. 65) singled out the "crucial assumption that production takes place under conditions of fixed proportions". Instead of imposing a fixed output capital ratio, equation (1) can be used to determine the value of the technical coefficient  $\sigma$  that is consistent with the equalization of natural and warranted growth rates. A smooth neoclassical production function ensures the existence of this growth path, provided the range of possible output capital ratios is sufficiently wide.<sup>1</sup>

The ubiquitous neoclassical aggregate production function carries a heavy load in most of contemporary macroeconomics. Yet, the justification for the production function is extremely weak, both theoretically and empirically. The issues have been analyzed thoroughly in a voluminous literature and should be well known. Yet, they seem to be forgotten or simply brushed under the rug; textbooks simply introduce the production function and the standard assumptions that go with it

The Cambridge capital controversy highlighted the theoretical weaknesses.<sup>2</sup> Indeed, Samuelson (1966) conceded the theoretical case in his "summing up", concluding that

If all this causes headaches for those nostalgic for the old time parables of neoclassical writing, we must remind ourselves that scholars are not born to live an easy existence. We must respect, and appraise, the facts of life. (p. 583)

Despite their theoretical weaknesses the 'old-time parables' go unquestioned in most contemporary macroeconomics. There appears to be a general perception that the neoclassical production function remains a useful tool, that it has empirical support, and that it is safe to ignore the theoretical possibilities and anomalies brought up by the capital controversy.<sup>3</sup>

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<sup>1</sup>The existence of the solution is guaranteed if the production function satisfies the Inada conditions. It may be worth noting that these conditions are restrictive: they fail to be met for all CES production functions, with the exception of the Cobb-Douglas case. The dynamics may imply that the capital labor ratio converges to zero if the elasticity of substitution is below 1; with high substitution elasticities and high saving rates, conversely, the system may become so productive and save so much that "perpetual full employment will increase the capital-labor ratio (and also output per head) beyond all limits" (Solow 1956, p. 72). In the latter case an increase in the saving rate raises the long-run growth rate; the model produces endogenous growth.

<sup>2</sup>See Harcourt (1972), Cohen and Harcourt (2003) and Felipe and Fisher (2003) for surveys of the capital controversy and aggregation in production functions. Some of the key contributions have been collected in Harcourt and Laing (1971).

<sup>3</sup>Solow (1966, pp. 1259-1260) expressed this pragmatic and instrumentalist defense explicitly when he declared that

I have never thought of the macroeconomic production function as a rigorously justifiable concept. In my mind it is either an illuminating parable, or else a mere device for handling data, to be used as long as it gives good empirical results, and to be abandoned as it doesn't, or as soon as something better comes along.

The empirical case also faces serious problems. Econometric regressions sometimes seem to find support for the neoclassical production function, but an underlying accounting relation links output to wage and capital income:  $Y = wL + rK$ . This accounting relation implies that a Cobb-Douglas production function will provide a good fit as long as the shares of wages and profits are roughly constant, a condition that can be met for reasons that have nothing to do with perfect markets, Cobb-Douglas production functions and factor prices that are equal to marginal products; see Fisher (1971) and Shaikh (1974); Felipe and McCombie 2009 and Felipe and Fisher 2003 provide useful surveys.

## 2.2 A Kaldor-Solow solution

All long-run macroeconomic models contain some kind of production function that links current investment to future capacity. But the Cambridge capital controversy and the literature on aggregation make it preferable to avoid models that rely heavily on the movements along a smooth neoclassical production function. A Leontief production function represents a simple, neutral starting point in much the same way that linear functions may be preferred as a benchmark specification if there are no good arguments for introducing non-linearities. Some other mechanism is needed, however, to reconcile the warranted and natural growth rates if capital intensity does not accommodate smoothly.

Endogenous adjustment of the saving rate is the obvious candidate. DSGE models and the basic Ramsey model that they build upon typically include a smooth aggregate production function but do not depend on this assumption: intertemporal optimization endogenizes the saving rate. Reacting to changes in the rate of return on capital, the optimizing representative household adjusts its saving rate, and the economy converges to full-employment growth, even if there are fixed coefficients in production.

The Ramsey solution is unconvincing, but there are other reasons for a dependence of the average saving rate on income distribution. This dependence means that the warranted rate may adjust to the natural rate if endogenous forces generate appropriate movements in the profit share.

Nicholas Kaldor subsequently changed his views, but in the 1950s he regarded steady growth at (near-) full employment as a good approximation to the experience of most rich economies. He presented his ‘Keynesian’ explanation of this stylized fact in Kaldor (1955-56). Leaving problems of the trade cycle outside the scope of his paper, he assumed that the natural growth rate governs the growth rate over longer periods. With a Leontief production function (and the utilization of capital at the desired rate) the share of profits was, Kaldor argued, the accommodating variable behind the equalization of the warranted and natural rates. Formally, if  $\omega$  denotes the share of wages in income and the saving propensities out of wages and profits are  $s_w$  and  $s_p$ , the share of wages ( $\omega$ ) must satisfy

$$[s_w\omega + s_p(1 - \omega)]\sigma = n + \delta$$

or

$$\omega = s_p - \frac{n + \delta}{(s_p - s_w)\sigma}$$

Kaldor (1955-56) focused on the steady growth path without any discussion of firms' investment decisions and of how the accumulation rate came to be adjusted to the natural rate. The argument merely established that it was possible for the warranted rate to adjust, even if the output capital ratio is exogenously given. Kaldor was quite clear about this limitation, stating that his argument "does not mean that there will be an *inherent* tendency to a smooth rate of growth in a capitalist economy, only that the causes of cyclical movements lie elsewhere – not in the lack of an adjustment mechanism (p. 232)" to equalize natural and warranted rates. This important caveat is similar to Solow's explicit recognition that his model left out all Keynesian problems.<sup>4</sup>

For the reasons brought up by the capital controversy Kaldor explicitly rejected smooth neoclassical production functions and marginal productivity theory. But one could embed Kaldor's argument in a Solow type framework, replete with marginal productivity theory and dynamic adjustments towards the steady growth path. Suppose, as in the Solow model, that factor prices are equal to marginal products; that labor and capital are supplied inelastically; that output is at the technical maximum (given available factor supplies), and that saving is automatically invested. With a Leontief production function, the marginal product of labor and the wage share are zero when labor is in excess supply, while the gross profit rate will be zero when capital is in excess supply. Thus, the average saving rate will be  $s_w$  if  $\lambda N < \sigma K$  and  $s_p$  if  $\lambda N > \sigma K$ . It follows that

$$\hat{k} = \hat{K} - n = \begin{cases} s_w \lambda N - (n + \delta) & \text{it } \lambda N < \sigma K \\ s_p \sigma - (n + \delta) & \text{if } \lambda N > \sigma K \end{cases}$$

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<sup>4</sup>His analysis, Solow explains, represents

the neoclassical side of the coin. Most especially it is full employment economics – in the dual aspect of equilibrium condition and frictionless, competitive, causal system. All the difficulties and rigidities which go into modern Keynesian income analysis have been shunted aside. It is not my contention that these problems don't exist, nor that they of no significance in the long run. My purpose was to examine what might be called the tightrope view of economic growth and to see where more flexible assumptions about production would lead a simple model. (Solow 1956, p. 91)

He goes on to mention some Keynesian obstacles to full employment growth, including rigid wages and liquidity preference, and ends the paper by commenting on uncertainty (pp. 93-94):

No credible theory of investment can be built on the assumption of perfect foresight and arbitrage over time. There are only too many reasons why net investment should be at times insensitive to current changes in the real return to capital, at other times oversensitive. All these cobwebs and some others have been brushed aside throughout this essay. In the context, this is perhaps justifiable.

Unfortunately, the profession has paid little or no attention to these qualifications.

So long as  $s_w\sigma < n + \delta < s_p\sigma$ , the economy will converge to a steady growth path with full employment and  $\lambda N = \sigma K$ ,  $k^* = \frac{\lambda}{\sigma}$ . The analysis, which can be extended to cases with a narrow range of feasible output capital ratios, has affinities with that of Solow. Smooth production function are not required, however, and the emphasis is on the effects of changes in the distribution of income on saving and the rate of accumulation.

### 2.3 A Marx-Goodwin solution

Karl Marx discussed the relation between the warranted and natural growth rates in chapter 25 of *Capital*. Fast accumulation reduces the size of the ‘reserve army of labor’; a small reserve army strengthens workers and wages go up, but as the profit share decreases, accumulation falls, and low accumulation means that the reserve army is replenished. Or in Marx’s words,

If the quantity of unpaid labour supplied by the working class, and accumulated by the capitalist class, increases so rapidly that its conversion into capital requires an extraordinary addition of paid labour, then wages rise, and, all other circumstances remaining equal, the unpaid labour diminishes in proportion. But as soon as this diminution touches the point at which the surplus labour that nourishes capital is no longer supplied in normal quantity, a reaction sets in: a smaller part of revenue is capitalised, accumulation lags, and the movement of rise in wages receives a check. The rise of wages therefore is confined within limits that not only leave intact the foundations of the capitalistic system, but also secure its reproduction on a progressive scale. (Marx 1867 [1906, p. 680])

The language may seem convoluted and the terminology unfamiliar, but the basic argument has two elements: movements in the employment rate are determined by the accumulation of capital, while the employment rate affects income distribution and the rate of accumulation. These forces, Marx suggests, interact in a way that secures the reproduction of the capitalist system. Dynamic interactions can be tricky, however, which makes it useful to formalize the argument.

**A version with monotonic convergence** Workers saved little, if at all, in Marx’s time, and the saving rate out of wages is still much lower than that out of profit income. As a stylized version of this observation we may assume that workers spend what they earn, while capitalists save and invest a constant proportion ( $s$ ) of their profits,

$$I = S = s(1 - \omega)Y \tag{2}$$

Now add a fixed coefficient production function ( $Y = \min\{AL, \sigma K\}$ ) and assume full utilization of capital and the absence of labor hoarding,

$$\frac{Y}{K} = \sigma \quad (3)$$

$$L = \frac{1}{A}Y \quad (4)$$

Dividing through by  $K$  in equation (2) and using (3), we have

$$\hat{A} + \hat{L} = \hat{Y} = \hat{K} = s(1 - \omega)\sigma - \delta \quad (5)$$

where ‘hats’ over a variable are used to denote growth rates.

Wages, Marx argues, increase when high accumulation generates "an extraordinary addition of paid labor"; that is, when the reserve army of unemployed declines and workers' are strengthened in the battle over wages. The employment rate  $e = L/N$  can be used as an inverse indicator of the size of the reserve army of labor, and Marx's analysis can be interpreted as positing a positive relation between the employment rate and the real wage per efficiency unit of labor:

$$\omega = f(e); \quad f' > 0 \quad (6)$$

Combining equation (5)-(6), the dynamics of  $e$  can be written

$$\hat{e} = \hat{L} - \hat{N} = \hat{K} - n = s(1 - f(e))\sigma - (n + \delta) \quad (7)$$

where  $n = n_0 + a$  is the growth rate of the labor force in efficiency units ( $a = \hat{A}, n_0 = \hat{N}$ ).

The differential equation (7) has a (non-trivial) stationary solution with  $0 < e < 1$  if  $s\sigma(1 - f(0)) > n > s\sigma(1 - f(1))$ . This condition requires that when workers are very weak ( $e \rightarrow 0$ ), a low wage share ensures accumulation rates that exceed the natural rate of growth, while strong workers and low profitability as  $e \rightarrow 1$  cause accumulation to fall below the natural rate. If these plausible inequalities are satisfied, the employment rate converges to the non-trivial stationary point for any positive initial employment rate,

$$e \rightarrow e^* = f^{-1}\left(1 - \frac{n + \delta}{s\sigma}\right)$$

Endogenous changes in the distribution of income serve to align the warranted and natural rates. The convergence process is analogous to the Kaldor-Solow process but with a difference: the Marx version does not have an inelastic supply of labor, and marginal productivities do not determine factor prices. Instead, the wage share is determined by wage bargaining, or using a more Marxian terminology, by the balance of power in the class struggle between capital and labor.



### 2.3.1 A version with cycles

Monotonic convergence to steady growth is not a prediction usually associated with Karl Marx, and a minor modification of the model produces a very different outcome. Equation (6) stipulated a positive relation between the level of the employment rate and the *level* of the wage share. It may be more reasonable – and arguably a more accurate interpretation of Marx’s verbal argument – to view employment as determining the growth rate of the wage share: strong workers fight for and obtain *increases* in wages.

On this interpretation, equation (6) must be replaced by a dynamic version,

$$\dot{\omega} = f(e) - a; \quad f' > 0 \quad (8)$$

where the  $f(\cdot)$ -function describes a real-wage Phillips curve. Goodwin (1967) used this specification in a formalization of Marx’s general law in a model that has become the workhorse for numerous contributions in (post-) Keynesian and neo-Marxian macroeconomics. Retaining all other equations and, following Goodwin, using a linear specification of the  $f$ -function ( $f(e) = -\gamma + \rho e$ ), the model now yields a 2D system in the two state variables,  $e$  and  $\omega$ .<sup>5</sup>

$$\begin{aligned} \dot{e} &= e[s\sigma(1 - \omega) - (n + \delta)] \\ \dot{\omega} &= \omega[-(\gamma + a) + \rho e] \end{aligned}$$

The system has a unique non-trivial stationary solution with  $e > 0$  and  $\omega > 0$ ,

$$\begin{aligned} e^* &= \frac{\gamma + a}{\rho} \\ \omega^* &= 1 - \frac{n + \delta}{s\sigma} \end{aligned}$$

Evaluated at the stationary point the Jacobian is given by

$$J(e, \omega) = \begin{pmatrix} 0 & -e^*\sigma \\ \omega^*\rho & 0 \end{pmatrix}$$

The determinant ( $DetJ = e^*\sigma\omega^*\rho$ ) is unambiguously positive which rules out saddlepoints. The trace is zero, and a local analysis of the linearized system described by the Jacobian matrix leaves the stability properties undetermined.<sup>6</sup>

<sup>5</sup>Goodwin assumed that all profits are saved ( $s = 1$ ).

<sup>6</sup>A positive (negative) trace is sufficient for local instability (local stability) when the determinant is positive. In a nonlinear system, however, a zero trace may be associated with local stability, instability or conservative fluctuations of constant amplitude. The intuition behind this indeterminateness when the trace is zero can be illustrated by a 1D system. Let  $\dot{x} = g(x)$  and assume that this differential equation has a stationary point at  $x_0$  ( $g(x_0) = 0$ ). The derivative  $g'(x_0)$  is the trace of the 1x1 Jacobian matrix for this 1D system, and it is readily seen that if  $g'(x_0) = 0$ , the stationary point can be locally stable ( $g(x) = -(x - x_0)^3$ , for example), unstable ( $g(x) = (x - x_0)^3$ , for example), or neutrally stable ( $g(x) = 0$ , for example).

It can be shown, however, that this formalization of Marx's general law of accumulation yields endogenous growth cycles around the stationary point  $(e^*, \omega^*)$ : the system implies closed, counterclockwise loops in  $(e, \omega)$  space, as depicted in the phase diagram in figure 1. Starting from an initial position, employment and the wage share exhibit perpetual fluctuations with a constant amplitude; an exogenous shock that shifts the initial position changes the amplitude of the fluctuations as the system moves to a new closed loop. These properties of the dynamic system extend to formulations with nonlinear real wage Phillips curves and accumulation functions; see Appendix A for details.

The stationary value of the employment rate is determined by the parameters of the distributive function (the exogenous elements of relative power) while the parameters of the accumulation function (including capitalists' saving rate) and the natural rate of growth determine the wage share. Intuitively, if workers get stronger and more militant for any given employment rate, an increase in the reserve army will be needed to restore discipline in the labor market and prevent an ever-increasing wage share, an adjustment brought about by the effects of changes in the size of the reserve army of labor. Analogously, changes in the saving rate, the technical output-capital ratio or the growth rate of the labor force in efficiency units require adjustments in the wage share in order to maintain the equality between the rate of accumulation and the natural growth rates.

In short, a capitalist economy may be unstable – cyclical fluctuations are created endogenously – but homeostatic forces imply that the rise of wages "is confined within limits that not only leave intact the foundations of the capitalistic system, but also secure its reproduction on a progressive scale", in Marx's words.

Elegant as it is, the Goodwin model fails to capture essential features of the business cycle. Output is determined by capacity (as in the Solow model), and with fixed coefficients and unemployment, the capital stock represents the binding constraint. Empirically, by contrast, the utilization rate of capital exhibits large and systematic fluctuations over the cycle.

The source of this failure of the Goodwin model is the absence of Keynesian features. There is no separate investment function, full-capacity saving automatically gets invested, and no aggregate demand failures constrain output below full capacity. I have discussed these issues in detail in other work, embedding Marx-Goodwin effects of variations in employment in Keynesian models with Harrodian instability.<sup>7</sup> The purpose of this paper is different: the Marx-Goodwin mechanism – with or without Keynesian complications – may need significant modification in any application to developing economies.

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<sup>7</sup>Skott (1989, 2015, 2022) and Skott and Zipperer (2012). Related work that incorporates Goodwinian feedback effects from the labor market in a Keynesian setting includes work by Peter Flaschel and several coauthors (e.g. Chiarella and Flaschel 2000). Barrales et al. (2021) provide a useful survey of some of the theoretical issues and the evidence.

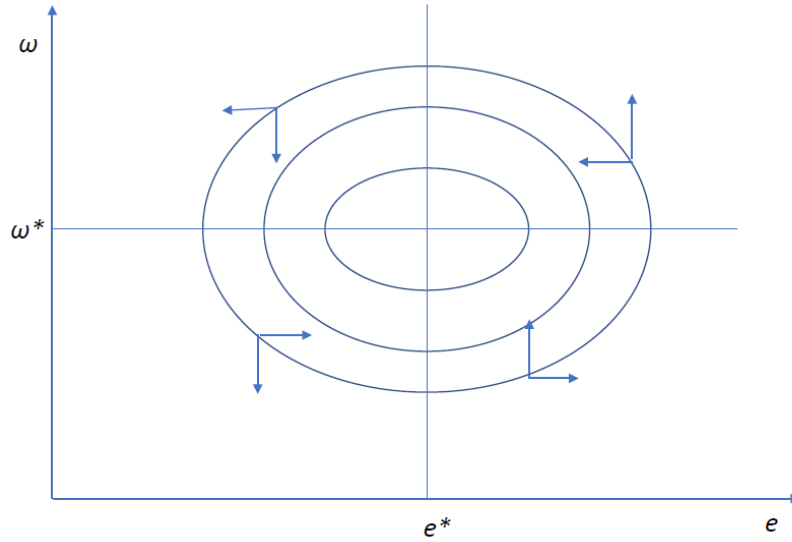


Figure 1: Phase diagram for the Goodwin model

### 3 Goodwin models for dual economies?

#### 3.1 A baseline version

Official unemployment rates are uninformative in dual economies. According to the ILO, India had unemployment rates below 6 percent from 1991 to 2019, but huge numbers of underemployed workers in informal sectors with low productivity and low average incomes render this official unemployment rate rather meaningless as an indicator of the state of the Indian labor market.

A simple adaptation of the Goodwin model to developing economies treats the informal sector as self-contained and uses the share of formal-sector employment as the measure of labor market conditions. If workers' average income in the informal sector is constant and below the wages in the formal sector, the modern sector now draws on the reserve army of informal-sector workers in much the same way that capitalists draw on the reserve army of the unemployed in Marx's analysis and the original Goodwin model, while changes in the wage share of the modern sector may depend on the sector's employment share.

Formally, let  $\theta = L_M/N$  denote the employment share of the modern sector and assume that all workers who fail to find a job in the modern sector move into the informal sector. As in Goodwin's original formulation, the technology in the modern sector is Leontief, the accumulation rate is proportional to the profit share, workers spend all income on consumption, and capitalists save and

invest a proportion of profits. These assumptions imply that

$$\hat{\theta} = \hat{L}_M - n = s\sigma(1 - \omega) - (n + \delta) \quad (9)$$

$$\hat{\omega} = -(\gamma + a) + \rho\theta \quad (10)$$

where  $\omega$  represents the wage share in the modern sector. The interpretation of the ‘employment rate’ and the wage share has changed, but the mathematical structure and formal properties of this dual-economy model are identical to those of the original model: the economy exhibits conservative fluctuations around a unique (non-trivial) stationary solution.

Although mathematically sound, this adaptation of the Goodwin model is economically implausible. Developing economies with small modern sectors and large reservoirs of underemployment in traditional and informal sectors may face temporary shortages of skilled workers, but many low-skill workers in the modern sector earn large wage premiums compared to the average incomes of similar workers in the informal sector (La Porta and Shleifer 2014). These observations imply that cyclical variations in the employment share of the modern sector are likely to have little effect on the bargaining power of workers in the formal sector.

Economic development, second, is associated with structural changes as underemployed workers move from informal to modern sectors, making Harrod’s first problem irrelevant: the very process of economic development is characterized by growth rates above the natural rate and a gradual reduction of underemployment. Successful developing economies may experience business cycles, but they do not fluctuate around a constant share of employment in the modern sector: the fluctuations take place around a trend of declining underemployment.

Formal and informal sectors, third, are not self-contained. Street vendors do not sell only to informal sector workers, and casual laborers are hired to do work for rich households. Workers in the informal sector, likewise, spend some of their income on TV sets and other goods and services produced in the formal sector. The sectors interact, and it becomes unreasonable to treat incomes in the informal sector as independent of the evolution of the formal sector.

Fast growth, fourth, is associated with technological catchup. The pace of this process of technological change is determined – at least in part – by the rate of expansion of the modern sector. Verdoorn’s law captures a simple formalization of this process of dynamic increasing returns to scale and semi-endogenous technical change.<sup>8</sup> It may do little harm to leave out induced technical change of this kind in an examination of cycles in a mature economy; in a dual economy, by contrast, the medium and long run growth rate of the economy becomes endogenous, potentially making induced technical change significant for the qualitative dynamics. The complications associated with induced technical change are likely to be less important, however, than those deriving from large informal sectors, the interaction between formal and informal sectors and the gradual

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<sup>8</sup>Kaldor (1966) introduced Verdoorn’s law, providing also the first empirical estimates of its magnitude. The law and the empirical assessment of its effects raise several issues; see Rowthorn (1979), Skott (1999), Ros (2013) and Basu and Budhiraja (2021).

shift towards the modern sector in the course of successful development. Thus, to simplify the analysis in the main text I disregard technical change completely (set the rate of technical change equal to zero); appendix B briefly outlines some implications of induced technical change for the Goodwin model.

### 3.2 Interactions between the sectors

The informal sectors in most contemporary developing economies do not resemble the self-contained subsistence agriculture in classical models of a dual economy. Underemployed workers in urban service sectors, who make up a large proportion of the informal sector, rely on demand from other informal workers as well as from the formal sector.

The informal sector produces a variety of goods and services, and workers in the informal sector have to sell these goods and services, e.g. as day laborers, street vendors, domestic workers or employees in small corner shops and construction activities. The sector may contain small-scale businesses that make a profit but, for simplicity, we may assume that all incomes in the informal sector go to workers and that output is produced using labor as the only input, a stylized representation of the low capital intensity that characterizes this sector. Thus, if  $p_U$  and  $w_U$  are the price of informal sector output ( $U$ -goods) and the average nominal income in the sector (the ‘wage rate’, for short), we have

$$p_U U = w_U L_U \quad (11)$$

where  $U$  is the real output and  $L_U = N - L_M$  the number of workers in the informal sector.

As a stylized formalization of the demand structure – and in line with the assumptions in the Goodwin model – it is assumed that workers in both sectors spend their income on consumption while capitalists save a fraction  $s$  of their profits, spending the rest on consumption. Thus, aggregate nominal consumption ( $C$ ) is given by

$$C = p_M C_M + p_U C_U = w_U L_U + w_M L_M + (1 - s)(p_M M - w_M L_M) \quad (12)$$

where  $C_U$  and  $C_M$  denote real consumption of goods from the two sectors. Nominal consumption is split between formal and informal goods, the proportion  $\alpha$  of consumption expenditure going to the formal sector. Thus,

$$p_M C_M = \alpha C \quad (13)$$

$$p_U C_U = (1 - \alpha)C \quad (14)$$

The informal sector produces a pure consumption good, and the equilibrium condition requires that

$$U = C_U \quad (15)$$

Formal-sector goods can be used for investment or consumption, and the equilibrium condition is given by

$$M = I + C_M \quad (16)$$

Still following the assumptions of the Goodwin model, capital is fully utilized and all saved profits are invested,  $I = s(M - w_M L_M)$ .

Total informal-sector income can be found by combining the consumption equations with the equilibrium condition for the informal sector (equations (11)-(15)):

$$w_U L_U = p_U U = \frac{1 - \alpha}{\alpha} w_M L_M [s + (1 - s) \frac{1}{\omega}] \quad (17)$$

Thus, the average income in the informal sector and the ‘wage ratio’  $w_U/w_M$  are given by

$$w_U = \frac{1 - \alpha}{\alpha} \frac{w_M L_M}{N - L_M} [s + (1 - s) \frac{1}{\omega}] = \frac{1 - \alpha}{\alpha} \frac{\theta}{1 - \theta} [s + (1 - s) \frac{1}{\omega}] w_M \quad (18)$$

$$\frac{w_U}{w_M} = \frac{1 - \alpha}{\alpha} \frac{\theta}{1 - \theta} [s + (1 - s) \frac{1}{\omega}] = \phi(\theta, \omega); \quad \phi_\theta > 0, \phi_\omega \leq 0 \quad (19)$$

Unlike in the baseline version, the average income in the informal sector depends on the size of the modern sector as well as on the wage rate and the wage share in this sector. The relative wage is determined by the size of modern sector and the wage share in this sector.

### 3.3 Relative-wage norms

The endogeneity of the relative wage leaves the modified Goodwin system (9)-(10) unchanged: the share of profits in the modern sector (and capitalists’ saving rate) still determine the modern-sector accumulation rate, and the endogeneity of relative wages is of no significance for the dynamics in the absence of feedback effects from relative wages to the distribution of income in the modern sector. The exclusion of feedback effects is implausible, however: there is substantial evidence that wage relativities exert a powerful influence on wage formation.<sup>9</sup>

Keynes (1936) emphasized workers’ resistance to cuts in relative wages as a source of nominal wage rigidity, while Akerlof (1982) and Akerlof and Yellen (1990) highlighted the importance of reciprocity and norms of fairness. Workers who feel they have been treated badly will tend to reciprocate; morale and labor productivity will suffer, giving firms an incentive to pay ‘fair wages’.

Fairness-based accounts of wage setting are singled out by Bewley (1999) as the only ones out of a large set of candidates that are consistent with the empirical evidence on wage stickiness in recessions. These accounts, it should be noted, can apply to labor markets without collective bargaining and labor unions (as in Akerlof and Yellen 1990) as well as to unionized labor markets; violations of prevailing pay norms induce aggressive wage demands and steel the rank and file to fight for these demands in a unionized economy. The precise forms may be different in unionized and non-unionized settings, but the basic mechanism is the same: the perception of fairness is important in wage setting, and fairness is judged in large part with reference to relative wages.

<sup>9</sup>Depending on parameters, the average income in the informal sector given by equation (19) could even exceed the wage in the formal sector. The economy becomes mature, however, and the modern sector faces labor constraints when the wage premium has been eliminated.

The presence of relative-wage norms suggests a respecification of the equation for the dynamics of wages in the modern sector. Formally, assume that the fair wage  $w_M^F$  is given by

$$w_M^F = \mu w_U$$

and that wage demands respond to deviations of the current wage ratio from the fair ratio,

$$\hat{\omega}_M = \lambda \left( \frac{w_M^F}{w_U} - \frac{w_M}{w_U} \right) = \lambda \left( \mu - \frac{w_M}{w_U} \right)$$

Using equation (19) and assuming, for simplicity, that  $a = 0$ , the dynamics of the wage share can now be written as

$$\hat{\omega} = \lambda \left( \mu - \frac{1}{\phi(\theta, \omega)} \right) \quad (20)$$

The accumulation rate and the dynamics of employment in the modern sector is unchanged, and equation (9) still holds. Equation (20) replaces (10), however.

The system (9) and (20) still has a unique (non-trivial stationary solution and the determinant remains positive. The trace, however, has turned negative, and the stationary solution is locally stable if  $\phi_\omega < 0$  (which happens if  $s < 1$ ). The economic intuition is straightforward. The change in the wage share is increasing in the relative wage  $w_U/w_M$ , and an increase in  $w_M$  raises incomes in the informal sector less than proportionately if capitalists spend some of their profits on the consumption of informal goods. Thus, the level of the wage share has a stabilizing, negative feedback effect on change in the wage share.

### 3.4 Endogenous norms

Wage aspirations and norms of fairness are predetermined in the short run but clearly differ across space and change over time. The real wage aspirations of auto workers in Germany, the Czech Republic and India are quite different, while wages that were considered fair by VW workers in 1960 would be deemed unacceptable in 2020. In short, wage aspirations are path dependent. Or as Marx put it, the value of labor power has a "historical and moral element".<sup>10</sup>

The historical and moral element also applies to relative wages. As noted by Hicks (1975), it can be difficult to achieve a general consensus on what is fair and what is not. No system of wages, Hicks argues,

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<sup>10</sup>The full quote is:

In contradistinction therefore to the case of other commodities, there enters into the determination of the value of labour-power a historical and moral element. Nevertheless, in a given country, at a given period, the average quantity of the means of subsistence necessary for the labourer is practically known. (Marx 1867 [1906], p. 190)

Marx's analysis focused on the real wage and the conflict between capitalists and workers. Equation (20) could be extended to include an effect of profit shares on target real wages.

when it is called into question, will ever be found to be fair. ... [To avoid the system being called into question, PS] the system of wages should be well established, so that it has the sanction of custom. It then becomes what is expected; and (admittedly on a low level of fairness) what is expected is fair (p. 65).

The gradual adjustment of notions of fairness finds support in social psychology and behavioral economics:

Psychological studies of adaptation suggest that any stable state of affairs tends to become accepted eventually, at least in the sense that alternatives to it no longer readily come to mind. ... Thus, the gap between the behaviour that people consider fair and the behavior that they expect in the market-place tends to be rather small. (Kahneman et al. 1986, pp. 730-1)

As a simple formal representation of these behavioral observations, suppose that the fair wage ratio changes over time in response to differences between actual and fair relative wages; that is,  $\mu$  changes in response to differences between  $w_M/w_U$  and  $\mu$ :<sup>11</sup>

$$\dot{\mu} = \nu \left( \frac{w_M}{w_U} - \mu \right) = \nu \left( \frac{1}{\phi(\theta, \omega)} - \mu \right) \quad (21)$$

where  $\theta$  is the adjustment speed for the target relative wage. The specification in equation (21) is quite mechanical and leaves out many factors that may influence workers' aspirations and their willingness and ability to fight for wage increases. Institutional factors and labor market legislation can be critical, and workers' militancy, more generally, cannot be separated from broader political and social movements. Aggressive wage demands and high and rising strike activity in the US, western Europe and many other countries in the late 1960s, for instance, did not develop independently of a general radicalization involving civil rights movements, anti-Vietnam war movements, student protests and rising opposition against dictatorships and oppression in many countries. With these caveats, however, equation (21) captures a systematic and potentially important mechanism in the formation of wage aspirations.

The three-dimensional system (9), (20) and (21) has a continuum of stationary points. The dynamic equation for  $\theta$  defines a unique stationary solution for  $\omega$  ( $\omega = \omega^*$ ), but the stationarity of  $\omega$  and  $\mu$  are ensured for any combination of  $\theta$  and  $\mu$  that satisfies  $\phi(\theta, \omega^*) = 1/\mu$ . The dynamic implications of the system becomes clearer by noting that  $\dot{\mu} = -\frac{\nu}{\lambda} \hat{\omega}$  and therefore, by integration,

$$\mu = -\frac{\nu}{\lambda} \log \omega + c \quad (22)$$

where  $c$ , the arbitrary constant of integration, is determined by initial conditions (the initial values of  $\omega$  and  $\mu$ ). Substituting (22) into (20), the dynamics of  $\omega$

<sup>11</sup>Similar specifications have been used by Skott (2005) and Martins and Skott (2021).



can be written as

$$\hat{\omega} = \lambda \left( -\frac{\nu}{\lambda} \log \omega + c - \frac{1}{\phi(\theta, \omega)} \right) \quad (23)$$

The two-dimensional system defined by (9) and (23) has a unique (non-trivial) and locally stable stationary solution (the Jacobian has a positive determinant and a negative trace). The stationary solution for  $\omega$  is  $\omega^* = 1 - \frac{n+\delta}{s\sigma}$ , while the stationary solution for the employment share of the modern sector is increasing in  $\omega$  and decreasing in  $c$ ;  $\theta^* = \theta^*(\omega, c)$  with  $\theta_\omega^* > 0, \theta_c^* < 0$ .<sup>12</sup>

The dependence of the share of employment in the modern sector,  $\theta$ , on the constant of integration captures the path dependency of long-run underemployment. In the standard Goodwin model a shock to the wage share leaves the stationary solution and the average value of the employment share of the modern sector unchanged.<sup>13</sup> In the version with endogenous norms, by contrast, positive shocks to  $\omega$  and/or  $\mu$  raise the constant  $c$ , generating a permanent increase in the degree of underemployment. There is no natural rate of underemployment.

The sensitivity of long-term outcomes to initial conditions is amplified by another empirically plausible modification of the wage equation. Workers react to deviations of the fair wage ratio from the actual ratio, but the fair ratio may be a little fuzzy. Workers, moreover, lack accurate information about average incomes in the informal sector; their perceptions of current relative wages derive from interactions with friends and family, observations of the spending behavior of neighbors and acquaintances, and news stories. Large and sudden shifts in relative wages will provoke a reaction, but slow and gradual changes may not be noticed. Much like a frog that fails to notice and react to the increasing temperature of water that is heated slowly, modern-sector workers may get accustomed to a decline in their relative wage without fully realizing the deterioration of their relative position.<sup>14</sup>

This argument is analogous to Rowthorn's (1977) suggestion that wage setters will ignore expected inflation as long as it stays below a threshold level but adjust their nominal wage demands fully to expected inflation rates above the threshold. Rowthorn's analysis focused on real wages and the extent to which price inflation affects the growth rate of nominal wages, but the same basic point also applies in the present setting: workers may not react to a slow erosion of the relative wage. Using a threshold formulation, this argument implies a respecification of the wage dynamics:

$$\hat{\omega} = \begin{cases} 0 & \text{if } -m < \mu - \frac{1}{\phi(\theta, \omega)} < m \\ \lambda \left( \mu - \frac{1}{\phi(\theta, \omega)} \right) & \text{elsewhere} \end{cases} \quad (24)$$

<sup>12</sup>Use the implicit function theorem and the fact that  $\theta^*$  satisfies  $-\frac{\nu}{\lambda} \log \omega + c = \frac{1}{\phi(\theta, \omega)}$ .

<sup>13</sup>There is a caveat to this statement. Non-linearities in the equations can make the average values of the variables differ from the stationary solution and cause the average values to depend on the amplitude of the fluctuations. Because of this dependence, shocks can have (minor) effects on average values.

<sup>14</sup>This frog metaphor does not capture the behavior of real frogs, according to modern biology. Frogs that do not change location in response to overheating would not survive in the wild. No similar physiological and evolutionary mechanisms are at play when it comes to social norms of fairness.

where  $m$  is the threshold beyond which deviations from the norm become apparent and lead to demands for an increased product real wage.

The full dynamics of the three dimensional model with threshold effects are complicated. To illustrate the amplification of the effects of initial positions, however, suppose that initially the actual relative wage conforms to the norm ( $\frac{1}{\phi(\theta, \omega)} = \mu$ ) and let  $\omega^*$  denote the wage share associated with a stationary share of employment in the modern sector ( $\omega^* = 1 - \frac{n}{s\sigma}$ ). If the initial value of the wage share falls below  $\omega^*$ , the modern sector is growing ( $\hat{\theta} > 0$ ) and the relative wage of modern-sector workers declines as the sector expands ( $\frac{1}{\phi(\theta, \omega)}$  is decreasing in  $\theta$ ). The emerging difference between the actual and the fair relative wage generates downward movements in  $\mu$ ; formally,  $\dot{\mu} = \nu(\frac{1}{\phi(\theta, \omega)} - \mu)$  turns negative. The wage share, however, does not react as long as the difference remains below the threshold, and the accumulation rate in the modern sector remains above the natural rate ( $\hat{\theta} > 0$ ).

The effect of increases in  $\theta$  on actual relative wages is highly non-linear – a small increase in  $\theta$  has a larger effect on the relative wage for high levels of  $\theta$  – and the threshold will be reached at some point as  $\theta$  grows. But the decline in the relative-wage norm may proceed for a long time before that happens; the rise in  $\omega$  is delayed, allowing the modern sector to expand further than in the case without a threshold for adjustment in  $\omega$ . Conversely, if the initial value of the wage share is above  $\omega^*$ , the employment share of the modern sector will be falling. The decline in the wage share  $\omega$  and the resulting increase in  $\hat{\theta}$  are now delayed by the non-linearity of the wage equation (24). Thus, the decline of the modern sector – the deindustrialization – can proceed further than in the case without the threshold.

## 4 Conclusions

The average medium and long run growth rate matches the natural rate in mature economies. Like Kaldor’s neo-Keynesian model, the Marx-Goodwin tradition explains this outcome by endogenizing the distribution of income and assuming that the accumulation rate of capital is increasing as a function of the profit share. The application of traditional labor-market based Goodwin cycles to developing economies may be hard to justify, however. A baseline dual-economy version of the model preserves the qualitative properties of the model but makes questionable assumptions. Small modern sectors and high rates of underemployment imply that small-scale cyclical fluctuations will have only minor effects on the degree of underemployment, making it unlikely that the fluctuations should exert a significant effect on the balance of power in the labor market. Perhaps even more important, economic development involves trend increases in the modern sector: the medium and long run pace of economic growth is not constrained by the natural rate, as suggested by the baseline adaptation of the Goodwin model to a dual economy.

The modified Goodwin models in this paper represent an attempt to over-

come these weaknesses of the baseline version. Relative-wage norms are central to wage setting in the modified models, and norms change endogenously. This endogeneity leads to path dependence (hysteresis) in the stationary solution for the employment share of the modern sector. The effects of shocks – the sensitivity of the long-run outcome to initial conditions – may be amplified by non-linearities in the adjustment of wages to deviations of actual wages from the norm, as exemplified by simple threshold formulations.

The presence of hysteresis with respect to the employment share of the modern sector in the dual economy model parallels the analysis of mature economies in Skott (2005): path dependency of wage norms creates employment hysteresis and undermines the notion of a natural rate of unemployment. But unlike in mature economies where the range of stationary solutions for the employment rate is relatively narrow, the employment share of the modern sector can stagnate at a very low level in a dual economy.

By definition a mature economy is close to something like full employment in the sense that labor supply constraints would make Chinese-style annual growth rates of 8-10 percent unsustainable for more than a couple of years. To maintain maturity, the average medium and long run growth rates of a mature economy must be approximately equal to the natural rate; they cannot exceed the natural rate without leading to labor constraints, and if growth falls systematically below the natural rate for a prolonged period, the economy ceases to be mature. The latter possibility cannot be excluded, and arguably there are cases where this has happened.<sup>15</sup> These issues are beyond the scope of this paper, however.

Disregarding transitions from mature to dual, a mature economy described by the Goodwin model exhibits fluctuations of the employment rate around a stationary solution that falls within a narrow range. Matters are quite different for a dual economy whose medium and long run growth rate need not be tethered to the natural rate. A successful development process involves secular increases in the share of the modern sector, increases that are made possible by the endogeneity of the relative wage target. The cycles predicted by the baseline dual-economy version of the Goodwin model cannot account for this possibility: the model gives a misleading picture of the constraints on and the dynamics of the development process.

The Marx-Goodwin tradition assumes that capital is fully utilized and that a decrease in the wage share raises the accumulation rate. Even in this context, the growth of the modern sector and the pace of economic development can be stimulated in ways that do not suppress the product real wage in the modern sector. As an obvious alternative, reductions in luxury consumption – increases in the saving rate  $s$  – have effects on accumulation that are similar to those of a fall in the wage share. Increasing labor productivity also boosts accumulation for

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<sup>15</sup>Skott (1989, section 6.4.3) noted that the stabilizing forces in a model with Harrodian local instability may be too weak to prevent cumulative downward divergence. Empirically, Taylor and Omer (2017), Storm (2017) and Mendieta Munoz et al. (2021) make a case that increasing wage inequality and the coexistence of high and low productivity sectors have given the US economy some characteristics of a dual economy.

any given wage, and productivity gains can be achieved by temporary subsidies to boost accumulation if the modern sector exhibits dynamic increasing returns.

Full capacity utilization (a constant degree of utilization), second, is a useful approximation for long-run analysis, but demand-induced fluctuations of the utilization rate are central to short-run fluctuations. The exclusion of aggregate demand issues – the potential for realization crisis, using Marxian terminology – represents a serious limitation of the analysis. Like their mature counterparts, developing economies may be subject to Harrodian instability, with another important source of instability coming from the external sector and the domestic policy response to external shocks (Martins and Skott 2021). These issues are beyond the scope of this paper, which has had a more limited purpose: to examine Goodwinian specifications of wage setting and their application to mature and dual economies.

## Appendix A: A generalized Goodwin model

Consider the generalized Goodwin system

$$\begin{aligned}\dot{x} &= \phi_1(y)\chi_1(x); & \phi_1' > 0, \chi_1 > 0 \\ \dot{y} &= \phi_2(x)\chi_2(y); & \phi_2' < 0, \chi_2 > 0\end{aligned}$$

where  $\phi_1, \phi_2, \chi_1, \chi_2$  are continuously differentiable and where we assume that the system has a stationary solution  $x^*, y^*$ . It is readily seen that this generalized system includes the simple Goodwin system as a special case: let  $\phi_1$  and  $\phi_2$  be affine and specify  $\chi_1(x) = x, \chi_2(y) = y$ .

A transformation of the generalized system brings it on the form

$$\dot{p} = f(q); f' > 0 \tag{25}$$

$$\dot{q} = g(p); g' < 0 \tag{26}$$

To see this, define  $p$  and  $q$  as

$$\begin{aligned}p &= \theta_1(x) = \int_k^x \frac{1}{\chi_1(v)} dv \\ q &= \theta_2(y) = \int_m^y \frac{1}{\chi_2(v)} du\end{aligned}$$

where  $m$  and  $k$  are arbitrarily chosen constants. Since  $\chi_1$  and  $\chi_2$  are both positive, the functions  $\theta_1$  and  $\theta_2$  are monotonically increasing. Furthermore,

$$\begin{aligned}\dot{p} &= \frac{1}{\chi_1(x)} \dot{x} = \phi_1(y) = \phi_1(\theta_2^{-1}(q)) = f(q) \\ \dot{q} &= \frac{1}{\chi_2(y)} \dot{y} = \phi_2(x) = \phi_2(\theta_1^{-1}(p)) = g(p)\end{aligned}$$

In order to show that the system (25)-(26) generates conservative fluctuations we multiply the left-hand side of ((25) by the right-hand side of (26) and

the right-hand side of ((25) with the left-hand side of (26) to get

$$g(p)\dot{p} = f(q)\dot{q}$$

or

$$g(p)\dot{p} - f(q)\dot{q} = 0 \quad (27)$$

The variables  $p$  and  $q$  (and their derivatives) are functions of  $t$ ; integrating (27) we get

$$\int g(p)\dot{p}dt - \int f(q)\dot{q}dt = \int g(p)dp - \int f(q)dq = C \quad (28)$$

where  $C$  is an arbitrary constant. Now define

$$H(p(t), q(t)) = F(q) - G(p) = \int f(q)dq - \int g(p)dp \quad (29)$$

From (27) and (28) it follows that the function  $H$  is constant,  $H(p, q) = C$ , the value of the constant being determined by the initial conditions (that is, by the initial values of  $p$  and  $q$ ). Furthermore,

$$H_p = -G'(p) = -g(p); H_q = F'(q) = f(q) \quad (30)$$

and

$$H_{pp} = -g' > 0, H_{qq} = f' > 0; H_{qp} = H_{pq} = 0 \quad (31)$$

It follows that

- the function  $H$  is convex
- $H$  has a global minimum at the stationary equilibrium,  $(p^*, q^*) = (\theta_1(x^*), \theta_2(y^*))$ . This follows since  $H_p = H_q = 0$  holds at the equilibrium.
- starting in some initial point away from the equilibrium,  $(p, q)$  will be circling around the level curve corresponding to the constant  $C$  (which in turn is determined by the initial values  $p_0, q_0$ ).

Conservative fluctuations of  $(p, q)$  around  $(p^*, q^*)$  imply conservative fluctuations of  $(x, y)$  around  $(x^*, y^*)$ . Furthermore, suitable specifications of the functions  $\phi_i$  and  $\chi_i$  ensure that the equilibrium values  $x^*$  and  $y^*$  belong to the unit interval and that if the initial values  $x_0$  and  $y_0$  are in the unit interval, then all trajectories for  $(x, y)$  will remain inside the unit box. As an example, choose  $\chi_1(x) = x(1 - x)$  and  $\chi_2(y) = y(1 - y)$ .

## Appendix B: Induced technical change in the Goodwin model

The benchmark Goodwin model assumes a constant rate of labor saving technical change. Suppose that, instead, the rate of technical change,  $a$ , is determined

by the growth rate of the modern sector, the share wages in income and the employment share of the modern sector. Formally,

$$a = a(\hat{M}, \omega, \theta); \quad 1 > a_{\hat{M}} > 0, a_{\omega} \geq 0, a_{\theta} \geq 0$$

This formulation includes the Verdoorn effect ( $1 > a_{\hat{M}} > 0$ ) well as the possibility that the pace of labor saving technical change may depend on share of wages ( $a_{\omega} \geq 0$ ) and the tightness of the labor market ( $a_{\theta} \geq 0$ ). The endogeneity of technical change affects the qualitative dynamics; the Goodwin system now becomes

$$\begin{aligned} \hat{\theta} &= s\sigma(1 - \omega) - (n_0 + a(\hat{\theta}, \omega, \theta)) \\ \hat{\omega} &= -(\gamma + a(\hat{\theta}, \omega, \theta)) + \rho\theta \end{aligned}$$

or

$$\begin{aligned} \hat{\theta} &= \phi(\theta, \omega); \quad \phi_{\theta} \leq 0, \phi_{\omega} < 0 \\ \hat{\omega} &= \psi(\theta, \omega); \quad \psi_{\theta} \geq 0, \psi_{\omega} \geq 0 \end{aligned}$$

The local stability properties are ambiguous. Local instability, for instance, arises in the simple Verdoorn case in which labor productivity growth depends only on the accumulation rate ( $a_{\omega} = a_{\theta} = 0, a_{\hat{\theta}} > 0$ ) This Verdoorn specification implies that both partials of  $\psi$  are positive,  $\psi_{\theta} > 0$  and  $\psi_{\omega} > 0$ , while  $\phi_{\omega} = 0$ . Thus, both the trace and determinant of the Jacobian become unambiguously positive. Without the Verdoorn effect –  $a_{\hat{\theta}} = 0$  – and with a weak effect of  $\omega$  on productivity growth, by contrast, the stationary point is locally stable: the determinant will be positive and the trace negative.

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